Why

We further drop conditions on the structure of the binary operations, and study only the algebraic structure of addition over the integers.

Definition

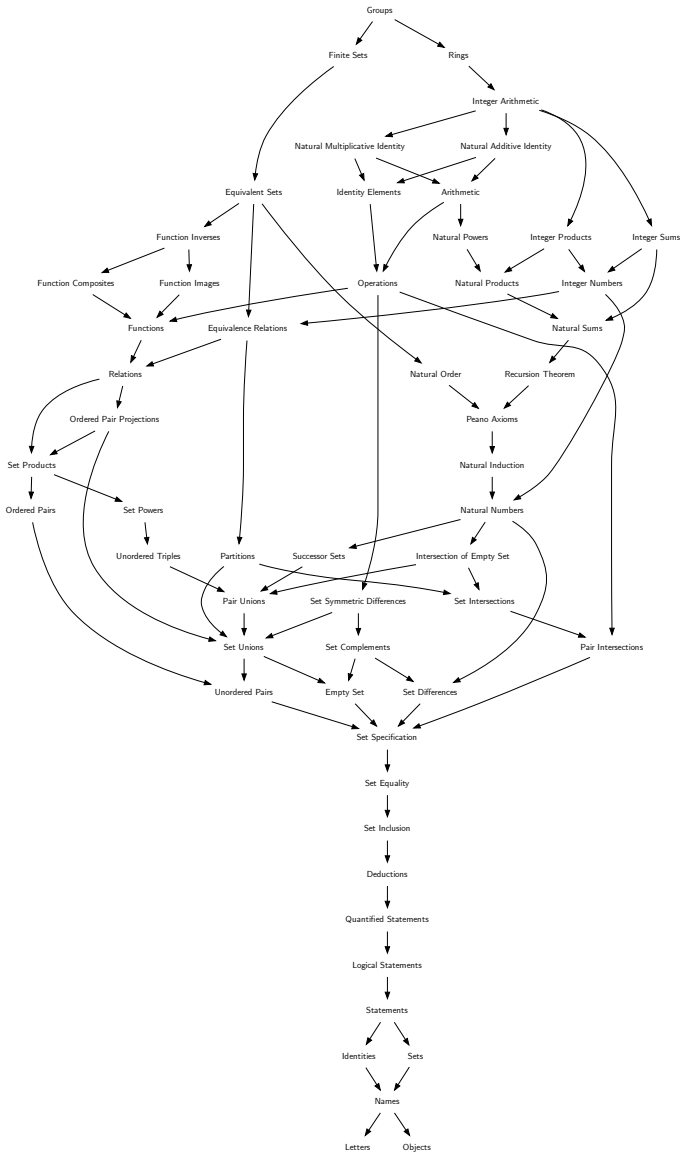
A *group* is an $\mathsf{algebra}(G, \circ)$ for which $\circ : G \times G \to G$ is associative, has an identity element in G, and has inverse elements. A group is a *commutative group* (or *abelian group*) if $\circ$ is commutative. A group is a *finite group* if G is a finite set.

Additive groups

Suppose that $(R, +, \cdot)$ is ring. Then $(R, +)$ is a commutative group. Conversely, suppose $(G, +)$ is a commutative group. Define multiplication on S by $a \cdot b = 0$ for all $a, b \in R$. Then $(S, +, \cdot)$ is a ring, called the *zero ring* of $(G, +)$. For this reason, it is customary to write $+$ for the operations $\circ$ when handling commutative groups.

Group Operations

Along with the group operation, we call the function which maps an element to its inverse element the *group operations*.


Groups
Finite Sets
Rings
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Identity Elements
Arithmetic
Function Inverses
Natural Powers
Integer Products
Integer Sums
Function Composites
Function Images
Operations
Natural Products
Integer Numbers
Functions
Equivalence Relations
Natural Sums
Relations
Natural Order
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Pair Unions
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects