Why

Given a vector $x \in \mathbf{R}^n$ and a subspace $S \subset \mathbf{R}^n$, what is the closest point in S to x.

Result

Proposition 1. *Suppose $U \in \mathbf{R}^{n \times k}$ with $U^\top U = I$. Then*

$$\mathrm{proj}_U(x) = UU^\top x$$


Projections On Subspaces
Real Subspace Representations
Real Matrices with Orthonormal Columns
Real Matrix-Matrix Products
Real Matrix-Vector Products
Real Linear Combinations
Vectors as Matrices
Orthonormal Set of Real Vectors
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Real Vector Projections
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Optimization Problems
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N-Dimensional Space
Plane Vector Sums and Differences
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Natural Multiplicative Identity
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Family Unions and Intersections
Orders
Function Inverses
Element Functions
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Arithmetic
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Set Powers
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Set Unions and Intersections
Set Dualities
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Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
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Names
Letters
Objects