



REAL INNER PRODUCT

Why

We want to measure angles in space.¹

Definition

The *real inner product* (or *dot product*, *scalar product*) of two real vectors $x, y \in \mathbf{R}^n$ is

$$x_1y_1 + x_2y_2 + \cdots + x_ny_n$$

We denote the inner product of x and y by $\langle x, y \rangle$.

Properties

The inner product has several important properties

1. $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$
2. $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$
3. $\langle x, y \rangle = \langle y, x \rangle$
4. $\langle x, x \rangle \geq 0$
5. $\langle x, x \rangle = 0 \iff x = 0$

Connection to norm

It is important to note that $\|x\| = \sqrt{\langle x, x \rangle}$.

¹Future editions will expand, and perhaps give the development for \mathbf{R}^2 first. Future editions will include pictures.

