

Set Unions

1 Why

We want to consider the elements of two sets together at one. Does a set exist which contains all elements which appear in either of one set or another?

2 Definition

We say yes. For every set of sets there exists a sets which contains all the elements that belong to at least one set of the given collection. We refer to this as the *axiom of unions*. If we have one set and another, the axiom of unions says that there exists a set which contains all the elements that belong to at least one of the former or the latter.

The set guaranteed by the axiom of unions may contain more elements than just those which are elements of a member of the the given set of sets. No matter: apply the axiom of specification to form the set which contains only those elements which are appear in at least one of any of the sets. As a result of the axiom of extension, this set is unique. We call it the *union* of the set of sets.

3 Notation

Let \mathcal{A} be a set of sets. We denote the union of \mathcal{A} by $\cup \mathcal{A}$.

4 Simple Facts

Proposition 1. $\cup \emptyset = \emptyset$

Proposition 2. $\cup \{A\} = A$