Why

The density of a random variable determines the cumulative distribution function.

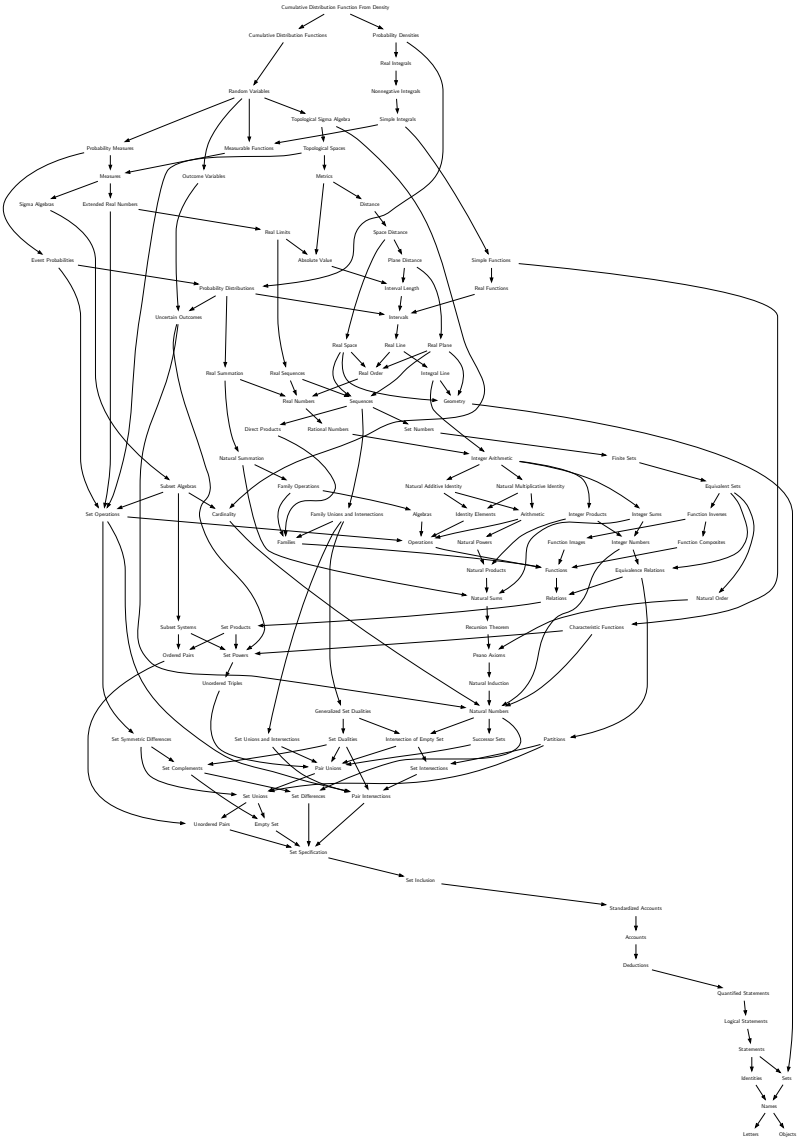
Result

Prop. 1. *A probability density function of a random variable, if it exists, characterizes the cumulative distribution function.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a probability space. Let f be a real-valued random variable on X. Let λ denote the cover length. Let g be a probability density of f: Then:

$$\begin{aligned} F_f(t) &= \mu(\{x \in X \mid f(x) \in (-\infty, t])\}) \\ &= \int_{(-\infty,t]} g d\lambda. \end{aligned}$$

□


Cumulative Distribution Function From Density
Cumulative Distribution Functions
Probability Densities
Real Integrals
Random Variables
Nonnegative Integrals
Topological Sigma Algebra
Simple Integrals
Probability Measures
Measurable Functions
Topological Spaces
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Probability Distributions
Interval Length
Real Functions
Uncertain Outcomes
Intervals
Real Space
Real Line
Real Plane
Real Summation
Real Sequences
Real Order
Integral Line
Real Numbers
Sequences
Geometry
Direct Products
Rational Numbers
Set Numbers
Finite Sets
Natural Summation
Integer Arithmetic
Subset Algebras
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Set Operations
Cardinality
Family Unions and Intersections
Algebras
Identity Elements
Arithmetic
Integer Products
Integer Sums
Function Inverses
Families
Operations
Natural Powers
Function Images
Integer Numbers
Function Composites
Natural Products
Functions
Equivalence Relations
Natural Sums
Relations
Natural Order
Subset Systems
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Recursion Theorem
Characteristic Functions
Ordered Pairs
Set Powers
Peano Axioms
Unordered Triples
Natural Induction
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Set Unions and Intersections
Set Dualities
Intersection of Empty Set
Successor Sets
Partitions
Pair Unions
Set Complements
Set Intersections
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects