Why

We are frequently interested in finding minimizers of real functions.[1]

Definition

An *optimization problem* is a pair $(\mathcal{X}, f)$ in which $\mathcal{X}$ is a nonempty set called the *constraint set* and $f : \mathcal{X} \to \mathbf{R}$ is called the *objective* (or *cost function*).

If $\mathcal{X}$ is finite we call the optimization problem *discrete*. If $\mathcal{X} \subset \mathbf{R}^d$ we call the optimization problem *continuous*.

We refer to all elements of the constraint set as *feasible*. We refer to an element $x \in \mathcal{X}$ of the constraint set as *optimal* if $f(x) = \inf_{z \in \mathcal{X}} f(z)$. We also refer to optimal elements as *solutions* of the optimization problem.

It is common for f and $\mathcal{X}$ to depend on some other, known, given objects. In this case, these objects are often called *parameters* or *problem data*.

Notation

We often write optimization problems as

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \mathcal{X}. \end{aligned}$$

In this case we call x the *decision variable*.

Extended reals

It is common to let $f : \mathcal{X} \to \bar{\mathbf{R}}$, and allow there to exist $x \in \mathcal{X}$ for which $f(x) = \infty$. This technique can be used to embed further constraints in the objective. For example, we interpret $f(x) = +\infty$ to mean x is *infeasible*.

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[1]Future editions will modify and expand.

Maximization

If we have some function $g : \mathcal{X} \to \bar{\mathbf{R}}$ that we wish to maximize, we can always convert it to an optimization problem by defining $f : \mathcal{X} \to \bar{\mathbf{R}}$ by $f(x) = -g(x)$. In this case g is often called a *reward* (or *utility, profit*).

Solvers

A *solver* (or *solution method, solution algorithm*) for a family of optimization problems is a function S mapping optimization problems to solutions.

Loosely speaking, the difficulty of "solving" the optimization problem $(\mathcal{X}, f)$ depends on the properties of $\mathcal{X}$ and f and the problem "size". For example, when $\mathcal{X} \subset \mathbf{R}^d$ the difficulty is related to the "dimension" d of $x \in \mathcal{X}$.

Optimization Problems

Real Functions

N-Dimensional Space

Distance

Extended Real Numbers

Space Distance

Real Limits

Plane Distance

Absolute Value

Interval Length

Optimizers

Real Space

Intervals

Greatest Lower Bounds

Real Plane

Real Line

Real Sequences

Real Order

Integral Line

Sequences

Geometry

Lists

Rational Order

Real Numbers

Set Numbers

Direct Products

Rational Numbers

Comparisons

Integer Order

Finite Sets

Integer Arithmetic

Orders

Equivalent Sets

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Natural Order

Function Inverses

Integer Sums

Family Unions and Intersections

Arithmetic

Identity Elements

Function Composites

Function Images

Families

Integer Numbers

Natural Powers

Operations

Converse Relations

Functions

Equivalence Relations

Natural Products

Relations

Natural Sums

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Set Products

Natural Induction

Set Powers

Natural Numbers

Generalized Set Dualities

Partitions

Unordered Triples

Intersection of Empty Set

Successor Sets

Set Dualities

Set Unions and Intersections

Set Symmetric Differences

Ordered Pairs

Set Intersections

Pair Unions

Set Complements

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects

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