



Why

The integrable functions are a vector space.

Definition

The *integrable function space* corresponding to a measure space is the set of real-valued functions which are integrable with respect to the measure. The term space is appropriate because this set is a real vector space. If we scale an integrable function, it remains integrable. If we add two integrable functions, the sum is integrable. Thus, a linear combination of integrable functions is integrable. The zero function is the zero element of the vector space.

The open question is: what elements of our geometric intuition can we bring to a space of functions. Do functions have a size? Are certain functions near each other?

Notation

Let (X, \mathcal{A}, μ) be a measure space.

We denote set the real-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, \mathbf{R})$, read aloud as “the real integrable functions on the measure space X script \mathcal{A} mu.” We denote set the complex-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, \mathbf{C})$, read aloud as “the complex integrable functions on the measure space X script \mathcal{A} mu.” When the field is irrelevant, we denote them by $\mathcal{I}(X, \mathcal{A}, \mu)$, read aloud as “integrable functions on the measure space X script \mathcal{A} mu.” The \mathcal{I} is a mnemonic for “integrable.”

