Why

What is an inner product if we take a vector space over the complex numbers.

Definition

An inner produce over a complex vector space is positive definite, Hermitian, and linear in the first argument.

Notation

Let $(V, \mathbf{C})$ be a complex vector space. Let $f : V \times V \to C$ be a function such that

1. $f(x, x) \geq 0$, $f(x, x) = 0 \Leftrightarrow x = 0$;

2. $f(x, y) = \overline{f(y, x)}$

3. $f(ax + by, z) = a(x, z) + b(y, z)$

Complex Inner Products

Real Inner Products

Real Inner Product

Space Inner Product — Real Norm — Vectors

Space Norm — Real Vector

N-Dimensional Space — Plane Inner Product — Plane Norm — Space Vectors — Plane Vector Sums and Differences — Complex Numbers

Distance — Right Triangle Sides Relation — Plane Vectors — Absolute Value

Space Distance — Real Square — Real Arithmetic

Plane Distance — Real Multiplicative Inverses

Interval Length

Real Space — Intervals — Square Number — Real Products

Real Plane — Real Line — Real Additive Inverses

Square — Integral Line — Real Order — Rational Multiplicative Inverses — Real Sums — Rational Arithmetic

Geometry — Rational Order — Real Numbers — Rational Products — Rational Sums — Fields

Rational Numbers — Rings

Integer Arithmetic

Lists — Set Numbers

Natural Additive Identity — Natural Multiplicative Identity — Integer Products

Direct Products — Finite Sets — Identity Elements — Arithmetic — Natural Powers

Inverse Elements — Equivalent Sets

Family Unions and Intersections — Function Inverses — Element Functions — Comparisons — Integer Order — Integer Sums — Natural Products

Families — Function Composites — Function Images — Operations — Orders — Integer Numbers — Natural Sums

Functions — Equivalence Relations — Converse Relations — Natural Order — Recursion Theorem

Relations — Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Partitions — Intersection of Empty Set — Set Dualities — Unordered Triples — Successor Sets — Set Symmetric Differences

Set Intersections — Ordered Pairs — Pair Unions — Set Complements

Pair Intersections — Set Union — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects