Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?

Definition

Two random variables are *independent* if the sigma algebras generated by the random variables are independent. In general, a family of random variables are *independent* if the sigma algebras generated by the random variables are independent.

Notation

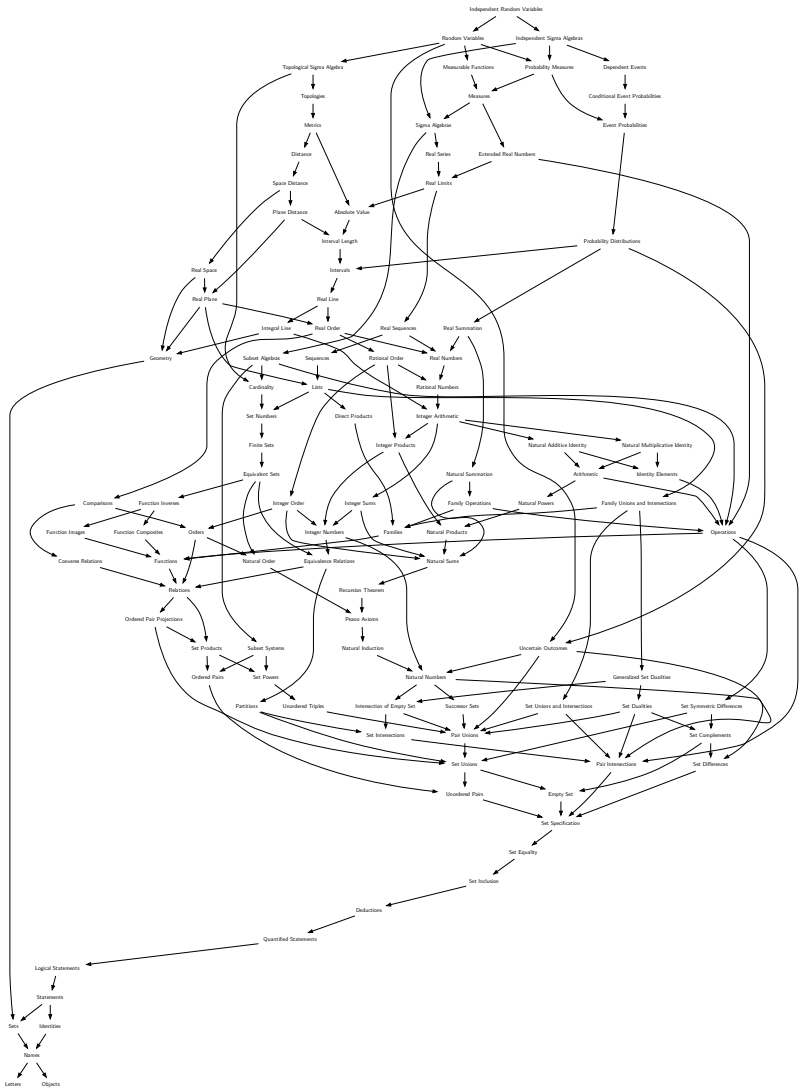
Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f_1, f_2 : X \to Y$ be random variables. If the random variables are independent we write $f_1 \perp f_2$.

Results

Proposition 1. *Let $f_1, \ldots, f_n$ be independent real-valued random variables defined on a probability space $(X, \mathcal{A}, \mu)$. Let $B_1, \ldots, B_n$ be Borel sets of real numbers and let $A_i = f_i^{-1}(B_i)$. Let $A = \cap_{i=1}^n f_i^{-1}(B_i)$. Then*

$$\mu(A) = \prod_{i=1}^{n} \mu(A_i)$$

Proof. Since f_i are independent, so are the sigma algebras they generate. A_i are in each of these sigma algebras, so by definition of independence the measure of the intersection is the product of the measures. $\square$


Independent Random Variables
Random Variables
Independent Sigma Algebras
Topological Sigma Algebra
Measurable Functions
Probability Measures
Dependent Events
Topologies
Measures
Conditional Event Probabilities
Metrics
Sigma Algebras
Event Probabilities
Distance
Real Series
Extended Real Numbers
Space Distance
Real Limits
Plane Distance
Absolute Value
Interval Length
Probability Distributions
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Real Sequences
Real Summation
Geometry
Subset Algebras
Sequences
Rational Order
Real Numbers
Cardinality
Unit
Rational Numbers
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Equivalent Sets
Natural Summation
Comparisons
Function Inverses
Integer Order
Integer Sums
Family Operations
Natural Powers
Family Unions and Intersections
Function Image
Function Composites
Orders
Integer Numbers
Families
Natural Products
Operations
Converse Relations
Functions
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Subset Systems
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Power
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Unions and Intersections
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Set Unions
Pair Intersections
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects