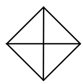


Definition

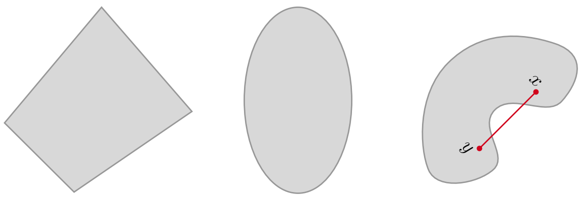
A set $C \subset \mathbf{R}^n$ is *convex* if it contains the closed line segment between every pair of points. In the notation of closed line segments, C is convex if

$$[x, y] \subset C \quad \text{for all } x, y \in C$$

In other words,

$$\lambda x + (1 - \lambda)y \in C \quad \text{for all } x, y \in C \text{ and } \lambda \in [0, 1].$$

Roughly speaking, C is convex if and only if its intersection with every line in $\mathbf{R}^n$ is either empty or a closed line segment.



convex convex not convex

Examples

The empty set, any singleton, any subspace, any affine set and any half-space.

Properties

Proposition 1 (closure under intersections). *Suppose $\mathcal{K} \subset \mathcal{P}(\mathbf{R}^d)$ is a set of convex sets. Then $\bigcap \mathcal{K}$ is convex.*

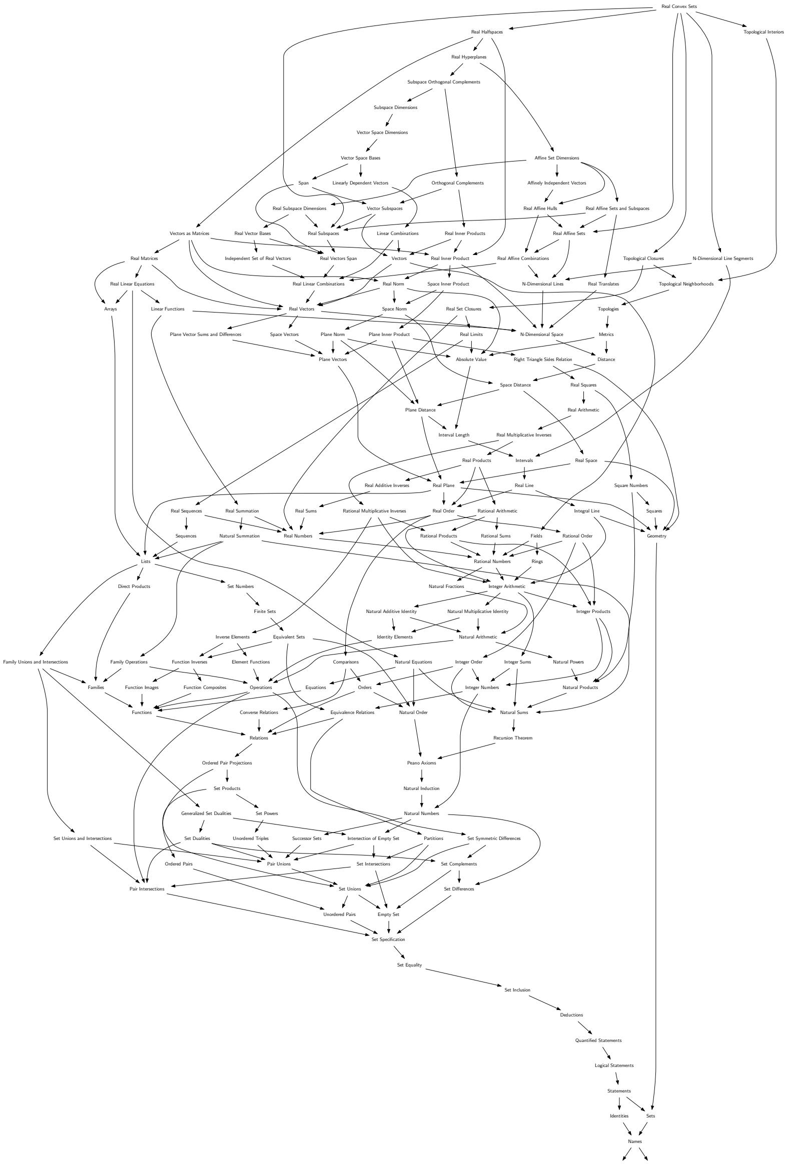
Proposition 2 (sums, differences, scales are convex). *Suppose $A, B \subset \mathbf{R}^d$ are convex sets. Then $A + B$, $A - B$ and λA for any real λ is convex.*

Proposition 3 (closure, interior)**.** *If* $A \subset \mathbf{R}^d$ *is convex, then* $\mathrm{cl}(A)$ *and* $\mathrm{Int}(A)$ *are convex.*[1]

Proposition 4 (interior line segments)**.** *Suppose* $A \subset \mathbf{R}^d$ *is convex,* $x \in A$ *and* $y \in \mathrm{Int}(a)$*. Then all points of the line segement between* x *and* y *are members of* $\mathrm{Int}(A)$*.*

Proposition 5 (images of affine maps)**.** *Suppose* $T : \mathbf{R}^d \to \mathbf{R}^d$ *is affine. If* $A \subset \mathbf{R}^d$ *is convex, then* $T(A)$ *is convex.*

[1]For the first, use $\mathrm{cl}(A) = \bigcap_{\mu>0}(A + \mu B)$ where B is unit ball of $\mathbf{R}^d$.


Real Convex Sets
Real Halfspaces
Topological Interiors
Real Hyperplanes
Subspace Orthogonal Complements
Subspace Dimensions
Vector Space Dimensions
Vector Space Bases
Affine Set Dimensions
Span
Linearly Dependent Vectors
Orthogonal Complements
Affinely Independent Vectors
Real Subspace Dimensions
Vector Subspaces
Real Affine Hulls
Real Affine Sets and Subspaces
Vectors as Matrices
Real Vector Bases
Real Subspaces
Linear Combinations
Real Inner Products
Real Affine Sets
Real Matrices
Independent Set of Real Vectors
Real Vectors Span
Vectors
Real Inner Product
Real Affine Combinations
Topological Closures
N-Dimensional Line Segments
Real Linear Equations
Real Linear Combinations
Real Norm
Space Inner Product
N-Dimensional Lines
Real Translates
Topological Neighborhoods
Arrays
Linear Functions
Real Vectors
Space Norm
Real Set Closures
Topologies
Plane Vector Sums and Differences
Space Vectors
Plane Norm
Plane Inner Product
Real Limits
N-Dimensional Space
Metrics
Plane Vectors
Absolute Value
Right Triangle Sides Relation
Distance
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Real Products
Intervals
Real Space
Real Additive Inverses
Real Plane
Real Line
Square Numbers
Real Sequences
Real Summation
Real Sums
Rational Multiplicative Inverses
Real Order
Rational Arithmetic
Integral Line
Squares
Sequences
Natural Summation
Real Numbers
Rational Products
Rational Sums
Fields
Rational Order
Geometry
Lists
Rational Numbers
Rings
Direct Products
Set Numbers
Natural Fractions
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Inverse Elements
Equivalent Sets
Identity Elements
Natural Arithmetic
Family Unions and Intersections
Family Operations
Function Inverses
Element Functions
Comparisons
Natural Equations
Integer Order
Integer Sums
Natural Powers
Families
Function Images
Function Composites
Operations
Equations
Orders
Integer Numbers
Natural Products
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects