Why

Linear equations are ubiquitous.

Definition

Given $a \in \mathbf{R}^n$ and $y \in \mathbf{R}$, suppose we want to find $x \in \mathbf{R}^n$ satisfying

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = y.$$

We refer to this expression as a *real linear equation* or *linear equation.* We treat each component $x_i \in \mathbf{R}$ as a variable and we treat each component $a_i \in \mathbf{R}$ and $y \in \mathbf{R}$ as constants. We call the pair (a, y) the *real linear equation constants.*[1]

The source of the terminology "linear" is by viewing the left hand side as a function. Define $f : \mathbf{R}^n \to \mathbf{R}$ by $f(x) = \sum_i a_ix_i$. We want to find $x \in \mathbf{R}^n$ to satisfy $f(x) = b$. Notice that f is a *linear* real function.[2]

Moreover, to each linear function $f : \mathbf{R}^d \to \mathbf{R}$ there exists a vector $a \in \mathbf{R}^d$ so that $f(x) = \sum_i a_ix_i$. For this reason, if we are given several linear function $f_1, \ldots, f_m$, then we can think of these as several vectors $a^1, \ldots, a^n$. If we are also given $b_i \in \mathbf{R}$ for each $i = 1, \ldots, m$, then we have the vector $b \in \mathbf{R}^m$

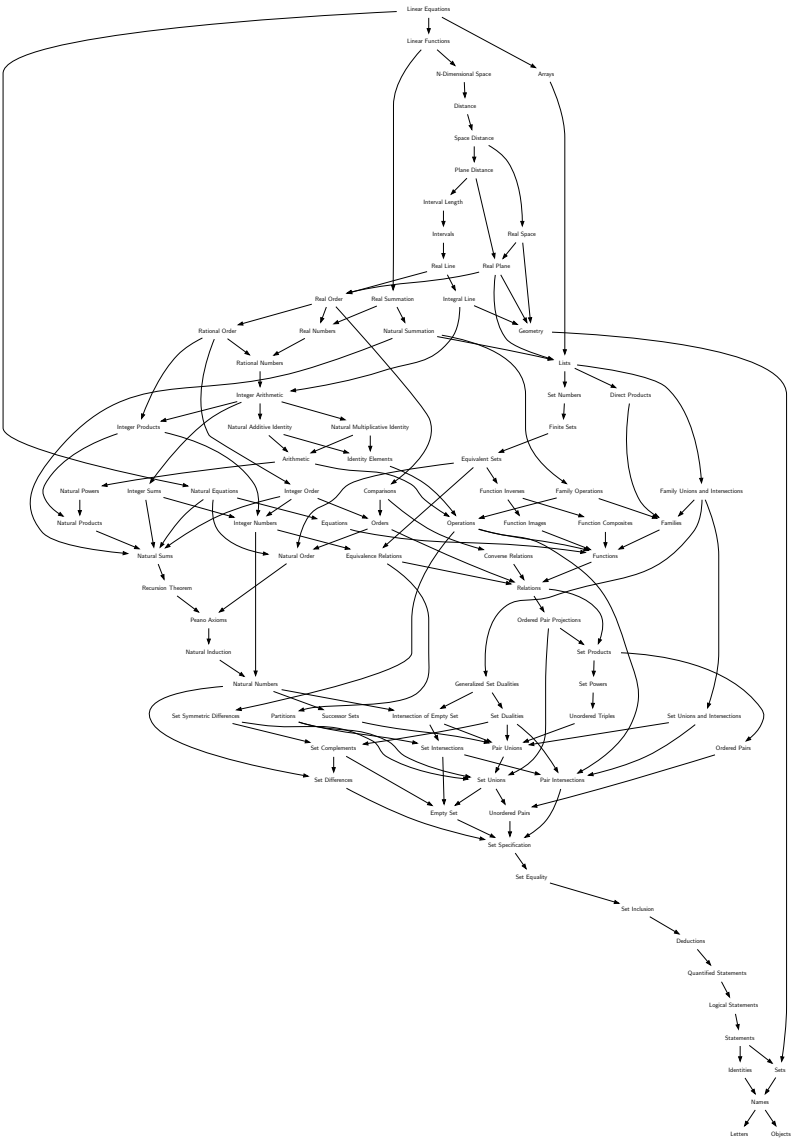
We can define the two-dimensional array $A \in \mathbf{R}^{m\times n}$ by $A_{ij} = a^i_j$. For this reason, a *linear system of equations* is a pair (A, b). A solution of a linear system of equations is a vector $x \in \mathbf{R}^n$ satisfying the equations

$$\begin{array}{rrcrl}
A_{11}x_1+ & A_{12}x_2+ & \cdots+ & A_{1n}x_n = & b_1 \\
A_{21}x_1+ & A_{22}x_2+ & \cdots+ & A_{2n}x_n = & b_2 \\
\vdots & \vdots & & \vdots & \vdots \\
A_{m1}x_1+ & A_{m2}x_2+ & \cdots+ & A_{mn}x_n = & b_n
\end{array}$$

Other terminology includes a *system of linear equations* or *linear system* or *simultaneous linear equations*

[1] Future editions will clarify.

[2] Future editions may require a sheet here.


Linear Equations
Linear Functions
N-Dimensional Space
Arrays
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Order
Real Summation
Integral Line
Geometry
Rational Order
Real Numbers
Natural Summation
Rational Numbers
Lists
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Natural Equations
Integer Order
Comparisons
Function Inverses
Family Operations
Family Unions and Intersections
Natural Products
Integer Numbers
Equations
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Natural Order
Equivalence Relations
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects