Definition

A conditional distribution (density) $q : Y \times X \to \mathbf{R}$ is *functionally parametrizable* if there exists a function $f : X \to \Theta$ and distribution (density) family $\{p^{(\theta)} : Y \to \mathbf{R}\}_{\theta \in \Theta}$ satisfying $q(y,x) \equiv p^{(f(x))}(\gamma)$ for all $x \in X$ and $y \in Y$.

In this case we call f the *parameterizer* and we call $\{p^{(\theta)}\}_{\theta \in \Theta}$ the *parameterized family*. A *parameterized conditional distribution* is an ordered pair whose first coordinate is a function from X to Θ and whose second coordinate is a family of distributions on X with parameter set Θ. For a particular choice of parameterizer and family, it induces a conditional distribution.

Since all conditional distributions are functionally parametrizable (consider $\{q(\cdot, \xi)\}_{\xi \in X}$ with parameters X and identity parameterizer), we are interested in parameterizers and parameterized families that are simple. Said differently, we are interested in approximating a conditional distribution by selecting an appropriate parameterizer and parameterized family.

If $\{f_\phi : X \to \Theta\}_{\phi \in \Phi}$ is a family of functions and $\{q^{(\theta)}\}$ is a family of distributions, then $\{p^{(\phi)} : X \times Z \to \mathbf{R}\}_\phi$ defined by $p^{(\phi)}(\cdot, \zeta) \equiv q^{f_\phi(\zeta)}$ is a conditional distribution family called a *functionally parameterized conditional distribution family*. In other words, by selecting some parameters ϕ, we induce a conditional distribution $p^{(\phi)} : X \times Z \to \mathbf{R}$

We similarly define *parameterized conditional densities* and *functionally parameterized conditional density families*.

Basic example

Let $Z = \{1, 2\}$ and $X = \mathbf{R}$. Let $f : \{1, 2\} \to \mathbf{R} \times \mathbf{R}_+$ be defined by $f(1) = (\mu_1, \sigma_1)$ and $f(2) = (\mu_2, \sigma_2)$. Let $\{g^{(\theta)}\}_\theta$ be the normal family. Then $(f, \{g^{(\theta)}\})$ is a *functionally parameterized conditional density*.[1]

---

[1] Future editions will modify.

Parameterized Distributions

Distribution Families

Probability Densities

Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions

Measure

Sigma Algebras

Real Series

Extended Real Numbers

Real Limits

Absolute Value

Simple Functions

Probability Distributions

Real Functions

Interval Length

Intervals

Real Line

Real Summation

Real Order

Real Sequences

Rational Order

Real Numbers

Sequence

Subset Algebras

Integral Line

Rational Numbers

Cardinality

Lists

Integer Arithmetic

Set Numbers

Direct Products

Natural Summation

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Arithmetic

Identity Elements

Equivalent Sets

Family Operations

Natural Power

Integer Sums

Integer Order

Comparisons

Operations

Function Inverses

Families

Family Unions and Intersections

Natural Products

Integer Numbers

Natural Sums

Natural Order

Converse Relations

Equivalence Relations

Function Images

Function Composites

Functions

Recursion Theorem

Orders

Relations

Peano Axioms

Ordered Pair Projections

Uncertain Outcomes

Natural Induction

Characteristic Functions

Set Products

Subset Systems

Natural Numbers

Set Powers

Generalized Set Dualities

Ordered Pairs

Set Symmetric Differences

Successor Sets

Unordered Triples

Set Dualities

Intersection of Empty Set

Partitions

Set Unions and Intersections

Set Complements

Pair Unions

Set Intersections

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects