Why

We have seen several concepts that consist of associating with a pair of sets a third set. For example, set unions and set intersections

Definition

An *operation* (or *binary operation*, *law of composition*) on a set A is a function from $A \times A$ to A.

Roughly speaking, operations *combine* (or *compose*) elements. We *operate* on ordered pairs.

Example: set operations

Let X be a set. Define $g : \mathcal{P}(X) \times \mathcal{P}(X) \to \mathcal{P}(X)$ by $g(A, B) = A \cup B$. Then g, the function which associates with two sets their union (see Pair Unions) is an operation on $\mathcal{P}(X)$. Likewise, define $h : \mathcal{P}(X) \times \mathcal{P}(X) \to \mathcal{P}(X)$ by $h(A, B) = A \cap B$.

Naming their properties

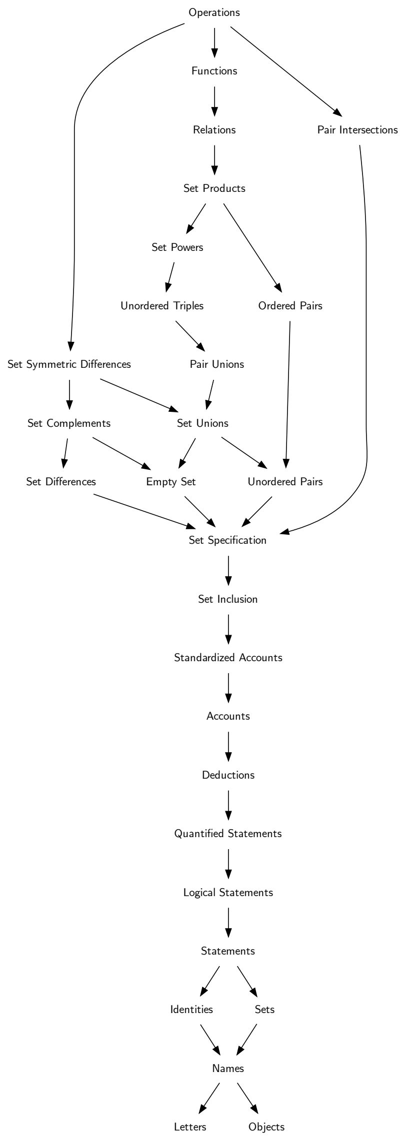
$\cup$ has several nice properties. For one $A \cup B = B \cup A$ and $(A \cup B) \cup C = A \cup (B \cup C)$.

An operation with the first property, that the ordered pair (A, B) and (B, A) have the same result is called *commutative.* An operation with the second property, that when given three objects the order in which we operate does not matter is called *associative.* $\cap$ shares these properties with $\cup$.

We call the operation of *forming unions* the function $(A, B) \mapsto A \cup B$. We call the operation of *forming intersections* the function $(A, B) \mapsto A \cap B$. We call the operation of *forming symmetric* *differences* the function $(A, B) \mapsto A + B$. Since forming unions commutes and is associative and likewise with forming intersections, forming symmetric differences also commutes.

Algebras

Of course, any operation is defined on some set. For this reason, we define an *algebra* as an ordered pair whose first element is a non-empty set and whose second element is an operation on that set. The *ground set* of the algebra is the set on which the operation is defined.


Operations
Functions
Relations
Pair Intersections
Set Products
Set Powers
Unordered Triples
Ordered Pairs
Set Symmetric Differences
Pair Unions
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects