



# Cumulative Distribution Functions

## 1 Why

TODO

## 2 Definition

The *cumulative distribution function* of a real-valued random variable is the function mapping real numbers to the measure of the set of outcomes for which the random variable takes value less than or equal to that real number. The range of the cumulative distribution function is the interval  $[0, 1]$ , since the measure of the base set is one and all measures are non-negative.

### 2.1 Notation

Let  $(X, \mathcal{A}, \mu)$  be a probability space. Let  $R$  denote the real numbers. Let  $f : X \rightarrow R$  be a measurable function, and so a real-valued random variable. We denote the cumulative distribution function of  $f$  by  $F_f : R \rightarrow [0, 1]$ . We defined it by

$$F_f(t) = \mu(\{x \in X \mid f(x) \leq t\}).$$

## 3 Properties

**Proposition 1.** *The cumulative distribution function of any real-valued random variable*

1. *is non-decreasing,*

2. *is right-continuous,*

3. *tends to one as its argument tends toward infinity, and*

4. *tends to zero as its argument tends towards negative infinity.*

*Proof.* TODO

□