Why

Definition

The *cumulative distribution function* of a real-valued random variable is the function mapping a real number to the measure of the set of outcomes for which the random variable takes value less than or equal to the number. Notation (below) helps.

The range of the cumulative distribution function is the interval $[0, 1]$, since the measure of the base set is one and all measures are non-negative.

We often abbreviate the words "cumulative distribution function" by *c.d.f.* or *cdf*.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space. Let $f : X \to \mathbf{R}$ be a measurable function (a real-valued random variable). We denote the cumulative distribution function of f by $F_f : \mathbf{R} \to [0, 1]$. We defined it by

$$F_f(t) = \mu(\{x \in X \mid f(x) \leq t\}).$$

Properties

Prop. 1. *The cumulative distribution function of any real-valued random variable*

1. *is non-decreasing,*

2. *is right-continuous,*

3. *tends to one as its argument tends toward infinity, and*

4. *tends to zero as its argument tends towards negative infinity.*

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[1] Future editions will include.

Proof. [2] □

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[2]Future editions will include the account.

Cumulative Distribution Functions

Random Variables

Measurable Functions   Probability Measures   Topological Sigma Algebras

Measures   Event Probabilities   Topologies

Sigma Algebras   Metrics

Extended Real Numbers   Real Series   Distance

Real Limits   Space Distance

Absolute Value   Plane Distance

Interval Length   Probability Distributions   Real Space

Intervals   Real Plane

Real Line   Integral Line

Real Summation   Real Sequences   Real Order   Geometry

Subset Algebras   Real Numbers   Sequences   Rational Order

Rational Numbers   Lists   Cardinality

Integer Arithmetic   Direct Products   Set Numbers

Natural Additive Identity   Natural Multiplicative Identity   Finite Sets   Integer Products

Natural Summation   Identity Elements   Arithmetic   Equivalent Sets

Family Operations   Family Unions and Intersections   Natural Powers   Function Inverses   Integer Sums   Integer Order   Comparisons

Operations   Families   Natural Products   Function Images   Function Composites   Integer Numbers   Orders

Functions   Natural Sums   Equivalence Relations   Converse Relations   Natural Order

Subset Systems   Set Products   Recursion Theorem   Relations

Ordered Pairs   Set Powers   Peano Axioms

Generalized Set Dualities   Unordered Triples   Natural Induction   Uncertain Outcomes

Set Symmetric Differences   Set Dualities   Set Unions and Intersections   Successor Sets   Intersection of Empty Set   Partitions

Set Complements   Pair Unions   Set Intersections

Pair Intersections   Set Unions   Set Differences

Empty Set   Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects