

## REAL SUBSPACE REPRESENTATIONS

## Why

How should we represent a subspace computationally?

## **Definition**

Given a subspace  $S \in \mathbf{R}^n$ , since it is finite dimensional, there exists a finite basis for the space. This basis can be made orthonormal. Therefore every subspace S has an orthonormal basis  $q_1, \ldots, q_k$ , where k is the dimension of the subspace. We can stack these as a matrix. Define  $Q \in \mathbf{R}^{n \times k}$  by

$$Q = \left[ \begin{array}{cccc} q_1 & q_2 & \cdots & q_k \end{array} \right].$$

For every  $x \in S$ , there exists unique coefficients  $z \in \mathbf{R}^k$  so that

$$x = Qz$$
.

Therefore we have a one-to-one correspondence between vectors  $x \in S$  and their coordinates  $z \in \mathbf{R}^k$ .

