

## COUNTABLE PROBABILITY DISTRIBUTIONS

## Why

Suppose we flip a coin until it lands heads. What is the probability that we see a head after one flip? Two flips? n flips? We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

## Why

Consider a set A.<sup>1</sup> If A has n elements, then a probability distribution on A is  $p:A\to \mathbf{R}$  where p(a)=1/n. There is a natural candidate.

What if A is the set of natural numbers  $\mathbb{N}$ . The principle difficulty is that not all sequences of real numbers  $a: \mathbb{N} \to \mathbb{R}$  are summable.

A (countable) probability distribution on  $\mathbb{N}$  is  $p: \mathbb{N} \to \mathbb{R}$  where  $p \geq 0$  and

$$\sum_{n=1}^{\infty} p(n) = 1.$$

<sup>&</sup>lt;sup>1</sup>Future editions will rework this sheet.

