



## Why

### Definition

Let  $(\Omega, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $I$  be an index set. A *normal process* (or *gaussian process*)<sup>1</sup>  $x : I \rightarrow (\Omega \rightarrow \mathbf{R})$  on  $I$  is a family of real-valued random variables with the property that any subset of the range of this family has a multivariate normal density. There exists a  $m : I \rightarrow \mathbf{R}$  and positive definite  $k : I \times I \rightarrow \mathbf{R}$  with the property that if  $J \subset I$ ,  $|J| = d$ , then  $x_J \sim \mathcal{N}(m(J), k(J \times J))$ . In other words, for each  $i \in I$ ,  $x_i : \Omega \rightarrow \mathbf{R}$  is a random variable. And  $x_J : \Omega \rightarrow \mathbf{R}^d$  is a Gaussian random variable. We call  $m$  is the *mean function* and  $k$  is the *covariance function*.

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<sup>1</sup>The choice of “normal” is a result of the Bourbaki project’s convention to eschew historical names. Though here, as in **Multivariate Normals** the language of the project is nonstandard. The community would seem to prefer Gaussian.



