# Multivariate Normal Conditionals

## 1 Why

What of the conditional densities of a multivariate normal density.

## 2 Result

**Proposition 1.** *Let*

$$\mu = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$$

*Then the conditional of $x$ given $y = \gamma$ is $\mathcal{N}(\bar{\mu}(\gamma), \bar{\Sigma},)$ with*

$$\bar{\mu}(\gamma) = \mu_1 + \Sigma_{xy}\Sigma_{yy}^{-1}(\gamma - \mu_y) \text{ and } \bar{\Sigma} = \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}.$$