Why

We want to talk about an algorithm which can learn from a hypothesis class, regardless of the underlying distribution.

Definition

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable input and output spaces.

A hypothesis class $\mathcal{H}$ of measurable functions from X to Y is *probably approximately correct learnable* (or *PAC learnable*) if (a) there exists an inductor $\mathcal{A} : (X \times Y)^n \to \mathcal{H}$, so that (b) for every underlying measure μ and labeling function $f : X \to Y$ (c) for every $\varepsilon, \delta \in (0, 1)$ (d) there exists $m_0 \in \mathbf{N}$ so that (e) for all $m \geq m_0$

$$\mu^m \left[\mu \left[\mathcal{A}((x_i, f(x_i))_{i=1}^n)(\xi) \neq f(\xi) \right] \leq \varepsilon \right] \geq 1 - \delta \tag{1}$$

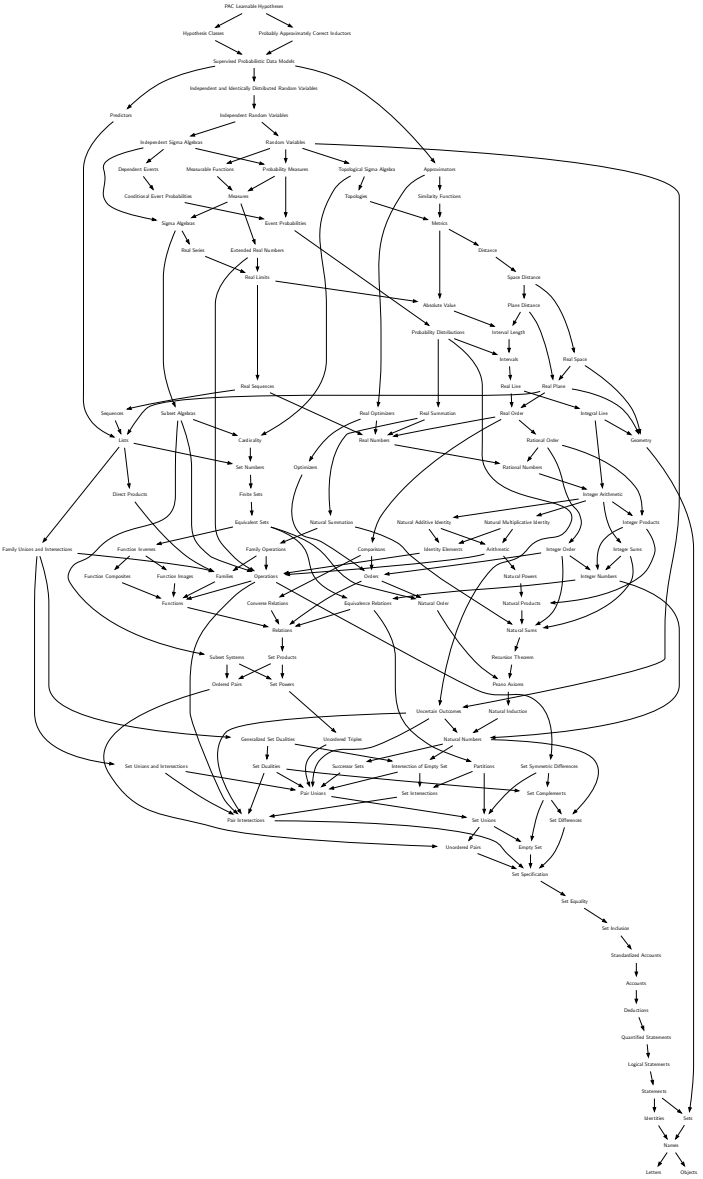
where $x \in X^m$ and $\xi \in X$. In this case we say that the inductor (or learning algorithm) $\mathcal{A}$ *PAC learns* $\mathcal{H}$.

We interpret this as follows: "no matter the underlying distribution and correct labeling function, if someone specifies an accuracy and confidence we can tell them the number of samples they need so that the inductor outputs a hypothesis which is probably approximately correct."

Some authors require that the hypothesis class be realizable with respect to the underlying distribution and correct labeling function. This is natural, because if the hypothesis class includes the correct labeling function f, then it is realizable. In this case they refer to the above definition as the *agnostic PAC model*. We emphasize here that there this definition includes the notion of realizability. In other words. We emphasize again that this definition contains to "approximation parameters." The accuracy parameter ε corresponds to the "approximately correct" piece and the confidence parameters δ corresponds to the "probably" piece.

Sample complexity

Note that the existence of an m_0 above for each ε and δ is equivalent to requiring that there exists $m_0 : (0,1)^2 \to \mathbf{N}$ so that for all $m \geq m_0(\varepsilon, \delta)$ the condition in Equation 1 holds. There may exist multiple such m_0, so we define $\tilde{m} : (0,1)^2 \to \mathbf{N}$ so that $\tilde{m}(\varepsilon, \delta)$ is the smallest integer so that Equation (1) holds. We call $\tilde{m}$ the *sample complexity*. Clearly it is a function of ε and δ. It also depends on the hypothesis class $\mathcal{H}$ and the learning algorithm $\mathcal{A}$.


PAC Learnable Hypotheses
Hypothesis Classes
Probably Approximately Correct Induction
Supervised Probabilistic Data Models
Independent and Identically Distributed Random Variables
Predictors
Independent Random Variables
Independent Sigma Algebras
Random Variables
Dependent Events
Measurable Functions
Probability Measures
Topological Sigma Algebras
Approximators
Conditional Event Probabilities
Measures
Topologies
Similarity Functions
Sigma Algebras
Event Probabilities
Metrics
Real Series
Extended Real Numbers
Distances
Real Limits
Space Distances
Absolute Value
Plane Distances
Probability Distributions
Interval Length
Intervals
Real Space
Real Sequences
Real Line
Real Plane
Sequences
Subset Algebras
Real Optimizers
Real Summation
Real Order
Integral Line
Lists
Cardinality
Real Numbers
Rational Order
Geometry
Set Numbers
Optimizers
Rational Numbers
Direct Products
Finite Sets
Integer Arithmetic
Equivalent Sets
Natural Summation
Natural Additive Identity
Natural Multiplication Identity
Integer Products
Family Unions and Intersection
Function Inverses
Family Operations
Comparisons
Identity Elements
Arithmetic
Integer Order
Integer Sums
Function Composites
Function Images
Families
Operations
Orders
Natural Powers
Integer Numbers
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Products
Natural Sums
Relations
Subset Symmetry
Set Products
Recursion Theorem
Ordered Pairs
Set Powers
Peano Axioms
Uncertain Outcomes
Natural Induction
Generalized Set Dualities
Unordered Triples
Natural Numbers
Set Dualities
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Difference
Set Unions and Intersections
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Nouns
Lemmas
Objects