



Why

There is a natural orientation of an ordered undirected graph.

Motivating Result

An ordered undirected graph can be converted into a directed graph by orienting the edges from lower to higher index. The *orientation* of an ordered undirected graph $((V, E), \sigma)$ is the directed graph (V, F) where

$$\{v, w\} \in E \longrightarrow (v, w) \in F \text{ and } \sigma^{-1}(v) < \sigma^{-1}(w).$$

In other words, we can “convert” the ordered undirected graph by “orienting” the edges from lower to higher index.

Proposition 1. *Let $G = ((V, E), \sigma)$ be an ordered undirected graph. The orientation of G is acyclic.*

Proof. Contradiction on the existence of a cycle.¹ □

Conversely, let (V, F) be directed acyclic. To each topological numbering σ of (V, F) (see **Directed Paths**) there exists an ordered undirected graph $((V, E), \sigma)$ where (V, E) is the skeleton of (V, F) .

Example

Let $G = ((V, E), \sigma)$ be an undirected graph with

$$V = \{a, b, c, d, e\},$$

¹Future editions will expand.

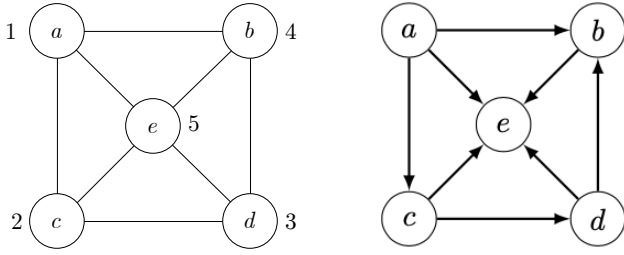


Figure 1: G and its (directed acyclic) orientation.

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\},$$

and

$$\sigma(1) = a \quad \sigma(2) = c \quad \sigma(3) = d \quad \sigma(4) = b \quad \sigma(e) = 5.$$

We visualize the ordered graph in Figure 1.

