

## Why

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## Definition

Let X be a set and let A be a finite set. We denote the set of all finite sequences (strings) in A by S(A). We read S(A) aloud as "the strings in A." The length zero string is  $\emptyset$ .

A code for X in A is a function from X to  $\mathcal{S}(A)$ . In this context, we refer to the finite set A as an alphabet and we call c(x) the codeword of x. The length of  $x \in X$ , with respect to a code  $c: X \to \mathcal{S}(A)$ , is the length of the sequence c(x) (its codeword). We call a code nonsingular if it is injective.

## **Examples**

Define 
$$c: \{\alpha, \beta\} \to \{0, 1\}$$
 by  $c(\alpha) = (0, 1)$  and  $c(\beta) = (1, 1)$ .

## Code extensions

Let  $s, t \in \mathcal{S}(A)$  of length m and n respectively. The concatenation of s with t is the length m + n string  $u \in \mathcal{S}(A)$  defined by  $u_1 = s_1, \ldots, u_m = s_m$  and  $u_{m+1} = t_1, \ldots, u_{m+n} = t_n$ . We denote the concatenation of s and t by st. Note, however, that  $st \neq ts$ , although s(tr) = (st)r.

<sup>&</sup>lt;sup>1</sup>Future editions will include, with perhaps discussion of encoding a representing text.

<sup>&</sup>lt;sup>2</sup>Future editions will include additional examples.

Given a code  $c: X \to \mathcal{S}(A)$ , we can produce a code for  $\mathcal{S}(X)$  in a natural way. The *extension* of c is the function  $C: \mathcal{S}(X) \to \mathcal{S}(A)$  defined, for  $\xi = (\xi_1, \dots, \xi_n) \in \mathcal{S}(X)$ , by

$$C(\xi) = c(\xi_1) \cdots c(\xi_n).$$

We call an code uniquely decodable if its extension is injective. In other words, given the code  $C(\xi)$  for a sequence  $\xi \in \mathcal{S}(X)$ , we can recover  $\xi$ . We call  $C(\xi)$  the encoding of  $\xi$ . We call  $\xi$  the decoding of  $C(\xi)$ .

