

INDUCED PROBABILITY DISTRIBUTIONS

Why

We can interpret the codomain of a random variable as a new sample space, since the underlying probability distribution induces a new probability distribution.

Definition

Let $p:\Omega\to \mathbf{R}$ be a probability distribution and $x:\Omega\to V$ an outcome variable. Define $q:V\to \mathbf{R}$ by

$$q(a) = \mathbf{P}[x = a].$$

Since events $x^{-1}(a)$ for $a \in V$ partition Ω , $\sum_{a \in A} q(a) = 1$. We call q the induced distribution (or induced probability mass function) of the random variable x. Thus we can think of V as a set of outcomes, which we call the outcomes induced by x.

Notation

It is common to denote it by p_x .

If $x:\Omega\to V$ is a random variable and $f:V\to U$, then if we define $y:\Omega\to V$ so that $y\equiv f(x),\,y$ is a random variable with induced distribution $p_y:\Omega\to \mathbf{R}$ satisfying

$$p_y(b) = \sum_{a \in V | y(a) = b} p_x(a).$$

As a matter of practical computation, we can evaluate probabilities having to do with the outcome variable x using p_x instead of p. For example with x as in the example above, $\mathbf{P}(x=4 \text{ or } x=5)=p_x(4)+p_x(5)$, rather than $\sum_{\omega\in\Omega|x(\omega)=4 \text{ or } x(\omega)=5}p(\omega)$.

