Why

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Definition

The *higher adjacency set* or *higher neighborhood* of a vertex v in an ordered undirected graph is all vertices in the neighborhood of v whose index is greater the v. Similarly, the *lower adjacency set* or *lower neighborhood* of v is all vertices in the neighborhood of v whose index is less the v. We call these *monotone neighborhoods*.

The *higher degree* of a vertex is the size of the higher adjacency set and the *lower degree* of a vertex is the size of its lower adjacency set.

The *closed monotone neighborhoods* are the *closed higher adjacency set*, the higher adjacency set of v union with the singleton $\{v\}$ and the *closed lower adjacency set*, the lower adjacency set of v union with the singleton $\{v\}$.
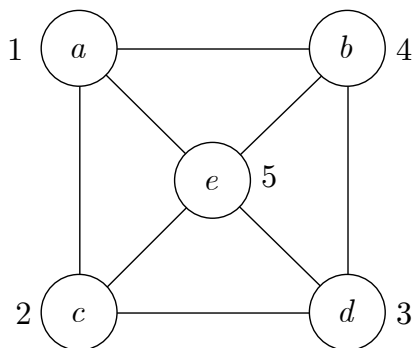
Notation

We denote the higher neighborhood of v by $\mathrm{adj}^+(v)$ and the lower neighborhood by $\mathrm{adj}^-(v)$.

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[1] Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.

Visualization

To help think about the monotone neighborhoods of the graph we visualize ordered graphs as triangular arrays with vertices along the diagonal and a bullet in row i and column j of the array if $i > j$ and the vertices $\sigma(i)$ and $\sigma(j)$ are adjacent.

An example is shown below for the ordered undirected graph in the figure (to understand this visualization, see Ordered Undirected Graphs) we use the

$$
\begin{bmatrix}
a & & & & \\
\bullet & c & & & \\
& \bullet & d & & \\
\bullet & & \bullet & b & \\
\bullet & \bullet & \bullet & \bullet & e
\end{bmatrix}
$$

In this array representation the higher and lower neighborhoods are easily identified. The indices of the elements of $\mathrm{adj}^+(v)$ are the column indices of the entries in row $\sigma^{-1}(v)$ of the array. For example, $\sigma^{-1}(d) = 3$, and the only bullet entry in row three is c so $\mathrm{adj}^-(d) = \{c\}$. Likewise, $\mathrm{adj}^-(c) = \{a\}$. And so on. Similalry, the indices of $\mathrm{adj}^+(v)$ are the row indices of the entries in column $\sigma^{-1}(v)$. For example, $\sigma^{-1}(d)$ is 3, and

there are indices 4 and 5 corresponding to b and e so $\mathrm{adj}^+(d) = \{b, e\}$. Likewise, $\mathrm{adj}^+(c) = \{d, e\}$.

For this reason, we use the notation $\mathrm{col}(v)$ and $\mathrm{row}(v)$ for the closed upper and lower neighborhoods. So $\mathrm{col}(v) = \mathrm{adj}^+(v) \cup \{v\}$ and $\mathrm{row}(v) = \mathrm{adj}^-(v) \cup \{v\}$.

Monotone Neighborhoods

Ordered Undirected Graphs • Arrays

Undirected Graphs • Lists

Set Numbers • Direct Products

Finite Sets

Equivalent Sets

Function Inverses • Natural Order • Family Unions and Intersections

Function Composites • Function Images • Families

Functions • Equivalence Relations • Peano Axioms

Relations • Natural Induction

Set Products

Set Powers • Natural Numbers • Generalized Set Dualities

Partitions • Unordered Triples • Successor Sets • Intersection of Empty Set • Set Dualities • Set Unions and Intersections

Ordered Pairs • Set Intersections • Pair Unions • Set Complements

Set Unions • Set Differences • Pair Intersections

Unordered Pairs • Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities • Sets

Names

Letters • Objects