Bounded Linear Norm

1 Why

We can give the set of bounded linear functions between two norm spaces a norm.

2 Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

2.1 Notation

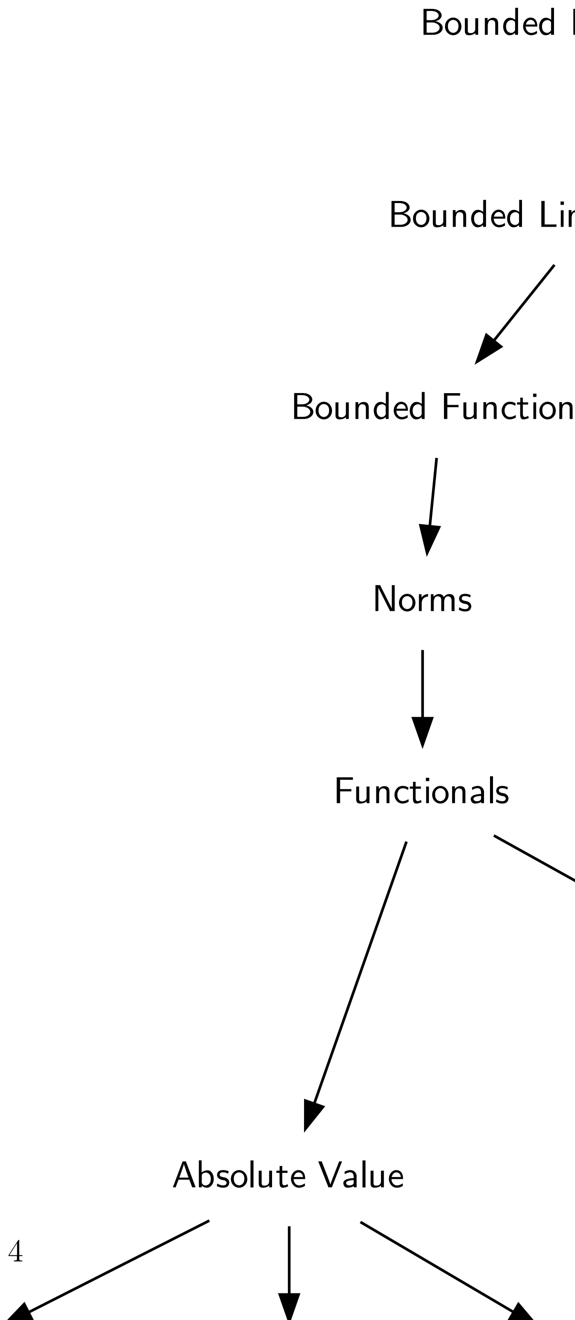
Let $((V_1, F_1), |\cdot|_1)$ and $((V_2, F_2), |\cdot|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ be linear and bounded. The norm of f is the smallest C so that

$$|f(v)|_2 \leq C|v|_1.$$

3 Equivalent Formulation

Proposition 1. *Let $((V_1, F_1), |\cdot|_1)$ and $((V_2, F_2), |\cdot|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ bounded and linear. The norm of f is*

$$\sup_{|x|_1 = 1} |f(x)|_2.$$


Bounded
Bounded Lin
Bounded Function
Norms
Functionals
Absolute Value