



## Why

The product of two sets is a (sub)set of ordered pairs. Is every set of ordered pairs a subset of a product of two sets?

## Result

The answer is easily seen to be yes. Let  $R$  denote a set of ordered pairs. So for  $x \in R$ ,  $x = \{\{a\}, \{a, b\}\}$ . First consider  $\bigcup R$ . Then  $\{a\} \in \bigcup R$  and  $\{a, b\} \in \bigcup R$ . Next consider  $\bigcup\bigcup R$ . Then  $a, b \in \bigcup\bigcup R$ . So if we want two sets—denote them by  $A$  and  $B$ —so that  $R \subset A \times B$ , we can take both  $A$  and  $B$  to be the set  $\bigcup\bigcup R$ .

## Projections

We often want to shrink the sets  $A$  and  $B$  to include only the *relevant* members. In other words, to include only those members which appear as either the first coordinate (for  $A$ ) or second coordinate (for  $B$ ) in an element of  $R$ . We can do this by specifying the elements of  $\bigcup\bigcup R$  which are actually a first coordinate or second coordinate for some ordered pair in the set  $R$ .

Define

$$A' = \{a \in A \mid (\exists b)((a, b) \in R)\},$$

and likewise

$$B' = \{b \in B \mid (\exists a)((a, b) \in R)\}.$$

We call  $A'$  the *projection onto the first coordinate* and  $B'$  the *projection onto the second coordinate*.



