



INTEGER NUMBERS

Why

We want to subtract numbers.¹

Definition

Consider the set $\omega \times \omega$. This set is the set of ordered pairs of ω . In other words, the ordered pairs of natural numbers.

We say that two of these ordered pairs (a, b) and (c, d) is *integer equivalent* the $a + d = b + c$. Briefly, the intuition is that (a, b) represents a less b , or in the usual notation “ $a - b$ ”.² So this equivalence relation says these two are the same if $a - b = c - d$ or else $a + d = b + c$.

Proposition 1. *Integer equivalence is an equivalence relation.*³

We define the *set of integer numbers* to be the set of equivalence classes (see **Equivalence Relations**) under integer equivalence on $\omega \times \omega$. We call an element of the set of integer numbers an *integer number* or an *integer*. We call the set of integer numbers the *set of integers* or *integers* for short.

Notation

We denote the set of integers by \mathbf{Z} . If we denote integer equivalence by \sim then $\mathbf{Z} = (\omega \times \omega) / \sim$.

¹Future editions will change this why. In particular, by referencing **Inverse Elements** and the lack thereof in ω .

²This account will be expanded in future editions.

³The proof is straightforward. It will be included in future editions.

