Why

Consider bounding the measure where two functions differ. We would look at their absolute value, and the measure of where this is greater than zero. The absolute value of their difference is a non-negative measurable random variable.

Result

We bound the measure that a non-negative measurable real-valued function exceeds some value by its integral.

Proposition 1. *Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $g : X \to [0, \infty]$ be measurable. Then for all $t > 0$,*

$$\mu(\{x \in X \mid g(x) \geq t\}) \leq \frac{\int g d\mu}{t}.$$

Proof. Let $A = \{x \in X \mid g(x) \geq t\}$. Define $h : X \to R$ by

$$h(x) = \begin{cases} 1 & \text{if } g(x) \geq t \\ 0 & \text{otherwise.} \end{cases}$$

First, $\mu(A) = \int h d\mu$. Second, $h \leq g$. So:

$$\mu(\{x \in X \mid g(x) \geq t\}) = \int h d\mu \leq \int g/t d\mu = \frac{\int g d\mu}{t}.$$

$\square$

This proposition is universally referred to as *Markov's inequality.*[1]

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[1] We eschew this terminology in accordance with the project's policy on naming.

Tail Measure Upper Bound

Variance

Expectation

Random Variable

Probability Measure   Measurable Functions   Topological Sigma Algebra   Outcome Variable Covariance

Outcome Variable Expectation   Measures   Topologies

Induced Probability Distributions   Sigma Algebras   Metrics

Outcome Variable Events   Real Series   Extended Real Numbers   Distance   Real Square Roots

Event Probabilities   Real Limits   Space Distance   Real Squares

Outcome Probabilities   Plane Distance   Real Arithmetic

Real Functions   Interval Length   Real Multiplicative Inverses

Absolute Value   Intervals   Real Space   Natural Square Roots   Real Products

Real Planes   Real Line   Real Additive Inverses   Square Numbers

Size of Direct Product   Real Sequences   Real Summation   Real Order   Real Sums   Rational Multiplicative Inverses   Rational Arithmetic   Integral Line   Squares

Sequences   Natural Summation   Subset Algebras   Real Numbers   Rational Order   Number of Set Products   Rational Sums   Rational Products   Geometry

Finite Set Examples   Lists   Cardinality   Number of Disjoint Unions   Rational Numbers

Direct Products   Set Number   Natural Functions   Integer Arithmetic   Integer Products

Inverse Elements   Finite Sets   Natural Multiplicative Identity   Natural Additive Identity

Uncertain Events   Family Operation   Element Functions   Equivalent Sets   Natural Arithmetic   Identity Elements   Natural Powers

Family Unions and Intersections   Families   Operation   Function Inverses   Comparisons   Natural Equations   Integer Order   Integer Sums   Natural Products

Function Images   Function Composite   Equations   Orders   Integer Numbers

Functions   Converse Relations   Natural Order   Equivalence Relations   Natural Sums

Relations   Peano Axioms   Recursion Theorem

Uncertain Outcomes   Ordered Pair Projections   Natural Induction

Subset Systems   Set Products   Generalized Set Dualities   Natural Numbers

Ordered Pairs   Set Powers   Succeeding Sets   Set Dualities   Intersection of Empty Set   Partitions   Set Symmetric Differences

Set Unions and Intersections   Unordered Triples   Pair Unions   Set Intersections   Set Complements

Pair Intersections   Set Unions   Set Differences

Unordered Pairs   Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects