



Real Integral Dominated Convergence

1 Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

2 Result

When context is clear, we refer to the following proposition as the *dominated convergence theorem*.

Proposition 1. *The integral of the almost everywhere limit of a sequence of measurable, extended-real-valued, almost-everywhere bounded functions is the limit of the sequence of integrals of the functions.*

Proof. Let (X, \mathcal{A}, μ) be a measure space. Let $f : X \rightarrow [-\infty, \infty]$ be a \mathcal{A} -measurable function. Let $f_n : X \rightarrow [-\infty, \infty]$ a \mathcal{A} -measurable function for every natural number n so that $(f_n)_n$ converges almost everywhere to f . Let $g : X \rightarrow [0, \infty]$ be an integrable function which dominates f_n almost everywhere for each n . We want to show that:

$$\int f d\mu = \lim_n \int f_n d\mu.$$

□