

## Definition

Let  $f: D \to R$  be a multivariate real-valued function where  $D \subset \mathbf{R}^d$ . The graph of f is the set in  $\mathbf{R}^{d+1}$  defined by

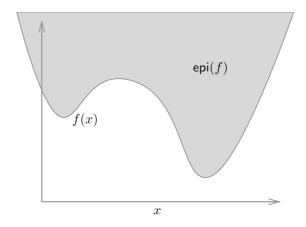
$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}$$

Of course, in these sheets, the graph is the same object as the function f (see discussion in Functions). The epigraph of f is the set in  $\mathbb{R}^{d+1}$  defined by

$$\{(x,\alpha)\in D\times \mathbf{R}\mid f(x)\leq \alpha\}.$$

The prefix "epi" is Greek, meaning "upon" or "above". It is merited (see the visualization below) by the fact that  $f \neq \text{epi } f$ .

## Visualization of epigraph



## Notation

We denote the epigraph of a function f by epi f.

## Extension to extended real numbers

We can extend this concept in the natural way to extended real value function  $f:D\to \bar{\mathbf{R}}.$ 

epi 
$$f\{(x,\alpha) \in D \times \mathbf{R} \mid f(x) \le \alpha\}$$

Caution, in this case  $f \not\subset \operatorname{epi} f$ .

