Why

[1]

Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\operatorname{tr} : \mathbf{R}^{n\times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n\times n}$. Then

$$\operatorname{tr} A = \sum_{i=1}^{n} A_{ii}.$$

Properties

Prop. 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n\times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\begin{aligned}\operatorname{tr} C = \sum_{i=1}^{n} C_{ii} &= \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii} \\ &= \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii} \\ &= \alpha \operatorname{tr} A + \beta \operatorname{tr} B.\end{aligned}$$

[1] Future editions will include, in the genetic tradition.

$\square$

Prop. 2. *Let* $A, B \in \mathsf{R}^{n \times n}$.

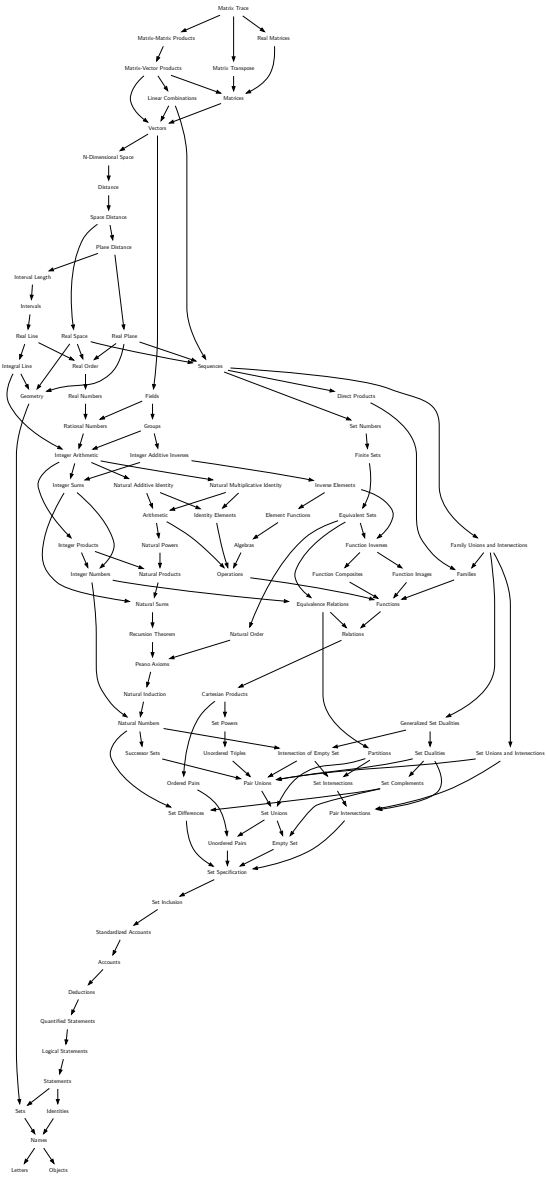
$$\operatorname{tr}(AB) = \operatorname{tr}(BA)$$

[2]

In other words, "matrices commute under the trace operator."

Prop. 3. *Let* $A \in \mathsf{R}^{n \times n}$. *Then* $\operatorname{tr} A = \operatorname{tr} A^{\top}$.

[2] Future editions will include an account.


Matrix Trace
Matrix-Matrix Products
Real Matrices
Matrix-Vector Products
Matrix Transpose
Linear Combinations
Matrices
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Space
Real Plane
Integral Line
Real Order
Sequences
Geometry
Real Numbers
Fields
Direct Products
Rational Numbers
Groups
Set Numbers
Integer Arithmetic
Integer Additive Inverses
Finite Sets
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Inverse Elements
Arithmetic
Identity Elements
Element Functions
Equivalence Sets
Integer Products
Natural Powers
Algebras
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Operations
Function Compositions
Function Images
Families
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Natural Order
Relations
Peano Axioms
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Doubles
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Doubles
Set Unions and Intersections
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects