Why

We want fractions.[1]

Rational equivalence

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We say that the elements (a, b) and (c, d) of this set are *rational equivalent* if $ad = bc$. Briefly, the intuition is that (a, b) represents a over b In the usual notation, (a, b) represents "a/b". So this equivalence relation says these two are the same if $a/b = c/d$ or else $ad = bc$.

Proposition 1. *Rational equivalence is an equivalence relation on* $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$.[2]

Definition

The *set of rational numbers* is the set of equivalence classes (see Equivalence Classes) of $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ under ratioanl equivalence. We call an element of the set of rational numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

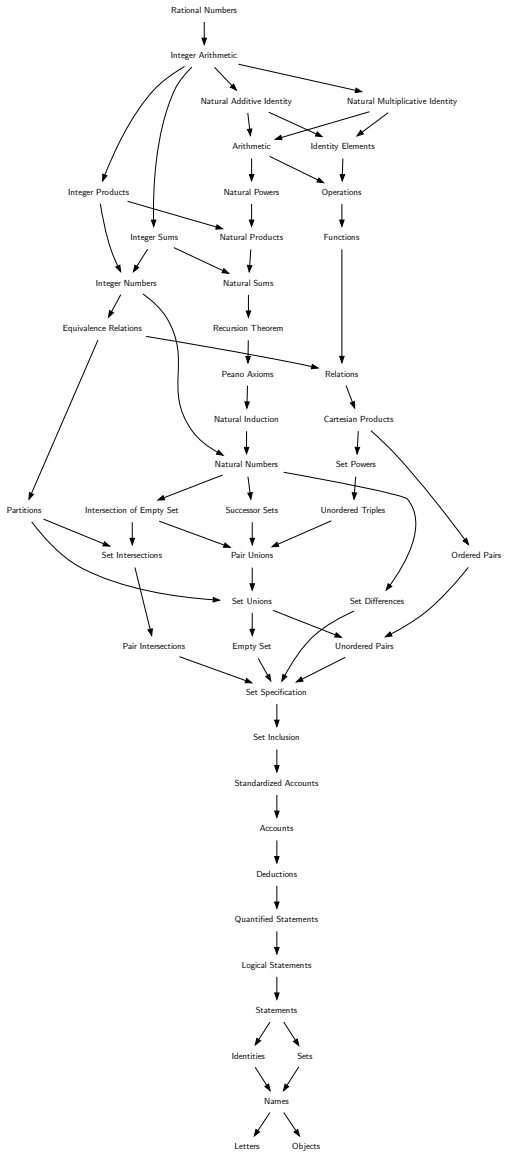
Notation

We denote the set of rationals by $\mathbf{Q}$.[3] If we denote rational equivalence by $\sim$ then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$.

[1] This why will be expanded in future editions.

[2] Future editions will include an account.

[3] From what we can tell so far, $\mathbf{Q}$ is a mnemonic for "quantity", from the latin "quantitas".


Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Integer Products
Natural Powers
Operations
Integer Sums
Natural Products
Functions
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects