



## Definition

A *distribution family* (*density family*) on  $X$  is a family of distributions (densities)  $\{p^{(\theta)}\}_{\theta \in \Theta}$  on  $X$ .

We call the index set  $\Theta$  (see **Families**) the *parameters*. Frequently  $\Theta \subset \mathbf{R}^p$ .

Similarly, a *conditional distribution family* (*conditional density family*) on  $Z$  from  $X$  is a family  $\{q^{(\theta)}\}_{\theta \in \Theta}$  whose terms  $q^{(\theta)} : Z \times X \rightarrow \mathbf{R}$  are such that  $q^{(\theta)}(\cdot, \xi) : Z \rightarrow \mathbf{R}$  is a distribution (density) for every  $\xi \in X$ .

## Examples

For example, let  $\Theta = [0, 1]$  and consider the family of distributions  $\{p^{(\theta)} : \{0, 1\} \rightarrow [0, 1]\}_{\theta \in [0, 1]}$  defined by, for each  $\theta \in [0, 1]$ ,

$$p^{(\theta)}(1) = \theta \text{ and } p^{(\theta)}(0) = 1 - \theta.$$

This family is called the *Bernoulli family* and  $p^{(\theta)}$  is called a *Bernoulli distribution* with parameter  $\theta$ .

For a second example, let  $\Theta = \mathbf{R} \times \mathbf{R}_+$  and consider the family of densities  $\{f^{(\theta)} : \mathbf{R} \rightarrow \mathbf{R}\}_{\theta \in \Theta}$  defined by, for each  $\theta = (\mu, \sigma) \in \Theta$ ,

$$f^{(\theta)}(x) = (1/\sqrt{2\pi}\sigma) \exp((x - \mu)/\sigma^2).$$

This family is called the *normal family* and  $f^{(\theta)}$  with  $\theta = (\mu, \sigma)$  is called a *normal density* with mean  $\mu$  and variance  $\sigma^2$ .

