



Nets



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## 1 Why

We generalize the notion of sequence to index sets beyond the naturals.

## 2 Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go “further out.”

To elaborate on property (b): if handed two natural numbers  $m$  and  $n$ , we can always find another, for example  $\max\{m, n\} + 1$ , larger than  $m$  and  $n$ . We might think of larger as “further out” from the first natural number: 1.

Combining these two observations, we define a directed set:

**Definition 1.** *A directed set is a set  $D$  with a partial order  $\preceq$*

*satisfying one additional property: for all  $a, b \in D$ , there exists  $c \in D$  such that  $a \preceq c$  and  $b \preceq c$ .*

**Definition 2.** *A net is a function on a directed set.*

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is  $m \preceq n$  if  $m \leq n$ .

## 2.1 Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter  $D$  as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net  $x : D \rightarrow A$  by  $\{a_\alpha\}$ , emulating notation for sequences. The use of  $\alpha$  rather than  $n$  reminds us that  $D$  need not be the set of natural numbers.