Why

For the difference of two (signed) measures to be well-defined, we need one of the two to be finite. Otherwise, the measure of the difference on the base set involves subtracting ∞ from ∞.

Definition

A *finite* signed measure is one for which the measure of every set is finite. This condition is equivalent to the base set having finite measure (see below).

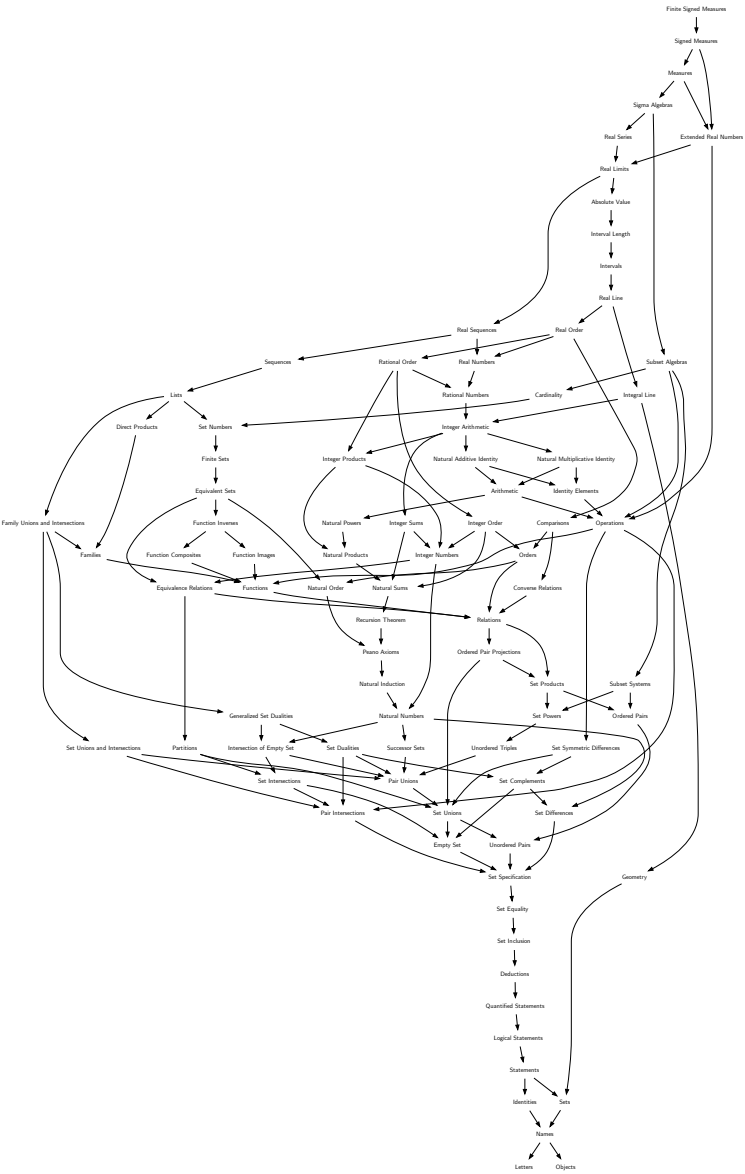
Result

Proposition 1. *A signed measure is finite if and only if it is finite on the base set.*

Proof. Let $(X, \mathcal{A})$ be a measurable space. Let $\mu : \mathcal{A} \to [-\infty, \infty]$ be a signed measure.

($\Rightarrow$) If μ is finite, then $\mu(X)$ is finite since $X \in \mathcal{A}$.

($\Leftarrow$) Next, suppose $\mu(X)$ is finite. Let $A \in \mathcal{A}$. Then $X = A \cup (X - A)$, with these sets disjoint, so by countable additivity of μ, $\mu(X) = \mu(A) + \mu(X - A)$. Since $\mu(X)$ finite, $\mu(A)$ and $\mu(X - A)$ are both finite. $\square$


Finite Signed Measures
Signed Measures
Measures
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Sequences
Real Order
Sequences
Rational Order
Real Numbers
Subset Algebras
Lists
Rational Numbers
Cardinality
Integral Line
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Arithmetic
Identity Elements
Family Unions and Intersections
Function Inverses
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Families
Function Composites
Function Images
Natural Products
Integer Numbers
Orders
Equivalence Relations
Functions
Natural Order
Natural Sums
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Subset Systems
Generalized Set Dualities
Natural Numbers
Set Powers
Ordered Pairs
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects