Why

We want language and notation involving order.[1]

Comparisons

A *comparison* is a statement (see Statements) involving a partial (which may or may not be total) order.

Notation

Let A be a set. We tend to denote an arbitrary partial order on A by $\preceq$. So $(A, \preceq)$ is a partially ordered set.

As usual (see Relations), we write $a \preceq b$ to mean $(a, b) \in A$. Alternatively, we write $b \succeq a$ to mean $a \preceq b$. In other words, $\succeq$ is the inverse relation (see Converse Relations) of $\preceq$.

Predecessors and successors

If $a \preceq b$ and $a \neq b$, we write $a \prec b$ and say that a *precedes* b. In this case we call a the *predecessor* of b. Alternatively, under the same conditions, we write $b \succ a$ and we say that b *succeeds* a. In this case we call b the *successor* of a.

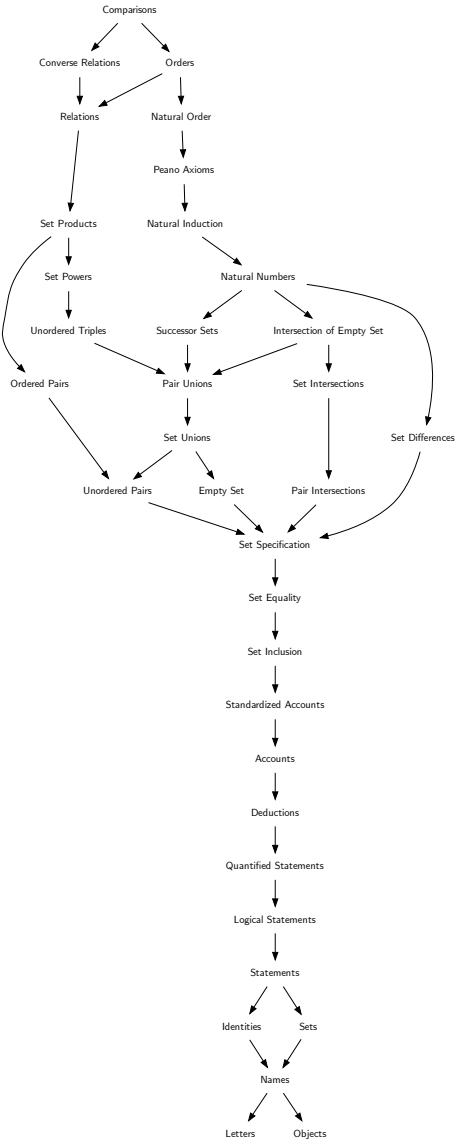
Induced partial orders

Of course, the object we have defined and denoted by $\prec$ is a relation on A. It satisfies (i) for no elements x and y do $x \prec y$ and $y \prec x$ hold simultaneously and (ii) if $x \prec y$ and $y \prec z$, then $x \prec z$ (i.e., $\prec$ is transitive). It is worthwhile to observe that if S is a relation satisfying (i) and (ii), then the relation R defined to mean $(a, b) \in S$ or $a = b$ is a partial order on A.

[1] In the present edition, this sheet can be thought of as an extended notation section for Orders.

Strict and weak relations

This connection between $\preceq$ and $\prec$ can be generalized. The *strict relation* corresponding to a relation R on a set A is the relation S on A defined by $(a, b) \in S$ if $(a, b) \in R$ and $a \neq b$. The *weak relation* corresponding to a relation S' on a set A is the relation R' defined by $(a, b) \in R'$ if $(a, b) \in S'$ or $a = b$. For this reason, a relation is said to *partially order* a set if it is a partial order or if its corresponding weak relation is one.


Comparisons
Converse Relations
Orders
Relations
Natural Order
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
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Standardized Accounts
Accounts
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Quantified Statements
Logical Statements
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Identities
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