



Why

We want to convert a subset of a vector space into a subspace.

Definition

The *span* of a subset of a vector space is the intersection of all subspaces which contain that subset. Since the intersection of a family of subspaces is a subspace, so too is the span. Indeed, the span of a subset is contained in every subspace containing the set.

A subset of a vector space *spans* a subspace if its span is that subspace. For example, the subspace may be a vector space, in which case the subset spans the entire space. In this case we say that the subset *spans the space*.

Notation

Let V be a vector space and let $x_1, \dots, x_n \in V$. We denote the span of these vectors by $\text{span}\{x_1, \dots, x_n\}$

Examples

The empty set is contained in every subspace, so the span of the empty set is the zero vector space.

