Why

We want to multiply real numbers.[1]

Definition

The *real product* of two real numbers R and S is defined

1. if R or S is $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$, then the $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$
2. otherwise,
   (a) if R or S is $0_{\mathbf{R}}$, then $0_{\mathbf{R}}$.
   (b) if $R, S \neq 0_{\mathbf{R}}$ and $0_{\mathbf{R}} \in R, S$, let T be
   $$\{t \in \mathbf{Q} \mid r \in R, s \in S, r, s \geq 0_{\mathbf{Q}}, t = r \cdot s\}$$
   then $T \cup \{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\}$[2]
   (c) If $R, S \neq 0_{\mathbf{R}}$, $0_{\mathbf{R}} \in R$ and $0_{\mathbf{R}} \notin S$, then the additive inverse of the product of $-R$ with S.
   (d) If $R, S \neq 0_{\mathbf{R}}$, $0_{\mathbf{R}} \notin R$ and $0_{\mathbf{R}} \in S$, then the additive inverse of the product of R with $-S$.
   (e) If $R, S \neq 0_{\mathbf{R}}$, and $0_{\mathbf{R}} \notin R, S$, then the product of $-R$ with $-S$.

Notation

We denote the product of two real numbers x and y by $x \cdot y$.

Properties

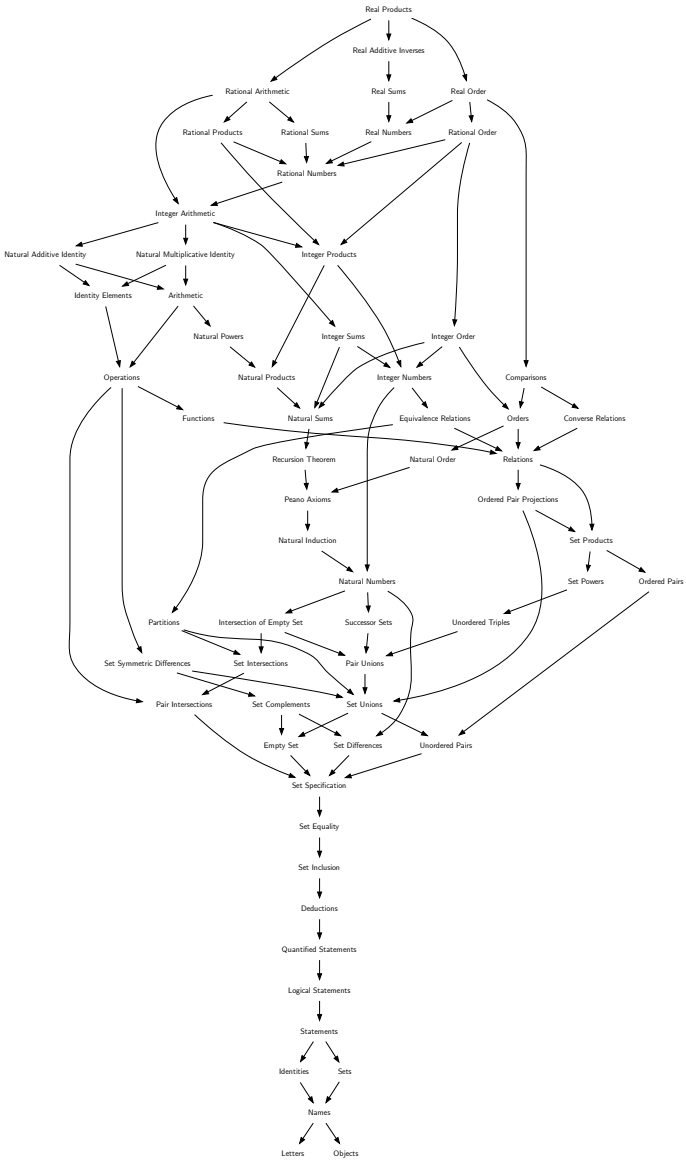
Proposition 1 (Associative)**.** $x + (y + z) = (x + y) + z$

Proposition 2 (Commutative)**.** $x + y = y + x$

Proposition 3 (Identity)**.** *The set of all rationals less than* $1_{\mathbf{Q}}$ *is the multiplicative identity.*

[1] Future editions will expand.
[2] We use $\geq$ in the usual way, it will be defined earlier in future editions.

We denote the the multiplicative identity by $1_{\mathbf{R}}$. When it is clear from context, we call $1_{\mathbf{R}}$ "one".


Real Products
Real Additive Inverses
Rational Arithmetic
Real Sums
Real Order
Rational Products
Rational Sums
Real Numbers
Rational Order
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Identity Elements
Arithmetic
Natural Powers
Integer Sums
Integer Order
Operations
Natural Products
Integer Numbers
Comparisons
Functions
Natural Sums
Equivalence Relations
Orders
Converse Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
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Names
Letters
Objects