Intervals

1 Why

We name and denote subsets of the set of real numbers which correspond to segments of a line?

2 Definition

Take two real numbers, with the first less than the second.

An **interval** is one of four sets:

1. the set of real numbers larger than the first number and smaller than the second; we call the interval **open**.
2. the set of real numbers larger than or equal to the first number and smaller than or equal to the second number; we call the interval **closed**.
3. the set of real numbers larger than the first number and smaller than or equal to the second; we call the interval **open on the left** and **closed on the right**.
4. the set of real numbers larger than or equal to the first number and smaller than the second; we call the interval

closed on the left and **open on the right**.

If an interval is neither open nor closed we call it **half-open** or **half-closed**

We call the two numbers the **endpoints** of the interval. An open interval does not contain its endpoints. A closed interval contains its endpoints. A half-open/half-closed interval contains only one of its endpoints. We say that the endpoints **delimit** the interval.

2.1 Notation

Let a, b be two real numbers which satisfy the relation $a < b$.

We denote the open interval from a to b by (a, b). This notation, although standard, is the same as that for ordered pairs; no confusion arises with adequate context.

We denote the closed interval from a to b by $[a, b]$. We record the fact $(a, b) \subset [a, b]$ in our new notation.

We denote the half-open interval from a to b, closed on the right, by $(a, b]$ and the half-open interval from a to b, closed on the left, by $[a, b)$.