



Why

Definition

A *normal random function* (or *normal process* or *gaussian process*)¹ is a real-valued random function with the property that any subset of results has a multivariate normal density.

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $x : I \rightarrow (\Omega \rightarrow \mathbf{R})$. Then x is a normal random function if there exists $m : I \rightarrow \mathbf{R}$ and positive definite $k : I \times I \rightarrow \mathbf{R}$ with the property that if $J \subset I$, $|J| = d$, then $x_J \sim \mathcal{N}(m(J), k(J \times J))$. In other words, $x_J : \Omega \rightarrow \mathbf{R}^d$ is a Gaussian random vector. We call m the *mean function* and k the *covariance function*.

Random function interpretation

Many authorities discuss a normal random function as “putting a prior” on a “space” (see, for example, *Real Function Space*). One can draw a sample from this space by first selecting $\omega \in \Omega$, and then defining a sample $f : I \rightarrow \mathbf{R}$ by $f(i) = x(i, \omega)$.

Multivariate normal special case

If the index set is finite, and can be ordered, then the normal random function is a multivariate normal random vector.

¹The choice of “normal” is a result of the Bourbaki project’s convention to eschew historical names. Though here, as in *Multivariate Normals* the language of the project is nonstandard.

