Nets

1 Why

We want to generalize the notion of sequence.

2 Definition

Recall that a sequence is a function on the naturals. The naturals are ordered and have the property that we can always go further out. If handed two natural numbers m and n, we can always find another, for example $\max\{m, n\} + 1$, larger than m and n. We might think of larger as being further out from the first natural number, namely 1. These observations motivate definining a directed set.

Definition 1 *A **directed set** is a set D with a partial order $\preceq$ satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.*

Definition 2 *A **net** is a function on a directed set.*

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

2.1 Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \to A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.