



## Why

Do the rational numbers correspond (in the sense Homomorphisms) to elements of the reals.

## Main result

Indeed, roughly speaking the rationals correspond to elements of the reals which are bounded above by that rational. Denote by  $\tilde{\mathbf{R}}$  the set  $\{q \in \mathbf{R} \mid \exists s \in \mathbf{Q}, q = \{t \in \mathbf{Q} \mid t < s\}\}$ .

**Proposition 1.** *The fields  $(\tilde{\mathbf{R}}, +_{\mathbf{R}} \mid \tilde{\mathbf{R}}, \cdot_{\mathbf{R}} \mid \tilde{\mathbf{R}})$  and  $(Q, +_{\mathbf{Q}}, \cdot_{\mathbf{Q}})$  are homomorphic.<sup>1</sup>*

*Proof.* The function is  $f : \mathbf{Q} \rightarrow \tilde{\mathbf{R}}$  with  $f(q) = \{r \in \mathbf{Q} \mid r < q\}$  □

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<sup>1</sup>Indeed, more is true and will be included in future editions. There is an *order perserving* field homomorphism.



