



## Why

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### Definition

Let  $(A, +, \cdot)$  be a ring. A *polynomial* in  $A$  of *degree*  $d$  is a finite sequence of length  $d + 1$ . We call the elements of the sequence the *coefficients* of the polynomial.

Let  $c = (c_0, c_1, \dots, c_{d-1}, c_d)$  be a polynomial of degree  $d$ . The *polynomial function* or *function of the polynomial*  $c$  is the function  $f : A \rightarrow A$  defined by

$$f(a) = c_0 + c_1a^1 + c_2a^2 + \dots + c_da^d.$$

In accordance with this terminology, we often call function  $f : A \rightarrow A$  a polynomial if there exists a polynomial  $c$  so that  $f$  is the polynomial function of  $c$ .

The function  $f : A \rightarrow A$  is a polynomial of degree 0 and order 1 if there exists  $c_0$  so that

$$f(a) = c_0$$

for all  $a \in A$ .

The function  $g : A \rightarrow A$  is a polynomial of degree 1 and order 2 if there exists  $c_0$  and  $c_1$  so that

$$g(a) = c_0 + c_1a$$

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<sup>1</sup>Future editions will include, and most likely will build on quadratics.

The function  $h : A \rightarrow A$  is a polynomial of degree 2 and order 3 if there exists  $c_0$  and  $c_1$  so that

$$h(a) = c_0 + c_1a + c_2a^2.$$

In other words, a second degree polynomial is a quadratic.

The function  $p : A \rightarrow A$  is a *polynomial* of degree  $d$  and order  $d+1$  if there exists a  $d+1$  length sequence  $(c_0, c_1, \dots, c_d)$  in  $A$  so that

$$p(a) = c_0 + c_1a + \dots + c_da^d.$$

