



Multivariate Normal Conditionals

1 Why

What of the conditional densities of a multivariate normal density.

2 Result

Proposition 1. *Let $f : \mathbf{R}^d \rightarrow \mathbf{R}$ be a normal density with mean $\mu \in \mathbf{R}^d$ and covariance $\Sigma \in \mathbb{S}^d$.*

$$\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$$

Then the conditional density $f_{x|y}(\xi, \gamma)$ is $\mathcal{N}(\bar{\mu}(\gamma), \bar{\Sigma})$ with

$$\bar{\mu}(\gamma) = \mu_x + \Sigma_{xy} \Sigma_{yy}^{-1} (\gamma - \mu_y) \quad \text{and} \quad \bar{\Sigma} = \Sigma_{xx} - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx}.$$