Why

It is natural to embed a dataset.

Definition

Let $(x : \Omega \to \mathbf{R}^d, A \in \mathbf{R}^{n \times d}, e : \Omega \to \mathbf{R}^n)$ be a probabilistic linear model over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Let $\phi : \mathbf{R}^d \to \mathbf{R}^{d'}$ be a feature map.

We call the sequence (x, A, e, ϕ) a *featurized probabilistic linear model* (also *embedded probabilistic linear model*). We interpret the model as a random field $h : \Omega \to (\mathbf{R}^d \to \mathbf{R})$ which is a linear function of the features

$$h_\omega(a) = \phi(a)^\top x(\omega).$$

Denote the data matrix of the embedded feature vectors by $\phi(A)$. In other words, $\phi(A) \in \mathbf{R}^{n \times d'}$ is a matrix whose rows are feature vectors. Then (x, A, e, ϕ) corresponds to the probabilistic linear model $(x, \phi(A), e)$.

Normal case

In the normal (Gaussian) case, the parameter posterior $g_{x|y}(\cdot, \gamma)$ is a normal density with mean

$$\Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x \phi(A)^\top + \Sigma_e\right)^{-1} \gamma$$

and covariance

$$\left({\Sigma_x}^{-1} + \phi(A)^\top {\Sigma_e}^{-1} \phi(A)\right)^{-1}.$$

The predictive density for $a \in \mathbf{R}^d$ is normal with mean

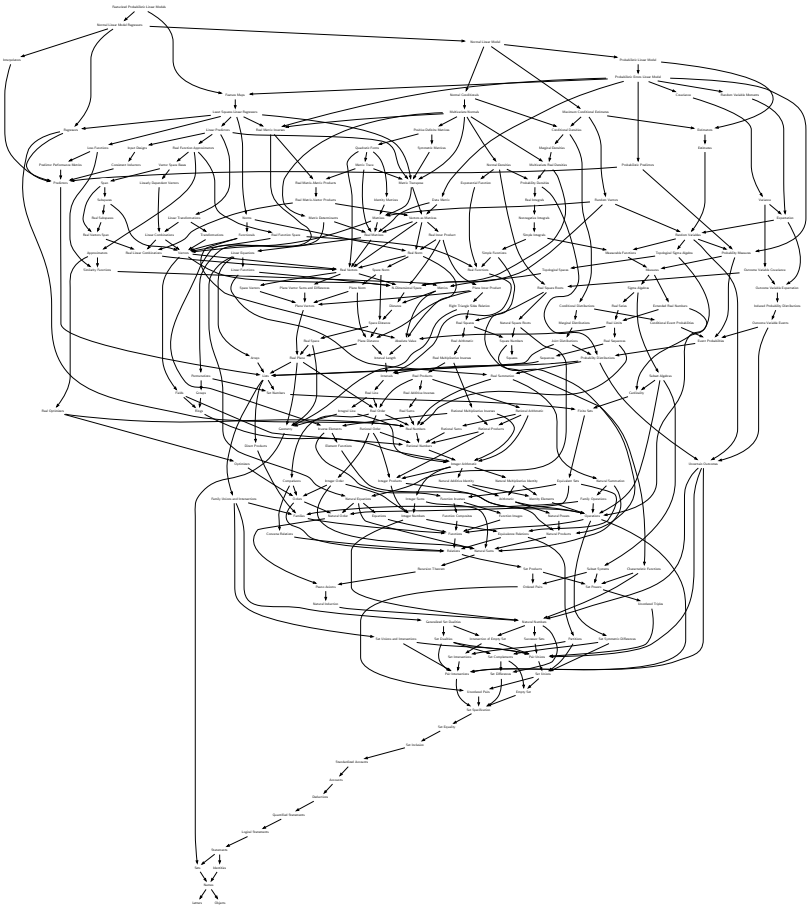
$$\phi(a)^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x \phi(A)^\top + \Sigma_e\right)^{-1} \gamma.$$

and covariance

$${\phi_a}^\top \Sigma_x \phi_a - {\phi_a}^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x \phi(A)^\top + \Sigma_e\right)^{-1} \phi(A)\Sigma_x \phi_a.$$

So the *featurized linear regressor* is the predictor $h : \mathbf{R}^d \to \mathbf{R}$ defined by

$$h(a) = \phi(a)^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x \phi(A)^\top + \Sigma_e\right)^{-1} \gamma.$$


Generalized Probabilistic Linear Models
Bernoulli Linear Model Regressors
Normal Linear Model
Probabilistic Linear Model
Probabilistic Error Linear Model
Gamma Model
Random Variable Matrices
Linear Regression
Normal Distribution
Probabilistic Functions
Sets
Set Operations
Set Equality
Set Intersection
Set Union
Set Difference
Set Complement
Set Symmetric Difference
Set Cartesian Product
Set Power
Set Equivalence
Set Inclusion
Set Exclusion
Set Equality
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