Why

Linear equations are ubiquitous.

Definition

Given $a \in \mathbf{R}^n$ and $y \in \mathbf{R}$, suppose we want to find $x \in \mathbf{R}^n$ satisfying

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = y.$$

We refer to this expression as a *real linear equation* or *linear equation*. We treat each component $x_i \in \mathbf{R}$ as a variable and we treat each component $a_i \in \mathbf{R}$ and $y \in \mathbf{R}$ as constants. We call the pair (a, y) the *real linear equation constants*.[1]

The source of the terminology "linear" is by viewing the left hand side as a function. Define $f : \mathbf{R}^n \to \mathbf{R}$ by $f(x) = \sum_i a_i x_i$. We want to find $x \in \mathbf{R}^n$ to satisfy $f(x) = b$. Notice that f is a *linear* real function.[2]

Moreover, to each linear function $f : \mathbf{R}^d \to \mathbf{R}$ there exists a vector $a \in \mathbf{R}^d$ so that $f(x) = \sum_i a_i x_i$. For this reason, if we are given several linear function $f_1, \ldots, f_m$, then we can think of these as several vectors $a^1, \ldots, a^n$. If we are also given $b_i \in \mathbf{R}$ for each $i = 1, \ldots, m$, then we have the vector $b \in \mathbf{R}^m$

We can define the two-dimensional array $A \in \mathbf{R}^{m \times n}$ by $A_{ij} = a^i_j$. For this reason, a *linear system of equations* is a pair (A, b). A solution of a linear system of equations is a vector $x \in \mathbf{R}^n$ satisfying the equations

$$
\begin{aligned}
A_{11}x_1 + & A_{12}x_2 + & \cdots + & A_{1n}x_n = & b_1 \\
A_{21}x_1 + & A_{22}x_2 + & \cdots + & A_{2n}x_n = & b_2 \\
\vdots \quad & \vdots \quad & & \vdots \quad & \vdots \\
A_{m1}x_1 + & A_{m2}x_2 + & \cdots + & A_{mn}x_n = & b_n
\end{aligned}
$$

Other terminology includes a *system of linear equations* or *linear system* or *simultaneous linear equations*

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[1] Future editions will clarify.

[2] Future editions may require a sheet here.