Why

It is often the case in considering set differences that all sets considered are subsets of one set.

Definition

Let A and B denote sets. In many cases, we take the difference between a set and one contained in it. In other words, we assume that $B \subset A$. In this case, we often take complements relative to the same set A. So we do not refer to it, and instead refer to the relative complement of B in A as the *complement* of B.

Notation

Let A denote a set, and let B denote a set for which $B \subset A$. We denote the relative complement of B in A by $C_A(B)$. When we need not mention the set A, and instead speak of the complement of B without qualification, we denote this complement by $C(B)$.

Complement of a complement

One nice property of a complement when $B \subset A$ is:

Proposition 1. $(B \subset A) \longleftrightarrow (C_A(C_A(B)) = B)$

Basic facts

Let E denote a set and let A and B denote sets satisfying $A, B \subset E$. Then take all complements with respect to E. Here are some immediate consequences of the definition.[1]

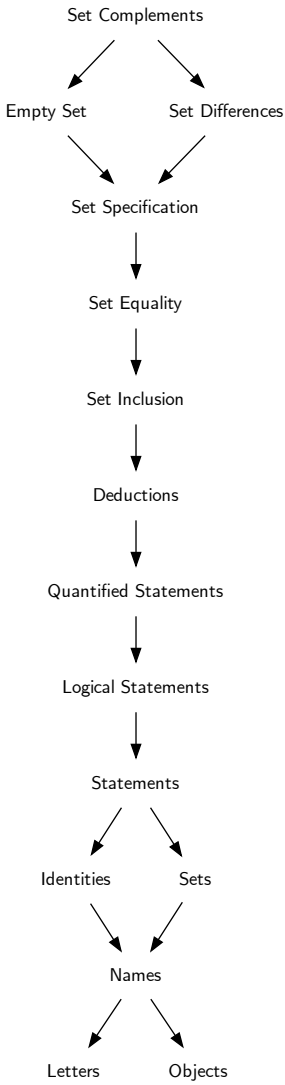
Proposition 2. $C(C(A)) = A$

Proposition 3. $C(\varnothing) = E$

Proposition 4. $C(E) = \varnothing$

[1] Future editions will include accounts.

Proposition 5. $A \subset B \longleftrightarrow C(B) \subset C(A)$


Set Complements
Empty Set
Set Differences
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects