

## Metrics

## 1 Why

We want to talk about a set with a prescribed quantitative degree of closeness (or distance) between its elements.

## 2 Definition

The correspondences which serve as a degree of closeness, or measure of distance, must satisfy our notions of distances previously developed.

A function on ordered pairs which does not depend on the order of the elements so considered is *symmetric*. A function into the real numbers which takes only non-negative values is *non-negative*. A repeated pair is an ordered pair of the same element twice. A function which satisfies a triangle inequality for any three elements is *triangularly transitive*.

A metric (or distance function) is a function on ordered pairs of elements of a set which is symmetric, non-negative, zero only on repeated pairs, and triangularly transitive. A metric space is an ordered pair: a nonempty set with a metric on the set.

In a metric space, we say that one pair of objects is *closer* together if the metric of the first pair is smaller than the metric value of the second pair.

Notice that a set can be made into different metric spaces by using different metrics.

## 2.1 Notation

Let A be a set and let R be the set of real numbers. We commonly denote a metric by the letter d, as a mnemonic for "distance." Let  $d: A \times A \to R$ . Then d is a metric if:

1. it is non-negative, which we tend to denote by

$$d(a,b) \ge 0, \quad \forall a,b \in A.$$

2. it is 0 only on repeated pairs, which we tend to denote by

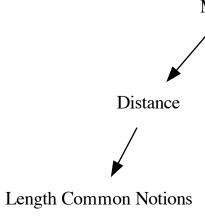
$$d(a,b) = 0 \Leftrightarrow a = b, \quad \forall a, b \in A.$$

3. it is symmetric, which we tend to denote by:

$$d(a,b) = d(b,a), \quad \forall a, b \in A.$$

4. it is triangularly transitive, which we tend to denote by

$$d(a,b) \le d(a,c) + d(c,b), \quad \forall a,b,c \in A.$$



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