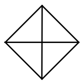


Why

We want to talk about influencing natural phenomena. [1].

Definition

Let $\mathcal{X}_0, \mathcal{X}_1, \dots, \mathcal{X}_T$ and $\mathcal{U}_0, \mathcal{U}_1, \dots, \mathcal{U}_{T-1}$ be sets. For $t = 0, \dots, T-1$, let $f_t : \mathcal{X}_t \times \mathcal{U}_t \to \mathcal{X}_{t+1}$. We call the sequence

$$\big((\mathcal{X}_t)_{t=0}^{T}), (\mathcal{U}_t)_{t=0}^{T-1}, (f_t)_{t=1}^{T-1}\big)$$

a *controlled kdeterministic discrete-time dynamical system.* We call the index t the *epoch*, the *stage* or the *period.*

Let $x_0 \in \mathcal{X}_0$. Let $u_0 \in \mathcal{U}_0, \dots, u_{T-1} \in \mathcal{U}_{T-1}$. Define a state sequence $x_1 \in \mathcal{X}_1, \dots, x_T \in \mathcal{X}_T$ by

$$x_{t+1} = f_t(x_t, u_t).$$

In this case we call x_0 the *initial state.* We call the x_t the *states.* We call the u_t a sequence of *inputs* (or *actions*, *decisions*, *choices*, or *controls*). We call f_t the *transition function* or *dynamics function.*

We call T the *horizon.* In the case that we have an infinite sequence of state sets, input sets, and dynamics, then we refer to a *infinite-horizon* dynamical system. To use language in

[1] Future editions will modify, and may restore former editions language: "We want to talk about making decisions over time." Though this language may also be used in a sheet on finite controlled dynamical systems

contrast with this case, we refer to the dynamical system when T is finite as a *finite-horizon* dynamical system.

State

The current action u_t affects future states x_s for $s > t$, but not the current or past states. The current state x_t depends on the initial state x_0 and the sequence of past actions $u_0, \ldots, u_{t-1}$. So the state is a "link" between the past and the future. Given x_t and $u_t, \ldots, u_{s-1}$, for $s > t$, we can compute x_s. In other words, the prior actions $u_0, \ldots, u_{t-1}$ are not relevant.

This nonrelevancy of prior actions and prior states simplifies the sequential decision problem (see Sequential Decisions).

Variations

The dynamical system is called *time-invariant* if $\mathcal{X}_t$, $\mathcal{U}_t$ and f_t do not depend on t. A simple variation is that $\mathcal{U}_t$ depends on x_t.[2]

Examples

Finite dynamical system

A dynamical system is finite if the state and action sets are finite. For example, $\mathcal{X} = \{1, \ldots, n\}$ and $\mathcal{U} = \{1, \ldots, m\}$. Then $f_t : \mathcal{X} \times \mathcal{U} \to \mathcal{U}$ is called a *transition map*.

Or else, let (V, E) be a directed graph, then $\mathcal{X} = V$, $\mathcal{U}_{x_t} =$

[2]Future editions will say more here.

$\{(u, v) \in E \mid u = x_t\}$ and $f_t(x_t, u_t) = v$ when $u_t = (x_t, v)$ is a dynamical system. Roughly this system models "moving" on the graph.

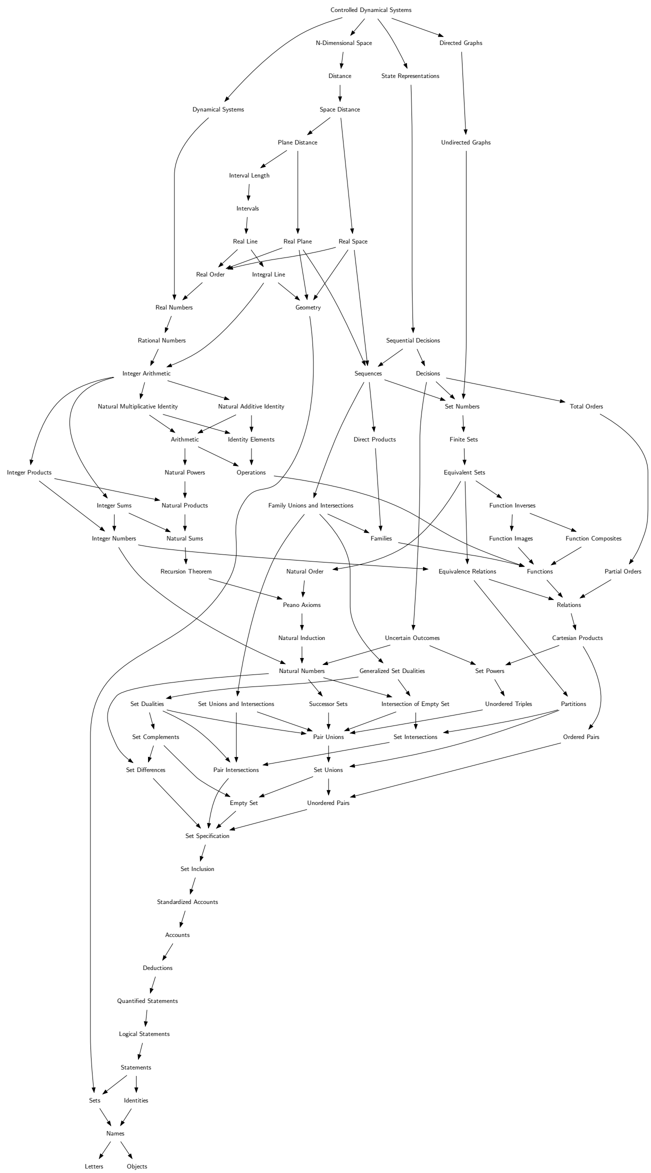
Discrete-time linear dynamical system

Let $\mathcal{X} = \mathbf{R}^n$ and $\mathcal{U} = \mathbf{R}^m$. Define $f_t : \mathcal{X} \times \mathcal{U} \to \mathcal{X}$ by

$$x_{t+1} = f_t(x_t, u_t) = A_t x_t + B_t u_t + c_t$$

for $y = 1, \ldots, T - 1$.[3]

[3] This very special form of dynamics arises in many applications. Future editions will say more.


Controlled Dynamical Systems
N-Dimensional Space
Directed Graphs
Distance
State Representations
Dynamical Systems
Space Distance
Plane Distance
Undirected Graphs
Interval Length
Intervals
Real Line
Real Plane
Real Space
Real Order
Integral Line
Real Numbers
Geometry
Rational Numbers
Sequential Decisions
Integer Arithmetic
Sequences
Decisions
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Total Orders
Arithmetic
Identity Elements
Direct Products
Finite Sets
Integer Products
Natural Powers
Operations
Equivalent Sets
Integer Sums
Natural Products
Family Unions and Intersections
Function Inverses
Integer Numbers
Natural Sums
Families
Function Images
Function Composites
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Partial Orders
Peano Axioms
Relations
Natural Induction
Uncertain Outcomes
Cartesian Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects