Why

Can permuting the rows or columns of a matrix be represented by matrix multiplication?

Definition

Let $\sigma : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ be a permutation of n. The *permutation matrix* of σ is the matrix P defined by $P_{ij} = 1$ if $\sigma(i) = j$ and 0 otherwise. This is sometimes called the *column representation* (in contrast to the row representation, in which $P_{ij} = 1$ if $\sigma(j) = i$.

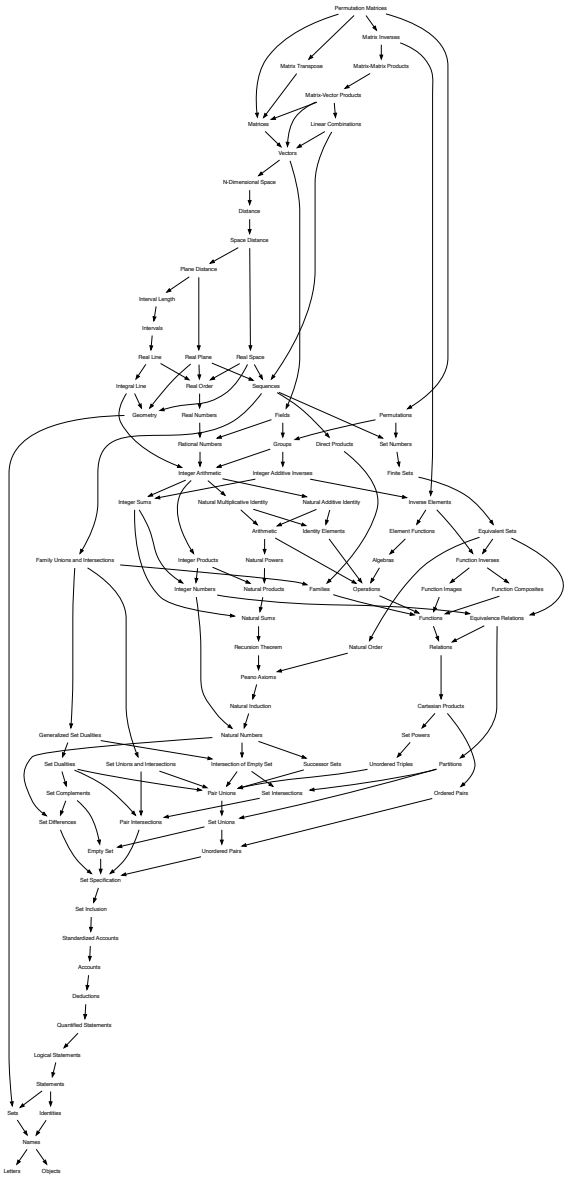
Let $A \in \mathbf{R}^{n \times n}$. Then pre-multipying A by P permutes the rows of A. In other words PA has the same rows as A but permuted according to σ. Similarly, post-multiplying by P permutes the columns of A. In other words, AP has the same columns as A but permuted according to σ. Clearly, we can also speak of permuting the components of a vector.

Composition

Let $\pi, \sigma \in S_n$ with corresponding permutation matrices P_σ and P_π. Then $P_\pi P_\sigma A$ has the same rows as A but permuted by $\pi\sigma$. Likewise, $AP_\pi P_\sigma$ has the same columns as A but permuted by $\pi\sigma$. Clearly, the identity permutation on $\{1, 2, \ldots, n\}$ is the identity $I \in \mathbf{R}^{n \times n}$.

Inverses

It is clear from the definition that $P_\sigma^{-1} = P_{\sigma^{-1}}$ and so if P is a permutation matrix then P^{-1} is $P^\top$.


Permutation Matrices
Matrix Inverses
Matrix Transpose
Matrix-Matrix Products
Matrix-Vector Products
Matrices
Linear Combinations
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Sequences
Geometry
Real Numbers
Fields
Permutations
Rational Numbers
Groups
Direct Products
Set Numbers
Integer Arithmetic
Integer Additive Inverses
Finite Sets
Integer Sums
Natural Multiplicative Identity
Natural Additive Identity
Inverse Elements
Arithmetic
Identity Elements
Element Functions
Equivalent Sets
Family Unions and Intersections
Integer Products
Natural Powers
Algebras
Function Inverses
Integer Numbers
Natural Products
Families
Operations
Function Images
Function Composites
Natural Sums
Functions
Equivalence Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Natural Induction
Cartesian Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects