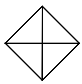


Why

We want to record natural numbers. Recall that we denote $\varnothing$ by 0, the successor of 0 by 1 and the sucessor of 1 by 2. 1 is the success of 0 and 2 is the successor of 1. We agree to continue 3, 4, 5, 6, 7, 9, each the successor if its predecessor in the list. Given these numbers, how should we denote the successor to 9?

Discussion

We could of course have a new symbol, and denote the successor of 9 by this symbol. But then we would need as many symbols are there are natural numbers. And the natural numbers are *infinite* (not finite!). The following is a trick around this, re-using our ten symbols. Why ten?... How many fingers do you have?

Definition

The *natural notation* (or *base-ten notation*, *base-10 notation*) for a natural number is a list of elements of the set

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

To a notation η corresponds a natural number, n as follows. Suppose η has length k, so that $\eta = (\eta_1, \dots, \eta_k)$. The natural corresponding to η is

$$\eta_1 + \eta_2 \cdot 9^+ + \eta_3 \cdot (9^+)^2 + \cdots + \eta_k \cdot (9^+)^{k-1}$$

We can use summation notation and the fact that $(9^+)^0 = 1$ to write

$$\sum_{i=1}^{k} \eta_i \cdot (9^+)^{i-1}$$

Notation

In a slightly different but universally standard way, we denote the natural notation $\eta = (\eta_1, \dots, \eta_k)$ by

$$\eta_k \eta_{k-1} \cdots \eta_2 \eta_1$$

Examples

Some examples will help, and illustrate the notation. Here is a natural notation for some number: $(0, 1)$. Which number? Well, we have agreed that it is the number

$$0 + 1 \cdot 9^{+} = 9^{+}$$

We denote the natural notation for 10. This is the natural notation for the sucessor of 9. The upshot is clear, we have denoted the sucessor to 9 using only ten symbols.

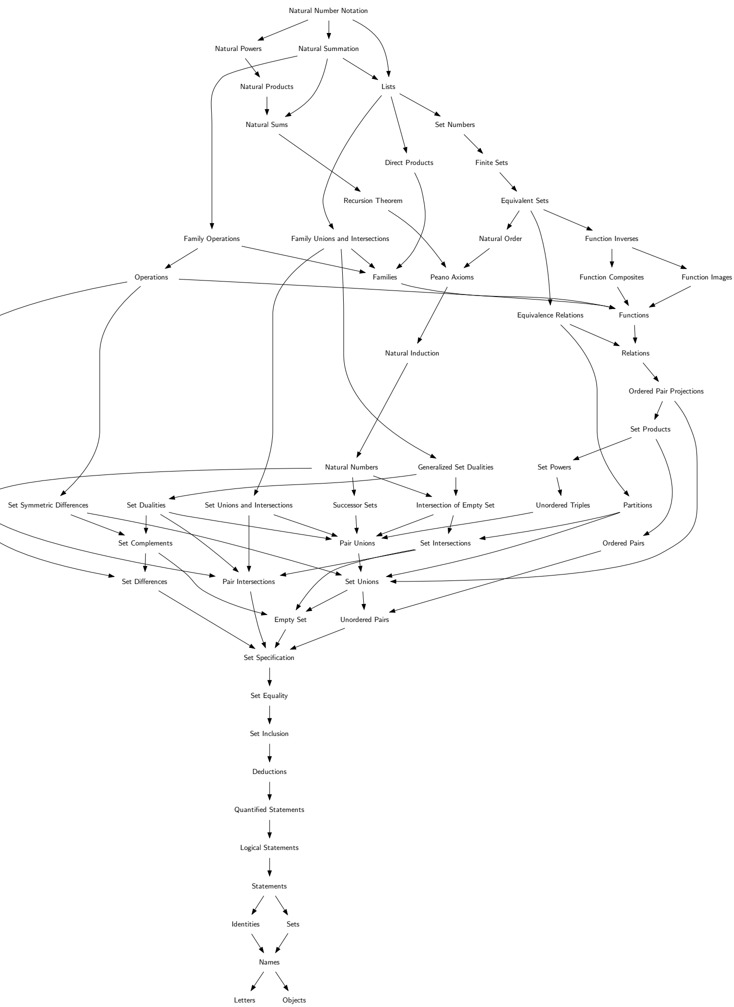
Here is a natural notation for some number: $(3, 2, 1)$. Which number? Well, we have agreed that it is the number

$$1 + 2 \cdot 9^{+} + 3 \cdot (9^{+})^{2}$$

Recall that we have notation for the successor to 9 now. We can write this number as

$$1 + 2 \cdot 10 + 3 \cdot (10)^{2}$$

We see that powers of 10 show up in the sum for the number corresponding to this notation. We name these powers. 100 is the number *hundred*. 1000 is the number *thousand*


Natural Number Notation
Natural Powers
Natural Summation
Natural Products
Lists
Natural Sums
Set Numbers
Direct Products
Finite Sets
Recursion Theorem
Equivalent Sets
Family Operations
Family Unions and Intersections
Natural Order
Function Inverses
Operations
Families
Peano Axioms
Function Composites
Function Images
Equivalence Relations
Functions
Natural Induction
Relations
Ordered Pair Projections
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects