

EVENT PROBABILITIES

Why

If we have some outcomes and a probability mass function, we can construct a function which assigns probabilities to events.

Definition

The probability of an event is the sum of the probabilities of the outcomes in the event. The event probability function is the correspondence assigning events to their probabilities.

Notation

Let A be a set of outcomes and p a probability mass function on A. Let $B \subset A$ be an event. Let $\mathbf{P}: 2^A \to \mathbf{R}$ be the event probability function, which is defined by

$$\mathbf{P}(B) = \sum_{b \in B} p(b).$$

The event probability function \mathbf{P} depends on the set of outcomes A and the probability mass function p. Sometimes we use notation indicating this dependence and will denote the event probability function by $\mathbf{P}_{A,p}$.

Properties

Proposition 1. Let P be the event probability function of a probability mass function p on a set of outcomes A. then

1.
$$\mathbf{P}(B) \geq 0$$
 for all $B \subset A$

- 2. P(A) = 1
- 3. $\mathbf{P}(B \cup C) = \mathbf{P}(B) + \mathbf{P}(C)$ for $B, C \subset A$ and $B \cap C = \emptyset$

