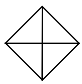


Why

Given an outcome variable $X : \Omega \to V$, and some probabilities on a sample space Ω, there is a natural set of probabilities to associate with outcomes in V.

Result

Suppose $X : \Omega \to V$ is a random variable on a finite sample space Ω. Given $p : \Omega \to \mathbf{R}$ is a probability distribution inducing probability measure $P : \mathcal{P}(\Omega) \to \mathbf{R}$, define the function $q : V \to \mathbf{R}$ by

$$q(x) = P(X = x)$$

It is easy to verify that q is nonnegative and normalized. The latter fact follows from the observation that the sets $\{X^{-1}(x)\}x \in V$ partition the set Ω.

The function q is sometimes called the *distribution of* X (the *induced distribution*, *induced probability distribution*) of the random variable X.

Notation

Given a random variable $X : \Omega \to V$, and some distribution $p : \Omega \to \mathbf{R}$, it is common to denote the induced distribution by p_X. Given a distribution $q : V \to \mathbf{R}$ and a random variable $X : \Omega \to V$ it is common to see the notation $X \sim q$ as an abbreviation of the sentence "The random variable X has distribution q.

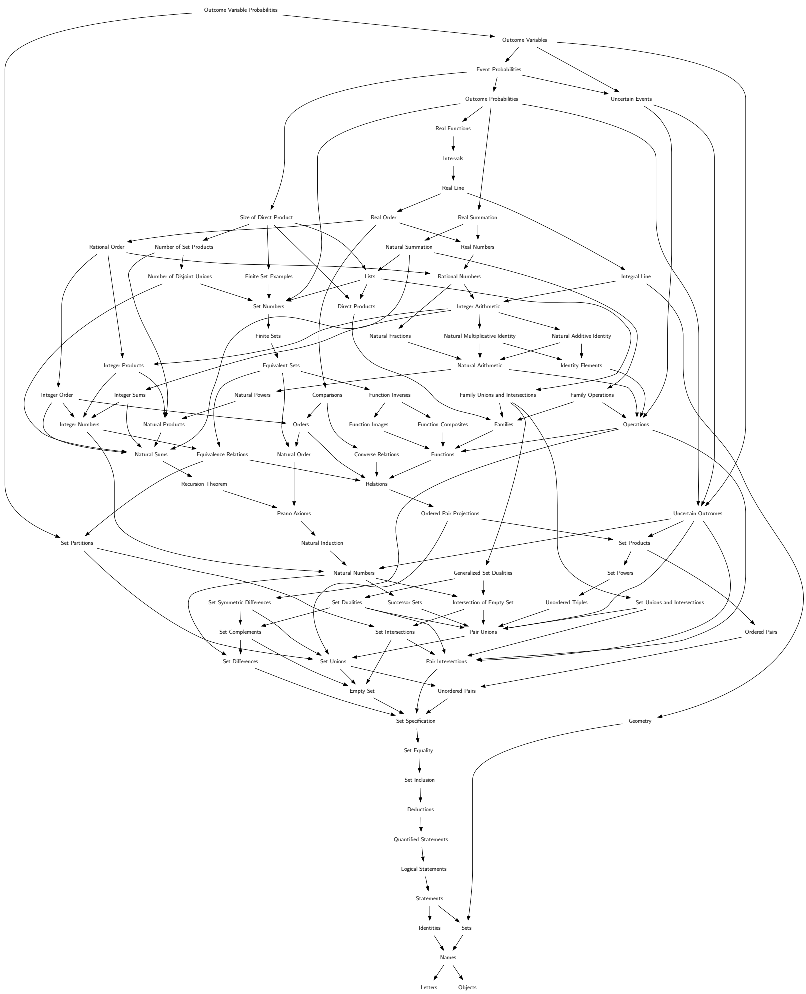
Computation

Suppose $X : \Omega \to V$ is a random variable and $f : V \to U$. Define $Y : \Omega \to V$ by $y(\omega) \equiv f(x(\omega))$ for every $\omega \in \Omega$. We frequently denote this by $Y = f(X)$. Y is a random variable with induced distribution $p_Y : \Omega \to \mathbf{R}$ satisfying

$$p_Y(y) = \textstyle\sum_{\omega \in \Omega \mid Y(\omega) = y} p(\omega) = \sum_{x \in V \mid f(x) = b} p_X(x).$$

Consequently, as a matter of practical computation, we can evaluate probabilities having to do with the outcome variable X using p_X instead of p and same with Y.[1]

[1]Future editions will give an example.


Outcome Variable Probabilities
Outcome Variables
Event Probabilities
Outcome Probabilities
Uncertain Events
Real Functions
Intervals
Real Line
Size of Direct Product
Real Order
Real Summation
Rational Order
Number of Set Products
Natural Summation
Real Numbers
Number of Disjoint Unions
Finite Set Examples
Lists
Rational Numbers
Integral Line
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Natural Fractions
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Equivalent Sets
Natural Arithmetic
Identity Elements
Integer Order
Integer Sums
Natural Powers
Comparisons
Function Inverses
Family Unions and Intersections
Family Operations
Integer Numbers
Natural Products
Orders
Function Images
Function Composites
Families
Operations
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Set Partitions
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects