

#### TREE DISTRIBUTIONS

## Why

It happens that if a distribution factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So the property of factoring according to a rooted tree is really property of factoring according to a tree.

#### Definition

## **Defining Result**

**Proposition 1.** Let  $A_1, \ldots, A_n$  be finite non-empty sets and define  $A = \prod_{i=1}^n A_i$ . Let  $p: A \to [0,1]$  be a distribution and let T be a tree on  $\{1,\ldots,n\}$ .

**Proposition 2.** If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.

#### Undirected definition

A distribution p factors according to the tree T if it factors according to the T rooted at any vertex.

# Existence and uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

#### Existence

A distribution p need not factor according to a tree.

### Uniqueness

A distribution p may factor according to multiple trees.

