



## Why

We want a set which corresponds to our notion of points on a line.<sup>1</sup>

## Rational cuts

We call a subset  $R$  of  $\mathbf{Q}$  a *rational cut* if (a)  $R \neq \emptyset$ , (b)  $R \neq \mathbf{Q}$ , (c) for all  $q \in R$ ,  $r \leq q \longrightarrow r \in R$ , and (d)  $R$  has no greatest element. Briefly, the intuition is that the point is the set of all rationals to less than (or, potentially, equal to) some particular rational number.<sup>2</sup>

## Definition

The *set of real numbers* is the set of all rational cuts. This set exists by an application of the principle of selection (see **Set Selection**) to the power set (see **Set Powers**) of  $\mathbf{Q}$ . We call an element of the set of real numbers a *real number* or a *real*. We call the set of real numbers the *set of reals* or *reals* for short.

## Notation

We follow tradition and denote the set of real numbers by  $\mathbf{R}$ , likely a mnemonic for “real.”

## Other terminology

Some authors call a real number a *quantity* or a *continuous quantity*. The real numbers, then, are said to be *continuous*. When contrasting (using this terminology) a finite set with the real numbers, one refers to the finite set as *discrete*.<sup>3</sup>

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<sup>1</sup>Future editions will modify and expand this justification.

<sup>2</sup>This brief intuition will be expanded upon in future sheets.

<sup>3</sup>Future editions may move this discussion later, to the discussion of the cardinality of the reals.

