

DISTORTION FUNCTIONS

Why

We want to quantify the error of compressing a real-valued random variable.

Definition

Let \mathcal{X} be a finite set and $q: \mathbf{R} \to \mathcal{X}$ a quantization (see Quantizations) of \mathbf{R} . Also, fix a probability space $(\Omega, \mathcal{A}, \mathbf{P})$ and a random variable $x: \Omega \to \mathbf{R}$.

The compression $\hat{x}: \Omega \to \mathcal{X}$ of x under q is $q \circ x$. A distortion function for x and \hat{x} is a function

$$d: (\Omega \to \mathbf{R}) \times (\Omega \to \mathcal{X}) \to \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

Examples

The expected mean-squared-error distortion d_{mse} between x and \hat{x} is

$$d_{\text{mse}}(x, \hat{x}) = \mathbf{E}[(x - \hat{x})^2]$$

The Kulback-Liebler distortion d_{kld} defined by

$$d_{\mathrm{kld}}(x,\hat{x}) = \mathsf{E}[d_{\mathrm{kl}}(\mathsf{P}(y \in \cdot \mid x,\hat{x}) \mid \mathsf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.¹

¹Future editions will clarify this sentence.

