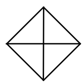


Why

We want a notion for a correspondence between two sets.

Definition

A *function* f (or *correspondence*, *mapping*, *map*) *from* a set X *to* a set Y is a relationwhose domain is X and whose range is a subset of Y, such that for each $x \in X$,

1. there exists $y \in Y$ so that $(x, y) \in f$
2. if $(x, y) \in f$ and $(x, z) \in f$, then $y = z$; where y and z are in Y

We often summarize these two conditions by saying: to every element $x \in X$ there corresponds a *unique* element $y \in Y$ so that $(x, y) \in f$.

We call this unique element $y \in Y$ the *result* of the function *at* the *argument* x. We call Y *a codomain*—notice our use of the word "a", since the codomain is not a property of the function. If the range is Y we say that f is a function from X *onto* Y (or call f *onto*, *surjective*). If distinct elements of X are mapped to distinct elements of Y, we say that the function is *one-to-one* (or *injective*).

We say that the function *maps* (or *takes*) elements from the domain to the codomain. Since the word "function" and the verb "maps" connote activity, some authors refer to the set of ordered pairs as the *graph* of a function and avoid defining the term "function" as we have, in terms of sets. Our use of the term here is in agreement with standard contemporary mathematical practice.

Notation

Given sets X and Y, we abbreviate the statement that the object denoted by f is a function whose domain is a X and whose codomain is a set Y by

$$f : X \to Y$$

We read the notation aloud as "f from X to Y." We emphasize again that the *range* of f need not be Y, but must necessarily be a subset.

We denote by Y^X the set of functions from X to Y. This set is contained in the power set $\mathcal{P}((X \times Y))$. A reasonable but nonstandard notation is $X \to Y$, read as "A to B." All the following three statements have the same meaning:

$$f : X \to Y, \quad f \in Y^X, \quad f \in (X \to Y).$$

We tend to denote functions by lower case latin letters; especially f, g, and h. f is a mnemonic for function and g and h are nearby in the usual ordering of the Latin letters.

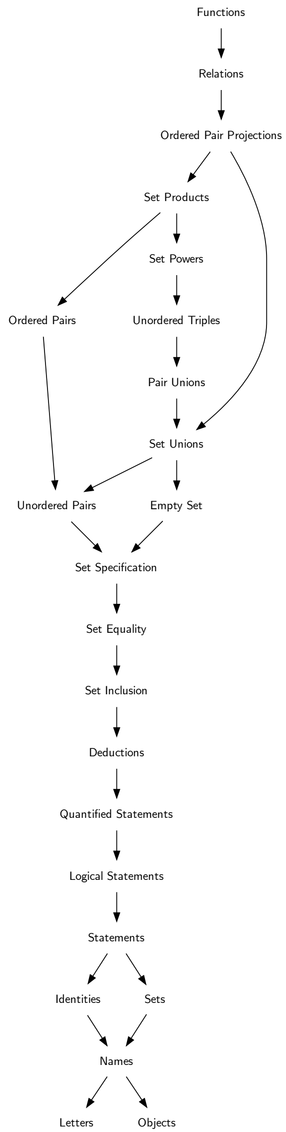
Suppose $f : A \to B$. For each element $a \in A$, we denote the result of applying f to a by $f(a)$, read aloud "f of a." We sometimes drop the parentheses, and write the result as f_a, read aloud as "f sub a." Let $g : A \times B \to C$. We often write $g(a, b)$ or g_{ab} instead of $g((a, b))$. We read $g(a, b)$ aloud as "g of a and b". We read g_{ab} aloud as "g sub a b."

Examples

If $X \subset Y$, the function $\{(x, y) \in X \times Y \mid x = y\}$ is the *inclusion function* of X into Y. We often introduce such a function as "the function from X to Y defined by $f(x) = y$". We mean by this that f is a function and that we are specifying the appropraite ordered pairs using the statement, called *argument-value notation.* The inclusion function of X into X is called the *identity function* of X. If we view the identity function as a relation on X, it is the relation of equality on X.

The functions $f : (X \times Y) \to X$ defined by $f(x, y) = x$ is the *pair projection* of $X \times Y$ ono X. Similarly $g : (X \times Y) \to Y$ defined by $g(x, y) = y$ is the pair projection of $X \times Y$ onto Y.

The identity function is one-to-one and onto, the inclusion functions are one-to-one but not always onto, and the pair projections are usually not one-to-one.



Functions
Relations
Ordered Pair Projections
Set Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects