



Why

We name those functions—an important set—whose range is contained in the real numbers.

Definition

A *real function* is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

Notation

Given *any* set A , $f : A \rightarrow \mathbf{R}$ is a real function. If $A = \mathbf{R}$, then $f \in \mathbf{R} \rightarrow \mathbf{R}$.

We often speak of functions defined on intervals. Given $a, b \in \mathbf{R}$, then $g : [a, b] \rightarrow \mathbf{R}$ is a real function defined on a closed interval. The function $h : (a, b) \rightarrow \mathbf{R}$ is a real function defined on an open interval.

We regularly declare the interval and the function at once. For example, “let $f : [a, b] \rightarrow \mathbf{R}$ ” is understood to mean “let a and b be real numbers with $a < b$, let $[a, b]$ be the closed interval with them as end-points, and let f be a real-valued function whose domain is this interval”. We read the notation $f : [a, b] \rightarrow \mathbf{R}$ aloud as “ f from closed a b to \mathbf{R} .” We use $f : (a, b) \rightarrow \mathbf{R}$ similarly (read aloud “ f from open a b to \mathbf{R} ”).

Examples

Example 1. Given $c \in \mathbf{R}$, define $f : \mathbf{R} \rightarrow \mathbf{R}$ by

$$f(x) = c \quad \text{for all } x \in \mathbf{R}$$

Example 2. Define $f : \mathbf{R} \rightarrow \mathbf{R}$ by

$$f(x) = 2x^2 + 1 \quad \text{for all } x \in \mathbf{R}$$

Example 3. Define $f : \mathbf{R} \rightarrow \mathbf{R}$ by

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{otherwise.} \end{cases}$$

