



## SET INCLUSION

### Why

We want language for all of the elements of a first set being the elements of a second set.

### Definition

Denote a set by  $A$  and a set by  $B$ . If every element of the set denoted by  $A$  is an element of the set denoted by  $B$ , then we say that the set denoted by  $A$  is a *subset* of the set denoted by  $B$ .

We say that the set denoted by  $A$  is *included* in the set denoted by  $B$ . We say that the set denoted by  $B$  is a *superset* of the set denoted by  $A$  or that the set denoted by  $B$  *includes* the set denoted by  $A$ .

Every set is included in and includes itself.

### Notation

Let  $A$  denote a set and  $B$  denote a set. We denote that the set  $A$  is included in the set  $B$  by  $A \subset B$ . In other words,  $A \subset B$  means  $(\forall x)((x \in A) \longrightarrow (x \in B))$ . We read the notation  $A \subset B$  aloud as “ $A$  is included in  $B$ ” or “ $A$  subset  $B$ ”. Or we write  $B \supset A$ , and read it aloud “ $B$  includes  $A$ ” or “ $B$  superset  $A$ ”.  $B \supset A$  also means  $(\forall x)((x \in A) \longrightarrow (x \in B))$ .

### Properties

There are some properties that our intuition suggests inclusion should have. First, every set should include itself. We describe this fact by saying that inclusion is *reflexive*.

**Proposition 1** (Reflexive). *Every set is included in itself*

*Proof.* (1) name  $A$ ; (2) have  $(\forall x)(x \in A \longrightarrow x \in A)$ ; (3) thus  $A \subset A$  by `SetInclusion:Definition`.  $\square$

Next, we expect that if one set is included in another, This fact is described by saying that inclusion is *transitive*

**Proposition 2** (Transitive). *If a set is included in another, and the latter in yet another, then the first is included in the last.*

*Proof.* (1) name  $A, B, C$ ; (2) have  $A \subset B$  (3) have  $B \subset C$  (4) thus  $A \subset C$  by `modus ponens`.  $\square$

Equality ( $=$ ) shares these two properties. Let  $A$  denote an object. Then  $A = A$ . Let  $B$  and  $C$  also denote objects. If  $A = B$  and  $B = C$ , then  $A = C$ . Of course, inclusion is not symmetric.. Belonging ( $\in$ ) may be, but need not be reflexive and transitive.

