

ABSOLUTE VALUE

Why

We want a notion of distance between elements of the real line.

Definition

The absolute value of a real number is the greater of itself and its additive inverse. In other words, if x is positive, then the absolute value of x is x. If x is negative, then the absolute value of x is -x (which would be a positive real number).

Notation

We denote the absolute value of a real number $x \in \mathbf{R}$ by |x|.

Distance

The absolute value can be interpreted as the distance between the point corresponding to the real number and the point corresponding to 0. We can generalize this idea. Consider $x, y \in \mathbb{R}$. If x > y, then x - y > 0 and so the distance between the corresponding points is x - y. If x < y then y - x > 0, and so the distance is y - x.

The observation is that |-x| = |x|. So

$$|y - x| = |-(x - y)| = |x - y|.$$

So if we just care about the distance between the points corresponding to y and x, we can consider |x-y|, without regard for their order. In other words, the function $(x,y) \mapsto |x-y|$ is symmetric in x and y.

