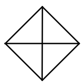

Why

When is a linear transformation *onto*? In other words, when is the range the whole space? This question is a bit more invovled, but we will start with observing in this sheet that the range of a linear map happens to be a subspace.

Definition

For a *linear* transformation $T \in \mathcal{L}(V, W)$, we refer to range(T) as the *range space* (or *image space*) of T.

Proposition 1. *Suppose* $T \in \mathcal{L}(V, W)$*. Then* $\operatorname{range} T$ *is a subspace of* W*.*

Proof. We verify that range(T) contains 0 and is closed under vector addition and scalar multiplication. Clearly $T(0) = 0$, so $0 \in \operatorname{range} T$. Next, suppose $w_1, w_2 \in \operatorname{range} T \subset W$ So there exists $v_1, v_2 \in V$ so that

$$Tv_1 = w_1 \text{ and } Tv_2 = w_2$$

We conclude $w_1 + w_2 = Tv_1 + Tv_2 = T(v_1 + v_2)$. So $w_1 + w_2 \in \operatorname{range} T$. Likeiwse, if $w \in \operatorname{range} T$, then there exists $v \in V$ such that $w = Tv$. So then $\lambda w = \lambda T v = T(\lambda v)$, and so $\lambda w \in \operatorname{range} T$. $\square$

Examples

To come.


Range Spaces of Linear Transformations
Linear Transformations
Real Polynomial Derivatives
Derivatives of Scalar Multiples
Derivatives of Sums
Real Integral Additivity
Real Integral Homogeneity
Real Linear Functions
Real Differentiable Functions
Simple Integral Additivity
Simple Integral Homogeneity
Transformations
Linear Combinations
Simple Integrals
Vector Space of Polynomials
Vectors
Real Linear Combinations
Simple Functions
Measurable Functions
Real Polynomials
Fields
Real Functions
Measures
Real Vectors
Length Measure
Sigma Algebras
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Real Series
Extended Real Numbers
Distance
Plane Vectors
Real Arithmetic
Rings
Real Limits
Space Distance
Absolute Value
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Real Sequences
Rational Products
Rational Order
Rational Sums
Real Numbers
Subset Algebras
Sequences
Geometry
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Inverse Elements
Natural Arithmetic
Identity Elements
Equivalent Sets
Element Functions
Integer Sums
Integer Order
Natural Powers
Operations
Function Inverses
Comparisons
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects