Why

Lots of things are (approximately) linear.[1]

Definition

A transformation is *linear* if the result of a linear combination of the two vectors is the linear combination of the results of the vectors (using the same coefficients). The transformation is linear *with respect to* the field of the two vector spaces.

We use the term transformation (Transformations) for emphasis and reminder that the function is defined on a vector space. Of course, R is a vector space and so a function $f : \mathsf{R} \to \mathsf{R}$ may be linear. It is, therefore, common to speak of *linear functions*.

Often authors will use the word *operator* for linear functions. It seems, generally, that this term is commonly reserved for the case in which the vector space discussed is a function space (or, at least, infinite dimensional).

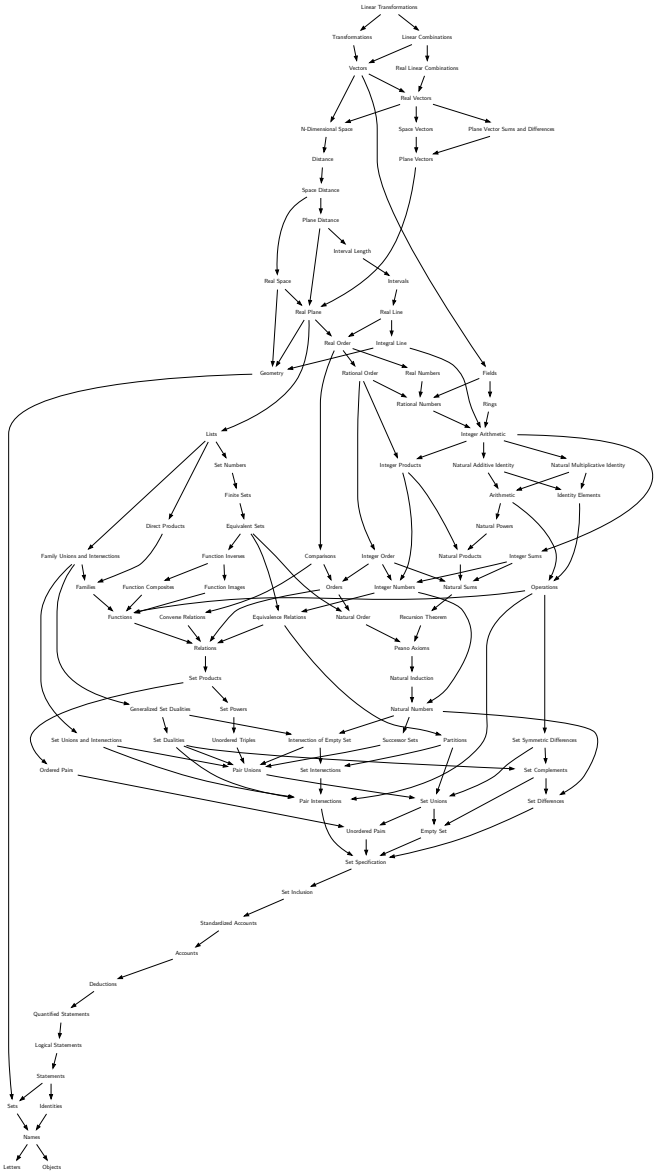
Notation

Let (V_1, F) and (V_2, F) be two vector spaces over the same field. Let $f : V_1 \to V_2$. f is linear means

$$f(au + bv) = af(u) + bf(v)$$

for all $a, b \in F$ and $u, v \in V_1$.

[1] Future editions will expand on this why. In particular, the intuition of proportionality.


Linear Transformations
Transformations
Linear Combinations
Vectors
Real Linear Combinations
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Positional Order
Real Numbers
Fields
Rational Numbers
Rings
Lists
Integer Arithmetic
Set Numbers
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Natural Products
Integer Sums
Families
Function Composition
Function Images
Orders
Integer Numbers
Natural Sums
Operations
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects