Optimization

1 Why

Given a correspondence between objects in a set with objects in an totally ordered set, we are interested in the objects which correspond to minimal or maximal elements of the ordered set.

2 Definition

Let A be a non-empty set and let $(C, \leq)$ be chain. Let $f : A \to C$.

The *minimization problem over* A associated with f is to find an element $a \in A$ so that $f(a)$ is minimal in $f(A)$. The *maximization problem over* A associated with f is to find an element $a \in A$ so $f(a)$ is maximal in $f(A)$. We call either of these an *optimization problem*.

We call f the *ordering* function. We call A the *feasible set* and we call $a \in A$ a *feasible element*. An element $a \in A$ is a *minimizer* of f if $f(a)$ is minimal in $f(A)$. An element $a \in A$ is a *maximizer* of f if $f(a)$ is maximal in $f(A)$.

3 Notation

Let $(C, \prec)$ be a chain. We denote the minimization problem to find an element $a \in A$ to minimize $f : A \to C$ by

$$\begin{array}{rl} \textbf{find} & a \in A \\ \textbf{to minimize} & f(a) \end{array}$$

We denote the minimizers by

$$\textsf{\textbf{minimizers}}(f).$$