Why

Can permuting the rows or columns of a matrix be represented by matrix multiplication?

Definition

Let $\sigma : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ be a permutation of n. The *permutation matrix* of σ is the matrix P defined by $P_{ij} = 1$ if $\sigma(i) = j$ and 0 otherwise. This is sometimes called the *column representation* (in contrast to the *row representation*, in which $P_{ij} = 1$ if $\sigma(j) = i$.

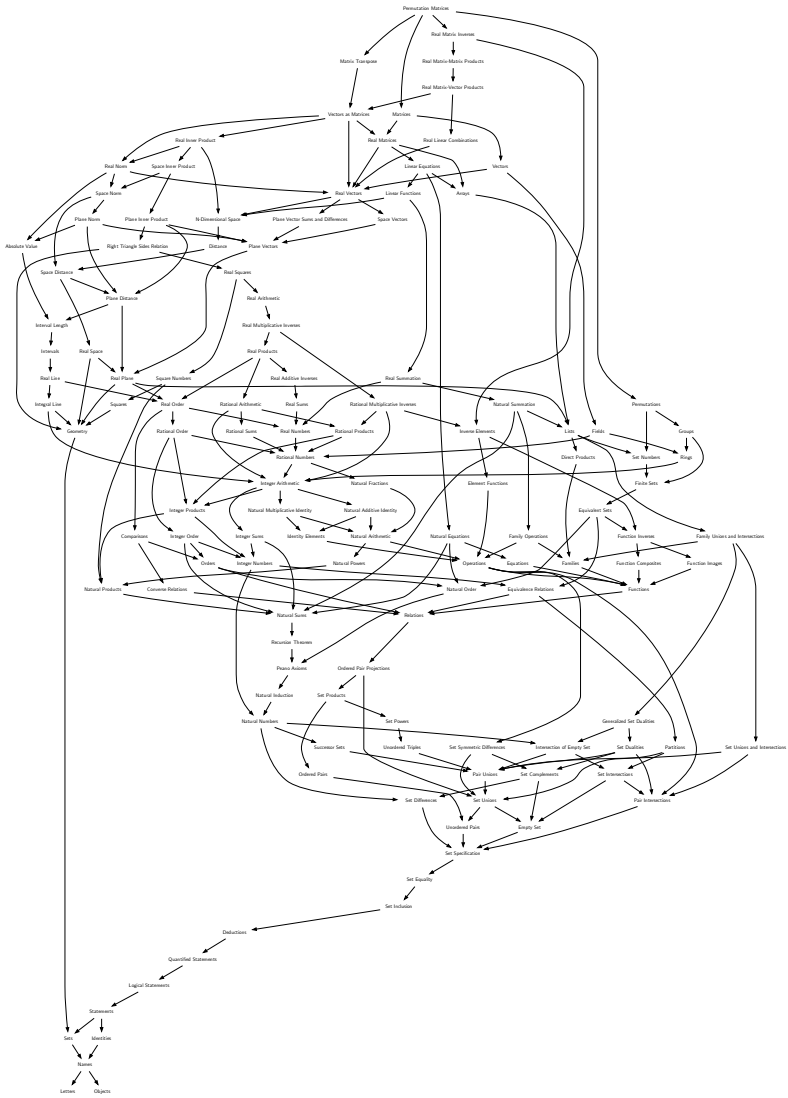
Let $A \in \mathbf{R}^{n \times n}$. Then pre-multipying A by P permutes the rows of A. In other words PA has the same rows as A but permuted according to σ. Similarly, post-multiplying by P permutes the columns of A. In other words, AP has the same columns as A but permuted according to σ. Clearly, we can also speak of permuting the components of a vector.

Composition

Let $\pi, \sigma \in S_n$ with corresponding permutation matrices P_σ and P_π. Then $P_\pi P_\sigma A$ has the same rows as A but permuted by $\pi\sigma$. Likewise, $AP_\pi P_\sigma$ has the same columns as A but permuted by $\pi\sigma$. Clearly, the identity permutation on $\{1, 2, \ldots, n\}$ is the identity $I \in \mathbf{R}^{n \times n}$.

Inverses

It is clear from the definition that $P_\sigma^{-1} = P_{\sigma^{-1}}$ and so if P is a permutation matrix then P^{-1} is $P^\top$.


Permutation Matrices
Real Matrix Inverse
Matrix Transpose
Real Matrix-Matrix Products
Real Matrix-Vector Products
Vectors as Matrices
Matrices
Real Inner Product
Real Matrices
Real Linear Combinations
Real Norm
Space Inner Product
Linear Equations
Vectors
Space Norm
Real Vectors
Linear Functions
Angle
Plane Norm
Plane Inner Product
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Absolute Value
Right Triangle Sides Relation
Distance
Plane Vectors
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Real Line
Real Plane
Square Numbers
Real Additive Inverses
Real Summation
Integral Line
Square
Real Order
Rational Arithmetic
Real Sums
Rational Multiplicative Inverses
Natural Summation
Permutations
Geometry
Rational Order
Rational Sums
Real Numbers
Rational Products
Inverse Elements
Lists
Fields
Groups
Rational Numbers
Direct Products
Set Numbers
Rings
Integer Arithmetic
Natural Fractions
Element Functions
Finite Sets
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Comparison
Integer Order
Integer Sums
Identity Elements
Natural Arithmetic
Natural Equations
Family Operations
Function Inverses
Family Unions and Intersections
Orders
Integer Numbers
Natural Powers
Operations
Equations
Families
Function Composites
Function Images
Natural Products
Converse Relations
Natural Order
Equivalence Relations
Functions
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Symmetric Differences
Intersection of Empty Set
Set Dualities
Partitions
Set Unions and Intersections
Ordered Pairs
Pair Unions
Set Complements
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects