



Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?¹

Definition

Two random variables are *independent* if the sigma algebras generated by the random variables are independent. In general, a family of random variables are *independent* if the sigma algebras generated by the random variables are independent.

Notation

Let (X, \mathcal{A}, μ) be a probability space and (Y, \mathcal{B}) be a measurable space. Let $f_1, f_2 : X \rightarrow Y$ be random variables. If the random variables are independent we write $f_1 \perp f_2$.

Results

PROPOSITION 1. *Let f_1, \dots, f_n be independent real-valued random variables defined on a probability space (X, \mathcal{A}, μ) .*

Let B_1, \dots, B_n be Borel sets of real numbers and let $A_i = f_i^{-1}(B_i)$. Let $A = \cap_{i=1}^n f_i^{-1}(B_i)$. Then

$$\mu(A) = \prod_{i=1}^n \mu(A_i)$$

Proof. Since f_i are independent, so are the sigma algebras they

¹Future editions will modify this.

generate. A_i are in each of these sigma algebras, so by definition of independence the measure of the intersection is the product of the measures. \square

