Why

1

Definition

A sequence of functions $(g_1, \ldots, g_\ell)$ is *composable* if g_i is composible with g_{i-1} for $i = 2, \ldots, \ell$. In this case we write $g_\ell \circ g_{\ell-1} \circ \cdots \circ g_2 \circ g_1$. For example, we write $g_3 \circ g_2 \circ g_1$ for (g_1, g_2, g_3).

A *neural network* (or *feedforward neural network*) from $\mathbf{R}^n$ to $\mathbf{R}^m$ is a sequence of composable functions $(g_1, \ldots, g_\ell)$, $\operatorname{dom} g_1 = \mathbf{R}^n$, $\operatorname{range} g_\ell \subset \mathbf{R}^m$, satisfying

$$g_i(\xi) = h_i(A_i \xi + b_i)$$

for some conforming matrices A_i, vectors b_i and functions h_i.

The ith *layer* of the neural network is the ith function g_i. The ith *activation* of the neural network is the function h_i. A *neural network* is called *deep* if its number of layers is larger than 3.

We call the composition of the layers of the neural network the *network predictor* (or just *predictor*). We also call it *the function* of the network.[2]

A *multi-layer perceptron* (*MLP*) is a neural network with 2 layers (1 *hidden layer*) and for which A_i and b_i have unrestricted nonzero entries.

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[1]Future editions will include. Future editions may change the name of this sheet to *computation networks*, or may add a prerequisite sheet on computation graphs.

[2]Many authorities refer to a neural network as a function. Strictly speaking that is true for us, as well, since a sequence is a function. But the meaning of the common use is in reference to the *network predictor*.

Neural Networks

Real Matrix-Vector Products

Real Linear Combinations — Vectors as Matrices

Real Matrices — Real Inner Product — Space Inner Product

Linear Equations — Real Norm — Space Norm

Arrays — Linear Functions — Real Vector — Space Vector

Plane Vector Sums and Differences — N-Dimensional Space — Plane Norm — Plane Inner Product

Plane Vectors — Distance — Absolute Value — Right Triangle Sides Relation

Space Distance — Real Square

Plane Distance — Real Arithmetic

Interval Length — Real Multiplicative Inverse

Intervals — Real Space — Real Products

Real Line — Real Plane — Real Additive Inverse — Square Number

Regression — Real Summation — Real Order — Real Sums — Rational Multiplicative Inverse — Integral Line — Rational Arithmetic — Squares

Predictors — Real Numbers — Rational Sums — Rational Order — Rational Products — Geometry

Lists — Rational Numbers

Direct Products — Set Numbers

Finite Sets — Integer Arithmetic

Natural Multiplicative Identity — Natural Additive Identity — Integer Products

Equivalent Sets — Inverse Elements — Natural Summation — Identity Elements — Arithmetic

Family Unions and Intersections — Function Inverses — Family Operations — Element Functions — Comparisons — Natural Equations — Integer Order — Integer Sums — Natural Powers

Families — Function Images — Function Composite — Operations — Equations — Order — Integer Numbers — Natural Products

Functions — Equivalence Relations — Converse Relations — Natural Order — Natural Sums

Relations — Recursion Theorem

Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Partitions — Intersection of Empty Set — Set Dualities — Unordered Triples — Successor Sets — Set Symmetric Differences

Set Intersections — Ordered Pairs — Pair Unions — Set Complements

Pair Intersections — Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects

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