Why

It happens that if a distribution factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So the property of factoring according to a rooted tree is really property of factoring according to a tree.

Definition

Defining Result

Proposition 1. *Let $A_1, \ldots, A_n$ be finite non-empty sets and define $A = \prod_{i=1}^{n} A_n$. Let $p : A \to [0, 1]$ be a distribution and let T be a tree on $\{1, \ldots, n\}$.*

Proposition 2. *If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

Undirected definition

A distribution p *factors according to the tree* T if it factors according to the T rooted at any vertex.

Existence and uniqueness

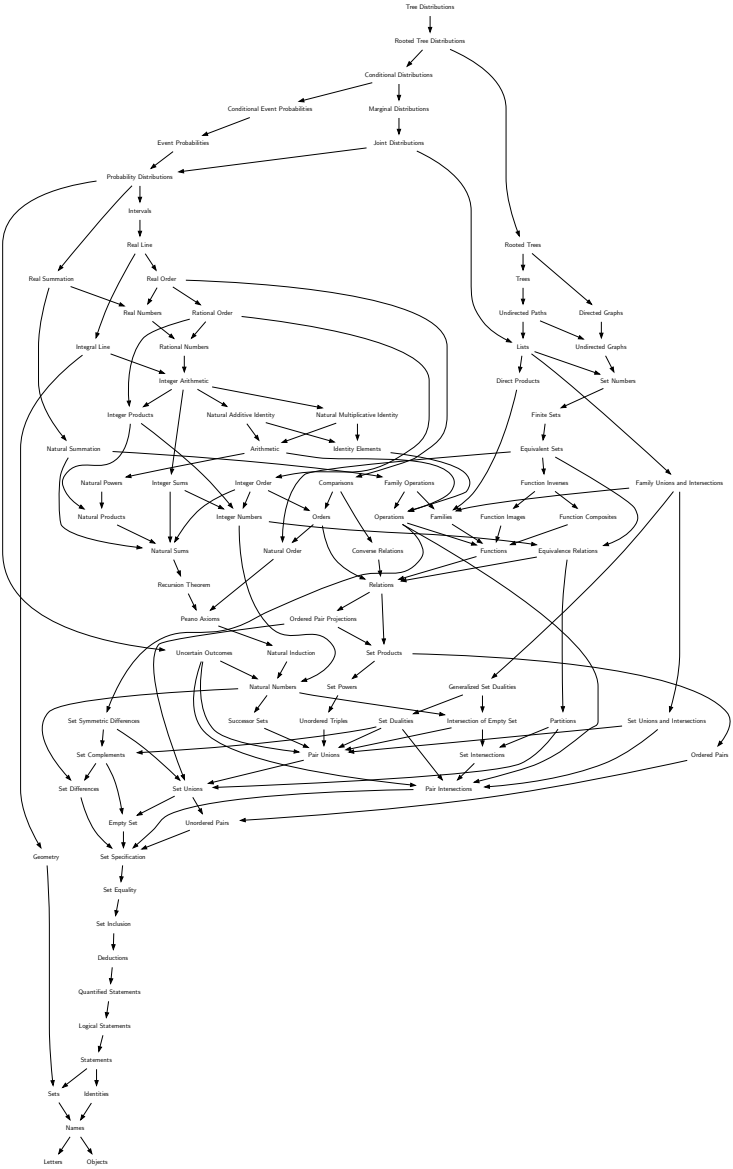
Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

Existence

A distribution p need not factor according to a tree.

Uniqueness

A distribution p may factor according to multiple trees.


Tree Distributions
Rooted Tree Distributions
Conditional Distributions
Conditional Event Probabilities
Marginal Distributions
Event Probabilities
Joint Distributions
Probability Distributions
Intervals
Real Line
Rooted Trees
Real Summation
Real Order
Trees
Real Numbers
Rational Order
Undirected Paths
Directed Graphs
Integral Line
Rational Numbers
Lists
Undirected Graphs
Integer Arithmetic
Direct Products
Set Numbers
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Natural Summation
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Family Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Families
Function Images
Function Composites
Natural Sums
Natural Order
Converse Relations
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects