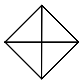


1 Why

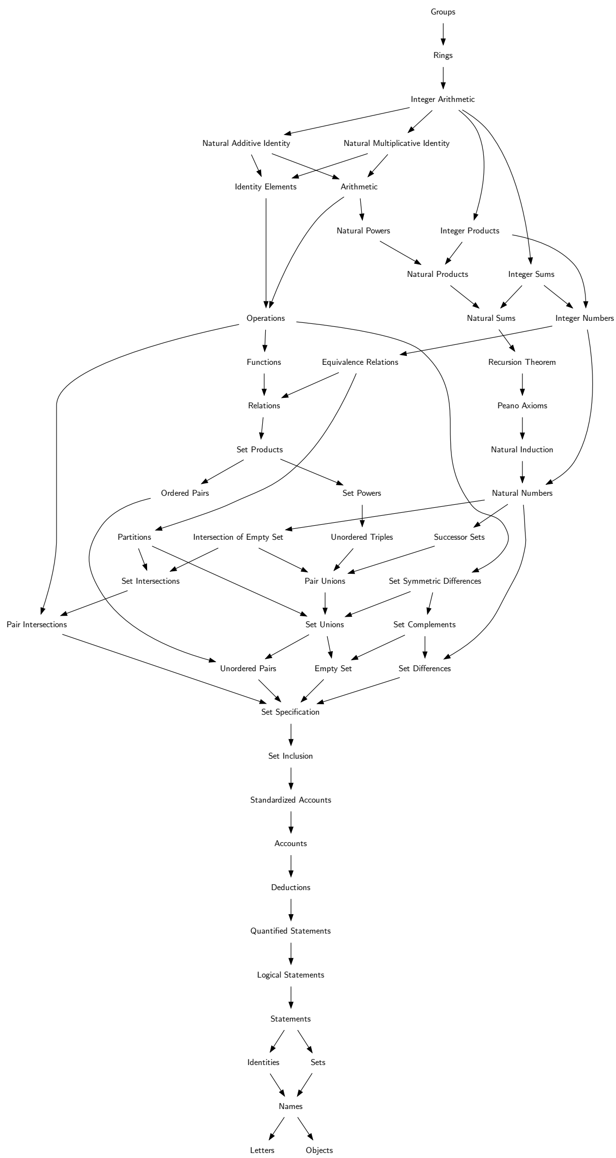
We further drop conditions on the structure of the binary operations, and study only the algebraic structure of addition over the integers.

2 Definition

A *group* is an *algebra* $(G, \circ)$ for which $\circ : G \times G \to G$ is associative, has an identity element in G, and has inverse elements. It is a *commutative group* (or *abelian group*) if $\circ$ is commutative.

3 Additive groups

Suppose that $(R, +, \cdot)$ is ring. Then $(R, +)$ is a commutative group. Conversely, suppose $(G, +)$ is a commutative group. Define multiplication on S by $a \cdot b = 0$ for all $a, b \in R$. Then $(S, +, \cdot)$ is a ring, called the *zero ring* of $(G, +)$. For this reason, it is customary to write $+$ for the operations $\circ$ when handling commutative groups.



Groups
Rings
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Identity Elements
Arithmetic
Natural Powers
Integer Products
Natural Products
Integer Sums
Operations
Natural Sums
Integer Numbers
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Partitions
Intersection of Empty Set
Unordered Triples
Successor Sets
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects