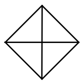

Why

We want to associate the natural numbers with bit strings for use on digital computers.[1]

Definition

A *digital natural* is a bit string. The set of *d-bit digtal natural numbers* is the set of length-d bit strings $\{0,1\}^d$. For example, the set of 8-bit digital naturals is the set $\{0,1\}^8$.

Correspondence with $\mathbf{N} \cup \{0\}$

We associate $x \in \{0,1\}^d$ corresponds to the number $\sum_{i=1}^{d} x_i 2^i$. For example, the bit string $(0,0,0) \in \{0,1\}^3$ corresponds to the natural number $0 \in \omega$. Likewise, $(1,0,0)$ corresponds to $1 \in \mathbf{N}$, $(0,1,0)$ corresponds to 2, $(1,1,0)$ corresponds to 3, etc.

Call the function so defined the *digital natural decoder*, and denote it by $f : \{0,1\}^d \to \mathbf{N} \cup \{0\}$. In other words $f((0,0,0)) = 0$, $f((0,1,0)) = 2$, etc. Call the set $f(\{0,1\}^d)$ the set of naturals *representable* by length-d bit strings.

Specifically, if, for $n \in \mathbf{N} \cup \{0,1\}$, there exists $x \in \{0,1\}^d$ so that $f(x) = n$, we say that x is *representable in d bits*.

Correspondence between d and $k > d$ bit naturals

Let $x \in \{0,1\}^d$ and $y \in \{0,1\}^k$ with $k > d$. Although $\{0,1\}^d \not\subset \{0,1\}^k$, $f(\{0,1\}^d) \subset f(\{0,1\}^k)$. We can identify

[1] Future editions will expand.

$x \in \{0,1\}^d$ with $x' \in \{0,1\}^k$ where $x' = (x_1, \ldots, x_d, 0, \ldots, 0)$ so that $f(x) = f(x')$. Clearly then, if x is representable in d bits, it is representable in $k > d$ bits.

Addition

We want to define addition $\oplus : \{0,1\}^d \times \{0,1\}^d \to \{0,1\}^d$ so that $f(x \oplus x') = f(x) + f(x')$. In general, we are stuck, because $x + x'$ may not be representable in d bits. Suppose, however and for the time being, that it is.[2]

[2] Future editions will complete.


Digital Naturals
Bit Strings
Natural Powers
Codes
Natural Products
Dot-Dash Code
Natural Sums
Flash Code
Alphabets
Recursion Theorem
Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Peano Axioms
Function Images
Function Composites
Functions
Equivalence Relations
Natural Induction
Relations
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects