Why

We generalize convex functions to arbitrary vector spaces.

Definition

Suppose X is a vector space over $\mathbf{R}$, $D \subset X$ and $f : D \to \bar{\mathbf{R}}$. As before, define

$$\operatorname{epi} f = \{(x, \alpha) \in X \times \mathbf{R} \mid f(x) \leq \alpha\}$$

f is convex if $\operatorname{epi} f$ is convex. It is straightforward that f is convex if and only if

$$f(tx + (1 - t)y) \leq tf(x) + (1 - t)f(y)$$

for all $t \in [0, 1]$ and $x, y \in D$.

Examples

Any norm on X is a convex function.