Why

Does a set exist of all the ordered pairs of elements from an ordered pair of sets?

Definition

Let A and B denote sets. Ordered pairs are sets of singletons and pairs. So to construct the set of all ordered pairs taken from two sets, we want to specify the elements of a set which contains all singletons $\{a\}$ and pairs $\{a, b\}$ for $a \in A$, $b \in B$.

Notice that $a \in A$ and $b \in A$ mean $a, b \in (A \cup B)$. In other words, $\{a\} \subset A$ and $\{b\} \subset B$ and $\{a\}, \{b\} \subset (A \cup B)$. In particular, $\{a\} \in \mathcal{P}((A \cup B))$. Similarly, $\{a, b\} \in \mathcal{P}((A \cup B))$. And so $\{\{a\}, \{a, b\}\} \in \mathcal{P}(\mathcal{P}((A \cup B)))$.

We define the set of "all ordered pairs" from A and B by specifying the appropriate pairs of this set.[1]

$$\{(a, b) \in \mathcal{P}(\mathcal{P}((A \cup B))) \mid a \in A \wedge b \in B\}$$

We name this set the *product* of the set denoted by A and the set denoted by B is the set of all ordered pairs. This set is also called the *set product* (or *cartesian product*[2]). If $A \neq B$, the ordering causes the product of A and B to differ from the product of B with A. If $A = B$, however, the symmetry holds.

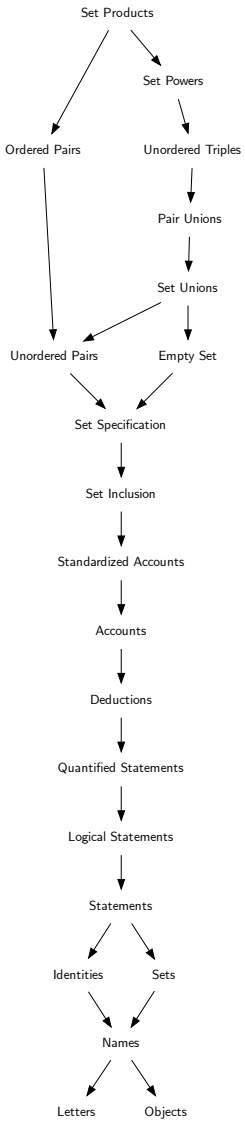
[1] The specific statement used here requires some translation. A discussion of this and the full statement will appear in a future edition.

[2] This second term is universal, but avoided in accordance with the project policy on naming.

Notation

We denote the product of A with B by $A \times B$, read aloud as "A cross B." In this notation, if $A \neq B$, then $A \times B \neq B \times A$.[3]

[3] Future editions may include a table figure visualizing the product.


Set Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects