



## Why

For each value of a random variable's codomain, the set of outcomes corresponding to that value is the inverse image of the random variable. We can speak of the probability that a random variable takes a value then, by assigning it the probability of the set of outcomes corresponding to that value.

## Definition

Let  $p : \Omega \rightarrow \mathbf{R}$  be a probability distribution with corresponding probability measure  $\mathbf{P} : \mathcal{P}(\Omega) \rightarrow \mathbf{R}$ . Suppose  $x : \Omega \rightarrow V$  is an outcome variable. The *probability*  $x = a$ , for  $a \in \Omega$ , is

$$\mathbf{P}(\{\omega \in \Omega \mid x(\omega) = a\}).$$

From the definition of  $\mathbf{P}$ , we express the above as

$$\sum_{\omega \in \Omega \mid x(\omega) = a} p(\omega).$$

We refer to the probability of the *event* that  $x = a$ .

## Notation

We denote the probability that  $x = a$  by  $\mathbf{P}[x = a]$ . Our square brackets deviate from the slightly slippery but universally standard notation  $\mathbf{P}(x = a)$ . We prefer the square brackets, since  $x = a$  is not itself an argument to  $\mathbf{P}$ , but shorthand for  $(\{\omega \in \Omega \mid x(\omega) = a\})$ .

There are many similar notations. For example,  $\mathbf{P}[x \in C]$  means  $\mathbf{P}(\{x \in \Omega \mid x(\omega) \in C\})$ . In particular, if  $x : \Omega \rightarrow \mathbf{R}$ ,  $\mathbf{P}[x \geq a]$  means  $\mathbf{P}(\{\omega \in \Omega \mid x(\omega) \geq a\})$ . Since the *event* that  $x = a$  is the inverse image of  $\{a\}$  under  $x$ , we also use the notations  $\mathbf{P}(x^{-1}(a))$  and  $\mathbf{P}(x^{-1}(C))$ .

## Example: sum of two dice

Define  $\Omega = \{1, \dots, 6\}^2$  and define  $p : \Omega \rightarrow \mathbf{R}$  with  $p(\omega) = 1/36$  for each  $\omega \in \Omega$ . Define  $x : \Omega \rightarrow \mathbf{N}$  by  $x(\omega_1, \omega_2) = \omega_1 + \omega_2$ . Then  $\mathbf{P}[x = 4] =$

$$p((2, 2)) + p(1, 3) + p(3, 1) = 1/12.$$

