## Why

Can permuting the rows or columns of a matrix be represented by matrix multiplication?

## Definition

Let $\sigma : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ be a permutation of $n$. The *permutation matrix* of $\sigma$ is the matrix $P$ defined by $P_{ij} = 1$ if $\sigma(i) = j$ and 0 otherwise. This is sometimes called the *column representation* (in contrast to the *row representation*, in which $P_{ij} = 1$ if $\sigma(j) = i$.

Let $A \in \mathbf{R}^{n \times n}$. Then pre-multipying $A$ by $P$ permutes the rows of $A$. In other words $PA$ has the same rows as $A$ but permuted according to $\sigma$. Similarly, post-multiplying by $P$ permutes the columns of $A$. In other words, $AP$ has the same columns as $A$ but permuted according to $\sigma$. Clearly, we can also speak of permuting the components of a vector.

## Composition

Let $\pi, \sigma \in S_n$ with corresponding permutation matrices $P_\sigma$ and $P_\pi$. Then $P_\pi P_\sigma A$ has the same rows as $A$ but permuted by $\pi\sigma$. Likewise, $AP_\pi P_\sigma$ has the same columns as $A$ but permuted by $\pi\sigma$. Clearly, the identity permutation on $\{1, 2, \ldots, n\}$ is the identity $I \in \mathbf{R}^{n \times n}$.

## Inverses

It is clear from the definition that $P_\sigma^{-1} = P_{\sigma^{-1}}$ and so if $P$ is a permutation matrix then $P^{-1}$ is $P^\top$.

Permutation Matrices

Real Matrix Inverse

Matrix Transpose

Real Matrix-Matrix Products

Real Matrix-Vector Products

Vector as Matrices

Matrices

Real Inner Product

Real Matrices

Real Linear Combinations

Real Norm

Space Inner Product

Real Linear Equations

Vectors

Space Norm

Real Vectors

Linear Function

Arrays

Plane Norm

Plane Inner Product

N-Dimensional Space

Plane Vector Sums and Differences

Space Vectors

Absolute Value

Right Triangle Sides Relation

Distance

Plane Vectors

Space Distance

Plane Distance

Real Squares

Interval Length

Real Arithmetic

Intervals

Real Space

Real Multiplication Inverse

Real Line

Real Plane

Space Numbers

Real Products

Real Additive Inverse

Real Summation

Integral Line

Squares

Real Order

Rational Arithmetic

Real Sums

Rational Multiplicative Inverse

Natural Summation

Permutation

Geometry

Rational Order

Rational Sums

Real Numbers

Rational Products

Inverse Elements

Lists

Fields

Groups

Rational Numbers

Direct Products

Set Numbers

Rings

Integer Arithmetic

Natural Fractions

Finite Sets

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Natural Equations

Family Operations

Element Functions

Equivalent Sets

Function Inverse

Family Unions and Intersections

Comparison

Integer Order

Integer Sums

Identity Elements

Natural Arithmetic

Operations

Equations

Families

Function Composites

Function Images

Orders

Integer Numbers

Natural Powers

Natural Order

Equivalence Relations

Functions

Natural Products

Converse Relation

Natural Sums

Relations

Recursion Theorem

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Successor Sets

Unordered Triples

Set Symmetric Differences

Intersection of Empty Set

Set Dualities

Partitions

Set Unions and Intersections

Ordered Pairs

Pair Unions

Set Complements

Set Intersection

Set Differences

Set Unions

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects