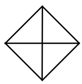


Why

We generalize the algebraic structure of addition and multiplication over the rationals.

Definition

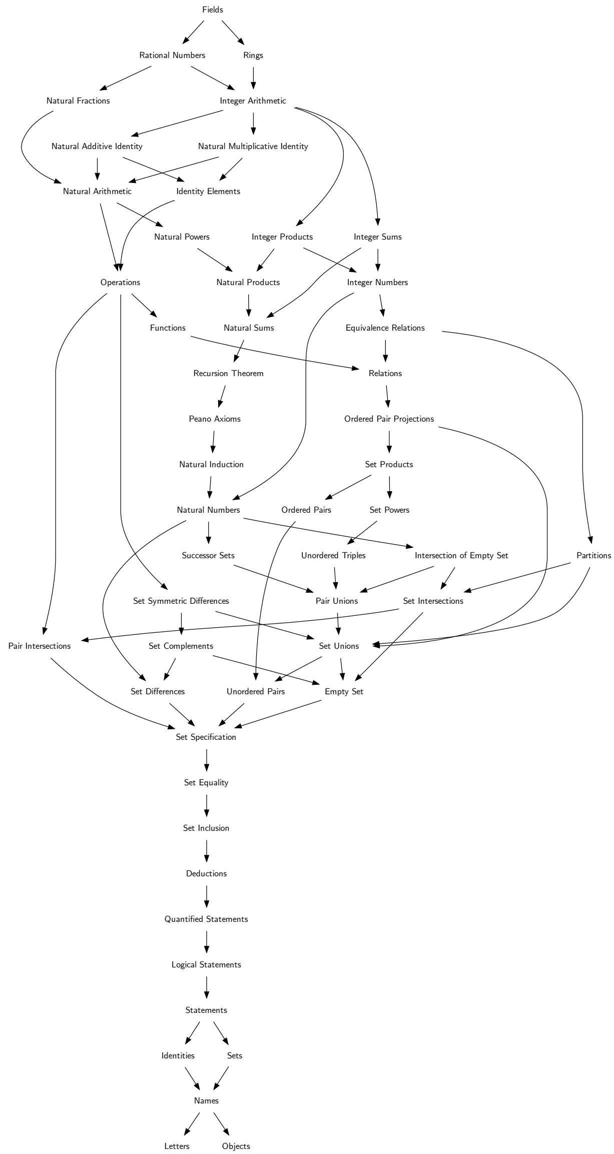
A *field* is a ring $(R, +, \cdot)$ for which $\cdot$ is commutative (i.e., $ab = ba$ for all $a, b \in R$) and $\cdot$ has inverses for all elements except 0. In this case, we refer to *field addition* and *field multiplication*.

Notation

Since our guiding example is the set of rationals $\mathbf{Q}$ with addition and multiplication defined in the usual manner, and we use a bold font for $\mathbf{Q}$, we tend to denote an arbitrary field by $\mathbf{F}$, a mnemonic for "field."

Field operations

Along with field addition and field multiplication, we call the function which takes an element of a field to its additive inverse and the function which takes an element of a field to its multiplicative inverse the *field operations*.


Fields
Rational Numbers
Rings
Natural Fractions
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Natural Arithmetic
Identity Elements
Natural Powers
Integer Products
Integer Sums
Operations
Natural Products
Integer Numbers
Functions
Natural Sums
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Ordered Pairs
Set Powers
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Pair Intersections
Set Complements
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects