



## Tree Densities

### 1 Why

As with distributions, if a density factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So, as before, the property of factoring according to a rooted tree is really a property of factoring according to a tree.

### 2 Definition

#### 2.1 Defining Result

**Proposition 1.** *If a density factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

*Proof.* TODO

□

#### 2.2 Undirected Definition

A density *factors according to a tree* if it factors according to the tree rooted at any vertex.

### 3 Existence and Uniqueness

Trees are not a property of densities, since there is no one-to-one correspondence, as demonstrated by the following propositions.

#### 3.1 Existence

**Proposition 2.** *A density  $f$  need not factor according to a tree.*

#### 3.2 Uniqueness

**Proposition 3.** *A density  $f$  may factor according to multiple trees.*