Why

We want to discuss making several decisions over time.

Definition

Let A^1 be a set of actions and $\mathcal{O}^1 = \{O^1_a\}$ a family of outcome sets. Define $H^1 = \{(a, o) \mid a \in A^1, o \in O^1_a\}$. Let $\mathcal{A}^2 = \{A^2_{h^1}\}_{h^1 \in H^1}$ be a family of action sets. Define

$$\tilde{H}^2 = \{(a_1, o_1, a_2) \mid (a_1, o_1) \in H^1, a_2 \in A^2_{a_1 o_1}\}.$$

Let $\mathcal{O}^2 = \{O^2_h\}_{h \in \tilde{H}^2}$ be a family of outcome sets. Define

$$H^2 = \{(a_1, o_1, a_2, o_2) \mid h = (a_1, o_1, a_2) \in \tilde{H}^2, o_2 \in O_h\}.$$

Let $\preceq$ be a total order on H^2. Then we call the sequence $((A^1, \mathcal{A}^2), (\mathcal{O}^1, \mathcal{O}^2), \preceq)$ a *two-stage decision problem*.

In general, let $\mathcal{A}^i = \{A^i_h\}_{h \in H^{i-1}}$ be a family of action sets, define $\tilde{H}^i$ by

$$\{(a_1, \ldots, o_{i-1}, a_i) \mid h = (a_1, \ldots, o_{i-1}) \in H^{i-1}, a_i \in A^i_h\}.$$

Let $\mathcal{O}^{i+1} = \{O^{i+1}_h\}_{h \in \tilde{H}^i}$ be a family of outcome sets. Define H^i by

$$\left\{(a_1, \ldots, a_i, o_i) \,\middle|\, h = (a_1, \ldots, a_i) \in \tilde{H}^i, o_i \in O^i_h\right\}.$$

For $i = 3, \ldots, n$. Let $\preceq$ be a total order on H^n. We call H^i the *histories until time i*. In this case we identify $\mathcal{A}^1 = A^1$ and call $((\mathcal{A}^i)^n_{i=1}, (\mathcal{O}^i)^n_{i=1}, \preceq)$ an *n-stage decision problem* (or *multi-stage decision problem* or *sequential decision problem*).[1]

Simplifications

Discussing n-stage decision problems is complicated, as the notation indicates. How can we simplify thinking about them?

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[1] Future editions will clarify these definitions.

One simplification occurs when the outcome sets at stage i do not depend on the action taken, or on the history of actions. In this case, we may discuss outcome sets O^i for $i = 1, \ldots, n$.

Another simplification occurs when the decisions to be made at each stage do not depend on the action taken or on the history of actions. In this case, we may discuss action sets A^i for $i = 1, \ldots, n$.

An even greater simplification occurs when the outcome and actions sets do not depend on the stage i. In this case, we speak of *the* action set and *the* outcome set.

State

2

Often we can summarize the history H^i with a set $\mathcal{X}_i$. In this case, we speak of the set of states S_i for $i = 1, \ldots, n$. In this case, we can associate the history H^1 with an element of S^1. We can associate the history H^2 with an element of S^2. And so on.

Naturally, the current action affects the future states, but not the current or past states, since the state is a "summary" of the history, of the "past".[3]

In this way, the state is a "link" between the "past" and the "future."

The idea is that the current action affects the future state, but not current or past states. The *O*ften our decisions do not depend so much on

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[2]Future editions will move this to State Representations.
[3]Other language includes "sufficient statistic", but we will avoid this for now. Future sheets may modify.

Sequential Decisions

Sequences

Lists   Decisions

Direct Products   Set Numbers

Finite Sets

Equivalent Sets   Orders

Family Unions and Intersections   Function Inverses   Natural Order

Families   Function Composites   Function Images

Equivalence Relations   Functions   Peano Axioms

Relations   Natural Induction

Uncertain Outcomes   Set Products

Generalized Set Dualities   Set Powers   Natural Numbers

Set Unions and Intersections   Partitions   Intersection of Empty Set   Set Dualities   Unordered Triples   Successor Sets

Set Intersections   Set Complements   Pair Unions   Ordered Pairs

Pair Intersections   Set Unions   Set Differences

Empty Set   Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects