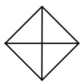


Why

We are regularly thinking of the set $\mathbf{C}$ as identified with the plane.

Definition

Since $\mathbf{C} = \mathsf{R}^2$, we can identify $z \in \mathbf{C}$ with a point in the plane, as we did in Real Plane. In this case, if $z = x + iy \in \mathbf{C}$ (i.e, $z = (x, y) \in \mathsf{R}^2$), we can visualize this identification in the following figure. We can identify the origin with the complex number $(0, 0) = 0 \in \mathbf{C}$. For this reason we call $0 \in \mathbf{C}$ the *complex origin*. Likewise, the imaginary number $(0, 1) = i \in \mathbf{C}$ corresponds to $(0, 1)$ Clearly, the horizontal axis corresponds to the purely real numbers and the vertical axis corresponds to the purely imaginary numbers. For these reasons, we refer to these axes as the *real axis* and *imaginar axis*, respectively.

Modulus and argument

The *modulus* of $z \in \mathbf{C}$ is the distance of z to the origin. If $z \in \mathbf{C}$, then the modulus of z is

$$\sqrt{\mathsf{Re}\, z^2 + \mathsf{Im}\, z^2}.$$

We denote the modulus of z by $|z|$.

The *argument* of $z \in \mathbf{C}$ is $\tan^{-1}(\mathsf{Im}\, z / \mathsf{Re}\, z)$. We denote the argument of z by $\arg z$.[1]

[1]Future editions will include the geometric interpretations.

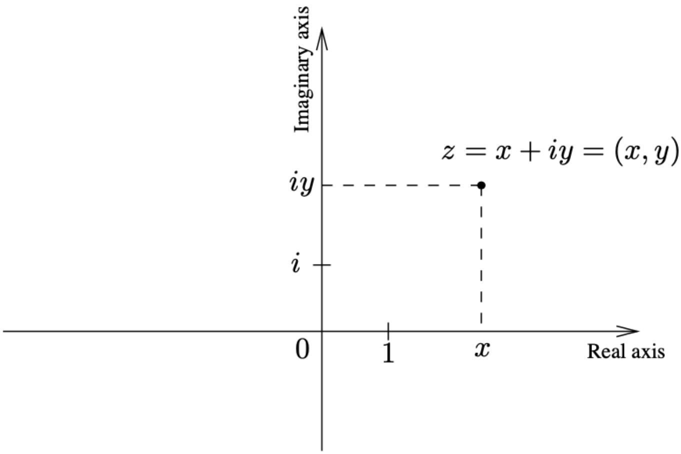
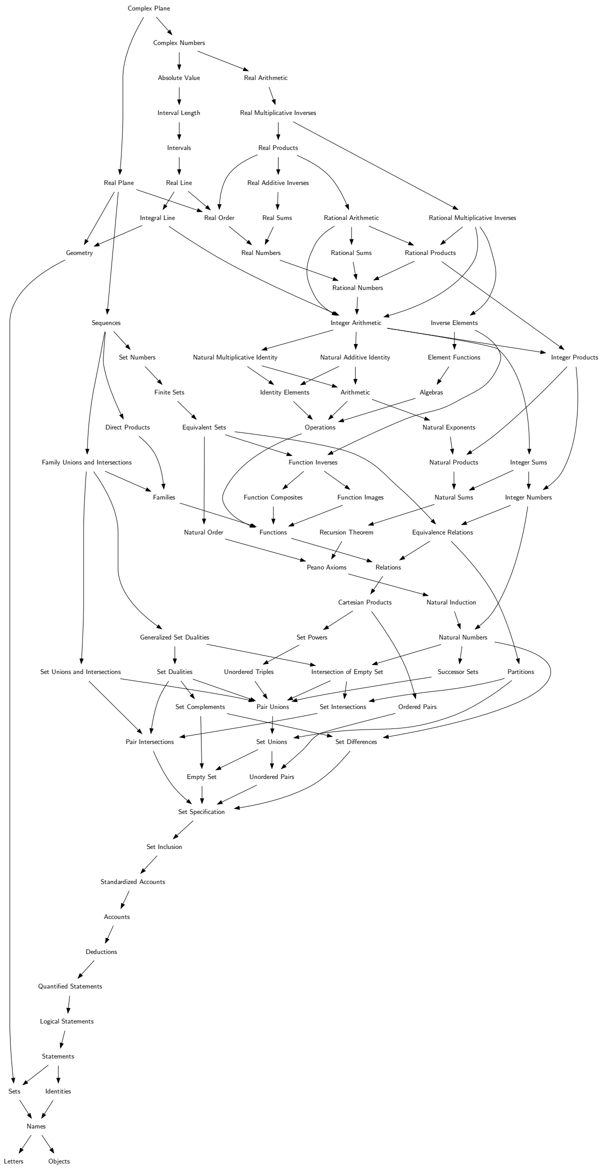


Figure 1: The complex plane

Complex disc

The *complex disc* is the set $\{z \in \mathbf{C} \mid |z| M1\}$. We denote it by $\mathbf{D}$, a mnemonic for disc. The *complex unit circle* is the set $\{z \in \mathbf{C} \mid |z| = 1\}$. We denote it by $\mathbf{T}$, a mnemonic for torus.


Complex Plane
Complex Numbers
Absolute Value
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Geometry
Real Numbers
Rational Sums
Rational Products
Rational Numbers
Sequences
Integer Arithmetic
Inverse Elements
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Element Functions
Integer Products
Finite Sets
Identity Elements
Arithmetic
Algebras
Direct Products
Equivalent Sets
Operations
Natural Exponents
Family Unions and Intersections
Function Inverses
Natural Products
Integer Sums
Families
Function Composites
Function Images
Natural Sums
Integer Numbers
Natural Order
Functions
Recursion Theorem
Equivalence Relations
Peano Axioms
Relations
Cartesian Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects