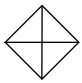


Why

It is natural to look for a class of structural equation models with favorable identifiability and properties.

1 Definition

A *d-dimensional rooted tree linear cascade* is a sequence of four objects: a tree on $\{1, \dots, d\}$, a vertex of the tree, a family of real numbers indexed by the edges of the tree, and a d-dimensional random vector whose covariance matrix is the identity matrix. The cascade is called "d-dimensional" because we associate it with a random vector (defined as a function of that in the form of its definition) whose codomain is R^d.

The tree together with the vertex form a rooted tree. The graph associated with the rooted tree and the family of real numbers together form a weighted graph.

The idea is to use the weights and the tree structure to recursively define a random vector in terms of elements of the given random vector. Let $C = (T, i, w, e)$ be a d-dimensional rooted tree linear cascade. So T is a tree on $\{1, \dots, d\}$, $i \in \{1, \dots, d\}$ and $w : T \to \mathsf{R}$, and $e : \Omega \to \mathsf{R}^d$ for some probability space $(A, \mathcal{A}, \mathbf{P})$. The random vector associated with C is the random variable $x : \Omega \to \mathsf{R}^d$ defined by

$$x_i = e_i \text{ and } x_j = w_{\{\mathsf{pa}\, j, j\}} x_{\mathsf{pa}\, j} + e_j \text{ for } j \neq i.$$

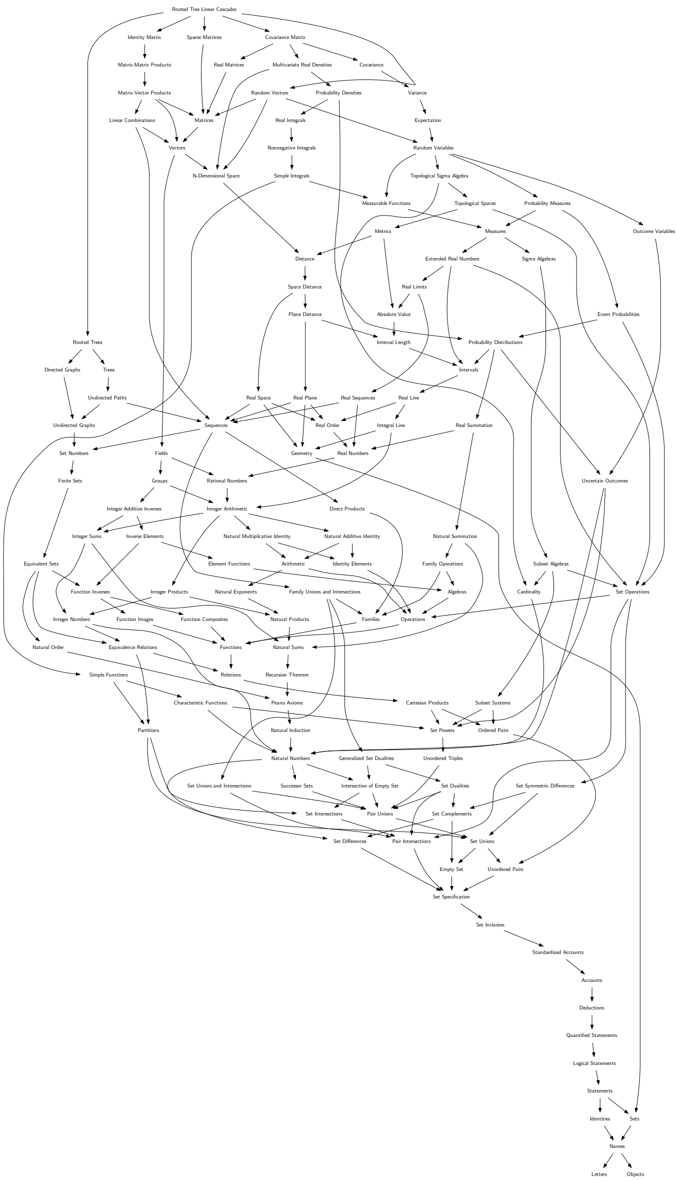
In other words,

$$e = Ax$$

where A is lower triangle and extremely sparse.

Notation

Let $(A, \mathcal{A}, \mathbf{P})$ be a probability space. Let $e : A \to \mathsf{R}^d$ be a random vector, let T be a tree on $\{1, \ldots, d\}$ with $a_{ij} = a_{ji}$ the weight on edge $\{i, j\} \in T$. We


Rooted Tree Linear Cascades
Identity Matrix
Sparse Matrices
Covariance Matrix
Matrix-Matrix Products
Real Matrices
Multivariate Real Densities
Covariance
Matrix-Vector Products
Random Vectors
Probability Densities
Variance
Linear Combinations
Matrices
Real Integrals
Expectation
Vectors
Nonnegative Integrals
Random Variables
N-Dimensional Space
Simple Integrals
Topological Sigma Algebra
Measurable Functions
Topological Spaces
Probability Measures
Metrics
Measures
Outcome Variables
Distance
Extended Real Numbers
Sigma Algebras
Space Distance
Real Limits
Plane Distance
Absolute Value
Event Probabilities
Rooted Trees
Interval Length
Probability Distributions
Directed Graphs
Trees
Intervals
Undirected Paths
Real Space
Real Plane
Real Sequences
Real Line
Undirected Graphs
Sequences
Real Order
Integral Line
Real Summation
Set Numbers
Fields
Geometry
Real Numbers
Finite Sets
Groups
Rational Numbers
Uncertain Outcomes
Integer Additive Inverses
Integer Arithmetic
Direct Products
Integer Sums
Inverse Elements
Natural Multiplicative Identity
Natural Additive Identity
Natural Summation
Equivalent Sets
Element Functions
Arithmetic
Identity Elements
Family Operations
Subset Algebras
Function Inverses
Integer Products
Natural Exponents
Family Unions and Intersections
Algebras
Cardinality
Set Operations
Integer Numbers
Function Images
Function Composites
Natural Products
Families
Operations
Natural Order
Equivalence Relations
Functions
Natural Sums
Simple Functions
Relations
Recursion Theorem
Characteristic Functions
Peano Axioms
Cartesian Products
Subset Systems
Partitions
Natural Induction
Set Powers
Ordered Pairs
Natural Numbers
Generalized Set Dualities
Unordered Triples
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects