



## Why

We want language and notation for selecting some of the entries (possibly, with reordering—i.e. permuting) from a list.

## Definition

An *index list* of order  $n$  and length  $r \leq n$  is a list of *distinct* elements of  $\{1, 2, \dots, n\}$ . Its  $i$ -*index* is the  $i$ th coordinate, where  $i = 1, \dots, r$ .

## Examples

Here are some index lists of order 5:  $(1, 2, 3)$ ,  $(3, 2, 1)$ ,  $(4, 5, 1)$ ,  $(5, 4, 3, 2, 1)$ ,  $(3, )$ . These have lengths 3, 3, 3, 7 and 1, respectively. The 3-index of the first is 3, and of the second is 1.

## Induced sublist

The *sublist* of an length- $n$  list  $x$  *induced* by a length- $r$  index list  $\alpha$  is the length- $r$  list  $y$  whose  $i$ th entry is the value  $x_{\alpha_i}$ . In other words,

$$y_i = x_{\alpha_i}$$

For example, define  $x = (6, 4, 5, 3, 9)$ . The sublists associated with the example index lists above are  $(6, 4, 5)$ ,  $(5, 4, 6)$ ,  $(3, 9, 6)$ ,  $(9, 3, 5, 4, 6)$  and  $(5, )$ .

For a particular case, the third holds because

$$(3, 9, 6) = (x_4, x_5, x_1) = (x_{\alpha_1}, x_{\alpha_2}, x_{\alpha_3})$$

## Notation

We denote the induced sublist of list  $x$  induced by index list  $\alpha$  by  $x_\alpha$ . This is a slight abuse of notation, since we have so far defined a list with a subscript symbol mean the subscript-symbol term of that list. This ambiguity is avoided in our discussion if we keep in mind the types of the objects.

## Index sets

An *index set*  $S \subset 1, \dots, n$  can be associated with an index list in a natural way. It corresponds to the length- $|S|$  index list which has the elements of  $S$  in their natural order. We denote the induced sublist by  $x_S$ .

