Definition

Let $f : D \to R$ be a multivariate real-valued function where $D \subset \mathbf{R}^d$. The *graph* of f is the set in $\mathbf{R}^{d+1}$ defined by
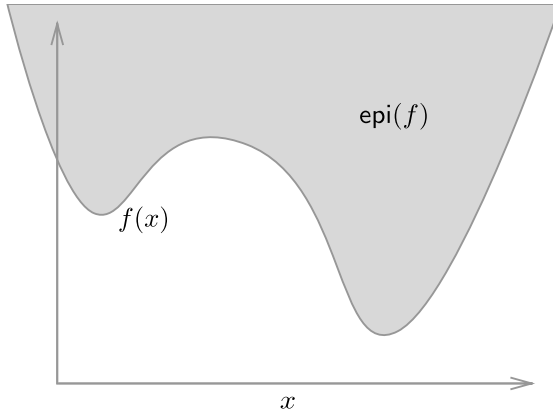
$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}$$

Of course, in these sheets, the graph is the *same object* as the function f (see discussion in Functions). The *epigraph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, \alpha) \in D \times \mathbf{R} \mid f(x) \leq \alpha\}.$$

The prefix "epi" is Greek, meaning "upon" or "above". It is merited (see the visualization below) by the fact that $f \neq \mathrm{epi}\, f$.

Visualization of epigraph

Notation

We denote the epigraph of a function f by $\mathrm{epi}\, f$.

Extension to extended real numbers

We can extend this concept in the natural way to extended real value function $f : D \to \bar{\mathbf{R}}$.

$$\text{epi } f \{(x, \alpha) \in D \times \mathbf{R} \mid f(x) \leq \alpha\}$$

Caution, in this case $f \not\subset \text{epi } f$.

Real Function Epigraph

Real Function Graphs

Real Convex Sets

Topological Interiors

Real Halfspaces

Real Hyperplanes

Real Functions

Subspace Orthogonal Complements

Multivariate Functions

Subspace Dimensions

Vector Space Dimensions

Affine Set Dimensions

Vector Space Bases

Affinely Independent Vectors

Linearly Dependent Vectors

Space

Orthogonal Complements

Affine Set Bases

Vector Subspaces

Real Subspace Dimensions

Real Affine Hulls

Real Affine Sets and Subspaces

Real Matrices

Real Subspaces

Real Vector Bases

Real Inner Products

Vectors as Matrices

Real Affine Sets

Real Affine Combinations

Real Affine Hulls

Real Transforms

Linear Combinations

Real Isometries

Independent Set of Real Vectors

N-Dimensional Line Segments

Real Inner Product

Real Inner Product

Topological Closures

Vectors

Linear Combination

Real Linear Combinations

Real Vectors

Real Norm

Space Inner Product

N-Dimensional Lines

Real Linear Equations

Topological Neighborhoods

Topology

Space Norm

Real Line Closures

Linear Function

Motion

Planar Inner Product

Planar Norm

Space Vectors

Planar Vector Sums and Differences

N-Dimensional Space

Extended Real Numbers

Arrays

Absolute Value

Right Triangle Sides Relation

Planar Union

Distance

Real Lines

Planar Distance

Space Distance

Real Square

Real Arithmetic

Real Multiplication Inverses

Interval Length

Intervals

Square Numbers

Real Line

Real Plane

Real Products

Real Addition Inverses

Real Sequences

Real Summation

Real Series

Real Numbers

Natural Arithmetic

Rational Multiplication Inverses

Real Order

Square

Integral Line

Geometry

Sequences

Natural Summation

Fields

Rational Sums

Rational Products

Rational Numbers

Natural Fractions

Set Sums

Rings

Rational Series

Integer Arithmetic

Integer Products

Direct Products

Natural Addition Identity

Natural Multiplicative Identity

Finite Sets

Inverse Elements

Integer Sums

Integer Order

Comparisons

Natural Arithmetic

Identity Elements

Element Function

Family Operations

Equivalent Sets

Natural Products

Integer Numbers

Family Unions and Intersections

Natural Powers

Natural Successors

Equivalence Relations

Equipollent Relations

Families

Function Composition

Function Images

Orders

Converse Relations

Natural Order

Functions

Relations

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Natural Induction

Set Products

Generalized Set Dualities

Natural Numbers

Set Powers

Set Unions and Intersections

Set Dualities

Set Symmetric Difference

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Ordered Pairs

Set Complements

Set Intersection

Pair Unions

Set Difference

Set Union

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Sentences

Identities

Sets

Spaces

Letters

Objects