Definition

Let $f : D \to \mathbf{R}$ be a multivariate real-valued function where $D \subset \mathbf{R}^d$. The *graph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}.$$

The *epigraph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, t) \in \mathbf{R}^d \mid x \in D \text{ and } f(x) \leq t\}.$$

Notation

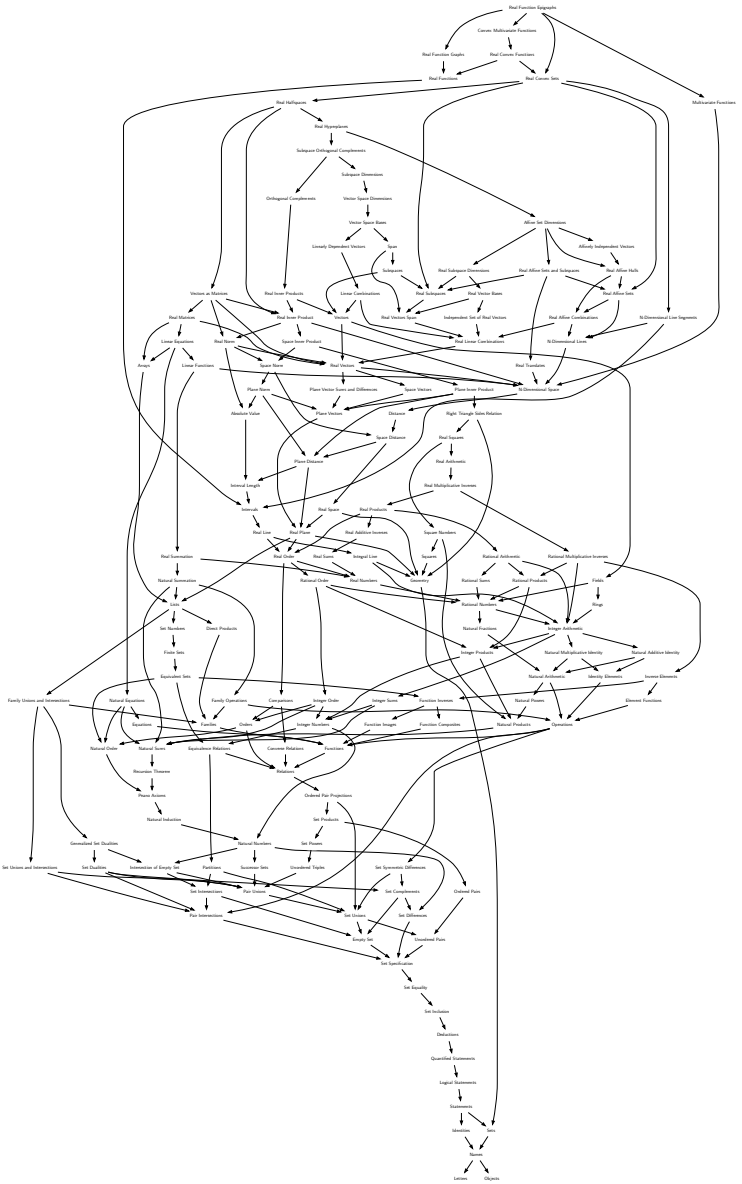
We denote the epigraph of a function f by $\operatorname{epi} f$.

Connecting convex sets and convex functions

A function $f : \mathbf{R}^d \to \mathbf{R}$ is convex if and only if $\operatorname{epi} f$ is a convex set.

Note on terminology

The prefix "epi" is Greek, meaning "upon" or "above".


Real Function Epigraphs
Convex Multivariate Functions
Real Function Graphs
Real Convex Functions
Real Functions
Real Convex Sets
Multivariate Functions
Real Halfspaces
Real Hyperplanes
Subspace Orthogonal Complements
Subspace Dimensions
Orthogonal Complements
Vector Space Dimensions
Affine Set Dimensions
Vector Space Bases
Affinely Independent Vectors
Linearly Dependent Vectors
Span
Real Subspace Dimensions
Real Affine Sets and Subspaces
Real Affine Hulls
Subspaces
Real Subspaces
Real Vector Bases
Real Affine Sets
Vectors as Matrices
Real Inner Products
Linear Combinations
Vectors
Real Vector Spaces
Independent Set of Real Vectors
Real Affine Combinations
N-Dimensional Line Segments
Real Matrices
Real Inner Product
Real Linear Combinations
N-Dimensional Lines
Linear Equations
Real Norms
Space Inner Product
Arrays
Linear Functions
Norm Axes
Real Vectors
Real Translates
N-Dimensional Space
Plane Norms
Plane Vector Sums and Differences
Space Vectors
Plane Inner Product
Absolute Value
Plane Vectors
Distance
Right Triangle Sides Relation
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Real Line
Real Plane
Real Additive Inverses
Square Numbers
Real Order
Real Sums
Integral Loci
Squares
Rational Arithmetic
Rational Multiplicative Inverses
Real Summation
Natural Summation
Rational Order
Real Numbers
Symmetry
Rational Sums
Rational Products
Fields
Logic
Rational Numbers
Rings
Set Numbers
Direct Products
Natural Fractions
Integer Arithmetic
Finite Sets
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Natural Arithmetic
Identity Elements
Inverse Elements
Family Unions and Intersections
Natural Functions
Family Operations
Concurrences
Integer Order
Integer Sums
Function Inverses
Natural Powers
Element Functions
Equations
Families
Orders
Integer Numbers
Function Images
Function Composites
Natural Products
Operations
Natural Order
Natural Sums
Equivalence Relations
Concrete Relations
Functions
Relations
Recursion Theorem
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Generalized Set Duality
Natural Numbers
Set Powers
Set Unions and Intersections
Set Duality
Intersection of Empty Set
Exercises
Sequence Sets
Unordered Triples
Set Symmetric Difference
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects