



Probability Distributions

1 Why

We want to talk about probability over finite sets.

2 Definition

A *probability distribution* or *probability mass function* is a real-valued function from a set of outcomes which is non-negative and normalized. A real-valued function on a finite set is *normalized* if the sum of its results is 1. We will refer to these as *distributions*. The *probability of an outcome* is the result of the outcome under the distribution.

2.1 Notation

Let A be a set of outcomes and Let $p : A \rightarrow \mathbf{R}$ be a distribution; then

- $p(a) > 0$ for each $a \in A$ and
- $\sum_{a \in A} p(a) = 1$

Proposition 1. *If $p : A \rightarrow \mathbf{R}$ is a distribution, then $p(A) \subset [0, 1]$.*

Proof. Let $a \in A$. First, $p(a) \geq 0$ by definition. Second, since p is normalized, $\sum_{b \in A} p(b) = 1$. And $p(a) \leq \sum_{b \in A} p(b)$, so $p(a) \leq 1$.

□