Why

We generalize the notion of angle between vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$ to vectors in $\mathbf{R}^n$.

Definition

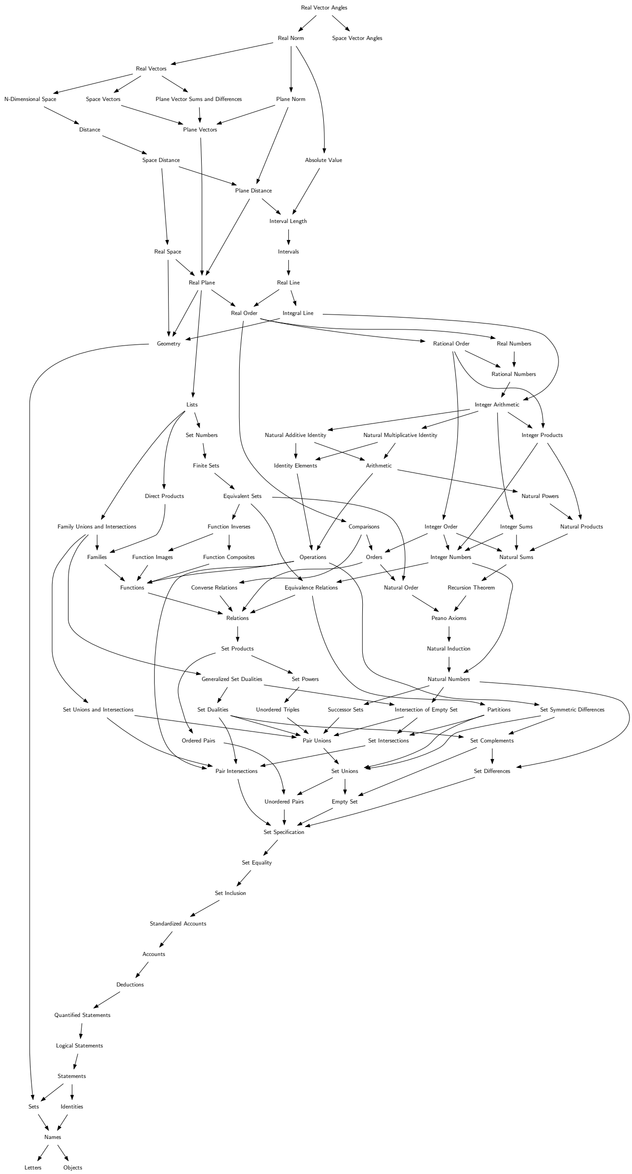
The *angle* (*unsigned angle*) between the nonzero vectors $x, y \in \mathbf{R}^n$, the number

$$\theta = \angle(x, y) = \cos^{-1} \frac{x^\top y}{\|x\|\|y\|}.$$

In the case that one (or both) of the vectors is zero, we define the angle between them to be 0. Thus, $x^\top y = \|x\|\|y\| \cos\theta$, which is a convenient way to remember the inner product norm inequality.

Terminology

x and y are *aligned* if $\theta = 0$, in which case $x^\top y = \|x\|\|y\|$. In the case that $x \neq 0$, x and y are aligned if $x = \alpha y$ for some $\alpha \geq 0$. x and y are *opposed* if $\theta = \pi$, in which case $x^\top y = \|x\|\|y\|$. In the case that $x \neq 0$, x and y are opposed if $x = -\alpha y$ for some $\alpha \geq 0$. Two nonnzero vectors x and y are *orthogonal* if $\theta = \pi/2$ or $-\pi/2$, in which case $x^\top y = \|x\|\|y\|$. The origin is orthogonal to every other vector. We denote that two vectors x and y are orthogonal by $x \perp y$.


Real Vector Angles
Real Norm
Space Vector Angles
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Plane Norm
Distance
Plane Vectors
Space Distance
Absolute Value
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects