



Nonnegative Integrals

1 Why

We want to define area under an extended real function. We use the infinite process to approximate the area under a nonnegative extended real function using simple functions.

2 Definition

Consider a measure space.

The *integral* of a measurable nonnegative function is the supremum of integrals over nonnegative simple functions pointwise less than or equal to the function.

2.1 Notation

Let (X, \mathcal{A}, μ) be a measure space. Let $f : X \rightarrow [0, \infty]$ be measurable. We denote the integral of f with respect to the measure μ by $\int f d\mu$. We defined:

$$\int f d\mu = \sup \left\{ \int g d\mu \mid g \in \mathcal{SF}_+(X) \text{ and } g \leq f \right\}.$$