### Why

Suppose I have three stones in my hands. If I have two in my left hand, how many are in my right hand?

### Discussion

Denote by $a$ the number of stones in my left hand. Denote by $b$ the number of stones in my right hand. Then the number of stones in both hands is $a + b$. Both $a$ and $b$ are natural numbers. In other words, $a$ is 0 or 1 or 2 or... Likewise for $b$.

We express that I have two stones in my left hand by the equation $a = 2$. We express that I have have five stones in total by the equation that $a + b = 5$. Because we have identified the names $a$ and $b$ with tangible objects in my hands, these equations are statements about tangible objects in my hands. But the *numbers* involved are intangible objects. In any case, we have two equations in two unknowns.

We express the question "how many stones do I have in my right hand?" by asking for a solution (see Equations) to the equation $a + b = 5$. We can start by trying natural numbers in order. Is $(a, b) = (0, 0)$ a solution? Well, $0 + 0 = 0$, and $0 \neq 5$, so it is not. Is $(a, b) = (1, 0)$ a solution? Well, $l + 0 = 1$, and $1 \neq 5$, so it is not. Well, $(a, b) = (0, 1)$ a solution? Well, $l + 0 = 1$, and $1 \neq 5$, so it is not. Likewise for $(0, 2)$, $(1, 2)$ and so on. Some people call this the process of *guess and check*.

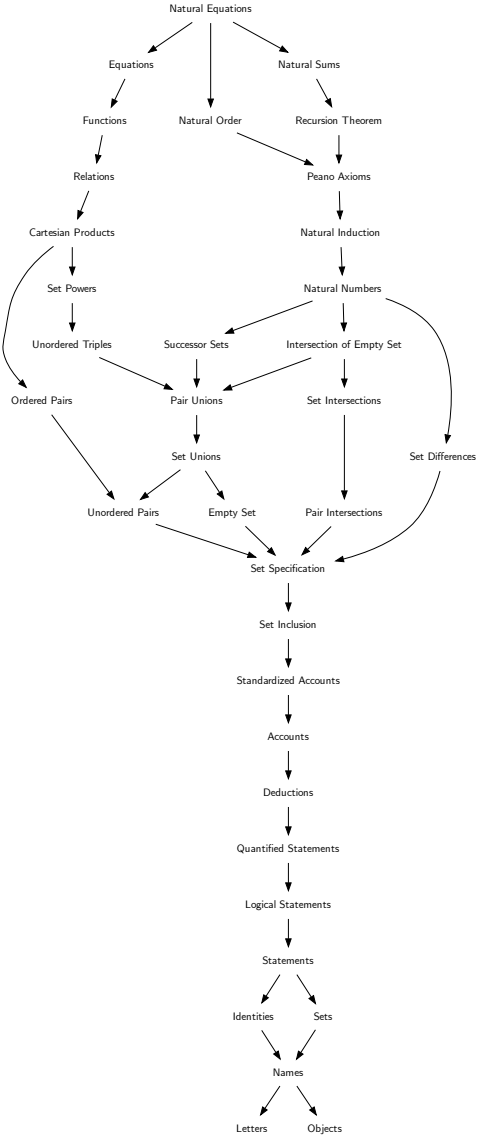
Continuing this way we find that $(3, 2)$ and $(2, 3)$ is a solu-

tion. Indeed, $3 + 2 = 2 + 3 = 5$. We are, however, interested in solutions to both equations

$$a = 3$$
$$a + b = 5$$

Both $(3, 2)$ and $(2, 3)$ satisfy the second equation, but only $(3, 2)$ satisfies both. Are there other solutions?[1]

[1] Future editions will expand


Natural Equations
Equations
Natural Sums
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects