



The Bourbaki Project

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Printed in Menlo Park, California

Editor's Preface

This project is one of the more ambitious with which I am affiliated. Its two-fold goal is to explain mathematics to the novice and provide standardized language for the expert. The reader should note that I have cut this edition under the pressure of time, in accordance with my annual goals for the project, and not because I felt we had reached a reasonable landmark, or that the content was particularly polished.

So then, what is here? An attempt to talk about language, symbols, intangible objects and logical reasoning enough to get to a few principles having to do with intangible objects called sets and a few things you can build out of these sets. The construction of real numbers and their relation to the lines of geometry becomes quite sparse toward the end, but the outline is included. The n -dimensional real space is touched upon, and barely metric spaces, barely topological spaces.

On that last point, I should mention that the original goal for this edition was to reach topological spaces. We agreed that this topic involved sufficiently abstract concepts which could test the project's assumptions. We all agree, now, that there was much more to be said (to the novice) about topics much preliminary to topological spaces. More than we anticipated. We could, early on, define a topological space in terms of sets. But we could not say why we cared. And this idea, that we might say why we wanted a new concept before we introduced it, was an assumption we were testing with this project.

What were the other assumptions to be tested? First, that the concepts and discussion could be so ordered that we only use prior concepts and discussion. Second, that we could structure the book so that topics are treated by short, two-page sheets. Third, that such a treatment would be useful as a reference. Fourth, that we could standardize language (perhaps formally) and use it in all theorems, definitions and proofs.

These traits would undoubtedly be useful. The sheets could serve both as a beginner's guide and a reference. When reaching for a particular

topic, the prerequisites would be clear, fine-grained, and each one only two-pages long. And a standardized language to facilitate understanding and communication is a centuries-old endeavor. That no such text exists, to our knowledge, must indicate that its construction is accompanied by great difficulty. But that is not to say impossible, and computers and screens may facilitate the process.

The text you hold is the first edition. And we might call it a first attempt. It is incomplete and with flaws. But that is not to say useless. There is visible in it the form of what is to come, if only you look at it properly. And, in any case, it is time that we have a first edition.

N.C.L.
16 July 2021
Menlo Park, California

To the Reader

The Bourbaki Project is a collection of documents describing mathematical concepts, terms, results and notation.

Sheets

We call these documents *sheets*. They are only ever two-pages long and sometimes shorter. They can be printed on a single sheet of paper, hence the name sheet. In a book, they occupy two facing pages. The decision to cap at two pages is arbitrary. But our experience suggests it is convenient.

Prerequisites

Each sheet is labeled with the names of those sheets which are its immediate prerequisites, with the names of those sheets for which it is an immediate prerequisite, and a diagram illustrating the dependencies between all its prerequisites.

For example, the sheet **Relations** needs the sheet **Ordered Pairs**. The reason, in this case, is that the concept of a relation is discussed using the concept of an ordered pair of objects. And since the phrase “ordered pair of objects” makes sense only if we know what is meant by object (discussed in the sheet **Objects**), the sheet **Relations** needs the sheet **Objects** also. The reader unacquainted with ordered pairs and objects must read (at least) these two sheets before the sheet on relations. In this case (and in every case) the prerequisites are naturally ordered. **Objects** ought to be read first, before **Ordered Pairs**, before **Relations**. Such an ordering always exists because we ensure that if a sheet X needs a sheet Y , then Y can not need X or any sheet that needs X . A sheet is an immediate prerequisite if it is not prerequisite to any other prerequisite.

Preface

The project is like a map. The landmarks are sheets, or really concepts. Walking is reading. And you must walk along the trails specified by the prerequisites.

Aims

Our primary aim is two-fold. First, to provide useful exposition to teach the concepts to an unacquainted reader (here the prerequisites help). And second, to serve as a reference for further work. It is a welcomed concomitant that we better understand and develop the mathematical concepts ourselves.

Caveats

There are two caveats. First, we give only one path to concepts. The point is that our way of structuring the concepts (and hence the prerequisites) is just one way, and there are many ways, since there are equivalent concepts, alternate proofs, and so on. The second caveat is a wink. These sheets are fiction. They contain only ideas. We have done our best to eliminate all false statements. The game for the practical cogitator is to fit these puzzle pieces to reality.

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Note on Printing

The font is *Computer Modern*. The document was typeset using L^AT_EX. This pamphlet was printed, folded, and stitched in Menlo Park, California.

