Convex Sets

Why

Definition

The *closed line segment between* two points in n-dimensional space is the set of points which can be expressed as the sum of the first point and a scalar multiple of the difference between the second point and the first; where the scalar is in the interval $[0, 1]$. Thus, the closed line segment between two points is a subset of the line though the two points. The *open line segment* between x and y is the closed line segment with the points x and y.

A *convex set* contains every closed line segement between any two points. Every affine set is convex. Thus, convex sets are more general.

Notation

Let x and y in $\mathbf{R}^n$. We can express the closed line segment between x and y as

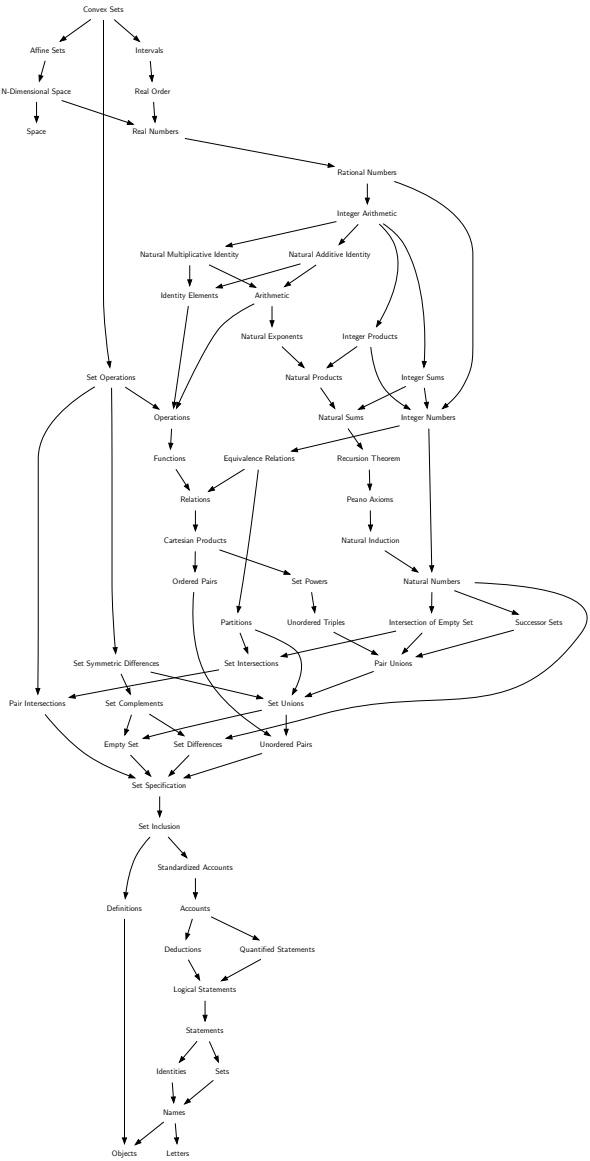
$$\{x + a(y - x) \mid 0 \leq a \leq 1, x, y \in \mathbf{R}^n\}.$$

Notice that $x + a(y - x) = (1 - a)x + ay$.

Proposition **1.** *Every affine set is convex.*

Proposition **2.** *The intersection of a family of convex sets is convex.*

Proposition **3.** *The translate of a convex set is convex. The scalar multiple of a convex set si convex.*


Convex Sets
Affine Sets
Intervals
N-Dimensional Space
Real Order
Space
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Identity Elements
Arithmetic
Natural Exponents
Integer Products
Natural Products
Integer Sums
Set Operations
Operations
Natural Sums
Integer Numbers
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters