## Why

We name a statement which involves an identity.[1]

## Definition

An *equation* is statement (see Statements) relating two terms by the relation of identity (see Identities). Some authors also call an equation an *equality*. The symbol "$=$" is called the (or an) *equals sign* or *equals symbol*.

## Variables

Let $X$ and $Y$ be sets and let $f, g : X \to Y$. In the statement

$$(\forall x)(f(x) = g(x)),$$

$f$ and $g$ are *free* names and $x$ is a *bound* name (see Quantified Statements).

For convenience, we often refer to the equation $f(x) = g(x)$ without the quantifier $\forall x$. In this case, $x$ appears free, but is not. In this context, the statement $f(x) = g(x)$ has $x$ implicitly bound. There are two senses here, though. The first is that $x$ is bound because it is "subordinate" to the quantifier $\forall$. The particular symbol is irrelevant; the symbol $y$ works just as well. In a second sense, though, the name is "free," as it is a placeholder and the choice of the symbol $x$ does not matter.

We use different terminology for this common case. In discussing $f(x) = g(x)$, we call the placeholder name $x$ a *variable* and we call the names $f$ and $g$ *constants*. The language is meant to convey $f$ and $g$ are *fixed* in the present discussion, as indicated by the usual language "Let $f$ and $g$ ...".

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[1]Future editions will modify this statement and sheet.

## Solutions

We are often interested in finding objects in some set to satisfy an equation. For example, we are interested in finding an object $\xi \in X$ to satisfy $f(\xi) = g(\xi)$. In this setting we call the variable $\xi$ in the equation an *unknown*.

We call an element $\xi \in X$ a *solution* of the equation if $f(\xi) = g(\xi)$. We call the set

$$\{\xi \in X \mid f(\xi) = g(\xi)\}$$

the *solution set*. If the solution set is non-empty, we say that a solution *exists*. If the solution set is a singleton, we say that the solution is *unique*.

We are often interested in solutions which satisfy several equations at once. For example, we have the equations $f_1(x) = g_1(x)$ and $h(x) = i(x)$ and so on. We want $x$ to satisfy these. Here it is *set of equations*, *simultaneous equations*, or a *system of equations*.

## Finding solutions

We often talk about *finding* or *searching* for solutions or *solving equations*. We say: "We want to *find* $x \in X$ to satisfy $f(x) = g(x)$." In addition to $f(x) = g(x)$, we may include other statements about $x$. The language is meant to convey that we are searching for an object which we will name, as a variable, $x$, and we want this object to satisfy the statements.

Equations

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Functions

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Relations

↓ ↘

Ordered Pair Projections

Set Products

↓ ↙

Set Powers

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Ordered Pairs

Unordered Triples

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Pair Unions

↓

Set Unions

↓ ↘

Unordered Pairs

Empty Set

↓ ↙

Set Specification

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Set Equality

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Set Inclusion

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Deductions

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Quantified Statements

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Logical Statements

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Statements

↙ ↘

Identities

Sets

↘ ↙

Names

↙ ↘

Letters

Objects