## Why

What is the best estimate for a random variable if we consider the square error?

## Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $x : \Omega \to \mathbf{R}$ a random variable. A *minimum mean squared error estimate* or *MMSE estimate* or *least square estimate* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}(x - \xi)^2$.

**Proposition 1.** *There is a unique MMSE estimate and it is given by* $\mathbf{E}(x)$.

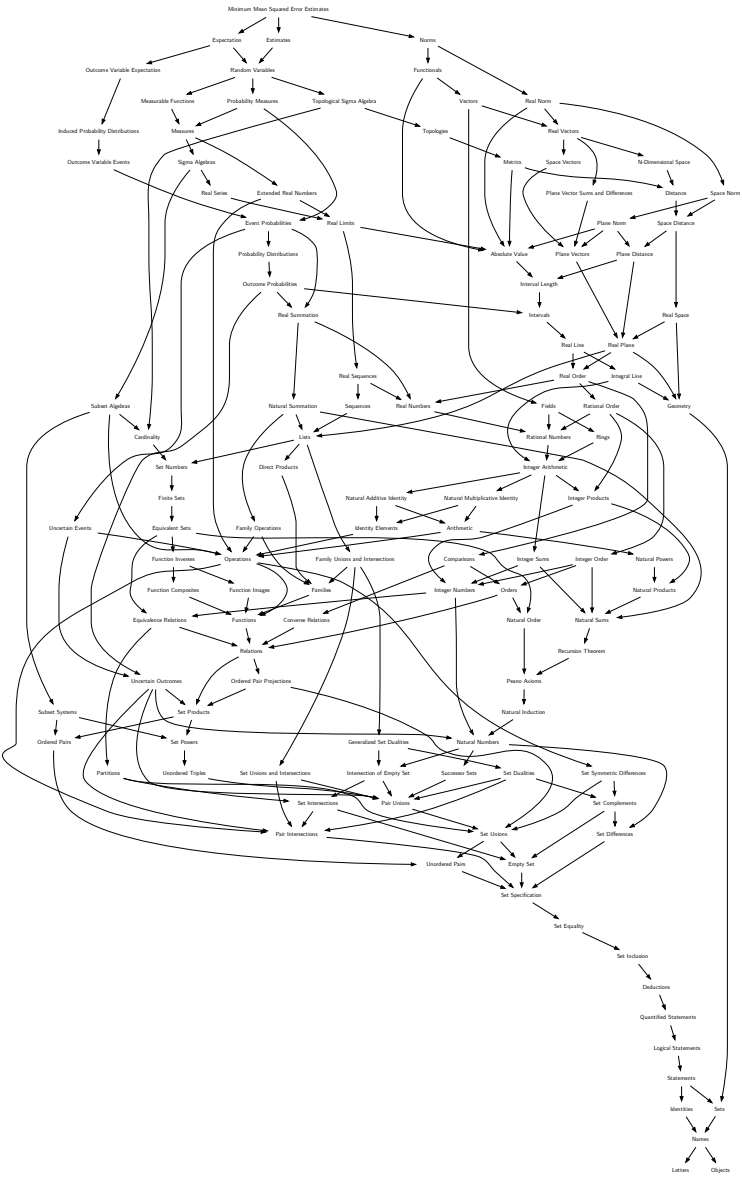
## Vector case

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $y : \Omega \to \mathbf{R}^n$ a random variable.[1]

A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}\|x - \xi\|^2$.

**Proposition 2.** *There is a unique MMSE estimator and it is given by* $\mathbf{E}(y)$.

[1] Future editions might collapse this into the previous case.


Minimum Mean Squared Error Estimates
Expectation
Estimates
Norms
Outcome Variable Expectation
Random Variables
Functionals
Measurable Functions
Probability Measures
Topological Signs Algebra
Vectors
Real Norm
Induced Probability Distributions
Measures
Topologies
Real Vectors
Outcome Variable Events
Signs Algebras
Metrics
Space Vectors
N-Dimensional Space
Real Series
Extended Real Numbers
Plane Vector Sums and Differences
Distance
Space Norm
Event Probabilities
Real Limits
Plane Norm
Space Distance
Probability Distributions
Absolute Value
Plane Vectors
Plane Distance
Outcome Probabilities
Interval Length
Real Summation
Intervals
Real Space
Real Line
Real Plane
Real Sequences
Real Order
Integral Line
Subset Algebras
Natural Summation
Sequences
Real Numbers
Fields
Rational Order
Geometry
Cardinality
Lists
Rational Numbers
Rings
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Uncertain Events
Equivalent Sets
Family Operations
Identity Elements
Arithmetic
Function Inverses
Operations
Family Unions and Intersections
Comparisons
Integer Sums
Integer Order
Natural Powers
Function Composites
Function Images
Families
Integer Numbers
Orders
Natural Products
Equivalence Relations
Functions
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Uncertain Outcomes
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Partitions
Unordered Triples
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects