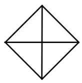


## Why

We want to estimate the weights of a linear function.[1]

## Definition

The *probabilistic linear model*; *linear model*; *linear regression*

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. We have $n$ precepts in $\mathsf{R}^d$. So let $a^1, \dots, a^n \in \mathsf{R}^d$ with data matrix $A \in \mathsf{R}^{n \times d}$. We are modeling a relation between $\mathsf{R}^d$ and $\mathsf{R}$.

Let $x : \Omega \to \mathsf{R}^d$ and $e : \Omega \to \mathsf{R}^n$ be independent random vectors with zero mean and covariances given by $\Sigma_x$ and $\Sigma_e$, respectively. For each $\omega \in \Omega$, define the map $f : \Omega \to (\mathsf{R}^d \to \mathsf{R})$ by $f(\omega)(a) = \sum_j a^i_j x_j(\omega) + e_i(\omega)$.

We call $x$ the *signal*. We call $e$ the *noise*. This class of models assumes the signal and noise are independent.

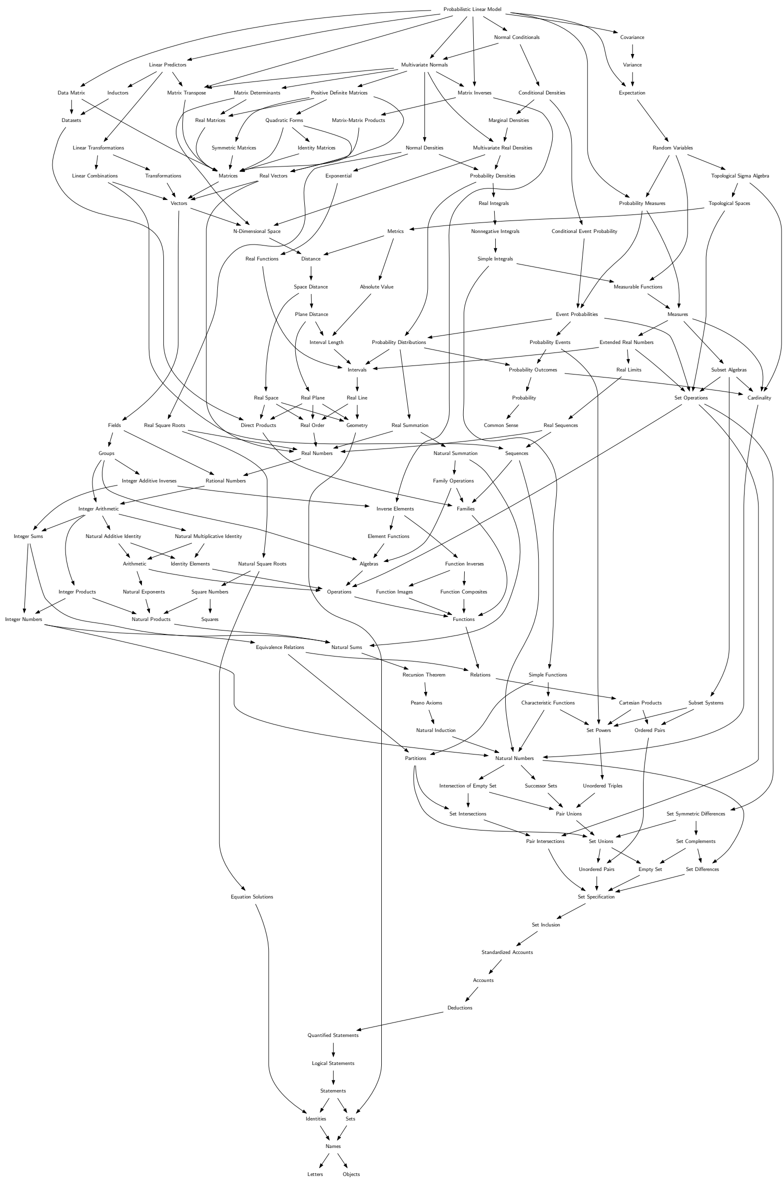
Define $y : \Omega \to \mathsf{R}^n$ by $y(\omega) = Ax(\omega) + e(\omega)$. So,

$$y = Ax + e.$$

**Proposition 1.** $\mathsf{E}(y) = A\,\mathsf{E}(x) + \mathsf{E}(w)$[2]

**Proposition 2.** $\mathbf{cov}((x, y)) = A\,\mathbf{cov}(x)A^\top + \mathbf{cov}\,e$[3]

[1] Future editions will include this.
[2] By linearity. Full account in future editions.
[3] Full account in future editions.


Probabilistic Linear Model
Normal Conditionals
Covariance
Linear Predictors
Multivariate Normals
Variance
Data Matrix
Inductors
Matrix Transpose
Matrix Determinants
Positive Definite Matrices
Matrix Inverses
Conditional Densities
Expectation
Datasets
Real Matrices
Quadratic Forms
Matrix-Matrix Products
Marginal Densities
Linear Transformations
Symmetric Matrices
Identity Matrices
Normal Densities
Multivariate Real Densities
Random Variables
Linear Combinations
Transformations
Matrices
Real Vectors
Exponential
Probability Densities
Topological Sigma Algebra
Vectors
Real Integrals
Probability Measures
Topological Spaces
N-Dimensional Space
Metrics
Nonnegative Integrals
Conditional Event Probability
Real Functions
Distance
Simple Integrals
Space Distance
Absolute Value
Measurable Functions
Plane Distance
Event Probabilities
Measures
Interval Length
Probability Distributions
Probability Events
Extended Real Numbers
Intervals
Probability Outcomes
Real Limits
Subset Algebras
Real Space
Real Plane
Real Line
Probability
Set Operations
Cardinality
Fields
Real Square Roots
Direct Products
Real Order
Geometry
Real Summation
Common Sense
Real Sequences
Groups
Real Numbers
Natural Summation
Sequences
Integer Additive Inverses
Rational Numbers
Family Operations
Integer Arithmetic
Inverse Elements
Families
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Element Functions
Arithmetic
Identity Elements
Natural Square Roots
Algebras
Function Inverses
Integer Products
Natural Exponents
Square Numbers
Operations
Function Images
Function Composites
Integer Numbers
Natural Products
Squares
Functions
Equivalence Relations
Natural Sums
Recursion Theorem
Relations
Simple Functions
Peano Axioms
Characteristic Functions
Cartesian Products
Subset Systems
Natural Induction
Set Powers
Ordered Pairs
Partitions
Natural Numbers
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Equation Solutions
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects