



# Real Series

## 1 Why

We want to sum infinitely many real numbers.

## 2 Definition

Let  $n$  be a natural number. The  $n$ th *partial sum* of a sequence of real numbers is the sum of first  $n$  elements of the sequence. The first partial sum is the first term of the sequence. The third partial sum is the sum of first three elements of the sequence.

The *series* of the sequence is the sequence of partial sums. The sequence is *summable* if the series converges.

Since there exist sequences which do not converge, there exist sequences which are not summable. Consider the sequence which alternates between  $+1$  and  $-1$ , and starts with  $+1$ . Its series alternates between  $+1$  and  $0$ , and so does not converge.

### 2.1 Notation

Let  $(a_n)_n$  be a sequence of real numnbers. For natural number  $n$ , define:

$$s_n = \sum_{k=1}^n a_k.$$

Then  $(s_n)_n$  is the series of  $(a_n)_n$ . If the series converges, then there exists a real number  $s$ , the limit, and we write:

$$s = \lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n a_k$$

We read these relations aloud as “ $s$  is the limit as  $n$  goes to infinity of  $s_n$ ” and “ $s$  is the limit as  $n$  goes to infinity of the sum of  $a_k$  from  $k$  equals 1 to  $n$ .”

To avoid referencing  $s_n$ , we write:

$$\sum_{k=1}^{\infty} a_k = s,$$

read aloud as the “the sum from 1 to infinity of  $a_k$  is  $s$ .” The notation is subtle, and requires justification by the algebra of series. TODO