



SET POWERS

Why

We want to consider all the subsets of a given set.

Definition

We do not yet have a principle stating that such a set exists, but our intuition suggests that it does.

Principle 1 (Powers). *For every set, there exists a set of its subsets.*

We call the existence of this set the *principles of powers* and we call the set the *power set*.¹ As usual, the principle of extension gives uniqueness (see *Set Equality*). The power set of a set includes the set itself and the empty set.

Notation

Let A denote a set. We denote the power set of A by A^* , read aloud as “powerset of A .” $A \in A^*$ and $\emptyset \in A^*$. However, $A \subset A^*$ is false.

Examples

Let a, b, c denote distinct objects. Let $A = \{a, b, c\}$ and $B = \{a, b\}$. Then $B \subset A$. In other notation, $B \in A^*$. We can walk through examples of power sets.

¹This terminology is standard, but unfortunate. Future editions may change these terms.

ℓEmpty Set

Proposition 1. $\emptyset^* = \{\emptyset\}$

ℓSingletons

Proposition 2. $\{a\}^* = \{\emptyset, \{a\}\}$

ℓPairs

Proposition 3. $\{a, b\}^* = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

ℓTriples

Proposition 4. $\{a, b, c\}^* = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

Properties

We can guess the following easy properties.²

Proposition 5. $\emptyset \in A^*$

Proposition 6. $A \in A^*$

We call A and \emptyset the *improper* subsets of A . All other subset we call *proper*.

Basic Fact

Proposition 7. $E \subset F \longrightarrow E^* \subset F^*$

²Future editions will expand this account.

