Set Operations

1 Why

We want to consider the elements of two sets together at once, and other sets created from two sets.

2 Definitions

Let A and B be two sets.

The *union* of A with B is the set whose elements are in either A *or* B *or* both. The key word in the definition is *or*.

The *intersection* of A with B is the set whose elements are in both A *and* B. The keyword in the definition is *and*.

Viewed as operations, both union and intersection commute; this property justifies the language “with." The intersection is a subset of A, of B, and of the union of A with B.

The *symmetric difference* of A and B is the set whose elements are in the union but not in the intersection. The symmetric difference commutes because both union and intersection commute; this property justifies the language “and." The symmetric difference is a subset of the union.

Let C be a set containing A. The *complement* of A in C is the symmetric difference of A and C. Since $A \subset C$, the union is C and the intersection is A. So the complement is the "left-over" elements of B after removing the elements of A.

We call these four operations *set-algebraic operations*.

2.1 Notation

Let A, B be sets. We denote the union of A with B by $A \cup B$, read aloud as "A union B." $\cup$ is a stylized U. We denote the intersection of A with B by $A \cap B$, read aloud as "A intersect B." We denote the symmetric difference of A and B by $A \Delta B$, read aloud as "A symdiff B." "Delta" is a mnemonic for difference.

Let C be a set containing A. We denote the complement of A in C by $C - A$, read aloud as "C minus A."

2.2 Results

Proposition 1. *For all sets A and B the operations $\cup$, $\cap$, and Δ commute.*

Proposition 2. *Let S a set. For all sets $A, B \subset S$,*

$$\begin{aligned} &(1) \quad S - (A \cup B) = (S - A) \cap (S - B) \\ &(2) \quad S - (A \cap B) = (S - A) \cup (S - B). \end{aligned}$$

Proposition 3. *Let S a set. For all sets $A, B \subset S$,*

$$A \Delta B = (A \cup B) \cap C_S(A \cap B)$$

$\boxed{TODO : notation}$