



## Why

We want to extend our notion of entropy (see [Discrete Entropy](#)) to real-valued (continuous) random variables.

## Definition

The *differential entropy* of a probability density function is the integral of the density against the negative log of the density. This definition made to be similar to the case of discrete entropy. If a real-valued random variable has a density, then we call the differential entropy of its density the *differential entropy* of the random variable.

## Notation

Let  $f : \mathbf{R}^n \rightarrow \mathbf{R}$  be a probability density function. The differential entropy of  $f$  is

$$-\int f \log f$$

We denote the differential entropy of  $f$  by  $h(f)$ .

## Example

Let  $x : \Omega \rightarrow \mathbf{R}$  be uniform on  $[0, 1/2]$ . Then  $h(x) = \log 1/2 < 0$ .

## Problems

We have  $h(ax) = h(x) + \log|a|$ . In general  $h(Ax) = h(x) + \log|A|$ .

## Differences still meaningful

Even though the value of the differential entropy is not necessarily a good analogy to discrete entropy, differences still are. In particular, the following holds

$$I(X; Y) = H(Y) - H(Y | X) = H(X) = H(X | Y)$$



