Why

We want to visualize (nonsymmetric) relations.

Definition

A *directed graph* is a pair (V, E) in which V is a finite nonempty set and E is a subset of $V \times V$. In other words, E is a relation on V. We call the elements of V *vertices* and the elements of E *edges*.

For example, define V and E by

$$V = \{1, 2, 3, 4\} \quad \text{and} \quad E = \{(1,2), (1,3), (2,4), (3,4)\}$$

It is worth drawing this graph.

Edge and vertex terminology

Let $(v, w) \in E$. We say that (v, w) is an edge *from v to w*, and that it is an *outgoing edge* of v and an *incoming edge* of w. We call v a *parent* of w and we call w a child of v. We say that the edge (v, w) is *incident* to v and w.

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. The *indegree* of a vertex is number parents it has and the *outdegree* is the number of children it has; c.f. Undirected Graphs.

A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

Notation

Let $\text{pa} : V \to \mathcal{P}(V)$ and $\text{ch} : V \to \mathcal{P}(V)$ be the functions associating to each vertex its set of parents and set of children, respectively. As usual,

we denote the parents of vertex v by pa_v and the children by ch_v.

Self-loops

If x is a vertex, and (x, x) is an edge, we call such an edge a *self-loop*. Many authors exclude self-loops in their definition of directed graphs, in order to make the situation correspond more closely to that of undirected graphs.

Skeletons

The *skeleton* of the directed graph (V, E) is the undirected graph (V, F) where
$$F = \{\{v, w\} \subset V \mid (v, w) \in E \text{ or } (w, v) \in E\}.$$
In other words, the skeleton is an undirected graph whose vertex set is V and whose edges are all (unordered) pairs which appear as an ordered pair in the directed graph.

In the case that (V, E) is a directed graph and E is a symmetric relation, the skeleton of (V, E) is a natural undirected graph to associate with (V, E). An *orientation* of an undirected graph G is a directed graph whose skeleton is G.

An *oriented graph* is a directed graph without self-loops satisfying the property for any two vertices x and y, either (x, y) or (y, x) is an edge, but not both. An oriented graph can be obtained from an undirected graph by selecting an "orientation" of the undirected edges.

Directed Graphs

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Undirected Graphs

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Set Numbers

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Finite Sets

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Equivalent Sets

Natural Order　　　　　Function Inverses

　　　　　　　　　　Function Composites　　Function Images

　　　Equivalence Relations　　Functions

Peano Axioms　　　　　Relations

Natural Induction　　　Ordered Pair Projections

　　　　　　　　　　Set Products

Natural Numbers　　　Set Powers

Intersection of Empty Set　　Successor Sets　　Unordered Triples

　　　　Pair Unions　　Partitions　　Ordered Pairs

Set Differences　　Set Intersections　　Set Unions

　　Pair Intersections　　Empty Set　　Unordered Pairs

　　　　Set Specification

　　　　Set Equality

　　　　Set Inclusion

　　　　Deductions

　　　Quantified Statements

　　　Logical Statements

　　　　Statements

Identities　　Sets

　　　Names

Letters　　Objects