Why

We want to talk about how a function of multiple real-valued arguments changes with respect to changes in its arguments.[1]

Definition

Let $f : \mathbf{R}^d \to \mathbf{R}$ For $i = 1, \ldots, d$, the *ith partial derivative* of f is the function $g_i : \mathbf{R}^d \to \mathbf{R}$ defined by

$$g_i(x) = \lim_{h \to 0} \frac{f(x + he_i) - f(x)}{h}$$

for each $x \in \mathbf{R}^d$.

Gradient

The *gradient* of a multivariate function is the vector-valued function whose ith component is the the partial derivative of the function with respect to its ith argument.

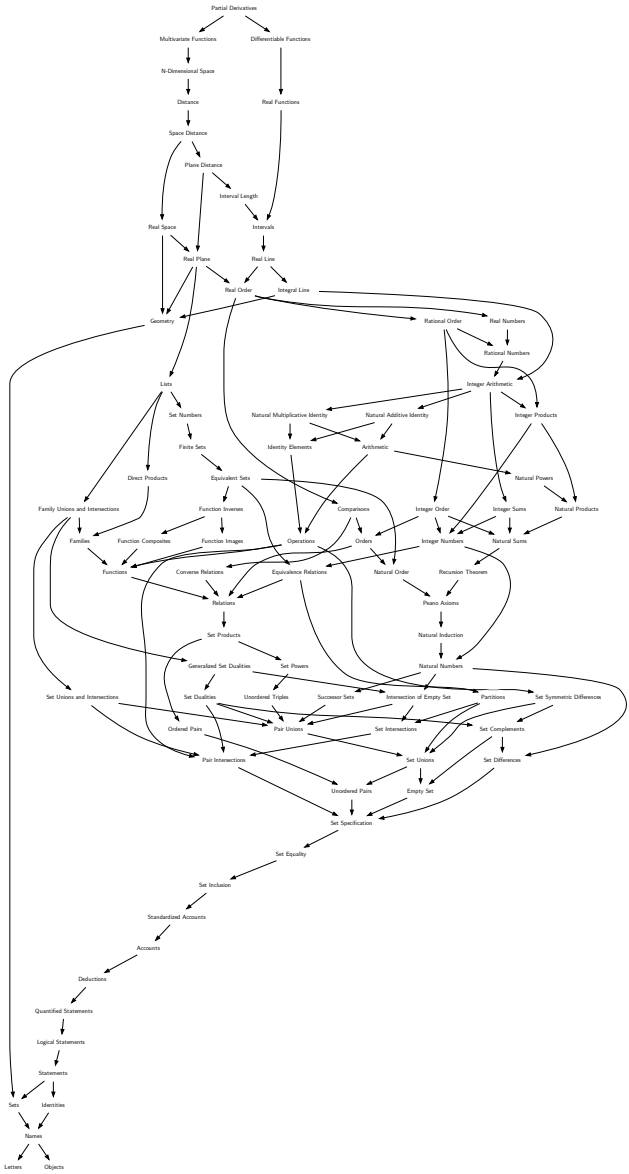
Notation

Let $f : \mathbf{R}^n \to \mathbf{R}$. The gradient of f is frequently denoted ∇f. It is understood that $(\nabla f) \in \mathbf{R}^d \to \mathbf{R}^d$. An alternative notation is to use that similar for single derivatives and to denote the gradient (sometimes called derivative) of f by f' (assuming it exists). It is important to here note that although when $g : \mathbf{R} \to \mathbf{R}$, $g' \in (\mathbf{R} \to \mathbf{R})$, (and so is another function from and to reals) when $f : \mathbf{R}^d \to \mathbf{R}$, $f' \in \mathbf{R}^d \to \mathbf{R}^d$, and so is a vector-valued (not a real-valued) function.

There is (unfortunately) much notation for the individual partial derivatives; most of which we shall not (fortunately) have occasion to use in these sheets. One popular usage is the use of the ∂ symbol, read aloud as "partial." For example, if $f : \mathbf{R}^2 \to \mathbf{R}$ is a function of two arguments, each being referred to as x and y, then $\partial_x f$ denotes the partial derivative

[1] Future editions will modify this sheet.

of f with respect to x and $\partial_y f$ denotes the partial derivative of f with respect to y. It is understood that $(\partial_x f) \in \mathbf{R}^d \to \mathbf{R}$. and likewise for $\partial_y f$. Another popular usage is $\partial f / \partial x$ for $\partial_x f$ and $\partial f / \partial y$ for $\partial_y f$. We will almost exclusively prefer the gradient notation.


Partial Derivatives
Multivariate Functions
Differentiable Functions
N-Dimensional Space
Distance
Real Functions
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
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Natural Additive Identity
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Arithmetic
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Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
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