Arithmetic

1 Why

Counting one by one is slow so we define an algebra on the naturals.

2 Sums and Addition

Let m and n be two natural numbers. If we apply the successor function to m n times we obtain a number. If we apply the successor function to n m times we obtain a number. Indeed, we obtain the same number in both cases. We call this number the **sum** of m and n. We say we **add** m to n, or vice versa. We call this correspondence, between (m, n) and the sum, **addition**.

2.1 Notation

We denote the function addition by $+$ and so denote the sum of the naturals m and n by $m + n$.

3 Products and Multiplication

Let m and n naturals. If we add n copies of m we obtain a number. If we add m copies of n we obtain a number. Indeed, we obtain the same number in both cases. We call this number the **product** of m and n. We say we **multiply** m to n, or vice versa. We call this symmetric operation mapping (m, n) to their product **multiplication**.

3.1 Notation

We denote the operation of multiplication by $\cdot$ and so denote the product of the naturals m and n by $m \cdot n$.