Why

[1]

Definition

An *index sequence* of *order* n is a finite sequence of distinct elements of $\{1, 2, \ldots, n\}$ whose length is less than or equal to n. We call the ith coordinate of an index sequence the *i-index* of the sequence. The *index matrix* associated with an index sequence is the $r \times n$ matrix whose i, jth entry is 1 if the index sequences's ith coordinate is j, and 0 otherwise. If $r = n$ then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an n-vector associated with a length-r index sequence is the product of the $r \times n$ index matrix with the n-dimensional vector. Its ith entry is the i-index entry of the vector.

Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions), they are not sets of indices, so we avoid this usage.

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[1]Future editions will include.

Notation

Let $r \leq n$ be natural numbers. Let $\alpha : \{1, 2, \ldots, r\} \rightarrow \{1, 2, \ldots, n\}$ be an index sequence. We denote the index matrix associated with α by P_α. This matrix P_α is an element of $\mathsf{N}^{r \times n}$ and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let A be a nonempty set and let $x \in A^n$. then the subvector of x associated with P_α (and so with α) is

$$P_\alpha x = \left(x_{\alpha(1)}, \ldots, x_{\alpha(r)} \right)$$

We denote the product $P_\alpha x$ by x_α.

We denote the product $P_\alpha X P_\alpha{}^\top$ by $X_{\alpha\alpha}$.

Multiplication

The product of the $n \times r$ transpose of an index matrix with an r vector is the n vector with The