



## Finite Measures

### 1 Why

Sometimes we want finite measures. TODO: which times?

### 2 Definition

A measurable set is *finite* if its measure is a real number. The measure space itself is *finite* if the base set is finite.

A measurable set is *sigma-finite* if there exists a sequence of finite measurable sets whose union is the set. The measure space itself is *sigma-finite* if the base set is sigma finite.

#### 2.1 Notation

We denote that a measure space is finite by saying "Let  $(A, \mathcal{A}, \mu)$  and  $\mu(A) < +\infty$ ."

**Example 1.** Let  $(A, \mathcal{A})$  be a measurable space.

*The counting measure on  $(A, \mathcal{A})$  is finite if and only if the base set is finite. It is sigma finite if and only if the base set is a union of a sequence of finite sets.*

*If  $\mathcal{A} = 2^A$ , then the counting measure is sigma finite if and only if  $A$  is countable.*

**Example 2.** A point mass measure is finite.

**Example 3.** *Let  $R$  be the set of real numbers. The Lebesgue measure on  $(R, \mathcal{B}(R))$  is sigma finite.*