Tree-Structured Density Approximation

1 Why

2 Problem

Tree-structured density approximation is an optimization problem in which we select a tree-structured density minimizing differential entropy relative to a given density over.

2.1 Notation

Let $g : \mathbf{R}^n \to \mathbf{R}$ be a density. Let d denote the differential relative entropy. We want to

find	density f and tree T on $\{1, \ldots, n\}$
minimizing	$d(g, f)$
subject to	f factors according to T.

3 Solution

Proposition 1. *Let $g : \mathbf{R}^n \to \mathbf{R}$ be a density. Let T be a tree on $\{1, \ldots, n\}$. Let p_j be the parent of vertex j for the T rooted*

at vertex i, $j = 1, \ldots, n$ *and* $j \neq i$. *Then the density* $f : \mathbf{R}^d \to \mathbf{R}$ *defined by*

$$f = g_i \prod_{j \neq i} g_{j|p_j}$$

achieves minimum differential entropy relative to g *among all densities which factor according to* T.

Proposition 2. *Let* $g : \mathbf{R}^n \to \mathbf{R}$ *be a density. A tree* T *is a solution to the problem above if and only if it is a minimum spanning tree of the differential mutual information graph of* g.