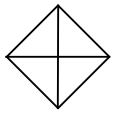

# Tree Densities

## 1 Why

We extend the definition of tree distributions to densities.

## 2 Definition

### 2.1 Rooted Definition

Let $f : \mathbf{R}^d \to \mathbf{R}$ be a density. *$f$ factors according to the rooted tree* on $\{1, \ldots, d\}$ rooted at a vertex $k$ if it can be written as a product of $f_k$ and the conditionals of $f_{i|j}$ for $i, j = 1, \ldots, d$ and $i \neq j$ and $i \neq k$ where $j$ is the parent of $i$ in the rooted tree.

### 2.2 Defining Result

**Proposition 1.** *If a density factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

### 2.3 Undirected Definition

A density $f$ *factors according to the tree* $T$ if it factors according to the $T$ rooted at any vertex.

# 3 Existence and Uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

## 3.1 Existence

A distribution $f$ need not factor according to a tree.

## 3.2 Uniqueness

A distribution $f$ may factor according to multiple trees.