

⊕ Bounded Linear Continuous

1 Why

All continuous functions between norm spaces are bounded linear functions.

2 Result

Proposition 1. Let $((V_1, F), |\cdot|_1)$ and Let $((V_2, F), |\cdot|_2)$ be two norm spaces. Let $f: V_1 \to V_2$ be a linear function between two norm spaces. Then

- 1. $\exists x \in V_1 \text{ such that } f \text{ is continuous at } x$,
- 2. f is continuous,
- 3. f is uniformly continuous, and
- 4. f is bounded,

are all equivalent conditions.

