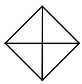


Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra. We so frequently work with real-valued random variables that if the range the random variable is not specified, it is assumed to be a subset of $\mathbf{R}$.

Notation

Suppose $(X, \mathcal{A}, \mathbf{P})$ is a probability space and $(Y, \mathcal{B})$ is a measurable space. A random variable is a measurable function $f : X \to Y$.

Some authors tend to denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. They reserve lower case letters x, y, z for the elements of the codomains of these functions. In such cases, they often denote the set of outcomes as Ω, which we have mentionedis a mnemonic for outcomes.

Special notation for common cases

Some authors use notation for the probability of particular, common sets. Suppose $(\Omega, \mathcal{A}, \mathbf{P})$ is a probability space with $X : \Omega \to \mathbf{R}$ a real-valued random variable. Many authors use $\mathbf{P}(X \in A)$ (or $P(X \in A)$, no bold) to denote

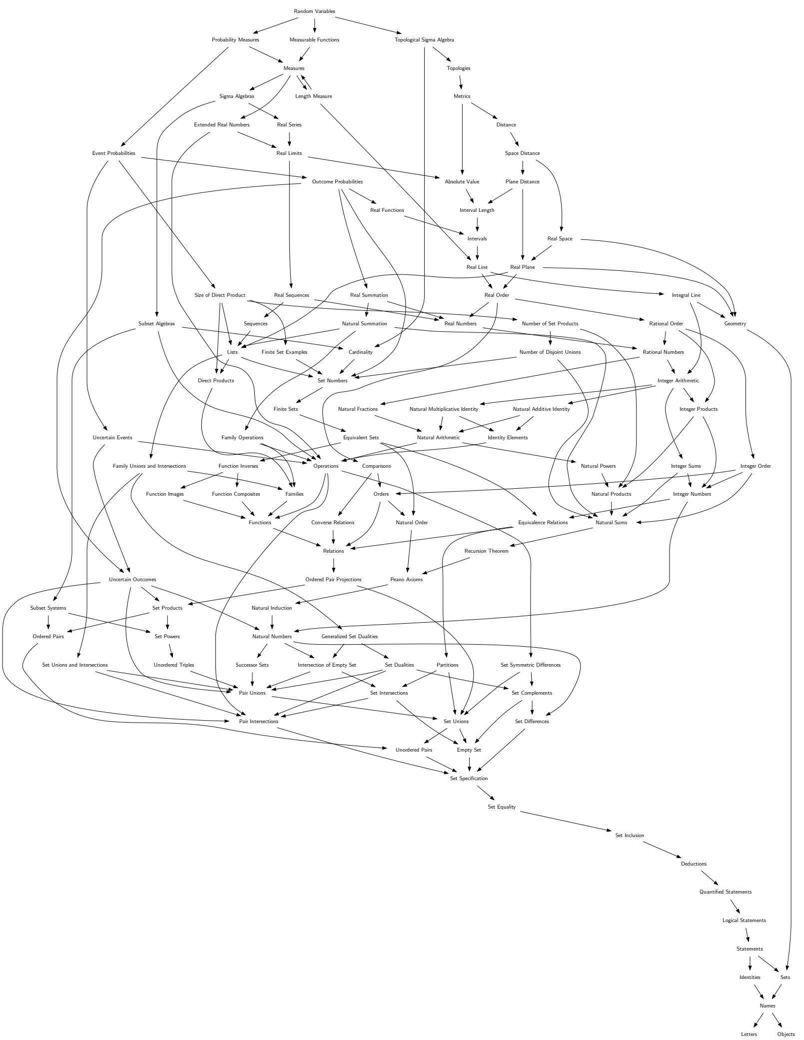
$$\mathbf{P}(X^{-1}(A)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\})$$

where $A \in \mathcal{B}(\mathbf{R})$, the Borel sigma algebra on $\mathbf{R}$. We will tend to use brackets in place of parentheses for clarity. So we will write $\mathbf{P}[X \in A]$.

Similar to the above, suppose $Y : \Omega \to \mathbf{R}$ is a random variable and $B \in \mathcal{B}(\mathbf{R})$. Then we will use $\mathbf{P}[X \in A, Y \in B]$ to denote

$$\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \dots, X_n : \Omega \to \mathbf{R}$, and Borel sets $A_1, \dots, A_n$, we will use $\mathbf{P}[X_1 \in A_1, \dots, X_n \in A_n]$ to denote $\mathbf{P}(\cap_{i=1}^{n} X_i^{-1}(A_i))$.


Random Variables
Probability Measures
Measurable Functions
Topological Sigma Algebra
Measures
Topologies
Sigma Algebras
Length Measure
Metrics
Extended Real Numbers
Real Series
Distance
Event Probabilities
Real Limits
Space Distance
Outcome Probabilities
Absolute Value
Plane Distance
Real Functions
Interval Length
Intervals
Real Space
Real Line
Real Plane
Size of Direct Product
Real Sequences
Real Summation
Real Order
Integral Line
Subset Algebras
Sequences
Natural Summation
Real Numbers
Number of Set Products
Rational Order
Geometry
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Finite Set Examples
Cardinality
Number of Disjoint Unions
Rational Numbers
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Natural Fractions
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Uncertain Events
Family Operations
Equivalent Sets
Natural Arithmetic
Identity Elements
Family Unions and Intersections
Function Inverses
Operations
Comparisons
Natural Powers
Integer Sums
Integer Order
Function Images
Function Composites
Families
Orders
Natural Products
Integer Numbers
Functions
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Uncertain Outcomes
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Set Unions and Intersections
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects