



CARTESIAN PRODUCTS

Why

Does a set exist of all the ordered pairs of elements from an ordered pair of sets?

Definition

Let A and B denote sets. Ordered pairs are sets of singletons and pairs. So to construct the set of all ordered pairs taken from two sets, we want to specify the elements of a set which contains all singletons $\{a\}$ and pairs $\{a, b\}$ for $a \in A, b \in B$.

Notice that $a \in A$ and $b \in A$ mean $a, b \in (A \cup B)$. In other words, $\{a\} \subset A$ and $\{b\} \subset B$ and $\{a\}, \{b\} \subset (A \cup B)$. In particular, $\{a\} \in (A \cup B)^*$. Similarly, $\{a, b\} \in (A \cup B)^*$. And so $\{\{a\}, \{a, b\}\} \in (A \cup B)^{**}$.

We define the set of “all ordered pairs” from A and B by specifying the appropriate pairs of this set.¹

$$\{(a, b) \in (A \cup B)^{**} \mid a \in A \wedge b \in B\}$$

We name this set the *product* of the set denoted by A and the set denoted by B is the set of all ordered pairs. This set is also called the *cartesian product*.² If $A \neq B$, the ordering causes the product of A and B to differ from the product of B with A . If $A = B$, however, the symmetry holds.

¹The specific statement used here requires some translation. A discussion of this and the full statement will appear in a future edition.

²This is the current name of the sheet, but may change in future editions, in accordance with the project policy on using names.

Notation

We denote the product of A with B by $A \times B$, read aloud as “A cross B.” In this notation, if $A \neq B$, then $A \times B \neq B \times A$.

