Why

We want to "combine" elements of a set.

Definition

Let A be a non-empty set. An *operation* on A is a function from ordered pairs of elements of the set to the same set. Operations *combine* elements. We *operate* on ordered pairs.

Notation

Let A be a set and $g : A \times A \to A$. We tend to forego the notation $g(a, b)$ and write $a \, g \, b$ instead. We call this *infix notation.*

Using lower case latin letters for elements and for operators confuses, so we tend to use special symbols for operations. For example, $+$, $-$, $\cdot$, $\circ$, and $\star$.

Let A be a non-empty set and $+ : A \times A \to A$ be an operation on A. According to the above paragraph, we tend to write $a + b$ for the result of applying $+$ to (a, b).

Example

A first example of an operation is if we consider the set A as the power set of some set X. Then the pair union (see Pair Unions) is an operation. For if $E \in X^*$ and $F \in X^*$ then $E \cup F \in F^*$ and so $\cup$ can be viewed as an operation on X^*.

Properties

Recall that $\cup$ has several nice properties. For one $A \cup B = B \cup A$ and $(A \cup B) \cup C = A \cup (B \cup C)$.

An operation with the first property, that the ordered pair (A, B) and (B, A) have the same result is called *commutative.* An operation with the second property, that when given three objects the order in which we operate does not matter is called *associative.*

Operations

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Functions

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Relations

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Cartesian Products

Set Powers

Ordered Pairs   Unordered Triples

Pair Unions

Set Unions

Unordered Pairs   Empty Set

Set Specification

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Set Inclusion

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Standardized Accounts

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Accounts

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Deductions

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Quantified Statements

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Logical Statements

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Statements

Identities   Sets

Names

Letters   Objects