Definition

A *quadratic form* is a multivariate polynomial each term of which has degree two.

Representation

Let $f : \mathbf{R}^n \to \mathbf{R}$ be a quadratic form. There exists a matrix $A \in \mathbf{R}^{n \times n}$ so that

$$x^\top A x = \sum_{i,j} A_{ij} x_i x_j.$$

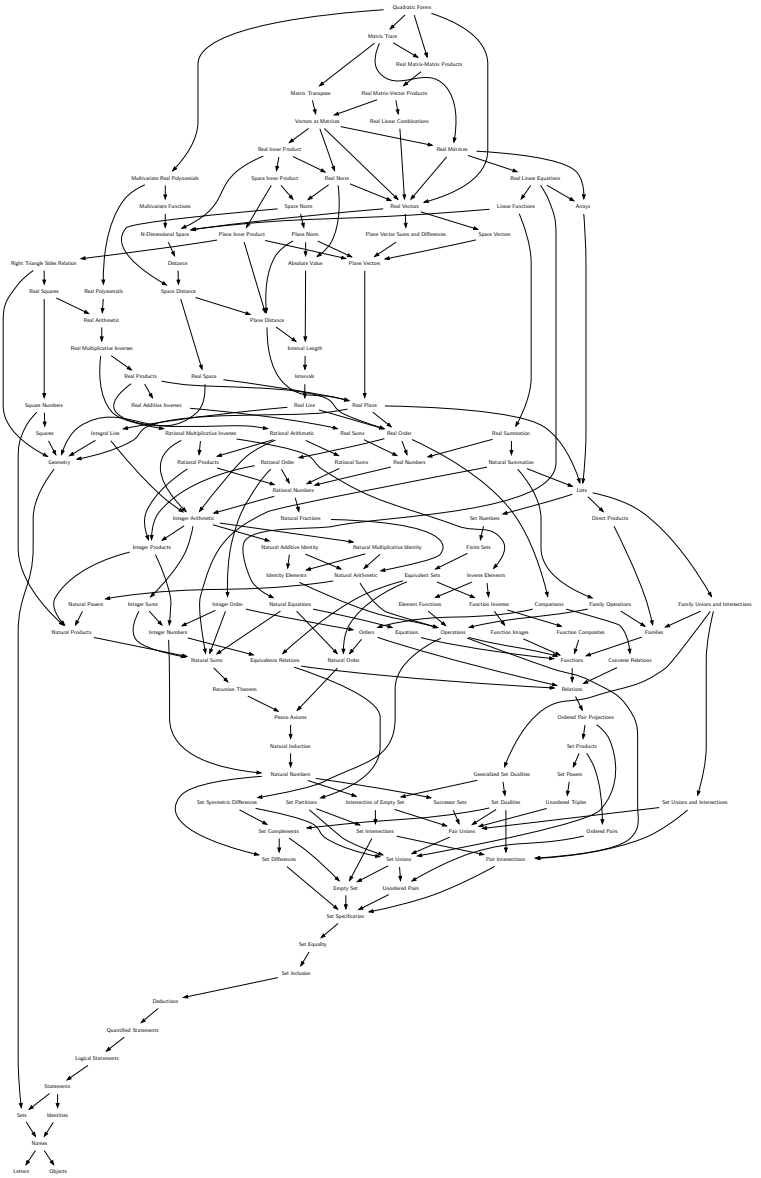
Suppose A is not symmetric. Then

$$f(x) = f(x)/2 + f(x)/2 = \frac{1}{2} x^\top A x + \frac{1}{2} x^\top A^\top x = x^\top (\frac{1}{2}(A + A^\top))x.$$

Define $B = 1/2(A^\top + A)$. We call B the *symmetric part* of A. Since every matrix A has a symmetric part, we can always assume that the matrix for a quadratic form is symmetric. If it is not, replace it with its symmetric part, obtaining the same function.

Under trace

Observe that $\operatorname{tr} x^\top A x = \operatorname{tr} A x x^\top = \operatorname{tr} x x^\top A$.


Quadratic Forms
Matric Trace
Real Matric-Matric Products
Matric Transpose
Real Matric-Vector Products
Vectors as Matrices
Real Linear Combinations
Real Inner Product
Real Matrices
Multivariate Real Polynomials
Space Inner Product
Real Norm
Real Linear Equations
Multivariate Functions
Space Norm
Real Vectors
Linear Functions
Arrays
N-Dimensional Space
Plane Inner Product
Plane Norm
Plane Vector Sums and Differences
Space Vectors
Right Triangle Sides Relation
Distance
Absolute Value
Plane Vectors
Real Squares
Real Polynomials
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Real Plane
Squares Numbers
Real Additive Inverses
Real Line
Squares
Integral Line
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Real Summation
Geometry
Rational Products
Rational Order
Rational Sums
Real Numbers
Natural Summation
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Integer Order
Natural Equations
Element Functions
Function Inverses
Comparisons
Family Operations
Family Unions and Intersections
Natural Products
Integer Numbers
Order
Equations
Operations
Function Images
Function Composites
Families
Natural Sums
Equivalence Relation
Natural Order
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Difference
Set Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects