ELIMINATION GRAPHS

Why

We want to model the progressive fill-in during Gauss elimination with graphs.

Definition

Let $G = ((V, E), \sigma)$ be an ordered undirected graph with $|V| = n$. Define $E_0 = E$. The *elimination edge sequence* of G is a sequence $(E_1, \ldots, E_{n-1})$ defined by

$$E_i = E_{i-1} \cup$$
$$\{\{v, w\} \mid v \succ \sigma(i), w \succ \sigma(i), \text{ and } \{\sigma(i), w\} \in E_{i-1}\}$$

for $i = 1, \ldots, n - 1$. The difference between E_{i-1} and E can be described by saying that the higher neighborhood of the intermediate graph $((V, E_{i-1}), \sigma)$ is made complete by the addition of edges between all non-adjacent vertices.

The *elimination graph* of G is the graph $((V, E_{n-1}), \sigma)$.

Elimination Graphs

Pivoted Row Reductions

Ordinary Row Reductions

Linear System Row Reductions

Linear Equation Solutions

Linear Equations

Linear Functions

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Arrays

Real Space

Real Line

Real Plane

Integral Line

Real Order

Real Summation

Geometry

Rational Order

Real Numbers

Ordered Undirected Graphs

Rational Numbers

Lists

Undirected Graphs

Integer Arithmetic

Set Numbers

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Arithmetic

Identity Elements

Direct Products

Finite Sets

Natural Powers

Natural Summation

Equivalent Sets

Function Inverse

Natural Equations

Integer Sums

Integer Order

Natural Products

Comparisons

Family Operations

Family Unions and Intersections

Function Composites

Function Images

Equations

Integer Numbers

Natural Sums

Order

Operations

Families

Recursion Theorem

Natural Order

Converse Relation

Functions

Equivalence Relations

Peano Axioms

Relations

Natural Induction

Set Products

Natural Numbers

Generalized Set Dualities

Set Powers

Set Symmetric Differences

Set Dualities

Successor Sets

Intersection of Empty Set

Unordered Triples

Partitions

Set Unions and Intersections

Set Complements

Pair Unions

Set Intersections

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects