Why

The identification of $\mathbf{C}$ with a plane leads $\mathbf{C}$ to naturally inherit $\mathbf{R}^2$'s notion of distance.

Definition

The *absolute value* or *modulus* of $z = (x, y) \in \mathbf{C}$ is the distance of z to the origin. If $z \in \mathbf{C}$, then the modulus of z is

$$\sqrt{x^2 + y^2}.$$

In other words, the modulus of z is the distance (in $\mathbf{R}^2$ of $z = (x, y)$ from the origin $(0, 0)$.

Notation

We denote the modulus of z by $|z|$.

Properties

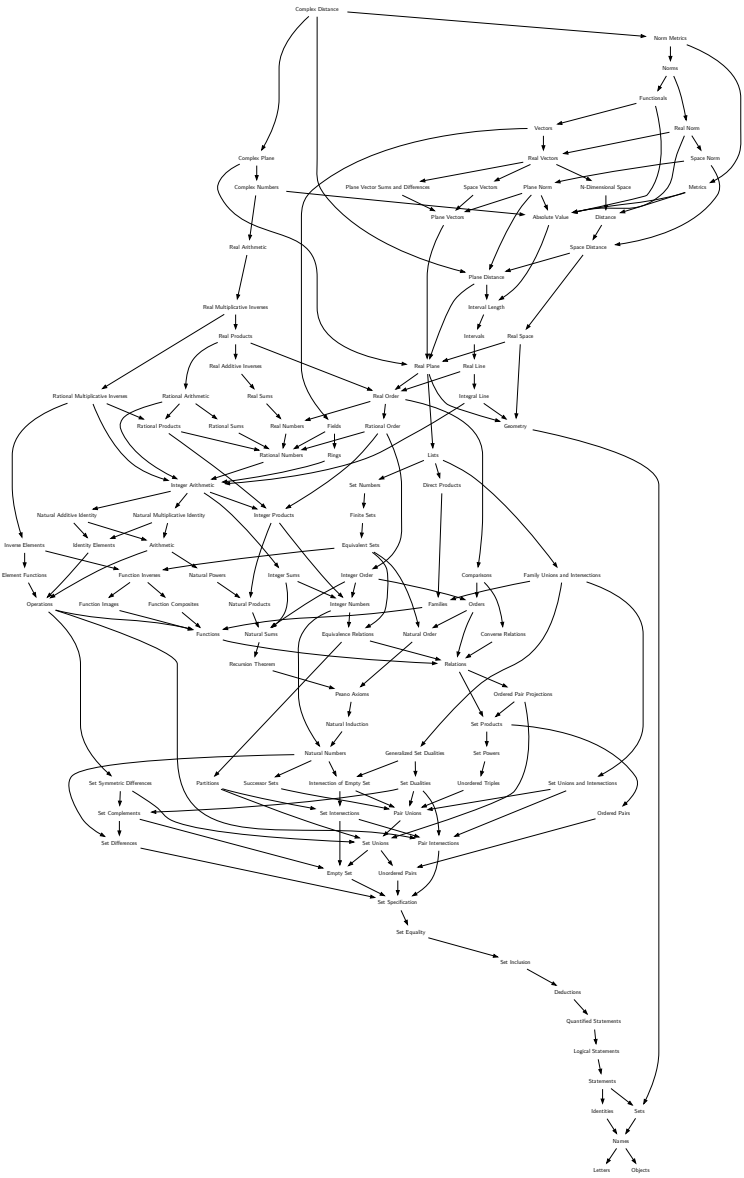
Proposition 1 (Triangle Inequality)**.** *For all $z, w \in \mathbf{C}$,*

$$|z + w| \leq |z| + |w|.$$

Also, for all $z \in \mathbf{C}$, we have $|\mathrm{Re}(z)| \leq |z|$ and $|\mathrm{Im}(z)| \leq |z|$, and for all $z, w \in \mathbf{C}$,[1]

$$||z| - |w|| \leq |z - w|.$$

[1] This follows from the triangle inequality. Future editions will include an account.


Complex Distance
Norm Metrics
Norms
Functionals
Vectors
Real Norm
Complex Plane
Real Vectors
Space Norm
Complex Numbers
Plane Vector Sums and Differences
Space Vectors
Plane Norm
N-Dimensional Space
Metrics
Plane Vectors
Absolute Value
Distance
Real Arithmetic
Space Distance
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Plane
Real Line
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Rational Products
Rational Sums
Real Numbers
Fields
Rational Order
Geometry
Rational Numbers
Rings
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Inverse Elements
Identity Elements
Arithmetic
Equivalent Sets
Element Functions
Function Inverses
Natural Powers
Integer Sums
Integer Order
Comparisons
Family Unions and Intersections
Operations
Function Images
Function Composites
Natural Products
Integer Numbers
Families
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Equivalence Relations
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Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
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Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
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Letters
Objects