Groups

1 Why

We generalize the algebraic structure of addition over the integers.

2 Definition

Let $(A, +)$ be an algebra.

We call $e \in A$ an **identity** if (1) $e + a = e$ and (2) $a + e = e$ for all $a \in A$. If only (1) holds, we call e a **left identity**. If only (2) holds, we call e a **right identity**.

We call $b \in A$ an **inverse** of $a \in A$ if (1) $b + a = e$ and (2) $a + b = e$. If only (1) holds, we call e a **left inverse**. If only (2) holds, we call e a **right inverse**.

A **group** is an algebra $(A, +)$ where $+$ is associative, there exists an identity element in A, and there exists an inverse for each element of A. A **commutative group** is a group $(A, +)$ where $+$ commutes. A commutative group is also called an **Abelian group**.

2.1 Notation

$\boxed{\mathit{TODO}}$