



# Measure Space

## 1 Why

We want to generalize the notions of length, area, and volume.

## 2 Definition

A *measurable space* is a sigma algebra. We call the distinguished subsets the *measurable sets*.

A *measure* on a measurable space is a function from the sigma algebra to the positive extended reals. A *measure space* is a measurable space and a measure.

### 2.1 Notation

### 2.2 Properties

**Proposition 1.** Let  $(A, \mathcal{A})$  be a measurable space and  $m : \mathcal{A} \rightarrow [0, \infty]$  be a measure.

If  $B \subset C \subset A$ , then  $m(B) \leq m(C)$ . We call this property the *monotonicity of measure*.

**Proposition 2.** *For a measure space  $(A, \mathcal{A}, m)$ .*

*If  $B \subset C \subset A$ , then  $m(B) \leq m(C)$ .*

*We call this property the monotonicity of measure.*

**Proposition 3.** *For a measure space  $(A, \mathcal{A}, m)$ .*

*If  $\{A_n\} \subset \mathcal{A}$  a countable family, then  $m(\cup A_n) \leq \sum_i m(A_i)$ .*

*We call this property the sub-additivity of measure.*

**Proposition 4.** *For a measure space  $(A, \mathcal{A}, m)$ .*

*If  $\{A_n\} \subset \mathcal{A}$  a countable family, then  $m(\cup A_n) \leq \sum_i m(A_i)$ .*

*We call this property the sub-additivity of measure.*

**Proposition 5.** *For a measure space  $(A, \mathcal{A}, m)$ .*

$$m(\cup_{n=1}^{\infty} A_n) = \lim_{n \rightarrow \infty} m(A_n)$$

**Proposition 6.** *For a measure space  $(A, \mathcal{A}, m)$ .*

$$m(\cap_{n=1}^{\infty} A_n) = \lim_{n \rightarrow \infty} m(A_n)$$

## 2.3 Examples

**Example 7.** *counting measure*