# Multivariate Gaussian Maximum Likelihood

## 1 Why

What of the generalization to a multivariate gaussian.

## 2 Result

**Proposition 1.** *Let* $(x^1, \ldots, x^n)$ *be a dataset in* $\mathbf{R}^d$*. Let* $f$ *be a multivariate gaussian density with mean*

$$\frac{1}{n}\sum_{k=1}^{d} x^k$$

*and covariance*

$$\frac{1}{n}\sum_{k=1}^{n}\left(x^k - \frac{1}{n}\sum_{k=1}^{n} x^k\right)\left(x^k - \frac{1}{n}\sum_{k=1}^{n} x^k\right)^\top.$$

*Then* $f$ *is a maximum likelihood multivariate gaussian density.*

*Proof.* We express the log likelihood

$$\sum_{k=1}^{n} -\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)$$

□

We call these two objects the **maximum likelihood mean** and **maximum likelihood covariance** of the dataset.