



### Definition

The *translate* of  $S \subset \mathbf{R}^n$  by the vector  $a \in \mathbf{R}^n$  is the set

$$\{z \in \mathbf{R}^n \mid \exists x \in S \text{ such that } z = x + a\}.$$

### Notation

We often use the abbreviated notation  $S + a$  for the translate of  $S$  by  $a$ . It is sometimes also convenient to extend set-builder notation and write

$$S + a = \{x + a \mid x \in M\}.$$

The right hand side is slick notation for the definition given above.

### Sums and differences

The *sum* (or *Minkowski sum*) of two sets  $S, T \subset \mathbf{R}^n$  is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x + y)\}.$$

Likewise, the *difference* (or *Minkowski difference*) of two sets  $S, T \subset \mathbf{R}^n$  is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x - y)\}.$$

### Notation

We denote the sum of  $S$  and  $T$  by  $S + T$ , and the difference by  $S - T$ .<sup>1</sup> We often use the slick notation

$$\{x + y \mid x \in S, y \in T\} \text{ and } \{x - y \mid x \in S, y \in T\},$$

for these two sets. Notice that in this notation

$$\{a\} + B = a + B$$

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<sup>1</sup>This second notation unfortunately conflicts with our notation for set differences. Future editions will correct.

## Scaled sets

Given a set  $A \subset \mathbf{R}^n$  and a  $\lambda \in \mathbf{R}$ , the set which is  $A$  *scaled by* (the *scaled set*) is

$$\{z \in \mathbf{R}^n \mid (\exists x \in A)(z = \lambda x)\}$$

We often denote this set by  $\lambda A$ . As before, we often use the slick notation

$$\lambda A = \{\lambda a \mid a \in A\}$$

The set  $(-1)A$  is denoted  $-A$

## Homothetic sets

A set  $A$  is *homothetic* to a set  $B$  if there is  $x \in \mathbf{R}^n$  and  $\lambda \neq 0$  so that

$$A = x + \lambda B$$

If  $\lambda > 0$ ,  $A$  is *positively homothetic* to  $B$ .



