



INTERVALS

Why

We name and denote subsets of the set of real numbers which correspond to segments of a line.

Definition

Take two real numbers, with the first less than the second.

An *interval* is one of four sets:

1. the set of real numbers larger than the first number and smaller than the second; we call the interval *open*.
2. the set of real numbers larger than or equal to the first number and smaller than or equal to the second number; we call the interval *closed*.
3. the set of real numbers larger than the first number and smaller than or equal to the second; we call the interval *open on the left* and *closed on the right*
4. the set of real numbers larger than or equal to the first number and smaller than the second; we call the interval *closed on the left* and *open on the right*.

If an interval is neither open nor closed we call it *half-open* or *half-closed*

We call the two numbers the *endpoints* of the interval. An open interval does not contain its endpoints. A closed interval

contains its endpoints. A half-open/half-closed interval contains only one of its endpoints. We say that the endpoints *delimit* the interval.

Notation

Let a, b be two real numbers which satisfy the relation $a < b$.

We denote the open interval from a to b by (a, b) . This notation, although standard, is the same as that for ordered pairs; no confusion arises with adequate context.¹

We denote the closed interval from a to b by $[a, b]$. We record the fact $(a, b) \subset [a, b]$ in our new notation.

We denote the half-open interval from a to b , closed on the right, by $(a, b]$ and the half-open interval from a to b , closed on the left, by $[a, b)$.²

The *unit interval* is the set $[0_{\mathbf{R}}, 1_{\mathbf{R}}]$ and we sometimes denote it by \mathbf{I} .

¹In future editions, we may use $\langle a, b \rangle$ or even $\{a, b\}$.

²Some authors use $]a, b]$, $[a, b[$ and $]a, b[$.

