Why

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Definition

Let $(I, \leq)$ be a totally ordered set. A family of sigma-algebras $\{F_i\}_{i \in I}$ is a *filtration* if $F_j \subset F_k$ for all $j \leq k$.

We call a filtration whose index set is the natural numbers with their usual order a *discrete-time filtration*. We call a filtration whose index set is the real numbers with their usual order a *continuous-time filtration*. The index set may also be finite, for example, $\{F_i\}_{i=1}^n$, in which case

$$F_1 \subset F_2 \subset \cdots \subset F_n.$$

Notation

It is extremely common to see filtrations written with the caligraphic $\mathcal{F}$. As in, let $\{\mathcal{F}_i\}_{i=1}^n$ be a filtration. This is in accordance with using caligraphic letters for sets of sets, and in accordance with the term sigma *field* for sigma algebra.

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[1]Future editions will include, and likely will need stochastic processes.

Filtrations

Sigma Algebras

Real Series

Real Limits

Absolute Value

Interval Length

Intervals

Real Line

Real Sequences

Real Order

Sequences

Real Numbers

Rational Order

Integral Line

Rational Numbers

Subset Algebra

Lists

Integer Arithmetic

Cardinality

Natural Multiplicative Identity

Natural Additive Identity

Set Numbers

Integer Products

Identity Elements

Arithmetic

Finite Sets

Natural Powers

Direct Products

Operations

Equivalent Sets

Integer Order

Integer Sums

Natural Products

Family Unions and Intersections

Function Inverse

Comparisons

Integer Numbers

Natural Sums

Families

Function Images

Function Composites

Orders

Functions

Equivalence Relations

Converse Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Set Products

Natural Induction

Subset Systems

Generalized Set Dualities

Set Powers

Natural Numbers

Ordered Pairs

Set Unions and Intersections

Set Dualities

Set Symmetric Differences

Unordered Triples

Successor Sets

Intersection of Empty Set

Partitions

Set Complements

Pair Unions

Set Intersections

Pair Intersections

Set Differences

Set Unions

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects