



Definition

Let $x : \Omega \rightarrow \mathbf{R}$ a random variable and $p : \Omega \rightarrow \mathbf{R}$ a distribution. The *mean square* of x is $\mathbf{E}(x^2)$. Define $\mu = \mathbf{E}(x)$. We can express $\mathbf{E}(x^2) = \mathbf{E}(x)^2 + \text{cov}(x)$ since

$$\begin{aligned} \text{cov}(x) &= \mathbf{E}((x - \mu)^2) = \mathbf{E}(x^2 - 2\mu x + \mu^2) \\ &= \mathbf{E}(x^2) - 2\mu\mathbf{E}(x) + \mu^2 \\ &= \mathbf{E}(x^2) - \mu^2. \end{aligned}$$

We refer to this relation as the *mean-variance decomposition* of x .¹

The *n'th moment* of x is $\mathbf{E}(x^n)$. The mean is the first moment. The covariance is the second moment minus the square of the first moment.

¹Future editions will modify this sheet, and likely motivate this decomposition in terms of minimum square error estimation.

