



Why

What are some simple functions? Here's one answer: those that only involve addition and multiplication.¹

Definition

A *real polynomial* (or *polynomial*) of degree d is a function $p : \mathbf{R} \rightarrow \mathbf{R}$ for which there exists a finite sequence $a = (a_0, a_1, \dots, a_d)$ so that

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_dx^d.$$

In particular, $q(x) = ax + b$ for $a, b \in \mathbf{R}$ is a polynomial of the first degree and $r(x) = ax^2 + bx + c$ for $a, b, c \in \mathbf{R}$ is a polynomial of the second degree.

In a sense, these are “simple” functions. We require addition (and subtraction) and multiplication; but no division.

Properties

Proposition 1. *Let $p : \mathbf{R} \rightarrow \mathbf{R}$ be a polynomial of degree d . Then p is continuous.*

Proposition 2. *Let $p : \mathbf{R} \rightarrow \mathbf{R}$ be a polynomial of degree d . Then p has derivatives of all orders. Every derivative of p is a polynomial.*

Proposition 3. *Let $p : \mathbf{R} \rightarrow \mathbf{R}$ be a polynomial of degree d . The every derivative of order $d + 1$ or greater is the constant 0 function.*

¹Future editions will modify and expand.

