



Why

We want to visualize the probabilistic relations between components of outcomes in probabilistic models over large (e.g., product) outcome sets.¹

Definition

Suppose X_1, \dots, X_k are sets and define $X = \prod_{i=1}^k X_i$. For $x \in X$ and $S \subset \{1, \dots, n\}$, denote the subvector of x indexed (in order) by S by x_S .²

A distribution $p : X \rightarrow [0, 1]$ *factors according to a directed graph* on $\{1, \dots, n\}$ with parent function $\text{pa} : \{1, \dots, n\} \rightarrow \mathcal{P}(\{1, \dots, n\})$ if

$$p(x) = \prod_{\text{pa}_i = \emptyset} g_i(x_i) \prod_{\text{pa}_i \neq \emptyset} g_i(x_i, x_{\text{pa}_i}),$$

where g_i is a distribution for all i which $\text{pa}_i = \emptyset$ and $g_i(\cdot, \xi)$ is a distribution for all $\xi \in \prod_{j \in \text{pa}_i} A_j$, i for which $\text{pa}_i \neq \emptyset$.

Proposition 1. *p so defined is a distribution, and the g_i are the marginals and conditionals.*³

Examples

Consider a rooted tree distribution (see [Rooted Tree Distributions](#)), or a memory chain (see [Memory Chains](#)), or a hidden memory chain (see [Hidden Memory Chains](#)).⁴

¹Future editions will modify and expand. The title of the sheet may change, since another interpretation for the words “directed graph distribution” is a distribution on directed graphs.

²Future editions will rework this treatment, perhaps combining it with the sheet [Index Matrices](#), which will possibly be split up.

³Future editions will be precise and give an account.

⁴Future editions will expand.

