



## Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object.<sup>1</sup>

## Definition

A *countably summable subset algebra* is a subset algebra for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of  $A_1, \dots, A_n$  coincides with the union of  $A_1, \dots, A_n, A_n, A_n \dots$ .

We call the set of distinguished sets a *sigma algebra* (or *sigma field*) on the base set. This language is justified (as for a regular subset algebra) by the closure properties of the sigma algebra under the usual set operations. We sometimes write  $\sigma$ -algebra and  $\sigma$ -field.

A *sub- $\sigma$ -algebra* (*sub-sigma-algebra*) is a subset of a sigma algebra which is itself a sigma algebra.

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<sup>1</sup>Future editions will make no reference to measure theory. The entire development will be for follow the historical development for handling integration.

## Notation

Let  $(A, \mathcal{A})$  be a countably summable subset algebra. We often say “let  $\mathcal{A}$  be a sigma algebra on  $A$ .” Since the largest element of the sigma algebra is the base set, we can also say (without ambiguity): “let  $\mathcal{A}$  be a sigma algebra.” In this last case, the base set is  $\cup \mathcal{A}$ .

## Examples

**Example 1.** *For any set  $A$ ,  $2^A$  is a sigma algebra.*

**Example 2.** *For any set  $A$ ,  $\{A, \emptyset\}$  is a sigma algebra.*

**Example 3.** *Let  $A$  be an infinite set. Let  $\mathcal{A}$  the collection of finite subsets of  $A$ .  $\mathcal{A}$  is not a sigma algebra.*

**Example 4.** *Let  $A$  be an infinite set. Let  $\mathcal{A}$  be the collection subsets of  $A$  such that the set or its complement is finite.  $\mathcal{A}$  is not a sigma algebra.*

**Proposition 1.** *The intersection of a family of sigma algebras is a sigma algebra.*

**Example 5.** *For any infinite set  $A$ , let  $\mathcal{A}$  be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

*$\mathcal{A}$  is an algebra; the countable/co-countable algebra.<sup>2</sup>*

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<sup>2</sup>Future editions will clean up and modify these examples.

