Why

If we treat the parameters of a linear function as a random variable, an inductor for the predictor is equivalent to an estimator for the parameters.[1]

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $x : \Omega \to \mathbf{R}^d$. Define $g : \Omega \to (\mathbf{R}^d \to \mathbf{R})$ by $g(\omega)(a) = a^\top x(\omega)$, for $a \in \mathbf{R}^d$. In other words, for each outcome $\omega \in \Omega$, $g_\omega : \mathbf{R}^d \to \mathbf{R}$ is a linear function with parameters $x(\omega)$. g_ω is the function of interest.

Let $a^1, \ldots, a^n \in \mathbf{R}^d$ a dataset with data matrix $A \in \mathbf{R}^{n \times d}$. Let $e : \Omega \to \mathbf{R}^n$ independent of x, and define $y : \Omega \to \mathbf{R}^n$ by

$$y = Ax + e.$$

In other words, $y_i = x^\top a^i + e_i$.

We call (x, A, e) a *probabilistic linear model*. Other terms include *linear model*, *statistical linear model*, *linear regression model*, *bayesian linear regression*, and *bayesian analysis of the linear model*.[2] We call x the parameters, A a *design*, e the *error* or *noise* vector, and y the *observation* vector.

One may want an estimator for the parameters x in terms of y or one may be modeling the function g and want to predict $g(a)$ for $a \in A$ not in the dataset.

Inconsistency

In this model, the dataset is assumed to be inconsistent as a result of the random errors. In these cases, the error vector e may model a variety

[1] Future editions will offer further discussion.

[2] The word bayesian is in reference to treating the object of interest—x—as a random variable.

of sources of error ranging from inaccuracies in the measurements (or measurement devices) to systematic errors from the "inapproriateness" of the use of a linear predictor.[3] In this case the linear part is sometimes called the *deterministic effect* of the response on the input $a \in A$.

Moment assumptions

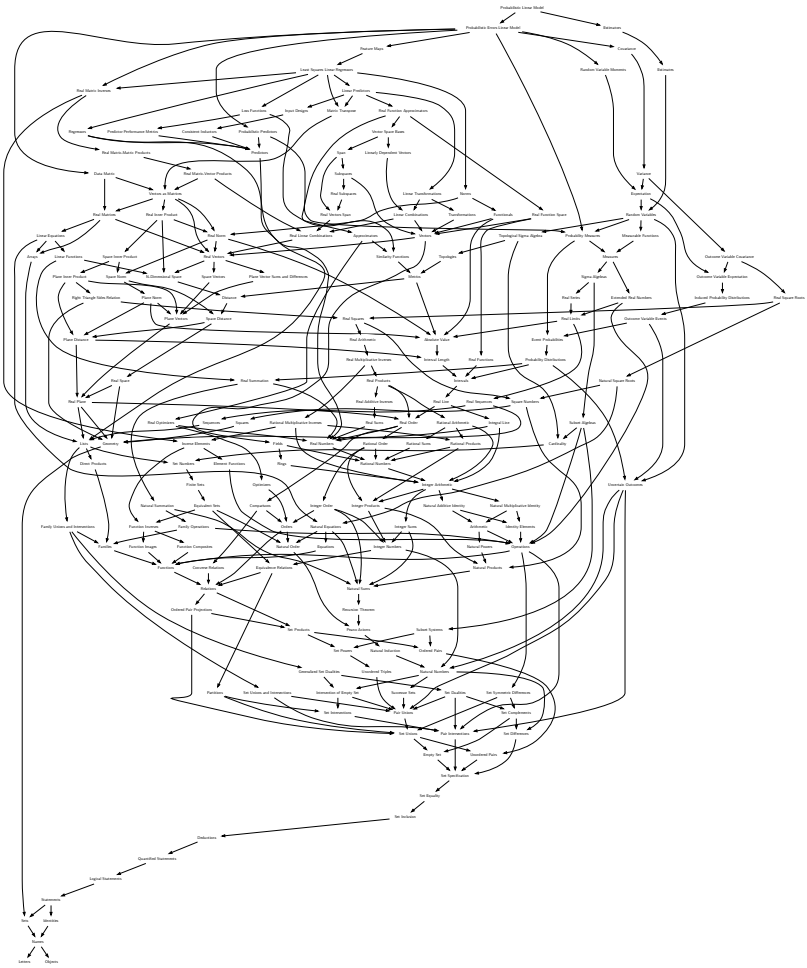
One route to be more specific about the underlying distribution of the random vector is give its mean and variance. It is common to give the mean of $\mathsf{E}(w)$

Mean and variance

Proposition 1. $\mathsf{E}(y) = A\mathsf{E}(x) + \mathsf{E}(w)$[4]

Proposition 2. $\operatorname{cov}((x, y)) = A \operatorname{cov}(x) A^\top + \operatorname{cov} e$[5]

[3] Future editions will clarify and may excise this sentence.
[4] By linearity. Full account in future editions.
[5] Full account in future editions.


Probabilistic Linear Model
Estimates
Probabilistic Error-Linear Model
Covariance
Feature Maps
Random Variable Moments
Estimates
Least Squares Linear Regressors
Real Matrix Inverses
Line Functions
Input Dimension
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Subspaces
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Linear Transformations
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Real Function Space
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Vector Addition
Vector Subtraction
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Similarity Functions
Topological Sigma Algebra
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Square Matrices
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Square Matrices
Plane Vector Norm and Differences
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Measure
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Plane Inner Product
Sigma Through Inline Relation
Exact Norm
Metrics
Real Sums
Extended Real Numbers
Indicator Probability Distributions
Real Square Roots
Plane Norm
Square Distance
Real Squares
Real Limits
Common Variable Average
Plane Distance
Real Addition
Metric Order
Event Probabilities
Real Multiplication Inverses
Interval Length
Real Functions
Probability Distributions
Real Sums
Real Summation
Real Subtraction
Natural Square Roots
Real Power
Real Inverses
Real Subtraction Inverses
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Cardinality
Inner Products
Set Numbers
Natural Numbers
Integer
Integer Addition
Quantifier Conversion
Natural Multiplication Identity
Natural Addition Identity
Natural Multiplication Identity
Natural Summation
Equivalence Sets
Comparison
Integer Order
Integer Product
Family Unions and Intersections
Family Operations
Integers
Natural Exponents
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Addition
Identity Elements
Function Inverses
Function Composition
Natural Order
Equations
Integer Numbers
Natural Power
Function Images
Function
Concrete Relations
Equivalence Relations
Natural Products
Relations
Natural Order
Sensible Theorems
Ordered Pair Projections
Set Products
Power Subsets
Subset Systems
Set Powers
Natural Indexing
Ordered Pairs
Generalized Set Doubles
Generalized Triples
Natural Sequences
Partitions
Set Unions and Intersections
Intersection of Empty Set
Set Doubles
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Set Unions
Set Intersections
Set Differences
Empty Set
Generalized Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Sums
Letters
Objects