Direct Products

Why

We generalize the idea of a produce of two sets to a product of n sets.

Direct Products

The *direct product* of a natural family is the set of ordered sequences of elements from each set in the family. We call the elements of the direct product *n-tuples*. We call the ith element in an n-tuple the ith coordinate. This language is meant to follow that used in defining ordered pairs. Two coordinates in a sequence are *consecutive* if their natural difference is 1.

Notation

Let $A_1, \ldots, A_n$ be a natural family of sets. We denote its direct product by

$$\prod_{i=1}^{n} A_i.$$

We read this notation as "product over alpha in I of A sub-alpha." We denote an element of $\prod_{i=1}^{n} A_i$ by $(a_1, a_2, \ldots, a_n)$ with the understanding that $a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n$.

If $A_i = A$ for $i = 1, \ldots, n$, then we denote $\prod_{i=1}^{n} A_i$ by A^n.