Why

We want to define area under an extended real function. We use the infinite process to approximate the area under a nonnegative extended real function using simple functions.

Definition

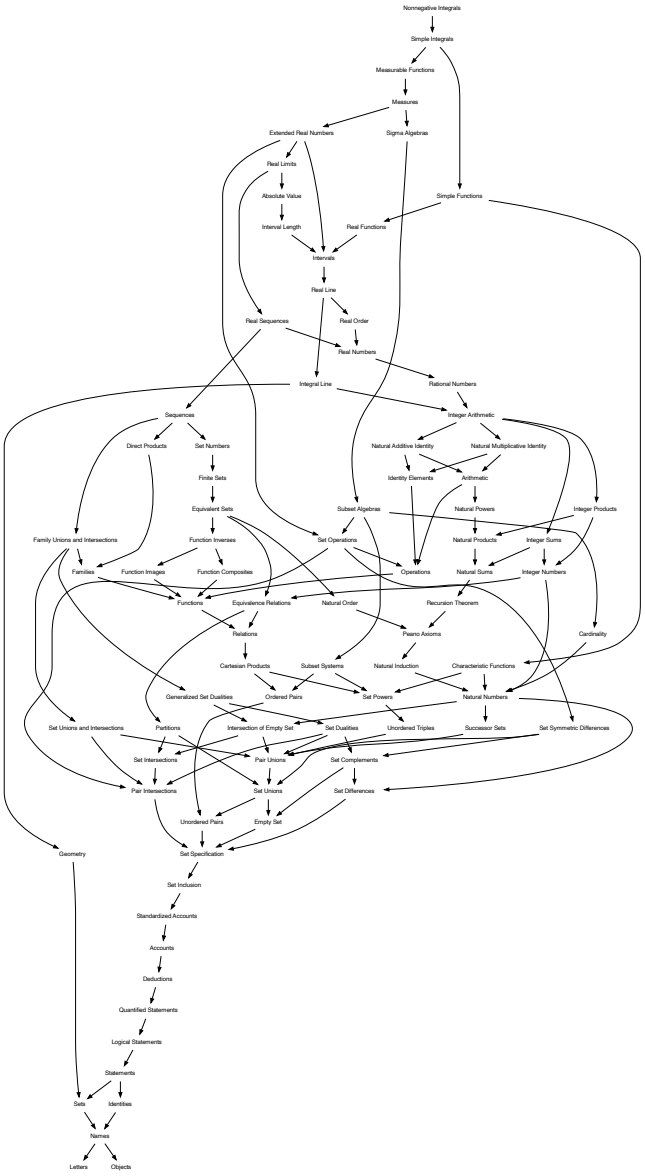
Consider a measure space.

The *integral* of a measurable nonnegative function is the supremum of integrals over nonnegative simple functions pointwise less than or equal to the function.

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f : X \to [0, \infty]$ be measurable. We denote the integral of f with respect to the measure μ by $\int f d\mu$. We defined:

$$\int f d\mu = \sup\left\{ \int g d\mu \;\middle|\; g \in \underset{+}{\mathsf{SF}}(X) \text{ and } g \leq f \right\}.$$


Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Extended Real Numbers
Sigma Algebras
Real Limits
Absolute Value
Simple Functions
Interval Length
Real Functions
Intervals
Real Line
Real Sequences
Real Order
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Sequences
Natural Additive Identity
Natural Multiplicative Identity
Direct Products
Set Numbers
Identity Elements
Arithmetic
Finite Sets
Equivalent Sets
Subset Algebras
Natural Powers
Integer Products
Family Unions and Intersections
Function Inverses
Set Operations
Natural Products
Integer Sums
Families
Function Images
Function Composites
Operations
Natural Sums
Integer Numbers
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Cardinality
Cartesian Products
Subset Systems
Natural Induction
Characteristic Functions
Generalized Set Dualities
Ordered Pairs
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects