

REAL INTEGRAL LIMIT INFERIOR BOUND

Result

Proposition 1. The integral of the limit inferior of a sequence of measurable, nonnegative, extended-real-valued functions is no larger than the limit inferior of the sequence of integrals.

Proof. Let (X, \mathcal{A}, μ) be a measure space, and let $f_n :\to [0, \infty]$ a \mathcal{A} -measurable function for every natural number n. We want to show that if

$$\int \liminf_n f_n d\mu \le \liminf_n \int f_n d\mu.$$

