



## NATURAL ORDER

### Why

We count in order.<sup>1</sup>

### Defining Result

We say that two natural numbers  $m$  and  $n$  are *comparable* if  $m \in n$  or  $m = n$  or  $n \in m$ .

**Proposition 1.** *Any two natural numbers are comparable.*<sup>2</sup>

In fact, more is true.

**Proposition 2.** *For any two natural numbers, exactly one of  $m \in n$ ,  $m = n$  and  $n \in m$  is true.*<sup>3</sup>

**Proposition 3.**  $m \in n \longleftrightarrow m \subset n$ .

If  $m \in n$ , then we say that  $m$  is *less than*  $n$ . If we know that  $m = n$  or  $m$  is less than  $n$ , we say that  $m$  is *less than or equal to*  $n$ .

### Notation

If  $m$  is less than  $n$  we write  $m < n$ , read aloud “ $m$  less than  $n$ .” If  $m$  is less than or equal to  $n$ , we write  $m \leq n$ , read aloud “ $m$  less than or equal to  $n$ .”

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<sup>1</sup>Future editions will expand.

<sup>2</sup>Future editions will include an account.

<sup>3</sup>Use the fact that no natural number is a subset of itself. Future editions will expand this account. See *Peano Axioms*).

## Properties

Notice that  $<$  and  $\leq$  are relations on  $\omega$  (see *Relations*).<sup>4</sup>

**Proposition 4** (Reflexivity).  $\leq$  is reflexive, but  $<$  is not.

**Proposition 5** (Symmetry). Both  $\leq$  and  $<$  are not symmetric.

**Proposition 6** (Transitivity). Both  $\leq$  and  $<$  are transitive.

**Proposition 7** (Antisymmetry). If  $m \leq n$  and  $n \leq m$ , then  $m = n$ .

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<sup>4</sup>Proofs of the following propositions will appear in future editions.

