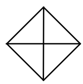


Why

We want to capture the useful properties of the standard basis vectors.

Definition

A set of vectors $\{v_1, \dots, v_k\} \subset \mathbf{R}^n$ is *independent* if

$$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_k v_k = 0 \Rightarrow \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0.$$

Notice that independence is a property of a set of vectors, not of any vector in particular. Another way of saying this is that no vector can be represented as a linear combination of another.

Unique representation

Suppose $v_1, \dots, v_k$ are independent and we have

$$x = \sum_{i=1}^{k} \alpha_i v_i \quad \text{and} \quad x = \sum_{i=1}^{k} \beta_i v_i.$$

Then

$$0 = x - x = \sum_{i=1}^{n} (\alpha_i - \beta_i) v_k.$$

Using the definition of independence, we conclude $\alpha_i - \beta_i = 0$ for $i = 1, \dots k$. Consequently, $\alpha_i = \beta_i$. In other words, if x can be represented as a linear combination of the vectors $v_1, \dots, v_k$, that representation is *unique*. We have shown that independence implies uniqueness? What of the converse?

We show that lack of independence gives a lack of uniqueness. Suppose there exists $\alpha_1, \dots, \alpha_k$, not all zero, so that

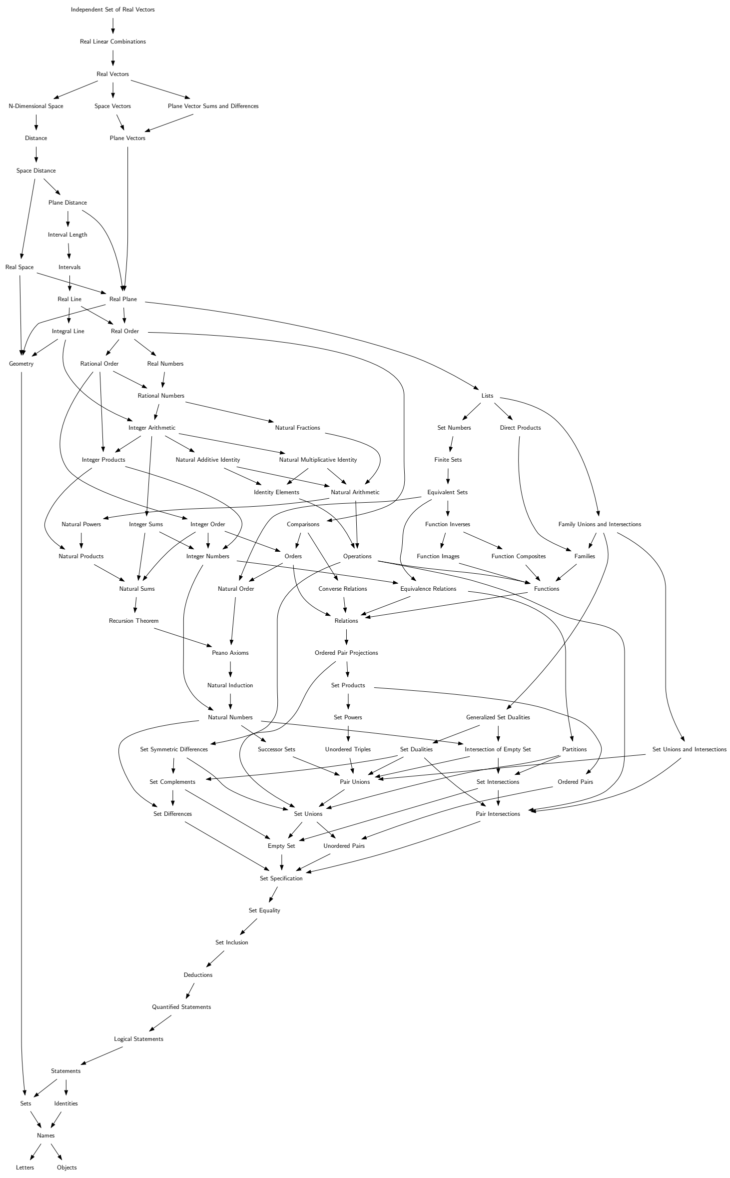
$$\alpha_1 v_1 + +\alpha_2 v_2 + \cdots + \alpha_k v_k = 0.$$

In particular, suppose $\alpha_i \neq 0$. Then we have

$$v_i = (1/\alpha_i) \sum_{j \neq i} \alpha_j v_j.$$

Suppose x can be written as a linear combination of $v_1, \ldots, v_k$. In other words, there are $\beta_1, \ldots, \beta_k$ so that

$$x = \beta_1 v_1 + \beta_2 v_2 + \cdots + \beta_k v_k$$


Independent Set of Real Vectors
Real Linear Combinations
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects