



Identity Elements

1 Why

We can construct functions on the ground set of an algebra by fixing an element in the ground set and defining a function which maps elements to the result of the operation applied to the fixed element and the given element.

2 Definition

Let $(A, +)$ be an algebra. For each $a \in A$, denote by $+_a : A \rightarrow A$ the function defined by

$$+_a(b) = a + b.$$

If $+_a$ is the identity function on A then we call a a *left identity element* of the algebra.

Similarly, denote by $+^a : A \rightarrow A$ the function defined by

$$+^a(b) = b + a.$$

If $+^a$ is the identity function on A then we call a a *right identity element* of the algebra.

An *identity element* of the algebra is an element which is both a left and right identity. If the operation commutes, then a left identity and right identities are the same.