Why

We compress the notation for linear equations.

Definition

A *real matrix* (*matrix, rectangular array*) is a two-dimensional array of real numbers. We denote the elements of a matrix in a grid between two rectangular braces, as in

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}.$$

We call m and n the *dimensions* of the matrix. We call m the *height* and n the *width*. If the height of the matrix is the same as the width of the matrix then we call the matrix *square*. If the height is larger than the width, we call the matrix *tall*. If the width is larger than the height, we call the matrix *wide*.

Linear equations

Recall that we are interested in solutions of the linear equations

$$y_1 = A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n,$$
$$y_2 = A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n,$$
$$\vdots$$
$$y_n = A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n.$$

We have suggestively used the notation A_{ij} for the coefficients of the equations, so they are the entries of $A \in \mathbf{R}^{m \times n}$.

A primer on matrix-vector products

Using the notation $A \in \mathbf{R}^{m \times n}$ and $x \in \mathbf{R}^n$ we want a compressed way to write the above system of linear equations. Define the *real matrix-vector product $z \in \mathbf{R}^m$* of A with x by

$$z_i = \sum_{j=1}^{n} A_{ij} x_j, \quad i = 1, \ldots, m.$$

We denote the matrix vector product z by Ax.

Notation

We express the above system of linear equations as

$$y = Ax,$$

where $y = (y_1, \ldots, y_n) \in \mathbf{R}^n$ and $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$. The compact notation $y = Ax$ is sometimes called the *matrix form* of the m linear equations and A the *coefficient matrix*.

Note on terminology

The word "matrix" is from the Latin "mater," meaning mother, and has an old sense in English similar "womb." The matrix is source of many determinants (discussed elsewhere).

Real Matrices

Linear Equations

Real Vector — Linear Function

Space Vectors — Plane Vector Sums and Differences — N-Dimensional Space

Plane Vectors

Distance

Arrays

Space Distance

Plane Distance — Real Space

Interval Length

Intervals

Real Plane — Real Line

Real Order — Real Summation — Integral Line

Rational Order — Real Numbers — Geometry

Rational Numbers

Lists — Integer Arithmetic

Set Numbers — Natural Multiplicative Identity — Natural Additive Identity — Integer Products

Direct Products — Finite Sets

Identity Elements — Arithmetic

Equivalent Sets

Natural Power

Natural Summation

Family Unions and Intersections — Function Inverses — Family Operations — Comparisons — Integer Order — Integer Sums — Natural Products — Natural Equations

Families — Function Composites — Function Images — Operations — Orders — Integer Numbers — Natural Sums — Equations

Functions — Converse Relations — Equivalence Relations — Natural Order — Recursion Theorem

Relations — Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Set Dualities — Unordered Triples — Intersection of Empty Set — Successor Sets — Partitions — Set Symmetric Differences

Ordered Pairs — Pair Unions — Set Intersections — Set Complements

Pair Intersections — Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects