

## NETS

## Why

We generalize the notion of sequence to index sets beyond the naturals.

## Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go "further out."

To elaborate on property (b): if handed two natural numbers m and n, we can always find another, for example  $\max\{m,n\}+1$ , larger than m and n. We might think of larger as "further out" from the first natural number: 1.

Combining these to observations, we define a directed set:

**Definition 1.** A directed set is a set D with a partial order  $\leq$  satisfying one additional property: for all  $a, b \in D$ , there exists  $c \in D$  such that  $a \leq c$  and  $b \leq c$ .

**Definition 2.** A net is a function on a directed set.

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is  $m \leq n$  if  $m \leq n$ .

## Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net  $x:D\to A$  by  $\{a_\alpha\}$ , emulating notation for sequences. The use of  $\alpha$  rather than n reminds us that D need not be the set of natural numbers.

