Definition

Suppose V is a subspace over $\mathbf{F}$. A vector space U over $\mathbf{F}$ is a *subspace* (or *linear subspace*, *vector subspace*) of V if $U \subset V$ and vector addition and scalar multiplication defined for U agree with those defined for V. In other words, a subspace is a subset of a vector space which is closed under vector addition and scalar multiplication.

For example, the entire set of vectors is a subspace. As a second example, the set consisting only of the zero vector is a subspace; we call this the *zero subspace*. These two subspaces are the *trivial subspaces*. A *nontrivial subspace* is a subspace that is not trivial.

Notation

Let $(V, \mathbf{F})$ be a vector space. Let $U \subset V$ with

$$\alpha u + \beta v \in U$$

for all $\alpha, \beta \in \mathbf{F}$ and $u, v \in U$. Then U is a subspace of $(V, \mathbf{F})$.

Characterization

Proposition 1. *Suppose V is a vector space over a field $\mathbf{F}$ and $U \subset V$. U is a subspace if and only if U satisifes*

1. $0 \in U$ *(contains additive identity)*
2. $u + w \in U$ *for all* $u, w \in U$ *(closed under addition)*
3. $\alpha u \in U$ *for all* $\alpha \in \mathbf{F}$ *and* $u \in U$ *(closed under scalar addition)*

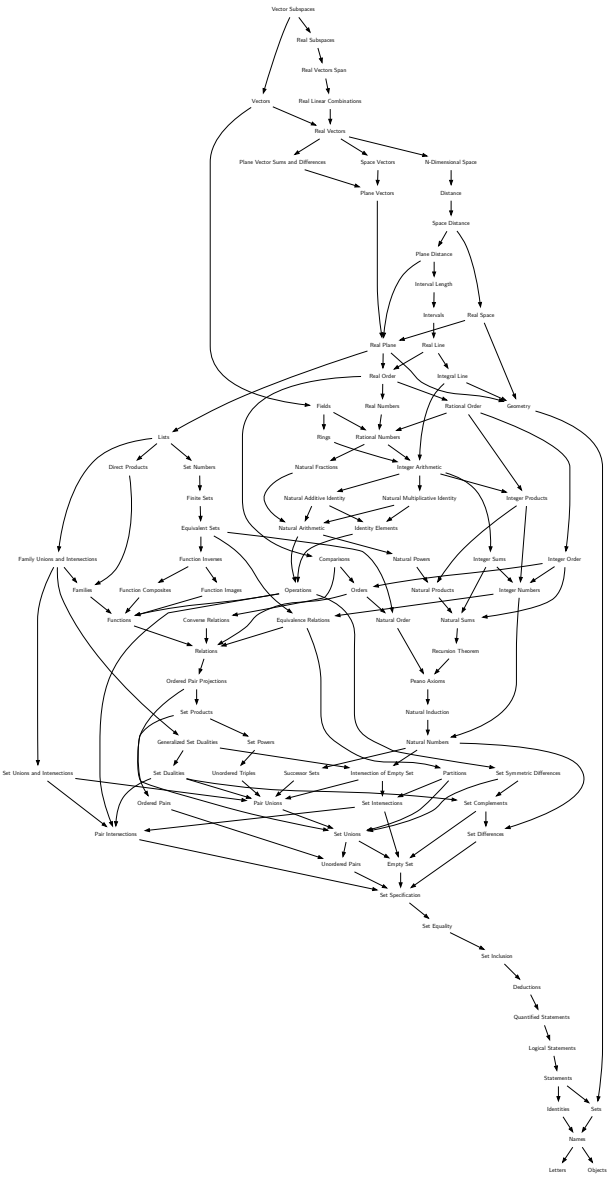
Properties

Proposition 2. *The intersection of a family of subspaces is a subspace.*

Proposition 3. *There exists a family of subspaces whose union is not a subspace;*

Remark 1. *In other words: the union of a family subspaces need not be a subspace.*

Proposition 4. *A subspace must contain the zero vector; in other words, every subspace is nonempty.*


Vector Subspaces
Real Subspaces
Real Vectors Span
Vectors
Real Linear Combinations
Real Vectors
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Real Order
Integral Line
Fields
Real Numbers
Rational Order
Geometry
Lists
Rings
Rational Numbers
Direct Products
Set Numbers
Natural Fractions
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Natural Arithmetic
Identity Elements
Family Unions and Intersections
Function Inverses
Comparisons
Natural Powers
Integer Sums
Integer Order
Families
Function Composites
Function Images
Operations
Orders
Natural Products
Integer Numbers
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects