Why

We generalize our notion of norm on real vectors to abstract vector spaces.

Definition

A *norm* is a real-valued functional that is (a) non-negative, (b) definite, (c) absolutely homogeneous, (d) and satisifies a triangle inequality. The triangle inequality property requires that the norm applied to the sum of any two vectors is less than the sum of the norms on those vectors.

A *normed space* (or *norm space*) is an ordered pair: a vector space whose field is the real or complex numbers and a norm on the space. We require the vector space to be over the field of real or complex numbers because of absolute homogeneity: the absolute value of a scalar must be defined.

Notation

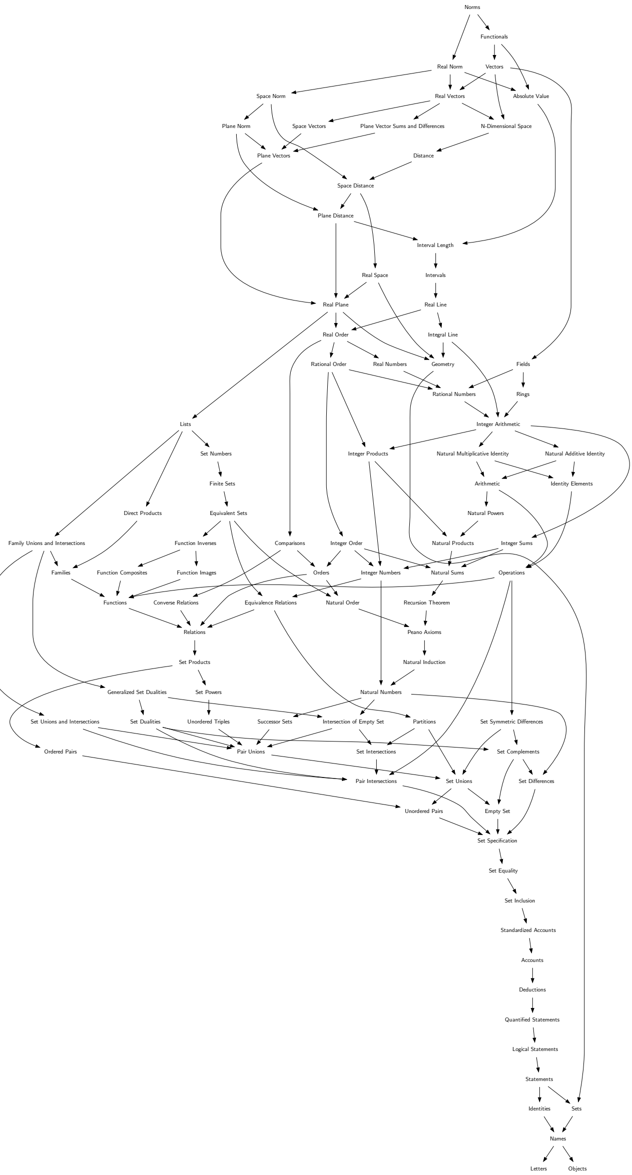
Let (X, F) be a vector space where F is the field of real numbers or the field of complex numbers. Let R denote the set of real numbers. Let $f : X \to R$. The functional f is a norm if

1. $f(v) \geq 0$ for all $x \in V$
2. $f(v) = 0$ if and only if $x = 0 \in X$.
3. $f(\alpha x) = |\alpha| f(x)$ for all $\alpha \in \mathsf{F}$, $x \in X$
4. $f(x + y) \leq f(x) + f(y)$ for all $x, y \in X$.

In this case, for $x \in X$, we denote $f(x)$ by $\|x\|$, read aloud "norm x". The notation follows the notation of absolute value as a norm. In some cases, we go further, and for a norm indexed by some parameter α or set A we write $\|x\|_\alpha$ or $\|x\|_A$.

Examples

The absolute value function is a norm on the vector space of real norms. In addition, the (Euclidean norm) is a norm on the vector space $\mathbf{R}^n$.


Norms
Functionals
Real Norm
Vectors
Space Norm
Real Vectors
Absolute Value
Plane Norm
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Rational Order
Real Numbers
Geometry
Fields
Rational Numbers
Rings
Lists
Integer Arithmetic
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Natural Products
Integer Sums
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Sums
Operations
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects