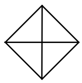


Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object (TODO).

Definition

A *countably summable subset algebra* is a subset space for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of $A_1, \dots, A_n$ coincides with the union of $A_1, \dots, A_n, A_n, A_n \dots$.

We say that the set of distinguished sets a *sigma algebra* on the base set; we justify this langauge, as for an algebra, by the closure properties under standard set operations.

Notation

The notation follows that of a subset space. Let $(A, \mathcal{A})$ be a countably summable subset algebra. We also say "let $\mathcal{A}$ be an sigma algebra on A." Moreover, since the largest element of the sigma algebra is the base set, we can say without ambiguity: "let $\mathcal{A}$ be a sigma algebra."

Examples

Example 1. *For any set A, 2^A is a sigma algebra.*

Example 2. *For any set* A*,* $\{A, \varnothing\}$ *is a sigma algebra.*

Example 3. *Let* A *be an infinite set. Let* $\mathcal{A}$ *the collection of finite subsets of* A*.* $\mathcal{A}$ *is not a sigma algebra.*

Example 4. *Let* A *be an infinite set. Let* $\mathcal{A}$ *be the collection subsets of* A *such that the set or its complement is finite.* $\mathcal{A}$ *is not a sigma algebra.*

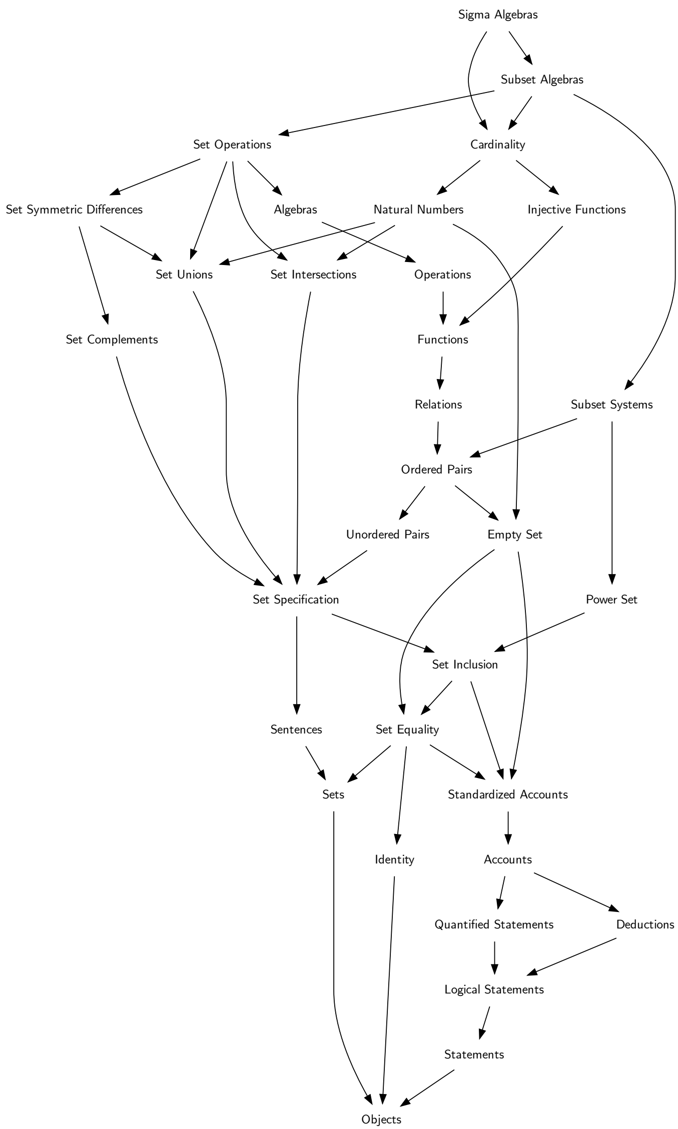
PROPOSITION **5.** *The intersection of a family of sigma algebras is a sigma algebra.*

Example 6. *For any infinite set* A*, let* $\mathcal{A}$ *be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

$\mathcal{A}$ *is an algebra; the countable/co-countable algebra.*

$\boxed{TOOD : cleanupexamples}$


Sigma Algebras
Subset Algebras
Set Operations
Cardinality
Set Symmetric Differences
Algebras
Natural Numbers
Injective Functions
Set Unions
Set Intersections
Operations
Set Complements
Functions
Relations
Subset Systems
Ordered Pairs
Unordered Pairs
Empty Set
Set Specification
Power Set
Set Inclusion
Sentences
Set Equality
Sets
Standardized Accounts
Identity
Accounts
Quantified Statements
Deductions
Logical Statements
Statements
Objects