Why

A probability event function is a measure on the set of outcomes.

Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The word "space" is natural, since the notion of a measure generalized the notion of volume in real space (see Real Space and N-Dimensional Space). The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra. The measure in a probability space corresponds to the event probability function.

Notation

Let $(A, \mathcal{A})$ be a measurable space.[1] We denote the sigma-algebra by $\mathcal{A}$, as usual. We denote a probability measure by $\mathbf{P}$, a mnemonic for "probability," and intended to remind of the event probabilty function. Thus, we often say "Let $(A, \mathcal{A}, \mathbf{P})$

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[1] Often, other authors will denote the set of outcomes (here denoted by A) by Ω, a mnemonic for "outcomes."

be a probability space."

Many authors associate an event $A \in \mathcal{A}$ with a function $\pi : \mathcal{X} \to \{0, 1\}$ so that $A = \{x \in \mathcal{X} \mid \pi(x) = 1\}$. In this context, it is common to write $\mu[\pi(x)]$ for $\mu(A)$.

Properties

Probability Measures

Measure

Extended Real Numbers

Sigma Algebras

Real Limits

Event Probabilities

Absolute Value

Probability Distributions

Interval Length

Intervals

Real Line

Real Summation

Real Order

Real Sequences

Real Numbers

Integral Line

Rational Numbers

Integer Arithmetic

Sequences

Natural Additive Identity

Natural Multiplicative Identity

Set Numbers

Direct Products

Natural Summation

Arithmetic

Identity Elements

Finite Sets

Integer Products

Natural Powers

Equivalent Sets

Family Operations

Subset Algebras

Family Unions and Intersections

Integer Sums

Natural Products

Algebras

Families

Operations

Function Inverses

Set Operations

Function Images

Function Composites

Natural Sums

Integer Numbers

Recursion Theorem

Natural Order

Equivalence Relations

Functions

Peano Axioms

Cardinality

Relations

Natural Induction

Uncertain Outcomes

Cartesian Products

Subset Systems

Generalized Set Dualities

Natural Numbers

Set Powers

Ordered Pairs

Set Unions and Intersections

Set Dualities

Intersection of Empty Set

Successor Sets

Unordered Triples

Partitions

Set Symmetric Differences

Pair Unions

Set Complements

Set Intersections

Set Differences

Pair Intersections

Set Unions

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects