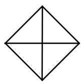


Why

What is the inverse element under matrix multiplication.

Definition

Recall that if $A \in \mathbf{R}^{m \times n}$ then $x \mapsto Ax$ is a function from $\mathbf{R}^n$ to $\mathbf{R}^m$. Clearly, if $m \neq n$, then the inverse of f can not exist.[1]

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that $BA = I$ we call B the *left inverse* of A and likewise if $AC = I$ we call C the *right inverse* of A. In the case that A is square, the right inverse and left inverse coincide.

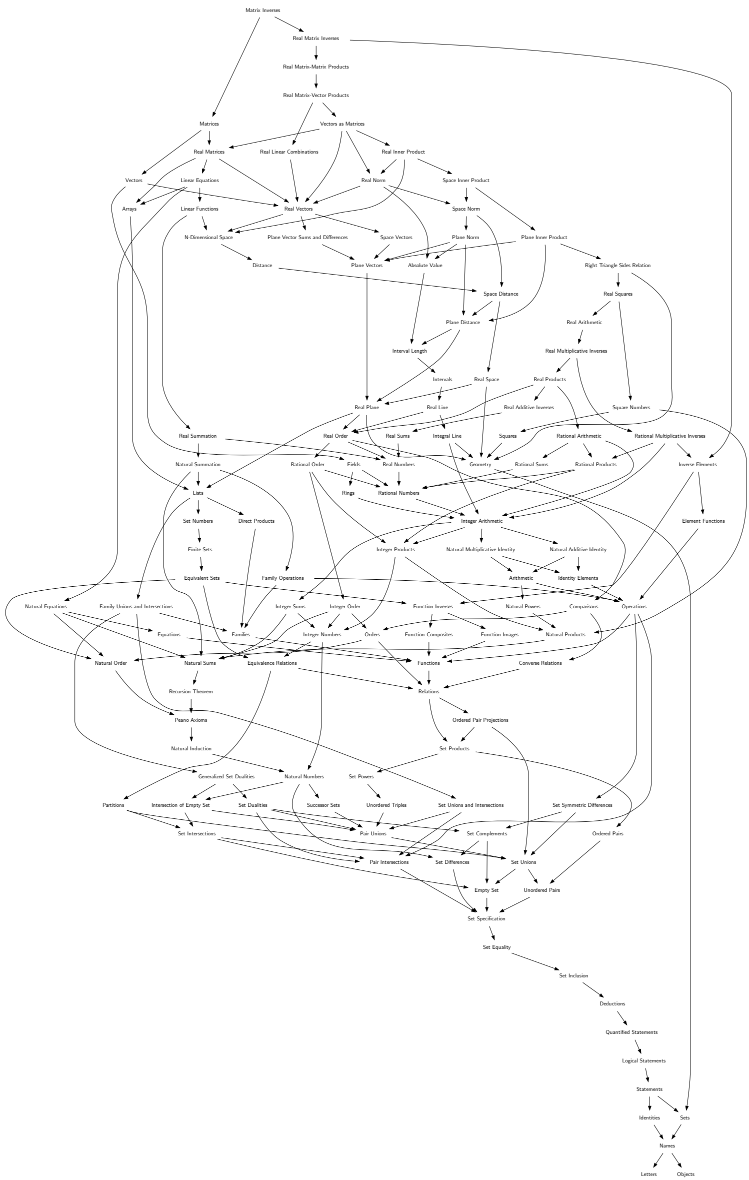
Proposition 1. *Let $A, B, C \in \mathbf{R}^{n \times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

Proof. Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. $\square$

Notation

Let $\mathbf{F}$ be a field. Let $A \in \mathbf{F}^{n \times n}$ be invertible. We follow the notation of inverse elements and denote the inverse of A by A^{-1}.

[1] Future editions will expand.


Matrix Inverses
Real Matrix Inverses
Real Matrix-Matrix Products
Real Matrix-Vector Products
Matrices
Vectors as Matrices
Real Matrices
Real Linear Combinations
Real Inner Product
Vectors
Linear Equations
Real Norm
Space Inner Product
Arrays
Linear Functions
Real Vectors
Space Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Plane Norm
Plane Inner Product
Distance
Plane Vectors
Absolute Value
Right Triangle Sides Relation
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Real Plane
Real Line
Real Additive Inverses
Square Numbers
Real Summation
Real Order
Real Sums
Integral Line
Squares
Rational Arithmetic
Rational Multiplicative Inverses
Natural Summation
Rational Order
Fields
Real Numbers
Geometry
Rational Sums
Rational Products
Inverse Elements
Lists
Rings
Rational Numbers
Set Numbers
Direct Products
Integer Arithmetic
Element Functions
Finite Sets
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Family Operations
Arithmetic
Identity Elements
Natural Equations
Family Unions and Intersections
Integer Sums
Integer Order
Function Inverses
Natural Powers
Comparisons
Operations
Equations
Families
Integer Numbers
Orders
Function Composites
Function Images
Natural Products
Natural Order
Natural Sums
Equivalence Relations
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects