Why

What does it mean for two random variables to be independent? What are the events associated with a random variable? TODO

Definition

Defining Result

Prop. 1. *The set of inverse images the distinguished sets of a measurable space under a function from a set to that space is a sigma algebra.*

If the first set and the function are measurable, the sigma algebra is a sub sigma algebra of the domain sigma algebra.

Proof. Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Let $f : X \to Y$ be a measurable function. Define

$$\mathcal{C} := \{ f^{-1}(B) \mid B \in \mathcal{B} \}.$$

First, since $f^{-1}(Y) = X$, $X \in \mathcal{C}$.

Second, let $C \in \mathcal{C}$. Then there is B such that $C = f^{-1}(B)$. Then $X - C = X - f^{-1}(B) = f^{-1}(Y - B)$. Since $\mathcal{B}$ is a sigma algebra, $B \in \mathcal{B}, Y - B \in \mathcal{B}$ and so $X - C \in \mathcal{C}$.

Finally, let $(C_n)_n \subset \mathcal{C}$. Then for every n there exists a $B_n \in \mathcal{B}$ so that $C_n = f^{-1}(B_n)$. Then:

$$\cup_n C_n = \cup_n f^{-1}(B_n) = f^{-1}(\cup B_n).$$

Since $\mathcal{B}$ is a sigma algebra, $\cup_n B_n \in \mathcal{B}$ and so $\cup_n C_n \in \mathcal{C}$.

Since f is measurable, $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}$, and so $\mathcal{C} \subset \mathcal{A}$. $\qquad\square$

The sigma algebra *generated by a random variable* is the sigma algebra consisting of the inverse images of every measurable set of the codomain.

The sigma algebra generated by a family of random variables is the sigma algebra generated by the union of the sigma algebras generated individually by each of the random variables.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f : X \to Y$ be a random variable. Denote by $\sigma(f)$ the sigma algebra generated by f.

Results

Prop. 2. *The sigma algebra generated by a family of random variables is the smallest sigma algebra for with respect to which each random variable is measurably.*

Random Variable Sigma Algebra

Random Variables

Measurable Functions

Topological Sigma Algebra

Outcome Variables

Probability Measures

Topological Spaces

Measures

Metrics

Sigma Algebras

Extended Real Numbers

Distance

Real Limits

Space Distance

Event Probabilities

Absolute Value

Plane Distance

Probability Distributions

Interval Length

Intervals

Real Sequences

Real Line

Real Plane

Real Space

Uncertain Outcomes

Real Summation

Integral Line

Real Order

Sequences

Set Numbers

Geometry

Real Numbers

Rational Numbers

Direct Products

Finite Sets

Natural Summation

Integer Arithmetic

Family Operations

Subset Algebras

Equivalent Sets

Natural Multiplicative Identity

Natural Additive Identity

Identity Elements

Algebras

Set Operations

Function Inverse

Cardinality

Family Unions and Intersections

Integer Sums

Integer Products

Arithmetic

Families

Function Image

Function Composites

Comparisons

Natural Powers

Integer Numbers

Operations

Functions

Orders

Converse Relations

Equivalence Relations

Natural Products

Natural Order

Relations

Natural Sums

Recursion Theorem

Subset Systems

Set Products

Set Powers

Peano Axioms

Ordered Pairs

Unordered Triples

Natural Induction

Natural Numbers

Generalized Set Dualities

Set Symmetric Differences

Successor Sets

Set Dualities

Intersection of Empty Set

Partitions

Set Unions and Intersections

Set Complements

Pair Unions

Set Intersections

Pair Intersections

Set Differences

Set Unions

Unordered Pairs

Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects