Why

We can give the set of bounded linear functions between two norm spaces a norm.

Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

Notation

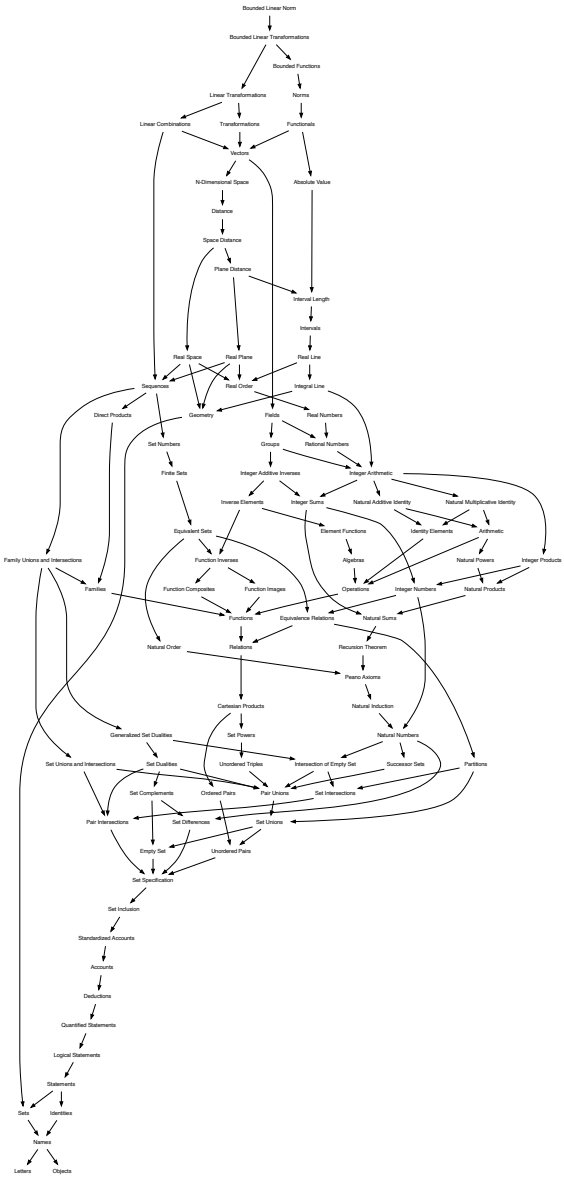
Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ be linear and bounded. The norm of f is the smallest C so that

$$\|f(v)\|_2 \leq C\|v\|_1.$$

Equivalent Formulation

Prop. 1. *Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ bounded and linear. The norm of f is*

$$\sup_{\|x\|_1=1} \|f(x)\|_2.$$


Bounded Linear Norm
Bounded Linear Transformations
Bounded Functions
Linear Transformations
Norms
Linear Combinations
Transformations
Functionals
Vectors
N-Dimensional Space
Absolute Value
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Sequences
Real Order
Integral Line
Direct Products
Geometry
Fields
Real Numbers
Set Numbers
Groups
Rational Numbers
Finite Sets
Integer Additive Inverses
Integer Arithmetic
Inverse Elements
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Element Functions
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Algebras
Natural Powers
Integer Products
Families
Function Composites
Function Images
Operations
Integer Numbers
Natural Products
Functions
Equivalence Relations
Natural Sums
Natural Order
Relations
Recursion Theorem
Peano Axioms
Cartesian Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Ordered Pairs
Pair Unions
Set Intersections
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects