



## Why

If we interpret a list of two numbers as displacement in a plane, and a list of three numbers as displacement in a space, what of a list of  $n$  numbers as displacement in  $\mathbf{R}^n$ ?

## Definition

A *real vector* (or *vector*, *n-dimensional vector*, *n-vector*) is a length- $n$  list of real numbers.

## Algebra

For  $x, y \in \mathbf{R}^n$ , we define the *real vector sum* (or *sum*) of  $x$  and  $y$  as the vector  $z \in \mathbf{R}^n$  where  $z_i = x_i + y_i$  for  $i = 1, \dots, n$ . As usual, we denote the sum by  $x + y$ , so

$$x + y = (x_1 + y_1, \dots, x_n + y_n).$$

For  $\alpha \in \mathbf{R}$  and  $x \in \mathbf{R}^n$ , *real scalar-vector product* (or *scalar product*, *product*)  $z \in \mathbf{R}^n$  is defined by  $z_i = \alpha x_i$  for  $i = 1, \dots, n$ . As usual, we denote the product  $\alpha x$ , and write

$$\alpha x = (\alpha x_1, \dots, \alpha x_n).$$

Our intuition for both of these *operations* comes from their special cases in  $\mathbf{R}^2$  and  $\mathbf{R}^3$ . As usual, the *real-vector difference* (or *difference*) of  $x$  and  $y$  is the vector  $z \in \mathbf{R}^n$  defined by  $z_i = x_i - y_i$  for  $i = 1, \dots, n$ . As usual, we denote it by  $x - y$ , and note that  $x - y = x + (-y)$ .

The algebra given here for vectors is natural in view of their generalization as  $n$ -dimensional *displacements*. However, we keep in mind that this algebra is over lists of numbers, and that these sums and products can be defined on these lists of numbers regardless of interpretation.



