Function Properties

1 Why

TODO

2 Definition

Let $f : A \to B$. The **image** of a set $C \subset A$ is the set $\{f(c) \in B \mid c \in C\}$. The **range** of f is the image of the domain. The **inverse image** of a set $D \subset B$ is the set $\{a \in A \mid f(a) \in B\}$.

The range need not equal the codomain; though it, like every other image, is a subset of the codomain. The function maps to domain **on** to the codomain if the range and codomain are equal; in this case we call the function **onto**. This language suggests that every element of the codomain is used by f. It means that for each element b of the codomain, we can find an element a of the domain so that $f(a) = b$.

An element of the codomain may be the result of several elements of the domain. This overlapping, using an element of the codomain more than once, is a regular occurrence. If a function is a unique correspondence in that every domain element has a different result, we call it **one-to-one**. This language is meant to suggest that each element of the domain corresponds to one and exactly one element of the codomain, and vice versa.

2.1 Notation

Let $f : A \to B$. We denote the image of $C \subset A$ by $f(C)$, read aloud as "f of C." This notation is overloaded: for $c \in C$, $f(c) \in A$, whereas $f(C) \subset A$. Read aloud, the two are indistinguishable, so we must be careful to specify whether we mean an element c or a set C. The property that f is onto can be written succintly as $f(A) = B$. We denote the inverse image of $D \subset B$ by $f^{-1}(D)$, read aloud as "f inverse D."