

## Complex Numbers

## Why

We want to find the roots of negative numbers.<sup>1</sup>

## Definition

A complex number is an ordered pair of real numbers. The real part of a complex number is its first coordinate. The imaginary part of a complex number is its second coordinate.

We can identify the imaginary numbers with no complex part (i.e.,  $\{(a,b) \in \mathbb{R}^2 \mid b=0\}$  with  $\mathbb{R}$  in the obvious way. For this reason, such a complex number is sometimes referred to as a *purely real number* Conversely, a complex number with zero imaginary part (i.e., an element of  $\{(a,b) \in \mathbb{R}^2 \mid a=0\}$ ) is said to be a *purely imaginary number*.

## Notation

When treating  $\mathbb{R}^2$  as the set of complex numbers, we denote it by  $\mathbb{C}$ . Let  $z \in \mathbb{C}$  with z = (a, b). The real part of z is a and its imaginary part is b. It is universal to denote z by a + ib, and to call i an (or the) *imaginary number*. Some authors use j, it is a matter of notation.

We denote the real part of z by  $\mathbf{Re}(z)$ , read "real of z," and the imaginary part by  $\mathbf{Im}(z)$ , read "imaginary of z." So, in particular,  $\mathbf{Re}(z) = a$  and  $\mathbf{Im}(z) = b$ .

 $<sup>^{1}</sup>$ Future editions will modify this, and will discuss the existence of solutions of algebraic equations.

