Why

It is natural to look for a class of structural equation models with favorable identifiability and properties.

Definition

A *d-dimensional rooted tree linear cascade* is a sequence of four objects: a tree on $\{1, \ldots, d\}$, a vertex of the tree, a family of real numbers indexed by the edges of the tree, and a d-dimensional random vector whose covariance matrix is the identity matrix. The cascade is called "d-dimensional" because we associate it with a random vector (defined as a function of that in the form of its definition) whose codomain is $\mathbf{R}^d$.

The tree together with the vertex form a rooted tree. The graph associated with the rooted tree and the family of real numbers together form a weighted graph.

The idea is to use the weights and the tree structure to recursively define a random vector in terms of elements of the given random vector. Let $C = (T, i, w, e)$ be a d-dimensional rooted tree linear cascade. So T is a tree on $\{1, \ldots, d\}$, $i \in \{1, \ldots, d\}$ and $w : T \to \mathbf{R}$, and $e : \Omega \to \mathbf{R}^d$ for some probability space $(A, \mathcal{A}, \mathbf{P})$. The random vector associated with C is the random variable $x : \Omega \to \mathbf{R}^d$ defined by

$$x_i = e_i \text{ and } x_j = w_{\{\mathrm{pa}\,j,j\}}x_{\mathrm{pa}\,j} + e_j \text{ for } j \neq i.$$

In other words,

$$e = Ax$$

where A is lower triangle and extremely sparse.

Notation

Let $(A, \mathcal{A}, \mathbf{P})$ be a probability space. Let $e : A \to \mathbf{R}^d$ be a random vector, let T be a tree on $\{1, \ldots, d\}$ with $a_{ij} = a_{ji}$ the weight on edge $\{i, j\} \in T$.