Why

We can generalize the real general linear groups to vector spaces over $\mathbf{C}$.

Definition

Suppose V is a vector space over the field $\mathbf{C}$ of complex numbers. The set of isomorphisms of V onto itself is a group, called the *general linear group*, under the operation of composition. If V has dimension n, then the general linear group can be identified with the invertible $n \times n$ complex matrices in the usual way.

Notation

We denote by $GL(V)$ the general linear group of isomorphisms of V onto itself. If $f \in GL(V)$, and V has a finite basis $e_1, \ldots, e_n \in V$, then f has corresponding matrix representation $A \in \mathbf{C}^{n \times n}$ given by

$$A = \begin{bmatrix} f(e_1) & \cdots & f(e_n) \end{bmatrix}.$$