Why

We want to model natural phenomena.[1]

Definition

Let $X_0, X_1, \ldots, X_T$ be a sequence of sets and let $f_t : X_t \to X_{t+1}$ for $t = 0, \ldots, T - 1$. We call $((X_0, \ldots, X_T), (f_1, \ldots, f_{T-1}))$ a *deterministic discrete-time dynamical system*.

We call the index t the *epoch*, the *stage* or the *period*. We call X_t the *state space* at period t. We call f_t the *transition function* or *dynamics function*.

Let $x_0 \in \mathcal{X}_0$. Define a state sequence $x_1 \in \mathcal{X}_1, \ldots, x_T \in \mathcal{X}_T$ by

$$x_{t+1} = f_t(x_t, u_t).$$

In this case we call x_0 the *initial state*. We call the x_t the *trajectory* associatd with initial state x_0.

We call T the *horizon*. In the case that we have an infinite sequence of state sets, input sets, and dynamics, then we refer to a *infinite-horizon* dynamical system. To use language in contrast with this case, we refer to the dynamical system when T is finite as a *finite-horizon* dynamical system.

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[1]Future editions will modify, and may develop dynamic systems via the genetic approach by appealing to their classical use in Newtonian physics for modeling celestial mechanics.

Dynamical Systems

Real Numbers

Rational Numbers

Integer Arithmetic

Natural Additive Identity

Natural Multiplicative Identity

Identity Elements

Arithmetic

Natural Powers

Integer Products

Natural Products

Integer Sums

Operations

Natural Sums

Integer Numbers

Functions

Equivalence Relations

Recursion Theorem

Relations

Peano Axioms

Set Products

Natural Induction

Ordered Pairs

Set Powers

Natural Numbers

Partitions

Intersection of Empty Set

Unordered Triples

Successor Sets

Set Intersections

Pair Unions

Set Symmetric Differences

Pair Intersections

Set Unions

Set Complements

Unordered Pairs

Empty Set

Set Differences

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects