



## REAL INNER PRODUCT

### Why

We want to measure angles in space.<sup>1</sup>

### Definition

The *real inner product* (or *dot product*, *scalar product*) of two real vectors  $x, y \in \mathbf{R}^n$  is

$$x_1y_1 + x_2y_2 + \cdots + x_ny_n$$

We denote the inner product of  $x$  and  $y$  by  $\langle x, y \rangle$ .

### Properties

The inner product has several important properties

1.  $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$
2.  $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$
3.  $\langle x, y \rangle = \langle y, x \rangle$
4.  $\langle x, x \rangle \geq 0$
5.  $\langle x, x \rangle = 0 \iff x = 0$

### Connection to norm

It is important to note that  $\|x\| = \sqrt{\langle x, x \rangle}$ .

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<sup>1</sup>Future editions will expand, and perhaps give the development for  $\mathbf{R}^2$  first. Future editions will include pictures.



