



## Why

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### Definition

Let  $(I, \leq)$  be a totally ordered set. A family of sigma-algebras  $\{F_i\}_{i \in I}$  is a *filtration* if  $F_j \subset F_k$  for all  $j \leq k$ .

We call a filtration whose index set is the natural numbers with their usual order a *discrete-time filtration*. We call a filtration whose index set is the real numbers with their usual order a *continuous-time filtration*. The index set may also be finite, for example,  $\{F_i\}_{i=1}^n$ , in which case

$$F_1 \subset F_2 \subset \cdots \subset F_n.$$

### Notation

It is extremely common to see filtrations written with the caligraphic  $\mathcal{F}$ . As in, let  $\{\mathcal{F}_i\}_{i=1}^n$  be a filtration. This is in accordance with using caligraphic letters for sets of sets, and in accordance with the term sigma *field* for sigma algebra.

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<sup>1</sup>Future editions will include, and likely will need stochastic processes.



