Why

We want to consider all the subsets of a given set.

Definition

We do not yet have a principle stating that such a set exists, but our intuition suggests that it does.

Principle 1 (Powers). *For every set, there exists a set of its subsets.*

We call the existence of this set the *principle of powers* and we call the set the *power set*.[1] As usual, the principle of extension gives uniqueness (see Set Equality). The power set of a set includes the set itself and the empty set.

Notation

Let A denote a set. We denote the power set of A by $\mathcal{P}(A)$, read aloud as "powerset of A." $A \in \mathcal{P}(A)$ and $\varnothing \in \mathcal{P}(A)$. However, $A \subset \mathcal{P}(A)$ is false.

Examples

Let a, b, c denote distinct objects. Let $A = \{a, b, c\}$ and $B = \{a, b\}$. Then $B \subset A$. In other notation, $B \in \mathcal{P}(A)$. Showing each of the following is straightforward.

1. The empty set: $\mathcal{P}(\varnothing) = \{\varnothing\}$

[1] This terminology is standard, but unfortunate. Future editions may change these terms.

2. Singletons: $\mathcal{P}(\{a\}) = \{\varnothing, \{a\}\}$

3. Pairs: $\mathcal{P}(\{a, b\}) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$

4. Triples:

$$\mathcal{P}(\{a, b, c\}) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$

Properties

We can guess the following easy properties.[2]

Proposition 1. $\varnothing \in \mathcal{P}(A)$

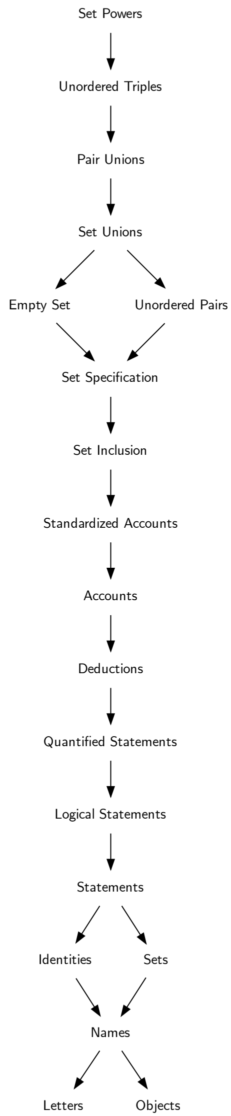
Proposition 2. $A \in \mathcal{P}(A)$

We call A and $\varnothing$ the *improper* subsets of A. All other subset we call *proper*.

Basic Fact

Proposition 3. $E \subset F \longrightarrow \mathcal{P}(E) \subset \mathcal{P}(F)$

[2]Future editions will expand this account.


Set Powers
Unordered Triples
Pair Unions
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects