



SUCCESSOR SETS

Why

We want numbers to count with.¹

Definition

The *successor* of a set is the set which is the union of the set with the singleton of the set. In other words, the successor of a set A is $A \cup \{A\}$. This definition has sense for any set, but is of interest only for those particular sets introduced here.

These sets are the following (and their successors): We call the empty set *zero*.² We call the successor of the empty set *one*. In other words, one is $\emptyset \cup \{\emptyset\} = \{\emptyset\}$. We call the successor of one *two*. In other words, two is $\{\emptyset\} \cup \{\{\emptyset\}\} = \{\emptyset, \{\emptyset\}\}$. Likewise, the successor of two we call *three* and the successor of three we call *four*. And we continue as usual,³ using the English language in the typical way.

A set is a *successor set* if it contains zero and if it contains the successor of each of its elements.

Notation

Let x be a set. We denote the successor of x by x^+ . We defined it by

$$x^+ := x \cup \{x\}$$

We denote one by 1. We denote two by 2. We denote three by

¹Future editions will expand on this sheet with a more justified why.

²In future editions, zero may be a separate sheet.

³Future editions will assume less in the introduction of natural numbers.

3. We denote four by 4. So

$$0 = \emptyset$$

$$1 = 0^+ = \{0\}$$

$$2 = 1^+ = \{0, 1\}$$

$$3 = 2^+ = \{0, 1, 2\}$$

$$4 = 3^+ = \{0, 1, 2, 3\}$$

