Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\operatorname{cov}(f, g)$. We defined it:

$$\operatorname{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\mathbf{E}(g).$$

Properties

Proposition 1. *Covariance is symmetric and billinear.*

Proposition 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\operatorname{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \operatorname{var}(f).$$

□

Proposition 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\begin{aligned} \operatorname{var} \textstyle\sum_{i=1}^n f_i &= \operatorname{cov}(\sum_{i=1}^n f_i, \sum_{i=1}^n f_i) \\ &= \sum_{i=1}^n \sum_{j=1}^n \operatorname{cov}(f_i, f_j) \end{aligned}$$

□

Covariance
Variance
Expectation
Random Variables
Outcome Variable Covariance
Topological Sigma Algebra
Measurable Functions
Probability Measures
Topologies
Measures
Outcome Variable Expectations
Metrics
Sigma Algebras
Indicator Probability Distributions
Real Square Roots
Distance
Extended Real Numbers
Real Series
Outcome Variable Events
Real Squares
Space Distance
Real Limits
Event Probabilities
Real Arithmetic
Plane Distance
Absolute Value
Outcome Probabilities
Real Multiplicative Inverses
Interval Length
Real Functions
Real Products
Natural Square Roots
Real Sums
Intervals
Square Numbers
Real Power
Real Line
Integers
Integral Line
Rational Multiplicative Inverses
Natural Arithmetic
Real Sums
Real Order
Real Summation
Real Sequences
Sum of Direct Product
Geometry
Rational Products
Rational Sums
Real Numbers
Natural Order
Number of Set Products
Natural Summation
Sequences
Cardinality
Set
Finite Set Examples
Natural Numbers
Number of Disjoint Unions
Set Numbers
Direct Products
Integer Arithmetic
Integer Products
Natural Addition Identity
Natural Multiplication Identity
Finite Sets
Inverse Elements
Arithmetic
Equivalent Sets
Identity Elements
Element Functions
Equally Cardinalities
Uncertain Events
Natural Powers
Integer Sums
Integer Order
Natural Equations
Comparison
Function Inverses
Equations
Equally Unions and Intersections
Natural Products
Integer Numbers
Order
Equations
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Binomium Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Unordered Set Doubles
Set Symmetric Differences
Partitions
Set Doubles
Accumulation of Empty Set
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Set Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Identification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Queues
Lemma
Objects