Linearly Dependent Vectors

1 Why

We want some notion building up spaces using a few vectors, and we want to pick vectors which are not redundant. This sheet is about redundancy when you are forming linear combinations of vectors.

2 Definition

A vector is *linearly dependent* on a sequence of vectors if it is identical to the result of some linear combination of them. For example, any vector is linearly independent on the singleton consisting of itself.

A sequence of vectors is *linearly dependent* if there is a coordinate which is linearly dependent on the sequence which is identical to the original sequence except at that coordinate, which is the zero vector.

We feel that the above definitions better capture the intuition. The usual definition of linear dependence is given in the following proposition.

Proposition 1. *A sequence of vectors is linearly dependent if and only if then there exists a nontrivial linear combination of sequence whose result is the zero vector.*

Proof. □

A finite sequence of vectors is *linearly dependent* if one is identical to a nontrivial linear combination of the others. If one can be written as a linearly combination

Two vectors, then are linearly dependent if one is a scalar multiple of the other. The sequence ($\mathbf{0}$) is linearly dependent since $a_1\mathbf{0} = \mathbf{0}$ for any scalar a_1.

An infinite sequence of vectors is *linearly dependent* if some finite subsequence is dependent. It is *linearly independent* if every finite subsequence is linearly independent.

Any sequence (finite or infinite) which contains the zero vector is linearly dependent.

If a finite sequence of vectors is linearly independent, then each of its coordinates is distinct.

A set of vectors is *linearly independent* if every sequence of distinct vectors in the set is linearly independent. The set is *linearly dependent* if somem finite sequence of distinct vectors is dependent.