Why

Definition

A *hyperplane* in n-dimensional space is an $(n-1)$-dimensional affine set.

Since the $n-1$-dimensional subspaces are the orthogonal complements of the one-dimensional subspaces, they are the sets which can be specified by

$$\{x \in \mathbf{R}^n \mid x \perp b\}$$

for $b \in \mathbf{R}^n$. The hyperplanes are translates of these,

$$\begin{aligned}\{x \in \mathbf{R}^n \mid x \perp b\} + a &= \{x + a \mid \langle x \rangle b = 0\} \\ &= \{y \mid \langle y - a \rangle b = 0\} = \{y \mid \langle y \rangle b = \beta\},\end{aligned}$$

where $\beta = \langle a \rangle b$.

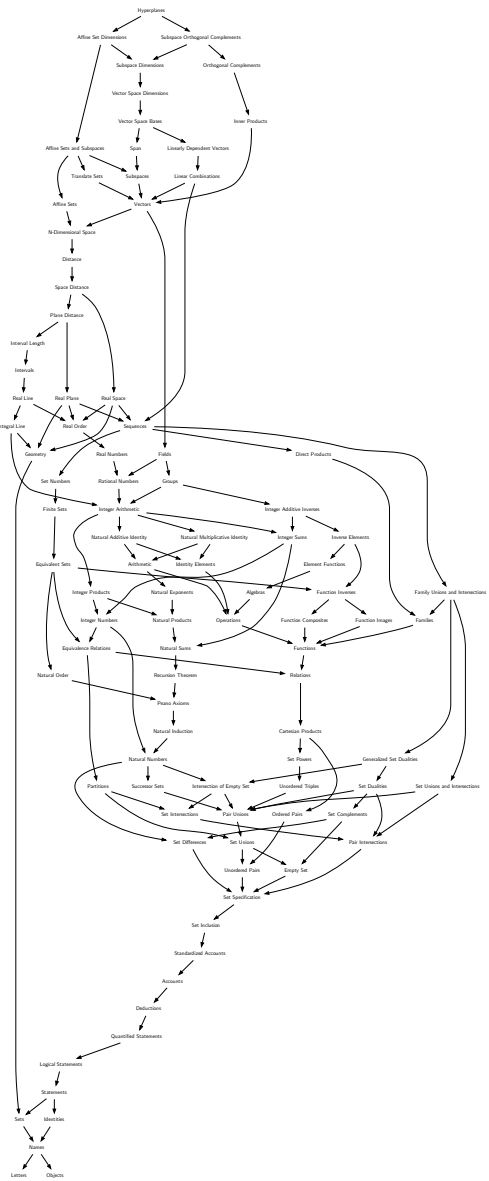
Characterization

Prop. 1. *$H \subset \mathbf{R}^n$ is a hyperplane if and only if there exists $\beta \in \mathbf{R}$ and nonzero $b \in \mathbf{R}^n$ so that*

$$H = \{x \in \mathbf{R}^n \mid \langle x \rangle b = \beta\}.$$

Remark 1. *b and β are unique up to a common nonzero multiple. For example, b, β and $2b, 2\beta$ give the same hyperplane.*

Remark 2. *The vector b is called a* normal *to the hyperplane.*


Hyperplanes
Affine Set Dimensions
Subspace Orthogonal Complements
Subspace Dimensions
Orthogonal Complements
Vector Space Dimensions
Vector Space Bases
Inner Products
Affine Sets and Subspaces
Span
Linearly Dependent Vectors
Translate Sets
Subspaces
Linear Combinations
Affine Sets
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Sequences
Geometry
Real Numbers
Fields
Direct Products
Set Numbers
Rational Numbers
Groups
Finite Sets
Integer Arithmetic
Integer Additive Inverses
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Inverse Elements
Equivalent Sets
Arithmetic
Identity Elements
Element Functions
Integer Products
Natural Exponents
Algebras
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Operations
Function Composition
Function Images
Families
Equivalence Relations
Natural Sums
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Latches
Objects