Why

We want to visualize the probabilistic relations between components of outcomes in probabilistic models over large (e.g., product) outcome sets.[1]

Definition

Suppose $X_1, \ldots, X_k$ are sets and define $X = \prod_{i=1}^{k} X_i$. For $x \in X$ and $S \subset \{1, \ldots, n\}$, denote the subvector of x indexed (in order) by S by x_S.[2]

A distribution $p : X \to [0, 1]$ *factors according to a directed graph* on $\{1, \ldots, n\}$ with parent function $\mathrm{pa} : \{1, \ldots, n\} \to \mathcal{P}(\{1, \ldots, n\})$ if

$$p(x) = \prod_{\mathrm{pa}_i = \varnothing} g_i(x_i) \prod_{\mathrm{pa}_i \neq \varnothing} g_i(x_i, x_{\mathrm{pa}_i}),$$

where g_i is a distribution for all i which $\mathrm{pa}_i = \varnothing$ and $g_i(\cdot, \xi)$ is a distribution for all $\xi \in \prod_{j \in \mathrm{pa}_i} A_j$, i for which $\mathrm{pa}_i \neq \varnothing$.

Proposition 1. *p so defined is a distribution, and the g_i are the marginals and conditionals.*[3]

Examples

Consider a rooted tree distribution (see Rooted Tree Distributions), or a memory chain (see Memory Chains), or a hidden memory chain (see Hidden Memory Chains).[4]

---

[1]Future editions will modify and expand. The title of the sheet may change, since another interpretation for the words "directed graph distribution" is a distribution on directed graphs.

[2]Future editions will rework this treatment, perhaps combining it with the sheet Index Matrices, which will possibly be split up.

[3]Future editions will be precise and give an account.

[4]Future editions will expand.

Directed Graph Distributions

Hidden Memory Chains

Rooted Tree Distributions

Memory Chains

Rooted Trees

Conditional Distributions

Directed Graphs

Trees

Marginal Distributions

Conditional Event Probabilities

Undirected Paths

Joint Distributions

Event Probabilities

Undirected Graphs

Probability Distributions

Intervals

Real Line

Real Summation

Real Order

Rational Order

Real Numbers

Rational Numbers

Integral Line

Lists

Integer Arithmetic

Set Numbers

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Finite Sets

Direct Products

Arithmetic

Identity Elements

Equivalent Sets

Natural Powers

Natural Summation

Family Unions and Intersections

Integer Sums

Integer Order

Natural Products

Comparisons

Function Inverses

Family Operations

Integer Numbers

Natural Sums

Orders

Function Images

Function Composites

Families

Operations

Equivalence Relations

Natural Order

Recursion Theorem

Converse Relations

Functions

Peano Axioms

Relations

Uncertain Outcomes

Natural Induction

Set Products

Generalized Set Dualities

Natural Numbers

Set Powers

Partitions

Intersection of Empty Set

Set Dualities

Successor Sets

Unordered Triples

Set Symmetric Differences

Set Unions and Intersections

Pair Unions

Set Complements

Ordered Pairs

Set Intersections

Set Differences

Pair Intersections

Set Unions

Empty Set

Unordered Pairs

Set Specification

Geometry

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects