Definition

A function $f : \mathbf{R}^n \to \mathbf{R}^m$ is *linear* if

1. $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbf{R}^n$, and
2. $f(\alpha x) = \alpha f(x)$ for all $\alpha \in \mathbf{R}$ and $x \in \mathbf{R}^n$.

Equivalently, $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$ for all $\alpha, \beta \in \mathbf{R}$ and $x, y \in \mathbf{R}^n$. In this case, some authors say that *superposition* holds for f.


Multivariate Vector Linear Functions
Vector-Valued Multivariate Functions
Linear Functions
Real Vectors
Multivariate Functions
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Real Summation
Real Order
Integral Line
Natural Summation
Real Numbers
Rational Order
Geometry
Lists
Rational Numbers
Direct Products
Set Numbers
Natural Fractions
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Natural Arithmetic
Identity Elements
Family Unions and Intersections
Family Operations
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Powers
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Natural Products
Functions
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Unordered Triples
Set Dualities
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects