



## Why

The relationship between the inclusion map and the identity map is characteristic of making small functions out of large ones.<sup>1</sup>

## Definition

Let  $X \subset Y$  and  $f : Y \rightarrow Z$ . There is a natural function  $g : X \rightarrow Z$ , namely the one defined by  $g(x) = f(x)$  for all  $x \in X$ . We call  $g$  the *restriction* of  $f$  to  $X$ . We call  $f$  an *extension* of  $g$  to  $Y$ . Clearly, there may be more than one extension of a function

## Notation

We denote the restriction of  $f : Y \rightarrow Z$  to the set  $X \subset Y$  by  $f \upharpoonright X$ .

## Example

A simple example is the that the inclusion mapping from  $X$  to  $Y$  with  $X \subset Y$  is a restriction of the identity map on  $X$

## An extension order

Here is a natural order involving set extensions and restrictions. Fix two sets  $A$  and  $B$ . Let  $F$  be the set of all functions  $f : X \rightarrow Y$  with  $X \subset A$  and  $Y \subset B$ . Define a relation  $R$  in  $F$  by  $(f, g) \in R$  if  $\text{dom } f \subset \text{dom } g$  and  $f(x) = g(x)$  for all  $x$  in  $\text{dom } f$ . In other words,  $(f, g) \in R$  if  $f$  is a restriction of  $g$  (or, equivalently,  $g$  is an extension of  $f$ ). We recognize that  $R$  is a special case of the inclusion partial order by recognizing the elements of  $F$  as subsets  $A \times B$ .

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<sup>1</sup>Future editions will modify this language.



