



# Probability Measures

## 1 Why

We use the language of measure theory to give a mathematical model for uncertain outcomes. TODO: probability intuition sheet.

## 2 Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. Since a finite measure can always be scaled to a probability measure, these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra.

### 2.1 Notation

We denote the set of outcomes by  $\Omega$ , a mnemonic for “outcomes.” We denote the sigma-algebra by  $\mathcal{A}$ , as usual. We denote a prob-

ability measure by  $p$ , a mnemonic for “probability.” Thus, we often say “Let  $(\Omega, \mathcal{A}, p)$  be a probability space.”

## 2.2 Properties