



## Simple Functions

### 1 Why

We want to define area under a real function. We start with defining functions for which this notion is obvious.

### 2 Definition

A *simple function* is a function whose range is a finite set.

Partition the range into the finite family of one-element sets. The family whose members consist of the inverse images of these sets is a partition of the domain. We call this the *simple partition* of the function.

A *real simple function* is a simple function whose codomain is real. In this case, we can write the simple function as a sum of the characteristic functions of the inverse images elements.

#### 2.1 Notation

Let  $A$  and  $B$  be non-empty sets. We denote the set of simple functions from  $A$  to  $B$  by  $\mathcal{SF}(A, B)$ .

We denote the set of simple real functions with domain  $A$  by  $\mathcal{SF}(A)$ . We denote subset of non-negative simple real functions with domain  $A$  by  $\mathcal{SF}_+(A)$ .

Let  $f \in \mathcal{SF}(A)$ . Order the members of the range of  $f$  from 1 to  $n$  as  $r_1, \dots, r_n$ . Define  $A_i = f^{-1}(\{r_i\})$ . Then  $f = \sum_{i=1}^n r_i \chi_{A_i}$ .