

## INTEGRABLE FUNCTION SPACES

## Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? And so on.<sup>1</sup>

## **Definition**

The integrable function spaces are a collection of function spaces, one for each real number  $p \ge 1$ , for which the pth power of the absolute value of the function is integrable.<sup>2</sup>

## Notation

Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $p \geq 1$ . We denote the integrable function space corresponding to p by  $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{R})$ . We have defined it by

$$\mathcal{L}^{p}(X, \mathcal{A}, \mu, \mathbf{R}) = \left\{ \text{ measurable } f: X \to R \mid \int |f|^{p} d\mu < \infty \right\}$$

Let  $\mathbf{C}$  denote the set of complex numbers. Similarly for complex-valued functions, we denote the pth space by  $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{C})$ .

<sup>&</sup>lt;sup>1</sup>Future sheets are likely to being with  $L^2$ .

<sup>&</sup>lt;sup>2</sup>Future editions will include the case where  $p = \infty$ .

