



**Definition**

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

**Notation**

Let  $f$  and  $g$  be two integrable random variables with  $fg$  integrable. Denote the covariance of  $f$  with  $g$  by  $\text{cov}(f, g)$ . We defined it:

$$\text{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\mathbf{E}(g).$$

**Properties**

**Proposition 1.** *Covariance is symmetric and bilinear.*

**Proposition 2.** *The covariance of a random variable with itself is its variance.*

*Proof.* Let  $f$  be a square-integrable real-valued random variable, then

$$\text{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \text{var}(f).$$

□

**Proposition 3.** *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

*Proof.* Let  $f_1, \dots, f_n$  be integrable random variables with  $f_i f_j$  integrable for all  $i, j = 1, \dots, n$ . Using the bilinearity,

$$\begin{aligned} \text{var} \sum_{i=1}^n f_i &= \text{cov}\left(\sum_{i=1}^n f_i, \sum_{i=1}^n f_i\right) \\ &= \sum_{i=1}^n \sum_{j=1}^n \text{cov}(f_i, f_j) \end{aligned}$$

□



