Why

If we interpret a list of two numbers as displacement in a plane, and a list of three numbers as displacement in a space, what of a list of n numbers as displacement in $\mathbf{R}^n$?

Definition

A *real vector* (or *vector*, *n-dimensional vector*, *n-vector*) is a length-n list of real numbers.

Algebra

For $x, y \in \mathbf{R}^n$, we define the *real vector sum* (or *sum*) of x and y as the vector $z \in \mathbf{R}^n$ where $z_i = x_i + y_i$ for $i = 1, \ldots, n$. As usual, we denote the sum by $x + y$, so

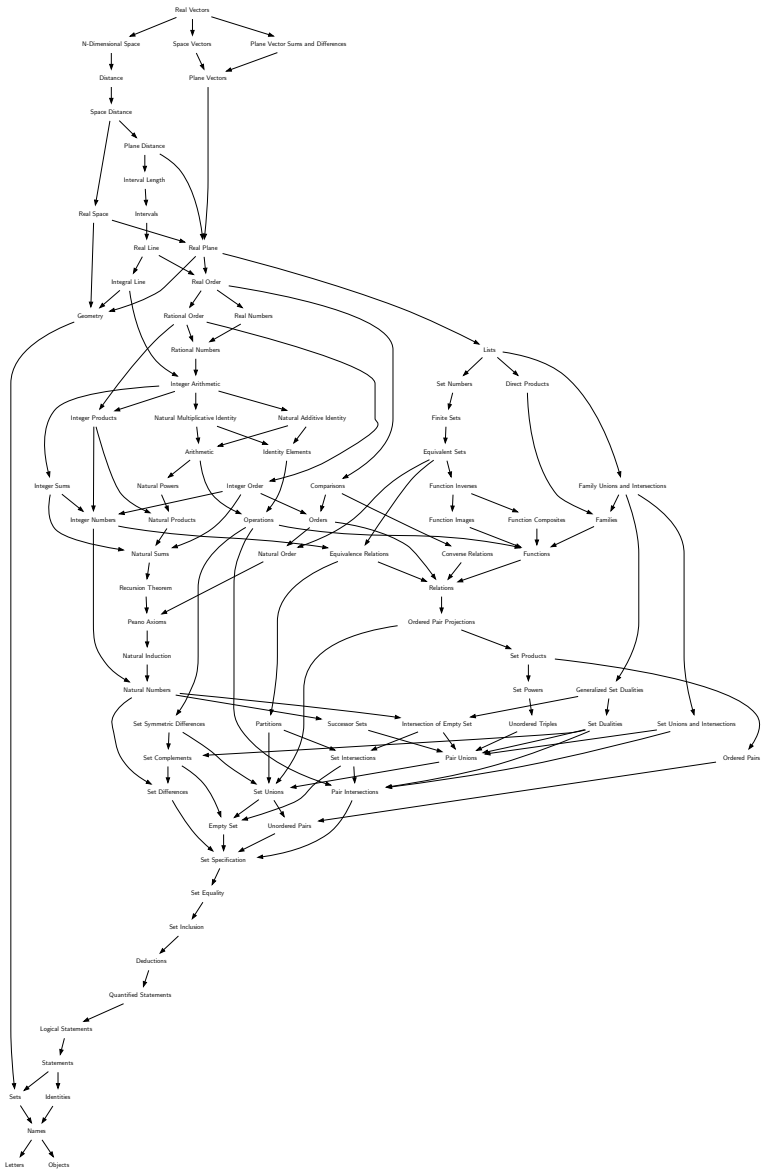
$$x + y = (x_1 + y_1, \ldots, x_n + y_n).$$

For $\alpha \in \mathbf{R}$ and $x \in \mathbf{R}^n$, *real scalar-vector product* (or *scalar product*, *product*) $z \in \mathbf{R}^n$ is defined by $z_i = \alpha x_i$ for $i = 1, \ldots, n$. As usual, we denote the product αx, and write

$$\alpha x = (\alpha x_1, \ldots, \alpha x_n).$$

Our intuition for both of these *operations* comes from their special cases in $\mathbf{R}^2$ and $\mathbf{R}^3$. As usual, the *real-vector difference* (or *difference*) of x and y is the vector $z \in \mathbf{R}^n$ defined by $z_i = x_i - y_i$ for $i = 1, \ldots, n$. As usual, we denote it by $x - y$, and note that $x - y = x + (-y)$.

The algebra given here for vectors is natural in view of their generalization as n-dimensional *displacements*. However, we keep in mind that this algebra is over lists of numbers, and that these sums and products can be defined on these lists of numbers regardless of interpretation.


Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Integer Sums
Natural Powers
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Operations
Orders
Function Images
Function Composites
Families
Natural Sums
Natural Order
Equivalence Relations
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects