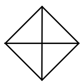


Why

We explore matrix-vector multiplication.

Definition

Given a matrix $A \in \mathbf{R}^{m \times n}$ and a vector $x \in \mathbf{R}^n$, the *product* of A *with* x is the vector $y \in \mathbf{R}^m$ defined by

$$y_i = \sum_{j=1}^{n} A_{ij} x_j, \quad i = 1, \dots, m.$$

Notation

We denote the product of A with x by Ax. With which we concisely write the system linear equations (A, b) as $b = Ax$.

This notation suggests both algebraic and geometric interpretations of solving systems of linear equations. The algebraic interpretation is that we are interested in the invertibility of the function $x \mapsto Ax$. In other words, we are interested in the existence of an inverse element of A. The geometric interpretation is that A transforms the vector x.

Conversely, we can view x as transforming (acting on) A. Let $a^j \in \mathbf{R}^m$ denote the jth column of A, then

$$Ax = \sum_{j=1}^{n} x_j a^j.$$

In other words, y is linear combination of the columns of A.

Properties

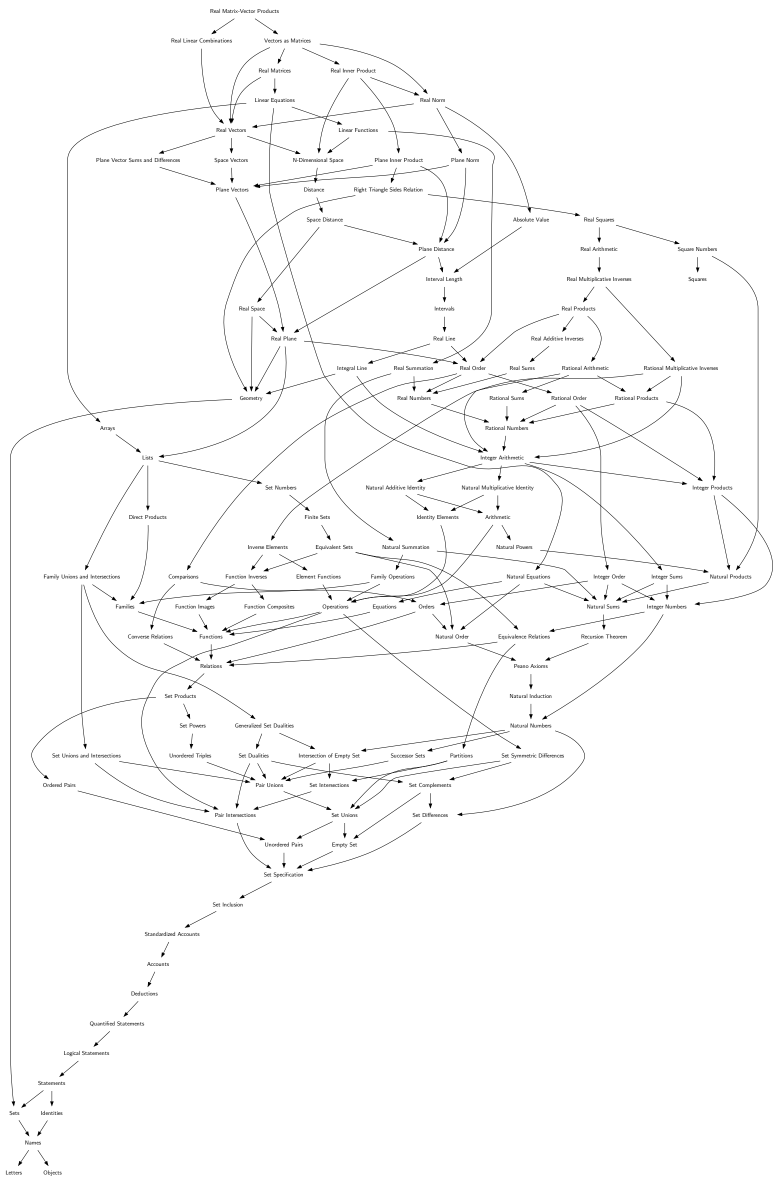
We call the function $f : \mathsf{R}^n \to \mathsf{R}^m$ defined by $f(x) = Ax$ the *matrix multiplication function* (or *matrix-vector product function*) associated with A. f is satisfies the following two important properties:

1. $A(x + y) = Ax + Ay$
2. $A(\alpha x) = \alpha Ax$.

We call such a function f *linear.* In other words, the matrix multiplication function is linear. Conversely, if $g : \mathsf{R}^n \to \mathsf{R}^m$ is linear, there exists a matrix inducing g.

Proposition 1. *Let $f : \mathsf{R}^n \to \mathsf{R}^m$ be linear. Then there exists a unique $A \in \mathsf{R}^{m\times n}$ satisfying $f(x) = Ax$ for all $x \in \mathsf{R}^n$.*

Proof. Evaluate f at the standard unit vectors e_i. The ith component of e_i is 1 and all other components are 0. $\square$



Real Matrix-Vector Products
Real Linear Combinations
Vectors as Matrices
Real Matrices
Real Inner Product
Linear Equations
Real Norm
Real Vectors
Linear Functions
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Inner Product
Plane Norm
Plane Vectors
Distance
Right Triangle Sides Relation
Space Distance
Absolute Value
Real Squares
Plane Distance
Real Arithmetic
Square Numbers
Interval Length
Real Multiplicative Inverses
Squares
Real Space
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Summation
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Geometry
Real Numbers
Rational Sums
Rational Order
Rational Products
Arrays
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Inverse Elements
Equivalent Sets
Natural Summation
Natural Powers
Family Unions and Intersections
Comparisons
Function Inverses
Element Functions
Family Operations
Natural Equations
Integer Order
Integer Sums
Natural Products
Families
Function Images
Function Composites
Operations
Equations
Orders
Natural Sums
Integer Numbers
Converse Relations
Functions
Natural Order
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Unordered Triples
Set Dualities
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects