



Why

The relationship between the inclusion map and the identity map is characteristic of making small functions out of large ones.¹

Definition

Let $X \subset Y$ and $f : Y \rightarrow Z$. There is a natural function $g : X \rightarrow Z$, namely the one defined by $g(x) = f(x)$ for all $x \in X$. We call g the *restriction* of f to X . We call f an *extension* of g to Y . Clearly, there may be more than one extension of a function

Notation

We denote the restriction of $f : Y \rightarrow Z$ to the set $X \subset Y$ by $f \upharpoonright X$ or $f|_X$.

Example

A simple example is the that the inclusion mapping from X to Y with $X \subset Y$ is a restriction of the identity map on X

An extension order

Here is a natural order involving set extensions and restrictions. Fix two sets A and B . Let F be the set of all functions $f : X \rightarrow Y$ with $X \subset A$ and $Y \subset B$. Define a relation R in F by $(f, g) \in R$ if $\text{dom } f \subset \text{dom } g$ and $f(x) = g(x)$ for all x in $\text{dom } f$. In other words, $(f, g) \in R$ if f is a restriction of g (or, equivalently, g is an extension of f). We recognize that R is a special case of the inclusion partial order by recognizing the elements of F as subsets $A \times B$.

¹Future editions will modify this language.

