Why

We want to talk rooting a tree at a given vertex.[1]

Definition

A *rooted tree* is an ordered pair $((V, T), r)$ where (V, T) is a tree and $r \in V$ is a distinguished vertex which we call the *root*. We visualize rooted trees with the root at the top (see the figure below).

Parents and children

Suppose w is the first vertex on the path from the root to a non-root vertex v. Since there is only one such path, w is unique and we call it *the parent* of v. Conversely, we call v *a child* of w. We denote the set of children of v by $\mathrm{ch}(v)$. A vertex may have no children or it may have many children. If it has no children we call it a *leaf*.

We define the *parent function* $\mathrm{pa} : V \to V$ with the convention that the *parent of the root* is the root. The *parent of degree k* where $k > 0$ is $\mathrm{pa}^k(x)$ where pa^k is the composite of pa with itself k times. So, in particular, $\mathrm{pa}^{k+1}(v) = \mathrm{pa}(\mathrm{pa}^k(v))$. We define the *parent of degree 0 of v* to be v, and denote it by $\mathrm{pa}^0(v) = v$. For the tree visualized in the figure below, $\mathrm{pa}(i) = g$, $\mathrm{pa}^2(i) = d$, $\mathrm{pa}^3(i) = a$.

If $w = \mathrm{pa}^k(v)$ for some $k \geq 0$, then w is a *ancestor* of v and v is a *descendent* of w. We use the term *proper ancestor* and *proper descendent* if $k > 0$ (i.e., $w \neq v$).

The *depth* or *level* of a vertex v is its distance (see Trees) to the root. We denote the level of a vertex v by $\mathrm{lev}(v)$. The level of the root is 0. If $\mathrm{lev}(v) = k > 0$, then $\mathrm{pa}^k(v)$ is the root. The level function lev satisfies $\mathrm{lev}(v) = \mathrm{lev}(\mathrm{pa}(v)) + 1$.

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[1]Future editions will expand this intuition.

Rooted Trees

Trees

Directed Graphs   Undirected Paths

Undirected Graphs   Lists

Set Numbers   Direct Products

Finite Sets

Equivalent Sets

Function Inverses   Natural Order   Family Unions and Intersections

Function Images   Function Composites   Families

Functions   Equivalence Relations

Relations   Peano Axioms

Ordered Pair Projections   Natural Induction

Set Products

Set Powers   Natural Numbers   Generalized Set Dualities

Partitions   Unordered Triples   Successor Sets   Intersection of Empty Set   Set Unions and Intersections   Set Dualities

Ordered Pairs   Set Intersections   Pair Unions   Set Complements

Set Unions   Set Differences   Pair Intersections

Unordered Pairs   Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects

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