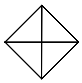

Why

Consider bounding the measure where two functions differ. We would look at their absolute value, and the measure of where this is greater than zero. The absolute value of their difference is a non-negative measurable random variable.

Result

We bound the measure that a non-negative measurable real-valued function exceeds some value by its integral.

PROPOSITION **1.** *Let* $(X, \mathcal{A}, \mu)$ *be a measure space. Let* $g : X \to [0, \infty]$ *be measurable. Then for all* $t > 0$*,*

$$\mu(\{x \in X \mid g(x) \geq t\}) \leq \frac{\int g d\mu}{t}.$$

Proof. Let $A = \{x \in X \mid g(x) \geq t\}$. Define $h : X \to R$ by

$$h(x) = \begin{cases} 1 & \text{if } g(x) \geq t \\ 0 & \text{otherwise.} \end{cases}$$

First, $\mu(A) = \int h d\mu$. Second, $h \leq g$. So:

$$\mu(\{x \in X \mid g(x) \geq t\}) = \int h d\mu \quad \leq \int g/t d\mu = \frac{\int g d\mu}{t}.$$

□

The above is also called *Markov's Inequality*


Tail Measure Upper Bound
Variance
Expectation
Random Variables
Probability Measures
Measurable Functions
Measures
Event Probabilities
Extended Real Numbers
Topological Sigma Algebra
Probability Distributions
Real Limits
Topological Spaces
Probability Events
Real Summation
Intervals
Real Sequences
Metrics
Probability Outcomes
Natural Summation
Real Numbers
Sequences
Subset Algebras
Distance
Probability
Rational Numbers
Family Operations
Direct Products
Cardinality
Set Operations
Length Common Notions
Common Sense
Integer Numbers
Algebras
Natural Families
Injective Functions
Natural Order
Zero
Families
Operations
Function Composites
Subset Systems
Natural Sums
Equation Solutions
Power Set
Functions
Natural Numbers
Set Symmetric Differences
Relations
Set Unions
Set Intersections
Ordered Pairs
Set Complements
Unordered Pairs
Empty Set
Equations
Set Specification
Set Inclusion
Set Equality
Sentences
Identity
Sets
Objects