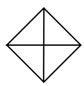


Why

We compress the notation for linear equations.

Definition

A *real matrix* (*matrix*, *rectangular array*) is a two-dimensional array of real numbers. We denote a matrix elements in a grid between two rectangular braces, as in

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}.$$

We call n and m the *dimensions* of the matrix. We call n the *height* and m the *width*. If the height of the matrix is the same as the width of the matrix then we call the matrix *square*. If the height is larger than the width, we call the matrix *tall*. If the width is larger than the height, we call the matrix *wide*.

Linear equations

Recall that we are interested in solutions of the linear equations

$$\begin{aligned} y_1 &= A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n, \\ y_2 &= A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n, \\ &\vdots \\ y_n &= A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n. \end{aligned}$$

We have suggestively used the notation A_{ij} for the coefficients of the equations, so they are the entries of $A \in \mathbf{R}^{m \times n}$.

A primer on matrix-vector products

Using the notation $A \in \mathbf{R}^{m \times n}$ and $x \in \mathbf{R}^n$ we want a compressed way to write the above system of linear equations. Define the *real matrix-vector product* $z \in \mathbf{R}^m$ of A with x by

$$z_i = \sum_{j=1}^{n} A_{ij} x_j, \quad i = 1, \dots, m.$$

We denote the matrix vector product z by Ax.

Notation

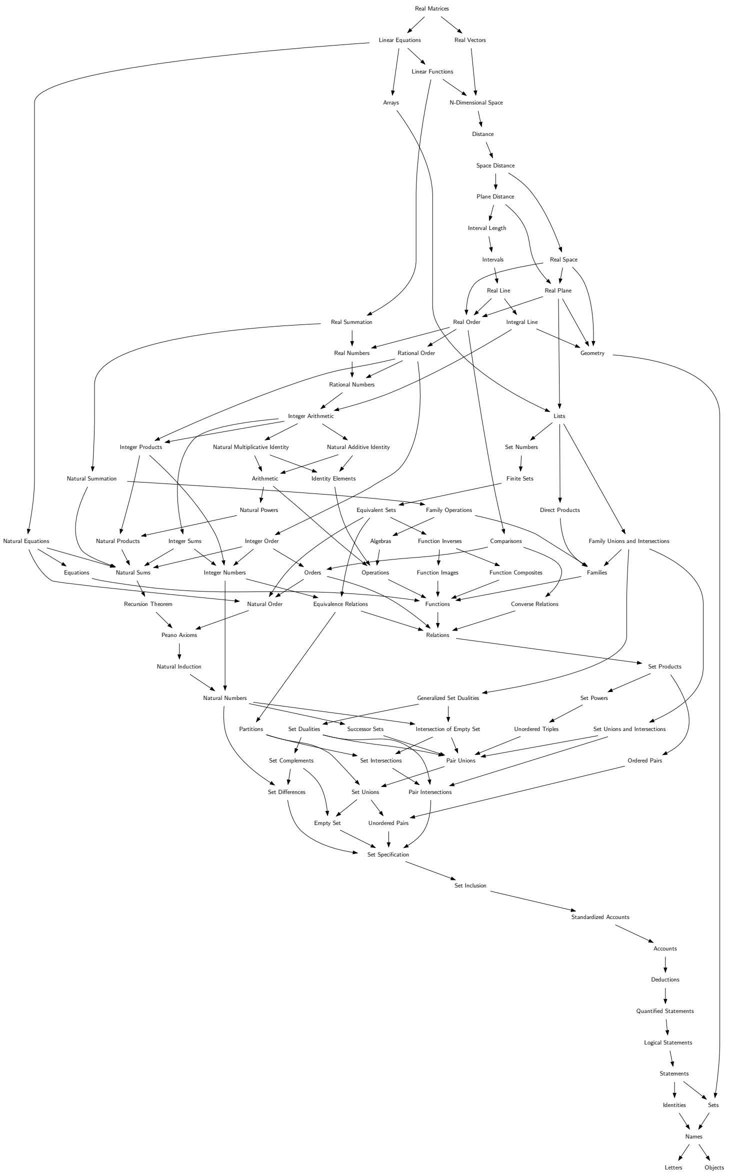
We express the above system of linear equations as

$$y = Ax,$$

where $y = (y_1, \dots, y_n) \in \mathbf{R}^n$ and $x = (x_1, \dots, x_n) \in \mathbf{R}^n$. The compact notation $y = Ax$ is sometimes called the *matrix form* of the m linear equations and A the *coefficient matrix*.

Note on terminology

The word "matrix" is from the Latin "mater," meaning mother, and has an old sense in English similar "womb." The matrix is source of many determinants (discussed elsewhere).



Real Matrices
Linear Equations
Real Vectors
Linear Functions
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Summation
Real Order
Integral Line
Real Numbers
Rational Order
Geometry
Rational Numbers
Integer Arithmetic
Lists
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Natural Summation
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Equivalent Sets
Family Operations
Direct Products
Natural Equations
Natural Products
Integer Sums
Integer Order
Algebras
Function Inverses
Comparisons
Family Unions and Intersections
Equations
Natural Sums
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Converse Relations
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Partitions
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Unions and Intersections
Set Complements
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Logical Statements
Statements
Identities
Sets
Names
Letters
Objects