Why

We can interpret the codomain of a random variable as a new sample space, since the underlying probability distribution induces a new probability distribution.

Definition

Let $p : \Omega \to \mathbf{R}$ be a probability distribution and $x : \Omega \to V$ an outcome variable. Define $q : V \to \mathbf{R}$ by

$$q(a) = \mathbf{P}[x = a].$$

Since events $x^{-1}(a)$ for $a \in V$ partition Ω, $\sum_{a \in A} q(a) = 1$. We call q the *induced distribution* (or *induced probability mass function*) of the random variable x. Thus we can think of V as a set of outcomes, which we call the outcomes *induced* by x.

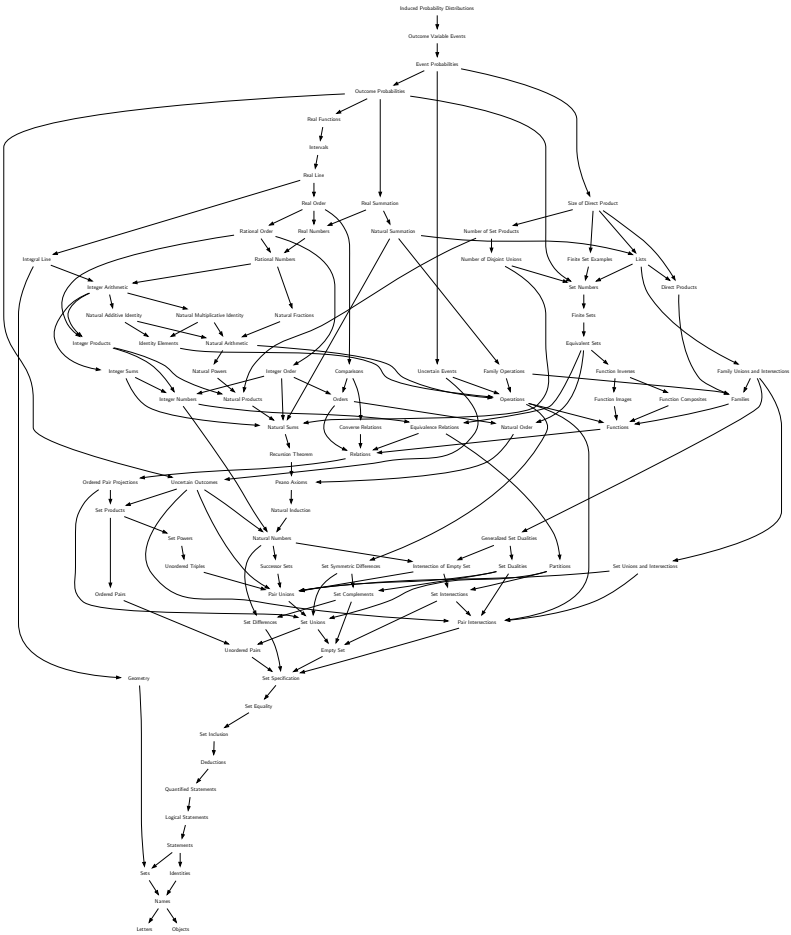
Notation

It is common to denote it by p_x.

If $x : \Omega \to V$ is a random variable and $f : V \to U$, then if we define $y : \Omega \to V$ so that $y \equiv f(x)$, y is a random variable with induced distribution $p_y : \Omega \to \mathbf{R}$ satisfying

$$p_y(b) = \textstyle\sum_{a \in V \mid y(a) = b} p_x(a).$$

As a matter of practical computation, we can evaluate probabilities having to do with the outcome variable x using p_x instead of p. For example with x as in the example above, $\mathbf{P}(x = 4 \text{ or } x = 5) = p_x(4) + p_x(5)$, rather than $\sum_{\omega \in \Omega \mid x(\omega) = 4 \text{ or } x(\omega) = 5} p(\omega)$.


Induced Probability Distributions
Outcome Variable Events
Event Probabilities
Outcome Probabilities
Real Functions
Intervals
Real Line
Real Order
Real Summation
Size of Direct Product
Rational Order
Real Numbers
Natural Summation
Number of Set Products
Integral Line
Rational Numbers
Number of Disjoint Unions
Finite Set Examples
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Additive Identity
Natural Multiplication Identity
Natural Fractions
Finite Sets
Integer Products
Identity Elements
Natural Arithmetic
Equivalent Sets
Integer Sums
Natural Powers
Integer Order
Comparisons
Uncertain Events
Family Operations
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Converse Relations
Equivalence Relations
Natural Order
Functions
Recursion Theorem
Relations
Ordered Pair Projections
Uncertain Outcomes
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Intersection of Empty Set
Set Dualities
Partitions
Set Unions and Intersections
Ordered Pairs
Pair Unions
Set Complements
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects