Partial Orders

1 Why

We want to handle elements of a set in a particular order.

2 Definition

Let R be a relation on a non-empty set A. R is a **partial order** if it is reflexive, transitive, and anti-symmetric. If $(a, b) \in R$ we say that a **precedes** b and that b **succeeds** a.

A **partially ordered set** is a set and a partial order. The language partial is meant to suggest that two elements need not be comparable. For example, suppose R is $\{(a, a) \mid a \in A\}$; we may justifiably call this no order at all and call A totally unordered, but it is a partial order by our definition.

2.1 Notation

We denote a partial order on a set A by $\preceq$. We read $\preceq$ aloud as “precedes or equal to" and so read $a \preceq b$ aloud as “a precedes or is equal to b." If $a \preceq b$ but $a \neq b$, we write $a \prec b$, read aloud as “a precedes b."