Result

Proposition 1. *Suppose* **P** *is a finite probability measure on a set of outcomes Ω. For any two events A, B with* $\mathbf{P}(A), \mathbf{P}(B) > 0$, *we have*

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(B \mid A)\mathbf{P}(A)}{\mathbf{P}(B)}.$$

Proof. By definition, we have

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}.$$

And also symmetrically,

$$P(B \mid A) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(A)}.$$

From this second equation we have $\mathbf{P}(A \cap B) = \mathbf{P}(B \mid A)\mathbf{P}(A)$, which we can substitute into the numerator of the first expression to obtain the result. $\square$

This result is known by many names including *Bayes' rule*, *Bayes rule* (no possessive), *Bayes' formula*, and *Bayes' theorem*.

It is a *basic* consequence of the *definition* of conditional probability, but it is *useful* in the case that we are given problem data in terms of the probabilities on the right hand side of the above equation.

Compound form

More is true.

Proposition 2. *Suppose* **P** *is a finite probability measure on a set of outcomes Ω. For any three events A, B, C with* $\mathbf{P}(A), \mathbf{P}(B), \mathbf{P}(C) > 0$, *we have*

$$\mathbf{P}(A \mid B \cap C) = \frac{\mathbf{P}(B \mid A \cap C)\mathbf{P}(A \mid C)}{\mathbf{P}(B \mid C)}.$$

Proof. Future editions will include, the strategy is the same as above. $\square$

Posterior Event Probabilities

Conditional Event Probabilities

Event Probabilities

Probability Distributions

Outcome Probabilities

Intervals

Real Summation

Real Line

Real Order

Real Numbers

Rational Order

Natural Summation

Integral Line

Rational Numbers

Lists

Integer Arithmetic

Set Numbers

Direct Products

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Finite Sets

Arithmetic

Identity Elements

Equivalent Sets

Function Inverses

Family Operations

Uncertain Events

Family Unions and Intersections

Natural Powers

Integer Sums

Integer Order

Comparisons

Natural Products

Integer Numbers

Orders

Operations

Function Composite

Function Images

Families

Functions

Natural Sums

Converse Relations

Equivalence Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Uncertain Outcomes

Ordered Pair Projections

Set Products

Natural Induction

Set Powers

Natural Numbers

Generalized Set Dualities

Partitions

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Symmetric Differences

Set Dualities

Set Unions and Intersections

Ordered Pairs

Set Intersections

Pair Unions

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Geometry

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects