Why

We can give the set of bounded linear functions between two norm spaces a norm.

Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

Notation

Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ be linear and bounded. The norm of f is the smallest C so that

$$\|f(v)\|_2 \leq C\|v\|_1.$$

Equivalent Formulation

Prop. 1. *Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ bounded and linear. The norm of f is*

$$\sup_{\|x\|_1=1} \|f(x)\|_2.$$

Bounded Linear Norm

Bounded Linear Transformations

Bounded Functions

Norms · Linear Transformations

Functionals · Transformations · Linear Combinations

Real Norm · Vectors · Real Linear Combinations

Absolute Value · Real Vectors

Plane Norm · N-Dimensional Space · Plane Vector Sums and Differences · Space Vectors

Distance · Plane Vectors

Space Distance

Plane Distance

Interval Length

Intervals · Real Space

Real Line · Real Plane

Integral Line · Real Order

Geometry · Rational Order · Real Numbers · Fields

Rational Numbers · Groups

Integer Arithmetic · Integer Additive Inverses · Lists

Integer Products · Natural Multiplicative Identity · Natural Additive Identity · Inverse Elements · Set Numbers

Arithmetic · Identity Elements · Element Function · Finite Sets · Direct Products

Natural Powers · Integer Sums · Algebra · Equivalent Sets

Natural Products · Integer Order · Comparisons · Operations · Function Inverses · Family Unions and Intersections

Natural Sums · Integer Numbers · Orders · Function Composites · Function Images · Families

Recursion Theorem · Natural Order · Converse Relations · Equivalence Relations · Functions

Peano Axioms · Relations

Natural Induction · Set Products · Generalized Set Dualities

Natural Numbers · Set Powers

Successor Sets · Unordered Triples · Set Dualities · Intersection of Empty Set · Partitions · Set Unions and Intersections

Ordered Pairs · Set Complements · Pair Union · Set Intersections

Set Differences · Set Unions · Pair Intersections

Unordered Pairs · Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets · Identities

Names

Letters · Objects