

Random Variables Joint Law

1 Why

We name the image measure of a collection of real-valued random variables.

2 Definition

The **joint law** of an ordered finite family of n real-valued random variables is the image measure of the tuple-valued function whose components are the individual random variables.

2.1 Notation

Let R denote the set of real numbers and let R^n denote set of n-tuples of real numbers. Let f_1, \ldots, f_n be random variables defined on a probability space (X, \mathcal{A}, μ) . Define $f: X \to R^n$ by $(f(x))_i = f_i(x)$. The joint law is the image measure of f.

We denote the joint law of $\{f_i\}$ by $\mu_{f_1,\dots,f_n}: \mathcal{A} \to [0,\infty]$. We defined it by

$$\mu_{f_1,\dots,f_n}(A) = \mu(\{x \in X \mid f(x) \in A\}).$$

for all A in the topological sigma algebra on \mathbb{R}^n .