Why

We can interpret the codomain of a random variable as a new sample space, since the underlying probability distribution induces a new probability distribution.

Definition

Let $p : \Omega \to \mathbf{R}$ be a probability distribution and $x : \Omega \to V$ an outcome variable. Define $q : V \to \mathbf{R}$ by

$$q(a) = \mathbf{P}[x = a].$$

Since events $x^{-1}(a)$ for $a \in V$ partition Ω, $\sum_{a \in A} q(a) = 1$. We call q the *induced distribution* (or *induced probability mass function*) of the random variable x. Thus we can think of V as a set of outcomes, which we call the outcomes *induced* by x.

Notation

It is common to denote it by p_x.

If $x : \Omega \to V$ is a random variable and $f : V \to U$, then if we define $y : \Omega \to V$ so that $y \equiv f(x)$, y is a random variable with induced distribution $p_y : \Omega \to \mathbf{R}$ satisfying

$$p_y(b) = \sum_{a \in V \mid y(a) = b} p_x(a).$$

As a matter of practical computation, we can evaluate probabilities having to do with the outcome variable x using p_x instead of p. For example with x as in the example above, $\mathbf{P}(x = 4 \text{ or } x = 5) = p_x(4) + p_x(5)$, rather than $\sum_{\omega \in \Omega \mid x(\omega) = 4 \text{ or } x(\omega) = 5} p(\omega)$.

Induced Probability Distributions

Outcome Variable Events

Event Probabilities

Probability Distributions

Intervals

Real Line

Real Summation

Real Order

Rational Order

Real Numbers

Rational Numbers

Integral Line

Integer Arithmetic

Integer Products

Natural Additive Identity

Natural Multiplicative Identity

Arithmetic

Identity Elements

Natural Summation

Integer Order

Integer Sums

Family Operations

Natural Powers

Integer Numbers

Families

Natural Products

Companions

Operations

Equivalence Relations

Natural Sums

Orders

Functions

Converse Relations

Recursion Theorem

Natural Order

Relations

Peano Axioms

Ordered Pair Projections

Set Products

Uncertain Outcomes

Natural Induction

Set Powers

Ordered Pairs

Natural Numbers

Successor Sets

Unordered Triples

Intersection of Empty Set

Partitions

Pair Unions

Set Intersections

Set Symmetric Differences

Pair Intersections

Set Unions

Set Complements

Unordered Pairs

Set Differences

Empty Set

Set Specification

Geometry

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects