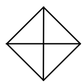


Why

We generalize random variables and random vectors to a map from the outcome space to a space of functions.

Definition

A *random function* is a map from the set of outcomes of a probability space to a set of functions.[1]

Notation

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let A and B be sets and let $x : \Omega \to (A \to B)$ be a random function. For each outcome $\omega \in \Omega$, $x_\omega : A \to B$ is a function from A to B.

A family of random variables

A *stochastic process* (or *random process* or *random field*) is a family of random variables. The term process is often used when the index set is associated with time and the term field is often used when the index set is associated with space.

We can associate to the random function $x : \Omega \to (A \to B)$ a family of random variables $y : A \to (\Omega \to B)$ defined by

$$y(a)(\omega) = x(\omega)(a).$$

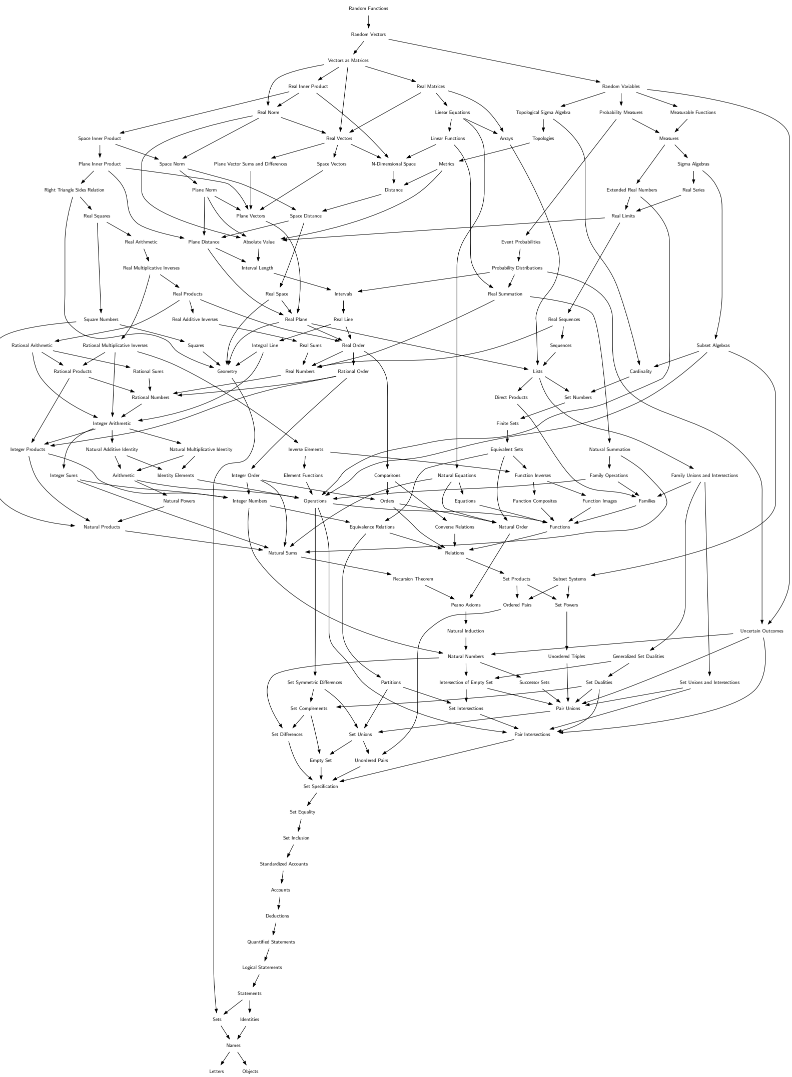
The index set of the family is the domain of the set of functions. The codomain of each random variable is the codomain of the set of functions.

The converse is also naturally true. Any family of random variables can be associated with a random function. In this way, a random function corresponds to a family of random variables, and any family of random variables corresponds to a random function.

For this reason many authors use the term random function or stochas-

[1] Future editions will make measurability precise.

tic function and random process or stochastic process interchangeably. In these sheets, we will be specific.


Random Functions
Random Vectors
Vectors as Matrices
Real Inner Product
Real Matrices
Random Variables
Real Norm
Linear Equations
Topological Sigma Algebra
Probability Measures
Measurable Functions
Space Inner Product
Real Vectors
Linear Functions
Arrays
Topologies
Measures
Plane Inner Product
Space Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Metrics
Sigma Algebras
Right Triangle Sides Relation
Plane Norm
Distance
Extended Real Numbers
Real Series
Real Squares
Plane Vectors
Space Distance
Real Limits
Real Arithmetic
Plane Distance
Absolute Value
Event Probabilities
Real Multiplicative Inverses
Interval Length
Probability Distributions
Real Products
Real Space
Intervals
Real Summation
Square Numbers
Real Additive Inverses
Real Plane
Real Line
Real Sequences
Rational Arithmetic
Rational Multiplicative Inverses
Squares
Integral Line
Real Sums
Real Order
Sequences
Subset Algebras
Rational Products
Rational Sums
Geometry
Real Numbers
Rational Order
Lists
Cardinality
Rational Numbers
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Inverse Elements
Equivalent Sets
Natural Summation
Integer Sums
Arithmetic
Identity Elements
Integer Order
Element Functions
Comparisons
Natural Equations
Function Inverses
Family Operations
Family Unions and Intersections
Natural Powers
Integer Numbers
Operations
Orders
Equations
Function Composites
Function Images
Families
Natural Products
Equivalence Relations
Converse Relations
Natural Order
Functions
Natural Sums
Relations
Recursion Theorem
Set Products
Subset Systems
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Uncertain Outcomes
Natural Numbers
Unordered Triples
Generalized Set Dualities
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects