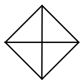


Why

[1]

Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\mathbf{cov}(f, g)$. We defined it:

$$\mathbf{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\,\mathbf{E}(g).$$

Properties

PROPOSITION **1.** *Covariance is symmetric and billinear.*[2]

PROPOSITION **2.** *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\mathbf{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\,\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \mathbf{var}(f).$$

□

[1]Future editions will include this.

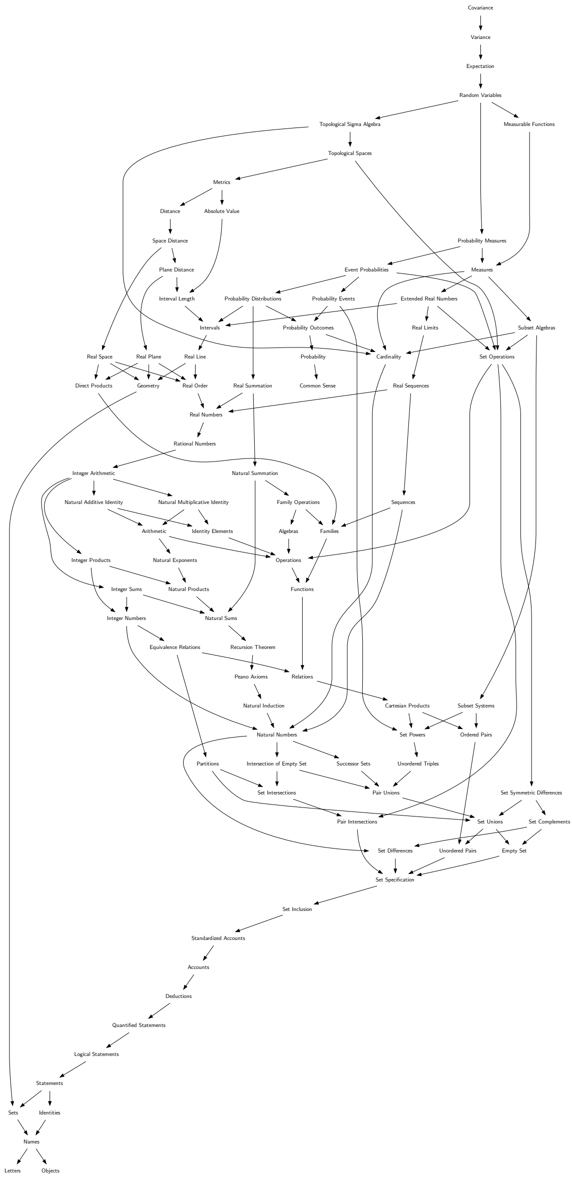
[2]Future editions will include an account.

Proposition 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\begin{aligned} \mathsf{var}\left(\sum_{i=1}^{n} f_i\right) &= \mathsf{cov}\left(\sum_{i=1}^{n} f_i, \sum_{i=1}^{n} f_i\right) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \mathsf{cov}(f_i, f_j) \end{aligned}$$

□


Covariance
Variance
Expectation
Random Variables
Topological Sigma Algebra
Measurable Functions
Topological Spaces
Metrics
Distance
Absolute Value
Space Distance
Probability Measures
Plane Distance
Event Probabilities
Measures
Interval Length
Probability Distributions
Probability Events
Extended Real Numbers
Intervals
Probability Outcomes
Real Limits
Subset Algebras
Real Space
Real Plane
Real Line
Probability
Cardinality
Set Operations
Direct Products
Geometry
Real Order
Real Summation
Common Sense
Real Sequences
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Summation
Natural Additive Identity
Natural Multiplicative Identity
Family Operations
Sequences
Arithmetic
Identity Elements
Algebras
Families
Integer Products
Natural Exponents
Operations
Integer Sums
Natural Products
Functions
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects