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Linearly Dependent Vectors

1 Why

We want to find a set of vectors so that there exists a unique linear combination of these vectors corresponding to each vector in the space.

2 Definition

First, we introduce terminology for when it is possible to write a vector as a linear combination of a set of vectors. A vector is *linearly dependent* on a set of vectors if it the result of some linear combination of them. For example, any vector is linearly dependent on the singleton consisting of itself: the linear combination consisting of that vector and the scalar 1 gives the vector.

Naturally, the interesting case is when the vector is not contained in the set it is dependent on. In this case, we imagine constructing it from those vectors in the set. For example, twice a vector is linearly dependent on the singleton set of that vector: it is the result of the linear combination consisting of that vector and the scalar 2. Even more interesting is when the dependent vector is not just a scalar multiple.

A set of vectors is a *linearly dependent set* if one element is linearly dependent on the set excluding it. The idea is that if a set is linearly dependent, we can drop one of its elements, but still express and still express The idea is the following: suppose that a vector is linearly dependent on this set. Then that vector can be written as a linear

combination of the set. Now suppose that the set is linearly dependent; then we can express one of the vectors as a linear combination of the others. The conclusion is that we can actually express the original vector as a linear combination of the set of vectors without the dependent vector. The upshot is that we can shrink the set of vectors, and still represent that vector.

We feel that the above definitions better capture the intuition. The usual definition of linear dependence is given in the following proposition.

Proposition 1. *A sequence of vectors is linearly dependent if and only if then there exists a nontrivial linear combination of sequence whose result is the zero vector.*

Proof. □

A finite sequence of vectors is *linearly dependent* if one is identical to a nontrivial linear combination of the others. If one can be written as a linearly combination

Two vectors, then are linearly dependent if one is a scalar multiple of the other. The sequence $(\mathbf{0})$ is linearly dependent since $a_1\mathbf{0} = \mathbf{0}$ for any scalar a_1.

An infinite sequence of vectors is *linearly dependent* if some finite subsequence is dependent. It is *linearly independent* if every finite subsequence is linearly independent.

Any sequence (finite or infinite) which contains the zero vector is linearly dependent.

If a finite sequence of vectors is linearly independent, then each of its coordinates is distinct.

A set of vectors is *linearly independent* if every sequence of distinct vectors in the set is linearly independent. The set is *linearly dependent* if somem finite sequence of distinct vectors is dependent.

Equival

Function Inverses

Function Composites   Injective Functions   S

Equivalence Relations   Functions

Relations