



### Definition

Let  $f : D \rightarrow \mathbf{R}$  be a multivariate real-valued function where  $D \subset \mathbf{R}^d$ . The *graph* of  $f$  is the set in  $\mathbf{R}^{d+1}$  defined by

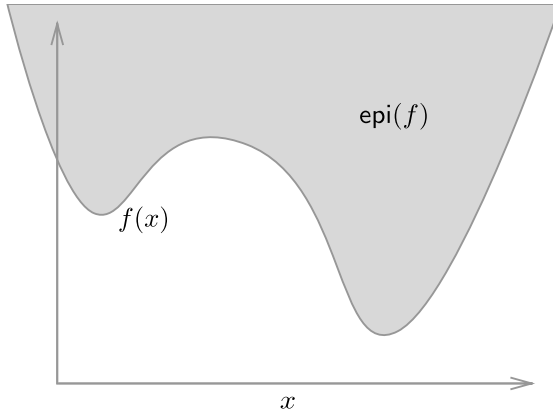
$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}$$

Of course, in these sheets, the graph is the *same object* as the function  $f$  (see discussion in **Functions**). The *epigraph* of  $f$  is the set in  $\mathbf{R}^{d+1}$  defined by

$$\{(x, \alpha) \in D \times \mathbf{R} \mid f(x) \leq \alpha\}.$$

The prefix “epi” is Greek, meaning “upon” or “above”. It is merited (see the visualization below) by the fact that  $f \neq \text{epi } f$ .

### Visualization of epigraph



### Notation

We denote the epigraph of a function  $f$  by  $\text{epi } f$ .

## Extension to extended real numbers

We can extend this concept in the natural way to extended real value function  $f : D \rightarrow \bar{\mathbf{R}}$ .

$$\text{epi } f \{ (x, \alpha) \in D \times \mathbf{R} \mid f(x) \leq \alpha \}$$

Caution, in this case  $f \not\in \text{epi } f$ .



