

INTERVALS

Why

We name and denote subsets of the set of real numbers which correspond to segments of a line.

Definition

Take two real numbers, with the first less than the second.

An *interval* is one of four sets:

- 1. the set of real numbers larger than the first number and smaller than the second; we call the interval *open*.
- 2. the set of real numbers larger than or equal to the first number and smaller than or equal to the second number; we call the interval closed.
- 3. the set of real numbers larger than the first number and smaller than or equal to the second; we call the interval open on the left and closed on the right
- 4. the set of real numbers larger than or equal to the first number and smaller than the second; we call the interval *closed on the left* and open on the right.

If an interval is neither open nor closed we call it $\mathit{half-open}$ or $\mathit{half-closed}$

We call the two numbers the *endpoints* of the interval. An open interval does not contain its endpoints. A closed interval contains its endpoints. A half-open/half-closed interval contains only one of its endpoints. We say that the endpoints *delimit* the interval.

Notation

Let a, b be two real numbers which satisfy the relation a < b.

We denote the open interval from a to b by (a, b). This notation,

although standard, is the same as that for ordered pairs; no confusion arises with adequate context. 1

We denote the closed interval from a to b by [a,b]. We record the fact $(a,b) \subset [a,b]$ in our new notation.

We denote the half-open interval from a to b, closed on the right, by (a,b] and the half-open interval from a to b, closed on the left, by [a,b).²

The unit interval is the set $[0_R, 1_R]$ and we sometimes denote it by I.

¹In future editions, we may use (a, b) or even (a, b).

²Some authors use]a, b], [a, b[and]a, b[.

