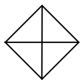


Why

Definition

Let X be nonempty (index) set. For example, X may be $\{1, 2, \dots, N\}$, Z, $[0, 1]$, R^d, $\{x \in \mathsf{R}^3 \mid \|x\| \leq 1\}$ (the unit sphere), or $\{x \in \mathsf{R}^3 \mid \alpha \leq \|x\| \leq \beta\}$ (the atmosphere, or volume between two concentric spheres).

A symmetric, real-valued function $k : X \times X \to \mathsf{R}$ of two variables is said to be *positive semidefinite* if for any $n \in \mathsf{N}$, for any real $a_1, \dots, a_n \in \mathsf{R}$ and $x_1, \dots, x_n \in X$, we have

$$\sum_{i,j=1}^{n} a_i a_j k(x_i, x_j) \geq 0,$$

and *positive definite* if the above holds with ">".[1]

Positive semidefinite kernels are useful for the following constructive reason:

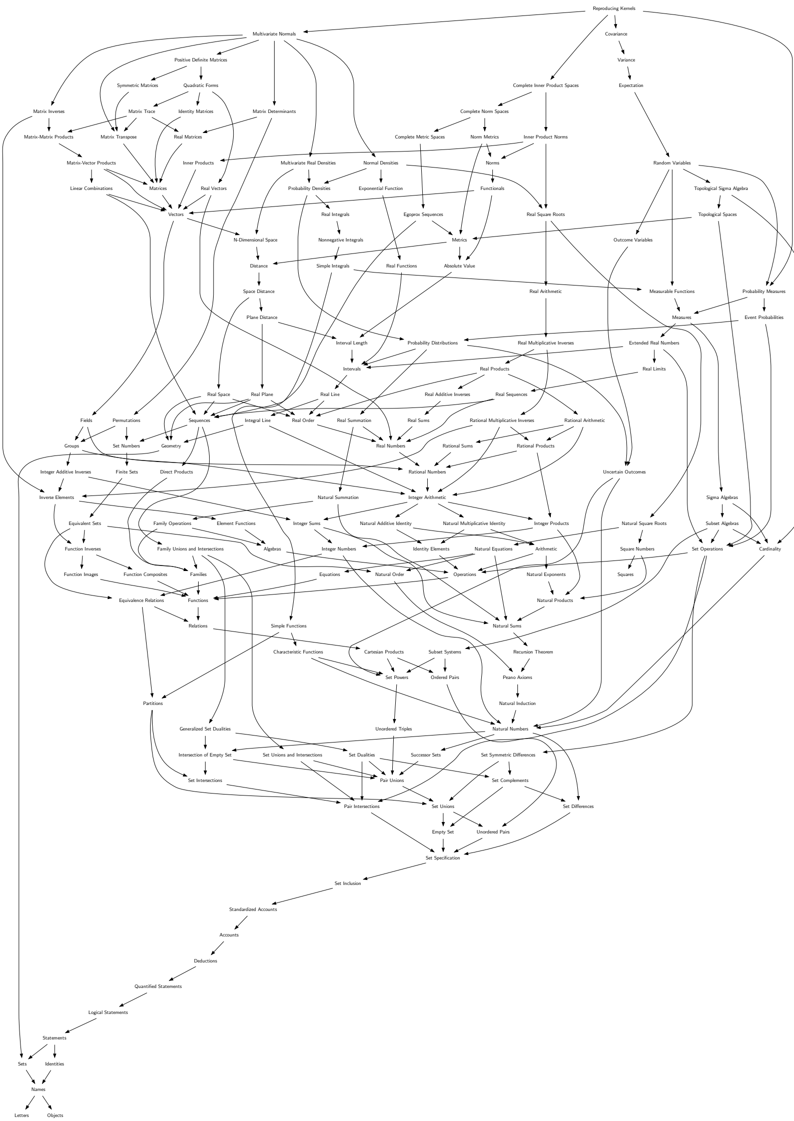
Proposition 1. *Let $X \neq \varnothing$ be a set. If $k : X \times X \to \mathsf{R}$ is positive semidefinite, then there exists a probability space $(\Omega, \mathcal{A}, \mathbf{P})$ and a family of zero-mean normal real-valued random variables $\{f_x : \Omega \to \mathsf{R}\}_{x \in X}$ with covariance function k, that is,*

$$\mathsf{E}\, f(a) f(b) = k(a, b), \qquad \textit{for all } a, b \in X.^2$$

[1] Some authors use the term "positive definite" for our term positive semidefinite and the term "*strictly positive definite*" for our term positive definite.

[2] Future editions will prove this result.

This result is known by the names *Kolmogorov extension theorem*, *Kolmogorov existence theorem*, *Kolmogorov consistency theorem* and *Daniell-Kolmogorov theorem*.


Reproducing Kernels
Multivariate Normals
Covariance
Positive Definite Matrices
Variance
Symmetric Matrices
Quadratic Forms
Complete Inner Product Spaces
Expectation
Matrix Inverses
Matrix Trace
Identity Matrices
Matrix Determinants
Complete Norm Spaces
Matrix-Matrix Products
Matrix Transpose
Real Matrices
Complete Metric Spaces
Norm Metrics
Inner Product Norms
Matrix-Vector Products
Inner Products
Multivariate Real Densities
Normal Densities
Norms
Random Variables
Linear Combinations
Matrices
Real Vectors
Probability Densities
Exponential Function
Functionals
Topological Sigma Algebra
Vectors
Real Integrals
Egoprox Sequences
Real Square Roots
Topological Spaces
N-Dimensional Space
Nonnegative Integrals
Metrics
Outcome Variables
Distance
Simple Integrals
Real Functions
Absolute Value
Space Distance
Real Arithmetic
Measurable Functions
Probability Measures
Plane Distance
Measures
Event Probabilities
Interval Length
Probability Distributions
Real Multiplicative Inverses
Extended Real Numbers
Intervals
Real Products
Real Limits
Real Space
Real Plane
Real Line
Real Additive Inverses
Real Sequences
Fields
Permutations
Sequences
Integral Line
Real Order
Real Summation
Real Sums
Rational Multiplicative Inverses
Rational Arithmetic
Groups
Set Numbers
Geometry
Real Numbers
Rational Sums
Rational Products
Integer Additive Inverses
Finite Sets
Direct Products
Rational Numbers
Uncertain Outcomes
Inverse Elements
Natural Summation
Integer Arithmetic
Sigma Algebras
Equivalent Sets
Family Operations
Element Functions
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Natural Square Roots
Subset Algebras
Function Inverses
Family Unions and Intersections
Algebras
Integer Numbers
Identity Elements
Natural Equations
Arithmetic
Square Numbers
Set Operations
Cardinality
Function Images
Function Composites
Families
Equations
Natural Order
Operations
Natural Exponents
Squares
Equivalence Relations
Functions
Natural Products
Relations
Simple Functions
Natural Sums
Characteristic Functions
Cartesian Products
Subset Systems
Recursion Theorem
Set Powers
Ordered Pairs
Peano Axioms
Partitions
Natural Induction
Generalized Set Dualities
Unordered Triples
Natural Numbers
Intersection of Empty Set
Set Unions and Intersections
Set Dualities
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects