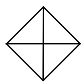


Why

The set of real numbers is large, and so we can often embed other sets within it. Also, we are often interested in modelling random quantities.

Definition

A *real outcome variable* (or *real random variable*, *random variable*) is an outcome variable whose codomain is a subset of the real numbers. Such variables are often called *quantitative*. Caution: many authorities *reserve* the term *random variable* for outcome variables whose domain is $\mathbf{R}$.

The *probability mass function* (or *p.m.f.*, *pmf*) of a random variable $X : \Omega \to \mathbf{R}$ is the function $f : \mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = P(X = x)$$

If Ω is finite, then $\operatorname{range} X$ is a finite set, and so the probability mass function is the extension to $\mathbf{R}$ of the induced distribution $p : \operatorname{range}(X) \to \mathbf{R}$ of X. If $\operatorname{range}(X)$ is finite or countable, we call X a *discrete random variable.*

Notation

For a real-valued random variable $X : \Omega \to \mathbf{R}$ and $\alpha \in \mathbf{R}$, we often abbreviate the sets

$$\{\omega \in \Omega \mid X(\omega) \leq \alpha\} \text{ and } \{\omega \in \Omega \mid X(\omega) \geq \alpha\}$$

by $\{X \leq \alpha\}$ and $\{X \geq \alpha\}$ respectively. Also, given a probability measure P, we denote the probabilities of these events by $P(X \leq \alpha)$ and $P(X \geq \alpha)$, respectively. Similar to before, the notation $X \sim f$ is shorthand for the random variable $X : \Omega \to \mathbf{R}$ has probability mass function $f : \mathbf{R} \to \mathbf{R}$.

Examples

Tossing a fair coin n times. Suppose we model n tosses of a fair coin as usual, so that $\Omega = \{0, 1\}^n$ and $p : \Omega \to \mathbf{R}$ is defined by

$$p(\omega) = 2^{-n} \quad \text{for all } \omega \in \Omega$$

Recall that we have calculuated $P(X = k)$ to be $\binom{n}{k}2^{-n}$. Thus, the probability mass function $f : \mathbf{R} \to \mathbf{R}$ of X satisifes

$$f(k) = \binom{n}{k}2^{-n} \quad \text{for } k = 0, \ldots, n$$

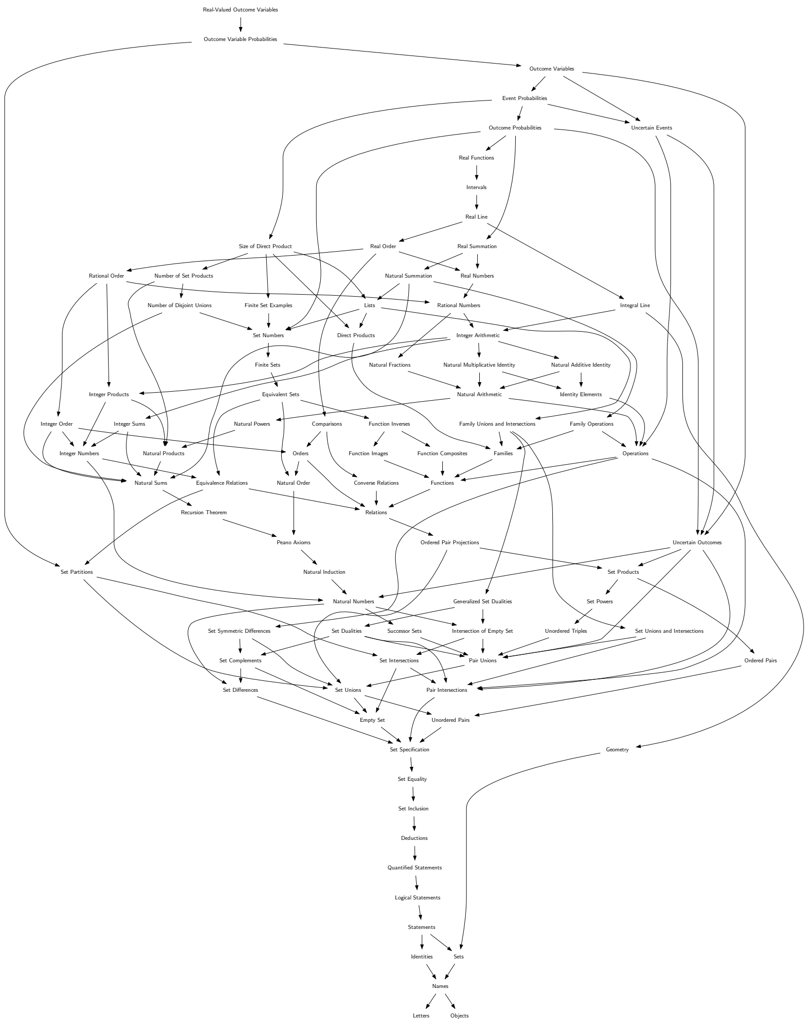
and $f(x) = 0$ for $x \neq 0, \ldots, n$.

Cumulative distribution function

Given a random variable $X : \Omega \to \mathbf{R}$ and probability measure P on $\mathcal{P}(\Omega)$, the function $F : \mathbf{R} \to \mathbf{R}$ defined by

$$F(x) = P(X \leq x)$$

is called the *cumulative distribution function* of X.


Real-Valued Outcome Variables
Outcome Variable Probabilities
Outcome Variables
Event Probabilities
Outcome Probabilities
Uncertain Events
Real Functions
Intervals
Real Line
Size of Direct Product
Real Order
Real Summation
Rational Order
Number of Set Products
Natural Summation
Real Numbers
Number of Disjoint Unions
Finite Set Examples
Lists
Rational Numbers
Integral Line
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Natural Fractions
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Equivalent Sets
Natural Arithmetic
Identity Elements
Integer Order
Integer Sums
Natural Powers
Comparisons
Function Inverses
Family Unions and Intersections
Family Operations
Integer Numbers
Natural Products
Orders
Function Images
Function Composites
Families
Operations
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Set Partitions
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects