Why

We want to solve linear equations.

Example

Suppose we want to find $x_1, x_2 \in \mathbf{R}$ to satisfy the system of linear equations

$$3x_1 + 2x_2\& = 10, \text{ and}$$
$$6x_1 + 5x_2\& = 20,$$

with constants $((3,2),10)$ and $((6,4),20)$.

We can associate to the first equation an *equation for x_1 in terms of x_2*. We call this *solving the first equation for x_1*.

$$3x_1 + 2x_2 = 10 \longleftrightarrow 3x_1 = 10 - 2x_2 \longleftrightarrow x_1 = {}^1\!/\!_3(10 - 2x_2).$$

Define $f_1 : \mathbf{R} \to \mathbf{R}$ as $f_1(y) = {}^1\!/\!_3(10 - 2y)$. Then $3x_1 + 2x_2 = 10$ if and only if $x_1 = f_1(x_2)$. We have written x_1 *as a function* of x_2 and obtained a new equation. The equation is not linear, however, as f_1 is not linear.

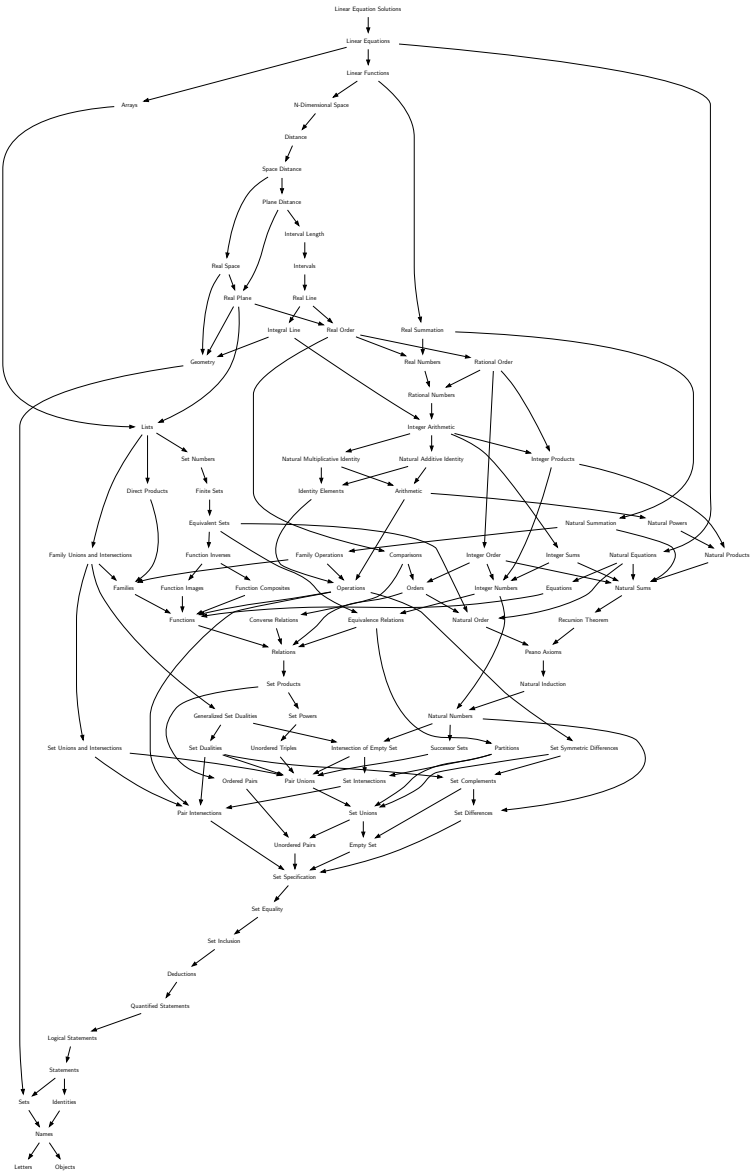
Using the equation for x_1 in terms of x_2, we can substitute this equation into our second linear equation. The two linear equations hold if and only if

$$6f(x_2) + 5x_2 = 20 \longleftrightarrow 20 - 4x_2 + 5x_2 = 20 \longleftrightarrow x_2 = 0.$$

So the equations are satisifed if and only if x_2 is 0. If $x_2 = 0$,then $3x_1 = 10$ and $6x_1 = 20$. Both of these are equivalent to $x_1 = {}^{10}\!/\!_3$. So we have that x_1 must be ${}^{10}\!/\!_3$ and x_2 must be 0.

Clearly this is *a solution*. Is it the only one?[1]

[1] Future editions will expand.


Linear Equation Solutions
Linear Equations
Linear Functions
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
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Integral Line
Real Order
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Arithmetic
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Family Unions and Intersections
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Intersection of Empty Set
Successor Sets
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Set Symmetric Differences
Ordered Pairs
Pair Unions
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Set Complements
Pair Intersections
Set Unions
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Unordered Pairs
Empty Set
Set Specification
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