Functions

1 Why

We want a notion for a correspondence between two sets.

2 Definition

A *functional* relation on two sets relates each element of the first set with a unique element of the second set. A *function* is a functional relation.

The *domain* of the function is the first set and *codomain* of the function is the second set. The function *maps* elements *from* the domain *to* the codomain. We call the codomain element associated with the domain element the *result* of *applying* the function to the domain element.

2.1 Notation

Let A and B be sets. If A is the domain and B the codomain, we denote the set of functions from A to B by $A \to B$, read aloud as “A to B".

We denote functions by lower case latin letters, especially f, g, and h. The letter f is a mnemonic for function; g and h follow f in the Latin alphabet. We denote that $f \in (A \to B)$ by $f : A \to B$, read aloud as “f from A to B".

Let $f : A \to B$. For each element $a \in A$, we denote the result of applying f to a by $f(a)$, read aloud “f of a." We sometimes drop the parentheses, and write the result as f_a, read aloud as “f sub a."

Let $g : A \times B \to C$. We often write $g(a, b)$ or g_{ab} instead of $g((a, b))$. We read $g(a, b)$ aloud as “g of a and b". We read g_{ab} aloud as “g sub a b."