



N-DIMENSIONAL SPACE

Why

If \mathbf{R} corresponds to a line, and \mathbf{R}^2 to a plane, and \mathbf{R}^3 to space, does \mathbf{R}^4 correspond to anything? What of \mathbf{R}^5 ?

Definition

Let n be a natural number. *n-dimensional space* is the set \mathbf{R}^n . We call elements of \mathbf{R}^n *points* and call the point associated with $x = (x_1, x_2, \dots, x_n) \in \mathbf{R}^n$ with $x_i = 0$ for $1 \leq i \leq n$ the *origin*.

Visualization

We can not visualize n -dimensional space. Thus, our intuition for it comes from real space (see **Real Space**).

Distance

A natural notion of distance for \mathbf{R}^n is the extension of the Euclidean distance. We define the distance between (x_1, x_2, \dots, x_n) , $(y_1, y_2, \dots, y_n) \in \mathbf{R}^n$ as

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$$

This is sometimes called the *Euclidean distance for n-dimensional space*. Does this have the properties that distance has in the plane and in space? We discussed these properties It does. Denote the function which associates to $x, y \in \mathbf{R}^n$ their distance

$d : \mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}$. So $d(x, y)$ is the distance between the points corresponding to x and y .

Proposition 1. *d is non-negative, symmetric, and the distance between two points is no larger than the sum of the distances with any third object.*¹

¹Future editions will include an account.

