



## Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? And so on.<sup>1</sup>

## Definition

The *integrable function spaces* are a collection of function spaces, one for each real number  $p \geq 1$ , for which the  $p$ th power of the absolute value of the function is integrable.<sup>2</sup>

## Notation

Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $p \geq 1$ . We denote the integrable function space corresponding to  $p$  by  $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{R})$ . We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{R}) = \left\{ \text{measurable } f : X \rightarrow \mathbf{R} \mid \int |f|^p d\mu < \infty \right\}$$

Let  $\mathbf{C}$  denote the set of complex numbers. Similarly for complex-valued functions, we denote the  $p$ th space by  $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{C})$ .

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<sup>1</sup>Future sheets are likely to begin with  $L^2$ .

<sup>2</sup>Future editions will include the case where  $p = \infty$ .



