Definition

Let $(X, \mathbf{F})$ be a vector space where $\mathbf{F}$ is $\mathbf{R}$ or $\mathbf{C}$. An inner product $\langle \cdot, \cdot \rangle : X \times X \to \mathbf{R}$ induces a norm $\|\cdot\| : X \to \mathbf{R}$ defined by $\|x\| = \sqrt{\langle x, x \rangle}$ and metric $d : X \times X \to \mathbf{R}$ defined by $d(x, y) = \|x - y\|$.

If (X, d) is a complete metric space, we call $((X, \mathbf{F}), \langle \cdot, \cdot \rangle)$ a *complete inner product space* (or *Hilbert space*).[1]

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[1] The term Hilbert space is universal, but in accordance with the Bourbaki project's guidelines on naming, we will tend to use the term complete inner product space, even though this is longer.

A directed dependency graph with the following node labels:

- Complete Inner Product Spaces
- Complete Real Inner Product Spaces
- Complete Normed Spaces
- Norm Metrics
- Real Inner Product Norm
- Complex Inner Products
- Complete Metric Spaces
- Norms
- Real Inner Products
- Real Inner Product
- Functionals
- Egyptian Sequences
- Real Norm
- Space Inner Product
- Vectors
- Metrics
- Real Vector
- Space Norm
- Real Egyptian Sequence
- N-Dimensional Space
- Space Vector
- Plane Vector Sums and Differences
- Plane Norm
- Plane Inner Product
- Real Limits
- Complex Numbers
- Distance
- Plane Vector
- Right Triangle Sides Relation
- Absolute Value
- Real Square Roots
- Space Distance
- Plane Distance
- Real Square
- Real Arithmetic
- Real Multiplication Inverse
- Interval Length
- Natural Square Roots
- Real Products
- Intervals
- Square Numbers
- Real Additive Inverse
- Real Space
- Real Plane
- Square
- Integral Line
- Real Sequences
- Real Order
- Real Sums
- Rational Arithmetic
- Rational Multiplicative Inverse
- Geometry
- Real Numbers
- Rational Order
- Rational Sums
- Fields
- Rational Products
- Sequences
- Rational Numbers
- Rings
- Integer Arithmetic
- Lists
- Set Numbers
- Finite Sets
- Natural Additive Identity
- Natural Multiplicative Identity
- Integer Products
- Direct Products
- Identity Elements
- Arithmetic
- Natural Powers
- Family Unions and Intersections
- Inverse Elements
- Equivalent Sets
- Element Functions
- Natural Equations
- Integer Order
- Integer Sums
- Comparisons
- Function Inverse
- Function Images
- Operations
- Equation
- Order
- Natural Products
- Integer Numbers
- Families
- Function Composite
- Function
- Natural Sums
- Converse Relations
- Relation
- Natural Order
- Equivalence Relation
- Recursion Theorem
- Set Products
- Peano Axioms
- Generalized Set Dualities
- Set Powers
- Natural Induction
- Set Unions and Intersections
- Set Dualities
- Unordered Triples
- Successor Sets
- Intersection of Empty Set
- Partitions
- Set Symmetric Differences
- Natural Numbers
- Ordered Pairs
- Pair Unions
- Set Intersections
- Set Complements
- Pair Intersections
- Set Union
- Set Differences
- Unordered Pairs
- Empty Set
- Set Specification
- Set Equality
- Set Inclusion
- Standardized Accounts
- Accounts
- Deductions
- Quantified Statements
- Logical Statements
- Statements
- Sets
- Identities
- Names
- Letters
- Objects