Why

1

Definition

The *higher adjacency set* or *higher neighborhood* of a vertex v in an ordered undirected graph is all vertices in the neighborhood of v whose index is greater the v. Similarly, the *lower adjacency set* or *lower neighborhood* of v is all vertices in the neighborhood of v whose index is less the v. We call these *monotone neighborhoods*.

The *higher degree* of a vertex is the size of the higher adjacency set and the *lower degree* of a vertex is the size of its lower adjacency set.

The *closed monotone neighborhoods* are the *closed higher adjacency set*, the higher adjacency set of v union with the singleton $\{v\}$ and the *closed lower adjacency set*, the lower adjacency set of v union with the singleton $\{v\}$.

Notation

We denote the higher neighborhood of v by $\mathbf{adj}^+(v)$ and the lower neighborhood by $\mathbf{adj}^-(v)$.

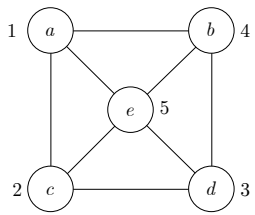


Figure 1: Ordered undirected graph.

Visualization

To help think about the monotone neighborhoods of the graph we visualize ordered graphs as triangular arrays with vertices along the diagonal and a bullet in row i and column j of the array if $i > j$ and the vertices $\sigma(i)$ and $\sigma(j)$ are adjacent.

An example is shown below for the ordered undirected graph in the figure (to understand this visualization, see Ordered Undirected Graphs) we use the

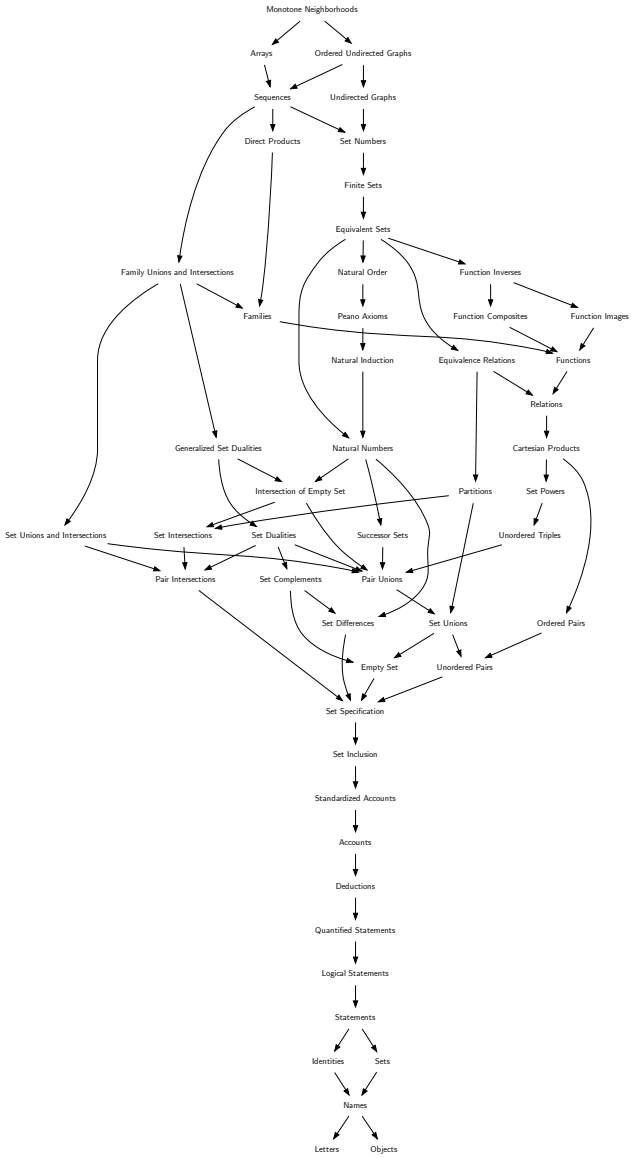
$$\begin{bmatrix} a & & & & \\ \bullet & c & & & \\ & \bullet & d & & \\ \bullet & & \bullet & b & \\ \bullet & \bullet & \bullet & \bullet & e \end{bmatrix}$$

In this array representation the higher and lower neighborhoods are easily identified. The indices of the elements of

[1]Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.

$\mathsf{adj}^+(v)$ are the column indices of the entries in row $\sigma^{-1}(v)$ of the array. For example, $\sigma^{-1}(d) = 3$, and the only bullet entry in row three is c so $\mathsf{adj}^-(d) = \{c\}$. Likewise, $\mathsf{adj}^-(c) = \{a\}$. And so on. Similalry, the indices of $\mathsf{adj}^+(v)$ are the row indices of the entries in column $\sigma^{-1}(v)$. For example, $\sigma^{-1}(d)$ is 3, and there are indices 4 and 5 corresponding to b and e so $\mathsf{adj}^+(d) = \{b, e\}$. Likewise, $\mathsf{adj}^+(c) = \{d, e\}$.

For this reason, we use the notation $\mathsf{col}(v)$ and $\mathsf{row}(v)$ for the closed upper and lower neighborhoods. So $\mathsf{col}(v) = \mathsf{adj}^+(v) \cup \{v\}$ and $\mathsf{row}(v) = \mathsf{adj}^-(v) \cup \{v\}$.


Monotone Neighborhoods
Arrays
Ordered Undirected Graphs
Sequences
Undirected Graphs
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Peano Axioms
Function Composites
Function Images
Natural Induction
Equivalence Relations
Functions
Relations
Generalized Set Dualities
Natural Numbers
Cartesian Products
Intersection of Empty Set
Partitions
Set Powers
Set Unions and Intersections
Set Intersections
Set Dualities
Successor Sets
Unordered Triples
Pair Intersections
Set Complements
Pair Unions
Set Differences
Set Unions
Ordered Pairs
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
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Logical Statements
Statements
Identities
Sets
Names
Letters
Objects