Why

What does it mean for two random variables to be independent? What are the events associated with a random variable? TODO

Definition

Defining Result

PROPOSITION **1.** *The set of inverse images the distinguished sets of a measurable space under a function from a set to that space is a sigma algebra.*

If the first set and the function are measurable, the sigma algebra is a sub sigma algebra of the domain sigma algebra.

Proof. Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Let $f : X \to Y$ be a measurable function. Define

$$\mathcal{C} := \{f^{-1}(B) \mid B \in \mathcal{B}\}.$$

First, since $f^{-1}(Y) = X$, $X \in \mathcal{C}$.

Second, let $C \in \mathcal{C}$. Then there is B such that $C = f^{-1}(B)$. Then $X - C = X - f^{-1}(B) = f^{-1}(Y - B)$. Since $\mathcal{B}$ is a sigma algebra, $B \in \mathcal{B}, Y - B \in \mathcal{B}$ and so $X - C \in \mathcal{C}$.

Finally, let $(C_n)_n \subset \mathcal{C}$. Then for every n there exists a $B_n \in \mathcal{B}$ so that $C_n = f^{-1}(B_n)$. Then:

$$\cup_n C_n = \cup_n f^{-1}(B_n) = f^{-1}(\cup B_n).$$

Since $\mathcal{B}$ is a sigma algebra, $\cup_n B_n \in \mathcal{B}$ and so $\cup_n C_n \in \mathcal{C}$.

Since f is measurable, $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}$, and so $\mathcal{C} \subset \mathcal{A}$. $\qquad\square$

The sigma algebra *generated by a random variable* is the sigma algebra consisting of the inverse images of every measurable set of the codomain.

The sigma algebra generated by a family of random variables is the sigma algebra generated by the union of the sigma algebras generated individually by each of the random variables.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f : X \to Y$ be a random variable. Denote by $\sigma(f)$ the sigma algebra generated by f.

Results

PROPOSITION **2.** *The sigma algebra generated by a family of random variables is the smallest sigma algebra for with respect to which each random variable is measurably.*

Random Variable Sigma Algebra

Random Variables

Measurable Functions

Topological Sigma Algebra

Topological Spaces

Metrics

Absolute Value

Distance

Space Distance

Plane Distance

Probability Measure

Measures

Event Probabilities

Extended Real Numbers

Probability Events

Probability Distributions

Interval Length

Subset Algebras

Real Limits

Probability Outcomes

Intervals

Real Plane

Real Space

Cardinality

Set Operations

Real Sequences

Probability

Real Summation

Real Order

Direct Products

Geometry

Common Sense

Natural Summation

Real Numbers

Sequences

Family Operations

Rational Numbers

Families

Integer Arithmetic

Algebras

Natural Multiplicative Identity

Natural Additive Identity

Identity Elements

Arithmetic

Operations

Integer Products

Natural Exponents

Functions

Integer Sums

Natural Products

Natural Sums

Integer Numbers

Recursion Theorem

Equivalence Relations

Relations

Peano Axioms

Subset Systems

Cartesian Products

Natural Induction

Set Powers

Ordered Pairs

Natural Numbers

Unordered Triples

Successor Sets

Intersection of Empty Set

Partitions

Set Symmetric Differences

Pair Unions

Set Intersections

Pair Intersections

Set Complements

Set Unions

Set Differences

Empty Set

Unordered Pairs

Set Specification

Set Inclusion

Standardand Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects

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