Why

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Definition

Let $\mathcal{D} = ((\mathcal{X}_t)_{t=0}^T), (\mathcal{U}_t)_{t=0}^{T-1}, (f_t)_{t=1}^{T-1})$ be a dynamical system. Let $g_t : \mathcal{X}_t \times \mathcal{U}_t \to \mathbf{R} \cup \{\infty\}$ for $t = 1, \ldots, T-1$ and let $g_T : \mathcal{X}_T \to \mathbf{R} \cup \{\infty\}$. Let $x_0 \in \mathcal{X}_0$.

We call the sequence $(x_0, \mathcal{D}, (g_t)_{t=1}^T)$ a *deterministic dynamic optimization problem*. We call x_0 the *initial state*. We call g_t the *stage cost function* for stage t and call g_T the *terminal cost function*.

A deterministic dynamic optimization problem corresponds to an optimization problem with variables $u_0 \in \mathcal{U}_0, \ldots, u_{T-1} \in \mathcal{U}_{T-1}$. Define $U = \mathcal{U}_0 \times \mathcal{U}_1 \times \mathcal{U}_{T-1}$. Define $J : U \to \mathbf{R} \cup \{\infty\}$ by

$$J(u) = \sum_{t=0}^{T-1} g_t(x_t, u_t) + g_T(x_T)$$

in which $x_{t+1} = f_t(x_t, u_t)$ for $t = 0, \ldots, T-1$. The optimization problem is (U, J). And so a dynamic optimization problem is just a (possibly big) optimization problem. We call $\sum_{t=0}^{T-1} g_t(x_t, u_t)$ the *total stage cost* and we call $g_T(x_T)$ the *terminal stage cost*.

Notation

We often write this problem as

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{T-1} g_t(x_t, u_t) + g_T(x_T) \\ \text{subject to} \quad & x_{t+1} = f_t(x_t, u_t), \quad t = 0, \ldots, T-1. \end{aligned}$$

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[1]Future editions will include.

Other terminology and comments

Dynamic optimization problems are frequently called *deterministic optimal control* problems or *classical* or *open-loop control* problems. These problems are said to address the dynamic effect of actions across time. Although these models include no notion of "uncertainty" (or "uncertain outcomes", see Uncertain Outcomes), they are frequently applied in situations with uncertain outcomes by ignoring the uncertainty in the application.

Dynamic Optimization Problems

Controlled Dynamical Systems

State Representations   Directed Graphs   Dynamical Systems

Sequential Decisions   Real Numbers

Sequences   Undirected Graphs   Rational Numbers

Lists   Decisions   Integer Arithmetic

Direct Products   Set Numbers   Natural Additive Identity   Natural Multiplicative Identity

Finite Sets   Identity Elements   Arithmetic

Equivalent Sets   Orders   Natural Powers   Integer Products

Family Unions and Intersections   Function Inverses   Natural Order   Natural Products   Integer Sums

Families   Function Images   Function Composites   Operations   Natural Sums   Integer Numbers

Functions   Equivalence Relations   Recursion Theorem

Relations   Peano Axioms

Set Products   Uncertain Outcomes   Natural Induction

Generalized Set Dualities   Set Powers   Natural Numbers

Set Unions and Intersections   Partitions   Intersection of Empty Set   Set Dualities   Unordered Triples   Successor Sets   Set Symmetric Differences

Ordered Pairs   Set Intersections   Pair Unions   Set Complements

Pair Intersections   Set Unions   Set Differences

Unordered Pairs   Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects