



Multivariate Normals

1 Why

We generalize the normal density to d -dimensional space.

2 Definition

Let $f : \mathbf{R}^d \rightarrow \mathbf{R}$ be a density such that

$$f(x) = \frac{1}{\sqrt{(2\pi)^d \mathbf{det} \Sigma}} \exp \left(-\frac{1}{2} (x - \mu)^\top \Sigma^{-1} (x - \mu) \right)$$

where $\mu \in \mathbf{R}^d$, $\Sigma \in \mathbf{S}^d$, and $\Sigma \succ 0$. We call f a *multivariate normal density*. A multivariate normal density with $d = 1$ is a normal density, so we refer to multivariate normal densities as *normal densities* without ambiguity. We frequently use the word normal as a substantive, and refer to *normals* when we mean multivariate normal densities. Some people call a multivariate normal distribution a *multivariate gaussian distribution* and speak of *gaussians* instead of normals.

We call μ the *mean* and Σ the *covariance matrix*. We call Σ^{-1} the *precision matrix*.