

### DISTRIBUTION EXPECTATION

## Why

If we model some measured value as a random variable with induced distribution  $p:V\to \mathbb{R}$ , then one interpretation of p(v) for  $v\in V$  is the *proportion* of times in a large number of trials that we *expect* to measure the value v.

## **Definition**

Given a distribution  $p: \Omega \to \mathbf{R}$  and a real-valued outcome variable  $x: \Omega \to \mathbf{R}$ , the expectation of x under p is  $\sum_{\omega \in \Omega} p(\omega) x(\omega)$ .

#### Notation

We denote the expectation of x under p by  $\mathbf{E}(x)$ . When there is no chance of ambiguity, we write  $\mathbf{E}(x)$ .

# **Properties**

Let  $x, y : \Omega \to \mathbb{R}$  be two outcome variables and  $p : \Omega \to \mathbb{R}$  a distribution. Let  $\alpha, \beta \in \mathbb{R}$ . Define  $z = \alpha x + \beta y$  by  $z(\omega) = \alpha x(\omega) + \beta y(\omega)$ . Then  $\mathbf{E}(z) = \alpha \mathbf{E}(x) + \beta \mathbf{E}(z)$ .

