



## Why

What is the inverse element under matrix multiplication.

## Definition

Recall that if  $A \in \mathbf{R}^{m \times n}$  then  $x \mapsto Ax$  is a function from  $\mathbf{R}^n$  to  $\mathbf{R}^m$ . Clearly, if  $m \neq n$ , then the inverse of  $f$  can not exist.<sup>1</sup>

Now suppose that  $A \in \mathbf{R}^{n \times n}$ . Of course, the inverse may not exist. Consider, for example if  $A$  was the  $n$  by  $n$  matrix of zeros. If there exists a matrix  $B$  so that  $BA = I$  we call  $B$  the *left inverse* of  $A$  and likewise if  $AC = I$  we call  $C$  the *right inverse* of  $A$ . In the case that  $A$  is square, the right inverse and left inverse coincide.

**Proposition 1.** *Let  $A, B, C \in \mathbf{R}^{n \times n}$ . Let  $BA = I$  and  $AC = I$ . Then  $B = C$ .*

*Proof.* Since  $BA = AC$  we have  $BBA = BAC$  so  $B = C$  since  $BA = I$ . □

## Notation

Let  $\mathbf{F}$  be a field. Let  $A \in \mathbf{F}^{n \times n}$  be invertible. We follow the notation of inverse elements and denote the inverse of  $A$  by  $A^{-1}$ .

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<sup>1</sup>Future editions will expand.



