Definition

The *range* of a matrix $A \in \mathbf{R}^{m \times n}$ is the set

$$\{y \in \mathbf{R}^m \mid (\exists z \in \mathbf{R}^n)(y = Az)\}.$$

We often denote this set by $\{Az \mid z \in \mathbf{R}^n\}$. It is a subset of $\mathbf{R}^m$, the output space.

The *nullspace* of a matrix $A \in \mathbf{R}^{m \times n}$ is the set

$$\{z \in \mathbf{R}^n \mid Az = 0\}.$$

It is a subset of $\mathbf{R}^n$, the input space.

Range and Null Space

Real Matrices

Linear Equations

Arrays

Real Vectors

Linear Functions

Space Vectors

Plane Vector Sums and Differences

N-Dimensional Space

Plane Vectors

Distance

Space Distance

Plane Distance

Interval Length

Real Space

Intervals

Real Plane

Real Line

Integral Line

Real Order

Real Summation

Geometry

Real Numbers

Rational Order

Rational Numbers

Lists

Integer Arithmetic

Set Numbers

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Direct Products

Finite Sets

Identity Elements

Arithmetic

Equivalent Sets

Natural Summation

Natural Powers

Family Unions and Intersections

Function Inverses

Family Operations

Comparisons

Integer Order

Integer Sums

Natural Equations

Natural Products

Families

Function Images

Function Composites

Operation

Orders

Integer Numbers

Equations

Natural Sums

Functions

Converse Relations

Equivalence Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Set Products

Natural Induction

Generalized Set Dualities

Set Powers

Natural Numbers

Set Unions and Intersections

Set Dualities

Unordered Triples

Intersection of Empty Set

Successor Sets

Partitions

Set Symmetric Differences

Ordered Pairs

Pair Unions

Set Intersections

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects

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