



## Why

We want to add repeatedly.

## Defining Result

**Proposition 1.** *For each natural number  $m$ , there exists a function  $p_m : \omega \rightarrow \omega$  which satisfies*

$$p_m(0) = 0 \quad \text{and} \quad p_m(n^+) = (p_m(n))^+ + m$$

*for every natural number  $n$ .*

*Proof.* The proof uses the recursion theorem (see Recursion Theorem).<sup>1</sup>

□

Let  $m$  and  $n$  be natural numbers. The value  $p_m(n)$  is the *product* of  $m$  with  $n$ .

## Notation

We denote the product  $p_m(n)$  by  $m \cdot n$ . We often drop the  $\cdot$  and write  $m \cdot n$  as  $mn$ .

## Properties

The properties of products are direct applications of the principle of mathematical induction (see Natural Induction).<sup>2</sup>

**Proposition 2** (Associativity). *Let  $k$ ,  $m$ , and  $n$  be natural numbers. Then*

$$(k \cdot m) \cdot n = k \cdot (m \cdot n).$$

**Proposition 3.** *Let  $m$  and  $n$  be natural numbers. Then*

$$m \cdot n = n \cdot m.$$

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<sup>1</sup>Future editions will give the entire account.

<sup>2</sup>Future editions will include the accounts.

