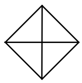


Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object.[1]

Definition

A *countably summable subset algebra* is a subset algebra for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of $A_1, \dots, A_n$ coincides with the union of $A_1, \dots, A_n, A_n, A_n \dots$.

We say that the set of distinguished sets is a *sigma algebra* or *sigma field* on the base set. This language is justified (as for a regular subset algebra) by the closure properties of the sigma algebra under the usual set operations. Other notations are σ-algebra and σ-field.

Notation

Let $(A, \mathcal{A})$ be a countably summable subset algebra. We often say "let $\mathcal{A}$ be a sigma algebra on A." Since the largest element of the sigma algebra is the base set, we can also say (without

[1]Future editions will make no reference to measure theory. The entire development will be for follow the historical development for handling integration.

ambiguity): "let $\mathcal{A}$ be a sigma algebra." In this last case, the base set is $\cup\mathcal{A}$.

Examples

Example 1. *For any set A, 2^A is a sigma algebra.*

Example 2. *For any set A, $\{A, \varnothing\}$ is a sigma algebra.*

Example 3. *Let A be an infinite set. Let $\mathcal{A}$ the collection of finite subsets of A. $\mathcal{A}$ is not a sigma algebra.*

Example 4. *Let A be an infinite set. Let $\mathcal{A}$ be the collection subsets of A such that the set or its complement is finite. $\mathcal{A}$ is not a sigma algebra.*

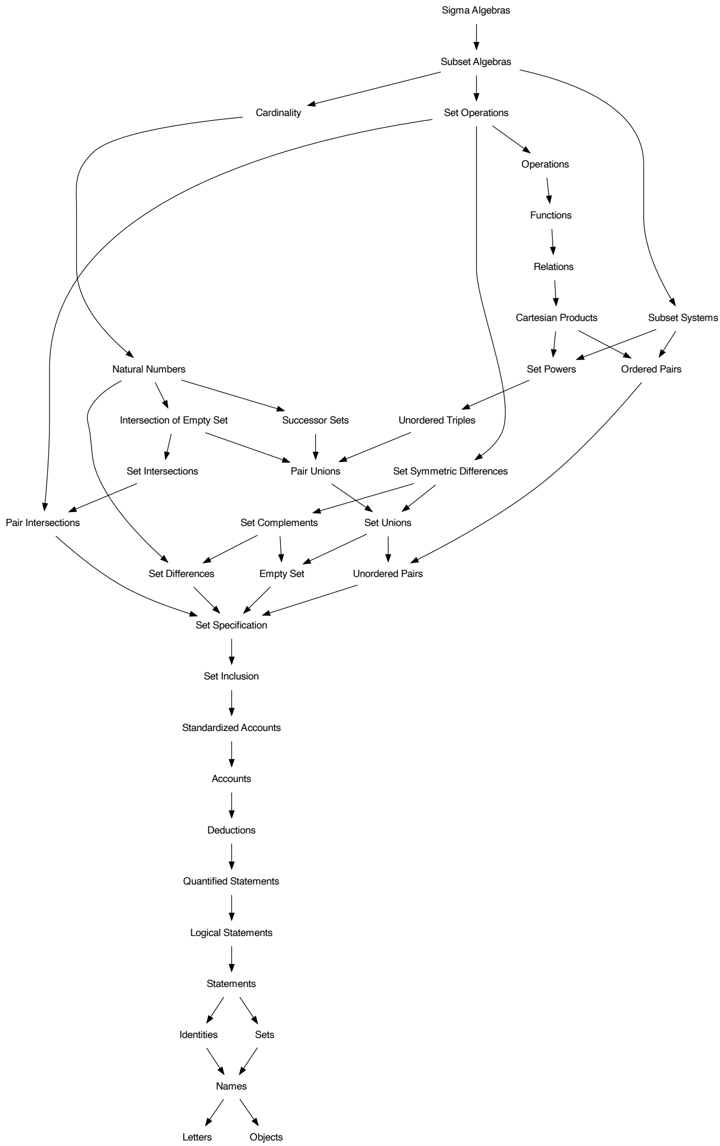
Prop. 1. *The intersection of a family of sigma algebras is a sigma algebra.*

Example 5. *For any infinite set A, let $\mathcal{A}$ be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

$\mathcal{A}$ is an algebra; the countable/co-countable algebra.[2]

[2]Future editions will clean up and modify these examples.



Sigma Algebras
Subset Algebras
Cardinality
Set Operations
Operations
Functions
Relations
Cartesian Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects