

## $\mathbf{Why}^1$

## **Definition**

An ordering of an undirected graph is an ordering (see Lists) of its vertices. An ordered undirected graph is an ordered pair  $((V, E), \sigma : \{1, 2, \dots, |V|\} \rightarrow V)$  where (V, E) is an undirected graph (see Undirected Graphs) and  $\sigma$  is an ordering of the vertex set V.

## Notation

Let  $((V, E), \sigma)$  be an ordered undirected graph. We commonly associate it with  $(V, E, \sigma)$  and call this ordered triple an undirected graph as well. But, throughout these sheets, an ordered undirected graph is an ordered pair.

We denote that  $\sigma^{-1}(v) < \sigma^{-1}(w)$  by  $v \prec_{\sigma} w$  and  $v \succeq_{\sigma} w$  by  $\sigma^{-1}(v) \le \sigma^{-1}(w)$ . We occasionally omit the subscripts in  $\prec_{\sigma}$  and  $\succeq_{\sigma}$  when clear from context.

## Visualization

We visualize an ordered undirected graph by labeling its nodes with the indices of each vertex. Let (V, E) be an undirected graph with  $V = \{a, b, c, d, e\}$  and

$$E = \{\{a,b\}, \{a,c\}, \{a,e\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,e\}, \{d,e\}\}.$$

Let  $\sigma: \{1, \ldots, 5\} \to V$  be an ordering with

$$\sigma(1) = a \quad \sigma(2) = c \quad \sigma(3) = d \quad \sigma(4) = b \quad \sigma(e) = 5.$$

We visualize the ordered graph in the figure.

<sup>&</sup>lt;sup>1</sup>Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.



