



## Why

We discuss learning (or inferring) relations from examples.

## Definition

Let  $X$  and  $Y$  be sets. A *relation inductor* (for a dataset of size  $n$  in  $X \times Y$ ) is a function mapping a dataset in  $(X \times Y)^n$  to a relation between  $X$  and  $Y$ . We frequently use the term inductor to refer to a family of inductors, indexed by  $n \in \mathbf{N}$ .

An inductor is *functional* if it produces functions. In this case, we call the elements of  $X$  the *inputs* and the elements of  $Y$  the *outputs*. We call a function from inputs to outputs a *predictor* and call the result of an input under a predictor a *prediction*. Using this language, a functional inductor maps datasets to predictors. A predictor maps inputs to outputs.

To every relation between  $X$  and  $Y$  corresponds a characteristic function on  $X \times Y$  and vice versa. For this reason, henceforth by *inductor* we mean a functional inductor. A relational inductor on a dataset in  $(X \times Y)^n$  can be modeled by a functional inductor on a dataset in  $((X \times Y) \times \{0, 1\})^n$ .

## Notation

Let  $D$  be a dataset of size  $n$  in  $X \times Y$ . Let  $g : X \rightarrow Y$ , a predictor, which makes prediction  $g(x)$  on input  $x \in X$ . Let  $G_n : (X \times Y)^n \rightarrow (X \times Y)$  be an inductor. Then  $G_n(D)$  is the predictor which the inductor associates with dataset  $D$ . And

$\{G_n : (X \times Y)^n \rightarrow \mathcal{P}((X \times Y))\}_{n \in \mathbf{N}}$  is a family of inductors.

### Consistent and complete datasets

Let  $D = ((x_i, y_i))_{i=1}^n$  be a dataset and  $R \subset X \times Y$  a relation.  $D$  is *consistent with*  $R$  if each  $(x_i, y_i) \in R$ .  $D$  is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take  $R = X \times Y$ ).  $D$  is *functionally consistent* if it is consistent with a function; in this case,  $x_i = x_j \longrightarrow y_i = y_j$ .  $D$  is *functionally complete* if  $\cup_i \{x_i\} = X$ . In this case, the dataset includes every element of the relation.

### Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent variables*, *explained variables*, *postcepts*, *targets*, *outcomes*, *labels* or *observational outcomes*.

Other terms for a predictor include *input-output mapping*, *prediction rule*, *hypothesis*, *concept*, or *classifier*. Since a predictor can be used to *guess* the output of an input, some authors call an inductor (or family of inductors) a *learner* or *learning algorithm* or *supervised learning algorithm* and refer to the argument as the *training dataset*. Often the word “supervised” is included, as in *supervised learning*. The language intends to indicate that inputs are given along with outputs, and these outputs “provide supervision to the algorithm.”

