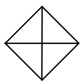


Why

[1]

Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

Notation

Let $(X, \mathcal{A}, \mathsf{P})$ be a probability space. Let $(Y, \mathcal{B})$ a measurable space. Then a random variable is a measurable function $f : X \to Y$.

Some authors denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. In this case, the base probability space is denoted by Ω, a mnemonic for "outcomes." Let $(\Omega, \mathcal{A}, \mathsf{P})$ be a probability space. Let $X : \Omega \to \mathsf{R}$ be measurable. Then X is a real-valued random variable.

Some authors use notation for the probability of particular, common sets. Although the authors often use regular parentheses, we will use [,] for precision. Let $A \in \mathcal{B}(R)$. Let

[1]Future editions will include this.

$\mathsf{P}[X \in A]$ denote $\mathsf{P}(X^{-1}(A))$. These are equivalent to

$$\mathsf{P}(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

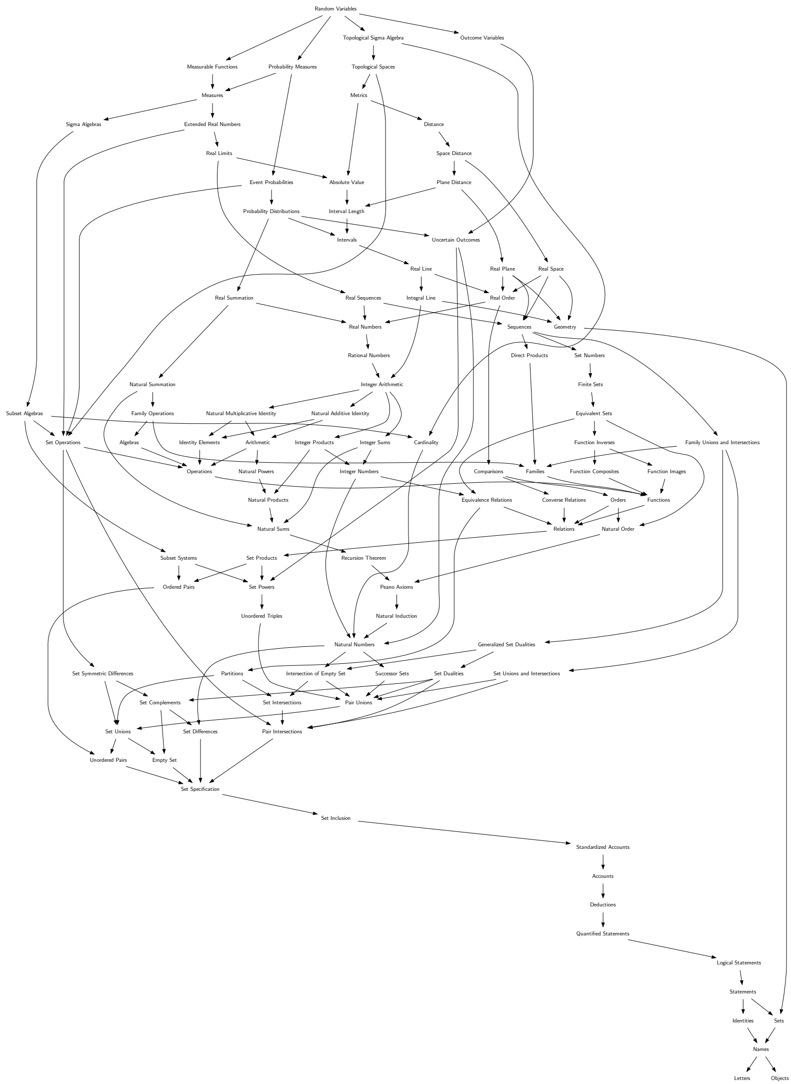
As we mentioned, some authors use $P(X \in A)$ for $P[X \in A]$, which is mostly harmless.

Next, let $Y : \Omega \to R$ a measurable function and let $B \in \mathcal{B}(R)$. Similar to the above, let $\mathsf{P}[X \in A, Y \in B]$ denote $\mathsf{P}(X^{-1}(A) \cap Y^{-1}(B))$. These are equivalent to

$$\mathsf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \ldots, X_n : \Omega \to \mathsf{R}$,

$$\mathsf{P}[X_1 \in A_1, \ldots, X_n \in A_n] = \mathsf{P}(\cap_{i=1}^n X_i^{-1}(A_i)).$$


Random Variables
Topological Sigma Algebra
Outcome Variables
Measurable Functions
Probability Measures
Topological Spaces
Measures
Metrics
Sigma Algebras
Extended Real Numbers
Distance
Real Limits
Space Distance
Event Probabilities
Absolute Value
Plane Distance
Probability Distributions
Interval Length
Intervals
Uncertain Outcomes
Real Line
Real Plane
Real Space
Real Summation
Real Sequences
Integral Line
Real Order
Real Numbers
Sequences
Geometry
Rational Numbers
Direct Products
Set Numbers
Natural Summation
Integer Arithmetic
Finite Sets
Subset Algebras
Family Operations
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Set Operations
Algebras
Identity Elements
Arithmetic
Integer Products
Integer Sums
Cardinality
Function Inverses
Family Unions and Intersections
Operations
Natural Powers
Integer Numbers
Comparisons
Families
Function Composites
Function Images
Natural Products
Equivalence Relations
Converse Relations
Orders
Functions
Natural Sums
Relations
Natural Order
Subset Systems
Set Products
Recursion Theorem
Ordered Pairs
Set Powers
Peano Axioms
Unordered Triples
Natural Induction
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects