Why

How many ways are there to split n objects into nonoverlapping groups, when the objects are indistinguishable?

Definition

Suppose n is a nonzero natural number. A *partition* of n is a *nonincreasing* list of *nonzero* natural numbers whose sum is n. The requirement that the list of numbers be nonincreasing makes the representation unique. The terms of the list are called the *parts* of the partition. The number n is sometimes called the *weight* of the partition. The number of times a particular number appears in the list is called the *multiplicity* of that part.

Examples

What are the partitions of the number 5?

$$\begin{aligned} 5 &= 5 \\ &= 4 + 1 \\ &= 3 + 2 \\ &= 3 + 1 + 1 \\ &= 2 + 2 + 1 \\ &= 2 + 1 + 1 + 1 \\ &= 1 + 1 + 1 + 1 + 1 \end{aligned}$$

These seven identities correspond to the seven partitions of 5, namely $(5,)$, $(4, 1)$, $(3, 2)$, $(3, 1, 1)$, $(2, 2, 1)$, $(2, 1, 1, 1)$, $(1, 1, 1, 1, 1)$. The multiplicity of 1 in these partitions is 0, 1, 0, 2, 1, 3, 5, respectively.

Notation

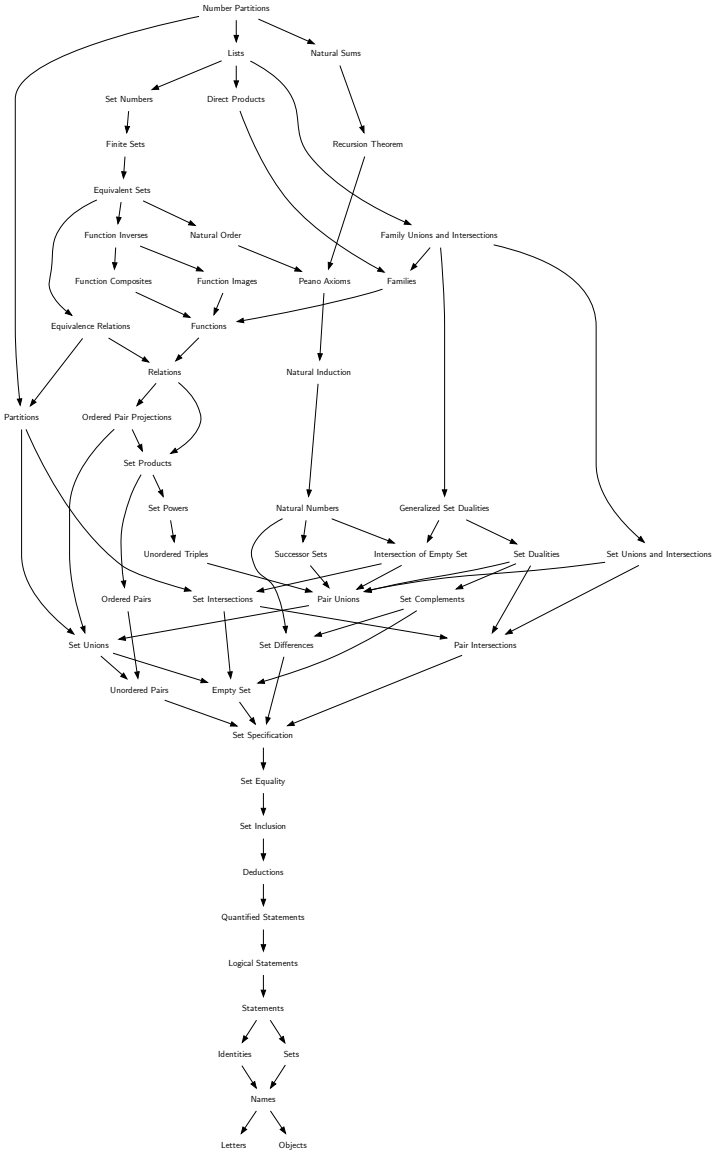
Suppose λ is a list in $\mathbf{N}$ of length $r \geq 1$. Then $\lambda = (\lambda_1, \ldots, \lambda_r)$ is a *partition* of $n \in \mathbf{N}$ if

$$\lambda_1 + \lambda_2 + \cdots + \lambda_r = n$$

and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r$. The terms of λ are the *parts* of the partition, so λ_i is the ith part, where $i = 1, \ldots, r$. Some authors denote the *weight* of λ by $|\lambda|$.

Partition function

How many partitions are there of the number n? We denote *this number* by $p(n)$. From the examples above, $p(5) = 7$.


Number Partitions
Lists
Natural Sums
Set Numbers
Direct Products
Finite Sets
Recursion Theorem
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Composites
Function Images
Peano Axioms
Families
Equivalence Relations
Functions
Relations
Natural Induction
Partitions
Ordered Pair Projections
Set Products
Set Powers
Natural Numbers
Generalized Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
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Objects