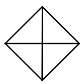


Why

We have seen that the matrices are a vector space. Are they an inner product space?

Definition

The *matrix scalar product* of $A \in \mathbf{R}^{n \times k}$ and $B \in \mathbf{R}^{n \times k}$ is the following product

$$\sum_{i=1}^{n} \sum_{j=1}^{k} a_{ij} b_{ij}.$$

Using the matrix trace, we can denote this as $\operatorname{tr} A^\top B$. Some authors call this the *Euclidean matrix scalar product*, *matrix inner product* or *Frobenius inner product.*

Proposition 1. *The matrix scalar product is an inner product.*[1]

For example, symmetry of the product is a consequence of the fact that a square matrix and its tranpose have identical traces. Commutativity of the trace yields $\operatorname{tr} A^\top B = \operatorname{tr} BA^\top$, where the LHS is the scalar product of $B^\top$ and $A^\top$. In other words, transposition "preserves" the matrix scalar product.

With this inner product, $\mathbf{R}^{n \times k}$ is a Euclidean vector space (see Inner Products) of dimension nk. For the case of $k = 1$, we recover a model[2] for the usual space $\mathbf{R}^n$.

Notation

We commonly denote the matrix inner product by $\langle A, B \rangle$.

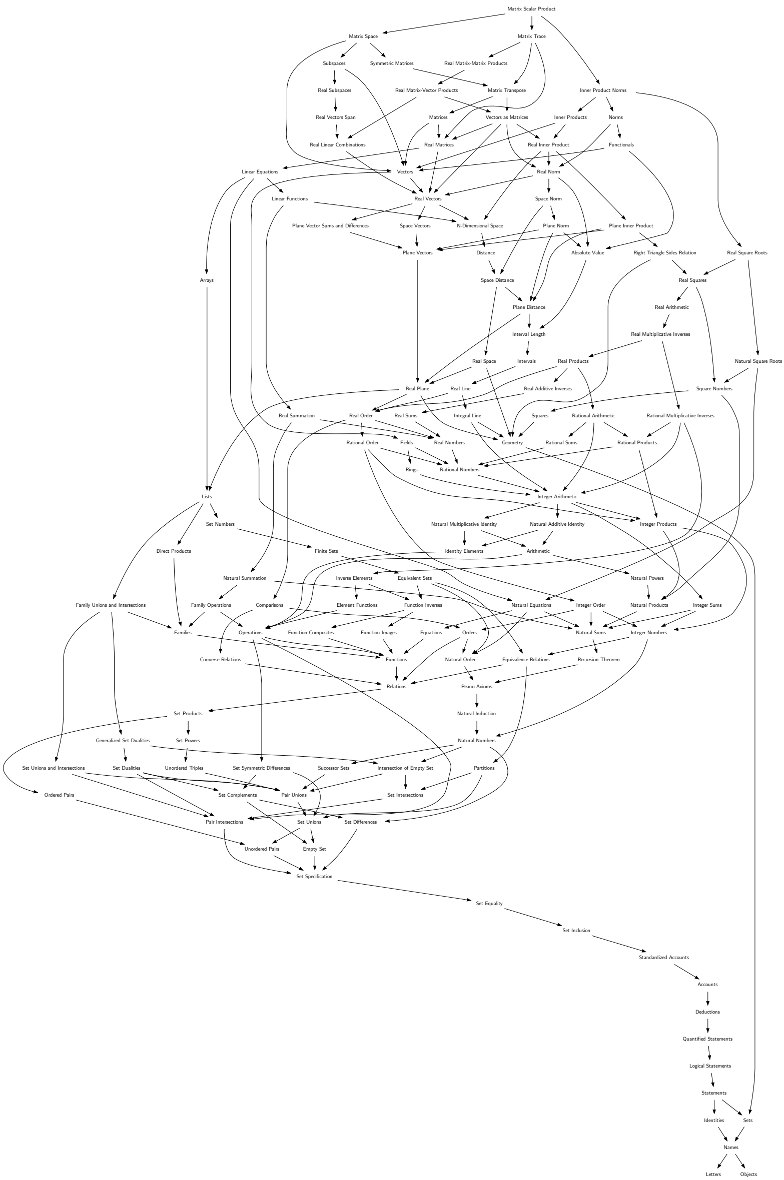
Induced norm

The matrix inner product induces a norm in the usual way. This norm is sometimes called the *Frobenius norm* and is often denoted for a matrix

[1]Future editions will provide an account.

[2]Future editions will define this term.

$A \in \mathbf{R}^{m \times n}$ by $\|A\|_F$.


Matrix Scalar Product
Matrix Space
Matrix Trace
Subspaces
Symmetric Matrices
Real Matrix-Matrix Products
Real Subspaces
Real Matrix-Vector Products
Matrix Transpose
Inner Product Norms
Real Vectors Span
Matrices
Vectors as Matrices
Inner Products
Norms
Real Linear Combinations
Real Matrices
Real Inner Product
Functionals
Linear Equations
Vectors
Real Norm
Linear Functions
Real Vectors
Space Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Norm
Plane Inner Product
Plane Vectors
Distance
Absolute Value
Right Triangle Sides Relation
Real Square Roots
Arrays
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Real Space
Intervals
Real Products
Natural Square Roots
Real Plane
Real Line
Real Additive Inverses
Square Numbers
Real Summation
Real Order
Real Sums
Integral Line
Squares
Rational Arithmetic
Rational Multiplicative Inverses
Rational Order
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Real Numbers
Geometry
Rational Sums
Rational Products
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Natural Summation
Inverse Elements
Equivalent Sets
Natural Powers
Family Unions and Intersections
Family Operations
Comparisons
Element Functions
Function Inverses
Natural Equations
Integer Order
Natural Products
Integer Sums
Families
Operations
Function Composites
Function Images
Equations
Orders
Natural Sums
Integer Numbers
Converse Relations
Functions
Natural Order
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
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Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
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Set Dualities
Unordered Triples
Set Symmetric Differences
Successor Sets
Intersection of Empty Set
Partitions
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
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Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects