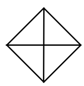


Why

We want to have random functions with simple marginal distributions.

Definition

A *normal random function* (or *normal process* or *gaussian process*)[1] is a real-valued random function whose family of random variables has the property that the image of any finite set of indices is a normal random vector.

Notation

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and A a set. Let $x : \Omega \to (A \to \mathsf{R})$ be a random function with family $y : A \to (\Omega \to \mathsf{R})$.

Then x is a normal random function if there exists $m : I \to \mathsf{R}$ and positive definite $k : I \times I \to \mathsf{R}$ with the property that if $J \subset I$, $|J| = d$, then $x_J \sim \mathcal{N}(m(J), k(J \times J))$. In other words, $x_J : \Omega \to \mathsf{R}^d$ is a Gaussian random vector. We call m the *mean function* and k the *covariance function*.[2]

Random function interpretation

Many authorities discuss a normal random function as "putting a prior" on a "space" (see, for example, Real Function Space) of

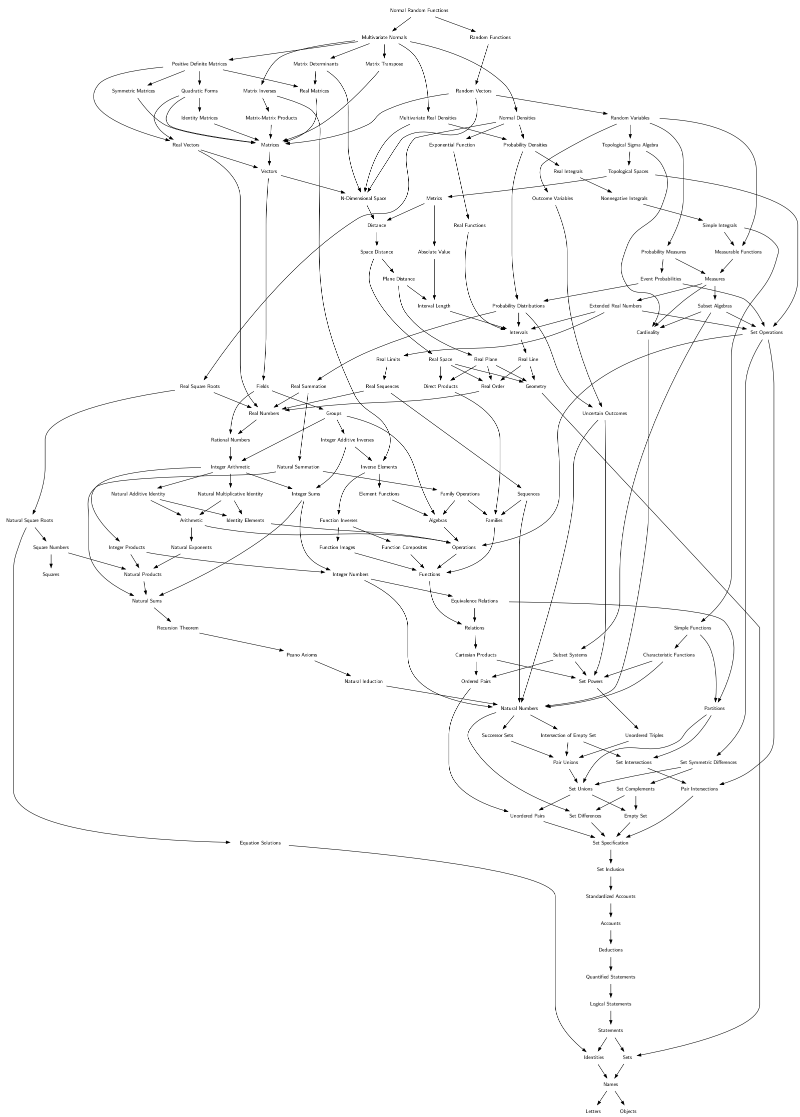
[1] The choice of "normal" is a result of the Bourbaki project's convention to eschew historical names. Though here, as in Multivariate Normals the language of the project is nonstandard.

[2] Future editions will change this description.

functions. One samples functions by drawing an outcome $\omega \in \Omega$, and then defining the sample $f : I \to \mathsf{R}$ by $f(i) = x(i)(\omega)$.

Multivariate normal special case

If the index set is finite, and can be ordered, then the normal random function is a multivariate normal random vector.


Normal Random Functions
Multivariate Normals
Random Functions
Positive Definite Matrices
Matrix Determinants
Matrix Transpose
Symmetric Matrices
Quadratic Forms
Matrix Inverses
Real Matrices
Random Vectors
Identity Matrices
Matrix-Matrix Products
Multivariate Real Densities
Normal Densities
Random Variables
Real Vectors
Matrices
Exponential Function
Probability Densities
Topological Sigma Algebra
Vectors
Real Integrals
Topological Spaces
N-Dimensional Space
Metrics
Outcome Variables
Nonnegative Integrals
Distance
Real Functions
Simple Integrals
Space Distance
Absolute Value
Probability Measures
Measurable Functions
Plane Distance
Event Probabilities
Measures
Interval Length
Probability Distributions
Extended Real Numbers
Subset Algebras
Intervals
Cardinality
Set Operations
Real Limits
Real Space
Real Plane
Real Line
Real Square Roots
Fields
Real Summation
Real Sequences
Direct Products
Real Order
Geometry
Real Numbers
Groups
Uncertain Outcomes
Rational Numbers
Integer Additive Inverses
Integer Arithmetic
Natural Summation
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Element Functions
Family Operations
Sequences
Natural Square Roots
Arithmetic
Identity Elements
Function Inverses
Algebras
Families
Square Numbers
Integer Products
Natural Exponents
Function Images
Function Composites
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Squares
Natural Products
Integer Numbers
Functions
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Relations
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Peano Axioms
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Subset Systems
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Ordered Pairs
Set Powers
Natural Numbers
Partitions
Successor Sets
Intersection of Empty Set
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