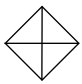


Why

Let $(A \in \mathbf{R}^{m\times m}, b \in \mathbf{R}^m)$ be an ordinarily reducible linear system with orindary reducer sequence $(L_1, \dots, L_{m-1})$. If $U = L_{m-1} \cdots L_2 L_1 A$, then

$$A = (L_{m-1} \cdots L_2 L_1)^{-1} U$$

is a factorization of U.

Inverting $L_{m-1} \cdots L_2 L_1$

Of course, assuming invertibility of the L_i,

$$(L_{m-1} \cdots L_2 L_1)^{-1} = L_1^{-1} L_2^{-1} \cdots L_{m-1}^{-1}.$$

So we are interested in the inverse of L_i for $i \leq m-1$.

Proposition 1. *L_i^{-1} is L_i with the subdiagonal entries negated.*

Proposition 2. *$L_k^{-1} L_{k+1}^{-1}$ is the unit lower-triangular matrix with the entries of both L_k^{-1} and L_{k+1}^{-1} in their usual places.*

Factorization perspective

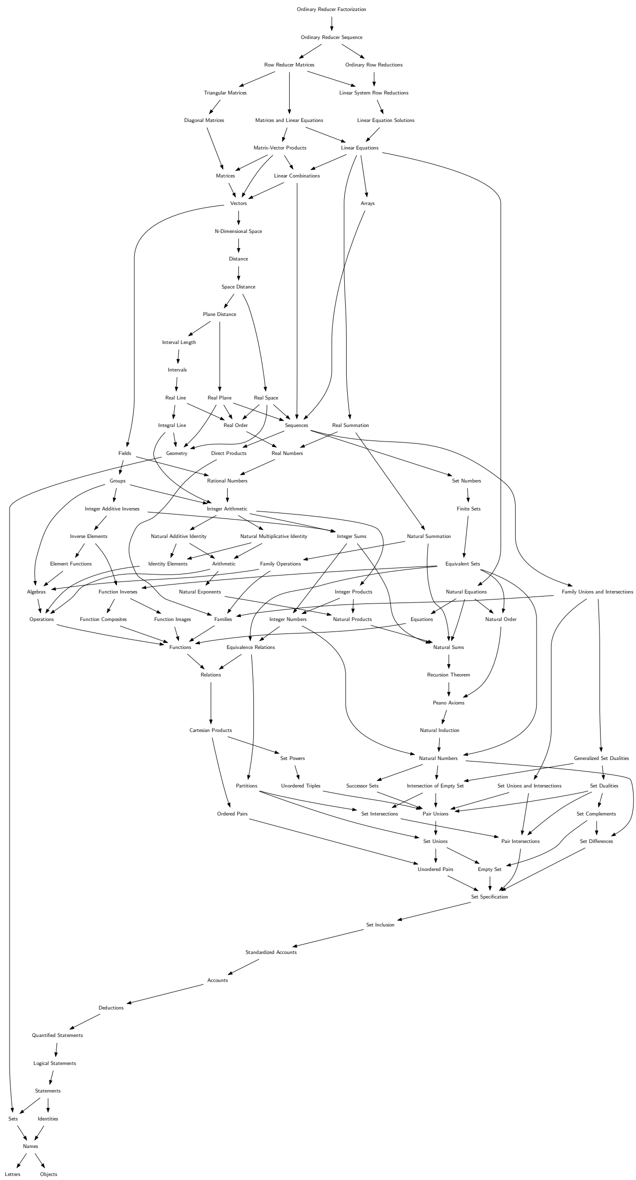
Since the matrix L_k has the form

$$L_k = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & -\ell_{k+1,k} & 1 & & \\ & & \vdots & & \ddots & \\ & & -\ell_{mk} & & & 1 \end{bmatrix}$$

where ℓ_{ij} are the row multipliers (see Ordinary Reducer Sequence), and immediate consequence Proposition 1 and Proposition 2 is that

$$L = L_1^{-1} L_2^{-1} \cdots L_{m-1}^{-1} = \begin{bmatrix} 1 & & & & \\ \ell_{21} & 1 & & & \\ \ell_{31} & \ell_{32} & 1 & & \\ \vdots & \vdots & \ddots & \ddots & \\ \ell_{m1} & \ell_{m2} & \cdots & \ell_{m,m-1} & 1 \end{bmatrix}$$

In other words, we have $A = LU$ where L is unit lower triangular and U is upper triangular. So we have factorized A in terms of a lower and upper triangular matrix.



Ordinary Reducer Factorization
Ordinary Reducer Sequence
Row Reducer Matrices
Ordinary Row Reductions
Triangular Matrices
Linear System Row Reductions
Diagonal Matrices
Matrices and Linear Equations
Linear Equation Solutions
Matrix-Vector Products
Linear Equations
Matrices
Linear Combinations
Vectors
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Sequences
Real Summation
Fields
Geometry
Direct Products
Real Numbers
Groups
Rational Numbers
Set Numbers
Integer Additive Inverses
Integer Arithmetic
Finite Sets
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Natural Summation
Element Functions
Identity Elements
Arithmetic
Family Operations
Equivalent Sets
Algebras
Function Inverses
Natural Exponents
Integer Products
Natural Equations
Family Unions and Intersections
Operations
Function Composites
Function Images
Families
Integer Numbers
Natural Products
Equations
Natural Order
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Unions and Intersections
Set Dualities
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Pair Intersections
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects