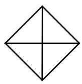


Why

Since one and only one outcome occurs, given a distribution on outcomes, we define the probability of a set of outcomes as the sum of their probabilities.

Definition

Given a distribution $p : \Omega \to \mathbf{R}$, the *probability of an event* $A \subset \Omega$ is $\sum_{a \in A} p(a)$, the sum of the probabilities of its outcomes.

Define $\mathbf{P} : \mathcal{P}(\Omega) \to \mathbf{R}$ by $\mathbf{P}(A) = \sum_{a \in A} p(a)$. We call $\mathbf{P}$ *the event probability function* (or *the probability measure*) induced by p. Clearly $\mathbf{P}$ depends on the set of outcomes Ω and the distribution $p : \Omega \to \mathbf{R}$. We sometimes denote this dependence by $\mathbf{P}_{\Omega,p}$ or $\mathbf{P}_p$.

Example: die

Define $p : \{1, \dots, 6\} \to \mathbf{R}$ by $p(\omega) = 1/6$ for $\omega = 1, \dots, 6$. Define the event $E = \{2, 4, 6\}$. Then

$$\mathbf{P}(E) = \sum_{\omega \in E} p(\omega) = p(2) + p(4) + p(6) = 1/2.$$

Properties of P

As a result of the conditions on p, $\mathbf{P}$ satisfies

1. $\mathbf{P}(A) \geq 0$ for all $A \subset \Omega$;
2. $\mathbf{P}(\Omega) = 1$ (and $\mathbf{P}(\varnothing) = 0$);
3. $\mathbf{P}(A) + \mathbf{P}(B)$ for all $A, B \subset \Omega$ and $A \cap B = \varnothing$. This statement follows from the more general identity

$$\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$$

for $A, B \subset \Omega$, by using $P(\varnothing) = 0$ of (2) above.

Do all such $\mathbf{P}$ satisfying (1)-(3) have a corresponding underlying probability distribution? In other words, suppose $f : \mathcal{P}(\Omega) \to \mathbf{R}$ satifies (1)-(3). These three conditions are sometimes called the *axioms of probability for finite sets.*

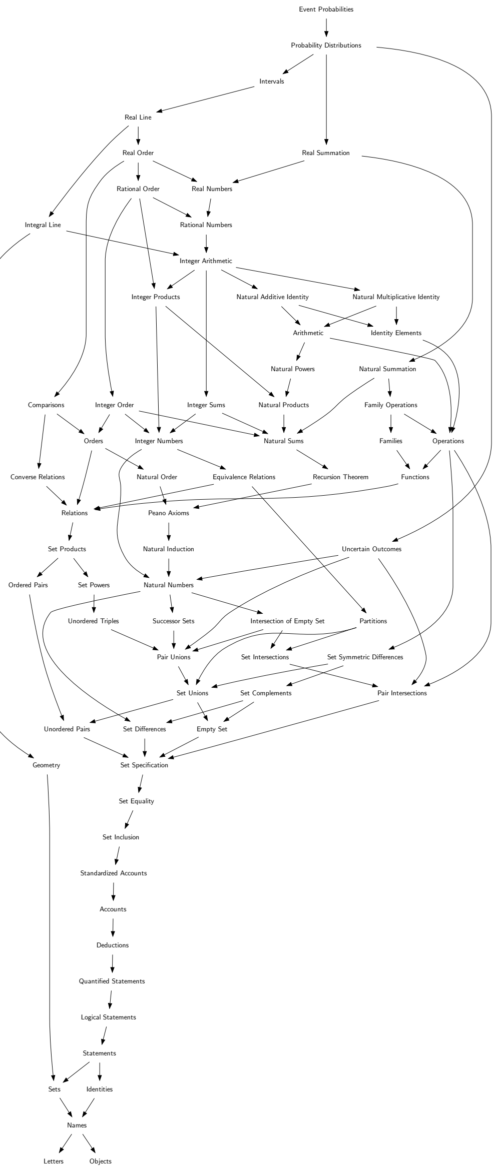
Define $q : \Omega \to \mathbf{R}$ by $q(\omega) = f(\{\omega\})$. If f satisfies the axioms, then q is a probability distribution. For this reason we call any function satisfying (i)-(iii) an *event probability function* (or a *probability measure*).

Probability by cases

Let $\mathbf{P}$ be a probability event function. Suppose $A_1, \ldots, A_n$ partition Ω. Then for any $B \subset \Omega$,

$$\mathbf{P}(B) = \textstyle\sum_{i=1}^n \mathbf{P}(A_i \cap B).$$

Some authors call this the *law of total probability.*


Event Probabilities
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Rational Order
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Natural Summation
Comparisons
Integer Order
Integer Sums
Natural Products
Family Operations
Orders
Integer Numbers
Natural Sums
Families
Operations
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Functions
Relations
Peano Axioms
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects