

LOWER UPPER TRIANGULAR FACTORIZATIONS

Why

If a system (A, b) is ordinarily reducible, then there exists L unit lower triangular and U upper triangular so that A = LU. When does such a factorization exist?

Definition

Let $A \in \mathbf{R}^{m \times m}$. A lower upper triangular factorization of A is a pair of matrices $(L \in \mathbf{R}^{m \times m}, U \in \mathbf{R}^{m \times m})$ where L is unit lower triangular, U is upper triangular and A = LU. Other terminology includes lower upper triangular decomposition, LU factorization, and LU decomposition.

Proposition 1. If (A,b) is ordinarily reducible, a LU factorization exists.

What about an LU-factorization when (A, b) is not ordinarily reducible? The main issue is that we may encounter a diagonal entry of some reduction of A which is zero.

