



Why

We want to combine two groups.¹

Defining Result

Proposition 1. *For each natural number m , there exists a function $s_m : \omega \rightarrow \omega$ which satisfies*

$$s_m(0) = m \quad \text{and} \quad s_m(n^+) = (s_m(n))^+$$

for every natural number n .

Proof. The proof uses the recursion theorem (see Recursion Theorem).² □

Let m and n be natural numbers. The value $s_m(n)$ is the *sum* of m with n .

Notation

We denote the sum $s_m(n)$ by $m + n$.

Properties

The properties of sums are direct applications of the principle of mathematical induction (see Natural Induction).³

Proposition 2 (Associative). *Let k , m , and n be natural numbers. Then*

$$(k + m) + n = k + (m + n).$$

Proposition 3 (Commutative). *Let m and n be natural numbers. Then*

$$m + n = n + m.$$

¹Future editions will change this section.

²Future editions will give the entire account.

³Future editions will include the accounts.

Relation to Addition

Proposition 4 (Distributive). *Let k , m , and n be natural numbers. Then*

$$k \cdot (m + n) = (k \cdot m) + (k \cdot n).$$

