



## INTERVALS

### Why

We name and denote subsets of the set of real numbers which correspond to segments of a line.

### Definition

Take two real numbers, with the first less than the second.

An *interval* is one of four sets:

1. the set of real numbers larger than the first number and smaller than the second; we call the interval *open*.
2. the set of real numbers larger than or equal to the first number and smaller than or equal to the second number; we call the interval *closed*.
3. the set of real numbers larger than the first number and smaller than or equal to the second; we call the interval *open on the left* and *closed on the right*
4. the set of real numbers larger than or equal to the first number and smaller than the second; we call the interval *closed on the left* and *open on the right*.

If an interval is neither open nor closed we call it *half-open* or *half-closed*

We call the two numbers the *endpoints* of the interval. An open interval does not contain its endpoints. A closed interval contains its endpoints. A half-open/half-closed interval contains only one of its endpoints. We say that the endpoints *delimit* the interval.

## Notation

Let  $a, b$  be two real numbers which satisfy the relation  $a < b$ .

We denote the open interval from  $a$  to  $b$  by  $(a, b)$ . This notation, although standard, is the same as that for ordered pairs; no confusion arises with adequate context.<sup>1</sup>

We denote the closed interval from  $a$  to  $b$  by  $[a, b]$ . We record the fact  $(a, b) \subset [a, b]$  in our new notation.

We denote the half-open interval from  $a$  to  $b$ , closed on the right, by  $(a, b]$  and the half-open interval from  $a$  to  $b$ , closed on the left, by  $[a, b)$ .<sup>2</sup>

The *unit interval* is the set  $[0_{\mathbf{R}}, 1_{\mathbf{R}}]$  and we sometimes denote it by  $\mathbf{I}$ .

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<sup>1</sup>In future editions, we may use  $\langle a, b \rangle$  or even  $\phi a, b \phi$ .

<sup>2</sup>Some authors use  $]a, b]$ ,  $[a, b[$  and  $]a, b[$ .

