



Why

Sums of non-negative functions are increasing, and workable with the monotone convergence theorem.

Result

Prop. 1. *The integral of the limit of the partial sums of a sequence of measurable, nonnegative, extended-real-valued functions is the limit of the partial sums of the integrals.*

Proof. Let (X, \mathcal{A}, μ) be a measure space, and let $f_n : \rightarrow [0, \infty]$ a \mathcal{A} -measurable function for every natural number n . We want to show that:

$$\int \sum_{k=1}^{\infty} f_k d\mu = \sum_{k=1}^{\infty} \int f_k d\mu.$$

We apply the monotone convergence theorem to the sequence $\{\sum_{i=1}^n f_i\}_n$. This sequence is nondecreasing because $f_n \geq 0$ for all n . □

