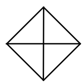


Definition

The *translate* of $S \subset \mathbf{R}^n$ *by* the vector $a \in \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid \exists x \in S \text{ such that } z = x + a\}.$$

Notation

We often use the abbreviated notation $S + a$ for the translate of S by a. It is sometimes also convenient to extend set-builder notation and write

$$S + a = \{x + a \mid x \in M\}.$$

The right hand side is slick notation for the definition given above.

Sums and differences

The *sum* (or *Minkowski sum*) of two sets $S, T \subset \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x + y)\}.$$

Likewise, the *difference* (or *Minkowski difference*) of two sets $S, T \subset \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x - y)\}.$$

Notation

We denote the sum of S and T by $S + T$, and the difference by $S - T$.[1] We often use the slick notation

$$\{x + y \mid x \in S, y \in T\} \text{ and } \{x - y \mid x \in S, y \in T\},$$

for these two sets. Notice that in this notation

$$\{a\} + B = a + B$$

[1] This second notation unfortunately conflicts with our notation for set differences. Future editions will correct.

Scaled sets

Given a set $A \subset \mathbf{R}^n$ and a $\lambda \in \mathbf{R}$, the set which is A *scaled by* (the *scaled set*) is

$$\{z \in \mathbf{R}^n \mid (\exists x \in A)(z = \lambda x)\}$$

We often denote this set by λA. As before, we often use the slick notation

$$\lambda A = \{\lambda a \mid a \in A\}$$

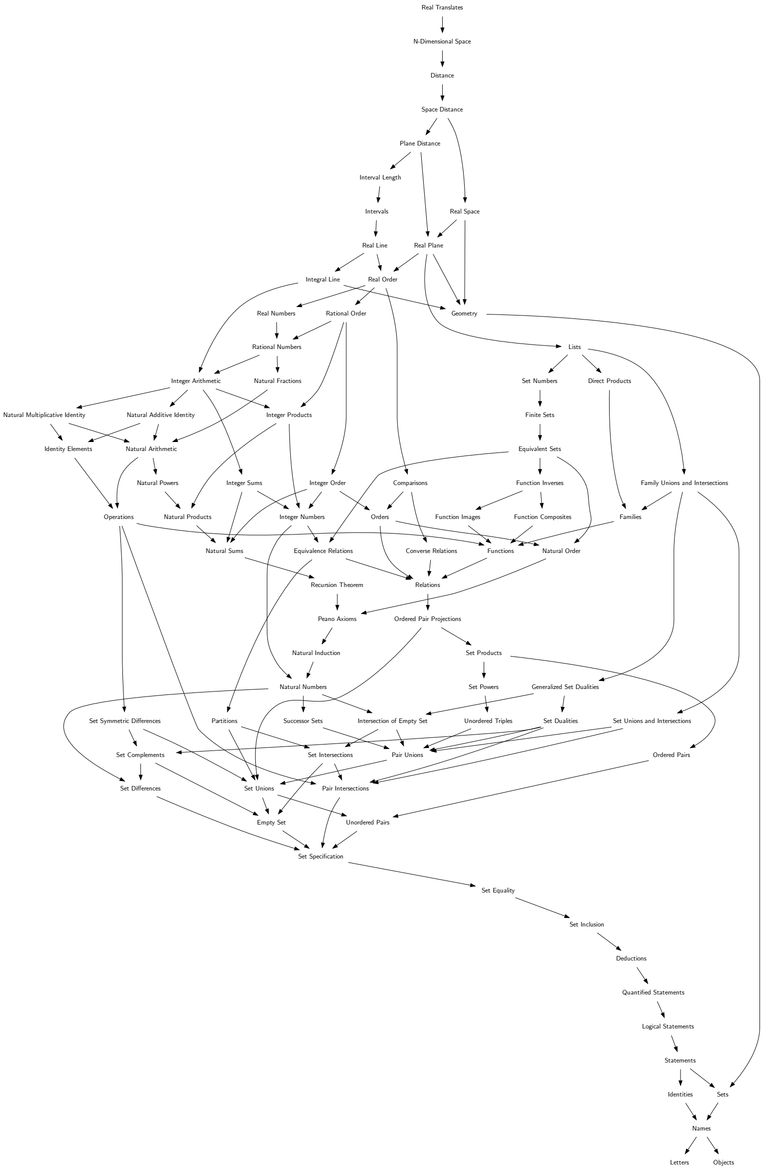
The set $(-1)A$ is denoted $-A$

Homothetic sets

A set A is *homothetic* to a set B if there is $x \in \mathbf{R}^n$ and $\lambda \neq 0$ so taht

$$A = x + \lambda B$$

If $\lambda > 0$, A is *positively homothetic* to B.


Real Translates
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Integral Line
Real Order
Real Numbers
Rational Order
Geometry
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Operations
Natural Products
Integer Numbers
Orders
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Converse Relations
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects