



## Why

We can use families to think about unions and intersections.

## Family unions

Let  $A : I \rightarrow \mathcal{P}(X)$  be a family of subsets. We refer to the union (see Set Unions) of the range (see Relations) of the family the *family union*. We denote it  $\cup_{i \in I} A_i$ .

**Proposition 1.**  $(x \in \cup_{i \in I} A_i) \longleftrightarrow (\exists i)(x \in A_i)$

If  $I = \{a, b\}$  is a pair with  $a \neq b$ , then  $\cup_{i \in I} A_i = A_a \cup A_b$ .

There is no loss of generality in considering family unions. Every set of sets is a family: consider the identity function from the set of sets to itself.

We can also show generalized associative and commutative law<sup>1</sup> for unions.

**Proposition 2.** Let  $\{I_j\}$  be a family of sets and define  $K = \cup_j I_j$ . Then  $\cup_{k \in K} A_k = \cup_{j \in J} (\cup_{i \in I_j} A_i)$ .<sup>2</sup>

## Family intersection

If we have a nonempty family of subsets  $A : I \rightarrow \mathcal{P}(X)$ , we call the intersection (see Set Intersections) of the range of the family the *family intersection*. We denote it  $\cap_{i \in I} A_i$ .

**Proposition 3.**  $x \in \cap_{i \in I} A_i \longleftrightarrow (\forall i)(x \in A_i)$

Similarly we can derive associative and commutative laws for intersection.<sup>3</sup> They can be derived as for unions, or from the facts of unions using generalized DeMorgan's laws (see Generalized Set Dualities).

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<sup>1</sup>The commutative law will appear in future editions.

<sup>2</sup>An account will appear in future editions.

<sup>3</sup>Statements of these will be given in future editions.

## Connections

The following are easy.<sup>4</sup>

Let  $\{A_i\}$  be a family of subsets of  $X$  and let  $B \subset X$ .

**Proposition 4.**  $B \cap \bigcup_i A_i = \bigcup_i (B \cap A_i)$

**Proposition 5.**  $B \cup \bigcap_i A_i = \bigcap_i (B \cup A_i)$

Let  $\{A_i\}$  and  $\{B_j\}$  be families of sets.<sup>5</sup>

**Proposition 6.**  $(\bigcup_i A_i) \cap (\bigcup_j B_j) = \bigcup_{i,j} (A_i \cap B_j)$

**Proposition 7.**  $(\bigcap_i A_i) \cup (\bigcap_j B_j) = \bigcap_{i,j} (A_i \cup B_j)$ .

**Proposition 8.**  $\bigcap_i X_i \subset X_j \subset \bigcup_i X_i$  for each  $j$ .

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<sup>4</sup>Nevertheless, full accounts will appear in future editions.

<sup>5</sup>An account of the notation used and the proofs will appear in future editions.

