Integrable Function Spaces

1 Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? TODO: perhaps do L^2 first then generalize.

2 Definition

The *integrable function spaces* are a collection of function spaces, one for each real number $p \geq 1$, for which the pth power of the absolute value of the function is integrable.

TODO: case ∞

2.1 Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $p \geq 1$. Let R denote the set of real numbers. We denote the integrable function space corresponding to p by $\mathcal{L}^p(X, \mathcal{A}, \mu, R)$. We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, R) = \left\{ \text{measurable } f : X \to R \;\middle|\; \int |f|^p d\mu < \infty \right\}$$

Let C denote the set of complex numbers. Similarly for complex-valued functions, we denote the pth space by $\mathcal{L}^p(X, \mathcal{A}, \mu, C)$.