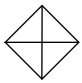


Why

We want to find matrix square roots.[1]

Definition

Let $A \in \mathsf{R}^{n \times n}$ be symmetric. A *lower upper triangular decomposition* A is a pair of matrices $(L, L^\top)$ where $L \in \mathsf{R}^{n \times n}$ is lower triangular, has nonnegative real diagonal entries, and satisfies

$$A = LL^\top.$$

Other terminology includes *lower upper triangular factorization*, *LU decomposition*, *LU factorization*. Define $R = L^\top$, then

$$A = R^\top R.$$

Basic properties

Proposition 1. *Let $A \in \mathsf{R}^{m \times m}$ be positive definite. Then there exists unique lower triangular matrix $L \in \mathsf{R}^{n \times n}$ so that*

$$A = LL^\top.$$

So, in the case that A is positive definite, a lower upper triangular decomposition exists and is unique. Therefore we refer to it as *the upper lower triangular decomposition* of A. It is also known (universally) as the *Cholesky decomposition* or *Cholesky factorization* of A.

[1]Future editions will expand.

Proposition 2. *If A is positive semidefinite, there exists a permutation matrix P for which there is a unique L so that*

$$P^\top AP = LL^\top.$$

This second proposition says that there is a unique Cholesky decomposition for a particular permutation (or pivoting) of A.

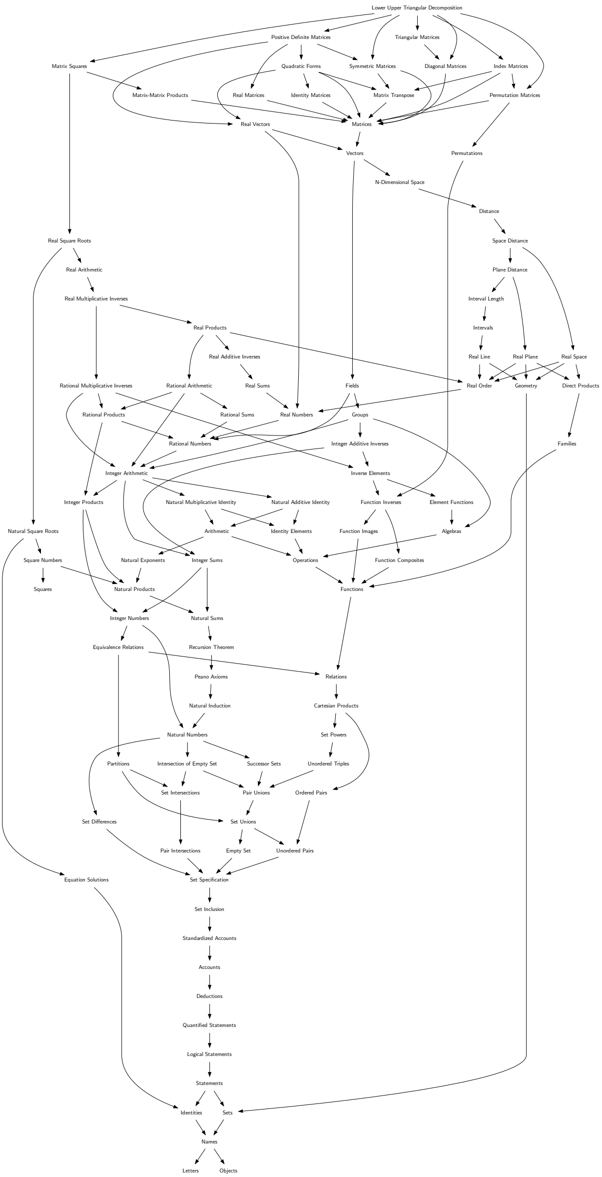
Unitriangular form

A *lower diagonal upper decomposition* (or *lower diagonal upper factorization*) of a matrix A a sequence $(L, D, L^\top)$ where $L \in \mathbf{R}^{n\times n}$ is unit lower triangular, $D \in \mathbf{R}^{n\times n}$ is diagonal with real nonnegative entries and

$$A = LDL^\top.$$

Other terminology includes *LDL decomposition*, *LDL factorization*, *LDU factorization*, *LDU decomposition*.

If $(L \in \mathbf{R}^{n\times n}, D \in \mathbf{R}^{n\times n}, L^\top)$ is a LDU decomposition of $A \in \mathbf{R}^{n\times n}$, then $(LD^{1/2}A = LDL^\top$ then $(\tilde{L}D^{1/2}, D^{1/2}L^\top)$ is a LU decomposition. Conversely, if $(B, B^\top)$ is a LU decomposition and S is the diagonal matrix satisfying $S_{ii} = B_{ii}$ for $i = 1, \dots, n$, then $(BS^{-1}, S^2, S^{-1}B^\top)$ is a LDU decomposition of A.


Lower Upper Triangular Decomposition
Positive Definite Matrices
Triangular Matrices
Matrix Squares
Quadratic Forms
Symmetric Matrices
Diagonal Matrices
Index Matrices
Matrix-Matrix Products
Real Matrices
Identity Matrices
Matrix Transpose
Permutation Matrices
Real Vectors
Matrices
Vectors
Permutations
N-Dimensional Space
Distance
Real Square Roots
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Additive Inverses
Real Line
Real Plane
Real Space
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Fields
Real Order
Geometry
Direct Products
Rational Products
Rational Sums
Real Numbers
Groups
Rational Numbers
Integer Additive Inverses
Families
Integer Arithmetic
Inverse Elements
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Function Inverses
Element Functions
Natural Square Roots
Arithmetic
Identity Elements
Function Images
Algebras
Square Numbers
Natural Exponents
Integer Sums
Operations
Function Composites
Squares
Natural Products
Functions
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Equation Solutions
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects