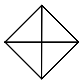


Why

We want to discuss real-valued random functions whose family of random variables have simple densities.[1]

Definition

A *normal random function* is a real-valued random function whose family of real-valued random variables has the property that any subfamily is jointly normal.

For this reason, we call the family of random variables corresponding to the random function a *normal process* or *Gaussian process*.[2]

Notation

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and A a set. Let $x : \Omega \to (A \to \mathsf{R})$ be a random function with family $y : A \to (\Omega \to \mathsf{R})$.

The random function x is a normal if, for all $a^1, \dots, a^m \in A$, $(y(a^1), \dots, y(a^m))$ is jointly normal.

Mean and covariance function

Proposition 1. *Let $x : \Omega \to (A \to \mathsf{R})$ be a normal random function with family $X : A \to (\Omega \to \mathsf{R})$. There exists unique*

[1] Future editions will expand.

[2] As usual, The choice of "normal" is a result of the Bourbaki project's convention to eschew historical names. Though here, as in Multivariate Normals the language of the project is nonstandard.

functions $m : A \to \mathsf{R}$ *and* $k : A \times A \to \mathsf{R}$ *so that the mean of the random variable* X_a *is* $m(a)$ *for all* A *and the covariance of the random variables* X_a *and* $X_{a'}$ *is* $k(a, a')$ *for all* $a, a' \in A$.[3]

For this reason, we call m the *mean function* and k the *covariance function* of the random function.

Conversely, suppose that we have function $m : A \to \mathsf{R}$ and $k : A \times A \to \mathsf{R}$. Then if k satisfies the property that for all $a^1, \ldots, a^m$, the $m \times m$ matrix

$$\begin{pmatrix} k(a^1, a^1) & \cdot k(a^1, a^m) & \\ \vdots & \ddots & \vdots \\ k(a^m, a^1) & \cdots k(a^m, a^m) & \end{pmatrix}$$

is positive semidefinite, then we can construct a Gaussian process from m and k as its mean and covariance function.

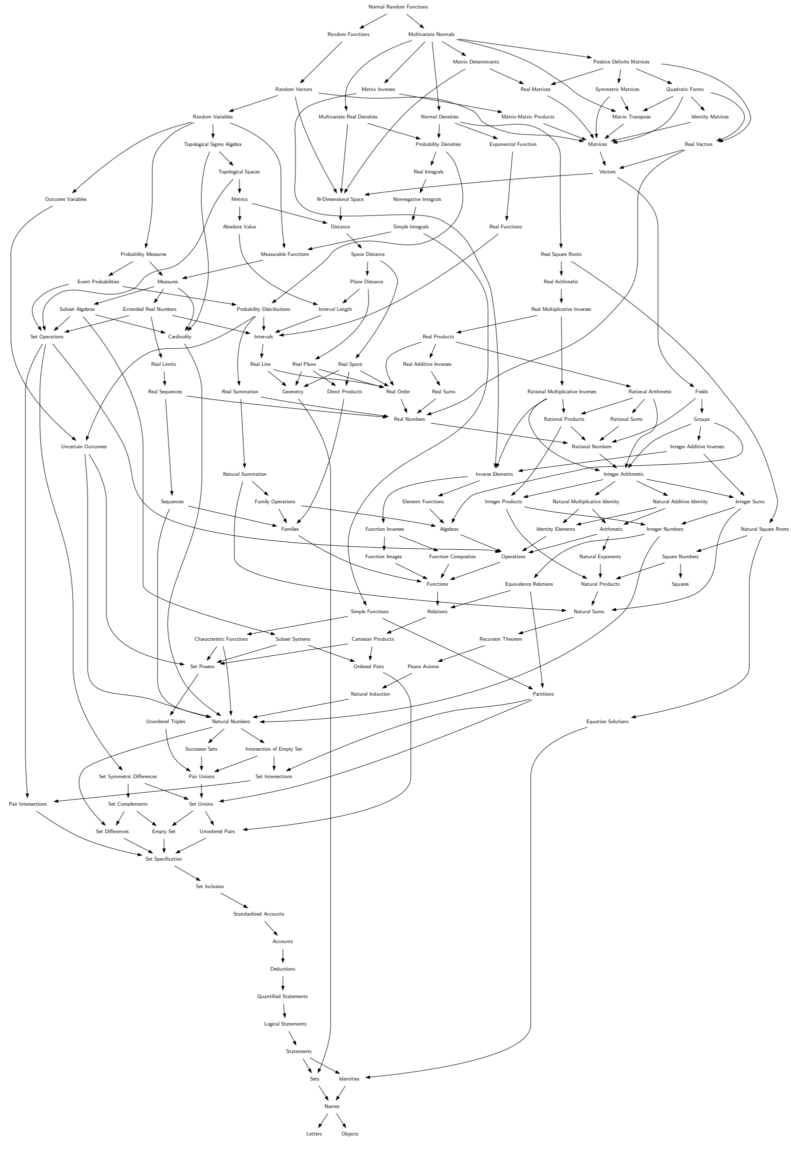
Random function interpretation

Many authorities discuss a normal random function as "putting a prior" on a "space" (see, for example, Real Function Space) of functions. One samples functions by drawing an outcome $\omega \in \Omega$, and then defining the sample $f : I \to \mathsf{R}$ by $f(i) = x(i)(\omega)$.

Multivariate normal special case

If the index set is finite, and can be ordered, then the normal random function is in one-to-one correspondence with the multivariate normal random vectors.

[3] Future editions may include an account.


Normal Random Functions
Random Functions
Multivariate Normals
Matrix Determinants
Positive Definite Matrices
Random Vectors
Matrix Inverses
Real Matrices
Symmetric Matrices
Quadratic Forms
Random Variables
Multivariate Real Densities
Normal Densities
Matrix-Matrix Products
Matrix Transpose
Identity Matrices
Topological Sigma Algebra
Probability Densities
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Matrices
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Real Integrals
Vectors
Outcome Variables
Metrics
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Nonnegative Integrals
Absolute Value
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Simple Integrals
Real Functions
Probability Measures
Measurable Functions
Space Distance
Real Square Roots
Event Probabilities
Measures
Plane Distance
Real Arithmetic
Subset Algebras
Extended Real Numbers
Probability Distributions
Interval Length
Real Multiplicative Inverses
Set Operations
Cardinality
Intervals
Real Products
Real Limits
Real Line
Real Plane
Real Space
Real Additive Inverses
Real Sequences
Real Summation
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Real Order
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