Sets

1 Why

We want to speak of a collection of objects, pre-specified or possessing some similar defining property.

2 Definition

A **set** is a collection of objects. We use the term object in the usual sense of the English language. So a set is itself an object, but of the peculiar nature that it contains other objects. In thinking of a set, then, we regularly consider the objects it contains. We call the objects contained in a set the **members** or **elements** of the set. So we say that an object contained in a set is a **member of** or an **element of** the set.

2.1 Notation

We denote sets by upper case latin letters: for example, A, B, and C. We denote elements of sets by lower case latin letters: for example, a, b, and c. We denote that an object A is an element of a set A by $a \in A$. We read the notation $a \in A$ aloud as "a in A." The $\in$ is a stylized ϵ, which possesses the mnemonic for element. We write $a \notin A$, read aloud as "a not in A," if a is not an element of A.

If we can write down the elements of A, we do so using a brace notation. For example, if the set A is such that it contains only the elements a, b, c, we denote A by $\{a, b, c\}$. If the elements of a set are well-known we introduce the set in English and name it; often we select the name mnemonically. For example, let L be the set of latin letters.

If the elements of a set are such that they satisfy some common condition, we use the braces and include the condition. For example, if V is the set of vowels we denote V by $\{l \in L \mid l \text{ is a vowel}\}$. The $\mid$ is read aloud as "such that," the notation reads aloud as "l in L such that l is a vowel." We call the notation $\{l \in L \mid l \text{ is a vowel}\}$ **set-builder notation**. Set-builder notation is indispensable for sets defiined implicitly by some condition. Here we could have alternatively denoted V by $\{``a", ``e", ``i", ``o", ``u"\}$. We prefer the former, slighly more concise notation.

3 Two Sets

If every element of a first set is also an element of second set, we say that the first set is a **subset** of or is **contained in** the second set. Conversely, we say that the second set is a **superset** of or **contains** the first set. If a first set is a subset of a second set and the second set is a subset of the first set, we say the two sets are **equal**.

We call the set of subsets of a set A the **powerset** of A. We call the set which has no members the **empty set**. The empty set is contained in every other set.

3.1 Notation

Let A and B be sets. We denote that A is a subset of B by $A \subset B$. We read the notation $A \subset B$ aloud as "A subset B". We denote that A is equal to B by $A = B$. We read the notation $A = B$ aloud as "A equals B". We denote the empty set by $\varnothing$, read aloud as "empty." We denote the power set of A by 2^A, read aloud as "two to the A."