



# Equivalence Classes

## 1 Why

Equivalence relations partition the base set into sets of “equivalent” elements. We can define an appropriate equivalence relation on a set and then work with the *set of sets* of equivalent objects.

## 2 Definition

We call all elements related to a particular element under an equivalence relation the *equivalence class* of the element. The key observation, recorded and proven below, is that the equivalence classes partition the base set. This will allow us to define appropriate equivalence relations on a set and then work with the set of equivalence classes instead.

We call the set of equivalence classes the *quotient set* of the set under the relation. An equally good name is the divided set of the set under the relation, but this terminology is not standard. The language in both cases reminds us that the relation partitions the set into equivalence classes.

### 2.1 Notation

Let  $A$  be a non-empty set and  $\sim$  be an equivalence relation on  $A$ . We denote the quotient set of  $A$  under  $\sim$  by  $A/\sim$ , read aloud as “A quotient sim”.

## 2.2 Results

TODO