

#### COMPLEX DISTANCE

## Why

The identification of C with a plane leads C to naturally inherit  $R^2$ 's notion of distance.

### **Definition**

The absolute value or modulus of  $z = (x, y) \in \mathbf{C}$  is the distance of z to the origin. If  $z \in \mathbf{C}$ , then the modulus of z is

$$\sqrt{x^2 + y^2}.$$

In other words, the modulus of z is the distance (in  $\mathbb{R}^2$  of z = (x, y) from the origin (0, 0).

#### Notation

We denote the modulus of z by |z|.

# **Properties**

**Proposition 1** (Triangle Inequality). For all  $z, w \in \mathbb{C}$ ,

$$|z+w| < |z| + |w|.$$

Also, for all  $z \in \mathbf{C}$ , we have  $|\text{Re}(z)| \leq |z|$  and  $|\text{Im}(z)| \leq |z|$ , and for all  $z, w \in \mathbf{C}$ ,

$$||z| - |w|| \le |z - w|.$$

<sup>&</sup>lt;sup>1</sup>This follows from the triangle inequality. Future editions will include an account.

