## Why

## Definition

Let $(V, E)$ be an undirected graph. An (undirected) *path* between vertex $v \in V$ and vertex $w \neq v$ is a finite sequence of distinct vertices, whose first coordinate is $v$ and whose last coordinate is $w$, and whose consecutive coordinates are adjacent in the graph. We call the first and last coordinate the *endpoints* of the path. We say the path is *between* its endpoints.

The *length* of a path is one less than the number of vertices: namely, the number of edges. Notice that this definition disagrees with the definition of the "length" of the sequence of vertices (namely, the number of vertices). The length of a path is always at least one: there exists a path of length one between any two adjacent vertices. If a path has length two or greater, we call a vertex which is not the first or last vertex an *interior vertex.*

Two vertices are *connected* in a graph if there exists at least one path between them. A graph is *connected* if each pair of vertices is connected. Recall that two vertices are *adjacent* if they are connected by a path of length one. In contrast, two vertices are connected if they are connected by a path of any length. In other words, all adjacent vertices are connected.

A *cycle* is a sequence whose first and last coordinate are identical, all other coordinates are distinct, and consecutive coordinates are adjacent. An undirected graph is *acyclic* if there are no cycles of its vertices.

### Other Terminology

Some authors allow paths to contain repeated vertices, and call a path with distinct vertices a *simple path.* Similarly, some authors allow a cycle

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[1]Future editions will include.

to contain repeated vertices, and call a path with distinct vertices a *simple cycle* or *circuit*. Some authors use the term *loop* instead of *cycle*.

**Notation**

Let $G = (V, E)$ be a graph. A path between $v$ and $w$ (with $v \neq w$) in $G$ is a sequence $(v_0, v_1, \ldots, v_k)$ where $v_0 = v$ and $v_k = w$ and $\{v_i, v_{i+1}\} \in E$ for $i = 0, \ldots, k-1$.

Undirected Paths

Undirected Graphs  Lists

Set Numbers  Direct Products

Finite Sets

Equivalent Sets

Function Inverses  Natural Order  Family Unions and Intersections

Function Images  Function Composites  Families

Equivalence Relations  Functions  Peano Axioms

Relations  Natural Induction

Set Products

Set Powers  Natural Numbers  Generalized Set Dualities

Partitions  Unordered Triples  Intersection of Empty Set  Successor Sets  Set Dualities  Set Unions and Intersections

Ordered Pairs  Set Intersections  Pair Unions  Set Complements

Set Unions  Set Differences  Pair Intersections

Unordered Pairs  Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities  Sets

Names

Letters  Objects