



## Why

How do we get a joint probability distribution function from marginals?<sup>1</sup>

## Definition

The *product distribution* of a sequence  $p_1, \dots, p_n : A_i \rightarrow [0, 1]$  of distributions is the function  $p : \prod_i A_i \rightarrow [0, 1]$  defined by

$$p(x) = \prod_{i=1}^n p_i(x_i).$$

**Proposition 1.** *Let  $p_i : A_i \rightarrow [0, 1]$  be probability distributions. Then the product distribution of  $p_1, \dots, p_n$  is a probability distribution with marginals  $p_1, \dots, p_n$ .<sup>2</sup>*

## Example: fair coin repeated flips

Suppose we want to model a coin flipped  $n$  times. Supposing the coin is fair (see **Probability Distributions**) we might use our probability distribution  $p : \{0, 1\} \rightarrow [0, 1]$  which assigned  $p(0) = p(1) = 1/2$ . Then the probability of obtaining a sequence of flips  $x \in \{0, 1\}$  is

$$p(x) = \prod_{x_i=1} p_i(1) \prod_{x_i=0} p_i(0).$$

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<sup>1</sup>Future editions will modify.

<sup>2</sup>Future editions will include.

Notice that if we know  $p_i(1) = \rho_i$ , then we know  $p(0) = (1 - \rho_i)$  and so we can write the above as

$$p(x) = \prod_{x_i=1} \rho_i \prod_{x_i=0} (1 - \rho_i).$$

