## Why

[1]

## Definition

A *subspace* of a vector space is a subset of vectors that is a vector space when restriction vector addition and scalar multiplication to the set. In other words, a subspace is a subset of a vector space which is closed under vector addition and scalar multiplication.

For example, the entire set of vectors is a subspace. As a second example, the set consisting only of the zero vector is a subspace; we call this the *zero subspace*. These two subspaces are the *trivial subspaces*. A *nontrivial subspace* is a subspace that is not trivial.

## Notation

Let $(V, \mathsf{F})$ be a vector space. Let $U \subset V$ with

$$\alpha u + \beta v \in U$$

for all $\alpha, \beta \in \mathsf{F}$ and $u, v \in U$. Then $U$ is a subspace of $(V, \mathsf{F})$.

## Properties

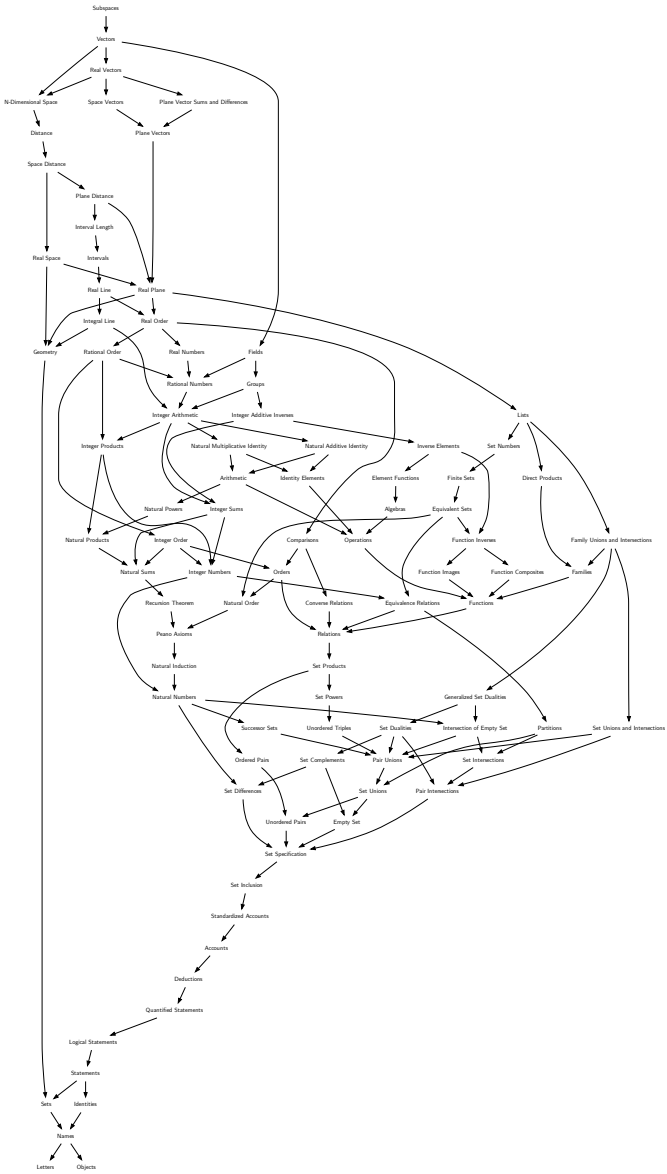
**Prop. 1.** *The intersection of a family of subspaces is a subspace.*

[1] Future editions will include.

**Prop. 2.** *There exists a family of subspaces whose union is not a subspace;*

**Remark 1.** *In other words: the union of a family subspaces need not be a subspace.*

**Prop. 3.** *A subspace must contain the zero vector; in other words, every subspace is nonempty.*


Subspaces
Vectors
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Fields
Rational Numbers
Groups
Integer Arithmetic
Integer Additive Inverses
Lists
Integer Products
Natural Multiplication Identity
Natural Additive Identity
Inverse Elements
Set Numbers
Arithmetic
Identity Elements
Element Functions
Finite Sets
Direct Products
Natural Powers
Integer Sums
Algebras
Equivalent Sets
Natural Products
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Orders
Function Images
Function Compositions
Families
Recursion Theorem
Natural Order
Converse Relations
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects