Why

Definition

Let $\mathcal{D} = ((\mathcal{X}_t)_{t=0}^T), (\mathcal{U}_t)_{t=0}^{T-1}, (f_t)_{t=1}^{T-1})$ be a dynamical system. Let $g_t : \mathcal{X}_t \times \mathcal{U}_t \to \mathbf{R} \cup \{\infty\}$ for $t = 1, \ldots, T-1$ and let $g_T : \mathcal{X}_T \to \mathbf{R} \cup \{\infty\}$. Let $x_0 \in \mathcal{X}_0$.

We call the sequence $(x_0, \mathcal{D}, (g_t)_{t=1}^T)$ a *deterministic dynamic optimization problem.* We call x_0 the *initial state.* We call g_t the *stage cost function* for stage t and call g_T the *terminal cost function.*

A deterministic dynamic optimization problem corresponds to an optimization problem with variables $u_0 \in \mathcal{U}_0, \ldots, u_{T-1} \in \mathcal{U}_{T-1}$. Define $U = \mathcal{U}_0 \times \mathcal{U}_1 \times \mathcal{U}_{T-1}$. Define $J : U \to \mathbf{R} \cup \{\infty\}$ by

$$J(u) = \sum_{t=0}^{T-1} g_t(x_t, u_t) + g_T(x_T)$$

in which $x_{t+1} = f_t(x_t, u_t)$ for $t = 0, \ldots, T-1$. The optimization problem is (U, J). And so a dynamic optimization problem is just a (possibly big) optimization problem. We call $\sum_{t=0}^{T-1} g_t(x_t, u_t)$ the *total stage cost* and we call $g_T(x_T)$ the *terminal stage cost.*

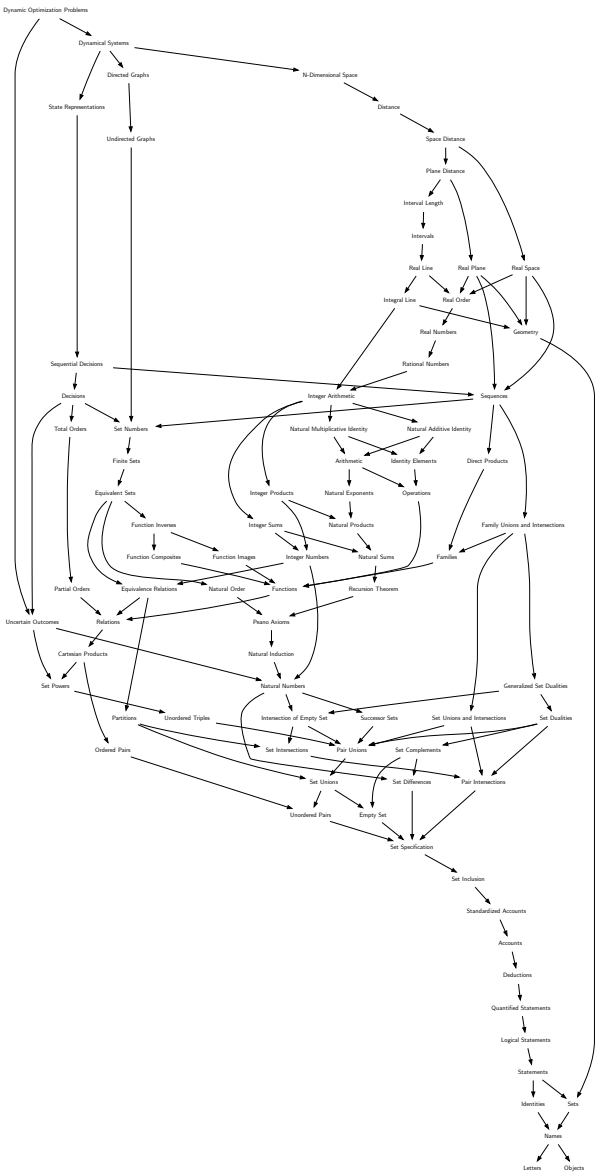
Notation

We often write this problem as

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{t=1}^{T-1} g_t(x_t, u_t) + g_T(x_T) \\ \text{subject to} & x_{t+1} = f_t(x_t, u_t), \quad t = 0, \ldots, T-1. \end{array}$$

Other terminology and comments

Dynamic optimization problems are frequently called *deterministic optimal control* problems or *classical* or *open-loop control* problems. These problems are said to address the dynamic effect of actions across time. Although these models include no notion of "uncertainty" (or "uncertain outcomes", see Uncertain Outcomes), they are frequently applied in situations with uncertain outcomes by ignoring the uncertainty in the application.


Dynamic Optimization Problems
Dynamical Systems
Directed Graphs
N-Dimensional Space
State Representations
Distance
Undirected Graphs
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
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Sequential Decisions
Integer Arithmetic
Sequences
Decisions
Total Orders
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Direct Products
Equivalent Sets
Integer Products
Natural Exponents
Operations
Function Inverses
Integer Sums
Natural Products
Family Unions and Intersections
Function Composites
Function Images
Integer Numbers
Natural Sums
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Recursion Theorem
Uncertain Outcomes
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Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Unions and Intersections
Set Dualities
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
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Statements
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Objects