Why

What is the natural generalization of a smooth function to functions defined on sets of $\mathbf{R}^k$.[1]

Definition

Let $U \subset \mathbf{R}^d$ be an open set (see Real Open Sets). A function $f : U \to \mathbf{R}$ is *smooth* if all its partial derivatives exists and are continuous.

More generally, let $X \subset \mathbf{R}^d$. A function $f : X \to R$ is *smooth* if there exists an open set $U \subset \mathbf{R}^d$ and a smooth $F : U \to \mathbf{R}$ so that $F(x) = f(x)$ for all $x \in U \cap X$.

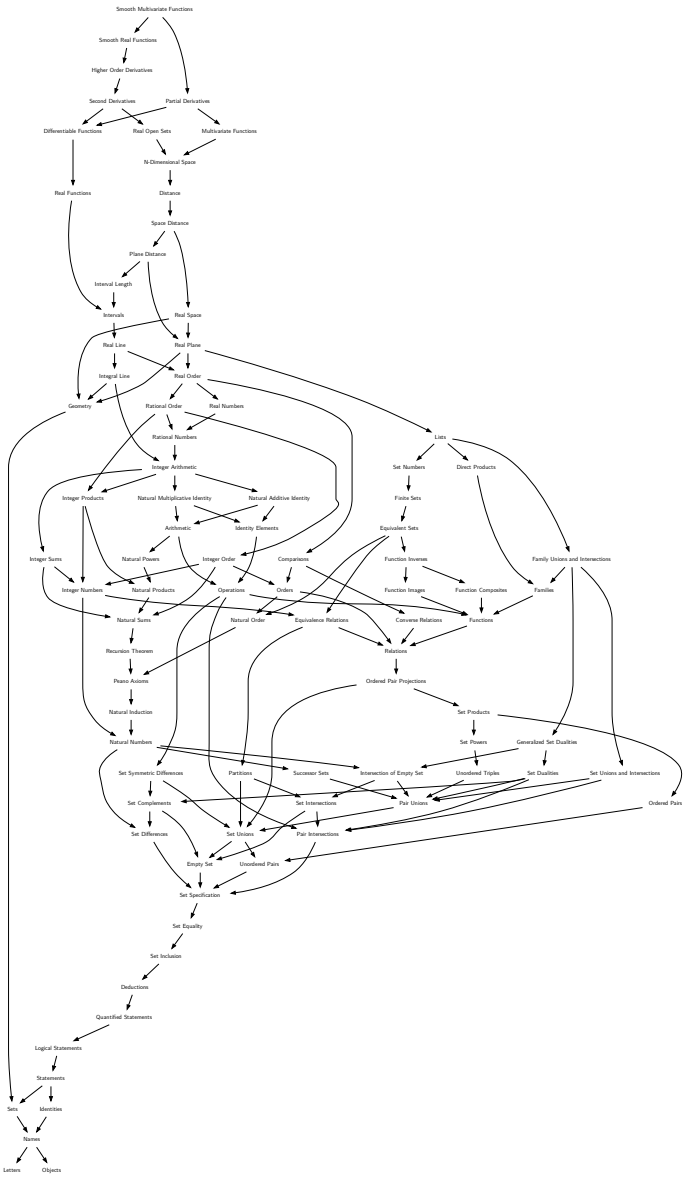
Example

The identity map is smooth. In other words, let $f : \mathbf{R}^d \to \mathbf{R}$ be so that $X \subset \mathbf{R}^d$. Then $f : X \to \mathbf{R}$ s

Properties

Proposition 1. *The composition of two smooth functions is smooth.*

[1] Future editions will expand.


Smooth Multivariate Functions
Smooth Real Functions
Higher Order Derivatives
Second Derivatives
Partial Derivatives
Differentiable Functions
Real Open Sets
Multivariate Functions
N-Dimensional Space
Real Functions
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Integer Sums
Natural Powers
Integer Order
Comparisons
Function Inverse
Family Unions and Intersections
Integer Numbers
Natural Products
Operations
Orders
Function Images
Function Composites
Families
Natural Sums
Natural Order
Equivalence Relations
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects