Definition

A *decision problem* is a pair (I, Y) of *instances* I and *yes-instances* $Y \subset I$.

Examples

Subgraph isomorphism

Let $\mathcal{G}(n)$ be the set of graphs with n vertices. For $m \leq n$, define $I = \mathcal{G}(n) \times \mathcal{G}(m)$. A graph $G_1 = (V_1, E_1)$ is *subgraph-isomorphic* to a graph $G_2 = (V_2, E_2)$ if there exists $V' \subset V_1$, $E' \subset E_1$ with $|V'| = |V_2|$, $|E'| = |E_2|$, and bijection $f : V' \to V_1$ so that

$$\{u, v\} \in E_2 \longleftrightarrow \{f(u), f(v)\} \in E'.$$

Define

$$Y = \{(G_1, G_2) \in I \mid G_1 \text{ is subgraph-isomorphic to } G_2\}$$

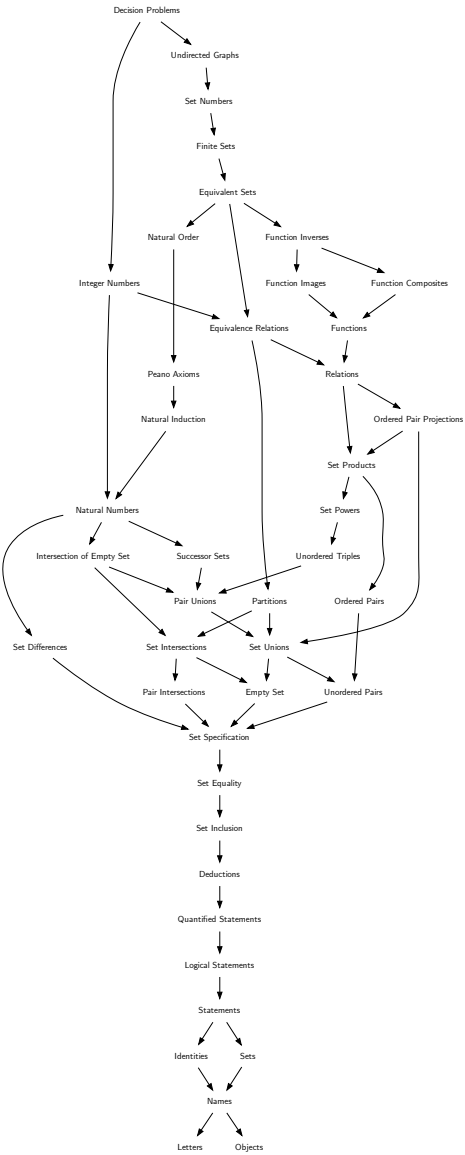
We call (Y, I) the *subgraph-isomorphism problem.*

Traveling Salesman

Denote by $S^n \subset \mathbf{Z}^{n \times n}$ the set of symmetric integer-valued matrices. Define $I = S^n \times \mathbf{Z}$. A *n-tour* is a numbering $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ and has *cost* $C(\pi)$ with respect to D defined by

$$C(\pi) = D_{\pi(n),\pi(1)} + \textstyle\sum_{j=1}^{n-1} D_{\pi(i),\pi(i+1)}$$

A tour is B-bounded if its cost is no greater than B. Define $Y = \{(D, B) \mid \text{ there is a } B - \text{bounded tour with respect to } D\}$.


Decision Problems
Undirected Graphs
Set Numbers
Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Integer Numbers
Function Images
Function Composites
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Pair Unions
Partitions
Ordered Pairs
Set Differences
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects