



## SIGMA ALGEBRAS

### Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object (TODO).

### Definition

A *countably summable subset algebra* is a subset space for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of  $A_1, \dots, A_n$  coincides with the union of  $A_1, \dots, A_n, A_n, A_n, \dots$ .

We say that the set of distinguished sets a *sigma algebra* on the base set; we justify this language, as for an algebra, by the closure properties under standard set operations.

### Notation

The notation follows that of a subset space. Let  $(A, \mathcal{A})$  be a countably summable subset algebra. We also say “let  $\mathcal{A}$  be a sigma algebra on  $A$ .” Moreover, since the largest element of the sigma algebra is the base set, we can say without ambiguity: “let  $\mathcal{A}$  be a sigma algebra.”

### Examples

**Example 1.** For any set  $A$ ,  $2^A$  is a sigma algebra.

**Example 2.** For any set  $A$ ,  $\{A, \emptyset\}$  is a sigma algebra.

**Example 3.** Let  $A$  be an infinite set. Let  $\mathcal{A}$  the collection of finite subsets of  $A$ .  $\mathcal{A}$  is not a sigma algebra.

**Example 4.** Let  $A$  be an infinite set. Let  $\mathcal{A}$  be the collection subsets of  $A$  such that the set or its complement is finite.  $\mathcal{A}$  is not a sigma algebra.

**PROPOSITION 5.** The intersection of a family of sigma algebras is a sigma algebra.

**Example 6.** For any infinite set  $A$ , let  $\mathcal{A}$  be the set

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

$\mathcal{A}$  is an algebra; the countable/co-countable algebra.

*TOOD : cleanuexamples*

