



**Why**

Suppose we flip a coin until it lands heads. What is the probability that we see a head after one flip? Two flips?  $n$  flips? We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

**Discussion**

Consider a set  $A$ .<sup>1</sup> If  $A$  has  $n$  elements, then a probability distribution on  $A$  is  $p : A \rightarrow \mathbf{R}$  where  $p(a) = 1/n$ . There is a natural candidate.

What if  $A$  is the set of natural numbers  $\mathbf{N}$ . The principle difficulty is that not all sequences of real numbers  $a : \mathbf{N} \rightarrow \mathbf{R}$  are summable.

**Definition**

A (*countable*) *probability distribution* on  $\mathbf{N}$  is  $p : \mathbf{N} \rightarrow \mathbf{R}$  where  $p \geq 0$  and

$$\sum_{n=1}^{\infty} p(n) = 1.$$

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<sup>1</sup>Future editions will rework this sheet.



