

Unordered Pairs

1 Why

Are there enough sets to ensure that every set is an element of some set? What of one set and another set — is there a set that they both belong to?

2 Definition

We will say that there is. For one set and another set, there exists a set that they both belong to. We refer to this as the axiom of pairing.

If there exists a set that contains both the sets we began with, then there exists a set which contains them and nothing else. First, use the axiom of pairing to obtain a set containing both sets, and then use the axiom of specification with a sentence that is true only if the element considered is one of the sets we began with. As a result of the axiom of extension, there can be only one set with this property. We call this set a pair or an unordered pair.

3 Notation

Let a and b be objects. We denote the set which contains a and b as elements and nothing else by $\{a, b\}$.

The pair of a with itself is the set $\{a, a\}$ which we will denote by $\{a\}$.

