



# Index Matrices

## 1 Why

TODO

## 2 Definition

An *index sequence* of order  $n$  is a finite sequence of distinct elements of  $\{1, 2, \dots, n\}$  whose length is less than or equal to  $n$ . We call the  $i$ th coordinate of an index sequence the  *$i$ -index* of the sequence. The *index matrix* associated with an index sequence is the  $r \times n$  matrix whose  $i, j$ th entry is 1 if the index sequence's  $i$ th coordinate is  $j$ , and 0 otherwise. If  $r = n$  then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an  $n$ -vector associated with a length- $r$  index sequence is the product of the  $r \times n$  index matrix with the  $n$ -dimensional vector. Its  $i$ th entry is the  $i$ -index entry of the vector.

### 2.1 Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions, and so relations, and so sets), they are not sets of indices, so we avoid this usage.

## 2.2 Notation

Let  $r \leq n$  be natural numbers. Let  $\alpha : \{1, 2, \dots, r\} \rightarrow \{1, 2, \dots, n\}$  be an index sequence. We denote the index matrix associated with  $\alpha$  by  $P_\alpha$ . This matrix  $P_\alpha$  is an element of  $\mathbf{N}^{r \times n}$  and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let  $A$  be a nonempty set and let  $x \in A^n$ . then the subvector of  $x$  associated with  $P_\alpha$  (and so with  $\alpha$ ) is

$$P_\alpha x = (x_{\alpha(1)}, \dots, x_{\alpha(r)})$$

We denote the product  $P_\alpha x$  by  $x_\alpha$ .

We denote the product  $P_\alpha X P_\alpha^\top$  by  $X_{\alpha\alpha}$ .

## 3 Multiplication

The product of the  $n \times r$  transpose of an index matrix with an  $r$  vector is the  $n$  vector with The





