

Complex Inner Products

1 Why

What is an inner product if we take a vector space over the complex numbers.

2 Definition

An inner produce over a complex vector space is positive defininte, Hermitian, and linear in the first argument.

2.1 Alternate Conventions

2.2 Notation

Let C be the set of complex numbers. Let (V,C) be a complex vector space. Let $f:V\times V\to C$ be a function such that

1.
$$f(x,x) \ge 0$$
, $f(x,x) = 0 \Leftrightarrow x = 0$;

2.
$$f(x,y) = \overline{f(y,x)}$$

3.
$$f(ax + by, z) = a(x, z) + b(y, z)$$