Set Inclusion

1 Why

We want language for all of the elements of a first set being the elements of a second set.

2 Definisions

If every element of a first set is an element of a second set we say that the first set is a *subset* of the second set. Conversely, we say that second set is a *superset* of the first set.

Every set is a subset of itself. Similarly, every set is a superset of itself. Thus, if two sets are equal, the first is a subset of the second and the second is a subset of the first. Because of our definition of set equality, the converse is also true.

The empty set is a subset of every set, since it has no elements and so satisfies our definition. Consider a set. We call the empty set and the set itself *improper subsets* of the set. All other subsets we call *proper subsets*.

2.1 Notation

Let A and B be sets. We denote that A is a subset of B by $A \subset B$. We read the notation $A \subset B$ aloud as "A subset B".

We can express the axiom of extension by

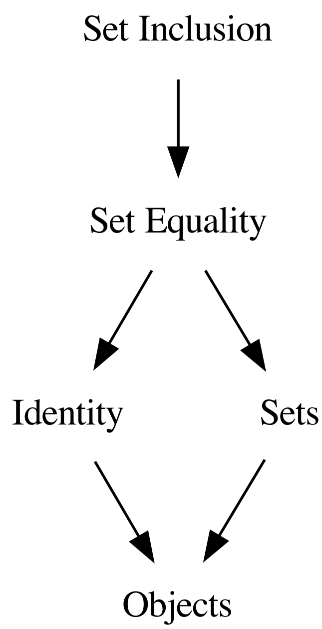
$$A = B \Leftrightarrow (A \subset B) \wedge (B \subset A)$$

The notation $A \subset B$ is a concise symbolism for the sentence "every element of A is an element of B." Or for the alternative notation $a \in A \implies a \in B$.

2.2 Immediate Results

Proposition 1. *If $A \subset B$ and $B \subset C$ then $A \subset C$.*

Proof. Let $a \in A$. Then $a \in B$ and so then $b \in C$. Thus $a \in C$. $\square$


Set Inclusion
Set Equality
Identity
Sets
Objects