



## Definition

An *affine combination* from  $\mathbf{R}^n$  is a linear combination whose scalars sum to 1. As with linear combinations, we say that  $y$  is *can be written as an affine combination of* the vectors  $x_1, \dots, x_k \in \mathbf{R}^n$  if there exists  $\lambda_1, \dots, \lambda_k \in \mathbf{R}$  so that

$$y = \sum_{i=1}^k \lambda_i x_i$$

and  $\sum_{i=1}^k \lambda_i = 1$ .

All affine combinations of two distinct vectors  $x, y \in \mathbf{R}^n$  is the line through  $x$  and  $y$ , which we denote as usual  $L(x, y)$ . In other words,

$$L(x, y) = \{(1 - \lambda)x + \lambda y \mid \lambda \in \mathbf{R}\}$$

A set of vectors  $\{v_1, \dots, v_k\}$  is *affinely dependent* if one can be written as an affine combination of the others. A set of vectors which is not affinely dependent is called an *affinely independent set of vectors*. An equivalent condition is that there exist an affine combination in these vectors in which at least one scalar is nonzero and the sum of all the scalars is 1.

**Proposition 1.** *Suppose  $y \in \mathbf{R}^n$ . The set  $X = \{x_1, \dots, x_k\} \subset \mathbf{R}^n$  is affinely dependent if and only if  $y \in X$  is.*

**Proposition 2.** *Suppose  $y \in \mathbf{R}^n$ . The set  $X = \{x_1, \dots, x_k\} \subset \mathbf{R}^n$  is affinely dependent if and only if  $y \in X$  is.*



