



# Multivariate Normal Conditionals

## 1 Why

What of the conditional densities of a multivariate normal density.

## 2 Result

**Proposition 1.** *Let  $f : \mathbf{R}^d \rightarrow \mathbf{R}$  be a normal density with mean  $\mu \in \mathbf{R}^d$  and covariance  $\Sigma \in \mathbb{S}^d$ .*

$$\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$$

*Then the conditional density  $f_{x|y}(\xi, \gamma)$  is  $\mathcal{N}(\bar{\mu}(\gamma), \bar{\Sigma})$  with*

$$\bar{\mu}(\gamma) = \mu_x + \Sigma_{xy} \Sigma_{yy}^{-1} (\gamma - \mu_y) \quad \text{and} \quad \bar{\Sigma} = \Sigma_{xx} - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx}.$$