Why

Since our predictions are often uncertain, we can use the language of probability distributions to characterize them.[1]

Definition

Denote the set of probability distributions on a set X by $\Delta(X)$.

A *probabilistic classifier* $G : A \to \Delta(B)$ is a function from inputs A to probability distributions over the classes B.

Given an input a, the *prediction* of G on a is a probability distribution $\hat{p}_a = G(a)$ on B.

Point classifier as probabilistic classifier

Given a point classifier $f : A \to B$, we can define a probabilistic classifier $G : A \to \Delta(B)$ corresponding to f by

$$\hat{p}_a(b) = \begin{cases} 1 & \text{if } f(a) = b \\ 0 & \text{otherwise.} \end{cases}$$

where $\hat{p}_a = G(a)$.

Probabilistic classifier from point classifier

On the other hand, given probabilistic classifier $G : A \to \Delta(B)$, we can define a point classifier $f : A \to B$ by

$$f(a) = \operatorname*{argmax}_{b \in B} \hat{p}_a(v)$$

We call f the *maximum likelihood classifier* corresponding to G. If there are ties, we can order the (finite) set B arbitrarily, and break ties accordingly.

We can extend this idea, and define a list classifier by sorting the outputs by their probability, from largest to smallest.

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[1]Future editions will improve this.

Judging probabilistic classifiers

Probabilistic Classifiers

Probability Distributions   List Classifiers

Intervals   Classifiers

Real Line   Predictors

Real Order

Real Summation

Factorials   Rational Order   Real Numbers

Sequences   Rational Numbers   Integral Line

Integer Arithmetic   Lists

Integer Products   Natural Multiplicative Identity   Natural Additive Identity   Set Numbers

Arithmetic   Identity Elements   Finite Sets   Direct Products

Natural Powers   Natural Summation   Equivalent Sets

Natural Products   Integer Sums   Integer Order   Comparisons   Family Operations   Function Inverses   Family Unions and Intersections

Natural Sums   Integer Numbers   Orders   Operations   Function Images   Function Composites   Families

Recursion Theorem   Natural Order   Converse Relations   Equivalence Relations   Functions

Peano Axioms   Relations

Uncertain Outcomes   Natural Induction   Set Products

Natural Numbers   Set Powers   Generalized Set Dualities

Set Symmetric Differences   Successor Sets   Unordered Triples   Set Dualities   Intersection of Empty Set   Partitions   Set Unions and Intersections

Set Complements   Pair Unions   Ordered Pairs   Set Intersections

Set Differences   Set Unions   Pair Intersections

Empty Set   Unordered Pairs

Set Specification   Geometry

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects