Why

What is the projection of a vector onto an affine set?

Result

Proposition 1. *Suppose $a \in \mathbf{R}^n$ and $U \in \mathbf{R}^{n \times k}$ with $U^\top U = I$. Define the affine set $W(a, U) = a + \mathrm{range}(U)$. Then*

$$\mathrm{proj}_{W(a,U)}(x) = UU^\top x + (I - UU^\top)a.$$

Proof. The minimizer of $J : \mathbf{R}^n \to \mathbf{R}$ defined by

$$J(z) = \|a + Uz - x\| = \|Uz - (x - a)\|,$$

is $z^\star = U^\top(x - a)$. So the projection of x onto $W(a, U)$ is

$$a + Uz^\star = a + UU^\top(x - a) = UU^\top x + (I - UU^\top)a.$$

$\square$

Projections On Affine Sets

Real Affine Set Representations

Affine Set Dimension

Subspace Dimension

Projections On Subspaces

Vector Space Dimension

Real Subspace Representations

Real Vector Projections

Real Matrices with Orthonormal Columns

Vector Space Basis

Affine Sets and Subspaces

Real Matrix-Matrix Products

Span

Linearly Dependent Vectors

Real Translation

Real Matrix-Vector Products

Subspace

Real Matrix Range

Orthonormal Set of Real Vectors

Vectors as Matrices

Real Subspaces

Real Inner Product

Real Vectors Span

Linear Combinations

Real Matrices

Affine Sets

Real Norm

Real Linear Combinations

Real Vectors

Linear Equations

N-Dimensional Lines

Space Norm

Real Vectors

Linear Functions

Optimization Problems

N-Dimensional Space

Plane Norm

Plane Inner Product

Plane Vector Sums and Differences

Space Vectors

Distance

Right Triangle Sides Relation

Plane Vectors

Extended Real Numbers

Space Distance

Real Squares

Real Functions

Real Limits

Arrays

Plane Distance

Absolute Value

Real Arithmetic

Real Space

Interval Length

Real Multiplication Inverses

Intervals

Real Products

Real Sequences

Square Numbers

Real Plane

Real Lines

Real Additive Inverses

Greatest Lower Bounds

Sequences

Square

Integral Law

Rational Multiplication Inverses

Rational Arithmetic

Real Order

Real Sums

Real Summation

Lists

Geometry

Rational Products

Rational Sums

Rational Order

Real Numbers

Fields

Set Number

Direct Products

Natural Numbers

Rings

Finite Sets

Equivalent Sets

Integer Arithmetic

Integer Products

Natural Multiplication Identity

Natural Addition Identity

Integer Order

Natural Summation

Inverse Elements

Optimizers

Comparisons

Natural Equations

Function Inverses

Family Union and Intersection

Integer Sums

Arithmetic

Identity Elements

Integer Numbers

Family Operations

Element Functions

Natural Order

Function Images

Function Composition

Family

Natural Powers

Equations

Natural Equation

Functions

Converse Relations

Equivalence Relations

Natural Products

Natural Sums

Relation

Recursion Theorem

Peano Axioms

Set Products

Natural Induction

Natural Numbers

Set Powers

Generalized Set Dualities

Partitions

Set Unions and Intersections

Set Symmetric Differences

Successor Sets

Unordered Triples

Set Dualities

Intersection of Empty Set

Set Complements

Pair Unions

Ordered Pairs

Set Intersections

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Nonrelational Accounts

Accounts

Deduction

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects