Why

We want to talk about optimally eliminating variables in a system of linear equations.[1]

Definition

An *ordered* undirected graph is *filled* or *monotone transitive* if all higher neighborhoods induce complete subgraphs. An ordering σ of an undirected graph (V, E) is a *perfect elimination ordering* if the ordered undirected graph $((V, E), \sigma)$ is filled.

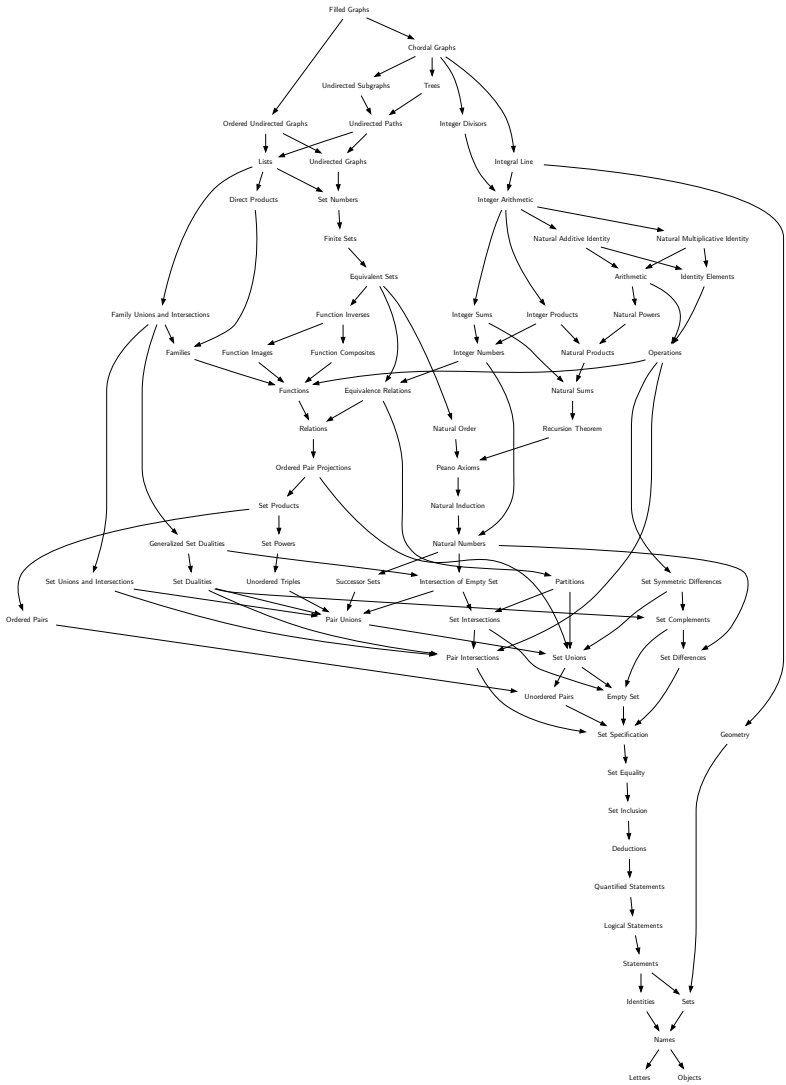
Let $G = ((V, E), \sigma)$ be an ordered undirected graph. G is filled if, for all $v \in V$, $w, z \in \mathrm{adj}^+(v) \longrightarrow \{w, z\} \in E$. Equivalently stated, G is filled if, for all $i < j < k$, $\{\sigma(i), \sigma(j)\} \in E$ and $\{\sigma(i), \sigma(k)\} \in E$ imply $\{\sigma(j), \sigma(k)\} \in E$.

Chordality

Proposition 1. *If (V, E, σ) is a filled graph, then (V, E) is chordal.*

Proof. Take the vertex with the lowest index on a cycle of length greater than three. Take $\square$

[1] Future editions will expand. For example, this sheet is needed for perfect elimination orderings.


Filled Graphs
Chordal Graphs
Undirected Subgraphs
Trees
Ordered Undirected Graphs
Undirected Paths
Integer Divisors
Lists
Undirected Graphs
Integral Line
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Equivalent Sets
Family Unions and Intersections
Function Inverses
Integer Sums
Integer Products
Natural Powers
Families
Function Images
Function Composites
Integer Numbers
Natural Products
Operations
Functions
Equivalence Relations
Natural Sums
Relations
Natural Order
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects