



# Relations

## 1 Why

How can we relate the elements of two sets?

## 2 Definition

A **relation** between two non-empty sets  $A$  and  $B$  is a subset of  $A \times B$ . A relation on a single set  $C$  is a subset of  $C \times C$ .

Let  $a \in A$  and  $b \in B$ . The pair  $(a, b)$  may or may not be in a relation on  $A$  and  $B$ . If  $A \neq B$ , then  $(b, a)$  is not a member of the product  $A \times B$ , and therefore not in any relation on  $A$  and  $B$ . If  $A = B$ , however, it may be that  $(b, a)$  is in the relation.

### 2.1 Notation

Let  $A$  and  $B$  be nonempty sets with  $a \in A$  and  $b \in B$ . Since relations are sets, we can use upper case Latin letters. Let  $R$  be a relation on  $A$  and  $B$ . We denote that  $(a, b) \in R$  by  $aRb$ , read aloud as "a in relation  $R$  to b."

When  $A = B$ , we tend to use other symbols instead of letters.

For example,  $\sim$ ,  $=$ ,  $<$ ,  $\leq$ ,  $\prec$ , and  $\preceq$ .