



DIRECT PRODUCTS

Why

We can generalize the product of two sets to a product of a family of sets.

Discussion for pairs

Let A and B be sets. There is a natural correspondence between $A \times B$ (see *Cartesian Products*) and a particular set of families. The particular set of families $z : \{i, j\} \rightarrow (A \cup B)$ with $z_i \in A$ and $z_j \in B$. The family z corresponds with the pair (z_i, z_j) . The pair (a, b) corresponds to the family $z : \{i, j\} \rightarrow (A \cup B)$ defined by $z(i) = a$ and $z(j) = b$. In other words, we can think about ordered pairs as special families. The generalization of Cartesian products to families generalizes the notion for families.

Direct Products

Let X be a set. Let $A : I \rightarrow X$ be a family of subsets of X . The *direct product* or *family Cartesian product* of A is the set of all families $a : I \rightarrow X$ which satisfy $a_i \in A_i$ for every $i \in I$. A function on a product is called a *function of several variables* and, in particular, a function on the product $X \times Y$ is called a *function of two variables*.

Notation

We denote the product of the family $\{A_i\}$ by

$$\prod_{i \in I} A_i$$

We read this notation as “product over i in I of A sub- i .”

Projections

The word “projection” is used in two senses with families. Let I be a set, and let $\{A_i\}$ be a family of sets. Define $A = \prod_{i \in I} A_i$. Then if $J \subset I$, there is a natural correspondence between the elements of X and those of $\prod_{j \in J} A_j$. To each element $x \in X$, we restrict x to J and this is an element of $\prod_{j \in J} A_j$. The correspondence is called the *projection* of X onto $\prod_{i \in J} X_i$. Also, the value of x at j is called the *projection of x onto index j* or the *j -coordinate* of x .

Repeated set

If $\{A_i\}$ is a finite sequence of sets and $A_i = A$ for some A for all i then we denote the product $\prod_{i=1}^n A_i$ by A^n . We call an element $a = (a_1, a_2, \dots, a_n) \in A$ an *n -tuple*.

