

### **TOPOLOGIES**

# Why

We want to generalize the notion of continuity.

### **Definition**

Let X be a set. A topology is a set of subsets of X for which (1) the empty set base set are distinguished (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the elements of the topology the  $open \ sets$ .

A topological space is an ordered pair: a base set and a set distinguished subsets of the base set which are a topology.

### Notation

Let X be a non-empty set. For the set of distinguished sets, we tend to use  $\mathcal{T}$ , a mnemonic for topology, read aloud as "script T". We tend to denote elements of  $\mathcal{T}$  by O, a mnemonic for open. We denote the topological space with base set X and topology  $\mathcal{T}$  by  $(X, \mathcal{T})$ . We denote the properties satisfied by elements of  $\mathcal{T}$ :

- 1.  $X, \emptyset \in \mathcal{T}$
- 2. if  $O_1, \ldots, O_n \in \mathcal{T}$ , then  $\bigcap_{i=1}^n O_i \in \mathcal{T}$
- 3. if  $O_{\alpha} \in \mathcal{T}$  for all  $\alpha \in I$ , then  $\bigcup_{\alpha \in I} \in \mathcal{T}$

## **Examples**

 $\boldsymbol{\mathsf{R}}$  with the open intervals as the open sets is a topological space.

