



# Product Sigma Algebras

## 1 Why

We want to generalize the construction of cover area as generated as a product of two cover lengths, and more generally for arbitrary measure spaces. TODO

## 2 Definition

Consider two measurable spaces. The *product base set* is the cartesian product of the first base set with the second base set. A first distinguished set is a distinguished set of the first measurable space, and likewise for a second distinguished set.

A *rectangle with measurable sides* is a set in product base set which is a product of a first distinguished set with a second distinguished set. The *product sigma algebra* is the sigma algebra generated by the rectangles with measurable sides.

The *product measurable space* is the measurable space whose base set is the product base set and whose sigma algebra is the product sigma algebra.

### 2.1 Notation

Let  $(X, \mathcal{A})$  and  $(Y, \mathcal{B})$  be measurable spaces. The product base set is  $X \times Y$ . A set  $R \in X \times Y$  is a rectangle with measurable sides if

$R = A \times B$  for  $A \in \mathcal{A}$  and  $B \in \mathcal{B}$ . We denote the product sigma algebra of  $\mathcal{A}$  and  $\mathcal{B}$  by  $\mathcal{A} \times \mathcal{B}$ .  $(X \times Y, \mathcal{A} \times \mathcal{B})$  is the product measurable space.