



## Definition

An operator  $T \in \mathcal{L}(V)$  is called *self-adjoint* (or *Hermitian*) if the adjoint of  $T$  is itself. In symbols,  $T$  is self-adjoint if  $T = T^*$ . In other words,  $T$  is self-adjoint if and only if

$$\langle Tv, w \rangle = \langle v, Tw \rangle \quad \text{for all } v, w \in V$$

## Properties

**Proposition 1.** *Suppose  $S, T \in \mathcal{L}(V)$  are self-adjoint. The  $S + T$  are self-adjoint. Also  $\lambda T$  is adjoint for all real  $\lambda$ .*

## Notation

We will see that the adjoint on  $\mathcal{L}(V)$  plays a role similar to complex conjugation on  $\mathbf{C}$ . The self-adjoint operators will seem to be analogous to the real numbers. A complex number is real if and only if  $z = z^*$ . Similarly, an operator is self-adjoint if and only if  $T = T^*$ .

## Characterization for complex space

**Proposition 2.** *Suppose  $V$  is a complex inner product space and let  $T \in \mathcal{L}(V)$ . Then*

$$T = T^* \quad \longleftrightarrow \quad (\forall v \in V) (\langle Tv, v \rangle \in \mathbf{R})$$



