Why

Do the integer numbers correspond (in the sense of Homomorphisms) to elements of the rationals.

Main Result

Indeed, roughly speaking the integers correspond to rationals whose denominator is 1. Define

$$\tilde{Q} := \{[(a, b)] \in \mathbf{Q} \mid b = 1_{\mathsf{Z}}\}.$$

Proposition 1. *The rings* $(\tilde{\mathbf{Q}}, +_{\mathbf{Q}} \mid \tilde{\mathbf{Q}}, \cdot_{\mathbf{Q}} \mid \tilde{\mathbf{Q}})$ *and* $(Z, +_{\mathsf{Z}}, \cdot_{\mathsf{Z}})$ *are homomorphic.*[1]

Proof. The function is $f : \mathbf{Z} \to \mathbf{Q}$ with $f(z) = [(z, 1)].$[2] □

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[1]Indeed, more is true and will be included in future editions. There is an *order perserving* ring homomorphism.

[2]The full account will appear in future editions.

Integer Rational Homomorphism

Rational Additive Inverses

Rational Multiplicative Inverses

Rational Arithmetic

Rational Sums

Rational Products

Integer Additive Inverses

Rational Numbers

Inverse Elements

Integer Arithmetic

Element Functions

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Function Inverses

Algebras

Identity Elements

Arithmetic

Function Images

Function Composites

Operations

Natural Powers

Functions

Integer Sums

Natural Products

Integer Numbers

Natural Sums

Equivalence Relations

Recursion Theorem

Relations

Peano Axioms

Cartesian Products

Natural Induction

Set Powers

Natural Numbers

Unordered Triples

Partitions

Successor Sets

Intersection of Empty Set

Ordered Pairs

Pair Unions

Set Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Pair Intersections

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects