



Why

Can a set have no elements?

Definition

Sure. A set exists by the principle of existence (see **Sets**); denote it by A . Specify elements (see **Set Specification**) of any set that exists using the universally false statement $x \neq x$. We denote that set by $\{x \in A \mid x \neq x\}$. It has no elements. In other words, $(\forall x)(x \notin A)$. The principle of extension (see **Set Equality**) says that the set obtained is unique (contradiction).¹ We call the unique set with no elements *the empty set*.

Notation

We denote the empty set by \emptyset . In other words, in all future accounts (see **Accounts**), there are two implicit lines. First, “**name** \emptyset ” and second “**have** $(\forall x)(x \notin \emptyset)$ ”.

Properties

It is immediate from our definition of the empty set and of the definition of inclusion (see **Set Inclusion**) that the empty set is included in every set (including itself).

Proposition 1. $(\forall A)(\emptyset \subset A)$

Proof. Suppose toward contradiction that $\emptyset \not\subset A$. Then there

¹This account will be expanded in the next edition.

exists $y \in \emptyset$ such that $y \notin A$. But this is impossible, since $(\forall x)(x \notin \emptyset)$. \square

