



# Tail Sigma Algebra

## 1 Why

## 2 Definition

The *tail sigma algebra* of a sequence of random variables is the sigma algebra which is the intersection of the sigma algebras of all final parts of the sequence. A *tail event* is an element of the tail sigma-algebra.

The tail sigma algebra coincides with the sigma algebra generated by the union of the sigma algebras of each of the random variables.

### 2.1 Notation

Let  $(f_n)_n$  be a sequence of random variables. Denote the tail sigma algebra by  $T((f_n)_n)$ . We defined it as:

$$T((f_n)_n) = \bigcap_{n=1}^{\infty} \sigma(\{X_{n+k}\}_k).$$

In other words, for all natural  $n$ , the event is in the sigma algebra of the final part of

### 3 Results

**Proposition 1.** *The tail sigma algebra of a sequence of random variables is the same equals the sigma algebra generated by the union of the sigma algebras of each of the random variables.*