Why

We want to sum infinitely many real numbers.

Definition

Let n be a natural number. The nth *partial sum* of a sequence of real numbers is the sum of first n elements of the sequence. The first partial sum is the first term of the sequence. The third partial sum is the sum of first three elements of the sequence.

The *series* of the sequence is the sequence of partial sums. The sequence is *summable* if the series converges.

Since there exist sequences which do not converge, there exist sequences which are not summable. Consider the sequence which alternates between $+1$ and -1, and starts with $+1$. Its series alternates between $+1$ and 0, and so does not converge.

Notation

Let $(a_n)_n$ be a sequence of real numnbers. For natural number n, define:

$$s_n = \sum_{k=1}^{n} a_k.$$

Then $(s_n)_n$ is the series of $(a_n)_n$. If the series converges, then there exists a real number s, the limit, and we write:

$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

We read these relations aloud as "s is the limit as n goes to infinity of s n" and "s is the limit as n goes to infinity of the sum of a k from k equals 1 to n."

To avoid referencing s_n, we write:

$$\sum_{k=1}^{\infty} a_k = s,$$

read aloud as the "the sum from 1 to infinity of a k is s." The notation is subtle, and requires justification by the algebra of series. TODO

Real Series

Real Convergence

Absolute Value

Real Sequences

Length

Natural Differences

Real Numbers

Sequences

Rational Numbers

Function Composites

Direct Products

Natural Sums

Integer Numbers

Families

Natural Numbers

Functions

Set Intersections

Successor Sets

Relations

Pair Intersections

Cartesian Products

Set Powers

Unordered Triples

Ordered Pairs

Pair Unions

Set Unions

Empty Set

Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Definitions

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Objects

Letters