

LEAST UPPER BOUNDS

Definition

Let A be a set and let \leq be an order¹ on A.

An upper bound for $B \subset A$ is an element $a \in A$ so that $b \leq a$ for all $b \in B$. A set is bounded from above if it has a least upper bound. A least upper bound for B is an element $c \in A$ so that c is an upper bound and c < a for all other upper bounds a.

Proposition 1. If there is a least upper bound it is unique.²

We call the unique least upper bound of a set (if it exists) the supre-mum.

Notation

We denote the supremum of a set $B \subset A$ by $\sup A$.

 $^{^1\}mathrm{To}$ be defined in future editions, but understood in the usual way. See Natural Orderor Integer Orderor Rational Orderetc.

²Proof in future editions.

