Why

We want to sum infinitely many real numbers.

Definition

Let n be a natural number. The nth *partial sum* of a sequence of real numbers is the sum of first n elements of the sequence. The first partial sum is the first term of the sequence. The third partial sum is the sum of first three elements of the sequence.

The *series* of the sequence is the sequence of partial sums. The sequence is *summable* if the series converges.

Since there exist sequences which do not converge, there exist sequences which are not summable. Consider the sequence which alternates between $+1$ and -1, and starts with $+1$. Its series alternates between $+1$ and 0, and so does not converge.

Notation

Let $(a_n)_n$ be a sequence of real numnbers. For natural number n, define:

$$s_n = \sum_{k=1}^{n} a_k.$$

Then $(s_n)_n$ is the series of $(a_n)_n$. If the series converges, then there exists a real number s, the limit, and we write:

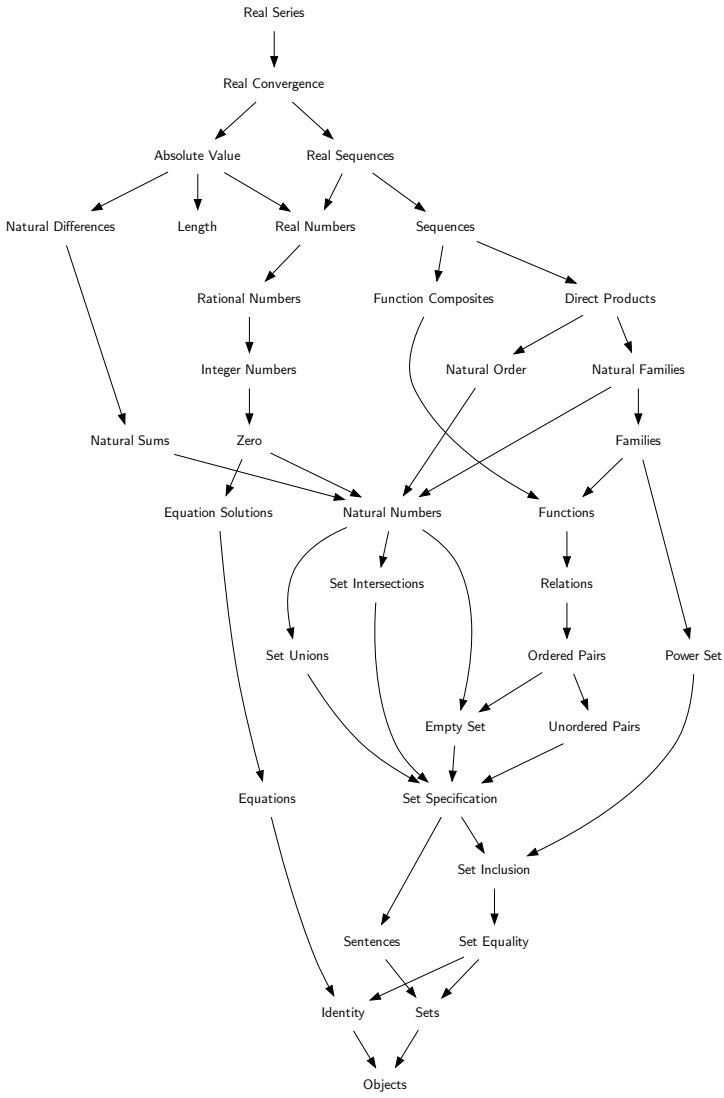
$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

We read these relations aloud as “s is the limit as n goes to infinity of s n" and “s is the limit as n goes to infinity of the sum of a k from k equals 1 to n."

To avoid referencing s_n, we write:

$$\sum_{k=1}^{\infty} a_k = s,$$

read aloud as the “the sum from 1 to infinity of a k is s." The notation is subtle, and requires justification by the algebra of series. TODO


Real Series
Real Convergence
Absolute Value
Real Sequences
Natural Differences
Length
Real Numbers
Sequences
Rational Numbers
Function Composites
Direct Products
Integer Numbers
Natural Order
Natural Families
Natural Sums
Zero
Families
Equation Solutions
Natural Numbers
Functions
Set Intersections
Relations
Set Unions
Ordered Pairs
Power Set
Empty Set
Unordered Pairs
Equations
Set Specification
Set Inclusion
Sentences
Set Equality
Identity
Sets
Objects