



Why

1

Definition

The *higher adjacency set* or *higher neighborhood* of a vertex v in an ordered undirected graph is all vertices in the neighborhood of v whose index is greater the v . Similarly, the *lower adjacency set* or *lower neighborhood* of v is all vertices in the neighborhood of v whose index is less the v . We call these *monotone neighborhoods*.

The *higher degree* of a vertex is the size of the higher adjacency set and the *lower degree* of a vertex is the size of its lower adjacency set.

The *closed monotone neighborhoods* are the *closed higher adjacency set*, the higher adjacency set of v union with the singleton $\{v\}$ and the *closed lower adjacency set*, the lower adjacency set of v union with the singleton $\{v\}$.

Notation

We denote the higher neighborhood of v by $\text{adj}^+(v)$ and the lower neighborhood by $\text{adj}^-(v)$.

Visualization

To help think about the monotone neighborhoods of the graph we visualize ordered graphs as triangular arrays with vertices along the diagonal and a bullet in row i and column j of the array if $i > j$

¹Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.

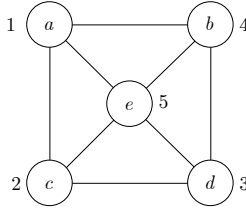


Figure 1: Ordered undirected graph.

and the vertices $\sigma(i)$ and $\sigma(j)$ are adjacent.

An example is shown below for the ordered undirected graph in the figure (to understand this visualization, see **Ordered Undirected Graphs**) we use the

$$\begin{bmatrix} a & & & & \\ \bullet & c & & & \\ & & \bullet & d & \\ \bullet & & & \bullet & b \\ \bullet & \bullet & \bullet & \bullet & e \end{bmatrix}$$

In this array representation the higher and lower neighborhoods are easily identified. The indices of the elements of $\text{adj}^+(v)$ are the column indices of the entries in row $\sigma^{-1}(v)$ of the array. For example, $\sigma^{-1}(d) = 3$, and the only bullet entry in row three is c so $\text{adj}^-(d) = \{c\}$. Likewise, $\text{adj}^-(c) = \{a\}$. And so on. Similarly, the indices of $\text{adj}^+(v)$ are the row indices of the entries in column $\sigma^{-1}(v)$. For example, $\sigma^{-1}(d)$ is 3, and there are indices 4 and 5 corresponding to b and e so $\text{adj}^+(d) = \{b, e\}$. Likewise, $\text{adj}^+(c) = \{d, e\}$.

For this reason, we use the notation $\text{col}(v)$ and $\text{row}(v)$ for the closed upper and lower neighborhoods. So $\text{col}(v) = \text{adj}^+(v) \cup \{v\}$

and $\text{row}(v) = \text{adj}^-(v) \cup \{v\}$.

