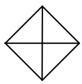


Why

We discuss inferring (or learning) functions from examples.

Definitions

A *predictor* $f : \mathcal{U} \to \mathcal{V}$ is a function from $\mathcal{U}$ to $\mathcal{V}$. An *inducer* is a function from finite datasets in $\mathcal{U} \times \mathcal{V}$ to predictors from $\mathcal{U}$ to $\mathcal{V}$. A *learner* is a function family of inducers, indexed by n, each defined for datasets of size n. We call $\mathcal{U}$ the *inputs*, $\mathcal{V}$ the *outputs*, and $f(u)$ the *prediction* of f on $u \in \mathcal{U}$.

Predicting relations

A *relation inducer* is a function from finite datasets in $\mathcal{U} \times \mathcal{V}$ to *relations* on $\mathcal{U} \times \mathcal{V}$. Since we can associate any relation R between $\mathcal{U}$ and $\mathcal{V}$ with a function $f : \mathcal{U} \times \mathcal{V} \to \{0, 1\}$, $f(u, v) = 1$ if and only if $(u, v) \in R$, the predictor case can accomodate learning general relations, beyond functions.

Notation

Let D be a dataset of size n in $\mathcal{U} \times \mathcal{V}$. Let $g : \mathcal{U} \to \mathcal{V}$, a predictor, which makes prediction $g(u)$ on input $u \in \mathcal{U}$. Let $G_n : (\mathcal{U} \times \mathcal{V})^n \to (\mathcal{U} \times \mathcal{V})$ be an inducer, so that $G_n(D)$ is the predictor which the inductor associates with dataset D. Then $\{G_n : (\mathcal{U} \times \mathcal{V})^n \to \mathcal{P}((\mathcal{U} \times \mathcal{V}))\}_{n \in \mathbf{N}}$ is a learner.

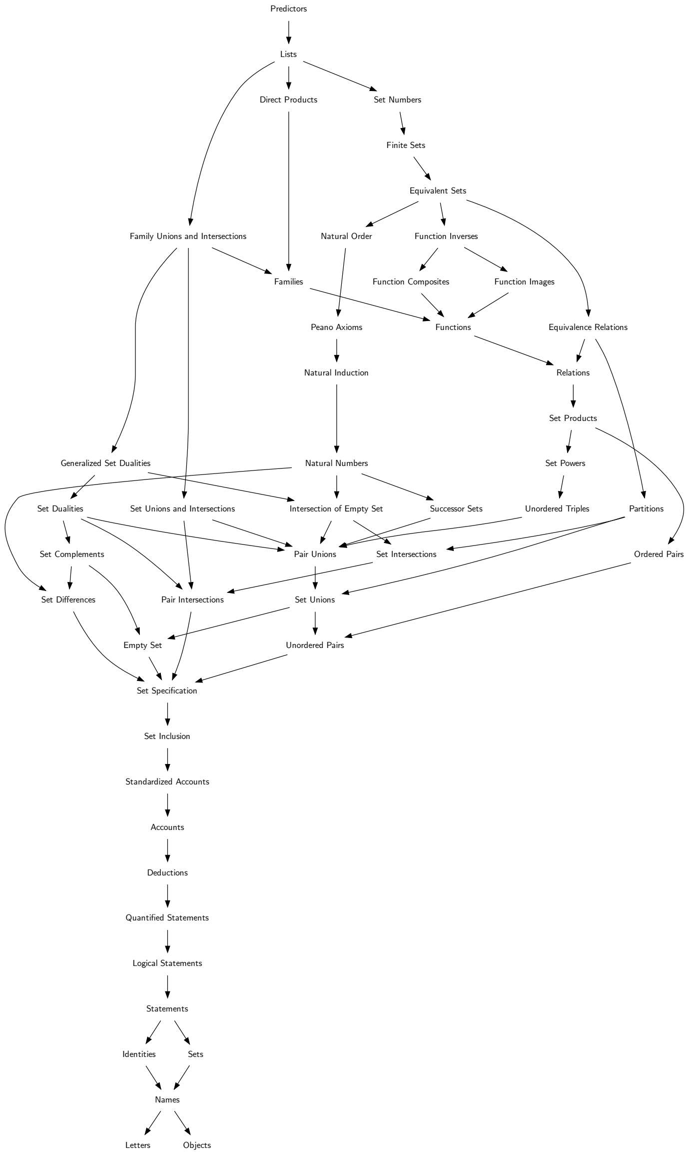
Consistent and complete datasets

Let $D = ((u_i, v_i))_{i=1}^n$ be a dataset and $R \subset X \times Y$ a relation. D is *consistent with* R if each $(u_i, v_i) \in R$. D is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take $R = \mathcal{U} \times \mathcal{V}$). D is *functionally consistent* if it is consistent with a function; in this case, $x_i = x_j \longrightarrow y_i = y_j$. D is *functionally complete* if $\cup_i \{x_i\} = X$. In this case, the dataset includes every element of the relation.

Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent variables*, *explained variables*, *postcepts*, *targets*, *outcomes*, *labels* or *observational outcomes*. An input, output pair is sometimes called a *record pair*.

Other terms for a learner include *learning algorithm*, or *supervised learning algorithm*. Other terms for a predictor include *input-output* mapping, *prediction rule*, *hypothesis*, *concept*, or *classifier*.


Predictors
Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Natural Induction
Relations
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects