Why

[1]

Definition

Let $(A, +, \cdot)$ be a ring.

A *polynomial* in A of *degree* d is a function $p : A \to A$ for which there exists a finite sequence $c = (c_0, c_1, \ldots, c_{d-1}, c_d) \in A^{d+1}$ satisfying

$$p(a) = c_0 + c_1 a^1 + c_2 a^2 + \cdots + c_d a^d,$$

for all $a \in A$. We call the sequence c the *polynomial coefficients*, and call the c_i the *coefficients* of p. We call $d + 1$ the *order* of the polynomial.

Clearly, to every polynomial in A of degree d there corresponds a sequence in A of length $d + 1$, and vice versa. For this reason, we can identify polynomials by their coefficients.

Examples

The function $f : A \to A$ is a polynomial of degree 0 and order 1 if there exists c_0 so that

$$f(a) = c_0$$

for all $a \in A$.

The function $g : A \to A$ is a polynomial of degree 1 and order 2 if there exists c_0 and c_1 so that

$$g(a) = c_0 + c_1 a$$

The function $h : A \to A$ is a polynomial of degree 2 and order 3 if there exists c_0 and c_1 so that

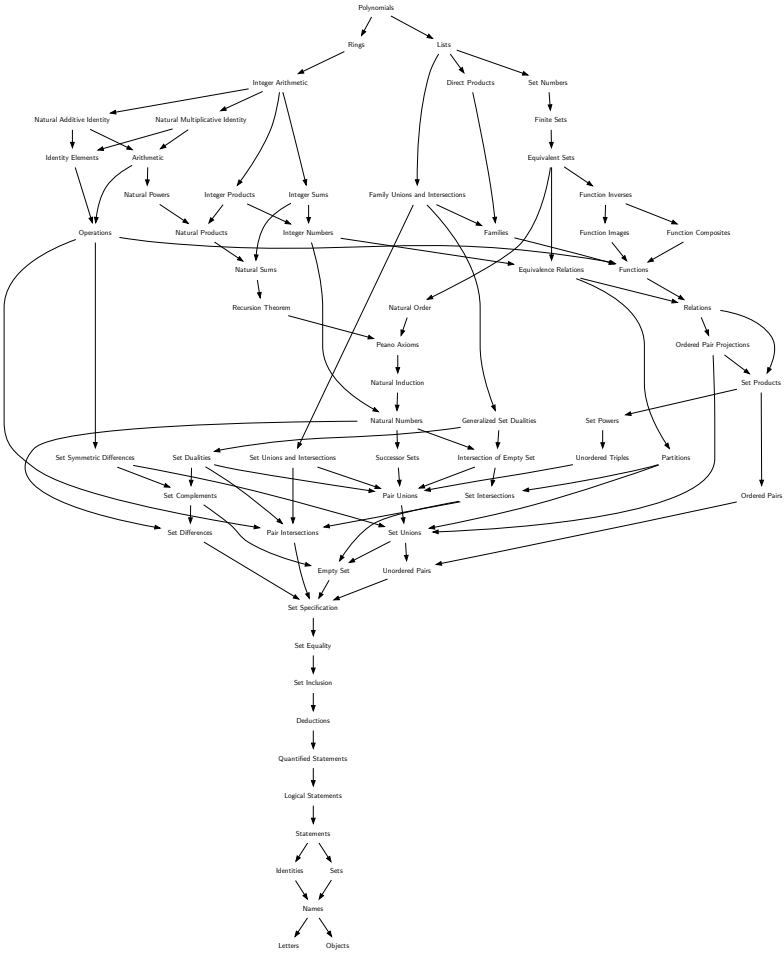
$$h(a) = c_0 + c_1 a + c_2 a^2.$$

[1] Future editions will include, and most likely will build on quadratics and an appeal to the simplicity of the "natural" algebraic operations.

In other words, a second degree polynomial is a quadratic.

The function $p : A \to A$ is a *polynomial* of degree d and order $d+1$ if there exists a $d+1$ length sequence $(c_0, c_1, \ldots, c_d)$ in A so that

$$p(a) = c_0 + c_1 a + \cdots + c_d a^d.$$


Polynomials
Rings
Lists
Integer Arithmetic
Direct Products
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Natural Powers
Integer Products
Integer Sums
Family Unions and Intersections
Function Inverses
Operations
Natural Products
Integer Numbers
Families
Function Images
Function Composites
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects