

WEIGHTED LEAST SQUARES LINEAR REGRESSORS

Why

What is the best linear regressor if we choose according to a weighted squared loss function.

Definition

Suppose we have a paired dataset of n records with inputs in \mathbf{R}^d and outputs in \mathbf{R} . A weighted least squares linear predictor for nonnegative weights $w \in \mathbf{R}^n$, $w \geq 0$, is a linear transformation $f: \mathbf{R}^d \to \mathbf{R}$ (the field is \mathbf{R}) which minimizes

$$\frac{1}{n} \sum_{i=1}^{n} w_i (y_i - x^{\top} a^i)^2.$$

Some authors refer to this process of selecting a linear predictor as the weighted least-squares problem.

Define $W \in \mathbf{R}^{n \times n}$ so that $W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix. We want to find x to minimize

$$||W(Ax-y)||$$

Solution

Proposition 1. There exists a unique weighted least squares linear predictor and its parameters are given by

$$(A^{\top}WA^{\top})^{-1}A^{\top}Wy.$$

