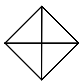


Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

PROPOSITION **1.** *Let f be a square-integrable real-valued random variable on the probability space $(X, \mathcal{A}, \mu)$. Then for $t > 0$,*

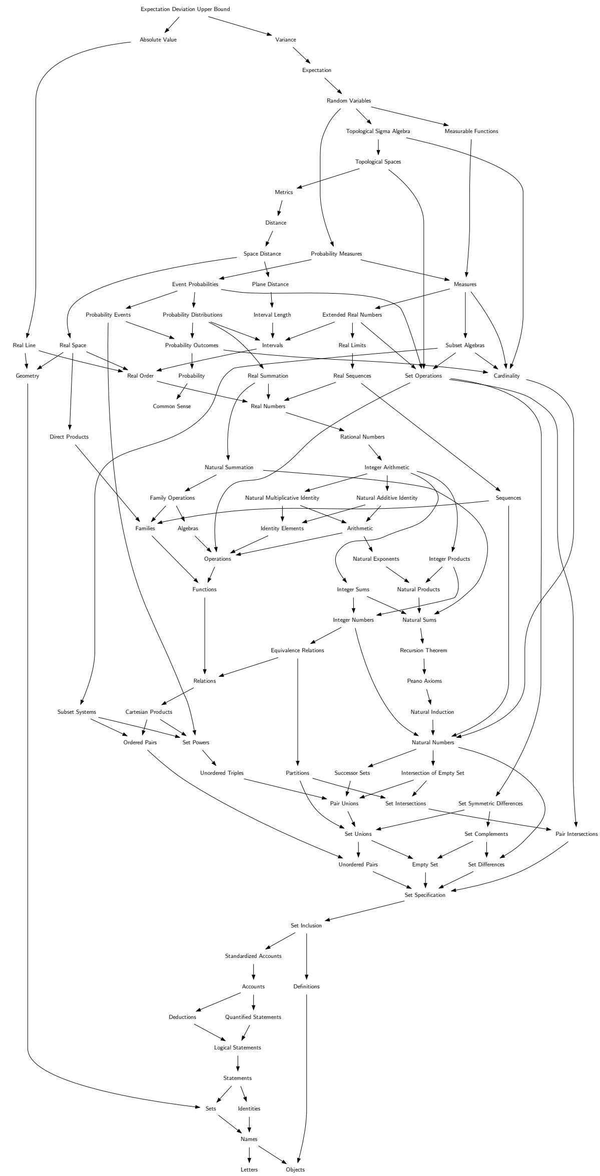
$$\mu(|f - \mathsf{E}(f)| \geq t) \leq \frac{\textsf{var}\, f}{t^2}.$$

Proof. The set $|f - \mathsf{E}(f)| \geq t$ is $\{x \in X \mid |f(x) - \mathsf{E}(f)| \geq t\}$. This set is $\{x \in X \mid (f(x) - \mathsf{E}(f))^2 \geq t^2\}$. By using the non-negative inequality

$$\mu(\left\{x \in X \;\middle|\; (f(x) - \mathsf{E}(f))^2 \geq t^2\right\}) \leq \frac{\mathsf{E}(f - \mathsf{E}(f))}{t^2}.$$

We recognize the numerator of the right hand side as the variance. □

The above is also called *Chebychev's Inequality*


Expectation Deviation Upper Bound
Absolute Value
Variance
Expectation
Random Variables
Topological Sigma Algebra
Measurable Functions
Topological Spaces
Metrics
Distance
Space Distance
Probability Measures
Event Probabilities
Plane Distance
Measures
Probability Events
Probability Distributions
Interval Length
Extended Real Numbers
Real Line
Real Space
Probability Outcomes
Intervals
Real Limits
Subset Algebras
Geometry
Real Order
Probability
Real Summation
Real Sequences
Set Operations
Cardinality
Common Sense
Real Numbers
Direct Products
Rational Numbers
Natural Summation
Integer Arithmetic
Family Operations
Natural Multiplicative Identity
Natural Additive Identity
Sequences
Families
Algebras
Identity Elements
Arithmetic
Operations
Natural Exponents
Integer Products
Functions
Integer Sums
Natural Products
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Cartesian Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Definitions
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects