



REAL FUNCTIONS

Why

We name functions whose domain is the real numbers.

Definition

A *real function* is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

Notation

Let A be a set. Let $f : A \rightarrow \mathbf{R}$. f is a real function. If $A = \mathbf{R}$, then $f \in \mathbf{R} \rightarrow \mathbf{R}$. To speak of functions defined on intervals, let $a, b \in \mathbf{R}$. Let $g : [a, b] \rightarrow \mathbf{R}$. Then g is a real function defined on a closed interval. Let $h : (a, b) \rightarrow \mathbf{R}$. Then h is a real function defined on an open interval.

We regularly declare the interval and the function at once. For example, “let $f : [a, b] \rightarrow \mathbf{R}$ ” is understood to mean “let a and b be real numbers with $a < b$, let $[a, b]$ be the closed interval with them as endpoints, and let f be a real-valued function whose domain is this interval”. We read the notation $f : [a, b] \rightarrow \mathbf{R}$ aloud as “ f from closed a b to \mathbf{R} .” We use $f : (a, b) \rightarrow \mathbf{R}$ similarly (read aloud “ f from open a b to \mathbf{R} ”).

Examples

Example 1. Let $c \in \mathbf{R}$. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be such that $f(x) = c$ for every $x \in \mathbf{R}$. f is a real function.

Example 2.

Let $f : \mathbf{R} \rightarrow \mathbf{R}$ with $f(x) = 2x^2 + 1$ for all $x \in \mathbf{R}$. f is a real function.

Example 3. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ with

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{otherwise.} \end{cases}$$

f is a real function.

