

MULTIVARIATE NORMALS

Why

We generalize the normal density to d-dimensional space.

Definition

Let $f: \mathbf{R}^d \to \mathbf{R}$ be a density such that

$$f(x) = \frac{1}{\sqrt{(2\pi)^d \det \Sigma}} \exp\left(-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\right)$$

where $\mu \in \mathbf{R}^d$, $\Sigma \in \mathbf{S}^d$, and $\Sigma \succ 0$. We call f a multivariate normal density. A multivariate normal density with d=1 is a normal density, so we refer to multivariate normal densities as normal densities without ambiguity. We frequently use the word normal as a substantive, and refer to normals when we mean multivariate normal densities. Many people call a multivariate normal distribution a multivariate gaussian distribution and speak of qaussians instead of normals.¹

We call μ the mean and Σ the covariance matrix. We call Σ^{-1} the precision matrix.

Maximum

The maximum of a normal density is its mean, $\mu \in \mathbf{R}^d$.

 $^{^1\}mathrm{We}$ avoid this usage in accordance with the project's policy on historical names.

