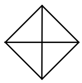

Why

We want to estimate the weights of a linear function.[1]

Definition

The *probabilistic linear model*; *linear model*; *linear regression*

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. We have n precepts in R^d. So let $a^1, \dots, a^n \in \mathsf{R}^d$ with data matrix $A \in \mathsf{R}^{n \times d}$. We are modeling a relation between R^d and R.

Let $x : \Omega \to \mathsf{R}^d$ and $e : \Omega \to \mathsf{R}^n$ be independent random vectors with zero mean and covariances given by Σ_x and Σ_e, respectively. For each $\omega \in \Omega$, define the map $f : \Omega \to (\mathsf{R}^d \to \mathsf{R})$ by $f(\omega)(a) = \sum_j a^i_j x_j(\omega) + e_i(\omega)$.

We call x the *signal*. We call e the *noise*. This class of models assumes the signal and noise are independent.

Define $y : \Omega \to \mathsf{R}^n$ by $y(\omega) = Ax(\omega) + e(\omega)$. So,

$$y = Ax + e.$$

Proposition 1. $\mathsf{E}(y) = A\,\mathsf{E}(x) + \mathsf{E}(w)$[2]

Proposition 2. $\mathsf{cov}((x, y)) = A\,\mathsf{cov}(x)A^\top + \mathsf{cov}\,e$[3]

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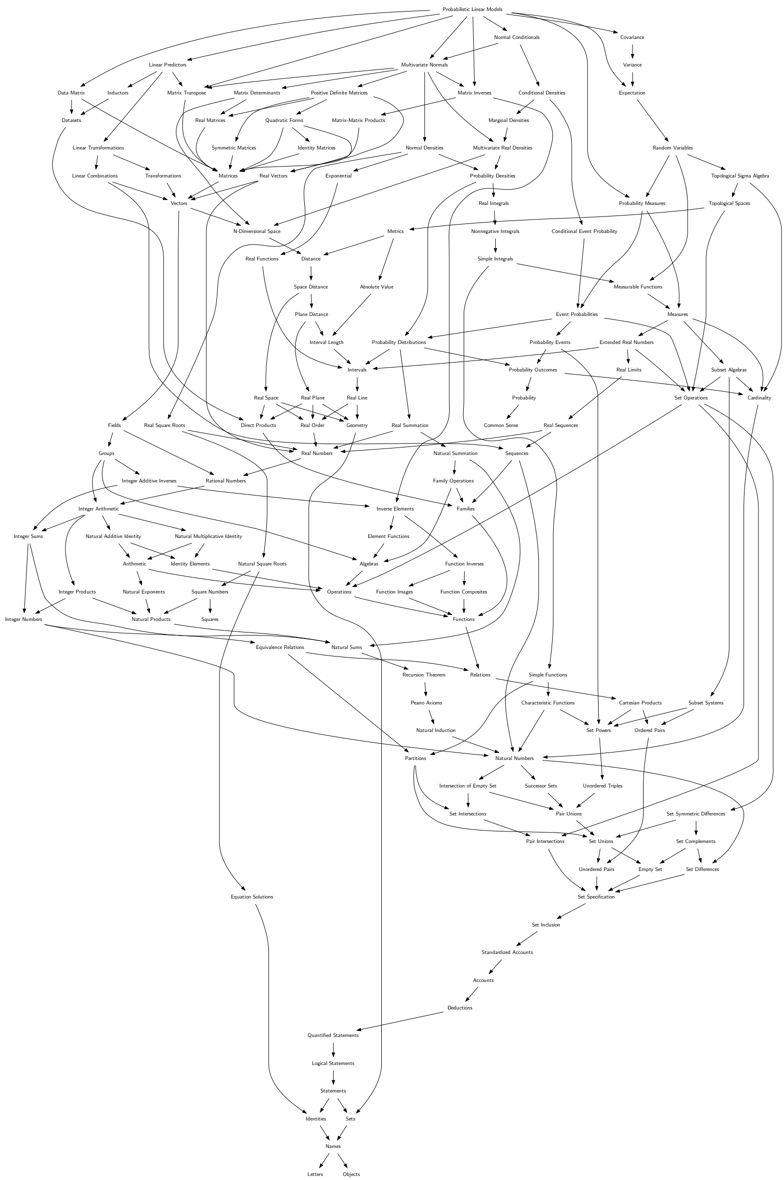
[1] Future editions will include this.

[2] By linearity. Full account in future editions.

[3] Full account in future editions.


Probabilistic Linear Models
Normal Conditionals
Covariance
Linear Predictors
Multivariate Normals
Variance
Data Matrix
Inductors
Matrix Transpose
Matrix Determinants
Positive Definite Matrices
Matrix Inverses
Conditional Densities
Expectation
Datasets
Real Matrices
Quadratic Forms
Matrix-Matrix Products
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Linear Transformations
Symmetric Matrices
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Multivariate Real Densities
Random Variables
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Topological Sigma Algebra
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Absolute Value
Measurable Functions
Plane Distance
Event Probabilities
Measures
Interval Length
Probability Distributions
Probability Events
Extended Real Numbers
Intervals
Probability Outcomes
Real Limits
Subset Algebras
Real Space
Real Plane
Real Line
Probability
Set Operations
Cardinality
Fields
Real Square Roots
Direct Products
Real Order
Geometry
Real Summation
Common Sense
Real Sequences
Groups
Real Numbers
Natural Summation
Sequences
Integer Additive Inverses
Rational Numbers
Family Operations
Integer Arithmetic
Inverse Elements
Families
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Element Functions
Arithmetic
Identity Elements
Natural Square Roots
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Cartesian Products
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Natural Induction
Set Powers
Ordered Pairs
Partitions
Natural Numbers
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Equation Solutions
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
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Logical Statements
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Names
Letters
Objects