Why

Since we have a partial order on the set of positive semidefinite matrices, we can study which familiar functions are have order-preserving or order-reversing properties.

Norms

It would be nice if the matrix norm induced by the matrix scalar produce (see Matrix Scalar Product) was an isotonic function. In other words, if $A, B \in \mathbf{S}^d$ satisfy $A \geq B$, does $\|A\| \geq \|B\|$?

Since $\|A\|^2 = \operatorname{tr} A^2$, we should study the trace first..

Trace

Proposition 1. *Let $f : \mathbf{S}^d \to \mathbf{R}$ defined by $f(A) = \operatorname{tr} A$.*

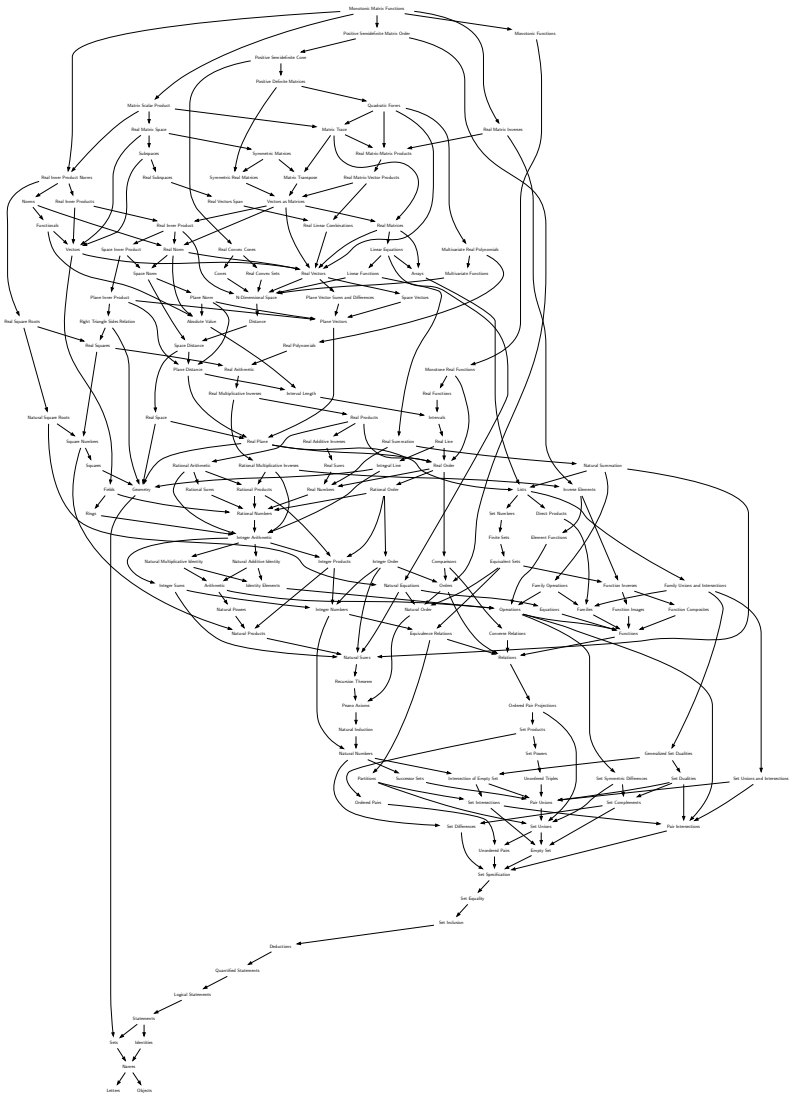
In other words, the function f is the restriction of the trace function onto the set of symmetric matrices.

Proposition 2. *Let $B \in \mathbf{S}^d$ Let $f_B : \mathbf{S}^d \to \mathbf{R}$ defined by $f(A) = \operatorname{tr} AB$.*

Inversion

Proposition 3. *Let $A \in \mathbf{S}^d_{++}$. Then the map $f : \mathbf{S}^d_{++} \to \mathbf{S}^d_{++}$ satisfying $f(A) = A^{-1}$ is an isotonic function mapping the (open) positive definite cone into itself.*[1]

[1] Future editions will include a proof.


Monotonic Matrix Function
Positive Semidefinite Matrix Order
Monotonic Functions
Positive Semidefinite Cone
Positive Definite Matrices
Matrix Scalar Product
Quadratic Forms
Real Matrix Space
Matrix Trace
Real Matrix Inverses
Subspaces
Symmetric Matrices
Real Matrix-Matrix Products
Real Inner Product Spaces
Real Subspaces
Symmetric Real Matrices
Matrix Transpose
Real Matrix-Vector Products
Norms
Real Inner Products
Real Vectors Space
Vectors as Matrices
Functionals
Real Inner Product
Real Linear Combinations
Real Matrices
Vectors
Space Inner Product
Real Norm
Real Convex Cones
Linear Equations
Multivariate Real Polynomials
Space Norm
Cones
Real Convex Sets
Real Vectors
Linear Functions
Arrays
Multivariate Functions
Plane Inner Product
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Real Square Roots
Right Triangle Ratio Relation
Absolute Value
Distance
Plane Distance
Real Polynomials
Real Square
Space Distance
Plane Distance
Real Arithmetic
Monotone Real Functions
Real Multiplication Inverses
Interval Length
Real Functions
Real Products
Intervals
National Square Roots
Real Sums
Real Plane
Real Addition Inverses
Real Summation
Real Line
Integer Numbers
Square
Rational Arithmetic
Rational Multiplication Inverses
Real Sums
Integral Line
Real Order
Natural Summation
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Rational Products
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Some Elements
Odd
Rational Numbers
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Direct Products
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Finite Sets
Element Functions
Integer Arithmetic
Equivalent Sets
Natural Multiplication Identity
Natural Additive Identity
Integer Products
Integer Order
Comparisons
Integer Sums
Arithmetic
Identity Elements
Natural Equations
Order
Family Operations
Function Inverses
Family Unions and Intersections
Natural Powers
Integer Numbers
Natural Order
Operations
Equations
Functions
Function Images
Function Composites
Natural Products
Equivalence Relations
Converse Relations
Functions
Relations
Recursion Theorem
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Set Powers
Natural Numbers
Generalized Set Unions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Symmetric Differences
Set Unions and Intersections
Set Doubles
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Numbers
Letters
Digits