



Why

We want to talk about how a function of multiple real-valued arguments changes with respect to changes in its arguments.¹

Definition

Let $f : \mathbf{R}^d \rightarrow \mathbf{R}$ For $i = 1, \dots, d$, The i th *partial derivative* of f is the function $g_i : \mathbf{R}^d \rightarrow \mathbf{R}$ defined by

$$g_i(x) = \lim_{h \rightarrow 0} \frac{f(x + he_i) - f(x)}{h}$$

for each $x \in \mathbf{R}^d$.

Notation

Gradient

The *gradient* of a multivariate function is the vector-valued function whose i th component is the the partial derivative of the function with respect to its i th argument.

Notation

Let $f : \mathbf{R}^n \rightarrow \mathbf{R}$. The gradient of f is frequently denoted ∇f . It is understood that $(\nabla f) \in \mathbf{R}^d \rightarrow \mathbf{R}^d$. An alternative notation is to use that similar for single derivatives and to denote the gradient (sometimes called derivative) of f by f' (assuming it exists). It is important to here note that although when $g : \mathbf{R} \rightarrow \mathbf{R}$, $g' \in (\mathbf{R} \rightarrow \mathbf{R})$, (and so is another function from and to reals) when $f : \mathbf{R}^d \rightarrow \mathbf{R}$, $f' \in \mathbf{R}^d \rightarrow \mathbf{R}^d$, and so is a vector-valued (not a real-valued) function.

¹Future editions will modify this sheet.

There is (unfortunately) much notation for the individual partial derivatives; most of which we shall not (fortunately) have occasion to use in these sheets. One popular usage is the use of the ∂ symbol, read aloud as “partial.” For example, if $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ is a function of two arguments, each being referred to as x and y , then $\partial_x f$ denotes the partial derivative of f with respect to x and $\partial_y f$ denotes the partial derivative of f with respect to y . It is understood that $(\partial_x f) \in \mathbf{R}^d \rightarrow \mathbf{R}$. and likewise for $\partial_y f$. Another popular usage is $\partial f / \partial x$ for $\partial_x f$ and $\partial f / \partial y$ for $\partial_y f$. We will almost exclusively prefer the gradient notation.

