Why

[1]

Definition

A *subspace* of a vector space is a subset of vectors that is a vector space when restriction vector addition and scalar multiplication to the set. In other words, a subspace is a subset of a vector space which is closed under vector addition and scalar multiplication.

For example, the entire set of vectors is a subspace. As a second example, the set consisting only of the zero vector is a subspace; we call this the *zero subspace*. These two subspaces are the *trivial subspaces*. A *nontrivial subspace* is a subspace that is not trivial.

Notation

Let (V, F) be a vector space. Let $U \subset V$ with

$$\alpha u + \beta v \in U$$

for all $\alpha, \beta \in \mathsf{F}$ and $u, v \in U$. Then U is a subspace of (V, F).

Properties

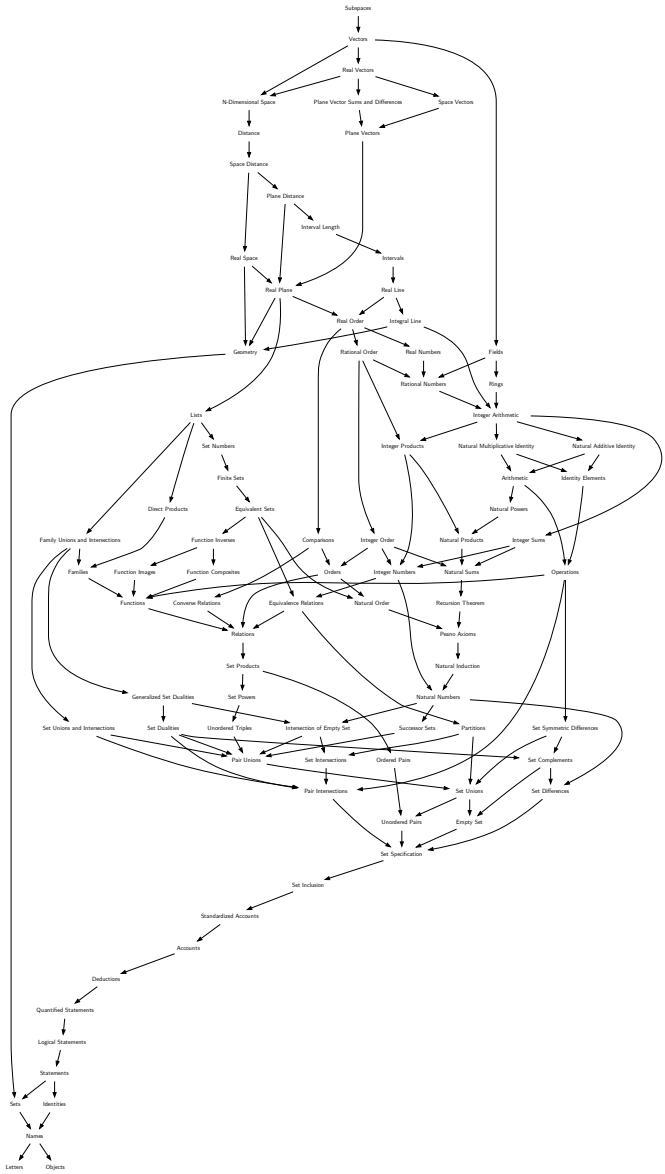
Prop. 1. *The intersection of a family of subspaces is a subspace.*

[1] Future editions will include.

Prop. 2. *There exists a family of subspaces whose union is not a subspace;*

Remark 1. *In other words: the union of a family subspaces need not be a subspace.*

Prop. 3. *A subspace must contain the zero vector; in other words, every subspace is nonempty.*


Subspaces
Vectors
Real Vectors
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Fields
Rational Numbers
Rings
Integer Arithmetic
Lists
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Natural Products
Integer Sums
Families
Function Images
Function Composites
Orders
Integer Numbers
Natural Sums
Operations
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Ordered Pairs
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects