

INVERSES UNIONS INTERSECTIONS AND COMPLEMENTS

Why

The inverse of a function interacts nicely with family unions, family intersections and complements.

Results

Let $f: X \to Y$. Throughout this sheet, let $f^{-1}: Y^* \to X^*$. And take $\{B_i\}$ to be a family of subsets of Y.

Proposition 1.
$$f^{-1}(\cup_i B_i) = \cup_i f^{-1}(B_i)$$

Proposition 2.
$$f^{-1}(\cup_i B_i) = \cap_i f^{-1}(B_i)$$

Proposition 3.
$$f^{-1}(Y - B) = X - f^{-1}(B)$$

Properties for Function Image

Notice that $f(\bigcup_i A_i) = \bigcup_i f(A_i)$ but not for interesctions. Nor is there a similar correspondence for complements. There are some relations, which we list below.²

Proposition 4. $f(A \cap B) = f(A) \cap f(B)$ if and only if f is one-to-one.

Proposition 5. For all $A \subset X$, f(X - A) = Y - f(A) if and only if f is one-to-one.

Proposition 6. For all $A \subset X$, $Y - f(A) \subset f(X - A)$ if and only if f is onto.

¹The proofs of the following will appear in future editions.

²Accounts of these facts will appear in future editions.

