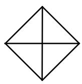


Why

We want to talk about influencing natural phenomena.[1]

Definition

Let $\mathcal{X}_0, \mathcal{X}_1, \dots, \mathcal{X}_T$ and $\mathcal{U}_0, \mathcal{U}_1, \dots, \mathcal{U}_{T-1}$ be sets. For $t = 0, \dots, T-1$, let $f_t : \mathcal{X}_t \times \mathcal{U}_t \to \mathcal{X}_{t+1}$. We call the sequence

$$\left((\mathcal{X}_t)_{t=0}^{T}), (\mathcal{U}_t)_{t=0}^{T-1}, (f_t)_{t=1}^{T-1}\right)$$

a *controlled kdeterministic discrete-time dynamical system.* We call the index t the *epoch*, the *stage* or the *period.*

Let $x_0 \in \mathcal{X}_0$. Let $u_0 \in \mathcal{U}_0, \dots, u_{T-1} \in \mathcal{U}_{T-1}$. Define a state sequence $x_1 \in \mathcal{X}_1, \dots, x_T \in \mathcal{X}_T$ by

$$x_{t+1} = f_t(x_t, u_t).$$

In this case we call x_0 the *initial state.* We call the x_t the *states.* We call the u_t a sequence of *inputs* (or *actions*, *decisions*, *choices*, or *controls*). We call f_t the *transition function* or *dynamics function.*

We call T the *horizon.* In the case that we have an infinite sequence of state sets, input sets, and dynamics, then we refer to a *infinite-horizon* dynamical system. To use language in contrast with this case, we refer to the dynamical system when T is finite as a *finite-horizon* dynamical system.

[1] Future editions will modify, and may restore former editions language: "We want to talk about making decisions over time." Though this language may also be used in a sheet on finite controlled dynamical systems.

State

The current action u_t affects future states x_s for $s > t$, but not the current or past states. The current state x_t depends on the initial state x_0 and the sequence of past actions $u_0, \ldots, u_{t-1}$. So the state is a "link" between the past and the future. Given x_t and $u_t, \ldots, u_{s-1}$, for $s > t$, we can compute x_s. In other words, the prior actions $u_0, \ldots, u_{t-1}$ are not relevant.

This nonrelevancy of prior actions and prior states simplifies the sequential decision problem (see Sequential Decisions).

Variations

The dynamical system is called *time-invariant* if $\mathcal{X}_t$, $\mathcal{U}_t$ and f_t do not depend on t. A simple variation is that $\mathcal{U}_t$ depends on x_t.[2]

Examples

Finite dynamical system

A dynamical system is finite if the state and action sets are finite. For example, $\mathcal{X} = \{1, \ldots, n\}$ and $\mathcal{U} = \{1, \ldots, m\}$. Then $f_t : \mathcal{X} \times \mathcal{U} \to \mathcal{U}$ is called a *transition map*.

Or else, let (V, E) be a directed graph, then $\mathcal{X} = V$, $\mathcal{U}_{x_t} = \{(u, v) \in E \mid u = x_t\}$ and $f_t(x_t, u_t) = v$ when $u_t = (x_t, v)$ is a dynamical system. Roughly this system models "moving" on the graph.

[2]Future editions will say more here.

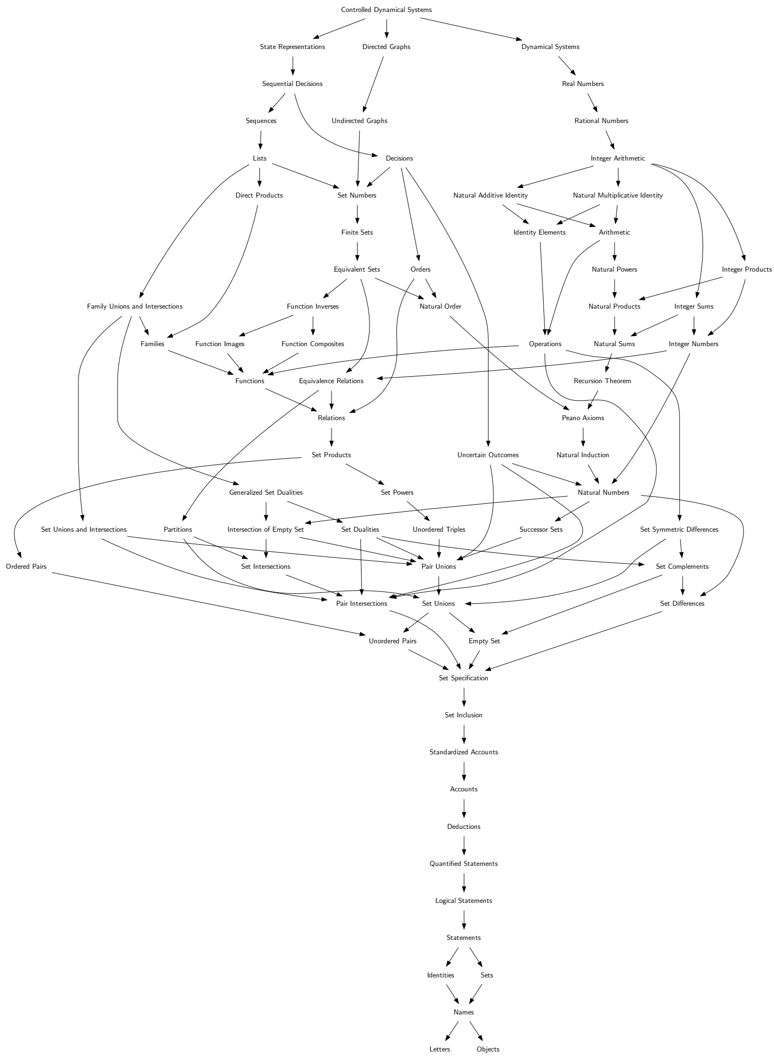
Discrete-time linear dynamical system

Let $\mathcal{X} = \mathbf{R}^n$ and $\mathcal{U} = \mathbf{R}^m$. Define $f_t : \mathcal{X} \times \mathcal{U} \to \mathcal{X}$ by

$$x_{t+1} = f_t(x_t, u_t) = A_t x_t + B_t u_t + c_t$$

for $y = 1, \ldots, T - 1$.[3]

[3] This very special form of dynamics arises in many applications. Future editions will say more.


Controlled Dynamical Systems
State Representations
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Dynamical Systems
Sequential Decisions
Real Numbers
Sequences
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Integer Arithmetic
Direct Products
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Natural Multiplicative Identity
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Family Unions and Intersections
Function Inverses
Natural Order
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Function Images
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Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
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Intersection of Empty Set
Set Dualities
Unordered Triples
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Set Symmetric Differences
Ordered Pairs
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