

## REAL INTEGRAL MONOTONE CONVERGENCE

## Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

## Result

When context is clear, we refer to the following proposition as the *monotone convergence theorem*.

**Prop. 1.** The integral of the almost everywhere limit of an almost-everywhere nondecreasing sequence of measurable, nonnegative, extended-real-valued functions is the limit of the sequence of integrals of the functions.

Proof. Let  $(X, \mathcal{A}, \mu)$  be a measure space, and let  $f_n : \to [-\infty, \infty]$  a  $\mathcal{A}$ -measurable function for every natural number n and let  $f: X \to [-\infty, \infty]$  a  $\mathcal{A}$ -measurable function. We want to show that if

$$f_n(x) \le f_{n+1}(x)$$
 and  $f(x) = \lim_{x \to 0} f_n(x)$ 

hold for all natural n and almost every x in X, then

$$\int f d\mu = \lim_{n} \int f_n d\mu.$$

