



## EMPTY SET

### Why

Can a set have no elements?

### Definition

Sure. A set exists by the principle of existence (see *Sets*); denote it by  $A$ . Specify elements (see *Set Specification*) of any set that exists using the universally false statement  $x \neq x$ . We denote that set by  $\{x \in A \mid x \neq x\}$ . It has no elements. In other words,  $(\forall x)(x \notin A)$ . The principle of extension (see *Set Equality*) says that the set obtained is unique (contradiction).<sup>1</sup>

**Definition 1** (Empty Set). We call the unique set with no elements *the empty set*.

### Notation

We denote the empty set by  $\emptyset$ . In other words, in all future accounts (see *Accounts*), there are two implicit lines. First, “name  $\emptyset$ ” and second “have  $(\forall x)(x \notin \emptyset)$ ”.

### Properties

It is immediate from our definition of the empty set and of the definition of inclusion (see *Set Inclusion*) that the empty set is included in every set (including itself).

**Proposition 1.**  $(\forall A)(\emptyset \subset A)$

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<sup>1</sup>This account will be expanded in the next edition.

*Proof.* Suppose toward contradiction that  $\emptyset \notin A$ . Then there exists  $y \in \emptyset$  such that  $y \notin A$ . But this is impossible, since  $(\forall x)(x \notin \emptyset)$ .  $\square$

