

## Graph Paths

## 1 Why

TODO

## 2 Definition

A path between a first and second distinct vertex of the graph is a sequence of distinct vertices, whose first coordinate is the first vertex and whose last coordinate is the second vertex, and whose consecutive coordinates are adjacent in the graph. We call the first and last coordinate the endpoints of the path. We say that the path is between its endpoints. The length of a path is one less than the number of vertices: namely, the number of edges. The length of a path is always at least one: there exists a path of length one between any two adjacent vertices. If a path has length two or greater, we call a vertex which is not the first or last vertex an interior vertex.

Two vertices are *connected* in a graph if there exists at least one path between them. A graph is *connected* if each pair of vertices is connected. A *connected component* of a graph is a maximal subset of vertices whose corresponding subgraph is connected.

A *cycle* is a sequence whose first and last coordinate are identical, all other coordinates are distinct, and consecutive coordinates are adjacent.

## 2.1 Notation

Let G = (V, E) be a graph. A path between v and w (with  $v \neq w$ ) in G is a sequence  $(v_0, v_1, \ldots, v_k)$  where  $v_0 = v$  and  $v_k = w$  and  $\{v_i, v_{i+1}\} \in E$  for  $i = 0, \ldots, k-1$ . The length of such a path is k. A coordinate of a path that is not the first or last coordinate