

LINEAR FUNCTIONALS

Why

Definition

A linear functional on a vector space V with field k is a function $f:V\to k$ which satisfies

$$f(\lambda x + \mu y) = \lambda f(x) + \mu f(y)$$

for all $\lambda, \mu \in k$ and $x, y \in V$.

Examples

1. Define $F: \mathbf{C}^n \to \mathbf{C}$ by

$$F(x_1,\ldots,x_n) = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

where $c_1, \ldots, c_n \in \mathbf{C}$.

2. Let $(c_n)_{n \in \mathbb{N}} \in \ell^{\infty}$. Define $F_c : \ell^1 \to \mathbf{C}$ by

$$F_c((x_n)_{n\in\mathbf{N}}) = \sum_{n=1}^{\infty} c_n x_n$$

.

 $\wp(A)$.

