

### Sparsity Patterns

## Why

Certain sparse matrices are easier to work with, especially those with chordal sparsity patterns.<sup>1</sup>

#### **Definition**

A sparsity pattern E of order n is a set of (unordered) pairs of  $V = \{1, \ldots, n\}$ . A sparsity pattern is *chordal* if the undirected graph (V, E) is chordal.

A symmetric matrix is said to have a sparsity pattern if its ijth entry is zero whenever  $\{i, j\}$  is not in the sparsity pattern. The diagonal entries and off-diagonal entries for pairs appearing in the sparsity pattern may or may not be zero.

The graph whose vertices are one through n and whose edge set is the sparsity patter is called the *sparsity graph* 

A sparsity pattern is not a property of a matrix because it is not unique (unless all off-diagonal entries are non-zero). If a matrix has a particular sparsity pattern it has every sparsity pattern which is a superset of it. In other words, every matrix has the sparsity pattern which is the set of all pairs of integers.

#### Notation

Let  $E \subset \{\{i,j\} \mid i,j \in \{1,2,\ldots,n\}\}$ . A symmetric matrix  $A \in \mathbf{S}^n$  is said to have sparsity pattern E if  $A_{ij} = A_{ji} = 0$ 

<sup>&</sup>lt;sup>1</sup>Future editions will expand.

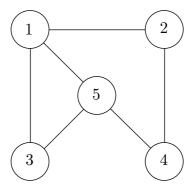


Figure 1: Sparsity graph for the matrix A.

wheneer  $i \neq j$  and  $\{i, j\} \notin E$ . The graph G = (V, E) where  $V = \{1, 2, ..., n\}$  is the sparsity graph associated with E.

We will denote the symmetric matrices of order n with sparsity pattern E by  $\mathbf{S}_{E}^{n}$ .

# Example

Figure 1 shows a sparsity graph for the matrix

$$A := \begin{bmatrix} A_{11} & A_{21} & A_{31} & 0 & A_{51} \\ A_{21} & A_{22} & 0 & A_{42} & 0 \\ A_{31} & 0 & A_{33} & 0 & A_{54} \\ 0 & A_{42} & 0 & A_{44} & A_{45} \\ A_{51} & 0 & A_{53} & A_{54} & A_{55} \end{bmatrix}.$$

