



Definition

A *decision problem* is a pair (I, Y) of *instances* I and *yes-instances* $Y \subset I$.

Examples

Subgraph isomorphism

Let $\mathcal{G}(n)$ be the set of graphs with n vertices. For $m \leq n$, define $I = \mathcal{G}(n) \times \mathcal{G}(m)$. A graph $G_1 = (V_1, E_1)$ is *subgraph-isomorphic* to a graph $G_2 = (V_2, E_2)$ if there exists $V' \subset V_1$, $E' \subset E_1$ with $|V'| = |V_2|$, $|E'| = |E_2|$, and bijection $f : V' \rightarrow V_2$ so that

$$\{u, v\} \in E_2 \iff \{f(u), f(v)\} \in E'.$$

Define

$$Y = \{(G_1, G_2) \in I \mid G_1 \text{ is subgraph-isomorphic to } G_2\}$$

We call (Y, I) the *subgraph-isomorphism problem*.

Traveling Salesman

Denote by $S^n \subset \mathbf{Z}^{n \times n}$ the set of symmetric integer-valued matrices. Define $I = S^n \times \mathbf{Z}$. A *n-tour* is a numbering $\pi : \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ and has *cost* $C(\pi)$ with respect to D defined by

$$C(\pi) = D_{\pi(n), \pi(1)} + \sum_{j=1}^{n-1} D_{\pi(j), \pi(j+1)}$$

A tour is *B-bounded* if its cost is no greater than B . Define $Y = \{(D, B) \mid \text{there is a } B\text{-bounded tour with respect to } D\}$.

