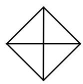


Why

We can identify any linear functional $F : \mathbf{R}^n \to \mathbf{R}$ with a vector $y \in \mathbf{R}^n$ so that $F(x) = \langle x, y \rangle$. We generalize this result to complete inner product spaces.

Motivating result

The following is known as the *Riesz representation theorem* (or *Riesz-Fréchet representation theorem*, or *Riesz theorem*, or *Riesz-Fréchet theorem*).

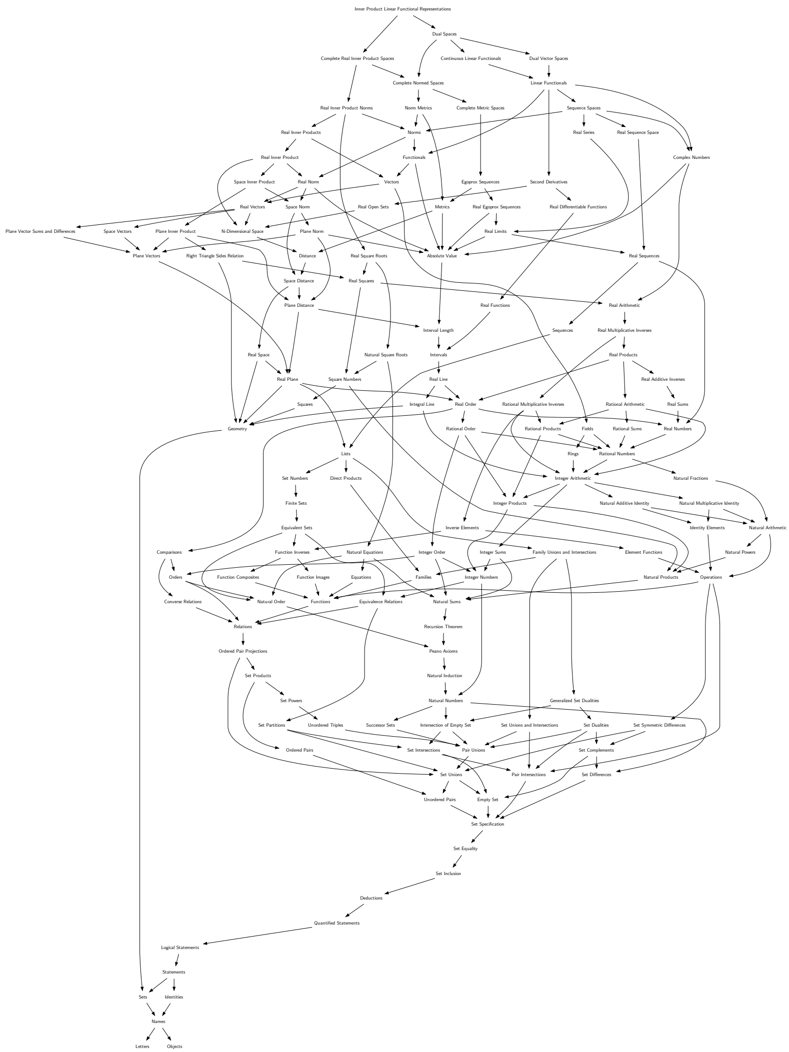
Proposition 1. *Let $((V, k), \langle \cdot, \cdot \rangle)$ be a complete inner product space and let $F : V \to k$ be a continuous linear functional on V. There exists a unique $y \in V$ so that*

$$F(x) = \langle x, y \rangle$$

for all $x \in V$. Moreover $\|y\| = \|F\|_*$.

Clearly $\mathbf{R}^n$ is a complete inner product space, and so this this theorem says the expected. We can identify linear functionals on $\mathbf{R}^n$ with elements (vectors) in $\mathbf{R}^n$.[1]

[1]Future editions will expand further.


Inner Product Linear Functional Representations
Dual Spaces
Complete Real Inner Product Spaces
Continuous Linear Functionals
Dual Vector Spaces
Complete Normed Spaces
Linear Functionals
Real Inner Product Norms
Norm Metrics
Complete Metric Spaces
Sequence Spaces
Real Inner Products
Norms
Real Series
Real Sequence Space
Real Inner Product
Functionals
Complex Numbers
Space Inner Product
Real Norm
Vectors
Egoprox Sequences
Second Derivatives
Real Vectors
Space Norm
Real Open Sets
Metrics
Real Egoprox Sequences
Real Differentiable Functions
Plane Vector Sums and Differences
Space Vectors
Plane Inner Product
N-Dimensional Space
Plane Norm
Real Limits
Plane Vectors
Right Triangle Sides Relation
Distance
Real Square Roots
Absolute Value
Real Sequences
Space Distance
Real Squares
Plane Distance
Real Functions
Real Arithmetic
Interval Length
Sequences
Real Multiplicative Inverses
Real Space
Natural Square Roots
Intervals
Real Products
Real Plane
Square Numbers
Real Line
Real Additive Inverses
Squares
Integral Line
Real Order
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Geometry
Rational Order
Rational Products
Fields
Rational Sums
Real Numbers
Lists
Rings
Rational Numbers
Set Numbers
Direct Products
Integer Arithmetic
Natural Fractions
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Inverse Elements
Identity Elements
Natural Arithmetic
Comparisons
Function Inverses
Natural Equations
Integer Order
Integer Sums
Family Unions and Intersections
Element Functions
Natural Powers
Orders
Function Composites
Function Images
Equations
Families
Integer Numbers
Natural Products
Operations
Converse Relations
Natural Order
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Unions and Intersections
Set Dualities
Set Symmetric Differences
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Pair Intersections
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects