



LOGICAL STATEMENTS

Why

We want symbols for “and”, “or”, “not”, and “implies”.¹

Overview

We call $=$ and \in *relational symbols*. They say how the objects denoted by a pair of placeholder names relate to each other in the sense of being or belonging. We call $_ = _$ and $_ \in _$ *simple statements*. They denote simple sentences “the object denoted by $_$ is the object denoted by $_$ ” and “the object denoted by $_$ belongs to the set denoted by $_$ ”. The symbols introduced here are *logical symbols* and statements using them are *logical statements*.

Conjunction

Consider the symbol \wedge . We will agree that it means “and”. If we want to make two simple statements like $a = b$ and $a \in A$ at once, we write $(a = b) \wedge (a \in A)$. The symbol \wedge is symmetric, reflecting the fact that a statement like $(a \in A) \wedge (a = b)$ means the same as $(a = b) \wedge (a \in A)$.

Disjunction

Consider the symbol \vee . We will agree that it means “or” in the sense of either one, the other, or both. If we want to say that at least one of the simple statements like $a = b$ and $a \in A$, we write $(a = b) \vee (a \in A)$. The symbol \vee is symmetric, reflecting the fact that a statement like $(a \in A) \vee (a = b)$ means the same as

¹This sheet does not explain logic. In the next edition there will be several more sheets serving this function.

$$(a = b) \vee (a \in A).$$

Negation

Consider the symbol \neg . We will agree that it means “not”. We will use it to say that one object “is not” another object and one object “does not belong to” another object. If we want to say the opposite of a simple statement like $a = b$ we will write $\neg(a = b)$. We read it aloud as “not a is b” or (the more desirable) “a is not b”. Similarly, $\neg(a \in A)$ we read as “not, the object denoted by a belongs to the set denoted by A ”. Again, the more desirable english expression is something like “the object denoted by a does not belong to the set A .” For these reasons, we introduce two new symbols \neq and \notin . $a \neq b$ means $\neg(a = b)$ and $a \notin A$ means $\neg(a \in A)$.

Implication

Consider the symbol \longrightarrow . We will agree that it means “implies”. For example $(a \in A) \longrightarrow (a \in B)$ means “the object denoted by a belongs to the object denoted by A implies the object denoted by a belongs to the set denoted by B ” It is the same as $(\neg(a \in A)) \vee (a \in B)$. In other words, if $a \in A$, then always $a \in B$. The symbol \longrightarrow is not symmetric, since implication is not symmetric. The symbol \longleftrightarrow means “if and only if”.

