Why

We can view the set of sequences as vector spaces and give them norms.

Bounded Sequences

Let $\mathbf{C}^{\mathsf{N}}$ denote the set of complex-valued sequences. Define $\ell^\infty \subset \mathbf{C}^{\mathsf{N}}$ to be the set of all *bounded sequences*. That is,

$$\ell^\infty = \left\{ x \in \mathbf{C}^{\mathsf{N}} \;\middle|\; \exists M \in \mathsf{R} \text{ with } |x_i| < M \text{ for all } i \right\}.$$

Then ℓ^∞ with componentwise addition and scalar multiplication is a vector space.

Exercise 1. *Define* $\|\cdot\|_\infty : \ell^\infty \to \mathsf{R}$ *by*

$$\|x\|_\infty = \sup_{n \in \mathsf{N}} |x_n|.$$

Then $\|\cdot\|_\infty$ *is a norm on* ℓ^∞*.*

Absolutely Summable Sequences

Let $\ell^1 \subset \mathbf{C}^{\mathsf{N}}$ denote the set of all *absolutely summable sequences.* In other words, for $x \in \mathbf{C}^{\mathsf{N}}$, $x \in \ell^1$ if

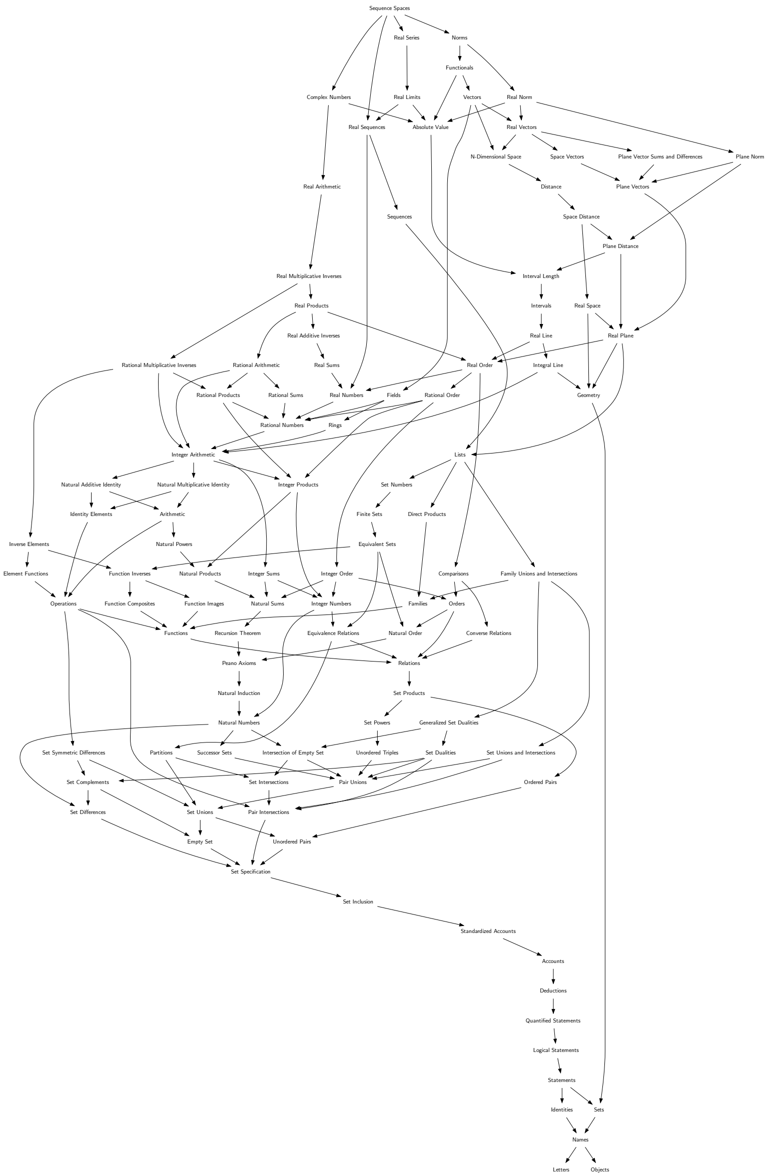
$$\sum_{n=1}^{\infty} |x_n| < \infty.$$

Then ℓ^1 is a vector space with componentwise addition and scalar multiplication.

Exercise 2. *Define* $\|\cdot\|_1 : \ell^1 \to \mathsf{R}$ *by*

$$\|x\|_1 = \sum_{n=1}^{\infty} |x_n|$$

Then $\|\cdot\|_1$ *is a norm on* ℓ^1*.*


Sequence Spaces
Real Series
Norms
Functionals
Complex Numbers
Real Limits
Vectors
Real Norm
Real Sequences
Absolute Value
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Plane Norm
Real Arithmetic
Distance
Plane Vectors
Sequences
Space Distance
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Rational Products
Rational Sums
Real Numbers
Fields
Rational Order
Geometry
Rational Numbers
Rings
Integer Arithmetic
Lists
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Set Numbers
Identity Elements
Arithmetic
Finite Sets
Direct Products
Inverse Elements
Natural Powers
Equivalent Sets
Element Functions
Function Inverses
Natural Products
Integer Sums
Integer Order
Comparisons
Family Unions and Intersections
Operations
Function Composites
Function Images
Natural Sums
Integer Numbers
Families
Orders
Functions
Recursion Theorem
Equivalence Relations
Natural Order
Converse Relations
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects