Why

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Definition

Consider an open interval of real numbers and a function on the interval. Take a real number in the interval. It corresponds to some element in the graph of the function. Next take a second real number in the interval. It too corresponds to some element in the graph of the function. We have two distinct elements of the interval corresponding to two elements of the graph.

Next, consider the line in the plane which passes through these two elements of the graph of the function. Consider the slope of this line, in other words, the tangent of the angle between the line and a horizontal line passing through the original point. Consider the tangent of the angle which the line makes with the horizontal. The quantity is the quotient of the result on the second real number less the result on the first real number with the second real number less the first real number.

For a selected first number, define a new function which maps a second distinct real number (in the interval) to this quantity, the slope of the line between them. Next, we consider if this second function has a limit as its argument ap-

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[1]Future editions will include.

proaches the first fixed number. If it does, we say that the original function is *differentiable at* the first number. We call the limiting real number the *derivative at* the first real number of the original function.

If the first function is differentiable at each number in the interval of its domain, we say that the function is *differentiable*. We call the function which assigns to each real number in the interval the real number which is the derivative at the argument of the function the *derivative of* the original function.

Notation

Let I be an open interval of $\mathbf{R}$. Let $f : I \to \mathbf{R}$ and $a \in I$. Define $A = I - \{a\}$, and define $g : A \to \mathbf{R}$ by $g(x) = (f(x) - f(a))/(x-a)$. We say that f is differentiable at a if $\lim_{x \to a} g(x)$ exists in $\mathbf{R}$. In other words, f is differentiable if

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists in $\mathbf{R}$.

If f is differentiable at a for each $a \in I$, then f is differentiable. In this case, the derivative of f is the function $h : I \to \mathbf{R}$ defined by

$$h(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

Differentiable Functions

Real Functions

Intervals

Real Line

Real Order

Real Numbers

Rational Order

Integral Line

Rational Numbers

Integer Arithmetic

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Identity Elements

Arithmetic

Natural Powers

Natural Products

Integer Sums

Integer Order

Comparisons

Operations

Natural Sums

Integer Numbers

Orders

Functions

Recursion Theorem

Equivalence Relations

Natural Order

Converse Relations

Peano Axioms

Relations

Natural Induction

Set Products

Natural Numbers

Set Powers

Ordered Pairs

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Symmetric Differences

Set Intersections

Pair Unions

Pair Intersections

Set Complements

Set Unions

Empty Set

Set Differences

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects