Why

A probability event function is a measure on the set of outcomes.

Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The word "space" is natural, since the notion of a measure generalized the notion of volume in real space (see Real Space and N-Dimensional Space). The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra. The measure in a probability space corresponds to the event probability function.

Notation

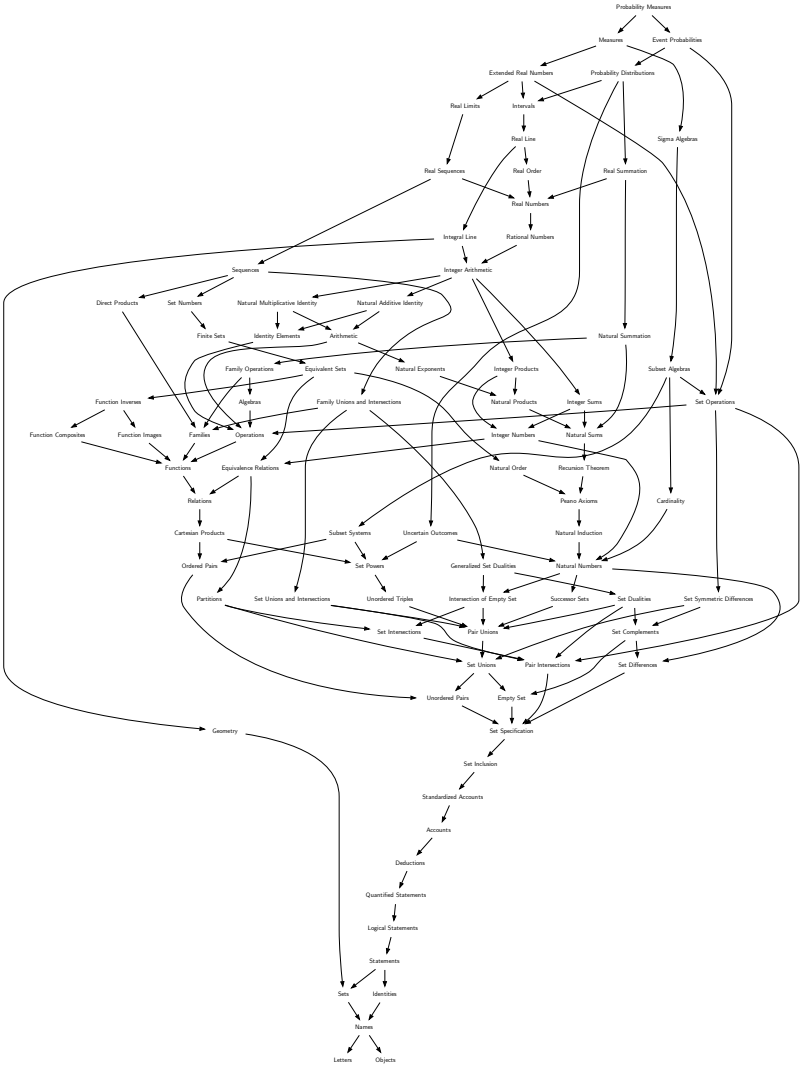
Let $(A, \mathcal{A})$ be a measurable space.[1] We denote the sigma-algebra by $\mathcal{A}$, as usual. We denote a probability measure by $\mathbf{P}$, a mnemonic for "probability," and intended to remind of the event probabilty function. Thus, we often say "Let $(A, \mathcal{A}, \mathbf{P})$

[1] Often, other authors will denote the set of outcomes (here denoted by A) by Ω, a mnemonic for "outcomes."

be a probability space."

Many authors associate an event $A \in \mathcal{A}$ with a function $\pi : \mathcal{X} \to \{0, 1\}$ so that $A = \{x \in \mathcal{X} \mid \pi(x) = 1\}$. In this context, it is common to write $\mu[\pi(x)]$ for $\mu(A)$.

Properties


Probability Measures
Measures
Event Probabilities
Extended Real Numbers
Probability Distributions
Real Limits
Intervals
Real Line
Sigma Algebras
Real Sequences
Real Order
Real Summation
Real Numbers
Integral Line
Rational Numbers
Sequences
Integer Arithmetic
Direct Products
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Identity Elements
Arithmetic
Natural Summation
Family Operations
Equivalent Sets
Natural Exponents
Integer Products
Subset Algebras
Function Inverses
Algebras
Family Unions and Intersections
Natural Products
Integer Sums
Set Operations
Function Composites
Function Images
Families
Operations
Integer Numbers
Natural Sums
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Cardinality
Cartesian Products
Subset Systems
Uncertain Outcomes
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Partitions
Set Unions and Intersections
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Differences
Set Unions
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects