Why

What is the best estimate for a random variable if we consider the square error?

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $x : \Omega \to \mathbf{R}$ a random variable. A *minimum mean squared error estimate* or *MMSE estimate* or *least square estimate* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}(x - \xi)^2$.

Proposition 1. *There is a unique MMSE estimate and it is given by* $\mathbf{E}(x)$.

Vector Case

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $y : \Omega \to \mathbf{R}^n$ a random variable.[1]

A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}\|x - \xi\|^2$.

Proposition 2. *There is a unique MMSE estimator and it is given by* $\mathbf{E}(y)$.

[1] Future editions might collapse this into the previous case.


Minimum Mean Squared Error Estimates
Norms
Expectation
Estimates
Random Variables
Functionals
Measurable Functions
Topological Sigma Algebra
Vectors
Topological Spaces
Probability Measures
N-Dimensional Space
Metrics
Measures
Distance
Extended Real Numbers
Outcome Variables
Space Distance
Real Limits
Sigma Algebras
Plane Distance
Absolute Value
Event Probabilities
Interval Length
Probability Distributions
Intervals
Uncertain Outcomes
Real Space
Real Plane
Real Line
Real Order
Integral Line
Real Sequences
Real Summation
Geometry
Sequences
Fields
Real Numbers
Set Numbers
Direct Products
Groups
Rational Numbers
Finite Sets
Integer Additive Inverses
Integer Arithmetic
Integer Sums
Natural Summation
Inverse Elements
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Family Operations
Element Functions
Arithmetic
Identity Elements
Subset Algebras
Family Unions and Intersections
Integer Products
Function Inverses
Algebras
Natural Powers
Set Operations
Cardinality
Integer Numbers
Natural Products
Function Images
Function Composition
Families
Operations
Equivalence Relations
Natural Sums
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Subset Systems
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Unordered Triples
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects