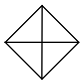


Why

We count in order.[1]

Defining Result

We say that two natural numbers m and n are *comparable* if $m \in n$ or $m = n$ or $n \in m$.

Proposition 1. *Any two natural numbers are comparable.*[2]

In fact, more is true.

Proposition 2. *For any two natural numbers, exactly one of $m \in n$, $m = n$ and $n \in m$ is true.*[3]

Proposition 3. $m \in n \longleftrightarrow m \subset n$.

If $m \in n$, then we say that m is *less than* n. We also say in this case that m is *smaller than* n. If we know that $m = n$ or m is less than n, we say that m is *less than or equal to* n.

Notation

If m is less than n we write $m < n$, read aloud "m less than n." If m is less than or equal to n, we write $m \leqq n$, read alout "m less than or equal to n."

[1] Future editions will expand.

[2] Future editions will include an account.

[3] Use the fact that no natural number is a subset of itself. Future editions will expand this account. See *Peano Axioms*).

Properties

Notice that $<$ and $\leqq$ are relations on ω (see *Relations*).[4]

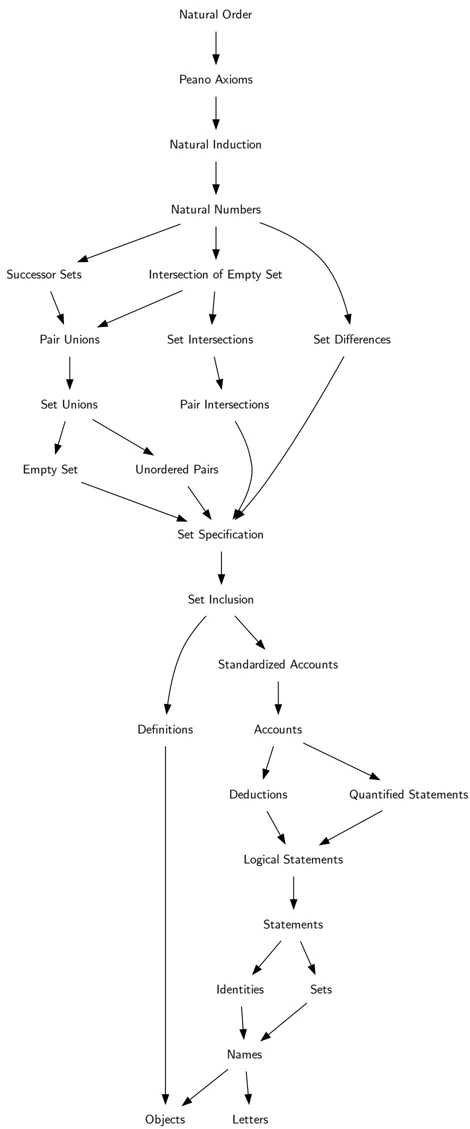
Proposition 4 (Reflexivity). *$\leqq$ is reflexive, but $<$ is not.*

Proposition 5 (Symmetry). *Both $\leqq$ and $<$ are not symmetric.*

Proposition 6 (Transitivity). *Both $\leqq$ and $<$ are transitive.*

Proposition 7 (Antisymmetry). *If $m \leqq n$ and $n \leqq n$, then $m = n$.*

[4]Proofs of the following propositions will appear in future editions.


Natural Order
Peano Axioms
Natural Induction
Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters