Why

We want to define the length of a subset of real numbers.

Notions

We take two common notions:

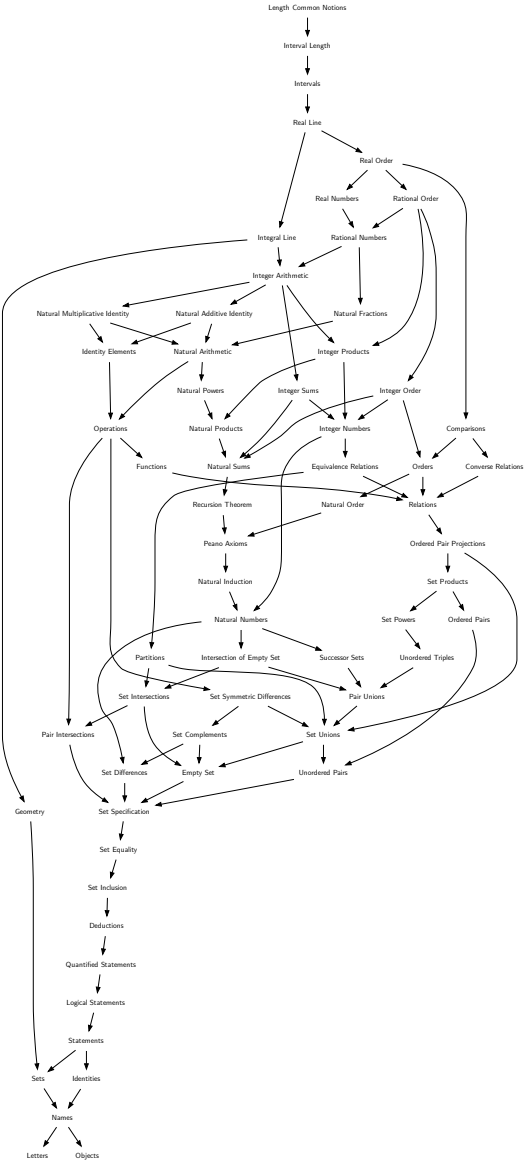
1. The length of a whole is the sum of the lengths of its parts; the *additivity principle*.
2. The length of a whole is the at least the length of any whole it contains the *containment principle*.

The task is to make precise the use of "whole," "parts," and "contains." We start with intervals.

Definition

By whole we mean set. By part we mean an element of a partition; in other words, a subset. By contains we mean set inclusion.

The length of an interval is the difference of its endpoints: the larger minus the smaller (see Interval Length). Two intervals are *non-overlapping* if their intersection is a single point or empty. The *length* of the union of two non-overlapping intervals is the sum of their lengths.


Length Common Notions
Interval Length
Intervals
Real Line
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Natural Fractions
Identity Elements
Natural Arithmetic
Integer Products
Natural Powers
Integer Sums
Integer Order
Operations
Natural Products
Integer Numbers
Comparisons
Functions
Natural Sums
Equivalence Relations
Orders
Converse Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects