Ordered Pairs

1 Why

We speak of objects composed of elements from different sets.

2 Definition

Let A and B be non-empty sets. Let $a \in A$ and $b \in B$. An **ordered pair** is the set $\{\{a\}, \{a, b\}\}$. The **cartesian product** of A and B is the set of all ordered pairs. The **first element** of $\{\{a\}, \{a, b\}\}$ is a and the **second element** is b.

We observe that two pairs are equal if they have equal elements in the same order. If $A \neq B$, the ordering causes the cartesian product of A and B to differ from the cartesian product of B with A. If $A = B$, however, the symmetry holds.

2.1 Notation

We denote the ordered pair $\{\{a\}, \{a, b\}\}$ by (a, b). We denote the cartesian product of A with B by $A \times B$, read aloud as "A cross B." In this notation, if $A \neq B$, then $A \times B \neq B \times A$.