



Real Functions

1 Why

We name functions whose domain is the real numbers.

2 Definition

A *real function* is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

2.1 Notation

$f : \mathbf{R} \rightarrow \mathbf{R}$. f is a real function. To speak of functions defined on intervals, let $a, b \in \mathbf{R}$. $g : [a, b] \rightarrow \mathbf{R}$. is a real function defined on a closed interval. $h : (a, b) \rightarrow \mathbf{R}$ is a real function defined on an open interval.

We regularly declare the interval and the function in one pass: Let $f : [a, b] \rightarrow \mathbf{R}$, read aloud as " f from closed a b to \mathbf{R} ." Or, let $f : (a, b) \rightarrow \mathbf{R}$ read aloud as " f from open a b to \mathbf{R} ".

3 Examples

Example 1. Let $c \in \mathbf{R}$. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be such that $f(x) = c$ for every $x \in \mathbf{R}$. f is a real function.

Example 2.

Let $f : \mathbf{R} \rightarrow \mathbf{R}$ with $f(x) = 2x^2 + 1$ for all $x \in \mathbf{R}$. f is a real function.

Example 3. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ with

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{otherwise.} \end{cases}$$

f is a real function.