Why

What is the best linear predictor if we choose according to a particular norm.

Definition

Suppose we have a paired dataset of n records with inputs in $\mathbf{R}^d$ and outputs in $\mathbf{R}$. A *norm weighted least squares linear predictor* for a norm $g : \mathbf{R}^n \to \mathbf{R}$ is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$g(y - Ax).$$

Weight matrix

Let $\|\cdot\|_W$ be the weighted norm for some positive semidefinite weight matrix W. We want to find x to minimize

$$\|y - AX\|_W.$$

This problem is referred to by many authors as *weighted least squares* or the *weighted least squares problem.*

Diagonal weight matrix

A special case of norm weighted least squares with a weighted norm is the usual weighted least squares problem (see Weighted Least Squares Linear Predictors). Consider weighted least squares with weights $w \in \mathbf{R}^n$, $w \geq 0$. Define $W \in \mathbf{R}^{n \times n}$ so that $W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix and

$$\|y - Ax\|_W = \sum_{i=1}^{n} w_i (y_i - x^\top a_i)^2.$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$\left(A^\top W A\right)^{-1} A^\top W y.$$

Noms Weighted Least Squares Linear Regression

Weighted Least Squares Linear Regression

Least Squares Linear Regression

Linear Prediction

Loss Function — Real Function Approximation — Matrix Transpose — Input Design — Consistent Inductive

Vector Space Basis — Real Matrix Inverse — Regression — Predictor Performance Metrics

Norms — Real Inner Products — Real Matrix-Matrix Products — Span — Linearly Dependent Vector — Prediction

Functionals — Real Function Space — Vectors as Matrices — Subspace — Linear Transformation — Linear Combination — Matrices

Real Inner Product — Real Subspace — Transformations — Linear Transformations — Real Matrices

Real Norm — Square Inner Product — Approximators — Real Vectors — Real Vector Space — Linear Equations

Similarity Functions — Space Norm — Real Linear Combinations — Linear Functions — Arrays

Metrics — Plane Norm — Plane Inner Product — N-Dimensional Space — Plane Vector Sums and Differences — Space Vectors

Absolute Value — Right Triangle Sides Relation — Distance — Plane Vector — Space Distance — Real Functions

Real Square — Space Distance — Plane Distance — Interval Length

Real Arithmetic — Interval Length

Real Multiplication Inverse — Real Space — Intervals — Lists

Square Numbers — Real Products — Real Addition Inverse — Real Line — Real Plane

Squares — Integral Lists — Real Sums — Real Order — Real Optimization — Real Summation — Inverse Elements

Geometry — Rational Multiplication Inverse — Natural Arithmetic — Real Numbers — Field — Set Numbers — Direct Products

Integer Products — Rational Products — Rational Sums — Rational Order — Rings — Optimizers — Natural Summation — Element Functions — Finite Sets

Integer Numbers — Rational Numbers — Integer Arithmetic — Comparison — Family Operations — Function Inverses — Equivalent Sets

Integer Sums — Integer Order — Natural Additive Identity — Natural Multiplicative Identity — Natural Equation — Operation — Equation — Function Composites — Function Image — Family Unions and Intersections

Arithmetic — Identity Elements — Orders — Natural Order — Functions — Families

Natural Power — Converse Relations — Equivalence Relations

Natural Products — Natural Sums — Relations

Recursion Theorem

Peano Axioms — Set Products

Natural Induction — Set Powers — Generalized Set Dualities

Natural Numbers — Set Symmetric Differences — Successor Sets — Unordered Triples — Set Dualities — Intersection of Empty Set — Partitions — Set Unions and Intersections

Set Complement — Ordered Pairs — Set Intersections

Set Differences — Set Unions — Pair Unions — Pair Intersections

Empty Set — Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deduction

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects

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