Definition

An *index set* of $\{1, 2, \ldots, n\}$ is a subset of $\{1, 2, \ldots, n\}$. A *submatrix* of an $m \times n$ matrix is a matrix whose are selected according to (TODO) an index set of $\{1, 2, \ldots, m\}$ and index set of $\{1, 2, \ldots, n\}$; we call the first index set the *row index set* and the second index set the *column index set*. A *principal submatrix* is the submatrix selected when the row and column index sets are identical.

A *sequential partition* of $\{1, 2, \ldots, n\}$ is a sequence of index sets such that all elements of a later piece of the partition are larger (in the natural order) than all elements in all previous pieces.

Submatrices

Matrices

Real Matrices

Linear Equations — Vectors

Arrays — Linear Functions — Real Vectors

N-Dimensional Space — Plane Vector Sums and Differences — Space Vectors

Distance — Plane Vectors

Space Distance

Plane Distance

Interval Length

Intervals — Real Space

Real Line — Real Plane

Real Summation — Real Order — Integral Line

Real Numbers — Fields — Rational Order — Geometry

Rings — Rational Numbers

Lists — Integer Arithmetic

Set Numbers — Natural Multiplicative Identity — Natural Additive Identity — Integer Products

Direct Products — Finite Sets — Identity Elements — Arithmetic

Equivalent Sets — Natural Summation — Natural Powers

Family Unions and Intersections — Comparisons — Function Inverse — Family Operations — Natural Equations — Natural Products — Integer Order — Integer Sums

Families — Function Image — Function Composites — Operations — Equations — Orders — Natural Sums — Integer Numbers

Converse Relations — Functions — Equivalence Relations — Natural Order — Recursion Theorem

Relations — Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Set Dualities — Unordered Triples — Intersection of Empty Set — Successor Sets — Partitions — Set Symmetric Differences

Ordered Pairs — Pair Unions — Set Intersections — Set Complements

Pair Intersections — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects

4