Why

Given that we know that one event has occured, we want language for what the new probabilities should be.[1]

Definition

Let $\mathbf{P} : \mathcal{P}(\Omega) \to \mathbf{R}$ be a finite probability measure. Let $A, B \subset \Omega$ and $\mathbf{P}(B) \neq 0$. The *conditional probability* of A *given* B is fraction of the probability of $A \cap B$ over the probability of B.

Notation

In a slightly slippery but universally standard notation, we denote the conditional probability of A given B by $\mathbf{P}(A \mid B)$. In other words, we define

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)},$$

whenever $A, B \subset \Omega$ and $\mathbf{P}(B) \neq 0$.

Induced conditional distribution

Conditioning on an event B induces a distribution. For $\mathbf{P}_p$, define $q : \Omega \to \mathbf{R}$ by

$$q(\omega) = \begin{cases} \frac{p(\omega)}{\mathbf{P}(B)} & \text{if } \omega \in B \\ 0 & \text{otherwise.} \end{cases}$$

In this case $\mathbf{P}_q(A) = \mathbf{P}_p(A \mid B)$. We call q the *conditional distribution* induced by *conditioning on* the event B.

---

[1]Future editions will improve.

Conditional Event Probability

Event Probabilities

Probability Distributions

Intervals

Real Line

Real Order

Real Summation

Rational Order

Real Numbers

Integral Line

Rational Numbers

Integer Arithmetic

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Arithmetic

Identity Elements

Natural Summation

Uncertain Outcomes

Natural Powers

Integer Sums

Natural Products

Integer Order

Family Operations

Comparisons

Algebras

Set Operations

Integer Numbers

Natural Sums

Orders

Operations

Families

Equivalence Relations

Recursion Theorem

Natural Order

Converse Relations

Functions

Partitions

Peano Axioms

Relations

Natural Induction

Set Products

Natural Numbers

Set Powers

Ordered Pairs

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Intersections

Pair Unions

Set Symmetric Differences

Pair Intersections

Set Unions

Set Complements

Unordered Pairs

Empty Set

Set Differences

Geometry

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects