Monotone Neighborhoods

Why

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Definition

The *higher adjacency set* or *higher neighborhood* of a vertex v in an ordered undirected graph is all vertices in the neighborhood of v whose index is greater the v. Similarly, the *lower adjacency set* or *lower neighborhood* of v is all vertices in the neighborhood of v whose index is less the v. We call these *monotone neighborhoods*.

The *higher degree* of a vertex is the size of the higher adjacency set and the *lower degree* of a vertex is the size of its lower adjacency set.

The *closed monotone neighborhoods* are the *closed higher adjacency set*, the higher adjacency set of v union with the singleton $\{v\}$ and the *closed lower adjacency set*, the lower adjacency set of v union with the singleton $\{v\}$.

Notation

We denote the higher neighborhood of v by $\mathrm{adj}^+(v)$ and the lower neighborhood by $\mathrm{adj}^-(v)$.

Visualization

To help think about the monotone neighborhoods of the graph we visualize ordered graphs as triangular arrays with vertices along the diagonal and a bullet in row i and column j of the array if $i > j$ and the vertices $\sigma(i)$ and $\sigma(j)$ are adjacent.

An example is shown below for the ordered undirected graph in the figure (to understand this visualization, see Ordered Undirected Graphs)

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[1]Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.
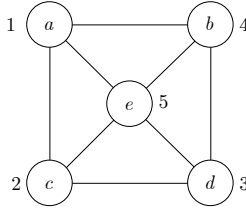
Figure 1: Ordered undirected graph.

we use the

$$
\begin{bmatrix}
a & & & & \\
\bullet & c & & & \\
& \bullet & d & & \\
\bullet & & \bullet & b & \\
\bullet & \bullet & \bullet & \bullet & e
\end{bmatrix}
$$

In this array representation the higher and lower neighborhoods are easily identified. The indices of the elements of $\mathrm{adj}^+(v)$ are the column indices of the entries in row $\sigma^{-1}(v)$ of the array. For example, $\sigma^{-1}(d) = 3$, and the only bullet entry in row three is c so $\mathrm{adj}^-(d) = \{c\}$. Likewise, $\mathrm{adj}^-(c) = \{a\}$. And so on. Similalry, the indices of $\mathrm{adj}^+(v)$ are the row indices of the entries in column $\sigma^{-1}(v)$. For example, $\sigma^{-1}(d)$ is 3, and there are indices 4 and 5 corresponding to b and e so $\mathrm{adj}^+(d) = \{b, e\}$. Likewise, $\mathrm{adj}^+(c) = \{d, e\}$.

For this reason, we use the notation $\mathrm{col}(v)$ and $\mathrm{row}(v)$ for the closed upper and lower neighborhoods. So $\mathrm{col}(v) = \mathrm{adj}^+(v) \cup \{v\}$ and $\mathrm{row}(v) = \mathrm{adj}^-(v) \cup \{v\}$.

Monotone Neighborhoods

Ordered Undirected Graphs

Arrays

Undirected Graphs

Lists

Set Numbers

Direct Products

Finite Sets

Equivalent Sets

Function Inverses

Natural Order

Family Unions and Intersections

Function Composites

Function Images

Families

Functions

Equivalence Relations

Peano Axioms

Relations

Natural Induction

Set Products

Set Powers

Natural Numbers

Generalized Set Dualities

Partitions

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Dualities

Set Unions and Intersections

Ordered Pairs

Set Intersections

Pair Unions

Set Complements

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects