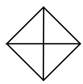


Why

We can extend the notion of independnence beyond pairs of uncertain events, to sets of events.

Definition

Suppose P is a event probability function on a finite sample space Ω. The events $A_1, \dots, A_n$ are *independent* (or *mutually independent*), if for all k between 1 and n, and distinct indicies $i_1, \dots, i_k$ between 1 and n,

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2}) \cdots P(A_{i_k}).$$

Similar to the case of pairs of events, one can show that this condition is equivalent to the statement that for any *distinct* indices $i_1, \dots, i_k, j_1, \dots, j_l$,

$$P(A_{j_1} \cap \cdots \cap A_{j_l} \mid A_{i_1} \cap \cdots \cap A_{i_k}) = P(A_{j_1} \cap \cdots \cap A_{j_l})$$

Examples

n tosses of a coin. As usual, model n tosses of a coin with $\{0, 1\}^n$ and put a distribution $p : \Omega \to [0, 1]$ so that

$$p(\omega) = 1/2^n \quad \text{for all } \omega \in \Omega$$

Now, for $i = 1, \dots, n$, define the event A_i by

$$A_i = \{\omega \in \Omega \mid \omega(i) = 1\}$$

We claim that the set $\{A_1, \dots, A_n\}$ is mutually independent. To see this, notice that for any distinct indices $i_1, \dots, i_k$,

$$|A_{i_1} \cap \cdots \cap A_{i_k}| = 2^{n-k}$$

This holds because, once k of the coin flips, there are 2^{n-k} ways for the remaining coins to land (using the fundamental principle of counting). Consequently,

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = \frac{2^{n-k}}{2^n} = 2^{-k}$$

We can use this result with one set $P(A_i) = 1/2$, and so we obtain

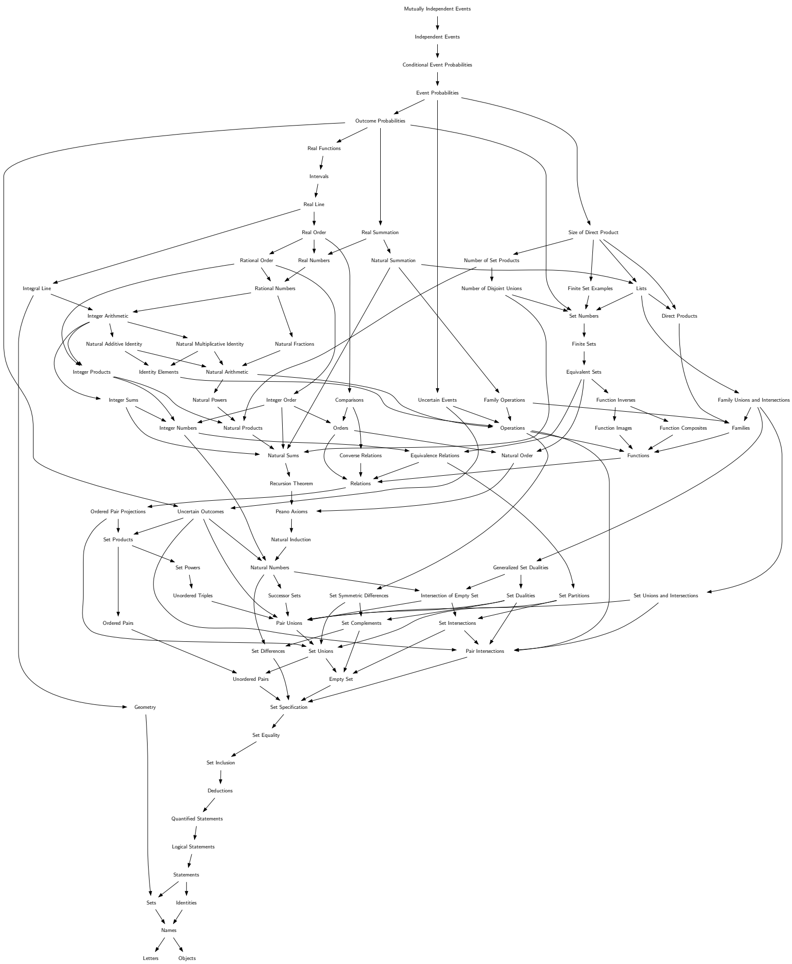
$$P(A_{i_1} \cap \cdots A_{i_k}) = P(A_{i_1}) \cdots P(A_{i_k}),$$

as desired.

Basic implications

It can be shown[1] that if $A_1, \ldots, A_n$ are indepnednet events, and $B_1, \ldots, B_n$ are events such that B_i is either A_i or A_i^c, then $B_1, \ldots, B_n$ are mutually independent.

[1] Future editions will.


Mutually Independent Events
Independent Events
Conditional Event Probabilities
Event Probabilities
Outcome Probabilities
Real Functions
Intervals
Real Line
Real Order
Real Summation
Size of Direct Product
Rational Order
Real Numbers
Natural Summation
Number of Set Products
Integral Line
Rational Numbers
Number of Disjoint Unions
Finite Set Examples
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Additive Identity
Natural Multiplicative Identity
Natural Fractions
Finite Sets
Integer Products
Identity Elements
Natural Arithmetic
Equivalent Sets
Integer Sums
Natural Powers
Integer Order
Comparisons
Uncertain Events
Family Operations
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Converse Relations
Equivalence Relations
Natural Order
Functions
Recursion Theorem
Relations
Ordered Pair Projections
Uncertain Outcomes
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Intersection of Empty Set
Set Dualities
Set Partitions
Set Unions and Intersections
Ordered Pairs
Pair Unions
Set Complements
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects