



## ALMOST EVERYWHERE MEASURABILITY

### Why

Does convergence almost everywhere of a sequence of measurable functions guarantee measurability of the limit function? It does on complete measure spaces, and we can use this result to “weaken” the hypotheses of many theorems.

### Results

A measure is *complete* if every subset of a measurable set of measure zero is measurable. If the measure is complete, then every negligible set must be measurable.

We begin with a transitivity property: almost everywhere equality of two functions allows us to infer measurability of one from the other.

**PROPOSITION 1.** *Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $f, g : X \rightarrow [-\infty, \infty]$  with  $f = g$  almost everywhere. If  $\mu$  is complete and  $f$  is  $\mathcal{A}$ -measurable, then  $g$  is  $\mathcal{A}$ -measurable.*

*Proof.* □

**PROPOSITION 2.** *Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $f_n : X \rightarrow [-\infty, \infty]$  for all natural numbers  $n$  and  $f : X \rightarrow [-\infty, \infty]$  with  $(f_n)_n$  converging to  $f$  almost everywhere. If  $\mu$  is complete and  $f_n$  is measurable for each  $n$ , then  $f$  is  $\mathcal{A}$ -measurable.*

*Proof.* □

