



DIRECTED GRAPHS

Why

We want to visualize (nonsymmetric) relations.

Definition

A *directed graph* is a pair (V, E) in which V is a finite nonempty set and E is a subset of $V \times V$. In other words, E is a relation on V . We call the elements of the first set the *vertices* of the graph and the elements of the second set the *edges*.

Let $(v, w) \in E$. We say that (v, w) is an edge *from* v *to* w , and that it is an *outgoing edge* of v and an *incoming edge* of w . We call v a *parent* of w and we call w a *child* of v . We say that the edge (v, w) is *incident* to v and w .

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

Notation

Let $\text{pa} : V \rightarrow \mathcal{P}(V)$ and $\text{ch} : V \rightarrow \mathcal{P}(V)$ be the functions associating to each vertex its set of parents and set of children, respectively. As usual, we denote the parents of vertex v by pa_v and the children by ch_v .

Skeletons

The *skeleton* of the directed graph (V, E) is the undirected graph (V, F) where

$$F = \{\{v, w\} \subset V \mid (v, w) \in E \text{ or } (w, v) \in E\}.$$

In other words, the skeleton is an undirected graph whose vertex set is V and whose edges are all (unordered) pairs which appear as an ordered pair in the directed graph. If the (V, E) is a directed graph and E is a symmetric relation, then the skeleton of (V, E) is a natural undirected graph to associate with (V, E) . An *orientation* of an undirected graph G is a directed graph whose skeleton is G .

Visualization

