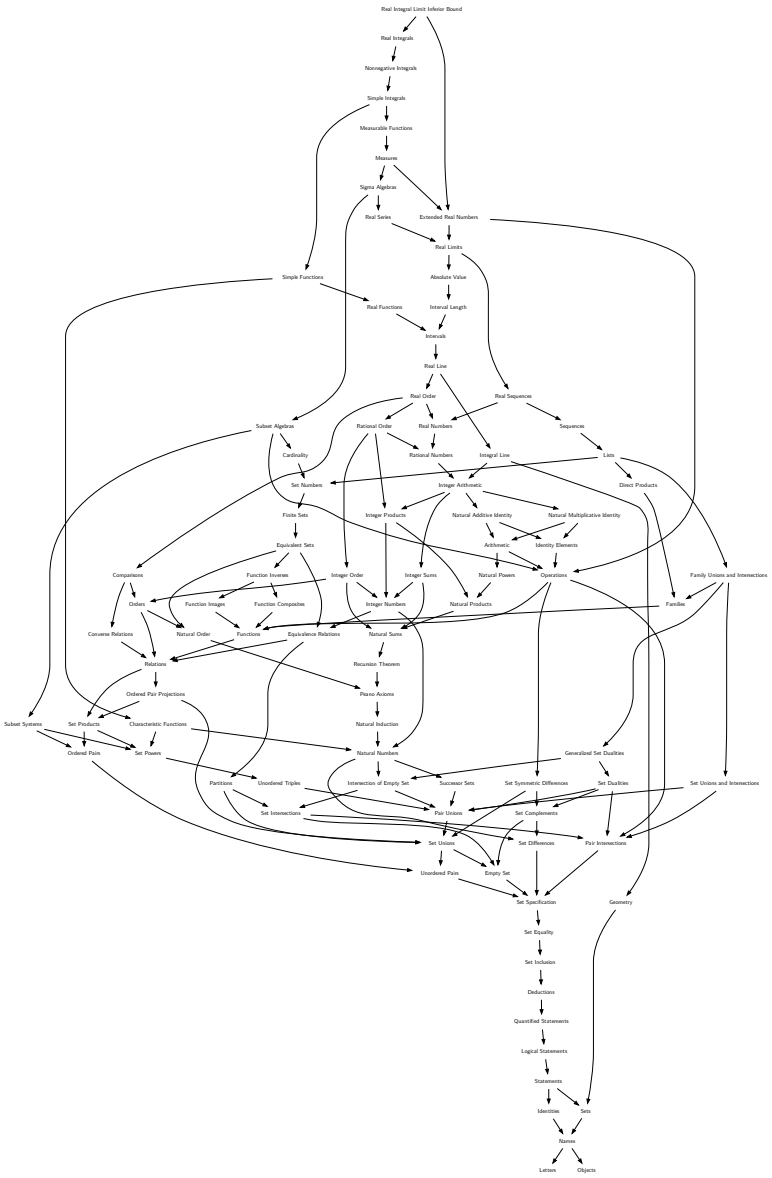
Result

Proposition 1. *The integral of the limit inferior of a sequence of measurable, nonnegative, extended-real-valued functions is no larger than the limit inferior of the sequence of integrals.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space, and let $f_n :\to [0, \infty]$ a $\mathcal{A}$-measurable function for every natural number n. We want to show that if

$$\int \liminf_n f_n d\mu \leq \liminf_n \int f_n d\mu.$$

$\square$


Real Integral Limit Infeior Bound
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Absolute Value
Simple Functions
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Subset Algebras
Rational Order
Real Numbers
Sequences
Cardinality
Rational Numbers
Integral Line
Lists
Set Numbers
Integer Arithmetic
Direct Products
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Equivalent Sets
Comparisons
Function Inverses
Integer Order
Integer Sums
Natural Powers
Operations
Family Unions and Intersections
Orders
Function Images
Function Composites
Integer Numbers
Natural Products
Families
Converse Relations
Natural Order
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects