

Normal Densities

1 Why

We want a density that is symmetric about some central value with some spread.

2 Definition

A normal density is one which can be written as a normalized

Let $f: \mathbb{R} \to \mathbb{R}$ be a density. If there exists $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}$ with $\sigma > 0$ so that for each $x \in \mathbb{R}$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2\right)$$

then f is a normal density. Some call a normal density a Gaussian density. We often drop the word density and use refer to these as normals or Gaussians, using these words as substantives.

We call the special case when $\mu=0$ and $\sigma=1$ the standard normal density.