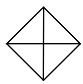


Definition

The *orthogonal complement* of a subset of an inner product space is the set of vectors which are orthogonal to every vector in the subset.

Here is the definition in symbols. Given an inner product space V and a *subset* $U \subset V$, the orthogonal complement of V is

$$\{v \in V \mid \langle v, u\rangle = 0 \text{ for every } u \in U\}$$

Notation

We denote the orthogonal complement of U by $U^\perp$.

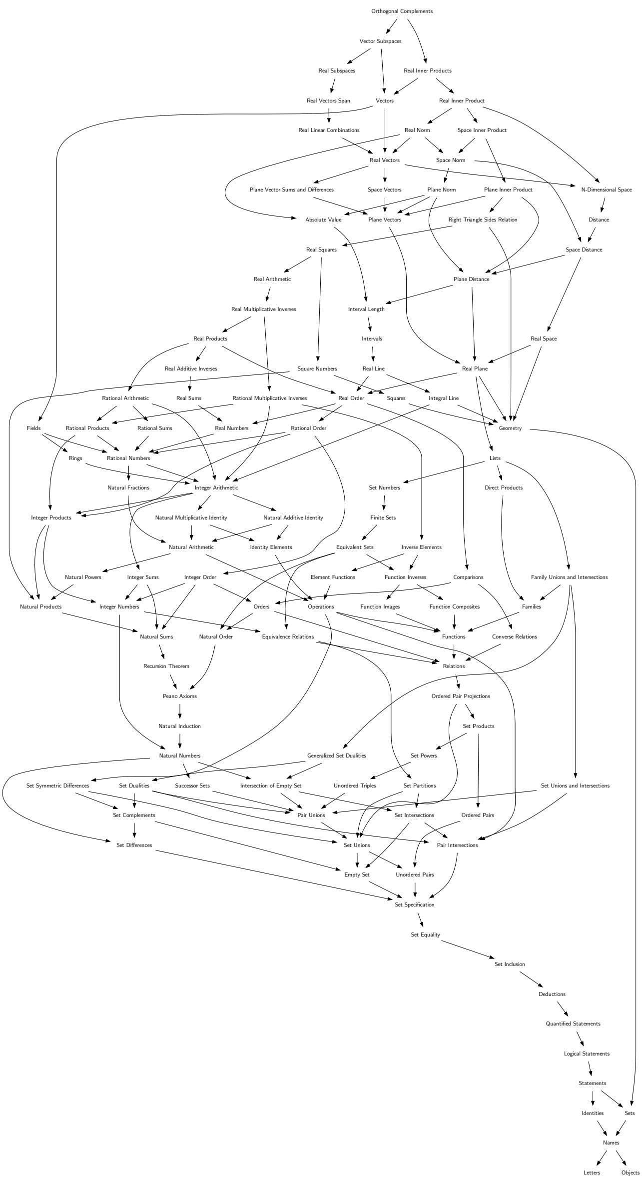
Examples

Complements of lines and planes in $\mathbf{R}^3$. If U is a line in $\mathbf{R}^3$, then $U^\perp$ is the plane containing the origin that is perpendicular to U. Conversely, if U is a plane in $\mathbf{R}^3$ containing the origin, then $U^\perp$ is the line containing the origin that is perpendicular to U.

Properties

Proposition 1. *Suppose V is an inner product space. Given any subset $U \subset V$,*

1. *$U^\perp$ is a subspace of V.*
2. *$\{0\}^\perp = V$*
3. *$V^\perp = \{0\}$*
4. *$U \cap U^\perp \subset \{0\}$*
5. *if $W \subset U$, then $W^\perp \subset U^\perp$*


Orthogonal Complements
Vector Subspaces
Real Subspaces
Real Inner Products
Real Vectors Span
Vectors
Real Inner Product
Real Linear Combinations
Real Norm
Space Inner Product
Real Vectors
Space Norm
Plane Vector Sums and Differences
Space Vectors
Plane Norm
Plane Inner Product
N-Dimensional Space
Absolute Value
Plane Vectors
Right Triangle Sides Relation
Distance
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Square Numbers
Real Line
Real Plane
Rational Arithmetic
Real Sums
Rational Multiplicative Inverses
Real Order
Squares
Integral Line
Fields
Rational Products
Rational Sums
Real Numbers
Rational Order
Geometry
Rings
Rational Numbers
Lists
Natural Fractions
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Arithmetic
Identity Elements
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Integer Order
Element Functions
Function Inverses
Comparisons
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Natural Order
Equivalence Relations
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects