## Why

We want fractions.[1]

## Rational equivalence

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_\mathbf{Z}\})$. We say that the elements $(a, b)$ and $(c, d)$ of this set are *rational equivalent* if $ad = bc$. Briefly, the intuition is that $(a, b)$ represents $a$ over $b$ In the usual notation, $(a, b)$ represents "$a/b$". So this equivalence relation says these two are the same if $a/b = c/d$ or else $ad = bc$.

**Proposition 1.** *Rational equivalence is an equivalence relation on* $\mathbf{Z} \times (\mathbf{Z} - \{0_\mathbf{Z}\})$.[2]

## Definition

The *set of rational numbers* is the set of equivalence classes (see Equivalence Classes) of $\mathbf{Z} \times (\mathbf{Z} - \{0_\mathbf{Z}\})$ under ratioanl equivalence. We call an element of the set of rational numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

## Notation

We denote the set of rationals by $\mathbf{Q}$.[3] If we denote rational equivalence by $\sim$ then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_\mathbf{Z}\}))/ \sim$.

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[1]This why will be expanded in future editions.

[2]Future editions will include an account.

[3]From what we can tell so far, $\mathbf{Q}$ is a mnemonic for "quantity", from the latin "quantitas".

Rational Numbers

Integer Arithmetic

Natural Additive Identity    Natural Multiplicative Identity

Identity Elements    Arithmetic

Natural Powers    Integer Products

Natural Products    Integer Sums

Operations    Natural Sums    Integer Numbers

Functions    Equivalence Relations    Recursion Theorem

Relations    Peano Axioms

Set Products    Natural Induction

Ordered Pairs    Set Powers    Natural Numbers

Partitions    Intersection of Empty Set    Unordered Triples    Successor Sets

Set Intersections    Pair Unions    Set Symmetric Differences

Pair Intersections    Set Unions    Set Complements

Unordered Pairs    Empty Set    Set Differences

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities    Sets

Names

Letters    Objects

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