Why

To talk about uncertain outcomes, we assign credibility to each outcome according to our intuition of proportion.[1]

Definition

A *probability distribution* (or *distribution*, *probability mass function*, *pmf*, *proportion distribution*) is a real-valued function from a finite set of outcomes which is non-negative and normalized. A real-valued function on a finite set is *normalized* if the sum of its results is 1.

The *probability of an outcome* is the result of the outcome under the distribution. The probability of an outcome is meant to indicate the credibility of an outcome. The word probabiliy has its roots in the English word probable, which has the Middle English sense "worthy of belief". The probability of an outcome models how worthy of belief it is, relative to other outcomes. In the case of flipping a coin, or rolling a die, we may assert that all outcomes are equally worthy of belief.

When an outcome has a higher probability than a second outcome we say that the first is *more probable* than the second. Similarly we say that the first outcome is *less probable* than the second outcome if it has higher probability than the second.

Notation

Let A be a set and let $p : A \to \mathbf{R}$ be a distribution. Then

$$p(a) \geq 0 \text{ for all } a \in A \text{ and } \sum_{a \in A} p(a) = 1.$$

Simple consequences of definition

Proposition 1. *Suppose* $p : A \to \mathbf{R}$ *is a distribution. For all* $a \in A$, $p(a) \leq 1$.

---

[1]Future editions may drop the dependence on real numbers, and use intuition of repeated trials to introduce *rational* probability distributions.

Proof. Let $a \in A$. By definition, $p(a) \geq 0$.. Second, since p is normalized, $p(a) \leq \sum_{b \in A} p(b) = 1$. □

Consequently, the range of p is contained in $[0, 1]$. For this reason, we will often introduce a distribution $p : A \to [0, 1]$.

Example: coin

When flipping a coin, both a heads and a tails are equally worthy of belief. Thus we say that the outcome 0 (tails) has probability $1/2$ and likewise for the outcome 1 (heads). In this case we say that we are modeling a *fair coin.*

Example: die

When rolling a die, all six sides are equally worthy of belief. Thus we say that the outcomes $1, 2, 3, 4, 5, 6$ each have probability $1/6$. Prior to roll, each is equally credible. In this case we say that we are modeing a *fair die.*

Probability Distributions

Intervals

Real Line

Real Order

Rational Order

Real Numbers

Real Summation

Integral Line

Rational Numbers

Integer Arithmetic

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Arithmetic

Identity Elements

Natural Summation

Integer Order

Integer Sums

Natural Powers

Family Operations

Comparisons

Integer Numbers

Natural Products

Families

Operations

Converse Relations

Orders

Equivalence Relations

Natural Sums

Functions

Relations

Natural Order

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Set Products

Natural Induction

Uncertain Outcomes

Ordered Pairs

Set Powers

Natural Numbers

Partitions

Intersection of Empty Set

Unordered Triples

Successor Sets

Set Intersections

Pair Unions

Set Symmetric Differences

Set Unions

Set Complements

Pair Intersections

Unordered Pairs

Empty Set

Set Differences

Geometry

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects