



Why

We want to model natural phenomena.¹

Definition

Let X_0, X_1, \dots, X_T be a sequence of sets and let $f_t : X_t \rightarrow X_{t+1}$ for $t = 0, \dots, T - 1$. We call $((X_0, \dots, X_T), (f_1, \dots, f_{T-1}))$ a *deterministic discrete-time dynamical system*.

We call the index t the *epoch*, the *stage* or the *period*. We call X_t the *state space* at period t . We call f_t the *transition function* or *dynamics function*.

Let $x_0 \in \mathcal{X}_0$. Define a state sequence $x_1 \in \mathcal{X}_1, \dots, x_T \in \mathcal{X}_T$ by

$$x_{t+1} = f_t(x_t, u_t).$$

In this case we call x_0 the *initial state*. We call the x_t the *trajectory* associated with initial state x_0 .

We call T the *horizon*. In the case that we have an infinite sequence of state sets, input sets, and dynamics, then we refer to a *infinite-horizon* dynamical system. To use language in contrast with this case, we refer to the dynamical system when T is finite as a *finite-horizon* dynamical system.

¹Future editions will modify, and may develop dynamic systems via the genetic approach by appealing to their classical use in Newtonian physics for modeling celestial mechanics.

