

### PROBABILITY MEASURES

## Why

A probability event function is a measure on the set of outcomes.

#### Definition

A probability measure is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A probability space is a measure space whose measure is a probability measure. The word "space" is natural, since the notion of a measure generalized the notion of volume in real space (see Real Space and N-Dimensional Space). The outcomes of a probability space are the elements of the base set. The set of outcomes is the base set. The events are the elements of the sigma algebra. The measure in a probability space corresponds to the event probability function.

#### **Notation**

Let (A, A) be a measurable space.<sup>1</sup> We denote the sigmaalgebra by A, as usual. We denote a probability measure by  $\mathbf{P}$ , a mnemonic for "probability," and intended to remind of the event probability function. Thus, we often say "Let  $(A, A, \mathbf{P})$ "

 $<sup>^1 \</sup>text{Often},$  other authors will denote the set of outcomes (here denoted by A) by  $\Omega,$  a mnemonic for "outcomes."

be a probability space."

Many authors associate an event  $A \in \mathcal{A}$  with a function  $\pi: \mathcal{X} \to \{0,1\}$  so that  $A = \{x \in \mathcal{X} \mid \pi(x) = 1\}$ . In this context, it is common to write  $\mu[\pi(x)]$  for  $\mu(A)$ .

# **Properties**

