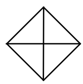


Definition

An operator $T \in \mathcal{L}(V)$ is called *self-adjoint* (or *Hermitian*) if the adjoint of T is itself. In symbols, T is self-adjoint if $T = T^*$. In other words, T is self-adjoint if and only if

$$\langle Tv, w \rangle = \langle v, Tw \rangle \quad \text{for all } v, w \in V$$

Properties

Proposition 1. *Suppose $S, T \in \mathcal{L}(V)$ are self-adjoint. The $S + T$ are self-adjoint. Also λT is adjoint for all real λ.*

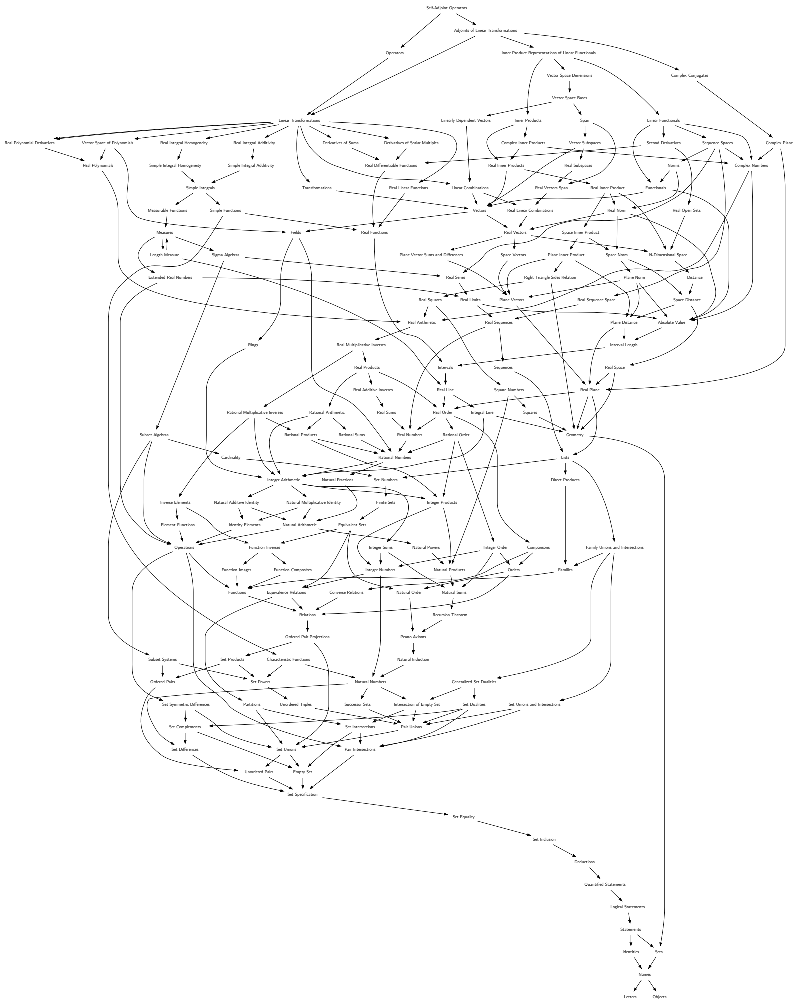
Notation

We will see that the adjoint on $\mathcal{L}(V)$ plays a role similar to complex conjugation on $\mathbf{C}$. The self-adjoint operators will seen to be analogous to the real numbers. A complex number is real if and only if $z = z^*$. Similarly, an operator is self-adjoint if and only if $T = T^*$.

Characterization for complex space

Proposition 2. *Suppose V is a complex inner product space and let $T \in \mathcal{L}(V)$. Then*

$$T = T^* \quad \longleftrightarrow \quad (\forall v \in V)\ (\langle Tv, v \rangle \in \mathbf{R})$$


Self-Adjoint Operators
Adjoints of Linear Transformations
Operators
Inner Product Representations of Linear Functionals
Vector Space Dimensions
Complex Conjugates
Vector Space Bases
Linear Transformations
Linearly Dependent Vectors
Inner Products
Span
Linear Functionals
Real Polynomial Derivatives
Vector Space of Polynomials
Real Integral Homogeneity
Real Integral Additivity
Derivatives of Sums
Derivatives of Scalar Multiples
Complex Inner Products
Vector Subspaces
Second Derivatives
Sequence Spaces
Complex Plane
Real Polynomials
Simple Integral Homogeneity
Simple Integral Additivity
Real Differentiable Functions
Real Inner Products
Real Subspaces
Norms
Complex Numbers
Simple Integrals
Transformations
Real Linear Functions
Linear Combinations
Real Vectors Span
Real Inner Product
Functionals
Measurable Functions
Simple Functions
Vectors
Real Linear Combinations
Real Norm
Real Open Sets
Measures
Fields
Real Functions
Real Vectors
Space Inner Product
Length Measure
Sigma Algebras
Plane Vector Sums and Differences
Space Vectors
Plane Inner Product
Space Norm
N-Dimensional Space
Extended Real Numbers
Real Series
Right Triangle Sides Relation
Plane Norm
Distance
Real Squares
Real Limits
Plane Vectors
Real Sequence Space
Space Distance
Real Arithmetic
Real Sequences
Plane Distance
Absolute Value
Rings
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Sequences
Real Space
Real Additive Inverses
Real Line
Square Numbers
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Squares
Subset Algebras
Rational Products
Rational Sums
Real Numbers
Rational Order
Geometry
Cardinality
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Integer Products
Element Functions
Identity Elements
Natural Arithmetic
Equivalent Sets
Operations
Function Inverses
Integer Sums
Natural Powers
Integer Order
Comparisons
Family Unions and Intersections
Function Images
Function Composites
Integer Numbers
Natural Products
Orders
Families
Functions
Equivalence Relations
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects