Measure Space

# 1 Why

We want to generalize the notions of length, area, and volume.

# 2 Definition

A *measurable space* is a sigma algebra. We call the distinguished subsets the *measurable sets*.

A *measure* on a measurable space is a function from the sigma algebra to the positive extended reals. A *measure space* is a measurable space and a measure.

## 2.1 Notation

## 2.2 Properties

**Proposition 1.** *Let $(A, \mathcal{A})$ be a measurable space and $m : \mathcal{A} \to [0, \infty]$ be a measure.*

*If $B \subset C \subset A$, then $m(B) \leq m(C)$. We call this property the of measures monotonicity of measure.*

**Proposition 2.** *For a measure space $(A, \mathcal{A}, m)$.*

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*We call this property the monotonicity of measure.*

**Proposition 3.** *For a measure space $(A, \mathcal{A}, m)$.*

*If* $\{A_n\} \subset \mathcal{A}$ *a countable family, then* $m(\cup A_n) \leq \sum_i m(A_i)$.

*We this property the sub-additivty of measure.*

**Proposition 4.** *For a measure space* $(A, \mathcal{A}, m)$.

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**Proposition 5.** *For a measure space* $(A, \mathcal{A}, m)$.

$$m(\cup_{n=1}^{\infty} A_i) = \lim_{n\to\infty} m(A_i)$$

**Proposition 6.** *For a measure space* $(A, \mathcal{A}, m)$.

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## 2.3 Examples

**Example 7.** *counting measure*