



## Why

Sums of non-negative functions are increasing, and workable with the monotone convergence theorem.

## Result

**Proposition 1.** *The integral of the limit of the partial sums of a sequence of measurable, nonnegative, extended-real-valued functions is the limit of the partial sums of the integrals.*

*Proof.* Let  $(X, \mathcal{A}, \mu)$  be a measure space, and let  $f_n : X \rightarrow [0, \infty]$  a  $\mathcal{A}$ -measurable function for every natural number  $n$ . We want to show that:

$$\int \sum_{k=1}^{\infty} f_k d\mu = \sum_{k=1}^{\infty} \int f_k d\mu.$$

We apply the monotone convergence theorem to the sequence  $\{\sum_{i=1}^n f_i\}_n$ . This sequence is nondecreasing because  $f_n \geq 0$  for all  $n$ . □



