



## RECURSION THEOREM

### Why

It is natural to want to define a sequence by giving its first term and then giving its later terms as functions of its earlier ones. In other words, we want to define sequences inductively.<sup>1</sup>

### Main Result

The following is often referred to as the *recursion theorem*.

**Proposition 1** (Recursion Theorem<sup>2</sup>). *Let  $X$  be a set, let  $a \in X$  and let  $f : X \rightarrow X$ . There exists a unique function  $u$  so that  $u(0) = a$  and  $u(\succ(n)) = f(u(n))$ .*<sup>3</sup>

When one uses the recursion theorem to assert the existence of a function with the desired properties, it is called *definition by induction*.

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<sup>1</sup>Future editions will expand on this. We are really headed toward natural addition, multiplication and exponentiation.

<sup>2</sup>Future editions will likely change this name.

<sup>3</sup>The account is somewhat straightforward, given a good understanding of the results of *Peano Axioms*. The full account will appear in future editions.



