



## Real Functions

### 1 Why

We name functions whose domain is the real numbers.

### 2 Definition

A *real function* is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

#### 2.1 Notation

$f : \mathbf{R} \rightarrow \mathbf{R}$ .  $f$  is a real function. To speak of functions defined on intervals, let  $a, b \in \mathbf{R}$ .  $g : [a, b] \rightarrow \mathbf{R}$ . is a real function defined on a closed interval.  $h : (a, b) \rightarrow \mathbf{R}$  is a real function defined on an open interval.

We regularly declare the interval and the function in one pass: Let  $f : [a, b] \rightarrow \mathbf{R}$ , read aloud as “ $f$  from closed  $a$   $b$  to  $\mathbf{R}$ .” Or, let  $f : (a, b) \rightarrow \mathbf{R}$  read aloud as “ $f$  from open  $a$   $b$  to  $\mathbf{R}$ ”.

### 3 Examples

**Example 1.** Let  $c \in \mathbf{R}$ . Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be such that  $f(x) = c$  for every  $x \in \mathbf{R}$ .  $f$  is a real function.

**Example 2.**

Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  with  $f(x) = 2x^2 + 1$  for all  $x \in \mathbf{R}$ .  $f$  is a real function.

**Example 3.** Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  with

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{otherwise.} \end{cases}$$

$f$  is a real function.