



Why

Suppose a manufacturer has raw materials, and production processes by which she can convert raw materials into finished goods. How should she *allocate* the raw materials among the finished goods to maximize revenue.

Model

We model the quantities of m raw materials available to the manufacturer by m real numbers, which we denote $q_1, \dots, q_m \in \mathbf{R}_+$. We suppose that there is a correspondence, $f_i : \mathbf{R}_+ \rightarrow \mathbf{R}_+^m$, which models the quantities of the m raw materials that will be needed for the i th finished good. In other words, $f_i(x)$ is the *bill of materials* to produce quantity x of the i th finished good. We suppose finished good i can be sold for a price p_i per unit.

We formulate the following optimization problem. Given a supply of raw materials q_1, \dots, q_m , find the quantities x_1, \dots, x_n to

$$\begin{aligned} & \text{maximize} && \sum_{i=1}^n p_i x_i \\ & \text{subject to} && \sum_{i=1}^n f_i(x_i) \leq q \\ & \text{and} && x \geq 0 \end{aligned}$$

This is sometimes called an *allocation problem* or *manufacturing problem*.

Linear simplification. In the case that f_i is modeled (or idealized) as a linear function, we obtain a *linear optimization problem*.

