



**Definition**

A function  $f : \mathbf{R}^n \rightarrow \mathbf{R}^m$  is *linear* if

1.  $f(x + y) = f(x) + f(y)$  for all  $x, y \in \mathbf{R}^n$ , and
2.  $f(\alpha x) = \alpha f(x)$  for all  $\alpha \in \mathbf{R}$  and  $x \in \mathbf{R}^n$ .

Equivalently,  $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$  for all  $\alpha, \beta \in \mathbf{R}$  and  $x, y \in \mathbf{R}^n$ .

In this case, we say that *superposition* holds.



