Why

We generalize the algebraic structure of addition and multiplication over the rationals.

Definition

A *field* is two algebras over the same ground set with: (1) both algebras are commutative groups (2) the operation of the second algebra distributes over the operation of the first algebra.

We call the operation of the first algebra *field addition*. We call the operation of the second algebra *field multiplication*.

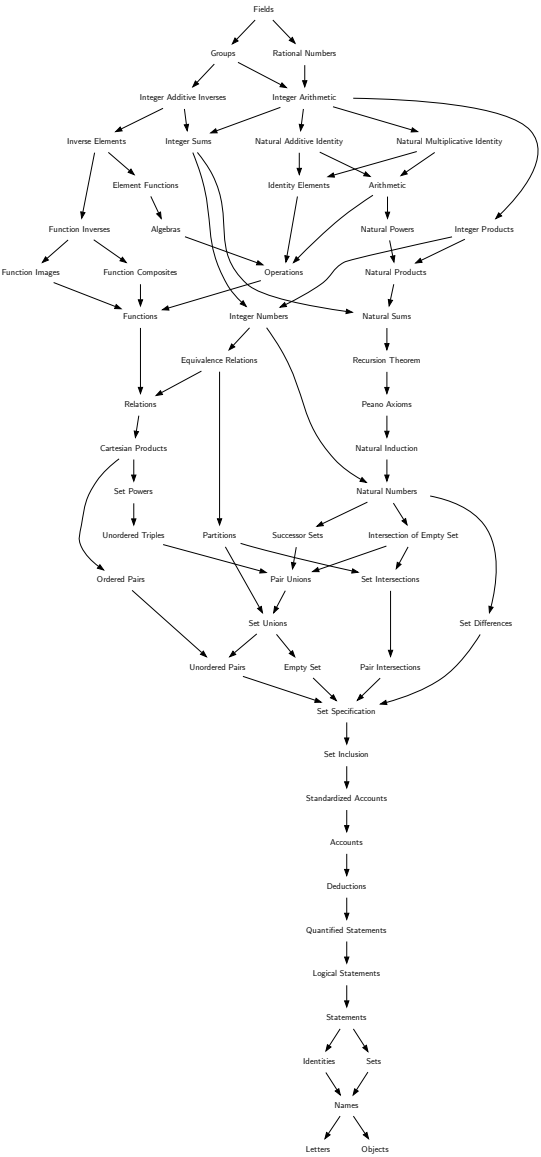
Notation

We tend to denote an arbitrary field by **F**, a mnemonic for "field."

1 Examples

Of course, **Q** with the usual addition (see Rational Sums) and multiplication (see Rational Products) and the inverse elements (see Rational Additive Inverse) and Rational Multiplicative Inverses) is a field.

Proposition 1. *The set of rational numbers with rational addition and multiplication is a field.*


Fields
Groups
Rational Numbers
Integer Additive Inverses
Integer Arithmetic
Inverse Elements
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Element Functions
Identity Elements
Arithmetic
Function Inverses
Algebras
Natural Powers
Integer Products
Function Images
Function Composites
Operations
Natural Products
Functions
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects