Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

Result

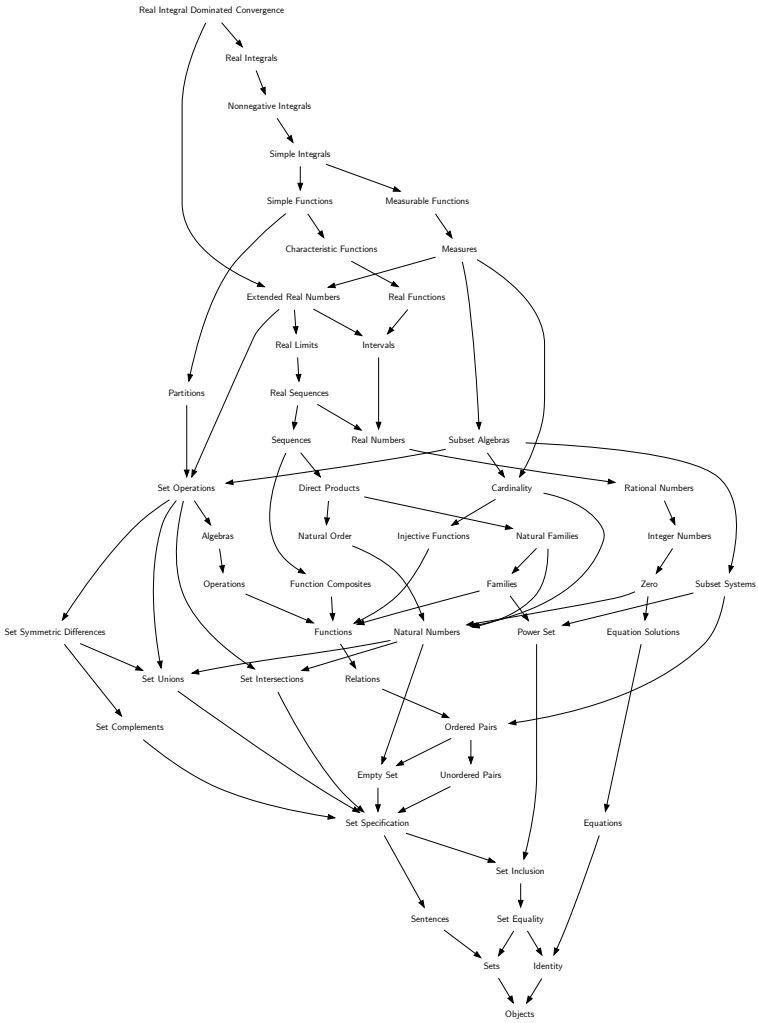
When context is clear, we refer to the following proposition as the *dominated convergence theorem*.

Proposition **1.** *The integral of the almost everywhere limit of a sequence of measurable, extended-real-valued, almost-everywhere bounded functions is the limit of the sequence of integrals of the functions.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f : X \to [-\infty, \infty]$ be a $\mathcal{A}$-measurable function. Let $f_n :\to [-\infty, \infty]$ a $\mathcal{A}$-measurable function for every natural number n so that $(f_n)_n$ converges almost everywhere to f. Let $g : X \to [0, \infty]$ be an integrable function which dominates f_n almost everywhere for each n. We want to show that:

$$\int f d\mu = \lim_n \int f_n d\mu.$$

$\square$


Real Integral Dominated Convergence
Real Integrals
Nonnegative Integrals
Simple Integrals
Simple Functions
Measurable Functions
Characteristic Functions
Measures
Extended Real Numbers
Real Functions
Real Limits
Intervals
Partitions
Real Sequences
Sequences
Real Numbers
Subset Algebras
Set Operations
Direct Products
Cardinality
Rational Numbers
Algebras
Natural Order
Injective Functions
Natural Families
Integer Numbers
Operations
Function Composites
Families
Zero
Subset Systems
Set Symmetric Differences
Functions
Natural Numbers
Power Set
Equation Solutions
Set Unions
Set Intersections
Relations
Set Complements
Ordered Pairs
Empty Set
Unordered Pairs
Set Specification
Equations
Set Inclusion
Sentences
Set Equality
Sets
Identity
Objects