



## FACTORIALS

### Why

How many ways are there to arrange  $n$  people at a table with  $n$  seats? In other words, how many ways can we order  $n$  objects?

### Discussion

There is one way to arrange a single person and there are two ways to arrange two people. What of three? First, we can pick a seat, and decide which person to put in that chair. We have three choices of people. Then, whomever we choose, there are only two ways to seat the other two people in the other two chairs. Since there were three ways of picking whom to sit in the first seat, there are  $3 \times 2 = 6$  ways to order the three people. We reason similarly for four people.

### Definition

Let  $f : \mathbf{N} \rightarrow \mathbf{N}$  be defined recursively by  $f(1) = 1$  and  $f(2) = 2f(1)$ , and  $f(n) = nf(n-1)$  for  $n \in \mathbf{N}$ .  $f$  exists by the recursion theorem (see Recursion Theorem). We call the result  $f(n)$  of  $n$  the *factorial* of  $n$ . We call  $f$  the *factorial function*.

### Notation

We denote the factorial of  $n$  by  $n!$ , read aloud “ $n$  factorial”.



