Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\mathrm{cov}(f, g)$. We defined it:

$$\mathrm{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\mathbf{E}(g).$$

Properties

Proposition 1. *Covariance is symmetric and billinear.*

Proposition 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\mathrm{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \mathrm{var}(f).$$

$\square$

Proposition 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\mathrm{var} \sum_{i=1}^n f_i = \mathrm{cov}\left(\sum_{i=1}^n f_i, \sum_{i=1}^n f_i\right)$$

$$= \sum_{i=1}^n \sum_{j=1}^n \mathrm{cov}(f_i, f_j)$$

$\square$

Covariance

Variance

Expectation

Random Variables

Topological Sigma Algebra — Measurable Function — Probability Measure

Measure — Topologies — Outcome Variable Covariance

Sigma Algebras — Metrics — Outcome Variable Expectation

Extended Real Numbers — Real Series — Real Square Roots

Distance — Induced Probability Distributions — Real Limits — Real Square

Outcome Variable Events — Space Distance

Event Probability — Plane Distance — Absolute Value — Real Arithmetic — Natural Square Roots

Probability Distributions — Interval Length — Real Multiplicative Inverses

Intervals — Real Space — Square Numbers

Real Products — Real Addition Inverses

Real Plane — Real Line — Real Sums

Real Sequences — Real Summation — Rational Multiplicative Inverses — Real Numbers — Real Order — Rational Arithmetic — Integral Line

Subset Algebras — Sequences — Rational Order — Rational Products — Rational Sums — Squares — Geometry

Cardinality — Lists — Uncertain Outcomes — Rational Numbers

Set Numbers — Direct Products — Integer Arithmetic

Finite Sets — Natural Additive Identity — Natural Multiplicative Identity — Integer Products

Equivalent Sets — Inverse Elements — Natural Summation

Family Unions and Intersections — Function Inverses — Family Operations — Element Functions — Comparison — Identity Elements — Arithmetic — Integer Sums — Natural Powers — Natural Equations

Families — Function Composites — Function Image — Operations — Order — Integer Order — Integer Sums — Natural Products — Equations

Functions — Converse Relations — Equivalence Relations — Integer Numbers — Natural Order — Natural Sums

Relations — Recursion Theorem

Subset Systems — Set Products — Peano Axioms

Ordered Pairs — Set Powers — Natural Induction

Generalized Set Duality — Unordered Triples — Natural Numbers

Set Unions and Intersections — Partitions — Intersection of Empty Set — Set Duality — Successor Sets — Set Symmetric Differences

Set Intersections — Set Complements

Pair Unions — Set Union — Set Differences

Pair Intersections — Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects