



Why

We want to speak of the pairwise conditional distributions of a particular joint distribution.¹

Definition

Suppose A_1, \dots, A_n is a list of finite sets and $p : \prod_{i=1}^n A_i \rightarrow [0, 1]$ is a distribution on the (finite) product $\prod_{i=1}^n A_i$.

For $i \neq j \in \{1, \dots, n\}$, the *conditional distribution* of i on j is the function $p_{i|j} : A_i \times A_j \rightarrow \mathbf{R}$ defined so that $p_{i|j}(\cdot, b)$ is the conditional distribution induced by conditioning on $\{a \in \prod_{i=1}^n A_i \mid a_j = b\}$.

For $i, j = 1, \dots, n$ and $i \neq j$, p_i , p_{ij} and $p_{i|j}$ satisfy

$$p_{i|j}(b, c)p_j(c) = p_{ij}(b, c) \quad \text{for all } b \in A_i, c \in A_j$$

¹Future editions will rework this sheet.

