



Definition

The *variance* of a square-integrable real-valued random variable is the expectation of its square less its expectation squared.

Notation

Let (X, \mathcal{A}, μ) be a probability space and f be a random variable. We denote the variance of f by $\mathbf{var} f$. We defined it by

$$\mathbf{var} f = \mathbf{E}(f^2) - (\mathbf{E}(f))^2.$$

Results

Prop. 1. *If a random variable on a probability space is square integrable then it is integrable.*

Proof. The L^p spaces are nested for finite measures. □

Prop. 2. *The variance of a square-integrable real-valued random variable is the expectation of the square of the difference between the random variable and its expectation.*

Proof.

$$\mathbf{var} f = \mathbf{E}((f - \mathbf{E}(f))^2)$$

□

