Prop. 1. *Let (V, F) be a complete inner product space. Let M be a closed subspace. then*

$$H = M \oplus M^{\perp}.$$

That is for all $x \in V$, there exists unique $z \in M$ and $q \in M^{\perp}$ such that $x = z + w$.

Complete Inner Product Decomposition

Orthogonal Complements

Inner Products

Real Inner Product

Real Norm — Vectors

Real Vectors — Plane Inner Product

Plane Norm — N-Dimensional Space — Space Vectors — Plane Vector Sums and Differences

Distance — Plane Vectors

Absolute Value — Space Distance

Plane Distance

Interval Length — Real Space

Intervals

Real Line — Real Plane

Integral Line — Real Order

Geometry — Rational Order — Real Numbers — Fields

Rational Numbers — Groups

Integer Arithmetic — Integer Additive Inverse — Lists

Integer Products — Natural Multiplicative Identity — Natural Additive Identity — Inverse Elements — Set Numbers

Arithmetic — Identity Elements — Element Functions — Finite Sets — Direct Products

Natural Powers — Integer Sums — Algebras — Equivalent Sets

Integer Order — Natural Products — Comparisons — Operations — Function Inverse — Family Unions and Intersections

Integer Numbers — Natural Sums — Order — Function Images — Function Composites — Families

Recursion Theorem — Natural Order — Equivalence Relations — Converse Relations — Functions

Peano Axioms — Relations

Natural Induction — Set Products

Natural Numbers — Set Powers — Generalized Set Dualities

Partitions — Successor Sets — Intersection of Empty Set — Unordered Triples — Set Dualities — Set Unions and Intersections

Set Intersections — Pair Unions — Ordered Pairs — Set Complements

Set Differences — Set Unions — Pair Intersections

Unordered Pairs — Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects

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