

REAL TRANSLATES

Definition

The translate of $S \subset \mathbf{R}^n$ by $a \in \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid \exists x \in S \text{ such that } z = x + a\}.$$

Notation

We often use the abbreviated notation S + a for the translate of S by a. It is sometimes also convenient to extend set-builder notation and write

$$S + a = \{x + a \mid x \in M\}.$$

The right hand side is slick notation for the definition given above.

Sums and differences

The sum of two sets $S,T\subset \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x + y)\}.$$

Likewise, the difference of two sets $S, T \subset \mathbb{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x - y)\}.$$

Notation

We denote the sum of S and T by S+T, and the difference by S-T. We often use the slick notation

$$\{x + y \mid x \in S, y \in T\} \text{ and } \{x - y \mid x \in S, y \in T\},\$$

for these two sets.

