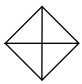


Why

What is the multiplicative inverse in the reals?

Result

We can show the following.[1]

Proposition 1. *The multiplicative inverse of $R \in \mathbf{R}$, $R \neq 0_{\mathbf{R}}$,*

1. *if $0_{\mathbf{Q}} \in R$, then*

$$S = \{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\} \cup \left\{r^{-1} \;\middle|\; \exists s < r, (r \notin R)\right\}$$

   is a multiplicative inverse of R.

2. *if $0_{\mathbf{Q}} \notin R$, then case (1) applies to $-R$. Let S be the multiplicative inverse of $-R$. Then the additive inverse of S, i.e., $-S$ is a multiplicative inverse of R.*

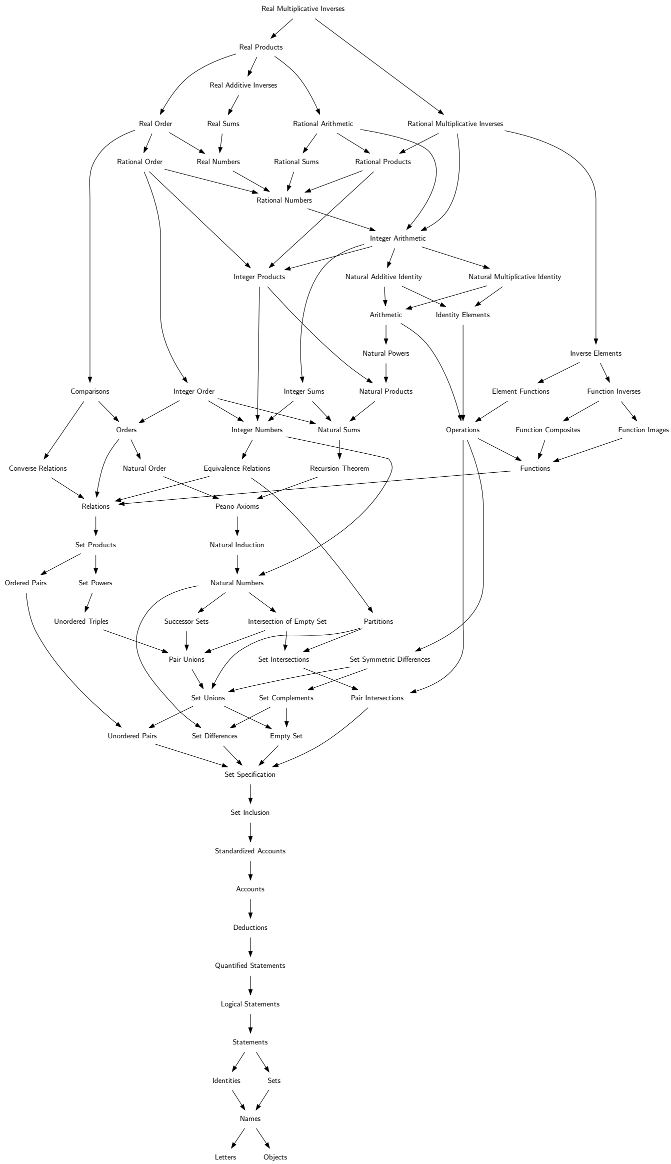
Notation

We denote the multiplicative inverse of $r \in \mathbf{R}$ by r^{-1}. We denote $q \cdot (r^{-1})$ by q/r.

Division

We call the operation $(a, b) \mapsto a/b$ *real division.* We call the product of a and the multiplicative inverse of b the *(real) quotient* of a and b.

[1] The account will appear in future editions.


Real Multiplicative Inverses
Real Products
Real Additive Inverses
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Rational Order
Real Numbers
Rational Sums
Rational Products
Rational Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Inverse Elements
Comparisons
Integer Order
Integer Sums
Natural Products
Element Functions
Function Inverses
Orders
Integer Numbers
Natural Sums
Operations
Function Composites
Function Images
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Functions
Relations
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects