



Why

We can generalize random variables and random vectors.

Definition

A *random function* (or *random process*, *stochastic process*¹ or *random field*²) is an outcome variable whose codomain is a set of functions.

We can associate a family of random variables to each random function. The index set of the family is the domain of the set of functions and the codomain of each random variable is the codomain of the set of functions. For this reason, some authors define a random function as a family of random variables.

Notation

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let A and B be sets and let $x : \Omega \rightarrow (A \rightarrow B)$. Then x is a random function. For each outcome $\omega \in \Omega$, $x_\omega : A \rightarrow B$ is a function from A to B .

We can associate to x a family of random variables indexed by A , $y : A \rightarrow (\Omega \rightarrow B)$, defined by

$$y(a)(\omega) = x(\omega)(a)$$

¹The word “process” is most frequently used when the index set is associated with time.

²The word “field” is most frequently used when the index set is associated with space.

