Why

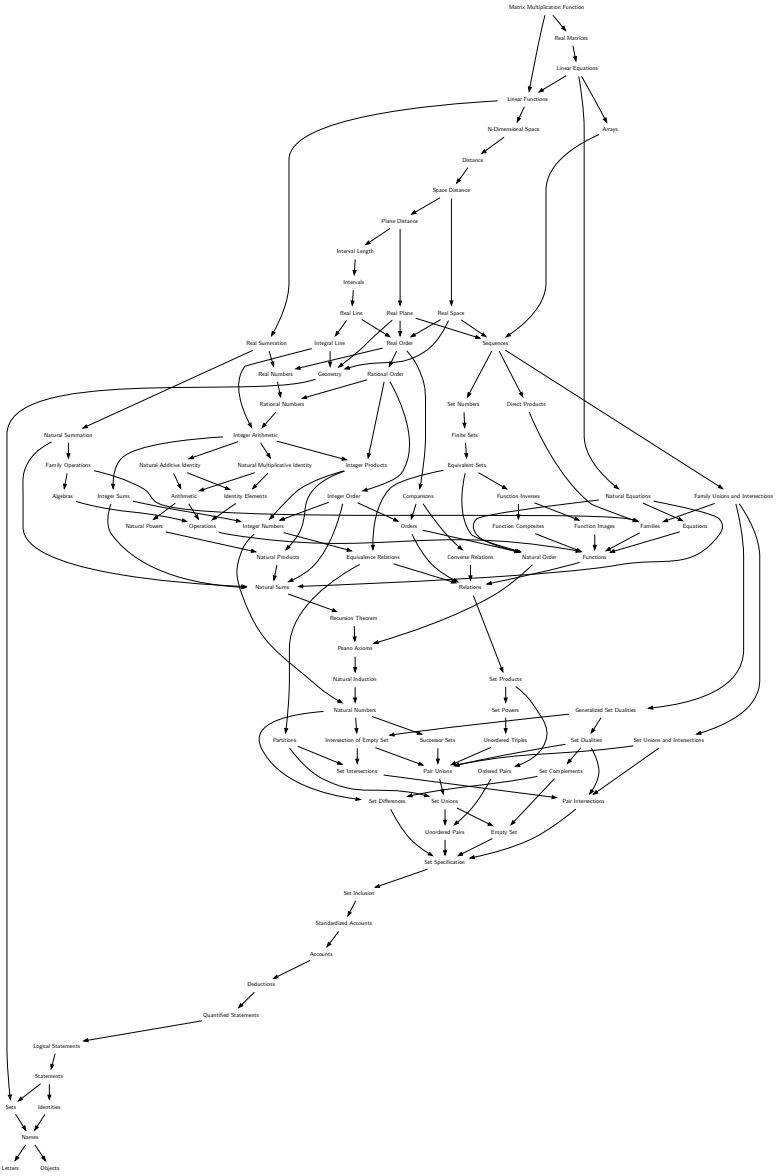
We view matrix-vector multiplication as a function mapping vectors to vectors.

Result

Define $f : \mathbf{R}^n \to \mathbf{R}^m$ by $f(x) = Ax$ where $A \in \mathbf{R}^{m \times n}$. We call f the *matrix multiplication function* associated with A. It is easy to verify that f is a linear function. The converse is true.

Proposition 1. *Let $f : \mathbf{R}^n \to \mathbf{R}^m$ be linear. Then there exists a unique $A \in \mathbf{R}^{m \times n}$ satisfying $f(x) = Ax$ for all $x \in \mathbf{R}^n$.*

Proof. Evaluate f at the standard basis vectors e_i. The ith component of e_i is 1 and all other components are 0. □


Matrix Multiplication Function
Real Matrices
Linear Equations
Linear Functions
N-Dimensional Space
Arrays
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Real Summation
Integral Line
Real Order
Sequences
Real Numbers
Geometry
Rational Order
Rational Numbers
Set Numbers
Direct Products
Natural Summation
Integer Arithmetic
Finite Sets
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Algebras
Arithmetic
Identity Elements
Integer Order
Comparisons
Function Inverses
Natural Equations
Family Unions and Intersections
Integer Sums
Natural Powers
Operations
Integer Numbers
Orders
Function Composites
Function Images
Families
Equations
Natural Products
Equivalence Relations
Converse Relations
Natural Order
Functions
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects