



## CARTESIAN PRODUCTS

### Why

Does a set exist of all the ordered pairs of elements from an ordered pair of sets?

### Definition

Let  $A$  and  $B$  denote sets. Ordered pairs are sets of singletons and pairs. So to construct the set of all ordered pairs taken from two sets, we want to specify the elements of a set which contains all singletons  $\{a\}$  and pairs  $\{a, b\}$  for  $a \in A$ ,  $b \in B$ .

Notice that  $a \in A$  and  $b \in A$  mean  $a, b \in (A \cup B)$ . In other words,  $\{a\} \subset A$  and  $\{b\} \subset B$  and  $\{a\}, \{b\} \subset (A \cup B)$ . In particular,  $\{a\} \in (A \cup B)^*$ . Similarly,  $\{a, b\} \in (A \cup B)^*$ . And so  $\{\{a\}, \{a, b\}\} \in (A \cup B)^{**}$ .

We define the set of “all ordered pairs” from  $A$  and  $B$  by specifying the appropriate pairs of this set.<sup>1</sup>

$$\{(a, b) \in (A \cup B)^{**} \mid a \in A \wedge b \in B\}$$

We name this set the *product* of the set denoted by  $A$  and the set denoted by  $B$  is the set of all ordered pairs. This set is also called the *cartesian product*.<sup>2</sup> If  $A \neq B$ , the ordering causes the product of  $A$  and  $B$  to differ from the product of  $B$  with  $A$ . If  $A = B$ , however, the symmetry holds.

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<sup>1</sup>The specific statement used here requires some translation. A discussion of this and the full statement will appear in a future edition.

<sup>2</sup>This is the current name of the sheet, but may change in future editions, in accordance with the project policy on using names.

## Notation

We denote the product of  $A$  with  $B$  by  $A \times B$ , read aloud as “A cross B.” In this notation, if  $A \neq B$ , then  $A \times B \neq B \times A$ .

