



# Filled Graphs

## 1 Why

TODO Needed for perfect elimination orderings.

## 2 Definition

An ordered graph is *filled* or *monotone transitive* if all higher neighborhoods induce complete subgraphs.

## 3 Notation

Let  $G_\sigma = (V, E, \sigma)$  be an ordered graph.  $G_\sigma$  is filled if

$$u, v \in \mathbf{adj}^+(v) \implies \{u, v\} \in E.$$

In other word, if  $i < j < k$  so that  $\{\sigma(i), \sigma(j)\} \in E$  and  $\{\sigma(i), \sigma(k)\} \in E$  then  $\{\sigma(j), \sigma(k)\} \in E$ .

## 4 Chordality

**Proposition 1.** *If  $(V, E, \sigma)$  is a filled graph, then  $(V, E)$  is chordal.*

*Proof.* Take the vertex with the lowest index on a cycle of length greater than three. Take □





