



## Definition

A set  $C \subset \mathbf{R}^n$  is *convex* if it contains the closed line segment between every pair of distinct points. In other words,

$$\lambda x + (1 - \lambda)y \in C \quad \text{for all } x, y \in C \text{ and } \lambda \in [0, 1].$$

Roughly speaking,  $C$  is convex if and only if its intersection with every line in  $\mathbf{R}^n$  is either empty or a closed line segment.

## Examples

The empty set, any singleton, any subspace, any affine set and any half-space.

## Properties

**Proposition 1** (closure under intersections). *Suppose  $\mathcal{K} \subset \mathcal{P}(\mathbf{R}^n)$  is a set of convex sets. Then  $\cap \mathcal{K}$  is convex.*

**Proposition 2** (sums, differences, scales are convex). *Suppose  $A, B$  are convex sets. Then  $A + B$ ,  $A - B$  and  $\lambda A$  for any real  $\lambda$  is convex.*



