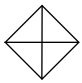


Why

Given that we know that one event has occured, we want language for what the new probabilities should be.[1]

Definition

Let $\mathsf{P} : \mathcal{P}(\Omega) \to \mathsf{R}$ be a finite probability measure. Let $A, B \subset \Omega$ and $\mathsf{P}(B) \neq 0$. The *conditional probability* of A *given* B is fraction of the probability of $A \cap B$ over the probability of B.

Notation

In a slightly slippery but universally standard notation, we denote the conditional probability of A given B by $\mathsf{P}(A \mid B)$. In other words, we define

$$\mathsf{P}(A \mid B) = \frac{\mathsf{P}(A \cap B)}{\mathsf{P}(B)},$$

whenever $A, B \subset \Omega$ and $\mathsf{P}(B) \neq 0$.

Induced conditional distribution

Conditioning on an event B induces a new distribution on the set of outcomes. For P_p, define $q : \Omega \to \mathsf{R}$ by

$$q(\omega) = \begin{cases} \frac{p(\omega)}{\mathsf{P}(B)} & \text{if } \omega \in B \\ 0 & \text{otherwise.} \end{cases}$$

In this case $\mathsf{P}_q(A) = \mathsf{P}_p(A \mid B)$. We call q the *conditional distribution* induced by *conditioning on* the event B.

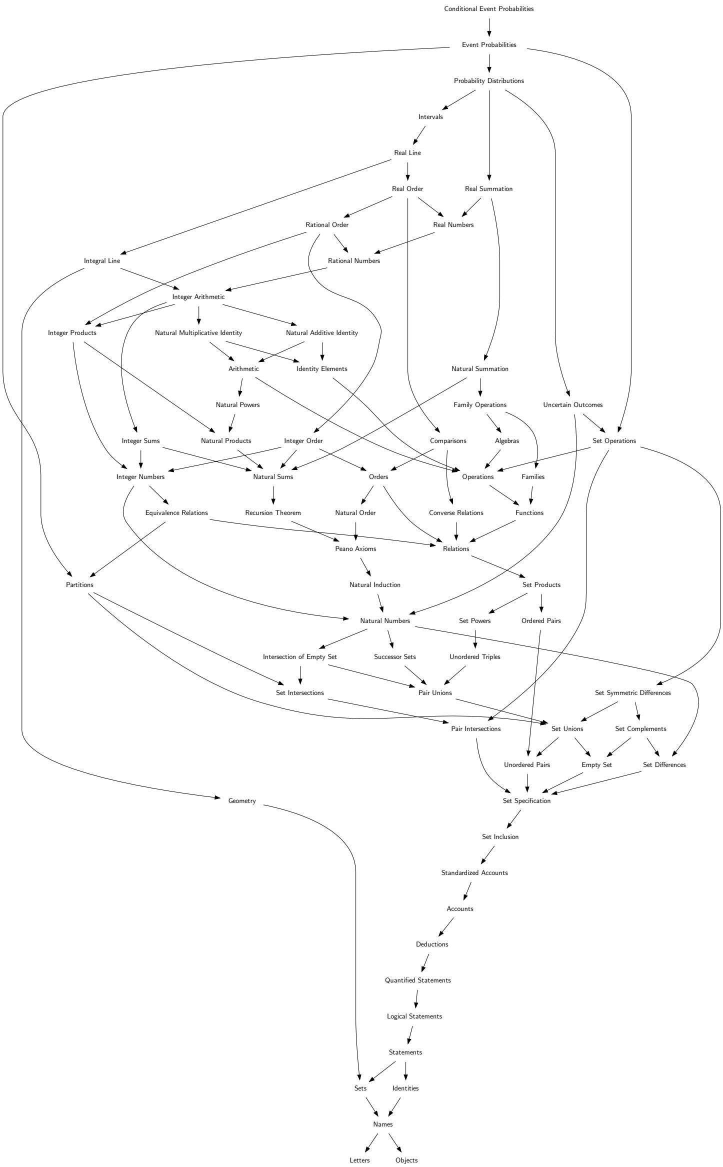
[1]Future editions will improve.

Total probability

Using the notation $\mathsf{P}(\cdot \mid \cdot)$, we can express the law of total probability for $\mathsf{P}(B)$, $B \subset \Omega$, as

$$\mathsf{P}(B) = \textstyle\sum_{i=1}^{n} \mathsf{P}(A_i)\mathsf{P}(B \mid A_i),$$

where $A_1, \ldots, A_n$ partition Ω.



Conditional Event Probabilities
Event Probabilities
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Rational Order
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Arithmetic
Identity Elements
Natural Summation
Natural Powers
Family Operations
Uncertain Outcomes
Integer Sums
Natural Products
Integer Order
Comparisons
Algebras
Set Operations
Integer Numbers
Natural Sums
Orders
Operations
Families
Equivalence Relations
Recursion Theorem
Natural Order
Converse Relations
Functions
Peano Axioms
Relations
Partitions
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects