Definition

Let $f : D \to \mathbf{R}$ be a multivariate real-valued function where $D \subset \mathbf{R}^d$. The *graph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}.$$

The *epigraph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, t) \in \mathbf{R}^d \mid x \in D \text{ and } f(x) \leq t\}.$$

Notation

We denote the epigraph of a function f by epi f.

Connecting convex sets and convex functions

A function $f : \mathbf{R}^d \to \mathbf{R}$ is convex if and only if epi f is a convex set.

Note on terminology

The prefix "epi" is Greek, meaning "upon" or "above".

Real Function Epigraphs

Multivariate Functions

Convex Multivariate Functions

Real Convex Functions

Real Function Graphs

Real Convex Sets

Real Functions

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Real Space

Intervals

Real Plane

Real Line

Integral Line

Real Order

Geometry

Real Numbers

Rational Order

Lists

Rational Numbers

Direct Products

Set Numbers

Integer Arithmetic

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Equivalent Sets

Identity Elements

Arithmetic

Family Unions and Intersections

Function Inverses

Comparisons

Integer Order

Integer Sums

Natural Powers

Families

Function Composites

Function Images

Operations

Orders

Integer Numbers

Natural Products

Functions

Converse Relations

Natural Order

Equivalence Relations

Natural Sums

Relations

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Set Products

Natural Induction

Generalized Set Dualities

Set Powers

Natural Numbers

Set Unions and Intersections

Set Dualities

Unordered Triples

Successor Sets

Intersection of Empty Set

Partitions

Set Symmetric Differences

Ordered Pairs

Pair Unions

Set Intersections

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects