



Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

Notation

Let $(X, \mathcal{A}, \mathbf{P})$ be a probability space. Let (Y, \mathcal{B}) a measurable space. A random variable is a measurable function $f : X \rightarrow Y$.

Some authors denote real-valued random variables by upper case Latin letters: for example, X, Y, Z . In this case, the base probability space is denoted by Ω , a mnemonic for “outcomes.” Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $X : \Omega \rightarrow \mathbf{R}$ be measurable. Then X is a real-valued random variable.

Some authors use notation for the probability of particular, common sets. Although the authors often use regular parentheses, we will use “[” and “]” for precision. Let $A \in \mathcal{B}(\mathbf{R})$. Let $\mathbf{P}[X \in A]$ denote $\mathbf{P}(X^{-1}(A))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

As we mentioned, some authors use $P(X \in A)$ for $P[X \in A]$, which is mostly harmless.

Next, let $Y : \Omega \rightarrow \mathbf{R}$ a measurable function and let $B \in \mathcal{B}(\mathbf{R})$. Similar to the above, let $\mathbf{P}[X \in A, Y \in B]$ denote $\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \dots, X_n : \Omega \rightarrow \mathbf{R}$,

$$\mathbf{P}[X_1 \in A_1, \dots, X_n \in A_n] = \mathbf{P}(\cap_{i=1}^n X_i^{-1}(A_i)).$$

