



Definition

A *distribution family* (*density family*) on X is a family of distributions (densities) $\{p^{(\theta)}\}_{\theta \in \Theta}$ on X .

We call the index set Θ (see **Families**) the *parameters*. Frequently $\Theta \subset \mathbf{R}^p$.

Similarly, a *conditional distribution family* (*conditional density family*) on Z from X is a family $\{q^{(\theta)}\}_{\theta \in \Theta}$ whose terms $q^{(\theta)} : Z \times X \rightarrow \mathbf{R}$ are such that $q^{(\theta)}(\cdot, \xi) : Z \rightarrow \mathbf{R}$ is a distribution (density) for every $\xi \in X$.

Examples

For example, let $\Theta = [0, 1]$ and consider the family of distributions $\{p^{(\theta)} : \{0, 1\} \rightarrow [0, 1]\}_{\theta \in [0, 1]}$ defined by, for each $\theta \in [0, 1]$,

$$p^{(\theta)}(1) = \theta \text{ and } p^{(\theta)}(0) = 1 - \theta.$$

This family is called the *Bernoulli family* and $p^{(\theta)}$ is called a *Bernoulli distribution* with parameter θ .

For a second example, let $\Theta = \mathbf{R} \times \mathbf{R}_+$ and consider the family of densities $\{f^{(\theta)} : \mathbf{R} \rightarrow \mathbf{R}\}_{\theta \in \Theta}$ defined by, for each $\theta = (\mu, \sigma) \in \Theta$,

$$f^{(\theta)}(x) = (1/\sqrt{2\pi}\sigma) \exp((x - \mu)/\sigma^2).$$

This family is called the *normal family* and $f^{(\theta)}$ with $\theta = (\mu, \sigma)$ is called a *normal density* with mean μ and variance σ^2 .

