Why

Certain sparse matrices are easier to work with, especially those with chordal sparsity patterns.[1]

Definition

A *sparsity pattern E* of *order n* is a set of (unordered) pairs of $V = \{1, \ldots, n\}$. A sparsity pattern is *chordal* if the undirected graph (V, E) is chordal.

A symmetric matrix is said to *have a sparsity pattern* if its ijth entry is zero whenever $\{i, j\}$ is not in the sparsity pattern. The diagonal entries and off-diagonal entries for pairs appearing in the sparsity pattern may or may not be zero.

The graph whose vertices are one through n and whose edge set is the sparsity patter is called the *sparsity graph*

A sparsity pattern is not a property of a matrix because it is not unique (unless all off-diagonal entries are non-zero). If a matrix has a particular sparsity pattern it has every sparsity pattern which is a superset of it. In other words, every matrix has the sparsity pattern which is the set of all pairs of integers.

Notation

Let $E \subset \{\{i, j\} \mid i, j \in \{1, 2, \ldots, n\}\}$. A symmetric matrix $A \in \mathbf{S}^n$ is said to have sparsity pattern E if $A_{ij} = A_{ji} = 0$ wheneer $i \neq j$ and $\{i, j\} \notin E$. The graph $G = (V, E)$ where $V = \{1, 2, \ldots, n\}$ is the sparsity graph associated with E.

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[1]Future editions will expand.
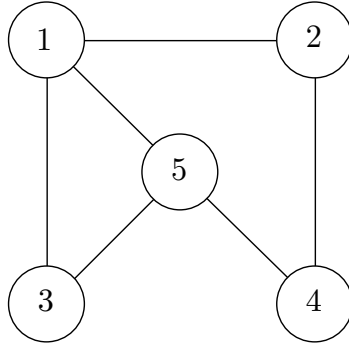
Figure 1: Sparsity graph for the matrix A.

We will denote the symmetric matrices of order n with sparsity pattern E by $\mathbf{S}_E^n$.

Example

Figure 1 shows a sparsity graph for the matrix

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A := \begin{bmatrix}
A_{11} & A_{21} & A_{31} & 0 & A_{51} \\
A_{21} & A_{22} & 0 & A_{42} & 0 \\
A_{31} & 0 & A_{33} & 0 & A_{54} \\
0 & A_{42} & 0 & A_{44} & A_{45} \\
A_{51} & 0 & A_{53} & A_{54} & A_{55}
\end{bmatrix} .
$$

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