Optimizers

1 Why

Given a correspondence between a set and a chain, we are interested in the objects which correspond to minimal or maximal elements of the chain.

2 Definition

Consider a non-empty set and a chain, with a function associating to each element of a set an element of the chain (a chain is a set with a total order).

A *minimizer* of the function is an element of its domain whose result under the function is a minimal element of the function's range. In other words, the result of the function on that element is less than the result of the function on any other element in its domain. Similarly, a *maximizer* of the function is an element of its domain whose result under the function is a maximal element of the function's range. In this context, we call elements of the function's domain *feasible*.

2.1 Notation

Let A be a non-empty set and $(C, \leq)$ a chain. Let $f : A \to C$. An element $a \in A$ is a minimizer of f if $f(a) \leq f(b)$ for all $b \in A$. Similarly, an element $a \in A$ is a maximizer of f if $f(a) \geq f(b)$ for all $b \in A$.

We denote the set of minimizers by f by $\mathsf{\mathbf{argmin}}\, f$ and the set of maximizers of f by $\mathsf{\mathbf{argmax}}\, f$. In other notation,

$$\mathsf{\mathbf{argmin}}\, f = \{a \in A \mid \forall b \in A, f(a) \leq f(b)\}$$

and

$$\mathbf{\mathsf{argmin}}\, f = \{a \in A \mid \forall b \in A, f(a) \geq f(b)\}.$$