Why

We want to discuss an inductor's performance on consistent (but possibly incomplete) datasets.

We take two steps. First, put a measure on the set of training sets and only consider high-measure subsets. Second, consider predictors performing well in some tolerance.

Definition

Let $(X, \mathcal{X}, \mu)$ be a probability space and $(Y, \mathcal{Y})$ a measurable space. Let $f : X \to Y$ measurable. We call the pair $((X, \mathcal{X}, \mu), f)$ a *(supervised) probabilistic data model*.

We interpret μ as the *data-generating distribution* or *underlying distribution* and f as the *correct labeling function*. Many authors refer to a supervised probabilistic data model as the *statistical learning (theory) framework*.

Probable datasets

We put a measure on the set of datasets by using the product measure $(X^n, \mathcal{X}^n, \mu^n)$. We interpret this as a model for a training set of independent and identically distributed inputs.

For $\delta \in (0, 1)$, $\mathcal{S} \subset X^n$ is $1 - \delta$*-representative* if $\mu^n(\mathcal{S}) \geq 1 - \delta$. If $\mathcal{S}$ is $1 - \delta$-representative for small δ, we think of $\mathcal{S}$ as a set of "probable" or "reasonable" datasets. We call δ the *confidence parameter*.

Prediction error

The *error* of (measurable) $h : X \to Y$ (under μ and f) is

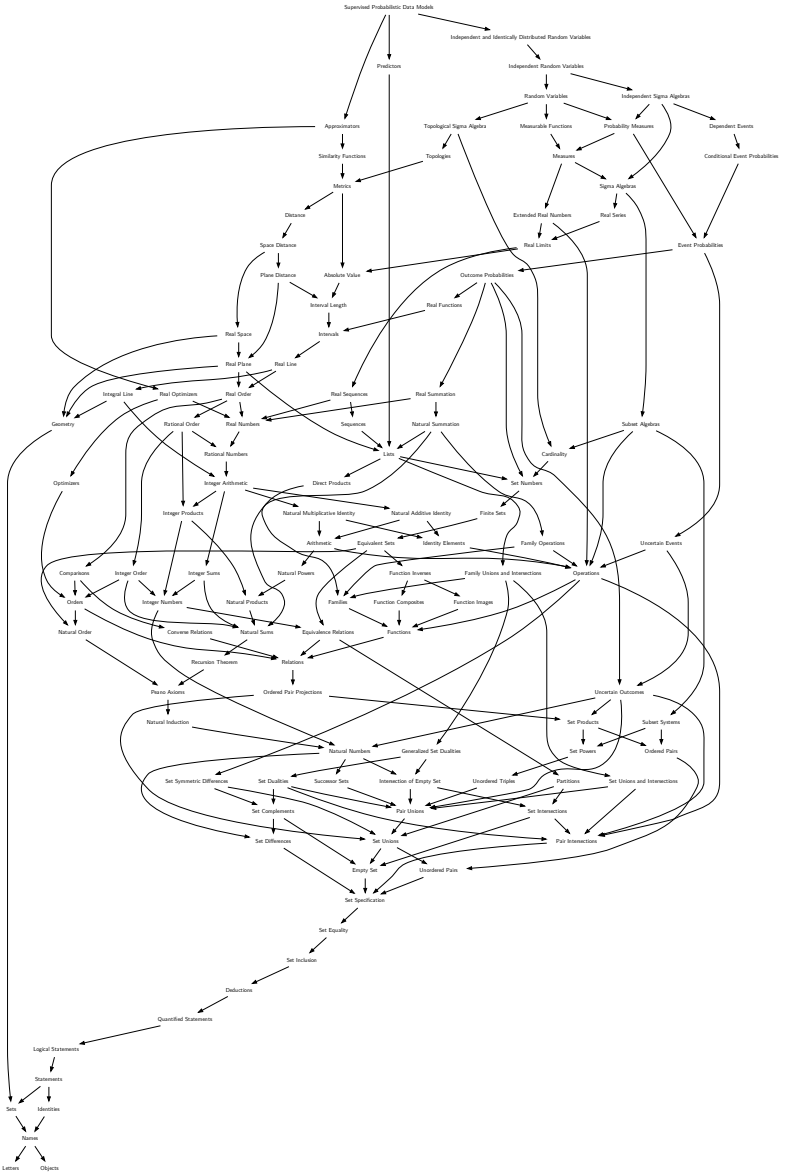
$$\operatorname*{error}_{\mu,f}(h) = \mu(\{x \in \mathcal{X} \mid h(x) \neq f(x)\}).$$

We interpret this as the probability that the predictor mislabels a point. The *accuracy* of h is $1 - \operatorname{error}_{\mu,f}(h)$.

Since $(f, g) \mapsto \mu[f(x) \neq g(x)]$ is a metric on $L^2(X, \mathcal{X}, \mu, Y)$ we can talk about the error as the "distance" from the correct label classifier. Thus we will say that $\varepsilon \in (0, 1)$, (measurable) $h : \mathcal{X} \to \mathcal{Y}$ *ε-approximates* the correct labeling function f if $\text{error}(h) \leq \varepsilon$. Roughly speaking, if $\varepsilon \ll 1$, the the error of the hypothesis is "fairly small." We call ε the *accuracy parameter*, since the accuracy of such a predictor is $1 - \varepsilon$.

Other terminology

A *hypothesis class* is a subset of the measurable functions from $X \to Y$. Other names for the error of a classifier include the *generalization error*, the *risk* or the *true error* or *loss*.


Supervised Probabilistic Data Models
Independent and Identically Distributed Random Variables
Predictors
Independent Random Variables
Random Variables
Independent Sigma Algebras
Approximators
Topological Sigma Algebras
Measurable Functions
Probability Measures
Dependent Events
Similarity Functions
Topologies
Measures
Conditional Event Probabilities
Metrics
Sigma Algebras
Distance
Extended Real Numbers
Real Series
Space Distance
Real Limits
Event Probabilities
Plane Distance
Absolute Value
Outcome Probabilities
Interval Length
Real Functions
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Optimizers
Real Order
Real Sequences
Real Summation
Geometry
Rational Order
Real Numbers
Sequences
Natural Summation
Subset Algebras
Rational Numbers
Lists
Cardinality
Optimizers
Integer Arithmetic
Direct Products
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Equivalent Sets
Identity Elements
Family Operations
Uncertain Events
Comparisons
Integer Order
Integer Sums
Natural Powers
Function Inverses
Family Unions and Intersections
Operations
Orders
Integer Numbers
Natural Products
Families
Function Composites
Function Images
Natural Order
Converse Relations
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Difference
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects