Why

We can give the set of bounded linear functions between two norm spaces a norm.

Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

Notation

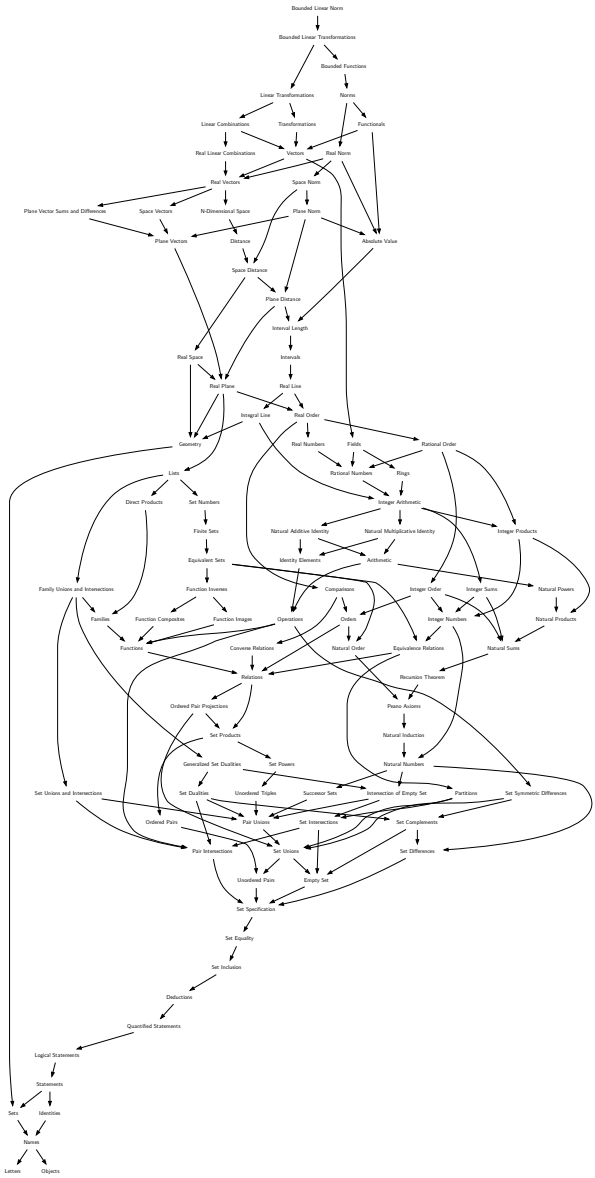
Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ be linear and bounded. The norm of f is the smallest C so that

$$\|f(v)\|_2 \leq C\|v\|_1.$$

Equivalent formulation

Proposition 1. *Let* $((V_1, F_1), \|\cdot\|_1)$ *and* $((V_2, F_2), \|\cdot\|_2)$ *be two norm spaces. Let* $f : V_1 \to V_2$ *bounded and linear. The norm of* f *is*

$$\sup_{\|x\|_1=1} \|f(x)\|_2.$$


Bounded Linear Norm
Bounded Linear Transformations
Bounded Functions
Linear Transformations
Norms
Linear Combinations
Transformations
Functionals
Real Linear Combinations
Vectors
Real Norm
Real Vectors
Space Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Norm
Plane Vectors
Distance
Absolute Value
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Geometry
Real Numbers
Fields
Rational Order
Rational Numbers
Rings
Lines
Integer Arithmetic
Direct Products
Set Numbers
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Powers
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Products
Functions
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Duality
Unordered Tuples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
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Objects