Operation Properties

1 Why

An operation **commutes** if the result of two elements is the same regardless of their order.

An operation **associates** if given any three elements in order it doesn't matter whether we first operate on the first two and then with the result of the first two the third, or the second two and with the result of the second two the first.

A first operation over a set **distributes** over a second operation over the same set if the result of applying the first operation to an element and a result of the second operation is the same as applying the second operation to the results of the first operation with the arguments of the second operation.

2 Definition

Let A be a non-empty set. An **operation** on A is a function from ordered pairs of elements in the set to the same set. We use operations to combine the elements. We operate on pairs.

2.1 Notation

Let A a set and $g : A \times A \to A$. We commonly forego the notation $g(a, b)$ and instead write $a\,g\,b$. We call this **infix notation**.

Using lower case latin letters for every the elements and for the operation is confusing, but we often have special symbols for particular operations. For example, $+$, $-$, $\cdot$, $\circ$, and $\star$.

If we had a set A and an operation $+ : A \times A \to A$, we would write $a + b$ for the result of applying $+$ to (a, b).