



# Functions

## 1 Why

We want a notion for a correspondence between two sets.

## 2 Definition

A *functional* relation on two sets relates each element of the first set with a unique element of the second set. A *function* is a functional relation.

The *domain* of the function is the first set and *codomain* of the function is the second set. The function *maps* elements *from* the domain *to* the codomain. We call the codomain element associated with the domain element the *result* of *applying* the function to the domain element.

### 2.1 Notation

Let  $A$  and  $B$  be sets. If  $A$  is the domain and  $B$  the codomain, we denote the set of functions from  $A$  to  $B$  by  $A \rightarrow B$ , read aloud as “A to B”.

We denote functions by lower case latin letters, especially  $f$ ,  $g$ , and  $h$ . The letter  $f$  is a mnemonic for function;  $g$  and  $h$  follow  $f$  in the Latin alphabet. We denote that  $f \in A \rightarrow B$  by  $f : A \rightarrow B$ , read aloud as “f from A to B”.

Let  $f : A \rightarrow B$ . For each element  $a \in A$ , we denote the result of applying  $f$  to  $a$  by  $f(a)$ , read aloud “f of a.” We sometimes drop the parentheses, and write the result as  $f_a$ , read aloud as “f sub a.” The set  $\{(a, f(a)) \in A \times B \mid a \in A\}$  of ordered pairs is the *graph* of  $f$ .

Let  $g : A \times B \rightarrow C$ . We often write  $g(a, b)$  or  $g_{ab}$  instead of  $g((a, b))$ . We read  $g(a, b)$  aloud as “g of a and b”. We read  $g_{ab}$  aloud as “g sub a b.”