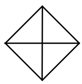

Why

It is natural to embed a dataset.

Definition

Let $(x : \Omega \to \mathbf{R}^d, A \in \mathbf{R}^{n \times d}, e : \Omega \to \mathbf{R}^n)$ be a probabilistic linear model over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Let $\phi : \mathbf{R}^d \to \mathbf{R}^{d'}$ be a feature map.

We call the sequence (x, A, e, ϕ) a *featurized probabilistic linear model* (also *embedded probabilistic linear model*). We interpret the model as a random field $h : \Omega \to (\mathbf{R}^d \to \mathbf{R})$ which is a linear function of the features

$$h_\omega(a) = \phi(a)^\top x(\omega).$$

Denote the data matrix of the embedded feature vectors by $\phi(A)$. In other words, $\phi(A) \in \mathbf{R}^{n \times d'}$ is a matrix whose rows are feature vectors. Then (x, A, e, ϕ) corresponds to the probabilistic linear model $(x, \phi(A), e)$.

Normal case

In the normal (Gaussian) case, the parameter posterior $g_{x|y}(\cdot, \gamma)$ is a normal density with mean

$$\Sigma_x \phi(A)^\top \left(\phi(A) \Sigma_x \phi(A)^\top + \Sigma_e\right)^{-1} \gamma$$

and covariance

$$\left({\Sigma_x}^{-1} + \phi(A)^\top {\Sigma_e}^{-1} \phi(A)\right)^{-1}.$$

The predictive density for $a \in \mathbf{R}^d$ is normal with mean

$$\phi(a)^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x\phi(A)^\top + \Sigma_e\right)^{-1} \gamma.$$

and covariance

$$\phi_a{}^\top \Sigma_x \phi_a - \phi_a{}^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x\phi(A)^\top + \Sigma_e\right)^{-1} \phi(A)\Sigma_x\phi_a.$$

So the *featurized linear regressor* is the predictor $h : \mathbf{R}^d \to \mathbf{R}$ defined by

$$h(a) = \phi(a)^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x\phi(A)^\top + \Sigma_e\right)^{-1} \gamma.$$


Featurized Probabilistic Linear Models
Normal Linear Model Regressors
Interpolators
Normal Linear Model
Probabilistic Linear Model
Normal Conditionals
Probabilistic Errors Linear Model
Multivariate Normals
Covariance
Random Variable Moments
Maximum Conditional Estimates
Feature Maps
Positive Definite Matrices
Variance
Estimators
Least Squares Linear Regressors
Symmetric Matrices
Quadratic Forms
Conditional Densities
Regressors
Expectation
Matrix Inverses
Linear Predictors
Matrix Trace
Marginal Densities
Loss Functions
Input Designs
Matrix-Matrix Products
Matrix Transpose
Identity Matrices
Multivariate Real Densities
Normal Densities
Probabilistic Predictors
Predictor Performance Metrics
Consistent Inductors
Random Vectors
Real Function Approximators
Real Matrix-Matrix Products
Matrix-Vector Products
Probability Densities
Predictors
Estimates
Vector Space Bases
Data Matrix
Real Matrix-Vector Products
Matrices
Real Integrals
Exponential Function
Linear Transformations
Span
Linearly Dependent Vectors
Norms
Matrix Determinants
Vectors as Matrices
Nonnegative Integrals
Random Variables
Transformations
Subspaces
Functionals
Linear Combinations
Real Function Space
Real Matrices
Real Inner Product
Conditional Distributions
Simple Integrals
Topological Sigma Algebra
Probability Measures
Measurable Functions
Approximators
Vectors
Real Linear Combinations
Conditional Event Probabilities
Linear Equations
Real Norm
Simple Functions
Topological Spaces
Measures
Similarity Functions
Event Probabilities
Real Vectors
Linear Functions
Real Functions
Sigma Algebras
Metrics
Plane Vector Sums and Differences
N-Dimensional Space
Space Vectors
Plane Norm
Plane Inner Product
Real Series
Extended Real Numbers
Distance
Plane Vectors
Right Triangle Sides Relation
Marginal Distributions
Real Square Roots
Real Limits
Space Distance
Arrays
Real Squares
Natural Square Roots
Real Sequences
Absolute Value
Real Space
Plane Distance
Joint Distributions
Real Arithmetic
Square Numbers
Sequences
Real Plane
Interval Length
Probability Distributions
Real Multiplicative Inverses
Squares
Permutations
Lists
Real Summation
Intervals
Real Products
Fields
Groups
Set Numbers
Real Line
Real Additive Inverses
Rings
Real Optimizers
Real Order
Real Sums
Integral Line
Rational Multiplicative Inverses
Rational Arithmetic
Subset Algebras
Inverse Elements
Real Numbers
Geometry
Rational Order
Rational Sums
Rational Products
Cardinality
Element Functions
Direct Products
Rational Numbers
Finite Sets
Optimizers
Integer Arithmetic
Equivalent Sets
Natural Summation
Comparisons
Integer Order
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Function Inverses
Family Operations
Family Unions and Intersections
Orders
Identity Elements
Arithmetic
Natural Equations
Integer Sums
Function Images
Function Composites
Families
Operations
Natural Order
Equations
Integer Numbers
Natural Powers
Functions
Converse Relations
Equivalence Relations
Natural Products
Relations
Natural Sums
Subset Systems
Set Products
Recursion Theorem
Characteristic Functions
Ordered Pairs
Set Powers
Peano Axioms
Uncertain Outcomes
Natural Induction
Generalized Set Dualities
Unordered Triples
Natural Numbers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Set Unions
Pair Intersections
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects