Why

What does it mean for two random variables to be independent? What are the events associated with a random variable? TODO

Definition

Defining Result

Proposition **1.** *The set of inverse images the distinguished sets of a measurable space under a function from a set to that space is a sigma algebra.*

If the first set and the function are measurable, the sigma algebra is a sub sigma algebra of the domain sigma algebra.

Proof. Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Let $f : X \to Y$ be a measurable function. Define

$$\mathcal{C} := \{ f^{-1}(B) \mid B \in \mathcal{B} \}.$$

First, since $f^{-1}(Y) = X$, $X \in \mathcal{C}$.

Second, let $C \in \mathcal{C}$. Then there is B such that $C = f^{-1}(B)$. Then $X - C = X - f^{-1}(B) = f^{-1}(Y - B)$. Since $\mathcal{B}$ is a sigma algebra, $B \in \mathcal{B}, Y - B \in \mathcal{B}$ and so $X - C \in \mathcal{C}$.

Finally, let $(C_n)_n \subset \mathcal{C}$. Then for every n there exists a $B_n \in \mathcal{B}$ so that $C_n = f^{-1}(B_n)$. Then:

$$\cup_n C_n = \cup_n f^{-1}(B_n) = f^{-1}(\cup B_n).$$

Since $\mathcal{B}$ is a sigma algebra, $\cup_n B_n \in \mathcal{B}$ and so $\cup_n C_n \in \mathcal{C}$.

Since f is measurable, $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}$, and so $\mathcal{C} \subset \mathcal{A}$. $\qquad\square$

The sigma algebra *generated by a random variable* is the sigma algebra consisting of the inverse images of every measurable set of the codomain.

The sigma algebra generated by a family of random variables is the sigma algebra generated by the union of the sigma algebras generated individually by each of the random variables.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f : X \to Y$ be a random variable. Denote by $\sigma(f)$ the sigma algebra generated by f.

Results

PROPOSITION **2.** *The sigma algebra generated by a family of random variables is the smallest sigma algebra for with respect to which each random variable is measurably.*

Random Variable Sigma Algebra

Random Variables

Measurable Functions Probability Measures

Measures

Topological Sigma Algebra Extended Real Numbers Event Probabilities

Topological Spaces Real Limits Probability Distributions

Metrics Real Sequences Intervals Real Summation Probability Events

Distance Real Numbers Subset Algebras Natural Summation Sequences Probability Outcomes

Length Common Notions Rational Numbers Set Operations Cardinality Family Operations Direct Products Probability

Integer Numbers Injective Functions Algebras Natural Families Common Sense

Zero Natural Sums Natural Order Operations Families Function Composites

Natural Numbers Functions

Set Intersections Relations

Subset Systems Cartesian Products

Pair Intersections Ordered Pairs Set Powers

Equation Solutions Unordered Triples

Set Symmetric Differences Pair Unions

Set Complements Set Unions

Set Differences Empty Set Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts Definitions

Deductions Quantified Statements

Logical Statements

Statements

Identities Sets

Names

Letters Objects