Why

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Definition

Let $(I, \leq)$ be a totally ordered set. A family of sigma-algebras $\{F_i\}_{i \in I}$ is a *filtration* if $F_j \subset F_k$ for all $j \leq k$.

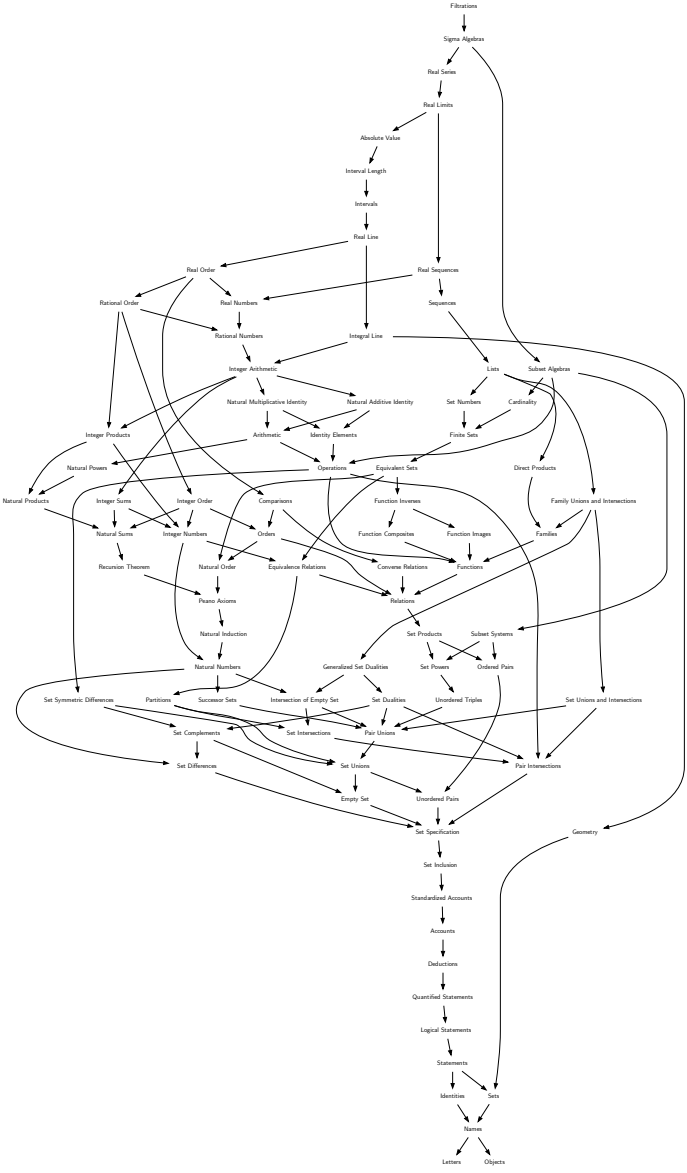
We call a filtration whose index set is the natural numbers with their usual order a *discrete-time filtration*. We call a filtration whose index set is the real numbers with their usual order a *continuous-time filtration*. The index set may also be finite, for example, $\{F_i\}_{i=1}^n$, in which case

$$F_1 \subset F_2 \subset \cdots \subset F_n.$$

Notation

It is extremely common to see filtrations written with the caligraphic $\mathcal{F}$. As in, let $\{\mathcal{F}_i\}_{i=1}^n$ be a filtration. This is in accordance with using caligraphic letters for sets of sets, and in accordance with the term sigma *field* for sigma algebra.

[1] Future editions will include, and likely will need stochastic processes.


Filtrations
Sigma Algebras
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Rational Numbers
Integral Line
Integer Arithmetic
Lists
Subset Algebras
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Cardinality
Integer Products
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Operations
Equivalent Sets
Direct Products
Natural Products
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Orders
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects