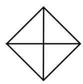


Why

We name those functions—and important set—whose range is contained in the real numbers.

Definition

A *real function* is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

Notation

Given *any* set A, $f : A \to \mathbf{R}$ is a real function. If $A = \mathbf{R}$, then $f \in \mathbf{R} \to \mathbf{R}$.

We often speak of functions defined on intervals. Given $a, b \in \mathbf{R}$, then $g : [a, b] \to \mathbf{R}$ is a real function defined on a closed interval. The function $h : (a, b) \to \mathbf{R}$ is a real function defined on an open interval.

We regularly declare the interval and the function at once. For example, "let $f : [a, b] \to \mathbf{R}$" is understood to mean "let a and b be real numbers with $a < b$, let $[a, b]$ be the closed interval with them as endpoints, and let f be a real-valued function whose domain is this interval". We read the notation $f : [a, b] \to \mathbf{R}$ aloud as "f from closed a b to $\mathbf{R}$." We use $f : (a, b) \to \mathbf{R}$ similarly (read aloud "f from open a b to $\mathbf{R}$").

Examples

Example 1. *Given $c \in \mathbf{R}$, define $f : \mathbf{R} \to \mathbf{R}$ by*

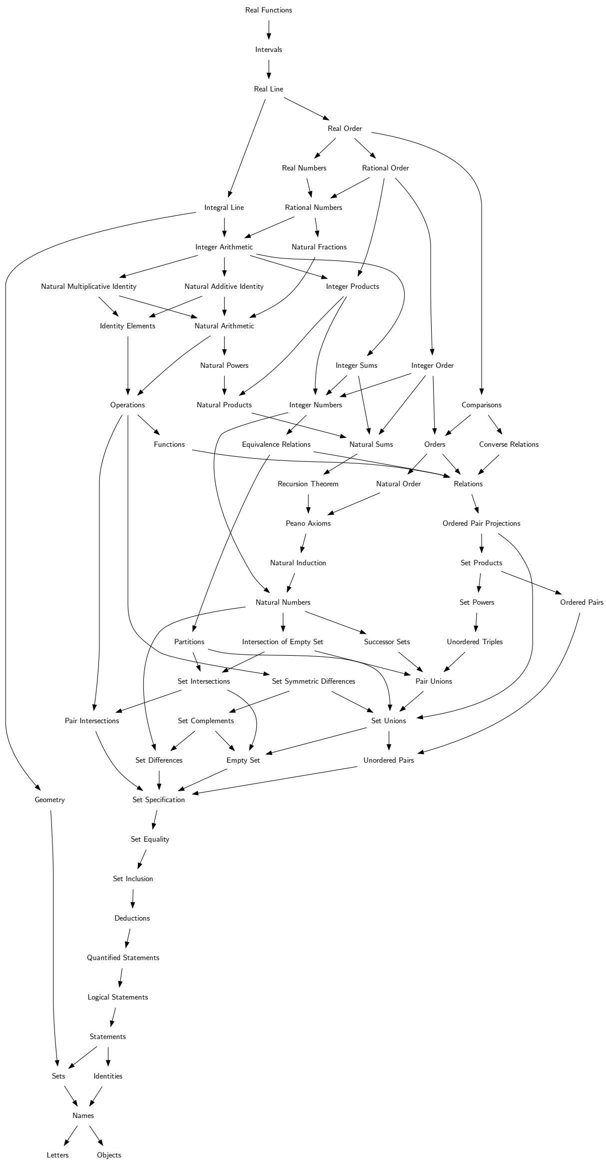
$$f(x) = c \quad \textit{for all } x \in \mathbf{R}$$

Example 2. *Define $f : \mathbf{R} \to \mathbf{R}$ by*

$$f(x) = 2x^2 + 1 \quad \textit{for all } x \in \mathbf{R}$$

Example 3. *Define $f : \mathbf{R} \to \mathbf{R}$ by*

$$f(x) = \begin{cases} 1 & \textit{if } x \in \mathbf{Q} \\ 0 & \textit{otherwise.} \end{cases}$$


Real Functions
Intervals
Real Line
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Fractions
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Identity Elements
Natural Arithmetic
Natural Powers
Integer Sums
Integer Order
Operations
Natural Products
Integer Numbers
Comparisons
Functions
Equivalence Relations
Natural Sums
Orders
Converse Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects