Why

We want to summarize the interaction between two binary traits.

Definition

The *contingency table* of a population $\{1, \ldots, n\}$ with respect to binary traits $a, b : \{1, 2, \ldots, n\} \to \{0, 1\}$ is the array $A \in \mathbf{N}^{2 \times 2}$ of natural numbers defined by

$$A = \begin{bmatrix} \left| a^{-1}(0) \cap b^{-1}(0) \right| & \left| a^{-1}(0) \cap b^{-1}(1) \right| \\ \left| a^{-1}(1) \cap b^{-1}(0) \right| & \left| a^{-1}(0) \cap b^{-1}(1) \right| \end{bmatrix}.$$

We interpret A_{11} as the number of individuals which have neither trait, A_{12} as the individuals which have trait b but not trait a, and so on.

Normalization

These four sets partition $\{1, 2, \ldots, n\}$, so that if we divide the elements by n, we obtain four numbers which sum to 1, the 2 by 2 table with these entries is called the *normalized contingency table*.

Contingency arrays

In general, we have k binary traits, each of which an individual may or may not have. We encode these traits using k functions

$$a_1, \ldots, a_k : \{1, 2, \ldots, n\} \to \{0, 1\}.$$

The contingency array is k-dimensional array A, with

$$A_x = \cap_{j=1}^{k} a_j^{-1}(x_j),$$

where $x \in \{0, 1\}^k$.

Contingency Tables

Arrays

Lists

Direct Products   Set Numbers

Finite Sets

Equivalent Sets

Function Inverses   Natural Order   Family Unions and Intersections

Function Composites   Function Images   Families

Functions   Equivalence Relations   Peano Axioms

Relations   Partitions

Ordered Pair Projections   Natural Induction

Set Products

Set Powers   Natural Numbers   Generalized Set Dualities

Unordered Triples   Successor Sets   Intersection of Empty Set   Set Dualities   Set Unions and Intersections

Ordered Pairs   Pair Unions   Set Complements   Set Intersections

Set Unions   Set Differences   Pair Intersections

Unordered Pairs   Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects