Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Proposition 1. *Suppose $f : \Omega \to \mathbf{R}$ is measurable on the probability space $(\Omega, \mathcal{F}, P)$. Then*

$$P[|f - \mathbf{E}(f)| \geq t] \leq \frac{\operatorname{var} f}{t^2} \qquad \textit{for all } t > 0$$

Proof. The symbols $|f - \mathbf{E}(f)| \geq t$ denote the set $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$. This set is the same as the set

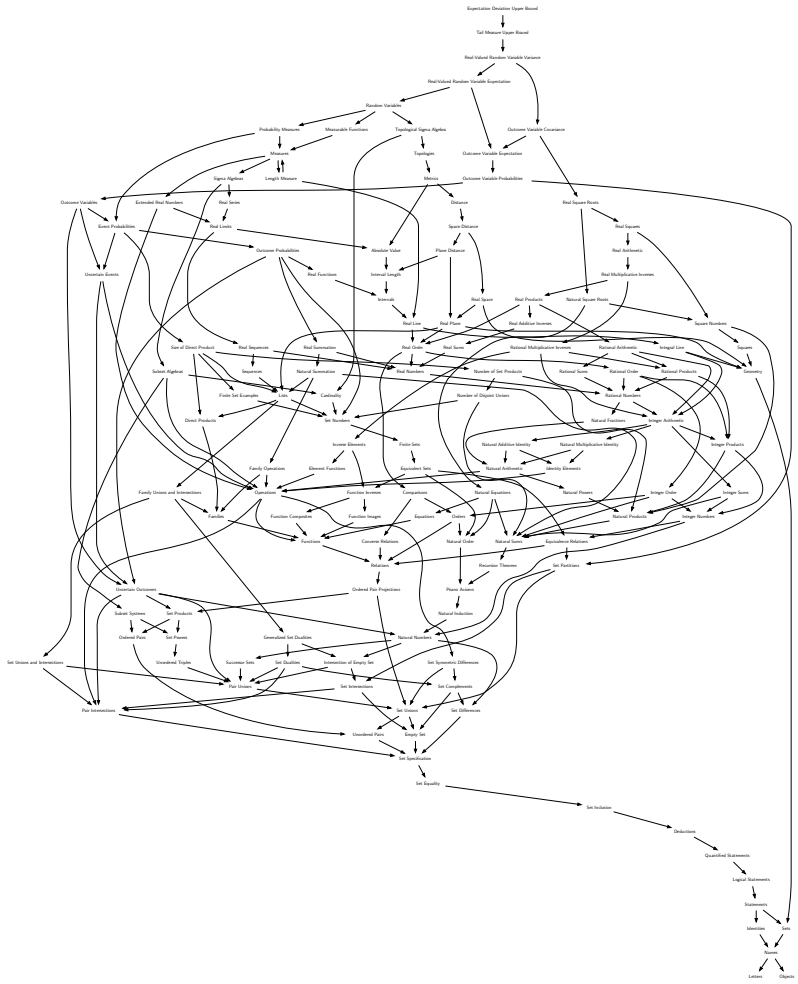
$$\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}.$$

By using the tail measure upper bound,

$$P(\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))^2}{t^2}.$$

We recognize the numerator of the right hand side as the varianceof f. $\square$

The above is also called *Chebychev's Inequality* (or the *Chebyshev inequality*).


Expectation Deviation Upper Bound
Tail Measure Upper Bound
Real-Valued Random Variable Variance
Real-Valued Random Variable Expectation
Random Variables
Probability Measure
Measurable Functions
Topological Sigma Algebra
Outcome Variable Covariance
Measures
Topologies
Outcome Variable Expectation
Sigma Algebras
Length Measure
Metrics
Outcome Variable Probabilities
Outcome Variables
Extended Real Numbers
Real Series
Distance
Real Square Roots
Event Probabilities
Real Limits
Space Distance
Real Squares
Absolute Value
Plane Distance
Real Arithmetic
Outcome Probabilities
Uncertain Events
Real Functions
Interval Length
Real Multiplication Inverses
Intervals
Real Space
Real Products
Natural Square Roots
Real Line
Real Plane
Real Additive Inverses
Square Roots
Real Order
Real Sums
Rational Multiplication Inverses
Rational Arithmetic
Integral Sum
Squares
Cart of Direct Product
Real Sequences
Real Enumeration
Real Numbers
Number of Set Products
Rational Sums
Rational Order
Rational Products
Symmetry
Subset Algebras
Sequences
Natural Enumeration
Finite Set Examples
Lists
Cardinality
Number of Disjoint Unions
Rational Numbers
Direct Products
Set Numbers
Natural Fractions
Integer Arithmetic
Integer Products
Inverse Images
Finite Sets
Natural Addition Identity
Natural Multiplication Identity
Family Operations
Element Functions
Equivalent Sets
Natural Arithmetic
Identity Elements
Family Unions and Intersections
Operations
Function Inverses
Comparisons
Natural Equations
Natural Powers
Integer Order
Integer Sums
Functions
Function Composition
Function Images
Equations
Sums
Natural Products
Integer Numbers
Functions
Natural Order
Natural Sums
Equivalence Relations
Concrete Relations
Relations
Recursion Theorem
Set Partitions
Uncertain Outcomes
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Set Unions and Intersections
Unordered Triples
Successor Sets
Set Dualities
Intersection of Empty Set
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Issues
Letters
Digraphs