

FILTRATIONS

Why

1

Definition

Let (I, \leq) be a totally ordered set. A family of sigma-algebras $\{F_i\}_{i\in I}$ is a filtration if $F_j \subset F_k$ for all $j \leq k$.

We call a filtration whose index set is the natural numbers with their usual order a discrete-time filtration. We call a filtration whose index set is the real numbers with their usual order a continuous-time filtration. The index set may also be finite, for example, $\{F_i\}_{i=1}^n$, in which case

$$F_1 \subset F_2 \subset \cdots \subset F_n$$
.

Notation

It is extremely common to see filtrations written with the caligraphic \mathcal{F} . As in, let $\{\mathcal{F}_i\}_{i=1}^n$ be a filtration. This is in accordance with using caligraphic letters for sets of sets, and in accordance with the term sigma *field* for sigma algebra.

¹Future editions will include, and likely will need stochastic processes.

