Why

Can permuting the rows or columns of a matrix be represented by matrix multiplication?

Definition

Let $\sigma : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ be a permutation of n. The *permutation matrix* of σ is the matrix P defined by $P_{ij} = 1$ if $\sigma(i) = j$ and 0 otherwise. This is sometimes called the *column representation* (in contrast to the row representation, in which $P_{ij} = 1$ if $\sigma(j) = i$.

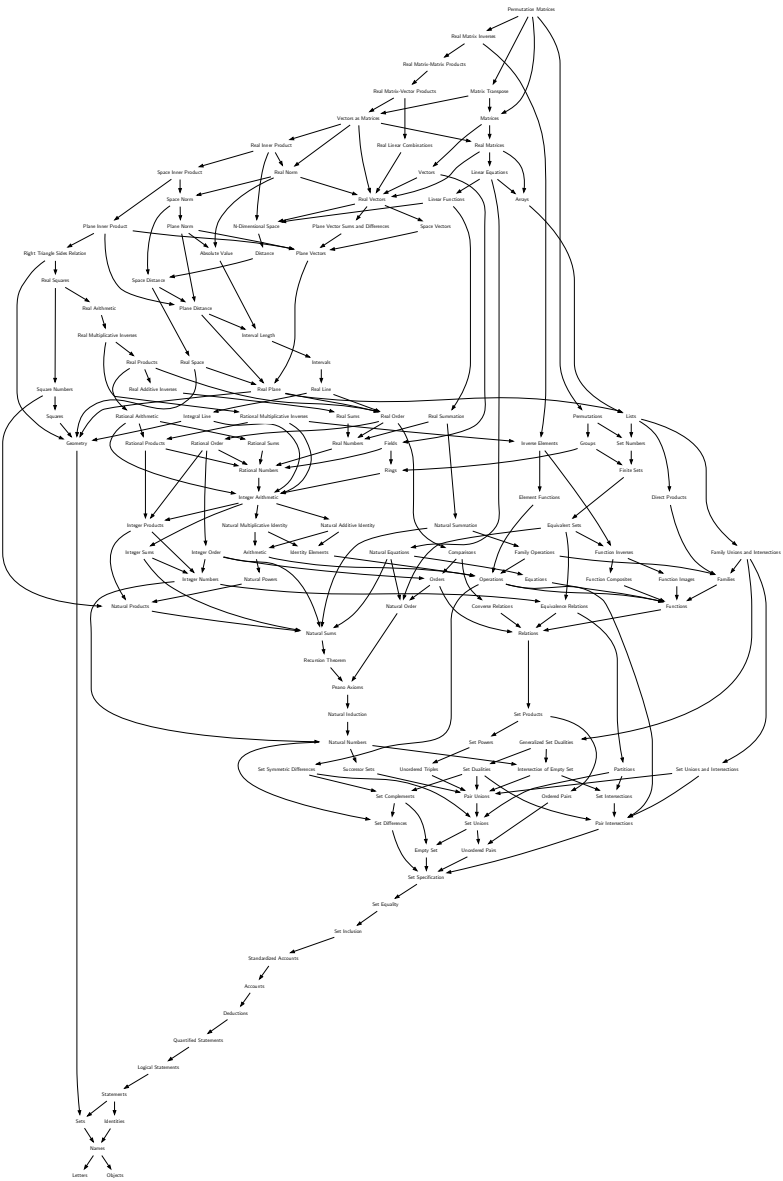
Let $A \in \mathbf{R}^{n \times n}$. Then pre-multipying A by P permutes the rows of A. In other words PA has the same rows as A but permuted according to σ. Similarly, post-multiplying by P permutes the columns of A. In other words, AP has the same columns as A but permuted according to σ. Clearly, we can also speak of permuting the components of a vector.

Composition

Let $\pi, \sigma \in S_n$ with corresponding permutation matrices P_σ and P_π. Then $P_\pi P_\sigma A$ has the same rows as A but permuted by $\pi\sigma$. Likewise, $AP_\pi P_\sigma$ has the same columns as A but permuted by $\pi\sigma$. Clearly, the identity permutation on $\{1, 2, \ldots, n\}$ is the identity $I \in \mathbf{R}^{n \times n}$.

Inverses

It is clear from the definition that $P_\sigma^{-1} = P_{\sigma^{-1}}$ and so if P is a permutation matrix then P^{-1} is $P^\top$.


Permutation Matrices
Real Matrix Inverses
Real Matrix-Matrix Products
Real Matrix-Vector Products
Matrix Transpose
Vectors as Matrices
Matrices
Real Inner Product
Real Linear Combinations
Real Matrices
Space Inner Product
Real Norm
Vectors
Linear Equations
Space Norm
Real Vectors
Linear Functions
Arrays
Plane Inner Product
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Right Triangle Sides Relation
Absolute Value
Distance
Plane Vectors
Real Square
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Square Numbers
Real Additive Inverses
Real Plane
Real Line
Square
Rational Arithmetic
Integral Line
Rational Multiplicative Inverses
Real Sums
Real Order
Real Summation
Permutations
Lists
Geometry
Rational Products
Rational Order
Rational Sums
Real Numbers
Fields
Inverse Elements
Groups
Set Numbers
Rational Numbers
Rings
Finite Sets
Integer Arithmetic
Element Functions
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Natural Summation
Equivalent Sets
Integer Sums
Integer Order
Arithmetic
Identity Elements
Natural Equations
Comparators
Family Operations
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Powers
Order
Operations
Equations
Function Composites
Function Images
Families
Natural Products
Natural Order
Converse Relations
Equivalence Relations
Functions
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Ordered Pairs
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Name
Letters
Objects