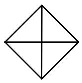


Why

We use a normal linear model to predict the function at inputs not included in the design.

Definition

Let $(x : \Omega \to \mathsf{R}^d, A \in \mathsf{R}^{n \times d}, e : \Omega \to \mathsf{R}^n)$ be a normal linear model over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$.

Predictive density

We are modeling $h_\omega : \mathsf{R}^d \to \mathsf{R}$ by $h_w(a) = x(\omega)^\top a$. The *predictive density* for a dataset $c^1, \dots, c^m \in \mathsf{R}^d$ is the conditional density of the random vector $(h_{(\cdot)}(c^1), \dots, h_{(\cdot)}(c^m))$ given y.

Proposition 1. *The predictive density for $c^1, \dots, c^m \in \mathsf{R}^d$ (with data matrix $C \in \mathsf{R}^{m \times d}$) is normal with mean*

$$g(a) = (C\Sigma_x A^\top) \left(A\Sigma_x A^\top + \Sigma_e\right)^{-1} \gamma.$$

and covariance

$$C\Sigma_x C^\top - C\Sigma_x A^\top \left(A\Sigma_x A^\top + \Sigma_e\right)^{-1} A\Sigma_x C^\top.$$

Proof. Define (as usual) $y : \Omega \to \mathsf{R}^n$ and $z : \Omega \to \mathsf{R}^m$ by

$$y = Ax + e$$
$$z = Cx.$$

Recognize (x, y, z) as jointly normal, and use Normal Conditionals). $\square$

Predictor

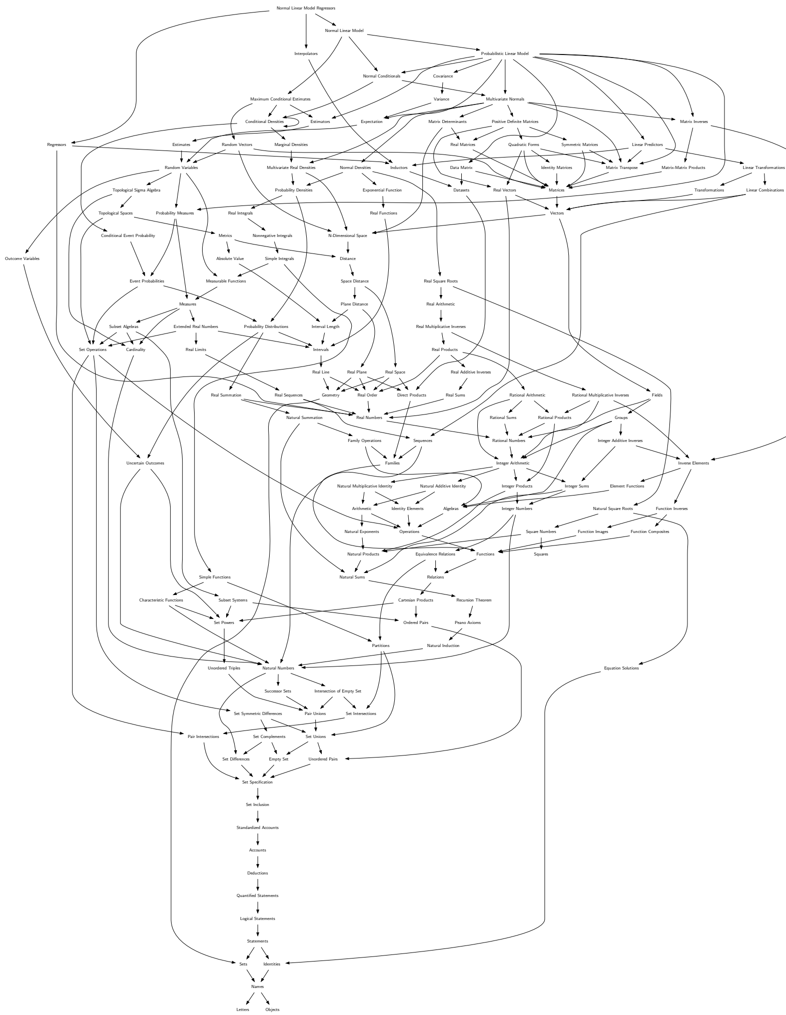
The *normal linear model predictor* or *normal linear model regressor* for the normal linear model (x, A, e) is the predictor which assigns to a new point $a \in \mathbf{R}^d$ the mean of the predictive density at a. That is, the predictor $g : \mathbf{R}^d \to \mathbf{R}$ defined by

$$g(a) = a^\top \Sigma_x A^\top \left(A \Sigma_x A^\top + \Sigma_e\right)^{-1} \gamma.$$

In the above we have substituted $a^\top$ for C. In the case of normal random vectors this corresponds with the MAP esetimate and the MMSE estimate.[1]

Use of a normal linear model predictor is often referred to as *Gaussian process regression.* The upside is that a gaussian process predicor interpolates the data, is smooth, and the so-called variance increases with the distance from the data. This is also called *Bayesian linear regression.*

[1] Future editions will have discussed this and include a reference to the sheet.


Normal Linear Model Regressors
Normal Linear Model
Interpolators
Probabilistic Linear Model
Normal Conditionals
Covariance
Maximum Conditional Estimates
Variance
Multivariate Normals
Conditional Densities
Estimators
Expectation
Matrix Determinants
Positive Definite Matrices
Matrix Inverses
Regressors
Estimates
Random Vectors
Marginal Densities
Real Matrices
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Symmetric Matrices
Linear Predictors
Random Variables
Multivariate Real Densities
Normal Densities
Inductors
Data Matrix
Identity Matrices
Matrix Transpose
Matrix-Matrix Products
Linear Transformations
Topological Sigma Algebra
Probability Densities
Exponential Function
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Transformations
Linear Combinations
Topological Spaces
Probability Measures
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Real Functions
Vectors
Conditional Event Probability
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Measurable Functions
Space Distance
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Measures
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Cardinality
Real Limits
Intervals
Real Products
Real Line
Real Plane
Real Space
Real Additive Inverses
Real Summation
Real Sequences
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Real Order
Direct Products
Real Sums
Rational Arithmetic
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Natural Additive Identity
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Function Inverses
Natural Exponents
Operations
Square Numbers
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Function Composites
Natural Products
Equivalence Relations
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Squares
Natural Sums
Relations
Simple Functions
Characteristic Functions
Subset Systems
Cartesian Products
Recursion Theorem
Set Powers
Ordered Pairs
Peano Axioms
Partitions
Natural Induction
Unordered Triples
Natural Numbers
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Intersection of Empty Set
Set Symmetric Differences
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Letters
Objects