Why

We want a notion of distance between elements of the real line.

Definition

The *absolute value* of a real number is the greater of itself and its additive inverse. In other words, if x is positive, then the absolute value of x is x. If x is negative, then the absolute value of x is $-x$ (which would be a positive real number).

Notation

We denote the absolute value of a real number $x \in \mathbf{R}$ by $|x|$.

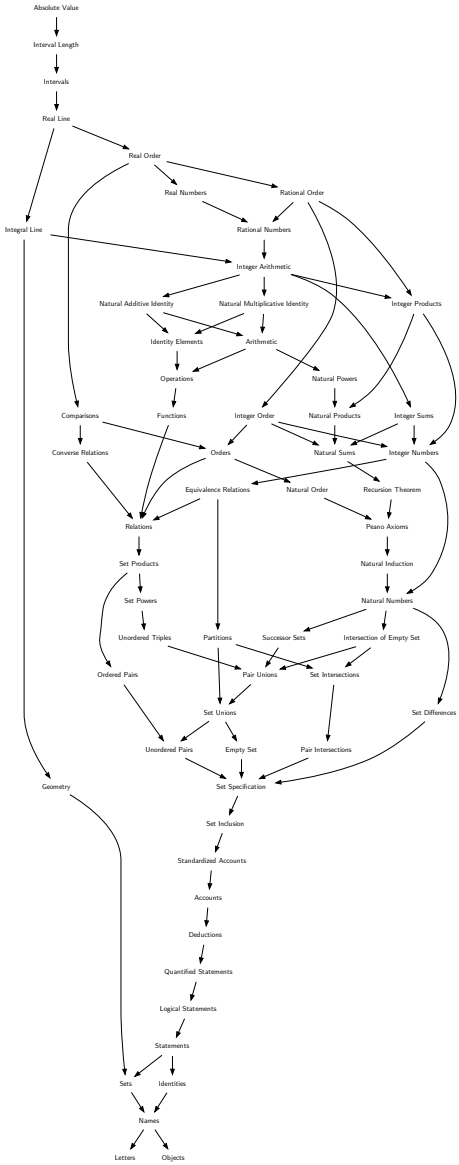
Distance

The absolute value can be interpreted as the distance between the point corresponding to the real number and the point corresponding to 0. We can generalize this idea. Consider $x, y \in \mathbf{R}$. If $x > y$, then $x - y > 0$ and so the distance between the corresponding points is $x - y$. If $x < y$ then $y - x > 0$, and so the distance is $y - x$.

The observation is that $|-x| = |x|$. So

$$|y - x| = |-(x - y)| = |x - y|.$$

So if we just care about the distance between the points corresponding to y and x, we can consider $|x - y|$, without regard for their order. In other words, the function $(x, y) \mapsto |x - y|$ is symmetric in x and y.


Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Identity Elements
Arithmetic
Operations
Natural Powers
Comparisons
Functions
Integer Order
Natural Products
Integer Sums
Converse Relations
Orders
Natural Sums
Integer Numbers
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects