Why

What is the best linear regressor if we choose according to a weighted squared loss function.

Definition

Suppose we have a paired dataset of n records with inputs in $\mathbf{R}^d$ and outputs in $\mathbf{R}$. A *weighted least squares linear predictor* for nonnegative weights $w \in \mathbf{R}^n$, $w \geq 0$, is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$\frac{1}{n} \sum_{i=1}^{n} w_i (y_i - x^\top a^i)^2.$$

Some authors refer to this process of selecting a linear predictor as the *weighted least-squares problem.*

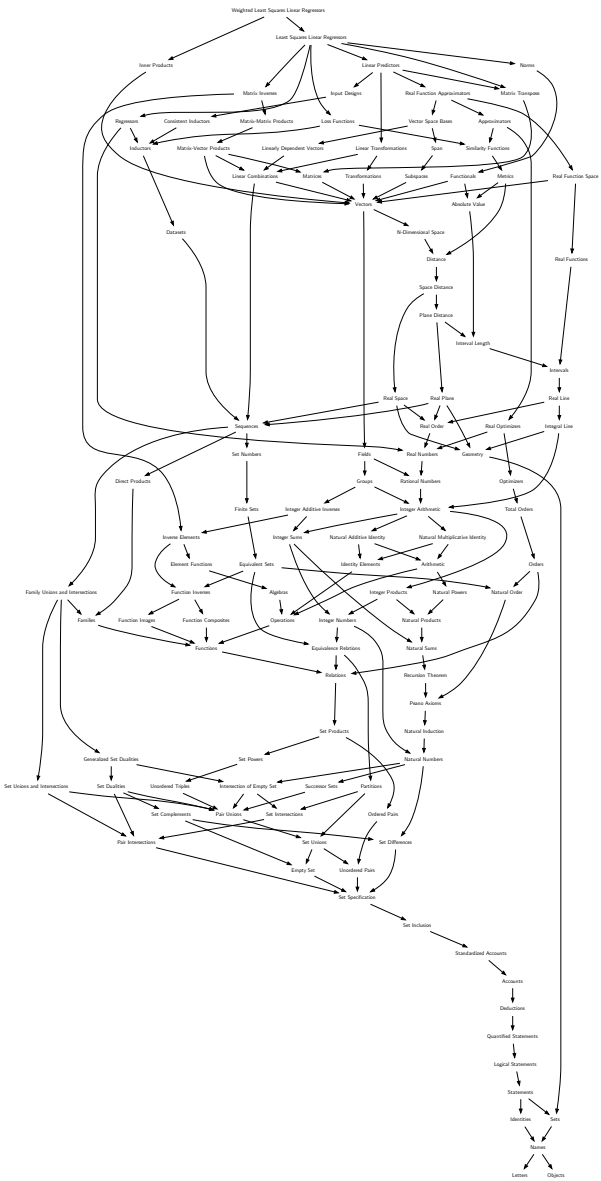
Define $W \in \mathbf{R}^{n \times n}$ so that $W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix. We want to find x to minimize

$$\|W(Ax - y)\|$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$\left(A^\top W A^\top\right)^{-1} A^\top W y.$$


Weighted Least Squares Linear Regressors
Least Squares Linear Regressors
Inner Products
Linear Predictors
Norms
Matrix Inverses
Input Designs
Real Function Approximators
Matrix Transpose
Regressors
Consistent Inductors
Matrix-Matrix Products
Loss Functions
Vector Space Bases
Approximators
Inductors
Matrix-Vector Products
Linearly Dependent Vectors
Linear Transformations
Span
Similarity Functions
Linear Combinations
Matrices
Transformations
Subspaces
Functionals
Metrics
Real Function Space
Vectors
Absolute Value
Datasets
N-Dimensional Space
Distance
Real Functions
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Sequences
Real Order
Real Optimizers
Integral Line
Set Numbers
Fields
Real Numbers
Geometry
Direct Products
Groups
Rational Numbers
Optimizers
Finite Sets
Integer Addition Inverses
Integer Arithmetic
Total Orders
Inverse Elements
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Element Functions
Equivalent Sets
Identity Elements
Arithmetic
Orders
Family Unions and Intersections
Function Inverses
Algebras
Integer Products
Natural Powers
Natural Order
Families
Function Images
Function Composites
Operations
Integer Numbers
Natural Products
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Name
Lemas
Objects