Why

We generalize the notion of sequence to index sets beyond the naturals.

Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go "further out."

To elaborate on property (b): if handed two natural numbers m and n, we can always find another, for example $\max\{m, n\} + 1$, larger than m and n. We might think of larger as "further out" from the first natural number: 1.

Combining these to observations, we define a directed set:

Definition 1. *A directed set is a set D with a partial order $\preceq$ satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.*

Definition 2. *A net is a function on a directed set.*

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \to A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.

Nets

Sequences

Partial Orders   Families

Functions

Relations

Cartesian Products

Set Powers   Natural Numbers

Unordered Triples   Successor Sets   Intersection of Empty Set

Ordered Pairs   Pair Unions   Set Intersections

Set Unions   Set Differences

Unordered Pairs   Empty Set   Pair Intersections

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects