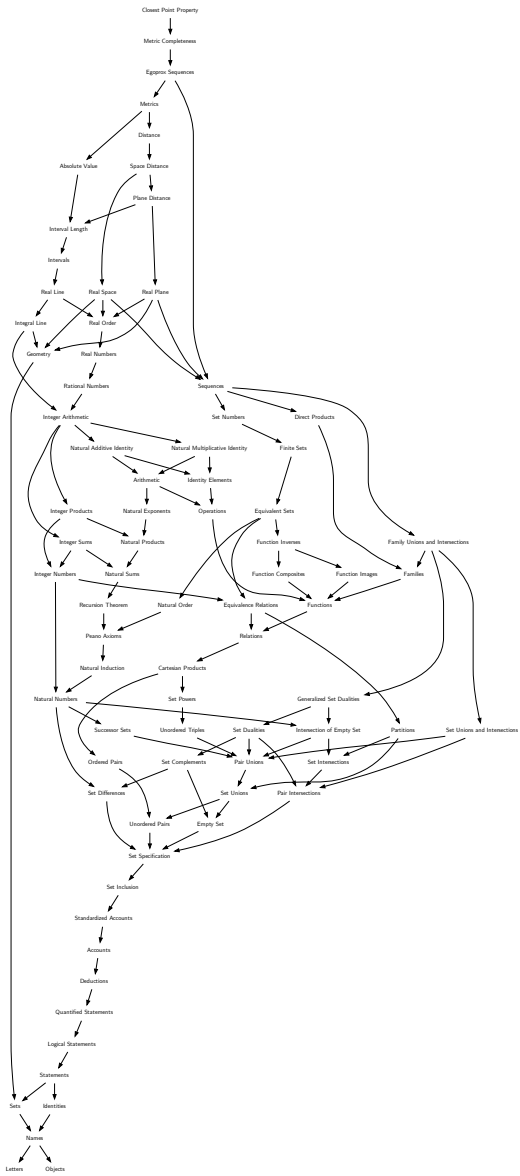
Suppose H is a Hilbert space $A \subset H$ closed and convex. then if $x \in H$, there exists a unique $z \in A$ closest to x. There exists a unique $z \in A$ closest to x such that

$$d(z, x) = \inf_{y \in A} d(y, x).$$

Proof. Take any sequence $(y_n)_n$ such that

$$d(y_n, x) \to d = \inf_{y \in A} d(y, x)$$

$\square$


Closest Point Property
Metric Completeness
Eggerox Sequences
Metrics
Distances
Absolute Value
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Space
Real Plane
Integral Line
Real Order
Geometry
Real Numbers
Rational Numbers
Sequences
Integer Arithmetic
Set Numbers
Direct Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Integer Products
Natural Exponents
Operations
Equivalent Sets
Integer Sums
Natural Products
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Function Compositions
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects