Definition

Let $A \in \mathbf{S}^n$ (i.e., $A \in \mathbf{R}^{n \times n}$ and symmetric). A is *positive definite* if, for all $x \neq 0 \in \mathbf{R}^d$,

$$x^\top A x > 0.$$

A is *positive semidefinite* (or *nonnegative definite*) if, for all $x \in \mathbf{R}^d$

$$x^\top A x \geq 0.$$

Notation

We denote the set of positive definite d by d matrices by $\mathbf{S}^d_{++}$. We denote the set of positive semidefinite d by d matrices by $\mathbf{S}^d_+$.

Positive Definite Matrices

Quadratic Form

Symmetric Real Matrices

Matrix Trace

Real Matrix-Matrix Products

Matrix Transpose

Real Matrix-Vector Products

Vectors as Matrices

Real Linear Combinations

Real Matrices

Real Inner Product

Multivariate Real Polynomials

Linear Equations

Real Norm

Space Inner Product

Multivariate Functions

Arrays

Linear Functions

Space Norm

Real Vectors

Plane Norm

N-Dimensional Space

Plane Inner Product

Plane Vector Sums and Differences

Space Vectors

Real Polynomials

Absolute Value

Distance

Right Triangle Sides Relation

Plane Vectors

Space Distance

Real Square

Real Arithmetic

Plane Distance

Real Multiplicative Inverse

Interval Length

Real Space

Real Products

Intervals

Square Number

Real Additive Inverse

Real Line

Real Plane

Squares

Real Summation

Rational Multiplicative Inverse

Rational Arithmetic

Real Sums

Real Order

Integral Line

Geometry

Rational Products

Rational Sums

Real Numbers

Rational Order

Lists

Rational Number

Direct Products

Set Number

Integer Arithmetic

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Natural Summation

Inverse Elements

Equivalent Sets

Identity Elements

Arithmetic

Natural Powers

Integer Sums

Integer Order

Comparisons

Family Unions and Intersections

Natural Equation

Family Operations

Function Inverse

Element Functions

Natural Products

Integer Numbers

Orders

Converse Relations

Families

Equations

Function Composite

Function Image

Operations

Natural Order

Natural Sums

Equivalence Relation

Functions

Relations

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Set Products

Natural Induction

Set Power

Generalized Set Dualities

Natural Number

Unordered Triples

Partitions

Set Unions and Intersections

Set Dualities

Set Symmetric Differences

Successor Sets

Intersection of Empty Set

Ordered Pair

Set Complements

Pair Unions

Set Intersections

Pair Intersections

Set Differences

Set Unions

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects

4