



## Why

We are constantly thinking of the integers as the endpoints of equal length segments of a line.

## Discussion

We commonly associate elements of the integers with the endpoints of equal-length segments of a real line. Take segment  $S_0$  of  $L$  with endpoints  $p$  and  $q$ . Associate the point  $p$  with 0. Associate the point  $q$  with 1. Take a segment  $S_1$  of equal length, non-overlapping with  $S_0$ , who shares the endpoint  $q$ . Associate the second endpoint of this segment 2. Continue with the rest.<sup>1</sup> We call the line so formed the *integral line* of unit  $S_0$ .

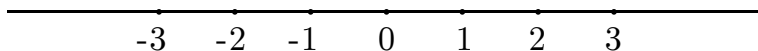


Figure 1: The integral segments.

## Integral Distance

Let  $f : \mathbf{Z} \rightarrow \mathbf{Z}$  be defined by  $f(a, b) = a - b$  if  $a > b$  and  $f(a, b) = b - a$  if  $b > a$ . Notice that  $f$  is symmetric:  $f(a, b) = f(b, a)$ . The (geometric) interpretation of  $f$  is the distance between the points associated with the two integers  $a, b \in \mathbf{Z}$  in some integral line. We call  $f$  the *integral distance*. Notice that  $f(a, b) > 0$  for all  $a, b \in \mathbf{Z}$ .

## Notation

We denote the distance between  $a, b \in \mathbf{Z}$  by  $|a - b|$ .

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<sup>1</sup>Future editions will expand.



