Why

If both signed measures are finite, then their difference is always well-defined. Is the difference a finite signed measure?

Preliminary result

Proposition 1. *A linear combination of finite signed measures is a finite signed measure.*

Proof. Let $(X, \mathcal{A})$ be a measurable space. Let μ and ν be finite signed measures. Let R denote the real numbers. Then $(\alpha\mu)(\varnothing) = \alpha \cdot \mu(\varnothing) = \alpha \cdot 0 = 0$. Also for $(A_n)_n \subset \mathcal{A}$ disjoint,

$$(\alpha\mu)(\cup A_n) = \alpha\mu(\cup A_n) = \alpha \sum_{n=1}^{\infty} \mu(A_n)$$

$$= \sum_{n=1}^{\infty} \alpha\mu(A_n) = (\alpha\mu)(A_n)$$

Similarly, $(\mu + \nu)(\varnothing) = \mu(\varnothing) + \nu(\varnothing) = 0$. And, for $(A_n)_n \subset \mathcal{A}$ disjoint,

$$(\mu + \nu)(\cup A_n) = \mu(\cup A_n) + \nu(\cup A_n) = \sum_{n=1}^{\infty} \mu(A_n) + \sum_{n=1}^{\infty} \nu(A_n)$$

$$= \sum_{n=1}^{\infty} \mu(A_n) + \nu(A_n) = \sum_{n=1}^{\infty} (\mu + \nu)(A_n)$$

$\square$

Main result

Proposition 2. *The set of finite signed measures is a vector space.*

Proof. Use the previous proposition. Observe that the function $\mu \equiv 0$ is a measure. And $\nu + \mu = \nu$ for all measures ν. $\square$

Notation

We denote the vector space of signed measures on measurable space $(X, \mathcal{A})$ by by $M(X, \mathcal{A}, \mathbf{R})$.

Measure Vector Space

Finite Signed Measures

Signed Measures

Measures

Sigma Algebras

Extended Real Numbers

Real Series

Real Limits

Absolute Value

Interval Length

Intervals

Real Line

Real Order

Real Numbers

Real Sequences

Sequences

Subset Algebra

Integral Line

Rational Order

Rational Numbers

Lists

Cardinality

Integer Arithmetic

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Direct Products

Set Numbers

Finite Sets

Arithmetic

Identity Elements

Equivalent Sets

Natural Powers

Integer Sums

Integer Order

Comparisons

Operations

Function Inverses

Family Unions and Intersections

Natural Products

Integer Numbers

Orders

Function Composite

Function Images

Families

Natural Sums

Natural Order

Converse Relations

Equivalence Relations

Functions

Recursion Theorem

Peano Axioms

Relations

Natural Induction

Ordered Pair Projections

Set Products

Subset Systems

Natural Numbers

Generalized Set Dualities

Set Powers

Ordered Pairs

Set Symmetric Differences

Set Dualities

Successor Sets

Intersection of Empty Set

Unordered Triples

Partitions

Set Unions and Intersections

Set Complements

Pair Unions

Set Intersections

Set Differences

Pair Intersections

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects