



# Sets

## 1 Why

We speak of a collection of objects, pre-specified or possessing a similar property.

## 2 Definition

A **set** is a collection of objects. We use **object** as usual in the English language. So a set is an object with the property that it contains other objects.

In thinking of a set, then, we regularly consider the objects it contains. We call the objects contained in a set the **members** or **elements** of the set. So we say that an object contained in a set is a **member of** or an **element of** the set.

### 2.1 Notation

We denote sets by upper case latin letters: for example,  $A$ ,  $B$ , and  $C$ . We denote elements of sets by lower case latin letters: for example,  $a$ ,  $b$ , and  $c$ . We denote that an object  $a$  is an element of a set  $A$  by  $a \in A$ . We read the notation  $a \in A$  aloud as “a

in  $A$ .” The  $\in$  is a stylized  $\epsilon$ , a mnemonic for “element of”. We write  $a \notin A$ , read aloud as “a not in  $A$ ,” if  $a$  is not an element of  $A$ .

If we can write down the elements of  $A$ , we do so using brace notation. For example, if the set  $A$  is such that it contains only the elements  $a, b, c$ , we denote  $A$  by  $\{a, b, c\}$ . If the elements of a set are well-known, then we introduce the set in English and name it; often we select the name mnemonically. For example, let  $L$  be the set of latin letters.

If the elements of a set satisfy some common condition, then we use the braces and include the condition. For example, let  $V$  be the set of vowels. We can denote  $V$  by  $\{l \in L \mid l \text{ is a vowel}\}$ . We read the symbol  $\mid$  aloud as “such that.” We read the whole notation aloud as “l in L such that l is a vowel.” We call the notation **set-builder notation**. Set-builder notation is indispensable for sets defined implicitly by some condition. Here we could have alternatively denoted  $V$  by  $\{“a”, “e”, “i”, “o”, “u”\}$ . We prefer the former, slightly more concise notation.

### 3 Two Sets

Let  $A$  a set. A **subset** of  $A$  is a set whose elements are also contained in  $A$ . A **superset** if  $A$  is a set which contains all the elements of  $A$ . Two sets are **equal** if they contain the same elements; equivalently if they are each subsets of each other.

The **power set** of  $A$  is the set of subsets of  $A$ . The **empty**

**set** is the set containing no elements. The empty set is subset of every set.

### 3.1 Notation

Let  $A$  and  $B$  be sets. We denote that  $A$  is a subset of  $B$  by  $A \subset B$ . We read the notation  $A \subset B$  aloud as “A subset B”. We denote that  $A$  is equal to  $B$  by  $A = B$ . We read the notation  $A = B$  aloud as “A equals B”. We denote the empty set by  $\emptyset$ , read aloud as “empty.” We denote the power set of  $A$  by  $2^A$ , read aloud as “two to the A.”