

INNER PRODUCT NORMS

Why

To every real-valued inner product there corresponds a norm, in a similar manner to the construction for \mathbb{R}^n .

Definition

Prop. 1. Let (V, F) be a vector space. Let $f: V \times V \to \mathsf{F}$ be an inner product with $f(x, x) \in \mathsf{R}$. Let $g: V \to \mathsf{R}$ such that

$$g(x) = \sqrt{f(x,x)}.$$

Then g is a norm.

The norm of a vector in an inner product space is the square root of the inner product of the vector with itself.

