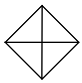


Why

Sums of non-negative functions are increasing, and workable with the monotone convergence theorem.

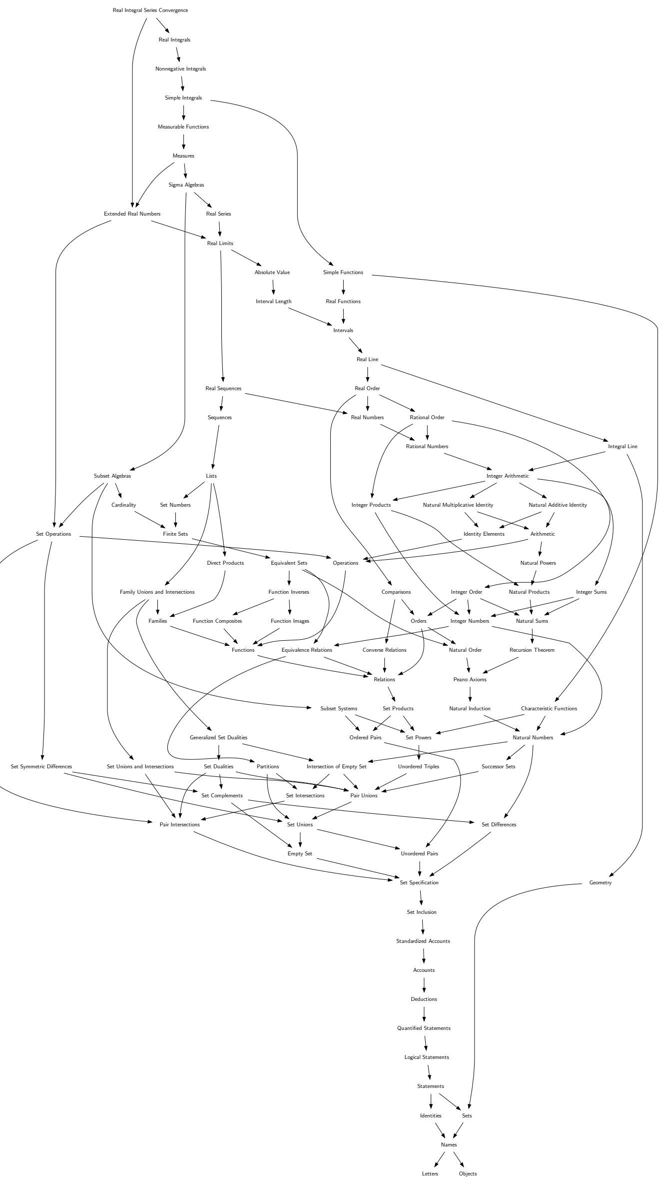
Result

Proposition 1. *The integral of the limit of the partial sums of a sequence of measurable, nonnegative, extended-real-valued functions is the limit of the partial sums of the integrals.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space, and let $f_n :\to [0, \infty]$ a $\mathcal{A}$-measurable function for every natural number n. We want to show that:

$$\int \sum_{k=1}^{\infty} f_k d\mu = \sum_{k=1}^{\infty} \int f_k d\mu.$$

We apply the monotone convergence theorem to the sequence $\{\sum_{i=1}^{n} f_i\}_n$. This sequence is nondecreasing because $f_n \geq 0$ for all n. $\square$


Real Integral Series Convergence
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Simple Functions
Interval Length
Real Functions
Intervals
Real Line
Real Sequences
Real Order
Sequences
Real Numbers
Rational Order
Rational Numbers
Integral Line
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Set Operations
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Operations
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Natural Products
Integer Sums
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Function Composites
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Integer Numbers
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Equivalence Relations
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Recursion Theorem
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Characteristic Functions
Generalized Set Dualities
Ordered Pairs
Set Powers
Natural Numbers
Set Symmetric Differences
Set Unions and Intersections
Set Dualities
Partitions
Intersection of Empty Set
Unordered Triples
Successor Sets
Set Complements
Set Intersections
Pair Unions
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects