

### COMPLEX PRODUCTS

## Definition

Let  $z_1, z_2 \in \mathbf{C}$  with  $z_1 = (x_1, y_1)$  and  $z_2 = (x_2, y_2)$ . The complex product of  $z_1$  and  $z_2$  is the complex number

$$(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2).$$

#### Notation

We denote the complex product of  $z_1$  and  $z_2$  by  $z_1 \cdot z_2$  or  $z_1 z_2$ .

The notation overloads that used for real numbers. This overloading is justified by the fact that the complex product of two purely real complex numbers  $z_1$  and  $z_2$  the purely real complex number whose real part is the product of the real parts of  $z_1$  and  $z_2$ .

Recall that we denote  $z_1 = x_1 + iy_1$  and  $z_2 = x_2 + iy_2$ . This notation is a mnemonic for the definition of a complex product if we treat  $i^2 = -1$ .

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2$$

$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2).$$

## **Properties**

**Proposition 1** (Commutativity). For all  $z_1, z_2 \in \mathbf{C}$ , we have  $z_1 z_2 = z_2 z_1$ .

**Proposition 2** (Associativity). For all  $z_1, z_2, z_3 \in \mathbb{C}$ , we have

$$z_1(z_2z_3) = (z_1z_2)z_3.$$

# Complex multiplication

We call the operation that associates a pair of complex numbers with their product *complex multiplication*. The operation is symmetric (commutative).

# Multiplicative identity and inverse

Notice that the complex number (1,0) is the multiplicative identity. It is unique,  $^1$  and so we call it the *complex multiplicative identity*.

We call the operation  $(z, w) \mapsto z/w$  complex division and we call z/w the (complex) quotient of z with w.

<sup>&</sup>lt;sup>1</sup>Future editions will include an account

