



# Topological Spaces

## 1 Why

We want to generalize the notion of continuity.

## 2 Definition

A *topological space* is a subset system for which: (1) the empty set and the base set are distinguished, (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the set of distinguished subsets the *topology*. We call the distinguished subsets the *open sets*.

### 2.1 Notation

Let  $A$  be a non-empty set. For the set of distinguished sets, we use  $\mathcal{T}$ , a mnemonic for topology, read aloud as “script T”. We denote elements of  $\mathcal{T}$  by  $O$ , a mnemonic for open. We denote the topological space with base set  $A$  and topology  $\mathcal{T}$  by  $(A, \mathcal{T})$ . We denote the properties satisfied by elements of  $\mathcal{T}$ :

1.  $X, \emptyset \in \mathcal{T}$
2.  $\{O_i\}_{i=1}^n \subset \mathcal{T} \implies \bigcap_{i=1}^n O_i \in \mathcal{T}$
3.  $\{O_\alpha\}_{\alpha \in I} \subset \mathcal{T} \implies \bigcup_{\alpha \in I} O_\alpha \in \mathcal{T}$