Simple Functions

1 Why

We want to define area under a real function. We start with defining functions for which this notion is obvious.

2 Definition

A *simple function* is a function whose range is a finite set.

Partition the range into the finite family of one-element sets The family whose members consist of the inverse images of these sets is a partition of the domain. We call this the *simple partition* of the function.

A *real simple function* is a simple function whose codomain is real. In this case, we can write the simple function as a sum of the characteristic functions of the inverse images elements.

2.1 Notation

Let A and B be non-empty sets. We denote the set of simple functions from A to B by $\mathcal{SF}(A, B)$.

We denote the set of simple real functions with domain A by by $\mathcal{SF}(A)$. We denote subset of non-negative simple real functions with domain A by by $\mathcal{SF}_+(A)$.

Let $f \in \mathcal{SF}(A,)$. Order the members of the range of f from 1 to n

as $r_1, \ldots, r_n$. Define $A_i = f^{-1}(\{r_i\})$. Then $f = \sum_{i=1}^{n} r_i \chi_{A_i}$.