Why

The integrable functions are a vector space.

Definition

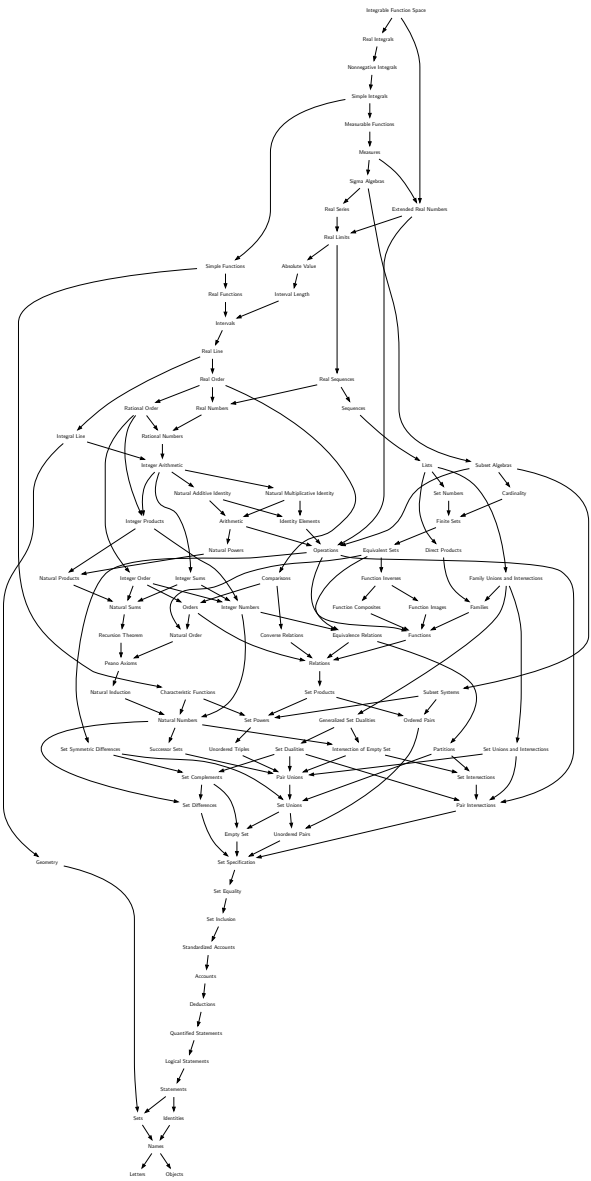
The *integrable function space* corresponding to a measure space is the set of real-valued functions which are integrable with respect to the measure. The term space is appropriate because this set is a real vector space. If we scale an integrable function, it remains integrable. If we add two integrable functions, the sum is integrable. Thus, a linear combination of integrable functions is integrable. The zero function is the zero element of the vector space.

TODO The open question is: what elements of our geometric intuition can we bring to a space of functions. Do functions have a size? Are certain functions near each other?

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let R denote the set of real numbers and let C denote the set of complex numbers.

We denote set the real-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, R)$, read aloud as "the real integrable functions on the measure space X script A mu." We denote set the complex-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, C)$, read aloud as "the complex integrable functions on the measure space X script A mu." When the field is irrelevant, we denote them by $\mathcal{I}(X, \mathcal{A}, \mu)$, read aloud as "integrable functions on the measure space X script A mu." The $\mathcal{I}$ is a mnemonic for "integrable."


Integrable Function Space
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Simple Functions
Absolute Value
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Integral Line
Rational Numbers
Integer Arithmetic
Lists
Subset Algebras
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Cardinality
Integer Products
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Operations
Equivalent Sets
Direct Products
Natural Products
Integer Order
Integer Sums
Comparisons
Function Inverses
Family Unions and Intersections
Natural Sums
Orders
Integer Numbers
Function Composition
Function Images
Families
Recursion Theorem
Natural Order
Converse Relations
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Set Powers
Generalized Set Duality
Ordered Pairs
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Duality
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Latters
Objects