

MATRIX INVERSES

Why

What is the inverse element under matrix multiplication.

Definition

Recall that if $A \in \mathbb{R}^{m \times n}$ then x - Ax is a function from \mathbb{R}^n to \mathbb{R}^m . Clearly, if $m \neq n$, then the inverse of f can not exist.¹

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that BA = I we call B the *left inverse* of A and likewise if AC = I we call C the *right inverse* of A. In the case that A is square, the right inverse and left inverse coincide.

Proposition 1. Let $A, B, C \in \mathbb{R}^{n \times n}$. Let BA = I and AC = I. Then B = C.

Proof. Since BA = AC we have BBA = BAC so B = C since BA = I.

Notation

Let F be a field. Let $A \in F^{n \times n}$ be invertible. We follow the notation of inverse elements and denote the inverse of A by A^{-1} .

 $^{^1{\}rm Future}$ editions will expand.

