Relations

1 Why

We want to speak of elements of one set in relation to elements of another set.

2 Definition

A **relation** between two non-empty sets A and B is a subset of $A \times B$. So then, naturally, a relation on a single set C is a subset of $C \times C$.

2.1 Notation

As relations are sets, our de facto protocol is to denote them by upper case capital letters, for example, the letter R. Let R a relation on A and B. If $(a, b) \in R$, we often write aRb, read aloud as "a in relation R to b."

In many cases, though, we eschew the set notation and use particular symbols. Often the symbols we use are meant to be suggestive of the relation. Some examples include $\sim$, $=$, $<$, $\leq$, $\prec$, and $\preceq$.