



RECURSION THEOREM

Why

It is natural to want to define a sequence by giving its first term and then giving its later terms as functions of its earlier ones. In other words, we want to define sequences inductively.¹

Main Result

The following is often referred to as the *recursion theorem*.

Proposition 1 (Recursion Theorem²). *Let X be a set, let $a \in X$ and let $f : X \rightarrow X$. There exists a unique function u so that $u(0) = a$ and $u(\succ(n)) = f(u(n))$.³*

When one uses the recursion theorem to assert the existence of a function with the desired properties, it is called *definition by induction*.

¹Future editions will expand on this. We are really headed toward natural addition, multiplication and exponentiation.

²Future editions will likely change this name.

³The account is somewhat straightforward, given a good understanding of the results of *Peano Axioms*. The full account will appear in future editions.

