Why

Suppose we flip a coin until it lands heads. What is the probability that we see a head after one flip? Two flips? n flips? We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

Discussion

Consider a set A.[1] If A has n elements, then a probability distribution on A is $p : A \to \mathbf{R}$ where $p(a) = 1/n$. There is a natural candidate.

What if A is the set of natural numbers $\mathbf{N}$. The principle difficulty is that not all sequences of real numbers $a : \mathbf{N} \to \mathbf{R}$ are summable.

Definition

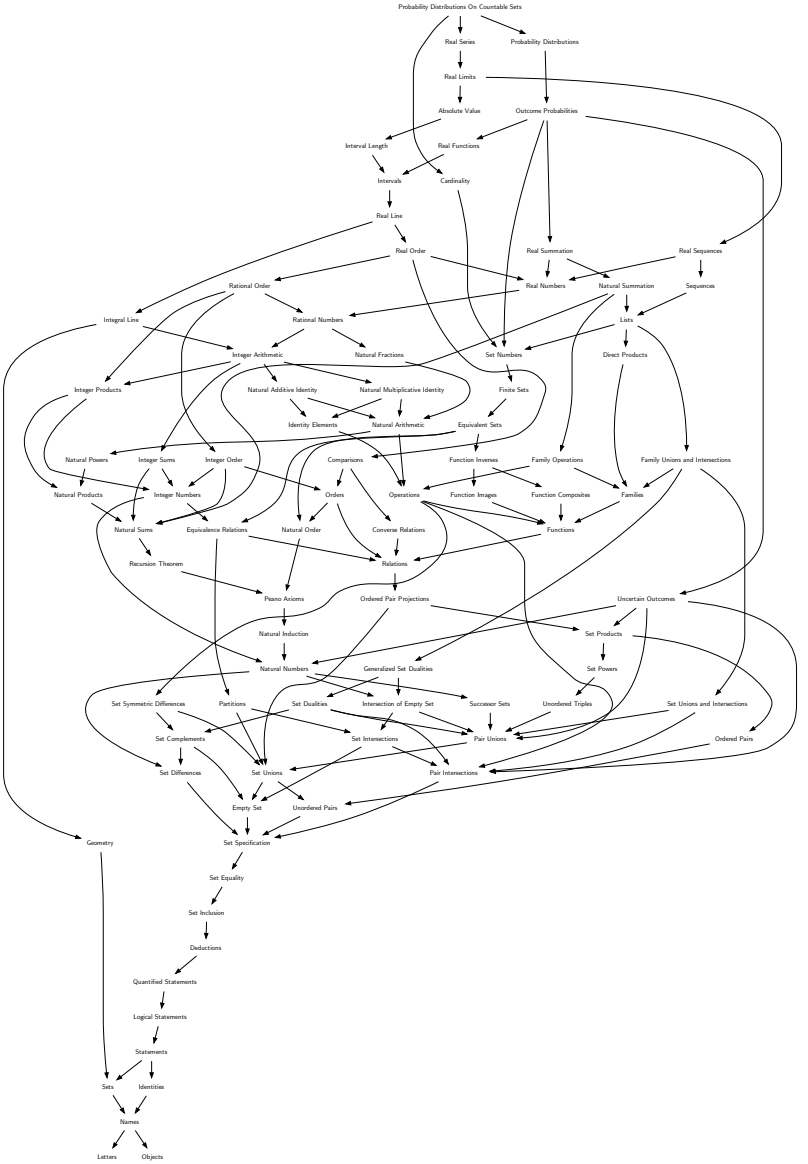
A *(countable) probability distribution* on $\mathbf{N}$ is $p : \mathbf{N} \to \mathbf{R}$ where $p \geq 0$ and

$$\sum_{n=1}^{\infty} p(n) = 1.$$

More generally, a probability distribution on some countable set C which we have numbered $c : \mathbf{N} \to C$, is a function $p : C \to [0, 1]$ such that

$$\sum_{i=1}^{n} p(c_i) = 1$$

[1] Future editions will rework this sheet.


Probability Distributions On Countable Sets
Real Series
Probability Distributions
Real Limits
Absolute Value
Outcome Probabilities
Interval Length
Real Functions
Intervals
Cardinality
Real Line
Real Order
Real Summation
Real Sequences
Rational Order
Real Numbers
Natural Summation
Sequences
Integral Line
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Operations
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Partitions
Set Dualities
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects