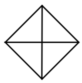


Why

Since our predictions are often uncertain, we can use the language of probability distributions to characterize them.[1]

Definition

Denote the set of probability distributions on a set X by $\Delta(X)$.

A *probabilistic classifier* $G : A \to \Delta(B)$ is a function from inputs A to probability distributions over the classes B.

Given an input u, the *prediction* of G on a is a probability distribution $\hat{p}_a = G(a)$ on B.

Point classifier as probabilistic classifier

Given a point classifier $f : A \to B$, we can define a probabilistic classifier $G : A \to \Delta(B)$ corresponding to f by

$$\hat{p}_a(b) = \begin{cases} 1 & \text{if } f(a) = b \\ 0 & \text{otherwise.} \end{cases}$$

where $\hat{p}_a = G(a)$.

Probabilistic classifier from point classifier

On the other hand, given probabilistic classifier $G : A \to \Delta(B)$, we can define a point classifier $f : A \to B$ by

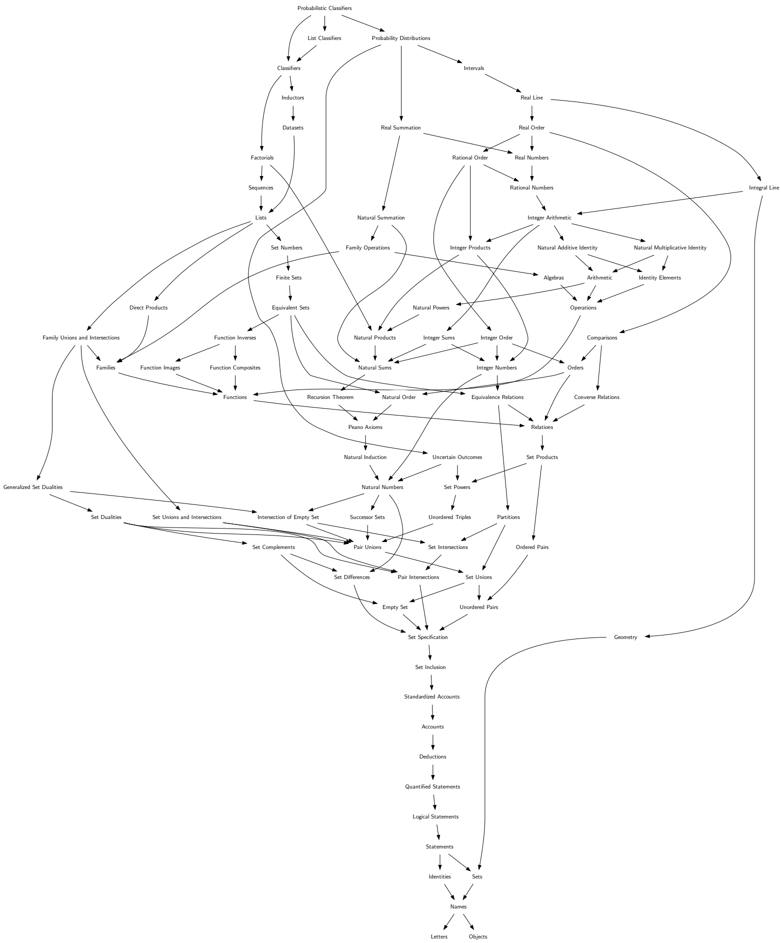
$$f(a) = \operatorname*{argmax}_{b \in B} \hat{p}_a(v)$$

[1]Future editions will improve this.

We call f the *maximum likelihood classifier* corresponding to G. If there are ties, we can order the (finite) set B arbitrarily, and break ties accordingly.

We can extend this idea, and define a list classifier by sorting the outputs by their probability, from largest to smallest.

1 Judging probabilistic classifiers


Probabilistic Classifiers
List Classifiers
Probability Distributions
Classifiers
Intervals
Inductors
Real Line
Datasets
Real Summation
Real Order
Factorials
Rational Order
Real Numbers
Sequences
Rational Numbers
Integral Line
Lists
Natural Summation
Integer Arithmetic
Set Numbers
Family Operations
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Algebras
Arithmetic
Identity Elements
Direct Products
Equivalent Sets
Natural Powers
Operations
Family Unions and Intersections
Function Inverses
Natural Products
Integer Sums
Integer Order
Comparisons
Families
Function Images
Function Composites
Natural Sums
Integer Numbers
Orders
Functions
Recursion Theorem
Natural Order
Equivalence Relations
Converse Relations
Peano Axioms
Relations
Natural Induction
Uncertain Outcomes
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects