



# Identity Elements

## 1 Why

We can construct functions on the ground set of an algebra by fixing an element in the ground set and defining a function which maps elements to the result of the operation applied to the fixed element and the given element.

## 2 Definition

Let  $(A, +)$  be an algebra. For each  $a \in A$ , denote by  $+_a : A \rightarrow A$  the function defined by

$$+_a(b) = a + b.$$

If  $+_a$  is the identity function on  $A$  then we call  $a$  a *left identity element* of the algebra.

Similarly, denote by  $+^a : A \rightarrow A$  the function defined by

$$+^a(b) = b + a.$$

If  $+^a$  is the identity function on  $A$  then we call  $a$  a *right identity element* of the algebra.

An *identity element* of the algebra is an element which is both a left and right identity. If the operation commutes, then a left identity and right identities are the same.