

SMOOTH FUNCTIONS

Why

What is the generalisation of smooth functions between Euclidean spaces.

Definition

Let $U \subset \mathbb{R}^n$ be open. A function $f: U \to \mathbb{R}^m$ is smooth if each of its components is smooth.

More generally, let $X \subset \mathbb{R}^n$ (not necessarily open). We call $g: X \to \mathbb{R}^m$ smooth if for each $x \in X$ there exists an open set $V \subset \mathbb{R}^n$ with $x \in V$ and smooth function $G: U \to \mathbb{R}^n$ so that G(y) = g(y) for all $y \in U \cap X$. In this case we say that g can be locally extended to a smooth map on open sets.

