



## Definition

Let  $(X, \mathbf{R})$  be a real vector space. An inner product  $\langle \cdot, \cdot \rangle : X \times X \rightarrow \mathbf{R}$  induces a norm  $\|\cdot\| : X \rightarrow \mathbf{R}$  defined by  $\|x\| = \sqrt{\langle x, x \rangle}$  and metric  $d : X \times X \rightarrow \mathbf{R}$  defined by  $d(x, y) = \|x - y\|$ .

If  $(X, d)$  is a complete metric space, we call  $((X, \mathbf{R}), \langle \cdot, \cdot \rangle)$  a *complete inner product space* (or *Hilbert space*).<sup>1</sup>

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<sup>1</sup>The term Hilbert space is universal, but in accordance with the Bourbaki project's guidelines on naming, we will tend to use the term complete inner product space, even though this is longer.



