Definition

A nonempty set $S \subset \mathbf{R}^n$ is called a *subspace* (*linear subspace*) if

1. $x + y \in S$ for all $x, y \in S$, and
2. $\alpha x \in S$ for all $\alpha \in \mathbf{R}$, $x \in S$.

We say that that S is (1) *closed under vector addition* and (2) *closed under scalar multiplication.*

Examples

The set $S_1 = \mathbf{R}^n$ is a subspace. In other words, the entire set is a subspace of itself. The set $S_2 = \{0\}$, consisting of a single point, the origin, is a subspace. S_1 is the biggest subspace. In other words, if S' is another subspace of $\mathbf{R}^n$, then $S' \subset S_1$. If S is a subspace, it is nonempty, so there is $x \in S$, and it is closed under scalar multiplication, so $0 \cdot x = 0 \in S$. In other words, every subspace contains the origin. So S_2 is the smallest subspace, in the sense that if S' is another subspace $S_2 \subset S'$.

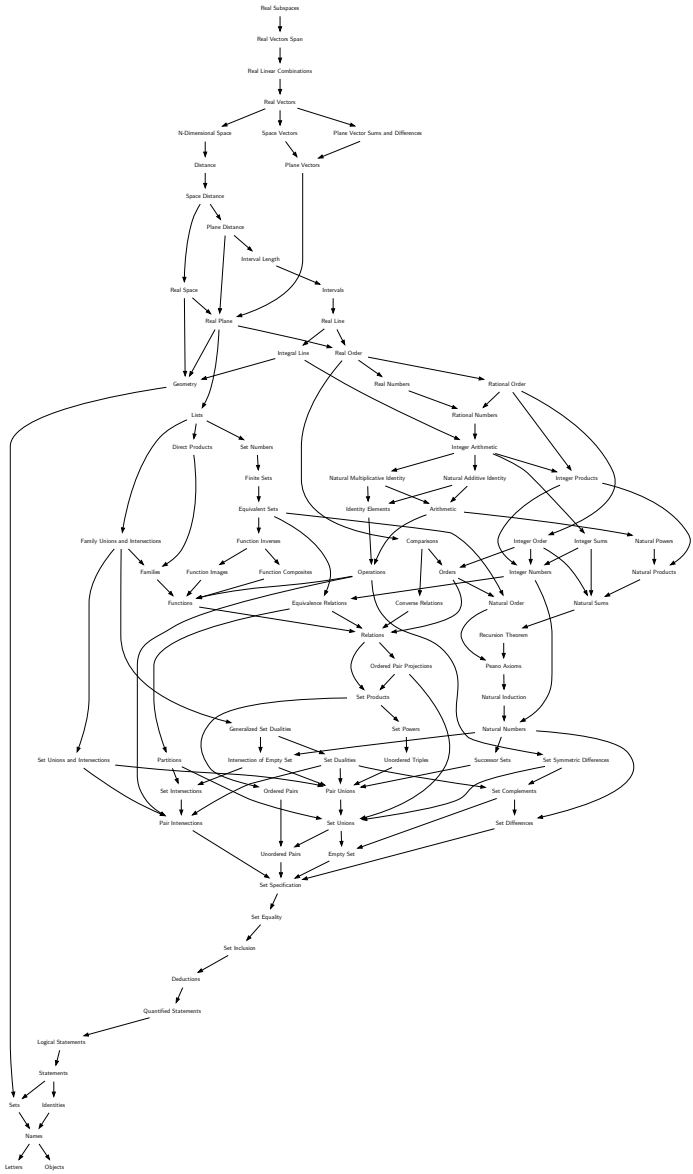
The span (see Real Vectors Span) of a set of vectors $v_1, \ldots, v_k$ is a subspace. For two subspaces $S, T \subset \mathbf{R}^n$, their sum

$$S + T = \{x + y \mid x \in S, y \in T\}$$

is a subspace.

Geometric intuition

Roughly speaking, a subspace S is a flat set which passes through the origin. In $\mathbf{R}^2$, the subspaces are the lines. In $\mathbf{R}^3$, the lines and the planes.


Real Subspaces
Real Vectors Span
Real Linear Combinations
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Geometry
Real Numbers
Rational Order
Rational Numbers
Lists
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Equivalent Sets
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Powers
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Natural Products
Functions
Equivalence Relations
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects