RELATIONS

Why

How can we relate the elements of two sets?

Definition

A *relation* is a set of ordered pairs (see Ordered Pairs). So if an object z is an element of a relation, there exists two other objects x, y so that $z = (x, y)$.

The *domain* of a relation is the set of all elements which appear as the first coordinate of some ordered pair of the relation (the projection onto the first coordinate, see Ordered Pair Projections) The *range* of a relation is the set of all elements which appear as the second coordinate of some ordered pair of the relation (the projection onto the second coordinate).

When the domain of a relation R is a subset of X and the range is a subset of Y, we say R is *from X to Y* or *between X and Y*. If $X = Y$, then R speak of a relation *in* or *on X*.

Notation

If R is a relation, we express that $(x, y) \in R$ by writing $x \, R \, y$, which we read as "x is in relation R to y". We denote the domain of R by dom R and the range of R by ran R.

Examples

For an uninteresting relation, consider the empty set. In the empty (set) relation, no object is related to any other. Both

the domain and range of $\varnothing$ are $\varnothing$. For another simple relation, consider the product of any two sets X and Y. In $X \times Y$, all objects are related. The domain is X and the range is Y.

For a more interesting example, consider the set

$$R := \{(x, y) \in X \times X \mid x = y\}.$$

This relation is the relation of equality (see Identities) between two objects. Here $x \, R \, y \longleftrightarrow x = y$. dom $R = \operatorname{ran} R = X$. Another similar example is if we consider the set X and $\mathcal{P}(X)$, and the relation

$$R := \{(x, y) \in X \times \mathcal{P}(X) \mid x \in y\}.$$

This relation is the relation of belonging (see Sets). Here $x \, R \, y \longleftrightarrow x \in y$. Here dom $R = X$ and ran $R = \mathcal{P}(X)$.

Properties

Often relations are defined over a single set, and there are a few useful properties to distinguish.

A relation is *reflexive* if every element is related to itself. A relation is *symmetric* if two objects are related regardless of their order. A relation is *transitive* if a first element is related to a second element and the second element is related to the third element, then the first and third element are related. Equality is reflexive, symmetric and transitive whereas belonging is neither. Exercise: what is inclusion?

Relations

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Set Products

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Set Powers

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Ordered Pairs

Unordered Triples

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Pair Unions

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Set Unions

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Unordered Pairs

Empty Set

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Set Specification

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Set Inclusion

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Standardized Accounts

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Accounts

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Deductions

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Quantified Statements

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Logical Statements

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Statements

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Identities

Sets

↘ ↙

Names

↙ ↘

Letters

Objects