Why

[1]

Definition

Let $(A, +, \cdot)$ be a ring.

A *polynomial* in A of *degree* d is a function $p : A \to A$ for which there exists a finite sequence $c = (c_0, c_1, \dots, c_{d-1}, c_d) \in A^{d+1}$ satisfying

$$p(a) = c_0 + c_1 a^1 + c_2 a^2 + \cdots + c_d a^d,$$

for all $a \in A$. We call the sequence c the *polynomial coefficients*, and call the c_i the *coefficients* of p. We call $d + 1$ the *order* of the polynomial.

Clearly, to every polynomial in A of degree d there corresponds a sequence in A of length $d+1$, and vice versa. For this reason, we can identify polynomials by their coefficients.

Examples

The function $f : A \to A$ is a polynomial of degree 0 and order 1 if there exists c_0 so that

$$f(a) = c_0$$

for all $a \in A$.

The function $g : A \to A$ is a polynomial of degree 1 and order 2 if there exists c_0 and c_1 so that

$$g(a) = c_0 + c_1 a$$

[1] Future editions will include, and most likely will build on quadratics and an appeal to the simplicity of the "natural" algebraic operations.

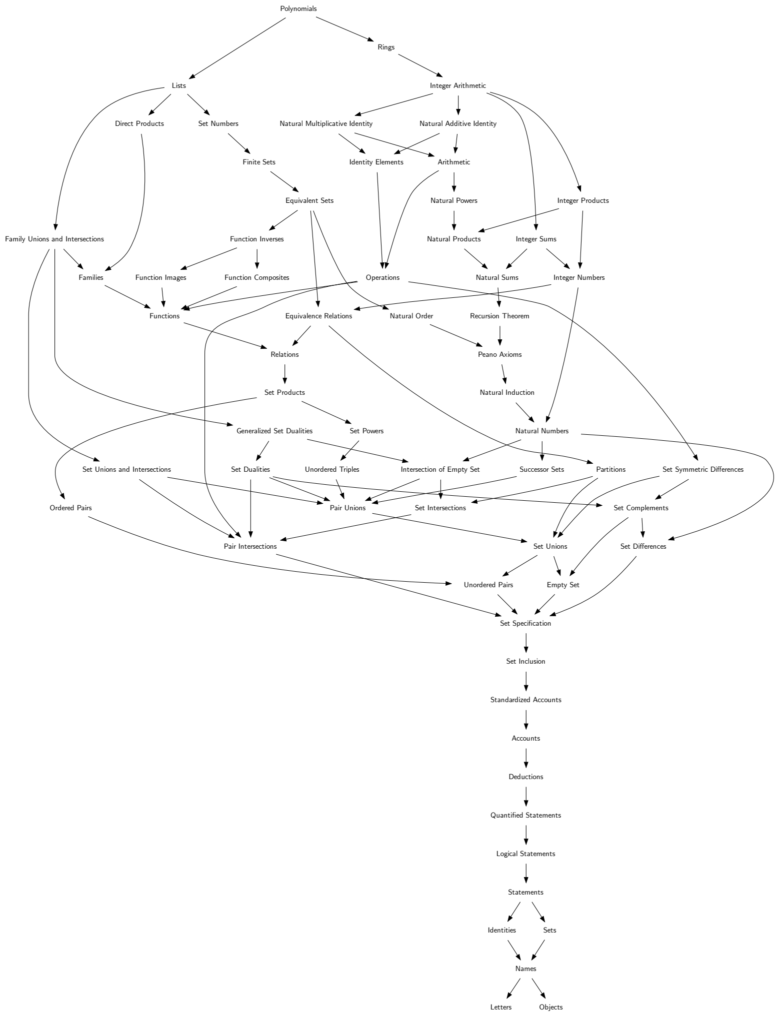
The function $h : A \to A$ is a polynomial of degree 2 and order 3 if there exists c_0 and c_1 so that

$$h(a) = c_0 + c_1 a + c_2 a^2.$$

In other words, a second degree polynomial is a quadratic.

The function $p : A \to A$ is a *polynomial* of degree d and order $d + 1$ if there exists a $d + 1$ length sequence $(c_0, c_1, \ldots, c_d)$ in A so that

$$p(a) = c_0 + c_1 a + \cdots + c_d a^d.$$


Polynomials
Rings
Lists
Integer Arithmetic
Direct Products
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Natural Powers
Integer Products
Family Unions and Intersections
Function Inverses
Natural Products
Integer Sums
Families
Function Images
Function Composites
Operations
Natural Sums
Integer Numbers
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects