Why

Since we have a partial order on the set of positive semidefinite matrices, we can study which familiar functions are have order-preserving or order-reversing properties.

Norms

It would be nice if the matrix norm induced by the matrix scalar produce (see Matrix Scalar Product) was an isotonic function. In other words, if $A, B \in \mathbf{S}^d$ satisfy $A \geq B$, does $\|A\| \geq \|B\|$?

Since $\|A\|^2 = \operatorname{tr} A^2$, we should study the trace first..

Trace

Proposition 1. *Let $f : \mathbf{S}^d \to \mathbf{R}$ defined by $f(A) = \operatorname{tr} A$.*

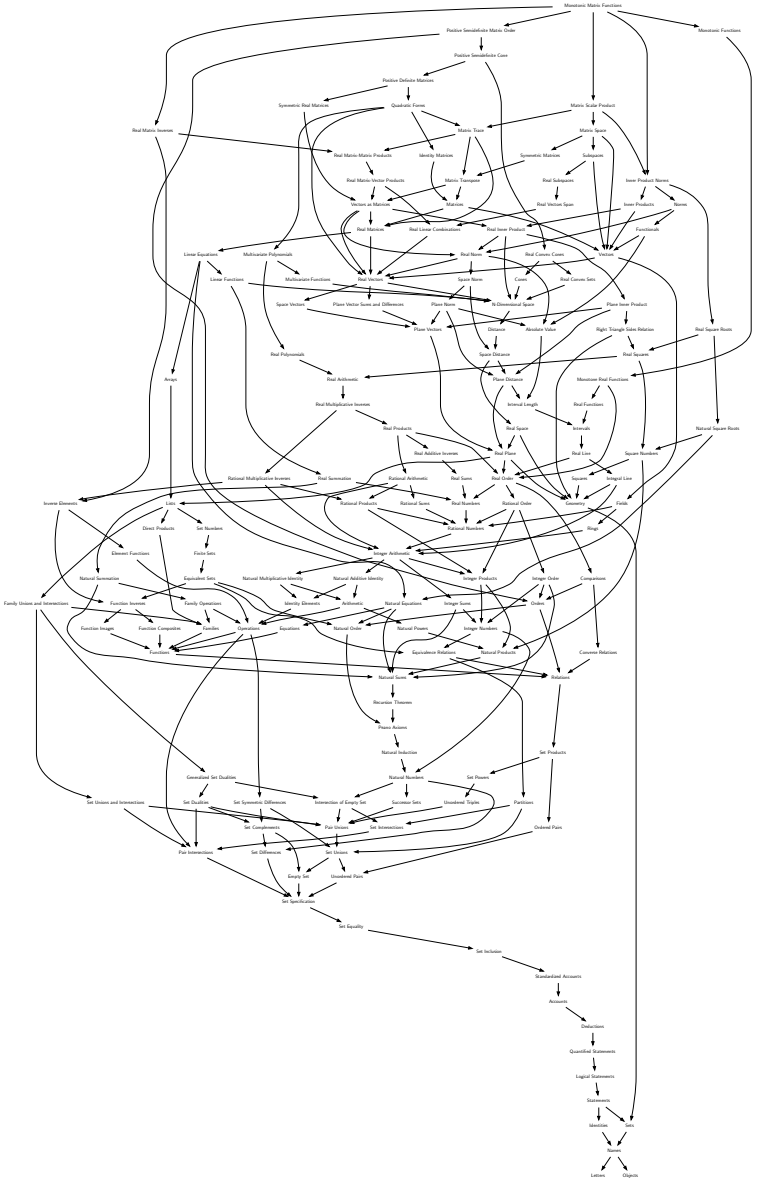
In other words, the function f is the restriction of the trace function onto the set of symmetric matrices.

Proposition 2. *Let $B \in \mathbf{S}^d$ Let $f_B : \mathbf{S}^d \to \mathbf{R}$ defined by $f(A) = \operatorname{tr} AB$.*

Inversion

Proposition 3. *Let $A \in \mathbf{S}^d_{++}$. Then the map $f : \mathbf{S}^d_{++} \to \mathbf{S}^d_{++}$ satisfying $f(A) = A^{-1}$ is an isotonic function mapping the (open) positive definite cone into itself.*[1]

[1] Future editions will include a proof.


Monotonic Matrix Functions
Positive Semidefinite Matrix Order
Monotonic Functions
Positive Semidefinite Cone
Positive Definite Matrices
Symmetric Real Matrices
Quadratic Forms
Matrix Scalar Product
Real Matrix Inverses
Matrix Trace
Matrix Space
Real Matrix-Matrix Products
Identity Matrices
Symmetric Matrices
Subspaces
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Inner Product Norms
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Real Vector Span
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Sparse Vectors
Plane Vector Sums and Differences
Plane Norm
N-Dimensional Spaces
Plane Inner Product
Plane Vectors
Distance
Absolute Value
Right Triangle Sides Relation
Real Square Roots
Real Polynomials
Sparse Distance
Real Squares
Arrays
Real Arithmetic
Plane Distance
Monotone Real Functions
Real Multiplicative Inverses
Interval Length
Real Functions
Real Products
Real Space
Intervals
Natural Square Roots
Real Additive Inverses
Real Plane
Real Line
Square Numbers
Rational Multiplicative Inverses
Real Summation
Rational Arithmetic
Real Sums
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Natural Additive Identity
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Functions Composites
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Natural Powers
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Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Set Symmetric Differences
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects