



Why

How do we get a joint probability distribution function from marginals?¹

Definition

The *product distribution* of a sequence $p_1, \dots, p_n : A_i \rightarrow [0, 1]$ of distributions is the function $p : \prod_i A_i \rightarrow [0, 1]$ defined by

$$p(x) = \prod_{i=1}^n p_i(x_i).$$

Proposition 1. *Let $p_i : A_i \rightarrow [0, 1]$ be probability distributions. Then the product distribution of p_1, \dots, p_n is a probability distribution with marginals p_1, \dots, p_n .*²

Example: fair coin repeated flips

Suppose we want to model a coin flipped n times. Supposing the coin is fair (see Probability Distributions) we might use our probability distribution $p : \{0, 1\} \rightarrow [0, 1]$ which assigned $p(0) = p(1) = 1/2$. Then the probability of obtaining a sequence of flips $x \in \{0, 1\}$ is

$$p(x) = \prod_{x_i=1} p_i(1) \prod_{x_i=0} p_i(0).$$

Notice that if we know $p_i(1) = \rho_i$, then we know $p(0) = (1 - \rho_i)$ and so we can write the above as

$$p(x) = \prod_{x_i=1} \rho_i \prod_{x_i=0} (1 - \rho_i).$$

¹Future editions will modify.

²Future editions will include.

