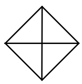


Why

We want to sum infinitely many real numbers.

Definition

Let n be a natural number. The nth *partial sum* of a sequence of real numbers is the sum of first n elements of the sequence. The first partial sum is the first term of the sequence. The third partial sum is the sum of first three elements of the sequence.

The *series* of the sequence is the sequence of partial sums. The sequence is *summable* if the series converges.

Since there exist sequences which do not converge, there exist sequences which are not summable. Consider the sequence which alternates between $+1$ and -1, and starts with $+1$. Its series alternates between $+1$ and 0, and so does not converge.

Notation

Let $(a_n)_n$ be a sequence of real numnbers. For natural number n, define:

$$s_n = \sum_{k=1}^{n} a_k.$$

Then $(s_n)_n$ is the series of $(a_n)_n$. If the series converges, then there exists a real number s, the limit, and we write:

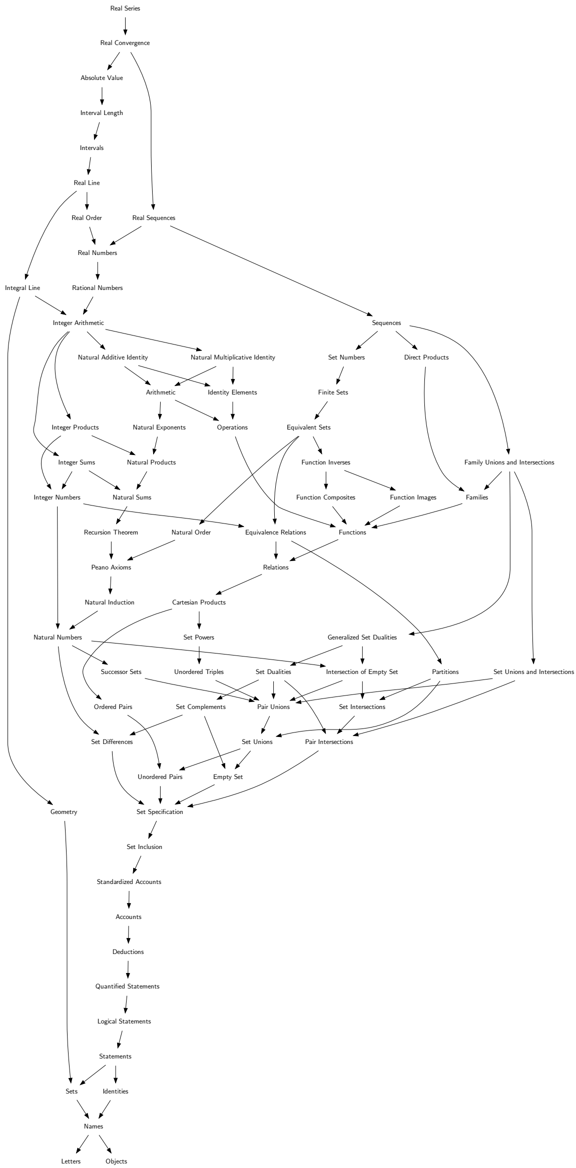
$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

We read these relations aloud as “s is the limit as n goes to infinity of s n” and “s is the limit as n goes to infinity of the sum of a k from k equals 1 to n.”

To avoid referencing s_n, we write:

$$\sum_{k=1}^{\infty} a_k = s,$$

read aloud as the “the sum from 1 to infinity of a k is s.” The notation is subtle, and requires justification by the algebra of series. TODO


Real Series
Real Convergence
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Sequences
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Finite Sets
Integer Products
Natural Exponents
Operations
Equivalent Sets
Integer Sums
Natural Products
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects