Why

We generalize the notion of angle between vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$ to vectors in $\mathbf{R}^n$.

Definition

The *angle* (*unsigned angle*) between the nonzero vectors $x, y \in \mathbf{R}^n$, is the real knumber

$$\theta = \angle(x, y) = \cos^{-1} \frac{x^\top y}{\|x\|\|y\|}.$$

In the case that one (or both) of the vectors is zero, we define the angle between them to be 0. Thus, $x^\top y = \|x\|\|y\| \cos\theta$, which is a convenient way to remember the inner product norm inequality.

Terminology

x and y are *aligned* if $\theta = 0$, in which case $x^\top y = \|x\|\|y\|$. In the case that $x \neq 0$, x and y are aligned if $x = \alpha y$ for some $\alpha \geq 0$. x and y are *opposed* if $\theta = \pi$, in which case $x^\top y = \|x\|\|y\|$. In the case that $x \neq 0$, x and y are opposed if $x = -\alpha y$ for some $\alpha \geq 0$. Two nonnzero vectors x and y are *orthogonal* if $\theta = \pi/2$ or $-\pi/2$, in which case $x^\top y = \|x\|\|y\|$. The origin is orthogonal to every other vector. We denote that two vectors x and y are orthogonal by $x \perp y$.

Real Vector Angles

Real Norm

Space Vector Angles

Real Vectors

Space Norm

Plane Vector Angles

N-Dimensional Space

Space Vectors

Plane Vector Sums and Differences

Plane Norm

Distance

Plane Vectors

Absolute Value

Space Distance

Plane Distance

Interval Length

Real Space

Intervals

Real Plane

Real Line

Real Order

Integral Line

Geometry

Rational Order

Real Numbers

Rational Numbers

Lists

Integer Arithmetic

Set Numbers

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Finite Sets

Identity Elements

Arithmetic

Direct Products

Equivalent Sets

Natural Powers

Family Unions and Intersections

Function Inverses

Comparisons

Integer Order

Integer Sums

Natural Products

Families

Function Composites

Function Images

Operations

Orders

Integer Numbers

Natural Sums

Functions

Converse Relations

Equivalence Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Set Products

Natural Induction

Generalized Set Dualities

Set Powers

Natural Numbers

Set Unions and Intersections

Set Dualities

Unordered Triples

Successor Sets

Intersection of Empty Set

Partitions

Set Symmetric Differences

Ordered Pairs

Pair Unions

Set Intersections

Set Complements

Pair Intersections

Set Unions

Set Difference

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Standardised Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects

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