



MONOTONE SEQUENCES

Why

If the base set of a sequence has a partial order, then we can discuss its relation to the order of sequence.

Definition

A sequence on a partially ordered set is *non-decreasing* if whenever a first index precedes a second index the element associated with the first index precedes the element associated with the second element. A sequence on a partially ordered set is *increasing* if it is non-decreasing and no two elements are the same.

A sequence on a partially ordered set is *non-increasing* if whenever a first index precedes a second index the element associated with the first index succeeds the element associated with the second element. A sequence on a partially ordered set is *decreasing* if it is non-increasing and no two elements are the same.

A sequence on a partially ordered set is *monotone* if it is non-decreasing, or non-increasing. An increasing sequence is non-decreasing. A decreasing sequence is non-increasing. A sequence on a partially ordered set is *strictly monotone* if it is decreasing, or increasing.

Notation

Let A a non-empty set with partial order \preceq . Let $(a_n)_n$ a sequence in A .

The sequence is non-decreasing if $n \leq m \longrightarrow a_n \preceq a_m$, and increasing if $n < m \longrightarrow a_n \prec a_m$. The sequence is non-increasing if $n \leq m \longrightarrow a_n \succeq a_m$, and decreasing if $n < m \longrightarrow a_n \succ a_m$.

Examples

Example 1. *Let A a non-empty set and $(A_n)_n$ a sequence of sets in $\mathcal{P}(A)$. Partially order elements of $\mathcal{P}(A)$ by the relation contained in.*

