Why

It is common to consider random functions whose domain is time, space, or n-dimensional space.

Definition

Let (X, d) be a metric space. A *distance covariance function* $k : X \times X \to \mathbf{R}$ is a covariance function satisfying

$$k(x, y) > k(x, y) \longleftrightarrow d(x, y) < d(x, y).$$

In other words, the covariance decreases as the distance between the arguments decreases.

Example: Squared Exponential

Let $k : X \times X \to \mathbf{R}$ be defined by

$$k(x, y) = \exp(-d(x, y)).$$

Then k is a distance covariance function. It is often called the *squared exponential covariance function.*

Let $\alpha, \sigma \in \mathbf{R}$. Define $k' : X \times X \to \mathbf{R}$ by

$$k'(x, y) = \alpha \exp(-d(x, y)/\sigma^2)$$

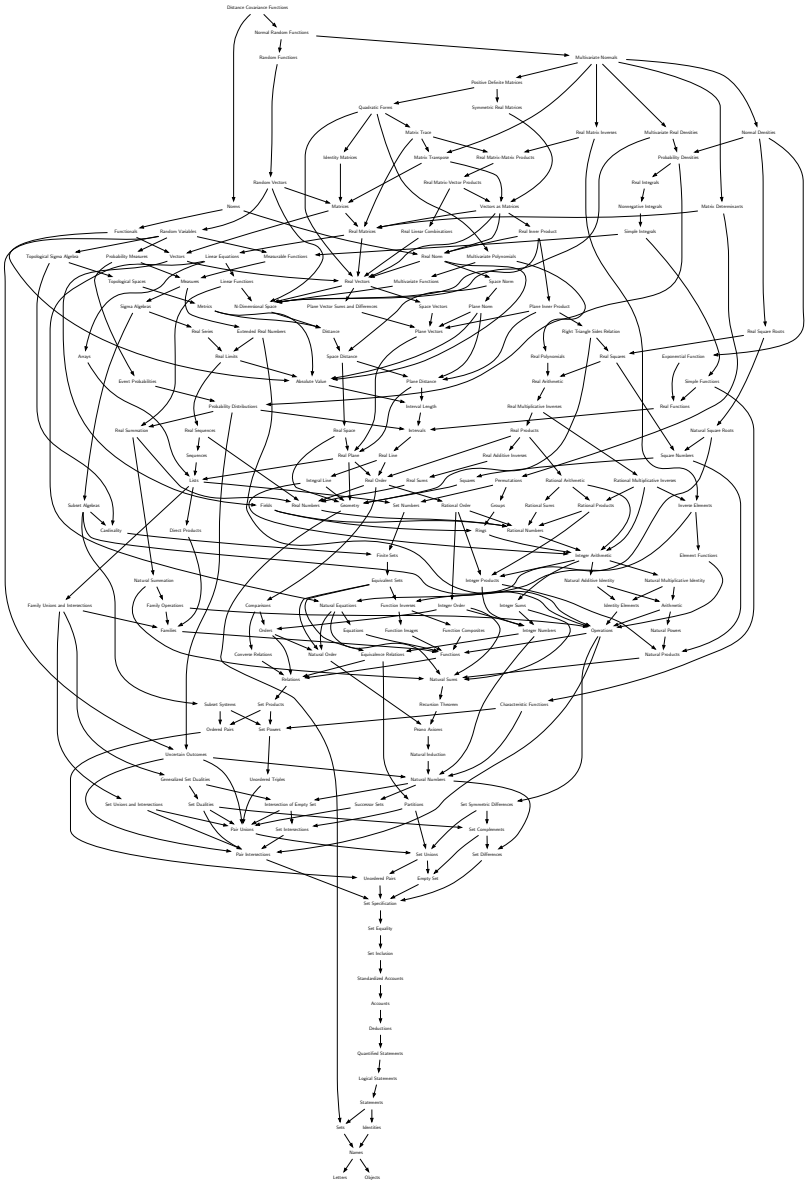
then k' is still a covariance function. In this context σ is often referred to as the *characteristic length-scale* of the process. The scalar α is sometimes called a "prefactor" that "controls" the "overall variance."

Suppose $(X, d) = (\mathbf{R}^n, \|\cdot\|)$. Then the squared exponential covariance function

$$\alpha \exp(-\|x - y\|/(2\sigma^2))$$

is sometimes called the *radial basis function* or *gaussian covariance function.*[1] Also called an *exponentiated quadratic kernel.*

[1] For reasons that will be included in future editions.


Distance Covariance Functions
Normal Random Functions
Random Functions
Multivariate Normals
Positive Definite Matrices
Symmetric Real Matrices
Quadratic Forms
Matrix Trace
Real Matrix Inverses
Multivariate Real Densities
Normal Densities
Identity Matrices
Matrix Transpose
Real Matrix-Matrix Products
Probability Densities
Real Matrix-Vector Products
Real Integrals
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Vectors as Matrices
Nonnegative Integrals
Matrix Determinants
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Topological Spaces
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Vector Norms
Signed Algebra
Metrics
N-Dimensional Spaces
Plane Vector Sums and Differences
Square Vectors
Plane Sums
Plane Inner Product
Real Series
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Distance
Plane Vectors
Right Triangle Sides Relation
Real Square Roots
Arrays
Real Limits
Space Distance
Real Polynomials
Real Squares
Exponential Function
Event Probabilities
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Real Arithmetic
Simple Functions
Probability Distributions
Interval Length
Real Multiplicative Inverses
Real Functions
Real Summations
Real Sequences
Real Spans
Intervals
Real Products
Natural Square Roots
Sequences
Real Plane
Real Line
Real Additive Inverses
Square Numbers
Lists
Integral Line
Real Order
Real Series
Squares
Permutations
Rational Arithmetic
Rational Multiplication Inverses
Subset Algebra
Cardinality
Real Numbers
Geometry
Set Numbers
Rational Order
Groups
Rational Sums
Rational Products
Inverse Elements
Direct Products
Real
Rational Numbers
Finite Sets
Integer Arithmetic
Elements Functions
Natural Summation
Equivalence Sets
Integer Products
Natural Addition Identity
Natural Multiplicative Identity
Family Unions and Intersections
Family Operations
Comparisons
Natural Equations
Function Images
Integer Order
Integer Sums
Identity Elements
Arithmetic
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Function Composition
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Natural Powers
Concrete Relations
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Equivalence Relations
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Relations
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Subset Systems
Set Products
Recursion Theorems
Characteristic Functions
Ordered Pairs
Set Powers
Peano Axioms
Uncertain Outcomes
Natural Induction
Unordered Triples
Natural Numbers
Generalized Set Duality
Set Unions and Intersections
Set Duality
Intersection of Empty Sets
Successor Sets
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Arguments
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects