Why

We want to quantify the error of compressing a real-valued random variable.

Definition

Let $\mathcal{X}$ be a finite set and $q : \mathbf{R} \to \mathcal{X}$ a quantization (see Quantizations) of $\mathbf{R}$. Also, fix a probability space $(\Omega, \mathcal{A}, \mathbf{P})$ and a random variable $x : \Omega \to \mathbf{R}$.

The *compression* $\hat{x} : \Omega \to \mathcal{X}$ of x under q is $q \circ x$. A *distortion function* for x and $\hat{x}$ is a function

$$d : (\Omega \to \mathbf{R}) \times (\Omega \to \mathcal{X}) \to \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

Examples

The *expected mean-squared-error distortion* d_{mse} between x and $\hat{x}$ is

$$d_{\mathrm{mse}}(x, \hat{x}) = \mathbf{E}[(x - \hat{x})^2]$$

The *Kulback-Liebler distortion* d_{kld} defined by

$$d_{\mathrm{kld}}(x, \hat{x}) = \mathbf{E}[d_{\mathrm{kl}}(\mathbf{P}(y \in \cdot \mid x, \hat{x}) \mid \mathbf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.[1]

[1] Future editions will clarify this sentence.

Distortion Functions
Differential Relative Entropy
Quantizations
Differential Cross Entropy
Differential Entropy
Multivariate Real Densities
N-Dimensional Space
Probability Densities
Real Integrals
Nonnegative Integrals
Expectation
Simple Integrals
Random Variables
Outcome Variable Expectation
Topological Signs Algebra
Relative Entropy
Measurable Functions
Probability Measures
Induced Probability Distributions
Topologies
Similarity Functions
Cross Entropy
Discrete Entropy
Measures
Outcome Variable Events
Metrics
Logarithm
Probability Distributions
Length Measure
Signs Algebras
Distance
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Real Series
Space Distance
Exponential Function
Real Limits
Event Probabilities
Plane Distance
Absolute Value
Simple Functions
Outcome Probabilities
Interval Length
Real Functions
Real Spaces
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Real Plane
Real Line
Integral Line
Real Order
Real Summation
Real Sequences
Size of Direct Product
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Natural Additive Identity
Natural Multiplicative Identity
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Equivalent Sets
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Uncertain Events
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Ordered Pairs
Set Symmetric Differences
Partitions
Set Duality
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deduction
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects