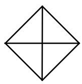


Definition

A subset of $\mathbf{R}^n$ is *affine* (or an *affine variety*, *linear variety*, or *flat*) if it contains the lines through each of its points.

Examples

The empty set is trivially an affine set. The entire set of points in n-dimensional space is an affine set. Any singleton is an affine set.

Our definition is meant to capture the intuitive idea of an infinite, uncurved set, like a line or a plane in space.

Notation

As usual, let $L(x, y)$ denote the line between $x, y \in \mathbf{R}^n$. The set $M \subset \mathbf{R}^n$ is affine if $L(a, b) \subset M$ for all $a, b \in M$. For any $x, y \in M$ and $\lambda \in \mathbf{R}$,

$$(1 - \lambda)x + \lambda y \in M.$$

Translations

The set of affine sets is closed under translation.

Proposition 1. *Let $M \subset \mathbf{R}^n$ affine. Then $M + a$ is affine for any $a \in \mathbf{R}$.*

Proof. Suppose $x, y \in M + a$. So there exists corresponding $\tilde{x}, \tilde{y} \in M$ with $x = \tilde{x} + a$ and $y = \tilde{y} + a$. For any $\lambda \in \mathbf{R}$,

$$\begin{aligned}(1 - \lambda)x + \lambda y &= (1 - \lambda)(\tilde{x} + a) + \lambda(\tilde{y} + a) \\ &= ((1 - \lambda)\tilde{x} + \lambda\tilde{y}) + a.\end{aligned}$$

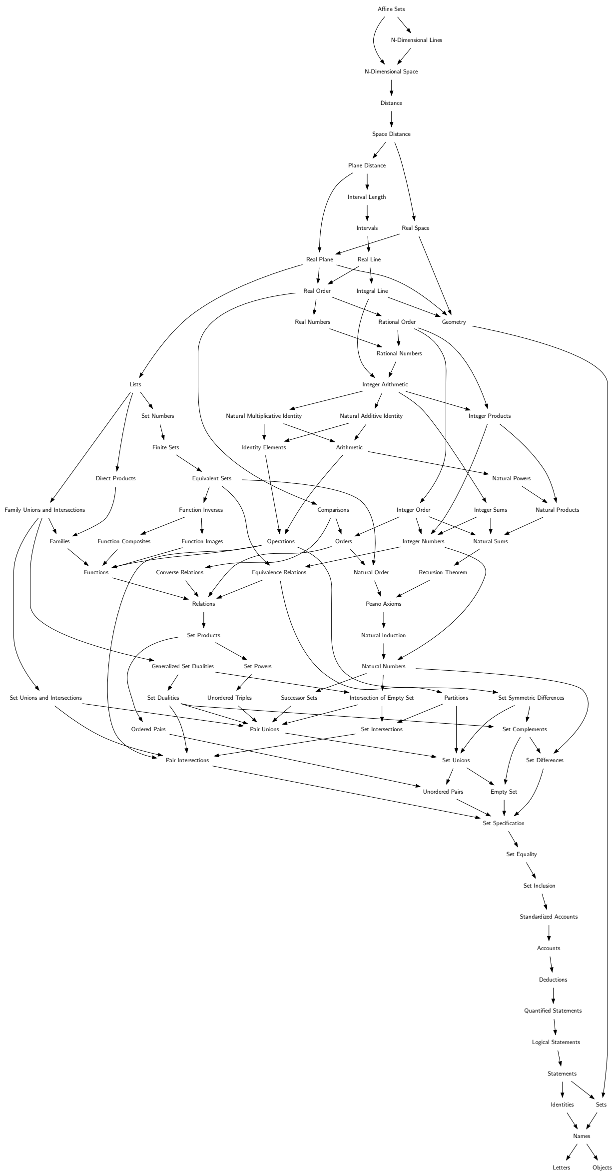
□

An affine set M is *parallel* to an affine set L if there exists $a \in \mathbf{R}^n$ so that $M = L + a$. The relation "M is parallel to L" on the set of affine sets is an equivalence relation. This notion of parallelism is more restrictive than the natural one, in which we may speak of a line being parallel to a plane. We must speak of a line which is parallel to another line within the given plane.

Properties

The set of affine sets is closed under intersection.

Proposition 2. *If $\mathcal{M}$ is a family of affine sets, then $\cap\mathcal{M}$ is affine.*


Affine Sets
N-Dimensional Lines
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Real Order
Integral Line
Real Numbers
Rational Order
Geometry
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects