

NATURAL EQUATIONS

Why

Suppose I have three stones in my hands. If I have two in my left hand, how many are in my right hand?

Discussion

Denote by a the number of stones in my left hand. Denote by b the number of stones in my right hand. Then the number of stones in both hands is a + b. Both a and b are natural numbers. In other words, a is 0 or 1 or 2 or... Likewise for b.

We express that I have two stones in my left hand by the equation a=2. We express that I have have five stones in total by the equation that a+b=5. Because we have identified the names a and b with tangible objects in my hands, these equations are statements about tangible objects in my hands. But the *numbers* involved are intangible objects. In any case, we have two equations in two unknowns.

We express the question "how many stones do I have in my right hand?" by asking for a solution (see Equations) to the equation a+b=5. We can start by trying natural numbers in order. Is (a,b)=(0,0) a solution? Well, 0+0=0, and $0 \neq 5$, so it is not. Is (a,b)=(1,0) a solution? Well, 1+0=1, and $1 \neq 5$, so it is not. Is (a,b)=(0,1) a solution? Well, 1+0=1, and $1 \neq 5$, so it is not. Likewise for (0,2), (1,2) and so on. Some people call this the process of guess and check.

Continuing this way we find that (3, 2) and (2, 3) is a solu-

tion. Indeed, 3 + 2 = 2 + 3 = 5. We are, however, interested in solutions to both equations

$$a = 3$$

$$a+b=5$$

Both (3,2) and (2,3) satisfy the second equation, but only (3,2) satisfies both. Are there other solutions?¹

¹Future editions will expand.

