Why

[1]

Definition

Let Ω be an open set in $\mathbf{C}$ and let $f : \Omega \to \mathbf{C}$. The function f is *holomorphic at the point* $z_0 \in \mathbf{C}$ if the complex quotient

$$\frac{f(z_0 + h) - f(z_0)}{h}$$

has a limit when $h \to \infty$, where $h \in \mathbf{C}$, $h \neq 0$ and $z_0 + h \in \Omega$ so that the quotient is well-defined.

This condition is similar to saying that a function is differentiable, except that the h is complex and so the condition above encomposes all limits approaching z (all angles) in the complex plane.[2] But we emphasize that h is a complex number approaching the complex number $(0, 0)$ from any direction. If the limit exists, then we call its value the *derivative of f at z_0*.

The function f is *holomorphic* on Ω if f is holomorphic at every point of Ω. If C is a closed subset of $\mathbf{C}$, we say that f is holomorphic on C if f is holomorphic on some open set containing c. If f is holomorphic on all of C then we call f *entire*. A holomorphic function is sometimes called *regular* or *complex differentiable*. The latter term is used in view of the similarities with the definition of a real derivative.

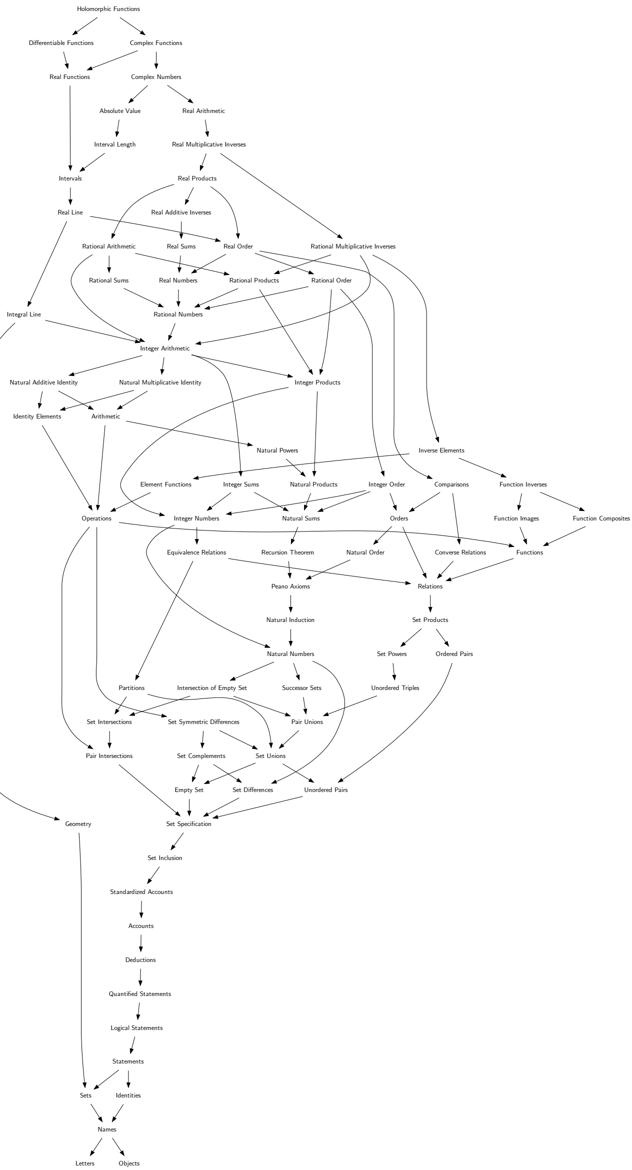
[1] Future notes will expand.

[2] Future editions will clarify.

Notation

In the case that $f : \Omega \to \mathbf{C}$ is holomorphic at z_0 we denote the derivative at z_0 by $f'(z_0)$. We have defined $f'(z_0)$ by

$$f'(z_0) = \lim_{h \to 0} \frac{f(z_0 + h) - f(z_0)}{h}.$$


Holomorphic Functions
Differentiable Functions
Complex Functions
Real Functions
Complex Numbers
Absolute Value
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Line
Real Additive Inverses
Rational Arithmetic
Real Sums
Real Order
Rational Multiplicative Inverses
Rational Sums
Real Numbers
Rational Products
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Identity Elements
Arithmetic
Natural Powers
Inverse Elements
Element Functions
Integer Sums
Natural Products
Integer Order
Comparisons
Function Inverses
Operations
Integer Numbers
Natural Sums
Orders
Function Images
Function Composites
Equivalence Relations
Recursion Theorem
Natural Order
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects