Why

We have seen that the matrices are a vector space. Are they an inner product space?

Definition

The *matrix scalar product* of $A \in \mathbf{R}^{n\times k}$ and $B \in \mathbf{R}^{n\times k}$ is the following product

$$\sum_{i=1}^{n}\sum_{j=1}^{k} a_{ij}b_{ij}.$$

Using the matrix trace, we can denote this as $\mathbf{tr}\, A^\top B$. Some authors call this the *Euclidean matrix scalar product*.

Proposition 1. *The matrix scalar product is an inner product.*[1]

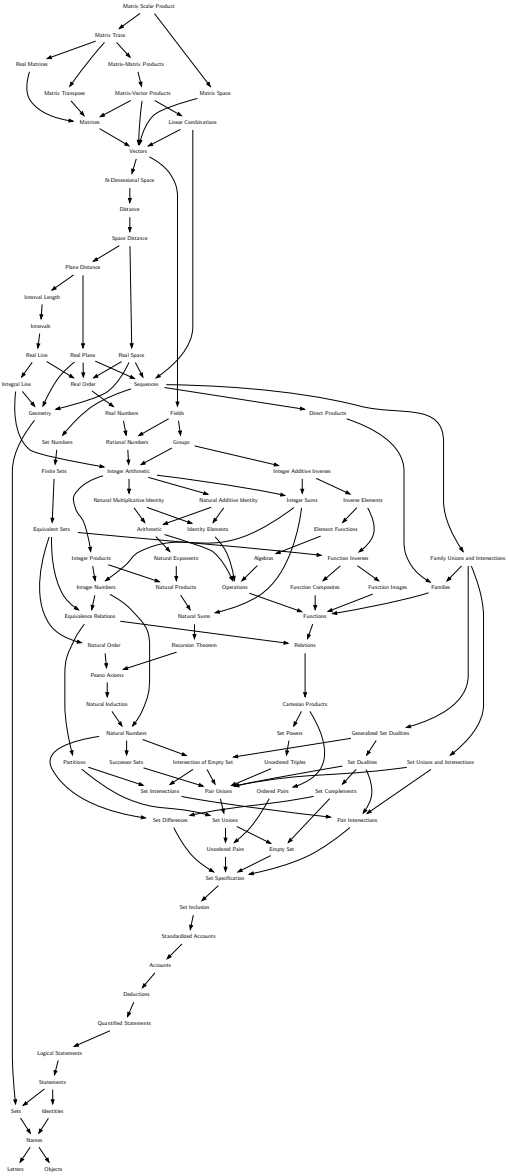
With this inner product, $\mathbf{R}^{n\times k}$ is a Euclidean vector space (see Inner Products) of dimension nk. For the case of $k = 1$, we recover a model[2] for the usual space $\mathbf{R}^n$.

Notation

We commonly denote the matrix inner product by $\langle A, B\rangle$.

[1] Future editions will provide an account.

[2] Future editions will define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Matric Transpose
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Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Sequences
Geometry
Real Numbers
Fields
Direct Products
Set Numbers
Rational Numbers
Groups
Finite Sets
Integer Arithmetic
Integer Additive Inverses
Natural Multiplicative Identity
Natural Additive Identity
Integer Sums
Inverse Elements
Equivalent Sets
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Integer Products
Natural Exponents
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Set Complements
Set Differences
Set Unions
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