



# Metric Continuity

## 1 Why

We define continuity for functions between metric spaces.

## 2 Definition

Our inspiration is continuity of functions from the set of real numbers to the set of real numbers. There we decided on a definition which codified our intuition that numbers which are sufficiently close to each other are mapped to numbers that are close to each other.

A function from a first metric space to a second metric space is *continuous at* an object of its domain if, for every positive real number (no matter how small), there is a second positive real number (possibly, though not necessarily, smaller) so that every element in the domain whose distance to the fixed object is less than the second positive number has a result under the function whose distance to the result of the fixed object is less than the first positive number.

A function between metric spaces is continuous if it is *continuous* at every object of its domain.

### 2.1 Notation

Let  $(A, d)$  and  $(B, d')$  be metric spaces. Let  $f : (A, d) \rightarrow (B, d')$ . Then  $f$  is continuous at  $\bar{a} \in A$ , if for all real numbers  $\varepsilon > 0$ , there exists a real number

$\delta > 0$  such that for all  $a \in A$ ,

$$d(\bar{a}, a) < \delta \implies d'(f(\bar{a}), f(a)) < \varepsilon.$$