

INTEGER ORDER

Why

We want to order the integers.

Definition

Consider $[(a,b)], [(b,c)] \in \mathbf{Z}$. If a+d < b+c, then we say that [(a,b)] is less than [(b,c)].¹ If [(a,b)] is less than [(b,c)] or equal, then we say that [(a,b)] is less than or equal to [(b,c)].

Notation

If $x, y \in \mathbf{Z}$ and x is less than y, then we write x < y. If x is less than or equal to y, we write $x \le y$.

Positive and Negative Integers

We call an integer z positive if z > 0 and we call z negative if $z < 0.^2$ We call an integer z nonnegative if z > 0 or z = 0 and nonpositive if z < 0 or z = 0.

Notation

We denote the set $\{z \in \mathbf{Z} \mid z \geq 0_Z\}$ by \mathbf{Z}_{++} .

¹One needs to show that this is well-defined. The account will appear in future editions.

²Some authors use the term positive for the case when z > 0 or z = 0. We use the term nonnegative in this case.

