

## Real Integral Dominated Convergence

## Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

## Result

When context is clear, we refer to the following proposition as the *dominated convergence theorem*.

**Proposition 1.** The integral of the almost everywhere limit of a sequence of measurable, extended-real-valued, almost-everywhere bounded functions is the limit of the sequence of integrals of the functions.

*Proof.* Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $f: X \to [-\infty, \infty]$  be a  $\mathcal{A}$ -measurable function. Let  $f_n : \to [-\infty, \infty]$  a  $\mathcal{A}$ -measurable function for every natural number n so that  $(f_n)_n$  converges almost everywhere to f. Let  $g: X \to [0, \infty]$  be an integrable function which dominates  $f_n$  almost everywhere for each n. We want to show that:

$$\int f d\mu = \lim_{n} \int f_n d\mu.$$

