



## METRIC SPACE EXAMPLES

### Why

We give examples of metric spaces.

### Example

**Example 1.** Let  $n$  be a natural number. Let  $A$  be  $\mathbf{R}^n$  and define  $d : \mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}$  by

$$d(a, b) = \sqrt{(a_1 - b_1)^2 + \cdots + (a_n - b_n)^2}.$$

$(A, d)$  is a metric space.

**Example 2.** Let  $A$  be the unit circle in  $\mathbf{R}^2$ . So  $A = \{x \in \mathbf{R}^2 \mid x_1^2 + x_2^2 = 1\}$ . Let  $d_1 : A \times A \rightarrow \mathbf{R}$  defined by

$$d_1(a, b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}.$$

Let  $d_2 : A \times A \rightarrow \mathbf{R}$  defined as the arc length between the two points. Both  $(A, d_1)$  and  $(A, d_2)$  are metric spaces.

**Example 3.** Let  $A = C([0, 1], \mathbf{R})$ . Let  $d_1 : A \times A \rightarrow \mathbf{R}$  be such that

$$d_1(a, b) = \max_{x \in [0, 1]} |a(x) - b(x)|.$$

Let  $\lambda$  be the outer cover measure. Let  $d_2 : A \times A \rightarrow \mathbf{R}$  be such that

$$d_2(a, b) = \int_{[0, 1]} |f - g| d\lambda.$$

Both  $(A, d_1)$  and  $(A, d_2)$  metric spaces.

