Real Vectors

Why

If we interpret a list of two numbers as displacement in a plane, and a list of three numbers as displacement in a space, what of a list of n numbers as displacement in $\mathbf{R}^n$?

Definition

A *real vector* (*vector, n-dimensional vector, n-vector*) is a length-n list of real numbers.

Algebra

For $x, y \in \mathbf{R}^n$, we define the *real vector sum* (or *sum*) of x and y as the vector $z \in \mathbf{R}^n$ where $z_i = x_i + y_i$ for $i = 1, \ldots, n$. As usual, we denote the sum by $x + y$, so

$$x + y = (x_1 + y_1, \ldots, x_n + y_n).$$

For $\alpha \in \mathbf{R}$ and $x \in \mathbf{R}^n$, *real scalar-vector product* (or *scalar product, product*) $z \in \mathbf{R}^n$ is defined by $z_i = \alpha x_i$ for $i = 1, \ldots, n$. As usual, we denote the product αx, and write

$$\alpha x = (\alpha x_1, \ldots, \alpha x_n).$$

Our intuition for both of these *operations* comes from their special cases in $\mathbf{R}^2$ and $\mathbf{R}^3$. As usual, the *real-vector difference* (or *difference*) of x and y is the vector $z \in \mathbf{R}^n$ defined by $z_i = x_i - y_i$ for $i = 1, \ldots, n$. As usual, we denote it by $x - y$, and note that $x - y = x + (-y)$.

The algebra given here for vectors is natural in view of their generalization as n-dimensional *displacements*. However, we keep in mind that this algebra is over lists of numbers, and that these sums and products can be defined on these lists of numbers regardless of interpretation.

Real Vectors

N-Dimensional Space   Space Vectors   Plane Vector Sums and Differences

Distance   Plane Vectors

Space Distance

Plane Distance

Interval Length

Intervals   Real Space

Real Line   Real Plane

Integral Line   Real Order

Geometry   Real Numbers   Rational Order

Rational Numbers

Integer Arithmetic   Lists

Integer Products   Natural Multiplicative Identity   Natural Additive Identity   Set Numbers

Arithmetic   Identity Elements   Finite Sets   Direct Products

Natural Powers   Operations   Equivalent Sets

Integer Sums   Natural Products   Integer Order   Comparisons   Function Inverses   Family Unions and Intersections

Integer Numbers   Natural Sums   Orders   Function Composites   Function Images   Families

Recursion Theorem   Natural Order   Equivalence Relations   Converse Relations   Functions

Peano Axioms   Relations

Natural Induction   Set Products

Natural Numbers   Set Powers   Generalized Set Dualities

Partitions   Successor Sets   Intersection of Empty Set   Unordered Triples   Set Dualities   Set Unions and Intersections

Set Intersections   Pair Unions   Ordered Pairs   Set Complements

Set Differences   Set Unions   Pair Intersections

Unordered Pairs   Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects