Why

We want to "combine" elements of a set.

Definition

Let A be a non-empty set. An *operation* on A is a function from ordered pairs of elements of the set to the same set. Operations *combine* elements. We *operate* on ordered pairs.

Notation

Let A be a set and $g : A \times A \to A$. We tend to forego the notation $g(a, b)$ and write $a\, g\, b$ instead. We call this *infix notation*.

Using lower case latin letters for elements and for operations confuses, so we tend to use special symbols for operations. For example, $+$, $-$, $\cdot$, $\circ$, and $\star$.

Let A be a non-empty set and $+ : A \times A \to A$ be an operation on A. According to the above paragraph, we tend to write $a + b$ for the result of applying $+$ to (a, b).

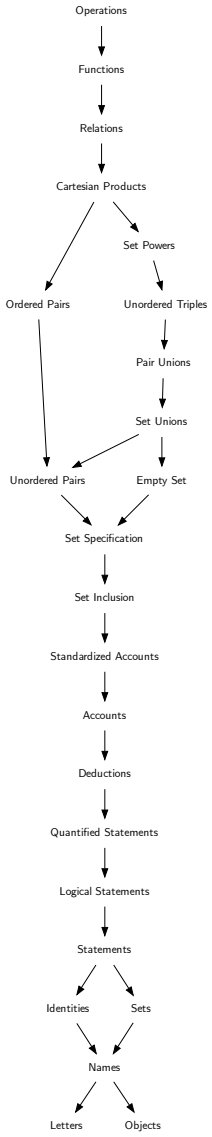
Example

A first example of an operation is if we consider the set A as the power set of some set X. Then the pair union (see Pair Unions) is an operation. For if $E \in \mathcal{P}(X)$ and $F \in \mathcal{P}(X)$ then $E \cup F \in \mathcal{P}(F)$ and so $\cup$ can be viewed as an operation on $\mathcal{P}(X)$.

Properties

Recall that $\cup$ has several nice properties. For one $A \cup B = B \cup A$ and $(A \cup B) \cup C = A \cup (B \cup C)$.

An operation with the first property, that the ordered pair (A, B) and (B, A) have the same result is called *commutative*. An operation with the second property, that when given three objects the order in which we operate does not matter is called *associative*.


Operations
Functions
Relations
Cartesian Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects