



## NATURAL INDUCTION

### Why

We want to show something holds for every natural number.<sup>1</sup>

### Definition

The most important property of the set of natural numbers is that it is the unique smallest successor set. In other words, if  $S$  is a successor set contained in  $\omega$  (see *Natural Numbers*), then  $S = \omega$ . This is useful for proving that a particular property holds for the set of natural numbers.

To do so we follow standard routine. First, we define the set  $S$  to be the set of natural numbers for which the property holds. This step uses the principle of selection (see *Set Selection*) and ensures that  $S \subset \omega$ . Next we show that this set  $S$  is indeed a successor set. The first part of this step is to show that  $0 \in S$ . The second part is to show that  $n \in S \longrightarrow n^+ \in S$ . These two together mean that  $S$  is a successor set, and since  $S \subset \omega$  by definition, that  $S = \omega$ . In other words, the set of natural numbers for which the property holds is the entire set of natural numbers. We call this the *principle of mathematical induction*.

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<sup>1</sup>Future editions will modify this superficial why.



