Why

How many ways are there to split n objects into nonoverlapping groups, when the objects are indistinguishable?

Definition

Suppose n is a nonzero natural number. A *partition* of n is a *nonincreasing* list of *nonzero* natural numbers whose sum is n. The requirement that the list of numbers be nonincreasing makes the representation unique. The terms of the list are called the *parts* of the partition. The number n is sometimes called the *weight* of the partition. The number of times a particular number appears in the list is called the *multiplicity* of that part.

Examples

What are the partitions of the number 5?

$$
\begin{aligned}
5 &= 5 \\
&= 4 + 1 \\
&= 3 + 2 \\
&= 3 + 1 + 1 \\
&= 2 + 2 + 1 \\
&= 2 + 1 + 1 + 1 \\
&= 1 + 1 + 1 + 1 + 1
\end{aligned}
$$

These seven identities correspond to the seven partitions of 5, namely $(5,)$, $(4, 1)$, $(3, 2)$, $(3, 1, 1)$, $(2, 2, 1)$, $(2, 1, 1, 1)$, $(1, 1, 1, 1, 1)$. The multiplicity of 1 in these partitions is 0, 1, 0, 2, 1, 3, 5, respectively.

Notation

Suppose λ is a list in **N** of length $r \geq 1$. Then $\lambda = (\lambda_1, \ldots, \lambda_r)$ is a *partition* of $n \in$ **N** if

$$\lambda_1 + \lambda_2 + \cdots + \lambda_r = n$$

and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r$. The terms of λ are the *parts* of the partition, so λ_i is the ith part, where $i = 1, \ldots, r$. Some authors denote the *weight* of λ by $|\lambda|$.

Partition function

How many partitions are there of the number n? We denote *this number* by $p(n)$. From the examples above, $p(5) = 7$.

Number Partitions

Lists

Natural Sums

Set Numbers

Direct Products

Finite Sets

Recursion Theorem

Equivalent Sets

Function Inverses

Natural Order

Family Unions and Intersections

Function Composites

Function Images

Peano Axioms

Families

Equivalence Relations

Functions

Relations

Natural Induction

Partitions

Ordered Pair Projections

Set Products

Set Powers

Natural Numbers

Generalized Set Dualities

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Dualities

Set Unions and Intersections

Ordered Pairs

Set Intersections

Pair Unions

Set Complements

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects