



Equivalence Relations

1 Why

We want to handle at once all elements which are indistinguishable or equivalent in some aspect.

2 Definition

A relation R on a set A is an *equivalence relation* if it is reflexive, symmetric, and transitive.

For an element $a \in A$, we call the set of elements in relation R to a the *equivalence class* of a . The key observation, recorded and proven below, is that the equivalence classes partition the set A . A frequent technique is to define an appropriate equivalence relation on a large set A and then to work with the set of equivalence classes of A .

We call the set of equivalence classes the *quotient set* of A under R . An equally good name is the divided set of A under R , but this terminology is not standard. The language in both cases reminds us that \sim partitions the set A into equivalence classes.

2.1 Notation

If R is an equivalence relation on a set A , we use the symbol \sim . When alone, \sim is read aloud as "sim," but we still read $a \sim b$ aloud as "a equivalent to b." We denote the quotient set of A under \sim by A/\sim , read aloud as "A quotient sim".

2.2 Results

TODO