Definition

A nonempty set $S \subset \mathbf{R}^n$ is called a *subspace* (*linear subspace*) if

1. $x + y \in S$ for all $x, y \in S$, and

2. $\alpha x \in S$ for all $\alpha \in \mathbf{R}$, $x \in S$.

We say that that S is (1) *closed under vector addition* and (2) *closed under scalar multiplication*.

Examples

The set $S_1 = \mathbf{R}^n$ is a subspace. In other words, the entire set is a subspace of itself. The set $S_2 = \{0\}$, consisting of a single point, the origin, is a subspace. S_1 is the biggest subspace. In other words, if S' is another subspace of $\mathbf{R}^n$, then $S' \subset S_1$. If S is a subspace, it is nonempty, so there is $x \in S$, and it is closed under scalar multiplication, so $0 \cdot x = 0 \in S$. In other words, every subspace contains the origin. So S_2 is the smallest subspace, in the sense that if S' is another subspace $S_2 \subset S'$.

The span (see Real Vectors Span) of a set of vectors $v_1, \ldots, v_k$ is a subspace. For two subspaces $S, T \subset \mathbf{R}^n$, their sum

$$S + T = \{x + y \mid x \in S, y \in T\}$$

is a subspace.

Geometric intuition

Roughly speaking, a subspace S is a flat set which passes through the origin. In $\mathbf{R}^2$, the subspaces are the lines. In $\mathbf{R}^3$, the lines and the planes.

Real Subspaces

Real Vectors Span

Real Linear Combinations

Real Vectors

N-Dimensional Space Space Vectors Plane Vector Sums and Differences

Distance Plane Vectors

Space Distance

Plane Distance

Interval Length

Real Space Intervals

Real Plane Real Line

Real Order Integral Line

Geometry Rational Order Real Numbers

Rational Numbers

Lists Integer Arithmetic

Set Numbers Natural Additive Identity Natural Multiplicative Identity Integer Products

Finite Sets Identity Elements Arithmetic

Direct Products Equivalent Sets Natural Powers

Family Unions and Intersections Function Inverses Comparisons Integer Order Integer Sums Natural Products

Families Function Composites Function Images Operations Orders Integer Numbers Natural Sums

Functions Converse Relations Equivalence Relations Natural Order Recursion Theorem

Relations Peano Axioms

Set Products Natural Induction

Generalized Set Dualities Set Powers Natural Numbers

Set Unions and Intersections Set Dualities Unordered Triples Successor Sets Intersection of Empty Set Partitions Set Symmetric Differences

Ordered Pairs Pair Unions Set Intersections Set Complements

Pair Intersections Set Union Set Differences

Unordered Pairs Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects