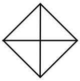

Why

What is the best estimator for a real-value random variable if we consider the squared loss.

Definition

We want to estimate a random variable $x : \Omega \to \mathbf{R}^n$ from a random variable $y : \Omega \to \mathbf{R}^n$ using an estimator $\phi : \mathbf{R}^m \to \mathbf{R}^n$.

Proposition 1. *The mmse estimator is the conditional mean.*

Let $x : \Omega \to \mathbf{R}^n$ and $y : \Omega \to \mathbf{R}^m$. A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* for x given y is an estimator $f : \mathbf{R}^m \to \mathbf{R}^n$ which minimizes $\mathbf{E}\|f(x) - y\|^2$.


Minimum Mean Squared Error Estimator
Estimators
Norms
Real-Valued Random Variable Expectation
Estimates
Functionals
Outcome Variable Expectation
Random Variables
Real Norm
Vectors
Topological Sigma Algebra
Induced Probability Distributions
Probability Measures
Measurable Functions
Real Vectors
Topologies
Outcome Variable Events
Measures
Space Vectors
N-Dimensional Space
Metrics
Length Measure
Sigma Algebras
Space Norm
Plane Vector Sums and Differences
Distance
Real Series
Extended Real Numbers
Space Distance
Plane Norm
Event Probabilities
Real Limits
Plane Distance
Plane Vectors
Absolute Value
Outcome Probabilities
Interval Length
Real Functions
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Real Summation
Real Sequences
Size of Direct Product
Geometry
Rational Order
Fields
Real Numbers
Number of Set Products
Natural Summation
Sequences
Subset Algebras
Rational Numbers
Rings
Number of Disjoint Unions
Lists
Finite Set Examples
Cardinality
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Fractions
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Arithmetic
Identity Elements
Equivalent Sets
Family Operations
Uncertain Events
Integer Order
Integer Sums
Natural Powers
Comparisons
Function Inverses
Operations
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Converse Relations
Natural Order
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Partitions
Set Symmetric Differences
Intersection of Empty Set
Successor Sets
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects