Why

We want to show something holds for every natural number.[1]

Definition

The most important property of the set of natural numbers is that it is the unique smallest successor set. In other words, if S is a successor set contained in ω (see Natural Numbers), then $S = \omega$. This is useful for proving that a particular property holds for the set of natural numbers.

To do so we follow standard routine. First, we define the set S to be the set of natural numbers for which the property holds. This step uses the principle of selection (see Set Selection) and ensures that $S \subset \omega$. Next we show that this set S is indeed a successor set. The first part of this step is to show that $0 \in S$. The second part is to show that $n \in S \longrightarrow n^+ \in S$. These two together mean that S is a successor set, and since $S \subset \omega$ by definition, that $S = \omega$. In other words, the set of natural numbers for which the property holds is the entire set of natural numbers. We call this the *principle of mathematical induction.*

---

[1]Future editions will modify this superficial why.

Natural Induction

↓

Natural Numbers

Successor Sets ← → Intersection of Empty Set → Set Differences

Pair Unions ← Set Intersections Set Differences

↓ ↓

Set Unions Pair Intersections

↓ ↘

Empty Set Unordered Pairs

↓ ↓

Set Specification

↓

Set Inclusion

↓

Standardized Accounts

↓

Accounts

↓

Deductions

↓

Quantified Statements

↓

Logical Statements

↓

Statements

↙ ↘

Identities Sets

↘ ↙

Names

↙ ↘

Letters Objects