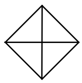


Why

We want to think about two abstract spaces as being equivalent. [1]

Definition

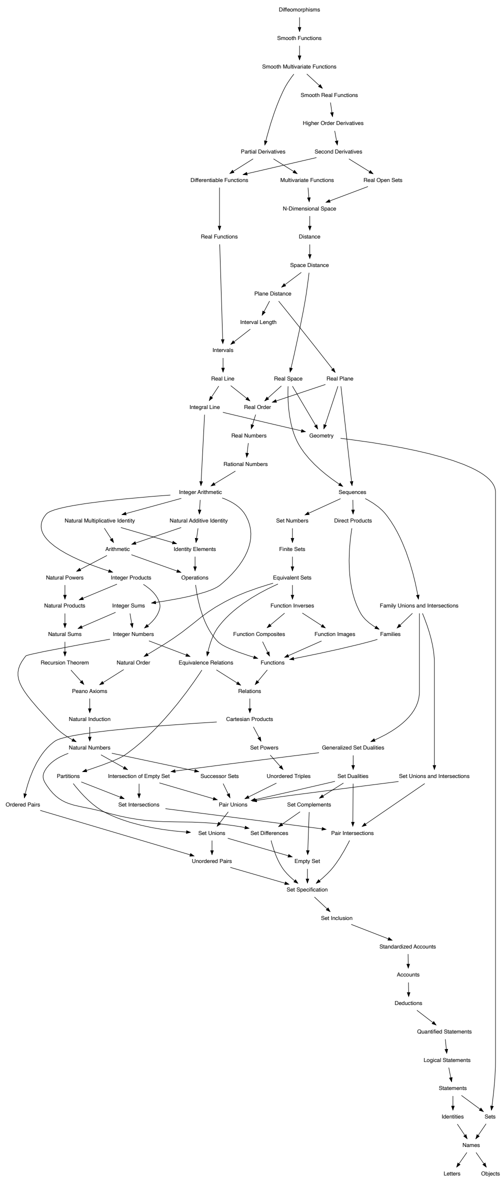
Let $X \subset \mathsf{R}^n$ and $Y \subset \mathsf{R}^m$. A smooth, invertible function $f : X \to Y$ is a *diffeomorphism* f^{-1} is smooth. X and Y are *diffeomorphic* if such a function exists.

The key is the relation diffeomorphic is an equivalence relation. It is reflexive because the identity map is smooth and invertible. It is symmetric since if f is a diffeomorphism from X to Y then f^{-1} is a diffeomorphism from Y to X. It is transitive because the composition of two smooth functions is smooth.

Differential Topology

Differential topology studies properties of $X \subset \mathsf{R}^n$ which do not change under diffeomorphism.

[1] Future editions will include.


Diffeomorphisms
Smooth Functions
Smooth Multivariate Functions
Smooth Real Functions
Higher Order Derivatives
Partial Derivatives
Second Derivatives
Differentiable Functions
Multivariate Functions
Real Open Sets
N-Dimensional Space
Real Functions
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Space
Real Plane
Integral Line
Real Order
Real Numbers
Geometry
Rational Numbers
Integer Arithmetic
Sequences
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Integer Products
Operations
Equivalent Sets
Natural Products
Integer Sums
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects