Why

Convex functions satisfy an interpolation property.

Discussion

By definition, given $S \subset \mathbf{R}^n$, $f : S \to \mathbf{R}$ is convex means that the point

$$(1-\lambda)(x\mu) + \lambda(x,\nu) = ((1-\lambda)x + \lambda y, (1-\lambda)\mu + \lambda\nu$$

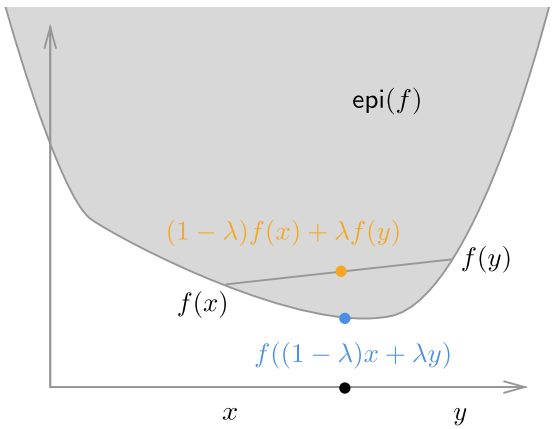
belongs to epi f whenever (x,μ) and (y,ν) belong to epi f and $0 \leq \lambda \leq 1$. Said differently, we have $(1-\lambda)x + \lambda y \in S$, and

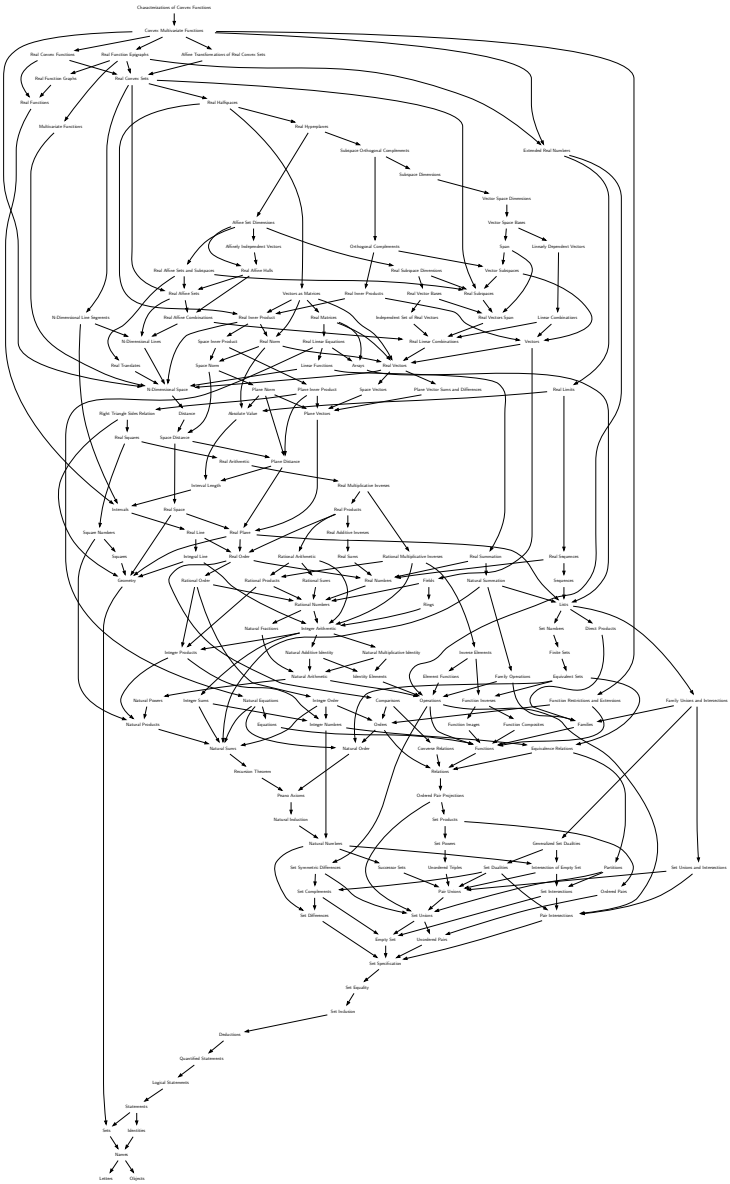
$$f((1-\lambda)x + \lambda y) \leq (1-\lambda)\mu + \lambda\nu$$

whenever $x \in S$, $y \in S$, $f(x) \leq \mu \in \mathbf{R}$, and $f(y) \leq \nu \in \mathbf{R}$.

Concave functions have a similar property, with the inequalities flipped, and affine functions satisfy with qualities. This shows that the functions $f : \mathbf{R}^n \to \mathbf{R}$ we are calling affine coincide exactly with the affine transformations from $\mathbf{R}^n$ to $\mathbf{R}$.

Visualization




Characterizations of Convex Functions
Convex Multivariate Functions
Real Convex Functions
Real Function Epigraphs
Affine Transformations of Real Convex Sets
Real Function Graphs
Real Convex Sets
Real Functions
Real Halfspaces
Real Hyperplanes
Multivariate Functions
Subspace Orthogonal Complements
Extended Real Numbers
Subspace Dimension
Vector Space Dimensions
Vector Space Basis
Affine Set Dimensions
Span
Linearly Dependent Vectors
Affinely Independent Vectors
Orthogonal Complements
Real Affine Sets and Subspaces
Real Affine Hulls
Real Subspace Dimension
Vector Subspaces
Real Affine Sets
Vectors as Matrices
Real Inner Products
Real Vector Norms
Real Subspaces
N-Dimensional Line Segments
Real Affine Combinations
Real Inner Product
Real Matrices
Independent Set of Real Vectors
Real Vectors Span
Linear Combinations
N-Dimensional Lines
Space Inner Product
Real Norm
Real Linear Equations
Real Linear Combinations
Vectors
Real Translation
Space Norm
Linear Functions
Array
Real Vectors
N-Dimensional Space
Plane Norm
Plane Inner Product
Space Vectors
Plane Vector Norm and Differences
Real Limits
Right Triangle Sides Relation
Distance
Absolute Value
Plane Vectors
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Interval Length
Real Multiplication Inverses
Intervals
Real Space
Real Products
Square Numbers
Real Line
Real Plane
Real Addition Inverses
Squares
Integral Line
Real Order
Rational Arithmetic
Real Norm
Rational Multiplication Inverses
Real Summation
Real Sequences
Geometry
Rational Order
Rational Products
Rational Norm
Real Numbers
Fields
Natural Summation
Sequences
Rational Numbers
Rings
Lists
Natural Functions
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Addition Identity
Natural Multiplication Identity
Inverse Elements
Finite Sets
Natural Arithmetic
Identity Elements
Element Functions
Freely Operations
Equivalent Sets
Natural Powers
Integer Norm
Natural Equations
Integer Order
Commutation
Association
Function Image
Function Restrictions and Archetypes
Freely Union and Intersections
Natural Products
Equations
Integer Numbers
Union
Function Images
Function Composition
Functions
Natural Norm
Natural Order
Converse Relations
Functions
Equivalence Relations
Euclidean Theorem
Relations
Plane Norms
Ordered Pair Projections
Natural Induction
Set Products
Natural Number
Set Powers
Generalized Set Unions
Set Symmetric Differences
Numerator Sets
Unordered Triples
Set Unions
Cardinality of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deduction
Quantified Statements
Logical Statements
Statement
Sets
Identities
Names
Letters
Objects