



SET OPERATIONS

Why

We want to consider the elements of two sets together at once, and other sets created from two sets.

Definitions

Let A and B be two sets.

The *union* of A with B is the set whose elements are in either A or B or both. The key word in the definition is *or*.

The *intersection* of A with B is the set whose elements are in both A and B . The keyword in the definition is *and*.

Viewed as operations, both union and intersection commute; this property justifies the language “with.” The intersection is a subset of A , of B , and of the union of A with B .

The *symmetric difference* of A and B is the set whose elements are in the union but not in the intersection. The symmetric difference commutes because both union and intersection commute; this property justifies the language “and.” The symmetric difference is a subset of the union.

Let C be a set containing A . The *complement* of A in C is the symmetric difference of A and C . Since $A \subset C$, the union is C and the intersection is A . So the complement is the “left-over” elements of B after removing the elements of A .

We call these four operations *set-algebraic operations*.

Notation

Let A, B be sets. We denote the union of A with B by $A \cup B$, read aloud as “A union B.” \cup is a stylized U. We denote the intersection of A with B by $A \cap B$, read aloud as “A intersect B.” We denote the symmetric difference of A and B by $A + B$, read aloud as “A symdiff B.” “Delta” is a mnemonic for difference.

Let C be a set containing A . We denote the complement of A in C by $C - A$, read aloud as “C minus A.”

Results

PROPOSITION 1. *For all sets A and B the operations \cup , \cap , and $+$ commute.*

PROPOSITION 2. *Let S a set. For all sets $A, B \subset S$,*

$$(1) \quad S - (A \cup B) = (S - A) \cap (S - B)$$

$$(2) \quad S - (A \cap B) = (S - A) \cup (S - B).$$

PROPOSITION 3. *Let S a set. For all sets $A, B \subset S$,*

$$A + B = (A \cup B) \cap C_S(A \cap B)$$

TODO : notation

Exercises

If A , B , X , and Y are sets, then

$$1. \quad (A \cup B) \times X = (A \times X) \cup (B \times X),$$

$$2. (A \cap B) \times (A \cap Y) = (A \times X) \cap (B \times Y),$$

$$3. (A - B) \times X = (A \times X) - (B \times X).$$

If either $A = \emptyset$ or $B = \emptyset$, then $A \times B = \emptyset$, and conversely.

If $A \subset X$ and $B \subset Y$, then $A \times B \subset X \times Y$, and (provided $A \times B \neq \emptyset$) conversely.

