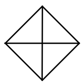


Why

We have seen that the matrices are a vector space. Are they an inner product space?

Definition

The *matrix scalar product* of $A \in \mathbf{R}^{n \times k}$ and $B \in \mathbf{R}^{n \times k}$ is the following product

$$\sum_{i=1}^{n} \sum_{j=1}^{k} a_{ij} b_{ij}.$$

Using the matrix trace, we can denote this as $\operatorname{tr} A^\top B$. Some authors call this the *Euclidean matrix scalar product*, *matrix inner product* or *Frobenius inner product*.

Proposition 1. *The matrix scalar product is an inner product.*[1]

For example, symmetry of the product is a consequence of the fact that a square matrix and its tranpose have identical traces. Commutativity of the trace yields $\operatorname{tr} A^\top B = \operatorname{tr} BA^\top$, where the LHS is the scalar product of $B^\top$ and $A^\top$. In other words, transposition "preserves" the matrix scalar product.

With this inner product, $\mathbf{R}^{n \times k}$ is a Euclidean vector space (see Inner Products) of dimension nk. For the case of $k = 1$, we recover a model[2] for the usual space $\mathbf{R}^n$.

Notation

We commonly denote the matrix inner product by $\langle A, B \rangle$.

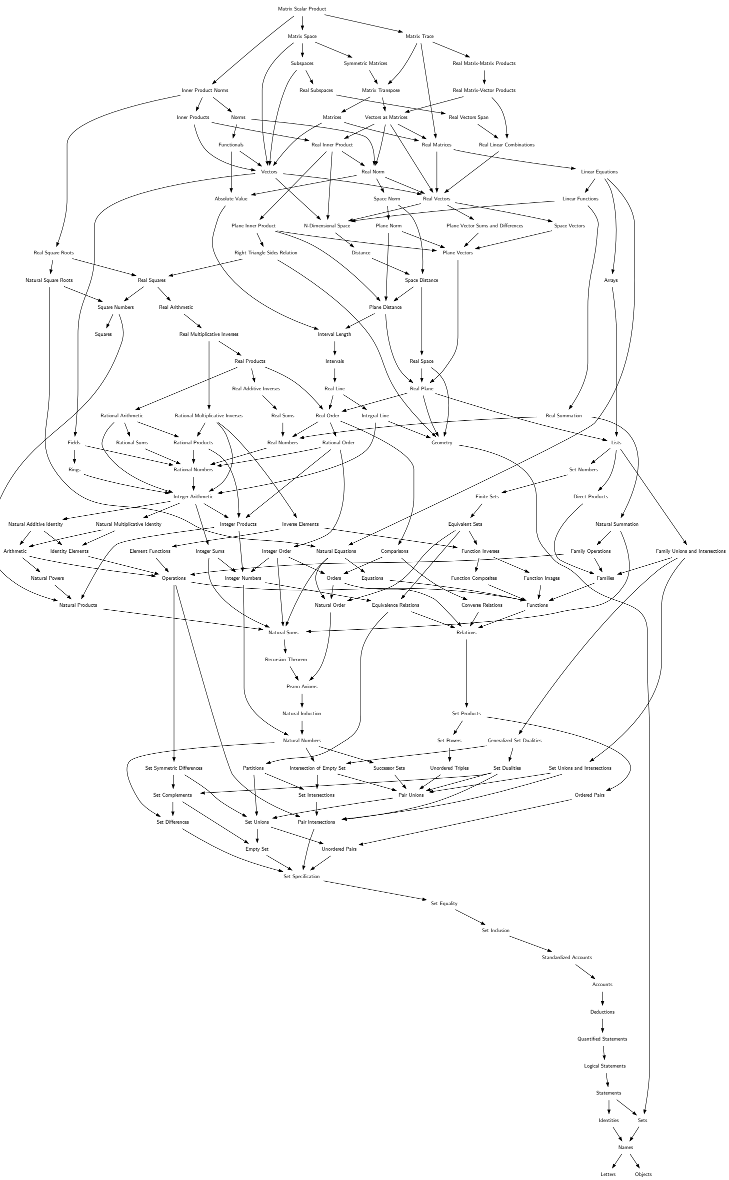
Induced norm

The matrix inner product induces a norm in the usual way. This norm is sometimes called the *Frobenius norm* and is often denoted for a matrix

[1] Future editions will provide an account.

[2] Future editions will define this term.

$A \in \mathbf{R}^{m \times n}$ by $\|A\|_F$.


Matrix Scalar Product
Matrix Space
Matrix Trace
Subspaces
Symmetric Matrices
Real Matrix-Matrix Products
Inner Product Norms
Real Subspaces
Matrix Transpose
Real Matrix-Vector Products
Inner Products
Norms
Matrices
Vectors as Matrices
Real Vectors Span
Functionals
Real Inner Product
Real Matrices
Real Linear Combinations
Vectors
Real Norm
Linear Equations
Absolute Value
Space Norm
Real Vectors
Linear Functions
Plane Inner Product
N-Dimensional Space
Plane Norm
Plane Vector Sums and Differences
Space Vectors
Real Square Roots
Right Triangle Sides Relation
Distance
Plane Vectors
Natural Square Roots
Real Squares
Space Distance
Arrays
Square Numbers
Real Arithmetic
Plane Distance
Squares
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Line
Real Plane
Rational Arithmetic
Rational Multiplicative Inverses
Real Sums
Real Order
Integral Line
Real Summation
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Rational Sums
Rational Products
Real Numbers
Rational Order
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Rings
Rational Numbers
Set Numbers
Integer Arithmetic
Finite Sets
Direct Products
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Inverse Elements
Equivalent Sets
Natural Summation
Arithmetic
Identity Elements
Element Functions
Integer Sums
Integer Order
Natural Equations
Comparisons
Function Inverses
Family Operations
Family Unions and Intersections
Natural Powers
Operations
Integer Numbers
Orders
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Function Images
Families
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Natural Order
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Converse Relations
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Natural Induction
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Set Powers
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Set Symmetric Differences
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Intersection of Empty Set
Successor Sets
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects