Why

We want to visualize (nonsymmetric) relations.

Definition

A *directed graph* is a pair (V, E) in which V is a finite nonempty set and E is a subset of $V \times V$. In other words, E is a relation on V. We call the elements of V *vertices* and the elements of E *edges*.

For example, define V and E by

$$V = \{1, 2, 3, 4\} \quad \text{and} \quad E = \{(1, 2), (1, 3), (2, 4), (3, 4)\}$$

It is worth drawing this graph.

Edge and vertex terminology

Let $(v, w) \in E$. We say that (v, w) is an edge *from v to w*, and that it is an *outgoing edge* of v and an *incoming edge* of w. We call v a *parent* of w and we call w a child of v. We say that the edge (v, w) is *incident* to v and w.

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. The *indegree* of a vertex is number parents it has and the *outdegree* is the number of children it has; c.f. Undirected Graphs.

A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

Notation

Let $\mathrm{pa} : V \to \mathcal{P}(V)$ and $\mathrm{ch} : V \to \mathcal{P}(V)$ be the functions associating to each vertex its set of parents and set of children, respectively. As usual,

we denote the parents of vertex v by pa_v and the children by ch_v.

Self-loops

If x is a vertex, and (x, x) is an edge, we call such an edge a *self-loop*. Many authors exclude self-loops in their definition of directed graphs, in order to make the situation correspond more closely to that of undirected graphs.

Skeletons

The *skeleton* of the directed graph (V, E) is the undirected graph (V, F) where

$$F = \{\{v, w\} \subset V \mid (v, w) \in E \text{ or } (w, v) \in E\}.$$

In other words, the skeleton is an undirected graph whose vertex set is V and whose edges are all (unordered) pairs which appear as an ordered pair in the directed graph.

In the case that (V, E) is a directed graph and E is a symmetric relation, the skeleton of (V, E) is a natural undirected graph to associate with (V, E). An *orientation* of an undirected graph G is a directed graph whose skeleton is G.

An *oriented graph* is a directed graph without self-loops satisfying the property for any two vertices x and y, either (x, y) or (y, x) is an edge, but not both. An oriented graph can be obtained from an undirected graph by selecting an "orientation" of the undirected edges.

Directed Graphs

Undirected Graphs

Set Numbers

Finite Sets

Equivalent Sets

Natural Order

Function Inverses

Function Composites

Function Images

Equivalence Relations

Functions

Peano Axioms

Relations

Natural Induction

Ordered Pair Projections

Set Products

Natural Numbers

Set Powers

Intersection of Empty Set

Successor Sets

Partitions

Unordered Triples

Set Intersections

Pair Unions

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects