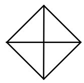


Why

[1]

Definition

The sigma algebra *generated by an event* is the sigma algebra consisting of the empty set, the event, the complement (in the base set) of the event, and the base set.

A family of events events are *independent* if the sigma algebras generated by the events are independent.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space. Let $A \in \mathcal{A}$ be an event. The sigma algebra generated by A is $\{\varnothing, A, X - A, X\}$. We denote it by $\sigma(A)$.

Let $B \in \mathcal{A}$. If A is independent of B we write $A \perp B$.

Equivalent Condition

Proposition 1. *Two events are independent if and only if the measure of their intersection is the product of their measures.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a probability space. Let $A, B \in \mathcal{A}$.

($\Rightarrow$) If $A \perp B$, then by definition $A \in \sigma(A)$ and $B \in \sigma(B)$ and so:

$$\mu(A \cap B) = \mu(A)\mu(B).$$

[1]Future editions will include

($\Leftarrow$) Conversely, let $a \in \sigma(A)$ and $b \in \sigma(B)$. If $a = \varnothing$ or $b = \varnothing$ then $a \cap b = \varnothing$. So

$$\mu(a \cap b) = \mu(\varnothing) = \mu(a)\mu(b),$$

since one of the two measures on the right hand side is zero. On the other hand, if $a = X$, then $a \cap b = b$ and so

$$\mu(a \cap b) = \mu(b) = \mu(a)\mu(b),$$

since $\mu(a) = \mu(X) = 1$. Likewise if $b = X$.

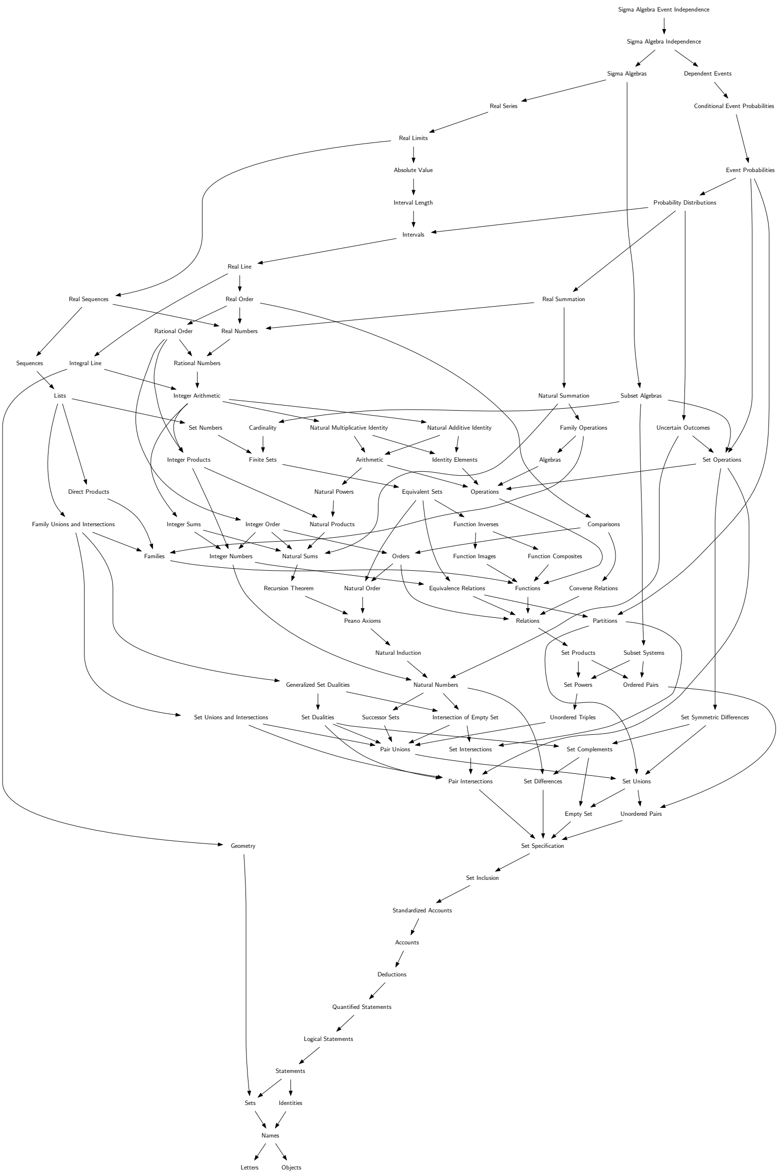
So it remains to verify $\mu(a \cap b) = \mu(a)\mu(b)$ for the cases $a \in \{A, X - A\}$ and $b \in \{B, X - B\}$. If $a = A$, and $b = B$, then the identity follows by hypothesis. Next, observe that $A \cap (X - B) = A - (A \cap B)$ and $(A \cap B) \subset A$ so $\mu(X) < \infty$ allows us to deduce:

$$\begin{aligned}
\mu(A \cap (X - B)) &= \mu(A - (A \cap B)) \\
&= \mu(A) - \mu(A \cap B) \\
&= \mu(A)(1 - \mu(B)) \\
&= \mu(A)\mu(X - A).
\end{aligned}$$

Similar for $X - A$ and B. Finally, recall that $\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B)$. So then,

$$\begin{aligned}
\mu((X - A) \cap (X - B)) &= 1 - \mu(A \cup B) \\
&= 1 - \mu(A) - \mu(B) + \mu(A \cap B) \\
&= 1 - \mu(A) - \mu(B) + \mu(A)\mu(B) \\
&= (1 - \mu(A))(1 - \mu(B)) \\
&= \mu(X - A)\mu(X - B).
\end{aligned}$$

$\square$


Sigma Algebra Event Independence
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Sigma Algebras
Dependent Events
Real Series
Conditional Event Probabilities
Real Limits
Absolute Value
Event Probabilities
Interval Length
Probability Distributions
Intervals
Real Line
Real Sequences
Real Order
Real Summation
Rational Order
Real Numbers
Sequences
Integral Line
Rational Numbers
Lists
Integer Arithmetic
Natural Summation
Subset Algebras
Set Numbers
Cardinality
Natural Multiplicative Identity
Natural Additive Identity
Family Operations
Uncertain Outcomes
Integer Products
Finite Sets
Arithmetic
Identity Elements
Algebras
Set Operations
Direct Products
Natural Powers
Equivalent Sets
Operations
Family Unions and Intersections
Integer Sums
Integer Order
Natural Products
Function Inverses
Comparisons
Families
Integer Numbers
Natural Sums
Orders
Function Images
Function Composites
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Converse Relations
Peano Axioms
Relations
Partitions
Natural Induction
Set Products
Subset Systems
Generalized Set Dualities
Natural Numbers
Set Powers
Ordered Pairs
Set Unions and Intersections
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects