

ELIMINATION GRAPHS

Why

We want to model the progressive fill-in during Gauss elimination with graphs.

Definition

Let $G = ((V, E), \sigma)$ be an ordered undirected graph with |V| = n. Define $E_0 = E$. The *elimination edge sequence* of G is a sequence (E_1, \ldots, E_{n-1}) defined by

$$E_i = E_{i-1} \cup \{\{v, w\} \mid v \succ \sigma(i), w \succ \sigma(i), \text{ and } \{\sigma(i), w\} \in E_{i-1}\}$$

for i = 1, ..., n - 1. The difference between E_{i-1} and E can be described by saying that the higher neighborhood of the intermediate graph $((V, E_{i-1}), \sigma)$ is made complete by the addition of edges between all non-adjacent vertices.

The elimination graph of G is the graph $((V, E_{n-1}), \sigma)$.

