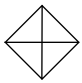


Why

We want a notion for a correspondence between two sets.

Definition

A *function* f *from* a set X *to* a set Y is a relation (see *Relations*) whose domain is X and whose range is a subset of Y such that for each $x \in X$, there exists a unique $y \in Y$ so that $(x, y) \in f$.

We call the unique $y \in Y$ the *result* of the function *at* the *argument* x. We call Y the *codomain*. If the range is Y we say that f is a function from X *onto* Y (or f is *surjective*). If distinct elements of X are mapped to distinct elements of Y, we say that the function is *one-to-one* (or f is *injective*).

We say that the function *maps* elements from the domain to the codomain. Since the word function and the verb "maps" connote activity, some authors refer to the concept that we have defined as a function as the *graph* of a function—namely, the set of ordered pairs which that function produces—and leave the concept of function undefined.

Notation

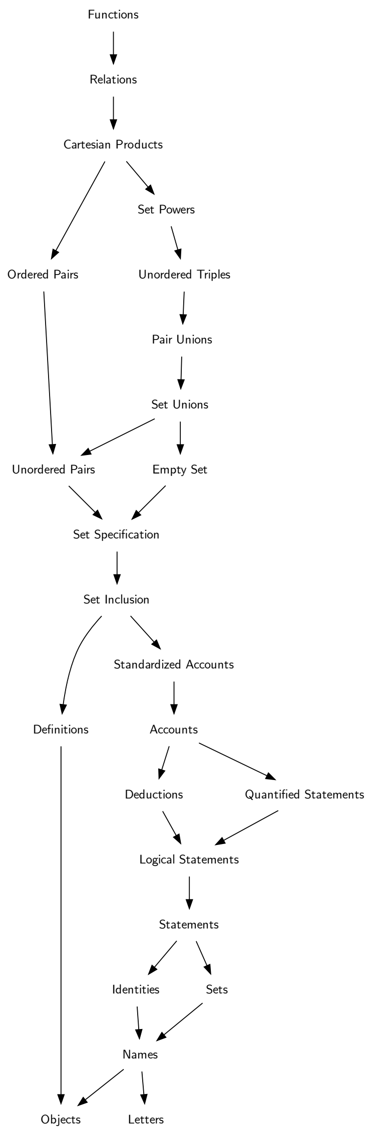
Let X and Y denote sets. We denote a function named f whose domain is X and whose codomain is Y by $f : X \to Y$. We read the notation aloud as "f from X to Y". We denote the set of all functions from X to Y (which is a subset of $(X \times Y)^*$) by Y^X. A less standard but equally good notation is $X \to Y$, read aloud as "A to B". Using the notations introduced so far, we denote that $f \in (A \to B)$ by $f : A \to B$. We tend to denote function by lower case latin letters, especially f, g, and h. f is a mnemonic for function and g and h are nearby.

Let $f : A \to B$. For each element $a \in A$, we denote the result of applying f to a by $f(a)$, read aloud "f of a." We sometimes drop the parentheses, and write the result as f_a, read aloud as "f sub a." Let $g : A \times B \to C$. We often write $g(a, b)$ or g_{ab} instead of $g((a, b))$. We read $g(a, b)$ aloud as "g of a and b". We read g_{ab} aloud as "g sub a b."

Examples

If $X \subset Y$, the function $\{(x, y) \in X \times Y \mid x = y\}$ is the *inclusion function* of X into Y. We often introduce such a function as "the function from X to Y defined by $f(x) = y$". We mean by this that f is a function and that we are specifying the appropraite ordered pairs using the statement, called *argument-value notation.* The inclusion function of X into X is called the *identity function* of X. If we view the identity function as a relation on X, it is the relation of equality on X.

The functions $f : (X \times Y) \to X$ defined by $f(x, y) = x$ is the *pair projection* of $X \times Y$ ono X. Similarly $g : (X \times Y) \to Y$ defined by $g(x, y) = y$ is the pair projection of $X \times Y$ onto Y. The identity function is one-to-one and onto, the inclusion functions are one-to-one but not always onto, and the pair projections are usually not one-to-one.



Functions
Relations
Cartesian Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters