



**Why**

A simple example of an embedding.<sup>1</sup>

**Definition**

Fix  $d \in \mathbf{N}$ . A *polynomial feature map* of degree  $d$  is a function  $\phi : \mathbf{R} \rightarrow \mathbf{R}^d$  with

$$\phi(x) = \begin{pmatrix} 1 & x & x^2 & \dots & x^d \end{pmatrix}^\top.$$

For  $x \in \mathbf{R}$ , we call  $\phi(x)$  the *polynomial embedding* of  $x$ .

A *polynomial regressor* is a least squares linear predictor using a polynomial feature embedding (of any degree, but to be precise one must specify the degree). The task of constructing a linear predictor is often referred to as *polynomial regression*.

Given a dataset of paired records  $(x^1, y^1), \dots, (x^n, y^n) \in \mathbf{R}^2$ , one can construct a predictor  $g : \mathbf{R} \rightarrow \mathbf{R}$  for  $y$  by embedding the dataset  $(\phi(x^1), \dots, \phi(x^n))$  and finding the least squares linear regressor  $f : \mathbf{R}^d \rightarrow \mathbf{R}$  for  $y$ . One defines the predictor  $g : \mathbf{R} \rightarrow \mathbf{R}$  by  $g(\phi(x))$ .

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<sup>1</sup>Future editions will expand, or perhaps collapse this sheet.



