Why

Do solutions exist to a linear optimization problem which is feasible and bounded? Yes.

Result

Proposition 1. *Suppose* $A \in \mathbf{R}^{m \times n}$, $b \in \mathbf{R}^n$, *and* $c \in \mathbf{R}^n$ *so that*

$$P = \{x \in \mathbf{R}^n \mid Ax \leq b\} \neq \varnothing$$

and

$$\delta = \inf\{c^\top x \mid x \in P\} > -\infty$$

Then there exists $x^\star \in \mathbf{R}^n$ *with* $c^\top x^\star = \delta$.

For this reason, a linear program is sometimes abbreviated $\min\{cx \mid Ax \leq b\}$ instead of $\inf\{c^\top x\} Ax \leq b$.


Linear Optimization Solutions
Linear Optimization Problems
Real Polyhedra
Real Halfspaces
Topological Interiors
Real Hyperplanes
Topological Neighborhoods
Real Balls
Subspace Orthogonal Complements
Topologies
Inner Product Linear Functional Representations
Subspace Dimensions
Vector Space Dimensions
Dual Spaces
Continuous Linear Functionals
Dual Vector Spaces
Complete Real Inner Product Spaces
Vector Space Bases
Affine Set Dimensions
Linear Functionals
Complete Normed Spaces
Orthogonal Complements
Linearly Dependent Vectors
Span
Affinely Independent Vectors
Sequence Spaces
Norm Metrics
Complete Metric Spaces
Real Inner Product Norms
Vector Subspaces
Real Subspace Dimensions
Real Affine Hulls
Real Affine Sets and Subspaces
Real Series
Norms
Vectors as Matrices
Real Inner Products
Real Subspaces
Real Vector Bases
Real Affine Sets
Real Translates
Second Derivatives
Functionals
Real Matrices
Real Inner Product
Linear Combinations
Real Vectors Span
Independent Set of Real Vectors
Real Affine Combinations
Optimization Problems
Real Differentiable Functions
Real Open Sets
Egoprox Sequences
Real Linear Equations
Space Inner Product
Real Norm
Vectors
Real Linear Combinations
N-Dimensional Lines
Extended Real Numbers
Real Functions
Real Egoprox Sequences
Metrics
Arrays
Linear Functions
Space Norm
Real Vectors
Real Sequence Space
Complex Numbers
Real Limits
Plane Inner Product
Plane Norm
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Real Sequences
Absolute Value
Real Square Roots
Right Triangle Sides Relation
Plane Vectors
Distance
Real Squares
Space Distance
Optimizers
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Sequences
Interval Length
Real Products
Natural Square Roots
Intervals
Real Space
Greatest Lower Bounds
Real Additive Inverses
Real Line
Square Numbers
Real Summation
Real Plane
Rational Multiplicative Inverses
Real Order
Real Sums
Rational Arithmetic
Integral Line
Natural Summation
Squares
Rational Order
Rational Products
Real Numbers
Rational Sums
Lists
Geometry
Fields
Rational Numbers
Set Numbers
Direct Products
Rings
Comparisons
Integer Order
Natural Fractions
Integer Arithmetic
Finite Sets
Orders
Inverse Elements
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Natural Equations
Function Inverses
Element Functions
Natural Arithmetic
Identity Elements
Integer Sums
Natural Order
Family Operations
Family Unions and Intersections
Integer Numbers
Function Composites
Function Images
Operations
Equations
Natural Powers
Families
Converse Relations
Equivalence Relations
Functions
Natural Products
Relations
Natural Sums
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Complements
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects