Why

It is common to consider random functions whose domain is time, space, or n-dimensional space.

Definition

Let (X, d) be a metric space. A *distance covariance function* $k : X \times X \to \mathbf{R}$ is a covariance function satisfying

$$k(x, y) > k(x, y) \longleftrightarrow d(x, y) < d(x, y).$$

In other words, the covariance decreases as the distance between the arguments decreases.

Example: squared exponential

Let $k : X \times X \to \mathbf{R}$ be defined by

$$k(x, y) = \exp(-d(x, y)).$$

Then k is a distance covariance function. It is often called the *squared exponential covariance function.*

Let $\alpha, \sigma \in \mathbf{R}$. Define $k' : X \times X \to \mathbf{R}$ by

$$k'(x, y) = \alpha \exp(-d(x, y)/\sigma^2)$$

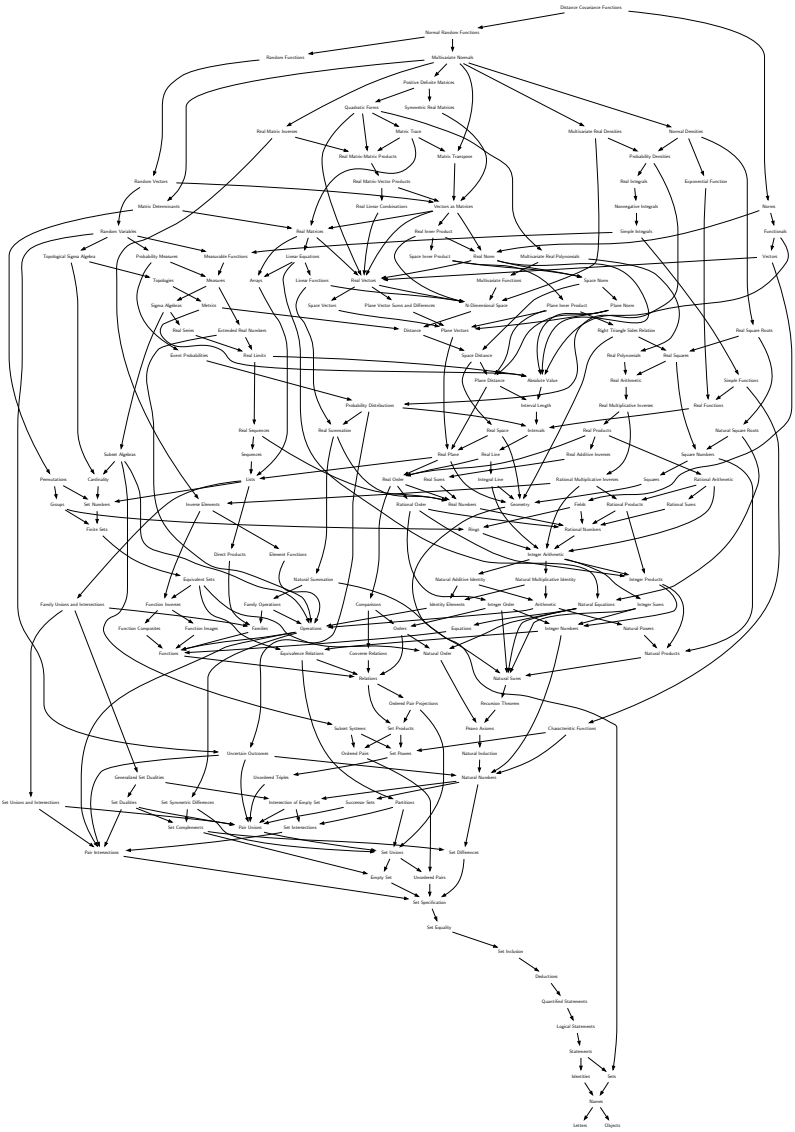
then k' is still a covariance function. In this context σ is often referred to as the *characteristic length-scale* of the process. The scalar α is sometimes called a "prefactor" that "controls" the "overall variance."

Suppose $(X, d) = (\mathbf{R}^n, \|\cdot\|)$. Then the squared exponential covariance function

$$\alpha \exp(-\|x - y\|/(2\sigma^2))$$

is sometimes called the *radial basis function* or *gaussian covariance function.*[1] Also called an *exponentiated quadratic kernel.*

[1] For reasons that will be included in future editions.


Distance Covariance Functions
Normal Random Functions
Random Functions
Multivariate Normals
Positive Definite Matrices
Quadratic Forms
Symmetric Real Matrices
Real Matrix Inverses
Matrix Trace
Multivariate Real Densities
Normal Densities
Real Matrix-Matrix Products
Matrix Transpose
Probability Densities
Random Vectors
Real Matrix-Vector Products
Real Integrals
Exponential Function
Matrix Determinants
Real Linear Combinations
Vectors as Matrices
Nonnegative Integrals
Norms
Random Variables
Real Matrices
Real Inner Product
Simple Integrals
Functionals
Topological Sigma Algebra
Probability Measures
Measurable Functions
Linear Equations
Space Inner Product
Real Norms
Multivariate Real Polynomials
Vectors
Topologies
Measures
Areas
Linear Functions
Real Vectors
Multivariate Functions
Space Norms
Sigma Algebras
Metrics
Space Vectors
Plane Vector Sums and Differences
n-Dimensional Space
Plane Inner Product
Plane Norms
Real Series
Extended Real Numbers
Distance
Plane Vectors
Right Triangle Side Relation
Real Square Roots
Real Limits
Space Distance
Real Polynomials
Real Squares
Event Probabilities
Plane Distance
Absolute Value
Real Arithmetic
Simple Functions
Probability Distributions
Interval Length
Real Multiplicative Inverses
Real Functions
Real Sequences
Real Summation
Real Spaces
Intervals
Real Products
Natural Square Roots
Subset Algebras
Sequences
Real Plane
Real Line
Real Addition Inverses
Square Numbers
Permutations
Cardinality
Lists
Real Order
Real Sums
Integral Line
Rational Multiplicative Inverses
Squares
Natural Arithmetic
Groups
Set Numbers
Inverse Elements
Rational Order
Real Numbers
Integrity
Fields
Rational Problems
Rational Sums
Finite Sets
Rings
Rational Numbers
Direct Products
Element Functions
Integer Arithmetic
Equivalence Sets
Natural Summation
Natural Multiplication Identity
Natural Addition Identity
Integer Products
Family Unions and Intersections
Function Inverses
Family Operations
Commutation
Identity Elements
Integer Order
Arithmetic
Natural Equations
Integer Sums
Function Composition
Function Images
Combos
Operations
Orders
Equations
Integer Numbers
Natural Powers
Functions
Equivalence Relations
Converse Relations
Natural Order
Natural Products
Relations
Natural Sums
Ordered Pair Projections
Recursion Theorem
Subset Systems
Set Products
Peano Axioms
Characteristic Functions
Uncertain Outcomes
Ordered Pairs
Set Powers
Natural Induction
Unordered Triple
Natural Numbers
Generalized Set Qualifiers
Set Union and Intersections
Set Qualifiers
Set Symmetric Differences
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Set Union
Set Intersection
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deduction
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects