Why

What is the projection of a vector onto an affine set?

Result

Proposition 1. *Suppose $a \in \mathbf{R}^n$ and $U \in \mathbf{R}^{n\times k}$ with $U^\top U = I$. Define the affine set $W(a,U) = a + \mathrm{range}(U)$. Then*

$$\mathrm{proj}_{W(a,U)}(x) = UU^\top x + (I - UU^\top)a.$$

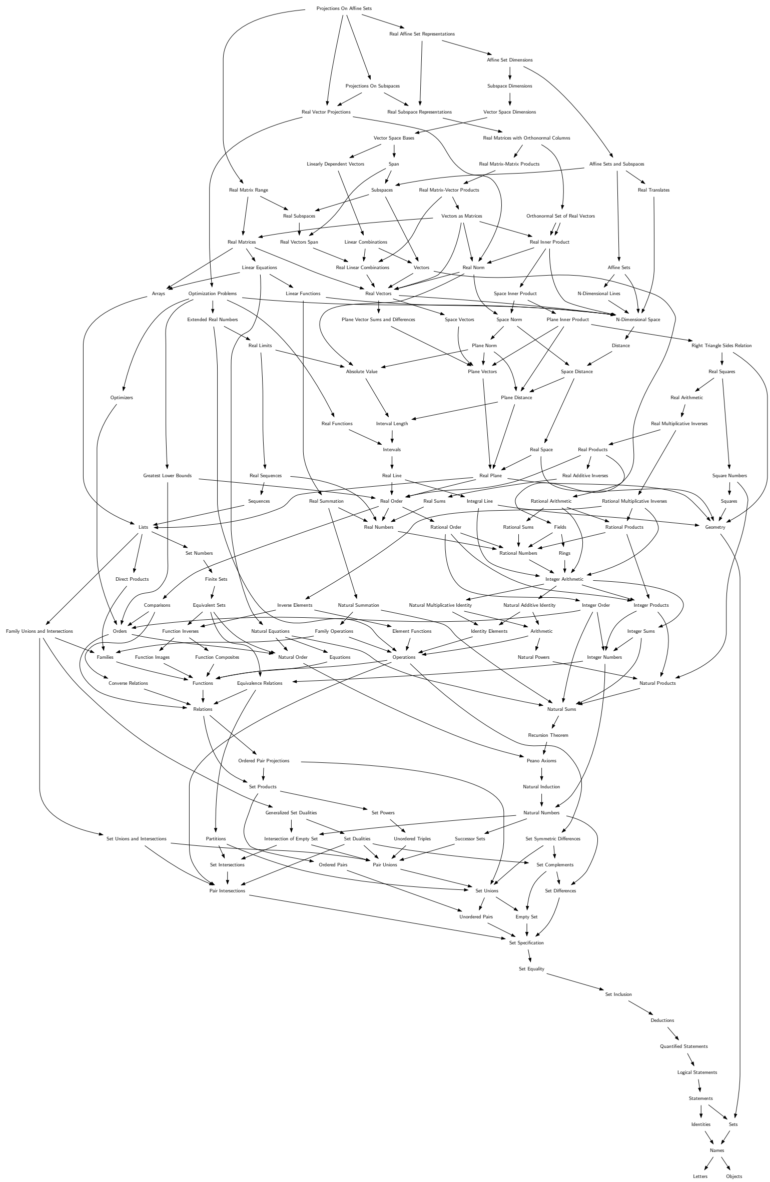
Proof. The minimizer of $J : \mathbf{R}^n \to \mathbf{R}$ defined by

$$J(z) = \|a + Uz - x\| = \|Uz - (x - a)\|,$$

is $z^\star = U^\top(x - a)$. So the projection of x onto $W(a,U)$ is

$$a + Uz^\star = a + UU^\top(x - a) = UU^\top x + (I - UU^\top)a.$$

□



Projections On Affine Sets
Real Affine Set Representations
Affine Set Dimensions
Projections On Subspaces
Subspace Dimensions
Real Vector Projections
Real Subspace Representations
Vector Space Dimensions
Vector Space Bases
Real Matrices with Orthonormal Columns
Linearly Dependent Vectors
Span
Real Matrix-Matrix Products
Affine Sets and Subspaces
Real Matrix Range
Subspaces
Real Matrix-Vector Products
Real Translates
Real Subspaces
Vectors as Matrices
Orthonormal Set of Real Vectors
Real Matrices
Real Vectors Span
Linear Combinations
Real Inner Product
Linear Equations
Real Linear Combinations
Vectors
Real Norm
Affine Sets
Arrays
Optimization Problems
Linear Functions
Real Vectors
Space Inner Product
N-Dimensional Lines
Extended Real Numbers
Plane Vector Sums and Differences
Space Vectors
Space Norm
Plane Inner Product
N-Dimensional Space
Real Limits
Plane Norm
Distance
Right Triangle Sides Relation
Absolute Value
Plane Vectors
Space Distance
Real Squares
Optimizers
Plane Distance
Real Arithmetic
Real Functions
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Greatest Lower Bounds
Real Sequences
Real Line
Real Plane
Real Additive Inverses
Square Numbers
Sequences
Real Summation
Real Order
Real Sums
Integral Line
Rational Arithmetic
Rational Multiplicative Inverses
Squares
Lists
Real Numbers
Rational Order
Rational Sums
Fields
Rational Products
Geometry
Set Numbers
Rational Numbers
Rings
Direct Products
Finite Sets
Integer Arithmetic
Comparisons
Equivalent Sets
Inverse Elements
Natural Summation
Natural Multiplicative Identity
Natural Additive Identity
Integer Order
Integer Products
Family Unions and Intersections
Orders
Function Inverses
Natural Equations
Family Operations
Element Functions
Identity Elements
Arithmetic
Integer Sums
Families
Function Images
Function Composites
Natural Order
Equations
Operations
Natural Powers
Integer Numbers
Converse Relations
Functions
Equivalence Relations
Natural Products
Relations
Natural Sums
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects