



## Definition

Let  $\Omega$  be an open set in  $\mathbf{C}$  and let  $f : \Omega \rightarrow \mathbf{C}$ . The function  $f$  is *holomorphic at the point*  $z_0 \in \mathbf{C}$  if the complex quotient

$$\frac{f(z_0 + h) - f(z_0)}{h}$$

has a limit when  $h \rightarrow 0$ , where  $h \in \mathbf{C}$ ,  $h \neq 0$  and  $z_0 + h \in \Omega$  so that the quotient is well-defined.

This condition is similar to saying that a function is differentiable, except that the  $h$  is complex and so the condition above encompasses all limits approaching  $z$  (all angles) in the complex plane.<sup>1</sup> But we emphasize that  $h$  is a complex number approaching the complex number  $(0, 0)$  from any direction. If the limit exists, then we call its value the *derivative of  $f$  at  $z_0$* .

The function  $f$  is *holomorphic* on  $\Omega$  if  $f$  is holomorphic at every point of  $\Omega$ . If  $C$  is a closed subset of  $\mathbf{C}$ , we say that  $f$  is holomorphic on  $C$  if  $f$  is holomorphic on some open set containing  $c$ . If  $f$  is holomorphic on all of  $C$  then we call  $f$  *entire*. A holomorphic function is sometimes called *regular* or *complex differentiable*. The latter term is used in view of the similarities with the definition of a real derivative.

## Notation

In the case that  $f : \Omega \rightarrow \mathbf{C}$  is holomorphic at  $z_0$  we denote the derivative at  $z_0$  by  $f'(z_0)$ . We have defined  $f'(z_0)$  by

$$f'(z_0) = \lim_{h \rightarrow 0} \frac{f(z_0 + h) - f(z_0)}{h}.$$

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<sup>1</sup>Future editions will clarify.



