Why

We want a set which corresponds to our notion of points on a line.[1]

Definition

First, call a subset R of $\mathbf{Q}$ a *rational cut* if $R \neq \varnothing$, $R \neq \mathbf{Q}$, for all $q \in R$, $r \leq q \longrightarrow r \in R$, and R has no greatest element. Briefly, the intuition is that the point is the set of all rationals to the left.[2]

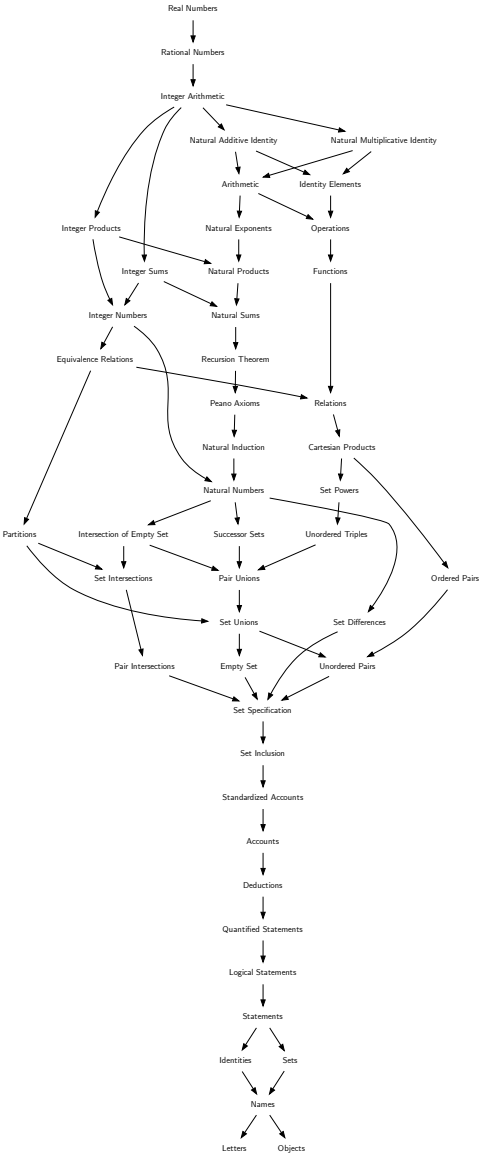
The *set of real numbers* is the set of all rational cuts. This set exists by an application of the principle of selection (see Sect Selection to the power set (see Set Powers) of $\mathbf{Q}$. We call an element of the set of real numbers a *real number* or a *real*. We call the set of real numbers the *set of reals* or *reals* for short.

Notation

We follow tradition and denote the set of real numbers by $\mathbf{R}$, likely a mnemonic for "real."

[1] Future editions will modify and expand this justification.

[2] This brief intuition will be expanded upon in future sheets.


Real Numbers
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Integer Products
Natural Exponents
Operations
Integer Sums
Natural Products
Functions
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Ordered Pairs
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects