



# Sets

## 1 Why

We want to talk about none, one, or several objects considered as an abstract whole, for which we will use the word *set*.

## 2 Definition

A *set* is an abstract object which we think of as several objects considered at once. We say that the set *contains* the objects so considered. We call these the *members* or *elements* of the set. We say that an object contained in a set is a *member* of or an *element* of the set.

We call a set which contains no objects an *empty*. Otherwise we call a set *nonempty*. We call a set which contains only a single object a *singleton*. A singleton is not the same as the object it contains. Besides these two cases, we think of sets as containing two or more objects.

The objects a set contains may be other sets. This may be subtle at first glance, but becomes familiar with experience.

## 2.1 Notation

Let us tend to denote sets by upper case Latin letters: for example,  $A$ ,  $B$ , and  $C$ . To aid our memory, let us tend to use the lower case form of the letter for an element of the set. For example, let  $A$  and  $B$  be nonempty sets. Let us tend to denote by  $a$  an element of  $A$ , and likewise, by  $b$  an element of  $B$ .

Let us denote that an object  $a$  is an element of a set  $A$  by  $a \in A$ . We read the notation  $a \in A$  aloud as "a in A." The  $\in$  is a stylized lower case Greek letter:  $\varepsilon$ . It is read aloud "ehp-sih-lawn" and is a mnemonic for "element of". We write  $a \notin A$ , read aloud as "a not in A," if  $a$  is not an element of  $A$ .

Suppose a set has few elements, and we can list them. If we give the objects names, then let us denote the set by listing the names of its elements between braces. For example, let  $a$ ,  $b$ , and  $c$  be three distinct objects. Denote by  $\{a, b, c\}$  the set containing these three objects and only these three objects. We can further compress notation, and denote this set of three objects by  $A$ : so,  $A = \{a, b, c\}$ . Then  $a \in A$ ,  $b \in A$ , and  $c \in A$ . Moreover, if  $d$  is an object and  $d \in A$ , then  $d = a$  or  $d = b$  or  $d = c$ .

Let  $a$  be an object. Note that  $a \neq \{a\}$ . The left hand side,  $a$ , is the object  $a$ . The right hand side,  $\{a\}$ , is the singleton whose element is the object  $a$ . We distinguish the set containing one element from the element itself.