## Why

It happens that if a distribution factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So the property of factoring according to a rooted tree is really property of factoring according to a tree.

## Definition

### Defining Result

**Proposition 1.** *Let $A_1, \ldots, A_n$ be finite non-empty sets and define $A = \prod_{i=1}^{n} A_n$. Let $p : A \to [0, 1]$ be a distribution and let $T$ be a tree on $\{1, \ldots, n\}$.*

**Proposition 2.** *If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

### Undirected definition

A distribution $p$ *factors according to the tree* $T$ if it factors according to the $T$ rooted at any vertex.

## Existence and uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

### Existence

A distribution $p$ need not factor according to a tree.

### Uniqueness

A distribution $p$ may factor according to multiple trees.


Tree Distributions
Rooted Tree Distributions
Conditional Distributions
Conditional Event Probabilities
Marginal Distributions
Event Probabilities
Joint Distributions
Probability Distributions
Intervals
Real Line
Real Summation
Real Order
Rational Order
Real Numbers
Rational Numbers
Integral Line
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Natural Summation
Natural Products
Integer Sums
Integer Order
Natural Sums
Integer Numbers
Comparisons
Orders
Family Operations
Operations
Families
Recursion Theorem
Natural Order
Convene Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Uncertain Outcomes
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Set Complements
Pair Unions
Ordered Pairs
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects
Rooted Trees
Trees
Undirected Paths
Directed Graphs
Lists
Undirected Graphs
Set Numbers
Direct Products
Finite Sets
Equivalent Sets
Function Inverses
Family Unions and Intersections
Function Composites
Function Images
Equivalence Relations
Partitions
Set Unions and Intersections