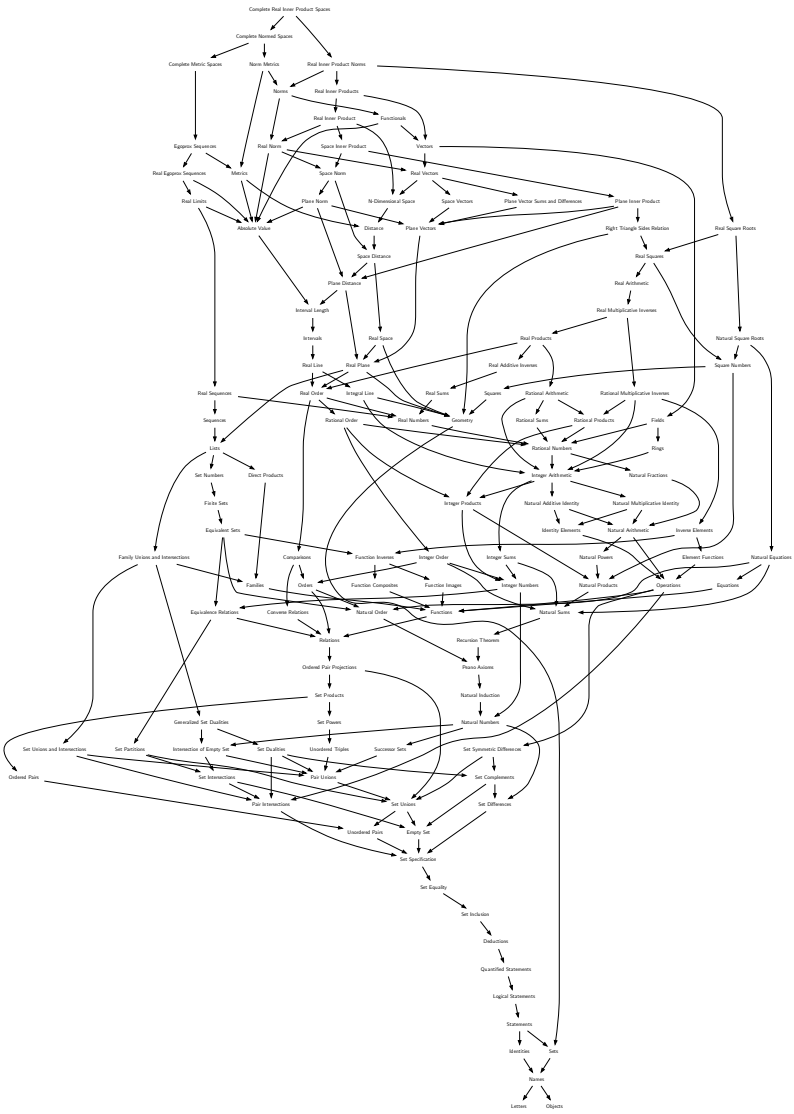
Definition

Let $(X, \mathbf{R})$ be a real vector space. An inner product $\langle \cdot, \cdot \rangle : X \times X \to \mathbf{R}$ induces a norm $\|\cdot\| : X \to \mathbf{R}$ defined by $\|x\| = \sqrt{\langle x, x \rangle}$ and metric $d : X \times X \to \mathbf{R}$ defined by $d(x, y) = \|x - y\|$.

If (X, d) is a complete metric space, we call $((X, \mathbf{R}), \langle \cdot, \cdot \rangle)$ a *complete real inner product space* (or *complete inner product space*—without specifying that it is real— or a *Hilbert space*).[1]

[1] The term Hilbert space is universal, but in accordance with the Bourbaki project's guidelines on naming, we will tend to use the term complete inner product space, even though this is longer.


Complete Real Inner Product Spaces
Complete Normed Spaces
Complete Metric Spaces
Norm Metrics
Real Inner Product Norms
Norms
Real Inner Products
Real Inner Product
Functionals
Epigraphs Sequences
Real Norms
Space Inner Product
Vectors
Real Epigraphs Sequences
Metrics
Space Norm
Real Vectors
Real Limits
Plane Norm
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Plane Inner Product
Absolute Value
Distance
Plane Vectors
Right Triangle Sides Relation
Real Square Roots
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Internal Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Natural Square Roots
Real Line
Real Plane
Real Additive Inverses
Square Numbers
Real Sequences
Real Order
Integral Line
Real Sums
Squares
Rational Arithmetic
Rational Multiplicative Inverses
Sequences
Rational Order
Real Numbers
Geometry
Rational Sums
Rational Products
Fields
Lists
Rational Numbers
Rings
Set Numbers
Direct Products
Integer Arithmetic
Natural Fractions
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplication Identity
Equivalent Sets
Identity Elements
Natural Arithmetic
Inverse Elements
Family Unions and Intersections
Comparisons
Function Inverses
Integer Order
Integer Sums
Natural Powers
Element Functions
Natural Equations
Families
Orders
Function Composites
Function Images
Integer Numbers
Natural Products
Operations
Equations
Equivalence Relations
Converse Relations
Natural Order
Functions
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Qualifiers
Set Powers
Natural Numbers
Set Unions and Intersections
Set Partitions
Intersection of Empty Set
Set Qualifiers
Unordered Triples
Successor Sets
Set Symmetric Differences
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specifications
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects