Why

We generalize the normal density to d-dimensional space.

Definition

Let $f : \mathbf{R}^d \to \mathbf{R}$ be a density such that

$$f(x) = \frac{1}{\sqrt{(2\pi)^d \det \Sigma}} \exp\left(-\frac{1}{2}(x - \mu)^\top \Sigma^{-1}(x - \mu)\right)$$

where $\mu \in \mathbf{R}^d$, $\Sigma \in \mathbf{S}^d$, and $\Sigma \succ 0$. We call f a *multivariate normal density*. A multivariate normal density with $d = 1$ is a normal density, so we refer to multivariate normal densities as *normal densities* without ambiguity. We frequently use the word normal as a substantive, and refer to *normals* when we mean multivariate normal densities. Many people call a multivariate normal distribution a *multivariate gaussian distribution* and speak of *gaussians* instead of normals.[1]

We call μ the *mean* and Σ the *covariance matrix*. We call Σ^{-1} the *precision matrix*.

Maximum

The maximum of a normal density is its mean, $\mu \in \mathbf{R}^d$.

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[1] We avoid this usage in accordance with the project's policy on historical names.