Why

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Definition

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex product* of z_1 and z_2 is the complex number $(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$.

Notation

We denote the complex product of z_1 and z_2 by $z_1 \cdot z_2$ or z_1z_2. The notation is justified because the complex product of two purely real complex numbers corresponds to the purely real complex number whose real part is the real product of the real parts of the first two numbers.

Recall that we denote $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. This notation is a mnemonic for the definition of a complex product if we treat $i^2 = -1$.

$$z_1z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2).$$

Properties

Proposition 1 (Commutativity). *For all $z_1, z_2 \in \mathbf{C}$, we have* $z_1z_2 = z_2z_1$.

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[1]Future editions will include.

Proposition 2 (Associativity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have and $z_1(z_2 z_3) = (z_1 z_2) z_3$.*

Complex multiplication

We call the operation that associates a pair of complex numbers with their product *complex multiplication.* The operation is symmetric (commutative).

Multiplicative identity and inverse

Notice that the complex number $(1, 0)$ is the multiplicative identity. It is unique,[2] and so we call it the *complex multiplicative identity.*

We call the operation $(z, w) \mapsto z/w$ *complex division* and we call z/w the *(complex) quotient* of z with w.

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[2]Future editions will include an account

Complex Products

Complex Sums

Complex Numbers

Absolute Value   Real Arithmetic

Interval Length   Real Multiplicative Inverses

Intervals

Real Line   Real Products

Real Additive Inverses

Rational Multiplicative Inverses   Real Order   Real Sums   Rational Arithmetic

Real Numbers   Rational Products   Rational Sums

Integral Line

Rational Numbers

Inverse Elements   Integer Arithmetic

Element Functions   Natural Additive Identity   Natural Multiplicative Identity   Integer Products

Function Inverses   Algebras   Identity Elements   Arithmetic

Function Images   Function Composites   Operations   Natural Powers

Functions   Natural Products   Integer Sums

Natural Sums   Integer Numbers

Equivalence Relations   Recursion Theorem

Relations   Peano Axioms

Cartesian Products   Natural Induction

Set Powers   Natural Numbers

Unordered Triples   Partitions   Successor Sets   Intersection of Empty Set

Ordered Pairs   Pair Unions   Set Intersections

Set Unions   Set Differences

Unordered Pairs   Empty Set   Pair Intersections

Geometry   Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects