Why

We want a notation for expressing the sum of several natural numbers.

Definition

Let $s = (m_1, \ldots, m_n)$ be a sequence of natural numbers. The *sequence sum* of s is the result of first summing the first two numbers, then summing the result with the third number, and so on, until we have summed all the numbers

Notation

We denote the sum of a sequence using by using the $\sum$ symbol. $\sum$ is the capital greek letter "sigma" and is a mnemonic for "sum."

Let $(m_1, \ldots, m_n)$ be a sequence of natural numbers. Let us denote by m_i an element of the sequence, where $i = 1, \ldots, n$.

We denote that the sum ranges over an (ordered) index set $\{1, \ldots, n\}$ by writing $\sum_{i=1}^{n}$. We denote the sequence sum

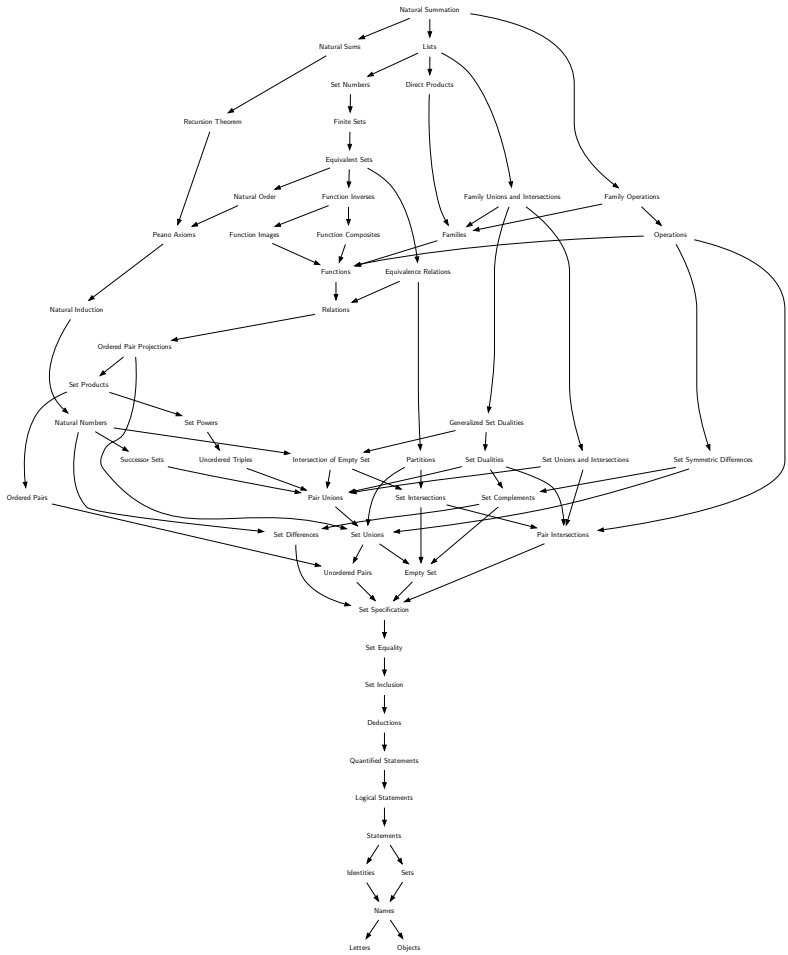
$$\sum_{i=1}^{n} m_i.$$

Summing over finite sets

Suppose A is a finite set and $f : A \to \mathbf{N}$ is a function. Then we the notation

$$\sum_{a \in A} f(a)$$

is notation for $\sum_{i=1}^{n} f(\sigma(i))$ where $\sigma : \{1, \ldots, |A|\} \to A$ is any numbering of A. The numbering is inconsequential because addition is commutative.


Natural Summation
Natural Sums
Lists
Set Numbers
Direct Products
Recursion Theorem
Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Family Unions and Intersections
Family Operations
Peano Axioms
Function Images
Function Composites
Families
Operations
Functions
Equivalence Relations
Natural Induction
Relations
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Dualities
Set Unions and Intersections
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects