Definition

A *decision problem* is a pair (I, Y) of *instances* I and *yes-instances* $Y \subset I$.

Examples

Subgraph isomorphism

Let $\mathcal{G}(n)$ be the set of graphs with n vertices. For $m \leq n$, define $I = \mathcal{G}(n) \times \mathcal{G}(m)$. A graph $G_1 = (V_1, E_1)$ is *subgraph-isomorphic* to a graph $G_2 = (V_2, E_2)$ if there exists $V' \subset V_1$, $E' \subset E_1$ with $|V'| = |V_2|$, $|E'| = |E_2|$, and bijection $f : V' \to V_1$ so that

$$\{u, v\} \in E_2 \longleftrightarrow \{f(u), f(v)\} \in E'.$$

Define

$$Y = \{(G_1, G_2) \in I \mid G_1 \text{ is subgraph-isomorphic to } G_2\}$$

We call (Y, I) the *subgraph-isomorphism problem*.

Traveling Salesman

Denote by $S^n \subset \mathbf{Z}^{n \times n}$ the set of symmetric integer-valued matrices. Define $I = S^n \times \mathbf{Z}$. A *n-tour* is a numbering $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ and has *cost* $C(\pi)$ with respect to D defined by

$$C(\pi) = D_{\pi(n), \pi(1)} + \sum_{j=1}^{n-1} D_{\pi(i), \pi(i+1)}$$

A tour is B-bounded if its cost is no greater than B. Define $Y = \{(D, B) \mid$ there is a $B - $ bounded tour with respect to $D\}$.

Decision Problems

Undirected Graphs

Set Numbers

Finite Sets

Equivalent Sets

Natural Order

Function Inverses

Integer Numbers

Function Images

Function Composites

Equivalence Relations

Peano Axioms

Functions

Natural Induction

Relations

Set Products

Set Powers

Natural Numbers

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Intersections

Pair Unions

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects