





Figure 1: A postordering of a rooted tree.
ROOTED TREE ORDERINGS

Why

We want to simplify common questions using orderings of a rooted tree.¹

Definition

A *topological ordering* of a rooted tree is an ordering σ for which $v \prec_{\sigma} \text{pa}(v)$. If there are n vertices, the root has index n and every other vertex has an index less than its parent.

A *postordering* is a topological ordering in which descendants of a vertex are given consecutive numbers. For the postordering σ , if $\sigma^{-1}(v) = j$ and v has k proper descendants then the proper descendants of v have are numbered consecutively from $j - k$ through $j - 1$. Figure 1 shows an example.

One can generate a postordering by numbering vertices in decreasing sequence (starting at n) in the order they are visited.²

Given an ordering, the *first descendent* of v (which we denote $\text{fdesc}(v)$) is the descendent with the lowest index. Given a postordering σ , one can check whether a vertex w is a proper descendent of v , by $\text{fdesc}(v) \preceq_{\sigma} w \prec_{\sigma} v$.

¹This sheet is incomplete and will be updated in future editions.

²Future editions will clarify.

