Why

Convex functions satisfy an interpolation property.

Discussion

By definition, given $S \subset \mathbf{R}^n$, $f : S \to \mathbf{R}$ is convex means that the point

$$(1 - \lambda)(x\mu) + \lambda(x, \nu) = ((1 - \lambda)x + \lambda y, (1 - \lambda)\mu + \lambda\nu$$

belongs to epi f whenever (x, μ) and (y, ν) belong to epi f and $0 \leq \lambda \leq 1$. Said differently, we have $(1 - \lambda)x + \lambda y \in S$, and

$$f((1 - \lambda)x + \lambda y) \leq (1 - \lambda)\mu + \lambda\nu$$

whenever $x \in S$, $y \in S$, $f(x) \leq \mu \in \mathbf{R}$, and $f(y) \leq \nu \in \mathbf{R}$.

Concave functions have a similar property, with the inequalities flipped, and affine functions satisfy with qualities. This shows that the functions $f : \mathbf{R}^n \to \mathbf{R}$ we are calling affine coincide exactly with the affine transformations from $\mathbf{R}^n$ to $\mathbf{R}$.

Visualization

Characterizations of Convex Functions

Convex Multivariate Functions

Real Convex Functions · Real Function Epigraphs · Affine Transformations of Real Convex Sets

Real Function Graphs · Real Convex Sets

Real Functions · Real Halfspaces

Extended Real Numbers · Real Hyperplanes · Multivariate Functions

Subspace Orthogonal Complements

Topological Interiors · Subspace Dimensions

Vector Space Dimensions

Vector Space Bases · Affine Set Dimension · Affinely Independent Vectors

Orthogonal Complements · Linearly Dependent Vectors · Span · Real Subspace Dimensions · Real Vector Bases · Real Affine Hulls

Real Inner Products · Vector Subspaces · Real Subspaces · Real Vector Bases · Real Translates · Real Affine Sets

Topological Closures · Vectors as Matrices · Linear Combinations · N-Dimensional Line Segments · Real Vector Space · Independent Set of Real Vectors · Real Affine Combinations · Real Matrices

Topological Neighborhoods · Real Inner Product · Vector · Space Inner Product · Real Norm · Real Linear Combinations · N-Dimensional Line · Real Linear Equations

Real Set Closures · Topologies · Space Norm · Real Vectors · Plane Vector Sums and Differences · N-Dimensional Space · Linear Functions · Maps

Real Limits · Metrics · Plane Norm · Plane Inner Product · Space Vector · Plane Vectors · Distance

Absolute Value · Right Triangle Sides Relation · Plane Vectors

Real Squares · Plane Distance · Space Distance

Real Arithmetic · Internal Length · Real Space

Real Multiplication Inverses · Intervals · Real Plane · Square Numbers

Real Products · Real Line · Squares

Real Additive Inverses · Real Sums · Real Order · Rational Arithmetic · Real Sequences · Integral List · Real Summation · Geometry

Rational Multiplicative Inverses · Fields · Real Numbers · Rational Sums · Rational Order · Rational Products · Integers · Natural Summation

Rings · Rational Numbers · Integer Arithmetic · Natural Functions · Direct Products · Set Numbers

Inverse Elements · Natural Multiplication Identity · Natural Additive Identity · Integer Products · Finite Sets

Element Functions · Family Operations · Identity Elements · Natural Arithmetic · Equivalent Sets

Compositions · Operations · Function Restrictions and Extensions · Integer Order · Integer Sums · Natural Powers · Natural Equations · Family Unions and Intersections

Converse Relations · Function Composition · Function Images · Orders · Integer Numbers · Families · Natural Products · Equations

Function Inverses · Functions · Equivalence Relations · Natural Sums · Natural Order

Ordered Pair Projections · Recursion Theorem

Relations · Peano Axioms

Natural Induction

Set Products · Natural Numbers · Generalized Set Dualities

Set Symmetric Differences · Set Powers · Unordered Triples · Intersection of Empty Set · Successor Sets · Set Dualities · Set Unions and Intersections

Ordered Pairs · Set Complements · Set Partitions · Set Intersections · Pair Unions

Set Differences · Pair Intersections

Unordered Pairs · Set Unions · Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities · Sets

Names

Letters · Objects