Why

We want a measure of the *spread* of a random variable.

Definition

The *covariance* (or *variance*) of a random variable $x : \Omega \to \mathsf{R}$ is $\mathsf{E}((x - \mathsf{E}(x))^2)$, the expectation of the square of the random variable's distance from its mean. The covariance measures the mean square difference from the mean.

Interpretation

The covariance of x summarizes how "wide" the induced distribution of x is. If the covariance is small, then the induced distribution is concentrated aroudn its mean.[1]

Notation

We denote the covariance of x by $\mathsf{cov}(x)$. Another common notation is $\mathsf{var}(x)$.

Standard deviation

If x has units meters, then $\mathsf{cov}(x)$ has units square meters. It can be useful to work instead with the *standard deviation* of x, defined as $\sqrt{\mathsf{cov}(x)}$, which has the same units as x. We denote the standard deviation of x by $\mathsf{std}(x)$.

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[1]Future editions will give example.

Outcome Variable Covariance

Outcome Variable Expectation

Induced Probability Distributions

Outcome Variable Events

Event Probabilities

Probability Distributions

Real Square Roots

Real Squares

Real Arithmetic

Real Multiplicative Inverses

Real Products

Real Additive Inverses

Intervals

Real Line

Real Order

Real Sums

Rational Arithmetic

Rational Multiplicative Inverses

Real Summation

Rational Order

Real Numbers

Rational Sums

Rational Products

Integral Line

Rational Numbers

Integer Arithmetic

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Natural Square Roots

Arithmetic

Identity Elements

Natural Summation

Square Numbers

Natural Powers

Squares

Natural Products

Inverse Elements

Natural Equations

Integer Order

Comparisons

Integer Sums

Family Operations

Element Functions

Function Inverses

Equations

Natural Sums

Orders

Integer Numbers

Families

Operations

Function Images

Function Composites

Recursion Theorem

Natural Order

Converse Relations

Equivalence Relations

Functions

Peano Axioms

Relations

Uncertain Outcomes

Natural Induction

Set Products

Natural Numbers

Set Powers

Ordered Pairs

Successor Sets

Unordered Triples

Intersection of Empty Set

Partitions

Pair Unions

Set Intersections

Set Symmetric Differences

Pair Intersections

Set Unions

Set Complements

Geometry

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects