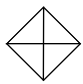


Why

Linear equations are ubiquitous.

Definition

Given $a \in \mathbf{R}^n$ and $y \in \mathbf{R}$, suppose we want to find $x \in \mathbf{R}^n$ satisfying

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = y.$$

We refer to this expression as a *real linear equation* or *linear equation*. We treat each component $x_i \in \mathbf{R}$ as a variable and we treat each component $a_i \in \mathbf{R}$ and $y \in \mathbf{R}$ as constants. We call the pair (a, y) the *real linear equation constants*.[1]

The source of the terminology "linear" is by viewing the left hand side as a function. Define $f : \mathbf{R}^n \to \mathbf{R}$ by $f(x) = \sum_i a_i x_i$. We want to find $x \in \mathbf{R}^n$ to satisfy $f(x) = b$. Notice that f is a *linear* real function.[2]

Moreover, to each linear function $f : \mathbf{R}^d \to \mathbf{R}$ there exists a vector $a \in \mathbf{R}^d$ so that $f(x) = \sum_i a_i x_i$. For this reason, if we are given several linear function $f_1, \dots, f_m$, then we can think of these as several vectors $a^1, \dots, a^n$. If we are also given $b_i \in \mathbf{R}$ for each $i = 1, \dots, m$, then we have the vector $b \in \mathbf{R}^m$

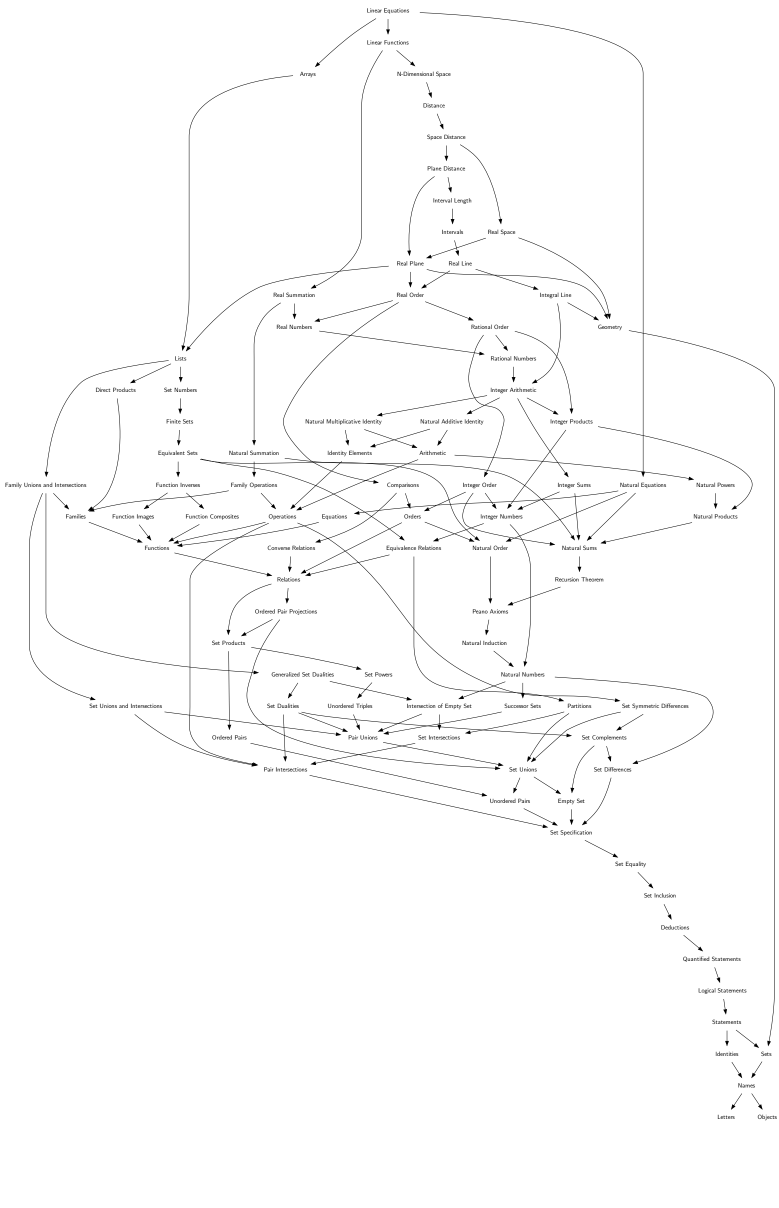
We can define the two-dimensional array $A \in \mathbf{R}^{m \times n}$ by $A_{ij} = a^i_j$. For this reason, a *linear system of equations* is a pair (A, b). A solution of a linear system of equations is a vector $x \in \mathbf{R}^n$ satisfying the equations

$$\begin{array}{ccccc} A_{11}x_1 + & A_{12}x_2 + & \cdots + & A_{1n}x_n = & b_1 \\ A_{21}x_1 + & A_{22}x_2 + & \cdots + & A_{2n}x_n = & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ A_{m1}x_1 + & A_{m2}x_2 + & \cdots + & A_{mn}x_n = & b_n \end{array}$$

Other terminology includes a *system of linear equations* or *linear system* or *simultaneous linear equations*

[1] Future editions will clarify.

[2] Future editions may require a sheet here.


Linear Equations
Linear Functions
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Real Summation
Real Order
Integral Line
Real Numbers
Rational Order
Geometry
Lists
Rational Numbers
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Equivalent Sets
Natural Summation
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Equations
Natural Powers
Families
Function Images
Function Composites
Operations
Equations
Orders
Integer Numbers
Natural Products
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects