



Why

We are regularly referring to a few common growth classes.

Definitions

Let $c \in \mathbf{R}$. Then we name the following growth classes

| growth class | name |
|------------------|-------------------------------------|
| $O(1)$ | <i>constant growth class</i> |
| $O(\log(x))$ | <i>logarithmic growth class</i> |
| $O((\log(x))^c)$ | <i>polylogarithmic growth class</i> |
| $O(x)$ | <i>linear growth class</i> |
| $O(x^2)$ | <i>quadratic growth class</i> |
| $O(x^c)$ | <i>polynomial growth class</i> |
| $O(c^x)$ | <i>exponential growth class</i> |

We have written these in order:

$$O(1) \subset O(\log(x)) \subset O((\log(x))^c) \subset \dots \subset O(x^c) \subset O(c^x).$$

A function that grows faster (is in the upper growth class) of a power of x is called *superpolynomial*. One that grows slower than c^n for some $c \in \mathbf{R}$ is called *subexponential*. The class $O(\log(x^c)) = O(\log(x))$ since $\log(x^c) = c \log x$. Similarly, for all $c_1, c_2 > 0$, $O(\log_{c_1}(x)) = O(\log_{c_2}(x))$.

This list is useful because of the following

Proposition 1. *Let $f, g : \mathbf{R} \rightarrow \mathbf{R}$ and defined $h : \mathbf{R} \rightarrow \mathbf{R}$ by $h = f + g$. If $O(f) \subset O(g)$, then $h \in O(g)$.*

In other words, if a function h is the sum of f and g and g is growing faster, then g (the one growing faster) determines the order of h .

