



## SET POWERS

### Why

We want to consider all the subsets of a given set.

### Definition

We do not yet have a principle stating that such a set exists, but our intuition suggests that it does.

**Principle 1** (Powers). *For every set, there exists a set of its subsets.*

We call the existence of this set the *principles of powers* and we call the set the *power set*.<sup>1</sup> As usual, the principle of extension gives uniqueness (see *Set Equality*). The power set of a set includes the set itself and the empty set.

### Notation

Let  $A$  denote a set. We denote the power set of  $A$  by  $A^*$ , read aloud as “powerset of  $A$ .”  $A \in A^*$  and  $\emptyset \in A^*$ . However,  $A \subset A^*$  is false.

### Examples

Let  $a, b, c$  denote distinct objects. Let  $A = \{a, b, c\}$  and  $B = \{a, b\}$ . Then  $B \subset A$ . In other notation,  $B \in A^*$ . We can walk through examples of power sets.

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<sup>1</sup>This terminology is standard, but unfortunate. Future editions may change these terms.

ℓEmpty Set

**Proposition 1.**  $\emptyset^* = \{\emptyset\}$

ℓSingletons

**Proposition 2.**  $\{a\}^* = \{\emptyset, \{a\}\}$

ℓPairs

**Proposition 3.**  $\{a, b\}^* = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

ℓTriples

**Proposition 4.**  $\{a, b, c\}^* = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

## Properties

We can guess the following easy properties.<sup>2</sup>

**Proposition 5.**  $\emptyset \in A^*$

**Proposition 6.**  $A \in A^*$

We call  $A$  and  $\emptyset$  the *improper* subsets of  $A$ . All other subset we call *proper*.

## Basic Fact

**Proposition 7.**  $E \subset F \longrightarrow E^* \subset F^*$

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<sup>2</sup>Future editions will expand this account.

