

# TOPOLOGICAL SPACES

# Why

We want to generalize the notion of continuity.

### **Definition**

A topological space is a base set and a set distinguished subsets of this set for which: (1) the empty set base set are distinguished (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the set of distinguished subsets the topology and we call its members the open sets.

#### Notation

Let X be a non-empty set. For the set of distinguished sets, we tend to use  $\mathcal{T}$ , a mnemonic for topology, read aloud as "script T". We tend to denote elements of  $\mathcal{T}$  by O, a mnemonic for open. We denote the topological space with base set X and topology  $\mathcal{T}$  by  $(X, \mathcal{T})$ . We denote the properties satisfied by elements of  $\mathcal{T}$ :

1. 
$$X, \emptyset \in \mathcal{T}$$

2. 
$$\{O_i\}_{i=1}^n \subset \mathcal{T} \longrightarrow \bigcap_{i=1}^n O_i \in \mathcal{T}$$

3. 
$$\{O_{\alpha}\}_{{\alpha}\in I}\subset\mathcal{T}\longrightarrow \cup_{{\alpha}\in I}\in\mathcal{T}$$

### **Examples**

 ${f R}$  with the open intervals as the open sets is a topological space.

