Why

[1]

Definition

Let $(I, \leq)$ be a totally ordered set. A family of sigma-algebras $\{F_i\}_{i \in I}$ is a *filtration* if $F_j \subset F_k$ for all $j \leq k$.

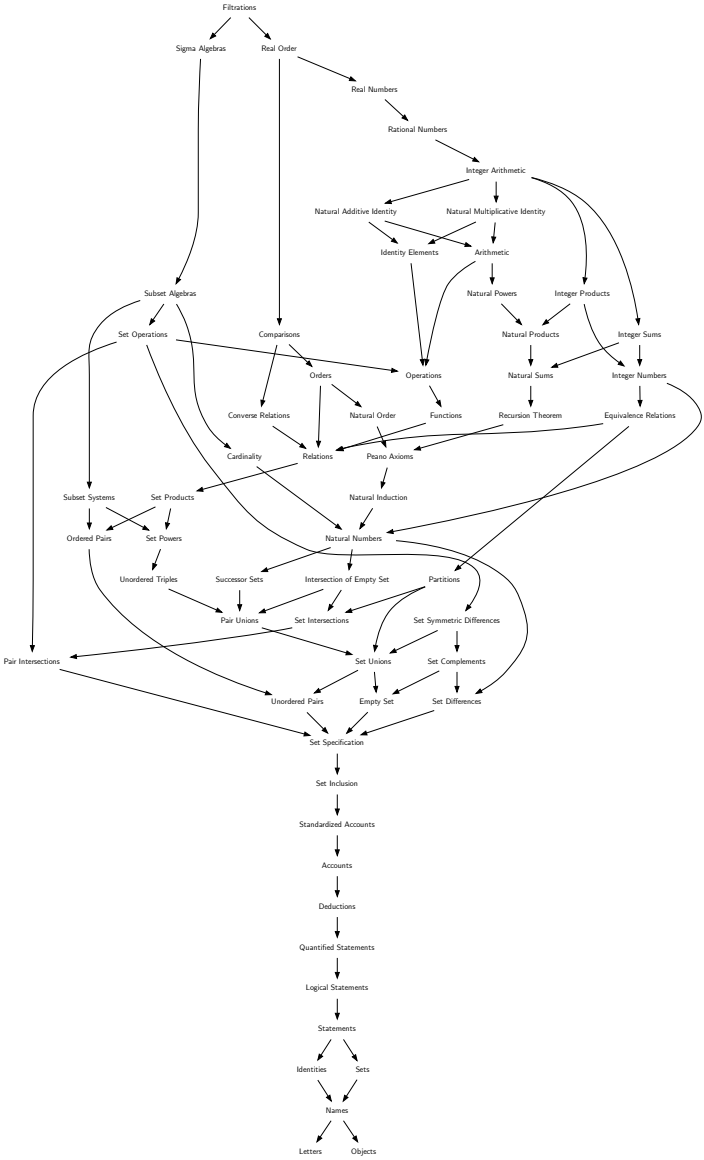
We call a filtration whose index set is the natural numbers with their usual order a *discrete-time filtration*. We call a filtration whose index set is the real numbers with their usual order a *continuous-time filtration*. The index set may also be finite, for example, $\{F_i\}_{i=1}^n$, in which case

$$F_1 \subset F_2 \subset \cdots \subset F_n.$$

Notation

It is extremely common to see filtrations written with the caligraphic $\mathcal{F}$. As in, let $\{\mathcal{F}_i\}_{i=1}^n$ be a filtration. This is in accordance with using caligraphic letters for sets of sets, and in accordance with the term sigma *field* for sigma algebra.

[1] Future editions will include, and likely will need stochastic processes.


Filtrations
Sigma Algebras
Real Order
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Identity Elements
Arithmetic
Subset Algebras
Natural Powers
Integer Products
Set Operations
Comparisons
Natural Products
Integer Sums
Operations
Orders
Natural Sums
Integer Numbers
Converse Relations
Natural Order
Functions
Recursion Theorem
Equivalence Relations
Cardinality
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects