Definition

Given two distinct points x and y in $\mathbf{R}^n$, the *line* through x and y is the set of points expressable as the sum of x and $\alpha(y - x)$ where $\alpha \in \mathbf{R}$.

In other words, the line through x and y is the set

$$\{z \in \mathbf{R}^n \mid \exists \alpha \in \mathbf{R}, z = x + \alpha(y - x)\} = \{x + \alpha(y - x) \mid \alpha \in \mathbf{R}\}$$

The second expression is notation for the first, and is called the *set-generator notation*. Notice that if $z = x + \alpha(y - x)$, then

$$z = (1 - \alpha)x + \alpha y,$$

where $\alpha \in \mathbf{R}$ and $x, y \in \mathbf{R}^n$. This highlights the obvious (and obviously desirable) property that the definition is symmetric in x and y.

Visualization in the plane

Notation

We denote the line through x and y by $L(x, y)$.

Properties

There are a few nice properties. If $x \neq y$, then $L(x, y) = L(y, x)$. Also, if v and w are in $L(x, y)$ and $v \neq w$, then the line through v and w is the same as the line through x and y. In symbols

$$L(v, w) = L(x, y)$$

N-Dimensional Lines

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space

Real Line

Real Plane

Integral Line

Real Order

Geometry

Real Numbers

Rational Order

Lists

Rational Numbers

Set Numbers

Direct Products

Integer Arithmetic

Natural Fractions

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Equivalent Sets

Family Unions and Intersections

Identity Elements

Natural Arithmetic

Natural Powers

Integer Sums

Integer Order

Comparisons

Function Inverses

Families

Operations

Natural Products

Integer Numbers

Orders

Function Images

Function Composites

Natural Sums

Equivalence Relations

Converse Relations

Functions

Natural Order

Recursion Theorem

Relations

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Set Symmetric Differences

Partitions

Successor Sets

Intersection of Empty Set

Unordered Triples

Set Dualities

Set Unions and Intersections

Set Complements

Set Intersections

Pair Unions

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects