Definition

Suppose $T \in \mathcal{L}(V, W)$. In other words, T is a linear map from a vector space V to a vector space W where V and W are over the same field of scalars.

An *adjoint* of T is a function $S : W \to V$ satisfying

$$\langle Tv, w \rangle = \langle v, Sw \rangle \quad \text{for every } v \in V \text{ and every } w \in W$$

It is not hard to see that there always exists an adjoint, and that this adjoint is unique. Thus, we speak of *the adjoint* of T.

Notation

We denote *the* adjoint of T by T^*. This notation is meant to remind of complex conjugation, for reasons which will become apparent shortly.

Examples

Space to the plane. Define $T : \mathbf{R}^3 \to \mathbf{R}^2$ by

$$T(x_1, x_2, x_3) = (x_2 + 3x_3, 2x_1)$$

We claim that the adjoint of T is $T^* : \mathbf{R}^2 \to \mathbf{R}^3$ defined by

$$T^*(y_1, y_2) = (2y_2, y_1, 3y_1)$$

Properties

Proposition 1 (Adjoint is Linear)**.** *Suppose $T \in \mathcal{L}(V, W)$. The adjoint of T is linear.*

Proposition 2 (Adjoint properties)**.** *Suppose V and W are finite dimensional inner product spaces over a field $\mathbf{F}$, which is $\mathbf{R}$ or $\mathbf{C}$. Suppose $S, T \in \mathcal{L}(V, W)$. Then*

1. $(S + T)^* = S^* + T^*$

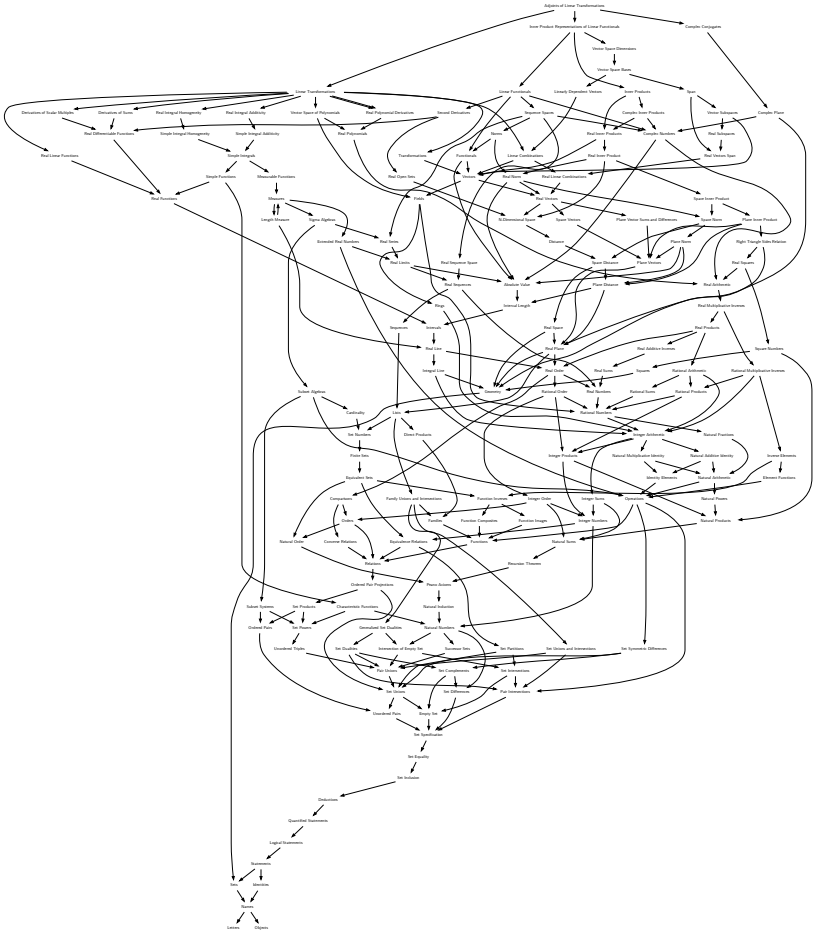
2. $(\lambda T)^* = \lambda^* T^*$ *for all* $\lambda \in \mathbf{F}$

3. $(T^*)^* = T$

4. $I^* = I$

Proposition 3. *Suppose* V, W, U *are finite dimensional inner product spaces over* $\mathbf{R}$ *or* $\mathbf{C}$. *For all* $T \in \mathcal{L}(V, W)$ *and* $S \in \mathcal{L}(W, U)$,

$$(ST)^* = T^* S^*$$


Adjoints of Linear Transformations
Inner Product Representation of Linear Functionals
Complex Conjugates
Vector Space Dimension
Vector Space Basis
Linear Transformations
Derivatives of Scalar Multiples
Derivatives of Sums
Real Integral Homogeneity
Real Integral Additivity
Vector Spaces of Polynomials
Real Polynomial Derivatives
Second Derivatives
Linear Functionals
Linearly Dependent Vectors
Inner Products
Bar
Vector Subspaces
Complex Plane
Real Differentiable Functions
Simple Integral Homogeneity
Simple Integral Additivity
Real Polynomials
Subspace Spaces
Complex Inner Products
Real Inner Products
Complex Numbers
Real Subspaces
Real Linear Functions
Simple Integrals
Transformations
Functionals
Linear Combinations
Real Inner Products
Real Orthoro Tuples
Simple Functions
Measurable Functions
Real Open Sets
Sets
Real Vectors
Real Linear Combinations
Real Functions
Measures
Cells
Real Vectors
Space Inner Products
Length/Distance
Metric Algebra
Bi-dimensional Space
Space Vectors
Plane Vector Sums and Differences
Inner Norm
Norm Inner Product
Extended Real Numbers
Real Limits
Real Sequence Space
Discrete
Plane Norm
Right Triangle Sides Relation
Real Limits
Space Distances
Plane Distances
Real Sequences
Absolute Value
Plane Distances
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Sets
Sequences
Intervals
Real Space
Real Products
Real Sum
Real Addition Inverses
Square Roots
Integral Line
Real Sum
Squares
Rational Arithmetic
Rational Multiplicative Inverses
Real Sum
Real Sequences
Rational Sum
Rational Products
Sums
Rational Sum
Rational Sum
Rational Sum
Rational Products
Simple Algebra
Linearity
Sets
Elem Products
Integer Arithmetic
Rational Fractions
Set Operations
Integer Fractions
Rational Multiplicative Identity
Rational Addition Identity
Inverse Elements
Finite Sets
Rational Arithmetic
Rational Arithmetic
Element Functions
Equivalent Sets
Comparison
Family Orders and Isomorphisms
Function Inverse
Integer Order
Integer Sets
Integers
Rational Powers
Orders
Families
Function Composition
Function Images
Integer Numbers
Rational Products
Natural Order
Converse Relations
Equivalence Relations
Function
Rational Sets
Induction
Recursion Theorem
Ordered Pair Polynomials
Peano Axioms
Subset Systems
Set Products
Organizations Functions
Natural Inductions
Ordered Pairs
Set Powers
Cartesian Set Quotient
Natural Numbers
Unordered Triples
Set Quotient
Intersection of Empty Set
Sentence Sets
Set Partitions
Set Union and Intersections
Set Symmetric Differences
Set Union
Set Intersections
Set Intersections
Set Union
Set Differences
Set Intersections
Unordered Pairs
Empty Set
Set Specifications
Set Equality
Set Inclusion
Statements
Quantified Statements
Logical Statements
Sets
Identities
Spaces
Letters
Objects