Why

What is the best linear predictor if we choose according to a particular norm.

Definition

Suppose we have a paired dataset of n records with inputs in $\mathbf{R}^d$ and outputs in $\mathbf{R}$. A *norm weighted least squares linear predictor* for a norm $g : \mathbf{R}^n \to \mathbf{R}$ is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$g(y - Ax).$$

Weight matrix

Let $\|\cdot\|_W$ be the weighted norm for some positive semidefinite weight matrix W. We want to find x to minimize

$$\|y - AX\|_W.$$

This problem is referred to by many authors as *weighted least squares* or the *weighted least squares problem*.

Diagonal weight matrix

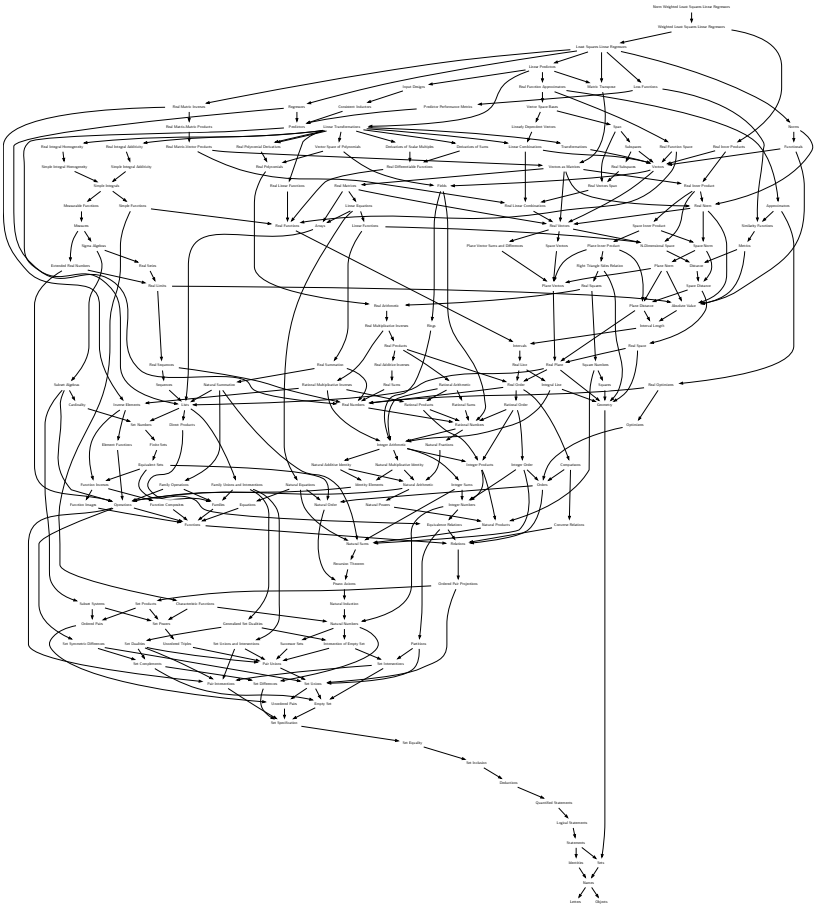
A special case of norm weighted least squares with a weighted norm is the usual weighted least squares problem (see Weighted Least Squares Linear Predictors). Consider weighted least squares with weights $w \in \mathbf{R}^n$, $w \geq 0$. Define $W \in \mathbf{R}^{n \times n}$ so that $W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix and

$$\|y - Ax\|_W = \sum_{i=1}^{n} w_i (y_i - x^\top a_i)^2.$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$(A^\top W A)^{-1} A^\top W y.$$


Sum Weighted Least Squares Linear Regressors
Weighted Least Squares Linear Regressors
Linear Least Squares Linear Regressors
Linear Predictors
Sum Functions
Sum Functions
Real Function Representations
Sum Functions
Sum Weights
Real Matrix Inverses
Regressors
Predictor Performance Metrics
Real Matrix Products
Functions
Vector Spaces
Vector Space Bases
Real Integral Homogeneity
Real Integral Additivity
Real Matrix-Vector Products
Real Polynomial Derivatives
Vector Spaces of Polynomials
Derivatives of Scalar Multiples
Derivatives of Sum
Linear Combinations
Linearizations
Subspaces
Real Vector Spaces
Real Inner Products
Sums
Pythagorean
Simple Integral Homogeneity
Simple Integral Additivity
Real Polynomials
Real Differentiable Functions
Vectors as Matrices
Real Subspaces
Real Vectors Space
Real Vectors Space
Real Inner Product
Length Functions
Real Linear Functions
Real Vector Spaces
Real Linear Transformations
Real Vector Product
Measurable Functions
Simple Functions
Real Linear Functions
Real Linear Transformations
Real Norms
Measures
Real Functions
Norms
Linear Functions
Space Inner Products
Symmetric Functions
Approximations
Measure
Place Vector Sums and Differences
Space Lengths
Place Inner Products
2Dimensional Spaces
Space Spaces
Metrics
Signed Measures
Real Sets
Right Triangle Side Relations
Place Norms
Angles
Extended Real Functions
Real Lists
Place Lengths
Place Angles
Inner Distances
Real Arithmetic
Place Distances
Absolute Value
Real Multiplication Inverses
Sum
Metrics
Real Powers
Real Sums
Real Sums
Real Sums
Real Sums
Real Products
Real Arithmetic
Real Sums
Real Lists
Real Lists
Real Lists
Real Sums
Real Subtractions
Real Sums
Real Subtractions
Real Sums
Real Sums
Real Sums
Real Sums