Set Notation

1 Why

We want to write down statements about objects and sets.

2 Notation

To aid in discussing and denoting objects, let us tend to give them short names. A single Latin letter regularly suffices: for example, a, b or c. Let us denote that the object a and the object b are the same object by $a = b$, read aloud as "a is b."

For sets, let us tend to use upper case Latin letters: for example, A, B, and C. To aid our memory, let us tend to use the lower case form of the letter for an element of the set. For example, if A is a set, we tend to denote by a an element of A. Likewise, if B is a set, we tend to denote by b an element of B.

Let us denote that an object a is an element of a set A by $a \in A$. We read the notation $a \in A$ aloud as "a in A." The $\in$ is a stylized lower case Greek letter: ϵ. It is read aloud "ehp-sih-lawn" and is a mnemonic for "element of". We write $a \notin A$, read aloud as "a not in A," if a is not an element of A.

If we have named the elements of a set, and can list them, let us do so between braces. For example, let a, b, and c be three distinct objects. Denote by $\{a, b, c\}$ the set containing theses three objects and only these three objects. We can further compress notation, and denote this set of three objects by A: so, $A = \{a, b, c\}$. Then $a \in A$, $b \in A$, and $c \in A$. Moreover, if d is an object and $d \in A$, then $d = a$ or $d = b$ or $d = c$.

If the elements of a set are so well-known that we can avoid ambiguityi, then we can describe the set in English. To aid our memory, let us tend to name such sets mnemonically. For example, let L be the set of Latin letters.

Often to be more precise, we should explicitly deal with objects which satisfy several conditions. If the elements of a set satisfy some common condition, then we use the braces and include the condition. For example, let V be the set of Latin vowels. We can denote V by $\{l \in L \mid l \text{ is a vowel}\}$. We read the symbol $\mid$ aloud as "such that." We read the whole notation aloud as "l in L such that l is a vowel." We call the notation **set-builder notation**. Set-builder notation is indispensable for sets defined implicitly by some condition. Here we could have alternatively denoted V by $\{$"a", "e", "i", "o", "u"$\}$. We prefer the former, slightly more concise notation.