Why

We want to talk about how a function of multiple real-valued arguments changes with respect to changes in its arguments.[1]

Definition

Let $f : \mathbf{R}^d \to \mathbf{R}$ For $i = 1, \ldots, d$, the *ith partial derivative* of f is the function $g_i : \mathbf{R}^d \to \mathbf{R}$ defined by

$$g_i(x) = \lim_{h \to 0} \frac{f(x + he_i) - f(x)}{h}$$

for each $x \in \mathbf{R}^d$.

Gradient

The *gradient* of a multivariate function is the vector-valued function whose ith component is the the partial derivative of the function with respect to its ith argument.

Notation

Let $f : \mathbf{R}^n \to \mathbf{R}$. The gradient of f is frequently denoted ∇f. It is understood that $(\nabla f) \in \mathbf{R}^d \to \mathbf{R}^d$. An alternative notation is to use that similar for single derivatives and to denote the gradient (sometimes called derivative) of f by f' (assuming it exists). It is important to here note that although when $g : \mathbf{R} \to \mathbf{R}$, $g' \in (\mathbf{R} \to \mathbf{R})$, (and so is another function from and to reals) when $f : \mathbf{R}^d \to \mathbf{R}$, $f' \in \mathbf{R}^d \to \mathbf{R}^d$, and so is a vector-valued (not a real-valued) function.

There is (unfortunately) much notation for the individual partial derivatives; most of which we shall not (fortunately) have occasion to use in these sheets. One popular usage is the use of the ∂ symbol, read aloud as "partial." For example, if $f : \mathbf{R}^2 \to \mathbf{R}$ is a function of two arguments, each being referred to as x and y, then $\partial_x f$ denotes the partial derivative

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[1]Future editions will modify this sheet.

of f with respect to x and $\partial_y f$ denotes the partial derivative of f with respect to y. It is understood that $(\partial_x f) \in \mathbf{R}^d \to \mathbf{R}$. and likewise for $\partial_y f$. Another popular usage is $\partial f / \partial x$ for $\partial_x f$ and $\partial f / \partial y$ for $\partial_y f$. We will almost exclusively prefer the gradient notation.

Partial Derivatives

Differentiable Functions Multivariate Functions

N-Dimensional Space

Real Functions Distance

Space Distance

Plane Distance

Interval Length Real Space

Intervals Real Line Real Plane

Integral Line Real Order

Geometry Rational Order Real Numbers Lists

Rational Numbers Set Numbers Direct Products

Integer Arithmetic Finite Sets

Integer Products Natural Additive Identity Natural Multiplicative Identity Equivalent Sets

Arithmetic Identity Elements Function Inverses Family Unions and Intersections

Natural Powers Integer Sums Integer Order Comparisons Function Image Function Composites Families

Natural Products Integer Numbers Orders Operations Functions

Natural Sums Natural Order Converse Relations Equivalence Relations

Recursion Theorem Relations

Peano Axioms Ordered Pair Projections

Natural Induction Set Products

Natural Numbers Set Powers Generalized Set Dualities

Set Symmetric Differences Successor Sets Unordered Triples Intersection of Empty Set Set Dualities Partitions Set Unions and Intersections

Set Complements Pair Unions Set Intersections Ordered Pairs

Set Differences Set Unions Pair Intersections

Empty Set Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects