

Functional Programming in LISP

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- Data should be immutable

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- It has a very simple syntax which is based entirely on lists

```
> (+ 1 2)
```

```
3
```

```
> (* (+ 2 3) 5)
```

```
25
```


Variables and Functions

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> (* x x)
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- The substitution model for evaluation

Logic

- Finally, we need conditionals (if, then, else)

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  (cond ((> x 0) x)
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- Logical operands work as you'd expect:

```
> (and (> 3 0) (not (= 1 2)))
#t
> (or (< 1 0) (> 2 2))
#f
```

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```
(define (factorial n)
  (fact-iter 1 1 n))
(define (fact-iter product step max-count)
  (if (> step max-count)
      product
      (fact-iter (* step product)
                  (+ step 1)
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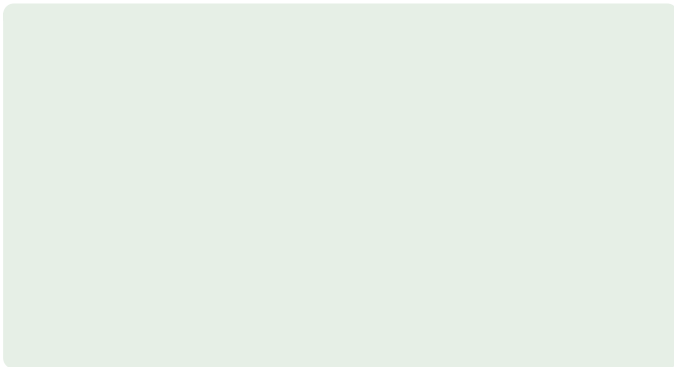
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```
(factorial 3)
(fact-iter 1 1 3)
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(fact-iter 2 3 3)
(fact-iter 6 4 3)
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- Write an iterative version of this!

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$$\begin{aligned} & (\text{number of ways to change } \pounds x \text{ using all but the first coin}) \\ & + (\text{number of ways to change } \pounds x - d \text{ using all } n \text{ coins}) \end{aligned}$$

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- It is difficult to turn this into an iterative process!

Counting Change (cont.)

```
(define (count-change x) (cc x 6))
(define (cc x kinds-of-coins)
  (cond ((= x 0) 1)
        ((or (< x 0) (= kinds-of-coins 0)) 0)
        (else (+ (cc x
                      (- kinds-of-coins 1))
                  (cc (- x (first-coin kinds-of-coins))
                      kinds-of-coins)))))
(define (first-coin n)
  (cond ((= n 1) 1)
        ((= n 2) 2)
        ((= n 3) 5)
        ((= n 4) 10)
        ((= n 5) 20)
        ((= n 6) 50)))
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- **References:** Abelson and Sussman, *Structure and Interpretation of Computer Programs* (SICP)