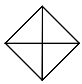


Why

We discuss inferring (or learning) functions from examples.

Definitions

Suppose $\mathcal{U}$ and $\mathcal{V}$ are two sets. A *predictor* from $\mathcal{U}$ to $\mathcal{V}$ is a function $f : \mathcal{U} \to \mathcal{V}$. We call $\mathcal{U}$ the *inputs*, $\mathcal{V}$ the *outputs*, and $f(u)$ the *prediction* of f on $u \in \mathcal{U}$.

An *inductor* is a function from datasets in $\mathcal{U} \times \mathcal{V}$ to predictors from $\mathcal{U}$ to $\mathcal{V}$. A *learner* (or *learning algorithm*) is a family of inductors whose index set is $\mathbf{N}$, and whose nth term is is an inductor for a dataset of size n.

Predicting relations

A *relation inductor* is a function from finite datasets in $\mathcal{U} \times \mathcal{V}$ to *relations* on $\mathcal{U} \times \mathcal{V}$. Since we can associate any relation R between $\mathcal{U}$ and $\mathcal{V}$ with a function $f : \mathcal{U} \times \mathcal{V} \to \{0, 1\}$, $f(u, v) = 1$ if and only if $(u, v) \in R$, the predictor case accomodates general relations, beyond functions.

Notation

Let D be a dataset of size n in $\mathcal{U} \times \mathcal{V}$. Let $g : \mathcal{U} \to \mathcal{V}$, a predictor, which makes prediction $g(u)$ on input $u \in \mathcal{U}$. Let $G_n : (\mathcal{U} \times \mathcal{V})^n \to (\mathcal{U} \times \mathcal{V})$ be an inductor, so that $G_n(D)$ is the predictor which the inductor associates with dataset D. Then $\{G_n : (\mathcal{U} \times \mathcal{V})^n \to \mathcal{P}((\mathcal{U} \times \mathcal{V}))\}_{n \in \mathbf{N}}$ is a learner.

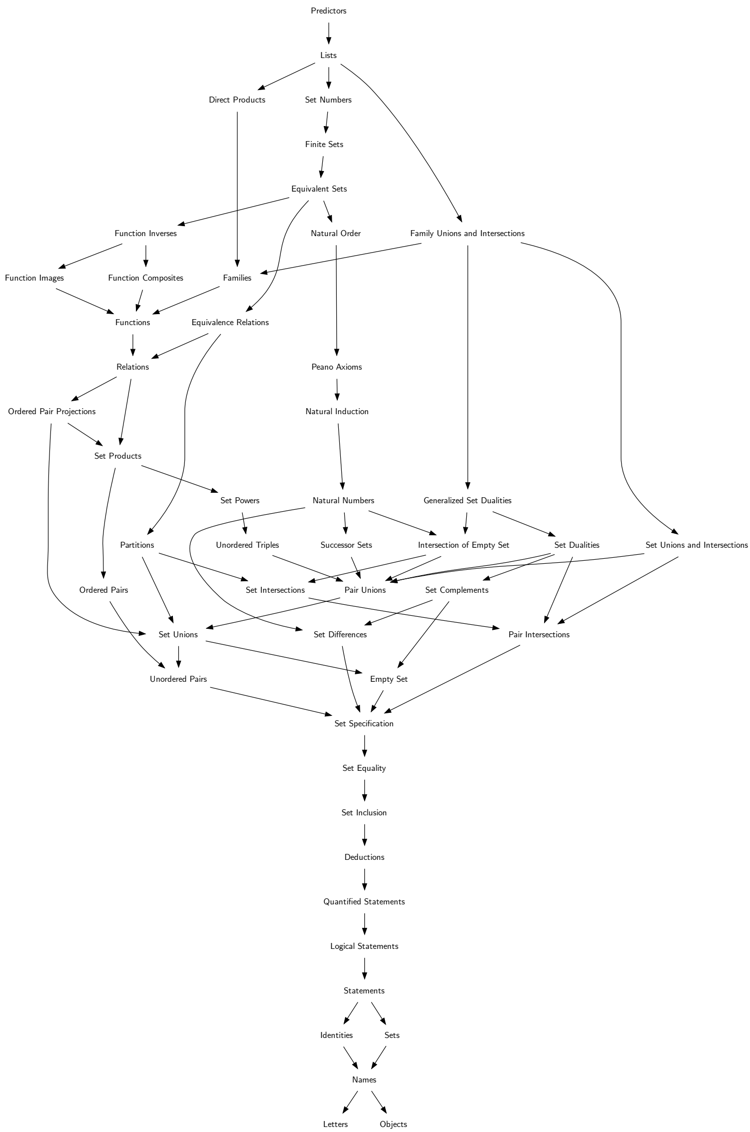
Consistent and complete datasets

Let $D = ((u_i, v_i))_{i=1}^n$ be a dataset and $R \subset X \times Y$ a relation. D is *consistent with* R if $(u_i, v_i) \in R$ for all $i = 1, \dots, n$. D is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take $R = \mathcal{U} \times \mathcal{V}$). D is *functionally consistent* if it is consistent with a function; in this case, $x_i = x_j \Rightarrow y_i = y_j$. D is *functionally* *complete* if $\cup_i \{x_i\} = X$. In this case, the dataset includes every element of the relation.

Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent variables*, *explained variables*, *postcepts*, *targets*, *outcomes*, *labels* or *observational outcomes*. An input-output pair is sometimes called a *record pair*.

Other terms for a learner include *learning algorithm*, or *supervised learning algorithm*. Other terms for a predictor include *input-output mapping*, *prediction rule*, *hypothesis*, *concept*, or *classifier*.


Predictors
Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Images
Function Composites
Families
Functions
Equivalence Relations
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects