Why

We can add and scale matrices, so the $m \times n$ matrices are a vector space over **R**.

Definition

The *matrix sum* of two matrices $A, B \in \mathbf{R}^{m \times n}$ is the matrix $C \in \mathbf{R}^{m \times n}$ defined by $C_{ij} = A_{ij} + B_{ij}$. In other words, the matrix C is given by summing the entries of A and B "entry-wise". We denote the matrix sum by $A + B$.

For $\alpha \in \mathbf{R}$, the *α-scaling* of $A \in \mathbf{R}^{m \times n}$ is the matrix $C \in \mathbf{R}^{m \times n}$ defined by $C_{ij} = \alpha A_{ij}$. In other words, the matrix C is given by scaling the entries of A "entry-wise". We denote the α-scaled version of A by αA. These two definitions are justified by the following.

The *matrix space* (or *matrix vector space*) is the vector space $\mathbf{R}^{m \times n}$ in which addition is given by the matrix sums and scalar multiplication by entry-wise scaling. Sometimes we reference explicitly the *$m \times n$ matrix space*.

Subspaces of symmetric matrices

Consider the space of square matrices $\mathbf{R}^{n \times n}$. It is obvious that the subset of symmetric matrices is a subspace. Adding two symmetric matrices gives a symmetric matrix. Scaling a symmetric matrix by a real number gives a symmetric matrix. The matrix all of whose entries are 0 is symmetric.

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