Signed Measure Examples

1 Why

Let us consider examples of signed measures. TODO

2 Examples

Consider an integrable function defined on some measurable space. The extended-real-valued funciton which assigns to each distinguished set the value of the integrating the function over that set is a signed measure. See Example 1.

Consider

Example 1. *Let $(X, \mathcal{A}, \mu)$ a measure space. Let R denote the set of real numbers. Let $f : X \to R$ an integrable function. Define $\nu : \mathcal{A} \to R$ by*

$$\nu(A) = \int_A f d\mu.$$

Then ν is a signed measure.

Proof. First,

$$\nu(\varnothing) = \int f \chi_\varnothing d\mu = \int 0 d\mu = 0.$$

Next, let $(A_n)_n$ disjoint. Notice that,

$$\chi_{\cup_{i=1}^n A_k} = \sum_{i=k}^n \chi_{A_k}$$

so for all n, □