Why

There is a natural orientation of an ordered undirected graph.

Motivating Result

An ordered undirected graph can be converted into a directed graph by orienting the eges from lower to higher index. The *orientation* of an ordered undirected graph $((V, E), \sigma)$ is the directed graph (V, F) where

$$\{v, w\} \in V \longrightarrow (v, w) \in F \text{ and } \sigma^{-1}(v) < \sigma^{-1}(w).$$

In other words, we can "convert" the ordered undirected graph by "orienting" the edges from lower to higher index.

Proposition 1. *Let $G = ((V, E), \sigma)$ be an ordered undirected graph. The orientation of G is acyclic.*

Proof. Contradiction on the existence of a cycle.[1] $\square$

Conversely, let (V, F) be directed acyclic. To each topological numbering σ of (V, F) (see Directed Paths) there exists an ordered undirected graph $((V, E), \sigma)$ where (V, E) is the skeleton of (V, F).

Example

Let $G = ((V, E), \sigma)$ be an undirected graph with

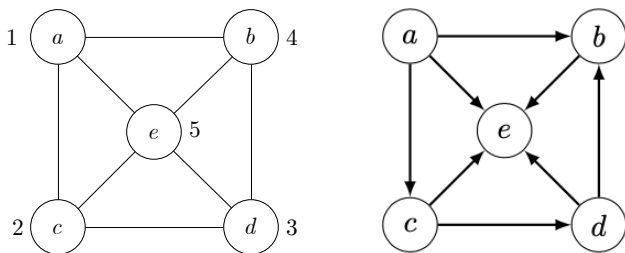$$V = \{a, b, c, d, e\},$$

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[1]Future editions will expand.

Figure 1: G and its (directed acyclic) orientation.

$$E = \{\{a,b\}, \{a,c\}, \{a,e\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,e\}, \{d,e\}\},$$

and

$$\sigma(1) = a \quad \sigma(2) = c \quad \sigma(3) = d \quad \sigma(4) = b \quad \sigma(e) = 5.$$

We visualize the ordered graph in Figure 1.

Ordered Undirected Graph Orientations

Directed Acyclic Graphs

Directed Paths

Directed Graphs   Ordered Undirected Graphs

Undirected Graphs   Sequences

Set Numbers   Direct Products

Finite Sets

Equivalent Sets

Function Inverses   Natural Order   Family Unions and Intersections

Function Images   Function Composites   Families

Functions   Equivalence Relations   Peano Axioms

Relations   Natural Induction

Cartesian Products

Set Powers   Natural Numbers   Generalized Set Dualities

Partitions   Unordered Triples   Successor Sets   Intersection of Empty Set   Set Dualities   Set Unions and Intersections

Ordered Pairs   Set Intersections   Pair Unions   Set Complements

Set Unions   Set Differences   Pair Intersections

Unordered Pairs   Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects

4