Why

We want to extend our notion of entropy (see Discrete Entropy) to real-valued (continuous) random variables.

Definition

The *differential entropy* of a probability density function is the integral of the density against the negative log of the density. This definition made to be similar to the case of discrete entropy. If a real-valued random variable has a density, then we call the differential entropy of its density the *differential entropy* of the random variable.

Notation

Let $f : \mathbf{R}^n \to \mathbf{R}$ be a probability density function. The differential entropy of f is

$$-\int f \log f$$

We denote the differential entropy of f by $h(f)$.

Example

Let $x : \Omega \to \mathbf{R}$ be uniform on $[0, 1/2]$. Then $h(x) = \log 1/2 < 0$.

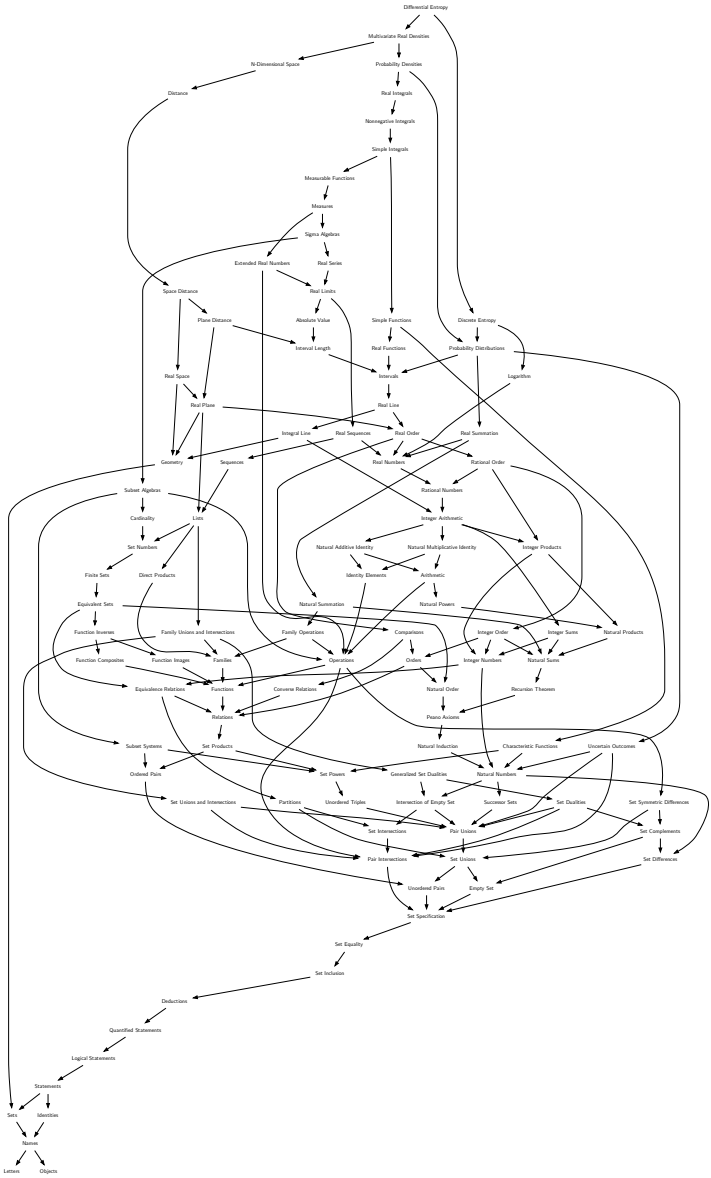
Problems

We have $h(ax) = h(x) + \log|a|$. In general $h(Ax) = h(x) + \log|A|$.

Differences still meaningful

Even though the value of the differential entropy is not necessarily a good analogy to discrete entropy, differences still are. In particular, the following holds

$$I(X;Y) = H(Y) - H(Y \mid X) = H(X) = H(X \mid Y)$$


Differential Entropy
Multivariate Real Densities
N-Dimensional Space
Probability Densities
Distance
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Space Distance
Real Limits
Plane Distance
Absolute Value
Simple Functions
Discrete Entropy
Interval Length
Real Functions
Probability Distributions
Real Space
Intervals
Logarithm
Real Plane
Real Line
Integral Line
Real Sequences
Real Order
Real Summation
Geometry
Sequences
Real Numbers
Rational Order
Subset Algebras
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Direct Products
Identity Elements
Arithmetic
Equivalent Sets
Natural Summation
Natural Powers
Function Inverses
Family Unions and Intersections
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Products
Function Composition
Function Images
Families
Operations
Orders
Integer Numbers
Natural Sums
Equivalence Relations
Functions
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Characteristic Functions
Uncertain Outcomes
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects