

## Partial Orders

## Why

We want to handle elements of a set in a particular order.

## **Definition**

Let R be a relation on a non-empty set A. R is a partial order if it is reflexive, transitive, and anti-symmetric. If  $(a,b) \in R$  we say that a precedes b and that b succeeds a.

A partially ordered set is a set and a partial order. The language partial is meant to suggest that two elements need not be comparable. For example, suppose R is  $\{(a, a) \mid a \in A\}$ ; we may justifiably call this no order at all and call A totally unordered, but it is a partial order by our definition.

## Notation

We denote a partial order on a set A by  $\leq$ . We read  $\leq$  aloud as "precedes or equal to" and so read  $a \leq b$  aloud as "a precedes or is equal to b." If  $a \leq b$  but  $a \neq b$ , we write  $a \prec b$ , read aloud as "a precedes b."

