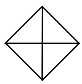


Why

We generalize the notion of a point in $\mathbf{R}$, a line in $\mathbf{R}^2$ and a plane in $\mathbf{R}^3$.

Definition

A *hyperplane* is a $(n-1)$-dimensional affine set in $\mathbf{R}^n$.

Discussion

Since the $n-1$-dimensional subspaces are the orthogonal complements of the one-dimensional subspaces, they are the sets which can be specified by

$$\{x \in \mathbf{R}^n \mid x \perp b\} + a$$

for $a, b \in \mathbf{R}^n$. The hyperplanes are translates of these,

$$\begin{aligned} \{x \in \mathbf{R}^n \mid x \perp b\} + a &= \{x + a \mid \langle x, b \rangle = 0\} \\ &= \{y \mid \langle y - a, b \rangle = 0\} = \{y \mid \langle y, b \rangle = \beta\}, \end{aligned}$$

where $\beta = \langle a, b \rangle$.

Characterization

Proposition 1. *$H \subset \mathbf{R}^n$ is a hyperplane if and only if there exists $\beta \in \mathbf{R}$ and nonzero $b \in \mathbf{R}^n$ so that*

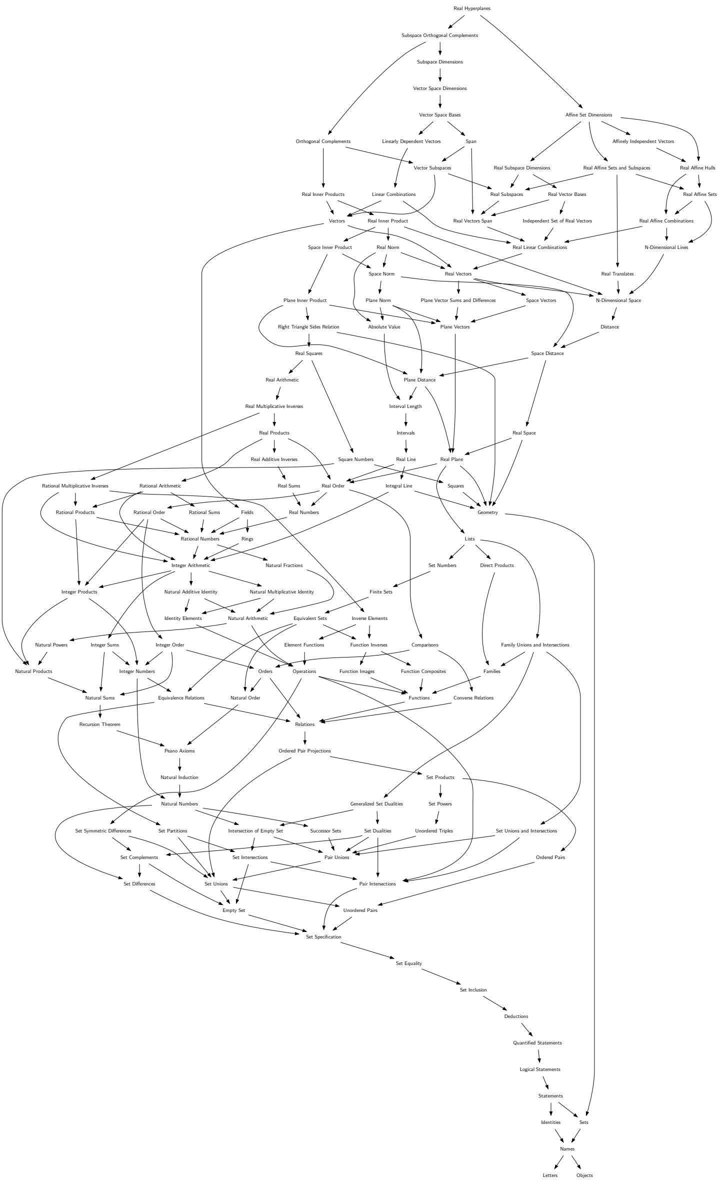
$$H = \{x \in \mathbf{R}^n \mid \langle x, b \rangle = \beta\}.$$

Remark 1. *b and β are unique up to a common nonzero multiple. For example, b, β and $2b, 2\beta$ give the same hyperplane.*

Remark 2. *Any such vector b is called a* normal *(or* normal vector*) to the hyperplane.*

Remark 3. *If H is a hyperplane not containing the origin, then there is a unique unit vecor u and $\alpha > 0$ so that*

$$H = \{x \in \mathbf{R}^n \mid \langle x, u \rangle = \alpha\}$$



Real Hyperplanes
Subspace Orthogonal Complements
Subspace Dimensions
Vector Space Dimensions
Vector Space Bases
Affine Set Dimensions
Orthogonal Complements
Linearly Dependent Vectors
Span
Affinely Independent Vectors
Vector Subspaces
Real Subspace Dimensions
Real Affine Sets and Subspaces
Real Affine Hulls
Real Inner Products
Linear Combinations
Real Subspaces
Real Vector Bases
Real Affine Sets
Vectors
Real Inner Product
Real Vectors Span
Independent Set of Real Vectors
Real Affine Combinations
Space Inner Product
Real Norm
Real Linear Combinations
N-Dimensional Lines
Space Norm
Real Vectors
Real Translates
Plane Inner Product
Plane Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Right Triangle Sides Relation
Absolute Value
Plane Vectors
Distance
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Square Numbers
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Squares
Rational Products
Rational Order
Rational Sums
Fields
Real Numbers
Geometry
Rational Numbers
Rings
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Integer Order
Element Functions
Function Inverses
Comparisons
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Set Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects