



## LENGTH COMMON NOTIONS

### Why

We want to define the length of a subset of real numbers.

### Notions

We take two common notions:

1. The length of a whole is the sum of the lengths of its parts;  
the *additivity principle*.
2. The length of a whole is the at least the length of any whole  
it contains the *containment principle*.

The task is to make precise the use of “whole,” “parts,” and “contains.” We start with intervals.

### Definition

By whole we mean set. By part we mean an element of a partition; in other words, a subset. By contains we mean set inclusion.

The length of an interval is the difference of its endpoints: the larger minus the smaller (see **Interval Length**). Two intervals are *non-overlapping* if their intersection is a single point or empty. The *length* of the union of two non-overlapping intervals is the sum of their lengths.



