

## Normal Densities

## 1 Why

We want a density that is symmetric about some central value with some spread.

## 2 Definition

A normal density is one which can be written as a normalized

Let  $f: \mathbb{R} \to \mathbb{R}$  be a density. If there exists  $\mu \in \mathbb{R}$  and  $\sigma \in \mathbb{R}$  with  $\sigma > 0$  so that for each  $x \in \mathbb{R}$ 

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2\right)$$

then f is a normal density. Some call a normal density a Gaussian density. We often drop the word density and use refer to these as normals or Gaussians, using these words as substantives.

We call the special case when  $\mu = 0$  and  $\sigma = 1$  the standard normal

