



# Graph Cliques

## 1 Why

We speak of the complete subgraphs of a graph.

## 2 Definition

A *complete* graph is one for which an edge exists between any two nodes.

A *subgraph* of a given graph is a graph whose vertex set is a subset of the given vertex set and whose edge set is the subset of given edges connecting vertices in the vertex subset. With reference to the underlying graph, then, a subgraph can be specified completely by its vertex set.

A *clique* of a given graph is a complete subgraph of that graph. When speaking of the cliques of a given graph, we identify the cliques with their vertex set. The relation contained in gives a partial order on cliques. A clique is *maximal* if it maximal with respect to this relation; i.e., it is contained in no other clique. As a convention, we include  $\emptyset$  as a clique.

### 2.1 Notation

Let  $(V, E)$  a graph. We denote a clique by  $C \subset V$ , a mnemonic for clique.