Probability Measures

1 Why

We use the language of measure theory to give a mathematical model for uncertain outcomes. TODO: probability intuition sheet.

2 Definition

A **probability measure**s a finite measure on a measurable space which assigns the value one to the base set. Since a finite measure can always be scaled to a probability measure, these measures are standard examples of finite measures.

A **probability space** is a measure space whose measure is a probability measure. The **outcomes** of a probability space are the elements of the base set. The **set of outcomes** is the base set. The **events** are the elements of the sigma algebra.

2.1 Notation

We denote the set of outcomes by Ω, a mnemonic for "outcomes." We denote the sigma-algebra by $\mathcal{A}$, as usual. We de-

note a probability measure by p, a mnemonic for "probability."
Thus, we often say "Let $(\Omega, \mathcal{A}, p)$ be a probability space."

2.2   Properties