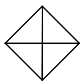


Why

We want a norm on the vector space of continuous functions.

Definition

Consider a function from a closed real interval to the real numbers. The *absolute supremum* of the function is the absolute value of its results on the interval. Since the function is continuous and defined on a closed interval, the supremum is finite.

Prop. 1. *The functional mapping $f \in C[a, b]$ to its absolute supremum is a norm.*

Proof. Let R denote the set of real numbers. Define $\phi : C[a, b] \to R$ by:

$$\phi(f) = \sup\{|f(x)| \mid x \in [a, b]\}.$$

1. $|f(x)| \geq 0$ for all $x \in [a, b]$, so $\phi(f) \geq 0$.
2. If $\phi(f) = 0$ then $|f(x)| \leq 0$ for all x and so $f(x) = 0$ for all $x \in [a, b]$. If $f = 0$, then $|f(x)| = 0$ for all $x \in [a, b]$
3. For all α real, $|\alpha f(x)| = |\alpha||f(x)|$. so $\phi(\alpha f) = |\alpha|\phi(f)$
4. For all $f, g \in C[a, b]$, and $x \in [a, b]$, $|f(x) + g(x)| \leq |f(x)| + |g(x)|$ by the triangle inequality for absolute value. Thus,

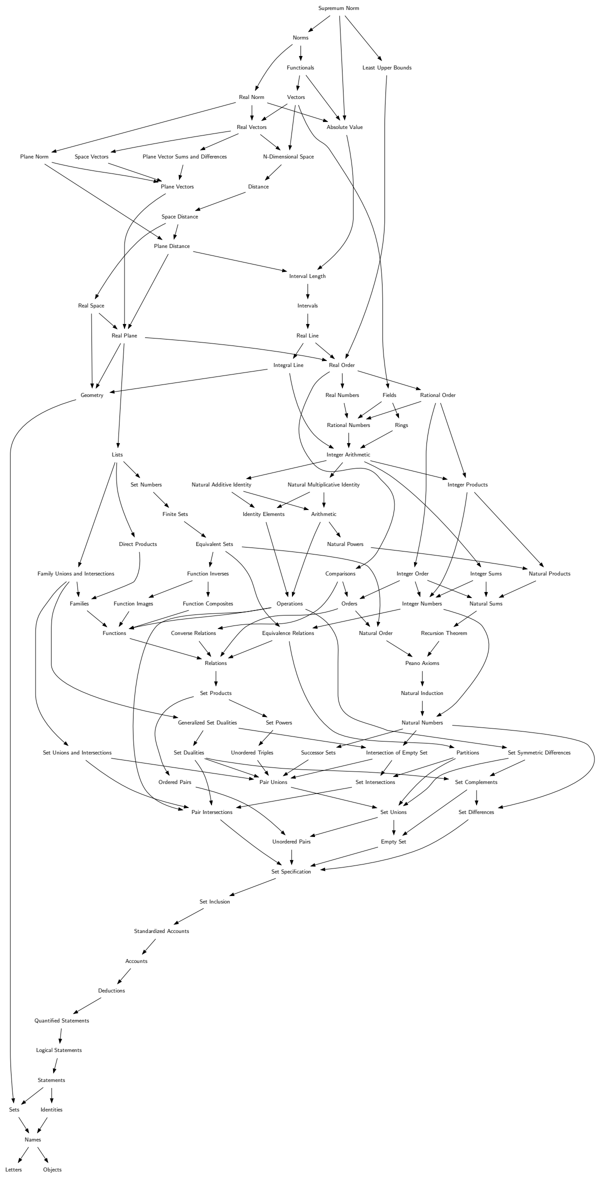
$$\begin{aligned}
\phi(f + g) &\leq \sup\{|f(x)| + |g(x)| \mid x \in [a, b]\} \\
&\leq \sup\{|f(x)| \mid x \in [a, b]\} + \sup\{|g(x)| \mid x \in [a, b]\} \\
&= \phi(f) + \phi(g)
\end{aligned}$$

□

We call the functional ϕ defined above the *supremum norm*.

Notation

Let $f \in C[a, b]$. We denote the supremum norm of f by $\|f\|_{\text{sup}}$.


Supremum Norm
Norms
Functionals
Least Upper Bounds
Real Norm
Vectors
Real Vectors
Absolute Value
Plane Norm
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Geometry
Real Numbers
Fields
Rational Order
Rational Numbers
Rings
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects