Convex Sets

1 Why

We speak of sets of real numbers which are like intervals.

2 Definition

A *convex combination* of two distinct real numbers is an element of the closed interval they delimit. A *convex set* of real numbers contains each convex combination of any two of its elements.

The *length* of a convex combination is the real number in $[0, 1]$ which is the ratio of the combination less the lower endpoint to the upper endpoint less the lower endpoint.

2.1 Notation

Denote the real numbers by R. Let $A \subset R$ be convex. Then for all $a, b \in A$, $[a, b] \subset A$.

Suppose A contains at least two element. Let $a, b \in A$ with $a < b$. If c is the combination of a and b, then the length of c is $(c - a)/(b - a)$.

If A is convex, then for each $a, b \in A$, and $\theta \in [0, 1]$,

$$\theta a + (1 - \theta)b \in A.$$

3 Examples

Example 1. *The real numbers are a convex set.*

Example 2. *Real intervals are convex.*

Example 3. *Let a, b be non-equal real numbers. The set $\{a, b\}$ is not convex.*

Example 4. *The empty set is convex*

Example 5. *Let a be a real number. The set $\{a\}$ is convex.*

Example 6. *Let $[a, b]$ and $[c, d]$ be two disjoint real intervals. The set $[a, b] \cup [c, d]$ is not convex.*