Why

Does a set exist containing the elements shared between two sets? How might we construct such a set?

Definition

Let A and B denote sets. Consider the set $\{x \in A \mid x \in B\}$. This set exists by the principle of specification (see Set Specification). Moreover $(y \in \{x \in A \mid x \in B\}) \longleftrightarrow (y \in A \wedge y \in B)$. In other words, $\{x \in A \mid x \in B\}$ contains all the elements of A that are also elements of B.

We can also consider $\{x \in B \mid x \in A\}$, in which we have swapped the positions of A and B. Similarly, the set exists by the principle of specification (see Set Specification) and again $y \in \{x \in B \mid x \in A\} \longleftrightarrow (y \in B \wedge y \in B)$. Of course, $y \in A \wedge y \in B$ means the same as[1] $y \in B \wedge y \in A$ and so by the principle of extension (see Set Equality)

$$\{x \in A \mid x \in B\} = \{x \in B \mid x \in A\}.$$

We call this set the *pair intersection* of the set denoted by A with the set denoted by B.

Notation

We denote the intersection fo the set denoted by A with the set denoted by B by $A \cap B$. We read this notation aloud as "A intersect B".

Basic properties

All the following results are immediate.[2]

Proposition 1. $A \cap \varnothing = \varnothing$

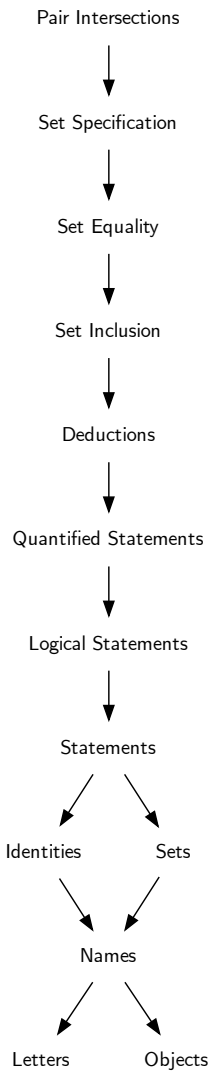
Proposition 2 (Commutativity)**.** $A \cap B = B \cap A$

[1] Future editions will name and cite this rule.
[2] Proofs of these results will appear in the next edition.

Proposition 3 (Associativity)**.** $(A \cap B) \cap C = A \cap (B \cap C)$

Proposition 4. $A \cap A = A$

Proposition 5. $(A \subset B) \longleftrightarrow (A \cap B = A)$.


Pair Intersections
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects