Why

We want a form of the dominated convergence theorem in terms of convergence in measure.[1]

Definition

A sequence of real-valued measurable functions *converges in measure* to a real-valued measurable limit function if for every positive number the measures of the set where the function deviates from the limit function by more than the positive number converges to zero.

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $(f_n)_n$ a sequence of real-valued measurable functions on X. Let f be a measurable real-valued function on X. If f_n converges in measure to f we write: $f_n \longrightarrow f$ in measure, read aloud as "f n goes to f in measure."

Suppose $f_n \longrightarrow f$ in measure. Then for every $\varepsilon > 0$,

$$\lim_{n \to \infty} \mu(\{x \in X \mid |f_n(x) - f(x)| > \varepsilon\}) = 0.$$

---

[1]Future editions will expand.

Convergence In Measure
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Sequences
Real Order
Real Numbers
Rational Order
Sequences
Subset Algebras
Rational Numbers
Lists
Integral Line
Cardinality
Integer Arithmetic
Direct Products
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Operations
Family Unions and Intersections
Natural Powers
Integer Sums
Integer Order
Function Inverses
Comparisons
Natural Products
Integer Numbers
Families
Function Composites
Function Images
Orders
Natural Sums
Equivalence Relations
Functions
Natural Order
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Subset Systems
Generalized Set Dualities
Natural Numbers
Set Powers
Ordered Pairs
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects