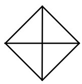


Why

We want to talk about particular attributes of an outcome, instead of the details of the outcomes themselves. These may be useful to specify events.

Definition

Given a sample space Ω, an *outcome variable* (or *random variable*) is any function on Ω. In this context, the range of the function is called the set of *values* of the random variable.

Notation

Standard convention denotes outcome random variables by capitals X, Y, Z and elements of their codomain by corresponding lower-case, x, y, z. Thus, if $X : \Omega \to V$ is a random variable then the lower case x is often reserved for an element of V. The event $\{\omega \in \Omega \mid X(\omega) = x\}$, where $x \in V$, is often abbreviated $\{X = x\}$. The probability of this event is often abbreviated $P(X = x)$.

Similarly, for a subset $A \subset V$, the event $\{\omega \in \Omega \mid X(\omega) \in A\}$ is often abbreviated $\{X \in A\}$ and its probability abbreviated $P(X \in A)$.[1] If $X : \Omega \to V_1$ and $Y : \Omega \to V_2$, and $A \subset V_1$ and $B \subset V_2$, then the event $\{X \in A\} \cap \{Y \in B\}$ is often written $\{X \in A, Y \in B\}$.

Examples

Sum of two dice. Suppose we model rolling two dice. We are interested in the sum of the pips shown facing up. Suppose we take as the set of outcome $\{1, \dots, 12\}$, whose elements correspond to the sum. We interpret $\{x \in \Omega \mid x \geq 10\}$ as the event that the sum of the two dice is greater than or equal to 10.

Alternatively, we may take the usual set of outcomes $\{1, \dots, 6\}^2$ and

[1] Occasionally, in the present edition of these sheets, we use the notation $P[X = a]$.

define an outcome variable $s : \{1, \ldots, 6\}^2 \to \{1, \ldots, 12\}$ by

$$s(d_1, d_2) = d_1 + d_2.$$

We interpret this natural-number-valued outcome variable s as sum of the two dice. The event that the sum of the two dice is greater than or equal to to 10 corresponds to the set $\{(d_1, d_2) \in \{1, \ldots, 6\} \mid s(d_1, d_2) \geq 10\}$.

As a third alternative, again take the usual set of outcomes $\Omega = \{1, \ldots, 6\}^2$. Define the outcome variable $X : \Omega \to \mathbf{N}$ to be the number of pips showing on the first die, $Y : \Omega \to \mathbf{N}$ to be the number of pips showing on the second die, and define $Z : \Omega \to \mathbf{N}$ by

$$Z(\omega) = X(\omega) + Y(\omega) \quad \text{for all } \omega \in \Omega$$

A standard notation for this relation between Z and X, Y by $Z = X + Y$. For example, if $\omega = (2, 5)$, $X(\omega) = 2$, $Y(\omega) = 5$, and $Z(\omega) = 7$. The event $\{Z = 4\}$ is the set $\{(2, 2), (1, 3), (3, 1)\}$ If we take the usual distribution p on Ω with $p(\omega) = 1/36$ for every ω, the the probability of this event is

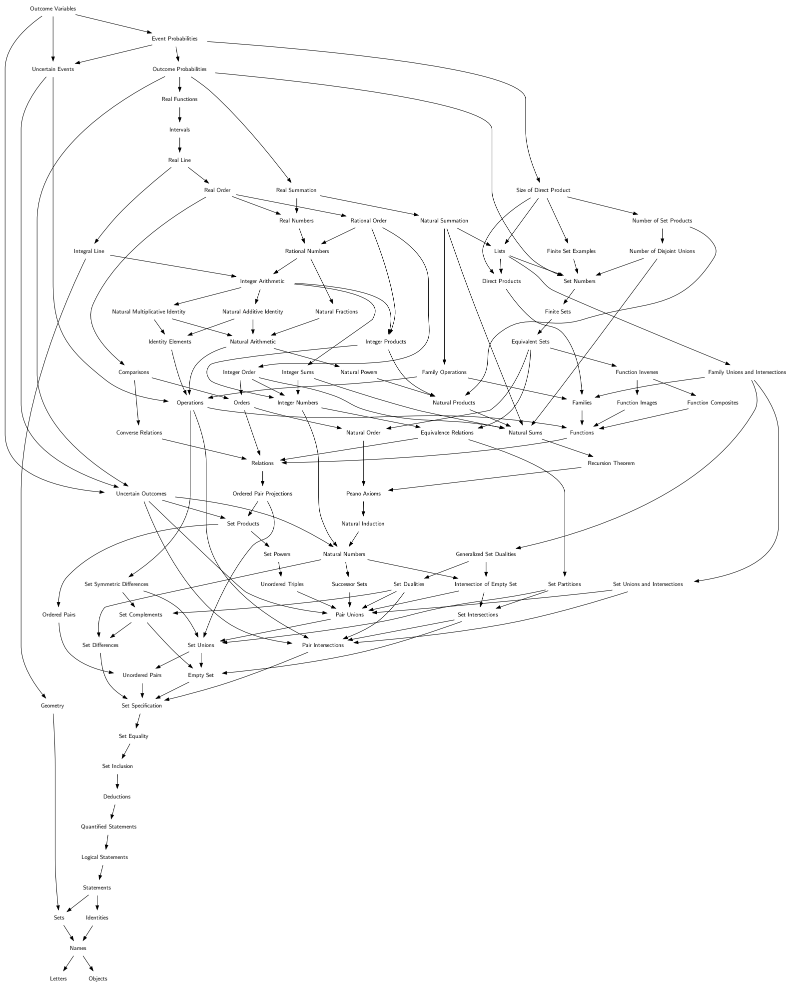
$$P(Z = 4) = p(2, 2) + p(1, 3) + p(3, 1) = 1/36 + 1/36 + 1/36 = 1/12$$

The preceding three paragraphs highlight that there are several ways of denoting the same situation.

Tossing a fair coin n times. As before, we model n tosses of a fair coin with the sample space $\Omega = \{0, 1\}^n$. Define $X_i : \Omega \to \{0, 1\}$ by $X_i(\omega) = \omega_i$. We interpret $\{\omega \in \Omega \mid X_i(\omega) = 1\}$ as the event that toss i turns up heads, and likewise $\{\omega \in \Omega \mid X_i(\omega) = 0\}$ as the event that toss i turns up tails, for $i = 1, \ldots, n$. We can define the function $X : \Omega \to 0, 1^n$ by $X(\omega) = \omega$ or by saying $X = (X_1, \ldots, X_n)$. Suppose we define $H : \Omega \to \mathbf{N}$ by

$$H(\omega) = \sum_{i=1}^{n} X_i(\omega)$$

Or, alternatively, $H = \sum_{i=1}^{n} X_i$. In this case, we interpret H as the number of heads observed in the n tosses.


Outcome Variables
Event Probabilities
Uncertain Events
Outcome Probabilities
Real Functions
Intervals
Real Line
Real Order
Real Summation
Size of Direct Product
Real Numbers
Rational Order
Natural Summation
Number of Set Products
Integral Line
Rational Numbers
Lists
Finite Set Examples
Number of Disjoint Unions
Integer Arithmetic
Direct Products
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Natural Fractions
Finite Sets
Identity Elements
Natural Arithmetic
Integer Products
Equivalent Sets
Comparisons
Integer Order
Integer Sums
Natural Powers
Family Operations
Function Inverses
Family Unions and Intersections
Operations
Orders
Integer Numbers
Natural Products
Families
Function Images
Function Composites
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Functions
Relations
Recursion Theorem
Uncertain Outcomes
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Unordered Triples
Successor Sets
Set Dualities
Intersection of Empty Set
Set Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects