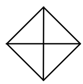


Definition

A nonempty set $S \subset \mathbf{R}^n$ is called a *subspace* (or *linear subspace*, *vector subspace*) if

1. $x + y \in S$ for all $x, y \in S$, and
2. $\alpha x \in S$ for all $\alpha \in \mathbf{R}$, $x \in S$.

We say that that S is (1) *closed under vector addition* and (2) *closed under scalar multiplication*.

Examples

The set $S_1 = \mathbf{R}^n$ is a subspace. In other words, the entire set is a subspace of itself. The set $S_2 = \{0\}$, consisting of a single point, the origin, is a subspace. S_1 is the biggest subspace. In other words, if S' is another subspace of $\mathbf{R}^n$, then $S' \subset S_1$. If S is a subspace, it is nonempty, so there is $x \in S$, and it is closed under scalar multiplication, so $0 \cdot x = 0 \in S$. In other words, every subspace contains the origin. So S_2 is the smallest subspace, in the sense that if S' is another subspace $S_2 \subset S'$.

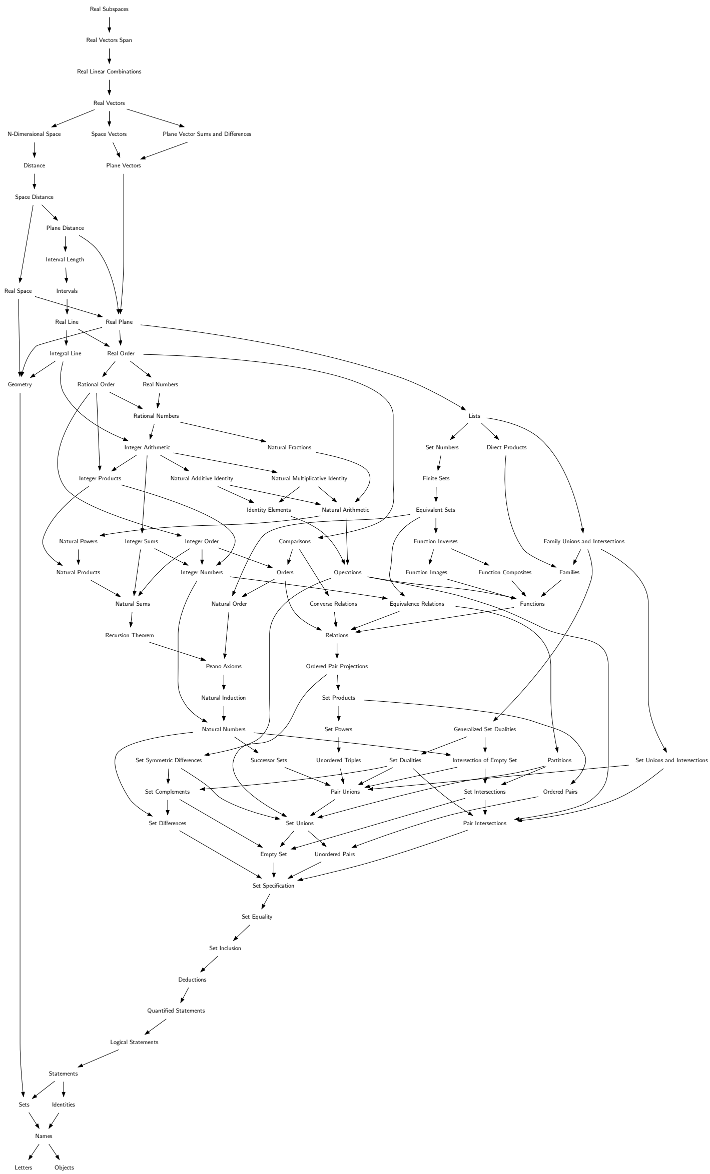
The span (see Real Vectors Span) of a set of vectors $v_1, \dots, v_k$ is a subspace. For two subspaces $S, T \subset \mathbf{R}^n$, their sum

$$S + T = \{x + y \mid x \in S, y \in T\}$$

is a subspace.

Geometric intuition

Roughly speaking, a subspace S is a flat set which passes through the origin. In $\mathbf{R}^2$, the subspaces are the lines. In $\mathbf{R}^3$, the lines and the planes.


Real Subspaces
Real Vectors Span
Real Linear Combinations
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects