

SET DUALITIES

Why

How does taking complements relate to forming unions and intersections.

Complements of unions or intersections

Let E denote a set. Let A and B denote sets and $A, B \subset E$. All complements are taken with respect to E. The following are known as $DeMorgan's\ Laws.^1$

Proposition 1. $C(A \cup B) = C(A) \cap C(B)$

Proposition 2. $C(A \cap B) = C(A) \cup C(B)$

Principle of duality

As a result of DeMorgan's Laws² and basic facts about complements (see Set Complements) theorems about sets often come in pairs. In other words, given an inclusion or identity relation involving complements, unions and intersections of some set (above E) if we replace all sets by their complements, swap unions and intersections, and flip all inclusions we obtain another, true, result. The correspondence is called the *principle* of duality for sets.

¹Proofs will appear in a future edition.

 $^{^2}$ Future editions will change the name to remove the reference to DeMorgan in accordance with the project's policy on naming.

