



Definition

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex product* of z_1 and z_2 is the complex number

$$(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2).$$

Notation

We denote the complex product of z_1 and z_2 by $z_1 \cdot z_2$ or z_1z_2 .

The notation overloads that used for real numbers. This overloading is justified by the fact that the complex product of two purely real complex numbers z_1 and z_2 the purely real complex number whose real part is the product of the real parts of z_1 and z_2 .

Recall that we denote $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. This notation is a mnemonic for the definition of a complex product if we treat $i^2 = -1$.

$$\begin{aligned} z_1z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2). \end{aligned}$$

Properties

Proposition 1 (Commutativity). *For all $z_1, z_2 \in \mathbf{C}$, we have $z_1z_2 = z_2z_1$.*

Proposition 2 (Associativity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have and $z_1(z_2z_3) = (z_1z_2)z_3$.*

Complex multiplication

We call the operation that associates a pair of complex numbers with their product *complex multiplication*. The operation is symmetric (commutative).

Multiplicative identity and inverse

Notice that the complex number $(1, 0)$ is the multiplicative identity. It is unique,¹ and so we call it the *complex multiplicative identity*.

We call the operation $(z, w) \rightarrow z/w$ *complex division* and we call z/w the *(complex) quotient* of z with w .

¹Future editions will include an account

