



Tree-Structured Distribution Approximation

1 Why

2 Problem

Tree-structured distribution approximation is a mathematical optimization problem in which we find a tree-structured distribution which minimizes its entropy relative to a given distribution over the set of all distributions which factor according to trees.

2.1 Notation

Let A_1, \dots, A_n be non-empty sets. Let q be a distribution on $\prod_{i=1}^n A_i$. Let d denote the relative entropy.

We want to find a distribution p on A and tree T on $\{1, \dots, n\}$ to

$$\begin{aligned} & \text{minimize} && d(q, p) \\ & \text{subject to} && p \text{ factors according to the tree } T \end{aligned}$$

3 Solution

Proposition 1. *Let q be a distribution on A . Let T be a tree on $\{1, \dots, d\}$. Let p_j be the parent of vertex j for the T rooted at vertex i , $j = 1, \dots, n$ and $j \neq i$. Then the distribution p on A defined by*

$$p = q_i \prod_{j \neq i} q_{j|p_j}$$

achieves minimum entropy relative to q among all distributions which factor according to T .

Proposition 2. *Let q be a distribution on A . Let T be a tree on $\{1, \dots, d\}$. Let p_j be the parent of vertex j for the T rooted at vertex i , $j = 1, \dots, n$ and $j \neq i$. Then the distribution p on A defined by*

$$p = q_i \prod_{j \neq i} q_{j|p_j}$$

achieves minimum entropy relative to q among all distributions which factor according to T .

Proposition 3. *Let q be a distribution on A . A tree T is a solution to the problem above if and only if it is a minimum spanning tree of the mutual information graph of q .*