



Why

We want a form of the dominated convergence theorem in terms of convergence in measure.¹

Definition

A sequence of real-valued measurable functions *converges in measure* to a real-valued measurable limit function if for every positive number the measures of the set where the function deviates from the limit function by more than the positive number converges to zero.

Notation

Let (X, \mathcal{A}, μ) be a measure space. Let $(f_n)_n$ a sequence of real-valued measurable functions on X . Let f be a measurable real-valued function on X . If f_n converges in measure to f we write: $f_n \rightarrow f$ in measure, read aloud as “f n goes to f in measure.”

Suppose $f_n \rightarrow f$ in measure. Then for every $\varepsilon > 0$,

$$\lim_{n \rightarrow \infty} \mu(\{x \in X \mid |f_n(x) - f(x)| > \varepsilon\}) = 0.$$

¹Future editions will expand.

