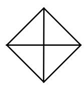


Why

We discuss learning (or inferring) relations from examples.

Definition

Let X and Y be sets. A *relation inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a relation between X and Y. We frequently use the term inductor to refer to a family of inductors, indexed by $n \in \mathbf{N}$.

An inductor is *functional* if it produces functions. In this case, we call the elements of X the *inputs* and the elements of Y the *outputs*. We call a function from inputs to outputs a *predictor* and call the result of an input under a predictor a *prediction*. Using this language, a functional inductor maps datasets to predictors. A predictor maps inputs to outputs.

To every relation between X and Y corresponds a characteristic function on $X \times Y$ and vice versa. For this reason, henceforth by *inductor* we mean a functional inductor. A relational inductor on a dataset in $(X \times Y)^n$ can be modeled by a functional inductor on a dataset in $((X \times Y) \times \{0, 1\})^n$.

Notation

Let D be a dataset of size n in $X \times Y$. Let $g : X \to Y$, a predictor, which makes prediction $g(x)$ on input $x \in X$. Let $G_n : (X \times Y)^n \to (X \times Y)$ be an inductor. Then $G_n(D)$ is the predictor which the inductor associates with dataset D. And

$\{G_n : (X \times Y)^n \to \mathcal{P}((X \times Y))\}_{n \in \mathbf{N}}$ is a family of inductors.

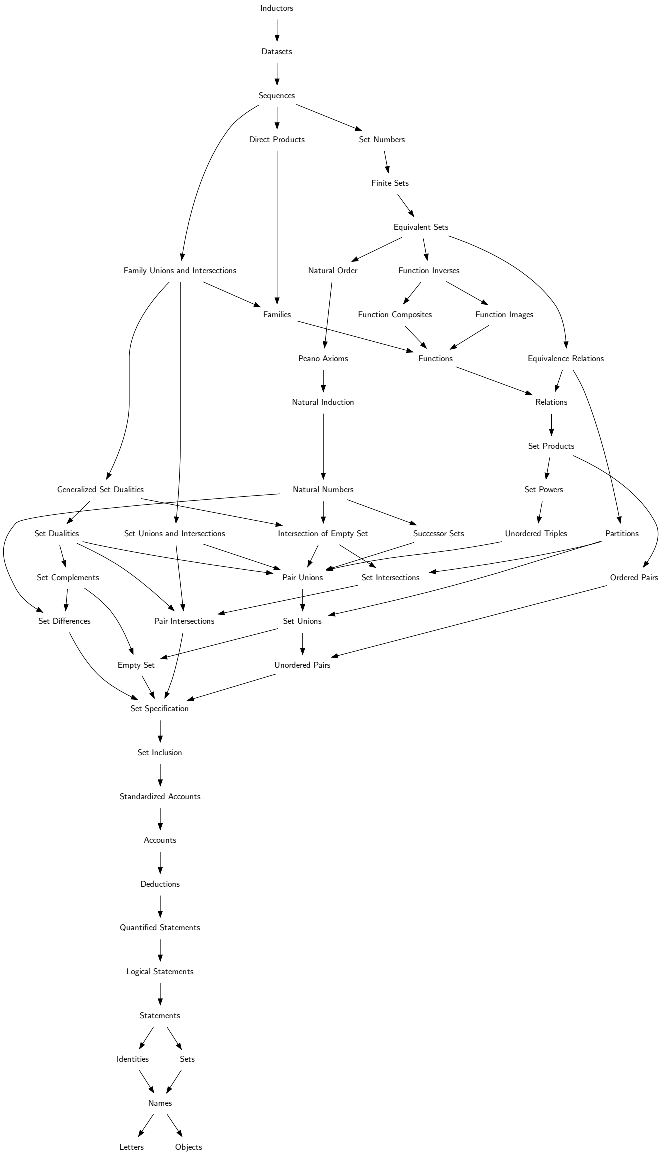
Consistent and complete datasets

Let $D = ((x_i, y_i))_{i=1}^n$ be a dataset and $R \subset X \times Y$ a relation. D is *consistent with* R if each $(x_i, y_i) \in R$. D is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take $R = X \times Y$). D is *functionally consistent* if it is consistent with a function; in this case, $x_i = x_j \longrightarrow y_i = y_j$. D is *functionally complete* if $\cup_i \{x_i\} = X$. If a dataset is complete, then it includes every element of the relation.

Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent variables*, *explained variables*, *postcepts*, *targets*, *outcomes*, *labels* or *observational outcomes*.

Other terms for a predictor include *input-output* mapping, *prediction rule*, *hypothesis*, *concept*, or *classifier*. Since a predictor can be used to *guess* the output of an input, some authors call an inductor (or family of inductors) a *learner* or *learning algorithm* or *supervised learning algorithm* and refer to the argument as the *training dataset*. Often the word "supervised" is included, as in *supervised learning*. The language intends to indicate that inputs are given along with outputs, and these outputs "provide supervision to the algorithm."


Inductors
Datasets
Sequences
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Natural Induction
Relations
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects