

## SMOOTH MANIFOLDS

## Definition

A subset  $M \subset \mathbf{R}^n$  is a *smooth manifold* of dimension d if for every  $x \in M$ , there exists a neighborhood V of x in X that is diffeomorphic to an open subset U of  $\mathbf{R}^d$ . In this case we say that the set is *locally diffeomorphic* to  $\mathbf{R}^d$ .

A diffeomorphism  $\phi:U\to V$  is called a parameterization of the neighborhood of V.

Its inverse diffeomorphism  $\phi^{-1}$  is called a *coordinate system* (or system of *coordinates*) on V.

## **Notation**

We denote the dimension of a manifold M by dim M.

## **Submanifolds**

If X and Z are both manifolds in  $\mathbb{R}^n$  and  $Z \subset X$ , then we call Z a submanifold of X. In particular, X is a submanifold of  $\mathbb{R}^n$ . Any open set of a manifold X is a submanifold X.

<sup>&</sup>lt;sup>1</sup>Future editions will expand.

