



Why

If A and B are two events in some finite sample space Ω and P is the event probability function of some distribution, we have seen that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

What if we have three events A, B, C ? What of a list of n events A_1, \dots, A_n ?

Case of $n = 3$.

Proposition 1. *Suppose A, B, C are events in a finite sample space and P is any event probability function. Then*

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &\quad - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &\quad + P(A \cap B \cap C) \end{aligned}$$

Proof. Express

$$\begin{aligned} P(A \cup B \cup C) &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\ &= P(A) + P(B) + P(C) - P(A \cap B) - P((A \cup B) \cap C) \end{aligned}$$

by using the inclusion-exclusion formula. Recall¹ that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

Now use the inclusion-exclusion formula again, and properties of set pair intersections to get

$$\begin{aligned} P((A \cup B) \cap C) &= P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C)) \\ &= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \end{aligned}$$

□

¹See Set Unions and Intersections.

Case of n .

Proposition 2. *Suppose A_1, \dots, A_n , are events in a finite sample space and P is any event probability function. Then*

$$\begin{aligned} P(A_1 \cup \dots \cup A_n) = & \sum P(A_i) - \sum P(A_i \cap A_j) \\ & \sum P(A_i \cap A_j \cap A_k) - \dots \\ & (-1)^{n-1} P(A_1 \cap \dots \cap A_n) \end{aligned}$$

Proof. This can be shown by induction on the number of events n . Future editions will include. □

Here there are $\binom{n}{r}$ terms in the r th sum, $r = 1, \dots, n$.

