



### Definition<sup>1</sup>

The *complex exponential* of a complex number  $z \in \mathbf{C}$  is the complex number

$$1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \cdots = \sum_{n=1}^{\infty} \frac{z^n}{n!}.$$

### Notation

We denote the complex exponential function  $\exp : \mathbf{C} \rightarrow \mathbf{C}$ , and so denote the complex exponential of  $z \in \mathbf{C}$  by  $\exp(z)$ , as usual. This overloaded notation is justified by the fact that the complex exponential agrees with the real exponential function on reals. It is also common to denote  $\exp(z)$  by  $e^z$ .

### Relation to power series of $\sin$ and $\cos$

It can be shown<sup>2</sup> that for  $x \in \mathbf{R}$ ,

$$\exp(ix) = \cos(x) + i \sin(x).$$

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<sup>1</sup>Future editions may modify this sheet, as there are several (equivalent) characterizations of the complex exponential function.

<sup>2</sup>And future editions will show it



