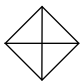


Nets

Why

We generalize the notion of sequence to index sets beyond the naturals.

Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go "further out."

To elaborate on property (b): if handed two natural numbers m and n, we can always find another, for example $\max\{m, n\} + 1$, larger than m and n. We might think of larger as "further out" from the first natural number: 1.

Combining these to observations, we define a directed set:

Definition 1. *A directed set is a set D with a partial order $\preceq$ satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.*

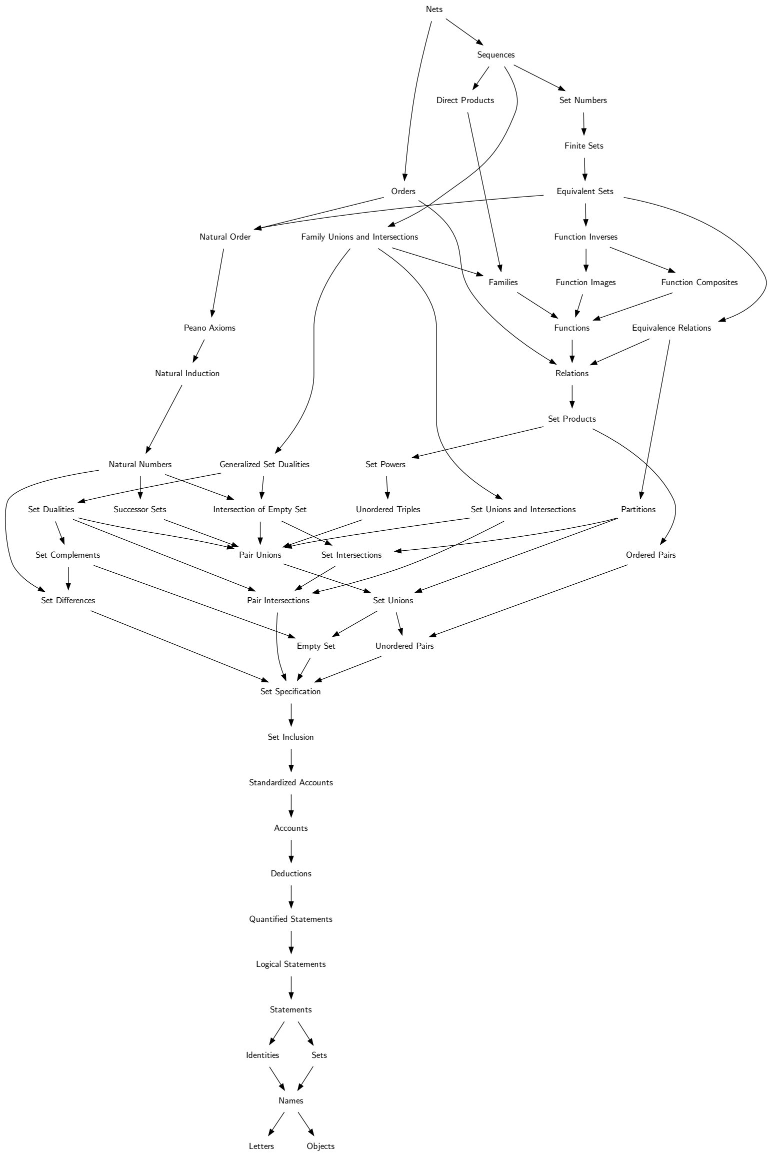
Definition 2. *A net is a function on a directed set.*

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \to A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.


Nets
Sequences
Direct Products
Set Numbers
Finite Sets
Orders
Equivalent Sets
Natural Order
Family Unions and Intersections
Function Inverses
Families
Function Images
Function Composites
Peano Axioms
Functions
Equivalence Relations
Natural Induction
Relations
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Unions and Intersections
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects