



# Integrable Function Space

## 1 Why

The integrable functions are a vector space.

## 2 Definition

The **integrable function space** corresponding to a measure space is the set of real-valued functions which are integrable with respect to the measure. The term space is appropriate because this set is a real vector space. If we scale an integrable function, it remains integrable. If we add two integrable functions, the sum is integrable. Thus, a linear combination of integrable functions is integrable. The zero function is the zero element of the vector space.

TODO The open question is: what elements of our geometric intuition can we bring to a space of functions. Do functions have a size? Are certain functions near each other?

## 2.1 Notation

Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $R$  denote the set of real numbers and let  $C$  denote the set of complex numbers.

We denote set the real-valued integrable functions on  $X$  by  $\mathcal{I}(X, \mathcal{A}, \mu, R)$ , read aloud as “the real integrable functions on the measure space  $X$  script  $\mathcal{A}$  mu.” We denote set the complex-valued integrable functions on  $X$  by  $\mathcal{I}(X, \mathcal{A}, \mu, C)$ , read aloud as “the complex integrable functions on the measure space  $X$  script  $\mathcal{A}$  mu.” When the field is irrelevant, we denote them by  $\mathcal{I}(X, \mathcal{A}, \mu)$ , read aloud as “integrable functions on the measure space  $X$  script  $\mathcal{A}$  mu.” The  $\mathcal{I}$  is a mnemonic for “integrable.”