

CONDITIONAL DISTRIBUTIONS

Why

We want to speak of the pairwise conditional distributions of a particular joint distribution.¹

Definition

Let $p:\prod_{i=1}^n A_i \to \mathbf{R}$ be a distribution on A.

For $i \neq j \in \{1, ..., n\}$, the conditional distribution of i on j is the function $p_{i|j}: A_i \times A_j \to \mathbf{R}$ defined so that that $p_{i|j}(\cdot, b)$ is the conditional distribution induced by conditioning on $\{a \in \prod_{i=1}^n A_i \mid a_j = b\}$.

For i, j = 1, ..., n and $i \neq j$, p_i and $p_{i,j}$ satisfy

$$p_{i|j}(b,c)p_j(c) = p_{ij}(b,c)$$

for $b \in A_i$ and $c \in A_j$.

¹Future editions will rework this sheet.

