



# Sigma Algebras

## 1 Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object (TODO).

## 2 Definition

A *countably summable subset algebra* is a subset space for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of  $A_1, \dots, A_n$  coincides with the union of  $A_1, \dots, A_n, A_n, A_n, \dots$ .

We say that the set of distinguished sets is a *sigma algebra* on the base set; we justify this language, as for an algebra, by the closure properties under standard set operations.

## 2.1 Notation

The notation follows that of a subset space. Let  $(A, \mathcal{A})$  be a countably summable subset algebra. We also say “let  $\mathcal{A}$  be an sigma algebra on  $A$ .” Moreover, since the largest element of the sigma algebra is the base set, we can say without ambiguity: “let  $\mathcal{A}$  be a sigma algebra.”

## 3 Examples

**Example 1.** *For any set  $A$ ,  $2^A$  is a sigma algebra.*

**Example 2.** *For any set  $A$ ,  $\{A, \emptyset\}$  is a sigma algebra.*

**Example 3.** *Let  $A$  be an infinite set. Let  $\mathcal{A}$  the collection of finite subsets of  $A$ .  $\mathcal{A}$  is not a sigma algebra.*

**Example 4.** *Let  $A$  be an infinite set. Let  $\mathcal{A}$  be the collection subsets of  $A$  such that the set or its complement is finite.  $\mathcal{A}$  is not a sigma algebra.*

**Proposition 5.** *The intersection of a family of sigma algebras is a sigma algebra.*

**Example 6.** *For any infinite set  $A$ , let  $\mathcal{A}$  be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

*$\mathcal{A}$  is an algebra; the countable/co-countable algebra.*

*TOOD : cleanuexamples*