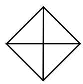


Why

We want to visualize symmetric relations.

Definition

An *undirected graph* is a pair (V, E) in which V is a finite nonempty set and E is a subset of unordered pairs of elements in V. We call the elements of V the *vertices* of the graph and the elements of E the *edges*. We call (V, E) an undirected graph *on* V.

Two vertices are *adjacent* if their pair is in the edge set. We say that the corresponding edge is *incident* to those vertices. The *adjacency set* of a vertex is the set of vertices adjacent to it. The *degree* of a vertex is the number of vertices adjacent to it; in other words, the size of its adjacency set. A graph is *complete* if each pair of two distinct vertices is adjacent.

The *complement* of (V, E) is the graph (V, F) where F is the complement of E in the set of pairs from V.

Other Terminology

Some authors call the adjacency set the *neighborhood* of the vertex. They call the union of the adjacency set of the vertex $v \in V$ with the singleton $\{v\}$ the *closed neighborhood* of v.

Notation

Let V be a nonempty set. Let $E \subset \{\{v, w\} \mid v, w \in V\}$. Then the pair (V, E) is an undirected graph. We regularly say "Let $G = (V, E)$" be a graph, in which the relevant properties of V and E are implicit.

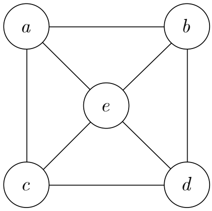


Figure 1: Undirected graph.

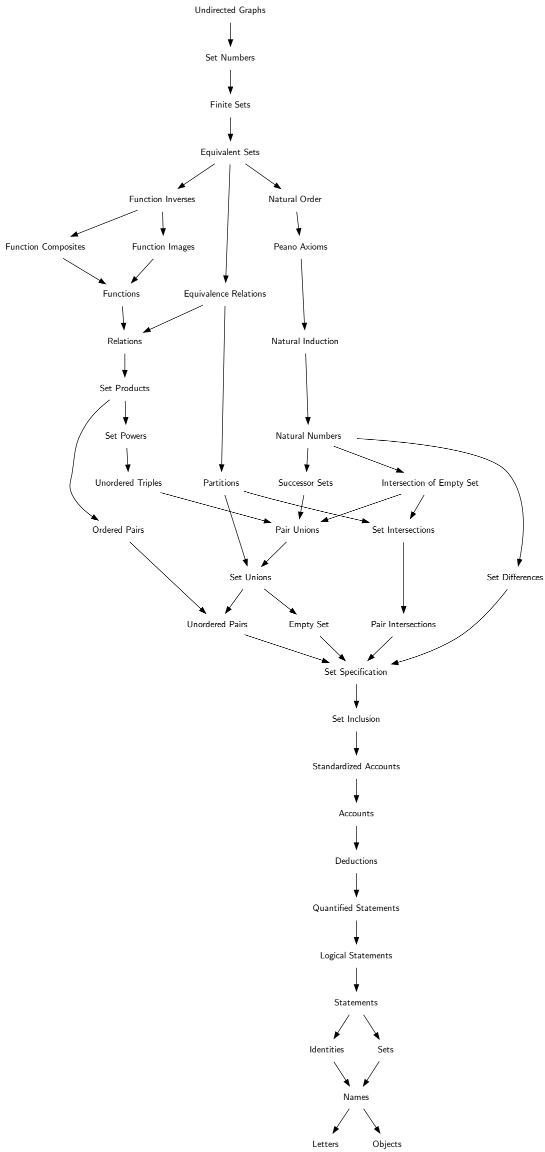
The notation $\{v, w\} \in E$ for an edge between vertices $v, w \in V$ reminds us that the edges are unordered pairs of distinct vertices. We denote the adjacency set of v by $\text{adj}(v)$ and the degree of v by $\deg(v)$.

Example

For example, let a, b, c, d, e be objects and consider an undirected graph (V, E) defind by $V = \{a, b, c, d, e\}$ and

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\}.$$

In visualizations of undirected graphs, the vertices are frequently represented as circles or rectangles in the plane and edges are shown as lines connecting the vertices.


Undirected Graphs
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Relations
Natural Induction
Set Products
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects