Why

Matrices with elements in a ring form a ring.

Definition

Suppose $(R, +, \cdot)$ is a ring. Given $A, B \in R^{n \times n}$, define the binary operation $\bar{+} : R^{n \times n} \times R^{n \times n} \to R^{n \times}$ by

$$[A \mathbin{\bar{+}} B]_{ij} = A_{ij} + B_{ij}$$

and define the binary operation $\bar{\cdot} : R^{n \times n} \times R^{n \times n} \to R^{n \times n}$ by

$$[A \mathbin{\bar{\cdot}} B]_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$$

Both of these definitions are similar to the case with real matrices. With these operations so defined, the object $(R^{n \times n}, \bar{+}, \bar{\cdot})$ is a ring. In other words, the set of $n \times n$ matrices whose elements are in some ring R is itself a ring, with the usual operations of addition and multiplication of matrices.

The additive identity of the ring is the matrix $0 \in R^{n \times n}$ for which $0_{ij} = 0 \in R$. The multiplicative identity the matrix I for which $I_{ii} = 1 \in R$ for $i = 1, \ldots, n$ and $I_{ij} = 0 \in R$ for $i \neq j = 1, \ldots, n$. As seen with real-valued matrices, multiplication on $R^{n \times n}$ need not be commutative even if R is.

Exercise 1. *Show that $R^{n \times n}$ is not a division ring when $n > 1$.*

Matrix Rings

Real Matrix-Matrix Products

Real Matrix-Vector Products

Vectors as Matrices   Real Linear Combinations   Real Matrices   Matrix Determinants

Real Inner Product   Real Norm   Real Vectors   Real Linear Equations   Arrays

Space Inner Product   Linear Functions

Space Norm

Plane Inner Product   Plane Norm   N-Dimensional Space   Space Vectors   Plane Vector Sums and Differences

Right Triangle Side Relation   Absolute Value   Distance   Plane Vector

Real Square   Space Distance

Real Arithmetic   Plane Distance

Real Multiplicative Inverse   Interval Length

Real Products   Real Space   Intervals

Square Numbers   Real Additive Inverse   Real Line   Real Plane   Real Summation

Square   Integral Lists   Rational Multiplicative Inverse   Rational Arithmetic   Real Sums   Real Order   Permutations   Natural Summation

Geometry   Rational Products   Rational Order   Rational Sums   Real Numbers   Groups   Lists

Rational Numbers   Rings   Set Numbers   Direct Products

Integer Arithmetic   Natural Fractions   Finite Sets

Natural Multiplication Identity   Natural Additive Identity   Equivalent Sets   Inverse Elements   Family Operations   Family Unions and Intersections

Integer Products   Identity Elements   Natural Arithmetic   Natural Equations   Comparisons   Element Functions   Function Inverses   Families

Integer Sums   Integer Order   Order   Operations   Equations   Function Images   Function Composites

Integer Numbers   Natural Powers   Natural Order   Converse Relations   Equivalence Relations   Functions

Natural Products   Relations

Natural Sums

Recursion Theorem

Peano Axioms   Ordered Pair Projections

Natural Induction   Set Products

Natural Numbers   Set Powers   Generalized Set Dualities

Set Symmetric Differences   Successor Sets   Unordered Triples   Intersection of Empty Set   Set Dualities   Partitions   Set Unions and Intersections

Set Complements   Pair Unions   Set Intersections   Ordered Pairs

Set Differences   Set Unions   Unordered Pairs   Pair Intersections

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects