Why

TODO

Definition

An *index sequence* of *order* n is a finite sequence of distinct elements of $\{1, 2, \ldots, n\}$ whose length is less than or equal to n. We call the ith coordinate of an index sequence the *i-index* of the sequence. The *index matrix* associated with an index sequence is the $r \times n$ matrix whose i, jth entry is 1 if the index sequences's ith coordinate is j, and 0 otherwise. If $r = n$ then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an n-vector associated with a length-r index sequence is the product of the $r \times n$ index matrix with the n-dimensional vector. Its ith entry is the i-index entry of the vector.

Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions, and so relations, and so sets), they are not sets of indices, so we avoid this usage.

Notation

Let $r \leq n$ be natural numbers. Let $\alpha : \{1, 2, \ldots, r\} \rightarrow \{1, 2, \ldots, n\}$ be an index sequence. We denote the index ma-

trix associated with α by P_α. This matrix P_α is an element of $\mathsf{N}^{r \times n}$ and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let A be a nonempty set and let $x \in A^n$. then the subvector of x associated with P_α (and so with α) is

$$P_\alpha x = \Big(x_{\alpha(1)}, \ldots, x_{\alpha(r)} \Big)$$

We denote the product $P_\alpha x$ by x_α.

We denote the product $P_\alpha X P_\alpha{}^\top$ by $X_{\alpha\alpha}$.

Multiplication

The product of the $n \times r$ transpose of an index matrix with an r vector is the n vector with The

Index Matrices

Permutation Matrices

Matrix Transpose   Matrices

Vectors

Permutations

N-Dimensional Space   Fields

Space   Real Numbers   Groups

Rational Numbers   Algebras

Function Inverses

Integer Numbers   Operations   Surjective Functions   Function Composites   Injective Functions

Functions

Relations

Cartesian Products

Set Powers   Ordered Pairs

Zero

Unordered Triples

Natural Numbers   Pair Unions

Set Intersections   Set Unions

Pair Intersections   Empty Set   Unordered Pairs

Equation Solutions   Set Specification

Set Inclusion

Standardized Accounts

Accounts   Definitions

Quantified Statements   Deductions

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects