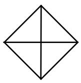


Why

Can we represent the function associating a linear system with its row reduction by matrix multiplication?

Main observation

Proposition 1. *Let* $(A \in \mathsf{R}^{n \times n}, b \in \mathsf{R}^n)$ *be a linear system with* $A_{ij} \neq 0$*. Let* (C, d) *be the* ij*-reduction of* (A, b)*.* $\exists L \in \mathsf{R}^{n \times n}$ *with*

$$C = LA \text{ and } d = Lb.$$

Proof. Define $L \in \mathsf{R}^{n \times n}$ by

$$L_{st} = \begin{cases} 1 & \text{if } s = t \\ -A_{sj}/A_{ij} & \text{if } s \neq i \\ 0 & \text{otherwise .} \end{cases}$$

□

For this reason, we call L in Proposition 1 a *row reducer matrix* or *row reducing matrix* or *row reducer.*

Example

For example, the $(1, 1)$-reduction of

$$S = (A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 6 & 7 & 9 & 5 \\ 8 & 7 & 9 & 8 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}).$$

is

$$S' = (A' = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix}, b' = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}).$$

The row reducer is $L \in R^{4 \times 4}$ defined by

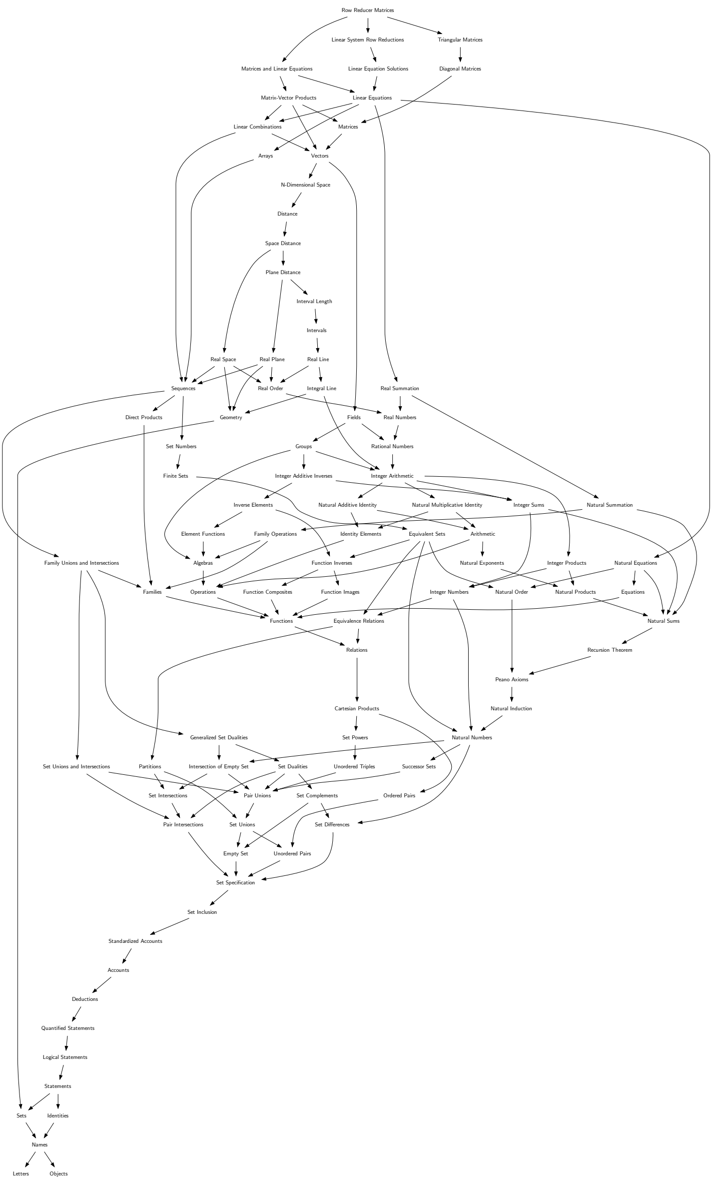
$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix}.$$

One can check that $A' = LA$ and $b' = Lb$. Notice that L here is lower triangular.

Lower triangular

Proposition 2. *For $i \geq j$, the row reducting matrix corresponding to an ij-reduction is lower unit triangular.*[1]

[1] Future editions will include an account.



Row Reducer Matrices
Linear System Row Reductions
Triangular Matrices
Matrices and Linear Equations
Linear Equation Solutions
Diagonal Matrices
Matrix-Vector Products
Linear Equations
Linear Combinations
Matrices
Arrays
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Natural Multiplicative Identity
Integer Sums
Natural Summation
Element Functions
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Identity Elements
Equivalent Sets
Arithmetic
Family Unions and Intersections
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Integer Products
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Recursion Theorem
Peano Axioms
Cartesian Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
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