Why

If both signed measures are finite, then their difference is always well-defined. Is the difference a finite signed measure?

Preliminary result

Proposition 1. *A linear combination of finite signed measures is a finite signed measure.*

Proof. Let $(X, \mathcal{A})$ be a measurable space. Let μ and ν be finite signed measures. Let R denote the real numbers. Then $(\alpha\mu)(\varnothing) = \alpha \cdot \mu(\varnothing) = \alpha \cdot 0 = 0$. Also for $(A_n)_n \subset \mathcal{A}$ disjoint,

$$\begin{aligned}(\alpha\mu)(\cup A_n) = \alpha\mu(\cup A_n) &= \alpha \sum_{n=1}^{\infty} \mu(A_n) \\ &= \sum_{n=1}^{\infty} \alpha\mu(A_n) = (\alpha\mu)(A_n)\end{aligned}$$

Similarly, $(\mu+\nu)(\varnothing) = \mu(\varnothing)+\nu(\varnothing) = 0$. And, for $(A_n)_n \subset \mathcal{A}$ disjoint,

$$\begin{aligned}(\mu+\nu)(\cup A_n) = \mu(\cup A_n) + \nu(\cup A_n) &= \sum_{n=1}^{\infty} \mu(A_n) + \sum_{n=1}^{\infty} \nu(A_n) \\ &= \sum_{n=1}^{\infty} \mu(A_n) + \nu(A_n) = \sum_{n=1}^{\infty} (\mu+\nu)(A_n)\end{aligned}$$

□

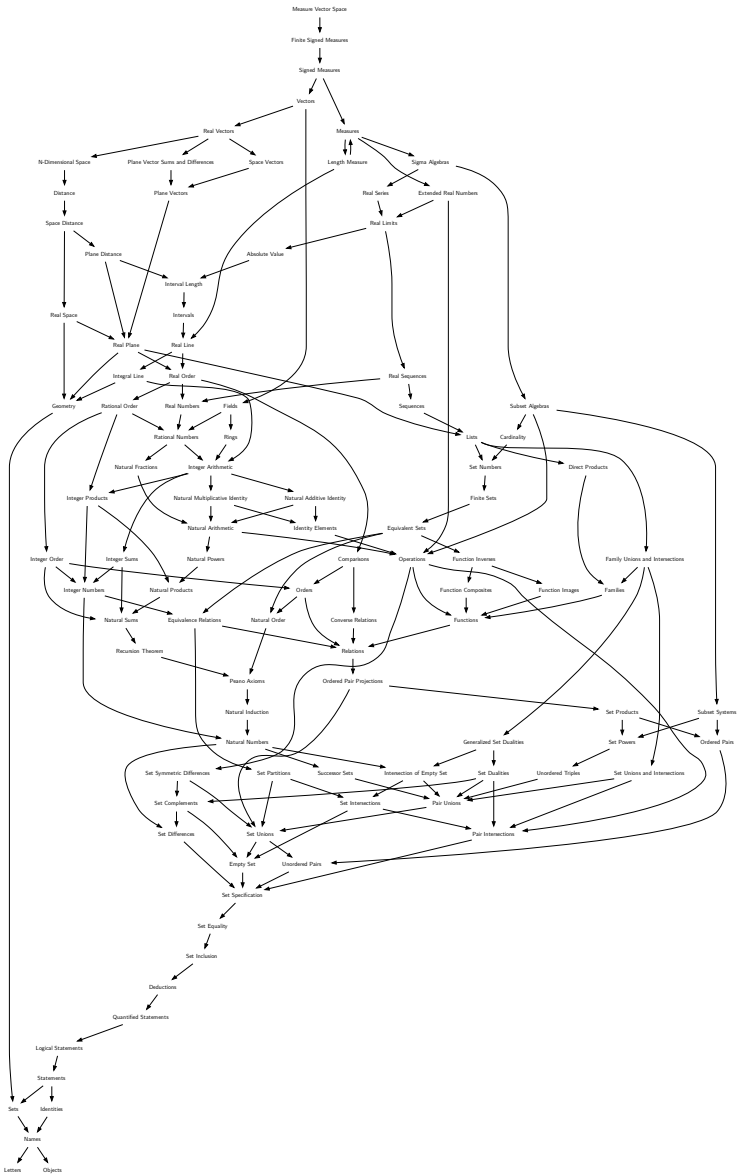
Main result

Proposition 2. *The set of finite signed measures is a vector space.*

Proof. Use the previous proposition. Observe that the function $\mu \equiv 0$ is a measure. And $\nu + \mu = \nu$ for all measures ν. □

Notation

We denote the vector space of signed measures on measurable space $(X, \mathcal{A})$ by by $M(X, \mathcal{A}, \mathbf{R})$.


Measure Vector Space
Finite Signed Measures
Signed Measures
Vectors
Real Vectors
Measures
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Length Measure
Sigma Algebras
Distance
Plane Vectors
Real Series
Extended Real Numbers
Space Distance
Real Limits
Plane Distance
Absolute Value
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Real Sequences
Geometry
Rational Order
Real Numbers
Fields
Sequences
Subset Algebras
Cardinality
Rational Numbers
Rings
Lists
Set Numbers
Direct Products
Natural Fractions
Integer Arithmetic
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Arithmetic
Identity Elements
Equivalent Sets
Integer Order
Integer Sums
Natural Powers
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Set Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects