Why

We want to do subtraction.[1]

Definition

Consider the set $\omega \times \omega$. This set is the set of ordered pairs of ω. In other words, the ordered pairs of natural numbers.

We say that two of these ordered pairs (a, b) and (c, d) is *integer equivalent* the $a + d = b + c$. Briefly, the intuition is that (a, b) represents a less b, or in the usual notation "$a - b$".[2] So this equivalence relation says these two are the same if $a - b = c - d$ or else $a + d = b + c$.

Proposition 1. *Integer equivalence is an equivalence relation.*[3]

We define the *set of integers* to be the set of equivalence classes (see *Equivalence Relations*) under integer equivalence on $\omega \times \omega$. We call an element of the set of integers an *integer* or an *integer number*. We call the set of integers the *integers* for short.

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[1]Future editions will change this why. In particular, by referencing *Inverse Elements* and the lack thereof in ω.

[2]This account will be expanded in future editions.

[3]The proof is straightforward. It will be included in future editions.

Notation

We denote the set of integers by Z. If we denote integer equivalence by $\sim$ then $\mathsf{Z} = (\omega \times \omega)/ \sim$.


Integer Numbers
Equivalence Relations
Relations
Cartesian Products
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters