

#### CHORDAL GRAPHS

### Why

Many problems simplify if the graph involved is chordal.<sup>1</sup>

#### **Paths**

Let G be an undirected graph. A *chord* in a path p of G is an edge between two non-consecutive vertices of p. So a chord of the path  $(v_1, v_1, \ldots, v_k)$  is an edge  $\{v_i, v_j\}$  with |j - i| > 1.

We interpret a chord as a "one-edge shortcut" between two vertices of a path. If a path p has a chord, it can be reduced to a shorter path p' by "skipping" vertices. In other words, the shortest path between two vertices is chordless. However, a chordless path need not be a shortest path. See the figure below.

### **Graphs**

A chord of a cycle  $(v_1, v_2, \ldots, v_{k-1}, v_1)$  is an edge  $\{v_i, v_j\}$  with (j-i) mod k > 1. An undirected graph G is *chordal* if every cycle with more than three edges has a chord.

If G is chordal, every cycle in G can be reduced to a cycle of length three. We sometimes call a cycle of length three a triangle. For this reason, chordal graphs are also sometimes called triangulated graphs. Other terminology includes rigid-circuit graphs, triangulated graphs, perfect elimination graphs, decomposable graphs.<sup>2</sup>

The last graph in the figure below is not chordal because the cycle (a, b, d, c, a) has length four and no chord. Adding the edge  $\{b, c\}$  or  $\{a, d\}$  would make the graph chordal An immediate consequence of the definition that G be chordal is that any subgraph of G is chordal.

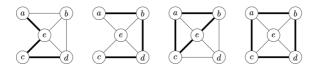
<sup>&</sup>lt;sup>1</sup>Future editions will expand.

<sup>&</sup>lt;sup>2</sup>See Vanenberghe and Anderson, 2014.

# Simple examples

Since trees and forests have no cycles, they are chordal. Similarly, any graph with no cycles longer than three edges are trivially chordal. Such graphs are sometimes called *cactus graphs*. The complete graphs are also trivially chordal.

## Specific example



The edge  $\{e,d\}$  is a chord in the path (a,e,c,d) of the first graph. The path (a,v,d,c) is chordless. The edge  $\{a,e\}$  is a chord in the cycle (a,v,e,c,a) of the second graph. The cycle (a,b,d,c,a) is chordless.

