Why

We can cast various common probabilistic regression models into the probabilistic errors linear model by mentioning the input space and feature maps. This unifies our analysis.

Definition

A *line fit model* has input space R and output space R. We use a regression function $\phi : \mathsf{R} \to \mathsf{R}^2$ defined by $\phi(t) = (1, t)^\top$.

We think of $t \in T \subset \mathsf{R}$ as a "dose level" (T is an interval). Given dose levels $t_1, \ldots, t_\ell$ and repetitions $n_1, \ldots, n_\ell$ we obtain the design matrix. Here the regression function generates a line segment embedded in the plane R^2. We call the parameters the *intercept parameter* and *slope parameter*.

A *parabola fit model* has input space R and output space R. We use a regression function $\phi : \mathsf{R} \to \mathsf{R}^3$ defined by $\phi(t) = (1, t, t^2)^\top$. Here the regression space is a segment of a parabola embedded in space R^3 (since $t \in T$ an interval).

These two are instance of *polynomial fit models* of degree $d \geq 1$, in which the regression function becomes $\phi : \mathsf{R} \to \mathsf{R}^{d+1}$ defined by $\phi(t) = (1, t, t^2, \ldots, t^d)^\top$. In this case, the regression range $\phi(T)$ is a one-dimensional curve embedded in R^{d+1}. In cases in which it is clear that the input space is a single real variable t, a linear model for a line fit (parabola fit, polynomial fit of degree d) is called a *first-degree model* (*second-degree model*, *dth degree model*).

m-way models

We can generalize to *m-way dth degree polynomial fit models* in which the input space is $X \subset \mathbf{R}^m$ and the regression function $\phi : \mathbf{R}^m \to \mathbf{R}^k$ (k is $d+m$ choose d) is the vector of all monomials of degree d in m variables.

For example, a two-way third-degree model has a regression function

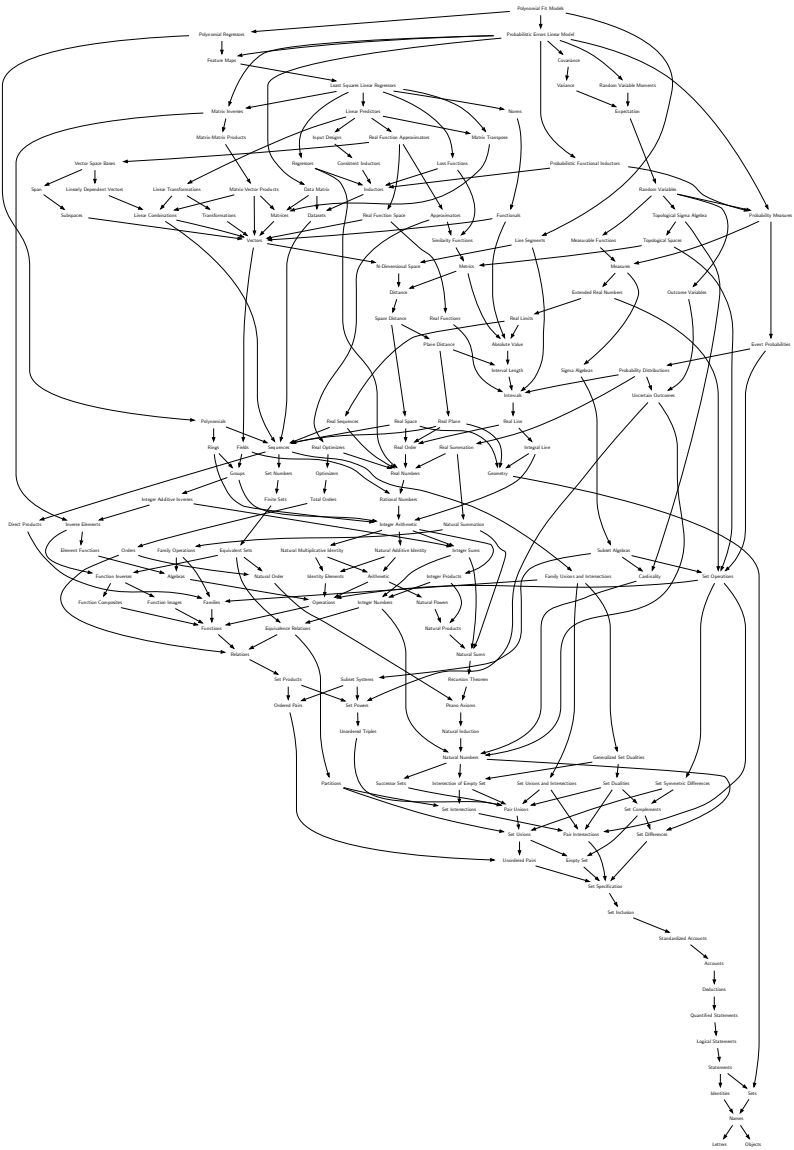
$$\phi(t_1, t_2) = \begin{bmatrix} 1 & t_1 & t_2 & t_1^2 & t_1 t_2 & t_2^2 & t_1^3 & t_1^2 t_2 & t_1 t_2^2 & t_2^3 \end{bmatrix}^\top.$$

