Why

We want to find a low-dimensional affine set into which we can project some high-dimensional data.

Problem

For $a \in \mathbf{R}^n$ and $U \in \mathbf{R}^{n \times k}$, the set $W(a, U) = \{a + Uz\} z \in \mathbf{R}^n$ is an affine set. Denote the projection of $x \in \mathbf{R}^n$ onto $W(a, U)$ by $\mathrm{proj}_{W(a,U)}(x)$.

Given $x^{(1)}, \dots, x^{(m)} \in \mathbf{R}^n$, and a dimension k, we want to choose a and U to minimize

$$\textstyle\sum_{i=1}^m \|x - \mathrm{proj}_{W(a,U)}(x)\|^2,$$

the sum of squares of the distance from x to its projection in $W(a, U)$.

Express $\mathrm{proj}_{W(a,U)}(x)$ as $UU^\top x + (I - UU^\top)a$ (see Projections on Affine Sets).