Why

Since we have a partial order on the set of positive semidefinite matrices, we can study which familiar functions are have order-preserving or order-reversing properties.

Norms

It would be nice if the matrix norm induced by the matrix scalar produce (see Matrix Scalar Product) was an isotonic function. In other words, if $A, B \in \mathsf{S}^d$ satisfy $A \geq B$, does $\|A\| \geq \|B\|$?

Since $\|A\|^2 = \operatorname{tr} A^2$, we should study the trace first..

Trace

Proposition 1. *Let* $f : \mathsf{S}^d \to \mathsf{R}$ *defined by* $f(A) = \operatorname{tr} A$.

In other words, the function f is the restriction of the trace function onto the set of symmetric matrices.

Proposition 2. *Let* $B \in \mathsf{S}^d$ *Let* $f_B : \mathsf{S}^d \to \mathsf{R}$ *defined by* $f(A) = \operatorname{tr} AB$.

Inversion

Proposition 3. *Let* $A \in \mathsf{S}^d_{++}$. *Then the map* $f : \mathsf{S}^d_{++} \to \mathsf{S}^d_{++}$ *satisfying* $f(A) = A^{-1}$ *is an isotonic function mapping the (open) positive definite cone into itself.*[1]

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[1]Future editions will include a proof.