



Why

We can use families to think about unions and intersections.

Family unions

Let $A : I \rightarrow \mathcal{P}(X)$ be a family of subsets. We refer to the union (see Set Unions) of the range (see Relations) of the family the *family union*. We denote it $\cup_{i \in I} A_i$.

Proposition 1. $(x \in \cup_{i \in I} A_i) \longleftrightarrow (\exists i)(x \in A_i)$

If $I = \{a, b\}$ is a pair with $a \neq b$, then $\cup_{i \in I} A_i = A_a \cup A_b$.

There is no loss of generality in considering family unions. Every set of sets is a family: consider the identity function from the set of sets to itself.

We can also show generalized associative and commutative law¹ for unions.

Proposition 2. Let $\{I_j\}$ be a family of sets and define $K = \cup_j I_j$. Then $\cup_{k \in K} A_k = \cup_{j \in J} (\cup_{i \in I_j} A_i)$.²

Family intersection

If we have a nonempty family of subsets $A : I \rightarrow \mathcal{P}(X)$, we call the intersection (see Set Intersections) of the range of the family the *family intersection*. We denote it $\cap_{i \in I} A_i$.

Proposition 3. $x \in \cap_{i \in I} A_i \longleftrightarrow (\forall i)(x \in A_i)$

Similarly we can derive associative and commutative laws for intersection³. They can be derived as for unions, or from the facts of unions using generalized DeMorgan's laws (see Generalized Set Dualities).

¹The commutative law will appear in future editions.

²An account will appear in future editions.

³Statements of these will be given in future editions.

Connections

The following are easy⁴

Let $\{A_i\}$ be a family of subsets of X and let $B \subset X$.

Proposition 4. $B \cap \bigcup_i A_i = \bigcup_i (B \cap A_i)$

Proposition 5. $B \cup \bigcap_i A_i = \bigcap_i (B \cup A_i)$

Let $\{A_i\}$ and $\{B_j\}$ be families of sets.⁵

Proposition 6. $(\bigcup_i A_i) \cap (\bigcup_j B_j) = \bigcup_{i,j} (A_i \cap B_j)$

Proposition 7. $(\bigcap_i A_i) \cup (\bigcap_j B_j) = \bigcap_{i,j} (A_i \cup B_j)$.

Proposition 8. $\bigcap_i X_i \subset X_j \subset \bigcup_i X_i$ for each j .

⁴Accounts will appear in future editions.

⁵An account of the notation used and the proofs will appear in future editions.

