

ALMOST EVERYWHERE MEASURABILITY

Why

Does convergence almost everywhere of a sequence of measurable functions guarantee measurability of the limit function? It does on complete measure spaces, and we can use this result to "weaken" the hypotheses of many theorems.

Results

A measure is *complete* if every subset of a measurable set of measure zero is measurable. If the measure is complete, then every negligible set must be measurable.

We begin with a transitivity property: almost everywhere equality of two functions allows us to infer measurability of one from the other.

Prop. 1. Let (X, \mathcal{A}, μ) be a measure space Let $f, g : X \to [-\infty, \infty]$ with f = g almost everywhere. If μ is complete and f is \mathcal{A} -measurable, then g is \mathcal{A} -measurable.

Proof.
$$\Box$$

Prop. 2. Let (X, \mathcal{A}, μ) be a measure space. Let $f_n : X \to [-\infty, \infty]$ for all natural numbers n and $f : X \to [-\infty, \infty]$ with $(f_n)_n$ converging to f almost everywhere. If μ is complete and and f_n is measurable for each n, then f is A-measurable.

Proof.
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¹Future editions will include.

