



Total Orders

1 Why

Often we want all elements of the set A to be comparable.

2 Definition

We call R *connexive* if for all $a, b \in A$, $(a, b) \in R$ or $(b, a) \in R$. If R is a partial order and connexive, we call it a *total order*.

A *totally ordered set* is a set together with a total order. The language is a faithful guide: we can compare any two elements. Still, we prefer one word to three, and so we will use the shorter term *chain* for a totally ordered set; other terms include *simply ordered set* and *linearly ordered set*.

Let $C = (A, R)$ be a chain. A *minimal element* of C is an element which precedes all other elements. A *maximal element* of C is an element which is preceded by all other elements.