Probability Distributions

Why

We want to talk about probability over finite sets.

Definition

A *probability distribution* or *probability mass function* is a real-valued function from a set of outcomes which is non-negative and normalized. A real-valued function on a finite set is *normalized* if the sum of its results is 1. We will refer to these as *distributions*. The *probability of an outcome* is the result of the outcome under the distribution.

Notation

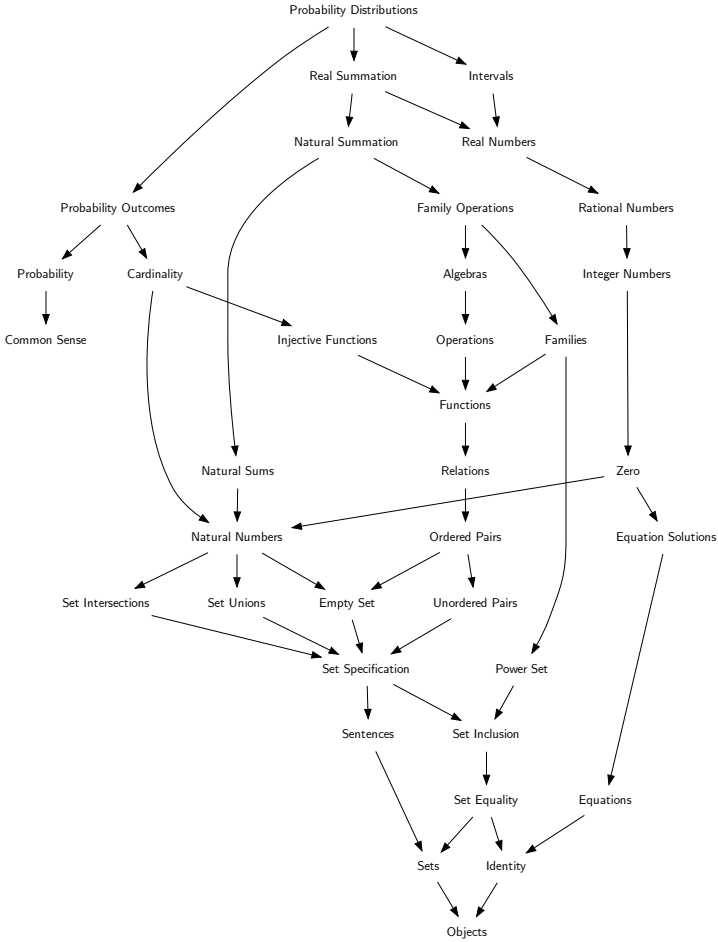
Let A be a set of outcomes and Let $p : A \to \mathbf{R}$ be a distribution; then

- $p(a) > 0$ for each $a \in A$ and
- $\sum_{a \in A} p(a) = 1$

Proposition **1.** *If $p : A \to \mathbf{R}$ is a distribution, then $p(A) \subset [0, 1]$.*

Proof. Let $a \in A$. First, $p(a) \geq 0$ by definition. Second, since p is normalized, $\sum_{b \in A} p(b) = 1$. And $p(a) \leq \sum_{b \in A} p(b)$, so $p(a) \leq 1$.

$\square$


Probability Distributions
Real Summation
Intervals
Natural Summation
Real Numbers
Probability Outcomes
Family Operations
Rational Numbers
Probability
Cardinality
Algebras
Integer Numbers
Common Sense
Injective Functions
Operations
Families
Functions
Natural Sums
Relations
Zero
Natural Numbers
Ordered Pairs
Equation Solutions
Set Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Power Set
Sentences
Set Inclusion
Set Equality
Equations
Sets
Identity
Objects