Why

Let $A \in \mathbf{R}^{m \times n}$ and define $f : \mathbf{R}^n \to \mathbf{R}^m$ by $f(x) = Ax$. Then f is a linear function from $\mathbf{R}^n$ to $\mathbf{R}^m$. Conversely, suppose $g : \mathbf{R}^n \to \mathbf{R}^m$ is a linear function. Then there exists a matrix $B \in \mathbf{R}^{m \times n}$ so that $g(z) = Bz$. Does this function have an inverse?

Derivation

If $A \in \mathbf{R}^{m \times n}$, with $m \neq n$, then the inverse of f can not exist. For a *square* matrix $A \in \mathbf{R}^{n \times n}$, $B \in \mathbf{R}^{n \times n}$ is a *left inverse* if $BA = I$. In other words, B is a left inverse element of A in the algebra of matrices with the operation of multiplication. $C \in \mathbf{R}^{n \times n}$ is a *right inverse* if $AC = I$.

Definition

We call a square matrix A *invertible* if there is $B \in \mathbf{R}^{n \times n}$ so that $BA = I$.

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that $BA = I$ we call B the *left inverse* of A and likewise if $AC = I$ we call C the *right inverse* of A. In the case that A is square, the right inverse and left inverse coincide.

Proposition 1. *Let $A, B, C \in \mathbf{R}^{n \times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

Proof. Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. $\qquad\square$

Real Matrix Inverses
Real Matrix-Matrix Products
Real Matrix-Vector Products
Real Linear Combinations
Vectors as Matrices
Real Matrices
Real Inner Product
Linear Equations
Real Norm
Space Norm
Real Vectors
Linear Functions
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Inner Product
Plane Norm
Plane Vectors
Distance
Right Triangle Sides Relation
Absolute Value
Arrays
Space Distance
Plane Distance
Real Square
Real Arithmetic
Real Multiplicative Inverses
Interval Length
Real Space
Real Products
Intervals
Real Plane
Real Additive Inverses
Real Line
Square Numbers
Real Summation
Real Order
Real Sums
Rational Arithmetic
Rational Multiplication Inverses
Integral Line
Squares
Inverse Elements
Rational Order
Real Numbers
Rational Sums
Rational Products
Geometry
Element Functions
Rational Numbers
Lists
Set Numbers
Integer Arithmetic
Direct Products
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Natural Summation
Natural Powers
Family Unions and Intersections
Function Inverses
Family Operation
Natural Equation
Comparisons
Integer Order
Natural Products
Integer Sums
Families
Function Composites
Function Images
Equations
Operations
Orders
Natural Sums
Integer Numbers
Function
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects