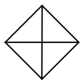


Why

What is the best linear predictor if we choose according to a squared loss function.

Definition

Let $X \in \mathbf{R}^{n \times d}$ and $y \in \mathbf{R}^d$. In other words, we have a paired dataset of records with inputs in $\mathbf{R}^d$ (the rows of X) and outputs in $\mathbf{R}$ (the elements of y).

A *least squares linear predictor* or *linear least squares predictor* is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$\frac{1}{n}\sum_{i=1}^{n}(f(x^i) - y_i)^2.$$

over the dataset of pairs $(x^1, y_1), \dots, (x^n, y_n) \in \mathbf{R}^d \times \mathbf{R}$where x^i is the ith row of X for $i = 1, \dots, n$.

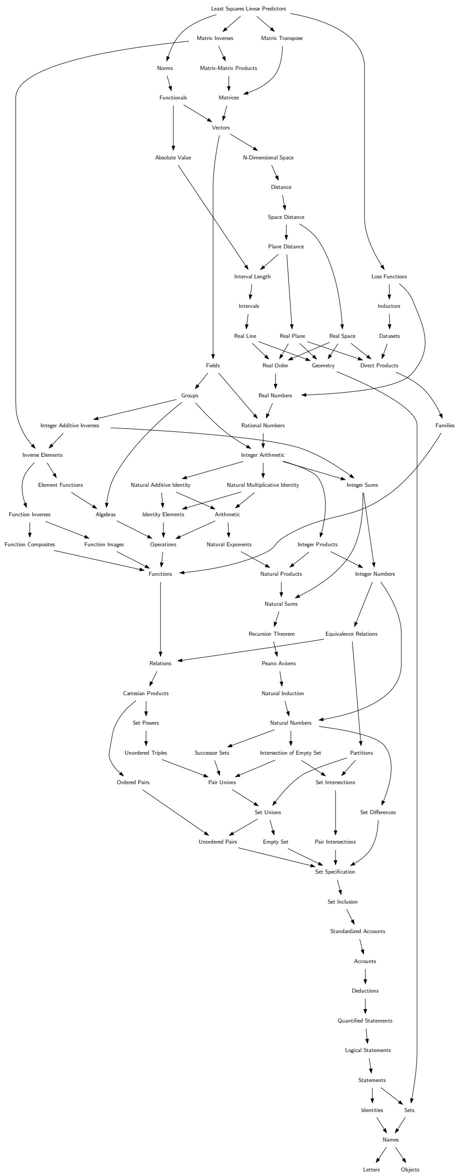
The set of linear functions from $\mathbf{R}^d$ to $\mathbf{R}$ is in one-to-one correspondence with $\mathbf{R}^d$. So we want to find $\theta \in \mathbf{R}^d$ to minimize

$$\frac{1}{n}\|X\theta - y\|^2.$$

Solution

Proposition 1. *There exists a unique linear least squares predictor and its parameters are given by* $\left(X^\top X\right)^{-1} X^\top y$.[1]

[1]Future editions will include an account.


Least Squares Linear Predictors
Matrix Inverses
Matrix Transpose
Norms
Matrix-Matrix Products
Functionals
Matrices
Vectors
Absolute Value
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Loss Functions
Intervals
Inductors
Real Line
Real Plane
Real Space
Datasets
Fields
Real Order
Geometry
Direct Products
Groups
Real Numbers
Integer Additive Inverses
Rational Numbers
Families
Inverse Elements
Integer Arithmetic
Element Functions
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Function Inverses
Algebras
Identity Elements
Arithmetic
Function Composites
Function Images
Operations
Natural Exponents
Integer Products
Functions
Natural Products
Integer Numbers
Natural Sums
Recursion Theorem
Equivalence Relations
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects