Why

We discuss learning (or inferring) relations from examples.

Definition

Let X and Y be sets. A *relation inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a relation between X and Y. We frequently use the term inductor to refer to a family of inductors, indexed by $n \in \mathsf{N}$.

An inductor is *functional* if it produces functions. In this case, we call the elements of X the *inputs* and the elements of Y the *outputs*. We call a function from inputs to outputs a *predictor* and call the result of an input under a predictor a *prediction*. Using this language, a functional inductor maps datasets to predictors. A predictor maps inputs to outputs.

To every relation between X and Y corresponds a characteristic function on $X \times Y$ and vice versa. For this reason, henceforth by *inductor* we mean a functional inductor. A relational inductor on a dataset in $(X \times Y)^n$ can be modeled by a functional inductor on a dataset in $((X \times Y) \times \{0, 1\})^n$.

Notation

Let D be a dataset of size n in $X \times Y$. Let $g : X \to Y$, a predictor, which makes prediction $g(x)$ on precept $x \in X$. Let $G_n : (X \times Y)^n \to (X \times Y)$ be an inductor. Then $G_n(D)$ is the predictor which the inductor associates with dataset D. And

$\{G_n : (X \times Y)^n \to (X \times Y)^*\}_{n \in \mathbb{N}}$ is a family of inductors.

Consistent and complete datasets

Let $D = ((x_i, y_i))_{i=1}^n$ be a dataset and $R \subset X \times Y$ a relation. D is *consistent with R* if each $(x_i, y_i) \in R$. D is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take $R = X \times Y$). D is *functionally consistent* if it is consistent with a function; in this case, $x_i = x_j \longrightarrow y_i = y_j$. D is *functionally complete* if $\cup_i \{x_i\} = X$. If a dataset is complete, then it includes every element of the relation.

Other terminology

Other terms for the inputs include *independent variables, explanatory variables, precepts, covariates, patterns, instances*, or *observations*. Other terms for the outputs include *dependent variables, explained variables, postcepts, targets, outcomes, labels* or *observational outcomes*.

Other terms for a predictor include *input-output* mapping, *prediction rule, hypothesis, concept*, or *classifier*. Since a predictor can be used to *guess* the output of an input, some authors call an inductor (or family of inductors) a *learner* or *learning algorithm* or *supervised learning algorithm* and refer to the argument as the *training dataset*. Often the word "supervised" is included, as in *supervised learning*. The language intends to indicate that inputs are given along with outputs, and these outputs "provide supervision to the algorithm."

Inductors

Datasets

Sequences

Direct Products

Set Numbers

Finite Sets

Equivalent Sets

Family Unions and Intersections

Natural Order

Function Inverses

Families

Function Composites

Function Images

Peano Axioms

Functions

Equivalence Relations

Natural Induction

Relations

Cartesian Products

Generalized Set Dualities

Natural Numbers

Set Powers

Set Dualities

Set Unions and Intersections

Intersection of Empty Set

Successor Sets

Unordered Triples

Partitions

Set Complements

Pair Unions

Set Intersections

Ordered Pairs

Set Differences

Pair Intersections

Set Unions

Empty Set

Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects