



**Definition**

A subset  $M \subset \mathbf{R}^n$  is a *smooth manifold* of dimension  $d$  if for every  $x \in M$ , there exists a neighborhood  $V$  of  $x$  in  $X$  that is diffeomorphic to an open subset  $U$  of  $\mathbf{R}^d$ . In this case we say that the set is *locally diffeomorphic* to  $\mathbf{R}^d$ .

A diffeomorphism  $\phi : U \rightarrow V$  is called a *parameterization* of the neighborhood of  $V$ .

Its inverse diffeomorphism  $\phi^{-1}$  is called a *coordinate system* (or system of *coordinates*) on  $V$ .

**Notation**

We denote the dimension of a manifold  $M$  by  $\dim M$ .

**Submanifolds**

If  $X$  and  $Z$  are both manifolds in  $R^n$  and  $Z \subset X$ , then we call  $Z$  a *submanifold* of  $X$ . In particular,  $X$  is a submanifold of  $R^n$ . Any open set of a manifold  $X$  is a submanifold  $X$ .<sup>1</sup>

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<sup>1</sup>Future editions will expand.



