



Why

We are constantly thinking of the integers as the endpoints of equal length segments of a line.

Discussion

We commonly associate elements of the integers with the endpoints of equal-length segments of a real line. Take segment S_0 of L with endpoints p and q . Associate the point p with 0. Associate the point q with 1. Take a segment S_1 of equal length, non-overlapping with S_0 , who shares the endpoint q . Associate the second endpoint of this segment 2. Continue with the rest.¹ We call the line so formed the *integral line* of unit S_0 .

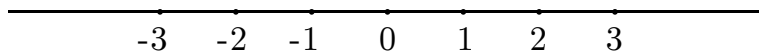


Figure 1: The integral segments.

Integral Distance

Let $f : \mathbf{Z} \rightarrow \mathbf{Z}$ be defined by $f(a, b) = a - b$ if $a > b$ and $f(a, b) = b - a$ if $b > a$. Notice that f is symmetric: $f(a, b) = f(b, a)$. The (geometric) interpretation of f is the distance between the points associated with the two integers $a, b \in \mathbf{Z}$ in some integral line. We call f the *integral distance*. Notice that $f(a, b) > 0$ for all $a, b \in \mathbf{Z}$.

Notation

We denote the distance between $a, b \in \mathbf{Z}$ by $|a - b|$.

¹Future editions will expand.

