Why

Given a vector $x \in \mathbf{R}^n$ and a subspace $S \subset \mathbf{R}^n$, what is the closest point in S to x.

Result

Proposition 1. *Suppose $U \in \mathbf{R}^{n \times k}$ with $U^\top U = I$. Then*

$$\mathrm{proj}_U(x) = UU^\top x$$

Projections On Subspaces

Real Subspace Representation

Real Matrices with Orthonormal Columns

Real Matrix-Matrix Products

Real Matrix-Vector Products

Vectors as Matrices — Orthonormal Set of Real Vectors

Real Matrices — Independent Set of Real Vectors — Real Inner Product — Real Vector Angles — Real Vector Projections

Real Linear Equations — Real Linear Combinations — Real Vectors — Space Vector Angles — Space Inner Product — Real Norm

Arrays — Linear Functions — Optimization Problems — N-Dimensional Space — Plane Vector Angles — Space Norm

Extended Real Numbers — Plane Vector Sums and Differences — Space Vectors — Plane Norm — Plane Inner Product

Distance — Plane Vectors

Real Limits — Right Triangle Sides Relation — Real Squares — Real Functions

Square Distance — Absolute Value — Real Arithmetic

Optimizers — Plane Distance — Real Multiplication Inverse — Interval Length

Real Summation — Real Sequences — Greatest Lower Bounds — Real Products — Real Square — Intervals — Square Numbers

Real Addition Inverse — Real Plane — Real Line — Square

Sequences — Natural Summation — Real Sums — Real Order — Rational Multiplication Inverses — Rational Arithmetic — Integral Line — Geometry

Lists — Set Number — Real Numbers — Rational Sums — Rational Order — Natural Products

Direct Products — Finite Sets — Natural Fractions — Integer Arithmetic — Natural Arithmetic

Inverse Elements — Equivalent Sets — Comparisons — Natural Multiplication Identity — Natural Addition Identity — Integer Order — Integer Products

Family Unions and Intersections — Family Operations — Function Inverses — Element Functions — Operations — Ordered — Identity Elements — Natural Equations — Integer Sums

Families — Function Composites — Function Images — Functions — Natural Order — Equations — Natural Power — Integer Numbers — Natural Products

Equivalence Relations — Converse Relations — Relations — Natural Sums

Recursion Theorem

Ordered Pair Projections — Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Partitions — Intersection of Empty Set — Set Dualities — Unordered Triples — Successor Sets — Set Symmetric Differences — Ordered Pair

Set Intersections — Pair Unions — Set Complements — Set Differences

Pair Intersections — Set Unions — Empty Set — Unordered Pair

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects