



**Definition**

For a nonzero  $b \in \mathbf{R}^n$  and  $\beta \in \mathbf{R}$  the sets

$$\{x \in \mathbf{R}^n \mid \langle x, b \rangle \leq \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x, b \rangle \geq \beta\},$$

are *closed halfspaces* and the sets

$$\{x \in \mathbf{R}^n \mid \langle x, b \rangle < \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x, b \rangle > \beta\},$$

are *open halfspaces*.

Each of these is nonempty and convex. As with hyperplanes, the same four sets appear if one uses  $\lambda\beta$  and  $\lambda b$  above, so the halfspaces depend only on the hyperplane  $H = \{x \mid \langle x \rangle b = \beta\}$ .



