



## FIELDS

### Why

We generalize the algebraic structure of addition and multiplication over the rationals.

### Definition

A *field* is two algebras over the same ground set with: (1) both algebras are commutative groups (2) the operation of the second algebra distributes over the operation of the first algebra.

We call the operation of the first algebra *field addition*. We call the operation of the second algebra *field multiplication*.

### Notation

We tend to denote an arbitrary field by  $\mathbf{F}$ , a mnemonic for “field.”

## 1 Examples

Of course,  $\mathbf{Q}$  with the usual addition (see Rational Sums) and multiplication (see Rational Products) and the inverse elements (see Rational Additive Inverse) and Rational Multiplicative Inverses) is a field.

**Proposition 1.** *The set of rational numbers with rational addition and multiplication is a field.*



