$$\sum_{i=1}^{N} 1 = N$$

$$Y = mx + b$$

① Show that $m(a+bX) = a + b \times m(x)$

$$\frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\sum_{i=1}^{N}(a+bX_i) = \frac{1}{N}\left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bX_i\right) = \frac{1}{N} a \sum_{i=1}^{N} 1 + \frac{1}{N} b \sum_{i=1}^{N} x_i$$

$$= \frac{1}{N}\left(aN + b\sum_{i=1}^{N} X_i\right) = a + b\underbrace{\frac{1}{N}\sum_{i=1}^{N} x_i}_{m(x)} = a + b \cdot m(x)$$

$$* \; Y_i = a + bX_i$$

② $$Cov(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(X_i - m(X))(Y_i - m(Y))^{*}$$

$$= \frac{1}{N}\sum_{i=1}^{N}(X_i - m(x))(a+bY - m(a+bY))$$

$$m(a+bY) = a + bm(Y)$$

$$= m[(X - m(x))\underbrace{((a+bY - m(a+bY)))}_{(\cancel{a}+bY)-(\cancel{a}+bm(Y)) \;\; b(Y - m(Y)}]$$

$$= m[(X - m(X))(b(Y - m(Y))]$$

$$= mb[(X - m(X)) \circ (Y - m(Y))]$$

$$= b \, Cov(X, Y)$$

$$* m(a+bX) = a + bm(x)$$

③ $$Cov[a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bX - m(a+bX)) \cancel{m(a+bX))} (a+bX - m(a+bX))$$

$$= \frac{1}{N}\sum_{i=1}^{N}[(\cancel{a}+bX - \cancel{a}+bm(x))(\cancel{a}+bX - (\cancel{a}+bm(x))]$$

$$= \frac{1}{N}\sum_{i=1}^{N}[(b(X - m(x)) \, b(X - m(X))]$$

$$= b \cdot b \, cov(X,X) = b^2 \underbrace{COV(X,X)}_{S^2}$$

$$* \; \frac{1}{N}\sum_{i=1}^{N} b\underbrace{(X - m(x))(X - m(x))}_{S^2}$$