Why

We want to quantify the error of compressing a real-valued random variable.

Definition

Let $\mathcal{X}$ be a finite set and $q : \mathbf{R} \to \mathcal{X}$ a quantization (see Quantizations) of $\mathbf{R}$. Also, fix a probability space $(\Omega, \mathcal{A}, \mathbf{P})$ and a random variable $x : \Omega \to \mathbf{R}$.

The *compression* $\hat{x} : \Omega \to \mathcal{X}$ of x under q is $q \circ x$. A *distortion function* for x and $\hat{x}$ is a function

$$d : (\Omega \to \mathbf{R}) \times (\Omega \to \mathcal{X}) \to \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

Examples

The *expected mean-squared-error distortion* d_{mse} between x and $\hat{x}$ is

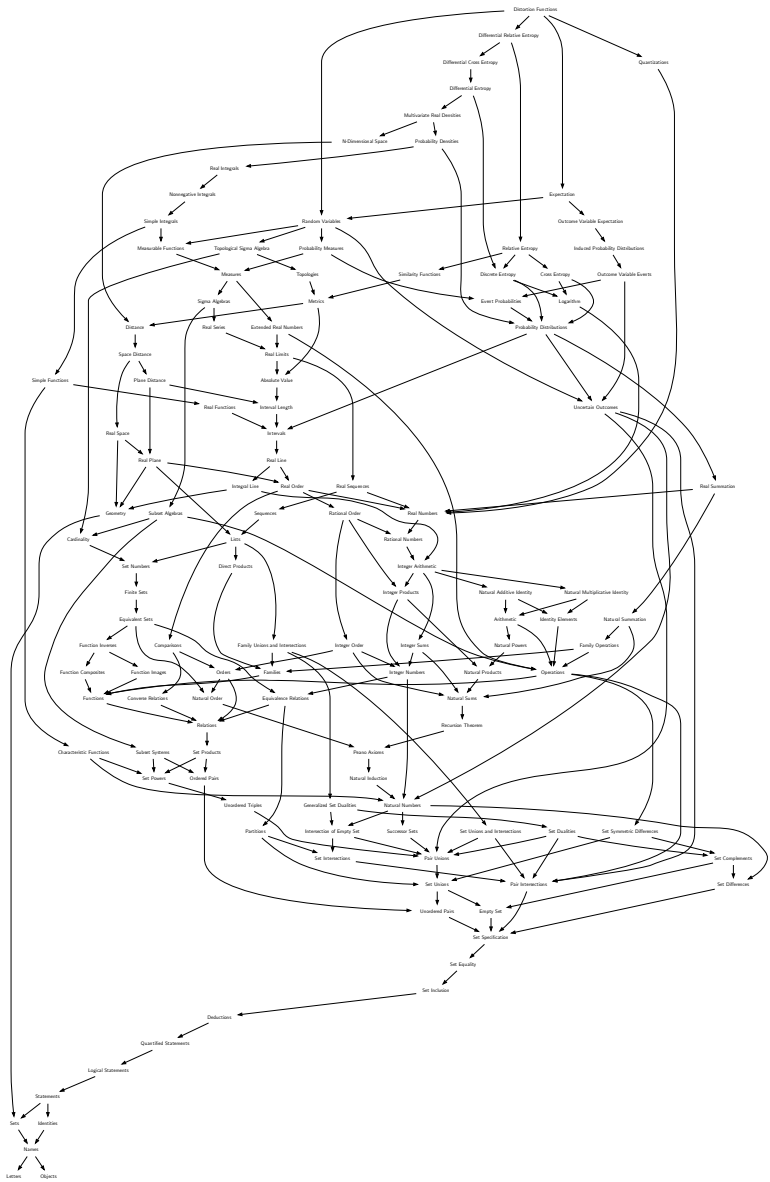
$$d_{\mathrm{mse}}(x, \hat{x}) = \mathbf{E}[(x - \hat{x})^2]$$

The *Kulback-Liebler distortion* d_{kld} defined by

$$d_{\mathrm{kld}}(x, \hat{x}) = \mathbf{E}[d_{\mathrm{kl}}(\mathbf{P}(y \in \cdot \mid x, \hat{x}) \mid \mathbf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.[1]

[1] Future editions will clarify this sentence.


Distortion Functions
Differential Relative Entropy
Quantizations
Differential Cross Entropy
Differential Entropy
Multivariate Real Densities
N-Dimensional Space
Probability Densities
Real Integrals
Nonnegative Integrals
Expectation
Simple Integrals
Random Variables
Outcome Variable Expectation
Measurable Functions
Topological Signed Algebra
Probability Measures
Relative Entropy
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Similarity Functions
Discrete Entropy
Cross Entropy
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Space Distance
Real Limits
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Absolute Value
Real Functions
Interval Length
Uncertain Outcomes
Real Space
Intervals
Real Plane
Real Line
Real Summation
Integral Line
Real Order
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Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
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Natural Summation
Function Inverses
Comparisons
Family Unions and Intersections
Integer Order
Integer Sums
Natural Powers
Family Operations
Function Composites
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Functions
Concrete Relations
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Characteristic Functions
Subset Systems
Set Products
Peano Axioms
Set Power
Ordered Pairs
Natural Induction
Unordered Triples
Generalized Set Duality
Natural Numbers
Partitions
Intersection of Empty Set
Successor Sets
Set Union and Intersections
Set Duality
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Set Unions
Pair Intersections
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Notes
Letters
Objects