Why

We compress the notation for linear equations.

Definition

A *real matrix* (*matrix*, *rectangular array*) is a two-dimensional array of real numbers. We denote the elements of a matrix in a grid between two rectangular braces, as in

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}.$$

We call m and n the *dimensions* of the matrix. We call m the *height* and n the *width*. If the height of the matrix is the same as the width of the matrix then we call the matrix *square*. If the height is larger than the width, we call the matrix *tall*. If the width is larger than the height, we call the matrix *wide*.

Linear equations

Recall that we are interested in solutions of the linear equations

$$\begin{aligned} y_1 &= A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n, \\ y_2 &= A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n, \\ &\vdots \\ y_n &= A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n. \end{aligned}$$

We have suggestively used the notation A_{ij} for the coefficients of the equations, so they are the entries of $A \in \mathbf{R}^{m \times n}$.

A primer on matrix-vector products

Using the notation $A \in \mathbf{R}^{m \times n}$ and $x \in \mathbf{R}^n$ we want a compressed way to write the above system of linear equations. Define the *real matrix-vector product* $z \in \mathbf{R}^m$ of A with x by

$$z_i = \sum_{j=1}^{n} A_{ij} x_j, \quad i = 1, \ldots, m.$$

We denote the matrix vector product z by Ax.

Notation

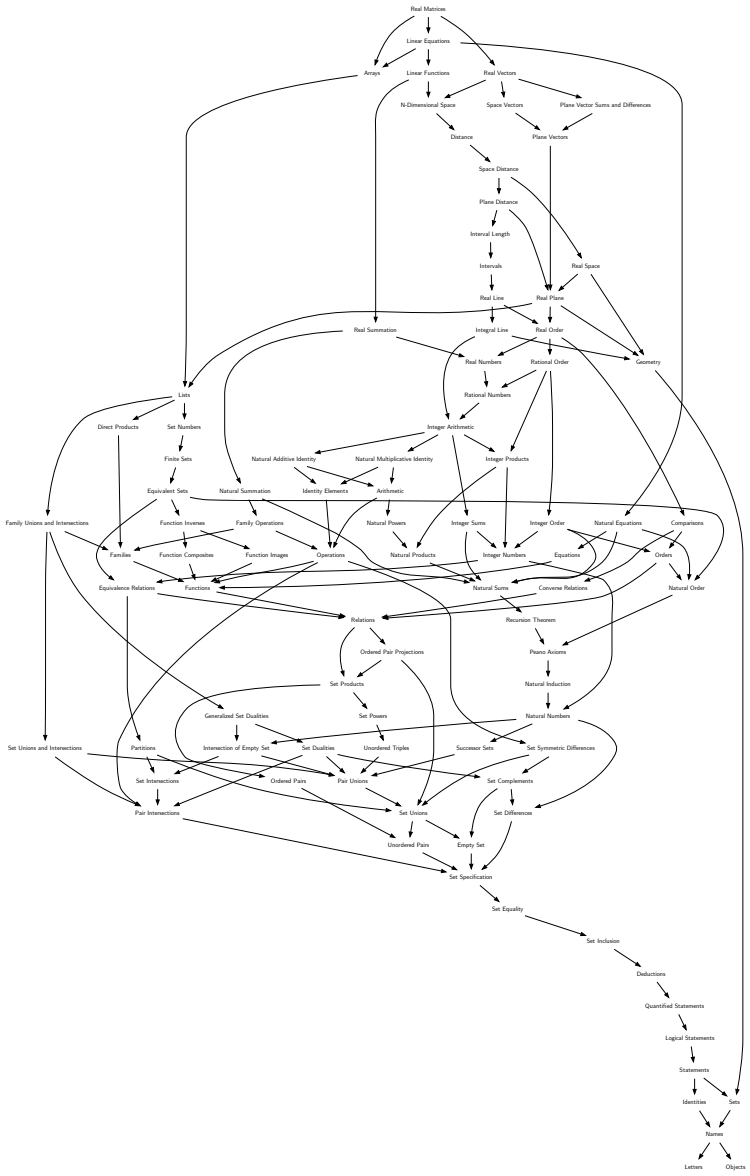
We express the above system of linear equations as

$$y = Ax,$$

where $y = (y_1, \ldots, y_n) \in \mathbf{R}^n$ and $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$. The compact notation $y = Ax$ is sometimes called the *matrix form* of the m linear equations and A the *coefficient matrix*.

Note on terminology

The word "matrix" is from the Latin "mater," meaning mother, and has an old sense in English similar "womb." The matrix is source of many determinants (discussed elsewhere).


Real Matrices
Linear Equations
Arrays
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Real Vectors
N-Dimensional Space
Space Vectors
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Distance
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Space Distance
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Natural Multiplicative Identity
Integer Products
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Family Operations
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Recursion Theorem
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Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
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