

## Why

We want to solve linear equations.

## **Example**

Suppose we want to find  $x_1, x_2 \in \mathbf{R}$  to satisfy the system of linear equations

$$3x_1 + 2x_2 &= 10$$
, and  $6x_1 + 5x_2 &= 20$ .

with constants ((3,2),10) and ((6,4),20).

We can associate to the first equation an equation for  $x_1$  in terms of  $x_2$ . We call this solving the first equation for  $x_1$ .

$$3x_1 + 2x_2 = 10 \longleftrightarrow 3x_1 = 10 - 2x_2 \longleftrightarrow x_1 = \frac{1}{3}(10 - 2x_2).$$

Define  $f_1: \mathbf{R} \to \mathbf{R}$  as  $f_1(y) = 1/3(10 - 2y)$ . Then  $3x_1 + 2x_2 = 10$  if and only if  $x_1 = f_1(x_2)$ . We have written  $x_1$  as a function of  $x_2$  and obtained a new equation. The equation is not linear, however, as  $f_1$  is not linear.

Using the equation for  $x_1$  in terms of  $x_2$ , we can substitute this equation into our second linear equation. The two linear equations hold if and only if

$$6f(x_2) + 5x_2 = 20 \longleftrightarrow 20 - 4x_2 + 5x_2 = 20 \longleftrightarrow x_2 = 0.$$

So the equations are satisfied if and only if  $x_2$  is 0. If  $x_2 = 0$ , then  $3x_1 = 10$  and  $6x_1 = 20$ . Both of these are equivalent to  $x_1 = \frac{10}{3}$ . So we have that  $x_1$  must be  $\frac{10}{3}$  and  $x_2$  must be 0.

Clearly this is a solution. Is it the only one? $^1$ 

 $<sup>^1</sup>$ Future editions will expand.

