



COMPLEX DISTANCE

Why

The identification of \mathbf{C} with a plane leads \mathbf{C} to naturally inherit \mathbf{R}^2 's notion of distance.

Definition

The *absolute value* or *modulus* of $z = (x, y) \in \mathbf{C}$ is the distance of z to the origin. If $z \in \mathbf{C}$, then the modulus of z is

$$\sqrt{x^2 + y^2}.$$

In other words, the modulus of z is the distance (in \mathbf{R}^2 of $z = (x, y)$ from the origin $(0, 0)$.

Notation

We denote the modulus of z by $|z|$.

Properties

Proposition 1 (Triangle Inequality). *For all $z, w \in \mathbf{C}$,*

$$|z + w| \leq |z| + |w|.$$

Also, for all $z \in \mathbf{C}$, we have $|\operatorname{Re}(z)| \leq |z|$ and $|\operatorname{Im}(z)| \leq |z|$, and for all $z, w \in \mathbf{C}$,

$$||z| - |w|| \leq |z - w|. ¹$$

¹This follows from the triangle inequality. Future editions will include an account.

