Why

The inverse of a function interacts nicely with family unions, family intersections and complements.

Results

Let $f : X \to Y$. Throughout this sheet, let $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$. And take $\{B_i\}$ to be a family of subsets of Y.[1]

Proposition 1. $f^{-1}(\cup_i B_i) = \cup_i f^{-1}(B_i)$

Proposition 2. $f^{-1}(\cup_i B_i) = \cap_i f^{-1}(B_i)$

Proposition 3. $f^{-1}(Y - B) = X - f^{-1}(B)$

Properties for Function Image

Notice that $f(\cup_i A_i) = \cup_i f(A_i)$ but not for interesctions. Nor is there a similar correspondence for complements. There are some relations, which we list below.[2]

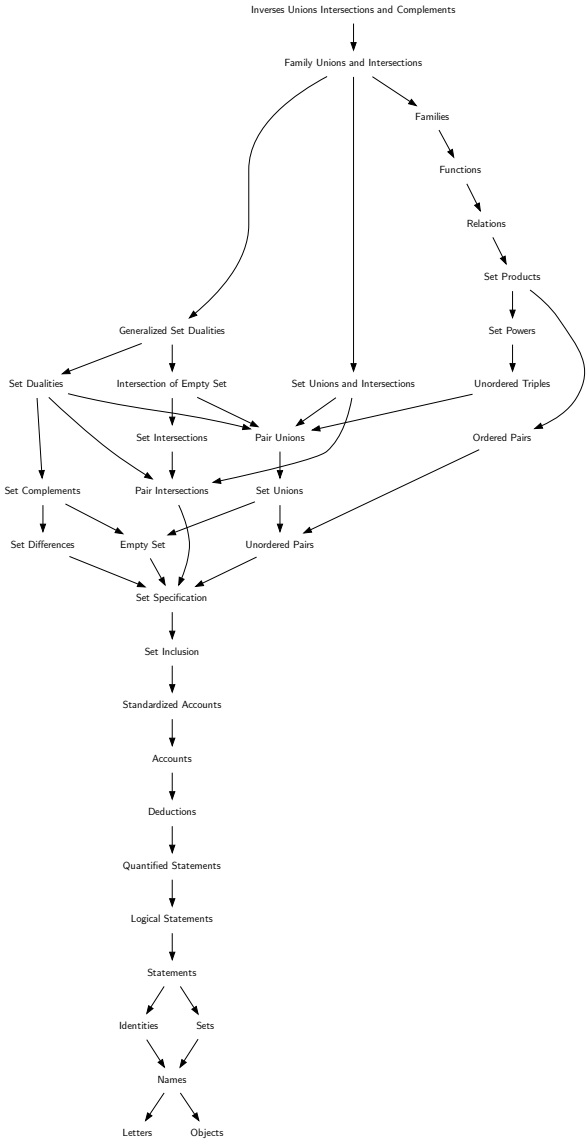
Proposition 4. $f(A \cap B) = f(A) \cap f(B)$ *if and only if* f *is one-to-one.*

Proposition 5. *For all* $A \subset X$, $f(X - A) = Y - f(A)$ *if and only if* f *is one-to-one.*

Proposition 6. *For all* $A \subset X$, $Y - f(A) \subset f(X - A)$ *if and only if* f *is onto.*

[1] The proofs of the following will appear in future editions.

[2] Accounts of these facts will appear in future editions.


Inverses Unions Intersections and Complements
Family Unions and Intersections
Families
Functions
Relations
Set Products
Generalized Set Dualities
Set Powers
Set Dualities
Intersection of Empty Set
Set Unions and Intersections
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects