

MATRIX TRACE

Why

TODO

Definition

The *trace* of a square real matrix is the sum of the elements on its diagonal.

Notation

We denote the function which associates a matrix with its trace by $\operatorname{tr}: \mathbb{R}^{n \times n} \to \mathbb{R}$. Let $A \in \mathbb{R}^{n \times n}$. Then

$$\operatorname{tr} A = \sum_{i=1}^{n} A_{ii}.$$

Properties

Proposition 1. The trace is a linear function on the vector space of $n \times n$ real matrices.

Proof. Let $A, B \in \mathbb{R}^{n \times n}$ and $\alpha, \beta \in \mathbb{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\operatorname{tr} C = \sum_{i=1}^{n} C_{ii} = \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii} = \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii} = \alpha \operatorname{tr} A + \beta \operatorname{tr} B.$$

Proposition 2. Let $A,B \in \mathbf{R}^{n \times n}$.

$$\mathsf{tr}\left(AB\right)=\mathsf{tr}\left(BA\right)$$

Proof. TODO

