

Why

We have considered the number of ways we can arrange 52 cards into a deck. What of n cards? A moment's reflection indicates this will also be the number of ways to arrange n objects in order (where the objects need not be cards).

Definition

By the fundamental principle of counting, there are n ways to select the first card, n-1 ways to select the second, and so on. Thus, the number of ways of stacking n cards in a deck is

$$n(n-1)(n-2)\cdots 1$$

We call this number the factorial of n, or n-factorial.

Factorial function. Define $f: \mathbf{N} \to \mathbf{N}$ recursively by f(1) = 1 and f(2) = 2f(1), and f(n) = nf(n-1) for $n \in \mathbf{N}$ (f exists by the the recursion theorem—see Recursion Theorem). f is defined such that f(n) is n factorial, for which reason we call f the factorial function. For convenience, we extend f to ω^1 by defining f(0) = 1.

Notation

We denote the factorial of n by n!, read aloud "n factorial". So for example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ and 0! = 1.

¹See Natural Numbers.

