1 Why

Since one and only one outcome occurs, given a distribution on outcomes, we define the probability of a set of outcomes as the sum of their probabilities.

2 Definition

Given a distribution $p : \Omega \to \mathsf{R}$, the *probability of an event* $A \subset \Omega$ is $\sum_{a \in A} p(a)$, the sum of probabilities of its outcomes.

3 Notation

Define define $\mathsf{P} : \mathcal{P}(\Omega) \to \mathsf{R}$ by

$$\mathsf{P}(A) = \textstyle\sum_{a \in A} p(a).$$

We call P the *event probability function* (or *probability measure*) of (or induced by) p.

4 Example: die

Define $p : \{1, \dots, 6\} \to \mathsf{R}$ by $p(\omega) = 1/6$ for $\omega = 1, \dots, 6$. Define the event $E = \{2, 4, 6\}$. Then

$$\mathsf{P}(E) = \textstyle\sum_{\omega \in E} p(\omega) = p(2) + p(4) + p(6) = 1/2.$$

5 Probability measures

Notice that for all $A \subset \Omega$, (i) $\mathsf{P}(A) \geq 0$. In particular, (ii) $\mathsf{P}(\Omega) = 1$ (and $\mathsf{P}(\varnothing) = 0$). For all $A, B \subset \Omega$, $\mathsf{P}(A \cup B) =$

$\mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$. In particular, if $A \cap B = \varnothing$, (iii) $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$.

Conversely, suppose $f : \mathcal{P}(\Omega) \to \mathbf{R}$ satifies (i), (ii), (iii). These three conditions are sometimes called the *axioms of probability* (for finite sets). Define $p : \Omega \to \mathbf{R}$ by

$$p(\omega) = f(\{\omega\}).$$

In case f satisfies the axioms, p is a probability distribution (nonnegative and sums to one). For this reason we call f satisfying (i)-(iii) an *event probability function* (or *probability measure*). In the case that we think of a probability event function $\mathbf{P}$ as induced by a distribution p, we write $\mathbf{P}_p$.

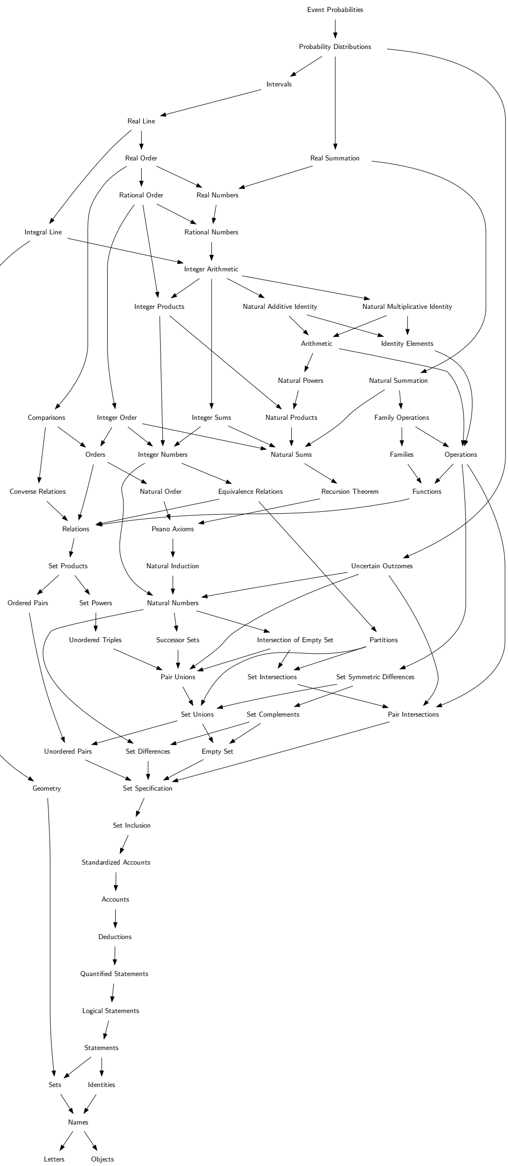
We conclude that p and $\mathbf{P}$ are two perspectives. We can think of elementary events (outcomes) and define their probabilities individually in a way that they sum to one and are nonnegative. Or we can think of the compound events, and define their probabilities in a way consistent with (i)-(iii).

6 Probability by cases

Let $\mathbf{P}$ be a probability event function. Suppose $A_1, \dots, A_n$ partition Ω. Then for any $B \subset \Omega$,

$$\mathbf{P}(B) = \textstyle\sum_{i=1}^n \mathbf{P}(A_i \cap B).$$

Some authors call this the *law of total probability.*


Event Probabilities
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Rational Order
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Natural Summation
Comparisons
Integer Order
Integer Sums
Natural Products
Family Operations
Orders
Integer Numbers
Natural Sums
Families
Operations
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Functions
Relations
Peano Axioms
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects