Why

Let $X = \{a, b\}$ and $Y = \{0, 1\}$. The dataset $((a, 0))$ is consistent, but it is not functionally complete. On the other hand, the dataset $((a, 0), (b, 0), (a, 0), (a, 0), (a, 0), (a, 1))$ is complete but it is not *functionally* consistent.

In general, if $y_i \neq y_j$ for some i and j where $x_i = x_j$, then the dataset is not functionally consistent. In the preceding example, both $(a, 0)$ and $(a, 1)$ appear.

If we emphasize the "predictive" aspect of a functional inductor, we interpret the input as an object we "see before" the output. And so treat $y \in Y$ as an uncertain outcome which is the element associated to $x \in X$.

In this case, we may use the language of probability to discuss this uncertain outcome. If, for example, Y is finite, we can associate a distribution with each input $x \in X$.

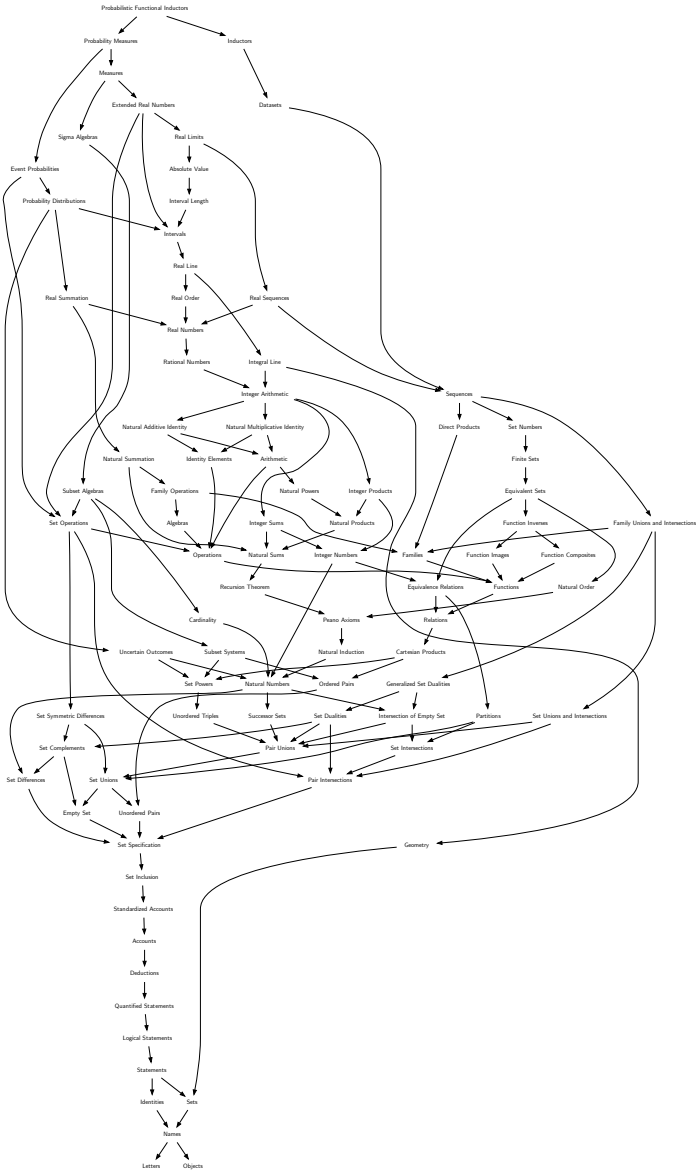
Definition

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable spaces.

A *probabilistic functional inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a family of measures on $(Y, \mathcal{Y})$, indexed by X. We call a function from inputs to output measures a *probabilistic predictor*. We call the distribution a *probabilistic prediction*.

Notation

Let $\mathcal{M}(Y, \mathcal{Y})$ be the set of measures on Y. Let D be a dataset in $(X \times Y)^n$. Let $g : X \to \mathcal{M}(Y, \mathcal{Y})$ a probabilistic predictor. Let $G_n(X \times Y)^n \to (X \to \mathcal{M}(Y, \mathcal{Y}))$ be a predictive probabilistic inductor. Then $G_n(D)$ is a family of measures $\{g_x : \mathcal{Y} \to [0, 1]\}_{x \in X}$.


Probabilistic Functional Inductors
Probability Measures
Inductors
Measures
Extended Real Numbers
Datasets
Sigma Algebras
Real Limits
Event Probabilities
Absolute Value
Probability Distributions
Interval Length
Intervals
Real Line
Real Summation
Real Order
Real Sequences
Real Numbers
Rational Numbers
Integral Line
Integer Arithmetic
Sequences
Natural Additive Identity
Natural Multiplicative Identity
Direct Products
Set Numbers
Natural Summation
Identity Elements
Arithmetic
Finite Sets
Subset Algebras
Family Operations
Natural Powers
Integer Products
Equivalent Sets
Set Operations
Algebras
Integer Sums
Natural Products
Function Inverses
Family Unions and Intersections
Operations
Natural Sums
Integer Numbers
Families
Function Images
Function Composites
Recursion Theorem
Equivalence Relations
Functions
Natural Order
Cardinality
Peano Axioms
Relations
Natural Induction
Uncertain Outcomes
Subset Systems
Cartesian Products
Set Powers
Natural Numbers
Ordered Pairs
Generalized Set Dualities
Set Symmetric Differences
Unordered Triples
Successor Sets
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects