

AFFINE SET DIMENSIONS

Why

Since every affine set is a translate of some (unique) subspace, it is natural to define the dimension of an affine set as the dimension of this subspace.

Definition

The dimension of a nonempty affine set is the dimension of the subspace parallel to it. By convention, \varnothing has dimension -1. Naturally, the points, lines and planes are affine sets of dimension 0, 1, and 2 respectively.

If an affine set has dimension r, then we often call it an r-flat.

For any $S \subset \mathbf{R}^n$, we define the dimension of A to be the dimension of the affine hull of A.

Notation

We denote the dimension of the set $S \subset \mathbb{R}^n$ by dim S We have defined it so that

$$\dim S = \dim \operatorname{aff} S$$

This makes sense if S is affine, since in this case aff S = S.

Result

Proposition 1. Any r-flat has r + 1 affinely independent points. Each of its sets of size r + 2 are affinely dependent.

