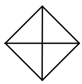


Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Prop. 1. *Let f be a square-integrable real-valued random variable on the probability space $(X, \mathcal{A}, \mu)$. Then for $t > 0$,*

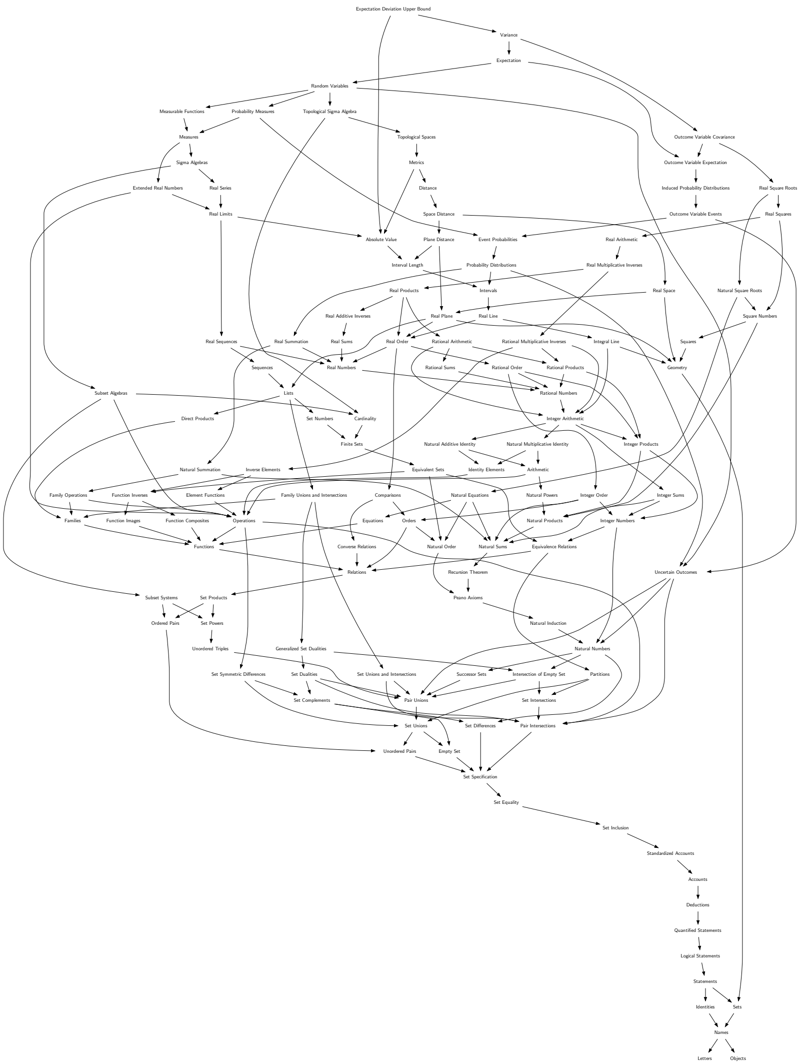
$$\mu(|f - \mathbf{E}(f)| \geq t) \leq \frac{\mathbf{var} f}{t^2}.$$

Proof. The set $|f - \mathbf{E}(f)| \geq t$ is $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$. This set is $\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}$. By using the non-negative inequality

$$\mu(\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))}{t^2}.$$

We recognize the numerator of the right hand side as the variance. □

The above is also called *Chebychev's Inequality*.


Expectation Deviation Upper Bound
Variance
Expectation
Random Variables
Measurable Functions
Probability Measures
Topological Sigma Algebra
Measures
Topological Spaces
Outcome Variable Covariance
Sigma Algebras
Metrics
Outcome Variable Expectation
Extended Real Numbers
Real Series
Distance
Induced Probability Distributions
Real Square Roots
Real Limits
Space Distance
Outcome Variable Events
Real Squares
Absolute Value
Plane Distance
Event Probabilities
Real Arithmetic
Interval Length
Probability Distributions
Real Multiplicative Inverses
Real Products
Intervals
Real Space
Natural Square Roots
Real Additive Inverses
Real Plane
Real Line
Square Numbers
Real Sequences
Real Summation
Real Sums
Real Order
Rational Arithmetic
Rational Multiplicative Inverses
Integral Line
Squares
Sequences
Real Numbers
Rational Sums
Rational Order
Rational Products
Geometry
Subset Algebras
Lists
Rational Numbers
Direct Products
Set Numbers
Cardinality
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Natural Summation
Inverse Elements
Equivalent Sets
Identity Elements
Arithmetic
Family Operations
Function Inverses
Element Functions
Family Unions and Intersections
Comparisons
Natural Equations
Natural Powers
Integer Order
Integer Sums
Families
Function Images
Function Composites
Operations
Equations
Orders
Natural Products
Integer Numbers
Functions
Converse Relations
Natural Order
Natural Sums
Equivalence Relations
Relations
Recursion Theorem
Uncertain Outcomes
Subset Systems
Set Products
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Unordered Triples
Generalized Set Dualities
Natural Numbers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects