Why

We want to generalize the notion of continuity.

Definition

Given a set X, a *topology* on X is a set of subsets of X for which (1) the empty set base set are distinguished (2) the intersection of a *finite* family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the elements of the topology the *open sets*.

A *topological space* is an ordered pair: a base set and a set distinguished subsets of the base set which are a topology.

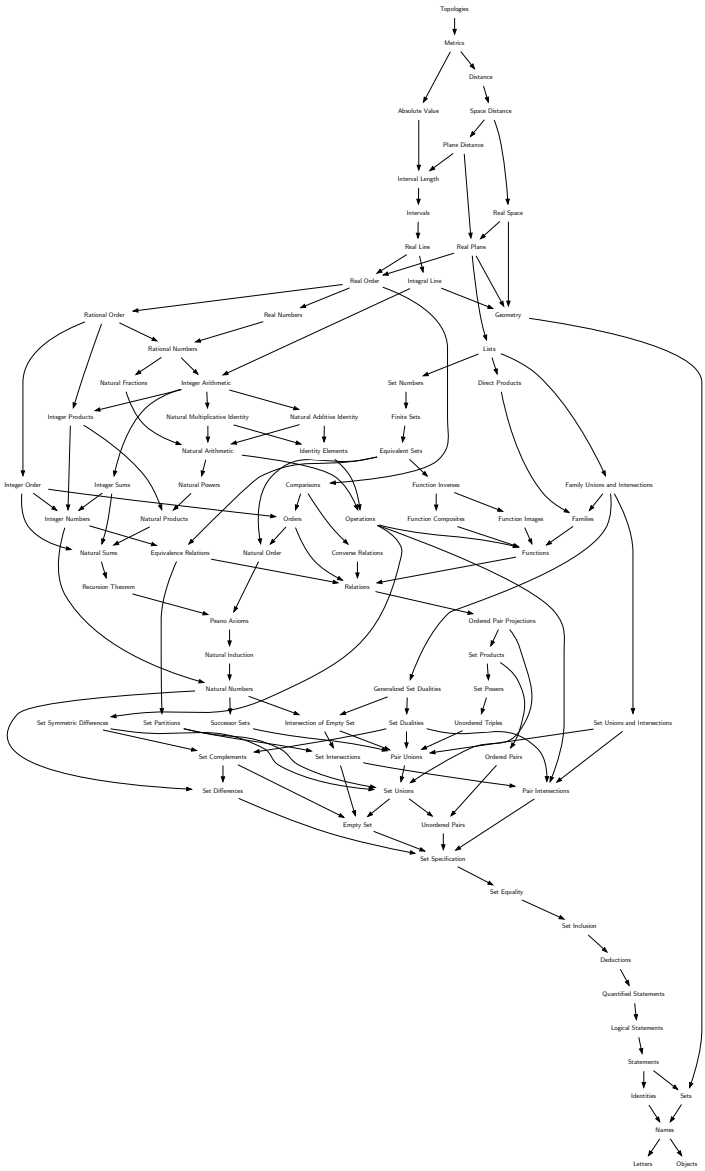
Notation

Let X be a non-empty set. For the set of distinguished sets, we tend to use $\mathcal{T}$, a mnemonic for topology, read aloud as "script T". We tend to denote elements of $\mathcal{T}$ by O, a mnemonic for open. We denote the topological space with base set X and topology $\mathcal{T}$ by $(X, \mathcal{T})$. We denote the properties satisfied by elements of $\mathcal{T}$:

1. $X, \varnothing \in \mathcal{T}$
2. if $O_1, \ldots, O_n \in \mathcal{T}$, then $\bigcap_{i=1}^{n} O_i \in \mathcal{T}$
3. if $O_\alpha \in \mathcal{T}$ for all $\alpha \in I$, then $\bigcup_{\alpha \in I} \in \mathcal{T}$

Examples

R with the open intervals as the open sets is a topological space.


Topologies
Metrics
Distance
Absolute Value
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Natural Fractions
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Arithmetic
Identity Elements
Equivalent Sets
Integer Order
Integer Sums
Natural Powers
Comparisons
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Operations
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Set Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects