



## Equivalent Sets

### 1 Why

We want to talk about the size of a set.

### 2 Definition

Two sets are *equivalent* if there exists a bijection between them.

**Proposition 1.** *Set equivalence in the sense defined above is an equivalence relation in the power set of a set.*

**Proposition 2.** *Every proper subset of a natural number is equivalent to some smaller natural number.*

*Proof.* TODO induction

□

TODO: smaller defined?

**Proposition 3.** *A set can be equivalent to a proper subset of itself.*

Halmos' example here is not a bijection, though...

**Proposition 4.** *If  $n$  is a natural number, then  $n$  is not equivalent to a proper subset of itself.*

**Proposition 5.** *A set can be equivalent to at most one natural number.*

**Proposition 6.** *The set of natural numbers is infinite.*

**Proposition 7.** *A finite set is never equivalent to a proper subset of itself.*

**Proposition 8.** *Every subset of a finite set is finite.*

**Proposition 9.** *Every subset of a natural number is equivalent to a natural number.*