

Why

We want language and notation for selecting some of the entries (possibly, with reordering—i.e. permuting) from a list.

Definition

An index list of order n and length $r \leq n$ is a list of distinct elements of $\{1, 2, ..., n\}$. Its *i-index* is the *i*th coordinate, where i = 1, ..., r.

Examples

Here are some index lists of order 5: (1,2,3), (3,2,1), (4,5,1), (5,4,3,2,1), (3,). These have lengths 3, 3, 3, 7 and 1, respectively. The 3-index of the first is 3, and of the second is 1.

Induced sublist

The *sublist* of an length-n list x induced by a length-r index list α is the length-r list y whose ith entry is the value x_{α_i} . In other words,

$$y_i = x_{\alpha_i}$$

For example, define x = (6, 4, 5, 3, 9). The sublists associated with the example index lists above are (6, 4, 5), (5, 4, 6), (3, 9, 6), (9, 3, 5, 4, 6) and (5,).

For a particular case, the third holds because

$$(3,9,6) = (x_4, x_5, x_1) = (x_{\alpha_1}, x_{\alpha_2}, x_{\alpha_2})$$

Notation

We denote the induced sublist of list x induced by index list α by x_{α} . This is a slight abuse of notation, since we have so far defined a list with a subscript symbol mean the subscript-symbol term of that list. This ambiguity is avoided in our discussion if we keep in mind the types of the objects.

Index sets

An index set $S \subset 1, \ldots, n$ can be associated with an index list in a natural way. It corresponds to the length-|S| index list which has the elements of S in their natural order. We denote the induced sublist by x_S .

