

Why

If R corresponds to a line, and R^2 to a plane, and R^3 to space, does R^4 correspond to anything? What of R^5 ?

Definition

Let n be a natural number. We call the set \mathbb{R}^n n-dimensional space (or Euclidean n-space). We call elements of \mathbb{R}^n points. We identify \mathbb{R}^1 with \mathbb{R} in the obvious way.

We call the point associated with $x = (x_1, x_2, ..., x_n) \in \mathbf{R}^n$ with $x_i = 0$ for $1 \le i \le n$ the *origin*. When clear from context, we denote the origin by 0. Similarly, we denote the point x with $x_i = 1$ for all i = 1, ..., n by 1.

Visualization

We can not visualize n-dimensional space. Thus, our intuition for it comes from real space (see Real Space).

Distance

There is a natural notion of distance for \mathbb{R}^n , which is generalize the definitions of distance in \mathbb{R}^2 and \mathbb{R}^3 . We define the distance (Euclidean distance) between $(x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \in \mathbb{R}^n$ as

$$\sqrt{(x_1-y_1)^2+(x_2-y_2)^2+\cdots+(x_n-y_n)^2}$$

Does this have the properties that distance has in the plane and in space? We discussed these properties It does. Denote the function which associates to $x, y \in \mathbb{R}^n$ their distance $d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$. So d(x, y) is the distance between the points corresponding to x and y.

Proposition 1. d is non-negative, symmetric, and the distance between two points is no larger than the sum of the distances with any third object.¹

¹Future editions will include an account.

Order

Let $x, y \in \mathbb{R}^n$. If $x_i < y_i$ for all i = 1, ..., n then we say x is less than y. Likewise, if $x_i \le y_i$ for all i = 1, ..., n then we say $x \le y$. Likewise for > and \ge .

Notation

If $x \in \mathbb{R}^n$ is less than $y \in \mathbb{R}^n$ then we write x < y. Similarly for $x \le y$, x > y and $x \ge y$.

