



Direct Products

1 Why

We generalize the notion of a cross product of two sets to a cross produce of n sets.

2 Direct Products

The *direct product* of a natural family is the set of ordered sequences of elements from each set in the family. We call the elements of the direct product *n-tuples*. We call the i th element in an n -tuple the i th coordinate. This language is meant to follow that used in defining ordered pairs. Two coordinates in a sequence are *consecutive* if their natural difference is 1.

2.1 Notation

Let A_1, \dots, A_n be a natural family of sets. We denote its direct product by

$$\prod_{i=1}^n A_i.$$

We read this notation as “product over alpha in I of A sub-alpha.” We denote an element of $\prod_{i=1}^n A_i$ by (a_1, a_2, \dots, a_n) with the understanding that $a_1 \in A_1, a_2 \in A_2, \dots, a_n \in A_n$.

If $A_i = A$ for $i = 1, \dots, n$, then we denote $\prod_{i=1}^n A_i$ by A^n .