Why

It is natural to look for a class of structural equation models with favorable identifiability and properties.

1 Definition

A *d-dimensional rooted tree linear cascade* is a sequence of four objects: a tree on $\{1, \ldots, d\}$, a vertex of the tree, a family of real numbers indexed by the edges of the tree, and a d-dimensional random vector whose covariance matrix is the identity matrix. The cascade is called "d-dimensional" because we associate it with a random vector (defined as a function of that in the form of its definition) whose codomain is $\mathbf{R}^d$.

The tree together with the vertex form a rooted tree. The graph associated with the rooted tree and the family of real numbers together form a weighted graph.

The idea is to use the weights and the tree structure to recursively define a random vector in terms of elements of the given random vector. Let $C = (T, i, w, e)$ be a d-dimensional rooted tree linear cascade. So T is a tree on $\{1, \ldots, d\}$, $i \in \{1, \ldots, d\}$ and $w : T \to \mathbf{R}$, and $e : \Omega \to \mathbf{R}^d$ for some probability space $(A, \mathcal{A}, \mathbf{P})$. The random vector associated with C is the random variable $x : \Omega \to \mathbf{R}^d$ defined by

$$x_i = e_i \text{ and } x_j = w_{\{\mathrm{pa}\, j, j\}} x_{\mathrm{pa}\, j} + e_j \text{ for } j \neq i.$$

In other words,

$$e = Ax$$

where A is lower triangle and extremely sparse.

Notation

Let $(A, \mathcal{A}, \mathbf{P})$ be a probability space. Let $e : A \to \mathbf{R}^d$ be a random vector, let T be a tree on $\{1, \ldots, d\}$ with $a_{ij} = a_{ji}$ the weight on edge $\{i, j\} \in T$. We

Rooted Tree Linear Cascades
Identity Matrix
Sparse Matrices
Covariance Matrix
Matrix-Matrix Products
Real Matrices
Multivariate Real Densities
Covariance
Probability Densities
Variance
Matrix-Vector Products
Random Vectors
Expectation
Real Integrals
Linear Combinations
Matrices
Random Variables
Nonnegative Integrals
Vectors
N Dimensional Space
Topological Sigma Algebra
Simple Integrals
Topological Spaces
Measurable Functions
Probability Measures
Metrics
Measures
Outcome Variables
Distances
Extended Real Numbers
Sigma Algebras
Space Distance
Real Limits
Absolute Value
Event Probabilities
Simple Functions
Plane Distance
Interval Length
Probability Distributions
Real Functions
Rooted Trees
Intervals
Trees
Directed Graphs
Real Spaces
Real Planes
Undirected Paths
Real Sequences
Real Line
Sequences
Undirected Graphs
Real Order
Paths
Integral Line
Real Summation
Geometry
Set Numbers
Groups
Real Numbers
Uncertain Outcomes
Finite Sets
Integer Additive Inverses
Rational Numbers
Integer Arithmetic
Inverse Elements
Direct Products
Natural Summation
Equivalent Sets
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Element Functions
Family Operations
Subset Algebras
Family Unions and Intersections
Integer Products
Function Inverse
Arithmetic
Identity Elements
Algebra
Set Operations
Cardinality
Integer Numbers
Comparison
Function Images
Natural Powers
Function Composition
Families
Operations
Equivalence Relations
Orders
Converse Relations
Natural Products
Functions
Natural Order
Relations
Natural Sums
Recursion Theorem
Set Products
Subset Systems
Characteristic Functions
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Unordered Triples
Generalized Set Duality
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Duality
Successor Sets
Set Symmetric Differences
Set Intersection
Pair Unions
Set Complements
Pair Intersection
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Notes
Letters
Objects