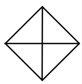


## Why

What are numbers? We want to count, forever. Does a set exist which contains zero, and one, and two, and three, and all the rest?

## Definition

In Successor Sets, we said "and we continue as usual using the English language..." in our definition of zero, and one and two and three. Can this really be carried on and on? We will say yes. We will say that there exists a set which contains zero and contains the successor of each of its elements.

**Principle 1** (Natural Numbers)**.** *A set which contains 0 and contains the successor of each of its elements exists.*

This principle is sometimes called the *principle of infinity* (or *axiom of infinity*).

We want this set to be unique. The principle says one successor set exists, but not that it is unique. To see that it is unique, notice that the intersection of a nonempty family of successor sets is a successor set.[1] Consider the intersection of the family of all successor sets. The intersection is nonempty by the principle of infinity (see Intersection of Empty Setfor this subtlety). The principle of extension guarantees that this intersection, which is a successor set contained in every other successor set, is unique. We summarize:

**Proposition 1** (Minimal Successor Set)**.** *There exists a unique smallest successor set.*

The *set of natural numbers* is the minimal successor set. A *natural number* (or *number*, *natural*) is an element of this minimal successor set.

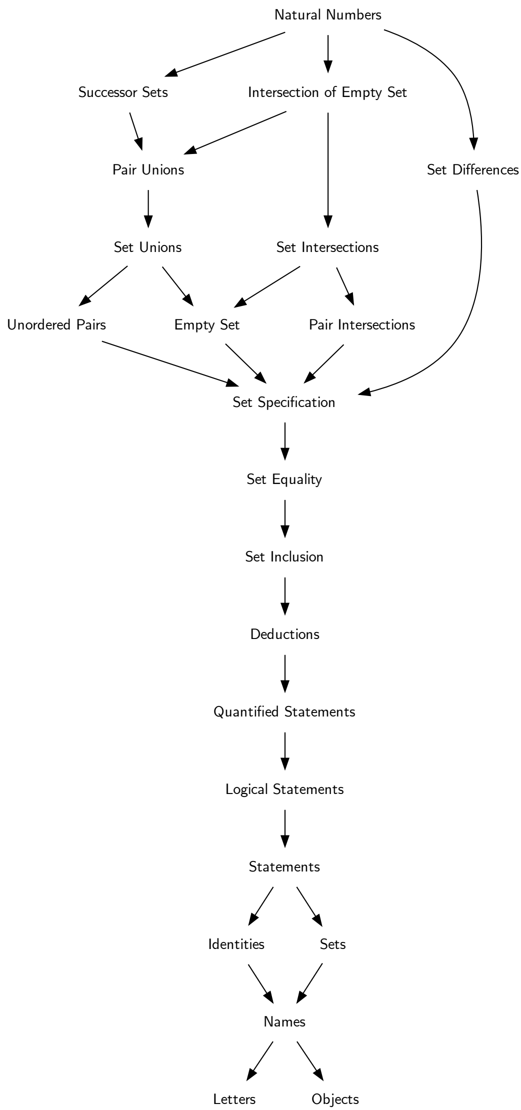
[1] This account will be expanded in future editions.

## Notation

We denote the unique smallest successor set by $\omega$.[2] We denote the set of natural numbers without 0 by $\mathsf{N}$, a mnemonic for natural. In other words $\mathsf{N} = \omega - \{0\}$. We often denote elements of $\omega$ or $\mathsf{N}$ by $n$, a mnemonic for number, or $m$, the letter before $m$ in the conventional ordering of the Latin alphabet(see Letters).

We denote the natural numbers up to $n$ by $\{1, 2, \dots, n\}$. Recall that $n$ *is a set*. In other words, we have defined $n$ so that $n - \{0\} = \{1, 2, \dots, n\}$.

[2] We use this notation to follow many authorities on the subject, and to meet the exigencies of time in producing this first edition. Future editions are likely to rework the treatment.


Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Differences
Set Unions
Set Intersections
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects