Why

We name a statement which involves an identity.[1]

Definition

An *equation* is statement (see Statements) relating two terms by the relation of identity (see Identities). Some authors also call an equation an *equality*. The symbol "=" is called the (or an) *equals sign* or *equals symbol*.

Variables

Let X and Y be sets and let $f, g : X \to Y$. In the statement

$$(\forall x)(f(x) = g(x)),$$

f and g are *free* names and x is a *bound* name (see Quantified Statements).

For convenience, we often refer to the equation $f(x) = g(x)$ without the quantifier $\forall x$. In this case, x appears free, but is not. In this context, the statement $f(x) = g(x)$ has x implicitly bound. There are two senses here, though. The first is that x is bound because it is "subordinate" to the quantifier $\forall$. The particular symbol is irrelevant; the symbol y works just as well. In a second sense, though, the name is "free," as it is a placeholder and the choice of the symbol x does not matter.

We use different terminology for this common case. In discussing $f(x) = g(x)$, we call the placeholder name x a *variable* and we call the names f and g *constants*. The language is meant to convey f and g are *fixed* in the present discussion, as indicated by the usual language "Let f and g ...".

[1] Future editions will modify this statement and sheet.

Solutions

We are often interested in finding objects in some set to satisfy an equation. For example, we are interested in finding an object $\xi \in X$ to satisfy $f(\xi) = g(\xi)$. In this setting we call the variable ξ in the equation an *unknown*.

We call an element $\xi \in X$ a *solution* of the equation if $f(\xi) = g(\xi)$. We call the set

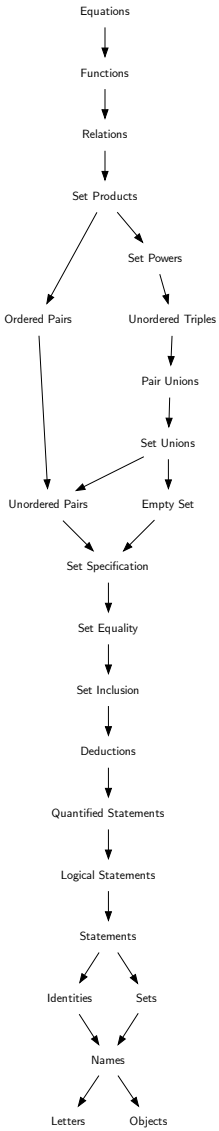
$$\{\xi \in X \mid f(\xi) = g(\xi)\}$$

the *solution set*. If the solution set is non-empty, we say that a solution *exists*. If the solution set is a singleton, we say that the solution is *unique*.

We are often interested in solutions which satisfy several equations at once. For example, we have the equations $f_1(x) = g_1(x)$ and $h(x) = i(x)$ and so on. We want x to satisfy these. Here it is *set of equations*, *simultaneous equations*, or a *system of equations*.

Finding solutions

We often talk about *finding* or *searching* for solutions or *solving equations*. We say: "We want to *find* $x \in X$ to satisfy $f(x) = g(x)$." In addition to $f(x) = g(x)$, we may include other statements about x. The language is meant to convey that we are searching for an object which we will name, as a variable, x, and we want this object to satisfy the statements.


Equations
Functions
Relations
Set Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects