Why

TODO

Definition

An *index sequence* of *order* n is a finite sequence of distinct elements of $\{1, 2, \ldots, n\}$ whose length is less than or equal to n. We call the ith coordinate of an index sequence the *i-index* of the sequence. The *index matrix* associated with an index sequence is the $r \times n$ matrix whose i, jth entry is 1 if the index sequences's ith coordinate is j, and 0 otherwise. If $r = n$ then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an n-vector associated with a length-r index sequence is the product of the $r \times n$ index matrix with the n-dimensional vector. Its ith entry is the i-index entry of the vector.

Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions, and so relations, and so sets), they are not sets of indices, so we avoid this usage.

Notation

Let $r \leq n$ be natural numbers. Let $\alpha : \{1, 2, \ldots, r\} \to \{1, 2, \ldots, n\}$ be an index sequence. We denote the index ma-

trix associated with α by P_α. This matrix P_α is an element of $\mathsf{N}^{r\times n}$ and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let A be a nonempty set and let $x \in A^n$. then the subvector of x associated with P_α (and so with α) is

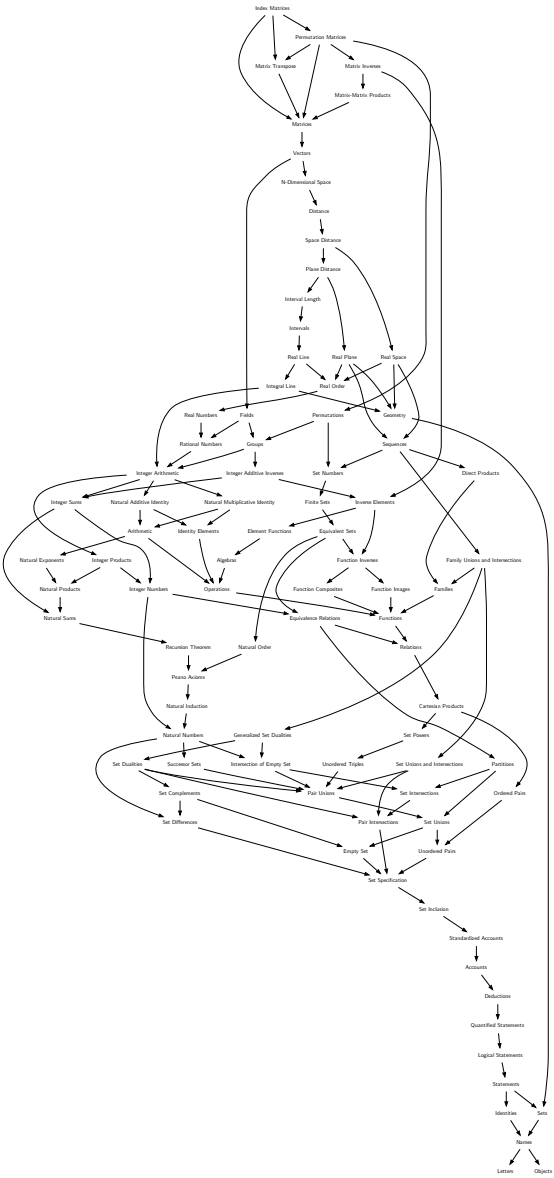
$$P_\alpha x = \left(x_{\alpha(1)}, \ldots, x_{\alpha(r)}\right)$$

We denote the product $P_\alpha x$ by x_α.

We denote the product $P_\alpha X {P_\alpha}^\top$ by $X_{\alpha\alpha}$.

Multiplication

The product of the $n \times r$ transpose of an index matrix with an r vector is the n vector with The


Index Matrices
Permutation Matrices
Matrix Transpose
Matrix Inverses
Matrix-Matrix Products
Matrices
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Real Numbers
Fields
Permutations
Geometry
Rational Numbers
Groups
Sequences
Integer Arithmetic
Integer Additive Inverse
Set Numbers
Direct Products
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Inverse Elements
Arithmetic
Identity Elements
Element Functions
Equivalent Sets
Natural Exponents
Integer Products
Algebras
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Operations
Function Composition
Function Images
Families
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Natural Order
Relations
Peano Axioms
Natural Induction
Cartesian Products
Natural Numbers
Generalized Set Duality
Set Powers
Set Duality
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Unions and Intersections
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Latrines
Objects