



## Why

Suppose I have three stones in my hands. If I have two in my left hand, how many are in my right hand?

## Discussion

Denote by  $a$  the number of stones in my left hand. Denote by  $b$  the number of stones in my right hand. Then the number of stones in both hands is  $a + b$ . Both  $a$  and  $b$  are natural numbers. In other words,  $a$  is 0 or 1 or 2 or... Likewise for  $b$ .

We express that I have two stones in my left hand by the equation  $a = 2$ . We express that I have five stones in total by the equation that  $a + b = 5$ . Because we have identified the names  $a$  and  $b$  with tangible objects in my hands, these equations are statements about tangible objects in my hands. But the *numbers* involved are intangible objects. In any case, we have two equations in two unknowns.

We express the question “how many stones do I have in my right hand?” by asking for a solution (see **Equations**) to the equation  $a + b = 5$ . We can start by trying natural numbers in order. Is  $(a, b) = (0, 0)$  a solution? Well,  $0 + 0 = 0$ , and  $0 \neq 5$ , so it is not. Is  $(a, b) = (1, 0)$  a solution? Well,  $1 + 0 = 1$ , and  $1 \neq 5$ , so it is not. Is  $(a, b) = (0, 1)$  a solution? Well,  $1 + 0 = 1$ , and  $1 \neq 5$ , so it is not. Likewise for  $(0, 2)$ ,  $(1, 2)$  and so on. Some people call this the process of *guess and check*.

Continuing this way we find that  $(3, 2)$  and  $(2, 3)$  is a solution. Indeed,  $3 + 2 = 2 + 3 = 5$ . We are, however, interested in solutions to both equations

$$a = 3$$

$$a + b = 5$$

Both  $(3, 2)$  and  $(2, 3)$  satisfy the second equation, but only  $(3, 2)$  satisfies both. Are there other solutions?<sup>1</sup>

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<sup>1</sup>Future editions will expand.



