



Product Metrics

1 Why

Given n sets each with metrics, there is a standard way of turning the direct product of the sets into a metric space. In other words, defining a distance on the tuples of elements from the sets.

2 Motivating Result

Proposition 1. *Let $(A_1, d_1), (A_2, d_2), \dots, (A_n, d_n)$ be metric spaces.*

Let A be $\prod_{i=1}^n A_i$ and let R be the set of real numbers. Define

$d : A \times A \rightarrow R$ by

$$d(a, b) = \max\{d_1(a_1, b_1), \dots, d_n(a_n, b_n)\}.$$

Then (A, d) is a metric space.