



## ORDER AND ARITHMETIC

### Why

How does arithmetic preserve order?

### Results

The following are standard useful results.<sup>1</sup>

**Proposition 1.** *If  $m < n$ , then  $m + k < n + k$  for all  $k$ .*

**Proposition 2.** *If  $m < n$  and  $k \neq 0$ , then  $m \cdot k < n \cdot k$ .*

**Proposition 3** (Least Element). *If  $E$  is a nonempty set of natural numbers, there exists  $k \in E$  such that  $k \leq m$  for all  $m \in E$ .*

**Proposition 4** (Greatest Element). *If  $E$  is a nonempty set of natural numbers, there exists  $k \in E$  such that  $m \leq k$  for all  $m \in E$ .*

---

<sup>1</sup>The accounts of which will appear in future editions.



