



Definition

A subset $M \subset \mathbf{R}^n$ is a *smooth manifold* of dimension d if for every $x \in M$, there exists a neighborhood V of x in X that is diffeomorphic to an open subset U of \mathbf{R}^d . In this case we say that the set is *locally diffeomorphic* to \mathbf{R}^d .

A diffeomorphism $\phi : U \rightarrow V$ is called a *parameterization* of the neighborhood of V .

Its inverse diffeomorphism ϕ^{-1} is called a *coordinate system* (or system of *coordinates*) on V .

Notation

We denote the dimension of a manifold M by $\dim M$.

Submanifolds

If X and Z are both manifolds in \mathbf{R}^n and $Z \subset X$, then we call Z a *submanifold* of X . In particular, X is a submanifold of \mathbf{R}^n . Any open set of a manifold X is a submanifold X .¹

¹Future editions will expand.

