



### Definition

The *variance* of a square-integrable real-valued random variable is the expectation of its square less its expectation squared.

### Notation

Let  $(X, \mathcal{A}, \mu)$  be a probability space and  $f$  be a random variable. We denote the variance of  $f$  by  $\text{var } f$ . We defined it by

$$\text{var } f = \mathbf{E}(f^2) - (\mathbf{E}(f))^2.$$

### Results

**Prop. 1.** *If a random variable on a probability space is square integrable then it is integrable.*

*Proof.* The  $L^p$  spaces are nested for finite measures. □

**Prop. 2.** *The variance of a square-integrable real-valued random variable is the expectation of the square of the difference between the random variable and its expectation.*

*Proof.*

$$\text{var } f = \mathbf{E}((f - \mathbf{E}(f))^2)$$

□



