Why

We have seen that the matrices are a vector space. Are they an inner product space?

Definition

The *matrix scalar product* of $A \in \mathbf{R}^{n\times k}$ and $B \in \mathbf{R}^{n\times k}$ is the following product

$$\sum_{i=1}^{n}\sum_{j=1}^{k} a_{ij}b_{ij}.$$

Using the matrix trace, we can denote this as $\operatorname{tr} A^\top B$. Some authors call this the *Euclidean matrix scalar product*, *matrix inner product* or *Frobenius inner product*.

Proposition 1. *The matrix scalar product is an inner product.*

For example, symmetry of the product is a consequence of the fact that a square matrix and its tranpose have identical traces. Commutativity of the trace yields $\operatorname{tr} A^\top B = \operatorname{tr} BA^\top$, where the LHS is the scalar product of $B^\top$ and $A^\top$. In other words, transposition "preserves" the matrix scalar product.

With this inner product, $\mathbf{R}^{n\times k}$ is a Euclidean vector space (see Inner procuts) of dimension nk. For the case of $k = 1$, we recover a model[1] for the usual space $\mathbf{R}^n$.

Notation

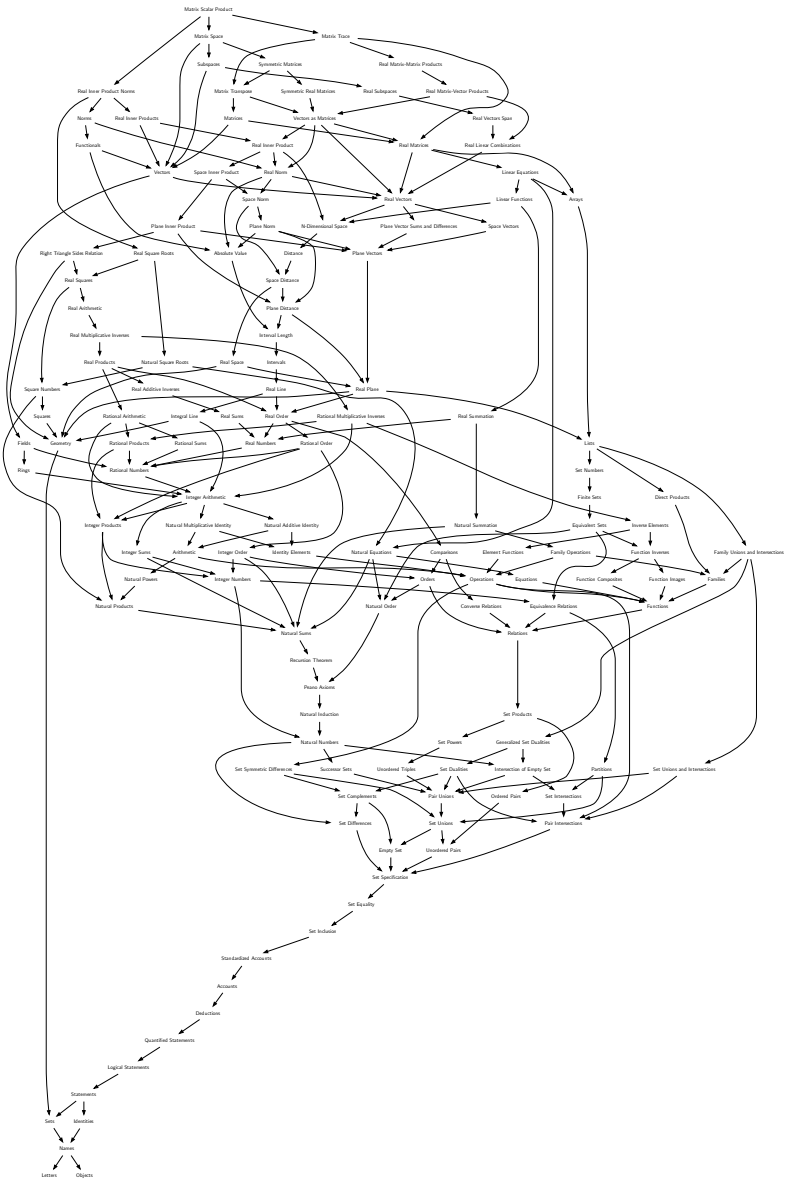
We commonly denote the matrix inner product by $\langle A, B\rangle$.

Induced norm

The matrix inner product induces a norm in the usual way. This norm is sometimes called the *matrix-vector norm* (or *Frobenius norm*) and is

[1] Future editions will define this term.

often denoted for a matrix $A \in \mathbf{R}^{m \times n}$ by $\|A\|_F$.


Matrix Scalar Product
Matrix Space
Matrix Trace
Subspaces
Symmetric Matrices
Real Matrix-Matrix Products
Real Inner Product Norms
Matrix Transpose
Symmetric Real Matrices
Real Subspaces
Real Matrix-Vector Products
Norms
Real Inner Products
Matrices
Vectors as Matrices
Real Vectors Span
Functionals
Real Inner Product
Real Matrices
Real Linear Combinations
Vectors
Space Inner Product
Real Norm
Linear Equations
Space Norm
Real Vectors
Linear Functions
Arrays
Plane Inner Product
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Right Triangle Sides Relation
Real Square Roots
Absolute Value
Distance
Plane Vectors
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Internal Length
Real Products
Natural Square Roots
Real Space
Intervals
Square Numbers
Real Additive Inverses
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Real Sums
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Rational Multiplicative Inverses
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Direct Products
Integer Arithmetic
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Natural Summation
Equivalent Sets
Inverse Elements
Integer Sums
Arithmetic
Integer Order
Identity Elements
Natural Equations
Comparisons
Element Functions
Family Operations
Function Inverses
Family Unions and Intersections
Natural Powers
Integer Numbers
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Operations
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Families
Natural Products
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Functions
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Doubles
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Doubles
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Ordered Pairs
Set Intersections
Set Differences
Set Union
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects