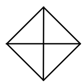


Why

We generalize the product of two sets to a product of a family of sets. To do so we discuss sets of families.

Discussion for pairs

Suppose X and Y are nonempty sets. There is a natural correspondence between the product $X \times Y$ (see Set Products) and the set of families

$$Z = \{z : \{i, j\} \to (A \cup B) \mid z_i \in A \text{ and } z_j \in B\}$$

where $\{i, j\}$ is any unordered pair with $i \neq j$.

The set Z can be put in one-to-one correspondence with $X \times Y$. The family $z \in Z$ corresponds with the pair (z_i, z_j). The pair (a, b) corresponds to the family $z \in Z$ defined by $z(i) = a$ and $z(j) = b$. So, ordered pairs can be put in one-to-one correspondence with families. The generalization of Cartesian products to more than two sets generalizes the notion for families.

Definition

Suppose $\{A_i\}_{i \in I}$ is a family of sets. The *direct product* (or *Cartesian product*, *family Cartesian product*) of A is the *set* of *all* functions (i.e., families) $a : I \to X$ which satisfy $a_i \in A_i$ for every $i \in I$.

A function on a product is called a *function of several variables* and, in particular, a function on the product $X \times Y$ is called a *function of two variables.*

Notation

We denote the product of the family $\{A_i\}_{i \in I}$ by

$$\textstyle\prod_{i \in I} A_i$$

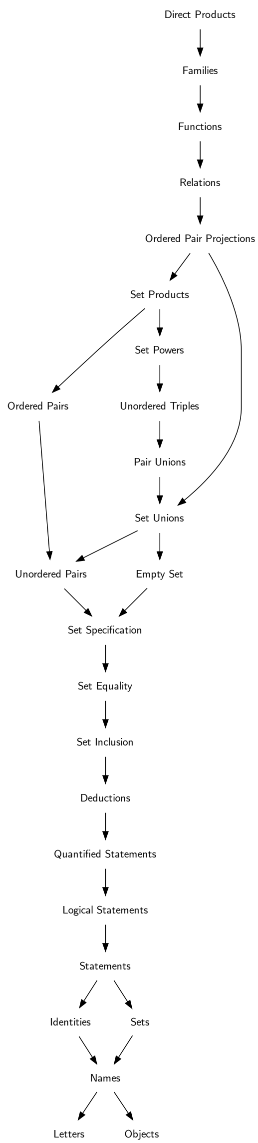
We read this notation as "product over i in I of A sub-i." Other notation in use includes $\times_{i \in I} A_i$.

Projections

The word "projection" is used in two senses with families. Let I be a set, and let $\{A_i\}_{i\in I}$ be a family of sets. Define $A = \prod_{i\in I} A_i$.

First, let $J \subset I$. There is a natural correspondence between the elements of A and those of $\prod_{j\in J} A_j$. To each element $a \in A$, we restrict a to J and this is restriction is an element of $\prod_{j\in J} A_j$. The correspondence is called the *projection* of A onto $\prod_{i\in J} A_i$. The projection in this sense is a set of families.

Second, consider the value of a family $a \in A$ at j. We call a_j the *projection of a onto index j* or the *j-coordinate* of a. This word *coordinate* is meant to follow the language used in defining ordered pairs. The projection in this sense is an element of A_j. The jth projection is a function mapping $\prod_{i\in I} X_i$ to X_j.


Direct Products
Families
Functions
Relations
Ordered Pair Projections
Set Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects