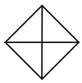


Why

We want to measure the size of vectors in $\mathbf{R}^n$.[1]

Definition

The *norm* (*Euclidean norm*) of $x \in \mathbf{R}^n$ is

$$\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

We denote the norm of x by $\|x\|$. In other words, $\|\cdot\| : \mathbf{R}^n \to \mathbf{R}$ is a function from vectors to real numbers.

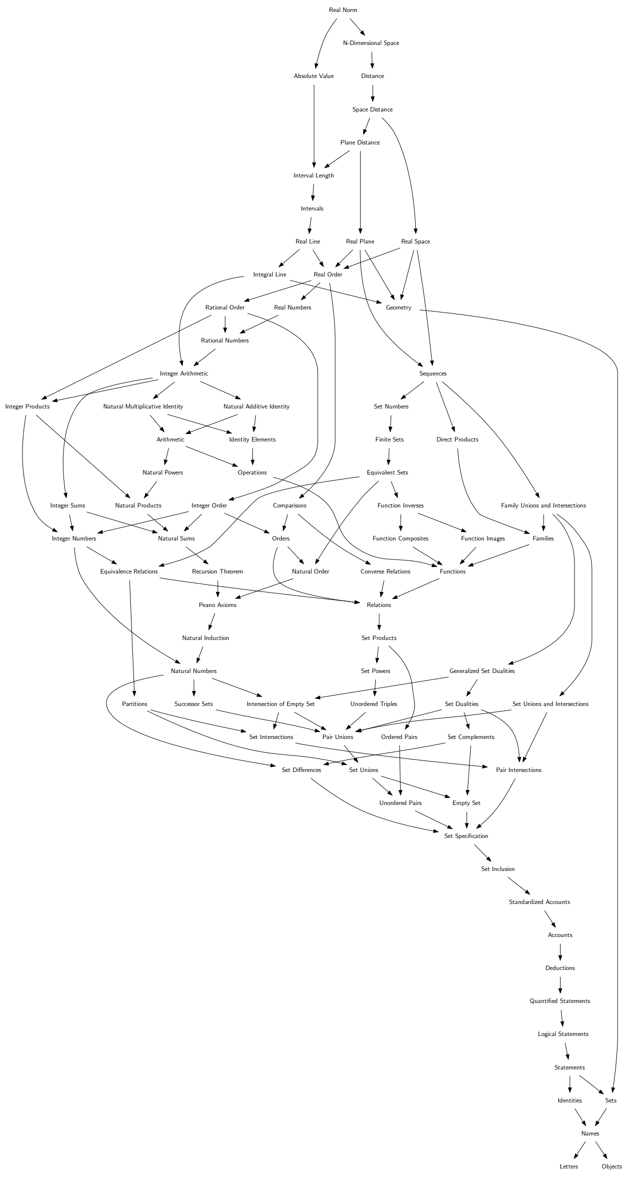
We understand the norm of x by comparison with the distance function $d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$. On one hand, the norm of x is $d(x, 0)$. So $\|x\|$ measures the length of the vector x from the origin 0. On the other hand, $d(x, y) = \|x - y\|$. So $\|x - y\|$ measures the distance between x and y.

Properties

The norm has several important properties

1. $\|\alpha x\| = |\alpha|\|x\|$, called *(absolute) homogeneity*
2. $\|x + y\| \leq \|x\| + \|y\|$, called the *triangle inequality*
3. $\|x\| \geq 0$, called *non-negativity*
4. $\|x\| = 0 \longleftrightarrow x = 0$, called *definiteness*

[1] Future editions may expand. They will certainly include images of the plane.


Real Norm
N-Dimensional Space
Absolute Value
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Rational Order
Real Numbers
Geometry
Rational Numbers
Integer Arithmetic
Sequences
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Arithmetic
Identity Elements
Finite Sets
Direct Products
Natural Powers
Operations
Equivalent Sets
Integer Sums
Natural Products
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Orders
Function Composites
Function Images
Families
Equivalence Relations
Recursion Theorem
Natural Order
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects