Definition

Let $A \in \mathbf{S}^n$ (i.e., $A \in \mathbf{R}^{n \times n}$ and symmetric). A is *positive definite* if, for all $x \neq 0 \in \mathbf{R}^d$,

$$x^\top A x > 0.$$

A is *positive semidefinite* (or *nonnegative definite*) if, for all $x \in \mathbf{R}^d$

$$x^\top A x \geq 0.$$

Notation

We denote the set of positive definite d by d matrices by $\mathbf{S}^d_{++}$. We denote the set of positive semidefinite d by d matrices by $\mathbf{S}^d_{+}$.