



Why

Let $X = \{a, b\}$ and $Y = \{0, 1\}$. The dataset $((a, 0))$ is consistent, but it is not functionally complete. On the other hand, the dataset $((a, 0), (b, 0), (a, 0), (a, 0), (a, 0), (a, 1))$ is complete but it is not *functionally* consistent.

In general, if $y_i \neq y_j$ for some i and j where $x_i = x_j$, then the dataset is not functionally consistent. In the preceding example, both $(a, 0)$ and $(a, 1)$ appear.

If we emphasize the “predictive” aspect of a functional inductor, we interpret the input as an object we “see before” the output. And so treat $y \in Y$ as an uncertain outcome which is the element associated to $x \in X$.

In this case, we may use the language of probability to discuss this uncertain outcome. If, for example, Y is finite, we can associate a distribution with each input $x \in X$.

Definition

Let (X, \mathcal{X}) and (Y, \mathcal{Y}) be measurable spaces.

A *probabilistic functional inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a family of measures on (Y, \mathcal{Y}) , indexed by X . We call a function from inputs to output measures a *probabilistic predictor*. We call the distribution a *probabilistic prediction*.

Notation

Let $\mathcal{M}(Y, \mathcal{Y})$ be the set of measures on Y . Let D be a dataset in $(X \times Y)^n$. Let $g : X \rightarrow \mathcal{M}(Y, \mathcal{Y})$ a probabilistic predictor. Let $G_n(X \times Y)^n \rightarrow (X \rightarrow \mathcal{M}(Y, \mathcal{Y}))$ be a predictive probabilistic inductor. Then $G_n(D)$ is a family of measures $\{g_x : \mathcal{Y} \rightarrow [0, 1]\}_{x \in X}$.

