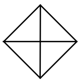


Why

It is natural to embed a dataset.

Definition

Let $(x : \Omega \to \mathsf{R}^d, A \in \mathsf{R}^{n \times d}, e : \Omega \to \mathsf{R}^n)$ be a probabilistic linear model over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Let $\phi : \mathsf{R}^d \to \mathsf{R}^{d'}$ be a feature embedding. Then (x, A, e, ϕ) is an *embedded probabilistic linear model*. We are modeling the function $h : \Omega \to (\mathsf{R}^d \to \mathsf{R})$ as linear in the features

$$h_\omega(a) = \phi(a)^\top x(\omega).$$

Correspondence to linear model

Denote the data matrix of the embedded feature vectors by $\phi(A)$. Then, of course, the embedded linear model (x, A, e, ϕ) corresponds to the linear model $(x, \phi(A), e)$.

Normal case

In the normal case, the parameter posterior $g_{x|y}(\cdot, \gamma)$ is a normal density with mean

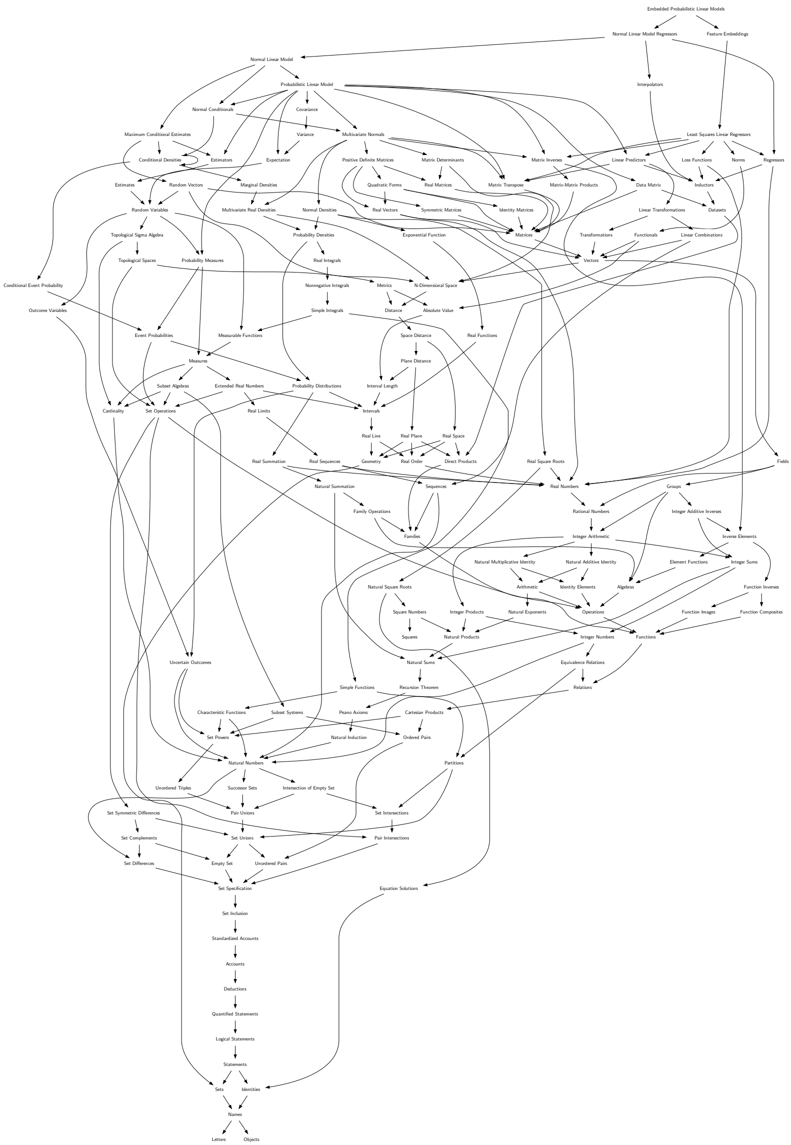
$$\Sigma_x \phi(A)^\top \left(\phi(A) \Sigma_x \phi(A)^\top + \Sigma_e\right)^{-1} \gamma$$

and covariance

$$\left(\Sigma_x{}^{-1} + \phi(A)^\top \Sigma_e{}^{-1} \phi(A)\right)^{-1}.$$

The predictive density for a point $a \in \mathsf{R}^d$ is normal with mean

$$(phi(a)^\top \Sigma_x \phi(A)^\top + \Sigma_{fe}) \left(\phi(a)^\top \Sigma_x a + \Sigma_f\right)^{-1} \gamma.$$


Embedded Probabilistic Linear Models
Normal Linear Model Regressors
Feature Embeddings
Normal Linear Model
Probabilistic Linear Model
Interpolators
Normal Conditionals
Covariance
Maximum Conditional Estimates
Variance
Multivariate Normals
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Conditional Densities
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Interval Length
Cardinality
Set Operations
Real Limits
Intervals
Real Line
Real Plane
Real Space
Real Summation
Real Sequences
Geometry
Real Order
Direct Products
Real Square Roots
Fields
Natural Summation
Sequences
Real Numbers
Groups
Family Operations
Rational Numbers
Integer Additive Inverses
Families
Integer Arithmetic
Inverse Elements
Natural Multiplicative Identity
Natural Additive Identity
Element Functions
Integer Sums
Natural Square Roots
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Function Composites
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Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Set Unions
Pair Intersections
Set Differences
Empty Set
Unordered Pairs
Set Specification
Equation Solutions
Set Inclusion
Standardized Accounts
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Logical Statements
Statements
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Identities
Names
Letters
Objects