Matrix Trace

1 Why

TODO

2 Definition

The *trace* of a square real matrix is the sum of the elements on its diagonal.

2.1 Notation

We denote the function which associates a matrix with its trace by $\mathbf{tr} : \mathbf{R}^{n \times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n \times n}$. Then

$$\mathbf{tr}\, A = \sum_{i=1}^{n} A_{ii}.$$

3 Properties

Proposition 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n \times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\mathbf{tr}\, C = \sum_{i=1}^{n} C_{ii} = \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii} = \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii} = \alpha\, \mathbf{tr}\, A + \beta\, \mathbf{tr}\, B.$$

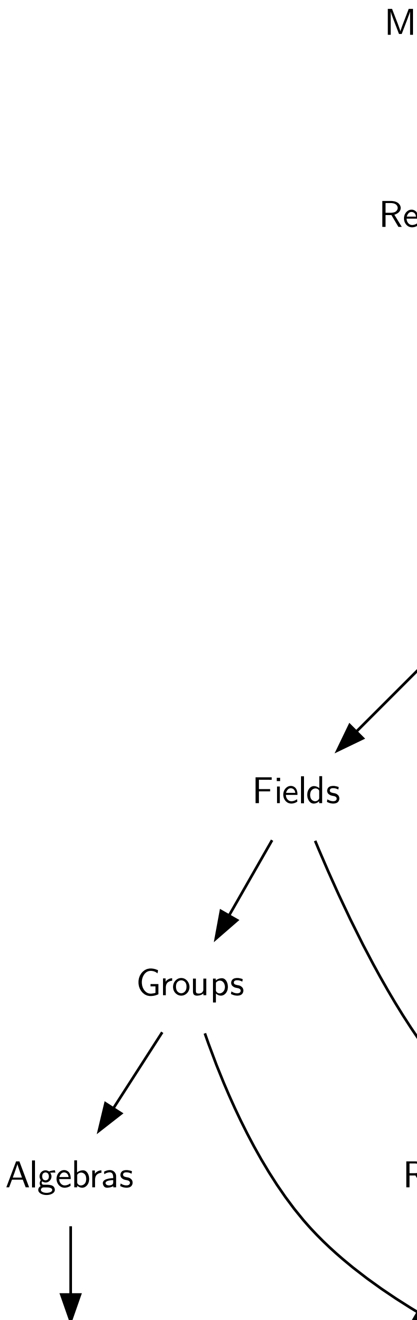
□

Proposition 2. *Let* $A, B \in \mathsf{R}^{n \times n}$.

$$\textsf{\textbf{tr}}\left(AB\right) = \textsf{\textbf{tr}}\left(BA\right)$$

Proof. TODO

$\square$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