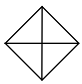


Why

We want to construct new sets out of old ones. So, can we always construct subsets?

Definition

We will say that we can. More specifically, if we have a set and some statement which may be true or false for the elements of that set, a set exists containing all and only the elements for which the statement is true.

Roughly speaking, the principle is like this. We have a set which contains some objects. Suppose the set of playing cards in a usual deck exists. We are taking as a principle that the set of all fives exists, so does the set of all fours, as does the set of all hearts, and the set of all face cards. Roughly, the corresponding statements are "it is a five", "it is a four", "it is a heart", and "it is a face card".

Principle 1 (Specification)**.** *For any statement and any set, there is a subset whose elements satisfy the statement.*

We call this the *principle of specification.* We call the second set (obtained from the first) the set obtained by *specifying* elements according to the sentence. The principle of extension (see *Set Equality*) says that this set is unique. All basic principles about sets (other than the principle of extension, see *Set*

Equality) assert that we can construct new sets out of old ones in reasonable ways.

Let A denote a set. Let s denote a statement in which the symbol x and A appear unbound. We assert that there is a set, denote it by B, for which belonging is equivalent to membership in A and s. In other words, regardless of the identity of A and s, there exists A' so that

$$(\forall x)((x \in B) \longleftrightarrow ((x \in A) \land s(x))).$$

Notation

Denote by A a set, and by $s(x)$ a statement with the unbound variable x. We denote the set of those elements of A which satisfy s by

$$\{a \in A \mid \; s(a) \; \}.$$

We read the symbol $|$ aloud as "such that." We read the whole notation aloud as "a in A such that..."

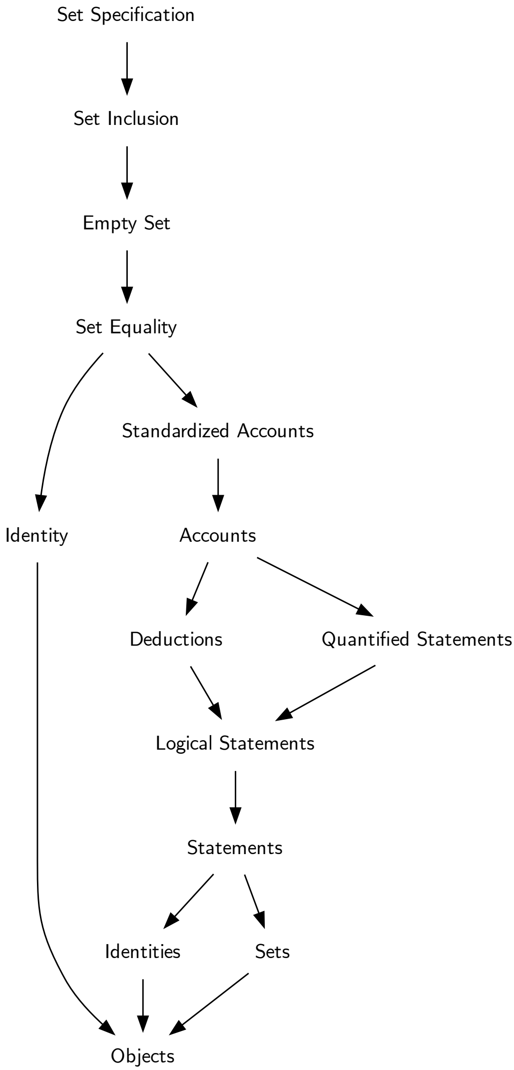
We call the notation *set-builder notation*. Set-builder notation avoids enumerating elements. This notation is really indispensable for sets which have many members, too many to reasonably write down.

Nothing contains everything

Account 1.

1-2	`name`	$A, B := \{x \in A \mid x \notin x\}$	
3	`have`	$(B \in B) \longleftrightarrow ((B \in A) \wedge B \notin B)$	
4	`thus`	$B \notin A$	`by 3`

In other words, if we had taken as principle "there is a set that contains all other sets" and used specification, we would have a paradox (sometimes called *Russell's paradox*)


Set Specification
Set Inclusion
Empty Set
Set Equality
Standardized Accounts
Identity
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Objects