Why

What does it mean for two random variables to be independent? What are the events associated with a random variable? TODO

Definition

Defining Result

Prop. 1. *The set of inverse images the distinguished sets of a measurable space under a function from a set to that space is a sigma algebra.*

If the first set and the function are measurable, the sigma algebra is a sub sigma algebra of the domain sigma algebra.

Proof. Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Let $f : X \to Y$ be a measurable function. Define

$$\mathcal{C} := \left\{ f^{-1}(B) \;\middle|\; B \in \mathcal{B} \right\}.$$

First, since $f^{-1}(Y) = X$, $X \in \mathcal{C}$.

Second, let $C \in \mathcal{C}$. Then there is B such that $C = f^{-1}(B)$. Then $X - C = X - f^{-1}(B) = f^{-1}(Y - B)$. Since $\mathcal{B}$ is a sigma algebra, $B \in \mathcal{B}, Y - B \in \mathcal{B}$ and so $X - C \in \mathcal{C}$.

Finally, let $(C_n)_n \subset \mathcal{C}$. Then for every n there exists a $B_n \in \mathcal{B}$ so that $C_n = f^{-1}(B_n)$. Then:

$$\cup_n C_n = \cup_n f^{-1}(B_n) = f^{-1}(\cup B_n).$$

Since $\mathcal{B}$ is a sigma algebra, $\cup_n B_n \in \mathcal{B}$ and so $\cup_n C_n \in \mathcal{C}$.

Since f is measurable, $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}$, and so $\mathcal{C} \subset \mathcal{A}$. $\square$

The sigma algebra *generated by a random variable* is the sigma algebra consisting of the inverse images of every measurable set of the codomain.

The sigma algebra generated by a family of random variables is the sigma algebra generated by the union of the sigma algebras generated individually by each of the random variables.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f : X \to Y$ be a random variable. Denote by $\sigma(f)$ the sigma algebra generated by f.

Results

Prop. 2. *The sigma algebra generated by a family of random variables is the smallest sigma algebra for with respect to which each random variable is measurably.*

Random Variable Sigma Algebra
Random Variables
Measurable Functions
Topological Sigma Algebra
Outcome Variables
Topological Spaces
Metrics
Distance
Absolute Value
Probability Measure
Space Distance
Measures
Event Probabilities
Plane Distance
Extended Real Numbers
Probability Distributions
Interval Length
Intervals
Uncertain Outcomes
Real Limits
Real Space
Real Line
Real Plane
Real Sequences
Real Summation
Real Order
Integral Line
Sequences
Real Numbers
Geometry
Sigma Algebras
Direct Products
Set Numbers
Rational Numbers
Finite Sets
Natural Summation
Integer Arithmetic
Subset Algebras
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Set Operations
Equivalent Sets
Cardinality
Algebras
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Operations
Natural Exponents
Integer Products
Integer Sums
Families
Function Images
Function Composites
Natural Products
Integer Numbers
Functions
Equivalence Relations
Natural Sums
Relations
Natural Order
Recursion Theorem
Subset Systems
Cartesian Products
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Generalized Set Dualities
Unordered Triples
Natural Numbers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects