

FINITE SETS

Why

As with introducing Equivalent Sets, we want to talk about the size of a set.¹

Definition

A finite set is one that is equivalent to some natural number; an infinite set is one which is not finite. From this we can show that ω is infinite. This justifies the language "principle of infinity" with Natural Numbers. The principle of infinity asserts the existence of a particular infinite set; namely ω .

Motivation for set number

It happens that if a set is equivalent to a natural number, it is equivalent to only one natural number.

Proposition 1. A set can be equivalent to at most one natural number ²

A consequence is that a finite set is never equivalent to a proper subset of itself. So long as we are considering finite sets, a piece (subset) is always less than than the whole (original set).

Proposition 2. A finite set is never equivalent to a proper subset of itself.

¹Will be expanded in future editions.

²Future edition will include proof, which uses comparability of numbers and the results of Equivalent Sets).

Subsets of finite sets

Every subset of a natural number is equivalent to a natural number.³ A consequence is:

Proposition 3. Every subset of a finite set is finite.⁴

Unions of finite sets

Proposition 4. if A and B are finite, then $A \cup B$ is finite.

Products of finite sets

Proposition 5. If A and B are finite, then $A \times B$ is finite.

Powers of finite sets

Proposition 6. If A is finite then $\mathcal{P}(A)$ is finite.

Functions between finite sets

Proposition 7. If A and B are finite, then A^B is finite.

³This requires proof, and may become a proposition in future editions.

 $^{^4\}mathrm{An}$ account will appear in future editions.

