Why

We can consider intersections of more than two sets.

Definition

Let $\mathcal{A}$ denote a set of sets. In other words, every element of $\mathcal{A}$ is a set. And suppose that $\mathcal{A}$ has at least one set (i.e., $\mathcal{A} \neq \varnothing$). Let C denote a set such that $C \in \mathcal{A}$. Then consider the set,

$$\{x \in C \mid (\forall A)(A \in \mathcal{A} \longrightarrow x \in A)\}.$$

This set exists by the principle of specification (see Set Specification). Moreover, the set does not depend on which set we picked. So the dependence on C does not matter. It is unique by the axiom of extension (see Set Equality). This set is called the *intersection* of $\mathcal{A}$.

Notation

We denote the intersection of $\mathcal{A}$ by $\bigcap \mathcal{A}$.

Equivalence with pair intersections

As desired, the the set denoted by $\mathcal{A}$ is a pair (see Unordered Pairs) of sets, the pair intersection (see Pair Intersections) coincides with intersection as we have defined it in this sheet.[1]

Proposition 1. $\bigcap\{A, B\} = A \cap B$

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[1] A full account of the proof will appear in future editions.

Set Intersections

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Pair Intersections

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Set Specification

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Set Equality

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Set Inclusion

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Standardized Accounts

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Accounts

↓

Deductions

↓

Quantified Statements

↓

Logical Statements

↓

Statements

Identities   Sets

Names

Letters   Objects