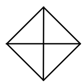


## Why

We abstract the properties of the natural numbers under natural addition and multiplication.

## Definition

A *semiring* $(S, +, \cdot)$ satisfies all the properties of a ring (see Rings) *except* that addition $+$ need not have additive inverses.

### Examples

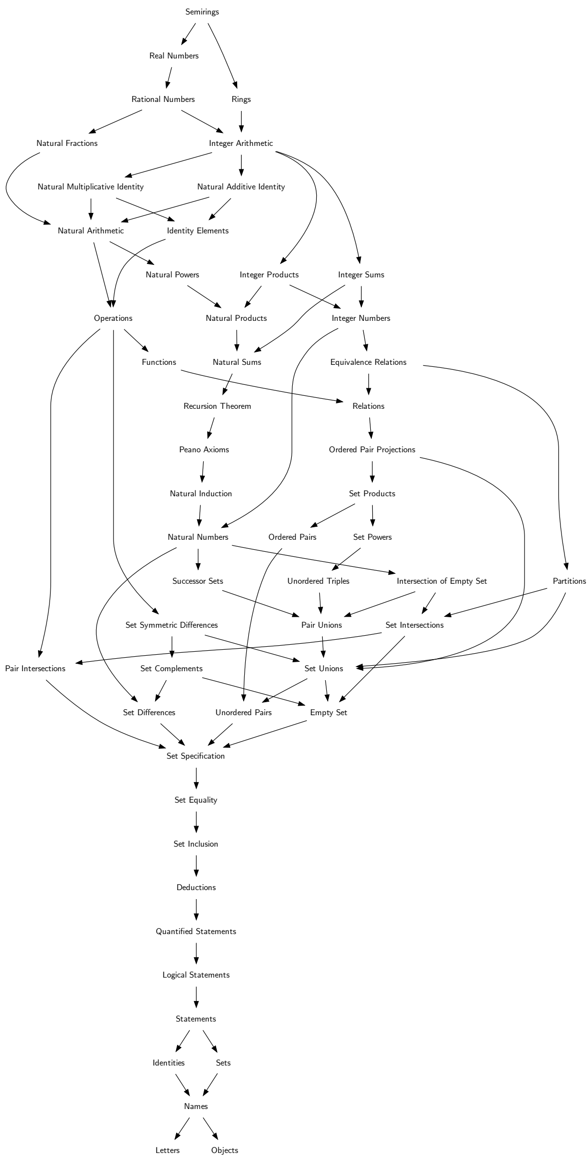
*Set of natural numbers.* The set $(\mathbf{N}, +, \cdot)$ where $+$ and $\cdot$ denote natural addition and multiplication respectively is a semiring.

*Nonnegative real numbers with max and multiplication* Notice that

$$\max(a, b) = \max(b, a) \quad \text{for all } a, b \in \mathbf{R}$$

$$\max(a, \max(b, c)) = \max(\max(a, b), c) \quad \text{for all } a, b, c \in \mathbf{R}$$

So $\max : \mathbf{R}^2 \to \mathbf{R}$ is a commutative and associative operation. The identity is 0, $\max(a, 0) = a$ for all $a \in \mathbf{R}_+$. Notice that there is no inverse element. Of course, $\cdot$ is associative and has identities.


Semirings
Real Numbers
Rational Numbers
Rings
Natural Fractions
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Natural Arithmetic
Identity Elements
Natural Powers
Integer Products
Integer Sums
Operations
Natural Products
Integer Numbers
Functions
Natural Sums
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Ordered Pairs
Set Powers
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Pair Intersections
Set Complements
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects