

BOUNDED KNAPSACK PROBLEMS

Why

We consider the knapsack problem in which the n items are considered to be types of items and we have a certain quantity of each.

Definition

Suppose we have zero-one knapsack problem data (p, w, c) where $p : [n] \to \mathbb{R}$ is the profit function, $w : [n] \to \mathbb{R}_+$ is the weight function, and $c \in \mathbb{R}_+$ is the capacity constraint. Given budgets $b_1, \ldots, b_n \in \mathbb{Z}_+$, find $x \in \mathbb{Z}_+^n$ to

minimize
$$\sum_i p_i x_i$$

subject to $\sum_i w_i x_i \le c$
 $0 \le x_i \le b_i, \quad i = 1, \dots, n,$
 $x_i \in \mathbf{Z} \quad i = 1, \dots, n$

The above is called the *bounded knapsack problem*. The problem above without the budget constraints, is called the *unbounded knapsack problem*.

