Event Probabilities

1 Why

If we have some outcomes and a probability mass function, we can construct a function which assigns probabilities to events.

2 Definition

The *probability* of an event is the sum of the probabilities of the outcomes in the event. The *event probability function* is the correspondence assigning events to their probabilities.

3 Notation

Let A be a set of outcomes and p a probability mass function on A. Let $B \subset A$ be an event. Let $\mathbf{P} : 2^A \to R$ be the event probability function:

$$\mathbf{P}(B) = \sum_{b \in B} p(b).$$

Notice that $\mathbf{P}$ depends on the set of outcomes A and the probability mass function p. Sometimes we will include this when denoting $\mathbf{P}$ and denote it by $\mathbf{P}_{A,p}$.

4 Properties

Proposition 1. *Let* $\mathbf{P}$ *be the event probability function of a probability mass function* p *on a set of outcomes* A*. then*

1. $\mathbf{P}(B) \geq 0$ *for all* $B \subset A$
2. $\mathbf{P}(A) = 1$
3. $\mathbf{P}(B \cup C) = \mathbf{P}(B) + \mathbf{P}(C)$ *for* $B, C \subset A$ *and* $B \cap C = \varnothing$