

PROJECTIONS ON SUBSPACES

Why

Given a vector $x \in \mathbf{R}^n$ and a subspace $S \subset \mathbf{R}^n$, what is the closest point in S to x.

Result

Proposition 1. Suppose $U \in \mathbb{R}^{n \times k}$ with $U^{\top}U = I$. Then

$$\operatorname{proj}_U(x) = UU^{\top}x$$

