Why

We want a set which corresponds to our notion of points on a line.[1]

Rational Cuts

We call a subset R of $\mathbf{Q}$ a *rational cut* if (a) $R \neq \varnothing$, (b) $R \neq \mathbf{Q}$, (c) for all $q \in R$, $r \leq q \longrightarrow r \in R$, and (d) R has no greatest element. Briefly, the intuition is that the point is the set of all rationals to less than (or, potentially, equal to) some particular rational number.[2]

Definition

The *set of real numbers* is the set of all rational cuts. This set exists by an application of the principle of selection (see Set Selection to the power set (see Set Powers) of $\mathbf{Q}$. We call an element of the set of real numbers a *real number* or a *real*. We call the set of real numbers the *set of reals* or *reals* for short.

Notation

We follow tradition and denote the set of real numbers by $\mathbf{R}$, likely a mnemonic for "real."

Other Terminology

Some authors call a real number a *quantity* or a *continuous quantity*. The real numbers, then, are said to be *continuous*. When contrasting (using this terminology) a finite set with the real numbers, one

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[1]Future editions will modify and expand this justification.

[2]This brief intuition will be expanded upon in future sheets.

refers to the finite set as *discrete*.[3]

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[3]Future editions may move this discussion later, to the discussion of the cardinality of the reals.

Real Numbers

Rational Numbers

Integer Arithmetic

Natural Additive Identity

Natural Multiplicative Identity

Identity Elements

Arithmetic

Natural Powers

Integer Products

Natural Products

Integer Sums

Operations

Functions

Equivalence Relations

Natural Sums

Integer Numbers

Recursion Theorem

Relations

Peano Axioms

Set Products

Natural Induction

Ordered Pairs

Set Powers

Natural Numbers

Partitions

Intersection of Empty Set

Unordered Triples

Successor Sets

Set Intersections

Pair Unions

Set Symmetric Differences

Pair Intersections

Set Unions

Set Complements

Unordered Pairs

Empty Set

Set Differences

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects

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