## Why

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## Definition

The *higher adjacency set* or *higher neighborhood* of a vertex $v$ in an ordered undirected graph is all vertices in the neighborhood of $v$ whose index is greater the $v$. Similarly, the *lower adjacency set* or *lower neighborhood* of $v$ is all vertices in the neighborhood of $v$ whose index is less the $v$. We call these *monotone neighborhoods*.

The *higher degree* of a vertex is the size of the higher adjacency set and the *lower degree* of a vertex is the size of its lower adjacency set.

The *closed monotone neighborhoods* are the *closed higher adjacency set*, the higher adjacency set of $v$ union with the singleton $\{v\}$ and the *closed lower adjacency set*, the lower adjacency set of $v$ union with the singleton $\{v\}$.

### Notation

We denote the higher neighborhood of $v$ by $\mathrm{adj}^+(v)$ and the lower neighborhood by $\mathrm{adj}^-(v)$.

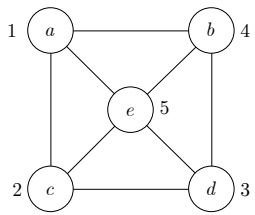



Figure 1: Ordered undirected graph.

### Visualization

To help think about the monotone neighborhoods of the graph we visualize ordered graphs as triangular arrays with vertices along the diagonal and a bullet in row $i$ and column $j$ of the array if $i > j$ and the vertices $\sigma(i)$ and $\sigma(j)$ are adjacent.

An example is shown below for the ordered undirected graph in the figure (to understand this visualization, see Ordered Undirected Graphs) we use the

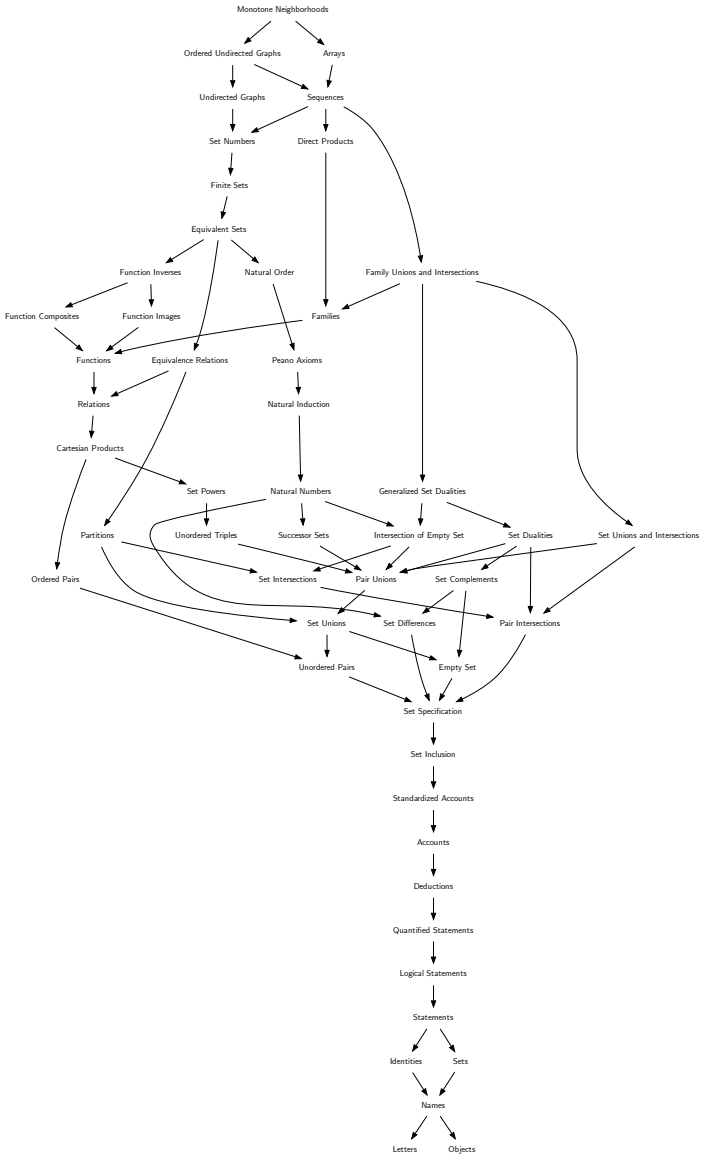
$$\begin{bmatrix} a & & & & \\ \bullet & c & & & \\ & \bullet & d & & \\ \bullet & & \bullet & b & \\ \bullet & \bullet & \bullet & \bullet & e \end{bmatrix}$$

In this array representation the higher and lower neighborhoods are easily identified. The indices of the elements of

[1]Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.

$\mathrm{adj}^+(v)$ are the column indices of the entries in row $\sigma^{-1}(v)$ of the array. For example, $\sigma^{-1}(d) = 3$, and the only bullet entry in row three is $c$ so $\mathrm{adj}^-(d) = \{c\}$. Likewise, $\mathrm{adj}^-(c) = \{a\}$. And so on. Similalry, the indices of $\mathrm{adj}^+(v)$ are the row indices of the entries in column $\sigma^{-1}(v)$. For example, $\sigma^{-1}(d)$ is 3, and there are indices 4 and 5 corresponding to $b$ and $e$ so $\mathrm{adj}^+(d) = \{b, e\}$. Likewise, $\mathrm{adj}^+(c) = \{d, e\}$.

For this reason, we use the notation $\mathrm{col}(v)$ and $\mathrm{row}(v)$ for the closed upper and lower neighborhoods. So $\mathrm{col}(v) = \mathrm{adj}^+(v) \cup \{v\}$ and $\mathrm{row}(v) = \mathrm{adj}^-(v) \cup \{v\}$.


Monotone Neighborhoods
Ordered Undirected Graphs
Arrays
Undirected Graphs
Sequences
Set Numbers
Direct Products
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Composites
Function Images
Families
Functions
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Cartesian Products
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
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Standardized Accounts
Accounts
Deductions
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Logical Statements
Statements
Identities
Sets
Names
Letters
Objects