

## MATRIX MULTIPLICATION FUNCTION

## Why

We view matrix-vector multiplication as a function mapping vectors to vectors.

## Result

Define  $f: \mathbb{R}^n \to \mathbb{R}^m$  by f(x) = Ax where  $A \in \mathbb{R}^{m \times n}$ . We call f the matrix multiplication function associated with A. It is easy to verify that f is a linear function. The converse is true.

**Proposition 1.** Let  $f : \mathbb{R}^n \to \mathbb{R}^m$  be linear. Then there exists a unique  $A \in \mathbb{R}^{m \times n}$  satisfying f(x) = Ax for all  $x \in \mathbb{R}^n$ .

*Proof.* Evaluate f at the standard basis vectors  $e_i$ . The ith component of  $e_i$  is 1 and all other components are 0.

