



## Why

We look at a particular subset of vertices and the edges involved between them.

## Definition

The *subgraph* of an undirected graph  $(V, E)$  *induced by* a subset of vertices  $W \subset V$  is the undirected graph with vertices  $W$  and all edges between vertices in  $W$ .

## Notation

Let  $G = (V, E)$  be an undirected graph. Let  $W \subset V$  and define  $F$  by

$$F = \{\{v, w\} \in E \mid v, w \in W\}.$$

The subgraph induced by  $W$  is the undirected graph  $(W, F)$ .

Some authors denote the subgraph induced by  $W$  by  $G(W)$  or  $(W, E(W))$ . We avoid this notation, as it abuses  $G$ , which is no longer an ordered pair, but (in our standard function notation) now indicates a function on subsets of  $V$  with a complicated codomain. Other authors occasionally refer to the “subgraph  $W$ ”, instead of “the subgraph  $G(W)$ ”. Again, we avoid this practice.

## Connected components

A set of vertices  $W$  in  $G$  is *connected* if there is a path between any two vertices  $v, w \in W$ . A set of vertices  $W$  in  $G$  is *maximally connected* if there is no other vertex  $v \notin W$  connected to a vertex in  $W$ . A *connected component* of  $G$  is the subgraph induced by a maximally connected set of vertices.

Since the vertex set of a graph can always be partitioned into sets maximally connected vertices, and a connected components is connect, we think of the connected components of  $G$  as the connected “pieces” of  $G$ .

## Cliques

A set of vertices is *complete* if the subgraph induced is complete. A set of vertices  $W$  is *maximally complete* if the subgraph induced is complete and there is no vertex  $v \notin W$  which is connected to every vertex in  $W$ . In other words, there is no other vertex which we can add to  $W$  so that the induced subgraph is still complete.

We call a *maximally complete* set of vertices a *clique*. Some authors define a clique in the way we have defined a complete set of vertices, without reference to the maximality.

