



Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Proposition 1. *Let f be a square-integrable real-valued random variable on the probability space (X, \mathcal{A}, μ) . Then for $t > 0$,*

$$\mu[|f - \mathbf{E}(f)| \geq t] \leq \frac{\text{var } f}{t^2}.$$

Proof. The symbols $|f - \mathbf{E}(f)| \geq t$ denote the set $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$. This set is the same as the set

$$\{x \in X \mid (f(x) - \mathbf{E}(f))^2 \geq t^2\}.$$

By using the tail measure upper bound,

$$\mu(\{x \in X \mid (f(x) - \mathbf{E}(f))^2 \geq t^2\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))^2}{t^2}.$$

We recognize the numerator of the right hand side as the **variance** of f . \square

The above is also called *Chebychev's Inequality*.

