Why

We define the area under an extended real function.

Definition

The *positive part* of an extended-real-valued function is the function mapping each element to the maximum of the function's result and zero. The *negative part* of an extended-real-valued function is the function mapping each element to the maximum of the additive inverse of function's result and zero.

We decompose an extended-real-valued function as the difference of its positive part and its negative part. Both the positive and negative parts are non-negative extended-real-valued functions.

Consider a measure space. An *integrable* function is a measurable extended real function for which the non-negative integral of the posititve part and the non-negative integral of the negative part of the function are finite. The *integral* of an integrable function is the difference of the non-negative integral of the posititive part and and the non-negative integral of the negative part.

If one but not both of the parts of the function are finite, we say that the integral *exists* and again define it as before. In this way we avoid arithmetic between two infinities.

Notation

Let A a non-empty set. Let $g : A \to [-\infty, \infty]$. We denote the positive part of g by g^+ and the negative part of g by g^-:

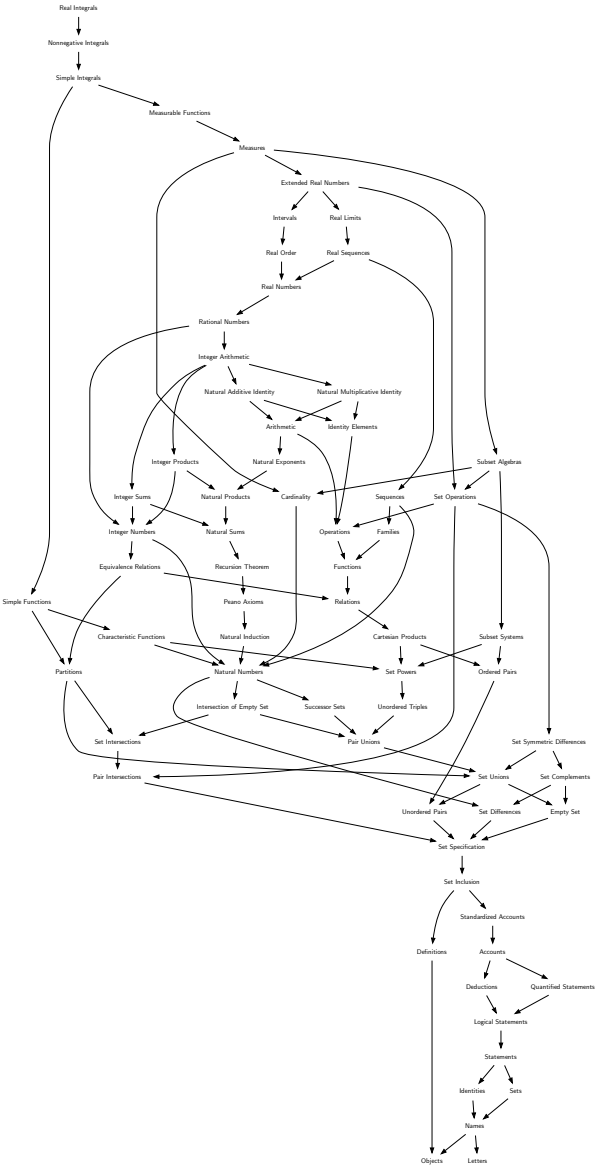
$$g^+(x) = \max\{g(x), 0\} \quad \text{and} \quad g^-(x) = \max\{-g(x), 0\}.$$

Moreover, we decompose g as $g = g^+ - g^-$. We observed that $g^+(x) \geq 0$ and $g^-(x) \geq 0$ for all $x \in X$.

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f : X \to [-\infty, +\infty]$ measurable and one of $\int f^+ d\mu$ or $\int f^- d\mu$ is finite (if both are finite, f is integrable).

We denote the integral of f with respect to the measure μ by $\int f d\mu$. We defined:

$$\int f d\mu = \left(\int f^+ d\mu\right) - \left(\int f^- d\mu\right).$$


Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Extended Real Numbers
Intervals
Real Limits
Real Order
Real Sequences
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Integer Products
Natural Exponents
Subset Algebras
Integer Sums
Natural Products
Cardinality
Sequences
Set Operations
Integer Numbers
Natural Sums
Operations
Families
Equivalence Relations
Recursion Theorem
Functions
Simple Functions
Peano Axioms
Relations
Characteristic Functions
Natural Induction
Cartesian Products
Subset Systems
Partitions
Natural Numbers
Set Powers
Ordered Pairs
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Set Differences
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters