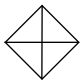


Why

We want to talk about several objects in order.

Definition

A *list* (or *finite sequence*, *string*, n-*tuple*) is a family (correspondence) whose index set is $\{1, \dots, n\}$ for $n \in \mathbf{N}$. The *length* (or *size*) of a list is the size of its index set, n. When the codomain of the sequence is a set A, we say that the sequence is *in* A or that it is a sequence *of* elements of A.

If $a : \{1, \dots, n\} \to A$ is a sequence, we refer to $a(k)$ as the kth *term* (or *entry*, or *element*[1]) of a.

Notation

Since the natural numbers are ordered, we regularly denote finite sequences from left to right between parentheses. For example, we denote $a : \{1, \dots, 4\} \to A$ by (a_1, a_2, a_3, a_4). $a(k)$ is the kth term. Following the convention with functions, we regularly usually denote $a(n)$ by a_n

Orderings and numberings

Let A be a set with $|A| = n$. A sequence $a : \{1, \dots, n\} \to A$ is an *ordering* of A if a is invertible. In this case, we call the inverse a *numbering* of A. An ordering associates with each number a unique object and a numbering associates with each

[1] We avoid this terminology because it conflicts with sets.

object a unique number (the object's *index*).

Relation to Direct Products

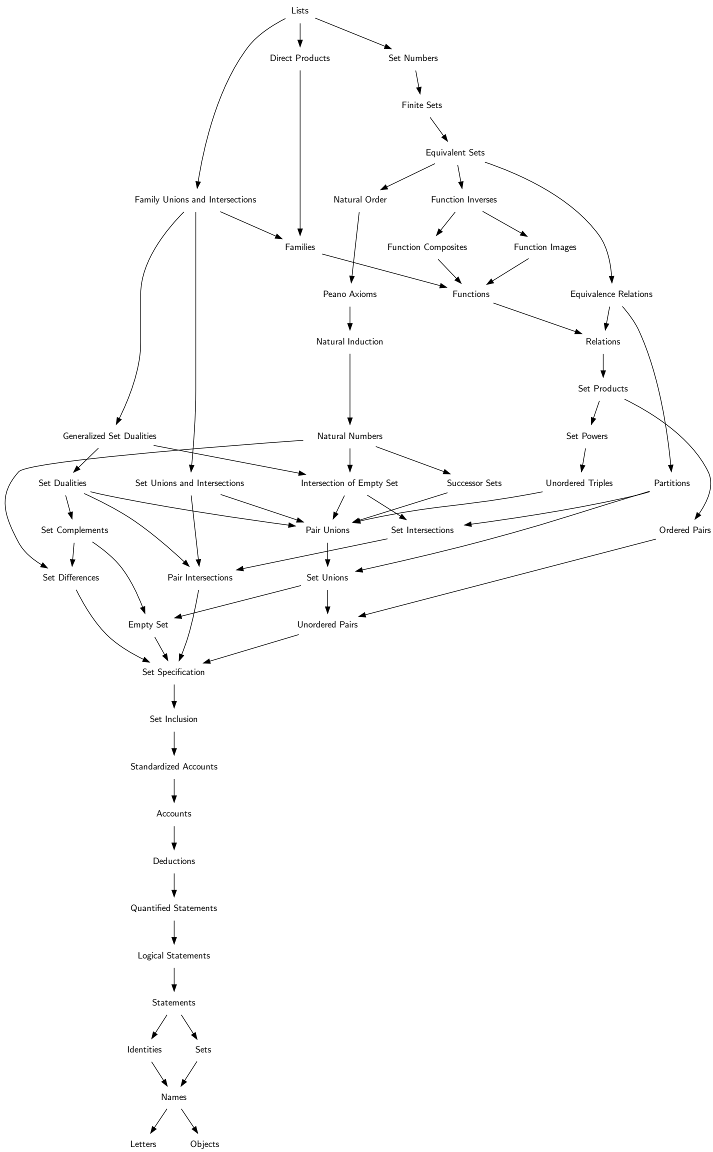
A *natural direct product* is a product of a sequence of sets. We denote the direct product of a sequence of sets $A_1, \dots, A_n$ by $\prod_{i=1}^n A_i$. If each A_i is the same set A, then we denote the product $\prod_{i=1}^n A_i$ by A^n. The set of sequences in a set A is the direct product A^n.

Natural unions and intersections

We denote the family union of the finite sequence of sets A_1, $\dots$, A_n by $\cup_{i=1}^n A_i$. Similarly, we denote the intersection by $\cap_{i=1}^n A_i$

Slices

An *index range* for a list s of length n is a pair (i, j) for which $1 \leq i < j \leq n$. The *slice* corresponding to the index range (i, j) is the length $j - i$ sequence s' defined by $s'_1 = s_i$, $s'_2 = s_{i+1}, \dots, s'_j = s_{i+j-1}$. We denote the (i, j)-slice of s by $s_{i:j}$. If $i = 1$ we use $s_{:j}$ and if $j = n$ we use $s_{i:}$ as shorthands for the slices $s_{1:j}$ and $si : n$.


Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Natural Induction
Relations
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects