Total Orders

1 Why

Often we want all elements of the set A to be comparable.

2 Definition

We call R **connexive** if for all $a, b \in A$, $(a, b) \in R$ or $(b, a) \in R$. If R is a partial order and connexive, we call it a **total order**.

A **totally ordered set** is a set together with a total order. The language is a faithful guide: we can compare any two elements. Still, we prefer one word to three, and so we will use the shorter term **chain** for a totally ordered set; other terms include **simply ordered set** and **linearly ordered set**.

Let $C = (A, R)$ be a chain. A **minimal element** of C is an element which precedes all other elements. A **maximial element** of C is an element which is preceded by all other elements.