



Why

We want to talk about several objects in order.

Definition

Suppose A is a set. A *list* (or *finite sequence*, *n -tuple*, *string*, *dataset*) in A (or *of* elements from or of A) is a function

$$a : \{1, \dots, n\} \rightarrow A.$$

In other words, a list is a *family* whose index set is $\{1, \dots, n\}$. The *length* (or *size*) of the list is the size of its domain. The *k th entry* (or *term*, *record*) of A is the result a_k of k ; here $k \in \{1, \dots, n\}$.

Notation

Since the natural numbers are **naturally ordered**, we denote lists using this order, from left to right, between parentheses. For example, we denote the function $a : \{1, \dots, 4\} \rightarrow A$ by (a_1, a_2, a_3, a_4) .

Orderings and numberings

Let A be a set with $|A| = n$. A sequence $a : \{1, \dots, n\} \rightarrow A$ is an *ordering* of A if a is invertible. In this case, we call the inverse a *numbering* of A . An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

Relation to Direct Products

A *natural direct product* is a product of a list of sets. We denote the direct product of a list of sets A_1, \dots, A_n by $\prod_{i=1}^n A_i$. If each A_i is the same set A , then we denote the product $\prod_{i=1}^n A_i$ by A^n . The direct product A^n is set of lists in A .

Natural unions and intersections

We denote the family union of the list of sets A_1, \dots, A_n by $\cup_{i=1}^n A_i$. Similarly, we denote the intersection by $\cap_{i=1}^n A_i$.

Slices

An *index range* for a list s of length n is a pair (i, j) for which $1 \leq i < j \leq n$. The *slice* corresponding to (i, j) is the length $j - i$ list s' defined by $s'_1 = s_i, s'_2 = s_{i+1}, \dots, s'_j = s_{i+j-1}$.

We denote the (i, j) -slice of s by $s_{i:j}$. If $i = 1$ we use $s_{:j}$ and if $j = n$ we use $s_{i:}$ as shorthands for the slices $s_{1:j}$ and $s_{i:n}$.

