



## LINEAR COMBINATIONS

### Why

We want to build vectors out of other vectors using scalar multiplication and vector addition.

### Definition

A *linear combination from* a vector space is an ordered pair: the first coordinate is a sequence of vectors and the second is a sequence of scalars. The *result* of a linear combination is the sum of the results of scaling each vector by the corresponding scalar in the sequence; itself a vector in the space. A *linear combination of a set* of vectors a linear combination consisting of a numbering of them and a scalar sequence.

A *trivial linear combination* is one whose sequence of scalars is zero at each coordinate. The result of any trivial linear combination is the zero vector. A *nontrivial linear combination* is one which is not trivial. In other words, to be nontrivial, there must exist at least one index of the scalar sequence whose corresponding value is nonzero.

We say that a given vector *can be written as a linear combination of* a sequence of vectors if there exists a sequence of scalars such that the result of the linear combination of that sequence of vectors and scalars is the given vector. In other words, a vector can be written as a linear combination of some other vectors if there exists scalars for those other vectors such that scaling them and adding the results gives the vector.

## Notation

Let  $(V, \mathbf{F})$  be a vector space. Let  $v = (v_1, \dots, v_n)$  be a sequence of vectors in  $V$  and  $a = (a_1, \dots, a_n)$  be a sequence of scalars in  $\mathbf{F}$ . Then  $(v, a)$  is a linear combination and we can express its result by

$$a_1v_1 + a_2v_2 + \dots + a_nv_n.$$

If  $(v, a)$  is trivial, then  $a_i = 0$  for  $i \in \{1, 2, \dots, n\}$  and the result of  $(v, a)$  is 0 (the zero vector). Otherwise, there exists  $i \in \{1, 2, \dots, n\}$  such that  $a_i \neq 0$ ; of course, the result of  $(v, a)$  may still be 0.

We say that a vector  $u$  can be written as a linear combination of the vectors  $v_1, v_2, \dots, v_n$  if there exists scalars  $a_1, a_2, \dots, a_n$  such that the result of the linear combination  $(v, a)$  is  $u$ . Which we express

$$u = a_1v_1 + a_2v_2 + \dots + a_nv_n.$$

If  $U$  is a finite set of vectors, then we often say “Let  $\{u_1, \dots, u_n\}$  be a (finite) set of vectors.” In this notation, we imply a numbering, and the set notation is intended to indicate that  $u_i \neq u_j$  if  $i \neq j$  for  $i, j \in \{1, 2, \dots, n\}$ . If  $u = (u_1, \dots, u_n)$  then a linear combination of  $\{u_1, \dots, u_n\}$  is any pair  $(u, a)$  where  $a = (a_1, \dots, a_n)$  is a scalar sequence.

## Relationships

TODO span equivalence

