



## Why

Many problems simplify if the graph involved is chordal.<sup>1</sup>

## Paths

Let  $G$  be an undirected graph. A *chord* in a path  $p$  of  $G$  is an edge between two non-consecutive vertices of  $p$ . So a chord of the path  $(v_1, v_1, \dots, v_k)$  is an edge  $\{v_i, v_j\}$  with  $|j - i| > 1$ .

We interpret a chord as a “one-edge shortcut” between two vertices of a path. If a path  $p$  has a chord, it can be reduced to a shorter path  $p'$  by “skipping” vertices. In other words, the shortest path between two vertices is chordless. However, a chordless path need not be a shortest path. See Figure 1.

## Graphs

A chord of a cycle  $(v_1, v_2, \dots, v_{k-1}, v_1)$  is an edge  $\{v_i, v_j\}$  with  $(j - i) \bmod k > 1$ . An undirected graph  $G$  is *chordal* if every cycle with more than three edges has a chord.

If  $G$  is chordal, every cycle in  $G$  can be reduced to a cycle of length three. We sometimes call a cycle of length three a *triangle*. For this reason, chordal graphs are also sometimes called *triangulated graphs*. Other terminology includes *rigid-circuit graphs*, *triangulated graphs*, *perfect elimination graphs*, *decomposable graphs*.<sup>2</sup>

The last graph in Figure 1 is not chordal because the cycle  $(a, b, d, c, a)$  has length four and no chord. Adding the edge  $\{b, c\}$  or  $\{a, d\}$  would make the graph chordal. An immediate consequence of the definition that  $G$  be chordal is that any subgraph of  $G$  is chordal.

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<sup>1</sup>Future editions will expand.

<sup>2</sup>See Vanenberghe and Anderson, 2014.

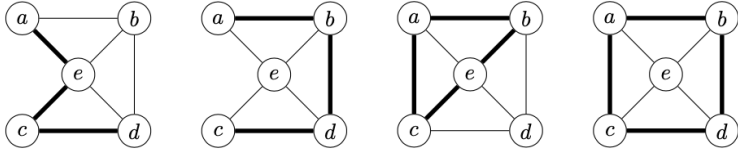


Figure 1: The edge  $\{e, d\}$  is a chord in the path  $(a, e, c, d)$  of the first graph. The path  $(a, v, d, c)$  is chordless. The edge  $\{a, e\}$  is a chord in the cycle  $(a, v, e, c, a)$  of the second graph. The cycle  $(a, b, d, c, a)$  is chordless.

### Simple Examples

Since trees and forests have no cycles, they are chordal. Similarly, any graph with no cycles longer than three edges are trivially chordal. Such graphs are sometimes called *cactus graphs*. The complete graphs are also trivially chordal.

