Why[1]

Definition

An *ordering* of an undirected graph is an ordering (see Lists) of its vertices. An *ordered undirected graph* is an ordered pair $((V, E), \sigma : \{1, 2, \ldots, |V|\} \to V)$ where (V, E) is an undirected graph (see Undirected Graphs) and σ is an ordering of the vertex set V.

Notation

Let $((V, E), \sigma)$ be an ordered undirected graph. We commonly associate it with (V, E, σ) and call this ordered triple an undirected graph as well. But, throughout these sheets, an ordered undirected graph is an ordered pair.

We denote that $\sigma^{-1}(v) < \sigma^{-1}(w)$ by $v \prec_\sigma w$ and $v \succeq_\sigma w$ by $\sigma^{-1}(v) \leq \sigma^{-1}(w)$. We occasionally omit the subscripts in $\prec_\sigma$ and $\succeq_\sigma$ when clear from context.

Visualization

We visualize an ordered undirected graph by labeling its nodes with the indices of each vertex. Let (V, E) be an undirected graph with $V = \{a, b, c, d, e\}$ and

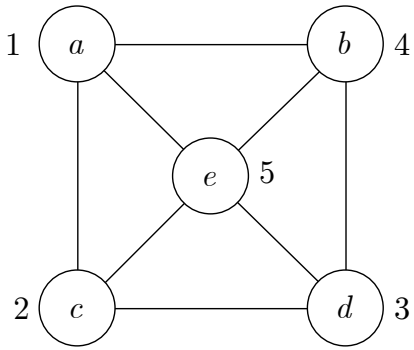
$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\}.$$

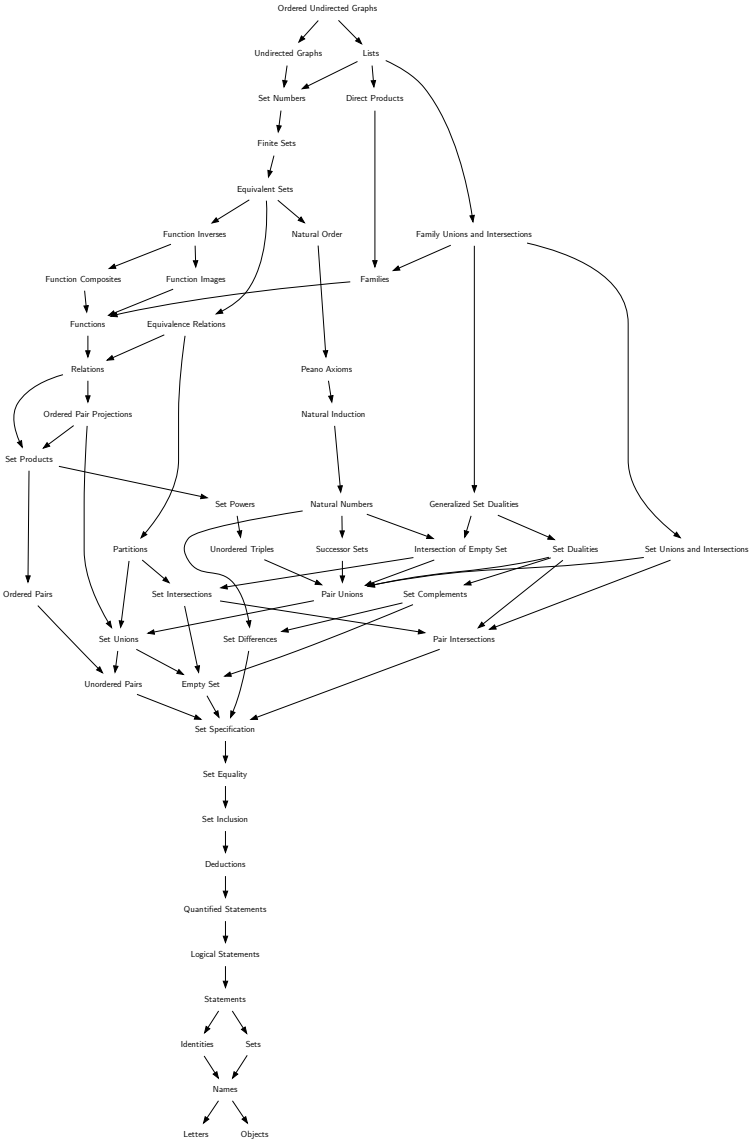
Let $\sigma : \{1, \ldots, 5\} \to V$ be an ordering with

$$\sigma(1) = a \quad \sigma(2) = c \quad \sigma(3) = d \quad \sigma(4) = b \quad \sigma(e) = 5.$$

We visualize the ordered graph in the figure.

[1] Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.


1
a
b
4
e
5
2
c
d
3


Ordered Undirected Graphs
Undirected Graphs
Lists
Set Numbers
Direct Products
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Composites
Function Images
Families
Functions
Equivalence Relations
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects