Why

TODO

Definition

An *index sequence* of *order* n is a finite sequence of distinct elements of $\{1, 2, \ldots, n\}$ whose length is less than or equal to n. We call the ith coordinate of an index sequence the *i-index* of the sequence. The *index matrix* associated with an index sequence is the $r \times n$ matrix whose i, jth entry is 1 if the index sequences's ith coordinate is j, and 0 otherwise. If $r = n$ then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an n-vector associated with a length-r index sequence is the product of the $r \times n$ index matrix with the n-dimensional vector. Its ith entry is the i-index entry of the vector.

Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions, and so relations, and so sets), they are not sets of indices, so we avoid this usage.

Notation

Let $r \leq n$ be natural numbers. Let $\alpha : \{1, 2, \ldots, r\} \to \{1, 2, \ldots, n\}$ be an index sequence. We denote the index ma-

trix associated with α by P_α. This matrix P_α is an element of $\mathsf{N}^{r\times n}$ and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let A be a nonempty set and let $x \in A^n$. then the subvector of x associated with P_α (and so with α) is

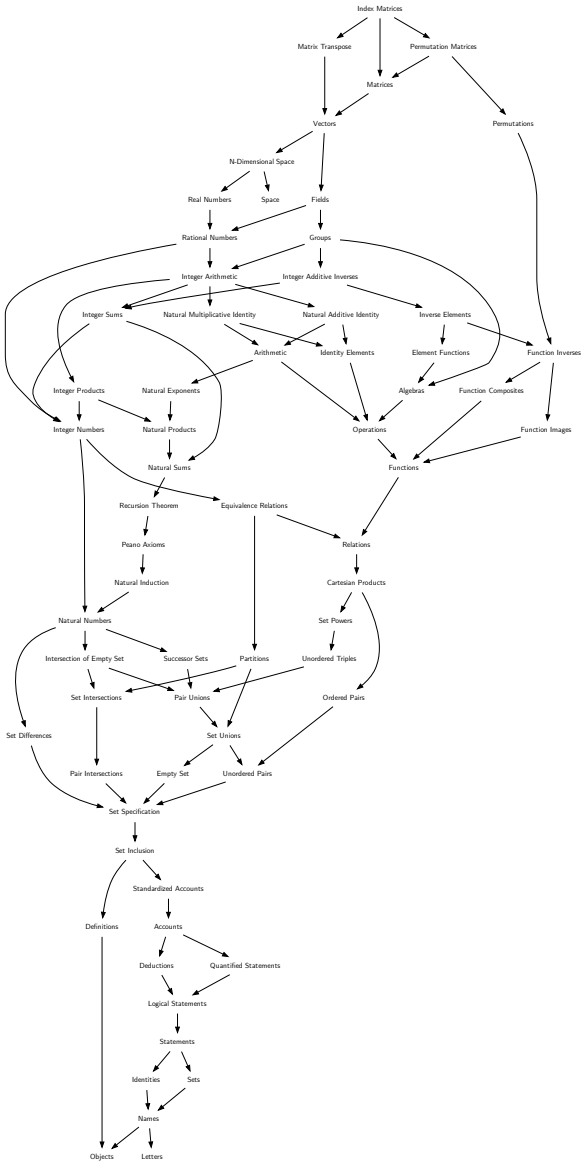
$$P_\alpha x = \left(x_{\alpha(1)}, \ldots, x_{\alpha(r)}\right)$$

We denote the product $P_\alpha x$ by x_α.

We denote the product $P_\alpha X {P_\alpha}^\top$ by $X_{\alpha\alpha}$.

Multiplication

The product of the $n \times r$ transpose of an index matrix with an r vector is the n vector with The


Index Matrices
Matrix Transpose
Permutation Matrices
Matrices
Vectors
Permutations
N-Dimensional Space
Real Numbers
Space
Fields
Rational Numbers
Groups
Integer Arithmetic
Integer Additive Inverses
Integer Sums
Natural Multiplicative Identity
Natural Additive Identity
Inverse Elements
Arithmetic
Identity Elements
Element Functions
Function Inverses
Integer Products
Natural Exponents
Algebras
Function Composites
Integer Numbers
Natural Products
Operations
Function Images
Natural Sums
Functions
Recursion Theorem
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Partitions
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters