



MINIMUM MEAN SQUARED ERROR ESTIMATOR

Why

What is the best estimator for a real-value random variable if we consider the squared loss.

Definition

We want to estimate a random variable $x : \Omega \rightarrow \mathbf{R}^n$ from a random variable $y : \Omega \rightarrow \mathbf{R}^n$ using an estimator $\phi : \mathbf{R}^n \rightarrow \mathbf{R}^n$.

Proposition 1. *The mmse estimator is the conditional mean.*

Let $x : \Omega \rightarrow \mathbf{R}^n$ and $y : \Omega \rightarrow \mathbf{R}^m$. A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* for x given y is an estimator $f : \mathbf{R}^m \rightarrow \mathbf{R}^n$ which minimizes $\mathbb{E}\|f(y) - x\|^2$.

