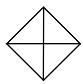


Why

We want to associate the natural numbers with bit strings for use on digital computers.[1]

Definition

A *digital natural* is a bit string. The set of *d-bit digtal natural numbers* is the set of length-d bit strings $\{0,1\}^d$. For example, the set of 8-bit digital naturals is the set $\{0,1\}^8$.

Correspondence with $\mathbf{N} \cup \{0\}$

We associate $x \in \{0,1\}^d$ corresponds to the number $\sum_{i=1}^d x_i 2^i$. For example, the bit string $(0,0,0) \in \{0,1\}^3$ corresponds to the natural number $0 \in \omega$. Likewise, $(1,0,0)$ corresponds to $1 \in \mathbf{N}$, $(0,1,0)$ corresponds to 2, $(1,1,0)$ corresponds to 3, etc.

Call the function so defined the *digital natural decoder*, and denote it by $f : \{0,1\}^d \to \mathbf{N} \cup \{0\}$. In other words $f((0,0,0)) = 0$, $f((0,1,0)) = 2$, etc. Call the set $f(\{0,1\}^d)$ the set of naturals *representable* by length-d bit strings.

Specifically, if, for $n \in \mathbf{N} \cup \{0,1\}$, there exists $x \in \{0,1\}^d$ so that $f(x) = n$, we say that x is *representable in d bits.*

Correspondence between d and $k > d$ bit naturals

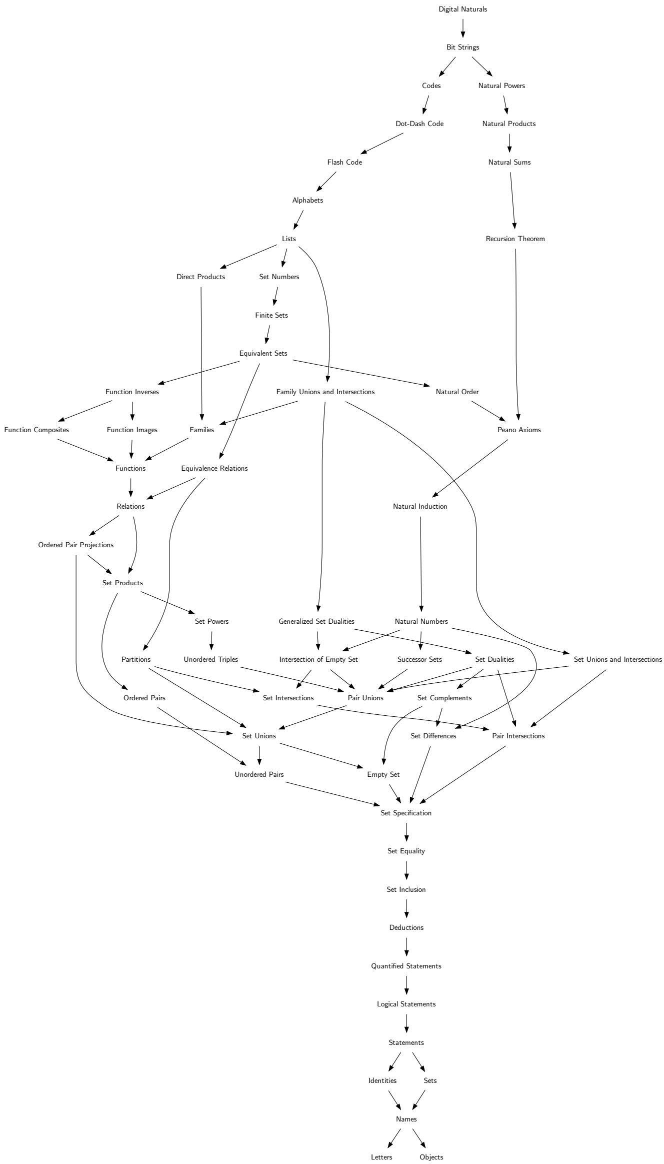
Let $x \in \{0,1\}^d$ and $y \in \{0,1\}^k$ with $k > d$. Although $\{0,1\}^d \not\subset \{0,1\}^k$, $f(\{0,1\}^d) \subset f(\{0,1\}^k)$. We can identify $x \in \{0,1\}^d$ with $x' \in \{0,1\}^k$ where $x' = (x_1, \dots, x_d, 0, \dots, 0)$ so that $f(x) = f(x')$. Clearly then, if x is representable in d bits, it is representable in $k > d$ bits.

[1] Future editions will expand.

Addition

We want to define addition $\oplus : \{0,1\}^d \times \{0,1\}^d \to \{0,1\}^d$ so that $f(x \oplus x') = f(x) + f(x')$. In general, we are stuck, because $x + x'$ may not be representable in d bits. Suppose, however and for the time being, that it is.[2]

[2]Future editions will complete.


Digital Naturals
Bit Strings
Codes
Natural Powers
Dot-Dash Code
Natural Products
Flash Code
Natural Sums
Alphabets
Lists
Recursion Theorem
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Family Unions and Intersections
Natural Order
Function Composites
Function Images
Families
Peano Axioms
Functions
Equivalence Relations
Relations
Natural Induction
Ordered Pair Projections
Set Products
Set Powers
Generalized Set Dualities
Natural Numbers
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects