



INNER PRODUCTS

Why

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Definition

Let (X, \mathbf{R}) be a vector space. A function $f : X \times X \rightarrow \mathbf{R}$ is an *inner product* on the vector space (X, \mathbf{R}) if

1. $f(x, x) \geq 0, = 0 \iff x = 0$,
2. $f(x + y, z) = f(x, z) + f(y, z)$,
3. $f(x, y) = f(y, x)$, and
4. $f(\alpha x, y) = \alpha f(x, y)$.

An *inner product space* is an ordered pair: a real vector space and an inner product.²

Examples

\mathbf{R}^n with the usual inner product is an inner product space. Some authors call any finite-dimensional inner product space over the real numbers a *Euclidean vector space*.

Examples

If $f : X \times X \rightarrow \mathbf{R}$ is an inner product we regularly denote $f(x, x)$ by $\langle x, x \rangle$.

¹Future editions will complete and rework this sheet.

²Future editions will discuss complex inner products.

Orthogonality

Two vectors in an inner product space are *orthogonal* if their inner product is zero. An *orthogonal family of vectors* in an inner product space is a family of vectors for which distinct family members are orthogonal.

A vector is *normalized* if its inner product with itself is one.

