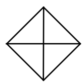


Why

If both signed measures are finite, then their difference is always well-defined. Is the difference a finite signed measure?

Preliminary result

Proposition 1. *A linear combination of finite signed measures is a finite signed measure.*

Proof. Let $(X, \mathcal{A})$ be a measurable space. Let μ and ν be finite signed measures. Let R denote the real numbers. Then $(\alpha\mu)(\varnothing) = \alpha \cdot \mu(\varnothing) = \alpha \cdot 0 = 0$. Also for $(A_n)_n \subset \mathcal{A}$ disjoint,

$$\begin{aligned}(\alpha\mu)(\cup A_n) = \alpha\mu(\cup A_n) &= \alpha \sum_{n=1}^{\infty} \mu(A_n) \\ &= \sum_{n=1}^{\infty} \alpha\mu(A_n) = (\alpha\mu)(A_n)\end{aligned}$$

Similarly, $(\mu+\nu)(\varnothing) = \mu(\varnothing)+\nu(\varnothing) = 0$. And, for $(A_n)_n \subset \mathcal{A}$ disjoint,

$$\begin{aligned}(\mu + \nu)(\cup A_n) = \mu(\cup A_n) + \nu(\cup A_n) &= \sum_{n=1}^{\infty} \mu(A_n) + \sum_{n=1}^{\infty} \nu(A_n) \\ = \sum_{n=1}^{\infty} \mu(A_n) + \nu(A_n) &= \sum_{n=1}^{\infty} (\mu + \nu)(A_n)\end{aligned}$$

□

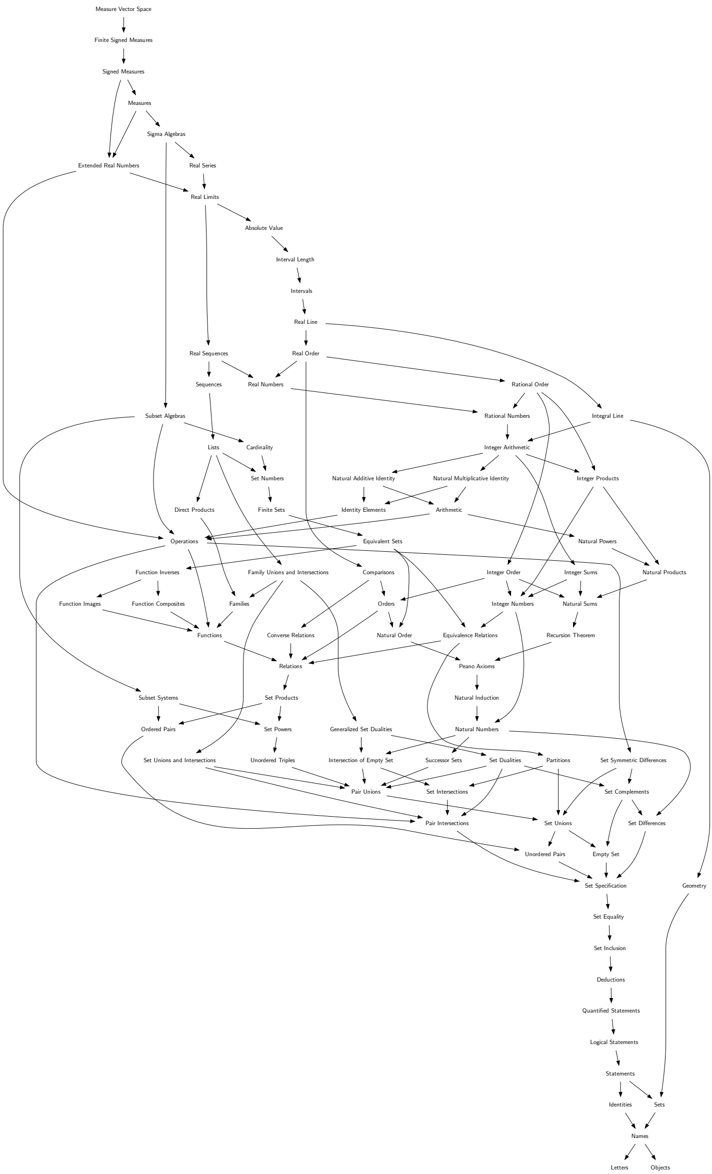
Main result

Proposition 2. *The set of finite signed measures is a vector space.*

Proof. Use the previous proposition. Observe that the function $\mu \equiv 0$ is a measure. And $\nu + \mu = \nu$ for all measures ν. □

Notation

We denote the vector space of signed measures on measurable space $(X, \mathcal{A})$ by by $M(X, \mathcal{A}, \mathbf{R})$.


Measure Vector Space
Finite Signed Measures
Signed Measures
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Sequences
Real Order
Sequences
Real Numbers
Rational Order
Subset Algebras
Rational Numbers
Integral Line
Lists
Cardinality
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Operations
Equivalent Sets
Natural Powers
Function Inverses
Family Unions and Intersections
Comparisons
Integer Order
Integer Sums
Natural Products
Function Images
Function Composites
Families
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects