



# ◈ Cumulative Distribution Function From Density

## 1 Why

The density of a random variable determines the cumulative distribution function.

## 2 Result

**Proposition 1.** *A probability density function of a random variable, if it exists, characterizes the cumulative distribution function.*

*Proof.* Let  $(X, \mathcal{A}, \mu)$  be a probability space. Let  $f$  be a real-valued random variable on  $X$ . Let  $\lambda$  denote the cover length. Let  $g$  be a probability density of  $f$ : Then:

$$\begin{aligned} F_f(t) &= \mu(\{x \in X \mid f(x) \in (-\infty, t]\}) \\ &= \int_{(-\infty, t]} g d\lambda. \end{aligned}$$

□





