



# Partial Orders

## 1 Why

We want to handle elements of a set in a particular order.

## 2 Definition

Let  $R$  be a relation on a non-empty set  $A$ .  $R$  is a *partial order* if it is reflexive, transitive, and anti-symmetric. If  $(a, b) \in R$  we say that  $a$  *precedes*  $b$  and that  $b$  *succeeds*  $a$ .

A *partially ordered set* is a set and a partial order. The language partial is meant to suggest that two elements need not be comparable. For example, suppose  $R$  is  $\{(a, a) \mid a \in A\}$ ; we may justifiably call this no order at all and call  $A$  totally unordered, but it is a partial order by our definition.

### 2.1 Notation

We denote a partial order on a set  $A$  by  $\preceq$ . We read  $\preceq$  aloud as "precedes or equal to" and so read  $a \preceq b$  aloud as "a precedes or is equal to b." If  $a \preceq b$  but  $a \neq b$ , we write  $a \prec b$ , read aloud as "a precedes b."