



Why

We have a space X and a family of probability measures \mathcal{P} on this space. Assume $x \in X$ drawn from a fixed, unknown measure $P \in \mathcal{P}$. Given x , how should we guess P ?

Definition

A *probabilistic model* (or *statistical model*, *parametric statistical model*, *statistical experiment*) is a family of probability measures over the same measurable space (X, \mathcal{F}) . Call the index set the *parameter set* or *set of parameters*. The set X is called the *sample space*. A *statistic* is any function on the sample space.

Notation

Let (X, \mathcal{F}) denote a measurable space. We usually denote the parameter by Θ , and denote the family

$$\mathcal{P} = \{\mathbf{P}_\theta : \mathcal{F} \rightarrow [0, 1] \mid \mathbf{P}_\theta \text{ a measure}, \theta \in \Theta\}.$$

Often $\Theta \subset \mathbf{R}^d$.

Example: coin flips

The usual model for n flips of a coin takes $X = \{0, 1\}^n$, the set of binary n -tuples. For $\theta \in [0, 1]$, a distribution $p_\theta(x) = \theta^t(1-\theta)^{n-t}$ where $t = t(x) = x_1 + \dots + x_n$ is defined on X . A probability measure \mathbf{P}_θ is defined on $\mathcal{P}(X)$ in the the usual way. Thus, the probabilistic model is $\{\mathbf{P}_\theta \mid \theta \in [0, 1]\}$. Given x , we are asked to guess θ .

Decisions

A *decision procedure* (*estimator*, *statistical procedure*) is a measurable function $A : \mathcal{X} \rightarrow \mathcal{A}$ where \mathcal{A} is a set, called the *actions* or *decisions*. Often $\mathcal{A} = \Theta$, in which case $A(x)$ gives an *estimate* of θ , which we denote $\hat{\theta}(x)$.

Judging decisions

Given a *loss function* $L : \mathcal{A} \times \Theta \rightarrow \bar{\mathbf{R}}$, the *risk* of A is

$$R(A, \theta) = \mathbf{E}L(A(x), \theta).$$

