Why

We want to discuss rearranging the order of some set.

Definition

Let X be a nonempty set. A *permutation* is a bijection from X to X.

As a group

It happens that the set of permutations with the operation of composition is a *group* (see Groups). To see this, suppose $\pi : X \to X$ and $\sigma : X \to X$ are two permutations. Then $\pi \circ \sigma$ is a permutation (the composition of two invertible functions is invertible). Also, the identity function $\mathrm{id}_X :$ $X \to X$ is a permutation (it has an inverse, *itself*). The identity *function* is the identity element, since $\pi\, \mathrm{id} = \mathrm{id}\, \pi$ for all permutations π. Since each permutation is invertible, inverse elements exist. And the associative law is valid for permutations since function composition is associative. We the group consisting of the set of permutations on X and the operation of composition the *symmetric group on X*.

Finite case

If X is finite and $|X| = n$, then we can associate each element of X with a number $\{1, 2, \ldots, n\}$ and in so doing consider permutations of the set $\{1, \ldots, n\}$. This special symmetric group is called the *symmetric group of degree n*.

Notation

We denote the symmetric group on X by $\mathrm{Sym}(X)$. It is common to denote the symmetric group of degree n by S_n.

Permutations

Set Numbers   Groups

Finite Sets   Rings

Integer Arithmetic

Natural Additive Identity   Natural Multiplicative Identity

Equivalent Sets   Arithmetic   Identity Elements

Function Inverses   Integer Sums   Integer Products   Natural Powers

Function Composites   Function Images   Integer Numbers   Natural Products   Operations

Functions   Equivalence Relations   Natural Sums

Relations   Natural Order   Recursion Theorem

Ordered Pair Projections   Peano Axioms

Set Products   Natural Induction

Ordered Pairs   Set Powers   Natural Numbers

Unordered Triples   Partitions   Successor Sets   Intersection of Empty Set

Pair Unions   Set Intersections   Set Symmetric Differences

Set Unions   Set Complements   Pair Intersections

Unordered Pairs   Empty Set   Set Differences

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects