

AFFINE SETS

Why

Definition

The *line through* two points in *n*-dimensional space is the set of points which can be expressed as the sum of the first point and a scaled multiple of the difference between the second point and the first. An *affine set* is a subset of *n*-dimensional space which contains the lines through each of its points.

Examples

The empty set is trivially an affine set. The entire set of points in n-dimensional space is an affine set. Any singleton is an affine set.

Notation

The line through two points x and y in \mathbb{R}^n is the set

$$\{x + a(y - x) \mid a \in \mathsf{R} \text{ and } x, y \in \mathsf{R}^n\}.$$

Notice that the expression x + a(y - x) is equivalent to (1 - a)x + ay.

Other Terminology

Some authors call affine sets affine varieties, linear varieties or flat.

Prop. 1. The intersection of a family of affine sets is affine.

