



Why

If a directed graph has no cycles, then it has a nice property.¹

Definition

Directed and acyclic graphs (sometimes *DAGs*) have some useful properties. Clearly, every subgraph induced on a directed acyclic graph is a directed acyclic graph.

Proposition 1. *Let (V, E) be a directed acyclic graph. Then there exists a vertex $v \in V$ which is a source and a vertex $w \in V$ which is a sink.*

Proof. There exists a directed path of maximum length. It must start at a source and end at a sink.² □

A *topological numbering*, *topological sort* or *topological ordering* of a directed graph (V, E) is a numbering $\sigma : \{1, \dots, |V|\} \rightarrow V$ satisfying

$$(v, w) \in E \longrightarrow \sigma^{-1}(v) < \sigma^{-1}(w).^3$$

Proposition 2. *There exists a topological sort for every acyclic graph.*

Proof. Let (V, E) be a directed acyclic graph. There exists a source vertex, v_1 . Set $\sigma(1) = v_1$. Take the subgraph induced

¹Future editions will expand this vague introduction.

²Future editions will expand.

³Future editions will further explain this concept.

by $V - \{v_1\}$. It is directed acyclic, and so has a source vertex, v_2 . Set $\sigma(2) = v_2$. Continue in this way.⁴ □

⁴Future editions will clarify and expand.

