

## **SMOOTH FUNCTIONS**

## Why

What is the generalisation of smooth functions between Euclidean spaces.

## Definition

Let  $U \subset \mathbb{R}^n$  be open. A function  $f: U \to \mathbb{R}^m$  is smooth if each of its components is smooth.

More generally, let  $X \subset \mathbb{R}^n$  (not necessarily open). We call  $g: X \to \mathbb{R}^m$  smooth if for each  $x \in X$  there exists an open set  $V \subset \mathbb{R}^n$  with  $x \in V$  and smooth function  $G: U \to \mathbb{R}^n$  so that G(y) = g(y) for all  $y \in U \cap X$ . In this case we say that g can be locally extended to a smooth map on open sets.

