

## Uncertain Outcomes

## Why

We would like to speak about an object, which is a member of some set, and some attributes of this object—without knowing the precise identity of the object. Such language would be especially useful in discussing games of chance.

## Definition

Suppose  $\Omega$  is a set which includes all possible objects. We call it the sample space. We call an element of this set an outcome. Other terms for outcome include possibility, sample, elementary event, simple event, and sample point.

Often, one imagines an *experiment* as having many possible outcomes. We leave this term undefined.

## **Examples of modeling**

A model for flipping a coin. We want to talk about the result of flipping a coin. The coin has two sides. When we flip the coin, it lands heads or tails. We may model these outcomes with the set  $\{0,1\}$ . If the coin lands tails, we say that outcome 0 has occurred. If the coin lands heads, we say that outcome 1 has occurred.

A model for rolling a die. We want to talk about the result of rolling a die. The die has six sides. When we roll the die, it lands with one of its six sides facing up. We may model this uncertain outcome as an element of the set  $\{1, 2, 3, 4, 5, 6\}$ . Here we have used the first six natural numbers. We say that event 1 occurs if there is one pip showing, that event 2 occurs if there are two pips showing, and so on.

A model for rolling two dice at once. We want to talk about the result of a simultaneous throw of two dice. When we throw the dice, each one lands with one of six sides facing up. To model this uncertain outcome, we first number the dice: 1 and 2. We may model the sample space with

the set  $\Omega = \{1, ..., 6\}^2$ . We interpret  $(\omega_1, \omega_2) \in \Omega$  as follows:  $\omega_1$  is the number of pips showing on the first die—the die numbered 1—and  $\omega_2$  is the number of pips showing on the second die—the die labeled 2.

A model for rolling one die twice. We want to talk about the result of rolling one die twice. To model this uncertain outcome, we agree to speak of the *first* and *second* rolls. We are interested in the number of pips showing face up on the first and second rolls.

As before, we may model this uncertain outcome as an element of the set  $\Omega = \{1, \dots, 6\}^2$ . We interpret  $(\omega_1, \omega_2) \in \Omega$  so that  $\omega_1$  is the number of pips showing on the first roll and  $\omega_2$  is the number of pips showing on the second roll. It would be natural to refer to  $\omega_1$  as the *first* outcome, and  $\omega_2$  as the *second* outcome.

We emphasize that we have used the same set of outcomes as in the previous case. In other words, we can use the same set of outcomes to model two different situations.

