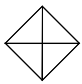


Definition

A *linear functional* on a vector space V over a field $\mathbf{F}$ is a linear function from V to $\mathbf{F}$. In other words, a linear function is an element of $\mathcal{L}(V, \mathbf{F})$.

Notation

We tend to denote linear functionals by $\phi : V \to \mathbf{F}$, a mnemonic for functional.

Examples

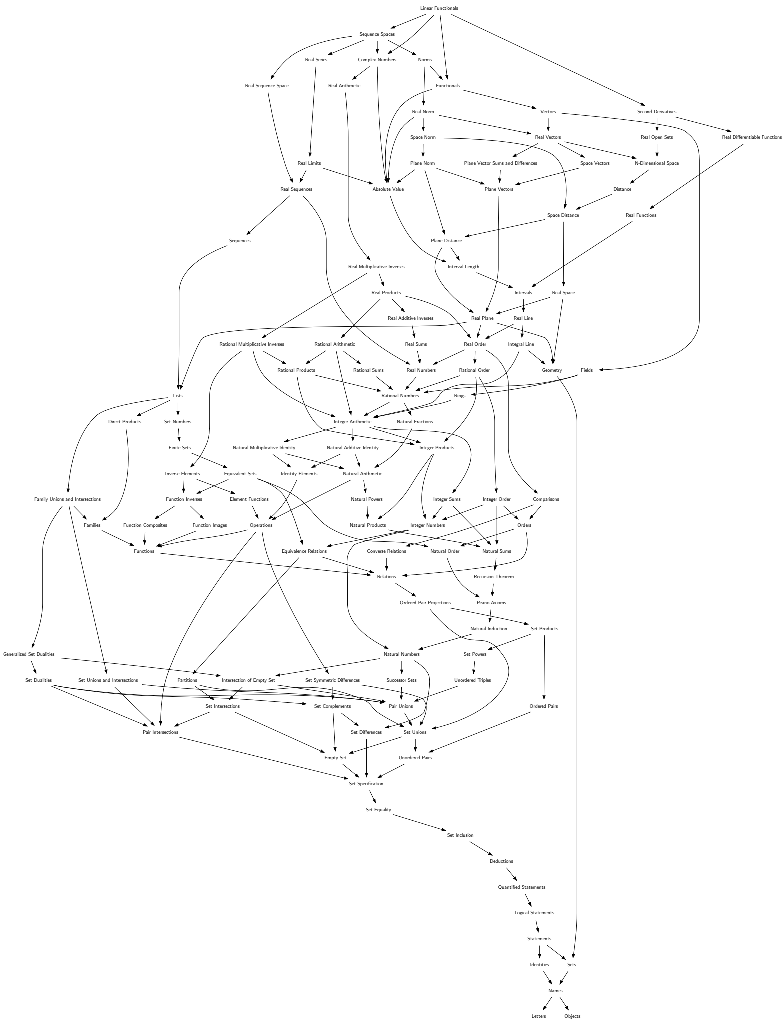
1. Define $\phi : \mathbf{R}^3 \to \mathbf{R}$ by
$$\phi(x, y, z) = 4x - 5y + 2z$$
ϕ is a linear function on $\mathbf{R}^3$

2. Define $\phi : \mathbf{C}^n \to \mathbf{C}$ by
$$\phi(x_1, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$
where $c_1, \ldots, c_n \in \mathbf{C}$. ϕ is a linear functional on $\mathbf{C}^n$.

3. Let $(c_n)_{n\in\mathbf{N}} \in \ell^\infty$. Define $F_c : \ell^1 \to \mathbf{C}$ by
$$F_c((x_n)_{n\in\mathbf{N}}) = \sum_{n=1}^{\infty} c_n x_n.$$

4. As usual, denote the set of real polynomials by $\mathcal{P}(\mathbf{R})$. Define $\phi : \mathcal{P}(\mathbf{R}) \to \mathbf{R}$ by
$$\phi(p) = 3p''(5) + 7p(4)$$
ϕ is a linear functional on $\mathcal{P}(\mathbf{R})$.

5. As usual, denote the set of real polynomials by $\mathcal{P}(\mathbf{R})$. Define $\phi : \mathcal{P}(\mathbf{R}) \to \mathbf{R}$ by
$$\phi(p) = \int_{[0,1]} p$$
ϕ is a linear functional on $\mathcal{P}(\mathbf{R})$.


Linear Functionals
Sequence Spaces
Real Series
Complex Numbers
Norms
Real Sequence Space
Real Arithmetic
Functionals
Real Norm
Vectors
Second Derivatives
Space Norm
Real Vectors
Real Open Sets
Real Differentiable Functions
Real Limits
Plane Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Real Sequences
Absolute Value
Plane Vectors
Distance
Space Distance
Real Functions
Sequences
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Plane
Real Line
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Rational Products
Rational Sums
Real Numbers
Rational Order
Geometry
Fields
Lists
Rational Numbers
Rings
Direct Products
Set Numbers
Integer Arithmetic
Natural Fractions
Finite Sets
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Inverse Elements
Equivalent Sets
Identity Elements
Natural Arithmetic
Family Unions and Intersections
Function Inverses
Element Functions
Natural Powers
Integer Sums
Integer Order
Comparisons
Families
Function Composites
Function Images
Operations
Natural Products
Integer Numbers
Orders
Functions
Equivalence Relations
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Natural Induction
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Intersections
Set Complements
Pair Unions
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects