Why

1

Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

Notation

Let $(X, \mathcal{A}, \mathbf{P})$ be a probability space. Let $(Y, \mathcal{B})$ a measurable space. Then a random variable is a measurable function $f : X \to Y$.

Some authors denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. In this case, the base probability space is denoted by Ω, a mnemonic for "outcomes." Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $X : \Omega \to \mathbf{R}$ be measurable. Then X is a real-valued random variable.

Some authors use notation for the probability of particular, common sets. Although the authors often use regular parentheses, we will use "[" and "]" for precision. Let $A \in \mathcal{B}(\mathbf{R})$. Let $\mathbf{P}[X \in A]$ denote $\mathbf{P}(X^{-1}(A))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

As we mentioned, some authors use $P(X \in A)$ for $P[X \in A]$, which is mostly harmless.

---

[1]Future editions will include this.

Next, let $Y : \Omega \to \mathbf{R}$ a measurable function and let $B \in \mathcal{B}(\mathbf{R})$. Similar to the above, let $\mathbf{P}[X \in A, Y \in B]$ denote $\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \ldots, X_n : \Omega \to \mathbf{R}$,

$$\mathbf{P}[X_1 \in A_1, \ldots, X_n \in A_n] = \mathbf{P}(\cap_{i=1}^n X_i^{-1}(A_i)).$$

Random Variables

Topological Sigma Algebra Measurable Functions Probability Measure

Topological Spaces Measures Event Probabilities

Sigma Algebras Metrics

Real Series Extended Real Numbers Distance

Real Limits Space Distance

Absolute Value Plane Distance

Interval Length Probability Distributions

Intervals Real Space

Real Plane Real Line

Real Summation Real Order Integral Line

Real Sequences Real Numbers Rational Order Geometry

Sequences Rational Numbers

Subset Algebras Lists Integer Arithmetic

Cardinality Set Numbers Natural Additive Identity Natural Multiplicative Identity Integer Products

Finite Sets Identity Elements Arithmetic

Equivalent Sets Direct Products Natural Summation Natural Powers

Function Inverses Family Unions and Intersections Family Operations Comparisons Natural Products Integer Order Integer Sums

Function Images Function Composites Families Operations Orders Natural Sums Integer Numbers

Equivalence Relations Functions Converse Relations Natural Order Recursion Theorem

Relations Peano Axioms

Subset Systems Set Products Generalized Set Dualities Natural Induction Uncertain Outcomes

Ordered Pair Set Powers Natural Numbers

Set Unions and Intersections Partitions Unordered Triples Intersection of Empty Set Successor Sets Set Dualities Set Symmetric Differences

Set Intersections Pair Unions Set Complements

Pair Intersections Set Unions Set Differences

Unordered Pair Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities Sets

Names

Letters Objects

4