Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\operatorname{tr} : \mathbf{R}^{n\times n} \to \mathbf{R}$. The trace of $A \in \mathbf{R}^{n\times n}$ is

$$\operatorname{tr} A = \sum _i = 1^n A_ii.$$

Properties

Proposition 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}Matnn$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\begin{aligned}\operatorname{tr} C = \sum _i = 1^n C_ii &= \sum _i = 1^n \alpha A_ii + \beta B_ii \\ &= \alpha \sum _i = 1^n A_ii + \beta \sum _i = 1^n B_ii \\ &= \alpha \operatorname{tr} A + \beta \operatorname{tr} B.\end{aligned}$$

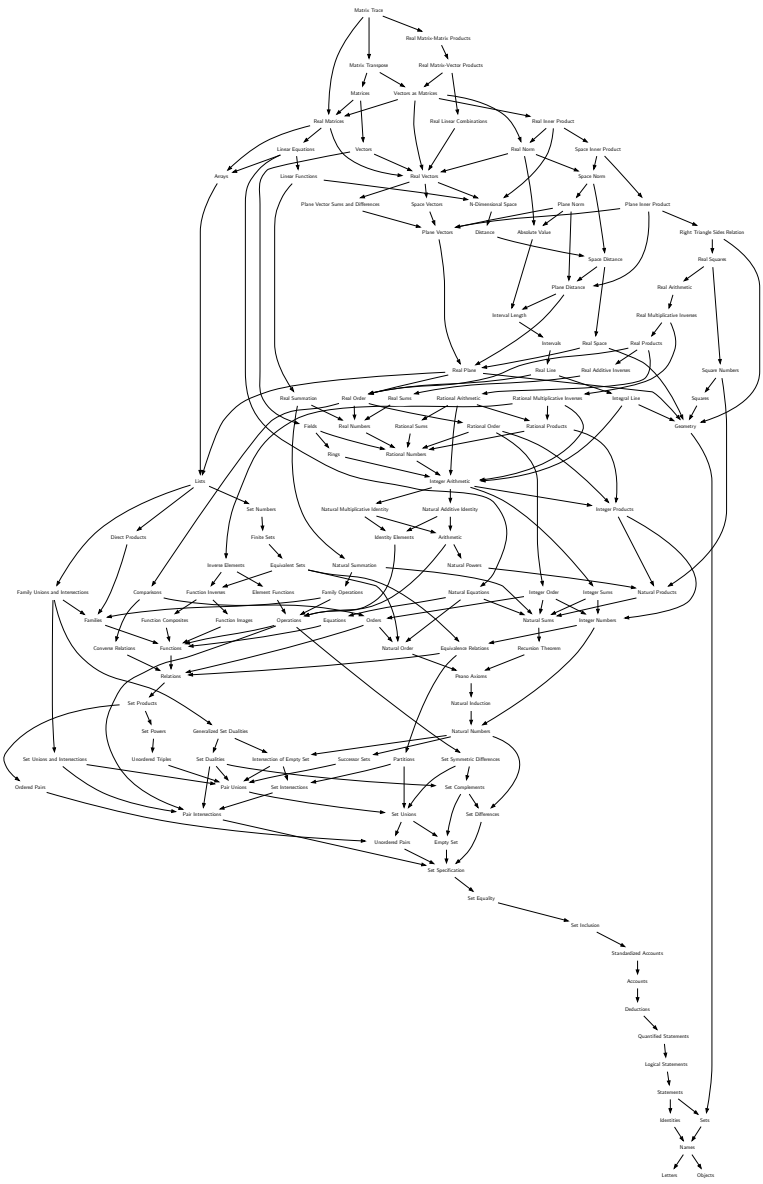
□

Proposition 2. *Let $A, B \in \mathbf{R}^{n\times n}$. Then*

$$\operatorname{tr}(AB) = \operatorname{tr}(BA).$$

In other words, "matrices commute under the trace operator."

Proposition 3. *Let $A \in \mathbf{R}^{n\times n}$. Then* $\operatorname{tr} A = \operatorname{tr} A^\top$.


Matrix Trace
Real Matrix-Matrix Products
Matrix Transpose
Real Matrix-Vector Products
Matrices
Vectors as Matrices
Real Matrices
Real Linear Combinations
Real Inner Product
Linear Equations
Vectors
Real Norm
Space Inner Product
Arrays
Real Vectors
Linear Functions
Space Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Norms
Plane Inner Product
Plane Vectors
Distance
Absolute Value
Right Triangle Sides Relation
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Real Plane
Real Line
Real Additive Inverses
Square Numbers
Real Summation
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Integral Line
Squares
Fields
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Integer Arithmetic
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Natural Multiplication Identity
Natural Additive Identity
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Direct Products
Finite Sets
Identity Elements
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Family Unions and Intersections
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Natural Equations
Integer Order
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Converse Relations
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Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Unordered Triples
Set Dualities
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Frames
Lenses
Objects