



## SIMPLE FUNCTIONS

### Why

We want to define area under a real function. We start with defining functions for which this notion is obvious.

### Definition

A *simple function* (or *step function*) is a real-valued function whose range is a finite set. We can write simple function as the sum of the characteristic functions for the inverse image elements.

We can partition the range of the function into a finite family of one-element sets. Then the family whose members are the inverse images of these sets partitions the domain. We call this the *simple partition* of the function.

### Notation

We denote the set of simple functions on  $A$  by  $SF(A)$ . We denote subset of non-negative simple real functions with domain  $A$  by  $SimpleF_+(A)$ .

Let  $f \in SF(A)$ . Let  $\{a_1, \dots, a_n\} = f(A)$ . Define  $A_i = f^{-1}(\{a_i\})$ . Then  $f = \sum_{i=1}^n a_i \chi_{A_i}$ .



