## Definition

An *orthonormal* (or *orthogonal*) matrix is a matrix whose columns are an orthonormal family of vectors.

Some authors call these *real orthogonal* or *unitary* matrices.

## Notation

Let $A \in \mathsf{F}^{m \times n}$. Something something

$$AA^\top = I.$$

## Characterizations

**Proposition 1.** *A matrix is orthonormal if and only if its transpose product with the matrix is the identity.*

**Proposition 2.** *A matrix is orthonormal if and only if its transpose is orthonormal.*

Orthogonal Matrices

Matrix Transpose

Orthonormal Set of Vectors

Real Inner Product Norms

Vectors as Matrices — Matrices — Orthonormal Set of Real Vectors — Real Inner Products — Norms

Real Matrices — Real Inner Product — Functionals

Linear Equations — Real Norm — Space Inner Product — Vectors

Arrays — Real Vectors — Linear Functions — Space Norm

Space Vectors — Plane Vector Sums and Differences — N-Dimensional Space — Plane Inner Product — Plane Norm

Plane Vectors — Distance — Right Triangle Sides Relation — Absolute Value — Real Square Roots

Space Distance — Real Squares

Plane Distance — Real Arithmetic

Interval Length — Real Multiplicative Inverses — Natural Square Roots

Real Space — Intervals — Real Products — Square Numbers

Real Plane — Real Line — Real Additive Inverses

Square — Integral Line — Real Summation — Real Order — Real Sums — Rational Multiplicative Inverse — Rational Arithmetic

Geometry — Real Numbers — Rational Order — Rational Products — Rational Sums — Fields

Lists — Set Numbers — Rational Numbers — Rings

Direct Products — Integer Arithmetic

Finite Sets — Natural Multiplicative Identity — Natural Additive Identity — Integer Products

Equivalent Sets — Inverse Elements — Natural Summation — Identity Elements — Arithmetic

Family Unions and Intersections — Function Inverses — Element Functions — Family Operations — Comparisons — Integer Order — Integer Sums — Natural Equations — Natural Powers

Families — Function Composites — Function Images — Operations — Orders — Integer Numbers — Equations — Natural Products

Equivalence Relations — Functions — Converse Relations — Natural Order — Natural Sums

Relations — Recursion Theorem

Ordered Pair Projections — Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Partitions — Intersection of Empty Set — Set Dualities — Unordered Triples — Successor Sets — Set Symmetric Differences

Ordered Pairs — Set Intersections — Pair Unions — Set Complements

Pair Intersections — Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Unions and Intersections

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects

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