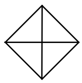


Why

[1]

Definition

Let (V, E) be an undirected graph. An (undirected) *path* between vertex $v \in V$ and vertex $w \neq v$ is a finite sequence of distinct vertices, whose first coordinate is v and whose last coordinate is w, and whose consecutive coordinates are adjacent in the graph. We call the first and last coordinate the *endpoints* of the path. We say the path is *between* its endpoints.

The *length* of a path is one less than the number of vertices: namely, the number of edges. Notice that this definition disagrees with the definition of the "length" of the sequence of vertices (namely, the number of vertices). The length of a path is always at least one: there exists a path of length one between any two adjacent vertices. If a path has length two or greater, we call a vertex which is not the first or last vertex an *interior vertex*.

Two vertices are *connected* in a graph if there exists at least one path between them. A graph is *connected* if each pair of vertices is connected. Recall that two vertices are *adjancent* if they are connected by a path of length one. In contrast, two vertices are connected if they are connected by a path of any length. In other words, all adjacent vertices are connected.

[1] Future editions will include.

A *cycle* is a sequence whose first and last coordinate are identical, all other coordinates are distinct, and consecutive coordinates are adjacent.

Other Terminology

Some authors allow paths to contain repeated vertices, and call a path with distinct vertices a *simple path*. Similarly, some authors allow a cycle to contain repeated vertices, and call a path with distinct vertices a *simple cycle* or *circuit*. Some authors use the term *loop* instead of *cycle*.

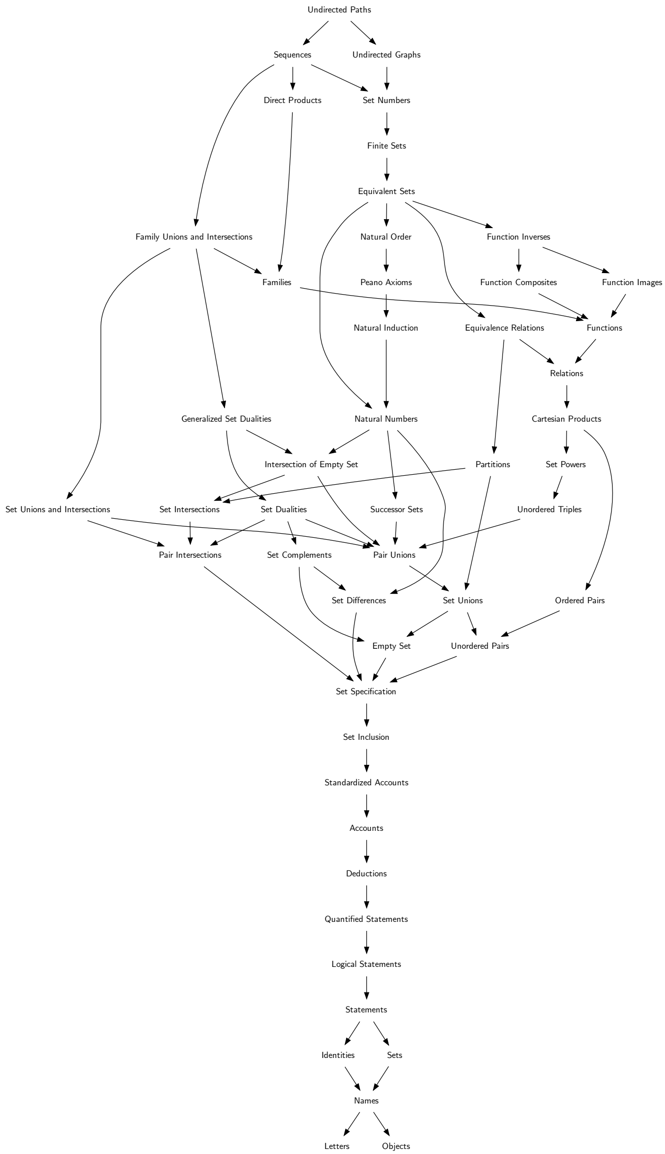
Notation

Let $G = (V, E)$ be a graph. A path between v and w (with $v \neq w$) in G is a sequence $(v_0, v_1, \ldots, v_k)$ where $v_0 = v$ and $v_k = w$ and $\{v_i, v_{i+1}\} \in E$ for $i = 0, \ldots, k-1$.

Connected Components

A set of vertices W in G is *connected* if there is a path between any two vertices $v, w \in W$. A set of vertices W in G is *maximimally connected* if there is no other vertex $v \notin W$ connected to a vertex in W. A *connected component* of G is the subgraph induced by a maximally connected set of vertices.

Since the vertex set of a graph can always be partitioned into sets maximally connected vertices, and a connected components is connect, we think of the connected components of G as the connected "pieces" of G.


Undirected Paths
Sequences
Undirected Graphs
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Peano Axioms
Function Composites
Function Images
Natural Induction
Equivalence Relations
Functions
Relations
Generalized Set Dualities
Natural Numbers
Cartesian Products
Intersection of Empty Set
Partitions
Set Powers
Set Unions and Intersections
Set Intersections
Set Dualities
Successor Sets
Unordered Triples
Pair Intersections
Set Complements
Pair Unions
Set Differences
Set Unions
Ordered Pairs
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects