



## Why

How do equivalence classes and functions relate

## Definition

We can associate to each element of a set its equivalence class under an equivalence relation. Let  $X$  denote a set and  $R$  an equivalence relation. We call the function  $f : X \rightarrow X/R$  defined by  $f(x) = x/R$  the *canonical map* from  $X$  to  $X/R$ .

Conversely, if  $f$  is an arbitrary function from  $X$  onto  $Y$ , we can naturally define an equivalence relation  $R$  in  $X$  so that for  $a, b \in X$ ,  $a R b \iff f(a) = f(b)$ . If  $f$  was onto, so for each  $y \in Y$ , there exists an  $x \in X$  with  $f(x) = y$ . Now let  $g : Y \rightarrow X/R$  be defined by  $g(y) = x/R$ . The values of  $g$  are the subset  $X/R$  which are mapped to the same value under  $f$ . Moreover, the function  $g$  is one-to-one.



