Why

As with distributions, if a density factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So, as before, the property of factoring according to a rooted tree is really a property of factoring according to a tree.

Defining result

Proposition 1. *If a density factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

Undirected definition

A density *factors according to a tree* if it factors according to the tree rooted at any vertex.

Existences and uniqueness

Trees are not a property of densities, since there is no one-to-one correspondence, as demonstrated by the following propositions.

Existence

Proposition 2. *A density f need not factor according to a tree.*

Uniqueness

Proposition 3. *A density f may factor according to multiple trees.*

Tree Densities

Rooted Tree Densities

Conditional Densities

Marginal Densities

Multivariate Real Densities

N-Dimensional Space — Probability Densities

Distance — Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions

Measures

Sigma Algebras

Extended Real Numbers — Real Series

Real Limits

Rooted Tree Distributions

Conditional Distributions

Conditional Event Probabilities — Marginal Distributions

Simple Functions — Space Distance

Absolute Value — Event Probabilities — Joint Distributions

Real Functions — Plane Distance

Interval Length — Probability Distributions

Rooted Trees

Real Sequences — Intervals — Real Space

Sequences — Trees

Undirected Paths — Directed Graphs

Real Line — Real Plane — Real Summation — Lists — Undirected Graphs

Subset Algebras — Real Order — Integral Line — Geometry

Real Numbers — Rational Order — Direct Products — Set Numbers

Cardinality — Rational Numbers — Finite Sets

Natural Summation

Integer Arithmetic

Natural Multiplicative Identity — Natural Additive Identity — Integer Products — Equivalent Sets — Family Operations — Family Unions and Intersections

Identity Elements — Arithmetic — Comparisons — Integer Sums — Integer Order — Function Inverses

Operations — Natural Powers — Concrete Relation — Integer Numbers — Natural Order — Function Composition — Function Images — Families

Natural Products — Natural Sums — Relations — Equivalence Relations — Functions

Characteristic Functions — Subset Systems — Set Products — Recursion Theorem

Set Powers — Ordered Pairs — Peano Axioms

Unordered Triples — Natural Induction — Uncertain Outcomes

Natural Numbers — Generalized Set Dualities

Set Symmetric Differences — Partitions — Successor Sets — Intersection of Empty Set — Set Dualities — Set Unions and Intersections

Set Complements — Set Intersections — Pair Unions

Set Unions — Set Differences — Pair Intersections

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects