## Why

Since our predictions are often uncertain, we can use the language of probability distributions to characterize them.[1]

## Definition

Denote the set of probability distributions on a set $X$ by $\Delta(X)$.

A *probabilistic classifier* $G : A \to \Delta(B)$ is a function from inputs $A$ to probability distributions over the classes $B$.

Given an input $a$, the *prediction* of $G$ on $a$ is a probability distribution $\hat{p}_a = G(a)$ on $B$.

### Point classifier as probabilistic classifier

Given a point classifier $f : A \to B$, we can define a probabilistic classifier $G : A \to \Delta(B)$ corresponding to $f$ by

$$\hat{p}_a(b) = \begin{cases} 1 & \text{if } f(a) = b \\ 0 & \text{otherwise.} \end{cases}$$

where $\hat{p}_a = G(a)$.

### Probabilistic classifier from point classifier

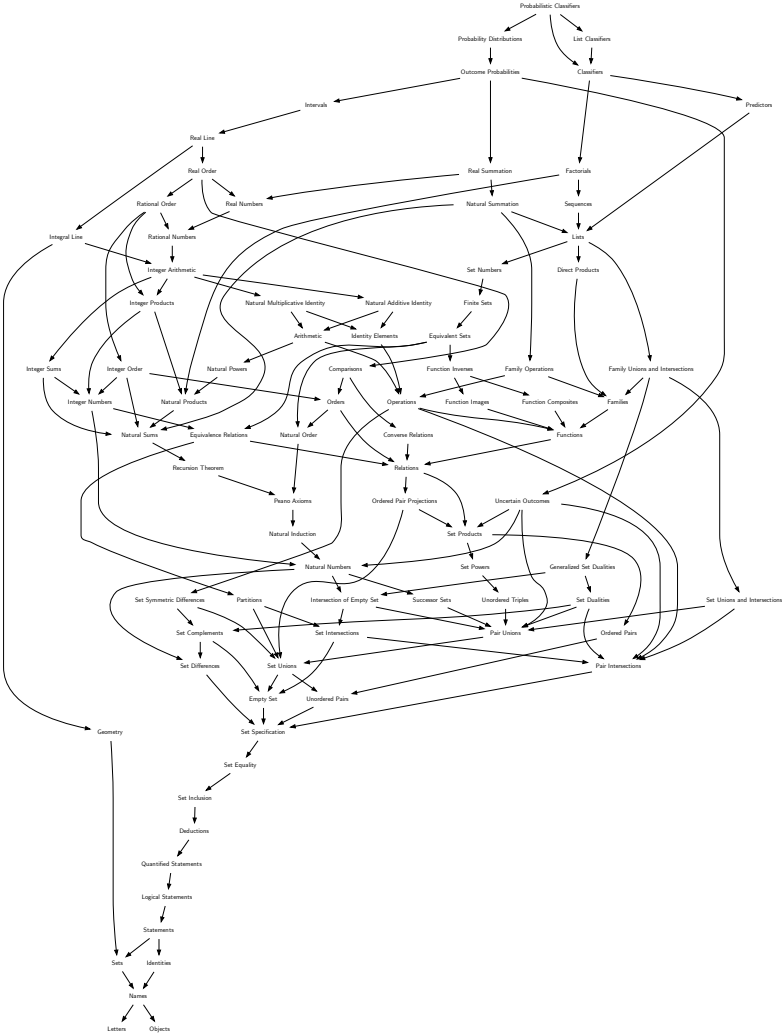
On the other hand, given probabilistic classifier $G : A \to \Delta(B)$, we can define a point classifier $f : A \to B$ by

$$f(a) = \operatorname*{argmax}_{b \in B} \hat{p}_a(v)$$

We call $f$ the *maximum likelihood classifier* corresponding to $G$. If there are ties, we can order the (finite) set $B$ arbitrarily, and break ties accordingly.

We can extend this idea, and define a list classifier by sorting the outputs by their probability, from largest to smallest.

[1] Future editions will improve this.


Probabilistic Classifiers
Probability Distributions
List Classifiers
Outcome Probabilities
Classifiers
Intervals
Predictors
Real Line
Real Order
Real Summation
Factorials
Rational Order
Real Numbers
Natural Summation
Sequences
Integral Line
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Integer Sums
Integer Order
Natural Powers
Comparisons
Function Inverses
Family Operations
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects