Metric Space Examples

1 Why

We give examples of metric spaces

2 Example

Example 1. *Let n be a natural number. Let A be R^n and define $d : R^n \times R^n \to R$ by*

$$d(a,b) = \sqrt{(a_1 - b_1)^2 + \cdots + (x_n - y_n)^2}.$$

(A, d) is a metric space.

Example 2. *Let A be the unit circle in R^2. So $A = \{x \in R^2 \mid x_1^2 + b^2 = 1\}$. Let $d_1 : A \times A \to R$ defined by*

$$d(a,b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}.$$

Let $d_2 : A \times A \to R$ defined as the arc length between the two points. Both (A, d_1) and (A, d_2) are metric spaces.

Example 3. *Let $A = C([0,1], R)$. Let $d_1 : A \times A \to R$ be such that*

$$d_1(a,b) = \max_{x \in [0,1]} |a(x) - b(x)|.$$

Let λ be the outer cover measure. Let $d_2 : A \times A \to R$ be such that

$$d_2(a,b) = \int_{[0,1]} |f - g| d\lambda.$$

Both (A, d_1) and (A, d_2) metric spaces.