

Positive Definite Matrices

Why

1

Definition

Let $A \in \mathbb{R}^{n \times n}$ be symmetric. A is positive definite if, for all $x \in \mathbb{R}^d$, $x \neq 0$, $x^{\top}Ax > 0$. A is positive semidefinite (or nonnegative definite) if, for all $x \in \mathbb{R}^d$, $x^{\top}Ax \geq 0$.

Notation

We denote the set of real-valued positive definite d by d matrices by \mathbf{S}_{++}^d . We denote the set of real-valued positive semidefinite d by d matrices by \mathbf{S}_{+}^d .

Characterizations

Proposition 1. Let $A \in \mathbf{S}^d$ and denote the smallest

¹Future editions will elaborate.

