

REAL BINOMIAL EXPANSIONS

Result

Proposition 1. Suppose x and y are real numbers. For any natural number n,

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

Proof. Roughly speaking, expand $(a+b)^n$ using the distributive law, and count the number of terms containing k x's and n-k y's. The number will be same as the number of ways of choosing k elements out of n, which is $\binom{n}{k}$. Each such term contributes $x^k y^{n-k}$ to the sum. Future editions will include a full proof, and perhaps some visualization.

The expression x+y is called a *binomial* and this result is often called the *binomial formula*, *binomial theorem*, *binomial identity*. For these reasons, $\binom{n}{k}$ is often called a *binomial coefficient*.

