



Why

We want to visualize the probabilistic relations between components of outcomes in probabilistic models over large (e.g., product) outcome sets.¹

Definition

Let $A = \prod_i A_i$. For $x \in A$ and $S \subset \{1, \dots, n\}$, denote the subvector of x indexed (in order) by S by x_S .²

A distribution $p : A \rightarrow [0, 1]$ *factors according to a directed graph* on $\{0, 1\}$ with parent function $\text{pa} : \{1, \dots, n\} \rightarrow \mathcal{P}(\{1, \dots, n\})$ if

$$p(x) = \prod_{\text{pa}_i = \emptyset} g_i(x_i) \prod_{\text{pa}_i \neq \emptyset} g_i(x_i, x_{\text{pa}_i}),$$

where g_i is a distribution for all i which $\text{pa}_i = \emptyset$ and $g_i(\cdot, \xi)$ is a distribution for all $\xi \in \prod_{j \in \text{pa}_i} A_j$, i for which $\text{pa}_i \neq \emptyset$.

Proposition 1. *p so defined is a distribution, and the g_i are the marginals and conditionals.*³

¹Future editions will modify and expand. The title of the sheet may change, since another interpretation for the words “directed graph distribution” is a distribution on directed graphs.

²Future editions will rework this treatment, perhaps combining it with the sheet **Index Matrices**, which will possibly be split up.

³Future editions will be precise and give an account.

Examples

Consider a rooted tree distribution (see [Rooted Tree Distributions](#)), or a memory chain (see [Memory Chains](#)), or a hidden memory chain (see [Hidden Memory Chains](#)).⁴

Conditional Distribution Network

We can associate a $p : \prod_i A_i \rightarrow [0, 1]$ which factors according to a directed graph $(\{1, \dots, n\}, E)$ with an ordered pair which $((\{1, \dots, n\}, E), (g_i)_{i=1}^n)$ where g_i are as above. We call this latter object, the graph and the conditionals, a *distribution network* (or *distribution graph*, *conditional distribution network*, *conditional distribution graph*, *bayesian network*⁵, *directed probabilistic graphical model*).

⁴Future editions will expand.

⁵Indeed, this term is near universal in certain literatures. We avoid it in these sheets as a result of the Bourbaki project's policy on naming.

