



## Definition

The *variance* of a square-integrable real-valued random variable is the expectation of its square less its expectation squared.

## Notation

Suppose  $f : X \rightarrow \mathbf{R}$  is a random variable on a probability space  $(X, \mathcal{A}, P)$ . We denote the variance of  $f$  by  $\text{var } f$ , so that

$$\text{var } f = \mathbf{E}(f^2) - (\mathbf{E}(f))^2.$$

## Results

**Proposition 1.** *If a random variable on a probability space is square integrable then it is integrable.*

*Proof.* The  $L^p$  spaces are nested for finite measures. □

**Proposition 2.** *The variance of a square-integrable real-valued random variable is the expectation of the square of the difference between the random variable and its expectation.*



