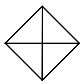


Why

We define the area under an extended real function.

Definition

The *positive part* of an extended-real-valued function is the function mapping each element to the maximum of the function's result and zero. The *negative part* of an extended-real-valued function is the function mapping each element to the maximum of the additive inverse of function's result and zero.

We decompose an extended-real-valued function as the difference of its positive part and its negative part. Both the positive and negative parts are non-negative extended-real-valued functions.

Consider a measure space. An *integrable* function is a measurable extended-real-valued function for which the non-negative integral of the posititve part and the non-negative integral of the negative part of the function are finite. The *integral* of an integrable function is the difference of the non-negative integral of the posititive part and and the non-negative integral of the negative part.

If one but not both of the parts of the function are finite, we say that the integral *exists* and again define it as before. In this way we avoid arithmetic between two infinities.

Notation

Let A a non-empty set. Let $g : A \to [-\infty, \infty]$. We denote the positive part of g by g^+ and the negative part of g by g^-:

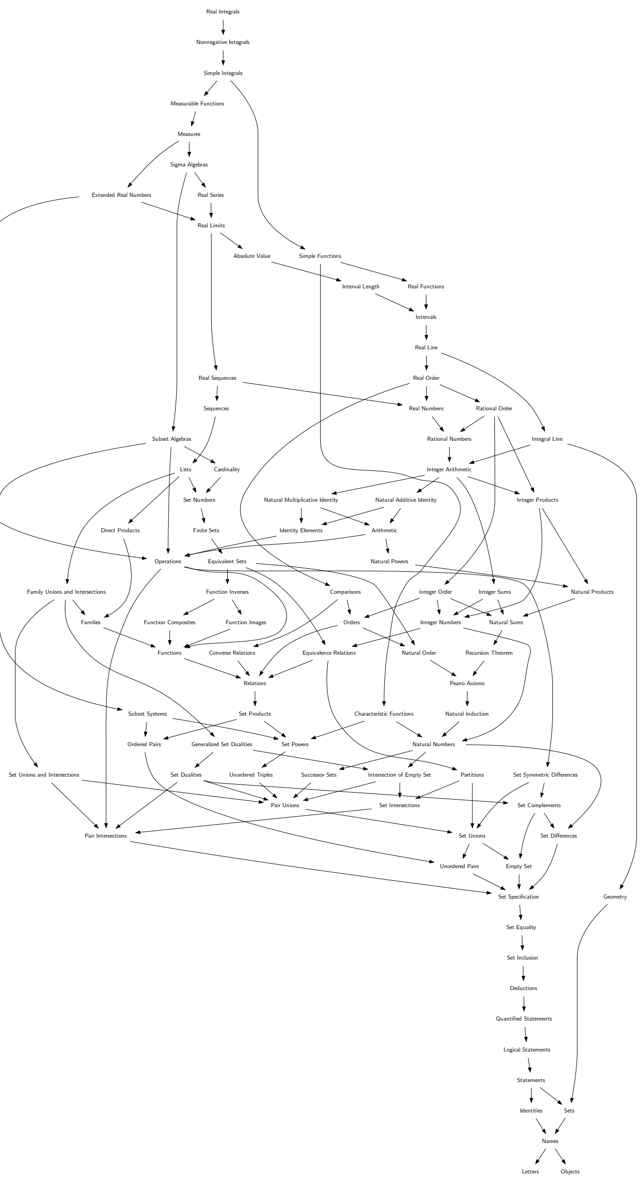
$$g^+(x) = \max\{g(x), 0\} \quad \text{and} \quad g^-(x) = \max\{-g(x), 0\}.$$

Moreover, we decompose g as $g = g^+ - g^-$. We observed that $g^+(x) \geq 0$ and $g^-(x) \geq 0$ for all $x \in X$.

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f : X \to [-\infty, +\infty]$ measurable and one of $\int f^+ d\mu$ or $\int f^- d\mu$ is finite (if both are finite, f is integrable).

We denote the integral of f with respect to the measure μ by $\int f d\mu$. We defined:

$$\int f d\mu = \left(\int f^+ d\mu\right) - \left(\int f^- d\mu\right).$$


Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Simple Functions
Interval Length
Real Functions
Intervals
Real Line
Real Sequences
Real Order
Sequences
Real Numbers
Rational Order
Subset Algebras
Rational Numbers
Integral Line
Lists
Cardinality
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Operations
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects