Why

We want to generalize the construction of cover area as generated as a product of two cover lengths, and more generally for arbitrary measure spaces.[1]

Definition

The *product measure* of the measures of two sigma finite measure spaces is the unique measure which assigns to every rectangle with measurable sides the product of the measures of the sides. We prove that such a measure exists, and is unique.

Defining Result

Prop. 1. *Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be sigma-finite measurable spaces. There is a unique measure π on $\mathcal{A} \times \mathcal{B}$ such that*

$$\pi(A \times B) = \mu(A) \times \nu(B)$$

for all $A \in \mathcal{A}$ and $B \in \mathcal{B}$. Furthermore, for any $E \in \mathcal{A} \times \mathcal{B}$.

$$\pi(E) = \int_X \nu(E_x) d\mu(x) = \int_Y \mu(E^y) d\nu(y).\text{[2]}$$

Notation

Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be sigma-finite measurable spaces. We denote the product measure by $\mu \times \nu$. For all $A \in \mathcal{A}$ and

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[1]Future editions will expand.

[2]Future editions will include a proof.

$B \in \mathcal{B}$,
$$(\mu \times \nu)(A \times B) = \mu(A) \times \nu(B).$$

Product Measures

Measures

Extended Real Numbers

Real Limits

Absolute Value

Interval Length

Intervals

Real Line

Real Order

Real Sequences

Real Numbers

Integral Line

Rational Numbers

Product Sigma Algebras

Integer Arithmetic

Sigma Algebras

Sequences

Natural Multiplicative Identity

Natural Additive Identity

Set Numbers

Direct Products

Arithmetic

Identity Elements

Finite Sets

Natural Powers

Integer Products

Subset Algebras

Equivalent Sets

Natural Products

Integer Sums

Set Operations

Function Inverse

Family Unions and Intersections

Natural Sums

Integer Numbers

Operations

Function Composites

Function Images

Families

Recursion Theorem

Natural Order

Equivalence Relations

Functions

Cardinality

Peano Axioms

Relations

Natural Induction

Subset Systems

Set Products

Natural Numbers

Ordered Pairs

Set Powers

Generalized Set Dualities

Set Symmetric Differences

Successor Sets

Unordered Triples

Set Dualities

Intersection of Empty Set

Partitions

Set Unions and Intersections

Set Complements

Pair Unions

Set Intersections

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Geometry

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects