Why

Since every affine set is a translate of a unique subspace, we can represent them by representing the vector and the subspace.

Definition

Recall that M is affine means $M = S + a$ for some subspace S and vector $a \in \mathbf{R}^n$. The dimension of M is the dimension of the subspace. Suppose $\dim S = k$, then there exists $Q \in \mathbf{R}^{n \times k}$ with $Q^\top Q = I$, so that for any $x \in S$, there exists unique $z \in \mathbf{R}^k$ with $x = Qz$. Since $M = S + a$, we have

$$M = \{y \in \mathbf{R}^n \mid (\exists z \in \mathbf{R}^k)(y = a + Qz)\}$$

We also denote this set $\{a + Qz \mid z \in \mathbf{R}^k\}$.

Real Affine Set Representations

Affine Set Dimensions

Subspace Dimension

Real Subspace Representation

Vector Space Dimensions

Vector Space Bases

Real Matrices with Orthonormal Columns

Linearly Dependent Vectors

Span

Real Matrix-Matrix Products

Affine Sets and Subspaces

Real Translates

Subspace

Real Matrix-Vector Products

Real Subspace

Vectors as Matrices

Orthonormal Set of Real Vectors

Real Matrices

Linear Combinations

Real Vector Span

Real Inner Product

Linear Equations

Vector

Real Linear Combinations

Real Norm

Space Inner Product

Affine Sets

Arrays

Linear Functions

Real Vector

Space Norm

N-Dimensional Lines

Space Vectors

Plane Vector Sums and Differences

Plane Norm

Plane Inner Product

N-Dimensional Space

Plane Vectors

Absolute Value

Distance

Right Triangle Sides Relation

Space Distance

Real Squares

Plane Distance

Real Arithmetic

Interval Length

Real Multiplicative Inverses

Intervals

Real Space

Real Products

Real Plane

Real Line

Real Addition Inverses

Integral Lines

Square Numbers

Real Summation

Real Order

Real Sums

Rational Arithmetic

Rational Multiplicative Inverses

Squares

Geometry

Natural Summation

Rational Order

Fields

Real Numbers

Rational Sums

Rational Products

Lists

Rings

Rational Numbers

Direct Products

Set Numbers

Integer Arithmetic

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Equivalent Sets

Inverse Elements

Identity Elements

Arithmetic

Family Unions and Intersections

Family Operations

Function Inverses

Comparisons

Element Function

Integer Order

Natural Equations

Integer Sums

Natural Powers

Families

Function Composition

Function Images

Operations

Orders

Equations

Natural Order

Natural Products

Functions

Equivalence Relations

Converse Relations

Integer Numbers

Natural Sums

Relations

Natural Numbers

Recursion Theorem

Ordered Pair Projection

Peano Axioms

Set Products

Natural Induction

Generalized Set Dualities

Set Powers

Set Unions and Intersections

Partition

Set Dualities

Intersection of Empty Set

Unordered Triples

Successor Sets

Set Symmetric Differences

Ordered Pairs

Set Intersection

Pair Unions

Set Complements

Pair Intersections

Unordered Pairs

Empty Set

Set Differences

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects