



Trace

## 1 Why

TODO

## 2 Definition

The *trace* of a square real matrix is the sum of the elements on its diagonal.

### 2.1 Notation

We denote the function which associates a matrix with its trace by  $\mathbf{tr} : \mathbf{R}^{n \times n} \rightarrow \mathbf{R}$ . Let  $A \in \mathbf{R}^{n \times n}$ . Then

$$\mathbf{tr} A = \sum_{i=1}^n A_{ii}.$$

## 3 Properties

**Proposition 1.** *The trace is a linear function on the vector space of  $n \times n$  real matrices.*

*Proof.* Let  $A, B \in \mathbf{R}^{n \times n}$  and  $\alpha, \beta \in \mathbf{R}$ . Define  $C = \alpha A + \beta B$ . Then  $C_{ii} = \alpha A_{ii} + \beta B_{ii}$ . So

$$\mathbf{tr} C = \sum_{i=1}^n C_{ii} = \sum_{i=1}^n \alpha A_{ii} + \beta B_{ii} = \alpha \sum_{i=1}^n A_{ii} + \beta \sum_{i=1}^n B_{ii} = \alpha \mathbf{tr} A + \beta \mathbf{tr} B.$$

□

**Proposition 2.** *Let  $A, B \in \mathbf{R}^{n \times n}$ .*

$$\mathbf{tr}(AB) = \mathbf{tr}(BA)$$

*Proof.* TODO

□