Why

We want to build vectors out of other vectors using scalar multiplication and vector addition.

Definition

A *linear combination from* a vector space is an ordered pair: the first coordinate is a sequence of vectors and the second is a sequence of scalars. The *result* of a linear combination is the sum of the results of scaling each vector by the corresponding scalar in the sequence; itself a vector in the space. A *linear combination of a set* of vectors a linear combination consisting of a numbering of them and a scalar sequence.

A *trivial linear combination* is one whose sequence of scalars is zero at each coordinate. The result of any trivial linear combination is the zero vector. A *nontrivial linear combination* is one which is not trivial. In other words, to be nontrivial, there must exist at least one index of the scalar sequence whose corresponding value is nonzero.

We say that a given vector *can be written as a linear combination of* a sequence of vectors if there exists a sequence of scalars such that the result of the linear combination of that sequence of vectors and scalars is the given vector. In other words, a vector can be written as a linear combination of some other vectors if there exists scalars for those other vectors such that scaling them and adding the results gives the vector.

Notation

Let $(V, \mathbf{F})$ be a vector space. Let $v = (v_1, \ldots, v_n)$ be a sequence of vectors in V and $a = (a_1, \ldots, a_n)$ be a sequence of scalars in $\mathbf{F}$. Then (v, a) is a linear combination and we can express its result by

$$a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

If (v, a) is trivial, then $a_i = 0$ for $i \in \{1, 2, \ldots, n\}$ and the result of (v, a) is 0 (the zero vector). Otherwise, there exists $i \in \{1, 2, \ldots, n\}$ such that $a_i \neq 0$; of course, the result of (v, a) may still be 0.

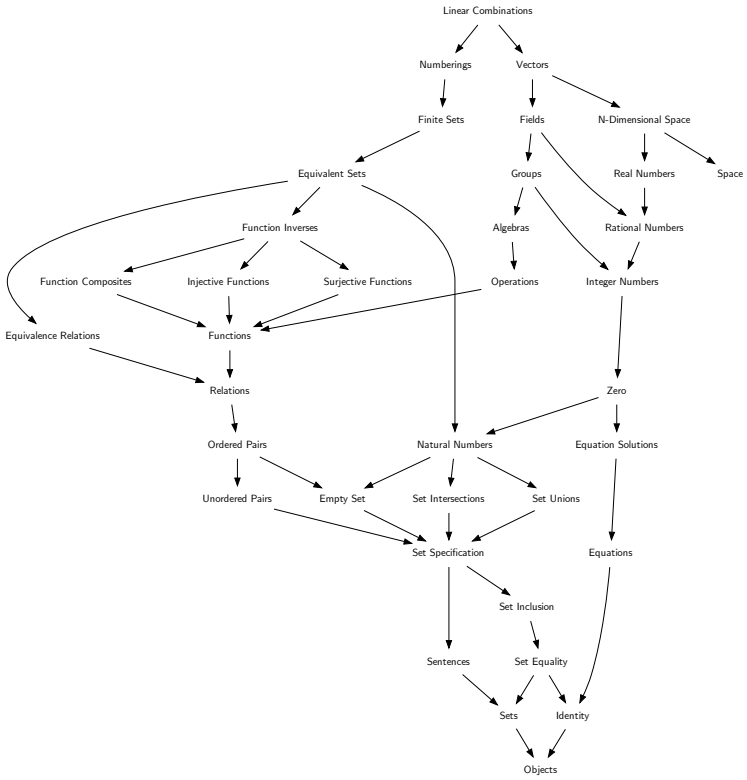
We say that a vector u can be written as a linear combination of the vectors $v_1, v_2, \ldots, v_n$ if there exists scalars $a_1, a_2, \ldots, a_n$ such that the result of the linear combination (v, a) is u. Which we express

$$u = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

If U is a finite set of vectors, then we often say "Let $\{u_1, \ldots, u_n\}$ be a (finite) set of vectors." In this notation, we imply a numbering, and the set notation is intended to indicate that $u_i \neq u_j$ if $i \neq j$ for $i, j \in \{1, 2, \ldots, n\}$. If $u = (u_1, \ldots, u_n)$ then a linear combination of $\{u_1, \ldots, u_n\}$ is any pair (u, a) where $a = (a_1, \ldots, a_n)$ is a scalar sequence.

Relationships

TODO span equivalence


Linear Combinations
Numberings
Vectors
Finite Sets
Fields
N-Dimensional Space
Equivalent Sets
Groups
Real Numbers
Space
Function Inverses
Algebras
Rational Numbers
Function Composites
Injective Functions
Surjective Functions
Operations
Integer Numbers
Equivalence Relations
Functions
Relations
Zero
Ordered Pairs
Natural Numbers
Equation Solutions
Unordered Pairs
Empty Set
Set Intersections
Set Unions
Set Specification
Equations
Set Inclusion
Sentences
Set Equality
Sets
Identity
Objects