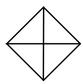


Definition

Consider two distributions on the same finite set. The *entropy* of the first distribution *relative* to the second distribution is the difference of the cross entropy of the first distribution relative to the second and the entropy of the second distribution. We call it the *relative entropy* of the first distribution with the second distribution. People also call the relative entropy the *Kullback-Leibler divergence* or *KL divergence*.

Notation

Let A be a non-empty finite set. Let $p : A \to \mathbf{R}$ and $q : A \to \mathbf{R}$ be distributions. Let $H(q, p)$ denote the cross entropy of p relative to q and let $H(q)$ denote the entropy of q. The entropy of p relative to q is

$$H(q, p) - H(q).$$

Herein, we denote the entropy of p relative to q by $d(q, p)$.

A similarity function

The relative entropy is a similarity function between distributions.

Proposition 1. *Let q and p be distributions on the same set. Then $d(q, p) \geq 0$ with equality if and only if $p = q$.*

So, d has a few of the properties of a metric. However, d is not a metric; for example, it is not symmetric.

Proposition 2. *There exist distributions $p : A \to \mathbf{R}$ and $q : A \to \mathbf{R}$ (with A a non-empty finite set) such that*

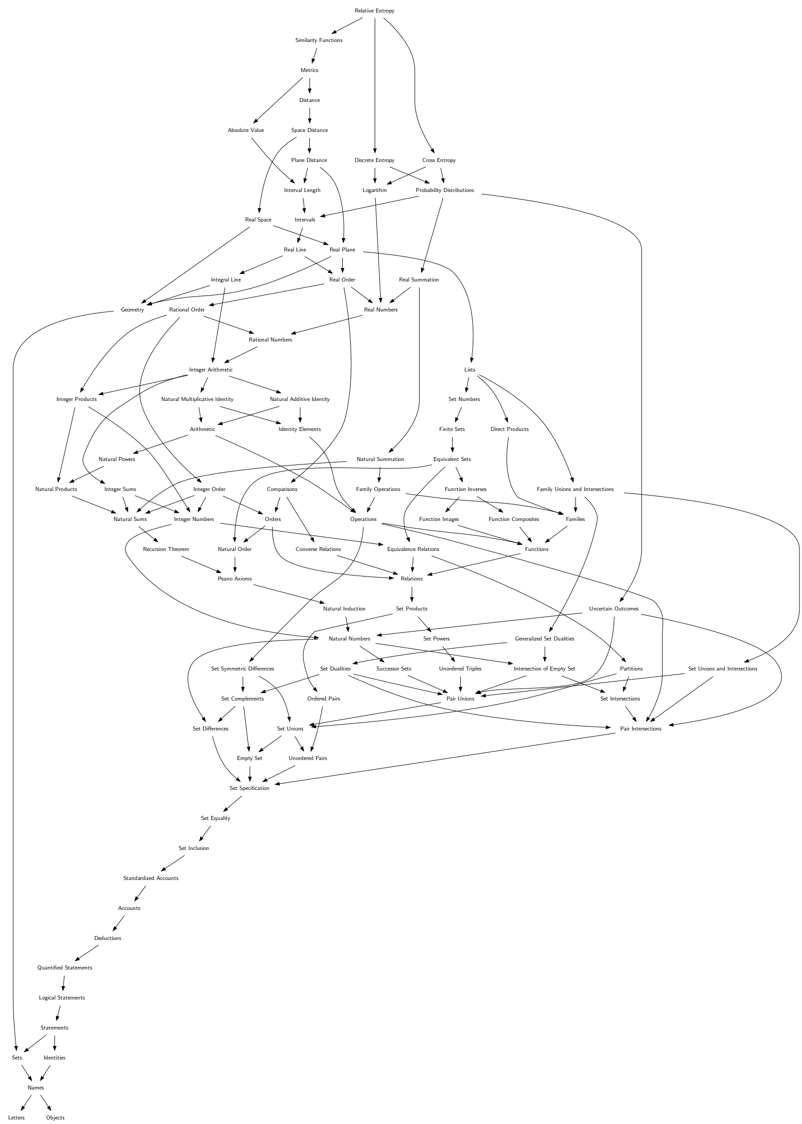
$$d(q, p) \neq d(p, q).$$

Optimization perspective

A solution to finding a distribution $p : A \to \mathbf{R}$ to

$$\text{minimize} \quad d(q, p),$$

is $p^\star = q$.


Relative Entropy
Similarity Functions
Metrics
Distance
Absolute Value
Space Distance
Plane Distance
Discrete Entropy
Cross Entropy
Interval Length
Logarithm
Probability Distributions
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Real Summation
Geometry
Rational Order
Real Numbers
Rational Numbers
Integer Arithmetic
Lists
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Arithmetic
Identity Elements
Finite Sets
Direct Products
Natural Powers
Natural Summation
Equivalent Sets
Natural Products
Integer Sums
Integer Order
Comparisons
Family Operations
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Recursion Theorem
Natural Order
Converse Relations
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Uncertain Outcomes
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Set Dualities
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Ordered Pairs
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects