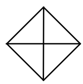


Why

We define continuity for functions between metric spaces.

Definition

Our inspiration is continuity of functions from the set of real numbers to the set of real numbers. There we decided on a definition which codified our intuition that numbers which are sufficiently close to each other are mapped to numbers that are close to each other.

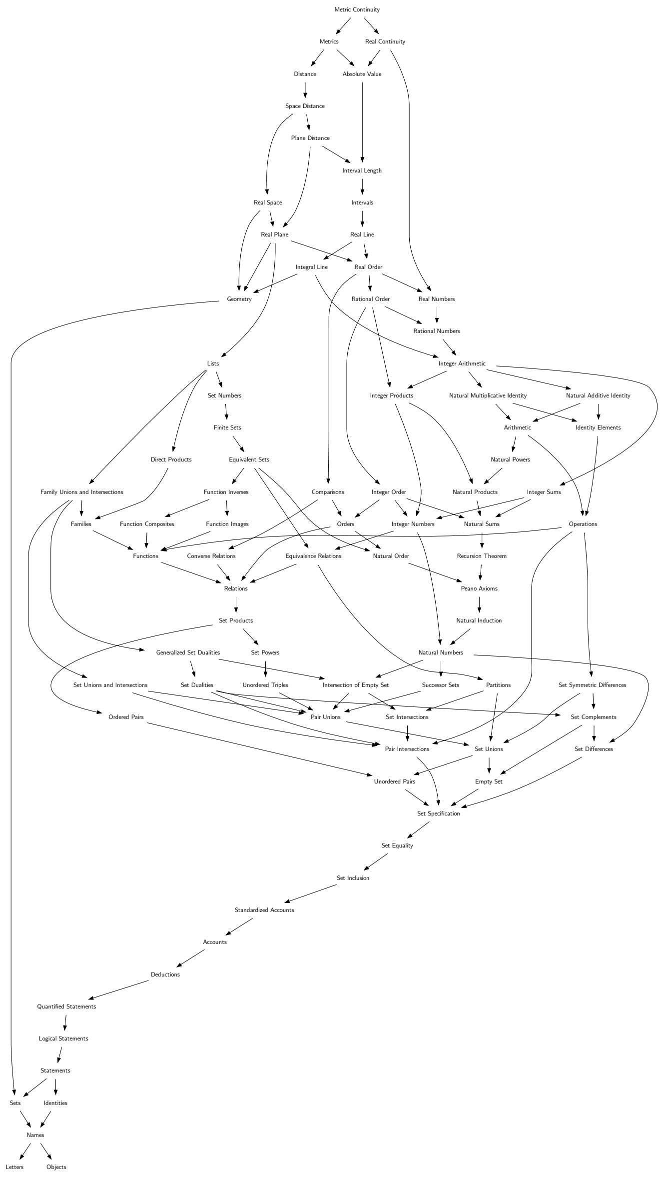
A function from a first metric space to a second metric space is *continuous at* an object of its domain if, for every positive real number (no matter how small), there is a second positive real number (possibly, though not necessarily, smaller) so that every element in the domain whose distance to the fixed object is less than the second positive number has a result under the function whose distance to the result of the fixed object is less than the first positive number.

A function between metric spaces is continuous if it is *continuous* at every object of its domain.

Notation

Let (A, d) and (B, d') be metric spaces. Let $f : (A, d) \to (B, d')$. Then f is continuous at $\bar{a} \in A$, if for all real numbers $\varepsilon > 0$, there exists a real number $\delta > 0$ such that for all $a \in A$,

$$d(\bar{a}, a) < \delta \longrightarrow d'(f(\bar{a}), f(a)) < \varepsilon.$$


Metric Continuity
Metrics
Real Continuity
Distance
Absolute Value
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Natural Products
Integer Sums
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Sums
Operations
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects