Definition

Let $f : D \to \mathbf{R}$ be a multivariate real-valued function where $D \subset \mathbf{R}^d$. The *graph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}.$$

The *epigraph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, t) \in \mathbf{R}^d \mid x \in D \text{ and } f(x) \leq t\}.$$

Notation

We denote the epigraph of a function f by $\operatorname{epi} f$.

Connecting convex sets and convex functions

A function $f : \mathbf{R}^d \to \mathbf{R}$ is convex if and only if $\operatorname{epi} f$ is a convex set.

Note on terminology

The prefix "epi" is Greek, meaning "upon" or "above".

Real Function Epigraphs

Multivariate Functions

Convex Multivariate Functions

Real Convex Functions

Real Function Graphs

Real Convex Sets

Real Functions

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space

Real Line

Real Plane

Integral Line

Real Order

Geometry

Real Numbers

Rational Order

Lists

Rational Numbers

Set Numbers

Direct Products

Integer Arithmetic

Finite Sets

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Arithmetic

Identity Elements

Equivalent Sets

Natural Powers

Integer Sums

Integer Order

Comparisons

Operations

Function Inverses

Family Unions and Intersections

Natural Products

Integer Numbers

Orders

Converse Relations

Equivalence Relations

Function Images

Function Composites

Families

Natural Sums

Natural Order

Functions

Recursion Theorem

Relations

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Set Symmetric Differences

Successor Sets

Unordered Triples

Set Dualities

Intersection of Empty Set

Partitions

Set Unions and Intersections

Set Complements

Pair Unions

Set Intersections

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects