

## ROOTED TREE LINEAR CASCADES

## Why

It is natural to look for a class of structural equation models with favorable identifiability and properties.

## **Definition**

A d-dimensional rooted tree linear cascade is a sequence of four objects: a tree on  $\{1, \ldots, d\}$ , a vertex of the tree, a family of real numbers indexed by the edges of the tree, and a d-dimensional random vector whose covariance matrix is the identity matrix. The cascade is called "d-dimensional" because we associate it with a random vector (defined as a function of that in the form of its definition) whose codomain is  $\mathbf{R}^d$ .

The tree together with the vertex form a rooted tree. The graph associated with the rooted tree and the family of real numbers together form a weighted graph.

The idea is to use the weights and the tree structure to recursively define a random vector in terms of elements of the given random vector. Let C = (T, i, w, e) be a d-dimensional rooted tree linear cascade. So T is a tree on  $\{1, \ldots, d\}$ ,  $i \in \{1, \ldots, d\}$  and  $w : T \to \mathbf{R}$ , and  $e : \Omega \to \mathbf{R}^d$  for some probability space  $(A, \mathcal{A}, \mathbf{P})$ . The random vector associated with C is the random variable  $x : \Omega \to \mathbf{R}^d$  defined by

$$x_i = e_i$$
 and  $x_j = w_{\{pa j, j\}} x_{pa j} + e_j$  for  $j \neq i$ .

In other words,

$$e = Ax$$

where A is lower triangle and extremely sparse.

## Notation

Let  $(A, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $e: A \to \mathbf{R}^d$  be a random vector, let T be a tree on  $\{1, \ldots, d\}$  with  $a_{ij} = a_{ji}$  the weight on edge  $\{i, j\} \in T$ .

