

## Random Variables Joint Law

## 1 Why

We name the image measure of a collection of real-valued random variables.

## 2 Definition

The *joint law* of an ordered finite family of n real-valued random variables is the image measure of the tuple-valued function whose components are the individual random variables.

## 2.1 Notation

Let R denote the set of real numbers and let  $R^n$  denote set of n-tuples of real numbers. Let  $f_1, \ldots, f_n$  be random variables defined on a probability space  $(X, \mathcal{A}, \mu)$ . Define  $f: X \to R^n$  by  $(f(x))_i = f_i(x)$ . The joint law is the image measure of f.

We denote the joint law of  $\{f_i\}$  by  $\mu_{f_1,\dots,f_n}: \mathcal{A} \to \mathbb{N}$ . We defined it by

$$\mu_{f_1,...,f_n}(A) = \mu(\{x \in X \mid f(x) \in A\}).$$

for all A in the topological sigma algebra on  $\mathbb{R}^n$ .