Why

We want to find the roots of negative numbers.[1]

Definition

A *complex number* is an ordered pair of real numbers. The *real part* of a complex number is its first coordinate. The *imaginary part* of a complex number is its second coordinate.

The *complex conjugate* (or *conjugate*) of a complex number z is the complex number whose real part matches z and whose imaginary part is the additive inverse of z. The complex conjugate of a real number (imaginary part is zero) is the real number. In other words, the complex conjugate of a complex number with no imaginary part is the same complex number.

Notation

When we think of R^2 as the set of complex numbers, we denote it by $\mathbf{C}$. Let $z \in \mathbf{C}$. We denote the real part of z by $\mathsf{Re}(z)$, read "real of z," and the imaginary part by $\mathsf{Im}(z)$, read "imaginary of z." If $z = (a, b)$ for $a, b \in \mathsf{R}$, then $\mathsf{Re}(z) = a$ and $\mathsf{Im}(z) = b$.

We denote the complex conjugate of the complex number $z \in \mathbf{C}$ by $z^* \in \mathbf{C}$. Another common notation, not used in these sheets is $\overline{z}$ or $\bar{z}$. If there exists $a, b \in \mathsf{R}$ so that $z = (a, b)$, then $z^* = (a, -b)$.

[1] Future editions will modify this, and will discuss the existence of solutions of algebraic equations.

Modulus and argument

The *modulus* of $z \in \mathbf{C}$ is the distance of z to the origin. If $z \in \mathbf{C}$, then the modulus of z is

$$\sqrt{\operatorname{\mathsf{Re}} z^2 + \operatorname{\mathsf{Im}} z^2}.$$

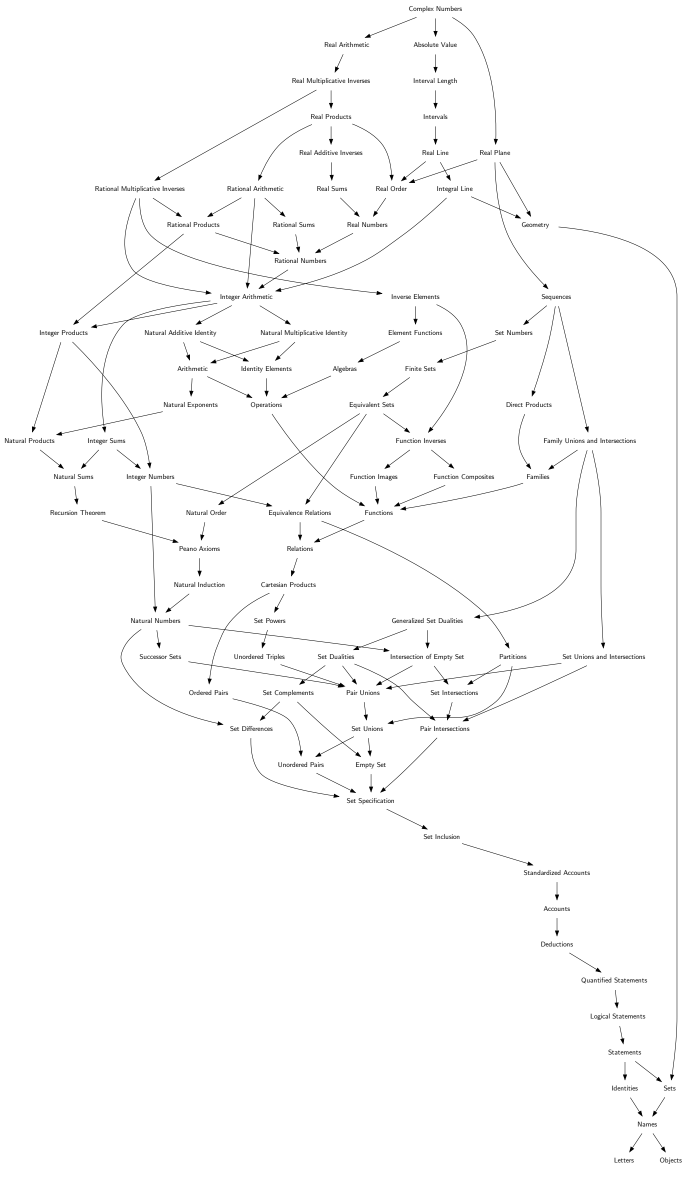
We denote the modulus of z by $|z|$.

The *argument* of $z \in \mathbf{C}$ is $\tan^{-1}(\operatorname{\mathsf{Im}} z / \operatorname{\mathsf{Re}} z)$. We denote the argument of z by $\arg z$.[2]

Complex disc

The *complex disc* is the set $\{z \in \mathbf{C} \mid |z| \leq 1\}$. We denote it by $\mathbf{D}$, a mnemonic for disc. The *complex unit circle* is the set $\{z \in \mathbf{C} \mid |z| = 1\}$. We denote it by $\mathbf{T}$, a mnemonic for torus.

[2] Future editions will include the geometric interpretations.


Complex Numbers
Real Arithmetic
Absolute Value
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Rational Products
Rational Sums
Real Numbers
Geometry
Rational Numbers
Integer Arithmetic
Inverse Elements
Sequences
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Element Functions
Set Numbers
Arithmetic
Identity Elements
Algebras
Finite Sets
Natural Exponents
Operations
Equivalent Sets
Direct Products
Natural Products
Integer Sums
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Function Images
Function Composites
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects