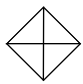


Why

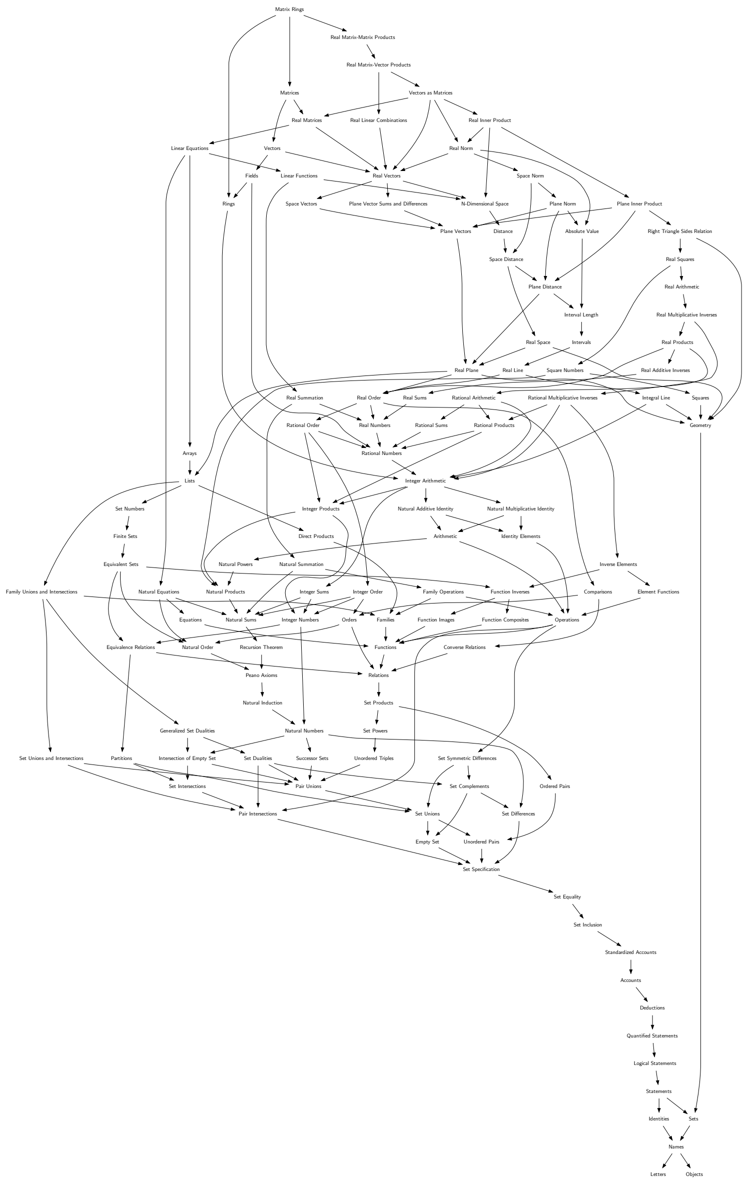
Matrices with elements in a ring form a ring.

Example

Let $(R, +, \cdot)$ be a ring. Define $C = A\bar{+}B$ by $C_{ij} = A_{ij} + B_{ij}$ and define $C = A\bar{\cdot}B$ by $C_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}$, as with real matrices, for $A, B \in R^{n\times n}$. Then $(R^{n\times n}, \bar{+}, \bar{\cdot})$ is a ring. In other words, the set of $n \times n$ matrices with elements in R is a ring, with the usual addition and multiplication of matrices.

The additive identity of the ring is the matrix $0 \in R^{n\times n}$ for which $0_{ij} = 0 \in R$. The multiplicative identity the matrix I for which $I_{ii} = 1 \in R$ for $i = 1, \dots, n$ and $I_{ij} = 0 \in R$ for $i \neq j = 1, \dots, n$. As seen with real-valued matrices, multiplication on $R^{n\times n}$ need not be commutative even if R is.

Exercise 1. *Show that $R^{n\times n}$ is not a division ring when $n > 1$.*


Matrix Rings
Real Matrix-Matrix Products
Real Matrix-Vector Products
Matrices
Vectors as Matrices
Real Matrices
Real Linear Combinations
Real Inner Product
Linear Equations
Vectors
Real Norm
Fields
Linear Functions
Real Vectors
Space Norm
Rings
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Norm
Plane Inner Product
Plane Vectors
Distance
Absolute Value
Right Triangle Sides Relation
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Real Space
Intervals
Real Products
Real Plane
Real Line
Square Numbers
Real Additive Inverses
Real Summation
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Integral Line
Squares
Rational Order
Real Numbers
Rational Sums
Rational Products
Geometry
Arrays
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Direct Products
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Natural Summation
Inverse Elements
Family Unions and Intersections
Natural Equations
Natural Products
Integer Sums
Integer Order
Family Operations
Function Inverses
Comparisons
Element Functions
Equations
Natural Sums
Integer Numbers
Orders
Families
Function Images
Function Composites
Operations
Equivalence Relations
Natural Order
Recursion Theorem
Functions
Converse Relations
Peano Axioms
Relations
Natural Induction
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects