



CANONICAL MAPS

Why

How do equivalence classes and functions relate

Definition

We can associate to each element of a set its equivalence class under an equivalence relation. Let X denote a set and R an equivalence relation. We call the function $f : X \rightarrow X/R$ defined by $f(x) = x/R$ the *canonical map* from X to X/R .

Conversely, if f is an arbitrary function from X onto Y , we can naturally define an equivalence relation R in X so that for $a, b \in X$, $a R b \iff f(a) = f(b)$. If f was onto, so for each $y \in Y$, there exists an $x \in X$ with $f(x) = y$. Now let $g : Y \rightarrow X/R$ be defined by $g(y) = x/R$. The values of g are the subset X/R which are mapped to the same value under f . Moreover, the function g is one-to-one.

