Matrix notation

Let $u_1 \ldots, u_k \in \mathbf{R}^n$. Define $U \in \mathbf{R}^{n \times k}$ so that

$$U = \begin{bmatrix} u_1 & u_2 & \cdots & u_k \end{bmatrix}.$$

Then $\{u_1, \ldots, u_k\}$ is an orthonormal set mean

$$U^\top U = I_k$$

Notice that if $k < n$, $UU^\top \neq I$, since its rank is at most k.

Real Matrices with Orthonormal Columns

Real Matrix-Matrix Products

Real Matrix-Vector Products

Vectors as Matrices — Orthonormal Set of Real Vectors

Real Linear Combinations — Real Matrices — Real Inner Product

Linear Equations

Real Norm

Linear Functions — Real Vectors — Space Norm

Plane Vector Sums and Difference — Space Vector — Plane Norm — N-Dimensional Space — Plane Inner Product

Plane Vector — Absolute Value — Distance — Right Triangle Sides Relation

Space Distance — Real Squares

Plane Distance — Real Arithmetic

Real Multiplicative Inverse

Interval Length — Real Space — Real Products

Intervals

Real Plane — Real Line — Real Additive Inverse — Integral Line — Square Numbers

Real Summation — Real Order — Real Sums — Rational Arithmetic — Rational Multiplicative Inverse

Real Number — Rational Order — Rational Sum — Rational Products — Squares

Arrays — Geometry

Rational Numbers

Lists — Integer Arithmetic

Set Numbers — Finite Sets — Natural Multiplication Identity — Natural Additive Identity — Integer Products

Direct Products — Inverse Elements — Identity Elements — Arithmetic — Equivalent Sets — Natural Summation — Natural Powers

Family Unions and Intersections — Comparisons — Function Inverses — Element Functions — Family Operations — Natural Equations — Integer Order — Integer Sums — Natural Products

Families — Function Composites — Function Images — Operations — Equations — Orders — Natural Sums — Integer Numbers

Converse Relations — Functions — Natural Order — Equivalence Relations — Recursion Theorem

Relations — Peano Axioms

Set Products — Natural Induction

Set Powers — Generalized Set Dualities — Natural Numbers

Set Unions and Intersections — Unordered Triples — Set Dualities — Intersection of Empty Set — Successor Sets — Partitions — Set Symmetric Differences

Ordered Pairs — Pair Unions — Set Intersections — Set Complements

Pair Intersections — Set Union — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects

4