



## Why

If we treat the parameters of a linear function as a random variable, an inductor for the predictor is equivalent to an estimator for the parameters.<sup>1</sup>

## Definition

Let  $(\Omega, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $x : \Omega \rightarrow \mathbf{R}^d$ . Define  $g : \Omega \rightarrow (\mathbf{R}^d \rightarrow \mathbf{R})$  by  $g(\omega)(a) = a^\top x(\omega)$ , for  $a \in \mathbf{R}^d$ . In other words, for each outcome  $\omega \in \Omega$ ,  $g_\omega : \mathbf{R}^d \rightarrow \mathbf{R}$  is a linear function with parameters  $x(\omega)$ .  $g_\omega$  is the function of interest.

Let  $a^1, \dots, a^n \in \mathbf{R}^d$  a dataset with data matrix  $A \in \mathbf{R}^{n \times d}$ . Let  $e : \Omega \rightarrow \mathbf{R}^n$  independent of  $x$ , and define  $y : \Omega \rightarrow \mathbf{R}^n$  by

$$y = Ax + e.$$

In other words,  $y_i = x^\top a^i + e_i$ .

We call  $(x, A, e)$  a *probabilistic linear model*. Other terms include *linear model*, *statistical linear model*, *linear regression model*, *bayesian linear regression*, and *bayesian analysis of the linear model*.<sup>2</sup> We call  $x$  the parameters,  $A$  a *design*,  $e$  the *error* or *noise* vector, and  $y$  the *observation* vector.

One may want an estimator for the parameters  $x$  in terms of  $y$  or one may be modeling the function  $g$  and want to predict  $g(a)$  for  $a \in A$  not in the dataset.

## Inconsistency

In this model, the dataset is assumed to be inconsistent as a result of the random errors. In these cases, the error vector  $e$  may model a variety

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<sup>1</sup>Future editions will offer further discussion.

<sup>2</sup>The word bayesian is in reference to treating the object of interest— $x$ —as a random variable.

of sources of error ranging from inaccuracies in the measurements (or measurement devices) to systematic errors from the “inappropriateness” of the use of a linear predictor.<sup>3</sup> In this case the linear part is sometimes called the *deterministic effect* of the response on the input  $a \in A$ .

## Moment assumptions

One route to be more specific about the underlying distribution of the random vector is give its mean and variance. It is common to give the mean of  $\mathbf{E}(w)$

## Mean and variance

**Proposition 1.**  $\mathbf{E}(y) = A\mathbf{E}(x) + \mathbf{E}(w)$ <sup>4</sup>

**Proposition 2.**  $\text{cov}((x, y)) = A \text{cov}(x) A^\top + \text{cov } e$ <sup>5</sup>

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<sup>3</sup>Future editions will clarify and may excise this sentence.

<sup>4</sup>By linearity. Full account in future editions.

<sup>5</sup>Full account in future editions.

