



## NATURAL EXPONENTS

### Why

We want to repeatedly multiply.

### Defining Result

**Proposition 1.** *For each natural number  $m$ , there exists a function  $e_m : \omega \rightarrow \omega$  which satisfies*

$$e_m(0) = 1 \quad \text{and} \quad e_m(n^+) = (e_m(n))^+ \cdot m$$

*for every natural number  $n$ .*

*Proof.* The proof uses the recursion theorem (see *Recursion Theorem*).<sup>1</sup> □

Let  $m$  and  $n$  be natural numbers. The value  $p_m(n)$  is the power of  $m$  with  $n$ . Or the  $n$ th power of  $m$

### Notation

We denote the  $n$ th power of  $m$  by  $m^n$ .

### Properties

Here are some basic properties of powers.

**Proposition 2.** *Let  $k$ ,  $m$ , and  $n$  be natural numbers. Then*

$$m^n m^k = m^{n+k}.$$

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<sup>1</sup>Future editions will give the entire account.

**Proposition 3.** *Let  $k$ ,  $m$ , and  $n$  be natural numbers. Then*

$$(m^n)^k = m^{nk}.$$

