



Why

Can flashlights speak?

Speech can be made to correspond to words. And words are sequences of letters. So, roughly, can we communicate letters with a flashlight? No sounds allowed.

Discussion

Well, what can we do with a flashlight? Turn it on and off. So we can show blinks of light. Here's a thought: One blink means a . Two blinks means b . Three blinks means c . Four blinks means d . And so on.

Here's the rub: how do we know when one letter stops, and another begins. Put another way, does 7 blinks mean (g) , or (b, a, d) , since $2 + 1 + 4 = 7$, or (c, d) , since $3 + 4 = 7$, or (d, c) and so on. If we change the game, maybe we can change the color of the flashlight. Or give a half-flash. Is a half-flash a "quick" flash, or is it a flash in which we have covered up half of the end of the flashlight?

The first question is a clue. The duration of the blink. How about the duration between blinks? A one second pause in between blinks for a letter. A three second pause for blinks in between letters. A five second pause for a space, to indicate a change of a word? Or an additional symbol, a "space", in our alphabet, communicated with 27 (!) blinks?

Definition

Let \star represent a blink. The *flash code*¹ for the alphabet A (the latin lower case letters) is the function $f : A \rightarrow \mathcal{S}(\{\star\})$ defined by $f(a) = \star$, $f(b) = \star\star$, $f(c) = \star\star\star$, $f(d) = \star\star\star\star$ and so on. We call $f(a)$ the *code word* of $a \in A$. It is a sequence of blinks.

The idea is that if we give someone the *code word* $f(a)$ for a letter of our alphabet $a \in A$, they will be able to tell us a . In the language of functions, we want f to have an inverse. Clearly, f is invertible.

It is natural to extend the flash code for A to strings of A . Let \square represent a long pause. Then we encode a sequence $x \in \mathcal{S}(A)$ by applying f to each element of x in turn, and including a \square in between. The *flash code* for the strings $\mathcal{S}(A)$ is the function $g : \mathcal{S}(A) \rightarrow \mathcal{S}(\{\star, \square\})$ defined recursively, as $g(x) = f(x_1')$ for length one strings x' , $g(x'') = f(x_1'')\square f(x_2)$ and for length n strings x by $f(x_1)\square g(x_{2:n})$. For example, we encode the word (string) (b, a, d) as

$$\star\star\square\star\square\star\star\star\star \tag{1}$$

Now suppose we are given a sequence as shown in (1). Well, so long as f is invertible, we look can decode the codewords (the blinks, in this case) in between the squares and recover the word.

¹The etymology of code is from the Latin codex, in the legal tradition of a list of statutes. We may well use the term correspondence, or function, but code is standard.

