



REAL MULTIPLICATIVE INVERSES

Why

What is the multiplicative inverse in the reals?

Result

We can show the following.¹

Proposition 1. *The multiplicative inverse of R is, if $R \neq 0_{\mathbf{R}}$,*

- 1. if $0_{\mathbf{Q}} \in \mathbf{R}$, then $\{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\} \cup \{r^{-1}\} \exists s < r, (r \notin \mathbf{R})$*
- 2. If $0_{\mathbf{Q}} \notin \mathbf{R}$, then the additive inverse of the multiplicative inverse of the additive inverse of R .*

Notation

We denote the multiplicative inverse of $r \in \mathbf{R}$ by r^{-1} . We denote $q \cdot (r^{-1})$ by q/r .

Division

We call the operation $(a, b) \mapsto a/b$ *real division*.

¹The account will appear in future editions.

