Why

We can interpret the codomain of a random variable as a new sample space, since the underlying probability distribution induces a new probability distribution.

Definition

Let $p : \Omega \to \mathbf{R}$ be a probability distribution and $x : \Omega \to V$ an outcome variable. Define $q : V \to \mathbf{R}$ by

$$q(a) = \mathbf{P}[x = a].$$

Since events $x^{-1}(a)$ for $a \in V$ partition Ω, $\sum_{a \in A} q(a) = 1$. We call q the *induced distribution* (or *induced probability mass function*) of the random variable x. Thus we can think of V as a set of outcomes, which we call the outcomes *induced* by x.

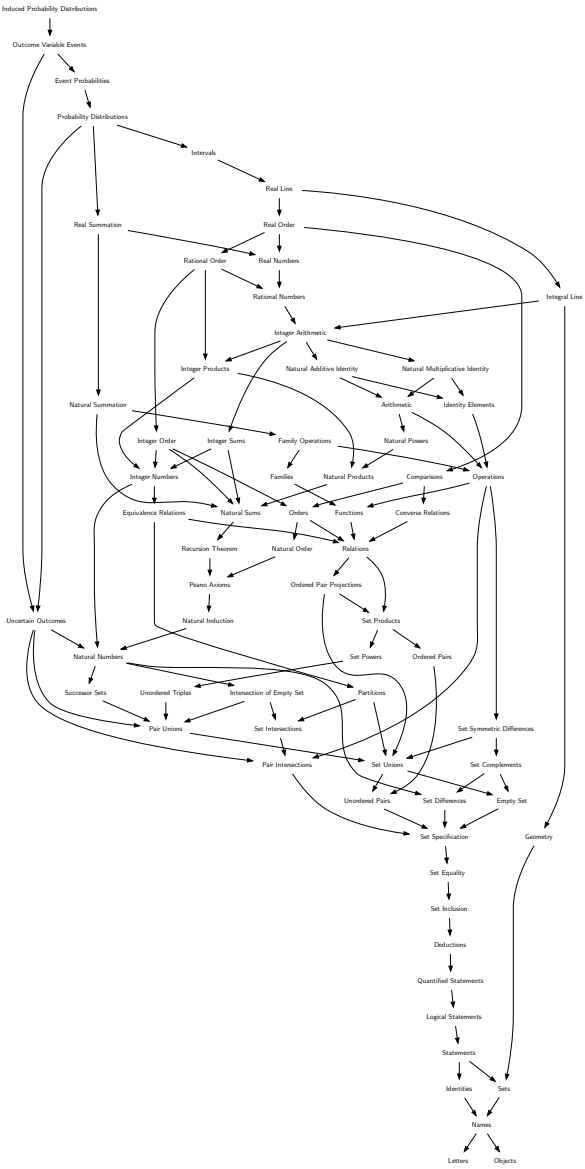
Notation

It is common to denote it by p_x.

If $x : \Omega \to V$ is a random variable and $f : V \to U$, then if we define $y : \Omega \to V$ so that $y \equiv f(x)$, y is a random variable with induced distribution $p_y : \Omega \to \mathbf{R}$ satisfying

$$p_y(b) = \textstyle\sum_{a \in V \mid y(a) = b} p_x(a).$$

As a matter of practical computation, we can evaluate probabilities having to do with the outcome variable x using p_x instead of p. For example with x as in the example above, $\mathbf{P}(x = 4 \text{ or } x = 5) = p_x(4) + p_x(5)$, rather than $\sum_{\omega \in \Omega \mid x(\omega) = 4 \text{ or } x(\omega) = 5} p(\omega)$.


Induced Probability Distributions
Outcome Variable Events
Event Probabilities
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Real Numbers
Rational Order
Rational Numbers
Integral Line
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Summation
Integer Order
Integer Sums
Family Operations
Natural Powers
Integer Numbers
Families
Natural Products
Comparisons
Operations
Equivalence Relations
Natural Sums
Orders
Functions
Converse Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Set Differences
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects