Why

We can extend the notion of independnence beyond pairs of uncertain events, to sets of events.

Definition

Suppose P is a event probability function on a finite sample space Ω. The events $A_1, \ldots, A_n$ are *independent* (or *mutually independent*), if for all k between 1 and n, and distinct indicies $i_1, \ldots, i_k$ between 1 and n,

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2}) \cdots P(A_{i_k}).$$

Similar to the case of pairs of events, one can show that this condition is equivalent to the statement that for any *distinct* indices $i_1, \ldots, i_k, j_1, \ldots, j_l$,

$$P(A_{j_1} \cap \cdots \cap A_{j_l} \mid A_{i_1} \cap \cdots \cap A_{i_k}) = P(A_{j_1} \cap \cdots \cap A_{j_l})$$

Examples

n tosses of a coin. As usual, model n tosses of a coin with $\{0,1\}^n$ and put a distribution $p : \Omega \to [0,1]$ so that

$$p(\omega) = 1/2^n \quad \text{for all } \omega \in \Omega$$

Now, for $i = 1, \ldots, n$, define the event A_i by

$$A_i = \{\omega \in \Omega \mid \omega(i) = 1\}$$

We claim that the set $\{A_1, \ldots, A_n\}$ is mutually independent. To see this, notice that for any distinct indices $i_1, \ldots, i_k$,

$$|A_{i_1} \cap \cdots \cap A_{i_k}| = 2^{n-k}$$

This holds because, once k of the coin flips, there are 2^{n-k} ways for the remaining coins to land (using the fundamental principle of counting). Consequently,

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = \frac{2^{n-k}}{2^n} = 2^{-k}$$

We can use this result with one set $P(A_i) = 1/2$, and so we obtain

$$P(A_{i_1} \cap \cdots A_{i_k}) = P(A_{i_1}) \cdots P(A_{i_k}),$$

as desired.

Mutually Independent Events

Independent Events

Conditional Event Probabilities

Event Probabilities

Outcome Probabilities

Real Functions

Intervals

Real Line

Real Order — Real Summation — Size of Direct Product

Rational Order — Real Numbers — Number of Set Products — Natural Summation

Rational Numbers — Number of Disjoint Unions — Integral Line — Finite Set Example — Lists

Integer Arithmetic — Set Number — Direct Products

Integer Products — Natural Multiplicative Identity — Natural Additive Identity

Natural Powers — Arithmetic — Identity Elements — Finite Sets

Equivalent Sets

Family Operations — Function Inverse — Uncertain Events — Family Unions and Intersections

Natural Products — Integer Sums — Integer Order — Comparisons — Operations — Families — Function Images — Function Composites

Integer Numbers

Natural Sums — Natural Order — Orders — Converse Relations — Functions — Equivalence Relations

Recursion Theorem — Relations

Peano Axioms — Ordered Pair Projections — Uncertain Outcomes

Natural Induction — Set Products

Natural Numbers — Set Powers — Generalized Set Dualities

Set Symmetric Differences — Successor Sets — Unordered Triples — Set Dualities — Intersection of Empty Set — Partitions — Set Unions and Intersections

Set Complements — Pair Unions — Set Intersections — Ordered Pairs

Set Differences — Set Unions — Unordered Pairs — Pair Intersections

Empty Set

Set Specification — Geometry

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects