



Definition

Let $\mathcal{D} = ((\mathcal{X}_t)_{t=0}^T, (\mathcal{U}_t)_{t=0}^{T-1}, (f_t)_{t=1}^{T-1})$ be a dynamical system. Let $g_t : \mathcal{X}_t \times \mathcal{U}_t \rightarrow \mathbf{R} \cup \{\infty\}$ for $t = 1, \dots, T-1$ and let $g_T : \mathcal{X}_T \rightarrow \mathbf{R} \cup \{\infty\}$. Let $x_0 \in \mathcal{X}_0$.

We call the sequence $(x_0, \mathcal{D}, (g_t)_{t=1}^T)$ a *deterministic dynamic optimization problem*. We call x_0 the *initial state*. We call g_t the *stage cost function* for stage t and call g_T the *terminal cost function*.

A deterministic dynamic optimization problem corresponds to an optimization problem with variables $u_0 \in \mathcal{U}_0, \dots, u_{T-1} \in \mathcal{U}_{T-1}$. Define $U = \mathcal{U}_0 \times \mathcal{U}_1 \times \mathcal{U}_{T-1}$. Define $J : U \rightarrow \mathbf{R} \cup \{\infty\}$ by

$$J(u) = \sum_{t=0}^{T-1} g_t(x_t, u_t) + g_T(x_T)$$

in which $x_{t+1} = f_t(x_t, u_t)$ for $t = 0, \dots, T-1$. The optimization problem is (U, J) . And so a dynamic optimization problem is just a (possibly big) optimization problem. We call $\sum_{t=0}^{T-1} g_t(x_t, u_t)$ the *total stage cost* and we call $g_T(x_T)$ the *terminal stage cost*.

Notation

We often write this problem as

$$\begin{aligned} & \text{minimize} && \sum_{t=1}^{T-1} g_t(x_t, u_t) + g_T(x_T) \\ & \text{subject to} && x_{t+1} = f_t(x_t, u_t), \quad t = 0, \dots, T-1. \end{aligned}$$

Other terminology and comments

Dynamic optimization problems are frequently called *deterministic optimal control* problems or *classical* or *open-loop control* problems. These problems are said to address the dynamic effect of actions across time. Although these models include no notion of “uncertainty” (or “uncertain outcomes”, see [Uncertain Outcomes](#)), they are frequently applied in

situations with uncertain outcomes by ignoring the uncertainty in the application.

