



Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object.¹

Definition

A *countably summable subset algebra* is a subset algebra for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of A_1, \dots, A_n coincides with the union of $A_1, \dots, A_n, A_n, A_n \dots$.

We call the set of distinguished sets a *sigma algebra* (or *sigma field*) on the base set. This language is justified (as for a regular subset algebra) by the closure properties of the sigma algebra under the usual set operations. We sometimes write σ -algebra and σ -field.

A *sub- σ -algebra* (*sub-sigma-algebra*) is a subset of a sigma algebra which is itself a sigma algebra.

¹Future editions will make no reference to measure theory. The entire development will be for follow the historical development for handling integration.

Notation

Let (A, \mathcal{A}) be a countably summable subset algebra. We often say “let \mathcal{A} be a sigma algebra on A .” Since the largest element of the sigma algebra is the base set, we can also say (without ambiguity): “let \mathcal{A} be a sigma algebra.” In this last case, the base set is $\cup \mathcal{A}$.

Examples

Example 1. *For any set A , 2^A is a sigma algebra.*

Example 2. *For any set A , $\{A, \emptyset\}$ is a sigma algebra.*

Example 3. *Let A be an infinite set. Let \mathcal{A} the collection of finite subsets of A . \mathcal{A} is not a sigma algebra.*

Example 4. *Let A be an infinite set. Let \mathcal{A} be the collection subsets of A such that the set or its complement is finite. \mathcal{A} is not a sigma algebra.*

Proposition 1. *The intersection of a family of sigma algebras is a sigma algebra.*

Example 5. *For any infinite set A , let \mathcal{A} be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

\mathcal{A} is an algebra; the countable/co-countable algebra.²

²Future editions will clean up and modify these examples.

