Why

Every matrix $A \in \mathbf{R}^{m \times n}$ maps the unit ball in $\mathbf{R}^n$ to an ellipsoid in $\mathbf{R}^m$.

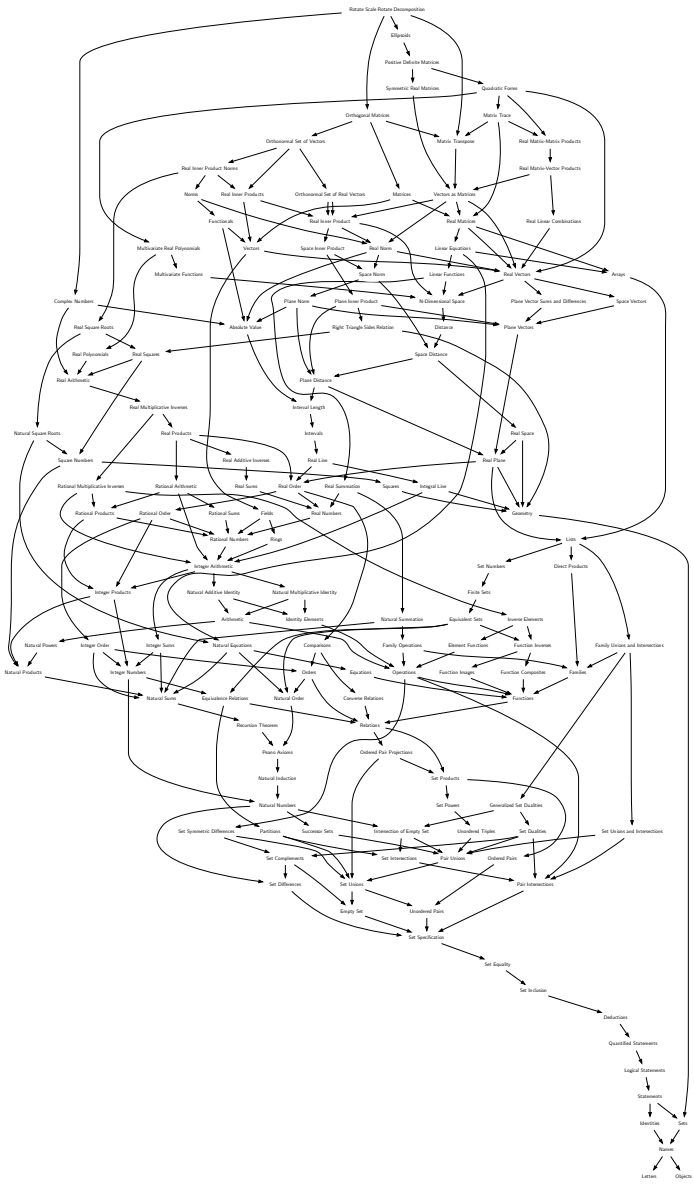
Definition

A *rotate scale rotate decomposition* (or *rotate scale rotate factorization*) of a matrix $A \in \mathbf{R}^{m \times n}$ is an ordered triple (U, S, V) where U and V are orthgonal and S is diagonal decreasing ($S_{11} \geq S_{22} \geq \cdots \geq S_{pp}$, where $p = \min\{m, n\}$) satisfying

$$A = USV^\top.$$

Other (universal) terminology includes the *singular value decomposition* or *SVD* of A. We call diagonal elements of S the *singular values* of A. We call the column vectors of U the *left singular vectors* or *output singular vectors*. We call the column vectors of V the *right singular vectors* or *input singular vectors*. We refer to them collectively as the *singular vectors*.

$$Av_i = \sigma_i u_i.$$


Rotate Scale Rotate Decomposition
Ellipsoids
Positive Definite Matrices
Symmetric Real Matrices
Quadratic Forms
Orthogonal Matrices
Matrix Trace
Orthonormal Set of Vectors
Matrix Transpose
Real Matrix-Matrix Products
Real Inner Product Norms
Real Matrix-Vector Products
Norms
Real Inner Products
Orthonormal Set of Real Vectors
Matrices
Vectors as Matrices
Functionals
Real Inner Product
Real Matrices
Real Linear Combinations
Multivariate Real Polynomials
Vectors
Space Inner Product
Real Norm
Linear Equations
Multivariate Functions
Space Norm
Linear Functions
Real Vectors
Arcsin
Complex Numbers
Plane Norm
Plane Inner Product
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Real Square Roots
Absolute Value
Right Triangle Sides Relation
Distance
Plane Vectors
Real Polynomials
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Natural Square Roots
Real Products
Intervals
Real Space
Square Numbers
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Real Summation
Squares
Integral Line
Rational Products
Rational Order
Rational Sums
Fields
Real Numbers
Geometry
Rational Numbers
Rings
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Natural Summation
Equivalent Sets
Inverse Elements
Natural Powers
Integer Order
Integer Sums
Natural Equations
Comparison
Family Operations
Element Functions
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Order
Equations
Operations
Function Images
Function Compositions
Families
Natural Sums
Equivalence Relations
Natural Order
Canonical Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Duality
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects