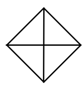


Why

[1]

Definition

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex product* of z_1 and z_2 is the complex number $(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$.

Notation

We denote the complex product of z_1 and z_2 by $z_1 \cdot z_2$ or z_1z_2. The notation is justified because the complex product of two purely real complex numbers corresponds to the purely real complex number whose real part is the real product of the real parts of the first two numbers.

Recall that we denote $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. This notation is a mnemonic for the definition of a complex product if we treat $i^2 = -1$.

$$\begin{aligned} z_1z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2). \end{aligned}$$

Properties

Proposition 1 (Commutativity)**.** *For all $z_1, z_2 \in \mathbf{C}$, we have $z_1z_2 = z_2z_1$.*

[1]Future editions will include.

Proposition 2 (Associativity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have and $z_1(z_2z_3) = (z_1z_2)z_3$.*

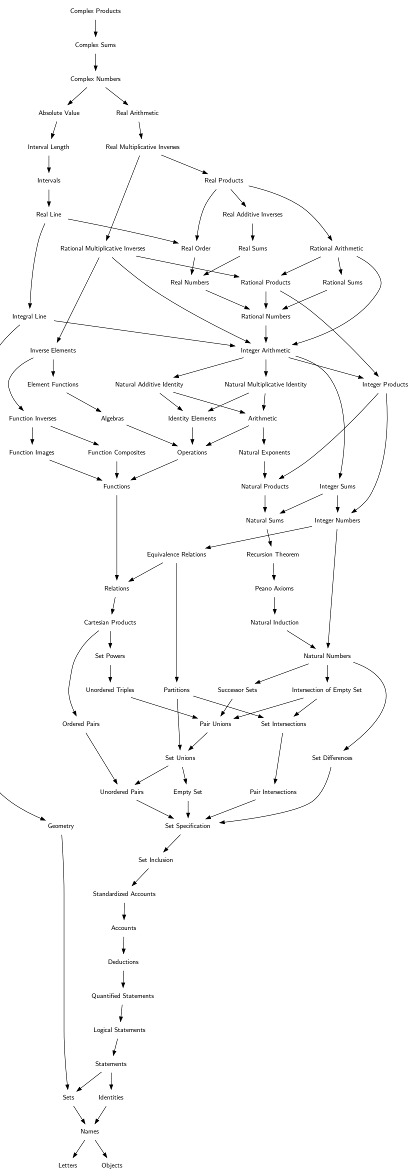
Complex multiplication

We call the operation that associates a pair of complex numbers with their product *complex multiplication.* The operation is symmetric (commutative).

Multiplicative identity and inverse

Notice that the complex number $(1, 0)$ is the multiplicative identity. It is unique,[2] and so we call it the *complex multiplicative identity.*

[2]Future editions will include an account


Complex Products
Complex Sums
Complex Numbers
Absolute Value
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Line
Real Additive Inverses
Rational Multiplicative Inverses
Real Order
Real Sums
Rational Arithmetic
Real Numbers
Rational Products
Rational Sums
Integral Line
Rational Numbers
Inverse Elements
Integer Arithmetic
Element Functions
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Function Inverses
Algebras
Identity Elements
Arithmetic
Function Images
Function Composites
Operations
Natural Exponents
Functions
Natural Products
Integer Sums
Natural Sums
Integer Numbers
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects