

Countable Probability Distributions

1 Why

We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

2 Why

Consider a set A. If A has n elements, then a probability distribution on A is $p: A \to \mathbb{R}$ where p(a) = 1/n. There is a natural candidate.

What if A is the set of natural numbers N. The principle difficulty is that not all sequences of real numbers $a: \mathbb{N} \to \mathbb{R}$ are summable.

A (countable) probability distribution on N is $p: \mathbb{N} \to \mathbb{R}$ where $p \geq 0$ and

$$\sum_{n=1}^{\infty} p(n) = 1.$$