Why

We want to visualize (nonsymmetric) relations.

Definition

A *directed graph* is a pair (V, E) in which V is a finite nonempty set and E is a subset of $V \times V$. In other words, E is a relation on V. We call the elements of the first set the *vertices* of the graph and the elements of the second set the *edges*.

Let $(v, w) \in E$. We say that (v, w) is an edge *from v to w*, and that it is an *outgoing edge* of v and an *incoming edge* of w. We call v a *parent* of w and we call w a child of v. We say that the edge (v, w) is *incident* to v and w.

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

Notation

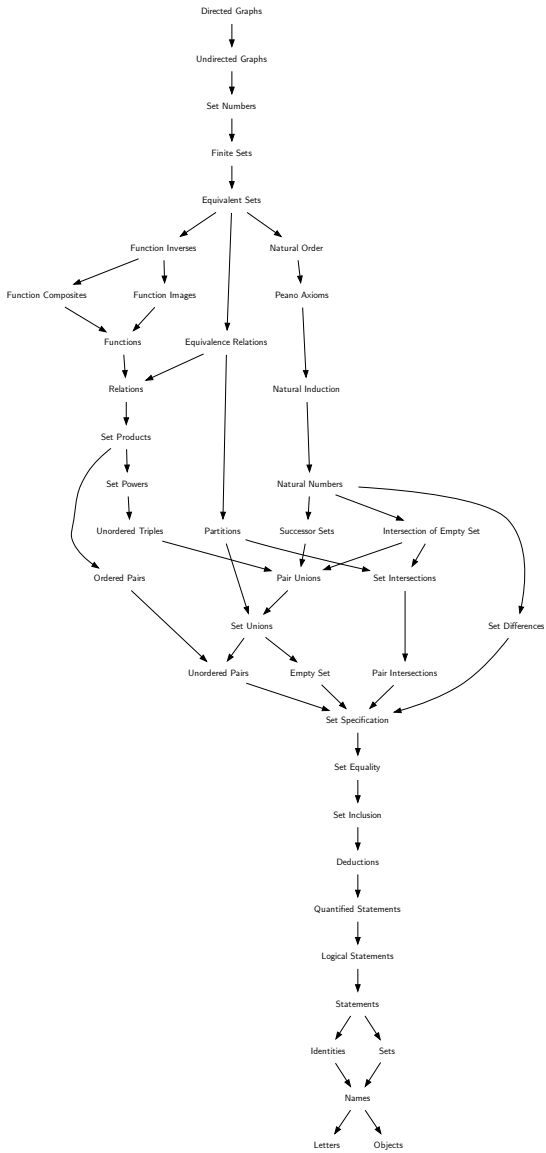
Let $\mathrm{pa} : V \to \mathcal{P}(V)$ and $\mathrm{ch} : V \to \mathcal{P}(V)$ be the functions associating to each vertex its set of parents and set of children, respectively. As usual, we denote the parents of vertex v by pa_v and the children by ch_v.

Skeletons

The *skeleton* of the directed graph (V, E) is the undirected graph (V, F) where

$$F = \{\{v, w\} \subset V \mid (v, w) \in E \text{ or } (w, v) \in E\}.$$

In other words, the skeleton is an undirected graph whose vertex set is V and whose edges are all (unordered) pairs which appear as an ordered pair in the directed graph. If the (V, E) is a directed graph and E is a symmetric relation, then we the skeleton of (V, E) is a natural undirected graph to associate with (V, E). An *orientation* of an undirected graph G is a directed graph whose skeleton is G.


Directed Graphs
Undirected Graphs
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Relations
Natural Induction
Set Products
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects