

## FINITE SETS

## Why

As with introducing Equivalent Sets, we want to talk about the size of a set.  $^1$ 

### Definition

A finite set is one that is equivalent to some natural number; an infinite set is one which is not finite. From this we can show that  $\omega$  is infinite. This justifies the language "principle of infinity" with Natural Numbers. The principle of infinity asserts the existence of a particular infinite set; namely  $\omega$ .

### Motivation for set number

It happens that if a set is equivalent to a natural number, it is equivalent to only one natural number.

**Proposition 1.** A set can be equivalent to at most one natural number.<sup>2</sup>

A consequence is that a finite set is never equivalent to a proper subset of itself. So long as we are considering finite sets, a piece (subset) is always less than than the whole (original set).

**Proposition 2.** A finite set is never equivalent to a proper subset of itself.

#### Subsets of finite sets

Every subset of a natural number is equivalent to a natural number.<sup>3</sup> A consequence is:

Proposition 3. Every subset of a finite set is finite.<sup>4</sup>

<sup>&</sup>lt;sup>1</sup>Will be expanded in future editions.

<sup>&</sup>lt;sup>2</sup>Future edition will include proof, which uses comparability of numbers and the results of Equivalent Sets.

<sup>&</sup>lt;sup>3</sup>This requires proof, and may become a proposition in future editions.

<sup>&</sup>lt;sup>4</sup>An account will appear in future editions.

# Unions of finite sets

**Proposition 4.** If A and B are finite, then  $A \cup B$  is finite.

## Products of finite sets

**Proposition 5.** If A and B are finite, then  $A \times B$  is finite.

## Powers of finite sets

**Proposition 6.** If A is finite then  $\mathcal{P}(A)$  is finite.

## Functions between finite sets

**Proposition 7.** If A and B are finite, then  $A^B$  is finite.

