



LOSS FUNCTIONS

Why

We compare inductors by comparing the predictors they produce. We compare predictors by judging predictions.

Definition

A *loss function* is a nonnegative real-valued function on pairs which is zero only on repeated pairs. It need not be symmetric. The first argument of the loss function is the predicted postcept and the second is the recorded (observed, true, recorded) postcept.

The *loss of a predictor on a pair* is the result of the loss function on the pair. Similarly, the *loss of a predictor on a sequence* of pairs is the sum of the losses on the pairs. The *average loss of a predictor on a sequence* is the loss divided by the length of the sequence. The average loss is also known as the *empirical risk* of the predictor on the dataset.

Notation

Let $(a, b) \in A \times B$; $A, B \neq \emptyset$. For a lost function $\ell : B \times B \rightarrow \mathbf{R}$ and predictor $f : A \rightarrow B$, the loss of f on (a, b) is

$$\ell(f(a), b).$$

Let $s = ((a^1, b^1), \dots, (a^n, b^n))$ be a record sequence. The loss of f on s is

$$\sum_{k=1}^n \ell(f(a^k), b^k).$$

The average loss of f on s is

$$\frac{1}{n} \sum_{k=1}^n \ell(f(a^k), b^k).$$

