



## FINITE SETS

### Why

We want to talk about the size of a set.<sup>1</sup>

### Definition

A *finite* set is one that is equivalent to some natural number; an infinite set is one which is not finite. From this we can show that  $\omega$  is infinite. This justifies the language we used in *Natural Numbers* about the principle of infinity. The principle of infinity asserts the existence of a particular infinite set; namely  $\omega$ .

### Motivation for set number

It happens that if a set is equivalent to a natural number, it is equivalent to only one natural number.

**Proposition 1.** *A set can be equivalent to at most one natural number.*<sup>2</sup>

A consequence is that a finite set is never equivalent to a proper subset of itself. So long as we are considering finite sets, a piece (subset) is always less than than the whole (original set).

**Proposition 2.** *A finite set is never equivalent to a proper subset of itself.*

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<sup>1</sup>Will be expanded in future editions.

<sup>2</sup>Future edition will include proof, which uses comparability of numbers and the results of *Equivalent Sets*).

## Subsets of finite sets

Every subset of a natural number is equivalent to a natural number.<sup>3</sup> A consequence is:

**Proposition 3.** *Every subset of a finite set is finite.*<sup>4</sup>

## Unions of finite sets

**Proposition 4.** *if  $A$  and  $B$  are finite, then  $A \cup B$  is finite.*

## Products of finite sets

**Proposition 5.** *If  $A$  and  $B$  are finite, then  $A \times B$  is finite.*

## Powers of finite sets

**Proposition 6.** *If  $A$  is finite then  $A^*$  is finite.*

## Functions between finite sets

**Proposition 7.** *If  $A$  and  $B$  are finite, then  $A^B$  is finite.*

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<sup>3</sup>This requires proof, and may become a proposition in future editions.

<sup>4</sup>An account will appear in future editions.

