Why

We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

Why

Consider a set A. If A has n elements, then a probability distribution on A is $p : A \to \mathsf{R}$ where $p(a) = 1/n$. There is a natural candidate.

What if A is the set of natural numbers N. The principle difficulty is that not all sequences of real numbers $a : \mathsf{N} \to \mathsf{R}$ are summable.

A *(countable) probability distribution* on N is $p : \mathsf{N} \to \mathsf{R}$ where $p \geq 0$ and

$$\sum_{n=1}^{\infty} p(n) = 1.$$

Countable Probability Distributions

Real Series

Real Convergence

Absolute Value

Probability Distributions

Real Line   Intervals

Probability Outcomes

Real Order   Real Sequences   Real Summation   Probability   Cardinality

Real Numbers

Common Sense

Geometry

Rational Numbers

Natural Summation

Integer Arithmetic

Natural Additive Identity   Natural Multiplicative Identity   Family Operations   Sequences

Arithmetic   Identity Elements   Algebras   Families

Integer Products   Natural Exponents   Operations

Integer Sums   Natural Products   Functions

Integer Numbers   Natural Sums

Equivalence Relations   Recursion Theorem

Relations   Peano Axioms

Cartesian Products   Natural Induction

Set Powers   Natural Numbers

Partitions   Intersection of Empty Set   Unordered Triples   Successor Sets

Set Intersections   Pair Unions   Ordered Pairs

Set Unions   Set Differences

Pair Intersections   Empty Set   Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts   Definitions

Deductions   Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects