

Closest Point Property

Suppose H is a Hilbert space $A \subset H$ closed and convex. then if $x \in H$, there exists a unique $z \in A$ closest to x. There exists a unique $z \in A$ closest to x such that

$$d(z,x) = \inf_{y \in A} d(y,x).$$

Proof. Take any sequence $(y_n)_n$ such that

$$d(y_n, x) \to d = \inf_{y \in A} d(y, x)$$