Why

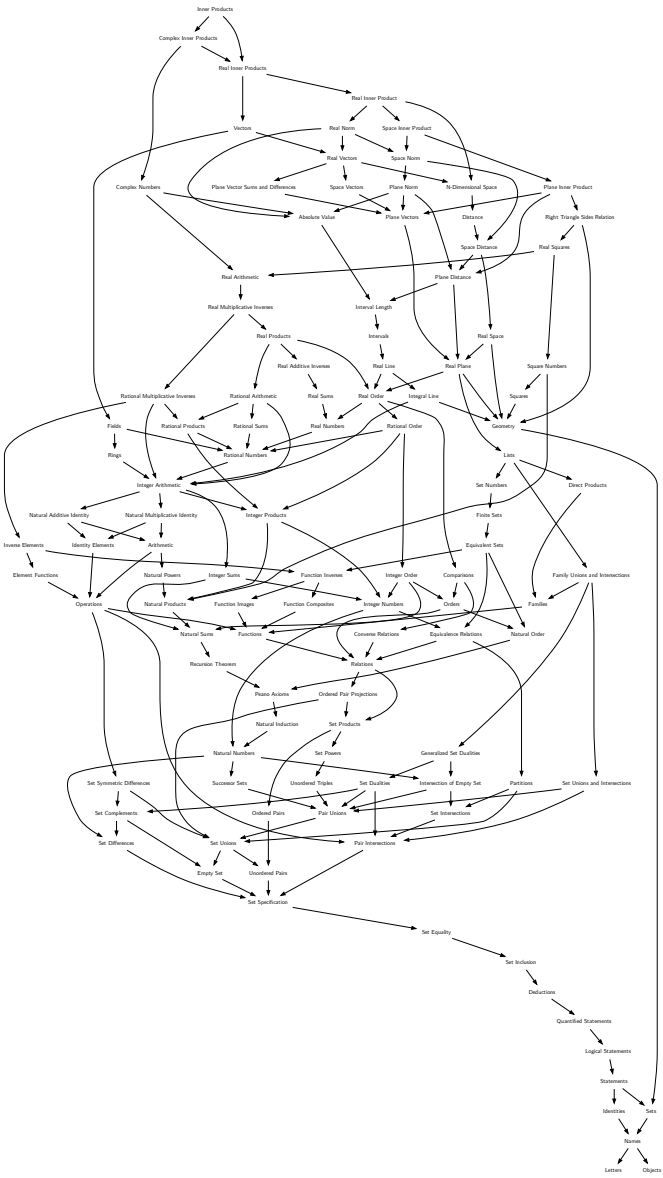
We abstract the notion of inner product to an arbitrary vector space.

Definition

Suppose $\mathbf{F}$ is a field which is either $\mathbf{R}$ or $\mathbf{C}$. Let $(V, \mathbf{F})$ be a vector space. Then a function $f : V \times V \to \mathbf{F}$ is an *inner product* on V if

1. $f(x, x) \geq 0$, $f(x, x) = 0 \Leftrightarrow x = 0$;
2. $f(x, y) = \overline{f(y, x)}$
3. $f(ax + by, z) = a(x, z) + b(y, z)$

A *inner product space* (or *pre-Hilbert space*) is a tuple: a vector space over $\mathbf{R}$ or $\mathbf{C}$ and an inner product.


Inner Products
Complex Inner Products
Real Inner Products
Real Inner Product
Vectors
Real Norm
Space Inner Product
Real Vectors
Space Norm
Complex Numbers
Plane Vector Sums and Differences
Space Vectors
Plane Norm
N-Dimensional Space
Plane Inner Product
Absolute Value
Plane Vectors
Distance
Right Triangle Sides Relation
Space Distance
Real Squares
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Line
Real Plane
Square Numbers
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Squares
Fields
Rational Products
Rational Sums
Real Numbers
Rational Order
Geometry
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Inverse Elements
Identity Elements
Arithmetic
Equivalent Sets
Element Functions
Natural Powers
Integer Sums
Function Inverses
Integer Order
Comparisons
Family Unions and Intersections
Operations
Natural Products
Function Images
Function Composites
Integer Numbers
Orders
Families
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Ordered Pairs
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects