Cumulative Distribution Function From Density

1 Why

The density of a random variable determines the cumulative distribution function.

2 Result

Proposition 1. *A probability density function of a random variable, if it exists, characterizes the cumulative distribution function.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a probability space. Let f be a real-valued random variable on X. Let λ denote the cover length. Let g be a probability density of f: Then:

$$\begin{aligned} F_f(t) &= \mu(\{x \in X \mid f(x) \in (-\infty, t])\}) \\ &= \int_{(-\infty,t]} g d\lambda. \end{aligned}$$

□