



BOUNDED LINEAR NORM

Why

We can give the set of bounded linear functions between two norm spaces a norm.

Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

Notation

Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \rightarrow V_2$ be linear and bounded. The norm of f is the smallest C so that

$$\|f(v)\|_2 \leq C\|v\|_1.$$

Equivalent Formulation

Prop. 1. *Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \rightarrow V_2$ bounded and linear. The norm of f is*

$$\sup_{\|x\|_1=1} \|f(x)\|_2.$$

