



### Definition

Recall that  $(\mathbb{C}, \mathbf{Cmod}\cdot)$  is a normed space, and so also a metric space. So, a sequence  $(z_n)_{n \in \mathbf{N}}$  of complex numbers is egoprox and convergent as usual. Both of these are equivalent to the corresponding conditions on the sequences of real and imaginary parts.

**Proposition 1.**  $(z_n)_{n \in \mathbf{N}} = (x_n, y_n)_{n \in \mathbf{N}}$  converges to  $z_0 = (x_0, y_0) \in \mathbb{C}$  if and only if  $x_n$  converges to  $x_0$  and  $y_n$  converges to  $y_0$ .

**Proposition 2.**  $(z_n)_{n \in \mathbf{N}} = (x_n, y_n)_{n \in \mathbf{N}}$  is egoprox if and only if  $x_n$  is egoprox and  $y_n$  is egoprox.

### Completeness

As a result of the second proposition, if  $z_n$  is egoprox then there is a limit  $x_0$  and  $y_0$  for its real and imaginary pieces, and so as a result of the first proposition,  $z_n$  converges. In other words, every cauchy sequence converges.

**Proposition 3.**  $\mathbb{C}$  with the metric induced by  $\mathbf{Cmod}\cdot$  is complete.



