Why

It is natural to want to define a sequence by giving its first term and then giving its later terms as functions of its earlier ones. In other words, we want to define sequences inductively.[1]

Main Result

The following is often referred to as the *recursion theorem.*

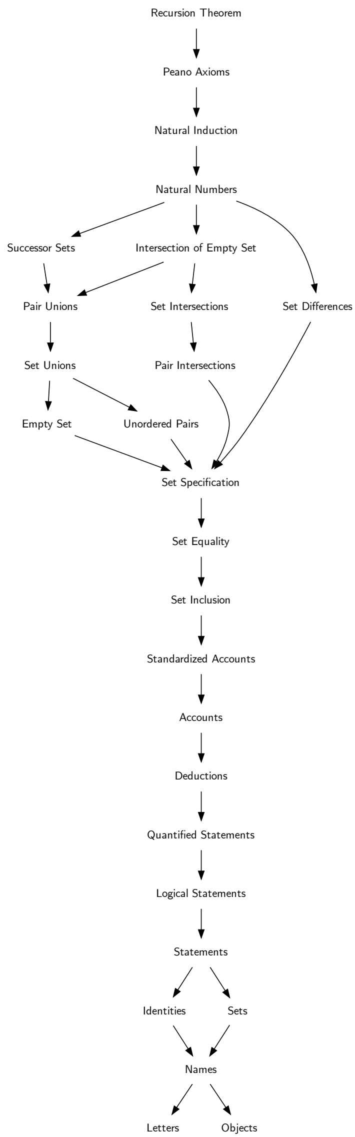
Proposition 1 (Recursion Theorem[2]). *Let X be a set, let $a \in X$ and let $f : X \to X$. There exists a unique function u so that $u(0) = a$ and $u(n^+) = f(u(n))$.*[3]

When one uses the recursion theorem to assert the existence of a function with the desired properties, it is called *definition by induction.*

[1] Future editions will expand on this. We are really headed toward natural addition, multiplication and exponentiation.

[2] Future editions will likely change this name.

[3] The account is somewhat straightforward, given a good understanding of the results of Peano Axioms. The full account will appear in future editions.


Recursion Theorem
Peano Axioms
Natural Induction
Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects