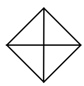


Why

What is the best linear predictor if we choose according to a particular norm.

Definition

Suppose we have a paired dataset of n records with inputs in $\mathbf{R}^d$ and outputs in $\mathbf{R}$. A *norm weighted least squares linear predictor* for a norm $g : \mathbf{R}^n \to \mathbf{R}$ is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$g(y - Ax).$$

Weight matrix

Let $\|\cdot\|_W$ be the weighted norm for some positive semidefinite weight matrix W. We want to find x to minimize

$$\|y - AX\|_W.$$

This problem is referred to by many authors as *weighted least squares* or the *weighted least squares problem.*

Diagonal weight matrix

A special case of norm weighted least squares with a weighted norm is the usual weighted least squares problem (see Weighted Least Squares Linear Predictors). Consider weighted least squares with weights $w \in \mathbf{R}^n$, $w \geq 0$. Define $W \in \mathbf{R}^{n \times n}$ so that

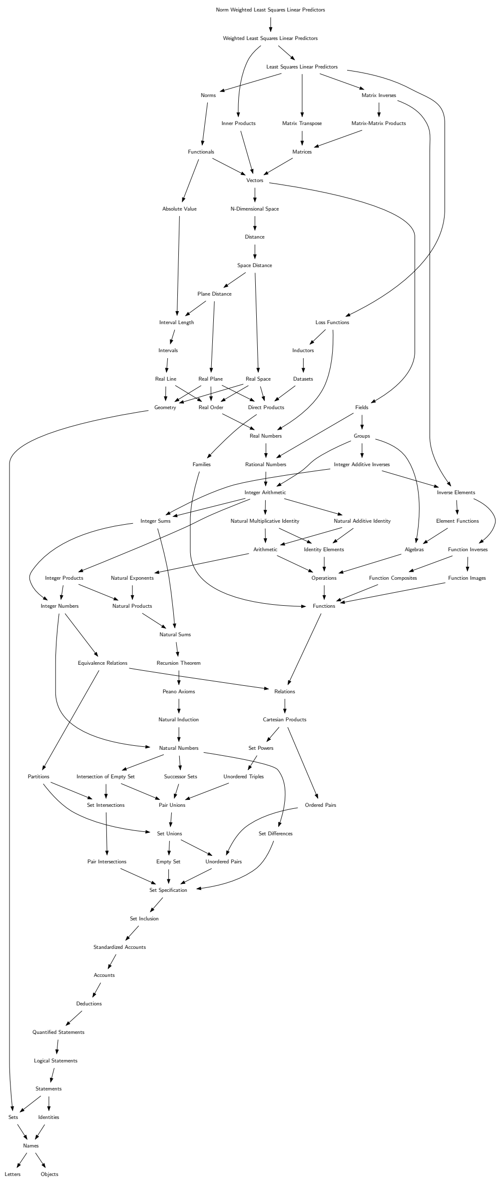
$W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix and

$$\|y - Ax\|_W = \sum_{i=1}^{n} w_i (y_i - x^\top a_i)^2.$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$\left(A^\top W A\right)^{-1} A^\top W y.$$


Norm Weighted Least Squares Linear Predictors
Weighted Least Squares Linear Predictors
Least Squares Linear Predictors
Norms
Matrix Inverses
Inner Products
Matrix Transpose
Matrix-Matrix Products
Functionals
Matrices
Vectors
Absolute Value
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Loss Functions
Intervals
Inductors
Real Line
Real Plane
Real Space
Datasets
Geometry
Real Order
Direct Products
Fields
Real Numbers
Groups
Families
Rational Numbers
Integer Additive Inverses
Integer Arithmetic
Inverse Elements
Integer Sums
Natural Multiplicative Identity
Natural Additive Identity
Element Functions
Arithmetic
Identity Elements
Algebras
Function Inverses
Integer Products
Natural Exponents
Operations
Function Composites
Function Images
Integer Numbers
Natural Products
Functions
Natural Sums
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
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Letters
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