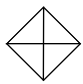


Why

We can use families to think about unions and intersections.

Family unions

Let $A : I \to \mathcal{P}(X)$ be a family of subsets. We refer to the union (see Set Unions) of the range (see Relations) of the family the *family union.* We denote it $\cup_{i \in I} A_i$.

Proposition 1. $(x \in \cup_{i \in I} A_i) \longleftrightarrow (\exists i)(x \in A_i)$

If $I = \{a, b\}$ is a pair with $a \neq b$, then $\cup_{i \in I} = A_a \cup A_b$.

There is no loss of generality in considering family unions. Every set of sets is a family: consider the identity function from the set of sets to itself.

We can also show generalized associative and commutative law[1] for unions.

Proposition 2. *Let $\{I_j\}$ be a family of sets and define $K = \cup_j I_j$. Then $\cup_{k \in K} A_k = \cup_{j \in J} (\cup_{i \in I_j} A_i)$.*[2]

Family intersection

If we have a nonempty family of subsets $A : I \to \mathcal{P}(X)$, we call the intersection (see Set Intersections) of the range of the family the *family intersection.* We denote it $\cap_{i \in I} A_i$.

Proposition 3. $x \in \cap_{i \in I} A_i \longleftrightarrow (\forall i)(x \in A_i)$

Similarly we can derive associative and commutative laws for intersection[3] . They can be derived as for unions, or from the facts of unions using generalized DeMorgan's laws (see Generalized Set Dualities).

[1] The commutative law will appear in future editions.
[2] An account will appear in future editions.
[3] Statements of these will be given in future editions.

Connections

The following are easy[4]

Let $\{A_i\}$ be a family of subsets of X and let $B \subset X$.

Proposition 4. $B \cap \bigcup_i A_i = \bigcup_i (B \cap A_i)$

Proposition 5. $B \cup \bigcap_i A_i = \bigcap_i (B \cup A_i)$

Let $\{A_i\}$ and $\{B_j\}$ be families of sets.[5]

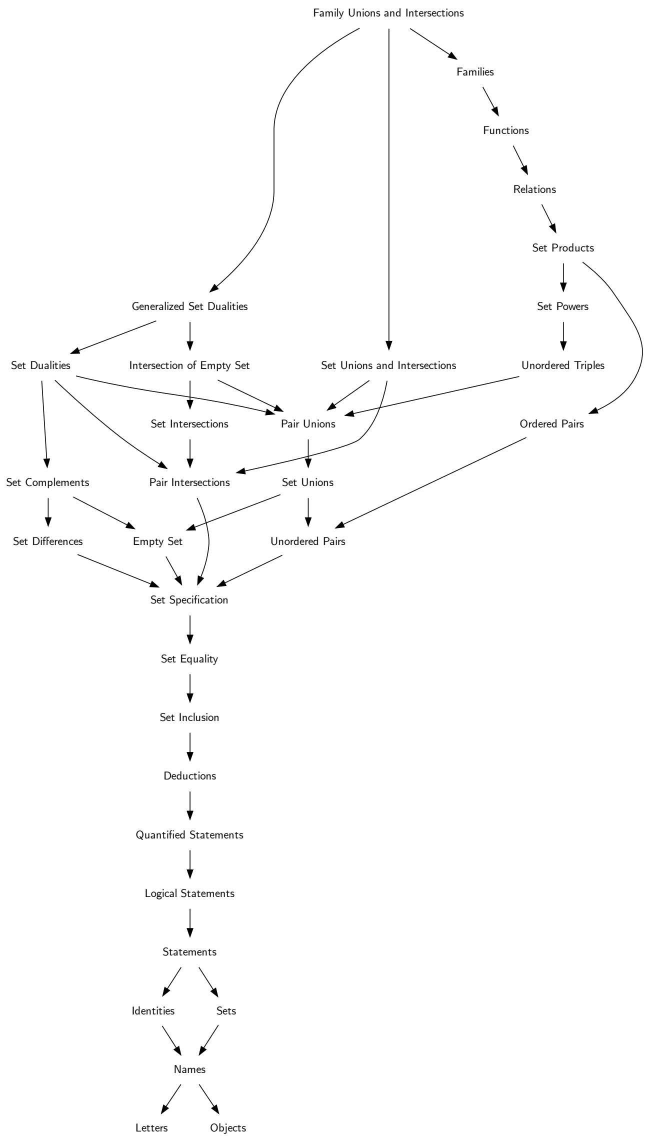
Proposition 6. $(\bigcup_i A_i) \cap (\bigcup_j B_j) = \bigcup_{i,j} (A_i \cap B_j)$

Proposition 7. $(\bigcap_i A_u) \cup (\bigcap_j B_j) = \bigcap_{i,j} (A_i \cup B_j)$.

Proposition 8. $\cap_i X_i \subset X_j \subset \cup_i X_i$ *for each* j.

[4] Accounts will appear in future editions.

[5] An account of the notation used and the proofs will appear in future editions.


Family Unions and Intersections
Families
Functions
Relations
Set Products
Generalized Set Dualities
Set Powers
Set Dualities
Intersection of Empty Set
Set Unions and Intersections
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects