Why

Is the set of real numbers a complete ordered field (in the sense of Complete Fields?

Main Result

Proposition 1. $(\mathbf{R}, +, \cdot, <)$ *is a complete ordered field.*[1]

Proof. The supremum of a set of nonempty real numbers bounded from above R is $\cup R$. $\qquad\square$

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[1] The account will appear in future editions.

Real Completeness

Complete Fields

Least Upper Bounds

Real Order

Fields — Real Numbers — Rational Order

Groups — Rational Numbers

Integer Additive Inverses — Integer Arithmetic

Inverse Elements — Natural Multiplicative Identity — Natural Additive Identity — Integer Sums — Integer Products — Integer Order

Element Functions — Identity Elements — Arithmetic — Integer Numbers

Function Inverses — Algebras — Natural Exponents

Function Composites — Function Images — Operations — Natural Products

Functions — Natural Sums

Recursion Theorem — Equivalence Relations

Relations — Peano Axioms

Cartesian Products — Natural Induction

Set Powers — Natural Numbers

Unordered Triples — Successor Sets — Intersection of Empty Set — Partitions

Ordered Pairs — Pair Unions — Set Intersections

Set Differences — Set Unions

Unordered Pairs — Empty Set — Pair Intersections

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects