



Why

We want to add repeatedly.

Definitiong result

Proposition 1. *For each natural number m , there exists a function $p_m : \omega \rightarrow \omega$ which satisfies*

$$p_m(0) = 0 \quad \text{and} \quad p_m(n^+) = (p_m(n))^+ + m$$

for every natural number n .

Proof. The proof uses the recursion theorem (see Recursion Theorem).¹

□

Let m and n be natural numbers. The value $p_m(n)$ is the *product* of m with n .

Notation

We denote the product $p_m(n)$ by $m \cdot n$. We often drop the \cdot and write $m \cdot n$ as mn .

Properties

The properties of products are direct applications of the principle of mathematical induction (see Natural Induction).²

Proposition 2 (Associativity). *Let k , m , and n be natural numbers. Then*

$$(k \cdot m) \cdot n = k \cdot (m \cdot n).$$

Proposition 3. *Let m and n be natural numbers. Then*

$$m \cdot n = n \cdot m.$$

¹Future editions will give the entire account.

²Future editions will include the accounts.

