Why

We generalize convex functions to $\mathbf{R}^n$.

Definition

Let A be a convex subset of $\mathbf{R}^n$. The function $f : A \to \mathbf{R}$ is *convex* if for any $a, b \in A$ and $t \in [0, 1]$,

$$f(ta + (1 - t)b) \leq tf(a) + (1 - t)f(b).$$

It is *concave* if $-f$ is convex.

Convex Multivariate Functions

Real Convex Functions

Real Convex Sets

N-Dimensional Space — Real Functions

Distance

Space Distance

Plane Distance

Interval Length

Real Space — Intervals

Real Plane — Real Line

Real Order — Integral Line

Geometry — Rational Order — Real Numbers

Rational Numbers

Lists — Integer Arithmetic

Set Numbers — Natural Multiplicative Identity — Natural Additive Identity — Integer Products

Finite Sets — Identity Elements — Arithmetic — Natural Powers

Direct Products — Equivalent Sets — Integer Order — Integer Sums — Natural Products

Family Unions and Intersections — Function Inverses — Comparisons — Integer Numbers — Natural Sums

Families — Function Composites — Function Images — Operations — Orders

Functions — Converse Relations — Equivalence Relations — Natural Order — Recursion Theorem

Relations — Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Set Dualities — Unordered Triples — Successor Sets — Intersection of Empty Set — Partitions — Set Symmetric Differences

Ordered Pairs — Pair Unions — Set Intersections — Set Complements

Pair Intersections — Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects

4