Why

We want to quantify the error of compressing a real-valued random variable.

Definition

Let $\mathcal{X}$ be a finite set and $q : \mathbf{R} \to \mathcal{X}$ a quantization (see Quantizations) of $\mathbf{R}$. Also, fix a probability space $(\Omega, \mathcal{A}, \mathbf{P})$ and a random variable $x : \Omega \to \mathbf{R}$.

The *compression* $\hat{x} : \Omega \to \mathcal{X}$ of x under q is $q \circ x$. A *distortion function* for x and $\hat{x}$ is a function

$$d : (\Omega \to \mathbf{R}) \times (\Omega \to \mathcal{X}) \to \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

Examples

The *expected mean-squared-error distortion* d_{mse} between x and $\hat{x}$ is

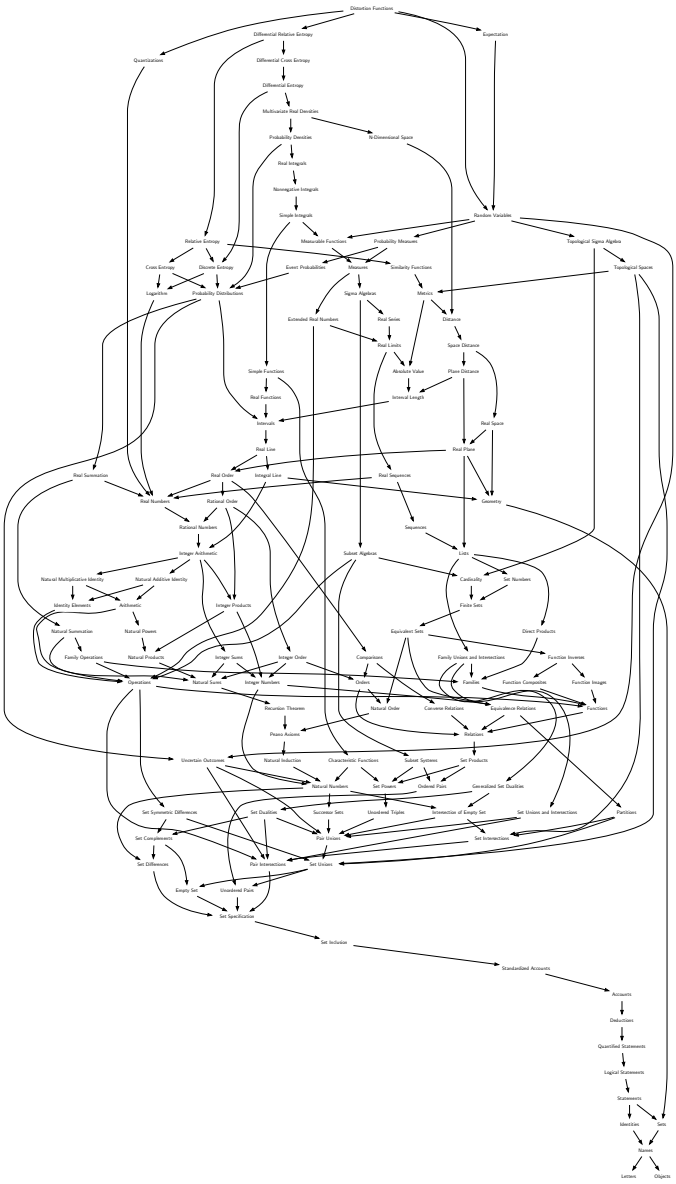
$$d_{\text{mse}}(x, \hat{x}) = \mathbf{E}[(x - \hat{x})^2]$$

The *Kulback-Liebler distortion* d_{kld} defined by

$$d_{\text{kld}}(x, \hat{x}) = \mathbf{E}[d_{\text{kl}}(\mathbf{P}(y \in \cdot \mid x, \hat{x}) \mid \mathbf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.[1]

[1] Future editions will clarify this sentence.


Distortion Functions
Differential Relative Entropy
Expectation
Quantizations
Differential Cross Entropy
Differential Entropy
Multivariate Real Densities
Probability Densities
N-Dimensional Space
Real Integrals
Nonnegative Integrals
Simple Integrals
Random Variables
Relative Entropy
Measurable Functions
Probability Measures
Topological Sigma Algebra
Cross Entropy
Discrete Entropy
Event Probabilities
Measures
Similarity Functions
Topological Spaces
Logarithms
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Sigma Algebras
Metrics
Extended Real Numbers
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Real Limits
Space Distance
Absolute Value
Plane Distance
Simple Functions
Interval Length
Real Functions
Real Space
Intervals
Real Line
Real Plane
Real Summation
Real Order
Integral Line
Real Sequences
Real Numbers
Rational Order
Geometry
Rational Numbers
Sequences
Integer Arithmetic
Subset Algebras
Lists
Natural Multiplicative Identity
Natural Additive Identity
Cardinality
Set Numbers
Identity Elements
Arithmetic
Integer Products
Finite Sets
Natural Summation
Natural Powers
Equivalent Sets
Direct Products
Family Operations
Natural Products
Integer Sums
Integer Order
Comparisons
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Natural Order
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Uncertain Outcomes
Natural Induction
Characteristic Functions
Subset Systems
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Generalized Set Dualities
Set Symmetric Differences
Set Dualities
Successor Sets
Unordered Triples
Intersection of Family Set
Set Unions and Intersections
Partitions
Set Complements
Pair Unions
Set Intersections
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects