



## Why

We are regularly referring to a few common growth classes.

## Definitions

Let  $c \in \mathbf{R}$ . Then we name the following growth classes

growth class	name
$O(1)$	<i>constant growth class</i>
$O(\log(x))$	<i>logarithmic growth class</i>
$O((\log(x))^c)$	<i>polylogarithmic growth class</i>
$O(x)$	<i>linear growth class</i>
$O(x^2)$	<i>quadratic growth class</i>
$O(x^c)$	<i>polynomial growth class</i>
$O(c^x)$	<i>exponential growth class</i>

We have written these in order:

$$O(1) \subset O(\log(x)) \subset O((\log(x))^c) \subset \dots \subset O(x^c) \subset O(c^x).$$

A function that grows faster (is in the upper growth class) of a power of  $x$  is called *superpolynomial*. One that grows slower than  $c^n$  for some  $c \in \mathbf{R}$  is called *subexponential*. The class  $O(\log(x^c)) = O(\log(x))$  since  $\log(x^c) = c \log x$ . Similarly, for all  $c_1, c_2 > 0$ ,  $O(\log_{c_1}(x)) = O(\log_{c_2}(x))$ .

This list is useful because of the following

**Proposition 1.** *Let  $f, g : \mathbf{R} \rightarrow \mathbf{R}$  and defined  $h : \mathbf{R} \rightarrow \mathbf{R}$  by  $h = f + g$ . If  $O(f) \subset O(g)$ , then  $h \in O(g)$ .*

In other words, if a function  $h$  is the sum of  $f$  and  $g$  and  $g$  is growing faster, then  $g$  (the one growing faster) determines the order of  $h$ .



