Why

We want to discuss rearranging the order of some set.

Definition

Let X be a nonempty set. A *permutation* is a bijection from X to X.

As a group

It happens that the set of permutations with the operation of composition is a *group* (see Groups). To see this, suppose $\pi : X \to X$ and $\sigma : X \to X$ are two permutations. Then $\pi \circ \sigma$ is a permutation (the composition of two invertible functions is invertible). Also, the identity function $\mathrm{id}_X : X \to X$ is a permutation (it has an inverse, *itself*). The identity *function* is the identity element, since $\pi\,\mathrm{id} = \mathrm{id}\,\pi$ for all permutations π. Since each permutation is invertible, inverse elements exist. And the associative law is valid for permutations since function composition is associative. We the group consisting of the set of permutations on X and the operation of composition the *symmetric group on X*.

Finite case

If X is finite and $|X| = n$, then we can associate each element of X with a number $\{1, 2, \ldots, n\}$ and in so doing consider permutations of the set $\{1, \ldots, n\}$. This special symmetric group is called the *symmetric group of degree n*.

There are $n!$ permutations (see Factorials).

Notation

We denote the symmetric group on X by $\mathrm{Sym}(X)$. It is common to denote the symmetric group of degree n by S_n.

Permutations

Set Numbers

Groups

Finite Sets

Rings

Integer Arithmetic

Natural Multiplicative Identity

Natural Additive Identity

Equivalent Sets

Natural Arithmetic

Identity Elements

Function Inverses

Integer Sums

Integer Products

Natural Powers

Function Images

Function Composites

Integer Numbers

Natural Products

Operations

Functions

Equivalence Relations

Natural Sums

Relations

Natural Order

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Set Products

Natural Induction

Ordered Pairs

Set Powers

Natural Numbers

Unordered Triples

Partitions

Successor Sets

Intersection of Empty Set

Pair Unions

Set Intersections

Set Symmetric Differences

Set Unions

Set Complements

Pair Intersections

Unordered Pairs

Empty Set

Set Differences

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects