Definition

A vector is *normalized* if its norm is 1. A set of vectors $\{u_1, \ldots, u_k\}$ *orthogonal* if $u_i^\top u_j = 0$ whenever $i \neq j$. A set of vectors is *orthonormal* if the set is orthogonal and each vector is normalized.

Basis

An orthonormal set of vectors is also an independent set. In other words, orthonormality is stronger than independence.

Orthonormal Set of Real Vectors

Independent Set of Real Vectors • Real Inner Product • Real Vector Angles

Real Linear Combinations • Real Norm • Space Inner Product • Space Vector Angles

Real Vectors • Space Norm • Plane Vector Angles

N-Dimensional Space • Space Vectors • Plane Vector Sums and Differences • Plane Norm • Plane Inner Product

Distance • Plane Vector • Absolute Value • Right Triangle Sides Relation

Space Distance • Real Squares

Plane Distance • Real Arithmetic

Interval Length • Real Multiplicative Inverse

Intervals • Real Products

Real Space • Square Numbers • Real Additive Inverses

Real Plane • Real Line • Squares • Real Order • Rational Multiplicative Inverse • Real Sums • Rational Arithmetic

Integral Line • Geometry • Rational Order • Real Numbers • Rational Products • Rational Sums

Lists • Rational Numbers

Direct Products • Set Numbers • Integer Arithmetic • Natural Fractions

Finite Sets • Natural Additive Identity • Natural Multiplicative Identity • Integer Products

Inverse Elements • Equivalent Sets • Identity Elements • Natural Arithmetic

Family Unions and Intersections • Function Inverses • Element Functions • Comparisons • Integer Order • Integer Sums • Natural Powers

Families • Function Images • Function Composite • Operations • Orders • Integer Numbers • Natural Products

Functions • Equivalence Relations • Converse Relations • Natural Order • Natural Sums

Relations • Recursion Theorem

Ordered Pair Projections • Peano Axioms

Set Products • Natural Induction

Generalized Set Dualities • Set Powers • Natural Numbers

Set Unions and Intersections • Set Partitions • Set Dualities • Intersection of Empty Set • Unordered Triples • Successor Sets • Set Symmetric Differences

Ordered Pairs • Set Intersections • Pair Unions • Set Complements

Pair Intersections • Set Unions • Set Differences

Unordered Pairs • Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets • Identities

Names

Letters • Objects