



**Why**

What is an inner product if we take a vector space over the complex numbers.

**Definition**

An inner product over a complex vector space is positive definite, Hermitian, and linear in the first argument.

**Notation**

Let  $(V, \mathbf{C})$  be a complex vector space. Let  $f : V \times V \rightarrow \mathbf{C}$  be a function such that

1.  $f(x, x) \geq 0$ ,  $f(x, x) = 0 \Leftrightarrow x = 0$ ;
2.  $f(x, y) = \overline{f(y, x)}$
3.  $f(ax + by, z) = a f(x, z) + b f(y, z)$

