



## Singular Measures

### 1 Why

TODO

### 2 Definition

A measure is *concentrated* on a set if the measure on the complement of the set is zero. A signed or complex measure is *concentrated* on a set if its variation is concentrated on the set.

Two measures (or signed or complex measures) are *mutually singular* if there exists a set on which one is concentrated and on whose complement the other is concentrated.

#### 2.1 Notation

Let  $(X, \mathcal{A})$  be a measurable space and let  $\mu$  be a measure. Then  $\mu$  is concentrated on a set  $C \in \mathcal{A}$  if  $\mu(X - C) = 0$ . If  $\nu$  is a signed or complex measure, then  $\nu$  is concentrated on  $C \in \mathcal{A}$  if  $|\nu|$  is concentrated on  $C$ ; in which case  $|\nu|(X - C) = 0$ .

Let  $\mu$  and  $\nu$  be measures on  $(X, \mathcal{A})$ . Then  $\mu$  and  $\nu$  are mutually singular if there exists a set  $A \in \mathcal{A}$  so that  $\mu$  is concentrated on  $A$  and  $\nu$  is concentrated on  $X - A$ . We denote that two measures are singular by  $\mu \perp \nu$ , read aloud as "mu perp nu".