



Definition

Recall that $(\mathbb{C}, \mathbf{Cmod}\cdot)$ is a normed space, and so also a metric space. So, a sequence $(z_n)_{n \in \mathbf{N}}$ of complex numbers is egoprox and convergent as usual. Both of these are equivalent to the corresponding conditions on the sequences of real and imaginary parts.

Proposition 1. $(z_n)_{n \in \mathbf{N}} = (x_n, y_n)_{n \in \mathbf{N}}$ converges to $z_0 = (x_0, y_0) \in \mathbb{C}$ if and only if x_n converges to x_0 and y_n converges to y_0 .

Proposition 2. $(z_n)_{n \in \mathbf{N}} = (x_n, y_n)_{n \in \mathbf{N}}$ is egoprox if and only if x_n is egoprox and y_n is egoprox.

Completeness

As a result of the second proposition, if z_n is egoprox then there is a limit x_0 and y_0 for its real and imaginary pieces, and so as a result of the first proposition, z_n converges. In other words, every cauchy sequence converges.

Proposition 3. \mathbb{C} with the metric induced by $\mathbf{Cmod}\cdot$ is complete.

