



## LEAST UPPER BOUNDS

### Why

#### Definition

Let  $A$  be a set and let  $\leq$  be an order<sup>1</sup> on  $A$ .

An *upper bound* for  $B \subset A$  is an element  $a \in A$  so that  $b \leq a$  for all  $b \in B$ . A set is *bounded from above* if it has a least upper bound. A *least upper bound* for  $B$  is an element  $c \in A$  so that  $c$  is an upper bound and  $c < a$  for all other upper bounds  $a$ .

**Proposition 1.** *If there is a least upper bound it is unique.*<sup>2</sup>

We call the unique least upper bound of a set (if it exists) the *supremum*.

#### Properties

We denote the supremum of a set  $B \subset A$  by  $\sup A$ .

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<sup>1</sup>To be defined in future editions, but understood in the usual way.  
See Natural Order or Integer Order or Rational Order etc.

<sup>2</sup>Proof in future editions.



