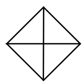


Why

Recall that if $A \in \mathbf{R}^{m \times n}$ then $x \quad Ax$ is a linear function from $\mathbf{R}^n$ to $\mathbf{R}^m$. And conversely, if $g : \mathbf{R}^n \to \mathbf{R}^m$ is a linear function, there is a matrix $B \in \mathbf{R}^{m \times n}$ so that $g(z) = Bz$. Does this function have an inverse?

Derivation

If $A \in \mathbf{R}^{m \times n}$, with $m \neq n$, then the inverse of f can not exist. For a *square* matrix $A \in \mathbf{R}^{n \times n}$, $B \in \mathbf{R}^{n \times n}$ is a *left inverse* if $BA = I$. In other words, B is a left inverse element of A in the algebra of matrices with the operation of multiplication. $C \in \mathbf{R}^{n \times n}$ is a *right inverse* if $AC = I$.

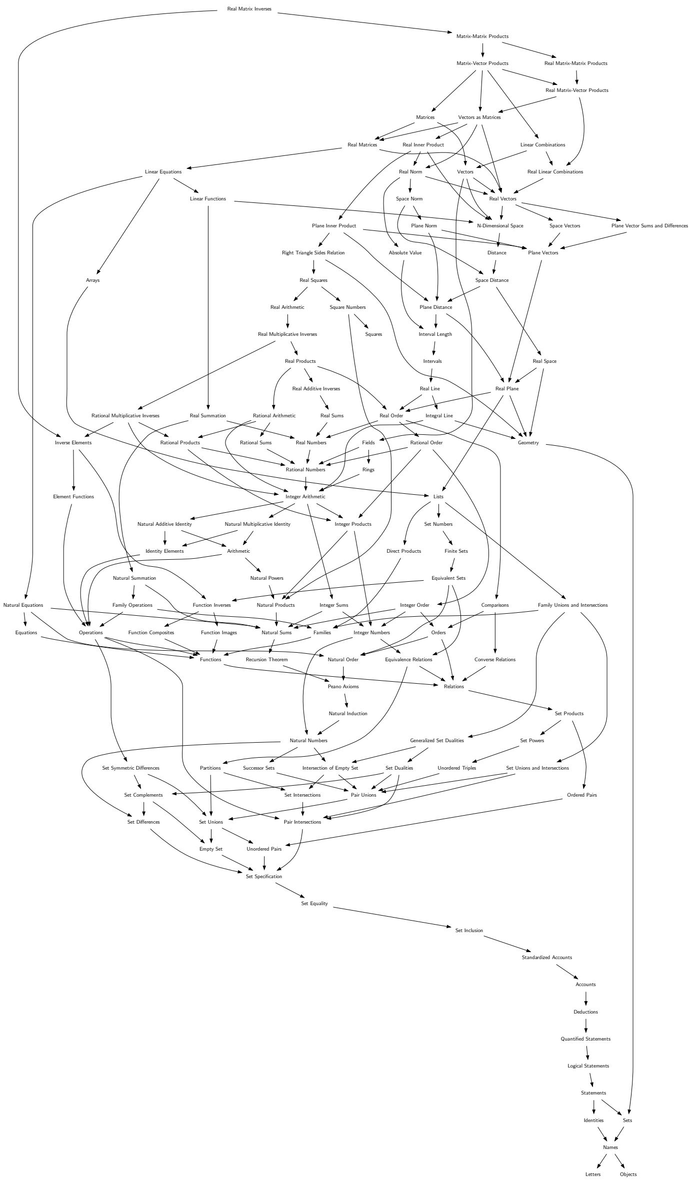
Definition

We call a square matrix A *invertible* if there is $B \in \mathbf{R}^{n \times n}$ so that $BA = I$.

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that $BA = I$ we call B the *left inverse* of A and likewise if $AC = I$ we call C the *right inverse* of A. In the case that A is square, the right inverse and left inverse coincide.

Proposition 1. *Let $A, B, C \in \mathbf{R}^{n \times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

Proof. Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. □


Real Matrix Inverses
Matrix-Matrix Products
Matrix-Vector Products
Real Matrix-Matrix Products
Real Matrix-Vector Products
Matrices
Vectors as Matrices
Real Matrices
Real Inner Product
Linear Combinations
Linear Equations
Real Norm
Vectors
Real Linear Combinations
Linear Functions
Space Norm
Real Vectors
Plane Inner Product
Plane Norm
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Right Triangle Sides Relation
Absolute Value
Distance
Plane Vectors
Arrays
Real Squares
Space Distance
Real Arithmetic
Square Numbers
Plane Distance
Real Multiplicative Inverses
Squares
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Real Summation
Rational Arithmetic
Real Sums
Real Order
Integral Line
Inverse Elements
Rational Products
Rational Sums
Real Numbers
Fields
Rational Order
Geometry
Rational Numbers
Rings
Element Functions
Integer Arithmetic
Lists
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Set Numbers
Identity Elements
Arithmetic
Direct Products
Finite Sets
Natural Summation
Natural Powers
Equivalent Sets
Natural Equations
Family Operations
Function Inverses
Natural Products
Integer Sums
Integer Order
Comparisons
Family Unions and Intersections
Equations
Operations
Function Composites
Function Images
Natural Sums
Families
Integer Numbers
Orders
Functions
Recursion Theorem
Natural Order
Equivalence Relations
Converse Relations
Peano Axioms
Relations
Set Products
Natural Induction
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects