Why

We want to multiply real numbers.[1]

Definition

The *real product* of two real numbers R and S is defined

1. if R or S is $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$, then the $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$
2. otherwise,
   (a) if R or S is $0_{\mathbf{R}}$, then $0_{\mathbf{R}}$.
   (b) if $R, S \neq 0_{\mathbf{R}}$ and $0_{\mathbf{R}} \in R, S$, let T be
   $$\{t \in \mathbf{Q} \mid r \in R, s \in S, r, s \geq 0_{\mathbf{Q}}, t = r \cdot s\}$$
   then $T \cup \{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\}$[2]
   (c) If $R, S \neq 0_{\mathbf{R}}$, $0_{\mathbf{R}} \in R$ and $0_{\mathbf{R}} \notin S$, then the additive inverse of the product of $-R$ with S.
   (d) If $R, S \neq 0_{\mathbf{R}}$, $0_{\mathbf{R}} \notin R$ and $0_{\mathbf{R}} \in S$, then the additive inverse of the product of R with $-S$.
   (e) If $R, S \neq 0_{\mathbf{R}}$, and $0_{\mathbf{R}} \notin R, S$, then the product of $-R$ with $-S$.

Notation

We denote the product of two real numbers x and y by $x \cdot y$.

Properties

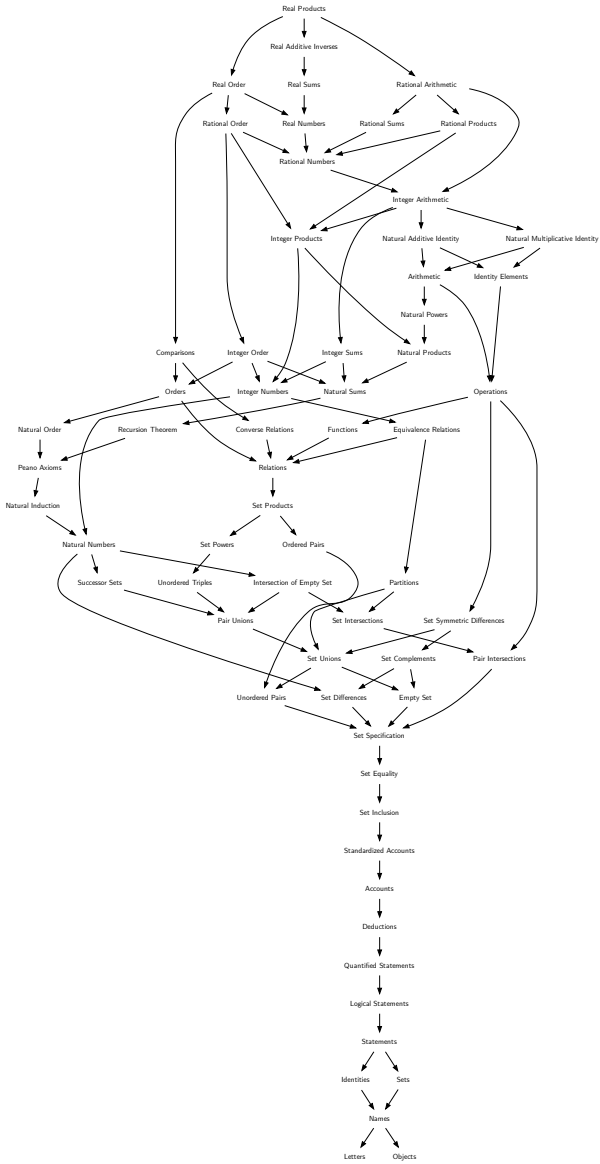
Proposition 1 (Associative)**.** $x + (y + z) = (x + y) + z$

Proposition 2 (Commutative)**.** $x + y = y + x$

Proposition 3 (Identity)**.** *The set of all rationals less than* $1_{\mathbf{Q}}$ *is the multiplicative identity.*

[1] Future editions will expand.
[2] We use $\geq$ in the usual way, it will be defined earlier in future editions.

We denote the the multiplicative identity by $1_{\mathbf{R}}$. When it is clear from context, we call $1_{\mathbf{R}}$ "one".


Real Products
Real Additive Inverses
Real Order
Real Sums
Rational Arithmetic
Rational Order
Real Numbers
Rational Sums
Rational Products
Rational Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Comparisons
Integer Order
Integer Sums
Natural Products
Orders
Integer Numbers
Natural Sums
Operations
Natural Order
Recursion Theorem
Converse Relations
Functions
Equivalence Relations
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Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects