Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Prop. 1. *Let f be a square-integrable real-valued random variable on the probability space $(X, \mathcal{A}, \mu)$. Then for $t > 0$,*

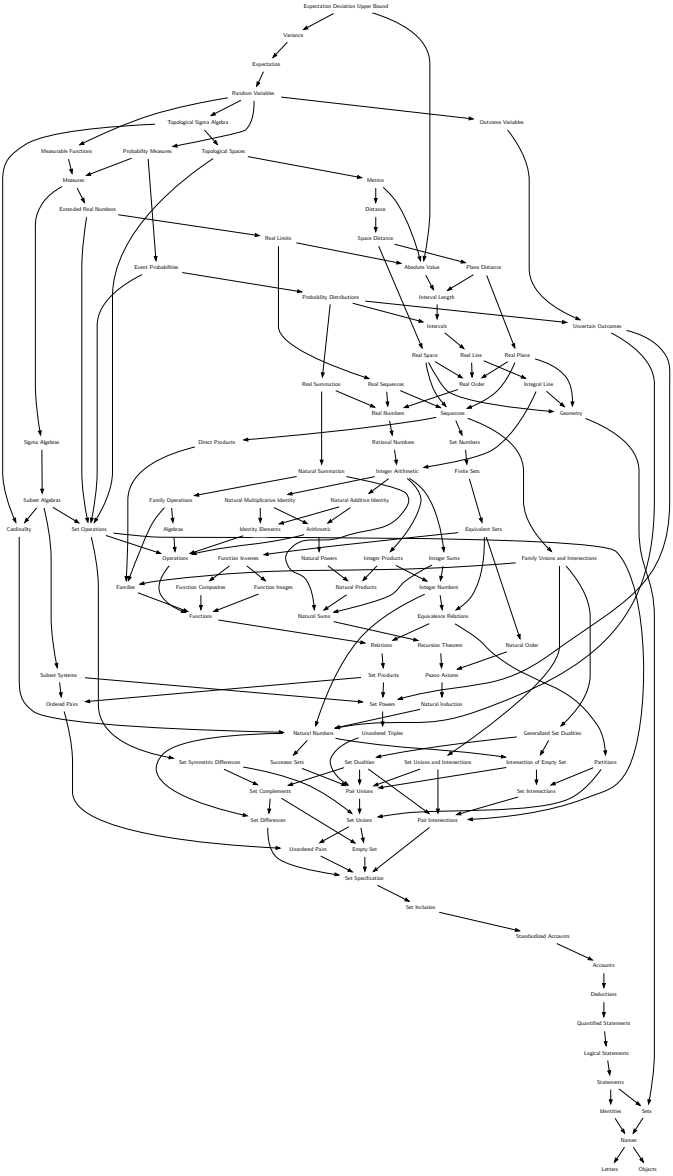
$$\mu(|f - \mathbf{E}(f)| \geq t) \leq \frac{\operatorname{var} f}{t^2}.$$

Proof. The set $|f - \mathbf{E}(f)| \geq t$ is $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$. This set is $\{x \in X \mid (f(x) - \mathbf{E}(f))^2 \geq t^2\}$. By using the non-negative inequality

$$\mu(\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))}{t^2}.$$

We recognize the numerator of the right hand side as the variance. □

The above is also called *Chebychev's Inequality.*


Expectation Deviation Upper Bound
Variance
Expectation
Random Variables
Outcome Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Topological Spaces
Measures
Metrics
Distance
Extended Real Numbers
Real Limits
Space Distance
Absolute Value
Plane Distance
Event Probabilities
Probability Distributions
Interval Length
Uncertain Outcomes
Intervals
Real Space
Real Line
Real Plane
Real Summation
Real Sequences
Real Order
Integral Line
Real Numbers
Sequences
Geometry
Sigma Algebras
Direct Products
Rational Numbers
Set Numbers
Natural Summation
Integer Arithmetic
Finite Sets
Subset Algebras
Family Operations
Natural Multiplicative Identity
Natural Additive Identity
Cardinality
Set Operations
Algebras
Identity Elements
Arithmetic
Equivalent Sets
Operations
Function Inverses
Natural Powers
Integer Products
Integer Sums
Family Unions and Intersections
Families
Function Composites
Function Images
Natural Products
Integer Numbers
Functions
Natural Sums
Equivalence Relations
Relations
Recursion Theorem
Natural Order
Subset Systems
Set Products
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Natural Numbers
Unordered Triples
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects