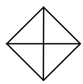


Why

We generalize the notion of sequence to index sets beyond the naturals.

Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go "further out."

To elaborate on property (b): if handed two natural numbers m and n, we can always find another, for example $\max\{m, n\} + 1$, larger than m and n. We might think of larger as "further out" from the first natural number: 1.

Now combine these two observations. A *directed set* is a set D with a partial order $\preceq$ satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.

A *net* is a function on a directed set.

Examples

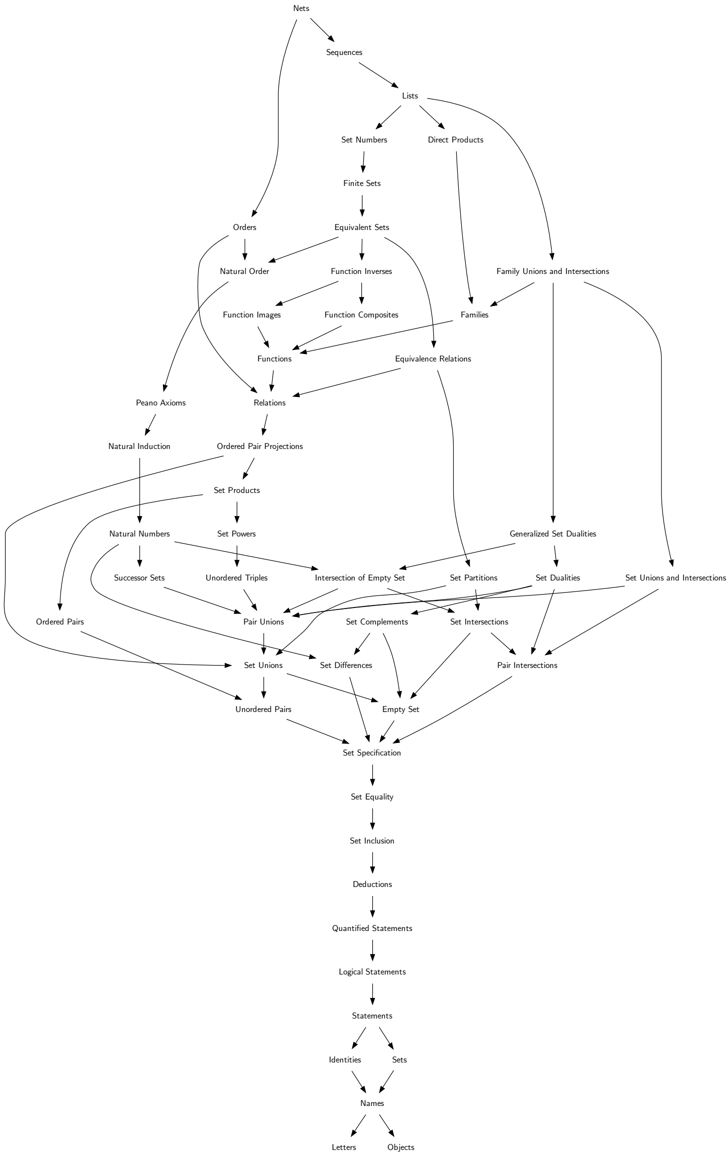
A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

Consider $\mathbf{N} \times \mathbf{N}$, and write $(a, b) \preceq (c, d)$ if $a \leq c$ *and* $b \leq d$. Clearly, $(\mathbf{N}^2, \preceq)$ so defined is a partially ordered set. Notice that given $a = (a_1, a_2)$ and $b = (b_1, b_2)$ the point $(\max(a_1, b_1), \max(a_2, b_2))$ succeeds or is equal to both a and b. Thus $(\mathbf{N}^2, \preceq)$ is a directed set on which we can define a net.

Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \to A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.


Nets
Sequences
Lists
Set Numbers
Direct Products
Finite Sets
Orders
Equivalent Sets
Natural Order
Function Inverses
Family Unions and Intersections
Function Images
Function Composites
Families
Functions
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Intersection of Empty Set
Set Partitions
Set Dualities
Set Unions and Intersections
Ordered Pairs
Pair Unions
Set Complements
Set Intersections
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects