Definition

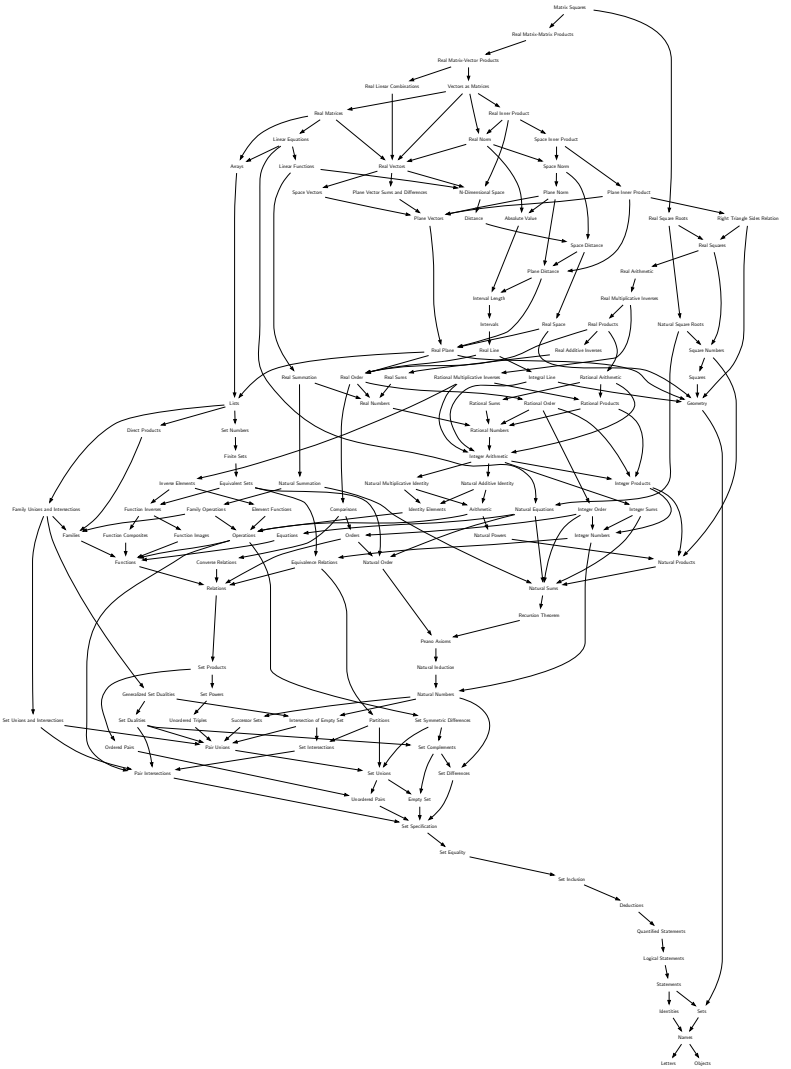
The *square* of a square matrix is the product of the matrix with itself. A *square root* (or *matrix square root*) of a given matrix is a matrix whose square is the given matrix. A matrix is *idempotent* if it is equal to its square.

Notation

Let $A \in \mathbf{R}^{n \times n}$. Then the square of A is AA. We denote the square of A by A^2. A is idempotent if $A^2 = A$. $B \in \mathbf{R}^{n \times n}$ is a square root of A if $A = B^2$.

Existence and uniqueness

Clearly a matrix can have a square root. Take for example the matrix in $\mathbf{R}^{1 \times 1}$ $\begin{bmatrix} 1 \end{bmatrix}$. A square root of this matrix is $\begin{pmatrix} 1 \end{pmatrix}$, but also $\begin{pmatrix} -1 \end{pmatrix}$. So matrix square roots do exist, but are not unique.


Matrix Squares
Real Matrix-Matrix Products
Real Matrix-Vector Products
Real Linear Combinations
Vectors as Matrices
Real Matrices
Real Inner Product
Linear Equations
Real Norm
Space Inner Product
Areas
Linear Functions
Real Vectors
Space Norm
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Norm
Plane Inner Product
Right Triangle Sides Relation
Plane Vectors
Distance
Absolute Value
Real Square Roots
Real Squares
Space Distance
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Natural Square Roots
Square Numbers
Real Plane
Real Line
Real Additive Inverses
Finite Summation
Real Order
Real Sums
Rational Multiplicative Inverses
Integral Loss
Rational Arithmetic
Squares
Lines
Real Numbers
Rational Sums
Rational Order
Rational Products
Geometry
Direct Products
Set Numbers
Rational Numbers
Finite Sets
Integer Arithmetic
Inverse Elements
Equivalent Sets
Natural Summation
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Family Unions and Intersections
Function Inverses
Family Operations
Element Functions
Comparisons
Identity Elements
Arithmetic
Natural Equations
Integer Order
Integer Sums
Families
Function Composites
Function Images
Operations
Equations
Orders
Natural Powers
Integer Numbers
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Products
Relations
Natural Sums
Recursion Theorem
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Unions and Intersections
Set Duality
Unordered Triple
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects