The Bourbaki Project

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Edition 1 — Summer 2021

Printed in Menlo Park, California

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Why

We want to communicate and remember.

Discussion

A *language* is a conventional correspondence of sounds to affections of mind. We deliberately leave the definition of *affections* vague. A *spoken word* is a succession of sounds. By using these sounds, our mind can communicate with other minds.

A *script* is a collection of written marks called *letters*. In *phonetic* languages, the letters correspond to sounds. A *written word* is a succession of letters. This succession of letters corresponds to a succession of sounds and so a written word corresponds to a spoken word. By making marks, we communicate with other minds—including our own—in the future.

To write this sheet, we use Latin letters arranged into *written words* which are meant to denote the *spoken words* of the English language. The written words on this page are several letters one after the other. For example, the word “word" is composed of the letters “w", “o", “r", “d".

These endeavors are at once obvious and remarkable. They are obvious by their prevalence, and remarkable by their success. We do not long forget the difficulty in communicating affections of the mind, however, and this leads us to be very particular about how we communicate throughout these sheets.

Latin letters

We will start by officially introducing the letters of the Latin language. These come in two kinds, or cases. The lower case latin letters.

a	b	c	d	e	f	g	h	i
j	k	l	m	n	o	p	q	r
s	t	u	v	w	x	y	z	

And the upper case latin letters.

A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	

So, A is the upper case of a, and a the lower case of A. Similarly with b and B, with c and C, and all the rest.

Arabic numerals

We will also use the following symbols. They are called the Arabic numberals.

0 1 2 3 4 5 6 7 8 9

Why

We want to talk and write about things.

Definition

We use the word *object* with its usual sense in the English language. Objects that we can touch we call *tangible*. Otherwise, we say that the object is *intangible*.

Examples

We pick up a pebble for an example of a tangible object. The pebble is an object. We can hold and touch it. And because we can touch it, the pebble is tangible.

We consider the color of the pebble as an example of an intangible object. The color is an object also, even though we can not hold it or touch it. Because we can not touch it, the color is intangible. These sheets discuss other intangible objects and little else besides.

Why

We (still) want to talk and write about things.

Names

We must use sounds to speak about objects. Likewise we must use symbols to write about objects. If we take some symbols like those in *Letters*, and we write them we say that they symbols *denote* the object. We call the collection of the symbols the *name* of the object. In these sheets, we will mostly tend to use the upper and lower case latin letters to denote objects. Sometimes, however, we will use the Arabic numerals, or add a mark like $'$ to latin letters, or we may use both letters and numerals to denote objects.

We are, however, using these same symbols on these pages for spoken words of the English language. So we need to distinguish when a symbol or group of symbols is meant to denote an object. We could box our symbols, and agree that everything in the box denotes the object. For example, $\boxed{\text{A}}$. Or we could underline our symbols, like $\underline{\text{A}}$. Either would work. The box would work particularly well for using two symbols to denote an object. For example, $\boxed{\text{AA}}$.And $\boxed{\text{A}}\!\boxed{\text{A}}$ is clearly different from $\boxed{\text{AA}}$.

Experience shows that using two letters twice is often confusing, and if accents are used, not needed. Rather than $\boxed{\text{AA}}$ why not use $\boxed{\text{A'}}$. Instead of $\boxed{\text{AAA}}$, use $\boxed{\text{A''}}$. Then experience also shows that the complications like boxes around symbols are unecessary. In other words, we agree never to use $\boxed{\text{A'B'}}$. If we have $\boxed{\text{A'}}\boxed{\text{B'}}$ there is really no confusion in dropping the boxes and writing A'B'. But we still want some way of distinguishing that we are talking about objects.

In these sheets, then, we will indicate that we are denoting an object by using italics. Instead of $\boxed{\text{A}}$, we will use A. Instead of $\boxed{\text{A'}}$, we will use A'. Experience shows that this practice is subtle, but easy enough to distinguish. This choice has the added benefit of agreeing with the traditional and modern practice. And the practice is several millenia old—so it ought to suit us in these sheets.

There is an odd aspect in these considerations. A may denote itself, that particular mark on the page. There is no helping it. As soon as we use some symbols to identify any object, pathological things like this may happen.

An interpretation of this peculiarity is that names are objects. In other words, the name is an abstract object, it is that which we use to refer to another object. It is the things which points to some object. It has associated with it the several places on the page where we write the name.

Why

We can give the same object two different names.

Definition

An object *is* itself. If the object denoted by one name is the same as the object denoted by a second name, then we say that the two names are *equal*.

Let A denote an object and let B denote an object. We say "A equals B" as a shorthand for "the object denoted by A is the same as the object denoted by B". In other words, A and B are two names for the same object.

"A equals B" means the same as "B equals A". This is because the identity of the object is not changed by the order in which the names are given.

Let A denote an object. Since every object is the same as itself, the object denoted by A is the same as the object denoted by A. We say "A equals A". In other words, every name equals itself.

Why

We want to talk about none, one, or several objects considered as an aggregate.

Definition

A *set* is an intangible object. We think of it as several objects considered as a whole. We say that these objects *belong* to the set. They are the set's *members* or *elements*.

The objects a set contains may be other sets. In other words, an element of a set may be another set. This may be subtle at first glance, but becomes familiar with experience.

We call a set which contains no objects *empty*. Otherwise we call a set *nonempty*.

Denoting a set

Let A denote a set. Then A is a name for an object. That object is a set. So A is a name for an object which is a grouping of other objects.

Why

We give some examples of objects and sets.

Examples

For familiar examples, let us start with some tangible objects. Find, or call to mind, a deck of playing cards.

First, consider the set of all the cards. This set contains fifty-two elements. Second, consider the set of cards whose suit is hearts. This set contains thirteen elements: the ace, two, three, four, five, six, seven, eight, nine, ten, jack, queen, and king of hearts. Third, consider the set of twos. This set contains four elements: the two of clubs, the two of spades, the two of hearts, and the two of diamonds.

We can imagine many more sets of cards. If we are holding a deck, each of these can be made tangible: we can touch the elements of the set. But the set itself is always abstract: we can not touch it. It is the idea of the group as distinct from any individual member.

Moreover, the elements of a set need not be tangible. First, consider the set consisting of the suits of the playing card: hearts, diamonds, spades, and clubs. This set has four elements. Each element is a suit, whatever that is.

Second, consider the set consisting of the card types. This set has thirteen elements: ace, two, three, four, five, six, seven, eight, nine, ten, jack, queen, king. The subtlety here is that this set is different than the set of hearts, namely those thirteen cards which are hearts. However these sets are similar: they both have thirteen elements, and there is a natural correspondence between their elements: the ace of hearts with the type ace, the two of hearts with the type two, and so on.

Of course, sets need have nothing to do with playing cards. For example, consider the set of seasons: autumn, winter, spring, and summer. This set has four elements. For another example, consider the set of Latin letters: a, b, c, . . . , x, y, z. This set has twenty-six elements. Finally, consider a pack of wolves, or a bunch of grapes, or a flock of pigeons.

Why

We want to write about objects belonging to sets.

Definition

Let A denote a set; in other words, an intangible object which has some objects as members. Let a denote an object. Recall that if two names refer to the same object, the names are equal. Similarly, if the object denoted by a is an element of the set denoted by A, then we say that the former name belongs to the latter name. We write that the name a belongs to the name A by $a \in A$.

We read this sequence of symbols aloud as "a in A." The symbol $\in$ is a stylized lower case Greek letter ε, which is a mnemonic for $\grave{\varepsilon}\sigma\tau\acute{\iota}$ which means "belongs" in ancient greek. Since in English, ε is read aloud "ehp-sih-lawn," $\in$ is also a mnemonic for "element of". Of course, we must take care. The first name is not an element on the second name. Rather, the object denoted by the first name is an element of the set (object) denoted by the second name.

We tend to denote sets by upper case latin letters: for example, A, B, and C. To aid our memory, we tend to use the lower case form of the letter for an element of the set. For example, let A and B denote nonempty sets. We tend to denote by a an object which is an element of A. And similarly, we tend to denote by b an object which is an element of B

Why

We want to succinctly and unambiguously record statements about names and the objects and sets of objects which the names refer to.

Definition

So far we have three kinds of sentences which we would like to record.

1. **Names** We want to declare that we are using a particular name. When we start with a blank slate, once we have declared a name we do not use the same name twice. So we will want to keep track of which names are in use.

2. **Identities** We want to declare that two names are equal. When we have names, we want to say when the identities of those names are the same.

3. **Belongings** We want to declare when the object represented by one name is an element of the set represented by a second name.

Our main purpose is to keep a list of the statements we have made. Experience suggests that we start with an example.

Suppose we want to summarize the following english language description of some names and objects. Denote an object by a. Denote the same object by a' In other words, a