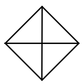


Why

The relationship between the inclusion map and the identity map is characteristic of making small functions out of large ones.[1]

Definition

Let $X \subset Y$ and $f : Y \to Z$. There is a natural function $g : X \to Z$, namely the one defined by $g(x) = f(x)$ for all $x \in X$. We call g the *restriction* of f to X. We call f an *extension* of g to Y. Clearly, there may be more than one extension of a function

Notation

We denote the restriction of $f : Y \to Z$ to the set $X \subset Y$ by $f|X$.

Example

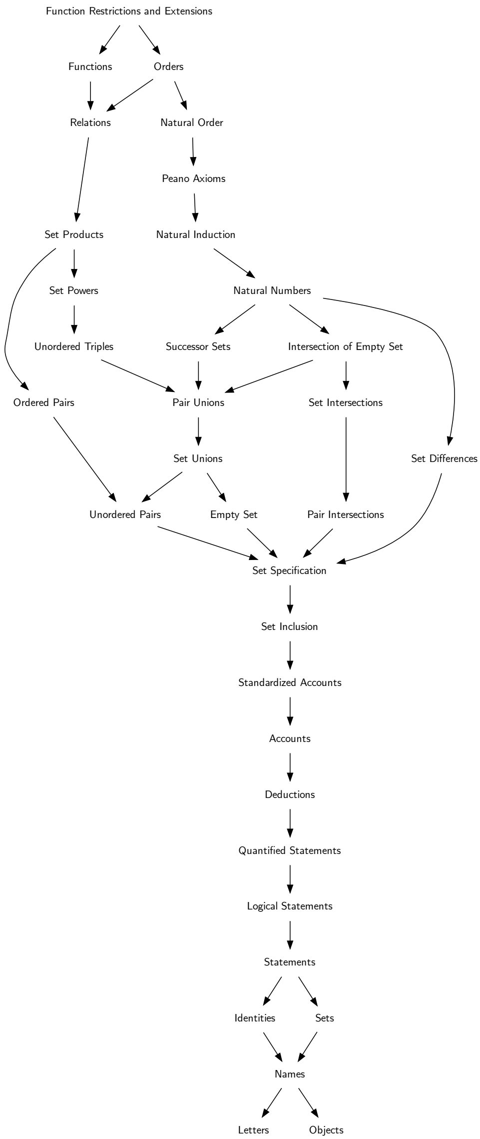
A simple example is the that the inclusion mapping from X to Y with $X \subset Y$ is a restriction of the identity map on X

An extension order

Here is a natural order involving set extensions and restrictions. Fix two sets A and B. Let F be the set of all functions $f : X \to Y$ with $X \subset A$ and $Y \subset B$. Define a relation R in

[1] Future editions will modify this language.

F by $(f, g) \in R$ if dom $f \subset$ dom g and $f(x) = g(x)$ for all x in dom f. In other words, $(f, g) \in R$ if f is a restriction of g (or, equivalently, g is an extension of f. We recognize that R is a special case of the inclusion partial order by recognizing the elements of F as subsets $A \times B$.


Function Restrictions and Extensions
Functions
Orders
Relations
Natural Order
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects