# Metric Continuity

## Why

We define continuity for functions between metric spaces.

## Definition

Our inspiration is continuity of functions from the set of real numbers to the set of real numbers. There we decided on a definition which codified our intuition that numbers which are sufficiently close to each other are mapped to numbers that are close to each other.

A function from a first metric space to a second metric space is *continuous at* an object of its domain if, for every positive real number (no matter how small), there is a second positive real number (possibly, though not necessarily, smaller) so that every element in the domain whose distance to the fixed object is less than the second positive number has a result under the function whose distance to the result of the fixed object is less than the first positive number.

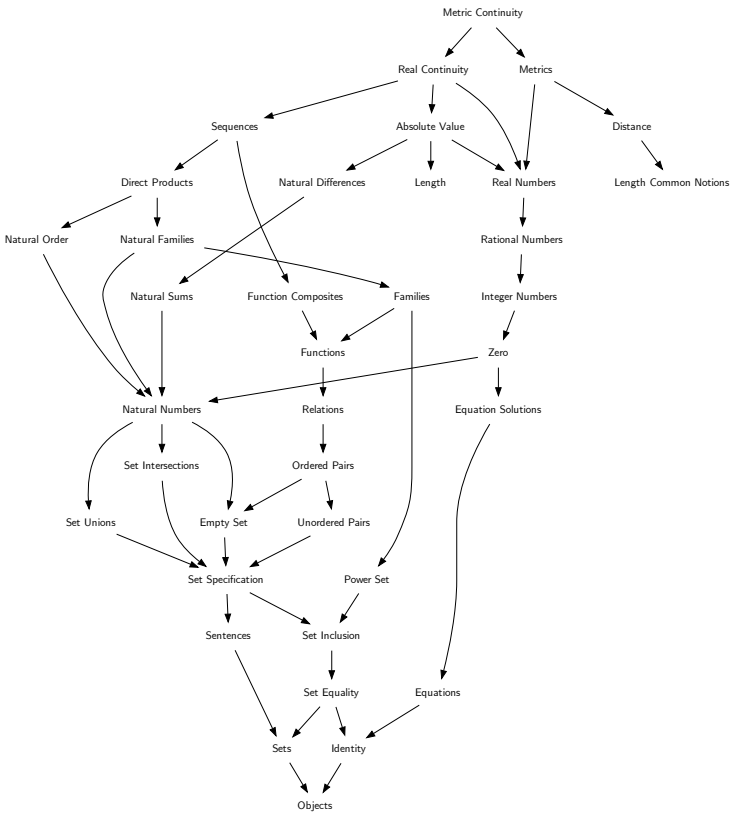
A function between metric spaces is continuous if it is *continuous* at every object of its domain.

## Notation

Let $(A, d)$ and $(B, d')$ be metric spaces. Let $f : (A, d) \to (B, d')$. Then $f$ is continuous at $\bar{a} \in A$, if for all real numbers

$\varepsilon > 0$, there exists a real number $\delta > 0$ such that for all $a \in A$,

$$d(\bar{a}, a) < \delta \implies d'(f(\bar{a}), f(a)) < \varepsilon.$$


Metric Continuity
Real Continuity
Metrics
Sequences
Absolute Value
Distance
Direct Products
Natural Differences
Length
Real Numbers
Length Common Notions
Natural Order
Natural Families
Rational Numbers
Natural Sums
Function Composites
Families
Integer Numbers
Functions
Zero
Natural Numbers
Relations
Equation Solutions
Set Intersections
Ordered Pairs
Set Unions
Empty Set
Unordered Pairs
Set Specification
Power Set
Sentences
Set Inclusion
Set Equality
Equations
Sets
Identity
Objects