

## Why

We want to combine two groups.<sup>1</sup>

## Defining result

**Proposition 1.** For each natural number m, there exists a function  $s_m$ :  $\omega \to \omega$  which satisfies

$$s_m(0) = m$$
 and  $s_m(n^+) = (s_m(n))^+$ 

for every natural number n.

 ${\it Proof.}$  The proof uses the recursion theorem (see Recursion Theorem).

Let m and n be natural numbers. The value  $s_m(n)$  is the sum of m with n.

#### Notation

We denote the sum  $s_m(n)$  by m+n.

## **Properties**

The properties of sums are direct applications of the principle of mathematical induction (see Natural Induction). $^3$ 

**Proposition 2** (Associative). Let k, m, and n be natural numbers. Then

$$(k+m) + n = k + (m+n).$$

**Proposition 3** (Commutative). Let m and n be natural numbers. Then

$$m+n=n+m$$
.

<sup>&</sup>lt;sup>1</sup>Future editions will change this section.

<sup>&</sup>lt;sup>2</sup>Future editions will give the entire account.

<sup>&</sup>lt;sup>3</sup>Future editions will include the accounts.

# Relation to addition

 $\textbf{Proposition 4} \ ( \textbf{Distributive}). \ \textit{Let} \ \textit{k, m, and n be natural numbers. Then}$ 

$$k \cdot (m+n) = (k \cdot m) + (k \cdot n).$$

