Signed Set Decomposition

1 Why

Given a signed measure, can we split the base set into two sets, one with positive measure and one with negative measure?

2 Definition

By ”positive” and ”negative” we mean ”non-negative” and ”non-positive.” Let $(X, \mathcal{A})$ be a measurable space. Let $\mu : \mathcal{A} \to [-\infty, \infty]$ be a signed measure.

A **positive set** is a measurable set with the property that each of its subsets have non-negative measure under μ. A **negative set** is a measurable set with the property that each of its subsets have non-positive measure under μ. A **signed-set decomposition** of X under μ is a partition of X into a positive and a negative set.

2.1 Existence

Proposition 1. *Let $(X, \mathcal{A})$ be a measurable space. Let $\mu : \mathcal{A} \to [-\infty, \infty]$ be a signed measure. There exists a signed-set decom-*

position of X under μ.

Proof. TODO $\square$

2.2 Uniqueness

2.3 Notation

We usually denote a positive set by P and a negative set by N. When we say "let (P, N) be a signed-set decomposition of X under μ", we mean that P is the positive set and N is the negative set.