Why

TODO

Definition

For a nonzero $b \in \mathbf{R}^n$ and $\beta \in \mathbf{R}$ the sets

$$\{x \in \mathbf{R}^n \mid \langle x \rangle b \leq \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x \rangle b \geq \beta\},$$

are *closed halfspaces* and the sets

$$\{x \in \mathbf{R}^n \mid \langle x \rangle b \leq \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x \rangle b \geq \beta\},$$

are *open halfspaces*.

Each of these is nonempty and convex. As with hyperplanes, the same four sets appear if one uses $\lambda\beta$ and λb above, so the halfspaces depend only on the hyperplane $H = \{x \mid \langle x \rangle b = \beta\}$.

Halfspaces

Hyperplanes

Affine Set Dimension — Subspace Orthogonal Complements

Subspace Dimensions — Orthogonal Complements

Vector Space Dimensions

Vector Space Bases — Inner Products

Affine Sets and Subspaces — Span — Linearly Dependent Vectors

Translate Sets — Subspaces — Linear Combinations

Affine Sets — Vectors

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Line — Real Plane — Real Space

Integral Line — Real Order — Sequences

Geometry — Real Numbers — Fields — Direct Products

Set Numbers — Rational Numbers — Groups

Finite Sets — Integer Arithmetic — Integer Additive Inverses

Natural Additive Identity — Natural Multiplicative Identity — Integer Sums — Inverse Elements

Equivalent Sets — Arithmetic — Identity Elements — Element Function

Integer Products — Natural Powers — Algebras — Function Inverses — Family Unions and Intersections

Integer Numbers — Natural Products — Operations — Function Composites — Function Images — Families

Equivalence Relations — Natural Sums — Functions

Natural Order — Recursion Theorem — Relations

Peano Axioms

Natural Induction — Set Products

Natural Numbers — Set Powers — Generalized Set Dualities

Partitions — Successor Sets — Intersection of Empty Set — Unordered Triples — Set Dualities — Set Unions and Intersections

Set Intersections — Pair Unions — Ordered Pairs — Set Complements

Set Differences — Set Unions — Pair Intersection

Unordered Pairs — Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects