Why

We want to estimate a random vector $x : \Omega \to \mathbf{R}^d$ from a random vector $y : \Omega \to \mathbf{R}^n$.

Definition

Denote by $g : \mathbf{R}^d \times \mathbf{R}^n \to \mathbf{R}$ the joint density for (x, y).[1] Denote the conditional density for x given y by $g_{x|y} : \mathbf{R}^d \times \mathbf{R}^n \to \mathbf{R}$. In this setting, $g_{x|y}$ is called the *posterior density*, g_x is called the *prior density*, and $g_{y|x}$ is called the *likelihood density* and g_y is called the *marginal likelihood density*.

As usual (and assuming $g_y > 0$), the posterior is related to the likelihood, prior and marginal likelihood by

$$g_{x|y} \equiv \frac{g_x g_{y|x}}{g_y}.$$

A *maximum conditional estimate* for $x : \Omega \to \mathbf{R}^n$ given that y has taken the value $\gamma \in \mathbf{R}^n$ is a maximizer $\xi \in \mathbf{R}^d$ of $g_{x|y}(\xi, \gamma)$. It is also called the *maximum a posteriori estimate* or *MAP estimate*. The maximum conditional estimate is natural, in part, because it also maximizes the joint density, since $g(\xi, \gamma) = g_y(\gamma)g_{x|y}(\xi, \gamma)$ for all $\xi \in \mathbf{R}^d$ and $\gamma \in \mathbf{R}^n$.

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[1] Future editions will comment on the existence of such a density.

Maximum Conditional Estimates

Conditional Densities — Estimators

Marginal Densities — Estimates

Multivariate Real Densities

Probability Densities

Real Integrals

Nonnegative Integrals

Simple Integrals — Random Variables — Conditional Distributions

Random Vectors

Matrices

Real Matrices

Topological Sigma Algebras — Measurable Functions — Probability Measure — Conditional Event Probabilities

Vectors — Linear Equations

Topological Spaces — Measure — Event Probabilities

Real Vectors — Linear Functions — Metrics — Sigma Algebras

Space Vectors — Plane Vector Sums and Differences — N-Dimensional Space

Real Series — Extended Real Numbers

Plane Vectors — Distance — Magical Densities

Space Distance — Absolute Value — Simple Functions — Real Limits — Joint Distributions

Plane Distance — Interval Length — Real Functions — Real Summation — Probability Distributions

Real Space — Arrays — Intervals — Real Sequences

Real Plane — Real Line — Sequences

Integral Line — Real Order

Fields — Geometry — Rational Order — Real Numbers — Subset Algebras

Rings — Rational Numbers — Limits — Set Numbers — Cardinality

Integer Arithmetic — Finite Sets — Natural Summation

Natural Equation — Integer Products — Natural Additive Identity — Natural Multiplication Identity — Direct Products — Equivalent Sets — Function Inverses — Family Operations

Integer Sums — Integer Order — Arithmetic — Comparison — Family Unions and Intersections — Identity Elements — Operations

Equations — Integer Numbers — Natural Powers — Orders — Function — Function Composites — Function Images

Natural Products — Natural Order — Equivalence Relations — Converse Relations — Functions

Natural Sums — Relations

Recursion Theorem — Set Products — Subset Systems — Characteristic Functions

Phase Actions — Ordered Pairs — Set Powers

Natural Induction

Natural Numbers — Generalized Set Dualities — Unordered Triples — Uncertain Outcomes

Partitions — Successor Sets — Intersection of Empty Set — Set Dualities — Set Unions and Intersections — Set Symmetric Differences

Set Complements

Set Intersection — Pair Unions — Pair Intersections

Set Differences — Set Unions

Empty Set — Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects