Why

Many problems simplify if the graph involved is chordal.[1]

Paths

Let G be an undirected graph. A *chord* in a path p of G is an edge between two non-consecutive vertices of p. So a chord of the path $(v_1, v_1, \dots, v_k)$ is an edge $\{v_i, v_j\}$ with $|j - i| > 1$.

We interpret a chord as a "one-edge shortcut" between two vertices of a path. If a path p has a chord, it can be reduced to a shorter path p' by "skipping" vertices. In other words, the shortest path between two vertices is chordless. However, a chordless path need not be a shortest path. See Figure 1.

Graphs

A chord of a cycle $(v_1, v_2, \dots, v_{k-1}, v_1)$ is an edge $\{v_i, v_j\}$ with $(j-i) \bmod k > 1$. An undirected graph G is *chordal* if every cycle with more than three edges has a chord.

If G is chordal, every cycle in G can be reduced to a cycle of length three. We sometimes call a cycle of length three a *triangle*. For this reason, chordal graphs are also sometimes called *triangulated graphs*. Other terminology includes *rigid-circuit graphs*, *triangulated graphs*, *perfect elimination graphs*, *decomposable graphs*.[2]

The last graph in Figure 1 is not chordal because the cycle (a, b, d, c, a) has length four and no chord. Adding the edge $\{b, c\}$ or $\{a, d\}$ would make the graph chordal An immediate consequence

[1] Future editions will expand.

[2] See Vanenberghe and Anderson, 2014.

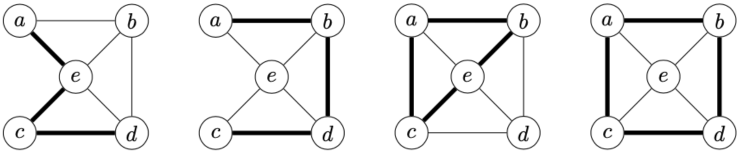
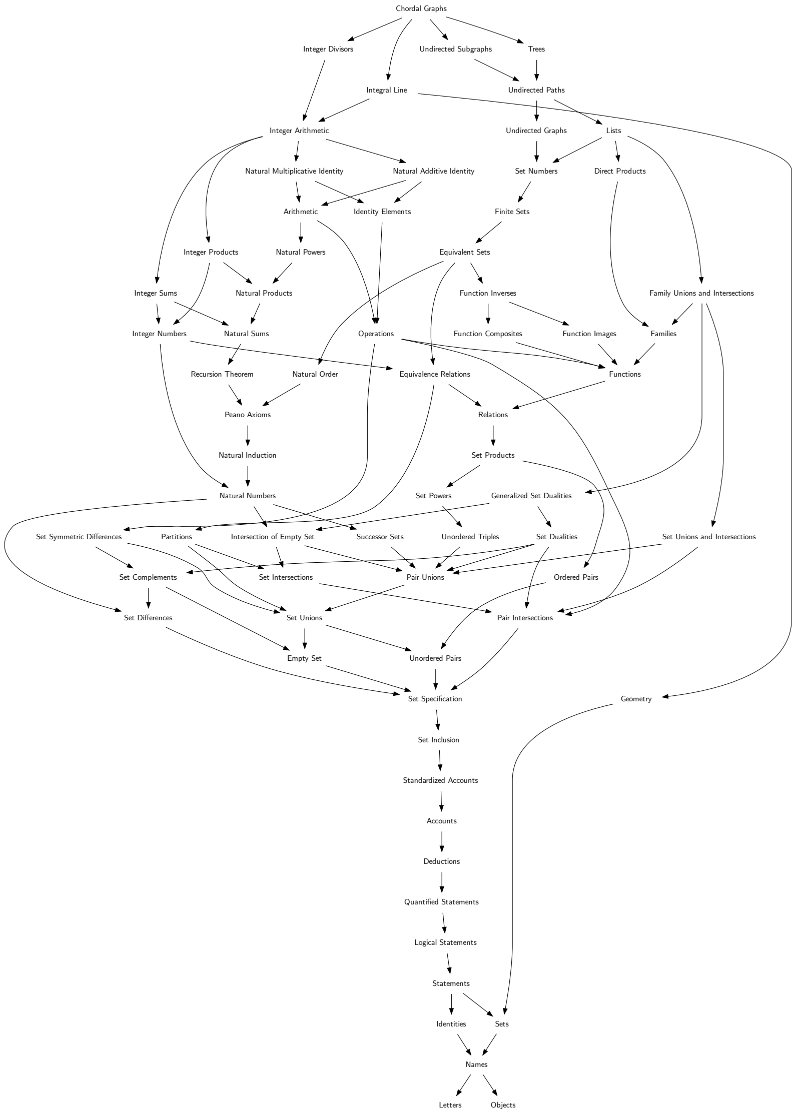


Figure 1: The edge $\{e, d\}$ is a chord in the path (a, e, c, d) of the first graph. The path (a, v, d, c) is chordless. The edge $\{a, e\}$ is a chord in the cycle (a, v, e, c, a) of the second graph. The cycle (a, b, d, c, a) is chordless.

of the definition that G be chordal is that any subgraph of G is chordal.

Simple Examples

Since trees and forests have no cycles, they are chordal. Similarly, any graph with no cycles longer than three edges are trivially chordal. Such graphs are sometimes called *cactus graphs*. The complete graphs are also trivially chordal.


Chordal Graphs
Integer Divisors
Undirected Subgraphs
Trees
Integral Line
Undirected Paths
Integer Arithmetic
Undirected Graphs
Lists
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Finite Sets
Integer Products
Natural Powers
Equivalent Sets
Integer Sums
Natural Products
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Operations
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects