Why

We use a normal random function model to make a regressor.

Definition

Let $F : \Omega \to (A \to \mathbf{R})$ be a normal random function with mean function $m : A \to \mathbf{R}$ and covariance function $k : A \times A \to \mathbf{R}$ over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Let the family of random variables (or stochastic process) of F be $f : A \to (\Omega \to \mathbf{R})$.

Let e be a normal random vector with mean zero and covariance Σ_e. Let $a^1, \ldots, a^n \in A$. We sometimes call the sequence $a^1, \ldots, a^n$ the *design*. Define $y : \Omega \to \mathbf{R}^d$ by

$$y_i = f(a^i) + e_i$$

We call y the *observation vector* or *observation random vector*. We call e the *error vector* or *noise vector*. In this context, $f(a^i)$ is sometimes called the *signal*.

Let $b^1, \ldots, b^m \in A$. Define $z : \Omega \to \mathbf{R}^d$ by $z_i = f(b^i)$ for $i = 1, \ldots, n$. So z_i is the random variable corresponding to the family at index $b^i \in A$. Then (y, z) is normal. We call the conditional density of z given y the *predictive density* for b given a.

Proposition 1. *Define $m_a \in \mathbf{R}^n$ by $(m(a^1), \cdots, m(a^n))^\top$ and define m_b by $(m(b^1), \cdots, m(b^m))^\top$.*[1] *Define $\Sigma_a \in \mathbf{R}^{n \times n}$ by*

$$\begin{pmatrix} k(a^1, a^1) & \cdots & k(a^1, a^n) \\ \vdots & \ddots & \vdots \\ k(a^n, a^1) & \cdots & k(a^n, a^n) \end{pmatrix}$$

and define $\Sigma_{ba} \in \mathbf{R}^{m \times n}$ by

$$\begin{pmatrix} k(b^1, a^1) & \cdots & k(b^1, a^n) \\ \vdots & \ddots & \vdots \\ k(b^m, a^1) & \cdots & k(b^m, a^n) \end{pmatrix}.$$

[1] Future editions will fix the re-use of the symbol m.

The predictive density $g_{z|y}(\cdot, \gamma) : \mathbf{R}^m \to \mathbf{R}$ *of* $b \in A$ *for design* $a^1, \ldots, a^n$ *is normal with mean.*

$$m_b + K_{ba}(K_a + \Sigma_e)^{-1}(\gamma - m_a)$$

and covariance

$$\Sigma_b - \Sigma_{ba}(\Sigma_a + \Sigma_e)^{-1}\Sigma_{ab}.$$

Normal Random Positive Probabilities Densities
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Square Inner Product
Vector Spaces
Matrix
Plane Vectors Sums and Differences
Space Vectors
Vector Algebra
Right Triangle Sides Relation
Distance
Real Square Roots
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Real Square
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Real Lines
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Real Arithmetic
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Partitions
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Successor Sets
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Intersections
Set Differences
Set Unions
Pair Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Sets
Identities
Sets
Lattices
Objects