

#### DIFFERENTIAL ENTROPY

# Why

We want to extend our notion of entropy (see Discrete Entropy) to real-valued (continuous) random variables.

## **Definition**

The differential entropy of a probability density function is the integral of the density against the negative log of the density. This definition made to be similar to the case of discrete entropy. If a real-valued random variable has a density, then we call the differential entropy of its density the differential entropy of the random variable.

#### Notation

Let  $f: \mathbf{R}^n \to \mathbf{R}$  be a probability density function. The differential entropy of f is

$$-\int f \log f$$

We denote the differential entropy of f by h(f).

### Example

Let  $x: \Omega \to \mathbf{R}$  be uniform on [0, 1/2]. Then  $h(x) = \log 1/2 < 0$ .

### **Problems**

We have  $h(ax) = h(x) + \log|a|$ . In general  $h(Ax) = h(x) + \log|A|$ .

## Differences still meaningful

Even though the value of the differential entropy is not necessarily a good analogy to discrete entropy, differences still are. In particular, the following holds

$$I(X;Y) = H(Y) - H(Y \mid X) = H(X) = H(X \mid Y)$$

