



## Monotone Sequences

### 1 Why

If the base set of a sequence has a partial order, then we can discuss its relation to the order of sequence.

### 2 Definition

A sequence on a partially ordered set is *non-decreasing* if whenever a first index precedes a second index the element associated with the first index precedes the element associated with the second element. A sequence on a partially ordered set is *increasing* if it is non-decreasing and no two elements are the same.

A sequence on a partially ordered set is *non-increasing* if whenever a first index precedes a second index the element associated with the first index succeeds the element associated with the second element. A sequence on a partially ordered set is *decreasing* if it is non-increasing and no two elements are the same.

A sequence on a partially ordered set is *monotone* if it is non-decreasing, or non-increasing. An increasing sequence is non-decreasing. A decreasing sequence is non-increasing. A sequence on a partially ordered set is *strictly monotone* if it is decreasing, or increasing.

## 2.1 Notation

Let  $A$  a non-empty set with partial order  $\preceq$ . Let  $(a_n)_n$  a sequence in  $A$ .

The sequence is non-decreasing if  $n \leq m \implies a_n \preceq a_m$ , and increasing if  $n < m \implies a_n \prec a_m$ . The sequence is non-increasing if  $n \leq m \implies a_n \succeq a_m$ , and decreasing if  $n < m \implies a_n \succ a_m$ .

## 3 Examples

**Example 1.** Let  $A$  a non-empty set and  $(A_n)_n$  a sequence of sets in  $A^*$ . Partially order elements of  $A^*$  by the relation contained in.