Why

We define integrals using an infinite process; in order for each step of the process to make sense, we need functions to be measurable.[1]

Definition

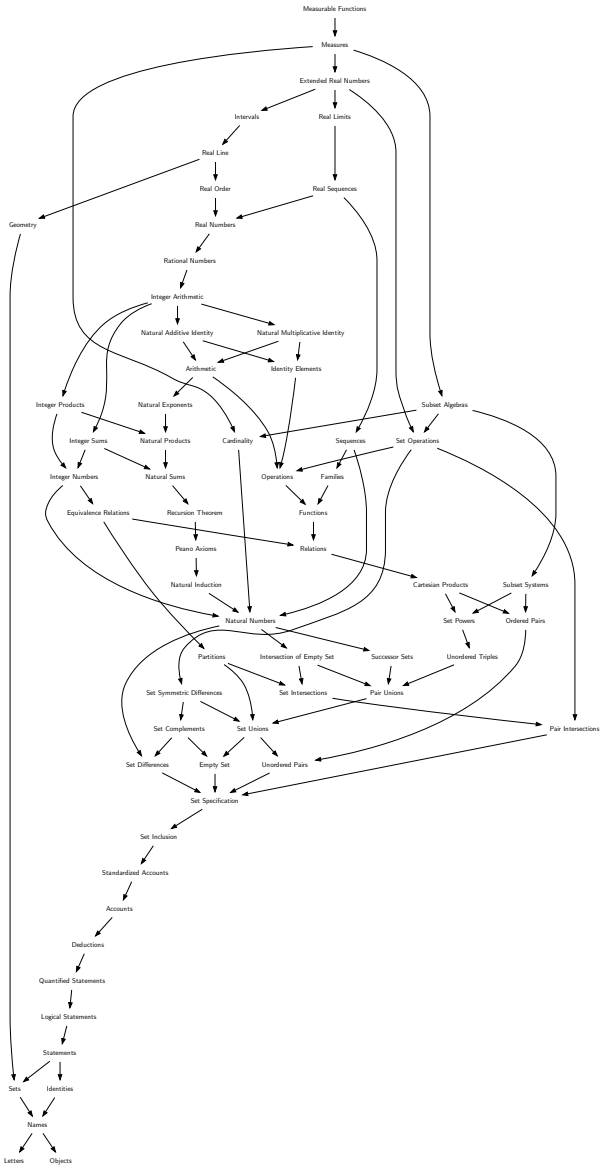
A function between the base sets of two measurable spaces is *measurable* with respect to the distinguished sets of the two spaces if the inverse image of every distinguished subset of the codomain is a distinguished subset of the domain.

Notation

Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Then a function $f : X \to Y$ is measurable if $B \in \mathcal{B}$ implies $f^{-1}(B) \in \mathcal{A}$. We say that f is measurable with respect to $\mathcal{A}$ and $\mathcal{B}$.

In this case, we sometimes say f is a measurable function from $(X, \mathcal{A})$ to $(Y, \mathcal{B})$. We say, $f : (X, \mathcal{A}) \to (Y, \mathcal{B})$ is measurable, read aloud as "f from X, A to Y, B is measurable."

[1] This statement contains a forward reference to Real Integrals, and so may be modified in future editions.


Measurable Functions
Measures
Extended Real Numbers
Intervals
Real Limits
Real Line
Real Order
Real Sequences
Geometry
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Integer Products
Natural Exponents
Subset Algebras
Integer Sums
Natural Products
Cardinality
Sequences
Set Operations
Integer Numbers
Natural Sums
Operations
Families
Equivalence Relations
Recursion Theorem
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects