



## Why

We want to multiply real numbers.<sup>1</sup>

## Definition

The *real product* of two real numbers  $R$  and  $S$  is defined

1. if  $R$  or  $S$  is  $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$ , then the  $\{q \in \mathbf{Q} \mid q < 0_{\mathbf{Q}}\}$
2. otherwise,
  - (a) if  $R$  or  $S$  is  $0_{\mathbf{R}}$ , then  $0_{\mathbf{R}}$ .
  - (b) if  $R, S \neq 0_{\mathbf{R}}$  and  $0_{\mathbf{S}} \in R, S$ , let  $T$  be  $\{t \in \mathbf{Q} \mid r \in R, s \in S, r, s \geq 0_{\mathbf{Q}}, t = r \cdot s\}$  then  $T \cup \{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\}$ <sup>2</sup>
  - (c) If  $R, S \neq 0_{\mathbf{R}}$ ,  $0_{\mathbf{R}} \in R$  and  $0_{\mathbf{R}} \notin S$ , then the additive inverse of the product of  $-R$  with  $S$ .
  - (d) If  $R, S \neq 0_{\mathbf{R}}$ ,  $0_{\mathbf{R}} \notin R$  and  $0_{\mathbf{R}} \in S$ , then the additive inverse of the product of  $R$  with  $-S$ .
  - (e) If  $R, S \neq 0_{\mathbf{R}}$ , and  $0_{\mathbf{R}} \notin R, S$ , then the product of  $-R$  with  $-S$ .

## Notation

We denote the product of two real numbers  $x$  and  $y$  by  $x \cdot y$ .

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<sup>1</sup>Future editions will expand.

<sup>2</sup>We use  $\geq$  in the usual way, it will be defined earlier in future editions.

## Properties

**Proposition 1** (Associative).  $x + (y + z) = (x + y) + z$

**Proposition 2** (Commutative).  $x + y = y + x$

**Proposition 3** (Identity). *The set of all rationals less than  $1_{\mathbf{Q}}$  is the multiplicative identity.*

We denote the the multiplicative identity by  $1_{\mathbf{R}}$ . When it is clear from context, we call  $1_{\mathbf{R}}$  “one”.

