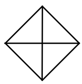


Why

We have a space X and a family of probability measures $\mathcal{P}$ on this space. Assume $x \in X$ drawn from a fixed, unknown measure $P \in \mathcal{P}$. Given x, how should we guess P?

Definition

A *probabilistic model* (or *statistical model*, *parametric statistical model*, *statistical experiment*) is a family of probability measures over the same measurable space $(X, \mathcal{F})$. Call the index set the *parameter set* or *set of parameters*. The set X is called the *sample space*. A *statistic* is any function on the sample space.

Notation

Let $(X, \mathcal{F})$ denote a measurable space. We usually denote the parameter by Θ, and denote the family

$$\mathcal{P} = \{\mathbf{P}_\theta : \mathcal{F} \to [0,1] \mid \mathbf{P}_\theta \text{ a measure}, \theta \in \Theta\}.$$

Often $\Theta \subset \mathbf{R}^d$.

Example: coin flips

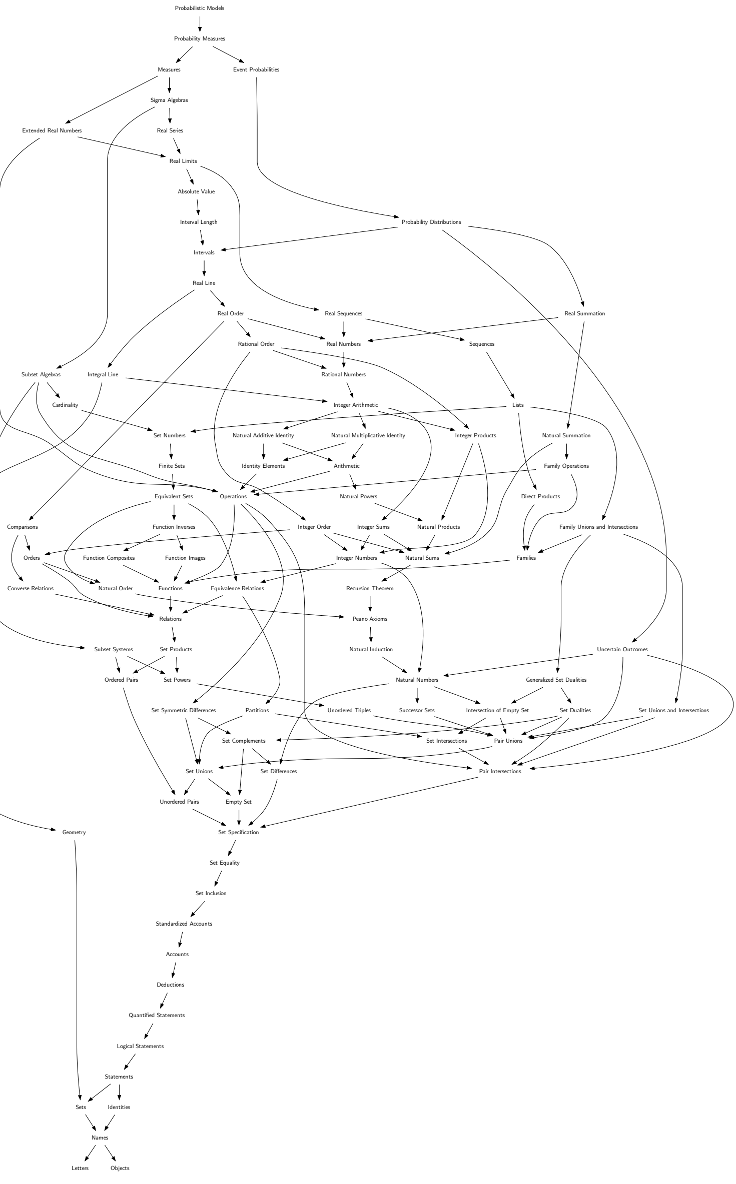
The usual model for n flips of a coin takes $X = \{0,1\}^n$, the set of binary n-tuples. For $\theta \in [0,1]$, a distribution $p_\theta(x) = \theta^t(1-\theta)^{n-t(x)}$ where $t(x) = x_1 + \cdots + x_n$ is defined on X. A probability measure $\mathbf{P}_\theta$ is defined on $\mathcal{P}(X)$ in the usual way. Thus, the probabilistic model is $\{\mathbf{P}_\theta \mid \theta \in [0,1]\}$. Given x, we are asked to guess θ.

Example: gaussian

A common model for quantities takes $X = \mathbf{R}^d$, the d-dimensional real space.

Decisions

A *decision procedure* (*estimator*, *statistical procedure*) is a measurable function that $A : \mathcal{X} \to \mathcal{A}$ where $\mathcal{A}$ is a set of possible *decisions*, or *actions*. A *loss function* is


Probabilistic Models
Probability Measures
Measures
Event Probabilities
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Interval Length
Probability Distributions
Intervals
Real Line
Real Order
Real Sequences
Real Summation
Rational Order
Real Numbers
Sequences
Subset Algebras
Integral Line
Rational Numbers
Cardinality
Integer Arithmetic
Lists
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Natural Summation
Finite Sets
Identity Elements
Arithmetic
Family Operations
Equivalent Sets
Operations
Natural Powers
Direct Products
Comparisons
Function Inverses
Integer Order
Integer Sums
Natural Products
Family Unions and Intersections
Orders
Function Composites
Function Images
Integer Numbers
Natural Sums
Families
Converse Relations
Natural Order
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects