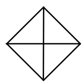


Definition

A subset of $\mathbf{R}^n$ is *affine* if it contains the lines through each of its points. Our definition is meant to capture the intuitive idea of an infinite, uncurved set, like a line or a plane in space.

Other terms in use include *affine manifold*, *affine variety*, *linear variety*, or *flat*. Some authors use the term *affine subspace* or *affine space*. We avoid this terminology because affine sets are not subspaces. Instead, the affine sets are the translated subspaces.

Examples

The empty set is trivially an affine set. The entire set of points in n-dimensional space is an affine set. Any singleton is an affine set.

Notation

As usual, let $L(x, y)$ denote the line between $x, y \in \mathbf{R}^n$. The set $M \subset \mathbf{R}^n$ is affine if $L(a, b) \subset M$ for all $a, b \in M$. For any $x, y \in M$ and $\lambda \in \mathbf{R}$,

$$(1-\lambda)x + \lambda y \in M.$$

Translations

The set of affine sets is closed under translation.

Proposition 1. *Let $M \subset \mathbf{R}^n$ affine. Then $M + a$ is affine for any $a \in \mathbf{R}$.*

Proof. Suppose $x, y \in M + a$. There exist $\tilde{x}, \tilde{y} \in M$ with $x = \tilde{x} + a$ and $y = \tilde{y} + a$. For any $\lambda \in \mathbf{R}$,

$$\begin{aligned}(1-\lambda)x + \lambda y &= (1-\lambda)(\tilde{x} + a) + \lambda(\tilde{y} + a) \\ &= ((1-\lambda)\tilde{x} + \lambda\tilde{y}) + a.\end{aligned}$$

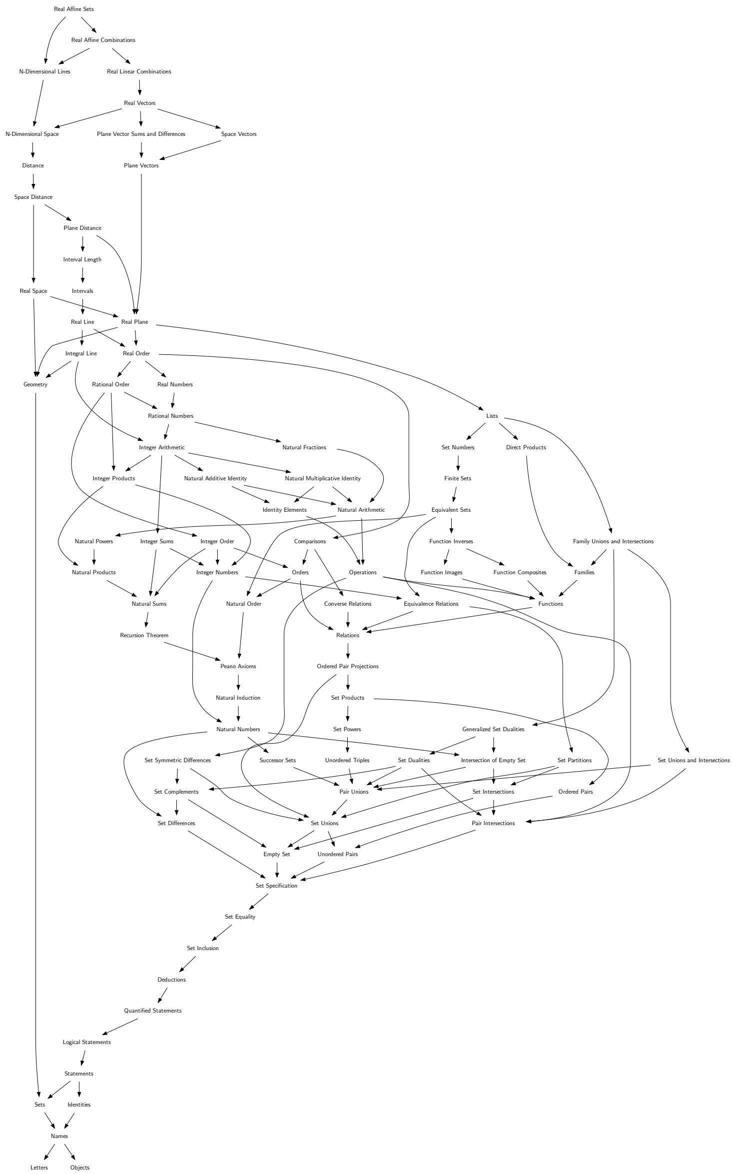
□

An affine set M is *parallel* to an affine set L if there exists $a \in \mathbf{R}^n$ so that $M = L + a$. The relation “M is parallel to L” on the set of affine sets is an equivalence relation. This notion of parallelism is more restrictive than the natural one, in which we may speak of a line being parallel to a plane. We must speak of a line which is parallel to another line within the given plane.

Properties

The set of affine sets is closed under intersection.

Proposition 2. *If $\mathcal{M}$ is a family of affine sets, then $\bigcap \mathcal{M}$ is affine.*


Real Affine Sets
Real Affine Combinations
N-Dimensional Lines
Real Linear Combinations
Real Vectors
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Set Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects