

ORDERED UNDIRECTED GRAPHS

Why

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Definition

An ordering of an undirected graph is an ordering (see Lists) of its vertices. An ordered undirected graph is an ordered pair $((V, E), \sigma : \{1, 2, ..., |V|\} \to V)$ where (V, E) is an undirected graph (see Undirected Graphs) and σ is an ordering of the vertex set V.

Notation

Let $((V, E), \sigma)$ be an ordered undirected graph. We commonly associate it with (V, E, σ) and call this ordered triple an undirected graph as well. But, throughout these sheets, an ordered undirected graph is an ordered pair.

We denote that $\sigma^{-1}(v) < \sigma^{-1}(w)$ by $v \prec_{\sigma} w$ and $v \succeq_{\sigma} w$ by $\sigma^{-1}(v) \leq \sigma^{-1}(w)$. We occasionally omit the subscripts in \prec_{σ} and \succeq_{σ} when clear from context.

Visualization

We visualize an ordered undirected graph by labeling its nodes with the indices of each vertex. Let (V, E) be an undirected graph with $V = \{a, b, c, d, e\}$ and

$$E = \{\{a,b\}, \{a,c\}, \{a,e\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,e\}, \{d,e\}\}.$$

¹Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.

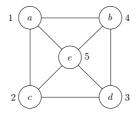


Figure 1: Ordered undirected graph.

Let $\sigma:\{1,\ldots,5\}\to V$ be an ordering with

$$\sigma(1) = a$$
 $\sigma(2) = c$ $\sigma(3) = d$ $\sigma(4) = b$ $\sigma(e) = 5$.

We visualize the ordered graph in the figure.

