Sets

1 Why

We speak of collections of objects which we explicitly specify or which we describe as possessing one or more defining properties.

2 Definition

We use the words **object** and **collection** with their usual sense in the English language. A **set** is a collection of objects. So a set is an object with the property that it contains other objects.

In thinking of a set, then, we regularly consider the objects it contains. We call the objects contained in a set the **members** or **elements** of the set. So we say that an object contained in a set is a **member of** or an **element of** the set.

For example, consider the set of seasons. This set has four elements: autumn, winter, spring and summer. Consider the set playing card suits: hearts, diamonds, spades, and clubs. Consider the set of cards for each suit: ace, two, three, four, so on, ten, jack, queen, king. Consider the set of fifty-two cards in a deck.

2.1   Notation

We denote sets by upper case latin letters: for example, A, B, and C. We denote elements of sets by lower case latin letters: for example, a, b, and c. We denote that an object a is an element of a set A by $a \in A$. We read the notation $a \in A$ aloud as "a in A." The $\in$ is a stylized ϵ, a mnemonic for "element of". We write $a \notin A$, read aloud as "a not in A," if a is not an element of A.

If we can write down the elements of A, we do so using brace notation. For example, if the set A is such that it contains only the elements a, b, c, we denote A by $\{a, b, c\}$. If the elements of a set are well-known, then we introduce the set in English and name it; often we select the name mnemonically. For example, let L be the set of latin letters.

If the elements of a set satisfy some common condition, then we use the braces and include the condition. For example, let V be the set of Latin vowels. We can denote V by $\{l \in L \mid l \text{ is a vowel}\}$. We read the symbol $\mid$ aloud as "such that." We read the whole notation aloud as "l in L such that l is a vowel." We call the notation **set-builder notation**. Set-builder notation is indispensable for sets defined implicitly by some condition. Here we could have alternatively denoted V by $\{\text{``}a\text{''}, \text{``}e\text{''}, \text{``}i\text{''}, \text{``}o\text{''}, \text{``}u\text{''}\}$. We prefer the former, slighly more concise notation.