Why

We want to define area under a real function. We start with defining functions for which this notion is obvious.

Definition

A *simple function* (or *step function*) is a real-valued function whose range is a finite set. We can write simple function as the sum of the characteristic functions fo the inverse image elements.

We can partition the range of the function into a finite family of one-elements sets. Then the family whose members are the inverse images of these sets partitions the domain. We call this the *simple partition* of the function.

Notation

We denote the set of simple functions on A by $\mathrm{SF}(A)$. We denote subset of non-negative simple real functions with domain A by by $\mathrm{SF}_+(A)$.

Let $f \in \mathrm{SF}(A)$. Let $\{a_1, \ldots, a_n\} = f(A)$. Define $A_i = f^{-1}(\{a_i\})$. Then $f = \sum_{i=1}^{n} a_i \chi_{A_i}$.

Simple Functions

Real Functions

Intervals

Real Line

Real Order

Real Numbers

Rational Order

Integral Line

Rational Numbers

Integer Arithmetic

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Identity Elements

Arithmetic

Natural Powers

Comparisons

Operations

Orders

Natural Products

Integer Order

Integer Sums

Converse Relations

Functions

Natural Order

Recursion Theorem

Equivalence Relations

Natural Sums

Integer Numbers

Relations

Peano Axioms

Set Products

Natural Induction

Characteristic Functions

Ordered Pairs

Set Powers

Natural Numbers

Partitions

Unordered Triples

Intersection of Empty Set

Successor Sets

Set Intersections

Set Symmetric Differences

Pair Unions

Pair Intersections

Set Unions

Set Complements

Unordered Pairs

Empty Set

Set Differences

Geometry

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects