



## Why

We look at a particular subset of vertices and the edges involved between them.

## Definition

Suppose  $\mathcal{G} = (V, E)$  is an undirected graph. An undirected graph  $(V', E')$  is a *subgraph* of  $\mathcal{G}$  if  $V' \subset V$  and  $E \subset E'$ . The *vertex-induced subgraph* of an undirected graph  $(V, E)$  induced by a subset of vertices  $W \subset V$  is the undirected graph with vertices  $W$  and all edges between vertices in  $W$ . The *edge-induced subgraph* of an undirected graph  $(V, E)$  induced by vertices  $F \subset E$  whose edge set is  $F$  and whose vertices are those vertices adjacent to the edges of  $F$ .

## Notation

Let  $W \subset V$  and define  $F$  by

$$F = \{\{v, w\} \in E \mid v, w \in W\}.$$

The subgraph induced by  $W$  is the undirected graph  $(W, F)$ .

Some authors denote the subgraph induced by  $W$  by  $G(W)$  or  $(W, E(W))$  or  $G[W]$ . We avoid this notation, as it abuses  $G$ , which is no longer an ordered pair, but (in our standard function notation) now indicates a function on subsets of  $V$  with a complicated codomain. Other authors occasionally refer to the “subgraph  $W$ ”, instead of “the subgraph  $G(W)$ ”. Again, we avoid this practice.

For  $D \subset E$ , the subgraph induced by  $D$  is the undirected graph  $(U, D)$  where

$$U = \{v \in V \mid \exists e \in D : v \in e\}$$

Similarly, people write  $G(D)$  or  $(V(D), D)$ . We avoid this.

## Connected components

A set of vertices  $W$  in  $G$  is *connected* if there is a path between any two vertices  $v, w \in W$ . A set of vertices  $W$  in  $G$  is *maximally connected* if there is no other vertex  $v \notin W$  connected to a vertex in  $W$ . A *connected component* of  $G$  is the subgraph induced by a maximally connected set of vertices.

Since the vertex set of a graph can always be partitioned into sets maximally connected vertices, and a connected components is connect, we think of the connected components of  $G$  as the connected “pieces” of  $G$ .

## Cliques

A set of vertices is *complete* if the subgraph induced is complete. A set of vertices  $W$  is *maximally complete* if the subgraph induced is complete and there is no vertex  $v \notin W$  which is connected to every vertex in  $W$ . In other words, there is no other vertex which we can add to  $W$  so that the induced subgraph is still complete.

We call a *maximally complete* set of vertices a *clique*. Some authors define a clique in the way we have defined a complete set of vertices, without reference to the maximality.

