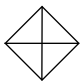

Why

We want to talk about making decisions over time.[1]

Definition

Let $\mathcal{X}_0, \mathcal{X}_1, \dots, \mathcal{X}_T$ and $\mathcal{U}_0, \mathcal{U}_1, \dots, \mathcal{U}_{T-1}$ be sets. For $t = 0, \dots, T-1$, let $f_t : \mathcal{X}_t \times \mathcal{U}_t \to \mathcal{X}_{t+1}$. We call the sequence

$$((\mathcal{X}_t)_{t=0}^{T}), (\mathcal{U}_t)_{t=0}^{T-1}, (f_t)_{t=1}^{T-1})$$

a *deterministic discrete-time dynamical system*. We call the index t the *epoch*, the *stage* or the *period*.

Let $x_0 \in \mathcal{X}_0$. Let $u_0 \in \mathcal{U}_0, \dots, u_{T-1} \in \mathcal{U}_{T-1}$. Define a state sequence $x_1 \in \mathcal{X}_1, \dots, x_T \in \mathcal{X}_T$ by

$$x_{t+1} = f_t(x_t, u_t).$$

In this case we call x_0 the *initial state*. We call the x_t the *states*. We call the u_t a sequence of *inputs* (or *actions*, *decisions*, *choices*, or *controls*). We call f_t the *transition function* or *dynamics function*.

We call T the *horizon*. In the case that we have an infinite sequence of state sets, input sets, and dynamics, then we refer to a *infinite-horizon* dynamical system. To use language in contrast with this case, we refer to the dynamical system when T is finite as a *finite-horizon* dynamical system.

[1] Future editions will expand, and may develop dynamic systems via the genetic approach by appealing to their classical use in Newtonian physics for modeling celestial mechanics.

State

The current action u_t affects future states x_s for $s > t$, but not the current or past states. The current state x_t depends on the initial state x_0 and the sequence of past actions $u_0, \ldots, u_{t-1}$. So the state is a "link" between the past and the future. Given x_t and $u_t, \ldots, u_{s-1}$, for $s > t$, we can compute x_s. In other words, the prior actions $u_0, \ldots, u_{t-1}$ are not relevant.

This nonrelevancy of prior actions and prior states simplifies the sequential decision problem (see Sequential Decisions).

Variations

The dynamical system is called *time-invariant* if $\mathcal{X}_t$, $\mathcal{U}_t$ and f_t do not depend on t. A simple variation is that $\mathcal{U}_t$ depends on x_t.[2]

Examples

Discrete dynamical system

Let $\mathcal{X} = \{1, \ldots, n\}$ and $\mathcal{U} = \{1, \ldots, m\}$. Then $f_t : \mathcal{X} \times \mathcal{U} \to \mathcal{U}$ is called a *transition map*. We can think of the transition map as a "table".

Discrete-time linear dynamical system

Let $\mathcal{X} = \mathsf{R}^n$ and $\mathcal{U} = \mathsf{R}^m$. Define $f_t : \mathcal{X} \times \mathcal{U} \to \mathcal{X}$ by

$$x_{t+1} = f_t(x_t, u_t) = A_t x_t + B_t u_t + c_t$$

[2] Future editions will say more here.

for $y = 1, \ldots, T-1$.[3]

[3] This very special form of dynamics arises in many applications. Future editions will say more.


Dynamical Systems
State Representations
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Integral Line
Real Order
Geometry
Real Numbers
Sequential Decisions
Rational Numbers
Decisions
Sequences
Integer Arithmetic
Total Orders
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Direct Products
Identity Elements
Arithmetic
Equivalent Sets
Operations
Natural Exponents
Integer Products
Function Inverses
Family Unions and Intersections
Natural Products
Integer Sums
Function Composites
Function Images
Families
Natural Sums
Integer Numbers
Partial Orders
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Uncertain Outcomes
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects