



CHARACTERISTIC FUNCTIONS

Why

We want to indicate membership in a set by a function.¹

Definition

The *characteristic function* of a set X is a function from X to 2 which is 1 if the argument is in A and 0 otherwise.

The function which assigns to each subset A of X to characteristic function of A is a one-to-one function from $\mathcal{P}(X)$ onto 2^X .

Notation

Let A be a non-empty set and $B \subset A$. We denote the characteristic function of B in A by $\chi_B : A \rightarrow R$. The Greek letter χ is a mnemonic for “characteristic”.

The subscript indicates the set on which the function is one. In other words, for all $B \subset A$, $\chi_B^{-1}(\{1\}) = B$.²

¹Future editions will elaborate, perhaps with forward-looking connections to Rectangular Functions.

²Another notation used, when referring to these as “indicator functions,” is $1_B : A \rightarrow \{0, 1\}$ or $\mathbf{1}_B$.

