



Why

We want to discuss making a single decision.¹ To discuss decisions, we first we speak of the choices to be made.

Definition

We have a set which includes all possible choices. The set is called the *actions*, *acts*, *decisions*, *choices* or *designs*.

For each decision, we have a set which includes all possible outcomes. Often the outcomes are uncertain, and associated with the future (see **Uncertain Outcomes**).

The intuition is that we will select an action prior to observing the outcome. We need some way to talk about which actions and outcomes are preferable to others. We call the *histories* all possible action-outcome pairs. A *preference* is a total order on the set of histories.

A *simple decision problem* is a triple $(A, \{O_a\}_{a \in A}, \preceq)$ in which A is a set of actions, $\{O_a\}$ is a family of sets of outcomes, and \preceq is a preference on the histories $\{(a, o) \mid a \in A, o \in O_a\}$.

Example: party

Consider deciding whether to host a party indoors or outdoors. We are unsure of the weather. We have a set of two actions $A = \{\text{IN}, \text{OUT}\}$ and a set of two outcomes (the same for each

¹Future editions will expand.

action) $O_a = W = \{\text{RAIN}, \text{SHINE}\}$ for $a \in A$. Although many orders on $A \times W$ exist, one such order is

$$(\text{OUT}, \text{SHINE}) \prec (\text{IN}, \text{RAIN}) \prec (\text{IN}, \text{SHINE}) \prec (\text{OUT}, \text{RAIN}).$$

In other words, having the party outside in the sun is preferred to having it inside when it is raining, but both of these are preferred to having it inside when the sun is shining, and all of these are preferred to having it outdoors in the rain.

Best actions

Let $(A, \{O_a\}, \preceq)$ be a simple decision problem in which $O_a = S$ for each $a \in A$. Let $o \in S$. An action $a \in A$ is *best for outcome* o if $(a, o) \preceq (a', o)$ for all $a' \in A$. An action $a \in A$ is *best for all outcomes* (or *uniformly best*) if, for all $o \in O$, $(a, o) \preceq (a', o')$ for all $a' \in A, b' \in O$.²

In the party example, the best action for RAIN is IN and the best action for SHINE is OUT. On the other hand, there is no uniformly best action. For the action OUT, there is an outcome RAIN, and action-outcome pair (IN, RAIN) so that $(\text{IN}, \text{RAIN}) \prec (\text{OUT}, \text{RAIN})$. In other words, OUT is not uniformly best, because there is an outcome, RAIN for which the action IN is preferred. Similarly for IN.

²Future editions will clarify, and may remove this section.

