Why

We can identify any linear functional $F : \mathbf{R}^n \to \mathbf{R}$ with a vector $y \in \mathbf{R}^n$ so that $F(x) = \langle x, y \rangle$. We generalize this result to complete inner product spaces.

Motivating result

The following is known as the *Riesz representation theorem* (or *Riesz-Fréchet representation theorem*, or *Riesz theorem*, or *Riesz-Fréchet theorem*).

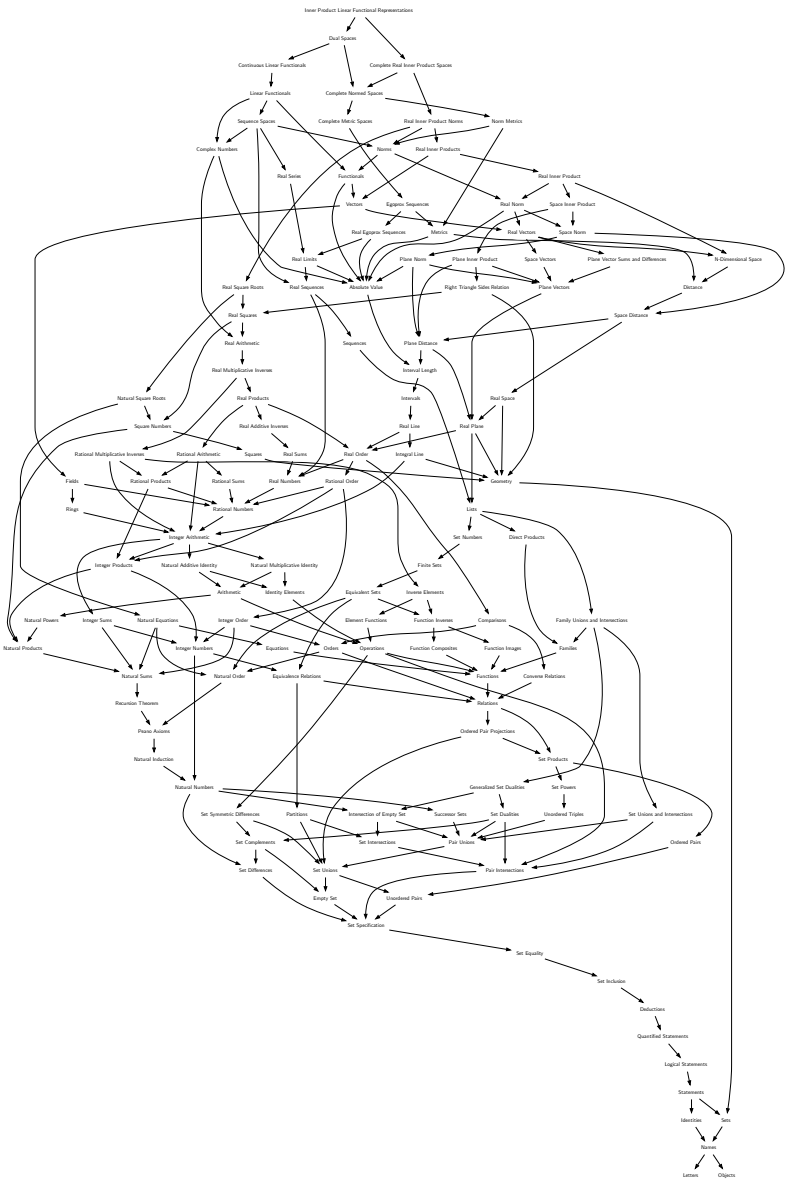
Proposition 1. *Let $((V, k), \langle \cdot, \cdot \rangle)$ be a complete inner product space and let $F : V \to k$ be a continuous linear functional on V. There exists a unique $y \in V$ so that*

$$F(x) = \langle x, y \rangle$$

for all $x \in V$. Moreover $\|y\| = \|F\|_*$.

Clearly $\mathbf{R}^n$ is a complete inner product space, and so this this theorem says the expected. We can identify linear functionals on $\mathbf{R}^n$ with elements (vectors) in $\mathbf{R}^n$.[1]

[1] Future editions will expand further.


Inner Product Linear Functional Representations
Dual Spaces
Continuous Linear Functionals
Complete Real Inner Product Spaces
Linear Functionals
Complete Normed Spaces
Sequence Spaces
Complete Metric Spaces
Real Inner Product Norms
Norm Metrics
Complex Numbers
Norms
Real Inner Products
Real Series
Functionals
Real Inner Product
Vectors
Equipoise Sequences
Real Norm
Space Inner Product
Real Equipoise Sequences
Metrics
Real Vectors
Space Norm
Real Limits
Plane Norm
Plane Inner Product
Space Vectors
Plane Vector Sums and Differences
n-Dimensional Space
Real Square Roots
Real Sequences
Absolute Value
Right Triangle Sides Relation
Plane Vectors
Distance
Real Squares
Space Distance
Real Arithmetic
Sequences
Plane Distance
Interval Length
Real Multiplicative Inverses
Natural Square Roots
Real Products
Intervals
Real Space
Square Numbers
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Squares
Real Sums
Real Order
Integral Line
Fields
Rational Products
Rational Sums
Real Numbers
Rational Order
Geometry
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Natural Equations
Integer Order
Element Functions
Function Inverses
Comparisons
Family Unions and Intersections
Natural Products
Integer Numbers
Equations
Orders
Operations
Function Composites
Function Images
Families
Natural Sums
Natural Order
Equivalence Relations
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Lemas
Objects