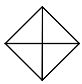


Why

If a directed graph has no cycles, then it has a nice property.[1]

Definition

Directed and acyclic graphs (or *directed acyclic graphs*, *DAGs*) have partial ordering property on vertices. We call a vertex s an *ancestor* of a vertex u if there is a directed path from s to u.

Partial Order

We call a vertex s and *ancestor* of a vertex t if there is a directed path from s to t. The relation R defined by $(s,t) \in R$ if s is an ancestor of t is a partial order.

Clearly, every subgraph induced on a directed acyclic graph is a directed acyclic graph.

Proposition 1. *Let (V, E) be a directed acyclic graph. Then there exists a vertex $v \in V$ which is a source and a vertex $w \in V$ which is a sink.*

Proof. There exists a directed path of maximum length. It must start at a source and end at a sink.[2] □

Topological numbering

We can choose a total ordering that is consistent with the partial order of ancestry.

A *topological numbering* (or *topological sort*, *topological ordering*) of a directed graph (V, E) is a numbering $\sigma : \{1, \dots, |V|\} \to V$ satisfying

$$(v, w) \in E \longrightarrow \sigma^{-1}(v) < \sigma^{-1}(w).$$

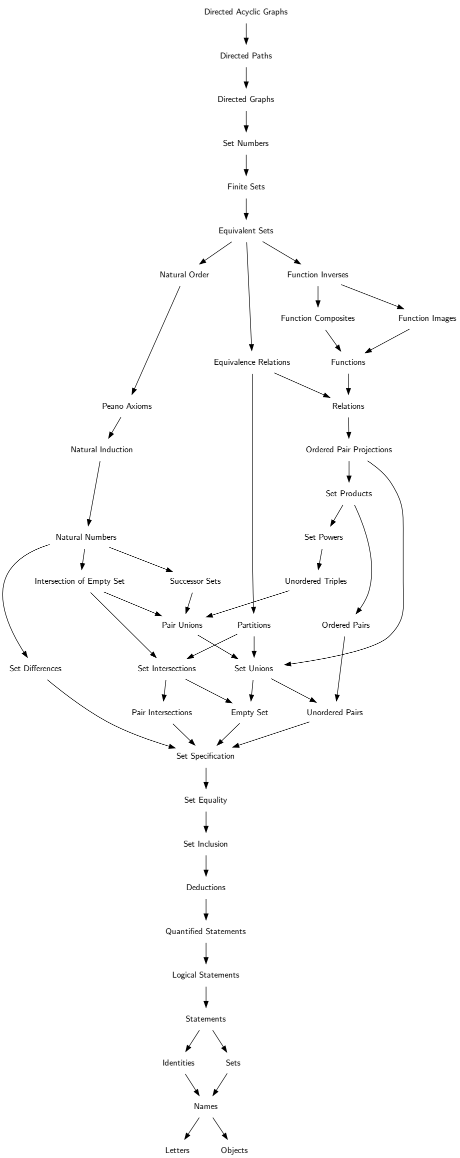
[3]

[1]Future editions will expand this vague introduction.
[2]Future editions will expand.
[3]Future editions will further explain this concept.

Proposition 2. *There exists a topological sort for every acyclic graph.*

Proof. Let (V, F) be a directed acyclic graph. There exists a source vertex, v_1. Set $\sigma(1) = v_1$. Take the subgraph induced by $V - \{v_1\}$. It is directed acyclic, and so has a source vertex, v_2. Set $\sigma(2) = v_2$. Continue in this way.[4] $\square$

[4] Future editions will clarify and expand.


Directed Acyclic Graphs
Directed Paths
Directed Graphs
Set Numbers
Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Function Composites
Function Images
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Pair Unions
Partitions
Ordered Pairs
Set Differences
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects