

COMPLEX PRODUCTS

Why

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Definition

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The complex product of z_1 and z_2 is the complex number $(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$.

Notation

We denote the complex product of z_1 and z_2 by $z_1 \cdot z_2$ or $z_1 z_2$. The notation is justified because the complex product of two purely real complex numbers corresponds to the purely real complex number whose real part is the real product of the real parts of the first two numbers.

Recall that we denote $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. This notation is a mnemonic for the definition of a complex product if we treat $i^2 = -1$.

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2$$

$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2).$$

Properties

Proposition 1 (Commutativity). For all $z_1, z_2 \in \mathbb{C}$, we have $z_1z_2 = z_2z_2$.

¹Future editions will include.

Proposition 2 (Associativity). For all $z_1, z_2, z_3 \in \mathbb{C}$, we have and $z_1(z_2z_3) = (z_1z_2)z_3$.

Complex multiplication

We call the operation that associates a pair of complex numbers with their product *complex multiplication*. The operation is symmetric (commutative).

Multiplicative identity and inverse

Notice that the complex number (1,0) is the multiplicative identity. It is unique,² and so we call it the *complex multiplicative identity*.

²Future editions will include an account

