



Why

Every matrix $A \in \mathbf{R}^{m \times n}$ maps the unit ball in \mathbf{R}^n to an ellipsoid in \mathbf{R}^m .

Definition

A *rotate scale rotate decomposition* (or *rotate scale rotate factorization*) of a matrix $A \in \mathbf{R}^{m \times n}$ is an ordered triple (U, S, V) where U and V are orthogonal and S is diagonal decreasing ($S_{11} \geq S_{22} \geq \dots \geq S_{pp}$, where $p = \min\{m, n\}$) satisfying

$$A = USV^\top.$$

Other (universal) terminology includes the *singular value decomposition* or *SVD* of A . We call diagonal elements of S the *singular values* of A . We call the column vectors of U the *left singular vectors* or *output singular vectors*. We call the column vectors of V the *right singular vectors* or *input singular vectors*. We refer to them collectively as the *singular vectors*.

$$Av_i = \sigma_i u_i.$$

