



**Definition**

Given a probability space  $(\Omega, \mathcal{F}, \mathbf{P})$ , a sequence of  $\mathcal{G}_1, \dots, \mathcal{G}_n$  of sub- $\sigma$ -algebras of  $\mathcal{F}$ , are *independent* if

$$\mathbf{P}(A_1 \cap A_2 \cap \dots \cap A_n) = \prod_{i=1}^n \mathbf{P}(A_i)$$

for any  $A_1 \in \mathcal{G}_1, A_2 \in \mathcal{G}_2, \dots, A_n \in \mathcal{G}_n$ . A family  $\{\mathcal{G}_i\}_{i \in I}$  is a family of sub- $\sigma$ -algebras of  $\mathcal{F}$  is *independent* if any finitely many of them are independent.



