Why

We look at a particular subset of vertices and the edges involved between them.

Definition

Suppose $\mathcal{G} = (V, E)$ is an undirected graph. An undirected graph (V', E') is a *subgraph* of $\mathcal{G}$ if $V' \subset V$ and $E \subset E'$. The *vertex-induced subgraph* of an undirected graph (V, E) *induced by* a subset of vertices $W \subset V$ is the undirected graph with vertices W and all edges between vertices in W. The *edge-induced subgraph* of an undirected graph (V, E) induced by vertices $F \subset E$ whose edge set is F and whose vertices are those vertices adjacent to the edges of F.

Notation

Let $W \subset V$ and define F by

$$F = \{\{v, w\} \in E \mid v, w \in W\}.$$

The subgraph induced by W is the undirected graph (W, F).

Some authors denote the subgraph induced by W by $G(W)$ or $(W, E(W))$ or $G[W]$. We avoid this notation, as it abuses G, which is no longer an ordered pair, but (in our standard function notation) now indicates a function on subsets of V with a complicated codomain. Other authors occasionally refer to the "subgraph W", instead of "the subgraph $G(W)$". Again, we avoid this practice.

For $D \subset E$, the subgraph induced by D is the undirected graph (U, D) where

$$U = \{v \in V \mid \exists e \in D : x \in e\}$$

Simiarly, people write $G(D)$ or $(V(D), D)$. We avoid this.

Connected components

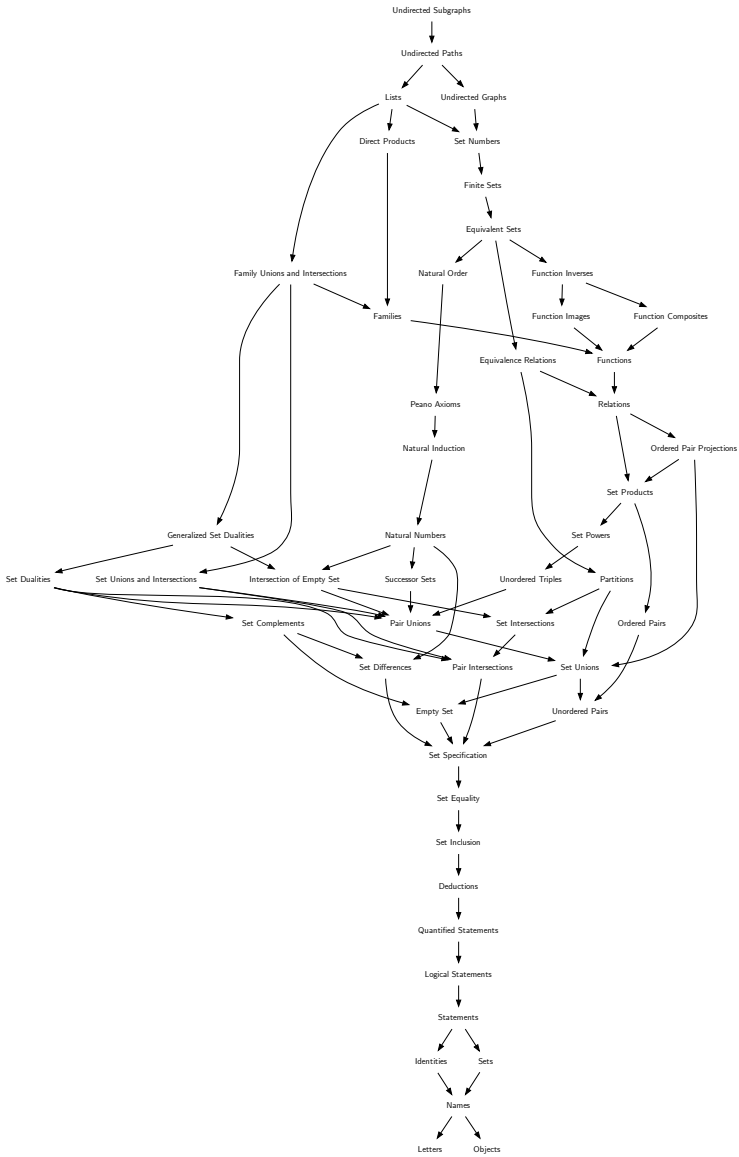
A set of vertices W in G is *connected* if there is a path between any two vertices $v, w \in W$. A set of vertices W in G is *maximimally connected* if there is no other vertex $v \notin W$ connected to a vertex in W. A *connected component* of G is the subgraph induced by a maximally connected set of vertices.

Since the vertex set of a graph can always be partitioned into sets maximally connected vertices, and a connected components is connect, we think of the connected components of G as the connected "pieces" of G.

Cliques

A set of vertices is *complete* if the subgraph induced is complete. A set of vertices W is *maximally complete* if the subgraph induced is complete and there is no vertex $v \notin W$ which is connected to every vertex in W. In other words, there is no other vertex which we can add to W so that the induced subgraph is still complete.

We call a *maximally complete* set of vertes a *clique*. Some authors define a clique in the way we have defined a complete set of vertices, without reference to the maximality.


Undirected Subgraphs
Undirected Paths
Lists
Undirected Graphs
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Images
Function Composites
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
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Statements
Identities
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Letters
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