Closest Point Property

Suppose H is a Hilbert space $A \subset H$ closed and convex. then if $x \in H$, there exists a unique $z \in A$ closest to x. There exists a unique $z \in A$ closest to x such that

$$d(z, x) = \inf_{y \in A} d(y, x).$$

Proof. Take any sequence $(y_n)_n$ such that

$$d(y_n, x) \to d = \inf_{y \in A} d(y, x)$$

$\square$

Closest Point Property

Complete Metric Spaces

Egorov Sequences

Metrics

Real Egorov Sequences

Distance

Real Limits

Space Distance

Absolute Value

Plane Distance

Interval Length

Intervals

Real Line

Real Space

Real Plane

Real Sequences

Integral Line

Real Order

Real Numbers

Geometry

Rational Numbers

Sequences

Integer Arithmetic

Set Numbers

Direct Products

Natural Additive Identity

Natural Multiplicative Identity

Finite Sets

Arithmetic

Identity Elements

Integer Products

Natural Powers

Operations

Equivalent Sets

Function Inverses

Family Unions and Intersections

Integer Sums

Natural Products

Function Images

Function Composites

Families

Integer Numbers

Natural Sums

Recursion Theorem

Natural Order

Equivalence Relations

Functions

Peano Axioms

Relations

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Dualities

Set Unions and Intersections

Set Intersections

Pair Unions

Ordered Pairs

Set Complements

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects