Naturals

1 Why

We want to count.

2 Definition

We assume that we know how to count. By count, we mean to how many distinct objects there are. We count by starting with a single object and then accumulating more objets, a single object at a time. We know, then, what having one object menas and we know what having one more object means.

We define the set of **natural numbers** implicitly. There is an element of the set which we call **one**. Then for each element of the set, we associate a unique corresponding element of the set. We call this corresponding element the **successor** of the first eelment. We call the function we definie implicitly in this way the **successor function**.

So we start by knowing that one is in the set, and the successor of one is in the set. We call the successor of one **two**. We call the successor of two **three**. And so on using the English language in the usual manner.

2.1 Notation

We denote the set of natural numbers by N, a mnemonic for natural. We often denote elements of N by n, a mnemonic for number, or m, a letter close to n. We denote the element one by 1.

3 Induction

We assert two additonal self-evident and practically indispensable properties of these natural numbers First, one is the successor of no other element. Second, if we have a subset of the naturals containing one with the property that it contains successors of its elements, then that set is equal to the natural numbers. We call this second property the **principle of mathematical induction.**

These two properties, along with the existence and uniqueness of successors are together called **Peano's axioms** for the natural numbers. When in familiar company, we refer to the set of natural numbers as the **naturals** and we freely assume Peano's axioms.

4 Notation

As an exercise in the notation assumed so far, we can write Peano's axioms: N is a set along with a function $s : N \to N$ such that

1. $s(n)$ is the successor of n for all $n \in N$.
2. s is one-to-one; $s(n) = s(m) \implies m = n$ for all $m, n \in N$.
3. There does not exist $n \in N$ such that $s(n) = 1$.
4. If $T \subset N$, $1 \in T$, and $s(n) \in T$ for all $n \in T$, then $T = N$.