Subset Algebra

1 Why

We speak of a subset space with standard set-algebraic properties.

2 Definition

A **subset algebra** a subset space for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of two distinguished sets is distinguished.

2.1 Notation

Let A be a set and $\mathcal{A} \subset 2^A$. We denote the subset algebra of A and $\mathcal{A}$ by $(A, \mathcal{A})$, read aloud as "A, script A."

3 Properties

Proposition 1 *For any subset algebra the empty set is distinguished.*

Proposition 2 *For any subset algebra* $(A, \mathcal{A})$*, if* $B, C \in \mathcal{A}$*, then (a)* $B \cap C \in \mathcal{A}$ *and (b)* $B \Delta C \in \mathcal{A}$*.*

Proposition 3 *For any subset algebra* $(A, \mathcal{A})$*. If* $A_1, \ldots, A_n \in \mathcal{A}$*, then (a)* $\cup_{i=1}^{n} A_i \in \mathcal{A}$ *and (b)* $\cap_{i=1}^{n} A_i \in \mathcal{A}$*.*

4 Examples

Example 4 *For any set* A*,* $(A, 2^A)$ *is a subset algebra.*

Example 5 *For any set* A*,* $(A, \{A, \varnothing\})$ *is a subset algebra.*

Example 6 *For any infinite set* A*, let* $\mathcal{A}$ *be the set*

$$\{B \subset A \mid \ |B| < \aleph_0 \vee |C_A(B)| < \aleph_0\}.$$

$(A, \mathcal{A})$ *is an algebra; the* ***finite/co-finite algebra****.*

Example 7 *For any infinite set* A*, let* $\mathcal{A}$ *be the set*

$$\{B \subset A \mid \ |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

$(A, \mathcal{A})$ *is an algebra; the* ***countable/co-countable algebra****.*

Example 8 *For any infinite set* A*, let* $\mathcal{A}$ *be the set*

$$\{B \subset A \mid \ |B| \leq \aleph_0\}.$$

$(A, \mathcal{A})$ *is not an algebra.*