

Event Probabilities

1 Why

If we have some outcomes and a probability mass function, we can construct a function which assigns probabilities to events.

2 Definition

The **probability** of an event is the sum of the probabilities of the outcomes in the event. The **event probability function** is the correspondence assigning events to their probabilities.

3 Notation

Let A be a set of outcomes and p a probability mass function on A. Let $B \subset A$ be an event. Then the probability of B is

$$\sum_{b \in B} p(b)$$

We can denote event probability function by $\mathbf{P}: 2^A \to R$. Notice that \mathbf{P} depends on the set of outcomes A and the probability mass function p. Sometimes we will include this when denoting \mathbf{P} and denote it by $\mathbf{P}_{A,p}$.

4 Properties

Proposition 1. Let P be the event probability function of a probability mass function p on a set of outcomes A. then

1.
$$\mathbf{P}(B) \geq 0$$
 for all $B \subset A$

2.
$$P(A) = 1$$

3. for
$$B, C \subset A$$
 with $B \cap C = \emptyset$, $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$.