



SET SPECIFICATION

Why

We want to construct new sets out of old ones. So, can we always construct subsets?

Definition

We will say that we can. Let A denote a set. Let s denote a statement in which the symbol x and A appear unbound.

We assert that there is a set, denote it by A' for which belonging is equivalent to the statement denoted by s . It is a consequence of the axiom of extension that this set is unique. This assertion is sometimes called the *axiom of specification* is this assertion. We call the second set (obtained from the first) the set obtained by *specifying* elements according to the sentence.

All the basic principles of set theory other than the axiom of extension assert that we can construct new sets out of old ones in reasonable ways.

For example:

Account 1. Example Specification

1		name	A, y	
2		thus	$(\exists A')((x \in A') \iff (x \neq y))$	by Axiom:Specification

Notation

Let A be a set. Let $S(a)$ be a sentence. We use the notation

$$\{a \in A \mid S(a)\}$$

to denote the subset of A specified by S . We read the symbol \mid aloud as “such that.” We read the whole notation aloud as “a in A such that...”

We call the notation *set-builder notation*. Set-builder notation avoids enumerating elements. This notation is really indispensable for sets which have many members, too many to reasonably write down.

Example

For example, let a, b, c, d be distinct objects. Let $A = \{a, b, c, d\}$. Then $\{x \in A \mid x \neq a\}$ is the set $\{b, c, d\}$

Now let B be an arbitrary set. The set $\{b \in B \mid b \neq b\}$ specifies the empty set. Since the statement $b \neq b$ is false for all objects b .

