



Sparsity Patterns

1 Why

TODO

2 Definition

A *sparsity pattern* of order n is a set of pairs of natural numbers from one to n .

A symmetric matrix is said to *have a sparsity pattern* if its ij th entry is zero whenever $\{i, j\}$ is not in the sparsity pattern. The diagonal entries and off-diagonal entries for pairs appearing in the sparsity pattern may or may not be zero.

The graph whose vertices are one through n and whose edge set is the sparsity pattern is called the *sparsity graph*.

A sparsity pattern is not a property of a matrix because it is not unique (unless all off-diagonal entries are non-zero). If a matrix has a particular sparsity pattern it has every sparsity pattern which is a superset of it. In other words, every matrix has the sparsity pattern which is the set of all pairs of integers.

2.1 Notation

Let $E \subset \{\{i, j\} \mid i, j \in \{1, 2, \dots, n\}\}$. A symmetric matrix $A \in \mathbf{S}^n$ is said to have sparsity pattern E if $A_{ij} = A_{ji} = 0$ whenever $i \neq j$ and $\{i, j\} \notin E$. The graph $G = (V, E)$ where $V = \{1, 2, \dots, n\}$ is the sparsity graph associated with E .



Figure 1: Sparsity graph for the matrix $()$.

We will denote the symmetric matrices of order n with sparsity pattern E by \mathbf{S}_E^n .

3 Example

Figure 1 shows a sparsity graph for the matrix

$$A := \begin{bmatrix} A_{11} & A_{21} & A_{31} & 0 & A_{51} \\ A_{21} & A_{22} & 0 & A_{42} & 0 \\ A_{31} & 0 & A_{33} & 0 & A_{54} \\ 0 & A_{42} & 0 & A_{44} & A_{45} \\ A_{51} & 0 & A_{53} & A_{54} & A_{55} \end{bmatrix}.$$



