



Why

We generalize the algebraic structure of addition and multiplication over the rationals.

Definition

A *field* is a ring $(R, +, \cdot)$ for which \cdot is commutative (i.e., $ab = ba$ for all $a, b \in R$) and \cdot has inverses for all elements except 0. In this case, we refer to *field addition* and *field multiplication*.

We also give names to the objects which have one of these additional properties or the other. A ring for which multiplication is commutative is called a *commutative ring*. Note that a ring is *always* commutative with respect to addition, here the term commutative refers to multiplication. A ring for which there are inverse elements, excepting 0, is called a *division ring*.

Notation

Since our guiding example is the set of rationals \mathbf{Q} with addition and multiplication defined in the usual manner, and we use a bold font for \mathbf{Q} , we tend to denote an arbitrary field by \mathbf{F} , a mnemonic for “field.”

