



## TREE DISTRIBUTIONS

### Why

It happens that if a distribution factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So the property of factoring according to a rooted tree is really property of factoring according to a tree.

### Definition

#### 0.1 Defining Result

**PROPOSITION 1.** *Let  $A_1, \dots, A_n$  be finite non-empty sets and define  $A = \prod_{i=1}^n A_i$ . Let  $p : A \rightarrow [0, 1]$  be a distribution and let  $T$  be a tree on  $\{1, \dots, n\}$ .*

**PROPOSITION 2.** *If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

### Undirected Definition

A distribution  $p$  factors according to the tree  $T$  if it factors according to the  $T$  rooted at any vertex.

### Existence and Uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

**Existence**

A distribution  $p$  need not factor according to a tree.

**Uniqueness**

A distribution  $p$  may factor according to multiple trees.

