Why

1

Definition

A sequence of functions $(g_1, \ldots, g_\ell)$ is *composable* if g_i is composible with g_{i-1} for $i = 2, \ldots, \ell$. In this case we write $g_\ell \circ g_{\ell-1} \circ \cdots \circ g_2 \circ g_1$. For example, we write $g_3 \circ g_2 \circ g_1$ for (g_1, g_2, g_3).

A *neural network* (or *feedforward neural network*) from $\mathbf{R}^n$ to $\mathbf{R}^m$ is a sequence of composable functions $(g_1, \ldots, g_\ell)$, $\operatorname{dom} g_1 = \mathbf{R}^n$, $\operatorname{range} g_\ell \subset \mathbf{R}^m$, satisfying

$$g_i(\xi) = h_i(A_i\xi + b_i)$$

for some conforming matrices A_i, vectors b_i and functions h_i.

The ith *layer* of the neural network is the ith function g_i. The ith *activation* of the neural network is the function h_i. A *neural network* is called *deep* if its number of layers is larger than 3.

We call the composition of the layers of the neural network the *network predictor* (or just *predictor*). We also call it *the function* of the network.[2]

A *multi-layer perceptron* (*MLP*) is a neural network with 2 layers (1 *hidden layer*) and for which A_i and b_i have unrestricted nonzero entries.

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[1]Future editions will include. Future editions may change the name of this sheet to *computation networks*, or may add a prerequisite sheet on computation graphs.

[2]Many authorities refer to a neural network as a function. Strictly speaking that is true for us, as well, since a sequence is a function. But the meaning of the common use is in reference to the *network predictor*.

Natural Networks

Real Matrix-Vector Products

Vectors as Matrices — Real Linear Combinations

Real Matrices — Real Inner Product

Linear Equations — Real Norm — Space Inner Product — Real Vectors

Arrays — Linear Functions — Space Norm — N-Dimensional Space — Plane Inner Product — Plane Vector Sums and Differences — Space Vectors

Plane Norm — Absolute Value — Distance — Right Triangle Sides Relation — Plane Vectors

Space Distance — Real Squares

Plane Distance — Real Arithmetic

Real Multiplicative Inverses — Interval Length — Real Space

Real Products — Intervals — Real Line — Real Plane — Square Numbers

Real Additive Inverses — Real Sums — Rational Arithmetic — Rational Multiplicative Inverses — Real Order — Integral Line — Squares

Regression — Real Summation — Real Sums — Rational Sums — Rational Products — Rational Order — Geometry

Prediction — Natural Summation — Real Numbers — Rational Numbers

Lists — Integer Arithmetic

Direct Products — Set Numbers — Integer Products

Natural Additive Identity — Natural Multiplicative Identity — Finite Sets — Integer Sums — Natural Powers — Integer Order — Companions

Identity Elements — Arithmetic — Inverse Elements — Equivalent Sets — Natural Equations — Integer Sums — Natural Products — Orders

Family Unions and Intersections — Family Operation — Element Functions — Function Inverses — Function Images — Equations — Natural Sums — Converse Relation

Operations — Families — Function Composites — Functions — Equivalence Relation — Natural Order — Relations

Recursion Theorem — Peano Axioms — Ordered Pair Projections

Natural Induction — Set Products

Generalized Set Dualities — Natural Numbers — Set Powers

Set Unions and Intersections — Set Dualities — Set Symmetric Difference — Partitions — Intersection of Empty Set — Successor Sets — Unordered Triples

Set Complements — Set Intersection — Pair Unions — Ordered Pairs

Pair Intersections — Set Differences — Set Unions — Unordered Pairs

Empty Set

Set Specification — Set Equality

Set Inclusion

Deduction

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects