## Why

We generalize convex functions to arbitrary vector spaces.

## Definition

Suppose $X$ is a vector space over $\mathbf{R}$, $D \subset X$ and $f : D \to \bar{\mathbf{R}}$. As before, define

$$\operatorname{epi} f = \{(x, \alpha) \in X \times \mathbf{R} \mid f(x) \leq \alpha\}$$

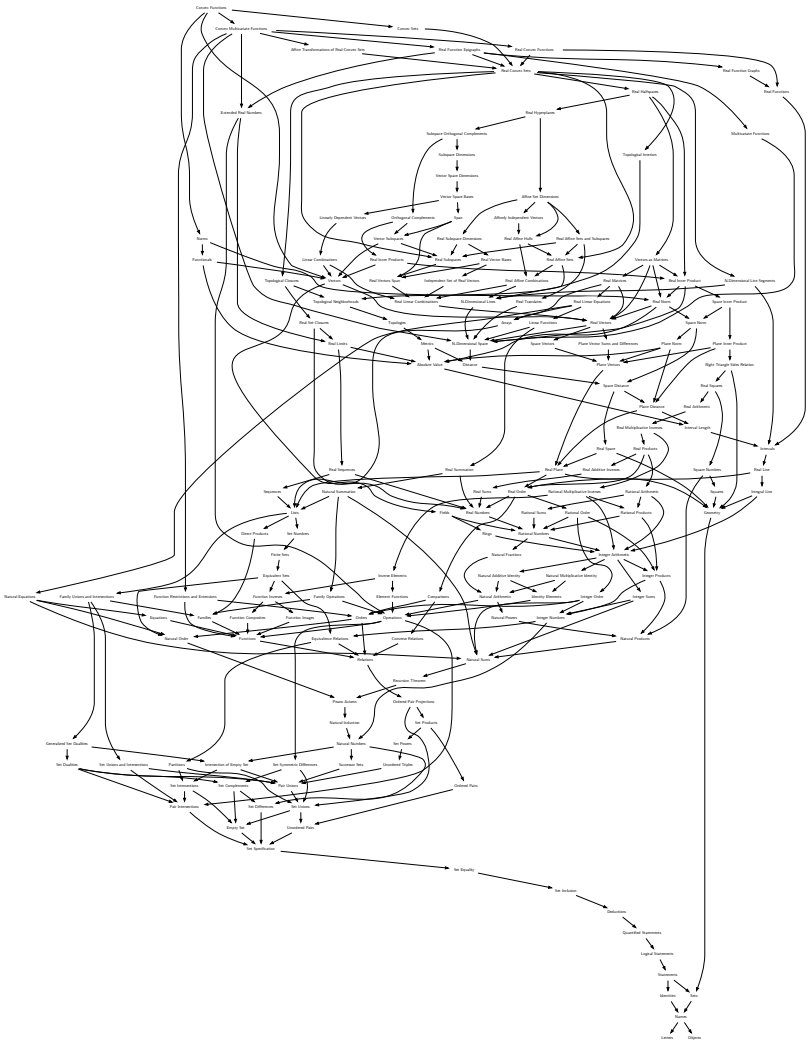
$f$ is convex if $\operatorname{epi} f$ is convex. It is straightforward that $f$ is convex if and only if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

for all $t \in [0, 1]$ and $x, y \in D$.

### Examples

Any norm on $X$ is a convex function.


Complex Functions
Complex Sets
Complex Multivariate Functions
Affine Transformations of Real Complex Sets
Real Function Equations
Real Complex Functions
Real Complex Sets
Real Function Sums
Real Functions
Real Subspaces
Extended Real Numbers
Real Hyperplanes
Multivariate Functions
Subspace Orthogonal Complements
Subspace Dimensions
Topological Sections
Vector Space Dimensions
Vector Space Bases
Affine Set dimensions
Linearly Dependent Vectors
Orthogonal Complements
Sets
Affinely Independent Vectors
Vector Sections
Real Subspace Dimensions
Real Affine Hulls
Real Affine Sets and Subspaces
Bases
Functionals
Linear Combinations
Real Vector Products
Real Subspaces
Real Affine Sets
Vectors as Matrices
Topological Sets
Sets
Real Vector Sums
Intersection Sets of Real Sets
Real Affine Combinations
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Real Scalar Product
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Topological Neighborhoods
Real Linear Combinations
2-Dimensional Lines
Real Translations
Real Linear Equations
Real Lines
Vector Sum Product
Real Line Segments
Real Line Segments
Lengths
Sets
Real Lines
Sets
2-Dimensional Rays
Space Vectors
Plane Vector Angles and Differences
Plane Rays
Plane Vector Product
Absolute Value
Distance
Real Vectors
Right-Triangle Side Relations
Space Distance
Real Squares
Real Distance
Real Arithmetic
Real Multiplication Sum
Vectors
Vectors Length
Real Values
Real Products
Lengths
Real Sequences
Real Summation
Real Power
Real Addition Sections
Square Numbers
Real Line
Integers
Real Sum
Natural Numbers
Real Sums
Rational Multiplication
Natural Multiplication
Natural Sums
Natural Order
Natural Products
Integer Line
Sets
Real Numbers
Prime
Rational Sums
Natural Order
Natural Products
Integer
Direct Products
Set Functions
Integer
Rational Numbers
Integer Sums
Finite Sets
Natural Factors
Integer Products
Equivalence Sets
Inverse Elements
Natural Addition Order
Natural Multiplication Identity
Integer Products
Natural Equivalence
Family Unions and Intersections
Function Restrictions and Extensions
Family Operations
Element Functions
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Integer Order
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Function Sections
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Natural Products
Relations
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Natural Sums
Recursive Theorems
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Ordered Pair Sets
Natural Exponents
Set Products
Natural Numbers
Set Powers
Generalized Set Qualifier
Set Qualifier
Set Unions and Intersections
Functions
Cardinalities of Empty Set
Set Symmetric Differences
Accessory Sets
Unordered Triples
Set Complements
Set Unions
Ordered Pairs
Set Intersections
Set Differences
Set Sums
Empty Set
Unordered Pairs
Set Equivalence
Set Equality
Set Inclusion
Definitions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Axioms
Lemmas
Origins