Why

Many problems simplify if the graph involved is chordal.[1]

Paths

Let G be an undirected graph. A *chord* in a path p of G is an edge between two non-consecutive vertices of p. So a chord of the path $(v_1, v_1, \ldots, v_k)$ is an edge $\{v_i, v_j\}$ with $|j - i| > 1$.

We interpret a chord as a "one-edge shortcut" between two vertices of a path. If a path p has a chord, it can be reduced to a shorter path p' by "skipping" vertices. In other words, the shortest path between two vertices is chordless. However, a chordless path need not be a shortest path. See the figure below.

Graphs

A chord of a cycle $(v_1, v_2, \ldots, v_{k-1}, v_1)$ is an edge $\{v_i, v_j\}$ with $(j - i) \bmod k > 1$. An undirected graph G is *chordal* if every cycle with more than three edges has a chord.

If G is chordal, every cycle in G can be reduced to a cycle of length three. We sometimes call a cycle of length three a *triangle*. For this reason, chordal graphs are also sometimes called *triangulated graphs*. Other terminology includes *rigid-circuit graphs*, *triangulated graphs*, *perfect elimination graphs*, *decomposable graphs*.[2]

The last graph in the figure below is not chordal because the cycle (a, b, d, c, a) has length four and no chord. Adding the edge $\{b, c\}$ or $\{a, d\}$ would make the graph chordal An immediate consequence of the definition that G be chordal is that any subgraph of G is chordal.

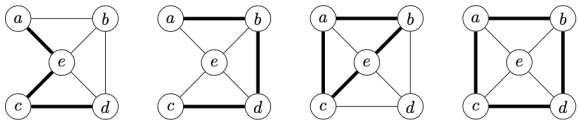
[1] Future editions will expand.

[2] See Vanenberghe and Anderson, 2014.

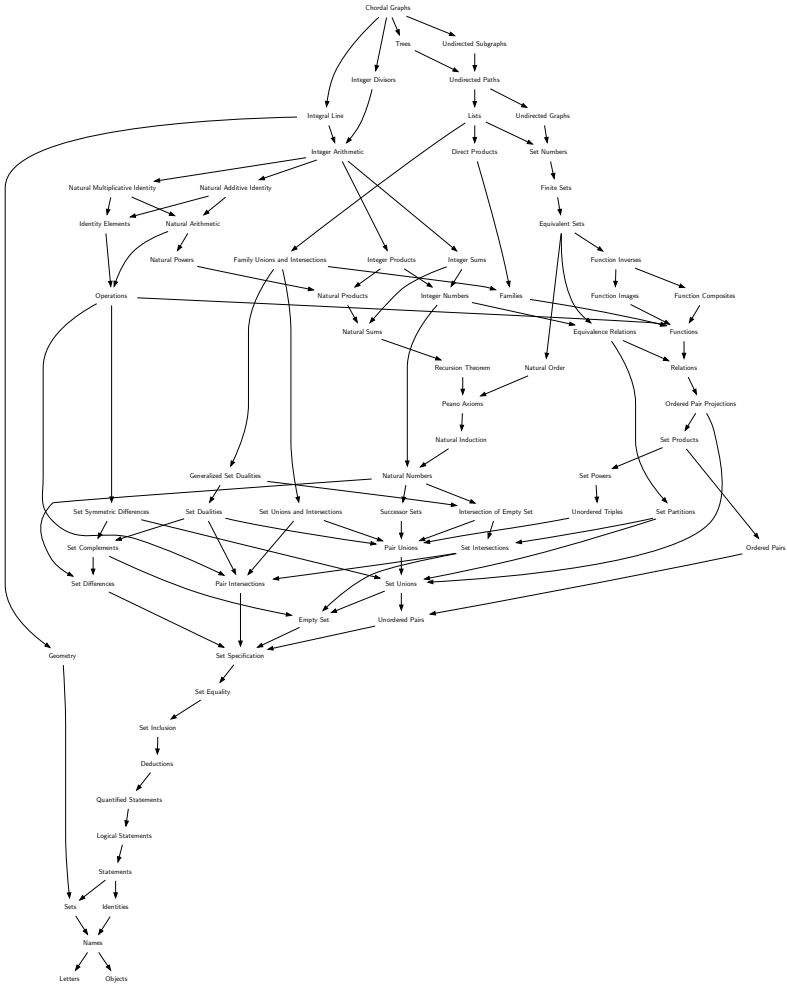
Simple examples

Since trees and forests have no cycles, they are chordal. Similarly, any graph with no cycles longer than three edges are trivially chordal. Such graphs are sometimes called *cactus graphs*. The complete graphs are also trivially chordal.

Specific example



The edge $\{e, d\}$ is a chord in the path (a, e, c, d) of the first graph. The path (a, v, d, c) is chordless. The edge $\{a, e\}$ is a chord in the cycle (a, v, e, c, a) of the second graph. The cycle (a, b, d, c, a) is chordless.


Chordal Graphs
Trees
Undirected Subgraphs
Integer Divisors
Undirected Paths
Integral Line
Lists
Undirected Graphs
Integer Arithmetic
Direct Products
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Family Unions and Intersections
Integer Products
Integer Sums
Function Inverses
Operations
Natural Products
Integer Numbers
Families
Function Images
Function Composites
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects