Why

How do equivalence classes and functions relate?

Definition

We can associate to each element of a set its equivalence class under an equivalence relation. Let X denote a set and R an equivalence relation. We call the function $f : X \to X/R$ defined by $f(x) = x/R$ the *canonical map* from X to X/R.

Conversely, if f is an arbitrary function from X onto Y, we can naturally define an equivalence relation R in X so that for $a, b \in X$, $a\,R\,b \longleftrightarrow f(a) = f(b)$ f was onto, so for each $y \in Y$, there exists an $x \in X$ with $f(x) = y$. Now let $g : Y \to X/R$ be defined by $g(y) = x/R$. The values of g are the subset X which are mapped to the same value under f. Moreover, the function g is one-to-one.

Canonical Maps

Equivalence Relations → Functions

Relations

Set Products

Set Powers   Ordered Pairs

Unordered Triples

Partitions   Pair Unions

Set Intersections   Set Unions

Pair Intersections Empty Set   Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects