



**Definition**

Let  $\Sigma$  be an alphabet. A language  $L \subset \mathbf{str}(\Sigma)$  is called *regular* if there exists a finite automaton that recognizes it.

**Regular operations**

Let  $A, B \subset \mathbf{str}(\Sigma)$  be languages in  $\Sigma$ .

**Union**

The *union* (*alternation*) of  $A$  and  $B$  is, as usual, the set  $A \cup B$ .

**Concatenation**

The *concatenation* of  $A$  and  $B$  is the set  $\{xy \mid x \in A \text{ and } y \in B\}$ , where  $xy$  denotes length  $|x| + |y|$  string which is the concatenation of  $x$  and  $y$

**Multi-concatenation**

The *star* (*Kleene star*, *multi-concatenation*) of  $A$  is the set

$$\{x \in \mathbf{str}(\Sigma) \mid \exists k \geq 0, x = y_1 y_2 \cdots y_k, y_i \in A\}.$$

By this definition we do mean to include the empty string  $\emptyset$  in  $A^*$ , regardless of  $A$ .

**Notation**

We denote the alternation of  $A$  and  $B$  by  $A \cup B$  as usual, but other notations include  $A + B$ ,  $A \mid B$ , and  $A \vee B$ . We denote the concatenation of  $A$  and  $B$  by  $AB$ , but other notations include  $A \circ B$ . We denote the star of  $A$  by  $A^*$ .



