

Complex Numbers

Why

We want to find the roots of negative numbers.¹

Definition

A complex number is an ordered pair of real numbers. The real part of a complex number is its first coordinate. The imaginary part of a complex number is its second coordinate.

We can identify the imaginary numbers with no complex part (i.e., $\{(a,b) \in \mathbb{R}^2 \mid b=0\}$ with \mathbb{R} in the obvious way. For this reason, such a complex number is sometimes referred to as a purely real number Conversely, a complex number with zero imaginary part (i.e., an element of $\{(a,b) \in \mathbb{R}^2 \mid a=0\}$) is said to be a purely imaginary number.

Notation

When treating \mathbb{R}^2 as the set of complex numbers, we denote it by \mathbb{C} . Let $z \in \mathbb{C}$ with z = (a, b). The real part of z is a and its imaginary part is b. It is universal to denote z by a + ib, and to call i an (or the) *imaginary number*. Some authors use j, it is a matter of notation.

We denote the real part of z by Re(z), read "real of z," and the imaginary part by Im(z), read "imaginary of z." So, in particular, Re(z) = a and Im(z) = b.

¹Future editions will modify this, and will discuss the existence of solutions of algebraic equations.

