Why

We want symbols for "and", "or", "not", and "implies".[1]

Overview

We call $=$ and $\in$ *relational symbols*. They say how the objects denoted by a pair of placeholder names relate to each other in the sense of being or belonging. We call $_ = _$ and $_ \in _$ *simple statements*. They denote simple sentences "the object denoted by $_$ is the object denoted by $_$" and "the object denoted by $_$ belongs to the set denoted by $_$". The symbols introduced here are *logical symbols* and statements using them are *logical statements*.

Conjunction

Consider the symbol $\wedge$. We will agree that it means "and". If we want to make two simple statements like $a = b$ and $a \in A$ at once, we write $(a = b) \wedge (a \in A)$. The symbol $\wedge$ is symmetric, reflecting the fact that a statement like $(a \in A) \wedge (a = b)$ means the same as $(a = b) \wedge (a \in A)$.

Disjunction

Consider the symbol $\vee$. We will agree that it means "or" in the sense of either one, the other, or both. If we want to say that at least one of the simple statements like $a = b$ and $a \in A$, we write write $(a = b) \vee (a \in A)$. The symbol $\vee$ is also symmetric, reflecting the fact that a statement like $(a \in A) \vee (a = b)$ means the same as $(a = b) \vee (a \in A)$.

Negation

Consider the symbol $\neg$. We will agree that it means "not". We will use it to say that one object "is not" another object and one object "does

[1] This sheet does not explain logic. In the next edition there will be several more sheets serving this function.

not belong to" another object. If we want to say the opposite of a simple statement like $a = b$ we will write $\neg(a = b)$. We read it aloud as "not a is b" or (the more desirable) "a is not b". Similarly, $\neg(a \in A)$ we read as "not, the object denoted by a belongs to the set denoted by A". Again, the more desirable pronunciation goes "the object denoted by a does not belong to the set A." For these reasons, we introduce two new symbols $\neq$ and $\notin$. $a \neq b$ means $\neg(a = b)$ and $a \notin A$ means $\neg(a \in A)$.

Implication

Consider the symbol $\longrightarrow$. We will agree that it means "implies". For example $(a \in A) \longrightarrow (a \in B)$ means "the object denoted by a belongs to the object denoted by A implies the object denoted by a belongs to the set denoted by B". It is the same as $(\neg(a \in A)) \vee (a \in B)$. In other words, if $a \in A$, then always $a \in B$. The symbol $\longrightarrow$ is not symmetric, since implication is not symmetric. The symbol $\longleftrightarrow$ means "if and only if".[2]

[2] Future editions will expand.

$$
\begin{tikzcd}
 & \text{Logical Statements} \arrow[d] & \\
 & \text{Statements} \arrow[dl] \arrow[dr] & \\
\text{Identities} \arrow[dr] & & \text{Sets} \arrow[dl] \\
 & \text{Names} \arrow[dl] \arrow[dr] & \\
\text{Letters} & & \text{Objects}
\end{tikzcd}
$$