Why

Lots of things are (approximately) linear.[1]

Definition

A transformation is *linear* if the result of a linear combination of the two vectors is the linear combination of the results of the vectors (using the same coefficients). The transformation is linear *with respect to* the field of the two vector spaces.

We use the term transformation (Transformations) for emphasis and reminder that the function is defined on a vector space. Of course, $\mathbf{R}$ is a vector space and so a function $f : \mathbf{R} \to \mathbf{R}$ may be linear. It is, therefore, common to speak of *linear functions*.

Often authors will use the word *operator* for linear functions. It seems, generally, that this term is commonly reserved for the case in which the vector space discussed is a function space (or, at least, infinite dimensional).

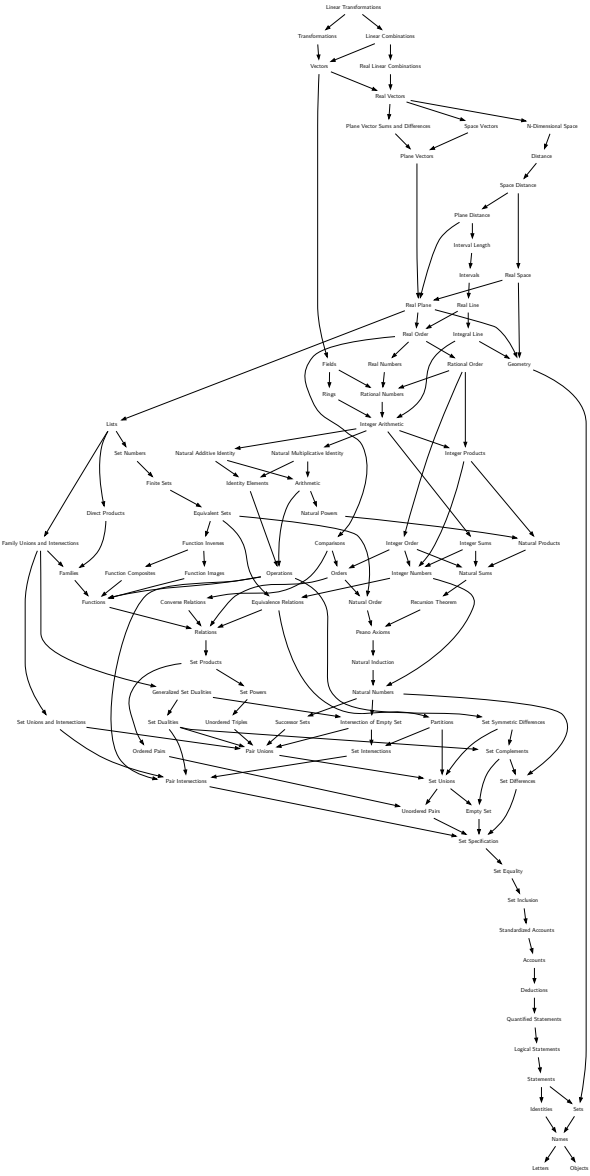
Notation

Let (V_1, F) and (V_2, F) be two vector spaces over the same field. Let $f : V_1 \to V_2$. f is linear means

$$f(au + bv) = af(u) + bf(v)$$

for all $a, b \in F$ and $u, v \in V_1$.

[1] Future editions will expand on this why. In particular, the intuition of proportionality.


Linear Transformations
Transformations
Linear Combinations
Vectors
Real Linear Combinations
Real Vectors
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Real Order
Integral Line
Fields
Real Numbers
Rational Order
Geometry
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Compositions
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
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Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects