

SMOOTH MULTIVARIATE FUNCTIONS

Why

What is the natural generalization of a smooth function to functions defined on sets of \mathbb{R}^{k} .¹

Definition

Let $U \subset \mathbb{R}^d$ be an open set (see Real Open Sets). A function $f: U \to \mathbb{R}$ is *smooth* if all its partial derivatives exists and are continuous. More

More generally, let $X \subset \mathbf{R}^d$. A function $f: X \to R$ is *smooth* if there exists an open set $U \subset \mathbf{R}^d$ and a smooth $F: U \to \mathbf{R}$ so that F(x) = f(x) for all $x \in U \cap X$.

Example

The identity map is smooth. In other words, let $f: \mathbb{R}^d \to \mathbb{R}$ be so that $X \subset \mathbb{R}^d$. Then $f: X \to \mathbb{R}$ s

Properties

Proposition 1. The composition of two smooth functions is smooth.

¹Future editions will expand.

