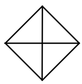


Why

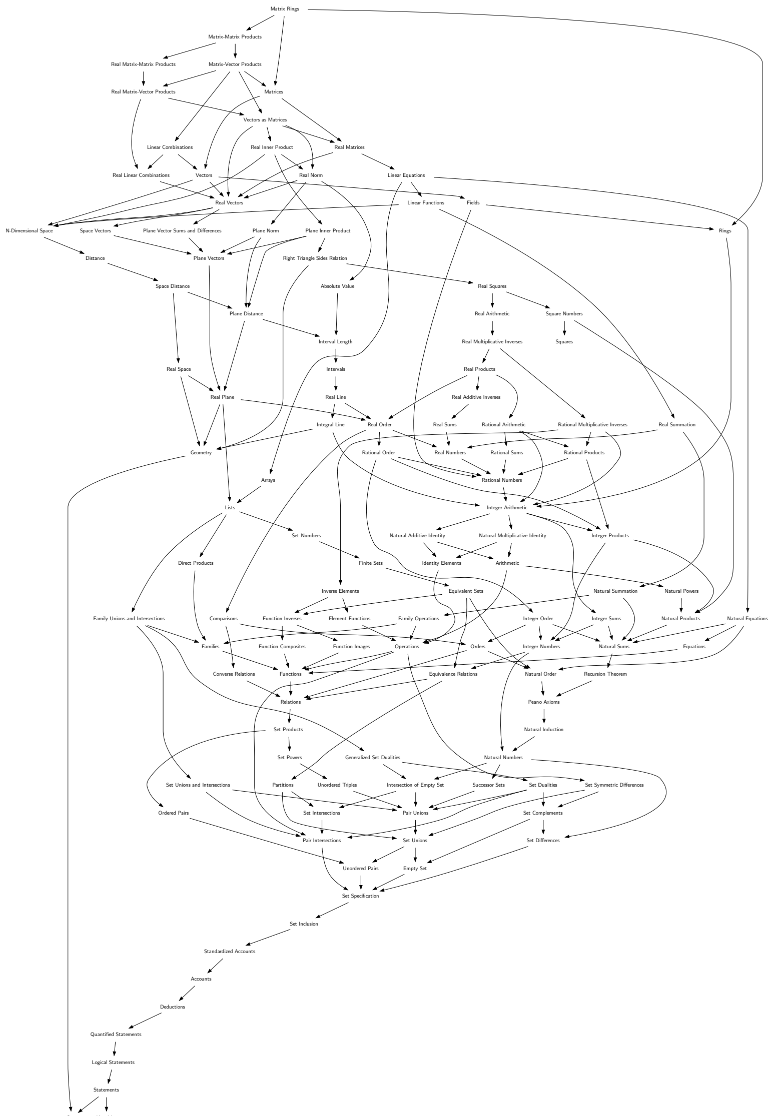
Matrices with elements in a ring form a ring.

Example

Let $(R, +, \cdot)$ be a ring. Define $C = A\bar{+}B$ by $C_{ij} = A_{ij} + B_{ij}$ and define $C = A\bar{\cdot}B$ by $C_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}$, as with real matrices, for $A, B \in R^{n\times n}$. Then $(R^{n\times n}, \bar{+}, \bar{\cdot})$ is a ring. In other words, the set of $n \times n$ matrices with elements in R is a ring, with the usual addition and multiplication of matrices.

The additive identity of the ring is the matrix $0 \in R^{n\times n}$ for which $0_{ij} = 0 \in R$. The multiplicative identity the matrix I for which $I_{ii} = 1 \in R$ for $i = 1, \dots, n$ and $I_{ij} = 0 \in R$ for $i \neq j = 1, \dots, n$. As seen with real-valued matrices, multiplication on $R^{n\times n}$ need not be commutative even if R is.

Exercise 1. *Show that $R^{n\times n}$ is not a division ring when $n > 1$.*


Matrix Rings
Matrix-Matrix Products
Real Matrix-Matrix Products
Matrix-Vector Products
Real Matrix-Vector Products
Matrices
Vectors as Matrices
Linear Combinations
Real Inner Product
Real Matrices
Real Linear Combinations
Vectors
Real Norm
Linear Equations
Real Vectors
Linear Functions
Fields
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Plane Norm
Plane Inner Product
Rings
Distance
Plane Vectors
Right Triangle Sides Relation
Space Distance
Absolute Value
Real Squares
Plane Distance
Real Arithmetic
Square Numbers
Interval Length
Real Multiplicative Inverses
Squares
Real Space
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Real Summation
Geometry
Rational Order
Real Numbers
Rational Sums
Rational Products
Arrays
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Inverse Elements
Equivalent Sets
Natural Summation
Natural Powers
Family Unions and Intersections
Comparisons
Function Inverses
Element Functions
Family Operations
Integer Order
Integer Sums
Natural Products
Natural Equations
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Equations
Converse Relations
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects