Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?

Definition

Two random variables are *independent* if the sigma algebras generated by the random variables are independent. In general, a family of random variables are *independent* if the sigma algebras generated by the random variables are independent.

Notation

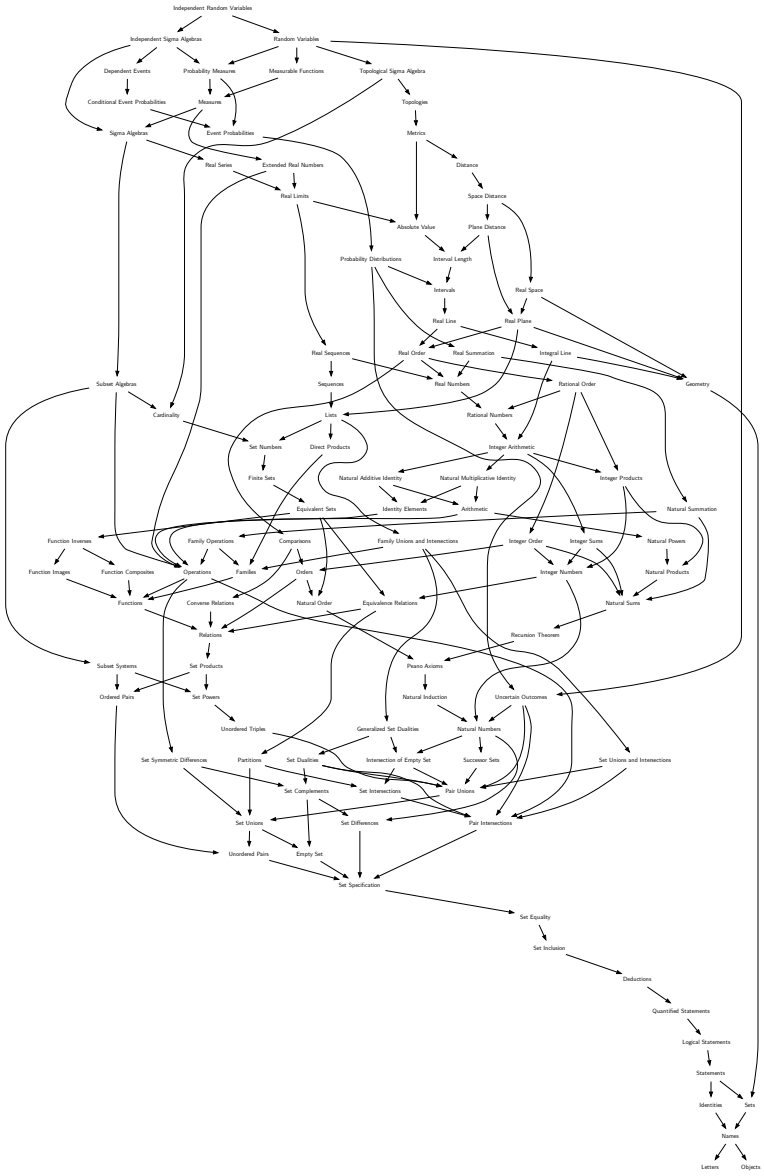
Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f_1, f_2 : X \to Y$ be random variables. If the random variables are independent we write $f_1 \perp f_2$.

Results

Proposition 1. *Let $f_1, \ldots, f_n$ be independent real-valued random variables defined on a probability space $(X, \mathcal{A}, \mu)$. Let $B_1, \ldots, B_n$ be Borel sets of real numbers and let $A_i = f_i^{-1}(B_i)$. Let $A = \cap_{i=1}^n f_i^{-1}(B_i)$. Then*

$$\mu(A) = \prod_{i=1}^{n} \mu(A_i)$$

Proof. Since f_i are independent, so are the sigma algebras they generate. A_i are in each of these sigma algebras, so by definition of independence the measure of the intersection is the product of the measures. $\square$


Independent Random Variables
Independent Sigma Algebras
Random Variable
Dependent Events
Probability Measures
Measurable Functions
Topological Sigma Algebra
Conditional Event Probabilities
Measures
Topologies
Sigma Algebras
Event Probabilities
Metrics
Real Series
Extended Real Numbers
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Probability Distributions
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Sequences
Real Order
Real Summation
Integral Line
Subset Algebras
Sequences
Real Numbers
Rational Order
Geometry
Cardinality
Lists
Rational Numbers
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Identity Elements
Arithmetic
Natural Summation
Function Inverses
Family Operations
Comparisons
Family Unions and Intersections
Integer Order
Integer Sums
Natural Powers
Function Images
Function Compositions
Operations
Families
Orders
Integer Numbers
Natural Products
Functions
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Subset Systems
Set Products
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Uncertain Outcomes
Unordered Triples
Generalized Set Dualities
Natural Numbers
Set Symmetric Differences
Partitions
Set Dualities
Intersection of Empty Set
Successor Sets
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects