



CONVEX SETS

Why

Definition

The *closed line segment between* two points in n -dimensional space is the set of points which can be expressed as the sum of the first point and a scalar multiple of the difference between the second point and the first; where the scalar is in the interval $[0, 1]$. Thus, the closed line segment between two points is a subset of the line through the two points. The *open line segment* between x and y is the closed line segment with the points x and y .

A *convex set* contains every closed line segment between any two points. Every affine set is convex. Thus, convex sets are more general.

Notation

Let x and y in \mathbf{R}^n . We can express the closed line segment between x and y as

$$\{x + a(y - x) \mid 0 \leq a \leq 1, x, y \in \mathbf{R}^n\}.$$

Notice that $x + a(y - x) = (1 - a)x + ay$.

PROPOSITION 1. *Every affine set is convex.*

PROPOSITION 2. *The intersection of a family of convex sets is convex.*

PROPOSITION 3. *The translate of a convex set is convex. The scalar multiple of a convex set is convex.*

