Sets

1 Why

We want to talk about things, for which we will use the word *object*, and we want to talk about none, one or several things considered as a whole, for which we will use the word *set*.

2 Definition

We use the word **object** with its usual sense in the English language. An object may be tangible, in that we can hold or touch it, or an object may be abstract, in that we can do neither.

A **set** is an abstract object which we think of as several objects considered at once. We say that the set **contains** the objects so considered. We call these the **elements** of the set.

We call the set which contains no objects the **empty set**. We call a set which contains only a single object a **singleton**. A singleton is not the same as the object it contains. Besides these two cases, we think of sets as containing two or more objects.

3 Examples

For familiar examples, let us start with some tangible objects. Find, or call to mind, a deck of playing cards.

First, consider the set of all the cards. This set contains fifty-two elements. Second, consider the set of cards whose suit is hearts. This set contains thirteen elements: the ace, two, three, four, five, six, seven, eight, nine, ten, jack, queen, and king of hearts. Third, consider the set of twos. This set contains four elements: the two of clubs, the two of spades, the two of hearts, and the two of diamonds.

We can imagine many more sets of cards. If we are holding a deck, each of these can be made tangible: we can touch the elements of the set. But the set itself is always abstract: we can not touch it. It is the idea of the group as distinct from any individual member.

Moreover, the elements of a set need not be tangible. First, consider the set consisting of the suits of the playing card: hearts, diamonds, spades, and clubs. This set has four elements. Each element is a suit. Second, consider the set consisting of the card types. This set has thirteen elements: ace, two, three, four, five, six, seven, eight, nine, ten, jack, queen, king. The subtlety here is that this set is different than the set of hearts, namely those thirteen cards which are hearts. However these sets are similar: they both have thirteen elements, and there is a natural correspondence between their elements.

Of course, sets need have nothing to do with playing cards. For example, consider the set of seasons: autumn, winter, spring, and summer. This set has four elements. For another example, consider the set of Latin letters: a, b, c, . . . , x, y, z. This set has twenty-six elements.

3.1 Notation

To aid in discussing and denoting objects, let us tend to give them short names. A single Latin letter regularly suffices: for example, a, b or c. Let us denote that the object a and the object b are the same object by $a = b$, read aloud as "a is b."

For sets, let us tend to use upper case Latin letters: for example, A, B, and C. To aid our memory, let us tend to use the lower case form of the letter for an element of the set. For example, if A is a set, let us tend to denote by a an element of A. Likewise, if B is a set, let us tend to denote by b an element of B.

Let us denote that an object a is an element of a set A by $a \in A$. We read the notation $a \in A$ aloud as "a in A." The $\in$ is a stylized lower case Greek letter: ϵ. It is read aloud "ehp-sih-lawn" and is a mnemonic for "element of". We write $a \notin A$, read aloud as "a not in A," if a is not an element of A.

If we have named the elements of a set, and can list them, let us do so between braces. For example, let a, b, and c be three distinct objects. Denote by $\{a, b, c\}$ the set containing theses three objects and only these three objects. We can further compress notation, and denote this set of three objects by A: so, $A = \{a, b, c\}$. Then $a \in A$, $b \in A$, and $c \in A$. Moreover, if d is an object and $d \in A$, then $d = a$ or $d = b$ or $d = c$.

If the elements of a set are so well-known that we can avoid ambiguityi, then we can describe the set in English. To aid our memory, let us tend to name such sets mnemonically. For example, let L be the set of Latin letters.

Often to be more precise, we should explicitly deal with objects which satisfy several conditions. If the elements of a set satisfy some common condition, then we use the braces and include the condition. For example, let V be the set of Latin vowels. We can denote V by $\{l \in L \mid l \text{ is a vowel}\}$. We read the symbol | aloud as "such that." We read the whole notation aloud as "l in L such that l is a vowel." We call the notation **set-builder notation**. Set-builder notation is indispensable for sets defined implicitly by some condition. Here we could have alternatively denoted V by $\{\text{“}a\text{”}, \text{“}e\text{”}, \text{“}i\text{”}, \text{“}o\text{”}, \text{“}u\text{”}\}$. We prefer the former, slightly more concise notation.