

#### **BOUNDED LINEAR NORM**

## Why

We can give the set of bounded linear functions between two norm spaces a norm.

### **Definition**

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

#### Notation

Let  $((V_1, F_1), \|\cdot\|_1)$  and  $((V_2, F_2), \|\cdot\|_2)$  be two norm spaces. Let  $f: V_1 \to V_2$  be linear and bounded. The norm of f is the smallest C so that

$$||f(v)||_2 \le C||v||_1.$$

# **Equivalent Formulation**

**Prop. 1.** Let  $((V_1, F_1), ||\cdot||_1)$  and  $((V_2, F_2), ||\cdot||_2)$  be two norm spaces. Let  $f: V_1 \to V_2$  bounded and linear. The norm of f is

$$\sup_{\|x\|_1=1} \|f(x)\|_2.$$

