



## Why

We generalize the notion of angle between vectors in  $\mathbf{R}^2$  and  $\mathbf{R}^3$  to vectors in  $\mathbf{R}^n$ .

## Definition

The *angle* (*unsigned angle*) between the nonzero vectors  $x, y \in \mathbf{R}^n$ , is the real knumber

$$\theta = \angle(x, y) = \cos^{-1} \frac{x^\top y}{\|x\| \|y\|}.$$

In the case that one (or both) of the vectors is zero, we define the angle between them to be 0. Thus,  $x^\top y = \|x\| \|y\| \cos \theta$ , which is a convenient way to remember the inner product norm inequality.

## Terminology

$x$  and  $y$  are *aligned* if  $\theta = 0$ , in which case  $x^\top y = \|x\| \|y\|$ . In the case that  $x \neq 0$ ,  $x$  and  $y$  are aligned if  $x = \alpha y$  for some  $\alpha \geq 0$ .  $x$  and  $y$  are *opposed* if  $\theta = \pi$ , in which case  $x^\top y = -\|x\| \|y\|$ . In the case that  $x \neq 0$ ,  $x$  and  $y$  are opposed if  $x = -\alpha y$  for some  $\alpha \geq 0$ . Two nonnzero vectors  $x$  and  $y$  are *orthogonal* if  $\theta = \pi/2$  or  $-\pi/2$ , in which case  $x^\top y = 0$ . The origin is orthogonal to every other vector. We denote that two vectors  $x$  and  $y$  are orthogonal by  $x \perp y$ .



