

REAL AFFINE COMBINATIONS

Definition

An affine combination from \mathbb{R}^n is a linear combination whose scalars sum to 1. As with linear combinations, we say that y is an can be written as an affine combination of of the vectors $x_1, \ldots, x_k \in \mathbb{R}^n$ if there exists $\lambda_1, \ldots, \lambda_k \in \mathbb{R}$ so that

$$y = \sum_{i=1}^{k} \lambda_i x_i$$

and $\sum_{i=1}^{k} \lambda_i = 1$.

All affine combinations of two distint vectors $x, y \in \mathbf{R}^n$ is the line through x and y, which we denote as usual L(x, y). In other words,

$$L(x,y) = \{(1-\lambda)x + \lambda y \mid \lambda \in \mathbf{R}\}\$$

A set of vectors $\{v_1, \ldots, v_k\}$ is affinely dependent if one can be written as an affine combination of the others. A set of vectors which is not affinely dependent is called an affinely independent set of vectors. An equivalent condition is that there exist an affine combination in these vectors in which at least one scalar is nonzero and the sum of all the scalars is 1.

Proposition 1. Suppose $y \in \mathbb{R}^n$. The set $X = \{x_1, \dots, x_k\} \subset \mathbb{R}^n$ is affinely dependent if and only if y + X is.

Proposition 2. Suppose $y \in \mathbb{R}^n$. The set $X = \{x_1, \dots, x_k\} \subset \mathbb{R}^n$ is affinely dependent if and only if y + X is.

