



REAL NUMBERS

Why

We want a set which corresponds to our notion of points on a line.¹

Definition

First, call a subset R of \mathbf{Q} a *rational cut* if $R \neq \emptyset$, $R \neq \mathbf{Q}$, for all $q \in R$, $r \leq q \longrightarrow r \in R$, and R has no greatest element. Briefly, the intuition is that the point is the set of all rationals to the left.²

The *set of real numbers* is the set of all rational cuts. This set exists by an application of the principle of selection (see *Sect Selection* to the power set (see *Set Powers*) of \mathbf{Q} . We call an element of the set of real numbers a *real number* or a *real*. We call the set of real numbers the *set of reals* or *reals* for short.

Notation

We follow tradition and denote the set of real numbers by \mathbf{R} , likely a mnemonic for “real.”

¹Future editions will modify and expand this justification.

²This brief intuition will be expanded upon in future sheets.