Or consider a three way second-degree model with regression function

$$\phi(t_1, t_2, t_3) = \begin{bmatrix} 1 & t_1 & t_2 & t_3 & t_1^2 & t_1 t_2 & t_1 t_3 & t_2^2 & t_2 t_3 & t_3^2 \end{bmatrix}^\top.$$

Both models will result in parameter vectors of size ten. We call these models *saturated* because they have every possible dth degree power or cross product of variables. In generally, a m-way dth degree model has $d+m$ choose d mean parameters.

In contrast to saturated models we can talk about *nonsaturated* models. For example, a nonsaturated two-way second-degree model has $\phi : \mathbf{R}^2 \to \mathbf{R}^4$ where $\phi(t_1, t_2) = (1, t_1, t_2, t_1^2)^\top$.


Polynomial Fit Models
Polynomial Regression
Probabilistic Ecoon Linear Model
Feature Maps
Covariance
Variance
Random Variable Moments
Least Squares Linear Regression
Expectation
Matrix Inverses
Linear Predictors
Noises
Matrix-Matrix Products
Input Designs
Real Function Approximation
Matrix Transpose
Vector Space Bases
Regressors
Constraint Inductors
Loss Functions
Probabilistic Functional Inductors
Span
Linearly Dependent Vectors
Linear Transformations
Matrix-Vector Products
Data Matrix
Inductors
Random Variables
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Transformations
Matrices
Datasets
Real Function Space
Approximators
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Topological Sigma Algebra
Probability Measures
Vectors
Similarity Functions
Line Segments
Measurable Functions
Topological Spaces
N-Dimensional Space
Metrics
Measures
Distances
Extended Real Numbers
Outcome Variables
Space Distance
Real Functions
Real Limits
Plane Distance
Absolute Value
Event Probabilities
Interval Length
Sigma Algebras
Probability Distributions
Intervals
Uncertain Outcomes
Polynomials
Real Sequences
Real Spaces
Real Planes
Real Lines
Rings
Fields
Sequences
Real Optimizers
Real Order
Real Summation
Integral Line
Integers
Set Numbers
Operators
Real Numbers
Countable
Integer Additive Inverses
Finite Sets
Total Orders
Rational Numbers
Direct Products
Inverse Elements
Integer Arithmetic
Natural Summation
Element Functions
Orders
Family Operations
Equivalent Sets
Natural Multiplication Identity
Natural Additive Identity
Integer Sums
Subset Algebras
Function Inverses
Algebras
Natural Order
Identity Elements
Arithmetic
Integer Products
Family Unions and Intersections
Cardinality
Set Operations
Function Composition
Function Images
Families
Operations
Integer Numbers
Natural Powers
Functions
Equivalence Relations
Natural Products
Relations
Natural Sums
Set Products
Subset Systems
Recursion Theorem
Ordered Pairs
Set Powers
Peano Axioms
Unordered Triples
Natural Induction
Natural Numbers
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Set Union and Intersection
Set Duality
Set Symmetric Differences
Set Intersection
Pair Union
Set Complements
Set Union
Pair Intersection
Set Difference
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Name
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Objects