Why

What is the best linear predictor if we choose according to a weighted squared loss function.

Definition

Suppose we have a paired dataset of n records with inputs in R^d and outputs in R. A *weighted least squares linear predictor* for nonnegative weights $w \in \mathsf{R}^n$, $w \geq 0$, is a linear transformation $f : \mathsf{R}^d \to \mathsf{R}$ (the field is R) which minimizes

$$\frac{1}{n} \sum_{i=1}^{n} w_i (y_i - x^\top a^i)^2.$$

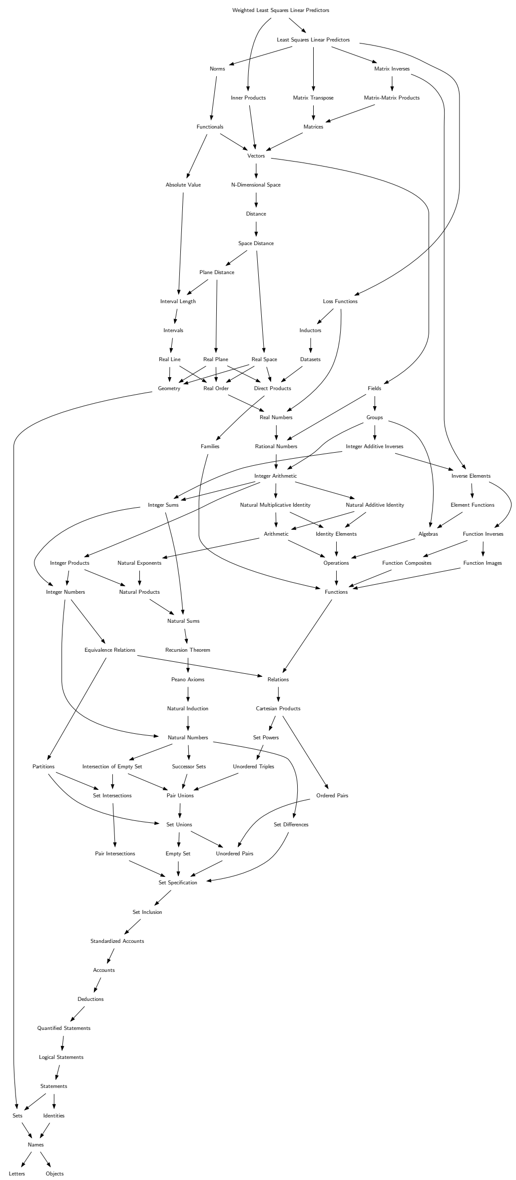
Some authors refer to this process of selecting a linear predictor as the *weighted least-squares problem.*

Define $W \in \mathsf{R}^{n \times n}$ so that $W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix. We are minimizing

$$|W(Ax - y)|$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by* $\left(A^\top W A^\top\right)^{-1} A^\top W y$.


Weighted Least Squares Linear Predictors
Least Squares Linear Predictors
Norms
Matrix Inverses
Inner Products
Matrix Transpose
Matrix-Matrix Products
Functionals
Matrices
Vectors
Absolute Value
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Loss Functions
Intervals
Inductors
Real Line
Real Plane
Real Space
Datasets
Geometry
Real Order
Direct Products
Fields
Real Numbers
Groups
Families
Rational Numbers
Integer Additive Inverses
Integer Arithmetic
Inverse Elements
Integer Sums
Natural Multiplicative Identity
Natural Additive Identity
Element Functions
Arithmetic
Identity Elements
Algebras
Function Inverses
Integer Products
Natural Exponents
Operations
Function Composites
Function Images
Integer Numbers
Natural Products
Functions
Natural Sums
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
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Letters
Objects