

### **OPERATIONS**

## Why

We want to "combine" elements of a set.

### Definition

Let A be a non-empty set. An *operation* on A is a function from ordered pairs of elements of the set to the same set. Operations *combine* elements. We *operate* on ordered pairs.

#### Notation

Let A be a set and  $g: A \times A \to A$ . We tend to forego the notation g(a,b) and write a g b instead. We call this *infix notation*.

Using lower case latin letters for elements and for operations confuses, so we tend to use special symbols for operations. For example, +, -,  $\cdot$ ,  $\circ$ , and  $\star$ .

Let A be a non-empty set and  $+: A \times A \to A$  be an operation on A. According to the above paragraph, we tend to write a + b for the result of applying + to (a, b).

# Example

A first example of an operation is if we consider the set A as the power set of some set X. Then the pair union (see Pair Unions) is an operation. For if  $E \in \mathcal{P}(X)$  and  $F \in \mathcal{P}(X)$  then  $E \cup F \in \mathcal{P}(F)$  and so  $\cup$  can be viewed as an operation on  $\mathcal{P}(X)$ .

## **Properties**

Recall that  $\cup$  has several nice properties. For one  $A \cup B = B \cup A$  and  $(A \cup B) \cup C = A \cup (B \cup C)$ .

An operation with the first property, that the ordered pair (A, B) and (B, A) have the same result is called *commutative*. An operation with the second property, that when given three objects the order in which we operate does not matter is called *associative*.

