Why

[1]

Definition

Recall that $(C, |\cdot|)$ is a normed space, and so also a metric space. So, a sequence $(z_n)_{n\in\mathbf{N}}$ of complex numbers is egoprox and convergent as usual. Both of these are equivalent to the corresponding conditions on the sequences of real and imaginary parts.

Proposition 1. $(z_n)_{n\in\mathbf{N}} = (x_n, y_n)_{n\in\mathbf{N}}$ *converges to* $z_0 = (x_0, y_0) \in \mathbf{C}$ *if and only if* x_n *converges to* x_0 *and* y_n *converges to* y_0.

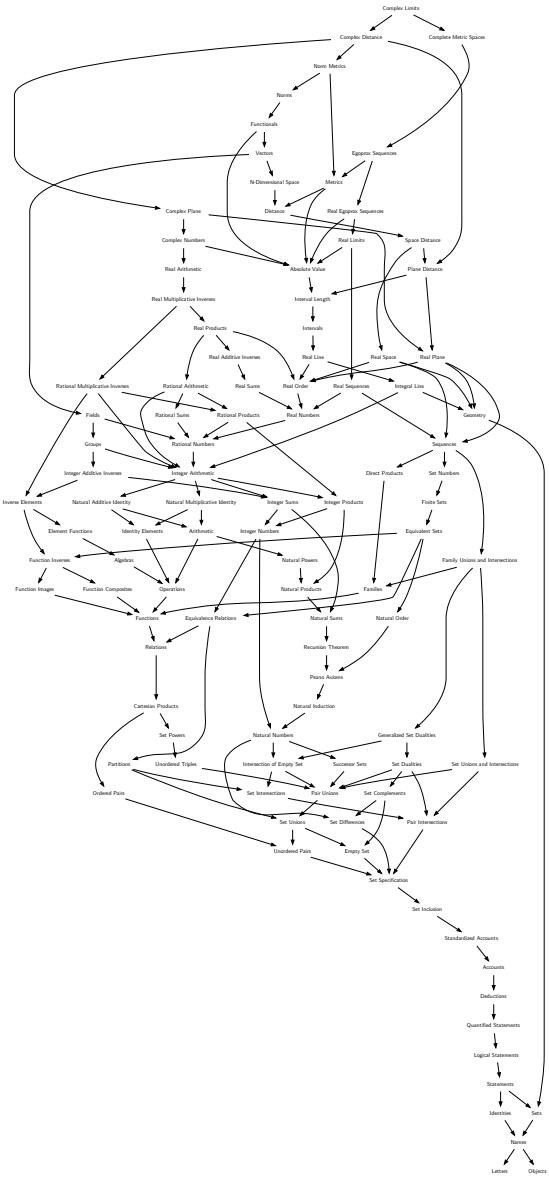
Proposition 2. $(z_n)_{n\in\mathbf{N}} = (x_n, y_n)_{n\in\mathbf{N}}$ *is egoprox if and only if* x_n *is egoprox and* y_n *is egoprox.*

Completeness

As a result of Proposition 2, if z_n is egoprox then there is a limit x_0 and y_0 for its real and imaginary pieces, and so as a result of Proposition 2 z_n convergeces. In other words, every cauchy sequence converges.

Proposition 3. $\mathbf{C}$ *with the metric induced by* $|\cdot|$ *is complete.*

[1] Future editions will include.


Complex Limits
Complex Distance
Complex Metric Spaces
Norm Metrics
Norms
Functionals
Vectors
Egoprox Sequences
N-Dimensional Space
Metrics
Distance
Real Egoprox Sequences
Complex Plane
Space Distance
Complex Numbers
Real Limits
Absolute Value
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Additive Inverses
Real Line
Real Space
Real Plane
Real Order
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Sequences
Integral Line
Fields
Rational Sums
Rational Products
Real Numbers
Geometry
Groups
Rational Numbers
Sequences
Integer Additive Inverses
Integer Arithmetic
Direct Products
Set Numbers
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Integer Products
Finite Sets
Element Functions
Identity Elements
Arithmetic
Integer Numbers
Equivalent Sets
Function Inverse
Algebras
Natural Powers
Family Unions and Intersections
Function Image
Function Composites
Operations
Natural Products
Families
Functions
Equivalence Relations
Natural Sums
Natural Order
Relations
Recursion Theorem
Peano Axioms
Natural Induction
Cartesian Products
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersection
Pair Unions
Set Complements
Set Differences
Pair Intersections
Set Union
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects