Why

What does it mean for two random variables to be independent? What are the events associated with a random variable? TODO

Definition

Defining Result

Proposition **1.** *The set of inverse images the distinguished sets of a measurable space under a function from a set to that space is a sigma algebra.*

If the first set and the function are measurable, the sigma algebra is a sub sigma algebra of the domain sigma algebra.

Proof. Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Let $f : X \to Y$ be a measurable function. Define

$$\mathcal{C} := \left\{ f^{-1}(B) \;\middle|\; B \in \mathcal{B} \right\}.$$

First, since $f^{-1}(Y) = X$, $X \in \mathcal{C}$.

Second, let $C \in \mathcal{C}$. Then there is B such that $C = f^{-1}(B)$. Then $X - C = X - f^{-1}(B) = f^{-1}(Y - B)$. Since $\mathcal{B}$ is a sigma algebra, $B \in \mathcal{B}, Y - B \in \mathcal{B}$ and so $X - C \in \mathcal{C}$.

Finally, let $(C_n)_n \subset \mathcal{C}$. Then for every n there exists a $B_n \in \mathcal{B}$ so that $C_n = f^{-1}(B_n)$. Then:

$$\cup_n C_n = \cup_n f^{-1}(B_n) = f^{-1}(\cup B_n).$$

Since $\mathcal{B}$ is a sigma algebra, $\cup_n B_n \in \mathcal{B}$ and so $\cup_n C_n \in \mathcal{C}$.

Since f is measurable, $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}$, and so $\mathcal{C} \subset \mathcal{A}$. $\square$

The sigma algebra *generated by a random variable* is the sigma algebra consisting of the inverse images of every measurable set of the codomain.

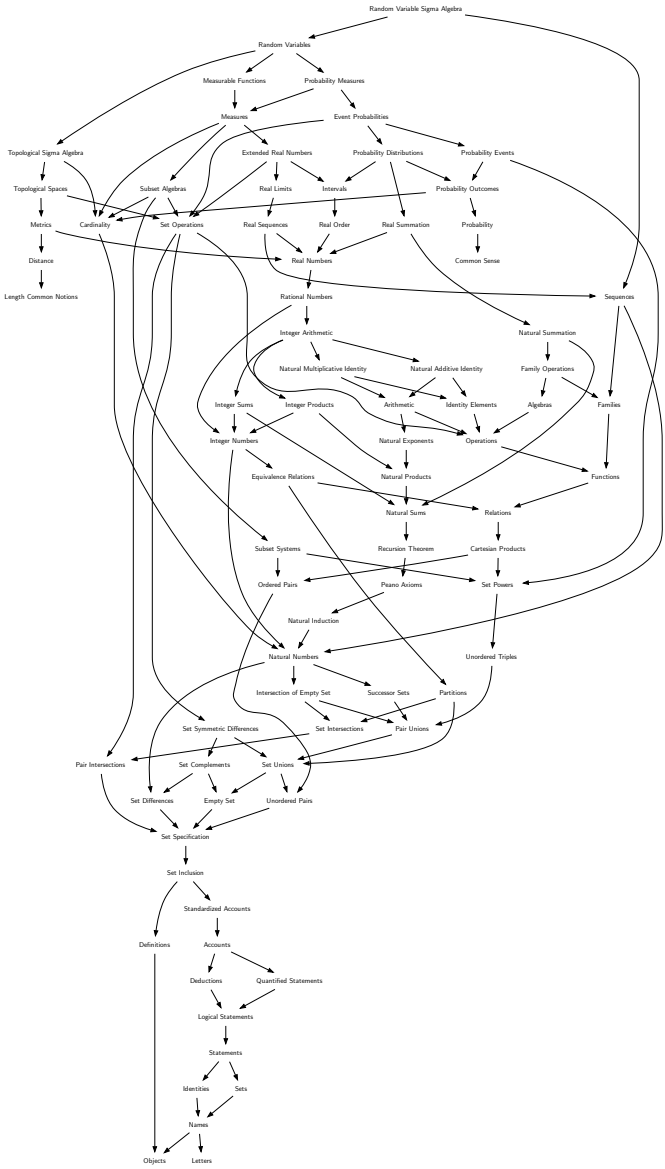
The sigma algebra generated by a family of random variables is the sigma algebra generated by the union of the sigma algebras generated individually by each of the random variables.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f : X \to Y$ be a random variable. Denote by $\sigma(f)$ the sigma algebra generated by f.

Results

Proposition **2.** *The sigma algebra generated by a family of random variables is the smallest sigma algebra for with respect to which each random variable is measurably.*


Random Variable Sigma Algebra
Random Variables
Measurable Functions
Probability Measures
Measures
Event Probabilities
Topological Sigma Algebra
Extended Real Numbers
Probability Distributions
Probability Events
Topological Spaces
Subset Algebras
Real Limits
Intervals
Probability Outcomes
Metrics
Cardinality
Set Operations
Real Sequences
Real Order
Real Summation
Probability
Distance
Real Numbers
Common Sense
Length Common Notions
Sequences
Rational Numbers
Natural Summation
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Family Operations
Integer Sums
Integer Products
Arithmetic
Identity Elements
Algebras
Families
Integer Numbers
Natural Exponents
Operations
Equivalence Relations
Natural Products
Functions
Natural Sums
Relations
Subset Systems
Recursion Theorem
Cartesian Products
Ordered Pairs
Peano Axioms
Set Powers
Natural Induction
Natural Numbers
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters