# Graphs

## 1 Why

We want to visualize relations.

## 2 Definition

A *graph* is a set and a relation on the set. The graph is *undirected* if the relation is symmetric; otherwise the graph is *directed*.

A *vertex* of the graph is an element of the set. The set is called the *vertex set*. An *edge* of the graph is an element of the relation. The relation is called the *edge set*.

If the graph is directed, we call the first element of an edge the *parent* of the second element. We call the second element of an edge the *child* of the first element. So we can discuss the set of parents or set of children of a particular vertex (these sets may be empty).

### 2.1 Notation

We denote the vertex set by $V$, a mnemonic for vertex. We denote the edge set by $E$, a mnemonic for edge. We denote a graph by $(V, E)$. If the vertex set is assumed, or if every vertex appears in $E$ we can unambiguously refer to the graph by $E$.

### 2.2 Visualization

We visualize a graph by drawing a point for each vertex. If two vertices $u$ and $v$ are in relation, we draw a line from the point corresponding to $u$ to the point corresponding to $v$ with an arrow at the point corresponding to $v$. If the graph is undirected, we omit arrows. Here are all undirected graphs on three vertices.

## 3 Paths

A path in a graph is a sequence of vertices with the property that consecutive vertices are related. A path *cycles* if a vertex appears more than once. A path is *finite* if the sequence is finite. A *loop* is a finite path that cycles once. A finite path from vertex $u$ to vertex $v$ is a path starting with $u$ and ending with $v$. The *length* of a finite path is the length of the sequence.

## 4 Properties

A graph is *connected* if there is a path between every pair of vertices. A graph is *acyclic* if none of its paths cycle.