Definition

An operator $T \in \mathcal{L}(V)$ is called *self-adjoint* (or *Hermitian*) if the adjoint of T is itself. In symbols, T is self-adjoint if $T = T^*$. In other words, T is self-adjoint if and only if

$$\langle Tv, w\rangle = \langle v, Tw\rangle \quad \text{for all } v, w \in V$$

Properties

Proposition 1. *Suppose $S, T \in \mathcal{L}(V)$ are self-adjoint. The $S + T$ are self-adjoint. Also λT is adjoint for all real λ.*

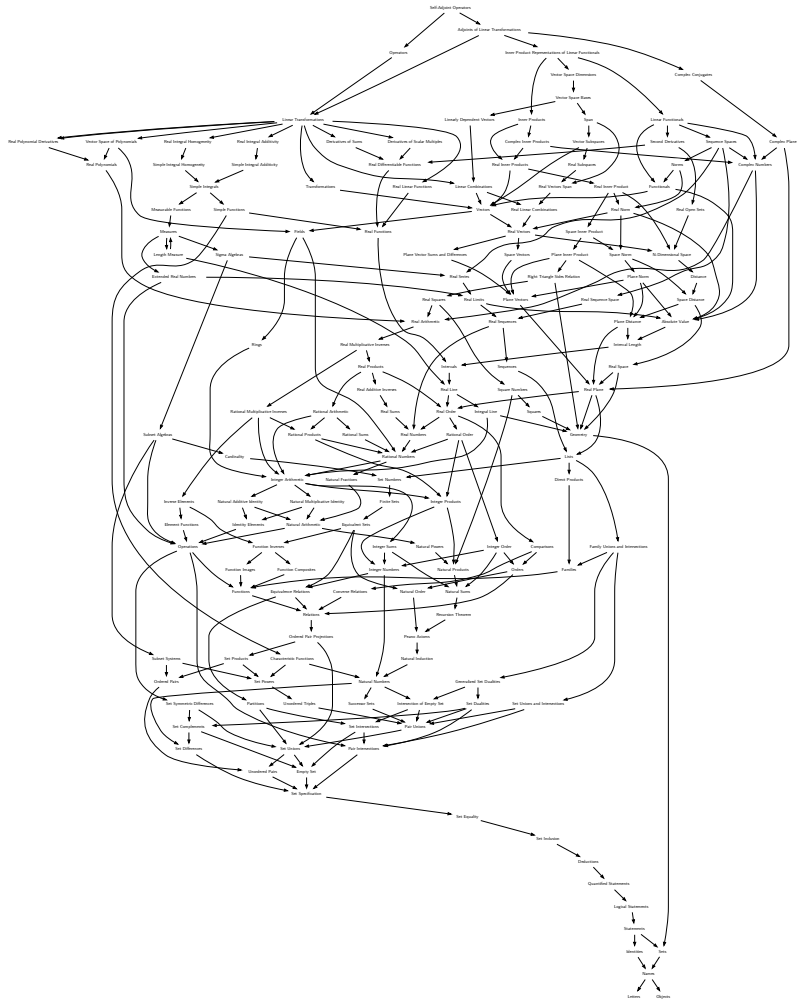
Notation

We will see that the adjoint on $\mathcal{L}(V)$ plays a role similar to complex conjugation on $\mathbf{C}$. The self-adjoint operators will seen to be analogous to the real numbers. A complex number is real if and only if $z = z^*$. Similarly, an operator is self-adjoint if and only if $T = T^*$.

Characterization for complex space

Proposition 2. *Suppose V is a complex inner product space and let $T \in \mathcal{L}(V)$. Then*

$$T = T^* \quad \longleftrightarrow \quad (\forall v \in V)\ (\langle Tv, v\rangle \in \mathbf{R})$$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