Why

Toward a theory of iterated integrals, we need to generalize rectangular strips to arbitrary products.

Definition

Consider the product of two non-empty sets.

First, consider a subset of this product. For a specified element in the first set, the *set section* of the subset with respect to that element is the set of elements in the second set for which the ordered pair of the specified element and that element is in the subset; the section is a subset of the second set. For elements of the second set, we define sections similarly.

Second, consider a function on the product. For a specified element in the first set, the *function section* of the function for that element is the function from the second set to the codomain of the function which maps elements of the second set to the result of the function applied to the ordered pair of the specified element and the element of the second set. For elements of the second set, we define sections similarly.

Notation

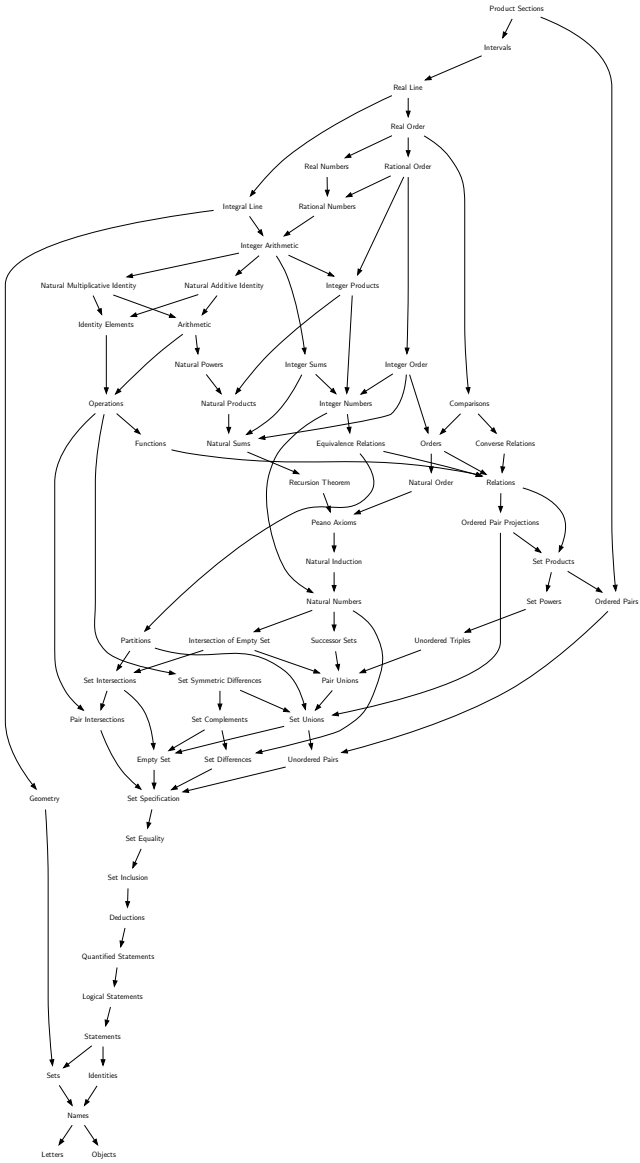
Let X, Y be non-empty sets.

Let $E \subset X \times Y$. For $x \in X$, we denote the section of E with respect to x by E_x. For $y \in Y$, we denote the section of E with respect to x by E^y. For every $x \in X$ and $y \in Y$,

$$E_x = \{y \in Y \mid (x, y) \in E\} \quad \text{and} \quad E^y = \{x \in X \mid (x, y) \in E\}.$$

$E_x \subset Y$ and $E_y \subset X$. Let $f : X \times Y \to Z$. For $x \in X$, we denote the section of f with respect to x by $f_x : Y \to Z$. For $y \in Y$, we denote the section of f with respect to x by $f^y : X \to Z$. For every $x \in X$ and $y \in Y$,

$$f_x(y) = f(x, y) \quad \text{and} \quad f^y(z) = f(x, y).$$


Product Sections
Intervals
Real Line
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Identity Elements
Arithmetic
Natural Powers
Integer Sums
Integer Order
Operations
Natural Products
Integer Numbers
Comparisons
Functions
Natural Sums
Equivalence Relations
Orders
Converse Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects