



## Undirected Subgraphs

### 1 Why

We often start with an undirected graph and want to talk about the graphs we obtain by only keeping the edges incident to particular vertices.

### 2 Definition

The *subgraph* of a graph corresponding to a subset of vertices is the undirected graph with those vertices and all edges between those vertices.

### 3 Notation

Let  $G = (V, E)$  be an undirected graph. Let  $W \subset V$ . The subgraph corresponding to  $W$  is the undirected graph  $(W, F)$  where

$$F = \{\{v, w\} \in E \mid v, w \in W\}.$$

TODO: maybe delete; this abuses  $G$ , which is no longer an ordered pair, but becomes a function on subsets of  $V$  with a complicated codomain. We denote the subgraph of  $G$  corresponding to  $W$  by  $G(W) = (W, E(W))$ .