Why

We name a statement which involves an identity.[1]

Definition

An *equation* is statement (see Statements) relating two terms
by the relation of identity (see Identities). Some authors also
call an equation an *equality*. The symbol "=" is called the (or
an) *equals sign* or *equals symbol*.

Variables

Let X and Y be sets and let $f, g : X \to Y$. In the statement
$(\forall x)(f(x) = g(x))$, f and g are *free* names and x is a *bound*
name (see Quantified Statements).

For convenience, we often refer to the equation $f(x) = g(x)$
without the quantifier $\forall x$. In this case, x appears free, but is
not. In this context, the statement $f(x) = g(x)$ has x implicitly
bound. There are two senses here, though. The first is that x
is bound because it is "subordinate" to the quantifier $\forall$. The
particular symbol is irrelevant; the symbol y works just as
well. In a second sense, though, the name is "free," as it is a
placeholder and the choice of the symbol x does not matter.

We use different terminology for this common case. In dis-
cussing $f(x) = g(x)$, we call the placeholder name x a *variable*
and we call the names f and g *constants*. The language is

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[1]Future editions will modify this statement and sheet.

meant to convey f and g are *fixed* in the present discussion, as indicated by the usual language "Let f and g ...".

Solutions

We are often interested in finding objects in some set to satisfy an equation. For example, we are interested in finding an object $\xi \in X$ to satisfy $f(\xi) = g(\xi)$. In this setting we call the variable ξ in the equation an *unknown.*

We call an element $\xi \in X$ a *solution* of the equation if $f(\xi) = g(\xi)$. We call the set $\{\xi \in X \mid f(\xi) = g(\xi)\}$ the *solution set.* If the solution set is non-empty, we say that a solution *exists.* If the solution set is a singleton, we say that the solution is *unique.*

We are often interested in solutions which satisfy several equations at once. For example, we have the equations $f_1(x) = g_1(x)$ and $h(x) = i(x)$ and so on. We want x to satisfy these. Here it is *set of equations*, *simultaneous equations*, or a *system of equations.*

Finding solutions

We often talk about *finding* or *searching* for solutions or *solving equations.* We say: "We want to *find* $x \in X$ to satisfy $f(x) = g(x)$." In addition to $f(x) = g(x)$, we may include other statements about x. The language is meant to convey that we are searching for an object which we will name, as a variable, x, and we want this object to satisfy the statements.

Equations

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Functions

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Relations

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Set Products

Set Powers

Ordered Pairs

Unordered Triples

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Pair Unions

↓

Set Unions

Unordered Pairs

Empty Set

Set Specification

↓

Set Equality

↓

Set Inclusion

↓

Standardized Accounts

↓

Accounts

↓

Deductions

↓

Quantified Statements

↓

Logical Statements

↓

Statements

Identities

Sets

Names

Letters

Objects