

## ♣ Normal Conditionals

## 1 Why

What of the conditional densities of a multivariate normal density.

## 2 Result

**Proposition 1.** Let  $f : \mathbb{R}^d \to \mathbb{R}$  be a normal density with mean  $\mu \in \mathbb{R}^d$  and covariance  $\Sigma \in \mathbb{S}^d$ .

$$\mu = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}$$
 and  $\Sigma = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$ 

The conditional density  $f_{x|y}(\xi,\gamma)$  is is  $\mathcal{N}(\bar{\mu}(\gamma),\bar{\Sigma})$  where

$$\bar{\mu}(\gamma) = \mu_1 + \Sigma_{xy} \Sigma_{yy}^{-1} (\gamma - \mu_y) \text{ and } \bar{\Sigma} = \Sigma_{xx} - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx}.$$

Conditional Event Probability