Why

We view matrix-vector multiplication as a function mapping vectors to vectors.

Result

Define $f : \mathbf{R}^n \to \mathbf{R}^m$ by $f(x) = Ax$ where $A \in \mathbf{R}^{m \times n}$. We call f the *matrix multiplication function* associated with A. It is easy to verify that f is a linear function. The converse is true.

Proposition 1. *Let $f : \mathbf{R}^n \to \mathbf{R}^m$ be linear. Then there exists a unique $A \in \mathbf{R}^{m \times n}$ satisfying $f(x) = Ax$ for all $x \in \mathbf{R}^n$.*

Proof. Evaluate f at the standard basis vectors e_i. The ith component of e_i is 1 and all other components are 0. $\qquad\square$

Matrix Multiplication Function

Real Matrices

Linear Equations

Linear Functions

Arrays · N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space · Real Plane · Real Line

Sequences · Integral Line · Real Order · Real Summation

Geometry · Real Numbers · Rational Order

Set Numbers · Direct Products · Rational Numbers · Integer Arithmetic · Natural Summation

Finite Sets

Equivalent Sets · Family Operations · Natural Multiplicative Identity · Natural Additive Identity · Integer Products · Natural Equations

Family Unions and Intersections · Function Inverses · Algebra · Comparisons · Identity Elements · Arithmetic · Integer Sums · Integer Order

Function Images · Function Composition · Families · Operations · Integer Numbers · Orders · Natural Powers · Equations · Natural Products

Equivalence Relations · Functions · Converse Relations · Natural Order · Natural Sums

Relations

Recursion Theorem

Peano Axioms

Set Products · Natural Induction

Generalized Set Dualities · Set Powers · Natural Numbers

Set Unions and Intersections · Partitions · Intersection of Empty Set · Set Dualities · Unordered Triples · Successor Sets

Set Intersections · Pair Unions · Set Complements · Ordered Pairs

Pair Intersections · Set Unions · Set Differences

Empty Set · Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets · Identities

Names

Letters · Objects

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