Definition

A *quadratic form* is a multivariate polynomial each term of which has degree two.

Representation

Let $f : \mathbf{R}^n \to \mathbf{R}$ be a quadratic form. There exists a matrix $A \in \mathbf{R}^{n \times n}$ so that

$$x^\top A x = \sum_{i,j} A_{ij} x_i x_j.$$

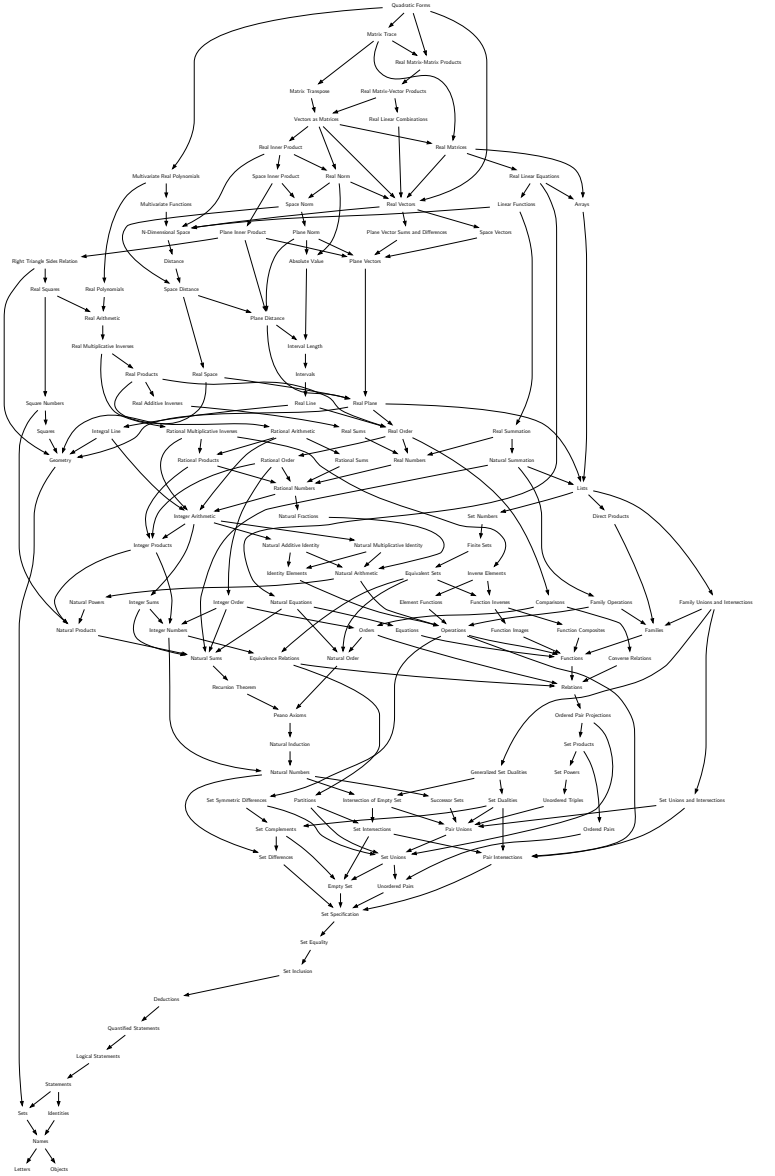
Suppose A is not symmetric. Then

$$f(x) = f(x)/2 + f(x)/2 = \frac{1}{2} x^\top A x + \frac{1}{2} x^\top A^\top x = x^\top (\frac{1}{2}(A + A^\top))x.$$

Define $B = 1/2(A^\top + A)$. We call B the *symmetric part* of A. Since every matrix A has a symmetric part, we can always assume that the matrix for a quadratic form is symmetric. If it is not, replace it with its symmetric part, obtaining the same function.

Under trace

Observe that $\operatorname{tr} x^\top A x = \operatorname{tr} A x x^\top = \operatorname{tr} x x^\top A$.


Quadratic Forms
Matrix Trace
Real Matrix-Matrix Products
Matrix Transpose
Real Matrix-Vector Products
Vectors as Matrices
Real Linear Combinations
Real Inner Product
Real Matrices
Multivariate Real Polynomials
Space Inner Product
Real Norm
Real Linear Equations
Multivariate Functions
Space Norm
Real Vectors
Linear Functions
Arrays
N-Dimensional Space
Plane Inner Product
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Plane Vector Sums and Differences
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Right Triangle Sides Relation
Distance
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Plane Vectors
Real Squares
Real Polynomials
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Squares Numbers
Real Additive Inverses
Real Line
Real Plane
Squares
Integral Line
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Real Summation
Geometry
Rational Products
Rational Order
Rational Sums
Real Numbers
Natural Summation
Rational Numbers
Lines
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Integer Order
Natural Equations
Element Functions
Function Inverses
Comparisons
Family Operations
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
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Operations
Function Images
Function Composites
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Natural Sums
Equivalence Relations
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Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Difference
Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects