



# Probability Measures

## 1 Why

We use the language of measure theory to give a mathematical model for uncertain outcomes. TODO: probability intuition sheet.

## 2 Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The word *space* is natural, since we developed measure theory partly as a generalization of volume in three-dimensional space. The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra.

The measure in a probability space corresponds to the even probability function.

### 2.1 Notation

Let  $(A, \mathcal{A})$  be a measurable space. We denote the set of outcomes by  $\Omega$ , a mnemonic for "outcomes." We denote the sigma-algebra by  $\mathcal{A}$ , as usual. We denote a probability measure by  $\mathbf{P}$ , a mnemonic for "probability," and intended to remind of the event probability function. Thus, we often say "Let  $(A, \mathcal{A}, \mathbf{P})$  be a probability space."

## 2.2 Properties