



## BOUNDED LINEAR NORM

### Why

We can give the set of bounded linear functions between two norm spaces a norm.

### Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

### Notation

Let  $((V_1, F_1), \|\cdot\|_1)$  and  $((V_2, F_2), \|\cdot\|_2)$  be two norm spaces. Let  $f : V_1 \rightarrow V_2$  be linear and bounded. The norm of  $f$  is the smallest  $C$  so that

$$\|f(v)\|_2 \leq C\|v\|_1.$$

### Equivalent Formulation

**Prop. 1.** *Let  $((V_1, F_1), \|\cdot\|_1)$  and  $((V_2, F_2), \|\cdot\|_2)$  be two norm spaces. Let  $f : V_1 \rightarrow V_2$  bounded and linear. The norm of  $f$  is*

$$\sup_{\|x\|_1=1} \|f(x)\|_2.$$

