Complex Inner Products

1 Why

What is an inner product if we take a vector space over the complex numbers.

2 Definition

An inner produce over a complex vector space is positive definte, Hermitian, and linear in the first argument.

2.1 Alternate Conventions

2.2 Notation

Let C be the set of complex numbers. Let (V, C) be a complex vector space. Let $f : V \times V \to C$ be a function such that

1. $f(x, x) \geq 0$, $f(x, x) = 0 \Leftrightarrow x = 0$;
2. $f(x, y) = \overline{f(y, x)}$
3. $f(ax + by, z) = a(x, z) + b(y, z)$