

RECURSION THEOREM

Why

It is natural to want to define a sequence by giving its first term and then giving its later terms as functions of its earlier ones. In other words, we want to define sequences inductively.¹

Proposition 1 (Recursion theorem). Let X be a set, let $a \in X$ and let $f: X \to X$. There exists a unique function u so that u(0) = a and $u(n^+) = f(u(n))^2$.

When one uses the recursion theorem to assert the existence of a function with the desired properties, it is called *definition by induction*.

¹Future editions will expand on this. We are really headed toward natural addition, multiplication and exponentiation.

²The account is somewhat straightforward, given a good understanding of the results of Peano Axioms. The full account will appear in future editions.

