Simple Integral Monotonicity

1 Why

If one rectangle contains another rectangle, the area of the first should be larger than the area of the second. Our definition of integral for simple functions carries this property. TODO: area sheet.

2 Result

Proposition 1. *The simple non-negative integral operator is monotone.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f, g \in \mathcal{SF}_+(X)$ with $f \leq g$. Then $f - g \in \mathcal{SF}_+(X)$, so

$$\begin{aligned}
\int g d\mu &= \int (f + (g - f)) d\mu \\
&\overset{(a)}{=} \int f d\mu + \int (g - f) d\mu \\
&\overset{(b)}{\geq} \int f d\mu
\end{aligned}$$

where (a) follows from linearity and (b) follows from non-negativity; properties of the non-negative simple integral operator.

□

Real F

Int