Trees

1 Why

Tree branches split and do not recombine. We formalize this property in the language of graphs.

2 Definition

A *tree* is a set of two-element sets with the following property: if we take the union over all the sets and define a relation on this union such that two elements are related if they appear as a two element set, then the ordered pair of the union and this relation is a graph that is connected and acyclic.

Thus every tree corresponds to an undirected, connected and acyclic graph. We avoid defining a tree to be that, though, because we want to keep around only the most important object: the set of two-element sets. From that object we can get the vertex set and edge set of the graph. We need the concepts from graphs to talk about which sets of two-element sets are trees, but we want the notation from trees to be amenable to the extreme structure in their nature. The notation we use will bear us out.

2.1 Notation

Let T be a tree, a mnemonic for “tree." Let V be the vertex set associated with T. In other words,

$$V = \cup_{e \in T} e$$

. Let E be so that $(u, v) \in E$ if and only if $\{u, v\} \in T$ for every $u, v \in V$. By construction, the graph (V, E) is undirected. T a tree means (V, E) is connected and acyclic. Let V be a non-empty finite set and $E \subset V \times V$ such that (V, E) is a tree.

A major motivator for our definition of trees is so that we can write $\{u, v\} \in T$. TODO

3 Properties

Proposition 1. *There is only one path between any two vertices in a tree.*

Proof. Suppose to the contrary that there were two paths from vertex u to vertex v. Then by combining these paths we obtain a cycle. But the tree has no cycles. So there must not be two paths between any two vertices. $\square$