



Definition

Proposition 1. *Let W be a positive semidefinite n by n matrix. Then $g : \mathbf{R}^n \rightarrow \mathbf{R}$ defined by $g(x) = \sqrt{x^\top W x}$ is a norm on \mathbf{R}^n .*

Notation

Let $W \in \mathbf{R}^{n \times n}$, positive semidefinite. Then we denote the norm corresponding to W by $\|\cdot\|_W$. So then, the norm of a vector $x \in \mathbf{R}^n$ is $\|x\|_W$. Notice that $\|x\|_W = \|W^{1/2}x\|_2$.

Visualization¹

We can compare the Euclidean norm on \mathbf{R}^2 with the weighted norm given by

$$W = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$

¹Future editions will visualize these norms as function on \mathbf{R}^2 , via contour plots.

