Why

We generalize the notion of angles in the plane to angles in space.

Definition

The *angle* (*unsigned angle*) between $x \in \mathbf{R}^3$ and $y \in \mathbf{R}^3$ is the real number

$$\theta = \angle(x, y) = \cos^{-1} \frac{x^\top y}{\|x\|\|y\|}$$

Space Vector Angles

Plane Vector Angles

Plane Norm

Absolute Value   Plane Distance   Plane Vectors

Interval Length

Intervals

Real Line   Real Plane

Integral Line   Real Order

Geometry   Rational Order   Real Numbers

Rational Numbers   Lists

Integer Arithmetic   Natural Fractions   Set Numbers   Direct Products

Integer Products   Natural Additive Identity   Natural Multiplicative Identity   Finite Sets

Identity Elements   Natural Arithmetic   Equivalent Sets

Integer Sums   Integer Order   Natural Powers   Comparisons   Function Inverses   Family Unions and Intersections

Integer Numbers   Natural Products   Orders   Operations   Function Composites   Function Images   Families

Natural Sums   Equivalence Relations   Natural Order   Converse Relations   Functions

Recursion Theorem   Relations

Peano Axioms   Ordered Pair Projections   Set Products

Natural Induction   Generalized Set Dualities   Set Powers

Natural Numbers   Successor Sets   Intersection of Empty Set   Set Dualities   Unordered Triples   Set Unions and Intersections

Set Symmetric Differences   Partitions   Pair Unions   Ordered Pairs

Set Complements   Set Intersections   Pair Intersections

Set Differences   Set Unions

Empty Set   Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects

4