Finite Measures

1 Why

Sometimes we want finite measures. TODO: which times?

2 Definition

A measurable set is *finite* if its measure is a real number. The measure space itself is *finite* if the base set is finite.

A measurable set is *sigma-finite* if there exists a sequence of finite measurable sets whose union is the set. The measure space itself is *sigma-finite* if the base set is sigma finite.

2.1 Notation

We denote that a measure space is finite by saying "Let $(A, \mathcal{A}, \mu)$ and $\mu(A) < +\infty$."

Example 1. *Let $(A, \mathcal{A})$ be a measurable space.*

The counting measure on $(A, \mathcal{A})$ is finite if and only if the base set is finite. It is sigma finite if and only if the base set is a union of a sequence of finite sets.

If $\mathcal{A} = 2^A$, then the counting measure is sigma finite if and only if A is countable.

Example 2. *A point mass measure is finite.*

Example 3. *Let R be the set of real numbers. The Lebesgue measure on $(R, \mathcal{B}(R))$ is sigma finite.*