Integrable Function Spaces

Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? TODO: perhaps do L^2 first then generalize.

Definition

The *integrable function spaces* are a collection of function spaces, one for each real number $p \geq 1$, for which the pth power of the absolute value of the function is integrable.

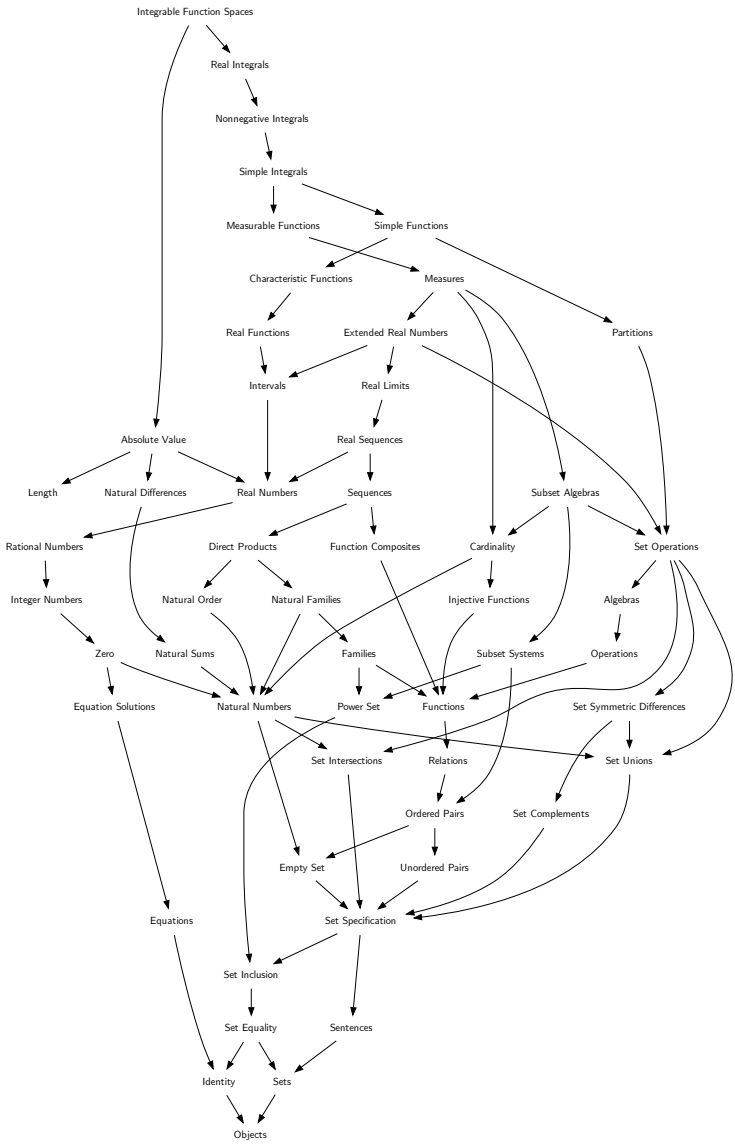
TODO: case ∞

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $p \geq 1$. Let R denote the set of real numbers. We denote the integrable function space corresponding to p by $\mathcal{L}^p(X, \mathcal{A}, \mu, R)$. We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, R) = \left\{ \text{ measurable } f : X \to R \;\middle|\; \int |f|^p d\mu < \infty \right\}$$

Let C denote the set of complex numbers. Similarly for complex-valued functions, we denote the pth space by $\mathcal{L}^p(X, \mathcal{A}, \mu, C)$.


Integrable Function Spaces
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Simple Functions
Characteristic Functions
Measures
Real Functions
Extended Real Numbers
Partitions
Intervals
Real Limits
Absolute Value
Real Sequences
Length
Natural Differences
Real Numbers
Sequences
Subset Algebras
Rational Numbers
Direct Products
Function Composites
Cardinality
Set Operations
Integer Numbers
Natural Order
Natural Families
Injective Functions
Algebras
Zero
Natural Sums
Families
Subset Systems
Operations
Equation Solutions
Natural Numbers
Power Set
Functions
Set Symmetric Differences
Set Intersections
Relations
Set Unions
Ordered Pairs
Set Complements
Empty Set
Unordered Pairs
Equations
Set Specification
Set Inclusion
Set Equality
Sentences
Identity
Sets
Objects