Algebra

1 Why

We want to combine set elements to get other set elements.

2 Basics

Let A be a non-empty set. An **operation** on A is a function $g : A \times A \to A$. Operations map ordered pairs of elements of a set to elements of the same set. An **algebra** is a set and an operation.

2.1 Notation

Let A a set and $g : A \times A \to A$. We commonly forego the notation $g(a, b)$ and instead write $a \, g \, b$. We call this style **infix notation**.

Using lower case latin letters for every the elements and for the operation is confusing, but we often have special symbols for particular operations. Examples of such symbols include $+$, $-$, $\cdot$, $\circ$, and $\star$.

If we had a set A and an operation $+ : A \times A \to A$, we would write $a + b$ for the result of applying $+$ to (a, b). In denoting the algebra, we would say let $(A, +)$ be an algebra.

3 Operation Properties

Let $(A, +)$ be an algebra. $+$ is **commutative** if $a + b = b + a$ for all $a, b \in A$. In this case we say that $+$ **commutes**. $+$ is **associative** if $(a + b) + c = a + (b + c)$ for all $a, b, c, \in A$.