Zero

Why

If I am holding three pebbles, and I have three in my left hand, how many might I have in my right hand? None, of course!

In the notation we have developed we find solutions of

$$3 + n = 3,$$

where n is a natural number. Unfortunately, any natural number added to three is a number different than three. So there is no natural number such that this equation holds.

How can we expand our algebra so that we can express this new, but common, situation in our language?

Definition

We consider a superset of the natural numbers with one additional element. We call this new elemenet *zero* and we call this new set the *natural numbers with zero*.

Extending Arithmetic

We extend the algebra on the natural numbers to an algebra on the natural numbers with zero.

We define an extension of addition, also called *addition*. Given two natural numbers, we define the result as the sum. Given a natural number and zero, we define the result to be

the natural number, and vice versa, so that addition still commutes. As a function, addition by zero is the identity.

We extend multiplication. Given two natural numbers, we define their product as before. Given a natural number and zero, we define the product to be the zero, and vice versa, so that addition still commutes. As a function, mutliplication by zero is constant.

A new solution

We have extended our algebra. We can search for solutions of:

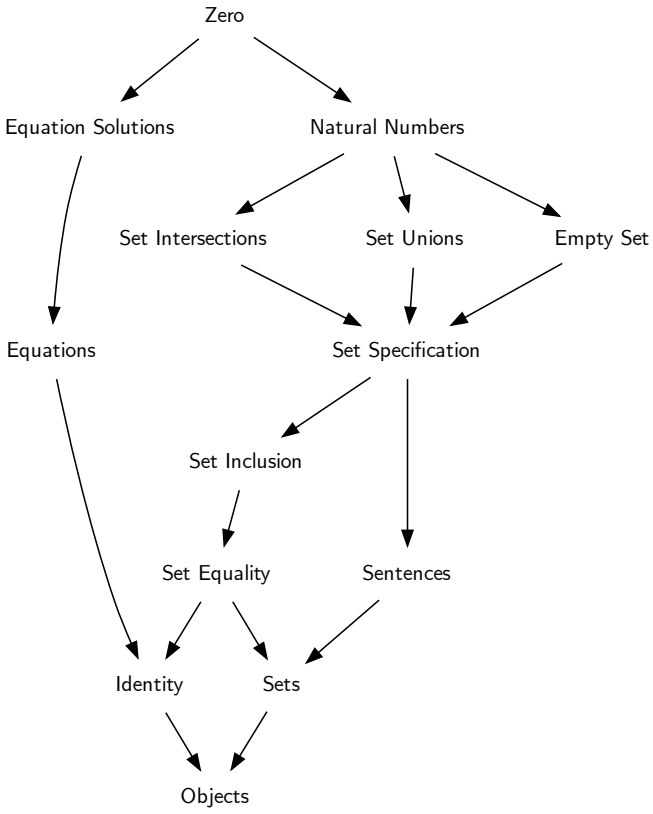
$$3 + n = 3$$

where n is a natural number or zero. Zero is a solution (by design)! Since no natural number solves, zero is the only solution.

Notation

We denote zero by 0. For every natural number n, we defined

$$n + 0 = 0 + n = n \quad \text{and} \quad n \cdot 0 = 0 \cdot n = 0$$


Zero
Equation Solutions
Natural Numbers
Set Intersections
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Set Specification
Set Inclusion
Set Equality
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