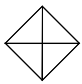


Why

We want to visualize the probabilistic relations between components of outcomes in probabilistic models over large (e.g., product) outcome sets.[1]

Definition

Suppose $X_1, \dots, X_k$ are sets. Define $X = \prod_{i=1}^{k} X_i$. For $x \in X$ and $S \subset \{1, \dots, n\}$, denote the subvector of x indexed (in order) by S by x_S.[2]

A distribution $p : X \to [0, 1]$ *factors according to a directed graph* on $\{1, \dots, n\}$ with parent function $\mathrm{pa} : \{1, \dots, n\} \to \mathcal{P}(\{1, \dots, n\})$ if

$$p(x) = \prod_{\mathrm{pa}_i = \varnothing} g_i(x_i) \prod_{\mathrm{pa}_i \neq \varnothing} g_i(x_i, x_{\mathrm{pa}_i}),$$

where g_i is a distribution for all i which $\mathrm{pa}_i = \varnothing$ and $g_i(\cdot, \xi)$ is a distribution for all $\xi \in \prod_{j \in \mathrm{pa}_i} A_j$, i for which $\mathrm{pa}_i \neq \varnothing$.

Proposition 1. *p so defined is a distribution, and the g_i are the marginals and conditionals.*[3]

Examples

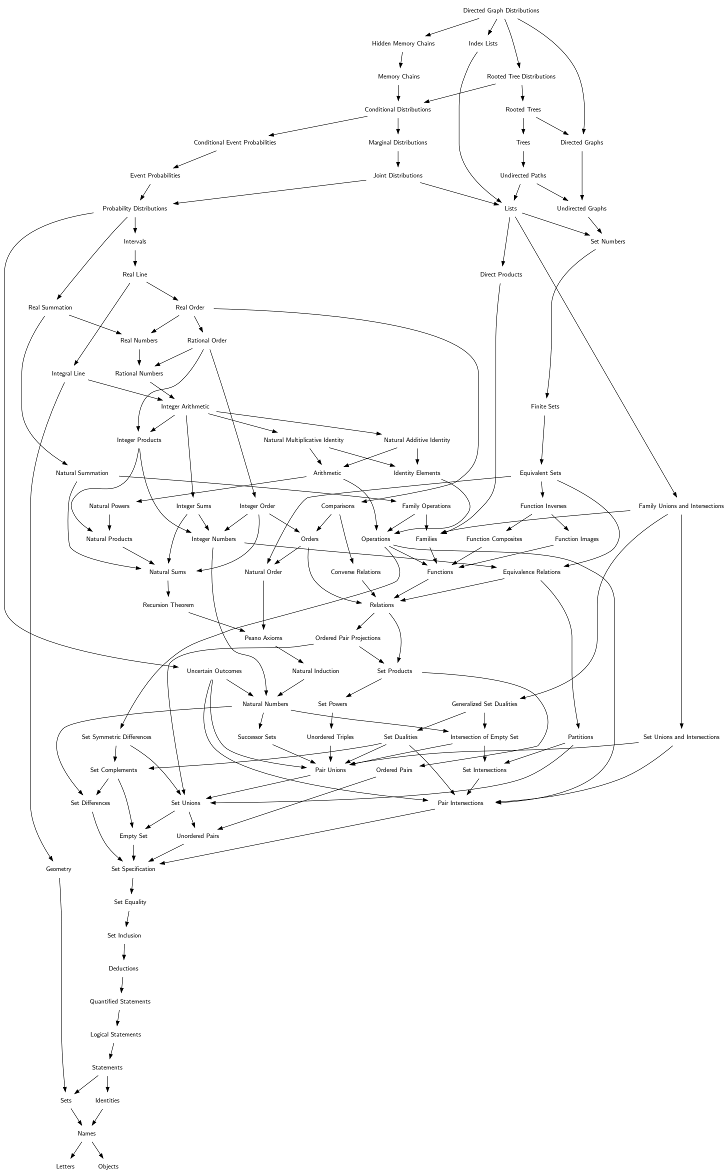
Consider a rooted tree distribution (see Rooted Tree Distributions), or a memory chain (see Memory Chains), or a hidden memory chain (see Hidden Memory Chains).[4]

[1] Future editions will modify and expand. The title of the sheet may change, since another interpretation for the words "directed graph distribution" is a distribution on directed graphs.

[2] Future editions will rework this treatment, perhaps combining it with the sheet Index Matrices, which will possibly be split up.

[3] Future editions will be precise and give an account.

[4] Future editions will expand.


Directed Graph Distributions
Hidden Memory Chains
Index Lists
Memory Chains
Rooted Tree Distributions
Conditional Distributions
Rooted Trees
Conditional Event Probabilities
Marginal Distributions
Trees
Directed Graphs
Event Probabilities
Joint Distributions
Undirected Paths
Probability Distributions
Lists
Undirected Graphs
Intervals
Set Numbers
Real Line
Direct Products
Real Summation
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Finite Sets
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Natural Summation
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Family Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Families
Function Composites
Function Images
Natural Sums
Natural Order
Converse Relations
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Ordered Pairs
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects