



Why

What is the generalisation of smooth functions between Euclidean spaces.

Definition

Let $U \subset \mathbf{R}^n$ be open. A function $f : U \rightarrow \mathbf{R}^m$ is *smooth* if each of its components is smooth.

More generally, let $X \subset \mathbf{R}^n$ (not necessarily open). We call $g : X \rightarrow \mathbf{R}^m$ smooth if for each $x \in X$ there exists an open set $V \subset \mathbf{R}^n$ with $x \in V$ and smooth function $G : V \rightarrow \mathbf{R}^m$ so that $G(y) = g(y)$ for all $y \in V \cap X$. In this case we say that g can be *locally extended* to a smooth map on open sets.

