



# Signed Measure Examples

## 1 Why

Let us consider examples of signed measures. TODO

## 2 Examples

Consider an integrable function defined on some measurable space. The extended-real-valued function which assigns to each distinguished set the value of the integrating the function over that set is a signed measure. See Example 1.

Consider

**Example 1.** Let  $(X, \mathcal{A}, \mu)$  a measure space. Let  $R$  denote the set of real numbers. Let  $f : X \rightarrow R$  an integrable function. Define  $\nu : \mathcal{A} \rightarrow R$  by

$$\nu(A) = \int_A f d\mu.$$

Then  $\nu$  is a signed measure.

*Proof.* First,

$$\nu(\emptyset) = \int f \chi_{\emptyset} d\mu = \int 0 d\mu = 0.$$

Next, let  $(A_n)_n$  disjoint. Notice that,

$$\chi_{\cup_{i=1}^n A_k} = \sum_{i=k}^n \chi_{A_k}$$

so for all  $n$ ,

□