



**Why**

We want a notion of distance between elements of the real line.

**Definition**

The *absolute value* of a real number is the greater of itself and its additive inverse. In other words, if  $x$  is positive, then the absolute value of  $x$  is  $x$ . If  $x$  is negative, then the absolute value of  $x$  is  $-x$  (a positive real number).

**Notation**

We denote the absolute value of a real number  $x \in \mathbf{R}$  by  $|x|$ .

**Distance**

The absolute value can be interpreted as the distance between the point corresponding to the real number and the point corresponding to 0. We can generalize this idea. Consider  $x, y \in \mathbf{R}$ . If  $x > y$ , then  $x - y > 0$  and so the distance between the corresponding points is  $x - y$ . If  $x < y$  then  $y - x > 0$ , and so the distance is  $y - x$ .

The observation is that  $|-x| = |x|$ . So

$$|y - x| = |-(x - y)| = |x - y|.$$

So if we just care about the distance between the points corresponding to  $y$  and  $x$ , we can consider  $|x - y|$ , without regard for their order. In other words, the function  $(x, y) \mapsto |x - y|$  is symmetric in  $x$  and  $y$ .



