



## EQUIVALENT SETS

### Why

We want to talk about the size of a set.

### Definition

Two sets are *equivalent* if there exists a bijection between them.

PROPOSITION 1. *Set equivalence in the sense defined above is an equivalence relation in the power set of a set.*

PROPOSITION 2. *Every proper subset of a natural number is equivalent to some smaller natural number.*

*Proof.* TODO induction

□

TODO: smaller defined?

PROPOSITION 3. *A set can be equivalent to a proper subset of itself.*

Halmos' example here is not a bijection, though...

PROPOSITION 4. *If  $n$  is a natural number, then  $n$  is not equivalent to a proper subset of itself.*

PROPOSITION 5. *A set can be equivalent to at most one natural number.*

PROPOSITION 6. *The set of natural numbers is infinite.*

PROPOSITION 7. *A finite set is never equivalent to a proper subset of itself.*

PROPOSITION 8. *Every subset of a finite set is finite.*

PROPOSITION 9. *Every subset of a natural number is equivalent to a natural number.*

