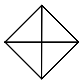


Why

Many problems simplify if the graph involved is chordal.[1]

Paths

Let G be an undirected graph. A *chord* in a path p of G is an edge between two non-consecutive vertices of p. So a chord of the path $(v_1, v_1, \ldots, v_k)$ is an edge $\{v_i, v_j\}$ with $|j - i| > 1$.

We interpret a chord as a "one-edge shortcut" between two vertices of a path. If a path p has a chord, it can be reduced to a shorter path p' skipping some vertices. In other words, the shortest path between two vertices is chordless. However, a chordless path need not be a shortest path. See Figure 1.

Graphs

A chord of a cycle $(v_1, v_2, \ldots, v_{k-1}, v_1)$ is an edge $\{v_i, v_j\}$ with $(j - i) \mod k > 1$. An undirected graph G is *chordal* if every cycle with more than three edges has a chord.

If G is chordal, every cycle in G can be reduced to a cycle of length three. We sometimes call a cycle of length three a *triangle*. For this reason, chordal graphs are also sometimes called *triangulated graphs*. Other terminology includes *rigid-circuit graphs*, *triangulated graphs*, *perfect elimination graphs*, *decomposable graphs*.[2]

[1] Future editions will expand.

[2] See Vanenberghe and Anderson, 2014.

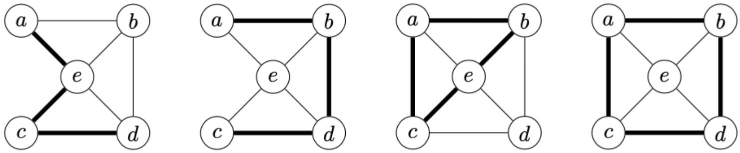
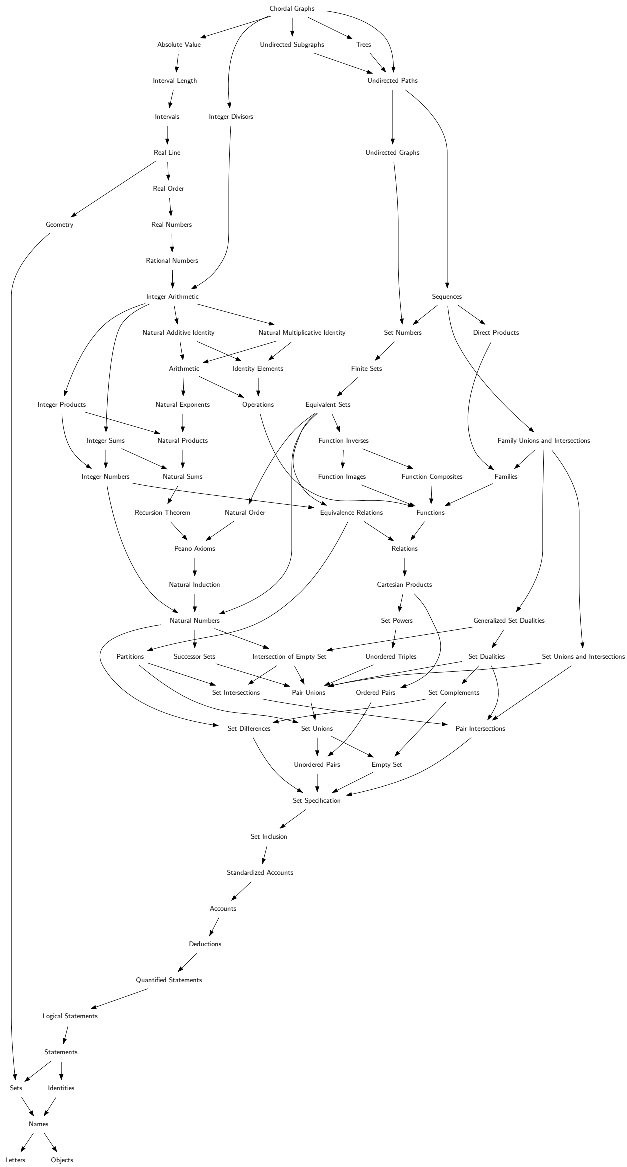


Figure 1: The edge $\{e, d\}$ is a chord in the path (a, e, c, d) of the first graph. The path (a, v, d, c) is chordless. The edge $\{a, e\}$ is a chord in the cycle (a, v, e, c, a) of the second graph. The cycle (a, b, d, c, a) is chordless.

The last graph in Figure 1 is not chordal because the cycle (a, b, d, c, a) has length four and no chord. Adding the edge $\{b, c\}$ or $\{a, d\}$ would make the graph chordal An immediate consequence of the definition that G be chordal is that any subgraph of G is chordal.

Simple Examples

Since trees and forests have no cycles, they are chordal. Similarly, any graph with no cycles longer than three edges are trivially chordal. Such graphs are sometimes called *cactus graphs*. The complete graphs are also trivially chordal.


Chordal Graphs
Absolute Value
Undirected Subgraphs
Trees
Interval Length
Undirected Paths
Intervals
Integer Divisors
Real Line
Undirected Graphs
Real Order
Geometry
Real Numbers
Rational Numbers
Integer Arithmetic
Sequences
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Finite Sets
Integer Products
Natural Exponents
Operations
Equivalent Sets
Integer Sums
Natural Products
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Function Images
Function Composites
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects