



OPERATIONS

Why

We want to “combine” elements of a set.

Definition

Let A be a non-empty set. An *operation* on A is a function from ordered pairs of elements of the set to the same set. Operations *combine* elements. We *operate* on ordered pairs.

Notation

Let A be a set and $g : A \times A \rightarrow A$. We tend to forego the notation $g(a, b)$ and write $a g b$ instead. We call this *infix notation*.

Using lower case latin letters for elements and for operators confuses, so we tend to use special symbols for operations. For example, $+$, $-$, \cdot , \circ , and \star .

Let A be a non-empty set and $+$: $A \times A \rightarrow A$ be an operation on A . According to the above paragraph, we tend to write $a + b$ for the result of applying $+$ to (a, b) .

Example

A first example of an operation is if we consider the set A as the power set of some set X . Then the pair union (see *Pair Unions*) is an operation. For if $E \in X^*$ and $F \in X^*$ then $E \cup F \in X^*$ and so \cup can be viewed as an operation on X^* .

Properties

Recall that \cup has several nice properties. For one $A \cup B = B \cup A$ and $(A \cup B) \cup C = A \cup (B \cup C)$.

An operation with the first property, that the ordered pair (A, B) and (B, A) have the same result is called *commutative*. An operation with the second property, that when given three objects the order in which we operate does not matter is called *associative*.

