



Why

We look at a particular subset of vertices and the edges involved between them.

Definition

Suppose $\mathcal{G} = (V, E)$ is an undirected graph. An undirected graph (V', E') is a *subgraph* of \mathcal{G} if $V' \subset V$ and $E \subset E'$. The *vertex-induced subgraph* of an undirected graph (V, E) induced by a subset of vertices $W \subset V$ is the undirected graph with vertices W and all edges between vertices in W . The *edge-induced subgraph* of an undirected graph (V, E) induced by vertices $F \subset E$ whose edge set is F and whose vertices are those vertices adjacent to the edges of F .

Notation

Let $W \subset V$ and define F by

$$F = \{\{v, w\} \in E \mid v, w \in W\}.$$

The subgraph induced by W is the undirected graph (W, F) .

Some authors denote the subgraph induced by W by $G(W)$ or $(W, E(W))$ or $G[W]$. We avoid this notation, as it abuses G , which is no longer an ordered pair, but (in our standard function notation) now indicates a function on subsets of V with a complicated codomain. Other authors occasionally refer to the “subgraph W ”, instead of “the subgraph $G(W)$ ”. Again, we avoid this practice.

For $D \subset E$, the subgraph induced by D is the undirected graph (U, D) where

$$U = \{v \in V \mid \exists e \in D : v \in e\}$$

Similarly, people write $G(D)$ or $(V(D), D)$. We avoid this.

Connected components

A set of vertices W in G is *connected* if there is a path between any two vertices $v, w \in W$. A set of vertices W in G is *maximally connected* if there is no other vertex $v \notin W$ connected to a vertex in W . A *connected component* of G is the subgraph induced by a maximally connected set of vertices.

Since the vertex set of a graph can always be partitioned into sets maximally connected vertices, and a connected components is connect, we think of the connected components of G as the connected “pieces” of G .

Cliques

A set of vertices is *complete* if the subgraph induced is complete. A set of vertices W is *maximally complete* if the subgraph induced is complete and there is no vertex $v \notin W$ which is connected to every vertex in W . In other words, there is no other vertex which we can add to W so that the induced subgraph is still complete.

We call a *maximally complete* set of vertices a *clique*. Some authors define a clique in the way we have defined a complete set of vertices, without reference to the maximality.

