



## Why

There is a natural orientation of an ordered undirected graph.

## Motivating Result

An ordered undirected graph can be converted into a directed graph by orienting the edges from lower to higher index. The *orientation* of an ordered undirected graph  $((V, E), \sigma)$  is the directed graph  $(V, F)$  where

$$\{v, w\} \in V \longrightarrow (v, w) \in F \text{ and } \sigma^{-1}(v) < \sigma^{-1}(w).$$

In other words, we can “convert” the ordered undirected graph by “orienting” the edges from lower to higher index.

**Proposition 1.** *Let  $G = ((V, E), \sigma)$  be an ordered undirected graph. The orientation of  $G$  is acyclic.*

*Proof.* Contradiction on the existence of a cycle.<sup>1</sup> □

Conversely, let  $(V, F)$  be directed acyclic. To each topological numbering  $\sigma$  of  $(V, F)$  (see **Directed Paths**) there exists an ordered undirected graph  $((V, E), \sigma)$  where  $(V, E)$  is the skeleton of  $(V, F)$ .

## Example

Let  $G = ((V, E), \sigma)$  be an undirected graph with

$$V = \{a, b, c, d, e\},$$

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\},$$

and

$$\sigma(1) = a \quad \sigma(2) = c \quad \sigma(3) = d \quad \sigma(4) = b \quad \sigma(e) = 5.$$

We visualize the ordered graph in Figure 1.

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<sup>1</sup>Future editions will expand.

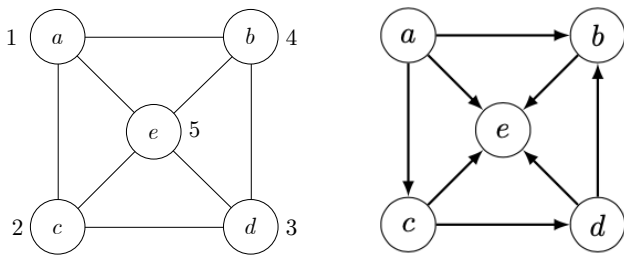


Figure 1:  $G$  and its (directed acyclic) orientation.

