



LEAST UPPER BOUNDS

Why

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Definition

Let A be a set and let \leq be an order² on A .

An *upper bound* for $B \subset A$ is an element $a \in A$ so that $b \leq a$ for all $b \in B$. A set is *bounded from above* if it has a least upper bound. A *least upper bound* for B is an element $c \in A$ so that c is an upper bound and $c < a$ for all other upper bounds a .

Proposition 1. *If there is a least upper bound it is unique.*³

We call the unique least upper bound of a set (if it exists) the *supremum*.

Notation

We denote the supremum of a set $B \subset A$ by $\sup A$.

¹To be given in future editions.

²To be defined in future editions, but understood in the usual way. See Natural Order or Integer Order or Rational Order etc.

³Proof in future editions.

