



Why

We generalize random variables and random vectors to a map from the outcome space to a space of functions.

Definition

A *random function* is a map from the set of outcomes of a probability space to a set of functions.¹

Notation

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let A and B be sets and let $x : \Omega \rightarrow (A \rightarrow B)$ be a random function. For each outcome $\omega \in \Omega$, $x_\omega : A \rightarrow B$ is a function from A to B .

A family of random variables

A *stochastic process* (or *random process* or *random field*) is a family of random variables. The term process is often used when the index set is associated with time and the term field is often used when the index set is associated with space.

We can associate to the random function $x : \Omega \rightarrow (A \rightarrow B)$ a family of random variables $y : A \rightarrow (\Omega \rightarrow B)$ defined by

$$y(a)(\omega) = x(\omega)(a).$$

The index set of the family is the domain of the set of functions. The codomain of each random variable is the codomain of the set of functions.

The converse is also naturally true. Any family of random variables can be associated with a random function. In this way, a random function corresponds to a family of random variables, and any family of random variables corresponds to a random function.

¹Future editions will make measurability precise.

For this reason many authors use the term random function or stochastic function and random process or stochastic process interchangeably. In these sheets, we will be specific.

