



COMPLEX NUMBERS

Why

We want to find the roots of negative numbers.¹

Definition

A *complex number* is an ordered pair of real numbers. The *real part* of a complex number is its first coordinate. The *imaginary part* of a complex number is its second coordinate.

We can identify the imaginary numbers with no complex part (i.e., $\{(a, b) \in \mathbf{R}^2 \mid b = 0\}$ with \mathbf{R} in the obvious way. For this reason, such a complex number is sometimes referred to as a *purely real number*. Conversely, a complex number with zero imaginary part (i.e., an element of $\{(a, b) \in \mathbf{R}^2 \mid a = 0\}$) is said to be a *purely imaginary number*.

The *complex conjugate* (or *conjugate*) of a complex number z is the complex number whose real part matches z and whose imaginary part is the additive inverse of z . The complex conjugate of a purely real number is the same purely real number. In other words, the complex conjugate of a complex number with no imaginary part is the same complex number.

Notation

When treating \mathbf{R}^2 as the set of complex numbers, we denote it by \mathbf{C} . Let $z \in \mathbf{C}$ with $z = (a, b)$. The real part of z is a and

¹Future editions will modify this, and will discuss the existence of solutions of algebraic equations.

its imaginary part is b . It is universal to denote z by $a + ib$, and to call i an (or the) *imaginary number*. Some authors use j , it is a matter of notation.

We denote the real part of z by $\mathbf{Re}(z)$, read “real of z ,” and the imaginary part by $\mathbf{Im}(z)$, read “imaginary of z .” So, in particular, $\mathbf{Re}(z) = a$ and $\mathbf{Im}(z) = b$.

We denote the complex conjugate of the complex number $z \in \mathbf{C}$ by z^* . Other common notation includes \bar{z} , read “ z bar”. If there exists $a, b \in \mathbf{R}$ so that $z = (a, b)$, then $z^* = (a, -b)$.

