



## Set Inclusion

### 1 Why

We want language for all of the elements of a first set being the elements of a second set.

### 2 Definitions

If every element of a first set is an element of a second set we say that the first set is a *subset* of the second set. Conversely, we say that second set is a *superset* of the first set.

Every set is a subset of itself. Similarly, every set is a superset of itself. Thus, if two sets are equal, the first is a subset of the second and the second is a subset of the first. Because of our definition of set equality, the converse is also true.

The empty set is a subset of every set, since it has no elements and so satisfies our definition. Consider a set. We call the empty set and the set itself *improper subsets* of the set. All other subsets we call *proper subsets*.

#### 2.1 Notation

Let  $A$  and  $B$  be sets. We denote that  $A$  is a subset of  $B$  by  $A \subset B$ . We read the notation  $A \subset B$  aloud as "A subset B".

We can express the axiom of extension by

$$A = B \Leftrightarrow (A \subset B) \wedge (B \subset A)$$

The notation  $A \subset B$  is a concise symbolism for the sentence "every element of  $A$  is an element of  $B$ ." Or for the alternative notation  $a \in A \Rightarrow a \in B$ .

## 2.2 Immediate Results

**Proposition 1.** *If  $A \subset B$  and  $B \subset C$  then  $A \subset C$ .*

*Proof.* Let  $a \in A$ . Then  $a \in B$  and so then  $b \in C$ . Thus  $a \in C$ . □