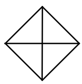


Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Prop. 1. *Let f be a square-integrable real-valued random variable on the probability space $(X, \mathcal{A}, \mu)$. Then for $t > 0$,*

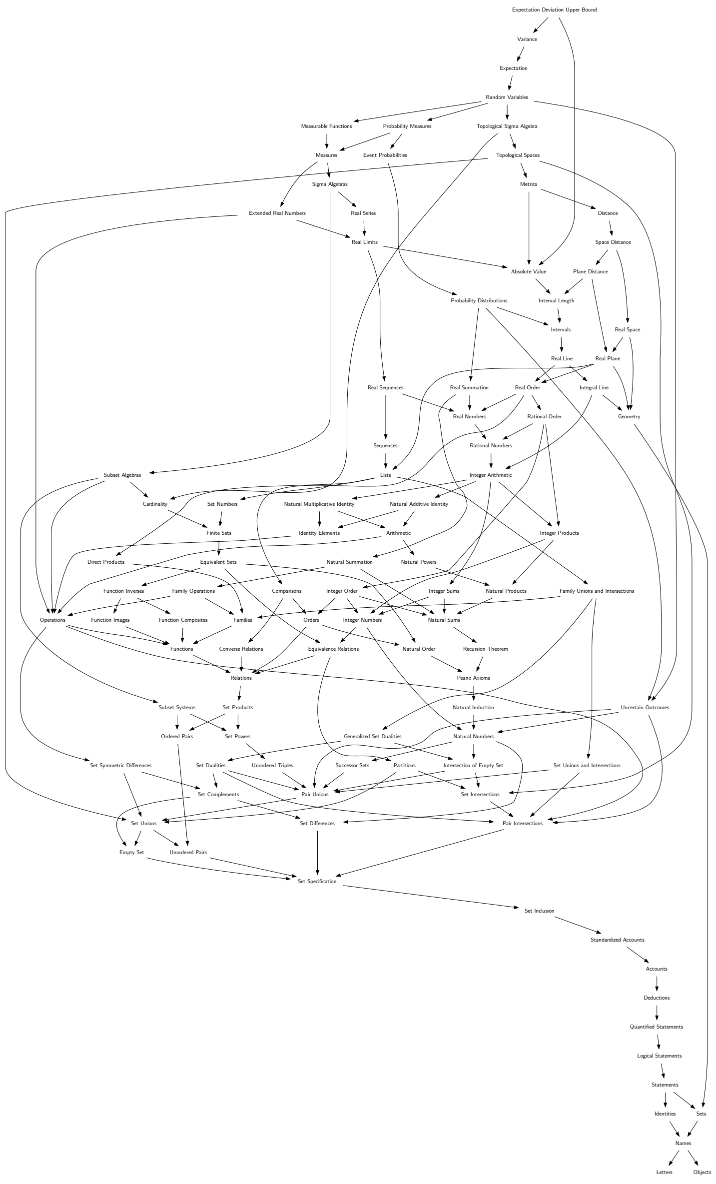
$$\mu(|f - \mathbf{E}(f)| \geq t) \leq \frac{\operatorname{var} f}{t^2}.$$

Proof. The set $|f - \mathbf{E}(f)| \geq t$ is $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$. This set is $\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}$. By using the non-negative inequality

$$\mu(\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))}{t^2}.$$

We recognize the numerator of the right hand side as the variance. □

The above is also called *Chebychev's Inequality.*


Expectation Deviation Upper Bound
Variance
Expectation
Random Variables
Measurable Functions
Probability Measures
Topological Sigma Algebra
Measures
Event Probabilities
Topological Spaces
Sigma Algebras
Metrics
Extended Real Numbers
Real Series
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Probability Distributions
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Sequences
Real Summation
Real Order
Integral Line
Real Numbers
Rational Order
Geometry
Sequences
Rational Numbers
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Identity Elements
Arithmetic
Integer Products
Direct Products
Equivalent Sets
Natural Summation
Natural Powers
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Products
Family Unions and Intersections
Operations
Function Images
Function Composites
Families
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Symmetric Differences
Set Dualities
Unordered Triples
Successor Sets
Partitions
Intersection of Empty Set
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects