Why

It is common to consider random functions whose domain is time, space, or n-dimensional space.

Definition

Let (X, d) be a metric space. A *distance covariance function* $k : X \times X \to \mathbf{R}$ is a covariance function satisfying

$$k(x, y) > k(x, y) \longleftrightarrow d(x, y) < d(x, y).$$

In other words, the covariance decreases as the distance between the arguments decreases.

Example: Squared Exponential

Let $k : X \times X \to \mathbf{R}$ be defined by

$$k(x, y) = \exp(-d(x, y)).$$

Then k is a distance covariance function. It is often called the *squared exponential covariance function.*

Let $\alpha, \sigma \in \mathbf{R}$. Define $k' : X \times X \to \mathbf{R}$ by

$$k'(x, y) = \alpha \exp(-d(x, y)/\sigma^2)$$

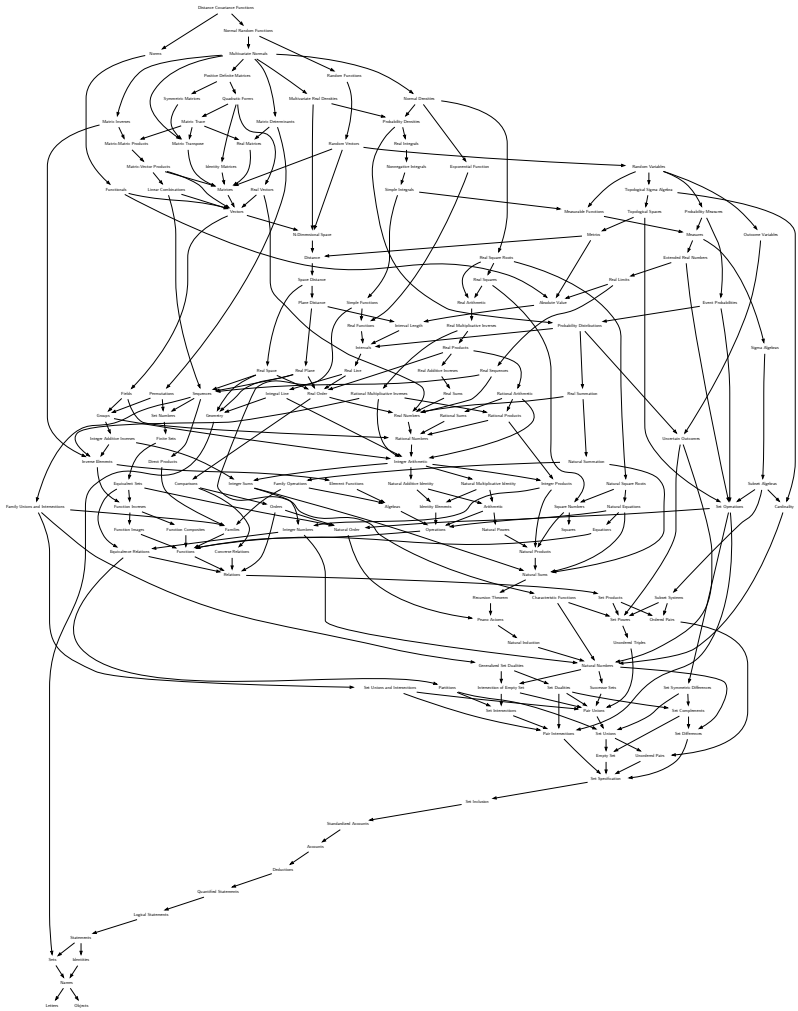
then k' is still a covariance function. In this context σ is often referred to as the *characteristic length-scale* of the process. The scalar α is sometimes called a "prefactor" that "controls" the "overall variance."

Suppose $(X, d) = (\mathbf{R}^n, \|\cdot\|)$. Then the squared exponential covariance function

$$\alpha \exp(-\|x - y\|/(2\sigma^2))$$

is sometimes called the *radial basis function* or *gaussian covariance function.*[1] Also called an *exponentiated quadratic kernel.*

[1]For reasons that will be included in future editions.


Distance Covariance Functions
Normal Random Functions
Norms
Multivariate Normals
Positive Definite Matrices
Random Functions
Symmetric Matrices
Quadratic Forms
Multivariate Real Densities
Normal Densities
Matrix Inverses
Matrix Trace
Matrix Determinants
Probability Densities
Matrix-Matrix Products
Matrix Transpose
Real Matrices
Random Vectors
Real Integrals
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Identity Matrices
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Exponential Functions
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Simple Integrals
Topological Spaces Algebra
Measurable Functions
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Metric Spaces
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Distance
Real Square Roots
Real Limits
Norm Distance
Real Inequlities
Absolute Value
Plane Distance
Simple Functions
Real Additions
Event Probabilities
Real Functions
Interval Length
Real Multiplicative Inverses
Probability Distributions
Intervals
Real Products
Signa Algebras
Real Spaces
Real Plane
Real Lies
Real Additions Inverses
Real Sequences
Finite
Permutations
Sequences
Integral Line
Real Order
Rational Multiplicative Inverses
Real Sums
Rational Inverses
Real Summation
Groups
Set Measures
Intervals
Real Numbers
Rational Sums
Rational Products
Uncertain Outcomes
Integer Addition Inverses
Finite Sets
Rational Numbers
Integer Subtractions
Natural Summation
Integer Divisors
Sum Products
Equivalent Sets
Comparisons
Integer Sums
Family Operations
Binomial Functions
Natural Addition Identity
Natural Multiplication Identity
Integer Products
Natural Square Roots
Subset Algebras
Family Unions and Intersections
Function Inverses
Algebras
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Function Compositions
Functions
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Natural Order
Integers
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Squares
Equations
Natural Products
Equivalence Relations
Inverses
Concrete Relations
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Natural Sums
Recursion Theorem
Characteristic Functions
Set Products
Subset Systems
Peano Axioms
Set Powers
Ordered Pairs
Natural Induction
Unordered Triples
Generalized Set Duality
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Duality
Symmetric Sets
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specifications
Set Inclusion
Standardized Axioms
Axioms
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Quantified Statements
Logical Statements
Statements
Sets
Identities
Theorems
Letters
Objects