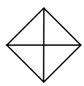


Why

What of the generalization to a multivariate normal.

Result

Prop. 1. *Let $(x^1, \dots, x^n)$ be a dataset in $\mathbf{R}^d$. Let f be a multivariate normal density with mean*

$$\frac{1}{n}\sum_{k=1}^{d} x^k$$

and covariance

$$\frac{1}{n}\sum_{k=1}^{n}\left(x^k - \frac{1}{n}\sum_{k=1}^{n} x^k\right)\left(x^k - \frac{1}{n}\sum_{k=1}^{n} x^k\right)^\top.$$

Then f is a maximum likelihood multivariate normal density.

Proof. We express the log likelihood

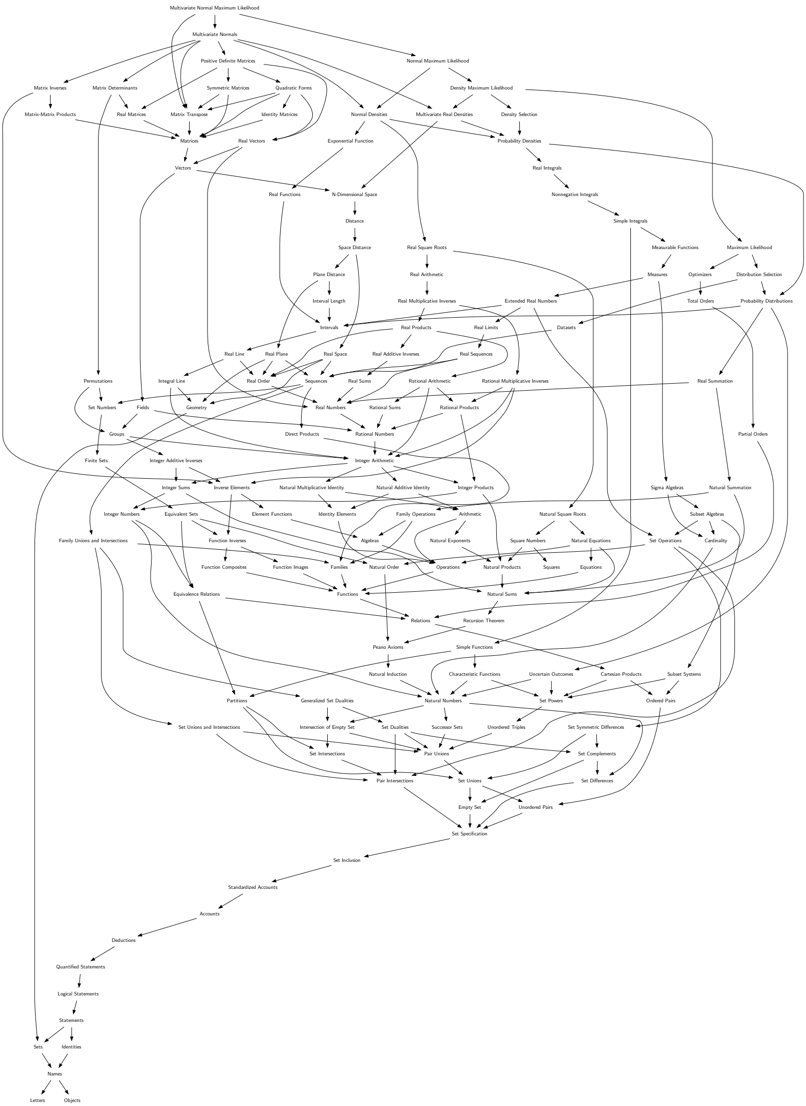
$$\sum_{k=1}^{n} -\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu) - \frac{1}{2}\log(2\pi)^d - \frac{1}{2}\log \mathbf{det}\,\Sigma$$

Let $P = \Sigma^{-1}$. The $\log \mathbf{det}\,\Sigma$ is $\log \mathbf{det}\, P^{-1}$ is $\log\left(\mathbf{det}\, P\right)^{-1}$ is $-\log \mathbf{det}\, P$. Use matrix calculus to get

$$\frac{\partial \ell}{\partial P} = \sum_{k=1}^{n}(x^k-\mu)(x^k-\mu)^\top - P^{-1}.$$

$\square$

We call these two objects the *maximum likelihood mean* or *empirical mean* and *maximum likelihood covariance* or *empirical covariance* of the dataset. We call the normal density with the empirical mean and empirical covariance the *empirical normal* of the dataset.


Multivariate Normal Maximum Likelihood
Multivariate Normals
Positive Definite Matrices
Normal Maximum Likelihood
Matrix Inverses
Matrix Determinants
Symmetric Matrices
Quadratic Forms
Density Maximum Likelihood
Matrix-Matrix Products
Real Matrices
Matrix Transpose
Identity Matrices
Normal Densities
Multivariate Real Densities
Density Selection
Matrices
Real Vectors
Exponential Function
Probability Densities
Vectors
Real Integrals
Real Functions
N-Dimensional Space
Nonnegative Integrals
Distance
Simple Integrals
Space Distance
Real Square Roots
Measurable Functions
Maximum Likelihood
Plane Distance
Real Arithmetic
Measures
Optimizers
Distribution Selection
Interval Length
Real Multiplicative Inverses
Extended Real Numbers
Total Orders
Probability Distributions
Intervals
Real Products
Real Limits
Datasets
Real Line
Real Plane
Real Space
Real Additive Inverses
Real Sequences
Permutations
Integral Line
Real Order
Sequences
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Real Summation
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Real Numbers
Rational Sums
Rational Products
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Direct Products
Rational Numbers
Partial Orders
Finite Sets
Integer Additive Inverses
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Inverse Elements
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Integer Products
Sigma Algebras
Natural Summation
Integer Numbers
Equivalent Sets
Element Functions
Identity Elements
Family Operations
Arithmetic
Natural Square Roots
Subset Algebras
Family Unions and Intersections
Function Inverses
Algebras
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Square Numbers
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Function Images
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Set Powers
Ordered Pairs
Set Unions and Intersections
Intersection of Empty Set
Set Dualities
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
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Logical Statements
Statements
Sets
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Letters
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