Why

What is the best estimate for a random variable if we consider the square error?

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $x : \Omega \to \mathbf{R}$ a random variable. A *minimum mean squared error estimate* or *MMSE estimate* or *least square estimate* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}(x - \xi)^2$.

Proposition 1. *There is a unique MMSE estimate and it is given by* $\mathbf{E}(x)$.

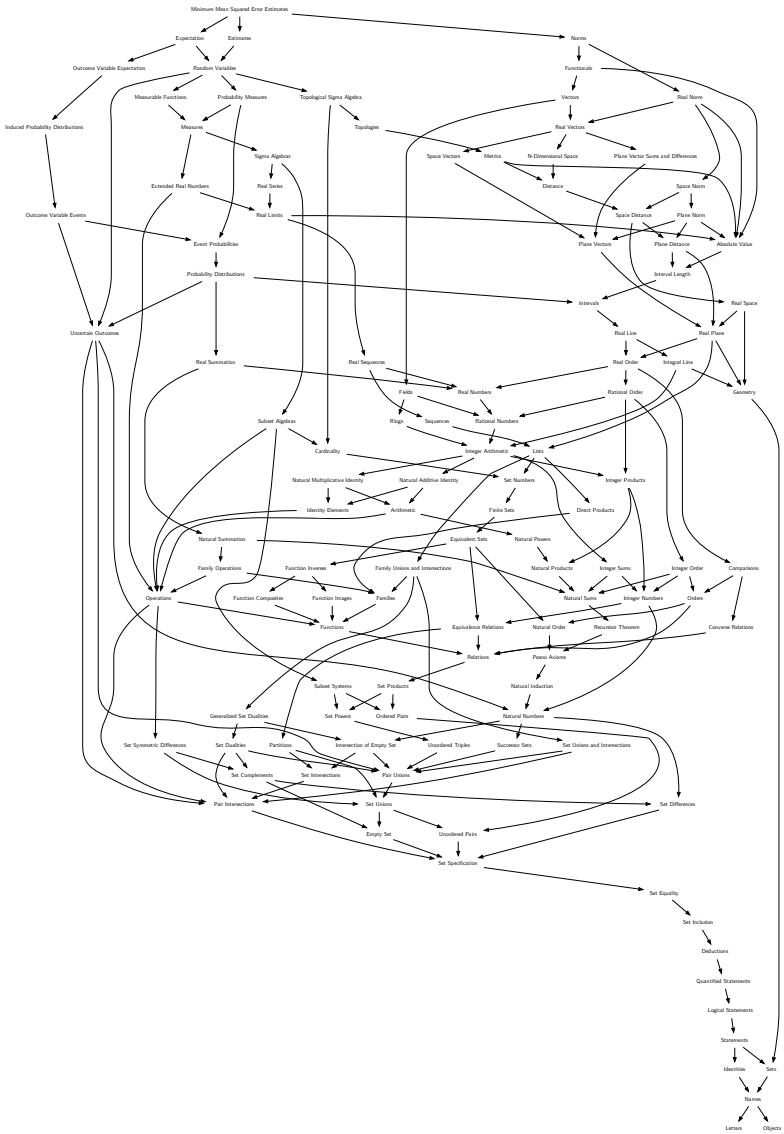
Vector case

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $y : \Omega \to \mathbf{R}^n$ a random variable.[1]

A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}\|x - \xi\|^2$.

Proposition 2. *There is a unique MMSE estimator and it is given by* $\mathbf{E}(y)$.

[1] Future editions might collapse this into the previous case.


Minimum Mean Squared Error Estimates
Expectation
Estimates
Norms
Outcome Variable Expectation
Random Variables
Functionals
Measurable Functions
Probability Measures
Topological Sigma Algebra
Vectors
Real Norm
Induced Probability Distributions
Measures
Topologies
Real Vectors
Sigma Algebras
Space Vectors
Metrics
N-Dimensional Space
Plane Vector Sums and Differences
Distance
Extended Real Numbers
Real Series
Space Norm
Space Distance
Outcome Variable Events
Real Limits
Plane Norm
Plane Vectors
Plane Distance
Absolute Value
Event Probabilities
Interval Length
Probability Distributions
Intervals
Real Space
Uncertain Outcomes
Real Line
Real Plane
Real Summation
Real Sequences
Real Order
Integral Line
Fields
Real Numbers
Rational Order
Geometry
Rings
Sequences
Rational Numbers
Subset Algebras
Integer Arithmetic
Lists
Cardinality
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Integer Products
Identity Elements
Arithmetic
Finite Sets
Direct Products
Natural Summation
Equivalent Sets
Natural Powers
Family Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Sums
Integer Order
Comparisons
Operations
Function Composites
Function Images
Families
Natural Sums
Integer Numbers
Orders
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Converse Relations
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Ordered Pairs
Natural Numbers
Set Symmetric Differences
Set Dualities
Partitions
Intersection of Empty Set
Unordered Triples
Successor Sets
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects