Why

What is the best linear regressor if we choose according to a weighted squared loss function.

Definition

Suppose we have a paired dataset of n records with inputs in $\mathbf{R}^d$ and outputs in $\mathbf{R}$. A *weighted least squares linear predictor* for nonnegative weights $w \in \mathbf{R}^n$, $w \geq 0$, is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$\frac{1}{n} \sum_{i=1}^{n} w_i (y_i - x^\top a^i)^2.$$

Some authors refer to this process of selecting a linear predictor as the *weighted least-squares problem.*

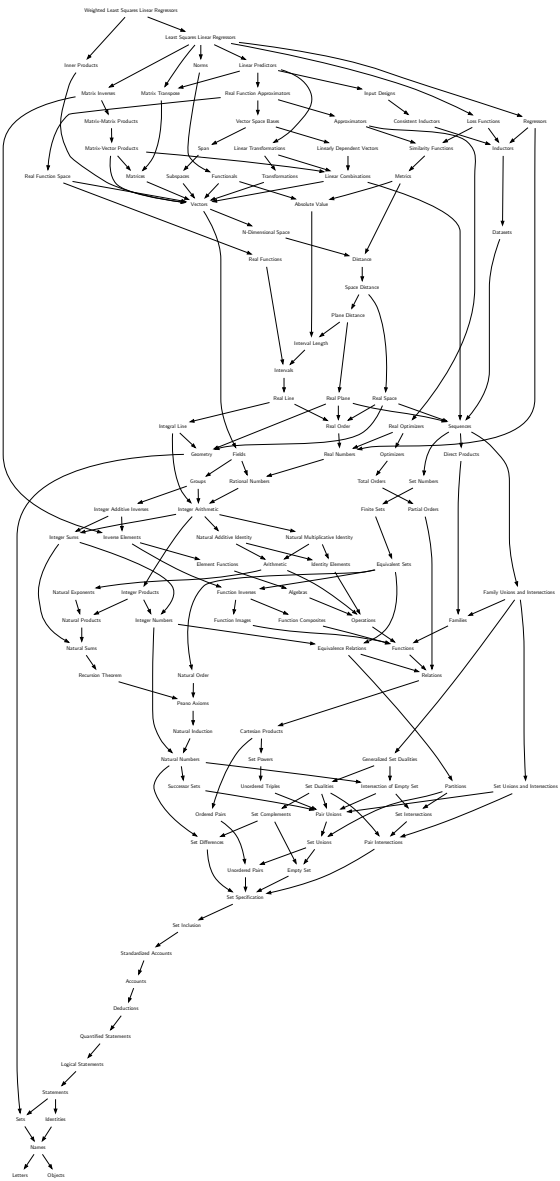
Define $W \in \mathbf{R}^{n \times n}$ so that $W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix. We want to find x to minimize

$$\|W(Ax - y)\|$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$\left(A^\top W A^\top\right)^{-1} A^\top W y.$$


Weighted Least Squares Linear Regressors
Least Squares Linear Regressors
Inner Products
Norms
Linear Predictors
Matrix Inverses
Matrix Transpose
Real Function Approximators
Input Design
Matrix-Matrix Products
Vector Space Bases
Approximators
Consistent Inductors
Loss Functions
Regressors
Matrix-Vector Products
Span
Linear Transformations
Linearly Dependent Vectors
Similarity Functions
Inductors
Real Function Space
Matrices
Subspaces
Functionals
Transformations
Linear Combinations
Metrics
Vectors
Absolute Value
N-Dimensional Space
Datasets
Real Functions
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Real Optimizers
Sequences
Geometry
Fields
Real Numbers
Optimizers
Direct Products
Groups
Rational Numbers
Total Orders
Set Numbers
Integer Additive Inverses
Integer Arithmetic
Finite Sets
Partial Orders
Integer Sums
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Element Functions
Arithmetic
Identity Elements
Equivalent Sets
Natural Exponents
Integer Products
Function Inverses
Algebras
Family Unions and Intersections
Natural Products
Integer Numbers
Function Images
Function Composites
Operations
Families
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Natural Order
Relations
Peano Axioms
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Stratified Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects