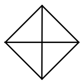


Why

We want to repeatedly multiply.

Defining Result

Proposition 1. *For each natural number m, there exists a function $e_m : \omega \to \omega$ which satisfies*

$$e_m(0) = 1 \quad \text{and} \quad e_m(n^+) = (e_m(n))^+ \cdot m$$

for every natural number n.

Proof. The proof uses the recursion theorem (see *Recursion Theorem*).[1] □

Let m and n be natural numbers. The value $p_m(n)$ is the *power* of m with n. Or the *nth power of m*

Notation

We denote the nth power of m by m^n.

Properties

Here are some basic properties of powers.

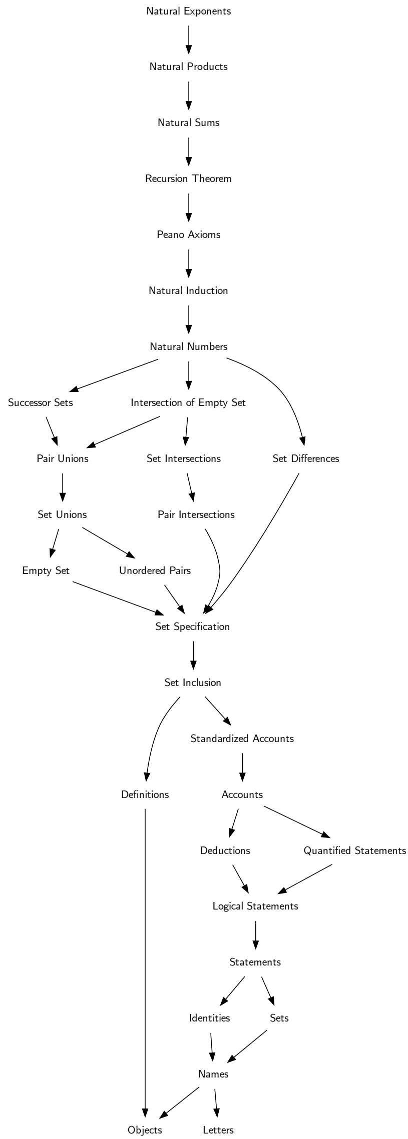
Proposition 2. *Let k, m, and n be natural numbers. Then*

$$m^n m^k = m^{k+k}.$$

[1] Future editions will give the entire account.

Proposition 3. *Let k, m, and n be natural numbers. Then*

$$(m^n)^k = m^{nk}.$$


Natural Exponents
Natural Products
Natural Sums
Recursion Theorem
Peano Axioms
Natural Induction
Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters