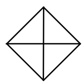


Definition

Suppose $(X, \mathcal{F})$ is a measurable space. A measure $\nu : \mathcal{F} \to \bar{\mathbf{R}}$ is said to have a density with respect to a measure $\mu : \mathcal{F} \to \bar{\mathbf{R}}$ if there exists a measurable function $f : X \to \mathbf{R}_+$

$$\nu(A) = \int_A f d\mu \quad \text{for all } A \in \mathcal{F}$$

In this case f is called a *density* of μ with respect to ν.

Examples

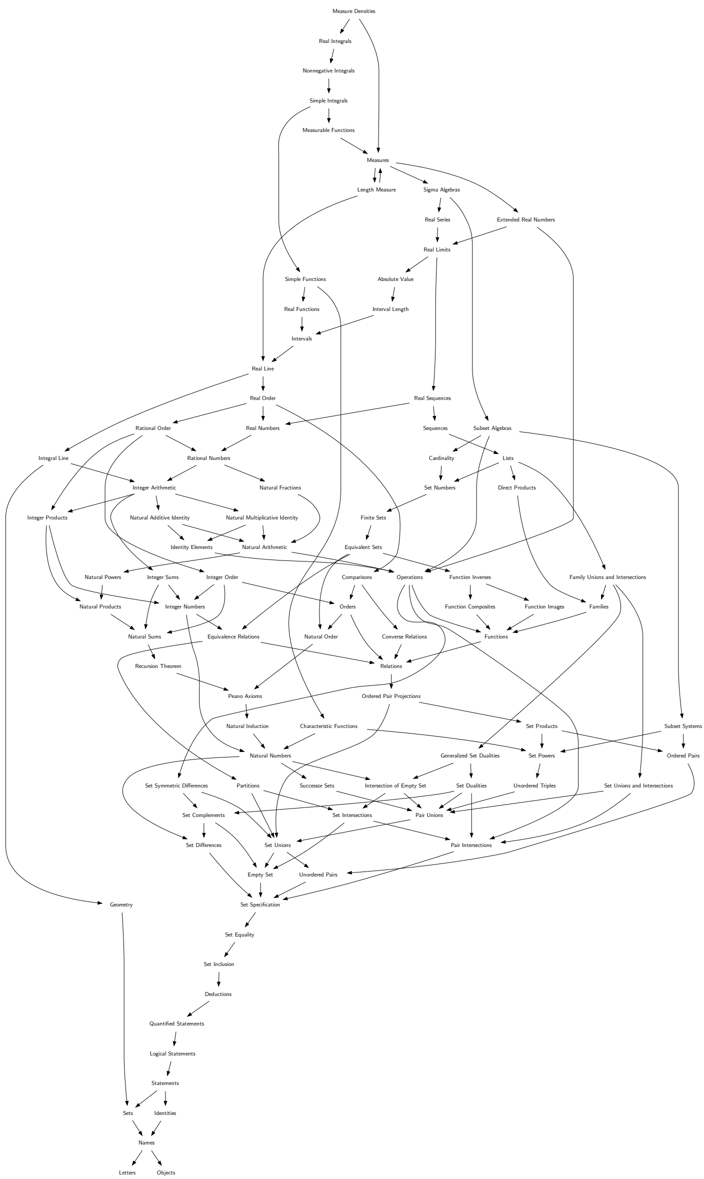
Probability on finite sets. Suppose P is a probability measure for a finite set Ω. Define $p : \Omega \to [0, 1]$ by

$$p(\omega) = P(\{\omega\}) \quad \text{for all } \omega \in \Omega$$

Then p is a probability distribution. Moreover, p is a density for P with respect to the counting measure $\# : \mathcal{P}(\Omega) \to \mathbf{R}$. Witness, for every $A \subset \Omega$,

$$\int_A p d\# = \sum_{a \in A} p(\omega)$$

We recognize the right hand side as $P(A)$ by using the additivity of P.


Measure Densities
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Length Measure
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Simple Functions
Absolute Value
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Subset Algebras
Integral Line
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects