



Real Series

1 Why

We want to sum infinitely many real numbers.

2 Definition

Let n be a natural number. The n th *partial sum* of a sequence of real numbers is the sum of first n elements of the sequence. The first partial sum is the first term of the sequence. The third partial sum is the sum of first three elements of the sequence.

The *series* of the sequence is the sequence of partial sums. The sequence is *summable* if the series converges.

Since there exist sequences which do not converge, there exist sequences which are not summable. Consider the sequence which alternates between $+1$ and -1 , and starts with $+1$. Its series alternates between $+1$ and 0 , and so does not converge.

2.1 Notation

Let $(a_n)_n$ be a sequence of real numbers. For natural number n , define:

$$s_n = \sum_{k=1}^n a_k.$$

Then $(s_n)_n$ is the series of $(a_n)_n$. If the series converges, then there exists a real number s , the limit, and we write:

$$s = \lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n a_k$$

We read these relations aloud as “ s is the limit as n goes to infinity of s_n ” and “ s is the limit as n goes to infinity of the sum of a_k from k equals 1 to n .”

To avoid referencing s_n , we write:

$$\sum_{k=1}^{\infty} a_k = s,$$

read aloud as the “the sum from 1 to infinity of a_k is s .” The notation is subtle, and requires justification by the algebra of series. TODO