

OUTCOME VARIABLE EVENTS

Why

For each value of a random variable's codomain, the set of outcomes corresponding to that value is the inverse image of the random variable. We can speak of the probability that a random variable takes a value then, by assigning it the probability of the set of outcomes corresponding to that value.

Definition

Let $p:\Omega\to \mathbf{R}$ be a probability distribution with corresponding probability measure $\mathbf{P}:\mathcal{P}(\Omega)\to \mathbf{R}$. Suppose $x:\Omega\to V$ is an outcome variable. The *probability* x=a, for $a\in\Omega$, is

$$\mathbf{P}(\{\omega \in \Omega \mid x(\omega) = a\}).$$

From the definition of \mathbf{P} , we express the above as

$$\sum_{\omega \in \Omega \mid x(\omega) = a} p(\omega).$$

We refer to the probability of the *event* that x = a.

Notation

We denote the probability that x=a by $\mathbf{P}[x=a]$. Our square brackets deviate from the slightly slippery but universally standard notation $\mathbf{P}(x=a)$. We prefer the square brackets, since x=a is not itself an argument to \mathbf{P} , but shorthand for the set $\{\omega \in \Omega \mid x(\omega)=a\}$.

There are many similar notations. For example, $\mathbf{P}[x \in C]$ means $\mathbf{P}(\{x \in \Omega \mid x(\omega) \in C\})$. In particular, if $x : \Omega \to \mathbf{R}$, $\mathbf{P}[x \geq a]$ means $\mathbf{P}(\{\omega \in \Omega \mid x(\omega) \geq a\})$. Since the *event* that x = a is the inverse image of $\{a\}$ under x, we also use the notations $\mathbf{P}(x^{-1}(a))$ and $\mathbf{P}(x^{-1}(C))$.

Example: sum of two dice

Define $\Omega=\{1,\ldots,6\}^2$ and define $p:\Omega\to \mathbf{R}$ with $p(\omega)=1/36$ for each $\omega\in\Omega$. Define $x:\Omega\to\mathbf{N}$ by $x(\omega_1,\omega_2)=\omega_1+\omega_2$. Then

$$\mathbf{P}[x=4] = p((2,2)) + p(1,3) + p(3,1) = 1/12.$$

