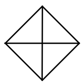


Why

We are frequently interested in finding minimizers of real functions.[1]

Definition

An *optimization problem* (or *extremum problem*) is a pair $(\mathcal{X}, f)$ in which $\mathcal{X}$ is a nonempty set called the *constraint set* and $f : \mathcal{X} \to \mathbf{R}$ is called the *objective* (or *cost function*).

If $\mathcal{X}$ is finite we call the optimization problem *discrete*. If $\mathcal{X} \subset \mathbf{R}^d$ we call the optimization problem *continuous*.

We refer to all elements of the constraint set as *feasible*. We refer to an element $x \in \mathcal{X}$ of the constraint set as *optimal* if $f(x) = \inf_{z \in \mathcal{X}} f(z)$. We also refer to optimal elements as *solutions* of the optimization problem.

It is common for f and $\mathcal{X}$ to depend on some other, known, given objects. In this case, these objects are often called *parameters* or *problem data*.

Notation

We often write optimization problems as

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathcal{X}. \end{array}$$

In this case we call x the *decision variable*.

Extended reals

It is common to let $f : \mathcal{X} \to \bar{\mathbf{R}}$, and allow there to exist $x \in \mathcal{X}$ for which $f(x) = \infty$. This technique can be used to embed further constraints in the objective. For example, we interpret $f(x) = +\infty$ to mean x is *infeasible*.

[1] Future editions will modify and expand.

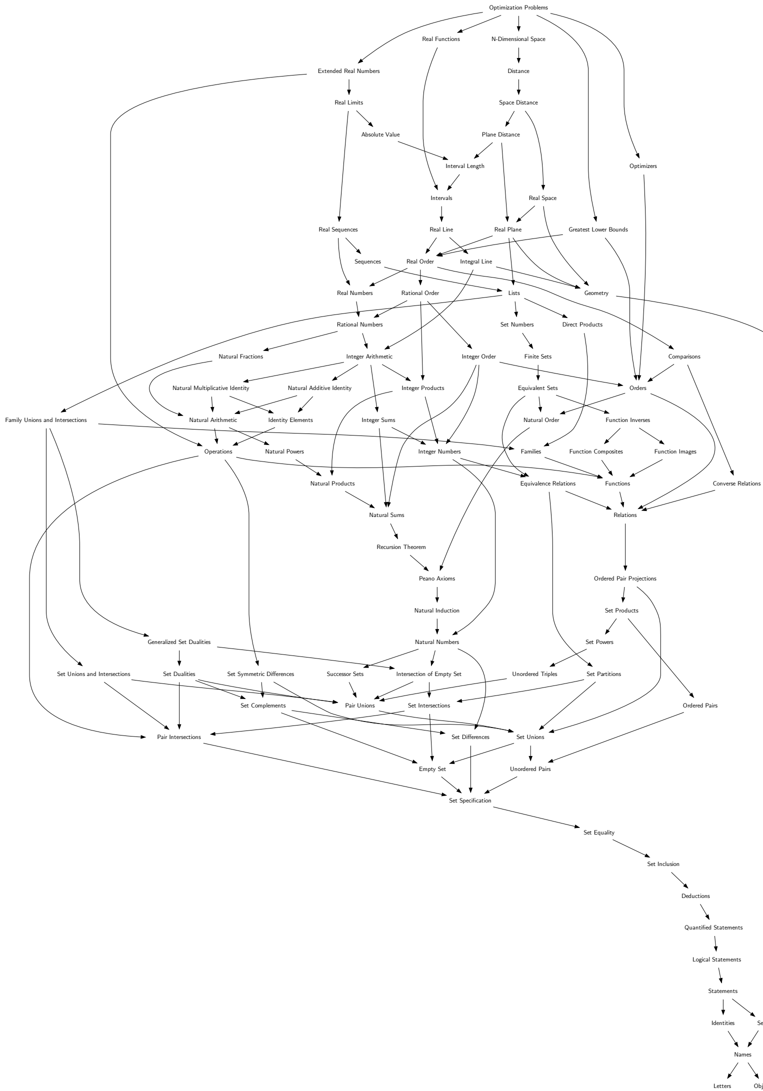
Maximization

If we have some function $g : \mathcal{X} \to \bar{\mathbf{R}}$ that we wish to maximize, we can always convert it to an optimization problem by defining $f : \mathcal{X} \to \bar{\mathbf{R}}$ by $f(x) = -g(x)$. In this case g is often called a *reward* (or *utility*, *profit*).

Solvers

A *solver* (or *solution method*, *solution algorithm*) for a family of optimization problems is a function S mapping optimization problems to solutions.

Loosely speaking, the difficulty of "solving" the optimization problem $(\mathcal{X}, f)$ depends on the properties of $\mathcal{X}$ and f and the problem "size". For example, when $\mathcal{X} \subset \mathbf{R}^d$ the difficulty is related to the "dimension" d of $x \in \mathcal{X}$.


Optimization Problems
Real Functions
N-Dimensional Space
Extended Real Numbers
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Interval Length
Optimizers
Intervals
Real Space
Real Sequences
Real Line
Real Plane
Greatest Lower Bounds
Sequences
Real Order
Integral Line
Real Numbers
Rational Order
Lists
Geometry
Rational Numbers
Set Numbers
Direct Products
Natural Fractions
Integer Arithmetic
Integer Order
Finite Sets
Comparisons
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Equivalent Sets
Orders
Family Unions and Intersections
Natural Arithmetic
Identity Elements
Integer Sums
Natural Order
Function Inverses
Operations
Natural Powers
Integer Numbers
Families
Function Composites
Function Images
Natural Products
Equivalence Relations
Functions
Converse Relations
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Set Symmetric Differences
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects