Why

We want to associate the natural numbers with bit strings for use on digital computers.[1]

Definition

A *digital natural* is a bit string. The set of *d-bit digtal natural numbers* is the set of length-d bit strings $\{0,1\}^d$. For example, the set of 8-bit digital naturals is the set $\{0,1\}^8$.

Correspondence with N $\cup$ $\{0\}$

We associate $x \in \{0,1\}^d$ corresponds to the number $\sum_{i=1}^{d} x_i 2^i$. For example, the bit string $(0,0,0) \in \{0,1\}^3$ corresponds to the natural number $0 \in \omega$. Likewise, $(1,0,0)$ corresponds to $1 \in$ **N**, $(0,1,0)$ corresponds to 2, $(1,1,0)$ corresponds to 3, etc.

Call the function so defined the *digital natural decoder*, and denote it by $f : \{0,1\}^d \to$ **N** $\cup \{0\}$. In other words $f((0,0,0)) = 0$, $f((0,1,0)) = 2$, etc. Call the set $f(\{0,1\}^d)$ the set of naturals *representable* by length-d bit strings.

Specifically, if, for $n \in$ **N** $\cup \{0,1\}$, there exists $x \in \{0,1\}^d$ so that $f(x) = n$, we say that x is *representable in d bits*.

Correspondence between d and $k > d$ bit naturals

Let $x \in \{0,1\}^d$ and $y \in \{0,1\}^k$ with $k > d$. Although $\{0,1\}^d \not\subset \{0,1\}^k$, $f(\{0,1\}^d) \subset f(\{0,1\}^k)$. We can identify $x \in \{0,1\}^d$ with $x' \in \{0,1\}^k$ where $x' = (x_1, \ldots, x_d, 0, \ldots, 0)$ so that $f(x) = f(x')$. Clearly then, if x is representable in d bits, it is representable in $k > d$ bits.

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[1]Future editions will expand.

Addition

We want to define addition $\oplus : \{0,1\}^d \times \{0,1\}^d \rightarrow \{0,1\}^d$ so that $f(x \oplus x') = f(x) + f(x')$. In general, we are stuck, because $x + x'$ may not be representable in d bits. Suppose, however and for the time being, that it is.[2]

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[2]Future editions will complete.

Digital Naturals

Bit Strings

Natural Powers   Codes

Natural Products   Dot-Dash Code

Natural Sums   Flash Code

                      Alphabets

Recursion Theorem   Lists

        Direct Products   Set Numbers

                               Finite Sets

                               Equivalent Sets

Family Unions and Intersections   Natural Order   Function Inverses

        Families   Peano Axioms   Function Images   Function Composites

                                   Functions   Equivalence Relations

                    Natural Induction   Relations

                                                 Set Products

Generalized Set Dualities   Natural Numbers   Set Powers

Set Unions and Intersections   Set Dualities   Successor Sets   Intersection of Empty Set   Unordered Triples   Partitions

                    Set Complements   Pair Unions   Set Intersections   Ordered Pairs

        Pair Intersections   Set Differences   Set Unions

                    Empty Set   Unordered Pairs

                    Set Specification

                    Set Equality

                    Set Inclusion

                    Standardized Accounts

                    Accounts

                    Deductions

                    Quantified Statements

                    Logical Statements

                    Statements

        Identities   Sets

                    Names

        Letters   Objects