## Why

We generalize the product of two sets to a product of a family of sets. To do so we discuss sets of families.

## Discussion for pairs

Let $A$ and $B$ be sets. There is a natural correspondence between the product set $A \times B$ (see Cartesian Products) and the set of families

$$Z = \{z : \{i, j\} \to (A \cup B) \mid z_i \in A \text{ and } z_b \in B\}.$$

The family $z \in Z$ corresponds with the pair $(z_i, z_i)$. The pair $(a, b)$ corresponds to the family $z \in Z$ defined by $z(i) = a$ and $z(j) = b$. So, ordered pairs can be put in one-to-one correspondence with families. The generalization of Cartesian products to more than two sets generalizes the notion for families.

## Definition

Let $X$ be a set. Let $A : I \to X$ be a family of subsets of $X$. The *direct product* or *family Cartesian product* of $A$ is the set of all families $a : I \to X$ which satisfy $a_i \in A_i$ for every $i \in I$.

A function on a product is called a *function of several variables* and, in particular, a function on the product $X \times Y$ is called a *function of two variables.*

### Notation

We denote the product of the family $\{A_i\}$ by

$$\prod_{i \in I} A_i$$

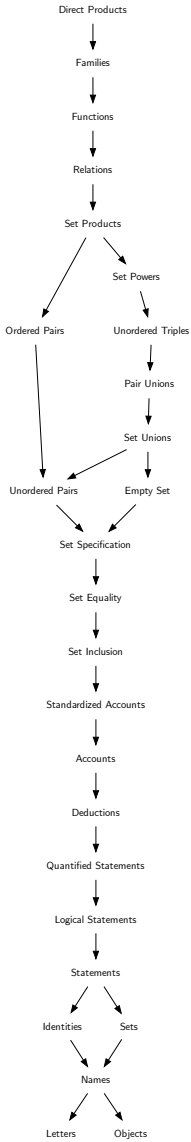
We read this notation as "product over i in I of A sub-i."

## Projections

The word "projection" is used in two senses with families. Let $I$ be a set, and let $\{A_i\}_{i \in I}$ be a family of sets. Define $A = \prod_{i \in I} A_i$.

First, let $J \subset I$. There is a natural correspondence between the elements of $A$ and those of $\prod_{j \in J} A_j$. To each element $a \in A$, we restrict $a$ to $J$ and this is restriction is an element of $\prod_{j \in J} A_j$. The correspondence is called the *projection* of $A$ onto $\prod_{i \in J} A_i$. The projection in this sense is a set of families.

Second, consider the value of a family $a \in A$ at $j$. We call $a_j$ the *projection of $a$ onto index $j$* or the *$j$-coordinate* of $a$. This word *coordinate* is meant to follow the language used in defining ordered pairs. The projection in this sense is an element of $A_j$. The $j$th projection is a function from $\prod_{i \in I} X_i \to X_j$.


Direct Products
Families
Functions
Relations
Set Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects