



## Why

The Hilbert space is one of the most important generalizations of the past century.<sup>1</sup>

## Definition

Let  $(X, \mathbf{R})$  be a real vector space. An inner product  $\langle \cdot, \cdot \rangle : X \times X \rightarrow \mathbf{R}$  induces a norm  $\|\cdot\| : X \rightarrow \mathbf{R}$  defined by  $\|x\| = \sqrt{\langle x, x \rangle}$  and metric  $d : X \times X \rightarrow \mathbf{R}$  defined by  $d(x, y) = \|x - y\|$ .

If  $(X, d)$  is a complete metric space, we call  $((X, \mathbf{R}), \langle \cdot, \cdot \rangle)$  a *complete inner product space* (or *Hilbert space*).<sup>2</sup>

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<sup>1</sup>Future editions will modify.

<sup>2</sup>The term Hilbert space is universal, but in accordance with the Bourbaki project's guidelines on naming, we will tend to use the term complete inner product space, even though this is longer.



