

RANDOM VARIABLES

Definition

A random variable is a measurable map from a probability space to a measurable space.

A real-valued random variable is a measurable map between the probability space and the set of real numbers with its topological sigma algebra. We so frequently work with real-valued random variables that if the range the random variable is not specified, it is assumed to be a subset of **R**.

Notation

Suppose $(X, \mathcal{A}, \mathbf{P})$ is a probability space and (Y, \mathcal{B}) is a measurable space. A random variable is a measurable function $f: X \to Y$.

Some authors tend to denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. They reserve lower case letters x, y, z for the elements of the codomains of these functions. In such cases, they often denote the set of outcomes as Ω , which we have mentioned a mnemonic for outcomes.

Special notation for common cases

Some authors use notation for the probability of particular, common sets. Suppose $(\Omega, \mathcal{A}, \mathbf{P})$ is a probability space with $X : \Omega \to \mathbf{R}$ a real-valued random variable. Many authors use $\mathbf{P}(X \in A)$ (or $P(X \in A)$, no bold) to denote

$$\mathbf{P}(X^{-1}(A)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\})$$

where $A \in \mathcal{B}(\mathbf{R})$, the Borel sigma algebra on \mathbf{R} . We will tend to use brackets in place of parentheses for clarity. So we will write $\mathbf{P}[X \in A]$.

Similar to the above, suppose $Y: \Omega \to \mathbf{R}$ is a random variable and $B \in \mathcal{B}(\mathbf{R})$. Then we will use $\mathbf{P}[X \in A, Y \in B]$ to denote

$$\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \ldots, X_n : \Omega \to \mathbf{R}$, and Borel sets A_1, \ldots, A_n , we will use $\mathbf{P}[X_1 \in A_1, \ldots, X_n \in A_n]$ to denote $\mathbf{P}(\cap_{i=1}^n X_i^{-1}(A_i))$.

