Definition

Suppose $T \in \mathcal{L}(V, W)$. In other words, T is a linear map from a vector space V to a vector space W where V and W are over the same field of scalars.

An *adjoint* of T is a function $S : W \to V$ satisfying

$$\langle Tv, w \rangle = \langle v, Sw \rangle \quad \text{for every } v \in V \text{ and every } w \in W$$

It is not hard to see that there always exists an adjoint, and that this adjoint is unique. Thus, we speak of *the adjoint* of T.

Notation

We denote *the* adjoint of T by T^*.

Examples

Space to the plane. Define $T : \mathbf{R}^3 \to \mathbf{R}^2$ by

$$T(x_1, x_2, x_3) = (x_2 + 3x_3, 2x_1)$$

We claim that the adjoint of T is $T^* : \mathbf{R}^2 \to \mathbf{R}^3$ defined by

$$T^*(y_1, y_2) = (2y_2, y_1, 3y_1)$$

Properties

Proposition 1 (Adjoint is Linear)**.** *Suppose $T \in \mathcal{L}(V, W)$. The adjoint of T is linear.*

Proposition 2 (Adjoint properties)**.** *Suppose V and W are finite dimensional inner product spaces over a field $\mathbf{F}$, which is $\mathbf{R}$ or $\mathbf{C}$. Suppose $S, T \in \mathcal{L}(V, W)$. Then*

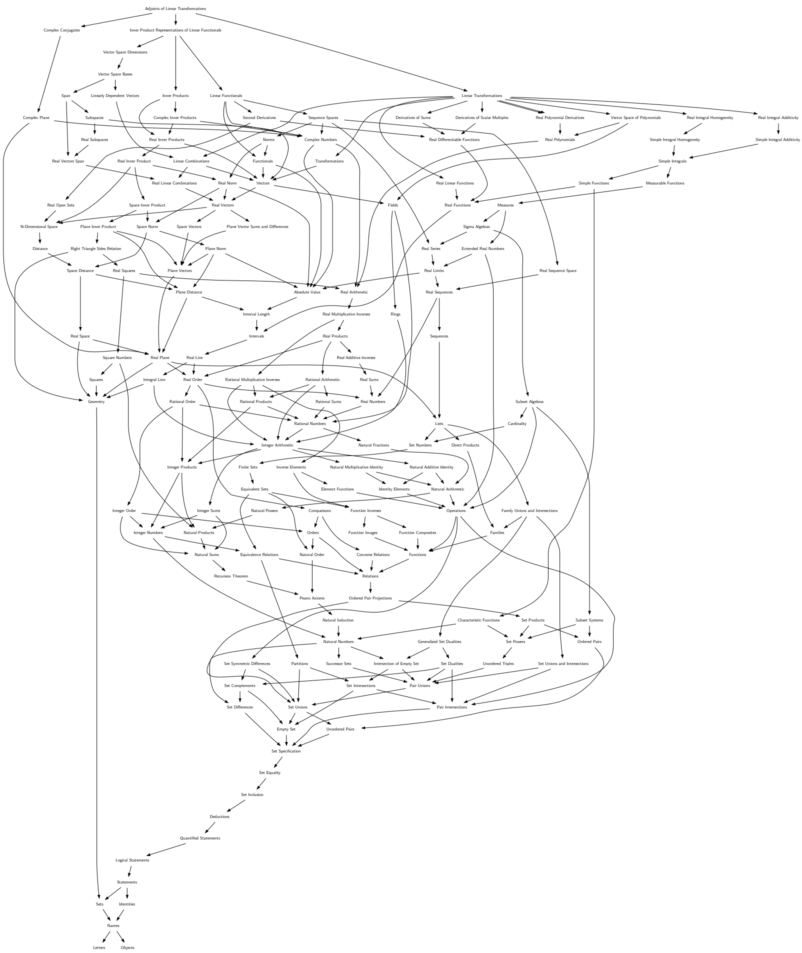
1. $(S + T)^* = S^* + T^*$
2. $(\lambda T)^* = \lambda^* T^*$ *for all* $\lambda \in \mathbf{F}$

3. $(T^*)^* = T$

4. $I^* = I$

Proposition 3. *Suppose V, W, U are finite dimensional inner product spaces over $\mathbf{R}$ or $\mathbf{C}$. For all $T \in \mathcal{L}(V, W)$ and $S \in \mathcal{L}(W, U)$,*

$$(ST)^* = T^* S^*$$


Adjoints of Linear Transformations
Complex Conjugates
Inner Product Representations of Linear Functionals
Vector Space Dimensions
Vector Space Bases
Span
Linearly Dependent Vectors
Inner Products
Linear Functionals
Linear Transformations
Complex Plane
Subspaces
Complex Inner Products
Second Derivatives
Sequence Spaces
Derivatives of Sums
Derivatives of Scalar Multiples
Real Polynomial Derivatives
Vector Space of Polynomials
Real Integral Homogeneity
Real Integral Additivity
Real Subspaces
Real Inner Products
Norms
Complex Numbers
Real Differentiable Functions
Real Polynomials
Simple Integral Homogeneity
Simple Integral Additivity
Real Vectors Span
Real Inner Product
Linear Combinations
Functionals
Transformations
Simple Integrals
Real Linear Combinations
Real Norm
Vectors
Real Linear Functions
Simple Functions
Measurable Functions
Real Open Sets
Space Inner Product
Real Vectors
Fields
Real Functions
Measures
N-Dimensional Space
Plane Inner Product
Space Norm
Space Vectors
Plane Vector Sums and Differences
Sigma Algebras
Distance
Right Triangle Sides Relation
Plane Norm
Real Series
Extended Real Numbers
Space Distance
Real Squares
Plane Vectors
Real Limits
Real Sequence Space
Plane Distance
Absolute Value
Real Arithmetic
Real Sequences
Interval Length
Real Multiplicative Inverses
Rings
Real Space
Intervals
Real Products
Sequences
Square Numbers
Real Plane
Real Line
Real Additive Inverses
Squares
Integral Line
Real Order
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Geometry
Rational Order
Rational Products
Rational Sums
Real Numbers
Subset Algebras
Rational Numbers
Lists
Cardinality
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Finite Sets
Inverse Elements
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Element Functions
Identity Elements
Natural Arithmetic
Integer Order
Integer Sums
Natural Powers
Comparisons
Function Inverses
Operations
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects