

#### RELATIVE ENTROPY

## **Definition**

Consider two distributions on the same finite set. The *entropy* of the first distribution *relative* to the second distribution is the difference of the cross entropy of the first distribution relative to the second and the entropy of the second distribution. We call it the *relative entropy* of the first distribution with the second distribution. People also call the relative entropy the *Kullback-Leibler divergence* or *KL divergence*.

### Notation

Let A be a non-empty finite set. Let  $p:A\to \mathbb{R}$  and  $q:A\to \mathbb{R}$  be distributions. Let H(q,p) denote the cross entropy of p relative to q and let H(q) denote the entropy of q. The entropy of p relative to q is

$$H(q,p) - H(q)$$
.

Herein, we denote the entropy of p relative to q by d(q, p).

# A similarity function

The relative entropy is a similarity function between distributions.

**Proposition 1.** Let q and p be distributions on the same set. Then  $d(q, p) \ge 0$  with equality if and only if p = q.

So, d has a few of the properties of a metric. However, d is not a metric; for example, it is not symmetric.

**Proposition 2.** There exist distributions  $p: A \to \mathbf{R}$  and  $q: A \to \mathbf{R}$  (with A a non-empty finite set) such that

$$d(q, p) \neq d(p, q)$$
.

### Optimization perspective

A solution to finding a distribution  $p: A \to \mathbf{R}$  to

minimize 
$$d(q, p)$$
,

is  $p^* = q$ .

