

CENTRAL LIMIT THEOREM

Why

The (normalized) sum of several independent and identically distributed random variables tends toward a normal distribution.¹

Result

Proposition 1. Let (X, \mathcal{A}, μ) be a probability space. Let $(f_n)_n$ be a sequence of independent and identically distributed real-valued random variables on X with $\mathsf{Expect}(f_n) = \mu < \infty$ and $\mathsf{var}(f_n) = \sigma^2 < \infty$ for all n. Define $s_n = \sum_{i=1}^n f_i$. For all real numbers t,

$$\lim_{n\to\infty}\mu\bigg(\bigg\{x\in X\ \bigg|\ \frac{s_n(x)-n\mu}{\sigma\sqrt{n}}\leq t\bigg\}\bigg)=\Phi(t).$$

 $^{^1\}mathrm{Future}$ editions may modify this statement and further specify the word "tends."

