



## Why

What is the multiplicative inverse in the reals?

## Result

We can show the following.<sup>1</sup>

**Proposition 1.** *The multiplicative inverse of  $R \in \mathbf{R}$ ,  $R \neq 0_{\mathbf{R}}$ ,*

1. *if  $0_{\mathbf{Q}} \in R$ , then*

$$S = \{q \in \mathbf{Q} \mid q \leq 0_{\mathbf{Q}}\} \cup \{r^{-1} \mid \exists s < r, (r \notin R)\}$$

*is a multiplicative inverse of  $R$ .*

2. *if  $0_{\mathbf{Q}} \notin R$ , then case (1) applies to  $-R$ . Let  $S$  be the multiplicative inverse of  $-R$ . Then the additive inverse of  $S$ , i.e.,  $-S$  is a multiplicative inverse of  $R$ .*

## Notation

We denote the multiplicative inverse of  $r \in \mathbf{R}$  by  $r^{-1}$ . We denote  $q \cdot (r^{-1})$  by  $q/r$ .

## Division

We call the operation  $(a, b) \mapsto a/b$  *real division*. We call the product of  $a$  and the multiplicative inverse of  $b$  the *(real) quotient* of  $a$  and  $b$ .

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<sup>1</sup>The account will appear in future editions.



