Why

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Definition

Let $(A, \leq)$ be a chain.

A *lower bound* for $B \subset A$ is an element $a \in A$ so that $a \leq b$ for all $b \in B$. A set is *bounded from below* if it has a least upper bound. A *greatest lower bound* for B is an element $c \in A$ so that c is a lower bound and $c < a$ for all other lower bounds a.

Proposition 1. *If there is a greatest lower bound it is unique.*[2]

We call the unique greatest lower bound of a set (if it exists) the *infimum*.

Notation

We denote the infimum of a set $B \subset A$ by $\inf A$.

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[1]To be given in future editions.

[2]Proof in future editions.

Greatest Lower Bounds

Real Order → Total Orders

Real Numbers

Rational Numbers → Partial Orders

Integer Arithmetic

Natural Additive Identity → Natural Multiplicative Identity

Arithmetic → Identity Elements

Integer Products → Natural Exponents → Operations

Integer Sums → Natural Products → Functions

Integer Numbers → Natural Sums

Equivalence Relations → Recursion Theorem

Peano Axioms → Relations

Natural Induction → Cartesian Products

Natural Numbers → Set Powers

Partitions → Intersection of Empty Set → Successor Sets → Unordered Triples

Set Intersections → Pair Unions → Ordered Pairs

Set Differences → Set Unions

Pair Intersections → Empty Set → Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities → Sets

Names

Letters → Objects