

### Inverses of Composite Relations

### Why

How do inverse and converse relations interact.

#### Results

Let R be a relation between X and Y and let S be a relation between Y and Z.

**Proposition 1.** 
$$(RS)^{-1} = S^{-1}R^{-1}$$

# **Identity Relations**

Recall that I is the identity relation on X if x I y if and only if x = y.

**Proposition 2.** Let R be a relation on X. Let I be the identity relation on X. Then RI = IR = R.

One would like  $RR^{-1} \supset I$ ,  $R^{-1}R \supset I$ . The father of the son is the father and the son of the father is the son. But the empty relation violates these claims.

# **Relation Properties**

**Proposition 3.** R is symmetric if and only if  $R \subset R^{-1}$ 

**Proposition 4.** R is reflextive if and only if  $I \subset R$ 

**Proposition 5.** R is transitive if and only if  $RR \subset R$ .

