Why

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Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\mathsf{cov}(f, g)$. We defined it:
$$\mathsf{cov}(f, g) = \mathsf{E}(fg) - \mathsf{E}(f)\,\mathsf{E}(g).$$

Properties

Prop. 1. *Covariance is symmetric and billinear.*[2]

Prop. 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then
$$\mathsf{cov}(f, f) = \mathsf{E}(ff) - \mathsf{E}(f)\,\mathsf{E}(f) = \mathsf{E}(f^2) - (\mathsf{E}(f))^2 = \mathsf{var}(f).$$

□

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[1]Future editions will include this.

[2]Future editions will include an account.

Prop. **3.** *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\mathsf{var}\left(\sum_{i=1}^{n} f_i\right) = \mathsf{cov}\left(\sum_{i=1}^{n} f_i, \sum_{i=1}^{n} f_i\right)$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \mathsf{cov}(f_i, f_j)$$

$\square$

Covariance

Variance

Expectation

Random Variables

Measurable Functions

Topological Sigma Algebras

Outcome Variables

Topological Spaces

Metrics

Distance

Absolute Value

Space Distance

Probability Measure

Plane Distance

Event Probabilities

Measure

Interval Length

Probability Distributions

Extended Real Numbers

Uncertain Outcomes

Intervals

Real Limits

Real Plane

Real Space

Real Line

Real Order

Integral Line

Real Sequence

Real Summation

Geometry

Sequences

Real Numbers

Set Numbers

Direct Products

Rational Numbers

Finite Sets

Natural Summation

Integer Arithmetic

Sigma Algebras

Family Operations

Natural Multiplication Identity

Natural Additive Identity

Subset Algebras

Equivalent Sets

Algebras

Arithmetic

Identity Elements

Set Operations

Cardinality

Family Unions and Intersections

Natural Exponents

Integer Products

Integer Sums

Function Inverses

Operations

Natural Products

Integer Numbers

Families

Function Image

Function Composites

Equivalence Relations

Natural Sums

Functions

Natural Order

Recursion Theorem

Relations

Peano Axioms

Cartesian Products

Subset Systems

Natural Induction

Set Powers

Ordered Pairs

Generalized Set Dualities

Unordered Triples

Natural Numbers

Set Unions and Intersections

Partitions

Intersection of Empty Set

Set Dualities

Successor Sets

Set Symmetric Differences

Set Intersections

Pair Unions

Set Complements

Pair Intersections

Set Unions

Set Differences

Empty Set

Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects