



## Why

It is common to consider random functions whose domain is time, space, or  $n$ -dimensional space.

## Definition

Let  $(X, d)$  be a metric space. A *distance covariance function*  $k : X \times X \rightarrow \mathbf{R}$  is a covariance function satisfying

$$k(x, y) > k(x, y) \iff d(x, y) < d(x, y).$$

In other words, the covariance decreases as the distance between the arguments decreases.

## Example: squared exponential

Let  $k : X \times X \rightarrow \mathbf{R}$  be defined by

$$k(x, y) = \exp(-d(x, y)).$$

Then  $k$  is a distance covariance function. It is often called the *squared exponential covariance function*.

Let  $\alpha, \sigma \in \mathbf{R}$ . Define  $k' : X \times X \rightarrow \mathbf{R}$  by

$$k'(x, y) = \alpha \exp(-d(x, y)/\sigma^2)$$

then  $k'$  is still a covariance function. In this context  $\sigma$  is often referred to as the *characteristic length-scale* of the process. The scalar  $\alpha$  is sometimes called a “prefactor” that “controls” the “overall variance.”

Suppose  $(X, d) = (\mathbf{R}^n, \|\cdot\|)$ . Then the squared exponential covariance function

$$\alpha \exp(-\|x - y\|/(2\sigma^2))$$

is sometimes called the *radial basis function* or *gaussian covariance function*.<sup>1</sup> Also called an *exponentiated quadratic kernel*.

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<sup>1</sup>For reasons that will be included in future editions.

