Successor Sets

Why

We want numbers to count with.[1]

Definition

The *successor* of a set is the set which is the union of the set with the singleton of the set. In other words, the successor of a set A is $A \cup \{A\}$. This definition has sense for any set, but is of interest only for those particular sets introduced here.

These sets are the following (and their successors): We call the empty set *zero*.[2] We call the successor of the empty set *one*. In other words, one is $\varnothing \cup \{\varnothing\} = \{\varnothing\}$. We call the successor of one *two*. In other words, two is $\{\varnothing\} \cup \{\{\varnothing\}\} = \{\varnothing, \{\varnothing\}\}$. Likewise, the successor of two we call *three* and the successor of three we call *four*. And we continue as usual,[3] using the English language in the typical way.

A set is a *successor set* if it contains zero and if it contains the successor of each of its elements.

[1] Future editions will expand on this sheet with a more justified why.

[2] In future editions, zero may be a separate sheet.

[3] Future editions will assume less in the introduction of natural numbers.

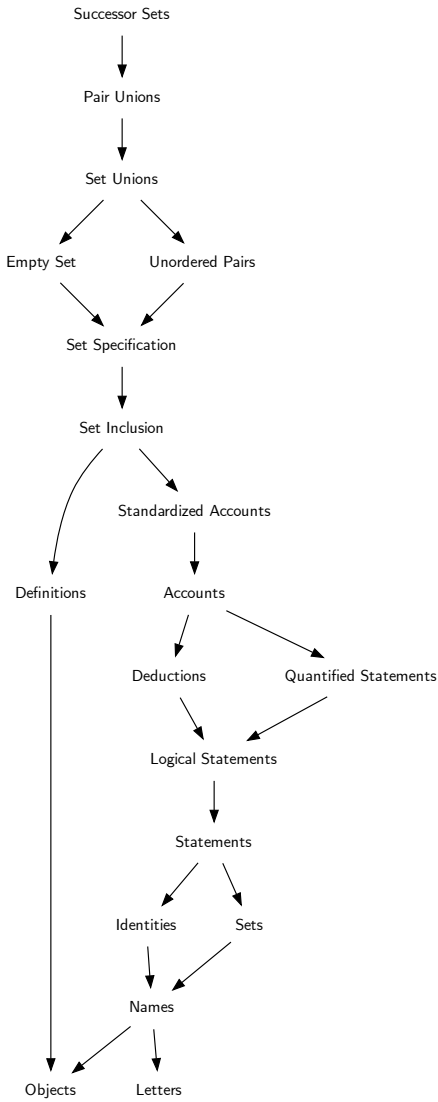
Notation

Let x be a set. We denote the successor of x by x^+. We defined it by

$$x^+ := x \cup \{x\}$$

We denote one by 1. We denote two by 2. We denote three by 3. We denote four by 4. So

$$\begin{aligned}
0 &= \varnothing \\
1 &= 0^+ = \{0\} \\
2 &= 1^+ = \{0, 1\} \\
3 &= 2^+ = \{0, 1, 2\} \\
4 &= 3^+ = \{0, 1, 2, 3\}
\end{aligned}$$


Successor Sets
Pair Unions
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters