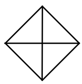


Why

We generalize the algebraic structure of addition and multiplication over the rationals.

Definition

A *field* is a ring $(R, +, \cdot)$ for which $\cdot$ is commutative (i.e., $ab = ba$ for all $a, b \in R$) and $\cdot$ has inverses for all elements except 0. In this case, we refer to *field addition* and *field multiplication*.

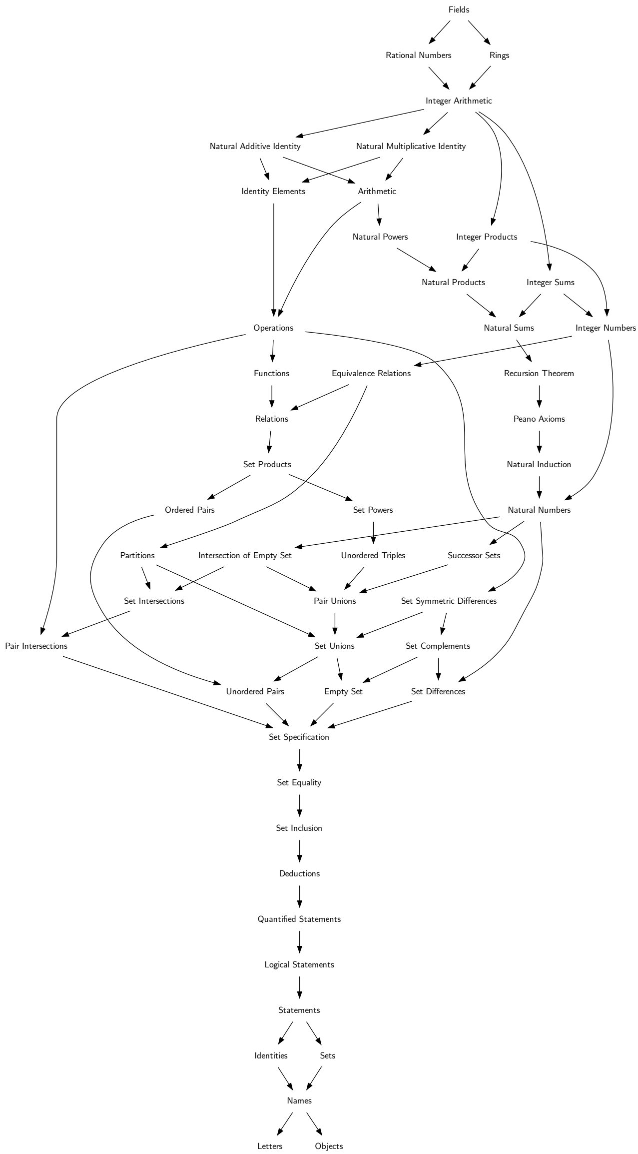
We also give names to the objects which have one of these additional properties or the other. A ring which for which multipliation is commutative is called a *commutative ring*. Note that a ring is *always* commutative with respect to addition, here the term commutative refers to multiplication. A ring for which there are inverse elements, excepting 0, is called a *division ring*).

Notation

Since our guiding example is the set of rationals **Q** with addition and multiplication defined in the usual manner, and we use a bold font for **Q**, we tend to denote an arbitrary field by **F**, a mnemonic for "field."

Field operations

Along with field addition and field multiplication, we call the function which takes an element of a field to its additive inverse and the function which takes an element of a field to its multiplicative inverse the *field operations*.


Fields
Rational Numbers
Rings
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Identity Elements
Arithmetic
Natural Powers
Integer Products
Natural Products
Integer Sums
Operations
Natural Sums
Integer Numbers
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Partitions
Intersection of Empty Set
Unordered Triples
Successor Sets
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects