



Why

We want to consider all the subsets of a given set.

Definition

We do not yet have a principle stating that such a set exists, but our intuition suggests that it does.

Principle 1 (Powers). *For every set, there exists a set of its subsets.*

We call the existence of this set the *principle of powers* and we call the set the *power set*.¹ As usual, the principle of extension gives uniqueness (see **Set Equality**). The power set of a set includes the set itself and the empty set.

Notation

Let A denote a set. We denote the power set of A by $\mathcal{P}(A)$, read aloud as “powerset of A .” $A \in \mathcal{P}(A)$ and $\emptyset \in \mathcal{P}(A)$. However, $A \subset \mathcal{P}(A)$ is false.

Examples

Let a, b, c denote distinct objects. Let $A = \{a, b, c\}$ and $B = \{a, b\}$. Then $B \subset A$. In other notation, $B \in \mathcal{P}(A)$. Showing each of the following is straightforward.

1. The empty set: $\mathcal{P}(\emptyset) = \{\emptyset\}$
2. Singletons: $\mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$
3. Pairs: $\mathcal{P}(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
4. Triples:

$$\mathcal{P}(\{a, b, c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$

¹This terminology is standard, but unfortunate. Future editions may change these terms.

Properties

We can guess the following easy properties.²

Proposition 1. $\emptyset \in \mathcal{P}(A)$

Proposition 2. $A \in \mathcal{P}(A)$

We call A and \emptyset the *improper* subsets of A . All other subset we call *proper*.

Basic Fact

Proposition 3. $E \subset F \longrightarrow \mathcal{P}(E) \subset \mathcal{P}(F)$

²Future editions will expand this account.

