Set Specification

1 Why

Can we always construct subsets?

2 Definition

We will say that we can. We assert that to every set and every sentence predicated of elements of the set there exists a second set (a subset of the first) whose elements satisfy the sentence. It is an consequence of the axiom of extension that this set is unique. The *axiom of specification* is this assertion. We call the second set (obtained from the first) the set obtained by *specifying* elements according to the sentence.

2.1 Notation

Let A be a set. Let $S(a)$ be a sentence. We use the notation

$$\{a \in A \mid \; S(a)\;\}$$

to denote the subset of A specified by S. We read the symbol $|$ aloud as "such that." We read the whole notation aloud as "a in A such that..."

We call the notation *set-builder notation*. Set-builder notation avoids enumerating elements. This notation is really indispensable for sets which have many members, too many to reasonably write down.

3 Example

For example, let a, b, c, d be distinct objects. Let $A = \{a, b, c, d\}$. Then $\{x \in A \mid x \neq a\}$ is the set $\{b, c, d\}$

Now let B be an arbitrary set. The set $\{b \in B \mid b \neq b\}$ specifies the empty set. Since the statement $b \neq b$ is false for all objects b.