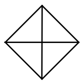


Why

Can a set have no elements?

Definition

Sure. A set exists by the principle of existence (see Sets); denote it by A. Specify elements (see Set Specification) of any set that exists using the universally false statement $x \neq x$. We denote that set by $\{x \in A \mid x \neq x\}$. It has no elements. In other words, $(\forall x)(x \notin A)$. The principle of extension (see Set Equality) says that the set obtained is unique (contradiction).[1]

Definition 1 (Empty Set). We call the unique set with no elements *the empty set.*

ßNotation

We denote the empty set by $\varnothing$. In other words, in all future accounts (see Accounts), there are two implicit lines. First, "`name` $\varnothing$" and second "`have` $(\forall x)(x \notin \varnothing)$".

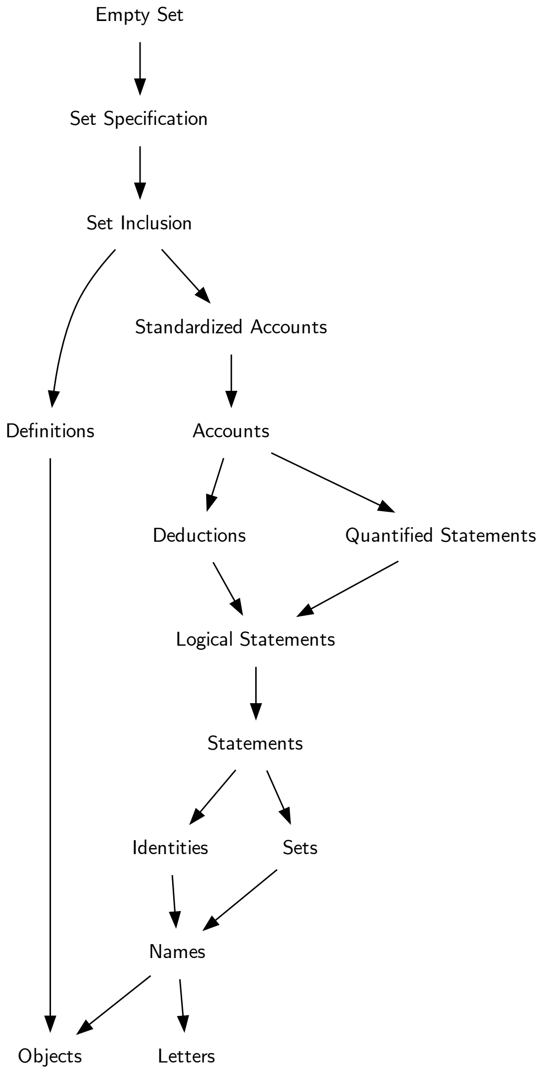
Properties

It is immediate from our definition of the empty set and of the definition of inclusion (see Set Inclusion) that the empty set is included in every set (including itself).

Proposition 1. $(\forall A)(\varnothing \subset A)$

[1] This account will be expanded in the next edition.

Proof. Suppose toward contradiction that $\varnothing \not\subset A$. Then there exists $y \in \varnothing$ such that $y \notin A$. But this is impossible, since $(\forall x)(x \notin \varnothing)$. $\square$


Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters