



Why

We want to discuss interactive decision making.

Example: rock paper scissors

We are interested in talking about situations in which there are several *decision makers*, *agents* or *players*, each of which are making decisions that will affect the outcome for all involved.

Consider the game “rock-paper-scissors” in which there are two players A and B . Each player may choose one of the three actions ROCK, PAPER, SCISSORS. To play the game, each player simultaneously selects an action, and these are compared.

So far we have a set of *players* or *agents* $P = \{A, B\}$ and a set of actions $\{\text{ROCK, PAPER, SCISSORS}\}$. In this case, both agents have the same set of actions, but they need not.

Example: tic-tac-toe

Consider the game “tic-tac-toe” in which there are two players. Denote the players by X and O . The game starts with an empty 3×3 array, which the players proceed to “fill.”

Player O starts and selects a cell in which to “mark her move.” From then on, that cell is “occupied,” Second, it is player X ’s turn to pick a cell, any one that is not already occupied. The play proceeds until either all cells are occupied or one of the player has three cells in a row, horizontally, verti-

cally, or diagonally.

1 Definition

In both these games there is a finite set of *players*, or *agents*, or *controllers*. Let \mathcal{I} be a finite set with $|\mathcal{I}| = n$, the players.

In rock-paper-scissors, for example, $\mathcal{I} = \{A, B\}$. There, each player could pick one of the three actions. Define $\mathcal{A}_A = \mathcal{A}_B = \{\text{ROCK}, \text{PAPER}, \text{SCISSORS}\}$. We call \mathcal{A}_A the actions of A and \mathcal{A}_B the actions of B .

We have a set of outcomes $\mathcal{O} = \{\text{A WINS}, \text{B WINS}, \text{TIE}\}$. Let $f : \mathcal{A}_A \times \mathcal{A}_B \rightarrow \mathcal{O}$ defined by $f(\text{ROCK}, \text{SCISSORS}) = \text{A WINS}$

the set of *players*. Let S be a finite set, the set of *states*. For $i = 1, \dots, n$, let $\{A_s^p\}_{s \in S}$ be a family of sets, the *action sets by state*. Define $\mathcal{A}^i = \cup_s A_s^p$ the set of *actions* for player $i = 1, \dots, n$.

Let $f : S \times \prod_i \mathcal{A}^i \rightarrow S$, the *game dynamics* or *transition function*.



Definition

The first thing to discuss is the set of players. Let P be a finite set with $|P| = n$. The set P is the set of players, and we

We begin with a single-player game.

Let S be a set.

