Metric Space Examples

Why

We give examples of metric spaces.

Example

Example 1. *Let n be a natural number. Let A be $\mathbf{R}^n$ and define $d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$ by*

$$d(a, b) = \sqrt{(a_1 - b_1)^2 + \cdots + (x_n - y_n)^2}.$$

(A, d) is a metric space.

Example 2. *Let A be the unit circle in $\mathbf{R}^2$. So $A = \{x \in R^2 \mid x_1^2 + b^2 = 1\}$. Let $d_1 : A \times A \to \mathbf{R}$ defined by*

$$d(a, b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}.$$

Let $d_2 : A \times A \to \mathbf{R}$ defined as the arc length between the two points. Both (A, d_1) and (A, d_2) are metric spaces.

Example 3. *Let $A = C([0, 1], \mathbf{R})$. Let $d_1 : A \times A \to \mathbf{R}$ be such that*

$$d_1(a, b) = \max_{x \in [0,1]} |a(x) - b(x)|.$$

Let λ be the outer cover measure. Let $d_2 : A \times A \to R$ be such that

$$d_2(a, b) = \int_{[0,1]} |f - g| d\lambda.$$

Both (A, d_1) and (A, d_2) metric spaces.

Metric Space Examples

Metrics

Distance — Absolute Value

Space Distance

Plane Distance

Interval Length

Real Space

Real Plane — Real Line

Intervals

Real Order — Integral Line

Geometry — Rational Order — Real Numbers

Rational Numbers

Lists — Integer Arithmetic

Set Numbers — Natural Multiplicative Identity — Natural Additive Identity — Integer Products

Finite Sets — Identity Elements — Arithmetic

Direct Products — Equivalent Sets — Natural Powers

Family Unions and Intersections — Function Inverses — Comparisons — Integer Order — Integer Sums — Natural Products

Families — Function Composites — Function Images — Operations — Orders — Integer Numbers — Natural Sums

Functions — Converse Relations — Equivalence Relations — Natural Order — Recursion Theorem

Relations — Peano Axioms

Set Products — Natural Induction

Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Set Dualities — Unordered Triples — Successor Sets — Intersection of Empty Set — Partitions — Set Symmetric Differences

Ordered Pairs — Pair Unions — Set Intersections — Set Complements

Pair Intersections — Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects