Definition

A function $f : \mathbf{R}^n \to \mathbf{R}^m$ is *linear* if

1. $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbf{R}^n$, and

2. $f(\alpha x) = \alpha f(x)$ for all $\alpha \in \mathbf{R}$ and $x \in \mathbf{R}^n$.

Equivalently, $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$ for all $\alpha, \beta \in \mathbf{R}$ and $x, y \in \mathbf{R}^n$. In this case, we say that *superposition* holds.

Multivariate Vector Linear Functions

Linear Functions

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Real Space

Intervals

Real Plane

Real Line

Integral Line

Real Order

Real Summation

Geometry

Rational Order

Real Numbers

Natural Summation

Rational Numbers

Lists

Set Numbers

Direct Products

Integer Arithmetic

Finite Sets

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Equivalent Sets

Arithmetic

Identity Elements

Comparisons

Function Inverses

Integer Order

Family Unions and Intersections

Natural Powers

Integer Sums

Family Operations

Orders

Function Images

Function Composite

Integer Numbers

Families

Natural Products

Operations

Converse Relations

Natural Order

Functions

Equivalence Relations

Natural Sums

Relations

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Set Products

Natural Induction

Set Powers

Natural Numbers

Generalized Set Dualities

Set Symmetric Differences

Partitions

Unordered Triples

Intersection of Empty Set

Successor Sets

Set Dualities

Set Unions and Intersections

Ordered Pairs

Set Intersections

Pair Unions

Set Complements

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects

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