



Tree Density Approximation

1 Why

Can we approximate a density by a tree density in a similar manner to how we handled distributions?

2 Definition

Tree density approximation is an optimization problem in which we select a tree density whose entropy relative to a given density is minimal.

2.1 Notation

Let $g : \mathbf{R}^n \rightarrow \mathbf{R}$ be a density. Let d denote the differential relative entropy. We want to

find density f and tree T on $\{1, \dots, n\}$
minimizing $d(g, f)$
subject to f factors according to T .

3 Solution

Proposition 1. *Let $g : \mathbf{R}^n \rightarrow \mathbf{R}$ be a density. Let T be a tree on $\{1, \dots, n\}$. Let p_j be the parent of vertex j for the T rooted at vertex i , $j = 1, \dots, n$ and*

$j \neq i$. Then the density $f : \mathbf{R}^d \rightarrow \mathbf{R}$ defined by

$$f = g_i \prod_{j \neq i} g_{j|p_j}$$

achieves minimum differential entropy relative to g among all densities which factor according to T .

Proposition 2. *Let $g : \mathbf{R}^n \rightarrow \mathbf{R}$ be a density. A tree T is a solution to the problem above if and only if it is a maximum spanning tree of the differential mutual information graph of g .*