Why

How should we represent a subspace computationally?

Definition

Given a subspace $S \in \mathbf{R}^n$, since it is finite dimensional, there exists a finite basis for the space. This basis can be made orthonormal. Therefore every subspace S has an orthonormal basis $q_1, \ldots, q_k$, where k is the dimension of the subspace. We can stack these as a matrix. Define $Q \in \mathbf{R}^{n \times k}$ by

$$Q = \begin{bmatrix} q_1 & q_2 & \cdots & q_k \end{bmatrix}.$$

For every $x \in S$, there exists unique coefficients $z \in \mathbf{R}^k$ so that

$$x = Qz.$$

Therefore we have a one-to-one correspondence between vectors $x \in S$ and their coordinates $z \in \mathbf{R}^k$.

Real Subspace Representation

Real Matrices with Orthonormal Columns

Real Matrix-Matrix Products

Real Matrix-Vector Products

Orthonormal Set of Real Vectors · Vectors as Matrices

Real Inner Product · Real Linear Combinations · Real Matrices

Square Inner Product · Real Norm · Linear Equations

Space Norm · Real Vector · Linear Function · Armijo

Plane Inner Product · Plane Norm · N-Dimensional Space · Plane Vector Sums and Differences · Space Vector

Right Triangle Sides Relation · Absolute Value · Distance · Plane Vectors

Real Square · Space Distance

Real Arithmetic · Plane Distance

Real Multiplicative Inverse · Interval Length

Real Products · Real Space · Intervals

Square Numbers · Real Additive Inverse · Real Line · Real Plane

Square · Integral Line · Rational Arithmetic · Rational Multiplicative Inverse · Real Sums · Real Order · Real Summation

Geometry · Rational Sums · Rational Products · Real Numbers · Rational Order · Natural Summation

Integer Arithmetic · Rational Numbers · Lists

Set Numbers · Direct Products

Integer Products · Finite Sets

Natural Addition Identity · Natural Multiplication Identity

Arithmetic · Identity Elements · Equivalent Sets · Inverse Elements · Comparisons · Family Operations

Natural Powers · Integer Sums · Integer Order · Natural Equations · Element Function · Function Inverse · Families · Family Unions and Intersections

Natural Products · Integer Numbers · Ordered · Equations · Operations · Function Composites · Functions · Function Images · Converse Relations

Natural Sums · Equivalence Relations · Natural Order · Relations

Recursion Theorem · Ordered Pair Projections

Peano Axioms

Natural Induction · Set Products

Natural Numbers · Set Powers · Generalized Set Dualities

Set Symmetric Differences · Partitions · Successor Sets · Intersection of Empty Set · Unordered Triples · Set Dualities · Set Unions and Intersections

Set Complements · Set Intersections · Pair Unions · Ordered Pairs

Set Differences · Set Unions · Pair Intersections

Empty Set · Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deduction

Quantified Statements

Logical Statements

Statements

Sets · Identities

Names

Letters · Objects