Why

It is natural to look for a class of structural equation models with favorable identifiability and properties.

Definition

A *d-dimensional rooted tree linear cascade* is a sequence of four objects: a tree on $\{1, \ldots, d\}$, a vertex of the tree, a family of real numbers indexed by the edges of the tree, and a d-dimensional random vector whose covariance matrix is the identity matrix. The cascade is called "d-dimensional" because we associate it with a random vector (defined as a function of that in the form of its definition) whose codomain is $\mathbf{R}^d$.

The tree together with the vertex form a rooted tree. The graph associated with the rooted tree and the family of real numbers together form a weighted graph.

The idea is to use the weights and the tree structure to recursively define a random vector in terms of elements of the given random vector. Let $C = (T, i, w, e)$ be a d-dimensional rooted tree linear cascade. So T is a tree on $\{1, \ldots, d\}$, $i \in \{1, \ldots, d\}$ and $w : T \to \mathbf{R}$, and $e : \Omega \to \mathbf{R}^d$ for some probability space $(A, \mathcal{A}, \mathbf{P})$. The random vector associated with C is the random variable $x : \Omega \to \mathbf{R}^d$ defined by

$$x_i = e_i \text{ and } x_j = w_{\{\mathrm{pa}\, j, j\}} x_{\mathrm{pa}\, j} + e_j \text{ for } j \neq i.$$

In other words,

$$e = Ax$$

where A is lower triangle and extremely sparse.[1]

Notation

Let $(A, \mathcal{A}, \mathbf{P})$ be a probability space. Let $e : A \to \mathbf{R}^d$ be a random vector, let T be a tree on $\{1, \ldots, d\}$ with $a_{ij} = a_{ji}$ the weight on edge $\{i, j\} \in T$.

[1] Future editions will clarify the meaning of the term sparse.

Rooted Tree Linear Cascades
Real Matrix-Matrix Products
Covariance-Matrix
Multivariate Real Densities
Covariance
Probability Densities
Real Matrix-Vector Products
Random Vectors
Real Integrals
Real Valued Random Variable Variance
Real Linear Combinations
Vectors as Matrices
Nonnegative Integrals
Real Valued Random Variable Expectation
Real Inner Product
Real Numbers
Simple Integrals
Random Variable
Real Norm
Real Linear Equations
Measurable Functions
Topological Sigma Algebra
Probability Measures
Outcome Variable Covariance
Space Inner Product
Real Vectors
Linear Functions
Zeroes
Topologies
Outcome Variable Expectation
Space Norm
N-Dimensional Space
Metrics
Plane Inner Product
Plane Vector Norm and Differences
Space Vectors
Length Measure
Sigma Algebra
Outcome Variable Probabilities
Distance
Plane Norm
Right Triangle Sides Ratios
Real Square Roots
Extended Real Numbers
Real Series
Outcome Variables
Space Distance
Plane Vectors
Real Squares
Real Values
Event Probabilities
Absolute Value
Plane Distance
Real Arithmetic
Simple Functions
Discrete Probabilities
Real Multiplication Inverses
Real Numbers
Real Subtractions
Natural Length
Natural Square Roots
Real Products
Rational Trees
Real Square
Integers
Real Powers
Real Additive Inverses
Zeros
External Grades
Real Sequences
Size of Direct Products
Square Roots
Square Numbers
Real Sums
Rational Multiplication Inverses
Number of Set Products
Unidentical Parts
Natural Cardinalities
Equivalences
Subset Algebra
Squares
Integer Unit
Rational Arithmetic
Real Units
Real Sums
Number of Direct Unions
Unidentical Grades
Set Numbers
Finite Set Remainders
Cardinality
Geometry
Rational Grades
Rational Sums
Real Numbers
Set Numbers
Other Products
Uncertain Events
Natural Sequences
Natural Numbers
Integer Arithmetic
Finite Sets
Natural Addition Identity
Natural Multiplication Identity
Equivalence Sets
Inverse Elements
Integer Products
Family Unions and Intersections
Integer Sums
Integer Order
Natural Exponents
Natural Arithmetic
Identity Elements
Compositions
Function Inverses
Element Functions
Family Operations
Integer Numbers
Natural Powers
Grades
Equations
Function Images
Function Compositions
Operations
Natural Products
Natural Grades
Concrete Relations
Equivalence Relations
Functions
Natural Sums
Measures
Set Partitions
Summation Theorem
Power Sets
Cartesian Projections
Discrete Processes
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Generalized Set Doubles
Natural Numbers
Set Powers
Ordered Pairs
Set Unions and Intersections
Intersections of Sets Set
Set Doubles
Separate Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Described Statements
Logical Statements
Statements
Sets
Identities
Zeroes
Letters
Objects