Expectation Deviation Upper Bound

Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Proposition **1.** *Let f be a square-integrable real-valued random variable on the probability space $(X, \mathcal{A}, \mu)$. Then for $t > 0$,*

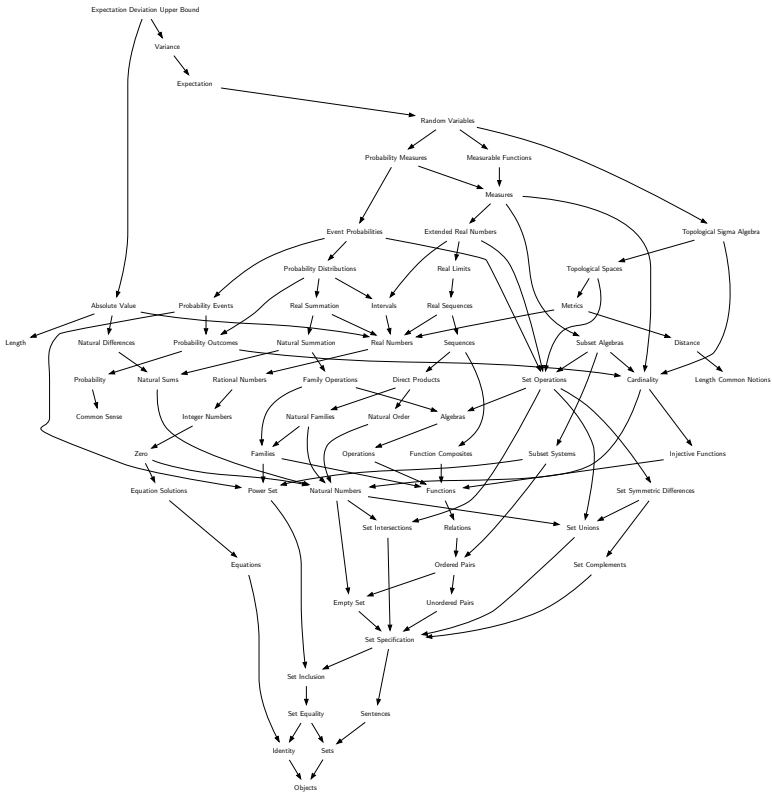
$$\mu(|f - \mathsf{E}(f)| \geq t) \leq \frac{\mathsf{var}\, f}{t^2}.$$

Proof. The set $|f - \mathsf{E}(f)| \geq t$ is $\{x \in X \mid |f(x) - \mathsf{E}(f)| \geq t\}$. This set is $\{x \in X \mid (f(x) - \mathsf{E}(f))^2 \geq t^2\}$. By using the non-negative inequality

$$\mu(\left\{x \in X \;\middle|\; (f(x) - \mathsf{E}(f))^2 \geq t^2\right\}) \leq \frac{\mathsf{E}(f - \mathsf{E}(f))}{t^2}.$$

We recognize the numerator of the right hand side as the variance. $\square$

The above is also called *Chebychev's Inequality*


Expectation Deviation Upper Bound
Variance
Expectation
Random Variables
Probability Measures
Measurable Functions
Measures
Event Probabilities
Extended Real Numbers
Topological Sigma Algebra
Probability Distributions
Real Limits
Topological Spaces
Absolute Value
Probability Events
Real Summation
Intervals
Real Sequences
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Natural Differences
Probability Outcomes
Natural Summation
Real Numbers
Sequences
Subset Algebras
Distance
Probability
Natural Sums
Rational Numbers
Family Operations
Direct Products
Set Operations
Cardinality
Length Common Notions
Common Sense
Integer Numbers
Natural Families
Natural Order
Algebras
Zero
Families
Operations
Function Composites
Subset Systems
Injective Functions
Equation Solutions
Power Set
Natural Numbers
Functions
Set Symmetric Differences
Set Intersections
Relations
Set Unions
Equations
Ordered Pairs
Set Complements
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Set Equality
Sentences
Identity
Sets
Objects