Why

Can we think of linear maps as vectors?

Definitions

Suppose V and W are some vector spaces over a field $\mathbf{F}$. Denote the linear maps from V to W by $\mathcal{L}(V, W)$ as usual.

Addition. Given $S, T \in \mathcal{L}(V, W)$ the *sum* of S and T is the linear map $R \in \mathcal{L}(V, W)$ defined by

$$Rv = Sv + Tv \quad \text{for all } v \in V$$

Scalar multiplication. Given $S \in \mathcal{L}(V, W)$ the *(scalar) product* of λ and T is the linear map $Q \in \mathcal{L}(V, W)$ defined by

$$Qv = \lambda Tv \quad \text{for all } v \in V$$

Proposition 1. *Suppose V and W are two vector spaces over the same field $\mathbf{F}$. Then $\mathcal{L}(V, W)$ is a vector space over the field $\mathbf{F}$ with respect to the operations of addition and scalar multiplication just defined.*

The additive identity of the vector space $\mathcal{L}(V, W)$ is the zero map $0 \in \mathcal{L}(V, w)$.

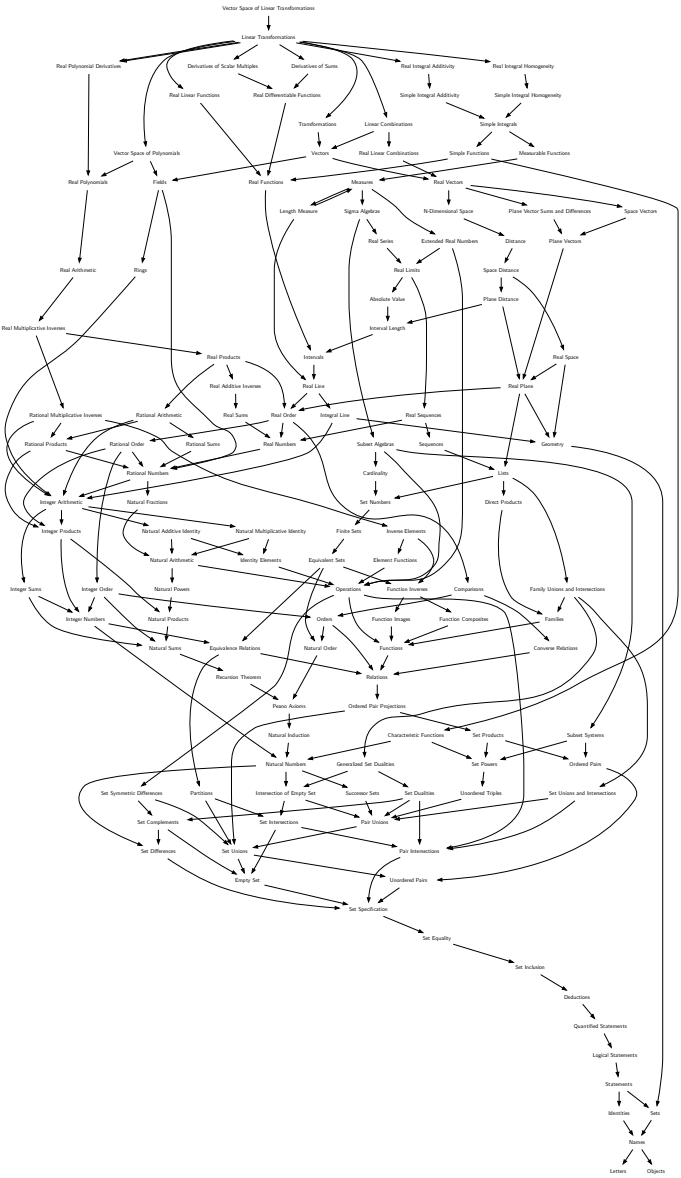
Notation

Given $S, T \in \mathcal{L}(V, W)$ the *sum* of S and T and $\lambda \in \mathbf{F}$, we denote the sum of S and T by $S + T$. Hence,

$$(S + T)(v) = Sv + Tv \quad \text{for all } v \in V$$

We denote the product of λ and T by λT. Hence,

$$(\lambda T)(v) = \lambda(Tv) \quad \text{for all } v \in V$$


Vector Space of Linear Transformations
Linear Transformations
Real Polynomial Derivatives
Derivatives of Scalar Multiples
Derivatives of Sums
Real Integral Additivity
Real Integral Homogeneity
Real Linear Functions
Real Differentiable Functions
Simple Integral Additivity
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Transformations
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Plane Vector Sums and Differences
Space Vectors
Real Series
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Distance
Plane Vectors
Real Arithmetic
Rings
Real Limits
Space Distance
Absolute Value
Plane Distance
Real Multiplicative Inverses
Interval Length
Intervals
Real Products
Real Space
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Real Sequences
Rational Products
Rational Order
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Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Inverse Elements
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Identity Elements
Equivalent Sets
Element Functions
Integer Sums
Integer Order
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Operations
Function Inverses
Comparisons
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects