



## Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

## Notation

Let  $(X, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $(Y, \mathcal{B})$  a measurable space. A random variable is a measurable function  $f : X \rightarrow Y$ .

Some authors tend to denote real-valued random variables by upper case Latin letters: for example,  $X, Y, Z$ . They reserve lower case letters  $x, y, z$  for the elements of the codomains of these functions. In such cases, they often denote the set of outcomes as  $\Omega$ , which we have mentioned is a mnemonic for outcomes.

## Special notation for common cases

Some authors use notation for the probability of particular, common sets. Suppose  $(\Omega, \mathcal{A}, \mathbf{P})$  is a probability space with  $X : \Omega \rightarrow \mathbf{R}$  a real-valued random variable. Many authors use  $\mathbf{P}(X \in A)$  (or  $P(X \in A)$ , no bold) to denote

$$\mathbf{P}(X^{-1}(A)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\})$$

where  $A \in \mathcal{B}(\mathbf{R})$ , the Borel sigma algebra on  $\mathbf{R}$ . We will tend to use brackets in place of parentheses for clarity. So we will write  $\mathbf{P}[X \in A]$ .

Similar to the above, suppose  $Y : \Omega \rightarrow \mathbf{R}$  is a random variable and  $B \in \mathcal{B}(\mathbf{R})$ . Then we will use  $\mathbf{P}[X \in A, Y \in B]$  to denote

$$\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for  $n$  random variables  $X_1, \dots, X_n : \Omega \rightarrow \mathbf{R}$ , and Borel sets  $A_1, \dots, A_n$ , we will use  $\mathbf{P}[X_1 \in A_1, \dots, X_n \in A_n]$  to denote  $\mathbf{P}(\cap_{i=1}^n X_i^{-1}(A_i))$ .



