



Why

We want to talk rooting a tree at a given vertex.¹

Definition

A *rooted tree* is an ordered pair $((V, T), r)$ where (V, T) is a tree and $r \in V$ is a distinguished vertex which we call the *root*. We visualize rooted trees with the root at the top (see Figure 1).

Parents and Children

Suppose w is the first vertex on the path from the root to a non-root vertex v . Since there is only one such path, w is unique and we call it *the parent* of v . Conversely, we call v a *child* of w . We denote the set of children of v by $\text{ch}(v)$. A vertex may have no children or it may have many children. If it has no children we call it a *leaf*.

We define the *parent function* $\text{pa} : V \rightarrow V$ with the convention that the *parent of the root* is the root. The *parent of degree* k where $k > 0$ is $\text{pa}^k(x)$ where pa^k is the composite of pa with itself k times. So, in particular, $\text{pa}^{k+1}(v) = \text{pa}(\text{pa}^k(v))$. We define the *parent of degree* 0 of v to be v , and denote it by $\text{pa}^0(v) = v$. For the tree visualized in Figure 1, $\text{pa}(i) = g$, $\text{pa}^2(i) = d$, $\text{pa}^3(i) = a$.

If $w = \text{pa}^k(v)$ for some $k \geq 0$, then w is a *ancestor* of v and v is a *descendent* of w . We use the term *proper ancestor* and *proper descendent* if $k > 0$ (i.e., $w \neq v$).

The *depth* or *level* of a vertex v is its distance (see *Trees*) to the root. We denote the level of a vertex v by $\text{lev}(v)$. The level of

¹Future editions will expand this intuition.

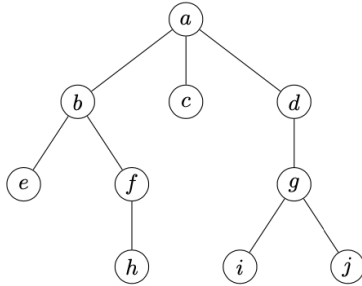


Figure 1: A rooted tree with root a .

the root is 0. If $\text{lev}(v) = k > 0$, then $\text{pa}^k(v)$ is the root. The level function lev satisfies $\text{lev}(v) = \text{lev}(\text{pa}(v)) + 1$.

