

#### COMPARISONS

# Why

We want language and notation involving order.<sup>1</sup>

# Comparisons

A *comparison* is a statement (see Statements) involving a partial (which may or may not be total) order.

#### Notation

Let A be a set. We tend to denote an arbitrary partial order on A by  $\leq$ . So  $(A, \leq)$  is a partially ordered set.

As usual (see Relations), we write  $a \leq b$  to mean  $(a, b) \in A$ . Alternatively, we write  $b \succeq a$  to mean  $a \leq b$ . In other words,  $\succeq$  is the inverse relation (see Converse Relations) of  $\preceq$ .

#### Predecessors and successors

If  $a \leq b$  and  $a \neq b$ , we write  $a \prec b$  and say that a precedes b. In this case we call a the predecessor of b. Alternatively, under the same conditions, we write  $b \succ a$  and we say that b succeeds a. In this case we call b the successor of a.

### Induced partial orders

Of course, the object we have defined and denoted by  $\prec$  is a relation on A. It satisfies (i) for no elements x and y do  $x \prec y$  and  $y \prec x$  hold simultaneously and (ii) if  $x \prec y$  and  $y \prec z$ , then  $x \prec z$  (i.e.,  $\prec$  is transitive). It is worthwhile to observe that if S is a relation satisfying (i) and (ii), then the relation R defined to mean  $(a,b) \in S$  or a=b is a partial order on A.

<sup>&</sup>lt;sup>1</sup>In the present edition, this sheet can be thought of as an extended notation section for Orders.

## Strict and weak relations

This connection between  $\leq$  and  $\prec$  can be generalized. The *strict relation* corresponding to a relation R on a set A is the relation S on A defined by  $(a,b) \in S$  if  $(a,b) \in R$  and  $a \neq b$ . The *weak relation* corresponding to a relation S' on a set A is the relation R' defined by  $(a,b) \in R'$  if  $(a,b) \in S'$  or a = b. For this reason, a relation is said to *partially order* a set if it is a partial order or if its corresponding weak relation is one.

