Definition

The *complex conjugate* (or *conjugate*) of a complex number z is the complex number whose real part matches z and whose imaginary part is the additive inverse of z. The complex conjugate of a purely real number is the same purely real number. In other words, the complex conjugate of a complex number with no imaginary part is the same complex number.

Notation

We denote the complex conjugate of the complex number $z \in \mathbf{C}$ by $\mathbf{C}conjz$. Other common notation includes $\bar{z}$, read "z bar". If there exists $a, b \in \mathbf{R}$ so that $z = (a, b)$, then $\mathbf{C}conjz = (a, -b)$.

Geometric interpretation

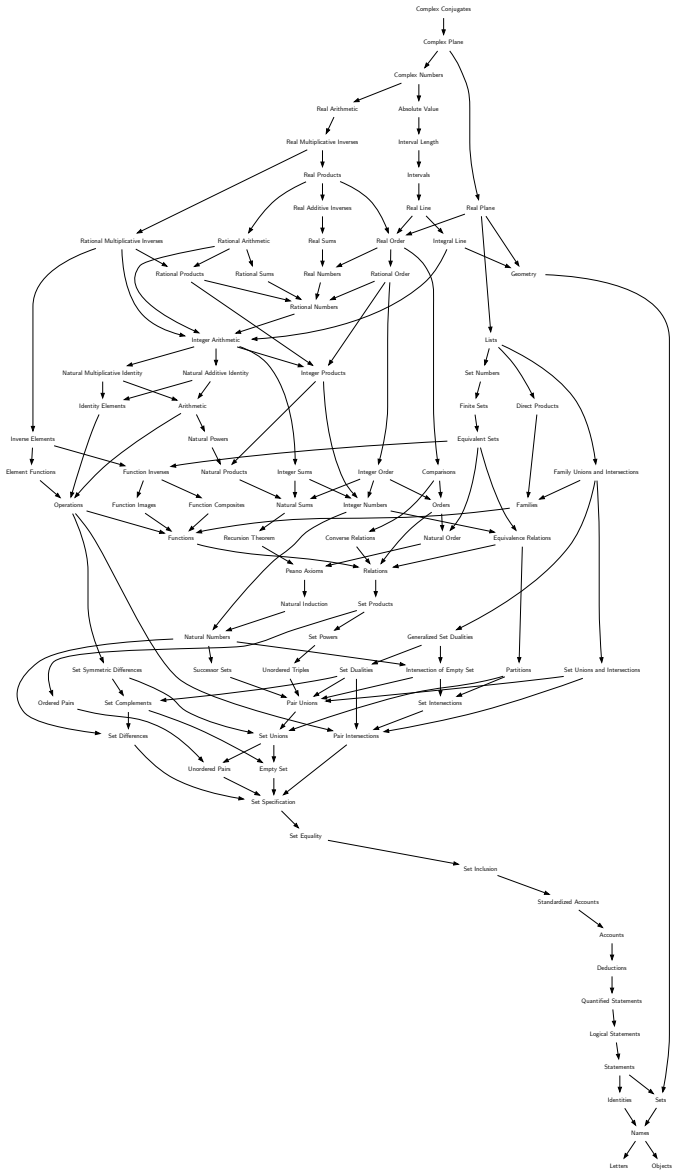
Taking the conjugate of a complex numbers corresponds to a reflection across the real axis in the plane.

Properties

A complex number z is real if and only if $z = \mathbf{C}conjz$ and it is imaginar if and only if $z = -\mathbf{C}conjz$.

Proposition 1. *For $z \in \mathbf{C}$, we have*

$$\mathrm{Re}(z) = \frac{z + z^*}{2} \quad \textit{and } \mathrm{Im}(z) = \frac{z - z^*}{2i}.$$


Complex Conjugates
Complex Plane
Complex Numbers
Real Arithmetic
Absolute Value
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Rational Products
Rational Sums
Real Numbers
Rational Order
Geometry
Rational Numbers
Integer Arithmetic
Lists
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Set Numbers
Identity Elements
Arithmetic
Finite Sets
Direct Products
Inverse Elements
Natural Powers
Equivalent Sets
Element Functions
Function Inverses
Natural Products
Integer Sums
Integer Order
Comparisons
Family Unions and Intersections
Operations
Function Images
Function Composites
Natural Sums
Integer Numbers
Orders
Families
Functions
Recursion Theorem
Converse Relations
Natural Order
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects