Families

1 Why

Halmos: "There are occasions when the range of a functionis deemed to be more important than the function itself. When that is the case, both the terminology and the notation undergo radical alterations." It is useful to have some language and notation for talking about a set of sets.

2 Definition

A *family* of sets is a set of sets. Experience shows that it is useful to have these associated with the elements of a well-known second set.

An *indexed family of sets* is a function from one set to the power set of a second set. We call the first set the *index set*. We call the second set the *base set*. The range of the indexed family of sets is, of course, a family.

2.1 Notation

Let A and I be be a non-empty sets. We use I as a mnemonic for "index" set. Let $a : I \to A^*$ be a family. For $i \in I$, we follow the function notation and denote the result of applying a to i by a_i.

We denote the range of the family by family of a_α indexed with I by $\{a_\alpha\}_{\alpha \in I}$, which is short-hand for set-builder notation. We read this notation "a sub-alpha, alpha in I."