



## Definition

The *translate* of  $S \subset \mathbf{R}^n$  by  $a \in \mathbf{R}^n$  is the set

$$\{z \in \mathbf{R}^n \mid \exists x \in S \text{ such that } z = x + a\}.$$

## Notation

We often use the abbreviated notation  $S + a$  for the translate of  $S$  by  $a$ . It is sometimes also convenient to extend set-builder notation and write

$$S + a = \{x + a \mid x \in M\}.$$

The right hand side is slick notation for the definition given above.

## Sums and differences

The *sum* of two sets  $S, T \subset \mathbf{R}^n$  is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x + y)\}.$$

Likewise, the *difference* of two sets  $S, T \subset \mathbf{R}^n$  is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x - y)\}.$$

## Notation

We denote the sum of  $S$  and  $T$  by  $S + T$ , and the difference by  $S - T$ . We often use the slick notation

$$\{x + y \mid x \in S, y \in T\} \text{ and } \{x - y \mid x \in S, y \in T\},$$

for these two sets.



