



**Why**

What is the generalisation of smooth functions between Euclidean spaces.

**Definition**

Let  $U \subset \mathbf{R}^n$  be open. A function  $f : U \rightarrow \mathbf{R}^m$  is *smooth* if each of its components is smooth.

More generally, let  $X \subset \mathbf{R}^n$  (not necessarily open). We call  $g : X \rightarrow \mathbf{R}^m$  smooth if for each  $x \in X$  there exists an open set  $V \subset \mathbf{R}^n$  with  $x \in V$  and smooth function  $G : U \rightarrow \mathbf{R}^m$  so that  $G(y) = g(y)$  for all  $y \in U \cap X$ . In this case we say that  $g$  can be *locally extended* to a smooth map on open sets.



