



## EQUIVALENT SETS

### Why

We want to talk about the size of a set.

### Definition

Two sets are *equivalent* if there exists a bijection between them.

**PROPOSITION 1.** *Set equivalence in the sense defined above is an equivalence relation in the power set of a set.*

**PROPOSITION 2.** *Every proper subset of a natural number is equivalent to some smaller natural number.*

*Proof.* TODO induction

□

TODO: smaller defined?

**PROPOSITION 3.** *A set can be equivalent to a proper subset of itself.*

Halmos' example here is not a bijection, though...

**PROPOSITION 4.** *If  $n$  is a natural number, then  $n$  is not equivalent to a proper subset of itself.*

**PROPOSITION 5.** *A set can be equivalent to at most one natural number.*

**PROPOSITION 6.** *The set of natural numbers is infinite.*

**PROPOSITION 7.** *A finite set is never equivalent to a proper subset of itself.*

PROPOSITION **8.** *Every subset of a finite set is finite.*

PROPOSITION **9.** *Every subset of a natural number is equivalent to a natural number.*

