



## Convex Sets

### 1 Why

We speak of sets of real numbers which are like intervals.

### 2 Definition

A *convex combination* of two distinct real numbers is an element of the closed interval they delimit. A *convex set* of real numbers contains each convex combination of any two of its elements.

The *length* of a convex combination is the real number in  $[0, 1]$  which is the ratio of the combination less the lower endpoint to the upper endpoint less the lower endpoint.

#### 2.1 Notation

Denote the real numbers by  $R$ . Let  $A \subset R$  be convex. Then for all  $a, b \in A$ ,  $[a, b] \subset A$ .

Suppose  $A$  contains at least two element. Let  $a, b \in A$  with  $a < b$ . If  $c$  is the combination of  $a$  and  $b$ , then the length of  $c$  is  $(c - a)/(b - a)$ .

If  $A$  is convex, then for each  $a, b \in A$ , and  $\theta \in [0, 1]$ ,

$$\theta a + (1 - \theta)b \in A.$$

### 3 Examples

**Example 1.** *The real numbers are a convex set.*

**Example 2.** *Real intervals are convex.*

**Example 3.** *Let  $a, b$  be non-equal real numbers. The set  $\{a, b\}$  is not convex.*

**Example 4.** *The empty set is convex*

**Example 5.** *Let  $a$  be a real number. The set  $\{a\}$  is convex.*

**Example 6.** *Let  $[a, b]$  and  $[c, d]$  be two disjoint real intervals. The set  $[a, b] \cup [c, d]$  is not convex.*