Sigma Algebras

1 Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object (TODO).

2 Definition

A *countably summable subset algebra* is a subset space for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of $A_1, \dots, A_n$ coincides with the union of $A_1, \dots, A_n, A_n, A_n \dots$.

We say that the set of distinguished sets a *sigma algebra* on the base set; we justify this langauge, as for an algebra, by the closure properties under standard set operations.

2.1 Notation

The notation follows that of a subset space. Let $(A, \mathcal{A})$ be a countably summable subset algebra. We also say “let $\mathcal{A}$ be an sigma algebra on A." Moreover, since the largest element of the sigma algebra is the base set, we can say without ambiguity: “let $\mathcal{A}$ be a sigma algebra."

3 Examples

Example 1. *For any set A, 2^A is a sigma algebra.*

Example 2. *For any set A, $\{A, \varnothing\}$ is a sigma algebra.*

Example 3. *Let A be an infinite set. Let $\mathcal{A}$ the collection of finite subsets of A. $\mathcal{A}$ is not a sigma algebra.*

Example 4. *Let A be an infinite set. Let $\mathcal{A}$ be the collection subsets of A such that the set or its complement is finite. $\mathcal{A}$ is not a sigma algebra.*

Proposition 5. *The intersection of a family of sigma algebras is a sigma algebra.*

Example 6. *For any infinite set A, let $\mathcal{A}$ be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

$\mathcal{A}$ is an algebra; the countable/co-countable algebra.

$\boxed{TOOD : cleanupexamples}$