Why

As with introducing Equivalent Sets, we want to talk about the size of a set.[1]

Definition

A *finite* set is one that is equivalent to some natural number; an infinite set is one which is not finite. From this we can show that ω is infinite. This justifies the language "principle of infinity" with Natural Numbers. The principle of infinity asserts the existence of a particular infinite set; namely ω.

Motivation for set number

It happens that if a set is equivalent to a natural number, it is equivalent to only one natural number.

Proposition 1. *A set can be equivalent to at most one natural number.*[2]

A consequence is that a finite set is never equivalent to a proper subset of itself. So long as we are considering finite sets, a piece (subset) is always less than than the whole (original set).

Proposition 2. *A finite set is never equivalent to a proper subset of itself.*

Subsets of finite sets

Every subset of a natural number is equivalent to a natural number.[3] A consequence is:

Proposition 3. *Every subset of a finite set is finite.*[4]

[1] Will be expanded in future editions.

[2] Future edition will include proof, which uses comparability of numbers and the results of Equivalent Sets.

[3] This requires proof, and may become a proposition in future editions.

[4] An account will appear in future editions.

Unions of finite sets

Proposition 4. *If A and B are finite, then $A \cup B$ is finite.*

Products of finite sets

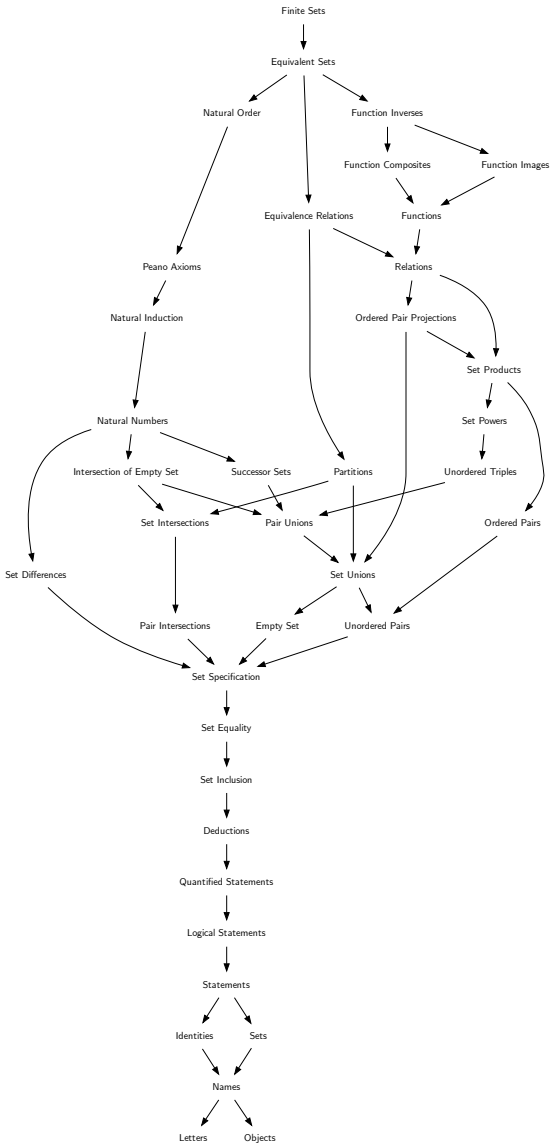
Proposition 5. *If A and B are finite, then $A \times B$ is finite.*

Powers of finite sets

Proposition 6. *If A is finite then $\mathcal{P}(A)$ is finite.*

Functions between finite sets

Proposition 7. *If A and B are finite, then A^B is finite.*


Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Function Composites
Function Images
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Partitions
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects