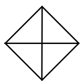


Why

We want to define area under a real function. We begin with functions whose area under the curve is self-evident.

Definition

Consider a measure space. The characteristic function of any measurable set is measurable. A simple function is measurable if and only if each element of its simple partition is measurable.

The *integral* of a measurable non-negative simple function is the sum of the products of the measure of each piece with the value of the function on that piece. For example, the integral of a measurable characteristic function of a subset is the measure of that subset.

The *integral operator* is the real-valued function which associates each measurable non-negative simple function with its integral. The simple integral is non-negative, so the integral operator is a non-negative function.

Notation

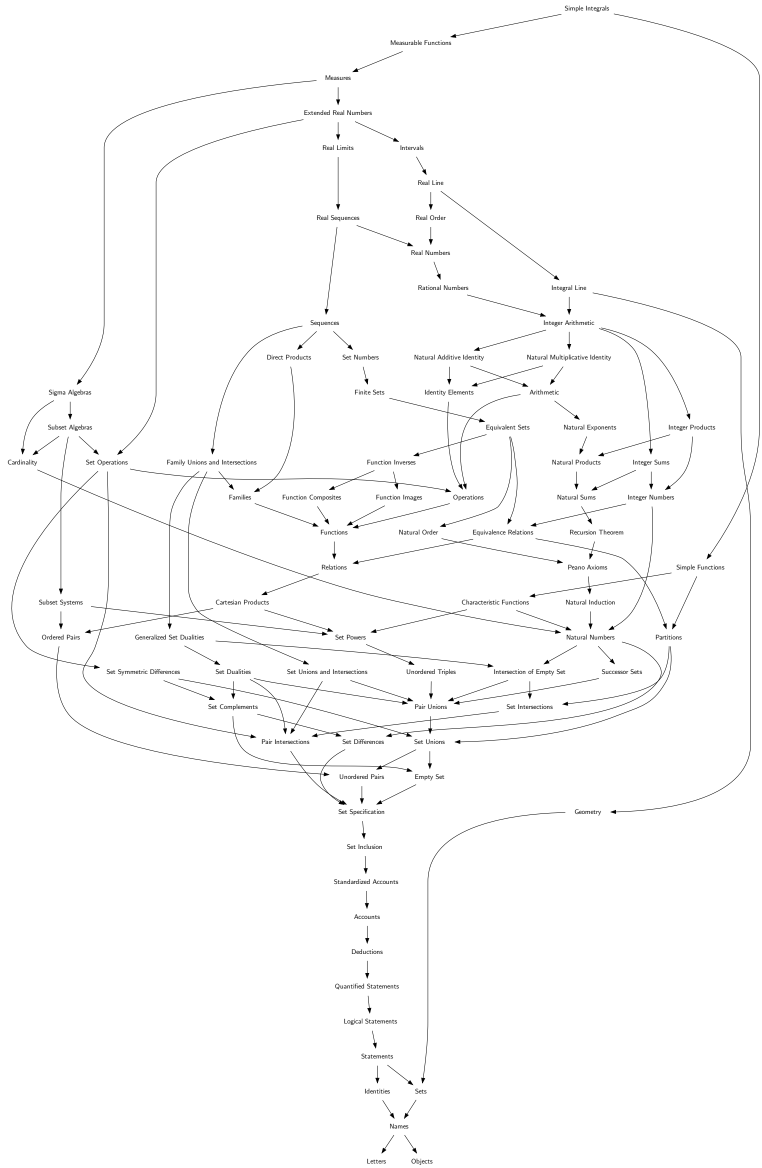
Let $(X, \mathcal{A}, \mu)$ be a measure space. Let R be the set of real numbers.

Let $f : X \to R$ be a measurable simple function. So there exist $A_1, \dots, A_n \in \mathcal{A}$ and $a_1, \dots, a_n \in R$ with:

$$f = \sum_{i=1}^{n} a_i \chi_{A_i}.$$

We denote the integral of f with respect to measure μ by $\int f d\mu$. We defined:

$$\int f d\mu = \sum_{i=1}^{n} a_i \mu(A_i).$$


Simple Integrals
Measurable Functions
Measures
Extended Real Numbers
Real Limits
Intervals
Real Line
Real Sequences
Real Order
Real Numbers
Rational Numbers
Integral Line
Sequences
Integer Arithmetic
Direct Products
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Sigma Algebras
Finite Sets
Identity Elements
Arithmetic
Subset Algebras
Equivalent Sets
Natural Exponents
Integer Products
Cardinality
Set Operations
Family Unions and Intersections
Function Inverses
Natural Products
Integer Sums
Families
Function Composites
Function Images
Operations
Natural Sums
Integer Numbers
Functions
Natural Order
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Simple Functions
Subset Systems
Cartesian Products
Characteristic Functions
Natural Induction
Ordered Pairs
Generalized Set Dualities
Set Powers
Natural Numbers
Partitions
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Differences
Set Unions
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects