Why

Matrices with elements in a ring form a ring.

Definition

Suppose $(R, +, \cdot)$ is a ring. Given $A, B \in R^{n \times n}$, define the binary operation $\bar{+} : R^{n \times n} \times R^{n \times n} \to R^{n \times}$ by

$$[A \bar{+} B]_{ij} = A_{ij} + B_{ij}$$

and define the binary operation $\bar{\cdot} : R^{n \times n} \times R^{n \times n} \to R^{n \times n}$ by

$$[A \bar{\cdot} B]_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$$

Both of these definitions are similar to the case with real matrices. With these operations so defined, the object $(R^{n \times n}, \bar{+}, \bar{\cdot})$ is a ring. In other words, the set of $n \times n$ matrices whose elements are in some ring R is itself a ring, with the usual operations of addition and multiplication of matrices.

The additive identity of the ring is the matrix $0 \in R^{n \times n}$ for which $0_{ij} = 0 \in R$. The multiplicative identity the matrix I for which $I_{ii} = 1 \in R$ for $i = 1, \ldots, n$ and $I_{ij} = 0 \in R$ for $i \neq j = 1, \ldots, n$. As seen with real-valued matrices, multiplication on $R^{n \times n}$ need not be commutative even if R is.

Exercise 1. *Show that $R^{n \times n}$ is not a division ring when $n > 1$.*