



# Real Continuity

## 1 Why

We try to precisely characterize the idea that a function is continuous, or uninterrupted.

## 2 Definition

Consider a function from the real numbers to the real numbers.

The function is *continuous at a point* in its domain if for every positive real number, there is a positive real number such that every point in the domain which is the second positive number close to the first element has result which is the first positive number close to the second.

A function is *continuous* if it is continuous at every point of its domain.

## 2.1 Notation

Let  $R$  denote the set of real numbers. Let  $f : R \rightarrow R$ . Then  $f$  is continuous at  $x \in R$  if

$$(\forall \varepsilon > 0)(\exists \delta > 0)(|x - y| < \delta \implies |f(x) - f(y)| < \varepsilon)$$

for all  $y \in R$ .

Then  $f$  is continuous.

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for all  $y \in R$ .