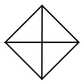


Why

We want to talk about how knowledge about an aspect of an outcome can give us knowledge about the another aspect of an outcome.

Definition

Let $\mathsf{P} : \mathcal{P}(\Omega) \to \mathsf{R}$ be a probability measure.

Two events $A, B \subset \Omega$ are *independent* if

$$\mathsf{P}(A \cap B) = \mathsf{P}(A)\mathsf{P}(B).$$

In other words, they are independent if the probability of their intersection is the product of their respective probabilities. Otherwise, we call A and B *dependent*.

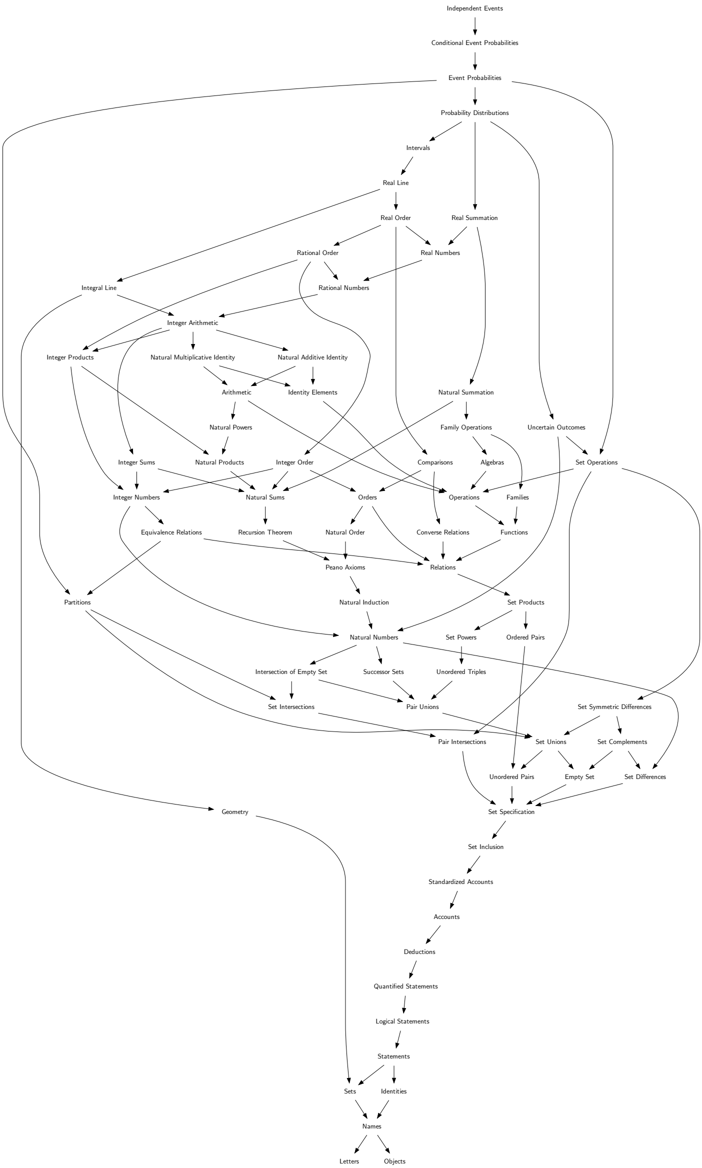
In the case that $\mathsf{P}(B) \neq 0$, then $\mathsf{P}(A \cap B) = \mathsf{P}(A)\mathsf{P}(B)$ is equivalent to $\mathsf{P}(A \mid B) = \mathsf{P}(A)$. We interpret this second expression as encoding the fact that the occurence of event B does not change the "credibility" of the event A.

Example: two dice

Define $\Omega = \{(\omega_1, \omega_2) \mid \omega_i \in \{1, \dots, 6\}\}$, and interpret $\omega \in \Omega$ as corresponding to pips face up after rolling two dice. Define $p : \Omega \to \mathsf{R}$ by $p(\omega) = 1/36$.

Two events are $A = \{\omega \in \Omega \mid \omega_1 + \omega_2 > 5\}$, "the sum is greater than 5", and $B = \{\omega \in \Omega \mid \omega_1 > 3\}$, "the number of pips on the first die in greater than 3". Then $\mathsf{P}(A) = 26/36$.

Also, $\mathsf{P}(A \mid B) = {}^{17}/_{16}$. So, these events are dependent. Roughly speaking, we say that knowing B tells us something about A. In this case, we say that it "makes A more probable."


Independent Events
Conditional Event Probabilities
Event Probabilities
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Rational Order
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Arithmetic
Identity Elements
Natural Summation
Natural Powers
Family Operations
Uncertain Outcomes
Integer Sums
Natural Products
Integer Order
Comparisons
Algebras
Set Operations
Integer Numbers
Natural Sums
Orders
Operations
Families
Equivalence Relations
Recursion Theorem
Natural Order
Converse Relations
Functions
Peano Axioms
Relations
Partitions
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects