



Definition

The *complex conjugate* (or *conjugate*) of a complex number z is the complex number whose real part matches z and whose imaginary part is the additive inverse of z . The complex conjugate of a purely real number is the same purely real number. In other words, the complex conjugate of a complex number with no imaginary part is the same complex number.

Notation

We denote the complex conjugate of the complex number $z \in \mathbf{C}$ by z^* . Other common notation includes \bar{z} , read “z bar”. If there exists $a, b \in \mathbf{R}$ so that $z = (a, b)$, then $z^* = (a, -b)$.

Geometric interpretation

Taking the conjugate of a complex numbers corresponds to a reflection across the real axis in the plane.

Properties

A complex number z is real if and only if $z = z^*$ and it is imaginary if and only if $z = -z^*$.

Proposition 1. For $z \in \mathbf{C}$, we have

$$\operatorname{Re}(z) = \frac{z + z^*}{2} \quad \text{and} \quad \operatorname{Im}(z) = \frac{z - z^*}{2i}.$$

