## Definition

A conditional distribution (density) $q : Y \times X \to \mathbf{R}$ is *functionally parametrizable* if there exists a function $f : X \to \Theta$ and distribution (density) family $\{p^{(\theta)} : Y \to \mathbf{R}\}_{\theta \in \Theta}$ satisfying $q(y, x) \equiv p^{(f(x))}(\gamma)$ for all $x \in X$ and $y \in Y$.

In this case we call $f$ the *parameterizer* and we call $\{p^{(\theta)}\}_{\theta \in \Theta}$ the *parameterized family.* A *parameterized conditional distribution* is an ordered pair whose first coordinate is a function from $X$ to $\Theta$ and whose second coordinate is a family of distributions on $X$ with parameter set $\Theta$. For a particular choice of parameterizer and family, it induces a conditional distribution.

Since all conditional distributions are functionally parametrizable (consider $\{q(\cdot, \xi)\}_{\xi \in X}$ with parameters $X$ and identity parameterizer), we are interested in parameterizers and parameterized families that are simple. Said differently, we are interested in approximating a conditional distribution by selecting an appropriate parameterizer and parameterized family.

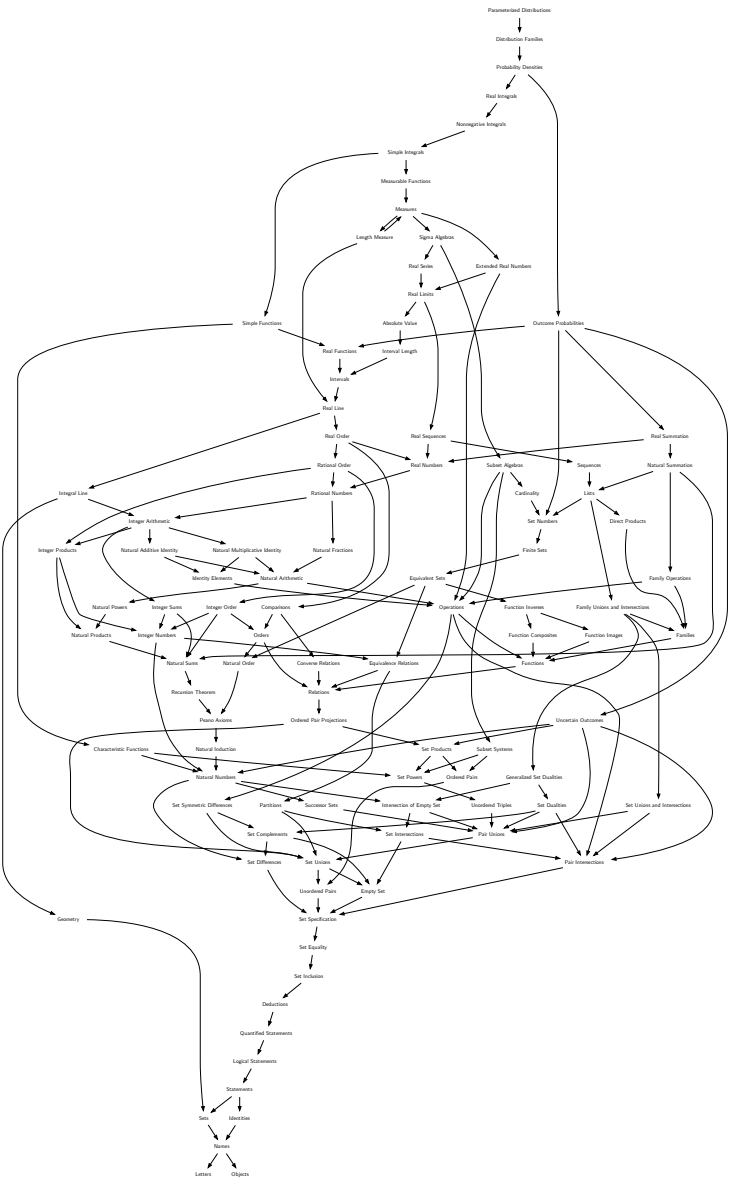
If $\{f_\phi : X \to \Theta\}_{\phi \in \Phi}$ is a family of functions and $\{q^{(\theta)}\}$ is a family of distributions, then $\{p^{(\phi)} : X \times Z \to \mathbf{R}\}_\phi$ defined by $p^{(\phi)}(\cdot, \zeta) \equiv q^{f_\phi(\zeta)}$ is a conditional distribution family called a *functionally parameterized conditional distribution family.* In other words, by selecting some parameters $\phi$, we induce a conditional distribution $p^{(\phi)} : X \times Z \to \mathbf{R}$

We similarly define *parameterized conditional densities* and *functionally parameterized conditional density families.*

### Basic example

Let $Z = \{1, 2\}$ and $X = \mathbf{R}$. Let $f : \{1, 2\} \to \mathbf{R} \times \mathbf{R}_+$ be defined by $f(1) = (\mu_1, \sigma_1)$ and $f(2) = (\mu_2, \sigma_2)$. Let $\{g^{(\theta)}\}_\theta$ be the normal family. Then $(f, \{g^{(\theta)}\})$ is a *functionally parameterized conditional density.*[1]

[1] Future editions will modify.


Parameterized Distributions
Distribution Families
Probability Densities
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Length Measure
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Absolute Value
Simple Functions
Outcome Probabilities
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Real Summation
Rational Order
Real Numbers
Subset Algebras
Sequences
Natural Summation
Integral Line
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Natural Fractions
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Family Operations
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Function Composites
Function Images
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Characteristic Functions
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects