

DIFFEOMORPHISMS

Why

We want to think about two abstract spaces as being equivalent.¹

Definition

Let $X \subset \mathbb{R}^n$ and $Y \subset \mathbb{R}^m$. A smooth, invertible function $f: X \to Y$ is a diffeomorphism f^{-1} is smooth. X and Y are diffeomorphic if such a function exists.

The key is the relation diffeomorphic is an equivalence relation. It is reflexive because the identity map is smooth and invertible. It is symmetric since if f is a diffeomorphism from X to Y then f^{-1} is a diffeomorphism from Y to X. It is transitive because the composition of two smooth functions is smooth.

Differential Topology

Differential topology studies properties of $X\subset \mathbf{R}^n$ which do not change under diffeomorphism.

¹Future editions will include.

