Why

We compress the notation for linear equations.

Definition

A *real matrix* (*matrix, rectangular array*) is a two-dimensional array of real numbers. We denote the elements of a matrix in a grid between two rectangular braces, as in

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}.$$

We call m and n the *dimensions* of the matrix. We call m the *height* and n the *width*. If the height of the matrix is the same as the width of the matrix then we call the matrix *square*. If the height is larger than the width, we call the matrix *tall*. If the width is larger than the height, we call the matrix *wide*.

Linear equations

Recall that we are interested in solutions of the linear equations

$$y_1 = A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n,$$
$$y_2 = A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n,$$
$$\vdots$$
$$y_n = A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n.$$

We have suggestively used the notation A_{ij} for the coefficients of the equations, so they are the entries of $A \in \mathbf{R}^{m \times n}$.

A primer on matrix-vector products

Using the notation $A \in \mathbf{R}^{m \times n}$ and $x \in \mathbf{R}^n$ we want a compressed way to write the above system of linear equations. Define the *real matrix-vector product* $z \in \mathbf{R}^m$ of A with x by

$$z_i = \sum_{j=1}^{n} A_{ij} x_j, \quad i = 1, \ldots, m.$$

We denote the matrix vector product z by Ax.

Notation

We express the above system of linear equations as

$$y = Ax,$$

where $y = (y_1, \ldots, y_n) \in \mathbf{R}^n$ and $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$. The compact notation $y = Ax$ is sometimes called the *matrix form* of the m linear equations. In this case, A is often called the *coefficient matrix*.

Note on terminology

The word "matrix" is from the Latin "mater," meaning mother, and has an old sense in English similar "womb." The matrix is source of many determinants (discussed elsewhere).

Real Matrices

Linear Equations

Arrays · Real Vectors · Linear Functions

Space Vectors · Plane Vector Sums and Differences · N-Dimensional Space

Plane Vectors · Distance

Space Distance

Plane Distance

Interval Length

Real Space · Intervals

Real Plane · Real Line

Integral Line · Real Summation · Real Order

Geometry · Natural Summation · Real Numbers · Rational Order

Lists · Rational Numbers

Set Numbers · Direct Products · Integer Arithmetic · Natural Fractions

Finite Sets · Natural Additive Identity · Natural Multiplicative Identity · Integer Products

Equivalent Sets · Identity Elements · Natural Arithmetic

Family Unions and Intersections · Function Inverse · Family Operations · Comparison · Integer Order · Integer Sums · Natural Powers · Natural Equations

Function Composites · Function Images · Families · Operations · Orders · Integer Numbers · Natural Products · Equations

Equivalence Relations · Functions · Converse Relations · Natural Order · Natural Sums

Relations · Recursion Theorem

Ordered Pair Projections · Peano Axioms

Set Products · Natural Induction

Generalized Set Dualities · Set Powers · Natural Numbers

Set Unions and Intersections · Partitions · Set Dualities · Intersection of Empty Set · Unordered Triples · Successor Sets · Set Symmetric Differences

Ordered Pairs · Set Intersections · Pair Unions · Set Complements

Pair Intersections · Set Unions · Set Differences

Unordered Pairs · Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets · Identities

Names

Letters · Objects