

LINEAR FUNCTIONALS

Why

Definition

A linear functional on a vector space V with field k is a function $f:V\to k$ which satisfies

$$f(\lambda x + \mu y) = \lambda f(x) + \mu f(y)$$

for all $\lambda, \mu \in k$ and $x, y \in V$.

Examples

1. Define $F: \mathbf{C}^n \to \mathbf{C}$ by

$$F(x_1, \dots, x_n) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

where $c_1, \ldots, c_n \in \mathbf{C}$.

2. Let $(c_n)_{n\in\mathbb{N}}\in\ell^{\infty}$. Define $F_c:\ell^1\to\mathbf{C}$ by

$$F_c((x_n)_{n\in\mathbf{N}}) = \sum_{n=1}^{\infty} c_n x_n$$

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 $\wp(A)$.

