Why

The identification of $\mathbf{C}$ with a plane leads $\mathbf{C}$ to naturally inherit $\mathbf{R}^2$'s notion of distance.

Definition

The *absolute value* or *modulus* of $z = (x, y) \in \mathbf{C}$ is the distance of z to the origin. If $z \in \mathbf{C}$, then the modulus of z is

$$\sqrt{x^2 + y^2}.$$

In other words, the modulus of z is the distance (in $\mathbf{R}^2$ of $z = (x, y)$ from the origin $(0, 0)$.

Notation

We denote the modulus of z by $|z|$.

Properties

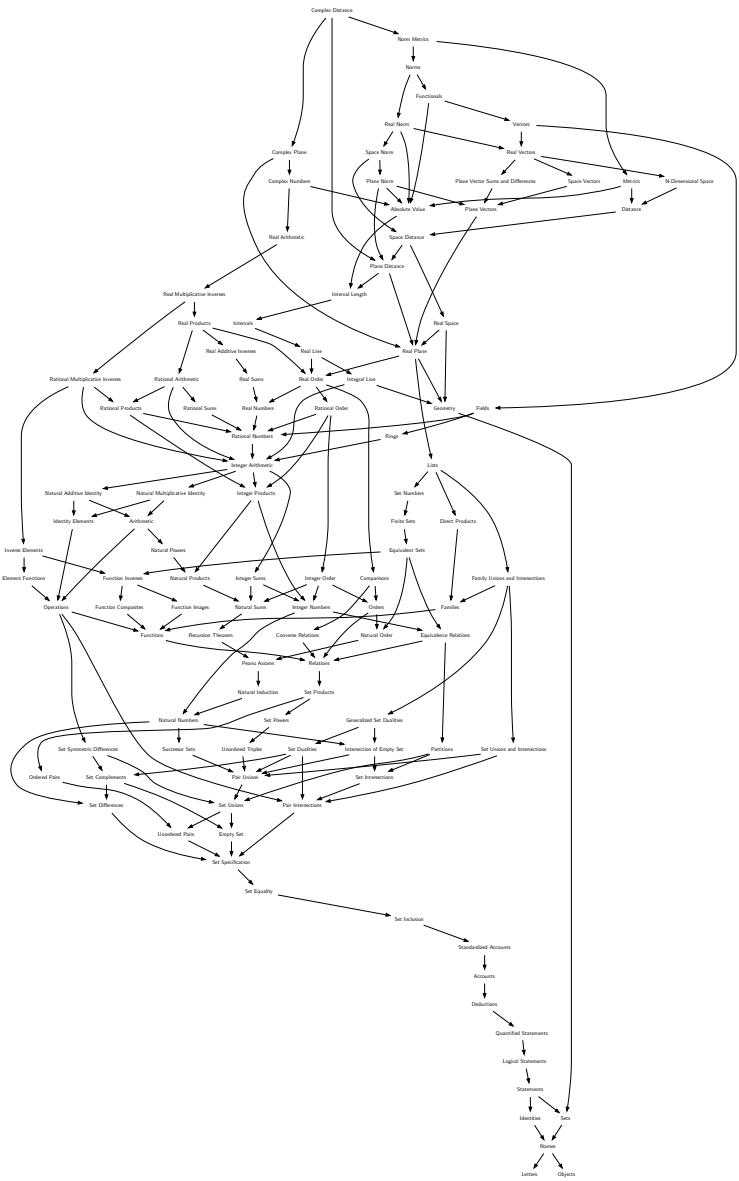
Proposition 1 (Triangle Inequality)**.** *For all $z, w \in \mathbf{C}$,*

$$|z + w| \leq |z| + |w|.$$

Also, for all $z \in \mathbf{C}$, we have $|\mathrm{Re}(z)| \leq |z|$ and $|\mathrm{Im}(z)| \leq |z|$, and for all $z, w \in \mathbf{C}$,[1]

$$||z| - |w|| \leq |z - w|.$$

[1] This follows from the triangle inequality. Future editions will include an account.


Complex Distance
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Complex Plane
Space Norm
Real Vectors
Complex Numbers
Plane Norm
Plane Vector Sums and Differences
Space Vectors
Metrics
N Dimensional Space
Absolute Value
Plane Vectors
Distance
Real Arithmetic
Space Distance
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Rational Products
Rational Sums
Real Numbers
Rational Order
Geometry
Fields
Rational Numbers
Rings
Integer Arithmetic
Lists
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Set Numbers
Identity Elements
Arithmetic
Finite Sets
Direct Products
Natural Powers
Equivalent Sets
Inverse Elements
Element Functions
Function Inverses
Natural Products
Integer Sums
Integer Order
Comparisons
Family Unions and Intersections
Operations
Function Compositions
Function Images
Natural Sums
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Recursion Theorem
Converse Relations
Natural Order
Equivalence Relations
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Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deduction
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects