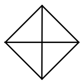


Why

[1]

Definition

Let X be a set and let A be a finite set. We denote the set of all finite sequences (strings) in A by $\mathcal{S}(A)$. We read $\mathcal{S}(A)$ aloud as "the strings in A."

A *code* for X *in* A is a function from X to $\mathcal{S}(A)$. In this context, we refer to the finite set A as an *alphabet*. The *length* of an object (w.r.t to a code $c : X \to \mathcal{S}(A)$) is the length of the sequence $c(x)$. We call a code *nonsingular* if it is injective.

Examples

Extensions

We can extend a code $c : X \to \mathcal{S}(A)$ to a code for $\mathcal{S}(X)$ in a naural way. The *extension* of c is the function $C : \mathcal{S}(X) \to \mathcal{S}(A)$ defined, for $\xi = (\xi_1, \dots, \xi_n) \in \mathcal{S}(X)$, by

$$\mathcal{S}(\xi) = (c(\xi_1), \dots, c(\xi_n)).$$

We call an code *uniquely decodable* if its extension is injective. In other words, given the code $C(\xi)$ for a sequence $\xi \in \mathcal{S}(X)$, we can recover ξ.

[1] Future editions will include.

Prefix-free codes

A code $C : X \to \mathcal{A}$ is *prefix-free* if, for all $x \in X$, $C(x)$ is not a prefix[2] of $C(x')$ for all $x' \neq x \in X$. Prefix-free codes are nice because they are uniquely decodable. The converse, is not true.

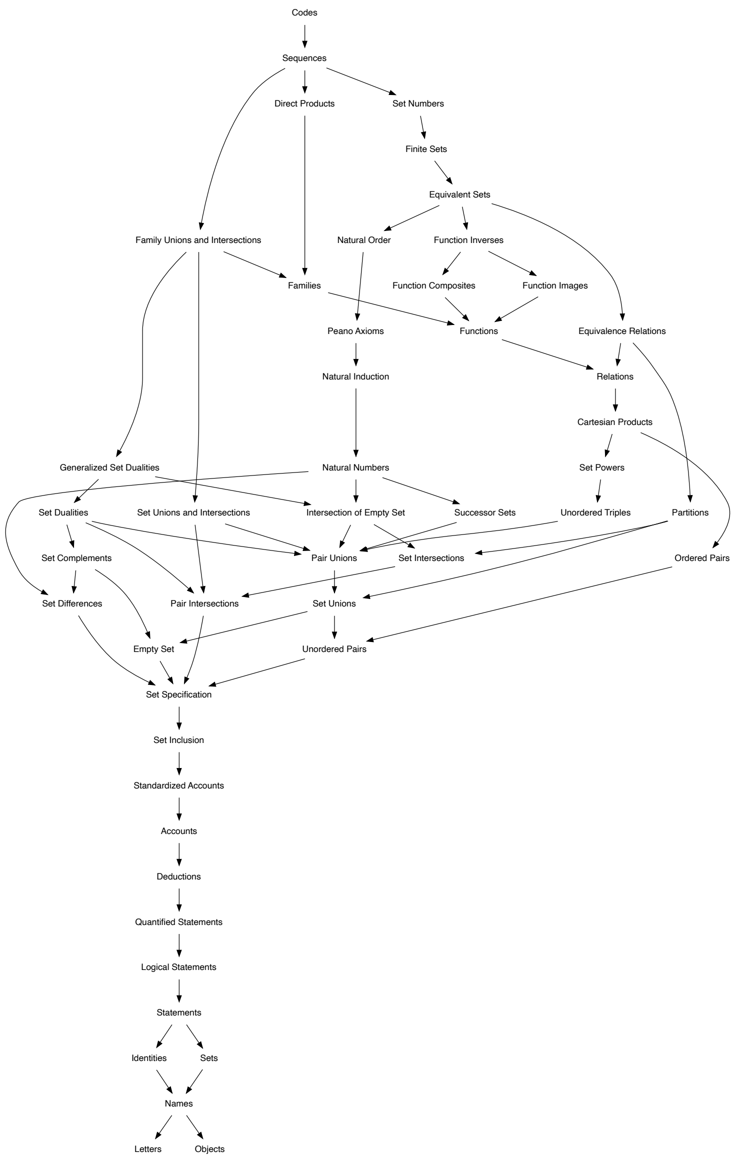
Proposition 1. *There exists a set X, alphabet A, and not prefix-free code $C : X \to \mathcal{A}$ such that C is uniquely decodable.*

Proof. Try $X = \{A, B\}$, $D = \{0, 1\}$ and $C(A) = (0)$, $C(B) = 01$. Proof by induction on the length of the sequence, base case length 1 and length 2 sequences.[3] $\square$

Example...

[2] To be defined.

[3] Future editions will expand on this account.


Codes
Sequences
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Natural Induction
Relations
Cartesian Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects