



## Why

We want to visualize (nonsymmetric) relations.

## Definition

A *directed graph* is a pair  $(V, E)$  in which  $V$  is a finite nonempty set and  $E$  is a subset of  $V \times V$ . In other words,  $E$  is a relation on  $V$ . We call the elements of the first set the *vertices* of the graph and the elements of the second set the *edges*.

Let  $(v, w) \in E$ . We say that  $(v, w)$  is an edge *from*  $v$  *to*  $w$ , and that it is an *outgoing edge* of  $v$  and an *incoming edge* of  $w$ . We call  $v$  a *parent* of  $w$  and we call  $w$  a *child* of  $v$ . We say that the edge  $(v, w)$  is *incident* to  $v$  and  $w$ .

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

## Notation

Let  $\text{pa} : V \rightarrow \mathcal{P}(V)$  and  $\text{ch} : V \rightarrow \mathcal{P}(V)$  be the functions associating to each vertex its set of parents and set of children, respectively. As usual, we denote the parents of vertex  $v$  by  $\text{pa}_v$  and the children by  $\text{ch}_v$ .

## Skeletons

The *skeleton* of the directed graph  $(V, E)$  is the undirected graph  $(V, F)$  where

$$F = \{\{v, w\} \subset V \mid (v, w) \in E \text{ or } (w, v) \in E\}.$$

In other words, the skeleton is an undirected graph whose vertex set is  $V$  and whose edges are all (unordered) pairs which appear as an ordered pair in the directed graph. If the  $(V, E)$  is a directed graph and  $E$  is a symmetric relation, then we the skeleton of  $(V, E)$  is a natural undirected graph to associate with  $(V, E)$ . An *orientation* of an undirected graph  $G$  is a directed graph whose skeleton is  $G$ .

