Definition

Let $(X, \mathbf{R})$ be a real vector space. An inner product $\langle \cdot, \cdot \rangle : X \times X \to \mathbf{R}$ induces a norm $\|\cdot\| : X \to \mathbf{R}$ defined by $\|x\| = \sqrt{\langle x, x \rangle}$ and metric $d : X \times X \to \mathbf{R}$ defined by $d(x, y) = \|x - y\|$.

If (X, d) is a complete metric space, we call $((X, \mathbf{R}), \langle \cdot, \cdot \rangle)$ a *complete real inner product space* (or *complete inner product space*—without specifying that it is real— or a *Hilbert space*).[1]

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[1] The term Hilbert space is universal, but in accordance with the Bourbaki project's guidelines on naming, we will tend to use the term complete inner product space, even though this is longer.

Complete Real Inner Product Spaces

Complete Normed Spaces

Real Inner Product Norm
Complete Metric Spaces
Space Metrics

Norm
Real Inner Products

Functionals
Real Inner Product

Egyptian Sequences
Vector
Real Norm
Space Inner Product

Real Egyptian Sequence
Metrics
Real Vector
Space Norm

Real Limits
Plane Norm
Plane Inner Product
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space

Real Square Roots
Absolute Value
Right Triangle Sides Relation
Plane Vectors
Distance

Real Square
Space Distance

Real Arithmetic
Plane Distance

Real Multiplicative Inverse
Interval Length

Natural Square Roots
Square Numbers
Real Products
Intervals
Real Space

Real Additive Inverse
Real Line
Real Plane

Rational Multiplicative Inverse
Rational Arithmetic
Real Sums
Real Order
Real Sequences
Integral Line
Square

Fields
Rational Products
Rational Sums
Rational Order
Real Numbers
Geometry

Rings
Rational Numbers
Sequences

Integer Arithmetic
Lists

Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Direct Products

Arithmetic
Identity Elements
Finite Sets

Natural Powers
Equivalent Sets
Inverse Elements

Natural Equations
Natural Products
Integer Sums
Integer Order
Element Functions
Function Inverses
Comparisons
Family Unions and Intersections

Equations
Natural Sums
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families

Recursion Theorem
Natural Order
Equivalence Relation
Functions
Converse Relations

Peano Axioms
Relations

Natural Induction
Set Products

Natural Numbers
Set Powers
Generalized Set Dualities

Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Dualities
Set Unions and Intersections

Set Complements
Set Intersections
Pair Unions
Ordered Pairs

Set Differences
Set Unions
Pair Intersections

Empty Set
Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deduction

Quantified Statements

Logical Statements

Statements

Identities
Sets

Names

Letters
Objects