Why

If A and B are two events in some finite sample space Ω and P is the event probability function of some distribution, we have seen that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

What if we have three events A, B, C? What of a list of n events $A_1, \ldots, A_n$?

Case of $n = 3$.

Proposition 1. *Suppose A, B, C are events in a finite sample space and P is any event probability function. Then*

$$\begin{aligned}
P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) \\
& - P(A \cap B) - P(A \cap C) - P(B \cap C) \\
& + P(A \cap B \cap C)
\end{aligned}$$

Proof. Express

$$\begin{aligned}
P(A \cup B \cup C) &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\
&= P(A) + P(B) + P(C) - P(A \cap B) - P((A \cup B) \cap C)
\end{aligned}$$

by using the inclusion-exclusion formula. Recall[1] that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

Now use the inclusion-exclusion formula again, and properties of set pair intersections to get

$$\begin{aligned}
P((A \cup B) \cap C) &= P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C)) \\
&= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)
\end{aligned}$$

$\square$

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[1] See Set Unions and Intersections.

Case of n.

Proposition 2. *Suppose $A_1, \ldots, A_n$, are events in a finite sample space and P is any event probability function. Then*

$$P(A_1 \cup \cdots \cup A_n) = \sum P(A_i) - \sum P(A_i \cap A_j)$$
$$\sum P(A_i \cap A_j \cap A_k) - \cdots$$
$$(-1)^{n-1} P(A_1 \cap \cdots A_n)$$

Proof. This can be shown by induction on the number of events n. Future editions will include. $\qquad\square$

Here there are $\binom{n}{r}$ terms in the rth sum, $r = 1, \ldots, n$.

Generalized Inclusion-Exclusion Formula

Event Probabilities

Outcome Probabilities

Real Functions

Intervals

Real Line

Real Order

Real Summation

Size of Direct Product

Rational Order Real Numbers

Natural Summation Number of Set Products

Integral Line Rational Numbers

Number of Disjoint Unions Finite Set Examples Lists

Integer Arithmetic

Family Operations Set Number Direct Products Family Unions and Intersections

Natural Additive Identity Natural Multiplicative Identity Natural Fractions

Finite Sets Families

Integer Products Identity Elements Natural Arithmetic

Equivalent Sets Function Inverse Uncertain Events

Comparisons Integer Order Integer Sums Natural Powers Function Image Function Composites

Orders Integer Numbers Natural Products Operations Functions

Converse Relations Natural Sums Natural Order Equivalence Relations

Recursion Theorem Relations Set Unions and Intersections

Peano Axioms Ordered Pair Projections Uncertain Outcomes

Natural Induction Set Products Set Powers Generalized Set Dualities

Natural Numbers Unordered Triples Intersection of Empty Set Set Dualities Partitions

Successor Sets Set Symmetric Difference Pair Union Set Complements Set Intersections

Ordered Pairs Set Differences Set Unions Empty Set Pair Intersections

Unordered Pairs

Set Specification

Geometry Set Equality

Set Inclusion

Deduction

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects