



# CODES

## Why

Can we generalize the idea of flash codes.<sup>1</sup>

## Definition

Let  $X$  be a set and a  $A$  be an alphabet set. We denote the set of all finite sequences (strings) in  $A$  by  $\mathcal{S}(A)$ . We read  $\mathcal{S}(A)$  aloud as “the strings in  $A$ .” The length zero string is  $\emptyset$ .

A *code* for  $X$  in  $A$  is a function from  $X$  to  $\mathcal{S}(A)$ . In this context, we refer to the finite set  $A$  as an *alphabet* and we call  $c(x)$  the *codeword* of  $x$ . The *length* of  $x \in X$ , with respect to a code  $c : X \rightarrow \mathcal{S}(A)$ , is the length of the sequence  $c(x)$  (its codeword). We call a code *nonsingular* if it is injective.

## Examples

Define  $c : \{\alpha, \beta\} \rightarrow \{0, 1\}$  by  $c(\alpha) = (0, )$  and  $c(\beta) = (1, )$ .<sup>2</sup>

## Code extensions

Let  $s, t \in \mathcal{S}(A)$  of length  $m$  and  $n$  respectively. The *concatenation* of  $s$  with  $t$  is the length  $m + n$  string  $u \in \mathcal{S}(A)$  defined by  $u_1 = s_1, \dots, u_m = s_m$  and  $u_{m+1} = t_1, \dots, u_{m+n} = t_n$ . We denote the concatenation of  $s$  and  $t$  by  $st$ . Note, however, that  $st \neq ts$ , although  $s(tr) = (st)r$ .

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<sup>1</sup>The reliance of this sheet on Flash Codes and Dot-Dash Codes is for this justification, and not for any of the terms presented.

<sup>2</sup>Future editions will include additional examples.

Given a code  $c : X \rightarrow \mathcal{S}(A)$ , we can produce a code for  $\mathcal{S}(X)$  in a natural way. The *extension* of  $c$  is the function  $C : \mathcal{S}(X) \rightarrow \mathcal{S}(A)$  defined, for  $\xi = (\xi_1, \dots, \xi_n) \in \mathcal{S}(X)$ , by

$$C(\xi) = c(\xi_1) \cdots c(\xi_n).$$

We call an code *uniquely decodable* if its extension is injective. In other words, given the code  $C(\xi)$  for a sequence  $\xi \in \mathcal{S}(X)$ , we can recover  $\xi$ . We call  $C(\xi)$  the *encoding* of  $\xi$ . We call  $\xi$  the *decoding* of  $C(\xi)$ .

