



Why

We want a notion of “uncertainty” for a real-valued (continuous) random variable.

Definition

The *relative entropy* of a probability density function is the integral of the density against the negative log of the density.

Notation

Let $f : \mathbf{R}^n \rightarrow \mathbf{R}$ be a probability density function. The differential entropy of f is

$$-\int f \log f$$

We denote the differential entropy of f by $h(f)$.

Example

Let $x : \Omega \rightarrow \mathbf{R}$ be uniform on $[0, 1/2]$. Then $h(x) = \log 1/2 < 0$.

Problems

We have $h(ax) = h(x) + \log |a|$. In generally $h(Ax) = h(x) + \log |A|$.

Differences still meaningful

Even though the value of the differential entropy is not necessarily a good analogy to discrete entropy, differences still are.

In particular, the following holds

$$I(X;Y) = H(Y) - H(Y \mid X) = H(X) = H(X \mid Y)$$

