



Why

Suppose I have three stones in my hands. If I have two in my left hand, how many are in my right hand?

Discussion

Denote by a the number of stones in my left hand. Denote by b the number of stones in my right hand. Then the number of stones in both hands is $a + b$. Both a and b are natural numbers. In other words, a is 0 or 1 or 2 or... Likewise for b .

We express that I have two stones in my left hand by the equation $a = 2$. We express that I have five stones in total by the equation that $a + b = 5$. Because we have identified the names a and b with tangible objects in my hands, these equations are statements about tangible objects in my hands. But the *numbers* involved are intangible objects. In any case, we have two equations in two unknowns.

We express the question “how many stones do I have in my right hand?” by asking for a solution (see **Equations**) to the equation $a + b = 5$. We can start by trying natural numbers in order. Is $(a, b) = (0, 0)$ a solution? Well, $0 + 0 = 0$, and $0 \neq 5$, so it is not. Is $(a, b) = (1, 0)$ a solution? Well, $1 + 0 = 1$, and $1 \neq 5$, so it is not. Is $(a, b) = (0, 1)$ a solution? Well, $1 + 0 = 1$, and $1 \neq 5$, so it is not. Likewise for $(0, 2)$, $(1, 2)$ and so on. Some people call this the process of *guess and check*.

Continuing this way we find that $(3, 2)$ and $(2, 3)$ is a solution. Indeed, $3 + 2 = 2 + 3 = 5$. We are, however, interested in solutions to both equations

$$\begin{aligned} a &= 3 \\ a + b &= 5 \end{aligned}$$

Both $(3, 2)$ and $(2, 3)$ satisfy the second equation, but only $(3, 2)$ satisfies both. Are there other solutions?¹

¹Future editions will expand.

