



## Direct Products

### 1 Why

We generalize the notion of a cross product of two sets to a cross produce of  $n$  sets.

### 2 Direct Products

The *direct product* of a natural family is the set of ordered sequences of elements from each set in the family. We call the elements of the direct product *n-tuples*. We call the  $i$ th element in an  $n$ -tuple the  $i$ th coordinate. This language is meant to follow that used in defining ordered pairs. Two coordinates in a sequence are *consecutive* if their natural difference is 1.

#### 2.1 Notation

Let  $A_1, \dots, A_n$  be a natural family of sets. We denote its direct product by

$$\prod_{i=1}^n A_i.$$

We read this notation as "product over alpha in I of A sub-alpha." We denote an element of  $\prod_{i=1}^n A_i$  by  $(a_1, a_2, \dots, a_n)$  with the understanding that  $a_1 \in A_1, a_2 \in A_2, \dots, a_n \in A_n$ .

If  $A_i = A$  for  $i = 1, \dots, n$ , then we denote  $\prod_{i=1}^n A_i$  by  $A^n$ .