



Why

1

Parameterizing distributions

Definition

A *variational autoencoder* is an ordered pair whose first coordinate with *latent distribution* (density) $p_z : Z \rightarrow \mathbf{R}$ and *observation distribution* (density) $p_x : X \rightarrow \mathbf{R}$ is a ordered pair () discrete (continuous) latent set Z and discrete (continuous) observation set X is a tuple

where (a) ν is an autoencoder (which need not be regular, see **Autoencoders**), (b) $q_{z|x} : Z \times X \rightarrow \mathbf{R}$ is a conditional distribution (density) called the *recognition distribution* (*recognition density*), (c) $p_z : Z \rightarrow \mathbf{R}$ is a distribution (density) called the *latent prior model*, and (d) $p_{x|z} : X \times Z \rightarrow \mathbf{R}$ is a conditional distribution (density) called the *generating model*.

In other words, for (a) $q_{z|x}(\cdot, \xi) : Z \rightarrow \mathbf{R}$ is a distribution (density) for each $\xi \in X$ and for (d) $p_{x|z}(\cdot, \zeta) : X \rightarrow \mathbf{R}$ is a distribution (density) on X for each $\zeta \in Z$.

If the model has discrete latent set and discrete observation set (or continuous latent set and continuous observation set),

¹Future editions will include. Future editions may also change the name of this sheet. It is also likely that there will be added prerequisite sheets on variational inference.

the *joint distribution (joint density)* $p_{zx} : Z \times X \rightarrow \mathbf{R}$ is defined by $p_{zx} = p_z p_{x|z}$. The *observation distribution*

A *continuous-continuous variational autoencoder family (discrete-discrete, discrete-continuous, continuous-discrete)* is a tuple

$$(\nu, \{(q^{(\theta)}, p_z^{(\theta)}, p_{x|z}^{(\theta)})\}_{\theta \in \Theta}),$$

where:

- ν is an autoencoder with encoder $f : \mathbf{R}^d \rightarrow \mathbf{R}^k$ and decoder $g : \mathbf{R}^\ell \rightarrow \mathbf{R}^m$. The autoencoder need not be regular, see [Autoencoders](#)).
- $\Theta \subset \mathbf{R}^p$. The *parameter set* (or *parameter space*).
- $q^{(\theta)} : \mathbf{R}^h \rightarrow \mathbf{R}$ is a density (distribution, density, distribution), for each $\theta \in \Theta$. We call $\{q^{(\theta)}\}_{\theta \in \Theta}$ the *recognition model family*.
- $p_z^\theta : \mathbf{R}^h \rightarrow \mathbf{R}$ is a density (distribution, distribution, density), for each $\theta \in \Theta$. We call $\{p_z^{(\theta)}\}_\theta$ the *latent prior model family*.
- $p_{x|z}^{(\theta)} : \mathbf{R}^d \times \mathbf{R}^h \rightarrow \mathbf{R}$ is a conditional density (distribution, density, distribution). In other words, $p_{x|z}^{(\theta)}(\cdot, \zeta) : \mathbf{R}^d \rightarrow \mathbf{R}$ is a density (distribution, density, distribution) for every $\zeta \in \mathbf{R}^d$. We call $\{p_{x|z}^{(\theta)}\}_{\theta \in \Theta}$ the *observation model family*.

A *variational autoencoder* (or *VAE*) may refer to any of the above. The convention we have adopted is “latent type”-“observation type”.

