

GAMES

Why

We want to discuss interactive decision making.

Discussion

We are interested in talking about situations in which there are several decision makers, agents or players, each of which are making decisions that will affect the outcome for all involved.

Example: rock paper scissors

Consider the game "rock-paper-scissors" in which there are two players A and B. Each player may choose one of the three actions ROCK, PAPER, SCISSORS. To play, each player simultaneously selects an action, and these are compared. Here, both agents have the same actions.

Definition

In both these games there is a finite set of *players*, or *agents*, or *controllers*. Let \mathcal{I} be a finite set with $|\mathcal{I}| = n$, the players.

In rock-paper-scissors, for example, $\mathcal{I} = \{A, B\}$. There, each player could pick one of the three actions. Define $\mathcal{A}_A = \mathcal{A}_B = \{\text{ROCK}, \text{PAPER}, \text{SCISSORS}\}$. We call \mathcal{A}_A the actions of A and \mathcal{A}_B the actions of B.

We have a set of outcomes $\mathcal{O} = \{A \text{ Wins}, B \text{ Wins}, \text{Tie}\}.$ Let $f : \mathcal{A}_A \times C_B \to \mathcal{O}$ defined by f(Rock, Scissors) = A Wins the set of players. Let S be a finite set, the set of states. For i = 1, ..., n, let $\{A_s^p\}_{s \in S}$ be a family of sets, the action sets by state. Define $\mathcal{A}^i = \bigcup_s A_s^p$ the set of actions for player i = 1, ..., n.

Let $f: S \times \prod_i A^i \to S$, the game dynamics or transition function.



Definition

The first thing to discuss is the set of players. Let P be a finite set with |P| = n. The set P is the set of players, and we

We begin with a single-player game.

Let S be a set.

