

MULTIWAY CLASSIFICATION MODELS

Definition

The two-sample model (or the two-sample problem) has input space $\{1,2\}$ and regression function $\phi: \{1,2\} \to \mathbf{R}^2$ where $\phi(1) = (1,0)^{\top}$ and $\phi(2) = (0,1)^{\top}$. In this case, we model two populations (corresponding to 1 and 2) with different means but a common variance. The regression range is the set $\{(1,0),(0,1)\}$.

Generalizing from two to a populations gives the one-way classification model. In that case the input space is $\{1,\ldots,a\}$ and the regression function is $\phi:\{1,\ldots,a\}\to \mathbf{R}^a$ defined by $\phi(i)=e_i$ where e_i is the standard unit vector in \mathbf{R}^a .¹ The regression range is $\{e_1,\ldots,e_a\}$. In this case we say that the factor population takes levels $1,\ldots,a$.

If there are more than one factors, then we have a multiway classification model. For example, the two-way classification model with no interaction has input domain $X = \{1, ..., a\} \times \{1, ..., b\}$ and the regression function $\phi: X \to \mathbf{R}^{a+b}$ is defined by $\phi(i, j) = (e_i, e_j)^{\top}$.

¹Future editions will define the standard unit vector.

