Why

We want to generalize the construction of cover area as generated as a product of two cover lengths, and more generally for arbitrary measure spaces.[1]

Definition

The *product measure* of the measures of two sigma finite measure spaces is the unique measure which assigns to every rectangle with measurable sides the product of the measures of the sides. We prove that such a measure exists, and is unique.

Defining result

Proposition 1. *Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be sigma-finite measurable spaces. There is a unique measure π on $\mathcal{A} \times \mathcal{B}$ such that*

$$\pi(A \times B) = \mu(A) \times \nu(B)$$

for all $A \in \mathcal{A}$ and $B \in \mathcal{B}$. Furthermore, for any $E \in \mathcal{A} \times \mathcal{B}$.[2]

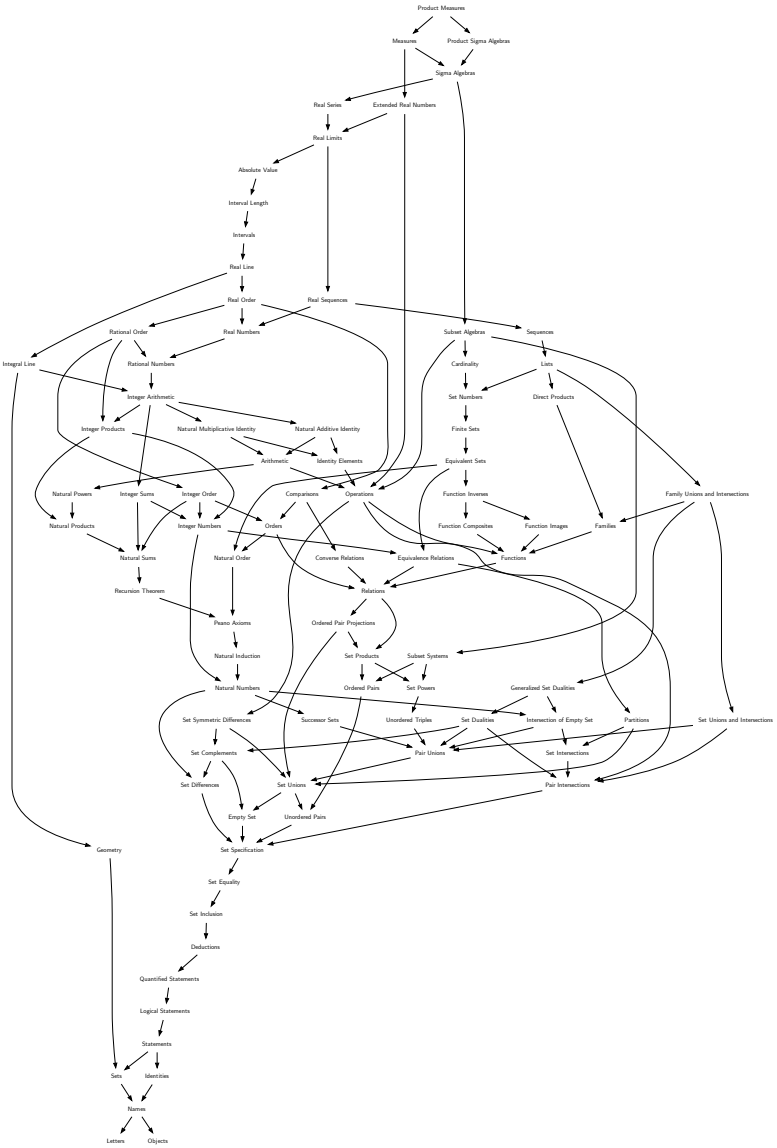
$$\pi(E) = \int_X \nu(E_x) d\mu(x) = \int_Y \mu(E^y) d\nu(y).$$

Notation

Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be sigma-finite measurable spaces. We denote the product measure by $\mu \times \nu$. For all $A \in \mathcal{A}$ and $B \in \mathcal{B}$,

$$(\mu \times \nu)(A \times B) = \mu(A) \times \nu(B).$$

[1] Future editions will expand.
[2] Future editions will include a proof.


Product Measures
Measures
Product Sigma Algebras
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Subset Algebras
Sequences
Integral Line
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Function Composites
Function Images
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Subset Systems
Natural Numbers
Ordered Pairs
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
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Letters
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