

## NEURAL NETWORKS

## Why

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## **Definition**

A sequence of functions  $(g_1, \ldots, g_\ell)$  is *composable* if  $g_i$  is composible with  $g_{i-1}$  for  $i = 2, \ldots, \ell$ . In this case we write  $g_\ell \circ g_{\ell-1} \circ \cdots \circ g_2 \circ g_1$ . For example, we write  $g_3 \circ g_2 \circ g_1$  for  $(g_1, g_2, g_3)$ .

A neural network (or feedforward neural network) from  $\mathbb{R}^n$  to  $\mathbb{R}^m$  is a sequence of composable functions  $(g_1, \ldots, g_\ell)$ , dom  $g_1 = \mathbb{R}^n$ , ran  $g_\ell \subset \mathbb{R}^m$ , satisfying

$$g_i(\xi) = h_i(A_i\xi + b_i)$$

for some conforming matrices  $A_i$ , vectors  $b_i$  and functions  $h_i$ .

The *i*th layer of the neural network is the *i*th function  $g_i$ . The *i*th activation of the neural network is the function  $h_i$ . A neural network is called deep if its number of layers is larger than 3.

We call the composition of the layers of the neural network the *network predictor* (or just *predictor*). We also call it *the* function of the network.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Future editions will include. Future editions may change the name of this sheet to *computation networks*, or may add prerequisite sheet on computation graphs.

<sup>&</sup>lt;sup>2</sup>Many authorities refer to a neural network as a function. Strictly speaking that is true for us, as well, since a sequence is a function. But the meaning of the common use is in reference to the *network predictor*.

A multi-layer perceptron (MLP) is a neural network with 2 layers  $(1 \ hidden \ layer)$  and for which  $A_i$  and  $b_i$  have unrestricted nonzero entries.

