Why

We use the language of measure theory to give a mathematical model for uncertain outcomes. TODO: probability intuition sheet.

Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The word *space* is natural, since we developed measure theory partly as a generalization of volume in three-dimensional space. The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra.

The measure in a probability space corresponds to the even probability function.

Notation

Let $(A, \mathcal{A})$ be a measurable space. We denote the set of outcomes by Ω, a mnemonic for "outcomes." We denote the sigma-algebra by $\mathcal{A}$, as usual. We denote a probability measure by $\mathbf{P}$, a mnemonic for "probability," and intended to remind of the

event probabilty function. Thus, we often say "Let $(A, \mathcal{A}, \mathbf{P})$ be a probability space."

Properties

Probability Measures

Event Probabilities · Measures

Probability Distributions · Extended Real Numbers

Intervals · Real Limits

Real Summation · Real Order · Real Sequences

Real Numbers

Rational Numbers

Probability Events · Natural Summation · Integer Arithmetic

Probability Outcomes · Subset Algebras · Natural Multiplicative Identity · Natural Additive Identity · Family Operations · Sequences

Probability · Set Operations · Cardinality · Integer Products · Integer Sums · Arithmetic · Identity Elements · Algebras · Families

Common Sense · Integer Numbers · Natural Exponents · Operations

Natural Products · Equivalence Relations · Functions

Natural Sums · Relations

Recursion Theorem · Subset Systems · Cartesian Products

Peano Axioms · Set Powers · Ordered Pairs

Natural Induction

Unordered Triples · Natural Numbers

Partitions · Intersection of Empty Set · Successor Sets

Set Symmetric Differences · Set Intersections · Pair Unions

Pair Intersections · Set Complements · Set Unions

Empty Set · Set Differences · Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Definitions · Accounts

Deductions · Quantified Statements

Logical Statements

Statements

Identities · Sets

Names

Objects · Letters