

Linear Functions

1 Why

Lots of things are (approximately) linear

2 Definition

A function between two vector spaces which share the same field is *linear* if the function applied to a linear combination of two vectors in the first space equals the linear combination of the results of the function (in the second place), using the same coefficients for the combination.

A linear function is always linear with respect to some field. The field is implicit, somewhat, in the definition but always present.

Linear functions are sometimes called operators

2.1 Notation

Let (V_1, F) and (V_2, F) be two vector spaces over the same field. Let $f: V_1 \to V_2$. We say that f is *linear* if

$$f(au + bv) = af(u) + bf(v)$$

for all $a, b \in F$ and $u, v \in V_1$.