Why

We want to extend addition to $\mathbf{C}$.

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex sum* of z_1 and z_2 is the complex number $(x_1 + x_2, y_1 + y_2)$.

Notation

For $z_1, z_2 \in \mathbf{C}$, we denote the complex sum of z_1 and z_2 by $z_1 + z_2$. The notation is justified because the complex sum of two purely real complex numbers corresponds to the purely real complex numbers whose real part is the real sum of the real parts of the first two numbers.

Recall that we denote $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. For example, we can express the definition of addition as

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2).$$

Properties

Proposition 1 (Commutativity). *For all $z_1, z_2 \in \mathbf{C}$, we have $z_1 + z_2 = z_2 + z_1$.*

Proposition 2 (Associativity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have and $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$.*

Complex addition

We call the operation that associates a pair of complex numbers with their sum *complex addition*. The operation is symmetric (commutative).

Additive identity and inverse

Notice that the complex number $(0,0)$ is the additive identity. It is unique,[1] and so we call it the *complex multiplicative identity*. Likewise, notice that additive inverse for (x, y) is $(-x, -y)$.

---

[1]Future editions will include an account

Complex Sums

Complex Numbers

Absolute Value

Real Arithmetic

Interval Length

Real Multiplicative Inverses

Intervals

Real Products

Real Line

Real Additive Inverses

Integral Line

Rational Multiplicative Inverses

Rational Arithmetic

Real Sums

Real Order

Rational Products

Rational Sums

Real Numbers

Rational Order

Rational Numbers

Integer Arithmetic

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Identity Elements

Arithmetic

Natural Powers

Inverse Elements

Function Inverses

Element Functions

Natural Products

Integer Sums

Integer Order

Comparisons

Function Composites

Function Images

Operations

Natural Sums

Integer Numbers

Orders

Functions

Recursion Theorem

Equivalence Relations

Natural Order

Converse Relations

Peano Axioms

Relations

Natural Induction

Set Products

Ordered Pairs

Natural Numbers

Set Powers

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Symmetric Differences

Set Intersections

Pair Unions

Pair Intersections

Set Complements

Set Unions

Empty Set

Set Differences

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects