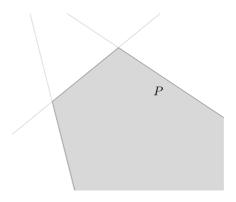


Definition

A polyhedron (or real polyhedron, or convex polyhedron) is a set $P \subset \mathbf{R}^n$ for which there exists $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$ satisfying

$$P = \{ x \in \mathbf{R}^n \mid Ax \le b \}.$$

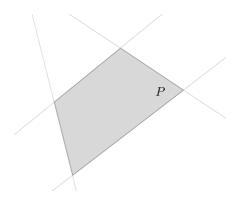
In other words, a polyhedron is an intersection of finitely many halfspaces. If A and b are rational, then P is called a rational polyhedron.



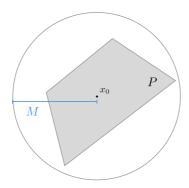
A polyhedron P is a polytope (a real polytope) if it is bounded. In other words, there exists $x_0 \in P$ and M > 0 such that

$$P \subset B_M(x_0) = \{x \mid ||x - x_0|| < M\}$$

Here $B_M(x_0)$ denotes the open ball of radius M, as usual.



As usual, the dimension of a polyhedron P is the dimension of the affine hull of P, which we denote by $\dim P$. P is called *full-dimensional* if $\dim P = n$. An equivalent condition for P to be full-dimensional is that there exist an interior point of P (as a subset of \mathbb{R}^n)



Terminology

Caution: some authors have a more relaxed notion of polyhedra, which does not require the polyhedra be convex.

