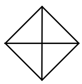


Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?[1]

Definition

Defining Result

Proposition 1. *The set of inverse images the distinguished sets of a measurable space under a function from a set to that space is a sigma algebra.*

If the first set and the function are measurable, the sigma algebra is a sub sigma algebra of the domain sigma algebra.

Proof. Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Let $f : X \to Y$ be a measurable function. Define

$$\mathcal{C} := \left\{ f^{-1}(B) \;\middle|\; B \in \mathcal{B} \right\}.$$

First, since $f^{-1}(Y) = X$, $X \in \mathcal{C}$.

Second, let $C \in \mathcal{C}$. Then there is B such that $C = f^{-1}(B)$. Then $X - C = X - f^{-1}(B) = f^{-1}(Y - B)$. Since $\mathcal{B}$ is a sigma algebra, $B \in \mathcal{B}, Y - B \in \mathcal{B}$ and so $X - C \in \mathcal{C}$.

Finally, let $(C_n)_n \subset \mathcal{C}$. Then for every n there exists a $B_n \in \mathcal{B}$ so that $C_n = f^{-1}(B_n)$. Then:

$$\cup_n C_n = \cup_n f^{-1}(B_n) = f^{-1}(\cup B_n).$$

[1] Future editions will expand.

Since $\mathcal{B}$ is a sigma algebra, $\cup_n B_n \in \mathcal{B}$ and so $\cup_n C_n \in \mathcal{C}$.

Since f is measurable, $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}$, and so $\mathcal{C} \subset \mathcal{A}$.

$\square$

The sigma algebra *generated by a random variable* is the sigma algebra consisting of the inverse images of every measurable set of the codomain.

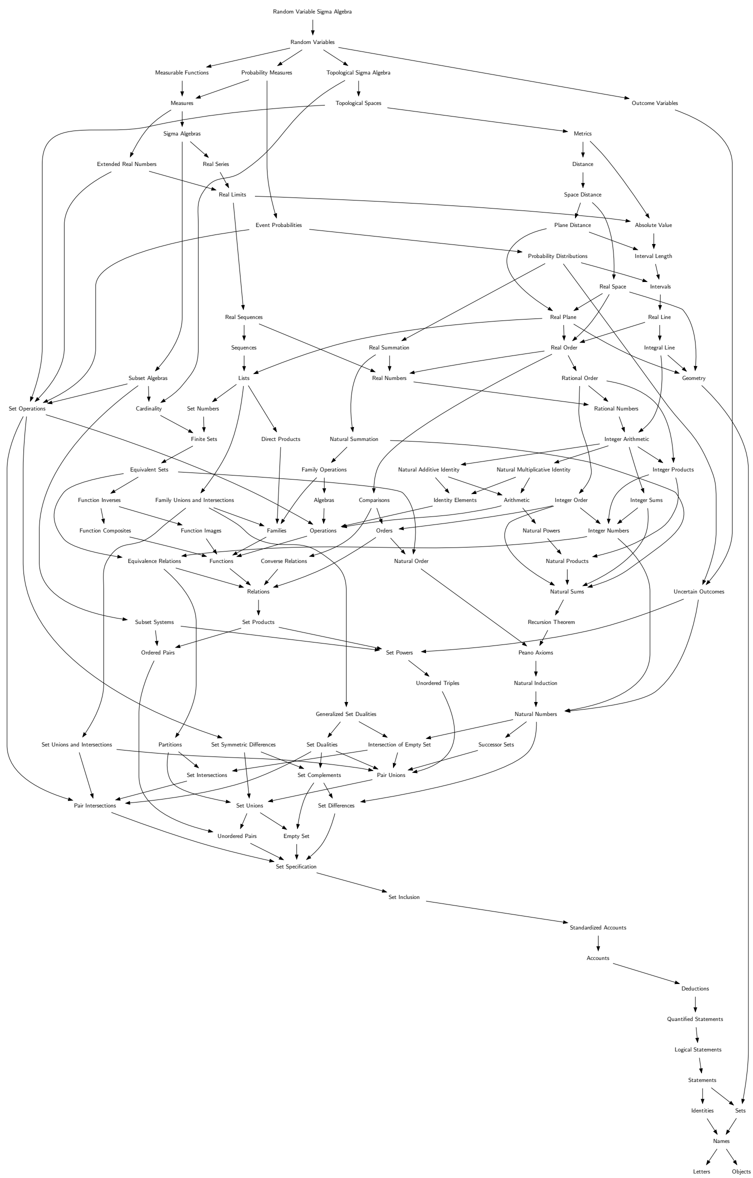
The sigma algebra generated by a family of random variables is the sigma algebra generated by the union of the sigma algebras generated individually by each of the random variables.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f : X \to Y$ be a random variable. Denote by $\sigma(f)$ the sigma algebra generated by f.

Results

Proposition 2. *The sigma algebra generated by a family of random variables is the smallest sigma algebra for with respect to which each random variable is measurably.*



Random Variable Sigma Algebra
Random Variables
Measurable Functions
Probability Measures
Topological Sigma Algebra
Measures
Topological Spaces
Outcome Variables
Sigma Algebras
Metrics
Extended Real Numbers
Real Series
Distance
Real Limits
Space Distance
Event Probabilities
Plane Distance
Absolute Value
Probability Distributions
Interval Length
Real Space
Intervals
Real Sequences
Real Plane
Real Line
Sequences
Real Summation
Real Order
Integral Line
Subset Algebras
Lists
Real Numbers
Rational Order
Geometry
Set Operations
Cardinality
Set Numbers
Rational Numbers
Finite Sets
Direct Products
Natural Summation
Integer Arithmetic
Equivalent Sets
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Function Inverses
Family Unions and Intersections
Algebras
Comparisons
Identity Elements
Arithmetic
Integer Order
Integer Sums
Function Composites
Function Images
Families
Operations
Orders
Natural Powers
Integer Numbers
Equivalence Relations
Functions
Converse Relations
Natural Order
Natural Products
Relations
Natural Sums
Uncertain Outcomes
Subset Systems
Set Products
Recursion Theorem
Ordered Pairs
Set Powers
Peano Axioms
Unordered Triples
Natural Induction
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Partitions
Set Symmetric Differences
Set Dualities
Intersection of Empty Set
Successor Sets
Set Intersections
Set Complements
Pair Unions
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects