

## REAL VECTORS

## Why

If we interpret a list of two numbers as displacement in a plane, and a list of three numbers as displacement in a space, what of a list of n numbers as displacement in  $\mathbb{R}^n$ ?

## **Definition**

A real vector (or vector, n-dimensional vector, n-vector) is a length-n list of real numbers.

## Algebra

For  $x, y \in \mathbf{R}^n$ , we define the *real vector sum* (or *sum*) of x and y as the vector  $z \in \mathbf{R}^n$  where  $z_i = x_i + y_i$  for i = 1, ..., n. As usual, we denote the sum by x + y, so

$$x + y = (x_1 + y_1, \dots, x_n + y_n).$$

For  $\alpha \in \mathbf{R}$  and  $x \in \mathbf{R}^n$ , real scalar-vector product (or scalar product, product)  $z \in \mathbf{R}^n$  is defined by  $z_i = \alpha x_i$  for i = 1, ..., n. As usual, we denote the product  $\alpha x$ , and write

$$\alpha x = (\alpha x_1, \dots, \alpha x_n).$$

Our intuition for both of these operations comes from their special cases in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ . As usual, the real-vector difference (or difference) of x and y is the vector  $z \in \mathbb{R}^n$  defined by  $z_i = x_i - y_i$  for i = 1, ..., n. As usual, we denote it by x - y, and note that x - y = x + (-y).

The algebra given here for vectors is natural in view of their generalization as n-dimensional displacements. However, we keep in mind that this algebra is over lists of numbers, and that these sums and products can be defined on these lists of numbers regardless of interpretation.

