Why

We want to talk about how knowledge of one aspect of an outcome can give us knowledge about another aspect.

Definition

Two events $A, B \subset \Omega$ are *independent* under a probability measure $\mathbf{P} : \mathcal{P}(\Omega) \to \mathbf{R}$ if

$$\mathbf{P}(A \cap B) = \mathbf{P}(A)\mathbf{P}(B).$$

In other words, they are independent if the probability of their intersection is the product of their respective probabilities. Otherwise, we call A and B *dependent*.

In the case that $\mathbf{P}(B) \neq 0$, then $\mathbf{P}(A \cap B) = \mathbf{P}(A)\mathbf{P}(B)$ is equivalent to $\mathbf{P}(A \mid B) = \mathbf{P}(A)$, which more clearly expresses the intuition captured by the definition. Roughly speaking, we interpret this second expression as encoding the fact that the occurence of event B does not change the probability—intuitively, the "credibility"—of the event A.

Example: two dice

Define $\Omega = \{(\omega_1, \omega_2) \mid \omega_i \in \{1, \dots, 6\}\}$, and interpret $\omega \in \Omega$ as corresponding to pips face up after rolling two dice. Define $p : \Omega \to \mathbf{R}$ by $p(\omega) = 1/36$.

Two events are $A = \{\omega \in \Omega \mid \omega_1 + \omega_2 > 5\}$, "the sum is greater than 5", and $B = \{\omega \in \Omega \mid \omega_1 > 3\}$, "the number of pips on the first die is greater than 3". Then $\mathbf{P}(A) = 26/36$. Also, $\mathbf{P}(A \mid B) = 17/16$. So, these events are dependent. Roughly speaking, we say that knowing B tells us something about A. In this case, we say that it "makes A more probable."

In the language used to describe the events, we say that knowledge that the number of pips on the first die is greater than three makes its more probable that the sum of the number of pips on each die is greater than 5.

Basic implications

Since $\mathsf{P}(\cdot \mid B)$ is a probability measure, and the events A and $C_\Omega(A)$ partition Ω, we have

$$\mathsf{P}(A \mid B) + \mathsf{P}(C_\Omega(A) \mid B) = 1.$$

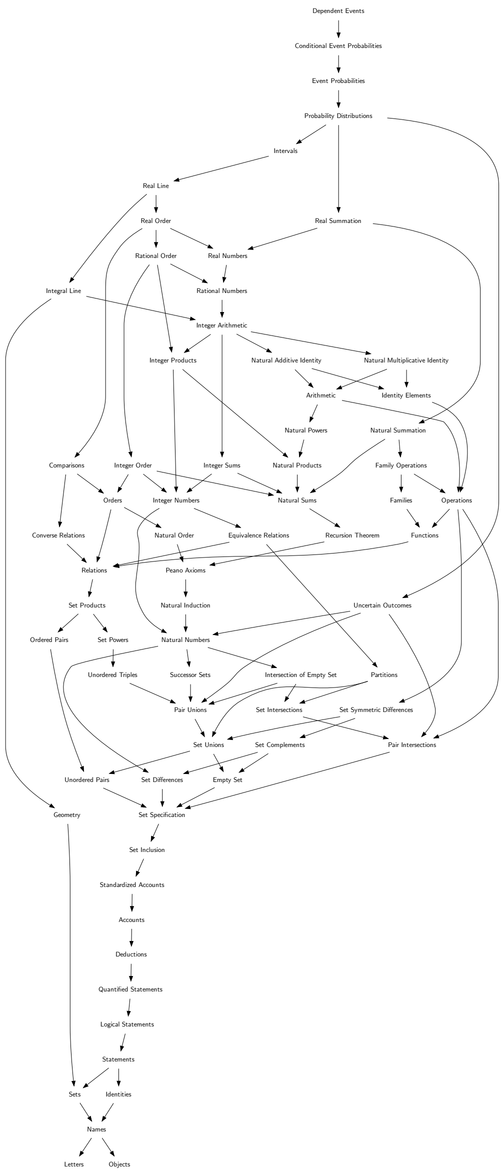
From which we deduce $\mathsf{P}(C_\Omega(A) \mid B) = 1 - \mathsf{P}(A) = \mathsf{P}(C_\Omega(A))$. Which is equivalent to $\mathsf{P}(C_\Omega(A) \cap B) = \mathsf{P}(C_\Omega(A))\mathsf{P}(B)$. In other words, B and $C_\Omega(A)$ are independent events. Similarly, A and $C_\Omega(B)$ are independent events. Since $(\Omega - A) \cap (\Omega - B) = \Omega - (A \cup B)$, we have

$$\mathsf{P}(A \cup B) + P(C_\Omega(A) \cap C_\Omega(B)) = 1.$$

Since $\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B) - \mathsf{P}(A \cap B)$, we obtain

$$P(C_\Omega(A) \cap C_\Omega(B)) = 1 - \mathsf{P}(A) - \mathsf{P}(B) - \mathsf{P}(A)\mathsf{P}(B).$$

We can express the right hand side as $(1 - \mathsf{P}(A))(1 - \mathsf{P}(B))$ or $\mathsf{P}(C_\Omega(A))\mathsf{P}(C_\Omega(B))$. In other words, $C_\Omega(A)$ and $C_\Omega(B)$ are independent.



Dependent Events
Conditional Event Probabilities
Event Probabilities
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Rational Order
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Natural Summation
Comparisons
Integer Order
Integer Sums
Natural Products
Family Operations
Orders
Integer Numbers
Natural Sums
Families
Operations
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Functions
Relations
Peano Axioms
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects