Sets

1 Why

We speak of a collection of objects, pre-specified or possessing a similar property.

2 Definition

A **set** is a collection of objects. We use **object** as usual in the English language. So a set is an object with the property that it contains other objects.

In thinking of a set, then, we regularly consider the objects it contains. We call the objects contained in a set the **members** or **elements** of the set. So we say that an object contained in a set is a **member of** or an **element of** the set.

2.1 Notation

We denote sets by upper case latin letters: for example, A, B, and C. We denote elements of sets by lower case latin letters: for example, a, b, and c. We denote that an object a is an element of a set A by $a \in A$. We read the notation $a \in A$ aloud as "a

in A." The $\in$ is a stylized ϵ, a mnemonic for "element of". We write $a \notin A$, read aloud as "a not in A," if a is not an element of A.

If we can write down the elements of A, we do so using brace notation. For example, if the set A is such that it contains only the elements a, b, c, we denote A by $\{a, b, c\}$. If the elements of a set are well-known, then we introduce the set in English and name it; often we select the name mnemonically. For example, let L be the set of latin letters.

If the elements of a set satisfy some common condition, then we use the braces and include the condition. For example, let V be the set of vowels. We can denote V by $\{l \in L \mid l \text{ is a vowel}\}$. We read the symbol $\mid$ aloud as "such that." We read the whole notation aloud as "l in L such that l is a vowel." We call the notation **set-builder notation**. Set-builder notation is indispensable for sets defined implicitly by some condition. Here we could have alternatively denoted V by $\{\text{“}a\text{”}, \text{“}e\text{”}, \text{“}i\text{”}, \text{“}o\text{”}, \text{“}u\text{”}\}$. We prefer the former, slighly more concise notation.

3 Two Sets

Let A a set. A **subset** of A is a set whose elements are also contained in A. A **superset** if A is a set which contains all the elements of A. Two sets are **equal** if they contain the same elements; equivalently if they are each subsets of each other.

The **power set** of A is the set of subsets of A. The **empty**

set is the set containing no elements. The empty set is subset of every set.

3.1 Notation

Let A and B be sets. We denote that A is a subset of B by $A \subset B$. We read the notation $A \subset B$ aloud as "A subset B". We denote that A is equal to B by $A = B$. We read the notation $A = B$ aloud as "A equals B". We denote the empty set by $\varnothing$, read aloud as "empty." We denote the power set of A by 2^A, read aloud as "two to the A."