Why

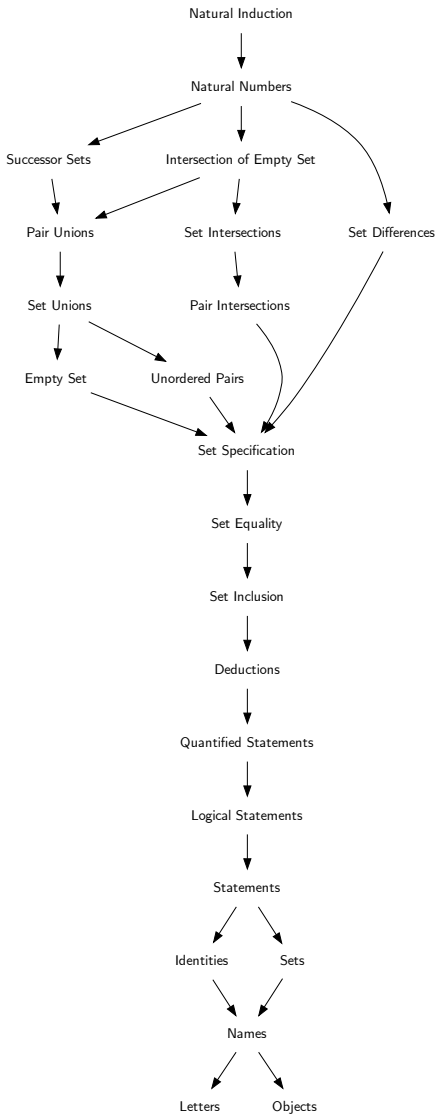
We want to show something holds for every natural number.[1]

Definition

The most important property of the set of natural numbers is that it is the unique smallest successor set. In other words, if S is a successor set contained in ω (see Natural Numbers), then $S = \omega$. This is useful for proving that a particular property holds for the set of natural numbers.

To do so we follow standard routine. First, we define the set S to be the set of natural numbers for which the property holds. This step uses the principle of selection (see Set Selection) and ensures that $S \subset \omega$. Next we show that this set S is indeed a successor set. The first part of this step is to show that $0 \in S$. The second part is to show that $n \in S \longrightarrow n^+ \in S$. These two together mean that S is a successor set, and since $S \subset \omega$ by definition, that $S = \omega$. In other words, the set of natural numbers for which the property holds is the entire set of natural numbers. We call this the *principle of mathematical induction.*

[1] Future editions will modify this superficial why.


Natural Induction
Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects