Definition

Suppose $(A, \leq)$ is a partially ordered set. A *lower bound* for $B \subset A$ is an element $a \in A$ satisfying

$$a \leq b \quad \text{for all } b \in B$$

In words, a is a predecessor of every element of B. A set is *bounded from below* if it has a lower bound. A *greatest lower bound* for B is an element $c \in A$ so that c is a lower bound and $c < a$ for all other lower bounds a.

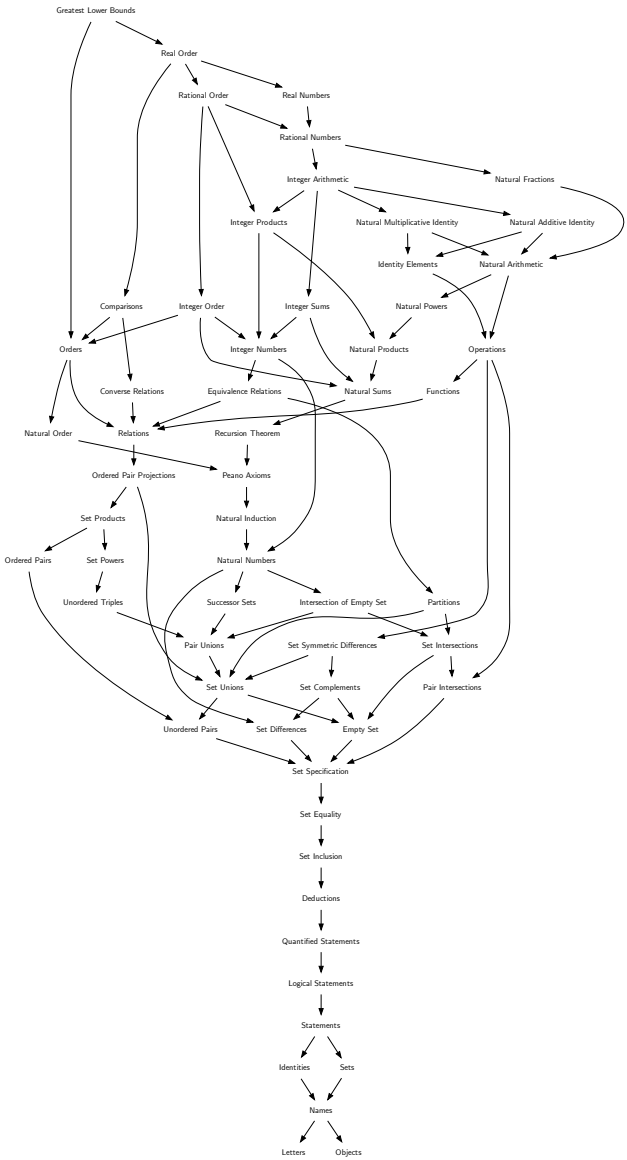
Proposition 1. *If there is a greatest lower bound it is unique.*[1]

We call the unique greatest lower bound of a set (if it exists) the *infimum*.

Notation

We denote the infimum of a set $B \subset A$ by $\inf A$.

[1] Proof in future editions.


Greatest Lower Bounds
Real Order
Rational Order
Real Numbers
Rational Numbers
Natural Fractions
Integer Arithmetic
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Identity Elements
Natural Arithmetic
Comparisons
Integer Order
Integer Sums
Natural Powers
Orders
Integer Numbers
Natural Products
Operations
Converse Relations
Equivalence Relations
Natural Sums
Functions
Natural Order
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects