Natural Numbers

1 Why

We want to count, forever.

2 Definition

We define the set of *natural numbers* implicitly. There is an element of the set which we call *one*. For each element of the set, there is a unique corresponding element, called the *successor* of the former, which is also in the set. We call the elements *numbers* and call the set itself the *naturals*.

The *successor function* is the correspondence between elements of the natural numbers and their successors. Its domain and codomain is the set of natural numbers. It is a one-to-one correspondence. It is not onto, however, since the element one has no successor.

To recap, we begin with a distinguished element, called one, in the set. There is a second element, the successor of one, in the set. We call the successor of one *two*. We call the successor of two *three*. And so on using the English language in the usual manner. We are saying, in the language of sets, that the essence of counting is starting with one and adding one repeatedly.

2.1 Notation

We denote the set of natural numbers by N, a mnemonic for natural. We often denote elements of N by n, a mnemonic for number, or m, a letter close to n. We denote the element called one by 1.