Why

We can identify any linear functional $F : \mathbf{R}^n \to \mathbf{R}$ with a vector $y \in \mathbf{R}^n$ so that $F(x) = \langle x, y \rangle$. We generalize this result to complete inner product spaces.

Motivating result

The following is known as the *Riesz representation theorem* (or *Riesz-Fréchet representation theorem*, or *Riesz theorem*, or *Riesz-Fréchet theorem*).

Proposition 1. *Let $((V, k), \langle \cdot, \cdot \rangle)$ be a complete inner product space and let $F : V \to k$ be a continuous linear functional on V. There exists a unique $y \in V$ so that*

$$F(x) = \langle x, y \rangle$$

for all $x \in V$. Moreover $\|y\| = \|F\|_*$.

Clearly $\mathbf{R}^n$ is a complete inner product space, and so this this theorem says the expected. We can identify linear functionals on $\mathbf{R}^n$ with elements (vectors) in $\mathbf{R}^n$.[1]

[1] Future editions will expand further.

Inner Product Linear Functional Representation
Dual Spaces
Complete Real Inner Product Spaces
Continuous Linear Functionals
Complete Normed Spaces
Linear Functionals
Real Inner Product Norms
Norm Metrics
Complete Metric Spaces
Sequence Spaces
Real Inner Products
Norms
Complex Numbers
Functionals
Real Inner Product
Real Series
Space Inner Product
Vectors
Real Norm
Sequence Sequences
Space Norm
Real Metrics
Metrics
Real Equence Sequences
Plane Inner Product
Plane Norm
Space Vectors
Plane Vector Sums and Differences
N Dimensional Space
Real Limits
Real Square Roots
Right Triangle Sides Relation
Plane Vectors
Distance
Absolute Value
Real Sequences
Real Squares
Space Distance
Plane Distance
Real Arithmetic
Sequences
Interval Length
Real Multiplicative Inverses
Natural Square Roots
Real Spaces
Intervals
Real Products
Square Numbers
Real Plane
Real Line
Real Additive Inverses
Squares
Integral Line
Real Order
Real Sums
Rational Arithmetic
Rational Multiplication Inverse
Geometry
Fields
Real Numbers
Rational Sums
Rational Products
Rational Order
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Multiplication Identity
Natural Addition Identity
Integer Products
Finite Sets
Arithmetic
Identity Elements
Equivalence Sets
Inverse Elements
Natural Equations
Comparison
Integer Order
Natural Powers
Integer Sums
Element Functions
Function Inverses
Family Unions and Intersections
Subset
Exponents
Natural Products
Integer Numbers
Operations
Function Images
Function Compositions
Functions
Natural Order
Converse Relations
Natural Sums
Equivalence Relations
Relations
Recursion Theorem
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Set Powers
Generalized Set Duality
Natural Numbers
Set Symmetric Differences
Set Unions and Intersections
Successor Sets
Unordered Triples
Set Duality
Intersection of Empty Set
Partitions
Set Complements
Ordered Pairs
Pair Unions
Set Intersections
Set Differences
Pair Intersections
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects