Index Matrices

1 Why

TODO

2 Definition

An *index sequence* is a length r sequence of distinct integers from $\{1, 2, \dots, n\}$, where $r \leq n$. The *index matrix* associated with an index sequence is the $r \times n$ matrix whose i, jth entry is 1 if the index's ith entry is j, and 0 otherwise. If $r = n$ then the index matrix is a permutation matrix.

2.1 Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions, and so relations, and so sets), they are not sets of indices, so we avoid this usage.

2.2 Notation

If $\alpha : \{1, 2, \dots, r\} \to \{1, 2, \dots, n\}$ is an index sequence of length $r \leq n$, then we denote the index matrix associated with α by P_α. This matrix P_α is an element of $\mathsf{N}^{r \times n}$ and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

3 Multiplication

Multiplying an n-vector by an $r \times n$ index matrix forms an r-vector with the entries indexed by the index sequence. In other words, multiplying a vector by an index matrix produces a permuted subvector.

A *principal submatrix* of a matrix is any matrix which can be formed by forming Multiplying an $n \times n$ matrix with an index matrix on the left and the transpose of the index matrix on the right extracts the