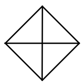


Why

We want to visualize symmetric relations.

Definition

An *undirected graph* (or *graph*) is a pair (V, E) in which V is a nonempty set and E is a subset of unordered pairs of elements in V. We call the elements of V the *vertices* of the graph and the elements of E the *edges*. We call (V, E) an undirected graph *on* V.

Two vertices are *adjacent* (or *neighboring*) if their pair is in the edge set. We say that the corresponding edge is *incident* to those vertices. The *adjacency set* of a vertex is the set of vertices adjacent to it. The *degree* of a vertex is the number of vertices adjacent to it; in other words, the size of its adjacency set. A graph is *complete* if each pair of two distinct vertices is adjacent.

The *complement* of (V, E) is the graph (V, F) where F is the complement of E in the set of pairs from V.

Other terminology

Some authors call the adjacency set the *neighborhood* of the vertex. They call the union of the adjacency set of the vertex $v \in V$ with the singleton $\{v\}$ the *closed neighborhood* of v.

When $\{x, y\} \in E$, some authors say that the edge "joins" the vertices, and call x and y the *end vertices* of the edge.

Some authors call two *edges adjacent* if they have exactly one common end vertex.

Self-loops

Some authors include a concept of a self-loop, which is meant to be an edge from of a "self-loop"—an edge from one node to itself. However, we do not allow this as the set $\{u, u\}$ is *not* a pair.

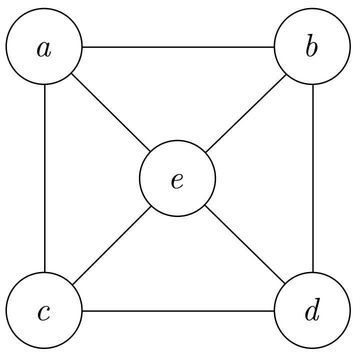
Notation

Let V be a nonempty set. Let $E \subset \{\{v, w\} \mid v, w \in V\}$. Then the pair (V, E) is an undirected graph. We regularly say "Let $G = (V, E)$" be a graph, in which the relevant properties of V and E are implicit.

The notation $\{v, w\} \in E$ for an edge between vertices $v, w \in V$ reminds us that the edges are unordered pairs of distinct vertices. We denote the adjacency set of v by $\mathrm{adj}(v)$ and the degree of v by $\deg(v)$.

Some authors will denote the vertex set of a graph they are denoting by G by $V(G)$ and the edges set by $E(G)$.

Example



Suppose $V = \{a, b, c, d, e\}$. Define

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\}.$$

In visualizations of graphs, the vertices are frequently represented as circles or rectangles in the plane and edges are shown as lines connecting the vertices.


Undirected Graphs
Set Numbers
Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Function Composites
Function Images
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Pair Unions
Partitions
Ordered Pairs
Set Differences
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects