





Figure 1: A postordering of a rooted tree.  
ROOTED TREE ORDERINGS

## Why

We want to simplify common questions using orderings of a rooted tree.<sup>1</sup>

## Definition

A *topological ordering* of a rooted tree is an ordering  $\sigma$  for which  $v \prec_{\sigma} \text{pa}(v)$ . If there are  $n$  vertices, the root has index  $n$  and every other vertex has an index less than its parent.

A *postordering* is a topological ordering in which descendants of a vertex are given consecutive numbers. For the postordering  $\sigma$ , if  $\sigma^{-1}(v) = j$  and  $v$  has  $k$  proper descendants then the proper descendants of  $v$  have are numbered consecutively from  $j - k$  through  $j - 1$ . Figure 1 shows an example.

One can generate a postordering by numbering vertices in decreasing sequence (starting at  $n$ ) in the order they are visited.<sup>2</sup>

Given an ordering, the *first descendent* of  $v$  (which we denote  $\text{fdesc}(v)$ ) is the descendent with the lowest index. Given a postordering  $\sigma$ , one can check whether a vertex  $w$  is a proper descendent of  $v$ , by  $\text{fdesc}(v) \preceq_{\sigma} w \prec_{\sigma} v$ .

<sup>1</sup>This sheet is incomplete and will be updated in future editions.

<sup>2</sup>Future editions will clarify.



