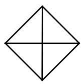


Why

We want to sum infinitely many real numbers.

Definition

Let $(a_k)_{k \in \mathbf{N}}$ be a sequence in $\mathbf{R}$. Define $(s_n)_{n \in \mathbf{N}}$ by

$$s_n = \sum_{k=1}^{n} a_k.$$

We call s_n the *nth partial sum* of (x_k). In other words, the first partial sum s_1 is a_1, the second partial sum s_2 is $a_1 + a_2$, the third partial sum s_3 is $a_1 + a_2 + a_3$ and so on.

We call (s_n) the *sequence of partial sums* or *series* of (a_k). If the *series* converges, then we say that (a_k) is *summable*. Clearly not every series is summable: consider, for example, $a_k = 1$ for all k. It has the divergent series $(1, 2, 3, 4, 5, \dots)$.

Notation

If the sequence is summable, then there exists a unique $s \in \mathbf{R}$ (the limit), which we denote

$$s = \lim_{n \to \infty} s_n = \lim_{n \to \infty} \sum_{k=1}^{n} a_k. \tag{1}$$

We read these relations aloud as "s is the limit as n goes to infinity of s n" and "s is the limit as n goes to infinity of the sum of a k from k equals 1 to n." We often avoid referencing s_n by abbreviating Equation (1) by

$$\sum_{k=1}^{\infty} a_k = s.$$

We read this notation aloud as "the sum from 1 to infinity of a k is s." The notation is subtle, and requires justification by the algebra of series.[1]

[1] Future editions will include such justification.

Convergence

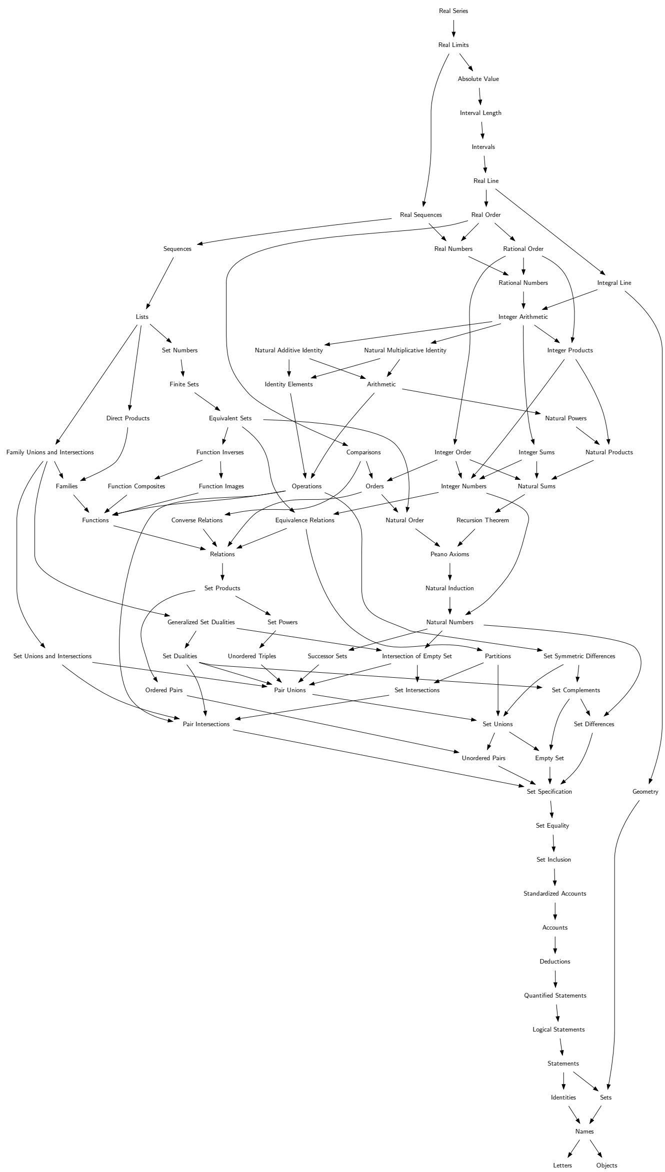
For a series to converge, intuition suggests that the additional terms added should be getting smaller and smaller. Indeed:

Proposition 1. *Let $(a_k)_{k \in \mathbf{N}}$ be a sequence of real numbers. If (a_k) is summable then a_k converges to 0.*[2]

The converse of this theorem has immediate relevance as a preliminary test for determining whether a series converges.

Proposition 2. *If (a_k) does not converge or converges to $a_0 \neq 0$, then it is not summable.*

[2] Future editions will include an account.


Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Sequences
Real Order
Sequences
Real Numbers
Rational Order
Rational Numbers
Integral Line
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects