

RATIONAL NUMBERS

Why

We want fractions.¹

Rational equivalence

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We say that the elements (a, b) and (c, d) of this set are rational equivalent if ad = bc. Briefly, the intuition is that (a, b) represents a over b In the usual notation, (a, b) represents "a/b". So this equivalence relation says these two are the same if a/b = c/d or else ad = bc.

Proposition 1. Rational equivalence is an equivalence relation on $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$.

Definition

The set of rational numbers is the set of equivalence classes (see Equivalence Classes) of $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ under rational equivalence. We call an element of the set of rational numbers a rational number or rational. We call the set of rational numbers the set of rationals or rationals for short.

Notation

We denote the set of rationals by \mathbf{Q} .³ If we denote rational equivalence by \sim then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$.

¹This why will be expanded in future editions.

²Future editions will include an account.

 $^{^3}$ From what we can tell so far, **Q** is a mnemonic for "quantity", from the latin "quantitas".

