Why

We want to quantify the error of compressing a real-valued random variable.

Definition

Let $\mathcal{X}$ be a finite set and $q : \mathsf{R} \to \mathcal{X}$ a quantization (see Quantizations) of R. Also, fix a probability space $(\Omega, \mathcal{A}, \mathsf{P})$ and a random variable $x : \Omega \to \mathsf{R}$.

The *compression* $\hat{x} : \Omega \to \mathcal{X}$ of x under q is $q \circ x$. A *distortion function* for x and $\hat{x}$ is a function

$$d : (\Omega \to \mathsf{R}) \times (\Omega \to \mathcal{X}) \to \mathsf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

Examples

The *expected mean-squared-error distortion* d_{mse} between x and $\hat{x}$ is

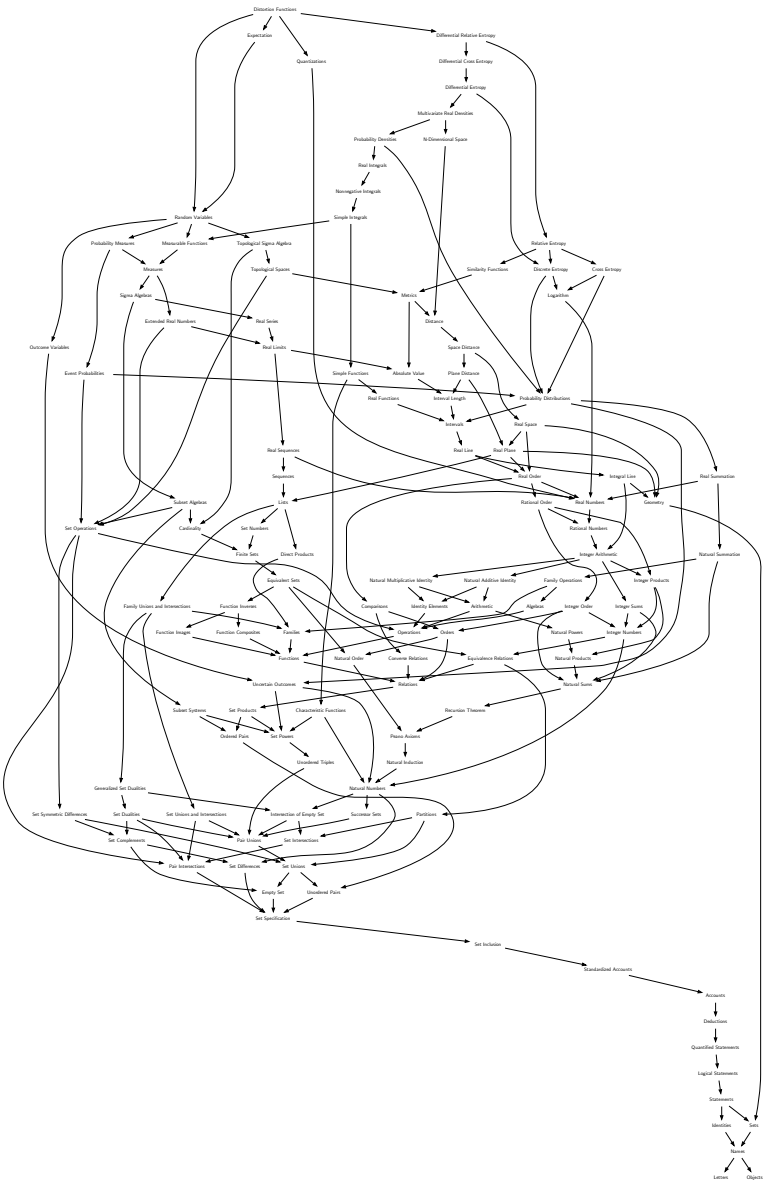
$$d_{\mathrm{mse}}(x, \hat{x}) = \mathsf{E}[(x - \hat{x})^2]$$

The *Kulback-Liebler distortion* d_{kld} defined by

$$d_{\mathrm{kld}}(x, \hat{x}) = \mathsf{E}[d_{\mathrm{kl}}(\mathsf{P}(y \in \cdot \mid x, \hat{x}) \mid \mathsf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.[1]

[1] Future editions will clarify this sentence.


Distortion Functions
Expectation
Quantizations
Differential Relative Entropy
Differential Cross Entropy
Differential Entropy
Multivariate Real Densities
Probability Densities
N-Dimensional Space
Real Integrals
Nonnegative Integrals
Simple Integrals
Random Variables
Probability Measures
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Topological Sigma Algebra
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Absolute Value
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Probability Distribution
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Real Space
Real Sequences
Real Line
Real Plane
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Real Order
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Subset Algebras
Set Operations
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Finite Sets
Direct Products
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Natural Summation
Equivalent Sets
Natural Multiplication Identity
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Characteristic Functions
Recursion Theorem
Ordered Pairs
Set Powers
Peano Axioms
Unordered Triples
Natural Induction
Generalized Set Dualities
Natural Numbers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Axioms
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Rules
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Objects