



Why

We want to add and multiply complex numbers.¹

Definition

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex product* of z_1 and z_2 is the complex number $(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$.

Notation

Properties

We have the following desirable properties.²

Proposition 1 (Commutativity). *For all $z_1, z_2 \in \mathbf{C}$, we have $z_1 + z_2 = z_2 + z_1$ and $z_1z_2 = z_2z_1$.*

Proposition 2 (Commutativity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ and $z_1(z_2z_3) = (z_1z_2)z_3$.*

Proposition 3 (Distributivity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have $z_1(z_2 + z_3)$ and $z_1z_2 + z_1z_3$*

Relation to \mathbf{R}^2

Addition in \mathbf{C} corresponds to the usual addition of the corresponding vectors in the plane \mathbf{R}^2 . On the other hand, multiplication

¹Future editions will expand in the genetic account for introducing complex numbers.

²Future editions will include accounts.

However multiplication in \mathbf{C}

