



Why

We define integrals using an infinite process; in order for each step of the process to make sense, we need functions to be measurable.¹

Definition

A function between the base sets of two measurable spaces is *measurable* with respect to the distinguished sets of the two spaces if the inverse image of every distinguished subset of the codomain is a distinguished subset of the domain.

Notation

Let (X, \mathcal{A}) and (Y, \mathcal{B}) be measurable spaces. Then a function $f : X \rightarrow Y$ is measurable if $B \in \mathcal{B}$ implies $f^{-1}(B) \in \mathcal{A}$. We say that f is measurable with respect to \mathcal{A} and \mathcal{B} .

In this case, we sometimes say f is a measurable function from (X, \mathcal{A}) to (Y, \mathcal{B}) . We say, $f : (X, \mathcal{A}) \rightarrow (Y, \mathcal{B})$ is measurable, read aloud as “f from X, A to Y, B is measurable.”

¹This statement contains a forward reference to Real Integrals, and so may be modified in future editions.

