



# Set Operations

## 1 Why

We want to consider the elements of two sets together at once, and other sets created from two sets.

## 2 Definitions

Let  $A$  and  $B$  be two sets.

The *union* of  $A$  with  $B$  is the set whose elements are in either  $A$  *or*  $B$  *or* both. The key word in the definition is *or*.

The *intersection* of  $A$  with  $B$  is the set whose elements are in both  $A$  *and*  $B$ . The keyword in the definition is *and*.

Viewed as operations, both union and intersection commute; this property justifies the language “with.” The intersection is a subset of  $A$ , of  $B$ , and of the union of  $A$  with  $B$ .

The *symmetric difference* of  $A$  and  $B$  is the set whose elements are in the union but not in the intersection. The symmetric difference commutes because both union and intersection commute; this property justifies the language “and.” The symmetric difference is a subset of the union.

Let  $C$  be a set containing  $A$ . The *complement* of  $A$  in  $C$  is the symmetric difference of  $A$  and  $C$ . Since  $A \subset C$ , the union is  $C$  and the intersection is  $A$ . So the complement is the "left-over" elements of  $B$  after removing the elements of  $A$ .

We call these four operations *set-algebraic operations*.

## 2.1 Notation

Let  $A, B$  be sets. We denote the union of  $A$  with  $B$  by  $A \cup B$ , read aloud as "A union B."  $\cup$  is a stylized U. We denote the intersection of  $A$  with  $B$  by  $A \cap B$ , read aloud as "A intersect B." We denote the symmetric difference of  $A$  and  $B$  by  $A \Delta B$ , read aloud as "A symdiff B." "Delta" is a mnemonic for difference.

Let  $C$  be a set containing  $A$ . We denote the complement of  $A$  in  $C$  by  $C - A$ , read aloud as "C minus A."

## 2.2 Results

**Proposition 1.** *For all sets  $A$  and  $B$  the operations  $\cup$ ,  $\cap$ , and  $\Delta$  commute.*

**Proposition 2.** *Let  $S$  a set. For all sets  $A, B \subset S$ ,*

$$(1) \quad S - (A \cup B) = (S - A) \cap (S - B)$$

$$(2) \quad S - (A \cap B) = (S - A) \cup (S - B).$$

**Proposition 3.** *Let  $S$  a set. For all sets  $A, B \subset S$ ,*

$$A \Delta B = (A \cup B) \cap C_S(A \cap B)$$

*TODO : notation*