Why

[1]

Definition

Define $S \in \mathbf{R}^{d \times d}$ by

$$S = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

For a vector $x \in \mathbf{R}^d$ the *down shift* of x is Sx.

Let $A \in \mathbf{R}^{d \times d}$ be a matrix with columsn $a_1, \dots, a_d$. A is a *circulant matrix* if $a_1 = Sa_d$, $a2 = Sa_2$, and $a_i = A_{i-1}$ for $i = 2, \dots, d$.

Example

For example, the matrix

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

is a circulant matrix.

Characterization

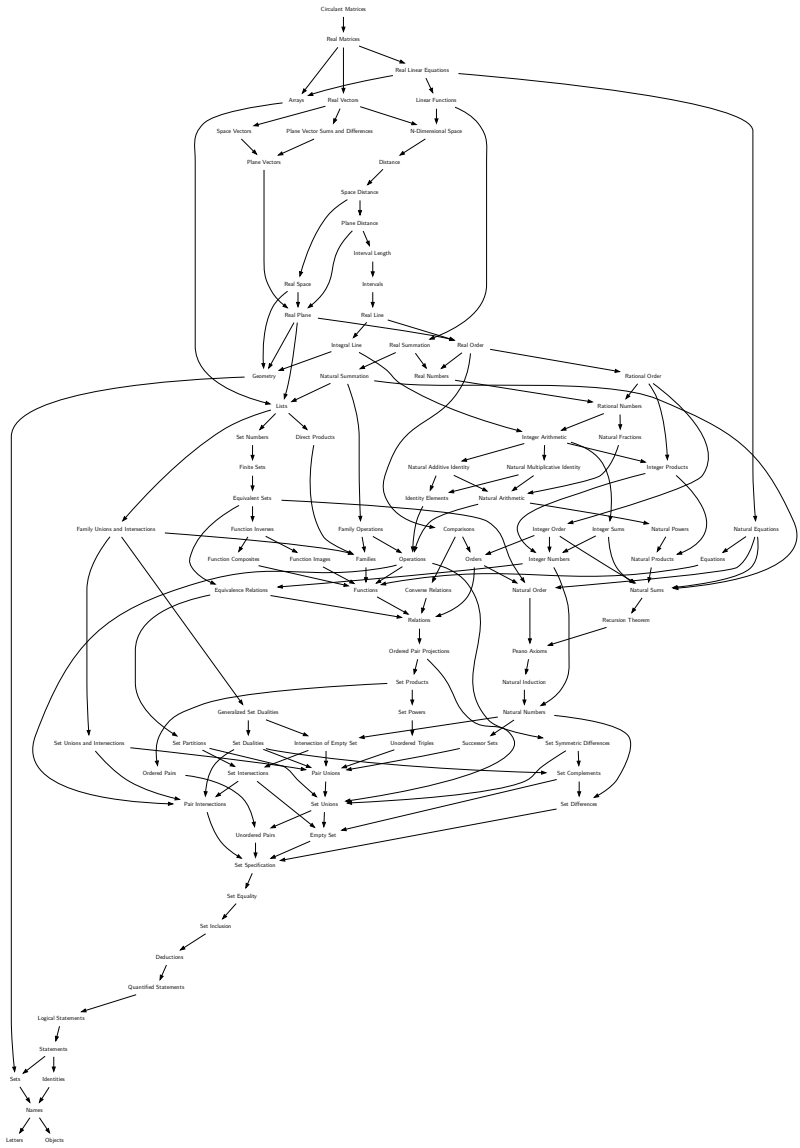
A matrix $C \in \mathbf{R}^{d \times d}$ is circulant if and only if there exists $c_0, \dots, c_{d-1}$ so that

$$C = c_0 I + c_1 S + c_2 S^2 + \cdot s + c_{n-1} S^{n-1}.$$

[1] Future sheets will include. These matrices arise in practice and each has the same eigenvectors.

Properties

The sum and product of any two circulant matrix is circulant. In other words, the circulant matrices with the usual matrix addition and multiplication form a commutative ring.


Circulant Matrices
Real Matrices
Real Linear Equations
Arrays
Real Vectors
Linear Functions
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Summation
Real Order
Geometry
Natural Summation
Real Numbers
Rational Order
Lists
Rational Numbers
Set Numbers
Direct Products
Integer Arithmetic
Natural Fractions
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Identity Elements
Natural Arithmetic
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Powers
Natural Equations
Function Composites
Function Images
Families
Operations
Orders
Integer Numbers
Natural Products
Equations
Equivalence Relations
Functions
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Partitions
Set Dualities
Intersection of Empty Set
Unordered Triples
Successor Sets
Set Symmetric Differences
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects