



Definition

A set of vectors $\{v_1, \dots, v_k\}$ is a *basis* for a subspace $S \subset \mathbf{R}^n$ if

$$S = \text{span}\{v_1, \dots, v_k\} \quad \text{and} \quad \{v_1, \dots, v_k\} \text{ is independent.}$$

This definition captures two competing properties. The first is that the set is large, in the sense that any vector in S can be represented as a linear combination of vectors in $\{v_1, \dots, v_k\}$. Simultaneously, the set is small, in the sense that no vector in the set is a linear combination of the others. In other words, there is no extra vector in the set.

Linear independence is equivalent to uniqueness of representation of the vectors representable as a linear combination of v_1, \dots, v_k . In other words, $\{v_1, \dots, v_k\}$ is a basis for S if each vector $x \in S$ can be uniquely expressed as

$$x = \alpha_1 v_1 + \dots + \alpha_k v_k.$$

