



## Why

We generalize our notion of *size* to  $n$ -dimensional space.

## Definition

The *norm* (or *Euclidean norm*) of  $x \in \mathbf{R}^n$  is

$$\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

## Notation

We denote the norm of  $x$  by  $\|x\|$ . In other words,  $\|\cdot\| : \mathbf{R}^n \rightarrow \mathbf{R}$  is a function from vectors to real numbers. The notation follows the notation of absolute value, the *magnitude* of a real number, and the double verticals remind us that  $x$  is a vector. A warning: some authors write  $|x|$  for the norm of  $x$  when it is understood that  $x \in \mathbf{R}^n$ .

We understand the norm of  $x$  by comparison with the distance function  $d : \mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}$ . On one hand, the norm of  $x$  is  $d(x, 0)$ . So  $\|x\|$  measures the length of the vector  $x$  from the origin 0. On the other hand,  $d(x, y) = \|x - y\|$ . So  $\|x - y\|$  measures the distance between  $x$  and  $y$ .

## Properties

The norm has several important properties

1.  $\|\alpha x\| = |\alpha| \|x\|$ , called (*absolute*) *homogeneity*,
2.  $\|x + y\| \leq \|x\| + \|y\|$ , called the *triangle inequality*,
3.  $\|x\| \geq 0$ , called *non-negativity*, and

4.  $\|x\| = 0 \longleftrightarrow x = 0$ , called *definiteness*.

**Visualization**



