



Why

We want a notion of distance between elements of the real line.

Definition

The *absolute value* of a real number is the greater of itself and its additive inverse. In other words, if x is positive, then the absolute value of x is x . If x is negative, then the absolute value of x is $-x$ (which would be a positive real number).

Notation

We denote the absolute value of a real number $x \in \mathbf{R}$ by $|x|$.

Distance

The absolute value can be interpreted as the distance between the point corresponding to the real number and the point corresponding to 0. We can generalize this idea. Consider $x, y \in \mathbf{R}$. If $x > y$, then $x - y > 0$ and so the distance between the corresponding points is $x - y$. If $x < y$ then $y - x > 0$, and so the distance is $y - x$.

The observation is that $|-x| = |x|$. So

$$|y - x| = |-(x - y)| = |x - y|.$$

So if we just care about the distance between the points corresponding to y and x , we can consider $|x - y|$, without regard for their order. In other words, the function $(x, y) \mapsto |x - y|$ is symmetric in x and y .

