Why

How should we represent a subspace computationally?

Definition

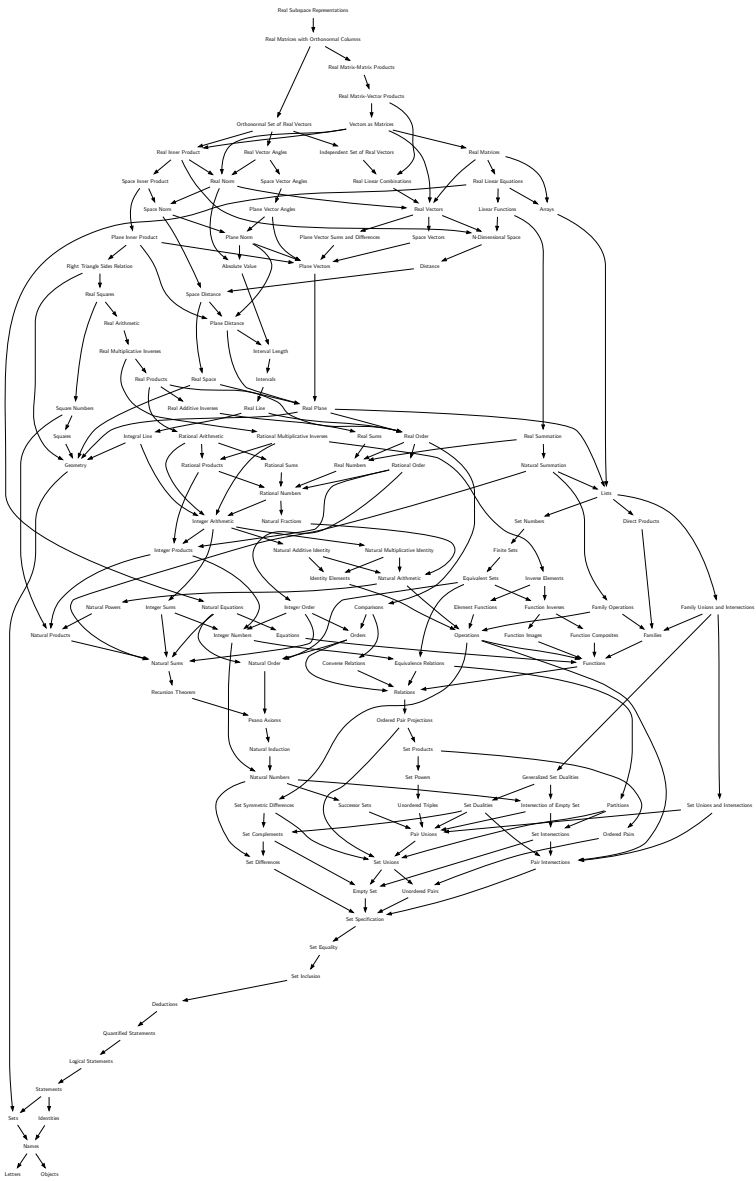
Given a subspace $S \in \mathbf{R}^n$, since it is finite dimensional, there exists a finite basis for the space. This basis can be made orthonormal. Therefore every subspace S has an orthonormal basis $q_1, \ldots, q_k$, where k is the dimension of the subspace. We can stack these as a matrix. Define $Q \in \mathbf{R}^{n \times k}$ by

$$Q = \begin{bmatrix} q_1 & q_2 & \cdots & q_k \end{bmatrix}.$$

For every $x \in S$, there exists unique coefficients $z \in \mathbf{R}^k$ so that

$$x = Qz.$$

Therefore we have a one-to-one correspondence between vectors $x \in S$ and their coordinates $z \in \mathbf{R}^k$.


Real Subspace Representations
Real Matrices with Orthonormal Columns
Real Matrix-Matrix Products
Real Matrix-Vector Products
Vectors as Matrices
Orthonormal Set of Real Vectors
Real Inner Product
Real Vector Angles
Independent Set of Real Vectors
Real Matrices
Space Inner Product
Real Norm
Space Vector Angles
Real Linear Combinations
Real Linear Equations
Space Norm
Plane Vector Angles
Real Vectors
Linear Functions
Arrays
Plane Inner Product
Plane Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Right Triangle Sine Relation
Absolute Value
Plane Vectors
Distance
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Square Numbers
Real Additive Inverses
Real List
Real Plane
Squares
Integral Line
Rational Arithmetic
Rational Multiplicative Inverses
Real Sums
Real Order
Real Summation
Geometry
Rational Products
Rational Sums
Real Numbers
Rational Order
Natural Summation
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Natural Equations
Integer Order
Comparisons
Element Functions
Function Inverses
Family Operations
Family Unions and Intersections
Natural Products
Integer Numbers
Equations
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Includes
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Spaces
Letters
Objects