Why

As with introducing Equivalent Sets, we want to talk about the size of a set.[1]

Definition

A *finite* set is one that is equivalent to some natural number; an infinite set is one which is not finite. From this we can show that ω is infinite. This justifies the language "principle of infinity" with Natural Numbers. The principle of infinity asserts the existence of a particular infinite set; namely ω.

Motivation for set number

It happens that if a set is equivalent to a natural number, it is equivalent to only one natural number.

Proposition 1. *A set can be equivalent to at most one natural number.*[2]

A consequence is that a finite set is never equivalent to a proper subset of itself. So long as we are considering finite sets, a piece (subset) is always less than than the whole (original set).

Proposition 2. *A finite set is never equivalent to a proper subset of itself.*

Subsets of finite sets

Every subset of a natural number is equivalent to a natural number.[3] A consequence is:

Proposition 3. *Every subset of a finite set is finite.*[4]

---

[1] Will be expanded in future editions.

[2] Future edition will include proof, which uses comparability of numbers and the results of Equivalent Sets).

[3] This requires proof, and may become a proposition in future editions.

[4] An account will appear in future editions.

Unions of finite sets

Proposition 4. *if A and B are finite, then $A \cup B$ is finite.*

Products of finite sets

Proposition 5. *If A and B are finite, then $A \times B$ is finite.*

Powers of finite sets

Proposition 6. *If A is finite then $\mathcal{P}(A)$ is finite.*

Functions between finite sets

Proposition 7. *If A and B are finite, then A^B is finite.*

Finite Sets

Equivalent Sets

Function Inverses   Natural Order

Function Composites   Function Images   Peano Axioms

Functions   Equivalence Relations

Relations   Natural Induction

Set Products

Set Powers   Natural Numbers

Unordered Triples   Partitions   Successor Sets   Intersection of Empty Set

Ordered Pairs   Pair Unions   Set Intersections

Set Unions   Set Differences

Unordered Pairs   Empty Set   Pair Intersections

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects

4