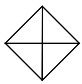


Why

We want to modify ordinary row reduction to handle the case in which a pivot is zero by selecting another suitable pivot.

Example

Let $A \in \mathbf{R}^{5\times 5}$. If $A_{11} \neq 0$, we may subtract multiples of row 1 from row $2, \dots, 5$ to eliminate variable x_1 from those equations. If A reduces to $C \in \mathbf{R}^{5\times 5}$ and $C_{22} \neq 0$, then step 2 moves from

$$\begin{bmatrix} \times & \times & \times & \times & \times \\ & C_{22} & \times & \times & \times \\ & \times & \times & \times & \times \\ & \times & \times & \times & \times \\ & \times & \times & \times & \times \end{bmatrix} \text{ to } \begin{bmatrix} \times & \times & \times & \times & \times \\ & C_{22} & \times & \times & \times \\ & \mathbf{0} & \times & \times & \times \\ & \mathbf{0} & \times & \times & \times \\ & \mathbf{0} & \times & \times & \times \end{bmatrix}.$$

What if $C_{22} = 0$? In this case suppose we pick a different row. For example, if $C_{42} \neq 0$ we can move from

$$\begin{bmatrix} \times & \times & \times & \times & \times \\ & \times & \times & \times & \times \\ & \times & \times & \times & \times \\ & C_{42} & \times & \times & \times \\ & \times & \times & \times & \times \end{bmatrix} \text{ to } \begin{bmatrix} \times & \times & \times & \times & \times \\ & \mathbf{0} & \times & \times & \times \\ & \mathbf{0} & \times & \times & \times \\ & C_{42} & \times & \times & \times \\ & \mathbf{0} & \times & \times & \times \end{bmatrix}.$$

Alternatively, we could introduce zeros in column 3 rather than column 2. For example, if we pick the pivot C_{43} we move from

$$\begin{bmatrix} \times & \times & \times & \times & \times \\ & \times & \times & \times & \times \\ & \times & \times & \times & \times \\ & \times & C_{43} & \times & \times \\ & \times & \times & \times & \times \end{bmatrix} \text{ to } \begin{bmatrix} \times & \times & \times & \times & \times \\ & \times & \mathbf{0} & \times & \times \\ & \times & \mathbf{0} & \times & \times \\ & \times & C_{43} & \times & \times \\ & \times & \mathbf{0} & \times & \times \end{bmatrix}.$$

We can choose any nonzero entry in $C_{k:m,k:m}$ as the pivot. Suppose we pick pivot $C_{st} \neq 0$ for $k \leq s, t \leq m$. Define $\tilde{C}$ by swapping row s of C with row k of C and column t of C with column k of C. Then $\tilde{C}_{kk} = C_{st} \neq 0$ and there exists an ordinary row reduction for $\tilde{C}$. We call this reduction of $(\tilde{C}, \tilde{d})$ a *pivoted row reduction* of C or the *st-reduction* of C.

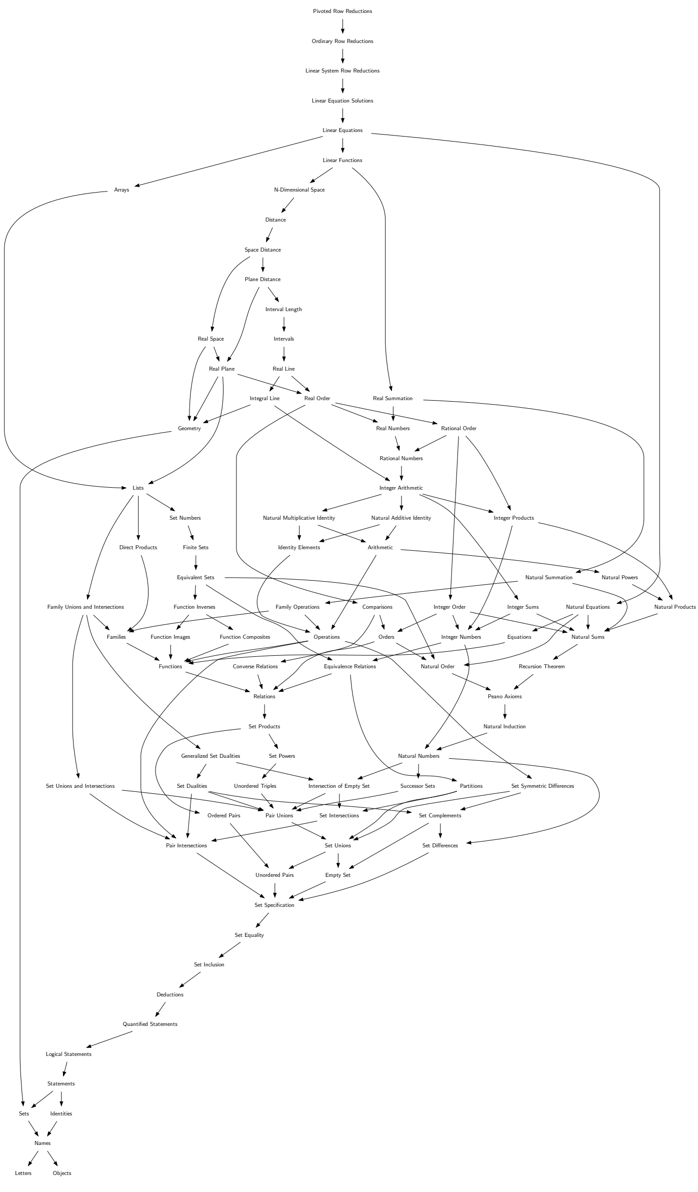
If all remaining pivots are zero, then there is no viable pivot. In this case, at least one variable is free and we do not have a unique solution. For convenience, in this case, we still call the system an st-reduction of itself.

Definition

At step k of ordinary elimination, multiples of row k are subtracted from rows $k+1, \ldots, m$ to introduce zeros in entry k of the rows. If we denote the matrix at the beginning of that step by X, then row k of X, column k of X and especially the pivot X_{kk} play a role. Ordinarily, we subtract from every entry in the submatrix $X_{k+1:m,k:m}$ the product of a number in row k and a number in column k, divided by the pivot X_{kk}. Generally, however, we can choose as pivot any nonzero entry of $X_{k:m,k:m}$.

An m-variable system (A, b) is *pivot reducible* (or *reducible*) if there exists a sequence of systems $S_1, \ldots, S_{m-1}$ so that S_1 is a reduction of (A, b) and S_i is a reduction of S_{i-1} for $i = 1, \ldots, m-1$. We call S_{m-1} the *final reduction* (or *reduction*) of (A, b). An immediate consequence of our definition is

Proposition 1. *All systems are reducible.*


Pivoted Row Reductions
Ordinary Row Reductions
Linear System Row Reductions
Linear Equation Solutions
Linear Equations
Linear Functions
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Real Summation
Geometry
Real Numbers
Rational Order
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Natural Summation
Natural Powers
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Equations
Natural Products
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Equations
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects