Why

It is common to consider random functions whose domain is time, space, or n-dimensional space.

Definition

Let (X, d) be a metric space. A *distance covariance function* $k : X \times X \to \mathbf{R}$ is a covariance function satisfying

$$k(x, y) > k(x, y) \longleftrightarrow d(x, y) < d(x, y).$$

In other words, the covariance decreases as the distance between the arguments decreases.

Example: squared exponential

Let $k : X \times X \to \mathbf{R}$ be defined by

$$k(x, y) = \exp(-d(x, y)).$$

Then k is a distance covariance function. It is often called the *squared exponential covariance function.*

Let $\alpha, \sigma \in \mathbf{R}$. Define $k' : X \times X \to \mathbf{R}$ by

$$k'(x, y) = \alpha \exp(-d(x, y)/\sigma^2)$$

then k' is still a covariance function. In this context σ is often referred to as the *characteristic length-scale* of the process. The scalar α is sometimes called a "prefactor" that "controls" the "overall variance."

Suppose $(X, d) = (\mathbf{R}^n, \|\cdot\|)$. Then the squared exponential covariance function

$$\alpha \exp(-\|x - y\|/(2\sigma^2))$$

is sometimes called the *radial basis function* or *gaussian covariance function.*[1] Also called an *exponentiated quadratic kernel.*

[1] For reasons that will be included in future editions.

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Random Functions
Multivariate Normals
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Symmetric Real Matrices
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Real Matrix Inverses
Metric Spaces
Multivariate Real Densities
Normal Densities
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Probability Distributions
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Real Matrix Inverse Products
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Intersect Every
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Set Differences
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Symmetric Differences
Set Intersections
Set Complements
Set Differences
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Set Differences
Set Complements
Set Differences
Set Equality
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