



Why

What is the multiplicative inverse in the reals?

Result

We can show the following.¹

Proposition 1. *The multiplicative inverse of $R \in \mathbf{R}$, $R \neq 0_{\mathbf{R}}$,*

1. if $0_{\mathbf{Q}} \in R$, then

$$S = \{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\} \cup \{r^{-1} \mid \exists s < r, (r \notin R)\}$$

is a multiplicative inverse of R .

2. if $0_{\mathbf{Q}} \notin R$, then case (1) applies to $-R$. Let S be the multiplicative inverse of $-R$. Then the additive inverse of S , i.e., $-S$ is a multiplicative inverse of R .

Notation

We denote the multiplicative inverse of $r \in \mathbf{R}$ by r^{-1} . We denote $q \cdot (r^{-1})$ by q/r .

Division

We call the operation $(a, b) \mapsto a/b$ *real division*. We call the product of a and the multiplicative inverse of b the *(real) quotient* of a and b .

¹The account will appear in future editions.

