



Why

We want to talk about optimally eliminating variables in a system of linear equations.¹

Definition

An *ordered* undirected graph is *filled* or *monotone transitive* if all higher neighborhoods induce complete subgraphs. An ordering σ of an undirected graph (V, E) is a *perfect elimination ordering* if the ordered undirected graph $((V, E), \sigma)$ is filled.

Let $G = ((V, E), \sigma)$ be an ordered undirected graph. G is filled if, for all $v \in V$, $w, z \in \text{adj}^+(v) \rightarrow \{w, z\} \in E$. Equivalently stated, G is filled if, for all $i < j < k$, $\{\sigma(i), \sigma(j)\} \in E$ and $\{\sigma(i), \sigma(k)\} \in E$ imply $\{\sigma(j), \sigma(k)\} \in E$.

Chordality

Prop. 1. *If (V, E, σ) is a filled graph, then (V, E) is chordal.*

Proof. Take the vertex with the lowest index on a cycle of length greater than three. Take □

¹Future editions will expand. For example, this sheet is needed for perfect elimination orderings.

