Why

We want to talk about several objects in order.

Definition

A *list* of *length* n with *entries* in A is a function $a : \{1, \dots, n\} \to A$. In other words, a list is a *family* whose index set is $\{1, \dots, n\}$ The result a_k is the *kth entry* of A.

Many authors refer to a list as a *finite sequence*, *string*, or *n-tuple*, and some refer to the length of the list as its *size*. Some authors say that the list is "*in*" A, or that it is a list "*of*" elements of A, and call an entry of k a *term*.

Notation

Since the natural numbers are ordered, we regularly denote lists from left to right between parentheses. For example, we denote $a : \{1, \dots, 4\} \to A$ by (a_1, a_2, a_3, a_4).

Orderings and numberings

Let A be a set with $|A| = n$. A sequence $a : \{1, \dots, n\} \to A$ is an *ordering* of A if a is invertible. In this case, we call the inverse a *numbering* of A. An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

Relation to Direct Products

A *natural direct product* is a product of a list of sets. We denote the direct product of a list of sets $A_1, \dots, A_n$ by $\prod_{i=1}^n A_i$. If each A_i is the same set A, then we denote the product $\prod_{i=1}^n A_i$ by A^n. The direct product A^n is set of lists in A .

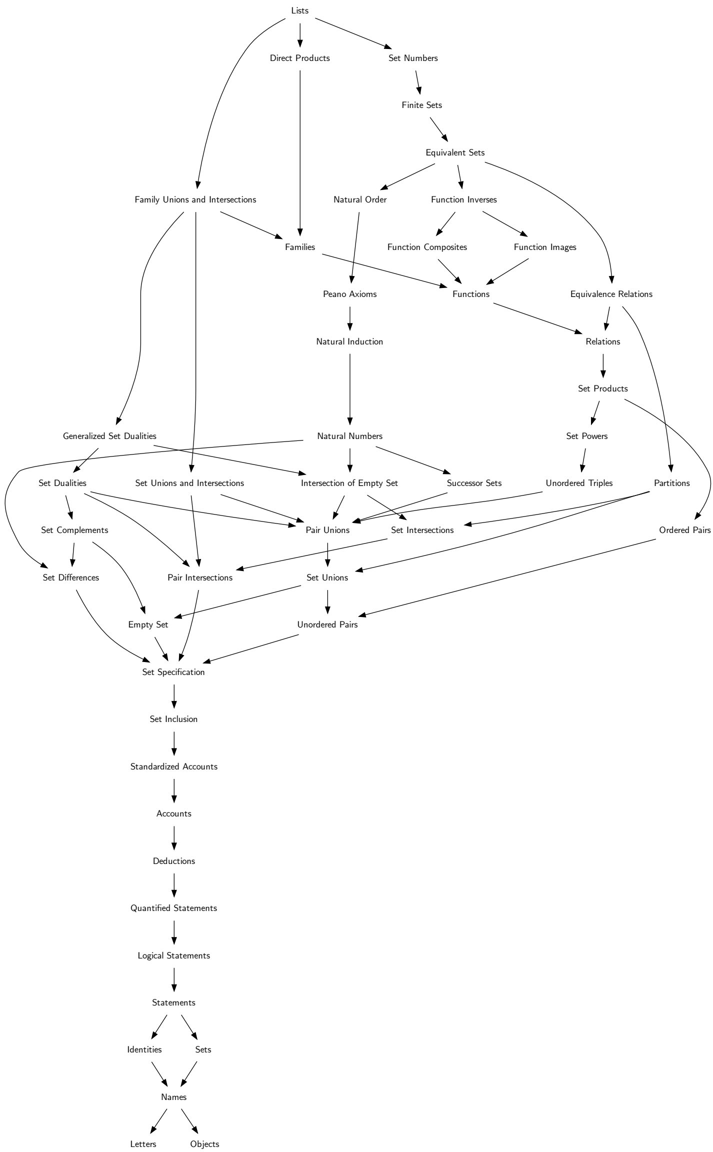
Natural unions and intersections

We denote the family union of the list of sets $A_1, \dots, A_n$ by $\cup_{i=1}^n A_i$. Similarly, we denote the intersection by $\cap_{i=1}^n A_i$.

Slices

An *index range* for a list s of length n is a pair (i, j) for which $1 \leq i < j \leq n$. The *slice* corresponding to (i, j) is the length $j - i$ list s' defined by $s'_1 = s_i$, $s'_2 = s_{i+1}, \dots, s'_j = s_{i+j-1}$.

We denote the (i, j)-slice of s by $s_{i:j}$. If $i = 1$ we use $s_{:j}$ and if $j = n$ we use $s_{i:}$ as shorthands for the slices $s_{1:j}$ and $s_{i:n}$.


Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Natural Induction
Relations
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects