Why

Since our predictions are often uncertain, we can use the language of probability distributions to characterize them.[1]

Definition

Denote the set of probability distributions on a set X by $\Delta(X)$.

A *probabilistic classifier* $G : A \to \Delta(B)$ is a function from inputs A to probability distributions over the classes B.

Given an input a, the *prediction* of G on a is a probability distribution $\hat{p}_a = G(a)$ on B.

Point classifier as probabilistic classifier

Given a point classifier $f : A \to B$, we can define a probabilistic classifier $G : A \to \Delta(B)$ corresponding to f by

$$\hat{p}_a(b) = \begin{cases} 1 & \text{if } f(a) = b \\ 0 & \text{otherwise.} \end{cases}$$

where $\hat{p}_a = G(a)$.

Probabilistic classifier from point classifier

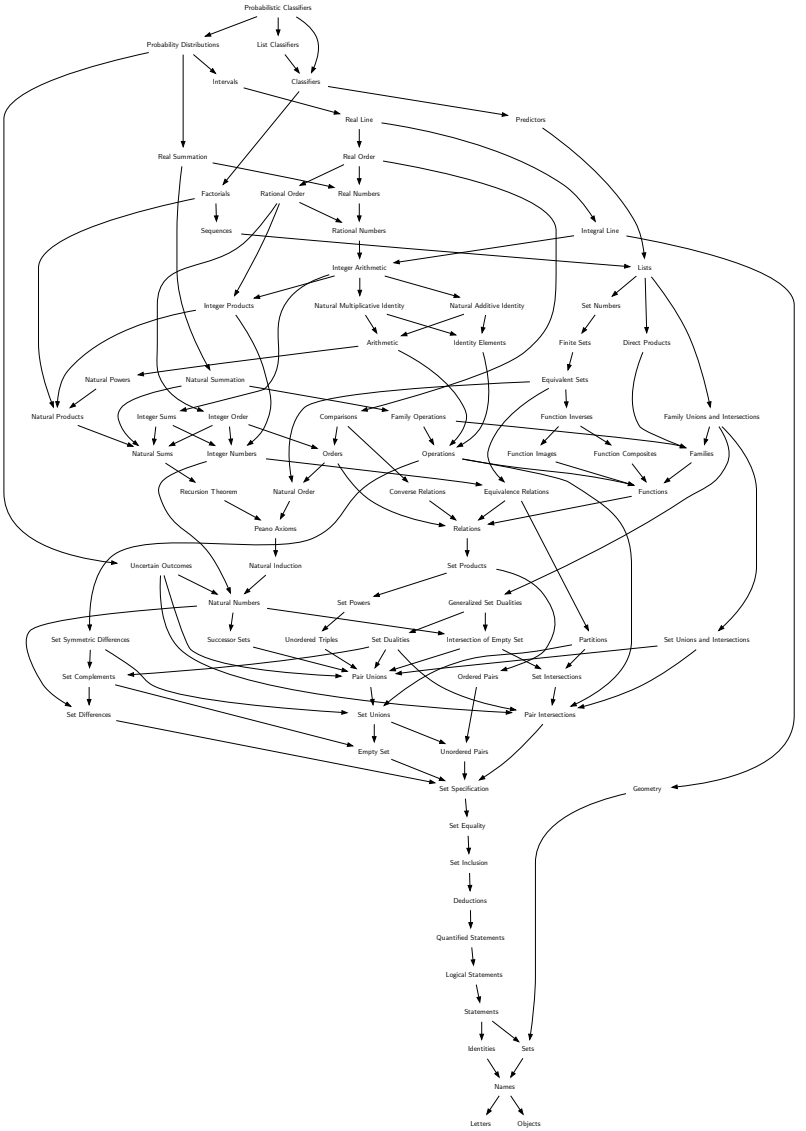
On the other hand, given probabilistic classifier $G : A \to \Delta(B)$, we can define a point classifier $f : A \to B$ by

$$f(a) = \operatorname*{argmax}_{b \in B} \hat{p}_a(v)$$

We call f the *maximum likelihood classifier* corresponding to G. If there are ties, we can order the (finite) set B arbitrarily, and break ties accordingly.

We can extend this idea, and define a list classifier by sorting the outputs by their probability, from largest to smallest.

[1] Future editions will improve this.


Probabilistic Classifiers
Probability Distributions
List Classifiers
Intervals
Classifiers
Real Line
Predictors
Real Summation
Real Order
Factorials
Rational Order
Real Numbers
Sequences
Rational Numbers
Integral Line
Integer Arithmetic
Lists
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Arithmetic
Identity Elements
Finite Sets
Direct Products
Natural Powers
Natural Summation
Equivalent Sets
Natural Products
Integer Sums
Integer Order
Comparisons
Family Operations
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Recursion Theorem
Natural Order
Converse Relations
Equivalence Relations
Functions
Peano Axioms
Relations
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Ordered Pairs
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects