



Why

We want to multiply real numbers.¹

Definition

The *real product* of two real numbers R and S is defined

1. if R or S is $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$, then the $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$
2. otherwise,
 - (a) if R or S is $0_{\mathbf{R}}$, then $0_{\mathbf{R}}$.
 - (b) if $R, S \neq 0_{\mathbf{R}}$ and $0_{\mathbf{S}} \in R, S$, let T be $\{t \in \mathbf{Q} \mid r \in R, s \in S, r, s \geq 0_{\mathbf{Q}}, t = r \cdot s\}$ then $T \cup \{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\}$ ²
 - (c) If $R, S \neq 0_{\mathbf{R}}$, $0_{\mathbf{R}} \in R$ and $0_{\mathbf{R}} \notin S$, then the additive inverse of the product of $-R$ with S .
 - (d) If $R, S \neq 0_{\mathbf{R}}$, $0_{\mathbf{R}} \notin R$ and $0_{\mathbf{R}} \in S$, then the additive inverse of the product of R with $-S$.
 - (e) If $R, S \neq 0_{\mathbf{R}}$, and $0_{\mathbf{R}} \notin R, S$, then the product of $-R$ with $-S$.

Notation

We denote the product of two real numbers x and y by $x \cdot y$.

Properties

Proposition 1 (Associative). $x + (y + z) = (x + y) + z$

Proposition 2 (Commutative). $x + y = y + x$

¹Future editions will expand.

²We use \geq in the usual way, it will be defined earlier in future editions.

Proposition 3 (Identity). *The set of all rationals less than $1_{\mathbf{Q}}$ is the multiplicative identity.*

We denote the the multiplicative identity by $1_{\mathbf{R}}$. When it is clear from context, we call $1_{\mathbf{R}}$ “one”.

