



Why

We want a set which corresponds to our notion of points on a line.¹

Rational Cuts

We call a subset R of \mathbf{Q} a *rational cut* if (a) $R \neq \emptyset$, (b) $R \neq \mathbf{Q}$, (c) for all $q \in R$, $r \leq q \longrightarrow r \in R$, and (d) R has no greatest element. Briefly, the intuition is that the point is the set of all rationals to less than (or, potentially, equal to) some particular rational number.²

Definition

The *set of real numbers* is the set of all rational cuts. This set exists by an application of the principle of selection (see **Set Selection** to the power set (see **Set Powers**) of \mathbf{Q} . We call an element of the set of real numbers a *real number* or a *real*. We call the set of real numbers the *set of reals* or *reals* for short.

Notation

We follow tradition and denote the set of real numbers by \mathbf{R} , likely a mnemonic for “real.”

Other Terminology

Some authors call a real number a *quantity* or a *continuous quantity*. The real numbers, then, are said to be *continuous*. When contrasting (using this terminology) a finite set with the real numbers, one refers to the finite set as *discrete*.³

¹Future editions will modify and expand this justification.

²This brief intuition will be expanded upon in future sheets.

³Future editions may move this discussion later, to the discussion of the cardinality of the reals.

