Why

[1]

Result

This result is called sometimes called the *probability inverse transform.*

Proposition 1. *Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space and let $X : \Omega \to \mathbf{R}$ be a random variable with cumulative distribution function $F_X : \mathbf{R} \to [0,1]$. Suppose ${F_X}^{-1} : [0,1] \to \mathbf{R}$ exists, then $Y = {F_X}^{-1} \circ X$ is a random variable with cumulative distribution function $F_Y : [0,1] \to [0,1]$ satisfying $F_Y(y) = y$.*

Remark 1. *The conclusion is equivalent to the following: Y has a density and that density is the the standard unform density (see* Uniform Densities*).*

Proof. Express $F_Y(\gamma) = \mathbf{P}[Y \leq \gamma] = \mathbf{P}(Y^{-1}([0,\gamma]))$ Notice

$$\begin{aligned} Y^{-1}([0,\gamma]]) &= \{\omega \in \Omega \mid Y(\omega) \leq \gamma\} \\ &= \{\omega \in \Omega \mid F_X(X(\omega)) \leq \gamma\} \\ &= \{\omega \in \Omega \mid X(\omega) \leq F_X^{-1}(\gamma)\}. \quad = X^{-1}(\cdots).^2 \end{aligned}$$

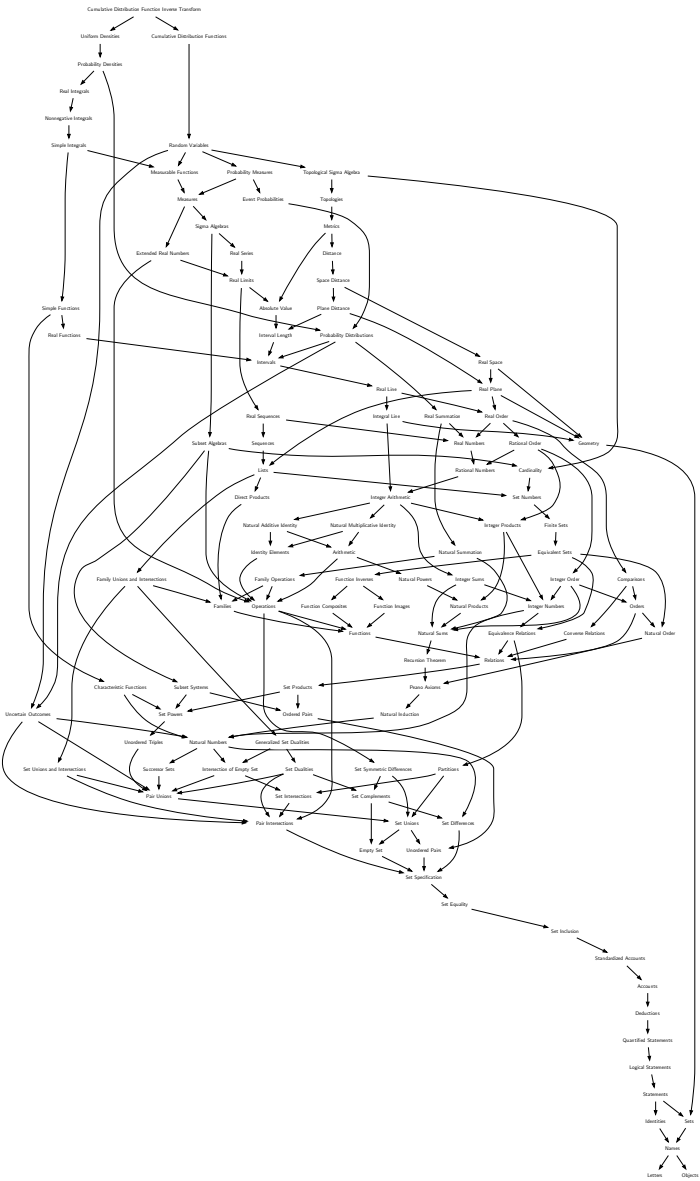
□

Remark 2. *Using different notation the above can be expressed succinctly as*

$$\begin{aligned} F_Y(\gamma) = \mathbf{P}[Y \leq \gamma] &= \mathbf{P}[F_X \circ X \leq \gamma] \\ &= \mathbf{P}[X \leq {F_X}^{-1}(\gamma)] = F_X({F_X}^{-1}(\gamma)) = \gamma. \end{aligned}$$

Future editions will discuss *inverse transform sampling.*

[1] Future editions will include.


Cumulative Distribution Function Inverse Transform
Uniform Densities
Cumulative Distribution Functions
Probability Densities
Real Integrals
Nonnegative Integrals
Simple Integrals
Random Variables
Measurable Functions
Probability Measures
Topological Signa Algebra
Measures
Event Probabilities
Topologies
Signa Algebras
Metrics
Extended Real Numbers
Real Series
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Simple Functions
Real Functions
Interval Length
Probability Distributions
Intervals
Real Space
Real Line
Real Plane
Real Sequences
Integral Line
Real Summation
Real Order
Sequences
Real Numbers
Rational Order
Geometry
Subset Algebras
Rational Numbers
Cardinality
Lists
Set Numbers
Direct Products
Integer Arithmetic
Integer Products
Finite Sets
Natural Additive Identity
Natural Multiplication Identity
Identity Elements
Arithmetic
Natural Summation
Equivalent Sets
Family Unions and Intersections
Family Operations
Function Inverses
Natural Powers
Integer Sums
Integer Order
Comparisons
Families
Operations
Function Composites
Function Images
Natural Products
Integer Numbers
Order
Functions
Natural Sums
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Characteristic Functions
Subset Systems
Set Products
Peano Axioms
Uncountable Datatypes
Set Powers
Ordered Pairs
Natural Induction
Unordered Triples
Natural Numbers
Generalized Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Set Dualities
Set Symmetric Differences
Partitions
Pair Unions
Set Intersections
Set Complements
Pair Intersection
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Name
Lemmas
Objects