Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Prop. 1. *Let f be a square-integrable real-valued random variable on the probability space $(X, \mathcal{A}, \mu)$. Then for $t > 0$,*

$$\mu(|f - \mathrm{E}(f)| \geq t) \leq \frac{\operatorname{var} f}{t^2}.$$

Proof. The set $|f - \mathrm{E}(f)| \geq t$ is $\{x \in X \mid |f(x) - \mathrm{E}(f)| \geq t\}$. This set is $\{x \in X \mid (f(x) - \mathrm{E}(f))^2 \geq t^2\}$. By using the non-negative inequality

$$\mu(\{x \in X \mid (f(x) - \mathrm{E}(f))^2 \geq t^2\}) \leq \frac{\mathrm{E}(f - \mathrm{E}(f))}{t^2}.$$

We recognize the numerator of the right hand side as the variance. □

The above is also called *Chebychev's Inequality*.

Expectation Deviation Upper Bound

Variance

Expectation

Random Variable

Topological Sigma Algebra — Outcome Variables

Measurable Functions — Probability Measures — Topological Spaces

Measures

Metrics

Extended Real Numbers

Distance

Real Limits — Space Distance

Event Probabilities — Absolute Value — Plane Distance

Probability Distributions — Interval Length

Intervals — Uncertain Outcomes

Real Line — Real Space — Real Plane

Integral Line — Real Summation — Real Order — Real Sequences

Geometry — Real Numbers — Sequences

Sigma Algebras — Rational Numbers — Direct Products — Set Numbers

Natural Summation — Integer Arithmetic — Finite Sets

Subset Algebras — Family Operations — Natural Additive Identity — Natural Multiplicative Identity

Cardinality — Set Operations — Algebras — Identity Elements — Arithmetic — Equivalent Sets

Operations — Function Inverse — Natural Powers — Integer Products — Integer Sums — Family Unions and Intersections

Families — Function Composites — Function Images — Natural Products — Integer Numbers

Functions — Natural Sums — Equivalence Relations

Relations — Recursion Theorem — Natural Order

Subset Systems — Cartesian Products — Peano Axioms

Ordered Pairs — Natural Induction — Set Powers

Natural Numbers — Unordered Triples — Generalized Set Dualities

Set Symmetric Differences — Successor Sets — Set Dualities — Set Unions and Intersections — Intersection of Empty Set — Partitions

Set Complements — Pair Unions — Set Intersections

Set Differences — Set Union — Pair Intersections

Unordered Pairs — Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects