



## Why

1

## Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

## Notation

Let  $(X, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $(Y, \mathcal{B})$  a measurable space. Then a random variable is a measurable function  $f : X \rightarrow Y$ .

Some authors denote real-valued random variables by upper case Latin letters: for example,  $X, Y, Z$ . In this case, the base probability space is denoted by  $\Omega$ , a mnemonic for “outcomes.” Let  $(\Omega, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $X : \Omega \rightarrow \mathbf{R}$  be measurable. Then  $X$  is a real-valued random variable.

Some authors use notation for the probability of particular, common sets. Although the authors often use regular parentheses, we will use “[” and “]” for precision. Let  $A \in \mathcal{B}(\mathbf{R})$ . Let  $\mathbf{P}[X \in A]$  denote  $\mathbf{P}(X^{-1}(A))$ . These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

As we mentioned, some authors use  $P(X \in A)$  for  $P[X \in A]$ , which is mostly harmless.

Next, let  $Y : \Omega \rightarrow \mathbf{R}$  a measurable function and let  $B \in \mathcal{B}(\mathbf{R})$ . Similar to the above, let  $\mathbf{P}[X \in A, Y \in B]$  denote  $\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B))$ . These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

---

<sup>1</sup>Future editions will include this.

Similarly for  $n$  random variables  $X_1, \dots, X_n : \Omega \rightarrow \mathbf{R}$ ,

$$\mathbf{P}[X_1 \in A_1, \dots, X_n \in A_n] = \mathbf{P}(\cap_{i=1}^n X_i^{-1}(A_i)).$$

