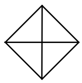


Why

Can a set have no elements?

Definition

Sure. A set exists by the principle of existence (see *Sets*); denote it by A. Specify elements (see *Set Specification*) of any set that exists using the universally false statement $x \neq x$. We denote that set by $\{x \in A \mid x \neq x\}$. It has no elements. In other words, $(\forall x)(x \notin A)$. The principle of extension (see *Set Equality*) says that the set obtained is unique (contradiction).[1]

Definition 1 (Empty Set). We call the unique set with no elements *the empty set.*

ßNotation

We denote the empty set by $\varnothing$. In other words, in all future accounts (see *Accounts*), there are two implicit lines. First, "`name` $\varnothing$" and second "`have` $(\forall x)(x \notin \varnothing)$".

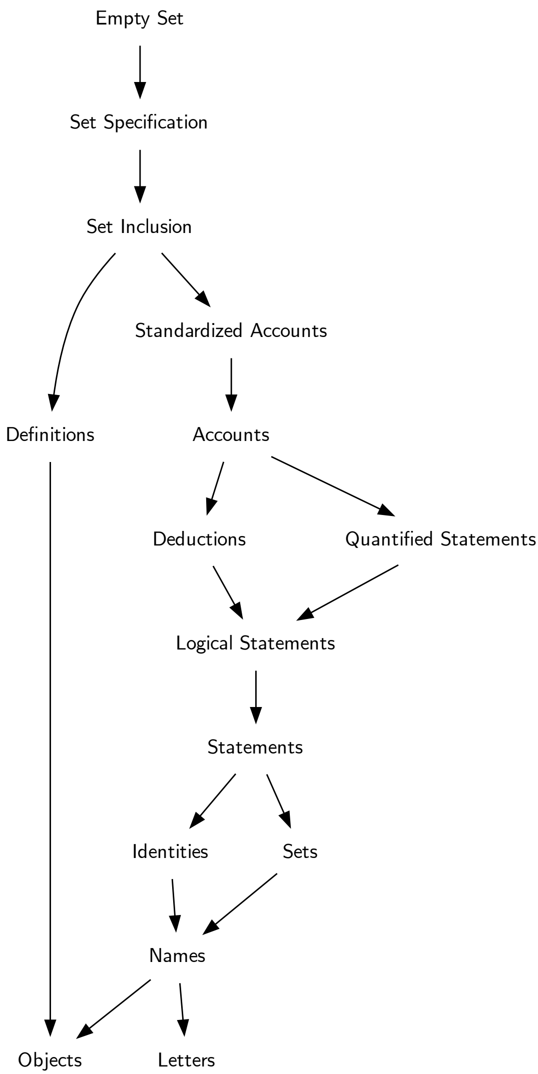
Properties

It is immediate from our definition of the empty set and of the definition of inclusion (see *Set Inclusion*) that the empty set is included in every set (including itself).

Proposition 1. $(\forall A)(\varnothing \subset A)$

[1] This account will be expanded in the next edition.

Proof. Suppose toward contradiction that $\varnothing \not\subset A$. Then there exists $y \in \varnothing$ such that $y \notin A$. But this is impossible, since $(\forall x)(x \notin \varnothing)$. $\square$


Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters