Why

[1]

Definition

Let Ω be an open set in $\mathbf{C}$ and let $f : \Omega \to \mathbf{C}$. The function f is *holomorphic at the point* $z_0 \in \mathbf{C}$ if the complex quotient

$$\frac{f(z_0 + h) - f(z_0)}{h}$$

has a limit when $h \to \infty$, where $h \in \mathbf{C}$, $h \neq 0$ and $z_0 + h \in \Omega$ so that the quotient is well-defined.

This condition is similar to saying that a function is differentiable, except that the h is complex and so the condition above encomposes all limits approaching z (all angles) in the complex plane.[2] But we emphasize that h is a complex number approaching the complex number $(0, 0)$ from any direction. If the limit exists, then we call its value the *derivative of* f *at* z_0.

The function f is *holomorphic* on Ω if f is holomorphic at every point of Ω. If C is a closed subset of $\mathbf{C}$, we say that f is holomorphic on C if f is holomorphic on some open set containing c. If f is holomorphic on all of C then we call f *entire*. A holomorphic function is sometimes called *regular* or *complex differentiable*. The latter term is used in view of the similarities with the definition of a real derivative.

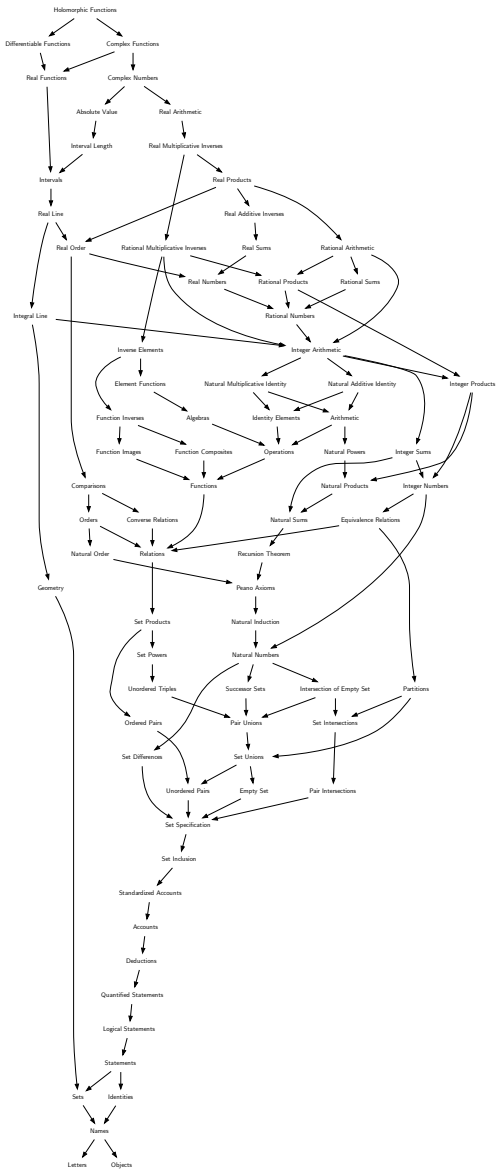
[1] Future notes will expand.

[2] Future editions will clarify.

Notation

In the case that $f : \Omega \to \mathbf{C}$ is holomorphic at z_0 we denote the derivative at z_0 by $f'(z_0)$. We have defined $f'(z_0)$ by

$$f'(z_0) = \lim_{h \to 0} \frac{f(z_0 + h) - f(z_0)}{h}.$$


Holomorphic Functions
Differentiable Functions
Complex Functions
Real Functions
Complex Numbers
Absolute Value
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Line
Real Additive Inverses
Real Order
Rational Multiplicative Inverses
Real Sums
Rational Arithmetic
Real Numbers
Rational Products
Rational Sums
Rational Numbers
Integral Line
Integer Arithmetic
Inverse Elements
Integer Products
Element Functions
Natural Multiplicative Identity
Natural Additive Identity
Function Inverses
Algebras
Identity Elements
Arithmetic
Function Images
Function Composites
Operations
Natural Powers
Integer Sums
Comparisons
Functions
Natural Products
Integer Numbers
Orders
Converse Relations
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Equivalence Relations
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Recursion Theorem
Geometry
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Ordered Pairs
Pair Unions
Set Intersections
Set Differences
Set Unions
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects