



## Why

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### Definition

Recall that  $(C, |\cdot|)$  is a normed space, and so also a metric space. So, a sequence  $(z_n)_{n \in \mathbf{N}}$  of complex numbers is egoprox and convergent as usual. Both of these are equivalent to the corresponding conditions on the sequences of real and imaginary parts.

**Proposition 1.**  $(z_n)_{n \in \mathbf{N}} = (x_n, y_n)_{n \in \mathbf{N}}$  converges to  $z_0 = (x_0, y_0) \in \mathbf{C}$  if and only if  $x_n$  converges to  $x_0$  and  $y_n$  converges to  $y_0$ .

**Proposition 2.**  $(z_n)_{n \in \mathbf{N}} = (x_n, y_n)_{n \in \mathbf{N}}$  is egoprox if and only if  $x_n$  is egoprox and  $y_n$  is egoprox.

### Completeness

As a result of Proposition 2, if  $z_n$  is egoprox then there is a limit  $x_0$  and  $y_0$  for its real and imaginary pieces, and so as a result of Proposition 2  $z_n$  converges. In other words, every cauchy sequence converges.

**Proposition 3.**  $\mathbf{C}$  with the metric induced by  $|\cdot|$  is complete.

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<sup>1</sup>Future editions will include.



