Why

We want to extend our notion of entropy (see Discrete Entropy) to real-valued (continuous) random variables.

Definition

The *differential entropy* of a probability density function is the integral of the density against the negative log of the density. This definition made to be similar to the case of discrete entropy. If a real-valued random variable has a density, then we call the differential entropy of its density the *differential entropy* of the random variable.

Notation

Let $f : \mathbf{R}^n \to \mathbf{R}$ be a probability density function. The differential entropy of f is

$$-\int f \log f$$

We denote the differential entropy of f by $h(f)$.

Example

Let $x : \Omega \to \mathbf{R}$ be uniform on $[0, 1/2]$. Then $h(x) = \log 1/2 < 0$.

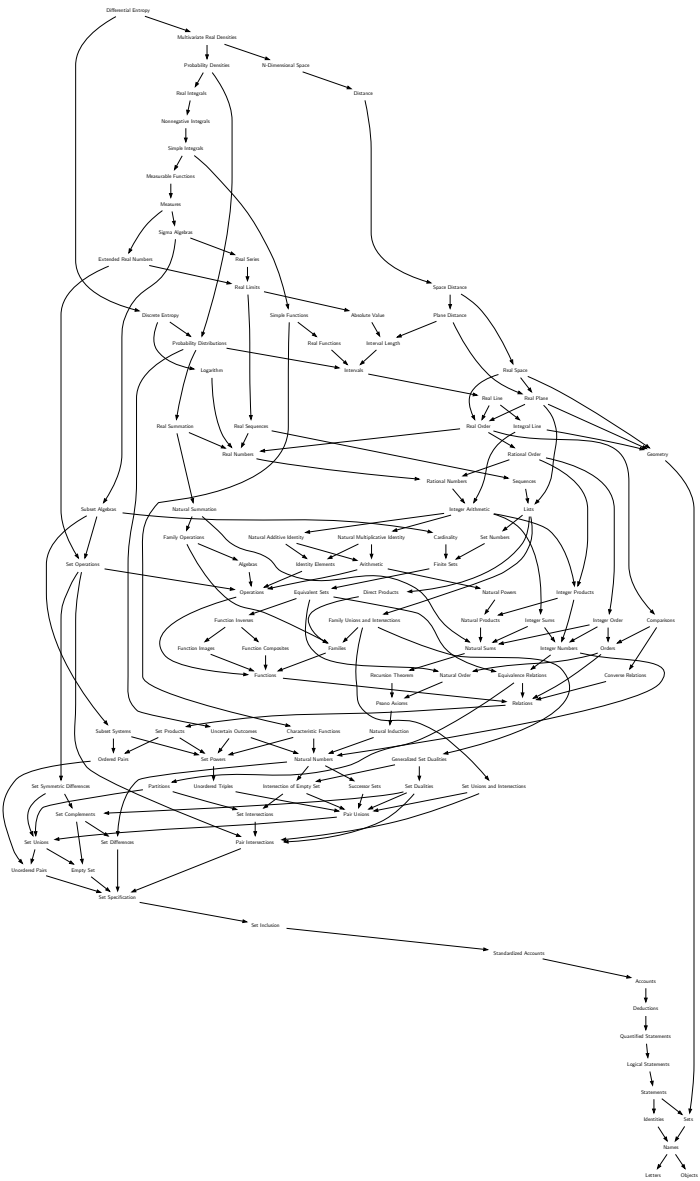
Problems

We have $h(ax) = h(x) + \log|a|$. In generaly $h(Ax) = h(x) + \log|A|$.

Differences still meaningful

Even though the value of the differential entropy is not necessarily a good analogy to discrete entropy, differences still are. In particular, the following holds

$$I(X;Y) = H(Y) - H(Y \mid X) = H(X) = H(X \mid Y)$$


Differential Entropy
Multivariate Real Densities
Probability Densities
N-Dimensional Space
Distance
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Space Distance
Discrete Entropy
Simple Functions
Absolute Value
Plane Distance
Probability Distributions
Real Functions
Interval Length
Intervals
Logarithm
Real Space
Real Line
Real Plane
Real Summation
Real Sequences
Real Order
Integral Line
Real Numbers
Rational Order
Geometry
Rational Numbers
Sequences
Subset Algebras
Natural Summation
Integer Arithmetic
Lists
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Cardinality
Set Numbers
Algebras
Identity Elements
Arithmetic
Finite Sets
Set Operations
Operations
Equivalent Sets
Direct Products
Natural Powers
Integer Products
Function Inverses
Family Unions and Intersections
Natural Products
Integer Sums
Integer Order
Comparisons
Function Images
Function Composites
Families
Natural Sums
Integer Numbers
Orders
Functions
Recursion Theorem
Natural Order
Equivalence Relations
Converse Relations
Peano Axioms
Relations
Subset Systems
Set Products
Uncertain Outcomes
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects