



**Why**

If one rectangle contains another rectangle, the area of the first should be larger than the area of the second. Our definition of integral for simple functions carries this property.

**Result**

**Prop. 1.** *The simple non-negative integral operator is monotone.*

*Proof.* Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $f, g \in \text{SF}_+(X)$  with  $f \leq g$ . Then  $f - g \in \text{SF}_+(X)$ , so

$$\begin{aligned} \int g d\mu &= \int (f + (g - f)) d\mu \\ &\stackrel{(a)}{=} \int f d\mu + \int (g - f) d\mu \\ &\stackrel{(b)}{\geq} \int f d\mu \end{aligned}$$

where (a) follows from linearity and (b) follows from non-negativity; properties of the non-negative simple integral operator.

□

