Definition

Suppose $(X, \mathcal{F})$ is a measurable space. A measure $\mu : \mathcal{F} \to \bar{\mathbf{R}}$ is said to have a density with respect to a measure $\nu : \mathcal{F} \to \bar{\mathbf{R}}$ if there exists a measurable function $f : X \to \mathbf{R}_+$

$$\mu(A) = \int_A f d\nu \quad \text{for all } A \in \mathcal{F}$$

In this case f is called a *density* of μ with respect to ν.

Examples

Probability on finite sets. Suppose P is a probability measure for a finite set Ω. Define $p : \Omega \to [0, 1]$ by

$$p(\omega) = P(\{\omega\}) \quad \text{for all } \omega \in \Omega$$

Then p is a probability distribution. Moreover, p is a density for P with respect to the counting measure $\# : \mathcal{P}(\Omega) \to \mathbf{R}$. Witness, for every $A \subset \Omega$,

$$\int_A p d\# = \sum_{a \in A} p(\omega)$$

We recognize the right hand side as $P(A)$ by using the additivity of P.

Measure Densities

Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions

Measures

Length Measure — Sigma Algebras — Extended Real Numbers

Real Series

Real Limits

Simple Functions — Absolute Value

Real Functions — Interval Length

Intervals

Real Line

Real Order — Real Sequences

Rational Order — Real Numbers — Sequences — Subset Algebras

Integral Line — Rational Numbers — Cardinality — Lists

Integer Arithmetic — Natural Fractions — Set Numbers — Direct Products

Integer Products — Natural Additive Identity — Natural Multiplicative Identity — Finite Sets

Identity Elements — Natural Arithmetic — Equivalent Sets

Natural Powers — Integer Sums — Integer Order — Comparisons — Operations — Function Inverses — Family Unions and Intersections

Natural Products — Integer Numbers — Orders — Function Composites — Function Images — Families

Natural Sums — Equivalence Relations — Natural Order — Converse Relations — Functions

Recursion Theorem — Relations

Peano Axioms — Ordered Pair Projections

Natural Induction — Characteristic Functions — Set Products — Subset Systems

Natural Numbers — Generalized Set Dualities — Set Powers — Ordered Pairs

Set Symmetric Differences — Partitions — Successor Sets — Intersection of Empty Set — Set Dualities — Unordered Triples — Set Unions and Intersections

Set Complements — Set Intersections — Pair Unions

Set Differences — Set Unions — Pair Intersections

Empty Set — Unordered Pairs

Geometry — Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects

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