# Cartesian Products

## 1 Why

We want to handle objects composed of elements from different sets.

## 2 Definition

Let $A$ and $B$ be sets. Construct a new set that is the ordered pairs of elements of $A$ and $B$: the first element of the pair is an element of $A$ and the second element of the pair is an element of $B$. The **cartesian product** of $A$ with $B$ is the set of all such ordered pairs.

We call an ordered pair **tuple**. Two tuples are equal if they have equal elements in the same order. Because of the ordering, if $A \neq B$, the cartesian product of $A$ with $B$ is not the same as the cartesian product of $B$ with $A$. Only in the case that $A = B$ does this symmetry hold.

### 2.1 Notation

For sets $A$ and $B$ we denote the cartesian product of $A$ with $B$ by $A \times B$. We read the notation $A \times B$ as "A cross B." We denote elements of $A \times B$ by $(a, b)$ with the understanding that $a \in A$ and $b \in B$. In this notation, we can write the observation that $A \times B \neq B \times A$, unless $A = B$.