

## NORMAL DENSITIES

## Why

We want a density that is symmetric about some center value with some spread.

## Definition

Let  $f : \mathbb{R} \to \mathbb{R}$  be a density. If there exists  $\mu \in \mathbb{R}$  and  $\sigma \in \mathbb{R}$  with  $\sigma > 0$  so that for each  $x \in \mathbb{R}$ 

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

then f is a normal density. A normal density is often called a Gaussian density.<sup>1</sup> We often drop the word density and use refer to these as normals or Gaussians, using these words as substantives.

We call the special case when  $\mu = 0$  and  $\sigma = 1$  the standard normal density or standard gaussian density.

## Maximum

The maximum of a normal density is  $\mu$ .

<sup>&</sup>lt;sup>1</sup>We do not use this term in accordance with the Bourbaki project's policy on historical names.

