



# Real Integral Dominated Convergence

## 1 Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

## 2 Result

When context is clear, we refer to the following proposition as the *dominated convergence theorem*.

**Proposition 1.** *The integral of the almost everywhere limit of a sequence of measurable, extended-real-valued, almost-everywhere bounded functions is the limit of the sequence of integrals of the functions.*

*Proof.* Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $f : X \rightarrow [-\infty, \infty]$  be a  $\mathcal{A}$ -measurable function. Let  $f_n : X \rightarrow [-\infty, \infty]$  a  $\mathcal{A}$ -measurable function for every natural number  $n$  so that  $(f_n)_n$  converges almost everywhere to  $f$ . Let  $g : X \rightarrow [0, \infty]$  be an integrable function which dominates  $f_n$  almost everywhere for each  $n$ . We

want to show that:

$$\int f d\mu = \lim_n \int f_n d\mu.$$

□