Why

TODO

Definition

An *index sequence* of *order* n is a finite sequence of distinct elements of $\{1, 2, \ldots, n\}$ whose length is less than or equal to n. We call the ith coordinate of an index sequence the *i-index* of the sequence. The *index matrix* associated with an index sequence is the $r \times n$ matrix whose i, jth entry is 1 if the index sequences's ith coordinate is j, and 0 otherwise. If $r = n$ then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an n-vector associated with a length-r index sequence is the product of the $r \times n$ index matrix with the n-dimensional vector. Its ith entry is the i-index entry of the vector.

Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions, and so relations, and so sets), they are not sets of indices, so we avoid this usage.

Notation

Let $r \leq n$ be natural numbers. Let $\alpha : \{1, 2, \ldots, r\} \to \{1, 2, \ldots, n\}$ be an index sequence. We denote the index ma-

trix associated with α by P_α. This matrix P_α is an element of $\mathsf{N}^{r\times n}$ and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let A be a nonempty set and let $x \in A^n$. then the subvector of x associated with P_α (and so with α) is

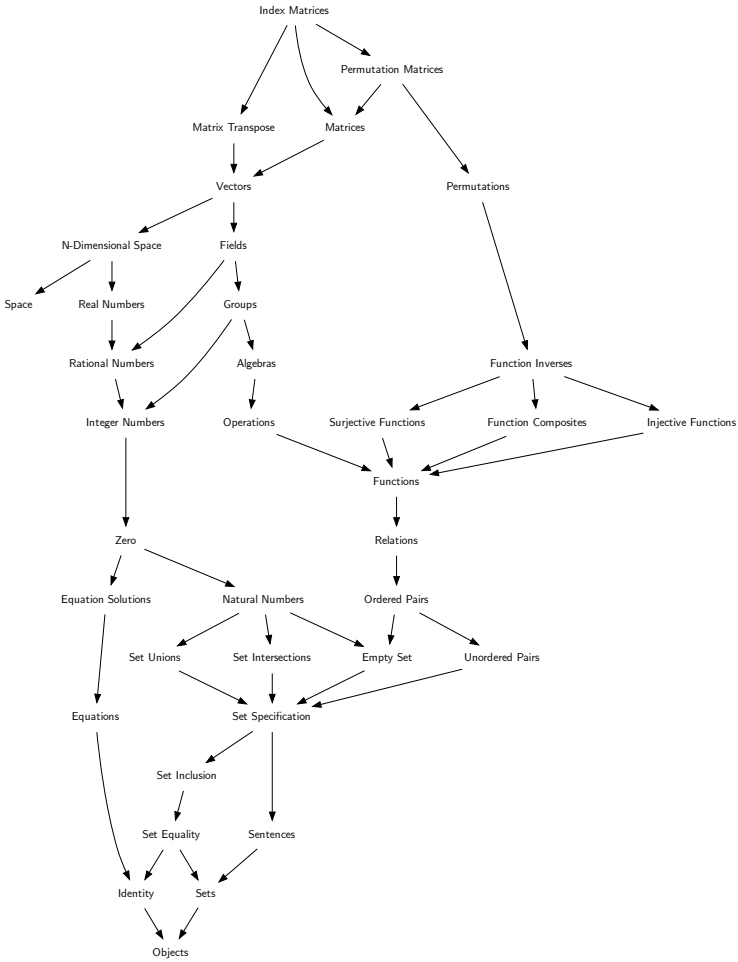
$$P_\alpha x = \left(x_{\alpha(1)}, \ldots, x_{\alpha(r)}\right)$$

We denote the product $P_\alpha x$ by x_α.

We denote the product $P_\alpha X {P_\alpha}^\top$ by $X_{\alpha\alpha}$.

Multiplication

The product of the $n \times r$ transpose of an index matrix with an r vector is the n vector with The


Index Matrices
Permutation Matrices
Matrix Transpose
Matrices
Vectors
Permutations
N-Dimensional Space
Fields
Space
Real Numbers
Groups
Rational Numbers
Algebras
Function Inverses
Integer Numbers
Operations
Surjective Functions
Function Composites
Injective Functions
Functions
Zero
Relations
Equation Solutions
Natural Numbers
Ordered Pairs
Set Unions
Set Intersections
Empty Set
Unordered Pairs
Equations
Set Specification
Set Inclusion
Set Equality
Sentences
Identity
Sets
Objects