Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object.[1]

Definition

A *countably summable subset algebra* is a subset algebra for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of $A_1, \ldots, A_n$ coincides with the union of $A_1, \cdots, A_n, A_n, A_n \cdots$.

We call the set of distinguished sets a *sigma algebra* (or *sigma field*) *on* the base set. This language is justified (as for a regular subset algebra) by the closure properties of the sigma algebra under the usual set operations. We sometimes write are σ-algebra and σ-field.

A *sub-σ-algebra* (*sub-sigma-algebra*) is a subset of a sigma algebra which is itself a sigma algebra.

Notation

Let $(A, \mathcal{A})$ be a countably summable subset algebra. We often say "let $\mathcal{A}$ be a sigma algebra on A." Since the largest element of the sigma algebra is the base set, we can also say (without ambiguity): "let $\mathcal{A}$ be a sigma algebra." In this last case, the base set is $\cup\mathcal{A}$.

Examples

Example 1. *For any set A, 2^A is a sigma algebra.*

[1] Future editions will make no reference to measure theory. The entire development will follow the genetic approach, and so roughly follow the historical development for handling integration.

Example 2. *For any set A, $\{A, \varnothing\}$ is a sigma algebra.*

Example 3. *Let A be an infinite set. Let $\mathcal{A}$ the collection of finite subsets of A. $\mathcal{A}$ is not a sigma algebra.*

Example 4. *Let A be an infinite set. Let $\mathcal{A}$ be the collection subsets of A such that the set or its complement is finite. $\mathcal{A}$ is not a sigma algebra.*

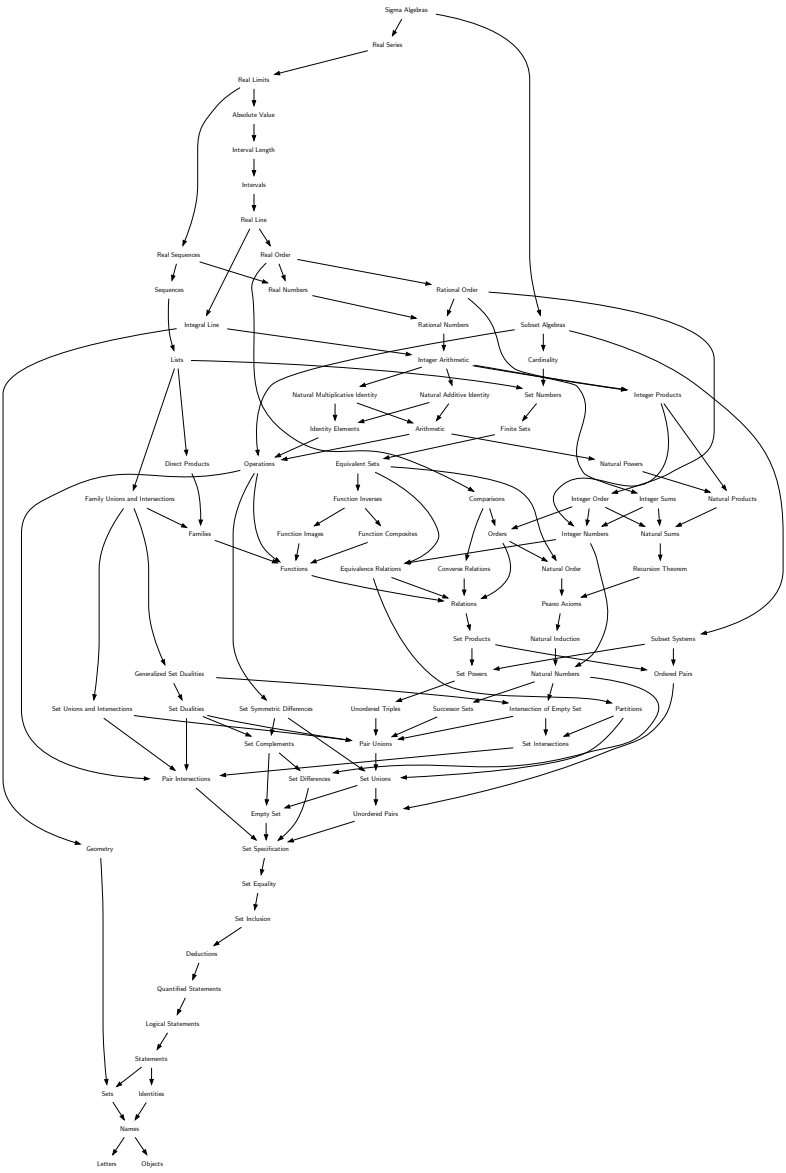
Proposition 1. *The intersection of a family of sigma algebras is a sigma algebra.*

Example 5. *For any infinite set A, let $\mathcal{A}$ be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

$\mathcal{A}$ is an algebra; the countable/co-countable algebra.[2]

[2]Future editions will clean up and modify these examples.


Sigma Algebras
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Sequences
Real Order
Sequences
Real Numbers
Rational Order
Rational Numbers
Subset Algebras
Integral Line
Integer Arithmetic
Cardinality
Lists
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Integer Products
Identity Elements
Arithmetic
Finite Sets
Direct Products
Operations
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Images
Function Composites
Orders
Integer Numbers
Natural Sums
Functions
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Subset Systems
Generalized Set Dualities
Set Powers
Natural Numbers
Ordered Pairs
Set Unions and Intersections
Set Dualities
Set Symmetric Differences
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects