



Why

How does the power set relate to an intersection?

Notation Preliminaries

First, if we have a set of sets—denote it \mathcal{C} —and all members are subsets of a fixed set—denote it E —then the set of sets is a subset of $\mathcal{P}(E)$. In this case, we can write

$$\bigcap \{X \in \mathcal{P}(E) \mid x \in \mathcal{C}\}$$

Which is a sort of justification for the notation

$$\bigcap_{X \in \mathcal{C}} X.$$

Basic Properties

Here are some basic interactions between the powerset and intersections.¹

Proposition 1. $\mathcal{P}(A) \cap \mathcal{P}(F) = \mathcal{P}((A \cap F))$

Proposition 2. $\bigcap_{X \in \mathcal{A}} \mathcal{P}(A) = \mathcal{P}((\bigcap_{X \in \mathcal{A}} A))$

Proposition 3. $\bigcap_{X \in \mathcal{P}(E)} X = \emptyset$

¹Future editions will expand on these propositions and provide accounts of them.

