Why

How can we construct distributions which factor according to a directed graph.[1]

Definition

Let $\bar{G} = (G, A)$ be a typed graph (see Typed Graphs) with directed and acyclic G. For source vertices i, let $g_i : A_i \to [0, 1]$ be a distribution and otherwise let $g_i : A \times A_{\mathrm{pa}_i} \to [0, 1]$ denote a function satisfying $g_i(\cdot, x)$ is a distribution for every $x \in X_{\mathrm{pa}_i}$.

We call the ordered pair $(\bar{G}, g)$ a *distribution graph* (or *distribution network, conditional distribution network, conditional distribution graph, bayesian network,*[2] *directed probabilistic graphical model, directed graphical model*).

The *distribution* of $(\bar{G}, g)$ is the function $p : \prod_i A_i \to [0, 1]$ defined by

$$p(a) = \prod_{\mathrm{pa}_i = \varnothing} g_i(a_i) \prod_{\mathrm{pa}_i \neq \varnothing} g_i(a_i, a_{\mathrm{pa}_i}).$$

It is, of course, a distribution. And it factors according to the directed and acyclic graph G. Also, the g_i, $i = 1, \ldots, n$, are the conditionals.[3]

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[1]Future editions might flip the order of this sheet with that of directed graph distributions since, in the genetic approach, in may be more natural to think of constructing such distributions before analyzing them. This is partially motivated by the acyclic constraint here, which restricts the graphs according to which a distribution can factor.

[2]Indeed, this term is near universal in certain literatures. We avoid it in these sheets as a result of the Bourbaki project's policy on naming.

[3]Future editions will elaborate and give a proof.