



# Sequences

## 1 Why

We introduce language for the steps of an infinite process.

## 2 Definition

Let  $A$  be a non-empty set. A **sequence in  $A$**  is a function from the natural numbers to the set. The  **$n$ th term** of a sequence is the result of  $n$ th natural number; it is an element of the set.

A **subindex** is a monotonically increasing function from and to the natural numbers. Roughly, it selects some ordered infinite subset of natural numbers. A **subsequence** of a first sequence is any second sequence which is the composition of the first sequence with a subindex.

Another way of describing a sequence is as an element of the direct product of a family of identical sets indexed by the natural numbers.

## 2.1 Notation

Keep  $A$  as a non-empty set. Denote the natural numbers by  $N$ . Let  $a : N \rightarrow A$ . Then  $a$  is a sequence and  $a(n)$  is the  $n$ th term. We denote  $a$  by  $\{a_n\}_n$  and  $a(n)$  by  $a_n$ .

Let  $i : N \rightarrow N$  such that  $n < m \implies i(n) < i(m)$ . Then  $i$  is a subindex. Let  $b = a \circ i$ . Then  $b$  is a subsequence of  $a$ . We denote it by  $\{b_{i(n)}\}_n$  and the  $n$ th term by  $b_{i(n)}$ .