



Why

What is the best estimate for a random variable if we consider the square error?

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $x : \Omega \rightarrow \mathbf{R}$ a random variable. A *minimum mean squared error estimate* or *MMSE estimate* or *least square estimate* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}(x - \xi)^2$.

Proposition 1. *There is a unique MMSE estimate and it is given by $\mathbf{E}(x)$.*

Vector case

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $y : \Omega \rightarrow \mathbf{R}^n$ a random variable.¹

A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}\|x - \xi\|^2$.

Proposition 2. *There is a unique MMSE estimator and it is given by $\mathbf{E}(y)$.*

¹Future editions might collapse this into the previous case.

