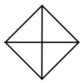


Why

We want to discuss an inductor's performance on consistent (but possibly incomplete) datasets.

We take two steps. First, put a measure on the set of training sets and only consider high-measure subsets. Second, consider predictors performing well in some tolerance.

Definition

Let $(X, \mathcal{X}, \mu)$ be a probability space and $(Y, \mathcal{Y})$ a measurable space. Let $f : X \to Y$ measurable. We call the pair $((X, \mathcal{X}, \mu), f)$ a *(supervised) probabilistic data model.*

We interpret μ as the *data-generating distribution* or *underlying distribution* and f as the *correct labeling function.* Many authors refer to a supervised probabilistic data model as the *statistical learning (theory) framework.*

Probable datasets

We put a measure on the set of datasets by using the product measure $(X^n, \mathcal{X}^n, \mu^n)$. We interpret this as a model for a training set of independent and identically distributed inputs.

For $\delta \in (0, 1)$, $\mathcal{S} \subset X^n$ is $1 - \delta$*-representative* if $\mu^n(\mathcal{S}) \geq 1 - \delta$. If $\mathcal{S}$ is $1 - \delta$-representative for small δ, we think of $\mathcal{S}$ as a set of "probable" or "reasonable" datasets. We call δ the *confidence parameter.*

Predictor error

The *error* of (measurable) $h : X \to Y$ (under μ and f) is

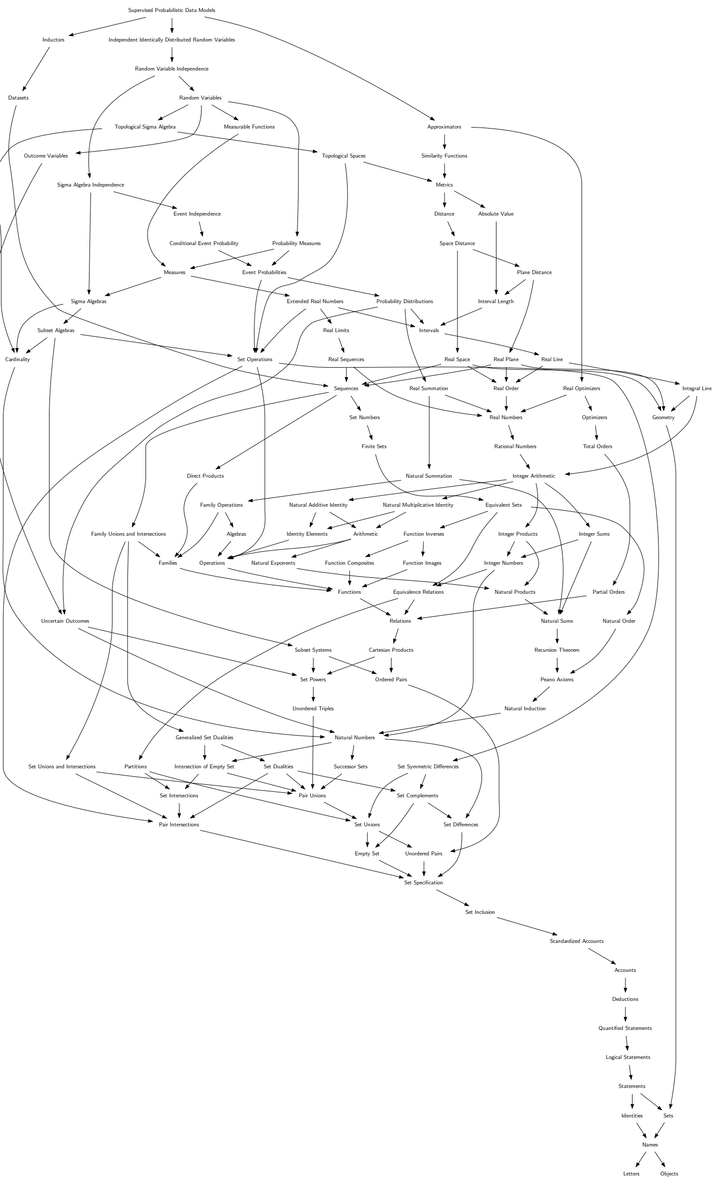
$$\underset{\mu,f}{\textsf{error}}(h) = \mu(\{x \in \mathcal{X} \mid h(x) \neq f(x)\}).$$

We interpret this as the probability that the predictor mislabels a point. The *accuracy* of h is $1 - \textsf{error}_{\mu,f}(h)$.

Since $(f, g) \mapsto \mu[f(x) \neq g(x)]$ is a metric on $L^2(X, \mathcal{X}, \mu, Y)$ we can talk about the error as the "distance" from the correct label classifier. Thus we will say that $\varepsilon \in (0, 1)$, (measurable) $h : \mathcal{X} \to \mathcal{Y}$ *ε-approximates* the correct labeling function f if $\textsf{error}(h) \leq \varepsilon$. Roughly speaking, if $\varepsilon \ll 1$, the the error of the hypothesis is "fairly small." We call ε the *accuracy parameter*, since the accuracy of such a predictor is $1 - \varepsilon$.

Other terminology

A *hypothesis class* is a subset of the measurable functions from $X \to Y$. Other names for the error of a classifier include the *generalization error*, the *risk* or the *true error* or *loss*.


Supervised Probabilistic Data Models
Inductors
Independent Identically Distributed Random Variables
Random Variable Independence
Datasets
Random Variables
Topological Sigma Algebra
Measurable Functions
Approximators
Outcome Variables
Topological Spaces
Similarity Functions
Sigma Algebra Independence
Metrics
Event Independence
Distance
Absolute Value
Conditional Event Probability
Probability Measures
Space Distance
Measures
Event Probabilities
Plane Distance
Sigma Algebras
Extended Real Numbers
Probability Distributions
Interval Length
Subset Algebras
Real Limits
Intervals
Cardinality
Set Operations
Real Sequences
Real Space
Real Plane
Real Line
Sequences
Real Summation
Real Order
Real Optimizers
Integral Line
Set Numbers
Real Numbers
Optimizers
Geometry
Finite Sets
Rational Numbers
Total Orders
Direct Products
Natural Summation
Integer Arithmetic
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Family Unions and Intersections
Algebras
Identity Elements
Arithmetic
Function Inverses
Integer Products
Integer Sums
Families
Operations
Natural Exponents
Function Composites
Function Images
Integer Numbers
Functions
Equivalence Relations
Natural Products
Partial Orders
Uncertain Outcomes
Relations
Natural Sums
Natural Order
Subset Systems
Cartesian Products
Recursion Theorem
Set Powers
Ordered Pairs
Peano Axioms
Unordered Triples
Natural Induction
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
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Standardized Accounts
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Logical Statements
Statements
Identities
Sets
Names
Letters
Objects