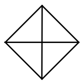


Why

Given a probability measure on the events of a set of outcomes, we can discuss probabilities of outcome variables.

Definition

Let $p : \Omega \to \mathsf{R}$ be a probability distribution with corresponding probability measure $\mathsf{P} : \mathcal{P}(\Omega) \to \mathsf{R}$. Suppose $x : \Omega \to V$ is an outcome variable. The *probability* $x = a$, for $a \in \Omega$, is

$$\mathsf{P}(\{\omega \in \Omega \mid x(\omega) = a\}).$$

From the definition of P, we express the above as

$$\textstyle\sum_{\omega \in \Omega | x(\omega) = a} p(\omega).$$

We refer to the probability of the *event* that $x = a$.

Notation

We denote the probability that $x = a$ by $\mathsf{P}[x = a]$. Our square brackets deviate from the slightly slippery but universally standard notation $\mathsf{P}(x = a)$. We prefer the square brakcets, since $x = a$ is not itself an argument to P, but shorthand for $(\{\omega \in \Omega \mid x(\omega) = a\})$.

Accordingly, there are many similar notations. For example if $V = \mathsf{N}$, $\mathsf{P}[x \geq a]$ is $\mathsf{P}(\{\omega \in \Omega \mid x(\omega) \geq a\})$. Or, $\mathsf{P}[x \in C]$ means $\mathsf{P}(\{x \in \Omega \mid x(\omega) \in C\})$. Since the *event* that $x = a$ is the inverse image of $\{a\}$ under x, we also use the notaion $x^{-1}(a)$. Or generally, $x^{-1}(C)$.

Example: sum of two dice

Define $\Omega = \{1,\ldots,6\}^2$ and define $p : \Omega \to \mathsf{R}$ with $p(\omega) = {}^1\!/_{36}$ for each $\omega \in \Omega$. Define $x : \Omega \to \mathsf{N}$ by $x(\omega_1, \omega_2) = \omega_1 + \omega_2$. Then $\mathsf{P}[x = 4] = p((2,2)) + p(1,3) + p(3,1) = {}^1\!/_{12}$.

Induced probability

For $x : \Omega \to V$, the events $x^{-1}(a)$ for $a \in V$ partition Ω. Define $q : V \to \mathsf{R}$ by

$$q(a) = \mathsf{P}[x = a].$$

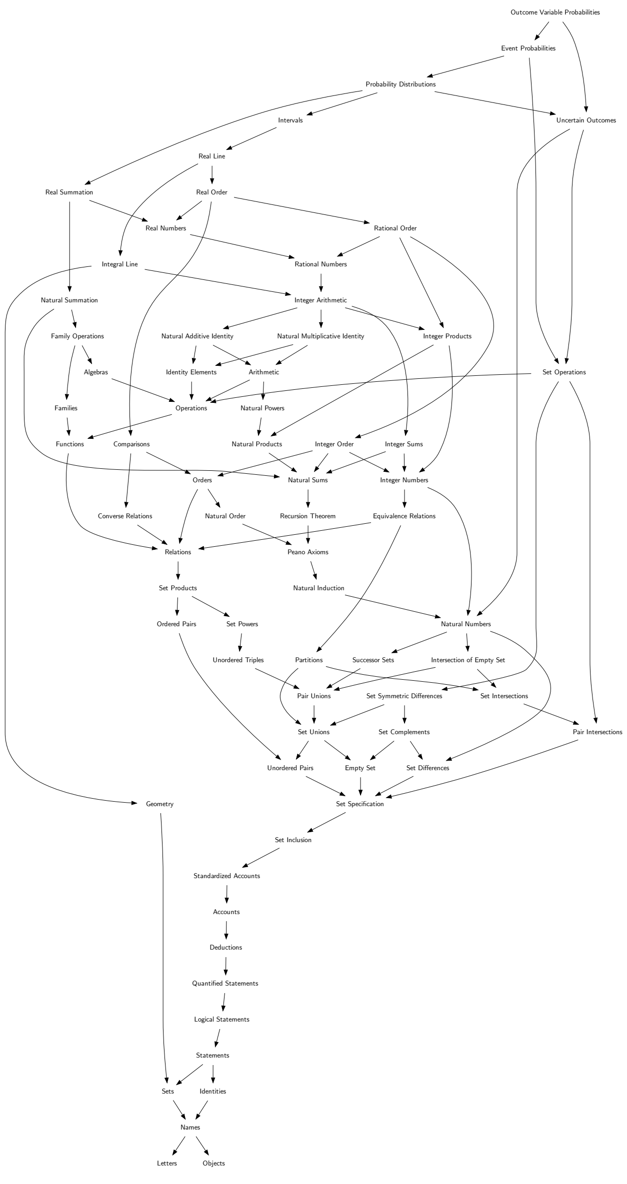
Since $\cup_{a \in V} x^{-1}(a) = \Omega$, $\sum_{a \in A} q(a) = 1$.

We call q the *induced distribution* (or *induced probability mass function*) of the random variable x. It is common to denote it by p_x. Thus we can think of V as a set of outcomes, which we call the outcomes *induced* by x.

If $x : \Omega \to V$ is a random variable and $f : V \to U$, then if we define $y : \Omega \to V$ so that $y \equiv f(x)$, y is a random variable with induced distribution $p_y : \Omega \to \mathsf{R}$ satisfying

$$p_y(b) = \textstyle\sum_{a \in V | y(a) = b} p_x(a).$$

As a matter of practical computation, we can evaluate probabilities having to do with the outcome variable x using p_x instead of p. For example with x as in the example above, $\mathsf{P}(x = 4 \text{ or } x = 5) = p_x(4) + p_x(5)$, rather than $\sum_{\omega \in \Omega | x(\omega) = 4 \text{ or } x(\omega) = 5} p(\omega)$.


Outcome Variable Probabilities
Event Probabilities
Probability Distributions
Intervals
Uncertain Outcomes
Real Line
Real Summation
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Natural Summation
Integer Arithmetic
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Algebras
Identity Elements
Arithmetic
Set Operations
Families
Operations
Natural Powers
Functions
Comparisons
Natural Products
Integer Order
Integer Sums
Orders
Natural Sums
Integer Numbers
Converse Relations
Natural Order
Recursion Theorem
Equivalence Relations
Relations
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Pair Unions
Set Symmetric Differences
Set Intersections
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Logical Statements
Statements
Sets
Identities
Names
Letters
Objects