

# Tail Sigma Algebra

#### 1 Why

### 2 Definition

The **tail sigma algebra** of a sequence of random variables is the sigma algebra which is the intersection of the sigma algebras of all final parts of the sequence. A **tail event** is an element of the tail sigma-algebra.

The tail sigma algebra coincides with the sigma algebra generated by the union of the sigma algebras of each of the random variables.

#### 2.1 Notation

Let  $\{f_n\}_n$  be a sequence of random variables. Denote the tail sigma algebra by  $T(\{f_n\}_n)$ . We defined it as:

$$T(\lbrace f_n \rbrace_n) = \bigcap_{n=1}^{\infty} \sigma(\lbrace X_{n+k} \rbrace_k).$$

In other words, for all natural n, the event is in the sigma algebra of the final part of

## 3 Results

**Proposition 1.** The tail sigma algebra of a sequence of random variables is the same equals the sigma algebra generated by the union of the sigma algebras of each of the random variables.