

## ROOTED TREE ORDERINGS

## Why

We want to simplify common questions using orderings of a rooted tree.<sup>1</sup>

## Definition

A topological ordering of a rooted tree is an ordering  $\sigma$  for which  $v \prec_{\sigma} \operatorname{pa}(v)$ . If there are n vertices, the root has index n and every other vertex has an index less than its parent.

A postordering is a topological ordering in which descendents of a vertex are given consecutive numbers. For the postordering  $\sigma$ , if  $\sigma^{-1}(v) = j$  and v has k proper descendents then the proper descendents of v have are numbered consecutively from j - k through j - 1. Figure 1 shows an example.

One can generate a postordering by numbering vertices in decreasing sequence (starting at n) in the order they are visited.<sup>2</sup>

Given an ordering, the *first descendent* of v (which we denote fdesc(v)) is the descendent with the lowest index. Given a postordering  $\sigma$ , one can check whether a vertex w is a proper descendent of v, by  $fdesc(v) \leq_{\sigma} w \prec_{\sigma} v$ .

<sup>&</sup>lt;sup>1</sup>This sheet is incomplete and will be updated in future editions.

<sup>&</sup>lt;sup>2</sup>Future editions will clarify.

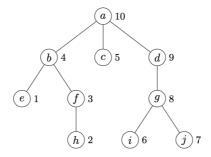


Figure 1: A postordering of a rooted tree.

