



## Why

We want to talk about how a function of multiple real-valued arguments changes with respect to changes in its arguments.<sup>1</sup>

## Definition

Let  $f : \mathbf{R}^d \rightarrow \mathbf{R}$ . For  $i = 1, \dots, d$ , the  *$i$ th partial derivative* of  $f$  is the function  $g_i : \mathbf{R}^d \rightarrow \mathbf{R}$  defined by

$$g_i(x) = \lim_{h \rightarrow 0} \frac{f(x + he_i) - f(x)}{h}$$

for each  $x \in \mathbf{R}^d$ .

## Gradient

The *gradient* of a multivariate function is the vector-valued function whose  $i$ th component is the the partial derivative of the function with respect to its  $i$ th argument.

## Notation

Let  $f : \mathbf{R}^n \rightarrow \mathbf{R}$ . The gradient of  $f$  is frequently denoted  $\nabla f$ . It is understood that  $(\nabla f) \in \mathbf{R}^d \rightarrow \mathbf{R}^d$ . An alternative notation is to use that similar for single derivatives and to denote the gradient (sometimes called derivative) of  $f$  by  $f'$  (assuming it exists). It is important to here note that although when  $g : \mathbf{R} \rightarrow \mathbf{R}$ ,  $g' \in (\mathbf{R} \rightarrow \mathbf{R})$ , (and so is another function from and to reals) when  $f : \mathbf{R}^d \rightarrow \mathbf{R}$ ,  $f' \in \mathbf{R}^d \rightarrow \mathbf{R}^d$ , and so is a vector-valued (not a real-valued) function.

There is (unfortunately) much notation for the individual partial derivatives; most of which we shall not (fortunately) have occasion to use in these sheets. One popular usage is the use of the  $\partial$  symbol, read aloud as “partial.” For example, if  $f : \mathbf{R}^2 \rightarrow \mathbf{R}$  is a function of two arguments, each being referred to as  $x$  and  $y$ , then  $\partial_x f$  denotes the partial derivative

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<sup>1</sup>Future editions will modify this sheet.

of  $f$  with respect to  $x$  and  $\partial_y f$  denotes the partial derivative of  $f$  with respect to  $y$ . It is understood that  $(\partial_x f) \in \mathbf{R}^d \rightarrow \mathbf{R}$ . and likewise for  $\partial_y f$ . Another popular usage is  $\partial f / \partial x$  for  $\partial_x f$  and  $\partial f / \partial y$  for  $\partial_y f$ . We will almost exclusively prefer the gradient notation.

