



Why

If we have some outcomes and a distribution, we can construct a function which assigns probabilities to events.

Definition

The *probability of an event* is the sum of the probabilities of the outcomes in the event. The *event probability function* is the correspondence assigning events to their probabilities.

Notation

Let A be a set of outcomes and p a distribution on A . Let $B \subset A$ be an event. Let $\mathbf{P} : 2^A \rightarrow \mathbf{R}$ be the event probability function, which is defined by

$$\mathbf{P}(B) = \sum_{b \in B} p(b).$$

The event probability function \mathbf{P} depends on the outcomes A and the distribution p . We sometimes indicate this dependence by writing $\mathbf{P}_{A,p}$.

Properties

Proposition 1. *Let \mathbf{P} be the event probability function of the distribution $p : A \rightarrow [0, 1]$.*¹

1. $\mathbf{P}(B) \geq 0$ for all $B \subset A$

¹Future editions will include an account.

2. $\mathbf{P}(A) = 1$, and $\mathbf{P}(\emptyset) = 0$
3. $\mathbf{P}(B \cup C) = \mathbf{P}(B) + \mathbf{P}(C) - \mathbf{P}(B \cap C)$ for $B, C \subset A$. In particular,
- (a) if $B \cap C = \emptyset$, then $\mathbf{P}(B \cup C) = \mathbf{P}(B) + \mathbf{P}(C)$.

