

REAL NEIGHBORHOODS

Why

Often when speaking of a set, we are interested in speaking of those elements which are close to it.

Definition

Let $x \in \mathbb{R}^d$ A subset $N \subset \mathbb{R}^d$ is a neighborhood of x if there is a $\delta > 0$ such that $B(x, \delta) \subset N$. The set \mathcal{N}_a of neighborhoods of x is called the *complete system of neighborhoods* of the point a.

We interpret a neighborhood of a point $x \in X$ as a set containing all the points of X that are sufficiently close to a. A neighborhood of x "encloses" x by virtue of it containing an open ball about a.¹

¹Future editions will continue to treatment, including pointing out that an open ball at x is a neighborhood of x and of all elements in the ball.

