Why

We want to quantify the error of compressing a real-valued random variable.[1]

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space, $x : \Omega \to \mathbf{R}$ a random variable, let $\mathcal{X}$ a finite set and $q : \mathbf{R} \to \mathcal{X}$ a quantization.

The *compression* $\hat{x} : \Omega \to \mathcal{X}$ of x under q is $q \circ x$. A *distortion function* for x and $\hat{x}$ is a function

$$d : (\Omega \to \mathbf{R}) \times (\Omega \to \mathcal{X}) \to \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

Examples

As a first example, consider the *mean-squared-error distortion* d_{mse} defined by

$$d_{\mathrm{mse}}(x, \hat{x}) = \mathrm{E}[(x - \hat{x})^2]$$

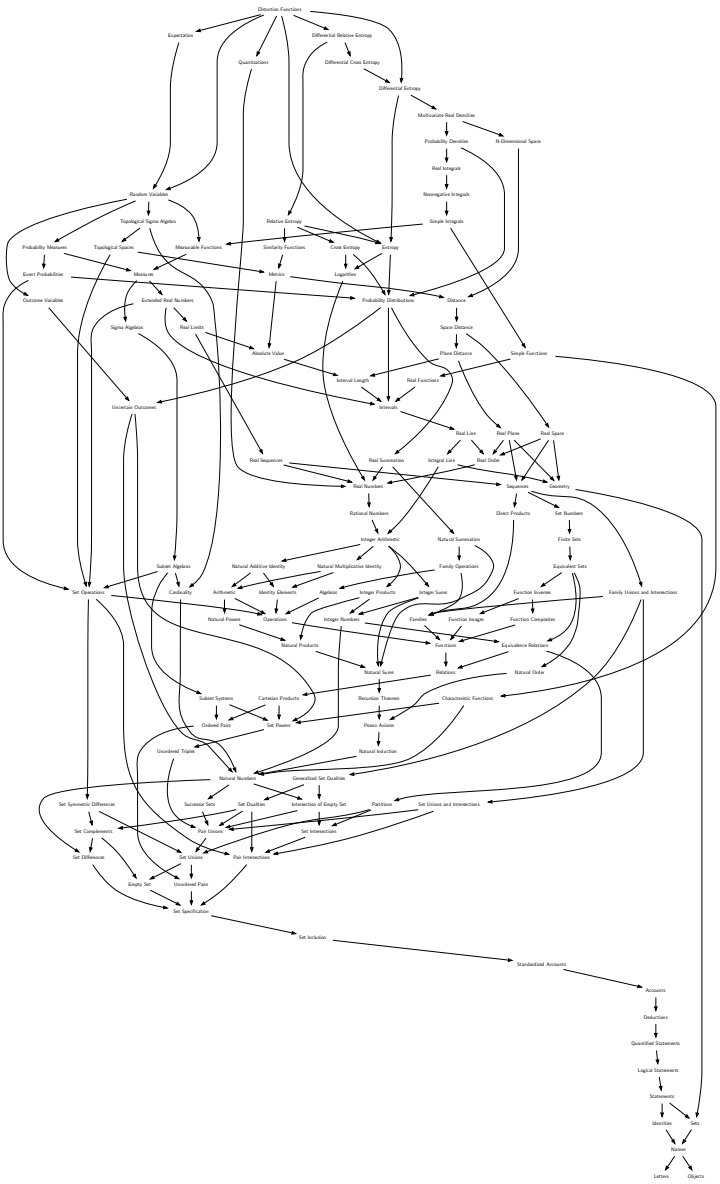
As a second example, consider the *Kulback-Liebler distortion* d_{kld} defined by

$$d_{\mathrm{kld}}(x, \hat{x}) = \mathrm{E}[d_{\mathrm{kl}}(\mathbf{P}(y \in \cdot \mid x, \hat{x}) \mid \mathbf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.[2]

[1] Future editions may expand.

[2] Future editions will clarify this sentence.


Distortion Functions
Expectation
Quantizations
Differential Relative Entropy
Differential Cross Entropy
Differential Entropy
Multivariate Real Densities
Probability Densities
N-Dimensional Space
Real Integrals
Nonnegative Integrals
Simple Integrals
Random Variables
Topological Sigma Algebra
Relative Entropy
Cross Entropy
Entropy
Probability Measure
Topological Spaces
Measurable Functions
Similarity Functions
Metrics
Logarithm
Event Probabilities
Measures
Outcome Variables
Extended Real Numbers
Probability Distributions
Distance
Sigma Algebras
Real Limits
Absolute Value
Space Distance
Plane Distance
Simple Functions
Interval Length
Real Functions
Uncertain Outcomes
Intervals
Real Line
Real Plane
Real Space
Real Sequences
Real Summation
Integral Line
Real Order
Real Numbers
Sequences
Geometry
Rational Numbers
Direct Products
Set Numbers
Integer Arithmetic
Natural Summation
Finite Sets
Subset Algebras
Natural Additive Identity
Natural Multiplication Identity
Family Operations
Equivalent Sets
Set Operations
Cardinality
Arithmetic
Identity Elements
Algebras
Integer Products
Integer Sums
Function Inverse
Family Unions and Intersections
Natural Powers
Operations
Integer Numbers
Families
Function Images
Function Composites
Natural Products
Functions
Equivalence Relations
Natural Sums
Relations
Natural Order
Subset Systems
Cartesian Products
Recursion Theorem
Characteristic Functions
Ordered Pairs
Set Powers
Peano Axioms
Unordered Triples
Natural Induction
Natural Numbers
Generalized Set Duality
Set Symmetric Differences
Successor Sets
Set Duality
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Lenses
Objects