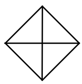


Why

Linear equations are ubiquitous.

Definition

Given $a \in \mathsf{R}^n$ and $y \in \mathsf{R}$, suppose we want to find $x \in \mathsf{R}^n$ satisfying

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = y.$$

We refer to this expression as a *real linear equation* or *linear equation*. We treat each component $x_i \in \mathsf{R}$ as a variable and we treat each component $a_i \in \mathsf{R}$ and $y \in \mathsf{R}$ as constants. We call the pair (a, y) the *real linear equation constants.*[1]

The source of the terminology "linear" is by viewing the left hand side as a function. Define $f : \mathsf{R}^n \to \mathsf{R}$ by $f(x) = \sum_i a_i x_i$. We want to find $x \in \mathsf{R}^n$ to satisfy $f(x) = b$. Notice that f is a *linear* real function.[2]

Moreover, to each linear function $f : \mathsf{R}^d \to \mathsf{R}$ there exists a vector $a \in \mathsf{R}^d$ so that $f(x) = \sum_i a_i x_i$. For this reason, if we are given several linear function $f_1, \dots, f_m$, then we can think of these as several vectors $a^1, \dots, a^n$. If we are also given $b_i \in \mathsf{R}$ for each $i = 1, \dots, m$, then we have the vector $b \in \mathsf{R}^m$

We can define the two-dimensional array $A \in \mathsf{R}^{m \times n}$ by $A_{ij} = a^i_j$. For this reason, a *linear system of equations* is a pair (A, b). A solution of a linear system of equations is a vector $x \in \mathsf{R}^n$ satisfying

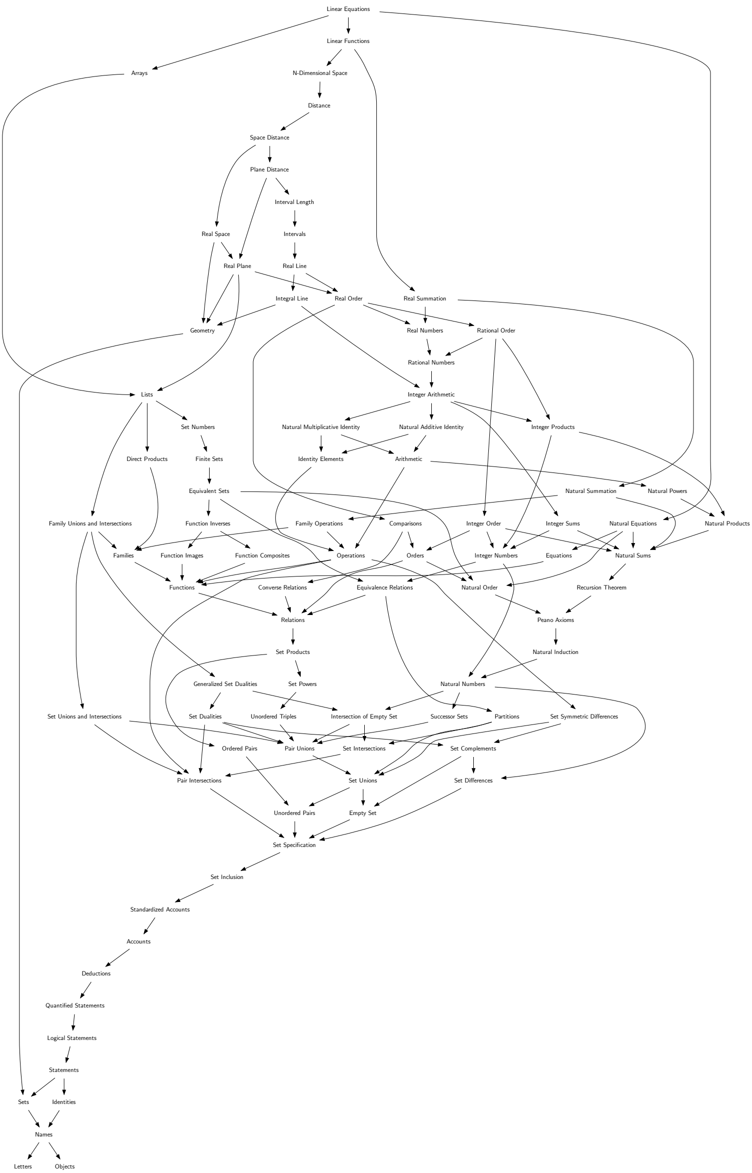
[1] Future editions will clarify.

[2] Future editions may require a sheet here.

the equations

$$\begin{array}{rrrrr}
A_{11}x_1+ & A_{12}x_2+ & \cdots+ & A_{1n}x_n = & b_1 \\
A_{21}x_1+ & A_{22}x_2+ & \cdots+ & A_{2n}x_n = & b_2 \\
\vdots & \vdots & & \vdots & \vdots \\
A_{m1}x_1+ & A_{m2}x_2+ & \cdots+ & A_{mn}x_n = & b_n
\end{array}$$

Other terminology includes a *system of linear equations* or *linear system* or *simultaneous linear equations*


Linear Equations
Linear Functions
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Real Summation
Geometry
Real Numbers
Rational Order
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Natural Summation
Natural Powers
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Equations
Natural Products
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Equations
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects