



## Why

We want to generalize the notion of continuity.

## Definition

A *topological space* is a base set and a set distinguished subsets of this set for which: (1) the empty set base set are distinguished (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the set of distinguished subsets the *topology* and we call its members the *open sets*.

## Notation

Let  $X$  be a non-empty set. For the set of distinguished sets, we tend to use  $\mathcal{T}$ , a mnemonic for topology, read aloud as “script T”. We tend to denote elements of  $\mathcal{T}$  by  $O$ , a mnemonic for open. We denote the topological space with base set  $X$  and topology  $\mathcal{T}$  by  $(X, \mathcal{T})$ . We denote the properties satisfied by elements of  $\mathcal{T}$ :

1.  $X, \emptyset \in \mathcal{T}$
2.  $\{O_i\}_{i=1}^n \subset \mathcal{T} \longrightarrow \bigcap_{i=1}^n O_i \in \mathcal{T}$
3.  $\{O_\alpha\}_{\alpha \in I} \subset \mathcal{T} \longrightarrow \bigcup_{\alpha \in I} O_\alpha \in \mathcal{T}$

## Examples

$\mathbf{R}$  with the open intervals as the open sets is a topological space.

