Subspaces

1 Why

TODO

2 Definition

A *subspace* of a vector space is a subset of vectors that is itself a vector space. In other words, a subspace is a subset of a vector space which is closed under vector addition and scalar multiplication.

3 Notation

Let (V, F) be a vector space. Let $U \subset V$ with

$$\alpha u + \beta v \in U$$

for all $\alpha, \beta \in \mathsf{F}$ and $u, v \in U$. Then U is a subspace of (V, F).

4 Examples

The entire set of vectors is a subspace. The set consisting only of the zero vector is a subspace; we call this the *zero vector space*. These two subspaces are called *trival subspaces*. A *nontrivial subspace* is a subspace that is not trivial.

5 Properties

Proposition 1. *The intersection of a family of subspaces is a subspace.*

Proposition 2. *There exists a family of subspaces whose union is not a subspace;*

Remark 3. *In other words: the union of a family subspaces need not be a subspace.*

Proposition 4. *A subspace must contain the zero vector; in other words, every subspace is nonempty.*