Optimization

1 Why

Given a correspondence between objects in a set with objects in an ordered set, we are interested in the objects which correspond to extremal elements of the ordered set.

2 Definition

Let A be a non-empty set and let $(C, \prec)$ be chain. Let $f : A \to C$.

The **minimization problem over** A associated with f is to find an element $a \in A$ so that $f(a) \leq f(b)$ for all $b \in A$. The **maximization problem over** A associated with f is to find an element $a \in A$ so that $f(a) \geq f(b)$ for all $b \in A$. We call either of these an **optimization problem**.

We call f the **ordering** function. We call A the **feasible set** and we call $a \in A$ a **feasible elemnet**. We call an element $a \in A$ so that $f(a) \leq f(b)$ for all $b \in A$ a **minimizer** of f over A. Similarly, we call $a \in A$ so that $f(a) \geq f(b)$ for all $b \in A$ a **maximizer** of f over A. There may be none, one, or several minimizers (or maximizers).

3 Notation

Let $(C, \prec)$ be a chain. We denote the minimization problem to find an element $a \in A$ to minimize $f : A \to C$ by

$$\begin{aligned} \text{find} \quad & a \in A \\ \text{to minimize} \quad & f(a) \end{aligned}$$

We denote the minimizers by

$$\textsf{\textbf{minimizers}}(f).$$