Why

Certain sparse matrices are easier to work with, especially those with chordal sparsity patterns.[1]

Definition

A *sparsity pattern* E of *order* n is a set of (unordered) pairs of $V = \{1, \ldots, n\}$. A sparsity pattern is *chordal* if the undirected graph (V, E) is chordal.

A symmetric matrix is said to *have a sparsity pattern* if its ijth entry is zero whenever $\{i, j\}$ is not in the sparsity pattern. The diagonal entries and off-diagonal entries for pairs appearing in the sparsity pattern may or may not be zero.

The graph whose vertices are one through n and whose edge set is the sparsity patter is called the *sparsity graph*

A sparsity pattern is not a property of a matrix because it is not unique (unless all off-diagonal entries are non-zero). If a matrix has a particular sparsity pattern it has every sparsity pattern which is a superset of it. In other words, every matrix has the sparsity pattern which is the set of all pairs of integers.

Notation

Let $E \subset \{\{i, j\} \mid i, j \in \{1, 2, \ldots, n\}\}$. A symmetric matrix $A \in \mathbf{S}^n$ is said to have sparsity pattern E if $A_{ij} = A_{ji} = 0$ wheneer $i \neq j$ and $\{i, j\} \notin E$. The graph $G = (V, E)$ where $V = \{1, 2, \ldots, n\}$ is the sparsity graph associated with E.

We will denote the symmetric matrices of order n with sparsity pattern E by $\mathbf{S}_E^n$.

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[1]Future editions will expand.

Example
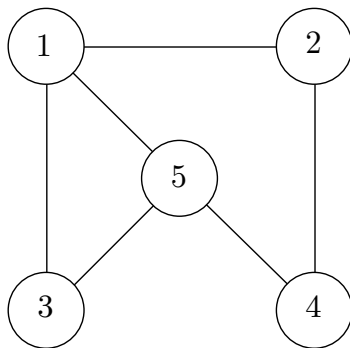
The figure above shows a sparsity graph for the matrix

$$A := \begin{bmatrix} A_{11} & A_{21} & A_{31} & 0 & A_{51} \\ A_{21} & A_{22} & 0 & A_{42} & 0 \\ A_{31} & 0 & A_{33} & 0 & A_{54} \\ 0 & A_{42} & 0 & A_{44} & A_{45} \\ A_{51} & 0 & A_{53} & A_{54} & A_{55} \end{bmatrix}.$$

Sparsity Patterns

Symmetric Matrices

Symmetric Real Matrices · Matrix Transpose

Vector as Matrices

Real Matrices · Real Inner Product · Space Inner Product

Linear Equations · Real Norm · Space Norm

Arrays · Linear Functions · Real Vector · Space Vector

Plane Vector Sums and Differences · N-Dimensional Space · Plane Norm · Plane Inner Product

Plane Vector · Distance · Absolute Value · Right Triangle Sides Relation

Space Distance · Real Squares

Plane Distance · Real Arithmetic

Interval Length · Real Multiplication Inverses

Intervals · Real Space · Real Products · Real Additive Inverses · Square Numbers

Chordal Graphs · Real Line · Real Plane · Real Sum · Squares · Rational Arithmetic · Rational Multiplicative Inverses

Undirected Subgraphs · Trees · Real Summation · Integral Line · Real Order · Geometry · Rational Sums · Rational Products

Undirected Paths · Natural Summation · Rational Order · Real Numbers · Rational Numbers

Undirected Graphs · Lists · Integer Divisors

Set Numbers · Direct Products · Integer Arithmetic

Finite Sets · Natural Additive Identity · Natural Multiplicative Identity · Integer Products

Equivalent Sets · Identity Elements · Inverse Elements · Arithmetic

Family Unions and Intersections · Family Operations · Comparisons · Natural Equations · Function Inverses · Integer Order · Element Functions · Integer Sums · Natural Powers

Families · Orders · Equations · Function Images · Function Composites · Operations · Integer Numbers · Natural Products

Natural Order · Equivalence Relations · Converse Relations · Functions · Natural Sums

Relations · Recursion Theorem

Ordered Pair Projections · Peano Axioms

Set Products · Natural Induction

Generalized Set Dualities · Set Powers · Natural Numbers

Set Unions and Intersections · Partitions · Intersection of Empty Set · Set Dualities · Unordered Triples · Successor Sets · Set Symmetric Differences

Set Intersection · Pair Unions · Ordered Pairs · Set Complements

Pair Intersections · Set Unions · Set Differences

Empty Set · Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities · Sets

Names

Letters · Objects

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