Why

We want to summarize the interaction between two binary traits.

Definition

The *contingency table* of a population $\{1, \ldots, n\}$ with respect to binary traits $a, b : \{1, 2, \ldots, n\} \to \{0, 1\}$ is the array $A \in \mathbf{N}^{2\times 2}$ of natural numbers defined by

$$A = \begin{bmatrix} \left|a^{-1}(0) \cap b^{-1}(0)\right| & \left|a^{-1}(0) \cap b^{-1}(1)\right| \\ \left|a^{-1}(1) \cap b^{-1}(0)\right| & \left|a^{-1}(0) \cap b^{-1}(1)\right| \end{bmatrix}.$$

We interpret A_{11} as the number of individuals which have neither trait, A_{12} as the individuals which have trait b but not trait a, and so on.

Normalization

These four sets partition $\{1, 2, \ldots, n\}$, so that if we divide the elements by n, we obtain four numbers which sum to 1, the 2 by 2 table with these entries is called the *normalized contingency table*.

Contingency arrays

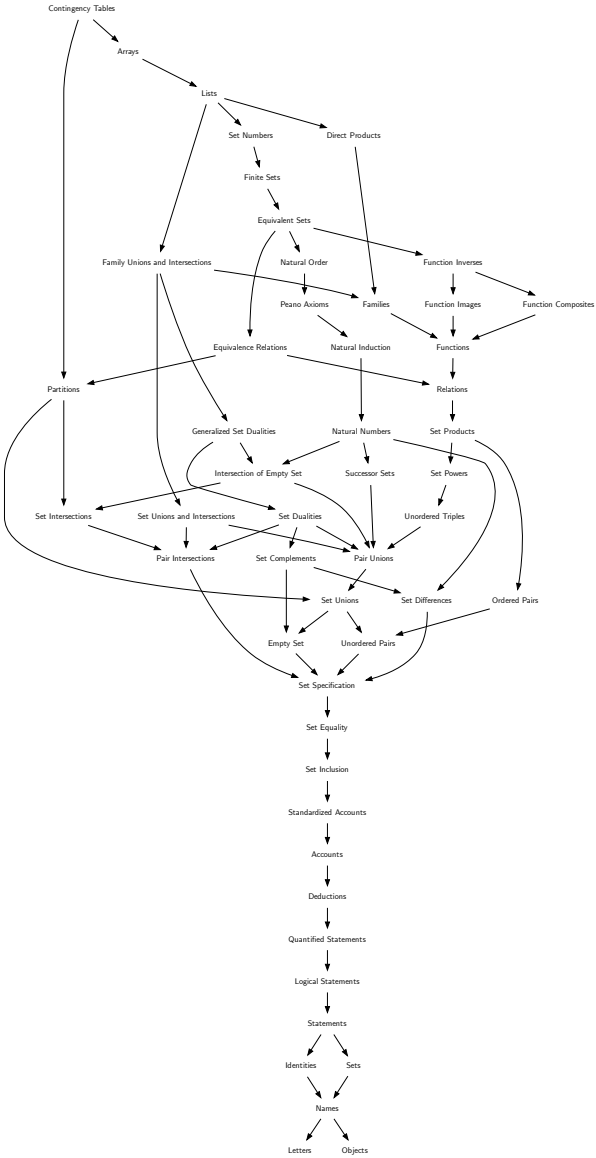
In general, we have k binary traits, each of which an individual may or may not have. We encode these traits using k functions

$$a_1, \ldots, a_k : \{1, 2, \ldots, n\} \to \{0, 1\}.$$

The contingency array is k-dimensional array A, with

$$A_x = \cap_{j=1}^{k} a_j^{-1}(x_j),$$

where $x \in \{0, 1\}^k$.


Contingency Tables
Arrays
Lists
Set Numbers
Direct Products
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Peano Axioms
Families
Function Images
Function Composites
Equivalence Relations
Natural Induction
Functions
Partitions
Relations
Generalized Set Dualities
Natural Numbers
Set Products
Intersection of Empty Set
Successor Sets
Set Powers
Set Intersections
Set Unions and Intersections
Set Dualities
Unordered Triples
Pair Intersections
Set Complements
Pair Unions
Set Unions
Set Differences
Ordered Pairs
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects