Why

We want to define area under an extended real function. We use the infinite process to approximate the area under a nonnegative extended real function using simple functions.

Definition

Consider a measure space.

The *integral* of a measurable nonnegative function is the supremum of integrals over nonnegative simple functions pointwise less than or equal to the function.

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f : X \to [0, \infty]$ be measurable. We denote the integral of f with respect to the measure μ by $\int f d\mu$. We defined:

$$\int f d\mu = \sup\left\{ \int g d\mu \;\middle|\; g \in \mathcal{SF}_+(X) \text{ and } g \leq f \right\}.$$


Nonnegative Integrals
Simple Integrals
Simple Functions
Measurable Functions
Characteristic Functions
Measures
Real Functions
Extended Real Numbers
Intervals
Real Limits
Real Sequences
Partitions
Real Numbers
Sequences
Subset Algebras
Rational Numbers
Direct Products
Cardinality
Set Operations
Integer Numbers
Natural Order
Natural Families
Injective Functions
Algebras
Zero
Families
Function Composites
Subset Systems
Operations
Equation Solutions
Natural Numbers
Power Set
Functions
Set Symmetric Differences
Set Intersections
Relations
Set Unions
Equations
Ordered Pairs
Set Complements
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Set Equality
Sentences
Identity
Sets
Objects