Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

Result

When context is clear, we refer to the following proposition as the *monotone convergence theorem*.

Prop. 1. *The integral of the almost everywhere limit of an almost-everywhere nondecreasing sequence of measurable, non-negative, extended-real-valued functions is the limit of the sequence of integrals of the functions.*

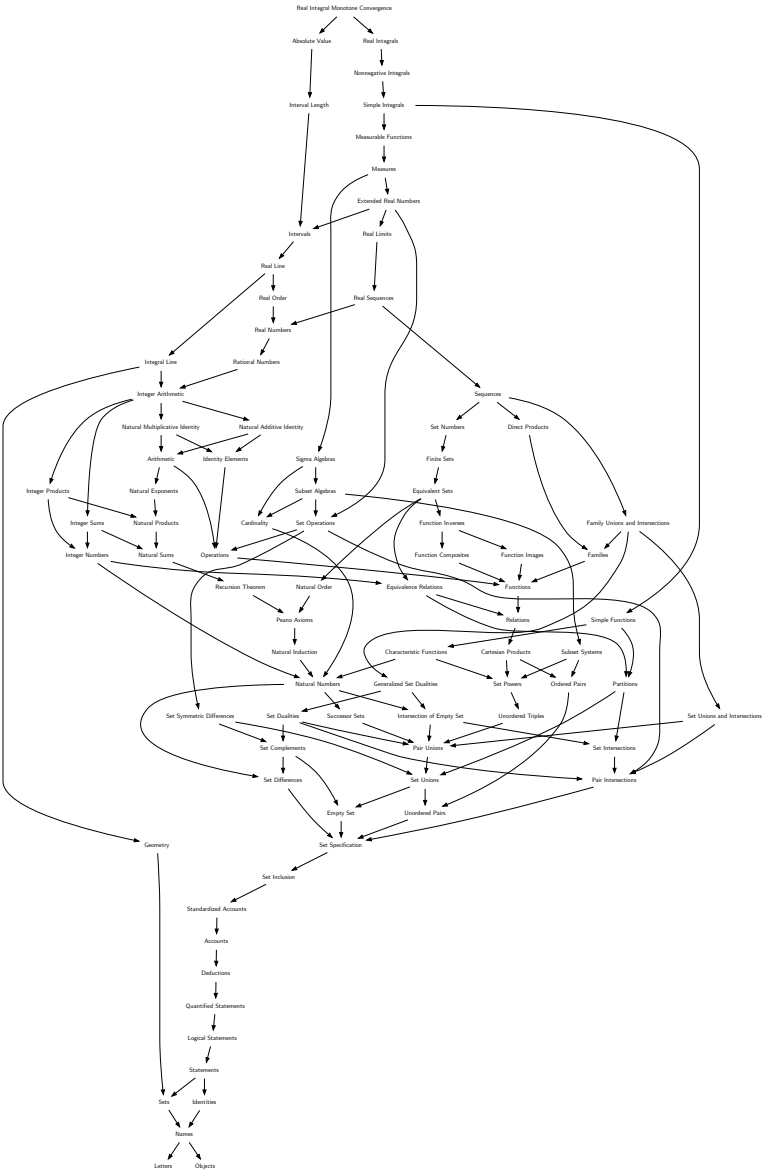
Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space, and let $f_n :\to [-\infty, \infty]$ a $\mathcal{A}$-measurable function for every natural number n and let $f : X \to [-\infty, \infty]$ a $\mathcal{A}$-measurable function. We want to show that if

$$f_n(x) \leq f_{n+1}(x) \quad \text{and} \quad f(x) = \lim_n f_n(x)$$

hold for all natural n and almost every x in X, then

$$\int f d\mu = \lim_n \int f_n d\mu.$$

$\square$


Real Integral Monotone Convergence
Absolute Value
Real Integrals
Nonnegative Integrals
Interval Length
Simple Integrals
Measurable Functions
Measures
Extended Real Numbers
Intervals
Real Limits
Real Line
Real Order
Real Sequences
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Sequences
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Sigma Algebras
Finite Sets
Integer Products
Natural Exponents
Subset Algebras
Equivalent Sets
Integer Sums
Natural Products
Cardinality
Set Operations
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Operations
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Simple Functions
Natural Induction
Characteristic Functions
Cartesian Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Partitions
Set Symmetric Differences
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects