Why

We generalize the normal density to d-dimensional space.

Definition

Let $f : \mathbf{R}^d \to \mathbf{R}$ be a density such that

$$f(x) = \frac{1}{\sqrt{(2\pi)^d \,\mathbf{det}\, \Sigma}} \exp\left(-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\right)$$

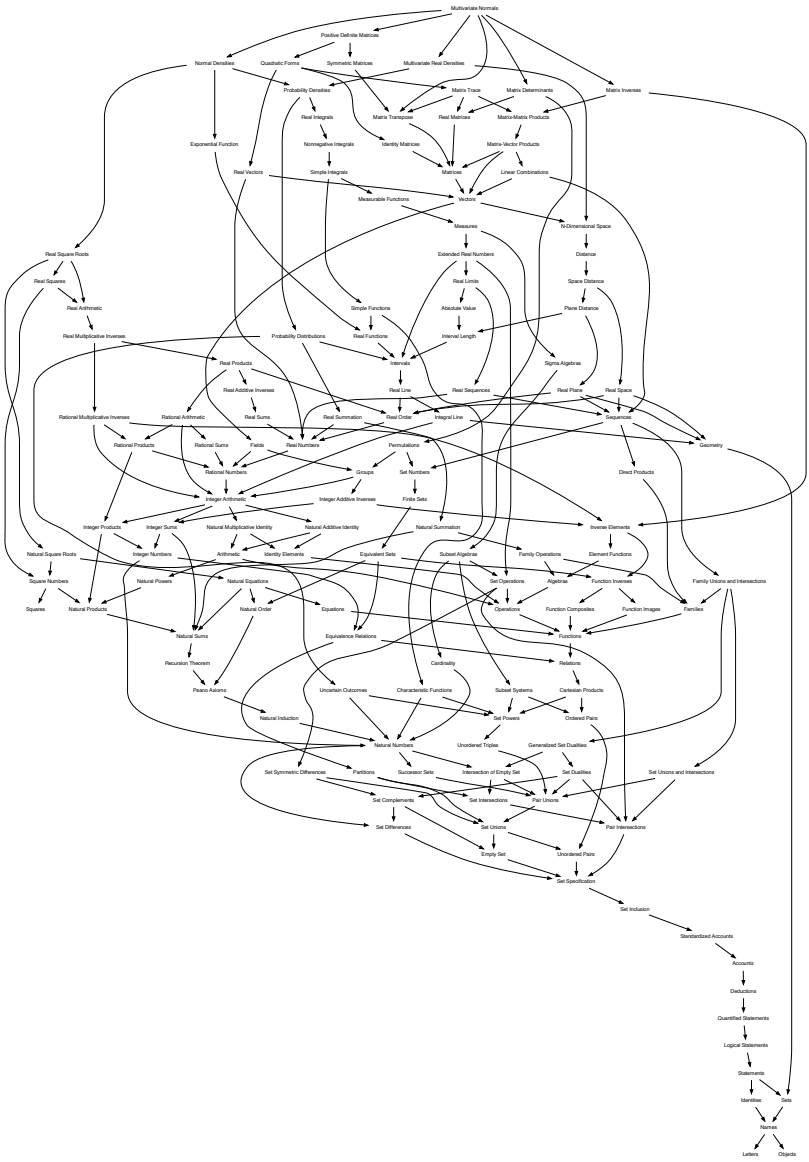
where $\mu \in \mathbf{R}^d$, $\Sigma \in \mathbf{S}^d$, and $\Sigma \succ 0$. We call f a *multivariate normal density.* A multivariate normal density with $d = 1$ is a normal density, so we refer to multivariate normal densities as *normal densities* without ambiguity. We frequently use the word normal as a substantive, and refer to *normals* when we mean multivariate normal densities. Many people call a multivariate normal distribution a *multivariate gaussian distribution* and speak of *gaussians* instead of normals.[1]

We call μ the *mean* and Σ the *covariance matrix.* We call Σ^{-1} the *precision matrix.*

Maximum

The maximum of a normal density is its mean, $\mu \in \mathbf{R}^d$.

[1] We avoid this usage in accordance with the project's policy on historical names.


Multivariate Normals
Positive Definite Matrices
Normal Densities
Quadratic Forms
Symmetric Matrices
Multivariate Real Densities
Probability Densities
Matrix Trace
Matrix Determinants
Matrix Inverses
Real Integrals
Matrix Transpose
Real Matrices
Matrix-Matrix Products
Exponential Function
Nonnegative Integrals
Identity Matrices
Matrix-Vector Products
Real Vectors
Simple Integrals
Matrices
Linear Combinations
Measurable Functions
Vectors
Measures
n-Dimensional Space
Real Square Roots
Extended Real Numbers
Distance
Real Squares
Real Limits
Space Distance
Real Arithmetic
Simple Functions
Absolute Value
Plane Distance
Real Multiplicative Inverses
Probability Distributions
Real Functions
Interval Length
Intervals
Sigma Algebras
Real Products
Real Additive Inverses
Real Line
Real Sequences
Real Plane
Real Space
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Summation
Real Order
Integral Line
Sequences
Rational Products
Rational Sums
Fields
Real Numbers
Permutations
Geometry
Rational Numbers
Groups
Set Numbers
Direct Products
Integer Arithmetic
Integer Additive Inverses
Finite Sets
Integer Products
Integer Sums
Natural Multiplicative Identity
Natural Additive Identity
Natural Summation
Inverse Elements
Natural Square Roots
Integer Numbers
Arithmetic
Identity Elements
Equivalent Sets
Subset Algebras
Family Operations
Element Functions
Square Numbers
Natural Powers
Natural Equations
Set Operations
Algebras
Function Inverses
Family Unions and Intersections
Squares
Natural Products
Natural Order
Equations
Operations
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Cardinality
Relations
Peano Axioms
Uncertain Outcomes
Characteristic Functions
Subset Systems
Cartesian Products
Natural Induction
Set Powers
Ordered Pairs
Natural Numbers
Unordered Triples
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects