Why

What is the best estimate for a random variable if we consider the square error?

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $x : \Omega \to \mathbf{R}$ a random variable. A *minimum mean squared error estimate* or *MMSE estimate* or *least square estimate* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}(x - \xi)^2$.

Proposition 1. *There is a unique MMSE estimate and it is given by* $\mathbf{E}(x)$.

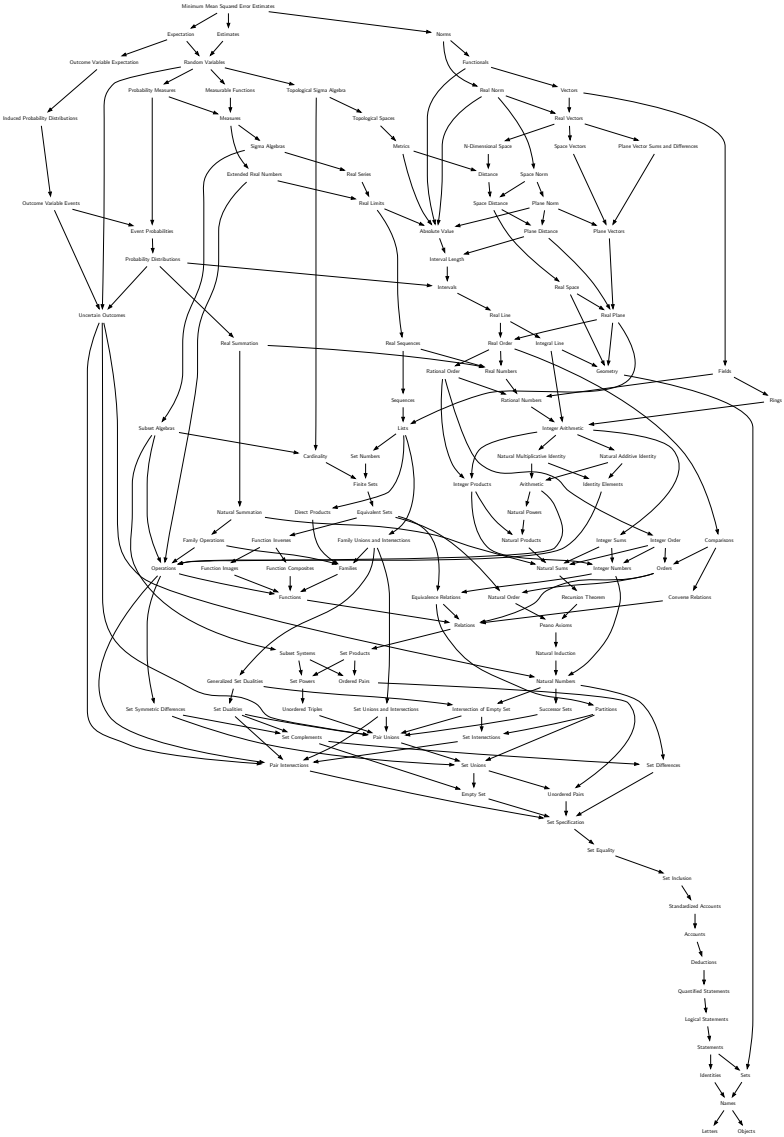
Vector Case

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $y : \Omega \to \mathbf{R}^n$ a random variable.[1]

A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}\|x - \xi\|^2$.

Proposition 2. *There is a unique MMSE estimator and it is given by* $\mathbf{E}(y)$.

[1] Future editions might collapse this into the previous case.


Minimum Mean Squared Error Estimates
Expectation
Estimates
Norms
Outcome Variable Expectation
Random Variables
Functionals
Probability Measure
Measurable Functions
Topological Signed Algebra
Real Norm
Vectors
Induced Probability Distributions
Measures
Topological Spaces
Real Vectors
Sigma Algebras
Metrics
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Extended Real Numbers
Real Series
Distance
Space Norm
Outcome Variable Events
Real Limits
Space Distance
Plane Norm
Event Probabilities
Absolute Value
Plane Distance
Plane Vectors
Probability Distributions
Interval Length
Intervals
Real Space
Uncertain Outcomes
Real Line
Real Plane
Real Summation
Real Sequences
Real Order
Integral Line
Rational Order
Real Numbers
Geometry
Fields
Sequences
Rational Numbers
Rings
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Integer Products
Arithmetic
Identity Elements
Natural Summation
Direct Products
Equivalent Sets
Natural Powers
Family Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Sums
Integer Order
Comparisons
Operations
Function Images
Function Composites
Families
Natural Sums
Integer Numbers
Orders
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Converse Relations
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Ordered Pairs
Natural Numbers
Set Symmetric Difference
Set Dualities
Unordered Triples
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects