Why

If one rectangle contains another rectangle, the area of the first should be larger than the area of the second. Our definition of integral for simple functions carries this property.

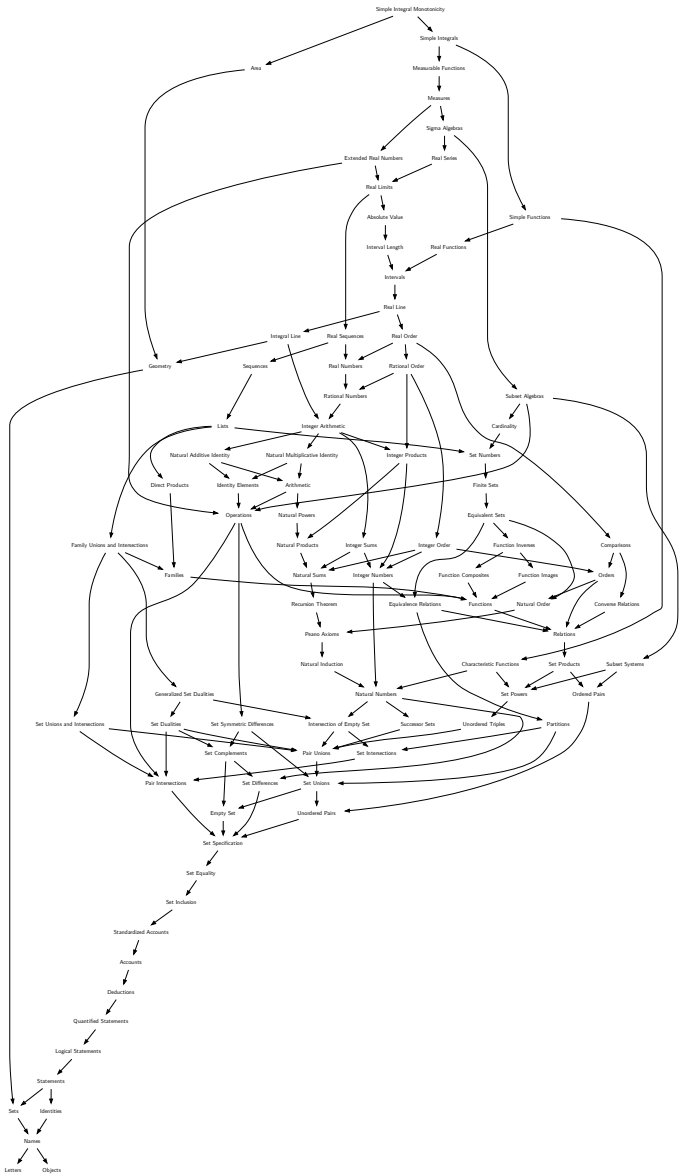
Result

Proposition 1. *The simple non-negative integral operator is monotone.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f, g \in \mathrm{SF}_+(X)$ with $f \leq g$. Then $f - g \in \mathrm{SF}_+(X)$, so

$$\begin{aligned}\int g d\mu &= \int (f + (g - f)) d\mu \\ &\overset{(a)}{=} \int f d\mu + \int (g - f) d\mu \\ &\overset{(b)}{\geq} \int f d\mu\end{aligned}$$

where (a) follows from linearity and (b) follows from non-negativity; properties of the non-negative simple integral operator. $\square$


Simple Integral Monotonicity
Simple Integrals
Area
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Simple Functions
Interval Length
Real Functions
Intervals
Real Line
Integral Line
Real Sequences
Real Order
Geometry
Sequences
Real Numbers
Rational Order
Rational Numbers
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Set Numbers
Direct Products
Identity Elements
Arithmetic
Finite Sets
Operations
Natural Powers
Equivalent Sets
Family Unions and Intersections
Natural Products
Integer Sums
Integer Order
Function Inverses
Comparisons
Families
Natural Sums
Integer Numbers
Function Composites
Function Images
Orders
Recursion Theorem
Equivalence Relations
Functions
Natural Order
Converse Relations
Relations
Peano Axioms
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Generalized Set Dualities
Natural Numbers
Set Powers
Ordered Pairs
Set Unions and Intersections
Set Dualities
Set Symmetric Differences
Intersection of Empty Set
Successor Sets
Unordered Triple
Partitions
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects