Why

What is the inverse element under matrix multiplication.

Definition

Recall that if $A \in \mathbf{R}^{m \times n}$ then $x \quad Ax$ is a function from $\mathbf{R}^n$ to $\mathbf{R}^m$. Clearly, if $m \neq n$, then the inverse of f can not exist.[1]

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that $BA = I$ we call B the *left inverse* of A and likewise if $AC = I$ we call C the *right inverse* of A. In the case that A is square, the right inverse and left inverse coincide.

Proposition 1. *Let $A, B, C \in \mathbf{R}^{n \times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

Proof. Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. □

Notation

Let F be a field. Let $A \in F^{n \times n}$ be invertible. We follow the notation of inverse elements and denote the inverse of A by A^{-1}.

[1] Future editions will expand.

Matrix Inverses
Real Matrix Inverses
Real Matrix-Matrix Products
Real Matrix-Vector Products
Vectors as Matrices
Matrices
Real Inner Product
Real Linear Combinations
Real Matrices
Real Norm
Vectors
Linear Equations
Space Norm
Real Vectors
Linear Functions
Plane Inner Product
N-Dimensional Space
Plane Norm
Plane Vector Sums and Differences
Space Vectors
Right Triangle Sides Relation
Distance
Absolute Value
Plane Vectors
Angle
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Square Numbers
Real Addition Inverses
Real Line
Real Plane
Squares
Integral Line
Rational Arithmetic
Rational Multiplicative Inverses
Real Sums
Real Order
Real Summation
Geometry
Rational Products
Rational Order
Rational Sums
Real Numbers
Zeros
Inverse Elements
Lists
Rational Numbers
Rings
Set Numbers
Direct Products
Integer Addition
Element Functions
Finite Sets
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Natural Summation
Equivalent Sets
Integer Sums
Integer Order
Arithmetic
Identity Elements
Natural Equations
Comparisons
Family Operations
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Powers
Orders
Operations
Equations
Function Composition
Function Images
Families
Natural Products
Natural Order
Equivalence Relations
Converse Relations
Functions
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Duality
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects