

REAL PLANE

Why

We are constantly thinking of the elements of ${\bf R}^2$ as points of a plane.¹

Discussion

We commonly associate elements of \mathbb{R}^2 with points on a plane. (see Geometry).

Principle 1 (Line Sets). Given a plane, there exists a set of its (infinite) lines.

Principle 2 (Real Plane Correspondence). Let L be the set of lines of a plane. Then $\cup L$ is the set of points of the plane. There exists a one-to-one correspondence mapping elements of $\cup L$ onto elements of \mathbb{R}^2 .

For this reason, we sometimes call elements of \mathbb{R}^2 points. We call the point associated with (0,0) the *origin*. We call the element of \mathbb{R}^2 which corresponds to a point the *coordinates* of the point.

Visualization

To visualize the correspondence we draw two perpendicular lines. We then associate a point of the line with $(0,0) \in \mathbb{R}^2$. We can label it so. We then pick a unit length. And proceed as usual.²

Given that we have identified a plane with \mathbb{R}^2 in this way, we call $(x,y) \in \mathbb{R}^2$ the *coordinates* of the point it corresponds to. Many authors refer to this identification as a *Cartesian coordinate system* (or *Rectangular coordinate system*, x-y coordinate system).

¹Future editions will modify this sheet.

 $^{^2}$ Future editions will expand this.

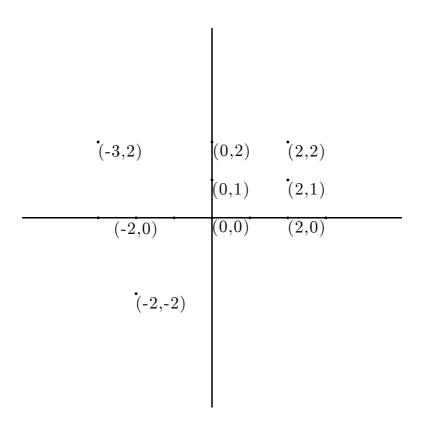


Figure 1: The real plane

