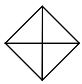


Why

Can we represent the function associating a linear system with its row reduction by matrix multiplication?

Main observation

The following proposition affirmatively answers the question.

Proposition 1. *Let* $(A \in \mathbf{R}^{m \times m}, b \in \mathbf{R}^m)$ *be a linear system with* $A_{kk} \neq 0$ *and* (C, d) *the* k*th reduction of* (A, b)*. Then there exists a matrix* $L \in \mathbf{R}^{m \times m}$ *so that* $C = LA$ *and* $d = Lb$*.*

Proof. Define $L \in \mathbf{R}^{m \times m}$ by $L_{st} = 1$ if $s = t$, $-A_{sj}/A_{ij}$ if $k < s \leq m$ and zero otherwise. □

For this reason, we call L in Proposition 1 a *row reducer matrix* or *row reducing matrix* or *row reducer*. The row reducer matrix for the kth reduction of (A, b) has the form

$$L_k = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & {}^{A_{ik}}/_{A_{kk}} & 1 & & \\ & & \vdots & & \ddots & \\ & & {}^{A_{mk}}/_{A_{kk}} & & & 1 \end{bmatrix}$$

So the following is immediate

Prop. 2. *Row reducing matrices are unit lower triangular.*

Example

For example, the $(1,1)$-reduction of (A,b) in which

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 6 & 7 & 9 & 5 \\ 8 & 7 & 9 & 8 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

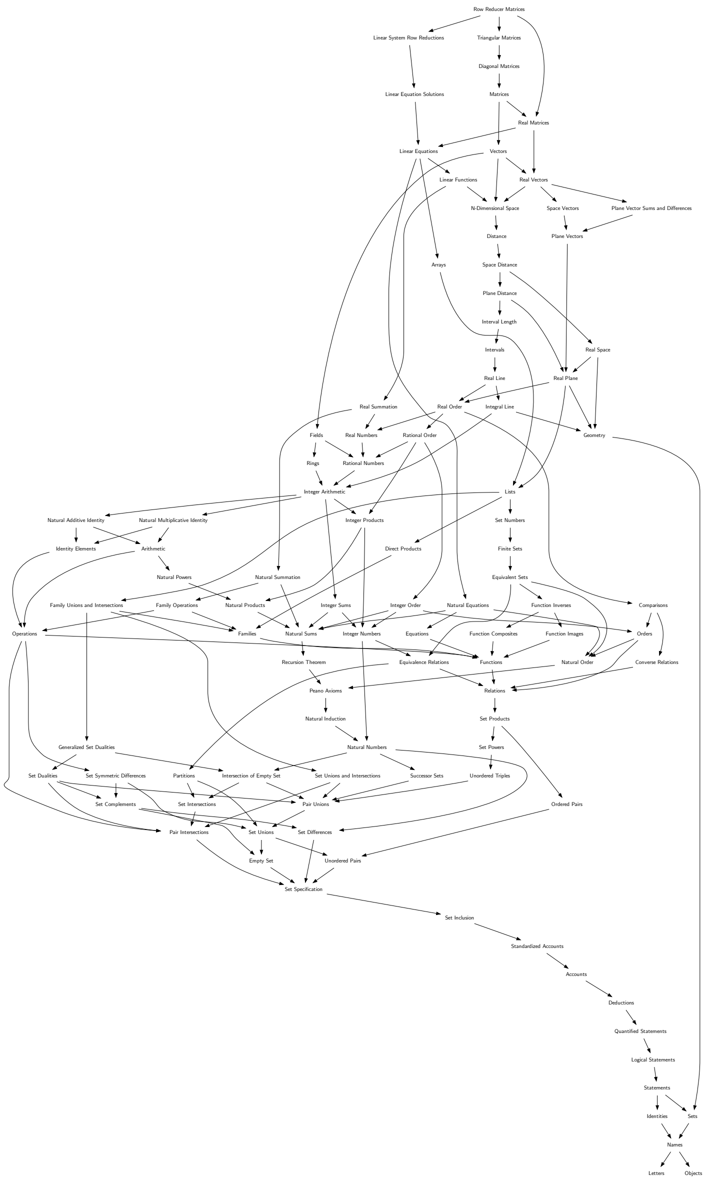
is the linear system

$$A' = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix} \text{ and } b' = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

The row reducer is $L \in R^{4\times 4}$ defined by

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix}.$$

One can check that $A' = LA$ and $b' = Lb$, and clearly L is unit lower triangular.


Row Reducer Matrices
Linear System Row Reductions
Triangular Matrices
Diagonal Matrices
Linear Equation Solutions
Matrices
Real Matrices
Linear Equations
Vectors
Linear Functions
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Arrays
Space Distance
Plane Distance
Interval Length
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Real Space
Real Line
Real Plane
Real Summation
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Natural Multiplicative Identity
Integer Products
Set Numbers
Identity Elements
Arithmetic
Direct Products
Finite Sets
Natural Powers
Natural Summation
Equivalent Sets
Family Unions and Intersections
Family Operations
Natural Products
Integer Sums
Integer Order
Natural Equations
Function Inverses
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Pair Intersections
Set Unions
Set Differences
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