



# Subspaces

## 1 Why

TODO

## 2 Definition

A *subspace* of a vector space is a subset of vectors that is a vector space when restriction vector addition and scalar multiplication to the set. In other words, a subspace is a subset of a vector space which is closed under vector addition and scalar multiplication.

For example, the entire set of vectors is a subspace. As a second example, the set consisting only of the zero vector is a subspace; we call this the *zero subspace*. These two subspaces are the *trivial subspaces*. A *nontrivial subspace* is a subspace that is not trivial.

## 3 Notation

Let  $(V, \mathbf{F})$  be a vector space. Let  $U \subset V$  with

$$\alpha u + \beta v \in U$$

for all  $\alpha, \beta \in \mathbf{F}$  and  $u, v \in U$ . Then  $U$  is a subspace of  $(V, \mathbf{F})$ .

## 4 Properties

**Proposition 1.** *The intersection of a family of subspaces is a subspace.*

**Proposition 2.** *There exists a family of subspaces whose union is not a subspace;*

**Remark 3.** *In other words: the union of a family subspaces need not be a subspace.*

**Proposition 4.** *A subspace must contain the zero vector; in other words, every subspace is nonempty.*





