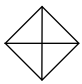


Why

Can we characterize positive (semi-)definite matrices in terms of their eigenvalues?

Main Result

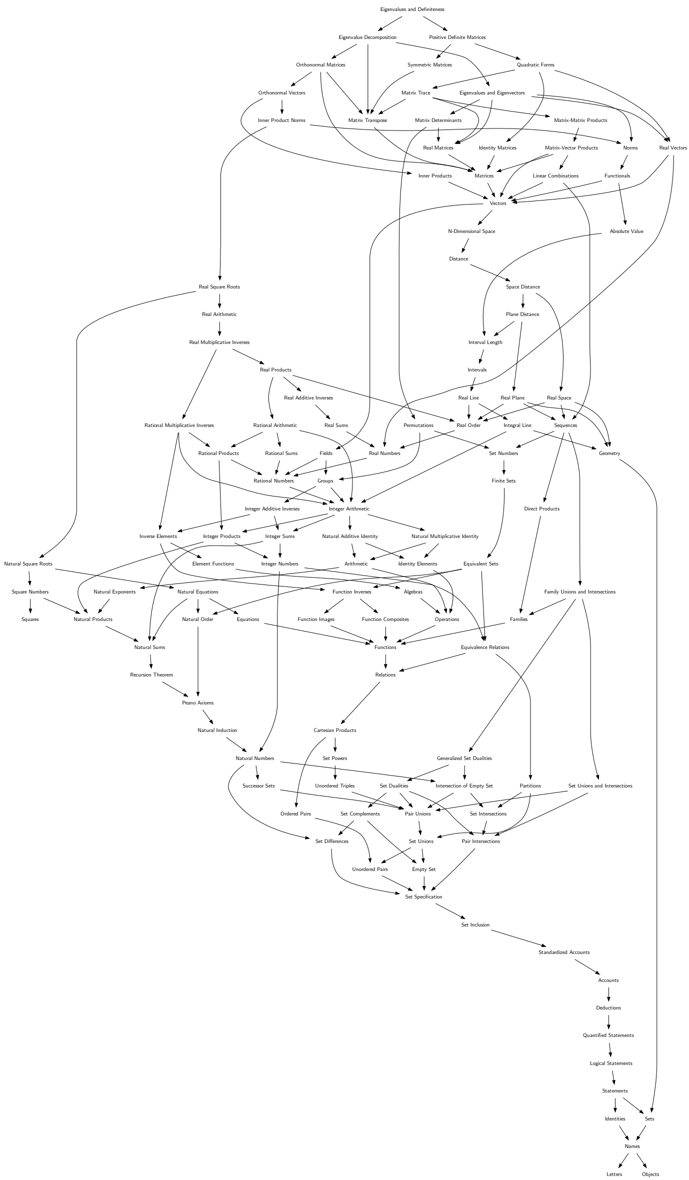
Using eigenvalue decompositions, we can answer in the affirmative.

Proposition 1. *Let* $A \in \mathbf{S}^d$ *with smallest eigenvalue* $\lambda_{\min}(A)$. *Then*

$$\begin{aligned} A \in \mathbf{S}^d_+ \quad &\longleftrightarrow \quad \lambda_{\min}(A) \geq 0 \\ &\longleftrightarrow \quad \mathbf{tr}\, AB \geq 0 \text{ for all } B \in \mathbf{S}^d_+. \end{aligned}$$

and

$$\begin{aligned} A \in \mathbf{S}^d_{++} \quad &\longleftrightarrow \quad \lambda_{\min}(A) > 0 \\ &\longleftrightarrow \quad \mathbf{tr}\, AB > 0 \text{ for all nonzero } B \in \mathbf{S}^d_{++}. \end{aligned}$$


Eigenvalues and Definiteness
Eigenvalue Decomposition
Positive Definite Matrices
Orthonormal Matrices
Symmetric Matrices
Quadratic Forms
Orthonormal Vectors
Matrix Trace
Eigenvalues and Eigenvectors
Inner Product Norms
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Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Additive Inverses
Real Line
Real Plane
Real Space
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Permutations
Real Order
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Sequences
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Fields
Real Numbers
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Geometry
Rational Numbers
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Integer Additive Inverses
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Direct Products
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Integer Products
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Natural Square Roots
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Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
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Statements
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