

### EMPTY SET

## Why

Can a set have no elements?

### Definition

Sure. A set exists by the principle of existence (see Sets); denote it by A. Specify elements (see Set Specification) of any set that exists using the universally false statement  $x \neq x$ . We denote that set by  $\{x \in A \mid x \neq x\}$ . It has no elements. In other words,  $(\forall x)(x \notin A)$ . The principle of extension (see Set Equality) says that the set obtained is unique (contradiction). We call the unique set with no elements the *empty set*. If a set is not the empty set, we call it *nonempty*.

### **Notation**

We denote the empty set by  $\varnothing$ . In other words, in all future accounts (see Accounts), there are two implicit lines. First, "name  $\varnothing$ " and second "have  $(\forall x)(x \notin \varnothing)$ ".

# **Properties**

It is immediate from our definition of the empty set and of the definition of inclusion (see Set Inclusion) that the empty set is included in every set (including itself).

**Proposition 1.**  $(\forall A)(\varnothing \subset A)$ 

*Proof.* Suppose toward contradiction that  $\varnothing \not\subset A$ . Then there exists  $y \in \varnothing$  such that  $y \notin A$ . But this is impossible, since  $(\forall x)(x \notin \varnothing)$ .

<sup>&</sup>lt;sup>1</sup>This account will be expanded in the next edition.

