## Why

We are constantly thinking of the integers as the endpoints of equal length segments of a line.

## Discussion

We commonly associate elements of the integers with the endpoints of equal-length segments of a real line. Take segment $S_0$ of $L$ with endpoints $p$ and $q$. Associate the point $p$ with 0. Associate the point $q$ with 1. Take a segment $S_1$ of equal length, non-overlapping with $S_0$, who shares the endpoint $q$. Associate the second endpoint of this segment 2. Continue with the rest.[1] We call the line so formed the *integral line* of unit $S_0$.

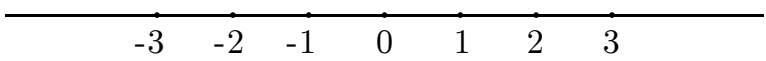



Figure 1: The integral segments.

## Integral Distance

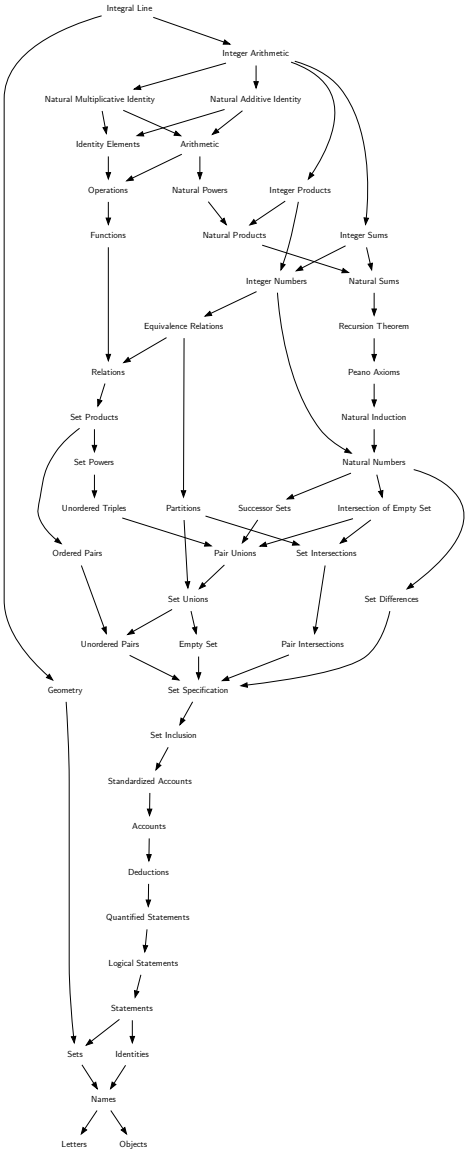
Let $f : \mathbf{Z} \to \mathbf{Z}$ be defined by $f(a, b) = a - b$ if $a > b$ and $f(a, b) = b - a$ if $b > a$. Notice that $f$ is symmetric: $f(a, b) = f(b, a)$. The (geometric) interpretation of $f$ is the distance between the points associated with the two integers $a, b \in \mathbf{Z}$

[1] Future editions will expand.

in some integral line. We call $f$ the *integral distance*. Notice that $f(a, b) > 0$ for all $a, b \in \mathbf{Z}$.

### Notation

We denote the distance between $a, b \in \mathbf{Z}$ by $|a - b|$.


Integral Line
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Identity Elements
Arithmetic
Operations
Natural Powers
Integer Products
Functions
Natural Products
Integer Sums
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects