



Why

We bound the probability that a random variable deviates from its mean using its variance.

Result

Prop. 1. *Let f be a square-integrable real-valued random variable on the probability space (X, \mathcal{A}, μ) . Then for $t > 0$,*

$$\mu(|f - \mathbf{E}(f)| \geq t) \leq \frac{\text{var } f}{t^2}.$$

Proof. The set $|f - \mathbf{E}(f)| \geq t$ is $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$. This set is $\{x \in X \mid (f(x) - \mathbf{E}(f))^2 \geq t^2\}$. By using the non-negative inequality

$$\mu(\{x \in X \mid (f(x) - \mathbf{E}(f))^2 \geq t^2\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))^2}{t^2}.$$

We recognize the numerator of the right hand side as the variance. □

The above is also called *Chebychev's Inequality*.

