Why

If we stack two rectangles, with equal base lengths but different heights, on top of each other, the additivity principle says that the area of the so-formed rectangle is the sum of the areas of the stacked rectangles. Our definition of integral for simple functions has this property, as it ought to.

Result

Proposition 1. *The simple non-negative integral operator is additive.*

Proof. Let $(X, \mathcal{A}, \mu)$be a measure space. Let $\mathrm{SF}_+(X)$ denote the non-negative real-valued simple functions on X. Define $s : \mathrm{SF}_+(X) \to [0, \infty]$ by $s(f) = \int f d\mu$ for $f \in \mathrm{SF}_+ X$.

In this notation, we want to show that$s(f + g) = s(f) + s(g)$ for all $f, g \in \mathrm{SF}_+(X)$. Toward this end, let $f, g \in \mathrm{SF}_+(X)$ with the simple partitions:

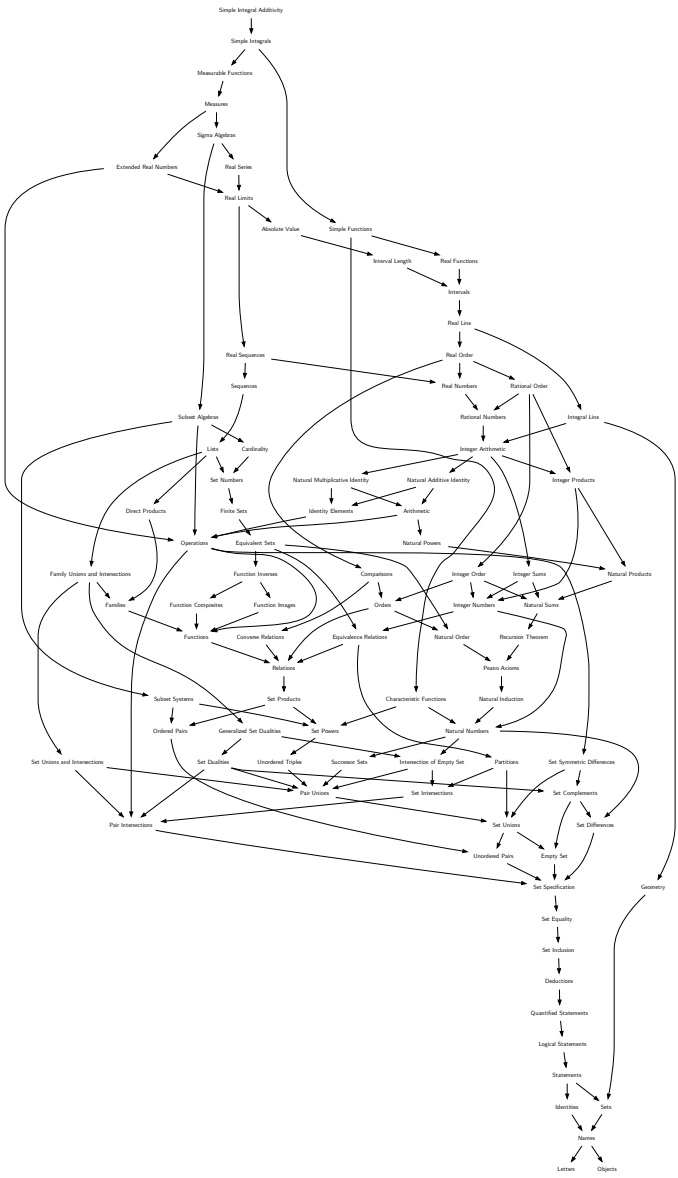
$$\{A_i\}_{i=1}^m, \{B_j\}_{j=1}^n \subset \mathcal{A} \quad \text{and} \quad \{a_i\}_{i=1}^m, \{b_j\}_{j=1}^n \subset [0, \infty].$$

We consider the refinement of the two partitions. TODO: this is why you don't do the unique maximalpartition business. $\{A_i \cap B_j\}_{i,j=1}^{i=m,j=n}$.

First, let$\alpha \in (0, \infty)$. Then $\alpha f \in \mathrm{SF}_+(X)$,with the simple partition$\{A_n\} \subset \mathcal{A}$ and $\{\alpha a_n\} \subset [0, \infty]$.

$$\begin{aligned} s(\alpha f) &= \sum_{i=1}^n \alpha a_n \mu(A_i) \\ &= \alpha \sum_{i=1}^n a_n \mu(A_i) \\ &= \alpha s(f). \end{aligned}$$

If $\alpha = 0$, then αf is uniformly zero; it is the non-negative simple with partition $\{X\}$ and $\{0\}$. Regardless of the measure of X, this non-negative simple function is zero. Recall that we define $0 \cdot \infty = \infty \cdot 0 = 0$. $\square$


Simple Integral Additivity
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Simple Functions
Interval Length
Real Functions
Intervals
Real Line
Real Sequences
Real Order
Sequences
Real Numbers
Rational Order
Rational Numbers
Integral Line
Subset Algebras
Lists
Cardinality
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Operations
Equivalent Sets
Natural Powers
Natural Products
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects