



Definition

A set $C \subset \mathbf{R}^n$ is *convex* if it contains the closed line segment between every pair of distinct points. In other words,

$$\lambda x + (1 - \lambda)y \in C \quad \text{for all } x, y \in C \text{ and } \lambda \in [0, 1].$$

Roughly speaking, C is convex if and only if its intersection with every line in \mathbf{R}^n is either empty or a closed line segment.

Examples

The empty set, any singleton, any subspace, any affine set and any half-space.

Properties

Proposition 1 (closure under intersections). *Suppose $\mathcal{K} \subset \mathcal{P}(\mathbf{R}^n)$ is a set of convex sets. Then $\cap \mathcal{K}$ is convex.*

Proposition 2 (sums, differences, scales are convex). *Suppose A, B are convex sets. Then $A + B$, $A - B$ and λA for any real λ is convex.*

