

#### INDUCTORS

# Why

We want to talk about learning associations between objects in time or space.

#### Definition

Let X and Y be sets. An *inductor* is a function mapping a dataset of paired records in  $X \times Y$  to a function from X to Y. We commonly speak of a family of inductors indexed by  $\mathbf{N}$ , one for each natural number n which is the size of the dataset.

We call the elements of X the *inputs* and the elements of Y the *outputs*. A *predictor* is a function from the inputs to the outputs and the result of an input under a predictor is a *prediction*. Using this language, an inductor maps datasets to predictors. A predictor maps inputs to outputs.

#### **Notation**

Let D be a dataset of size n in  $X \times Y$ . Let  $g: X \to Y$ , a predictor, which makes prediction g(x) on precept  $x \in X$ . Let  $G: (X \times Y)^n \to (A \to B)$  be an inductor. Then G(D) is the predictor which the inductor associates with dataset D.

## Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent* 

variables, explained variables, postcepts, targets, outcomes, labels or observational outcomes.

Other terms for a predictor include *input-output* mapping, prediction rule, hypothesis, concept, or classifier. Some authors refer to a prediction as a guess.

### (Supervised) learning algorithms

Since we use a predictor to guess inputs which do not necessarily appear in the dataset, some authors call an inductor a learner or learning algorithm. In accordance with this usage, they refer to the argument of an inductor as the training data or training dataset or training set. The word "set", however, may mislead since since we are speaking of a sequence.

It is common to refer to the construction a predictor from a dataset a *learning problem*. In this case, the learning problem is said to be *supervised learning*. By supervision, we mean to indicate that we have the outputs corresponding to the inputs. In line with this usage, the outputs are often called *labels* and the labels are said "to provide supervision."

### Consistent and complete datasets

Let  $D = ((x_i, y_i))_{i=1}^n$  and  $f: X \to Y$ . D is consistent with f if  $f(x_i) = y_i$  for all i = 1, ..., n. D is consistent if there exists a predictor with which it is consistent. If D is consistent, then  $x_i = x_j \longrightarrow y_i = y_j$ . D is complete if  $\bigcup_i \{x_i\} = X$ . If a dataset is complete, then it includes every input.

