Why

Can we represent the function associating a linear system with its row reduction by matrix multiplication?

Main observation

The following proposition answers the question in the affirmative.

Proposition 1. *Let* $(A \in \mathsf{R}^{n \times n}, b \in \mathsf{R}^n)$ *be a linear system with* $A_{ij} \neq 0$ *and* ij*-reduction Let* (C, d)*. Then there exists a matrix* $L \in \mathsf{R}^{n \times n}$ *so that* $C = LA$ *and* $d = Lb$*.*

Proof. Define $L \in \mathsf{R}^{n \times n}$ by

$$L_{st} = \begin{cases} 1 & \text{if } s = t \\ -A_{sj}/A_{ij} & \text{if } s \neq i \\ 0 & \text{otherwise .} \end{cases}$$

□

For this reason, we call L in Proposition 1 a *row reducer matrix* or *row reducing matrix* or *row reducer.*

Example

For example, the $(1,1)$-reduction of

$$S = (A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 6 & 7 & 9 & 5 \\ 8 & 7 & 9 & 8 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}).$$

is

$$S' = (A' = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix}, b' = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}).$$

The row reducer is $L \in R^{4 \times 4}$ defined by

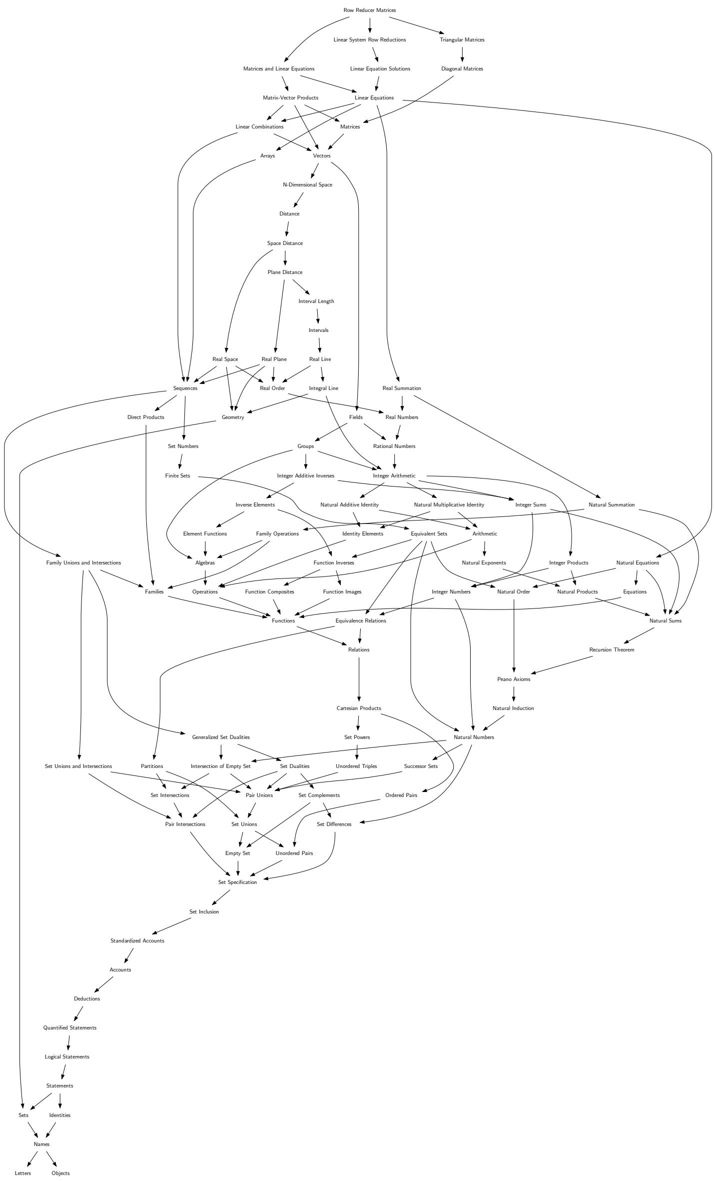
$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix}.$$

One can check that $A' = LA$ and $b' = Lb$. Notice that L here is lower triangular.

Lower triangular

Proposition 2. *For $i \geq j$, the row reducting matrix corresponding to an ij-reduction is lower unit triangular.*[1]

[1] Future editions will include an account.


Row Reducer Matrices
Linear System Row Reductions
Triangular Matrices
Matrices and Linear Equations
Linear Equation Solutions
Diagonal Matrices
Matrix-Vector Products
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Linear Combinations
Matrices
Arrays
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N-Dimensional Space
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Natural Summation
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Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
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Set Inclusion
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