



## INVERSES OF COMPOSITE RELATIONS

### Why

How do inverse and converse relations interact.

### Results

Let  $R$  be a relation between  $X$  and  $Y$  and let  $S$  be a relation between  $Y$  and  $Z$ .

**Proposition 1.**  $(RS)^{-1} = S^{-1}R^{-1}$

### Identity Relations

Recall that  $I$  is the identity relation on  $X$  if  $x I y$  if and only if  $x = y$ .

**Proposition 2.** *Let  $R$  be a relation on  $X$ . Let  $I$  be the identity relation on  $X$ . Then  $RI = IR = R$ .*

One would like  $RR^{-1} \supset I$ ,  $R^{-1}R \supset I$ . The father of the son is the father and the son of the father is the son. But the empty relation violates these claims.

### Relation Properties

**Proposition 3.**  *$R$  is symmetric if and only if  $R \subset R^{-1}$*

**Proposition 4.**  *$R$  is reflexive if and only if  $I \subset R$*

**Proposition 5.**  *$R$  is transitive if and only if  $RR \subset R$ .*



