Why

For the difference of two (signed) measures to be well-defined, we need one of the two to be finite. Otherwise, the measure of the difference on the base set involves subtracting ∞ from ∞.

Definition

A *finite* signed measure is one for which the measure of every set is finite. This condition is equivalent to the base set having finite measure (see below).

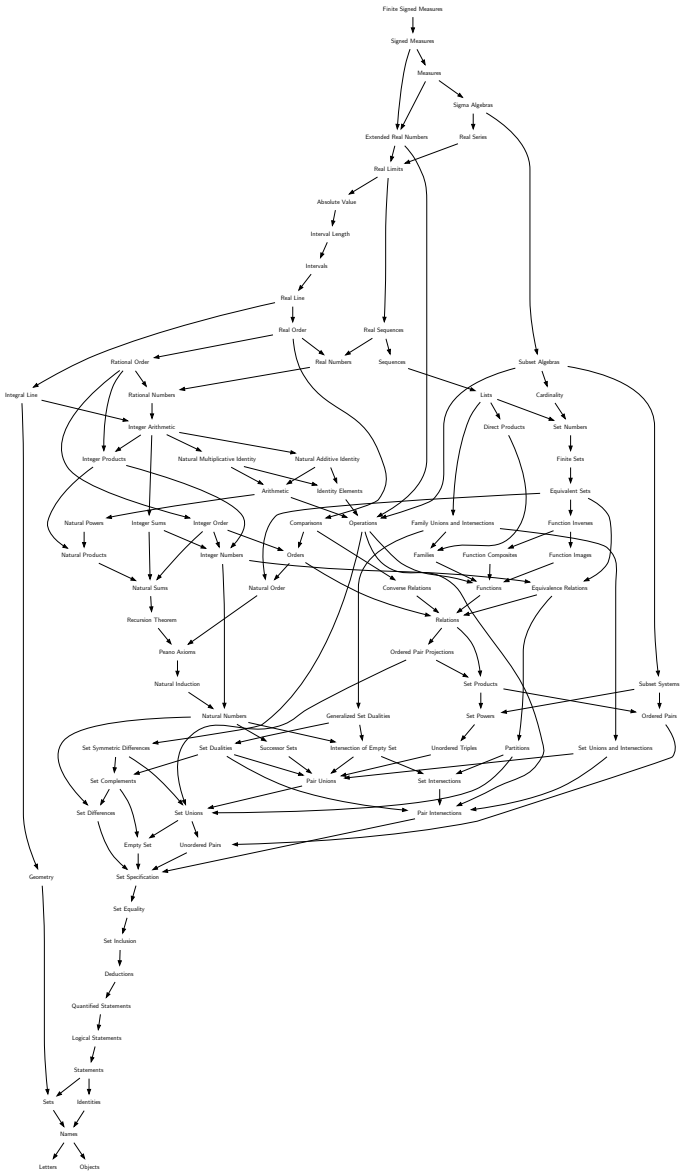
Result

Proposition 1. *A signed measure is finite if and only if it is finite on the base set.*

Proof. Let $(X, \mathcal{A})$ be a measurable space. Let $\mu : \mathcal{A} \to [-\infty, \infty]$ be a signed measure.

($\Rightarrow$) If μ is finite, then $\mu(X)$ is finite since $X \in \mathcal{A}$.

($\Leftarrow$) Next, suppose $\mu(X)$ is finite. Let $A \in \mathcal{A}$. Then $X = A \cup (X - A)$, with these sets disjoint, so by countable additivity of μ, $\mu(X) = \mu(A) + \mu(X - A)$. Since $\mu(X)$ finite, $\mu(A)$ and $\mu(X - A)$ are both finite. $\square$


Finite Signed Measures
Signed Measures
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Subset Algebras
Integral Line
Rational Numbers
Lists
Cardinality
Integer Arithmetic
Direct Products
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Family Unions and Intersections
Function Inverses
Natural Products
Integer Numbers
Orders
Families
Function Composites
Function Images
Natural Sums
Natural Order
Converse Relations
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects