



## COMPLEX CONJUGATES

### Definition

The *complex conjugate* (or *conjugate*) of a complex number  $z$  is the complex number whose real part matches  $z$  and whose imaginary part is the additive inverse of  $z$ . The complex conjugate of a purely real number is the same purely real number. In other words, the complex conjugate of a complex number with no imaginary part is the same complex number.

### Notation

We denote the complex conjugate of the complex number  $z \in \mathbf{C}$  by  $\mathbf{C}conjz$ . Other common notation includes  $\bar{z}$ , read “ $z$  bar”. If there exists  $a, b \in \mathbf{R}$  so that  $z = (a, b)$ , then  $\mathbf{C}conjz = (a, -b)$ .

### Geometric interpretation

Taking the conjugate of a complex numbers corresponds to a reflection across the real axis in the plane.

### Properties

A complex number  $z$  is real if and only if  $z = \mathbf{C}conjz$  and it is imaginär if and only if  $z = -\mathbf{C}conjz$ .

**Proposition 1.** *For  $z \in \mathbf{C}$ , we have*

$$\operatorname{Re}(z) = \frac{z + z^*}{2} \quad \text{and} \quad \operatorname{Im}(z) = \frac{z - z^*}{2i}.$$



