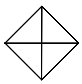


## Why

The most important families are those indexed by (subsets of) the natural numbers.

## Definition

A *finite sequence* is a family whose index set is a nonzero natural number. In other words, a finite sequence is a function whose domain is a nonzero natural number.

The *length* of a finite sequence is the size of its index set (which is the same as the domain of the sequence, see Set Numbers). A sequence whose codomain is some nonempty $A$ is said to be a *sequence in $A$*. Another term for a finite sequence, especially when $A$ is some finite set, is a *string*.

We call an invertible sequence of length $n$ in a finite set $A$ of size $n$ an *ordering* of $A$. We call the inverse of an ordering a *numbering*. An ordering associates to a number $n$ and object in $A$. A numbering associates to the set $A$ a number $n \in \mathbf{N}$

## Notation

Since the natural numbers are ordered, we often denote sequences from left to right between parentheses. For example, We denote $a : 4 \to A$ by listing its elements as $(a_1, a_2, a_3, a_4)$.

#### Relation to Direct Products

A *natural direct product* is a product of a sequence of sets. We denote the direct product of a sequence of sets $A_1, \ldots, A_n$ by $\prod_{i=1}^n A_i$. If each $A_i$ is the same set $A$, then we denote the product $\prod_{i=1}^n A_i$ by $A^n$. In this case, we call an element (the sequence $a = (a_1, a_2, \ldots, a_n) \in A^n$) an *n-tuple* or *tuple*. The set of sequences in a set $A$ is the direct product $A^n$.

#### Infinite Sequences

An *infinite sequence* is a family whose index set is $\mathsf{N}$ (the set of natural numbers without zero). The *nth term* or *coordinate* of a sequence is the result of the $n$th natural number, $n \in \mathsf{N}$.[1]

#### Notation

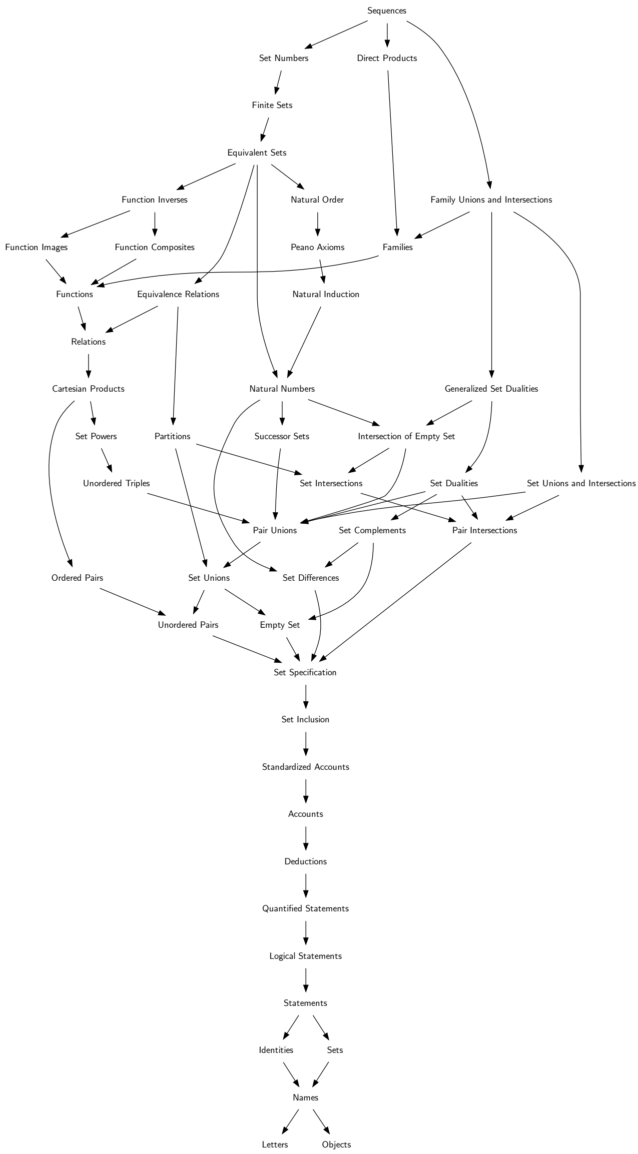
Let $A$ be a non-empty set and $a : \mathsf{N} \to A$. Then $a$ is a (infinite) sequence in $A$. $a(n)$ is the $n$th term. We also denote $a$ by $(a_n)_n$ and $a(n)$ by $a_n$. If $\{A_n\}_{n \in \mathsf{N}}$ is an infinite sequence of sets, then we denote the direct product of the sequence by $\prod_{i=1}^\infty A_i$.

#### Natural unions and intersections

We denote the family union of the finite sequence of sets $A_1$, $\ldots$, $A_n$ by $\cup_{i=1}^n A_i$. We denote the family of the infinite sequence of sets $(A_n)_n$ by $\cup_{i=1}^\infty A_i$. Similarly, we denote the intersections of a finite and infinite sequence of sets $\{A_i\}$ by

[1] Future editions may also comment that we are introducing language for the steps of an infinite process.

$\cap_{i=1}^{n} A_i$ and $\cap_{i=1}^{\infty} A_i$, respectively.


Sequences
Set Numbers
Direct Products
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Images
Function Composites
Peano Axioms
Families
Functions
Equivalence Relations
Natural Induction
Relations
Cartesian Products
Natural Numbers
Generalized Set Dualities
Set Powers
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Intersections
Set Dualities
Set Unions and Intersections
Pair Unions
Set Complements
Pair Intersections
Ordered Pairs
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects