## Why

The relationship between the inclusion map and the identity map is characteristic of making small functions out of large ones.[1]

## Definition

Let $X \subset Y$ and $f : Y \to Z$. There is a natural function $g : X \to Z$, namely the one defined by $g(x) = f(x)$ for all $x \in X$. We call $g$ the *restriction* of $f$ to $X$. We call $f$ an *extension* of $g$ to $Y$. Clearly, there may be more than one extension of a function

### Notation

We denote the restriction of $f : Y \to Z$ to the set $X \subset Y$ by $f \mid X$ or $f_{|X}$.

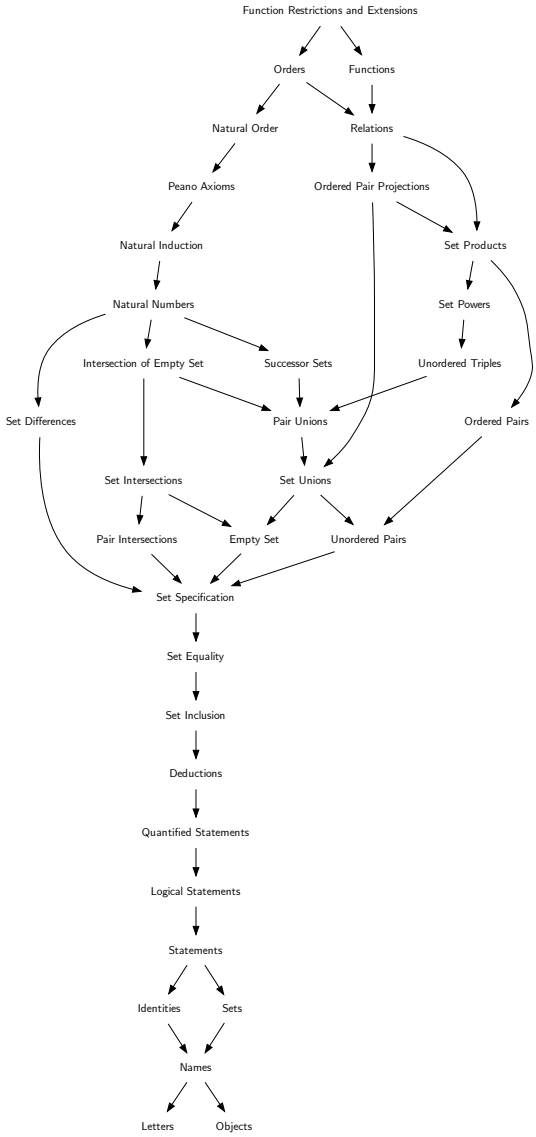
### Example

A simple example is the that the inclusion mapping from $X$ to $Y$ with $X \subset Y$ is a restriction of the identity map on $X$

## An extension order

Here is a natural order involving set extensions and restrictions. Fix two sets $A$ and $B$. Let $F$ be the set of all functions $f : X \to Y$ with $X \subset A$ and $Y \subset B$. Define a relation $R$ in $F$ by $(f, g) \in R$ if $\operatorname{dom} f \subset \operatorname{dom} g$ and $f(x) = g(x)$ for all $x$ in $\operatorname{dom} f$. In other words, $(f, g) \in R$ if $f$ is a restriction of $g$ (or, equivalently, $g$ is an extension of $f$. We recognize that $R$ is a special case of the inclusion partial order by recognizing the elements of $F$ as subsets $A \times B$.

[1] Future editions will modify this language.


Function Restrictions and Extensions
Orders
Functions
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Differences
Pair Unions
Ordered Pairs
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects