

Total Orders

1 Why

Often we want all elements of the set A to be comparable.

2 Definition

We call R connexive if for all $a, b \in A$, $(a, b) \in R$ or $(b, a) \in R$. If R is a partial order and connexive, we call it a total order.

A totally ordered set is a set together with a total order. The language is a faithful guide: we can compare any two elements. Still, we prefer one word to three, and so we will use the shorter term chain for a totally ordered set; other terms include simply ordered set and linearly ordered set.

Let C = (A, R) be a chain. A minimal element of C is an element which precedes all other elements. A maximial element of C is an element which is preceded by all other elements.

