



Why

Lots of things are (approximately) linear.¹

Definition

A transformation is *linear* if the result of a linear combination of the two vectors is the linear combination of the results of the vectors (using the same coefficients). The transformation is linear with respect to the field of the two vector spaces.

We use the term transformation (**Transformations**) to emphasize that the function is defined on a vector space. Of course, \mathbf{R} is a vector space and so $f : \mathbf{R} \rightarrow \mathbf{R}$ may be linear. It is also common, though, to speak of *linear functions*.

Often authors will use the word *operator* for linear functions. It seems, generally, that this term is commonly reserved for the case in which the vector space discussed is a function space (or, at least, infinite dimensional).

Notation

Let (V_1, F) and (V_2, F) be two vector spaces over the same field. Let $f : V_1 \rightarrow V_2$. f is linear means

$$f(au + bv) = af(u) + bf(v)$$

for all $a, b \in F$ and $u, v \in V_1$.

¹Future editions will expand on this why. In particular, the intuition of proportionality.

