Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

Notation

Let $(X, \mathcal{A}, \mathbf{P})$ be a probability space. Let $(Y, \mathcal{B})$ a measurable space. A random variable is a measurable function $f : X \to Y$.

Some authors denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. In this case, the base probability space is denoted by Ω, a mnemonic for "outcomes." Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $X : \Omega \to \mathbf{R}$ be measurable. Then X is a real-valued random variable.

Some authors use notation for the probability of particular, common sets. Although the authors often use regular parentheses, we will use "[" and "]" for precision. Let $A \in \mathcal{B}(\mathbf{R})$. Let $\mathbf{P}[X \in A]$ denote $\mathbf{P}(X^{-1}(A))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

As we mentioned, some authors use $P(X \in A)$ for $P[X \in A]$, which is mostly harmless.

Next, let $Y : \Omega \to \mathbf{R}$ a measurable function and let $B \in \mathcal{B}(\mathbf{R})$. Similar to the above, let $\mathbf{P}[X \in A, Y \in B]$ denote $\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \ldots, X_n : \Omega \to \mathbf{R}$,

$$\mathbf{P}[X_1 \in A_1, \ldots, X_n \in A_n] = \mathbf{P}(\cap_{i=1}^{n} X_i^{-1}(A_i)).$$

Random Variables

Measurable Functions · Probability Measures · Topological Sigma Algebras

Measures · Event Probabilities · Topologies

Sigma Algebras · Metrics

Extended Real Numbers · Real Series · Distance

Real Limits · Space Distance

Absolute Value · Plane Distance

Interval Length · Probability Distributions

Intervals · Real Space

Real Line · Real Plane

Real Summation · Real Sequences · Real Order · Integral Line · Geometry

Subset Algebra · Real Numbers · Sequences · Rational Order

Rational Numbers · Lists · Cardinality

Integer Arithmetic · Direct Products · Set Numbers

Natural Additive Identity · Natural Multiplicative Identity · Finite Sets · Integer Products

Natural Summation · Identity Elements · Arithmetic · Equivalent Sets

Family Operations · Family Unions and Intersections · Natural Powers · Function Inverse · Integer Sums · Integer Order · Comparisons

Operations · Families · Natural Products · Function Image · Function Composites · Integer Numbers · Orders

Functions · Natural Sums · Equivalence Relations · Converse Relations · Natural Order

Recursion Theorem · Relations

Subset Systems · Set Products · Peano Axioms

Ordered Pairs · Set Powers · Natural Induction · Uncertain Outcomes

Generalized Set Dualities · Unordered Triples · Natural Numbers

Set Symmetric Differences · Set Dualities · Set Unions and Intersections · Successor Sets · Intersection of Empty Set · Partitions

Set Complements · Pair Unions · Set Intersections

Pair Intersections · Set Unions · Set Differences

Empty Set · Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities · Sets

Names

Letters · Objects