Why

We consider a simple distribution on sequences.[1]

Definition

A *memory chain* (*markov chain*[2], *memory model*, *markov model*) *on* a set A of *length* n is a joint distribution $p : A^n \to [0,1]$ satisfying

$$p(a) = f(a_1) \prod_{i=2}^{n} g(a_i, a_{i-1})$$

for some distribution $f : A \to [0,1]$ and $g : A^2 \to [0,1]$ is a function for which $g(\cdot, \alpha) : A \to [0,1]$ is a distribution on A for every $\alpha \in A$.

Proposition 1. *p so defined is a distribution. The function g is the conditional distribition $p_{i|i-1}$ for $i = 2, \ldots, n$. The function f is the first marginal p_1.*

For this reason, we call g the *conditional distribution* of the Markov chain. We call f the *initial distribution.*

Terminology

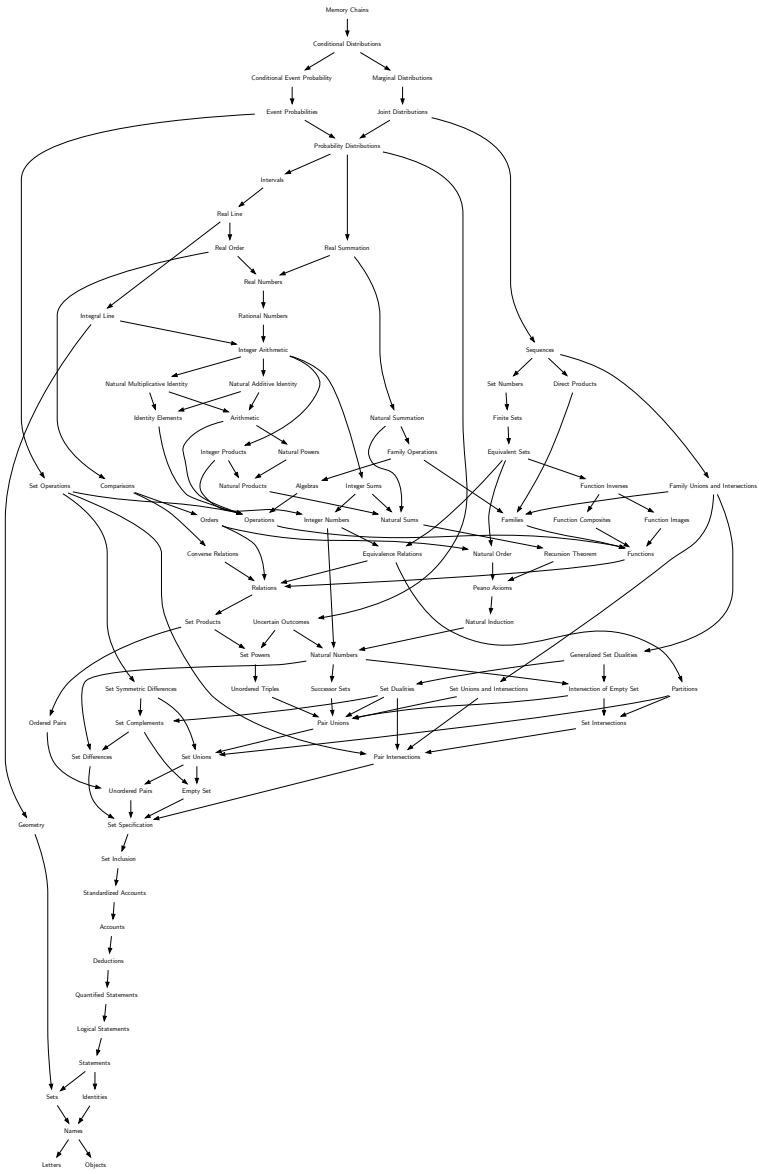
Let $A = \prod_i A_i$ and $p : A \to [0,1]$ be a distribution. On one hand, p is said to be *memoryless* (or have *zero-order mem-*

[1] Future editions will modify and expand.

[2] This term is universal. We avoid it in these sheets because of the Bourbaki project's policy on naming. The skeptical reader will note, at least, out term and this term terms have the same intials.

ory) if $p = \prod_i p_i$. In particular, $p_{i|i-1}(\alpha, \beta) = p_i(\alpha)$ for every $i = 2, \ldots, n$. On the other hand, p is said to have *first-order memory* if $p = p_1 \prod_{i=2}^{n} p_{i|i-1}$

If, in addition, $A_i = A_1$ for all $i = 1, \ldots, n$, then we say that p has *homogenous conditionals* if $p_{i|i-1} = p_{j|j-1}$ for $i \neq j = 2, \ldots, n$. In this language, a memory chain is a joint distribution with first-order memory and homogogenous conditionals.


Memory Chains
Conditional Distributions
Conditional Event Probability
Marginal Distributions
Event Probabilities
Joint Distributions
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Sequences
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Direct Products
Identity Elements
Arithmetic
Natural Summation
Finite Sets
Integer Products
Natural Powers
Family Operations
Equivalent Sets
Set Operations
Comparisons
Natural Products
Algebras
Integer Sums
Function Inverses
Family Unions and Intersections
Orders
Operations
Integer Numbers
Natural Sums
Families
Function Composites
Function Images
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Functions
Relations
Peano Axioms
Set Products
Uncertain Outcomes
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Unordered Triples
Successor Sets
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Partitions
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects