Why

We can generalize the real general linear groups to vector spaces over $\mathbf{C}$.

Definition

Suppose V is a vector space over the field $\mathbf{C}$ of complex numbers. The set of isomorphisms of V onto itself is a group, called the *general linear group*, under the operation of composition. If V has dimension n, then the general linear group can be identified with the invertible $n \times n$ complex matrices in the usual way.

Notation

We denote by $GL(V)$ the general linear group of isomorphisms of V onto itself. If $f \in GL(V)$, and V has a finite basis $e_1, \ldots, e_n \in V$, then f has corresponding matrix representation $A \in \mathbf{C}^{n \times n}$ given by

$$A = \begin{bmatrix} f(e_1) & \cdots & f(e_n) \end{bmatrix}.$$

General Linear Groups

Vector Space Isomorphisms

Real General Linear Groups — Vector Space Dimension

Real Matrix Inverses — Vector Space Bases

Complex Matrices — Real Matrix-Matrix Products — Linearly Dependent Vectors — Span

Complex Arithmetic — Matrices — Real Matrix-Vector Products — Linear Transformations — Subspaces

Complex Products — Real Matrices — Vectors as Matrices — Real Inner Product — Transformations — Linear Combination — Vectors — Real Subspaces

Linear Equations — Complex Sums — Space Inner Product — Real Norm — Real Vectors — Real Vector Span

Arrays — Linear Functions — Complex Numbers — Space Norm — N-Dimensional Space — Plane Vector Sums and Differences — Space Vectors — Real Linear Combinations

Plane Inner Product — Plane Norm — Absolute Value — Plane Vectors — Distance

Right Triangle Sides Relation — Real Squares — Plane Distance — Space Distance

Real Arithmetic — Interval Length — Real Space

Real Multiplicative Inverses — Intervals — Square Numbers — Real Plane

Real Products — Real Addition Inverses — Real Line — Squares — Geometry — Fields

Real Summation — Rational Multiplicative Inverses — Rational Arithmetic — Real Sums — Real Order — Integral Line

Lists — Inverse Elements — Rational Products — Rational Sums — Real Numbers — Rational Order — Rings

Direct Products — Set Numbers — Element Functions — Rational Numbers — Integer Arithmetic

Natural Summation — Finite Sets — Equivalent Sets — Natural Additive Identity — Natural Multiplicative Identity — Integer Products

Natural Equations — Family Unions and Intersections — Family Operations — Function Inverses — Function Image — Identity Elements — Arithmetic — Natural Powers — Integer Sums — Integer Order — Comparisons

Equations — Families — Function Composites — Functions — Operations — Natural Order — Natural Products — Equivalence Relations — Orders — Converse Relations

Natural Sums — Relations

Recursion Theorem

Peano Axioms — Set Products

Natural Induction — Set Names

Natural Numbers — Partitions

Generalized Set Dualities — Unordered Triples — Ordered Pairs

Set Dualities — Set Symmetric Differences — Successor Sets — Intersection of Empty Set

Set Union and Intersection — Set Complements — Pair Unions — Set Intersections

Set Differences — Set Unions

Empty Set — Unordered Pairs

Pair Intersection

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects