



# Linear Functions

## 1 Why

Lots of things are (approximately) linear

## 2 Definition

A function between two vector spaces which share the same field is *linear* if the function applied to a linear combination of two vectors in the first space equals the linear combination of the results of the function (in the second place), using the same coefficients for the combination.

A linear function is always linear with respect to some field. The field is implicit, somewhat, in the definition but always present.

Linear functions are sometimes called *operators*

### 2.1 Notation

Let  $(V_1, F)$  and  $(V_2, F)$  be two vector spaces over the same field. Let  $f : V_1 \rightarrow V_2$ . We say that  $f$  is *linear* if

$$f(au + bv) = af(u) + bf(v)$$

for all  $a, b \in F$  and  $u, v \in V_1$ .