



## Why

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## Definition

Let  $z_1, z_2 \in \mathbf{C}$  with  $z_1 = (x_1, y_1)$  and  $z_2 = (x_2, y_2)$ . The *complex product* of  $z_1$  and  $z_2$  is the complex number  $(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$ .

## Notation

We denote the complex product of  $z_1$  and  $z_2$  by  $z_1 \cdot z_2$  or  $z_1 z_2$ . The notation is justified because the complex product of two purely real complex numbers corresponds to the purely real complex number whose real part is the real product of the real parts of the first two numbers.

Recall that we denote  $z_1 = x_1 + iy_1$  and  $z_2 = x_2 + iy_2$ . This notation is a mnemonic for the definition of a complex product if we treat  $i^2 = -1$ .

$$\begin{aligned} z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2). \end{aligned}$$

## Properties

**Proposition 1** (Commutativity). *For all  $z_1, z_2 \in \mathbf{C}$ , we have  $z_1 z_2 = z_2 z_1$ .*

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<sup>1</sup>Future editions will include.

**Proposition 2** (Associativity). *For all  $z_1, z_2, z_3 \in \mathbf{C}$ , we have and  $z_1(z_2 z_3) = (z_1 z_2) z_3$ .*

## Complex multiplication

We call the operation that associates a pair of complex numbers with their product *complex multiplication*. The operation is symmetric (commutative).

## Multiplicative identity and inverse

Notice that the complex number  $(1, 0)$  is the multiplicative identity. It is unique,<sup>2</sup> and so we call it the *complex multiplicative identity*.

We call the operation  $(z, w) \mapsto z/w$  *complex division* and we call  $z/w$  the *(complex) quotient* of  $z$  with  $w$ .

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<sup>2</sup>Future editions will include an account



