



Definition

A *linear functional* on a vector space V with field k is a function $f : V \rightarrow k$ which satisfies

$$f(\lambda x + \mu y) = \lambda f(x) + \mu f(y)$$

for all $\lambda, \mu \in k$ and $x, y \in V$.

Examples

1. Define $F : \mathbf{C}^n \rightarrow \mathbf{C}$ by

$$F(x_1, \dots, x_n) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

where $c_1, \dots, c_n \in \mathbf{C}$.

2. Let $(c_n)_{n \in \mathbf{N}} \in \ell^\infty$. Define $F_c : \ell^1 \rightarrow \mathbf{C}$ by

$$F_c((x_n)_{n \in \mathbf{N}}) = \sum_{n=1}^{\infty} c_n x_n.$$

