Hyperplanes

1 Why

2 Definition

A *hyperplane* in n-dimensional space is an $(n-1)$-dimensional affine set.

Since the $n-1$-dimensional subspaces are the orthogonal complements of the one-dimensional subspaces, they are the sets which can be specified by

$$\{x \in \mathbf{R}^n \mid x \perp b\}$$

for $b \in \mathbf{R}^n$. The hyperplanes are translates of these,

$$\begin{aligned}\{x \in \mathbf{R}^n \mid x \perp b\} + a &= \{x + a \mid \langle x, b\rangle = 0\} \\ &= \{y \mid \langle y - a, b\rangle = 0\} = \{y \mid \langle y, b\rangle = \beta\},\end{aligned}$$

where $\beta = \langle a, b\rangle$.

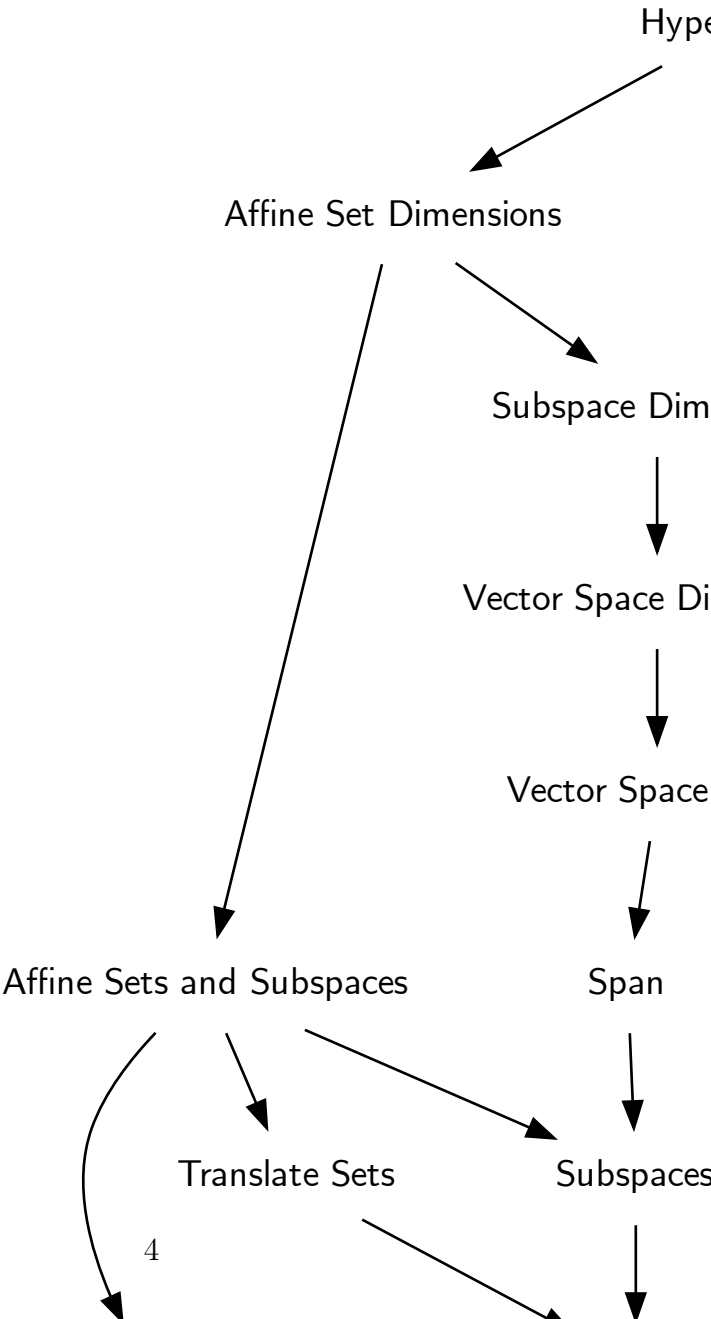
3 Characterization

Proposition 1. *$H \subset \mathbf{R}^n$ is a hyperplane if and only if there exists $\beta \in \mathbf{R}$ and nonzero $b \in \mathbf{R}^n$ so that*

$$H = \{x \in \mathbf{R}^n \mid \langle x, b\rangle = \beta\}.$$

Remark 2. *b and β are unique up to a common nonzero multiple. For example, b, β and $2b, 2\beta$ give the same hyperplane.*

Remark 3. *The vector b is called a* normal *to the hyperplane.*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