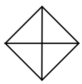


Why

If we stack two rectangles, with equal base lengths but different heights, on top of each other, the additivity principle says that the area of the so-formed rectangle is the sum of the areas of the stacked rectangles. Our definition of integral for simple functions has this property, as it ought to.

Result

Prop. 1. *The simple non-negative integral operator is additive.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $\mathcal{SF}_+(X)$ denote the non-negative real-valued simple functions on X. Define $s : \mathcal{SF}_+(X) \to [0, \infty]$ by $s(f) = \int f d\mu$ for $f \in \mathcal{SF}_+(X)$.

In this notation, we want to show that $s(f+g) = s(f)+s(g)$ for all $f, g \in \mathcal{SF}_+(X)$. Toward this end, let $f, g \in \mathcal{SF}_+(X)$ with the simple partitions:

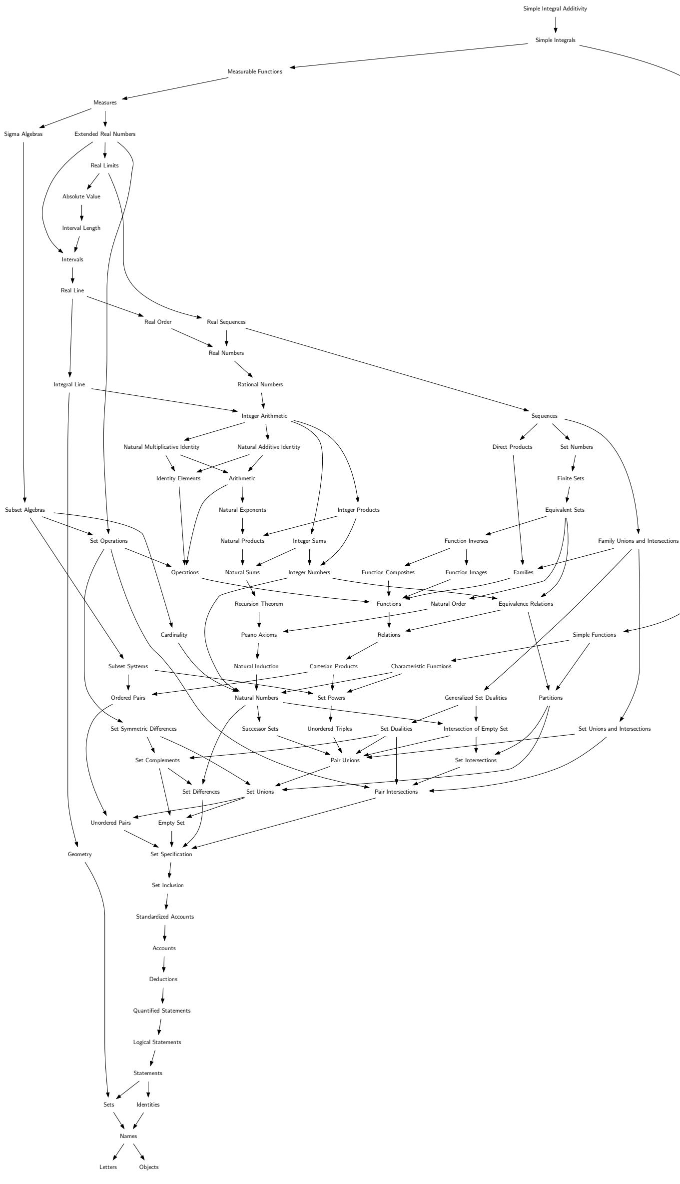
$$\{A_i\}_{i=1}^m, \{B_j\}_{j=1}^n \subset \mathcal{A} \quad \text{and} \quad \{a_i\}_{i=1}^m, \{b_j\}_{j=1}^n \subset [0, \infty].$$

We consider the refinement of the two partitions. TODO: this is why you don't do the unique maximal partition business. $\{A_i \cap B_j\}_{i,j=1}^{i=m,j=n}$.

First, let $\alpha \in (0, \infty)$. Then $\alpha f \in \mathcal{SF}_+(X)$, with the simple partition $\{A_n\} \subset \mathcal{A}$ and $\{\alpha a_n\} \subset [0, \infty]$.

$$s(\alpha f) = \sum_{i=1}^n \alpha a_n \mu(A_i) \quad = \alpha \sum_{i=1}^n a_n \mu(A_i) = \alpha s(f).$$

If $\alpha = 0$, then αf is uniformly zero; it is the non-negative simple with partition $\{X\}$ and $\{0\}$. Regardless of the measure of X, this non-negative simple function is zero Recall that we define $0 \cdot \infty = \infty \cdot 0 = 0$. $\square$


Simple Integral Additivity
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Sequences
Natural Multiplicative Identity
Natural Additive Identity
Direct Products
Set Numbers
Identity Elements
Arithmetic
Finite Sets
Subset Algebras
Natural Exponents
Integer Products
Equivalent Sets
Set Operations
Natural Products
Integer Sums
Function Inverses
Family Unions and Intersections
Operations
Natural Sums
Integer Numbers
Function Composites
Function Images
Families
Recursion Theorem
Functions
Natural Order
Equivalence Relations
Cardinality
Peano Axioms
Relations
Simple Functions
Subset Systems
Natural Induction
Cartesian Products
Characteristic Functions
Ordered Pairs
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects