Why

Given a subset of the real line, what is its length?

Background

Let $a, b \in R$ with $a \leq b$. The *length* of the closed interval of the real numbers $[a, b]$ is $b - a$. The length is non-negative.

A family $\{A_\alpha\}_{\alpha \in I}$ is *disjoint* if for $\alpha, \beta \in I$, $\alpha \neq \beta$, then $A_\alpha \cap A_\beta = \varnothing$. A set A can be *partioned* into a family if there exists a disjoint family whose union is A. A set $A \subset R$ is *simple* if it can be partitioned into a countable family whose members are closed intervals. The above discussion suggests that we should define the length of a simple set as the sum of the lengths of sets which parition it.

The above discussion suggests that if we wish to define a function length : $2^R \to R \cup \{-\infty, \infty\}$, we should ask that (1) length$(A) \geq 0$, (2) length$([a, b]) = b - a$, (3) for disjoint closed intervals $\{A_n\}_{n \in N}$, length$(A_i) = \sum_i$ length(A_i), and (4) for all $A \subset R$ and $a \in R$, length$(A + x) =$ length(A).

Converse

Define the equivalence relation $\sim$ on R by by $x \sim y$ if $x \sim y \in Q$

Notation

Let A be a set and $\mathcal{A} \subset \mathcal{P}(A)$. We denote the subset algebra of A and $\mathcal{A}$ by $(A, \mathcal{A})$, read aloud as "A, script A."

Properties

Prop. 1. *For any set A, 2^A is a sigma algebra.*

Prop. 2. *The intersection of a family of sigma algebras is a sigma algebra.*

Generation

Prop. 3. *Let A a set and $\mathcal{B}$ a set of subsets. There is a unique smallest sigma algebra $(A, \mathcal{A})$ with $\mathcal{B} \subset \mathcal{A}$.*

We call the unique smallest sigma algebra containing B the *generated sigma algebra* of B.

Real Length Impossible

Real Numbers

Rational Numbers

Integer Arithmetic

Natural Additive Identity

Natural Multiplicative Identity

Identity Elements

Arithmetic

Natural Powers

Integer Products

Natural Products

Integer Sums

Operations

Natural Sums

Integer Numbers

Equivalence Relations

Functions

Recursion Theorem

Relations

Peano Axioms

Partitions

Set Products

Natural Induction

Ordered Pairs

Set Powers

Natural Numbers

Intersection of Empty Set

Unordered Triples

Successor Sets

Set Symmetric Differences

Set Intersections

Pair Unions

Pair Intersections

Set Complements

Set Unions

Empty Set

Set Differences

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects