

## LINEAR FUNCTIONS

## Why

Lots of things are (approximately) linear.<sup>1</sup>

## Definition

A function between two vector spaces which share the same field is *linear* if the function applied to a linear combination of two vectors in the first space is the linear combination of the results of the function (in the second space), using the same coefficients for the combination.

A linear function is always linear with respect to some field. The field is implicit, somewhat, in the definition but always present. Linear functions are sometimes called *operators*.

## Notation

Let  $(V_1, F)$  and  $(V_2, F)$  be two vector spaces over the same field F. Let  $f: V_1 \to V_2$ . f is linear if

$$f(au + bv) = af(u) + bf(v)$$

for all  $a, b \in F$  and  $u, v \in V_1$ .

<sup>&</sup>lt;sup>1</sup>Future editions will expand on this why. In particular, the intuition of proportionality.

