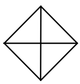


## Why

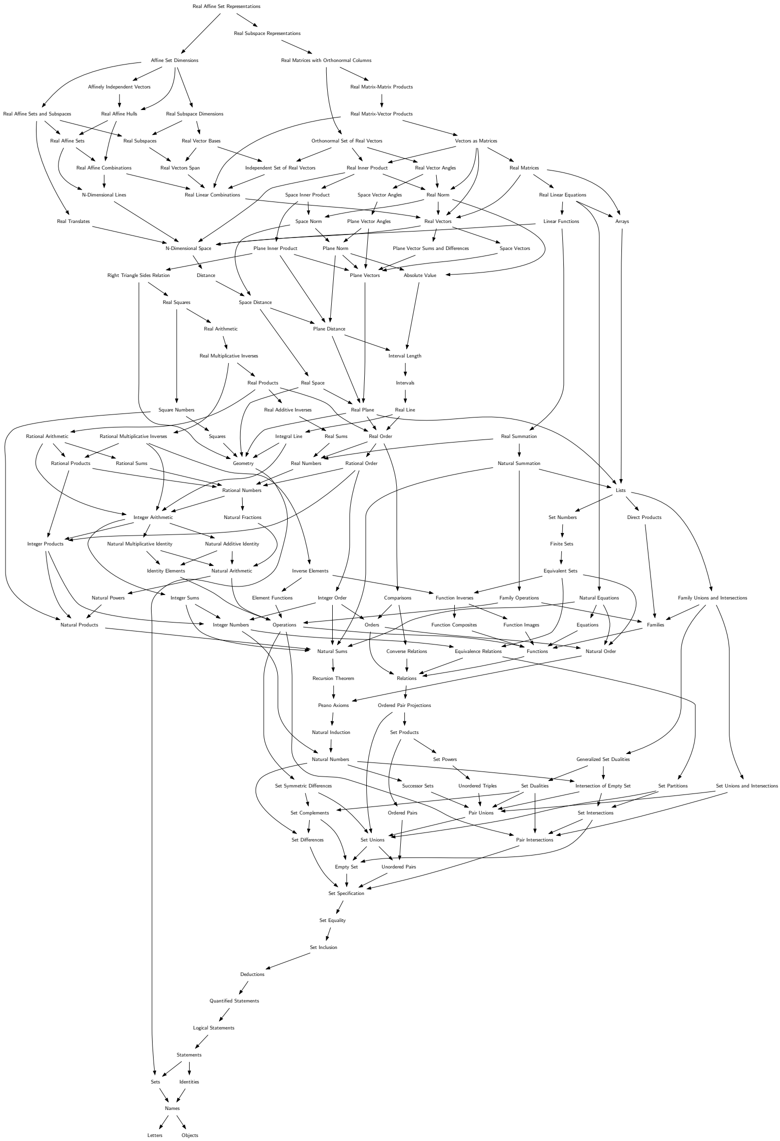
Since every affine set is a translate of a unique subspace, we can represent them by representing the vector and the subspace.

## Definition

Recall that $M$ is affine means $M = S + a$ for some subspace $S$ and vector $a \in \mathbf{R}^n$. The dimension of $M$ is the dimension of the subspace. Suppose $\dim S = k$, then there exists $Q \in \mathbf{R}^{n \times k}$ with $Q^\top Q = I$, so that for any $x \in S$, there exists unique $z \in \mathbf{R}^k$ with $x = Qz$. Since $M = S + a$, we have

$$M = \{y \in \mathbf{R}^n \mid (\exists z \in \mathbf{R}^k)(y = a + Qz)\}$$

We also denote this set $\{a + Qz \mid z \in \mathbf{R}^k\}$.


Real Affine Set Representations
Real Subspace Representations
Affine Set Dimensions
Real Matrices with Orthonormal Columns
Affinely Independent Vectors
Real Matrix-Matrix Products
Real Affine Sets and Subspaces
Real Affine Hulls
Real Subspace Dimensions
Real Matrix-Vector Products
Real Affine Sets
Real Subspaces
Real Vector Bases
Orthonormal Set of Real Vectors
Vectors as Matrices
Real Affine Combinations
Real Vectors Span
Independent Set of Real Vectors
Real Inner Product
Real Vector Angles
Real Matrices
N-Dimensional Lines
Real Linear Combinations
Space Inner Product
Space Vector Angles
Real Norm
Real Linear Equations
Real Translates
Space Norm
Plane Vector Angles
Real Vectors
Linear Functions
Arrays
N-Dimensional Space
Plane Inner Product
Plane Norm
Plane Vector Sums and Differences
Space Vectors
Right Triangle Sides Relation
Distance
Plane Vectors
Absolute Value
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Square Numbers
Real Additive Inverses
Real Plane
Real Line
Rational Arithmetic
Rational Multiplicative Inverses
Squares
Integral Line
Real Sums
Real Order
Real Summation
Rational Products
Rational Sums
Geometry
Real Numbers
Rational Order
Natural Summation
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Identity Elements
Natural Arithmetic
Inverse Elements
Equivalent Sets
Natural Powers
Integer Sums
Element Functions
Integer Order
Comparisons
Function Inverses
Family Operations
Natural Equations
Family Unions and Intersections
Natural Products
Integer Numbers
Operations
Orders
Function Composites
Function Images
Equations
Families
Natural Sums
Converse Relations
Equivalence Relations
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Set Partitions
Set Unions and Intersections
Set Complements
Ordered Pairs
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects