MATRIX TRACE

Why

TODO

Definition

The *trace* of a square real matrix is the sum of the elements on its diagonal.

Notation

We denote the function which associates a matrix with its trace by $\mathbf{tr} : \mathbf{R}^{n \times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n \times n}$. Then

$$\mathbf{tr}\, A = \sum_{i=1}^{n} A_{ii}.$$

Properties

PROPOSITION **1.** *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n \times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\mathbf{tr}\, C = \sum_{i=1}^{n} C_{ii} = \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii} = \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii} = \alpha\, \mathbf{tr}\, A + \beta\, \mathbf{tr}\, B.$$

$\square$

PROPOSITION **2.** *Let $A, B \in \mathbf{R}^{n \times n}$.*

$$\mathbf{tr}\,(AB) = \mathbf{tr}\,(BA)$$

Proof. TODO

$\square$

Matrix Trace
↓
Real Matrices
↓
Matrices
↓
Vectors
↓ ↓
Fields   N-Dimensional Space
↓   ↓   ↓
Groups   Real Numbers   Space
↓   ↓
Algebras   Rational Numbers
↓   ↓
Operations   Integer Numbers
↓   ↓
Functions   Zero
↓   ↓   ↓
Relations   Natural Numbers   Equation Solutions
↓
Ordered Pairs
↓   ↓   ↓   ↓
Unordered Pairs   Empty Set   Set Intersections   Set Unions   Equations
↓
Set Specification
↓   ↓
Sentences   Set Inclusion
↓
Set Equality
↓   ↓
Sets   Identity
↓
Objects