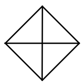


Why

What of lower upper triangular factorizations when the system is symmetric?

Definition

A *symmetric lower upper triangular factorization* of A is a pair of matrices $(L \in \mathbf{R}^{m \times m}, U \in \mathbf{R}^{m \times m})$ where L is unit lower triangular, U is upper triangular and $A = LU$.

$$A = LL^\top.$$

Other terminology includes *lower upper triangular factorization*, *LU decomposition*, *LU factorization.* Define $R = L^\top$, then

$$A = R^\top R.$$

Let $A \in \mathbf{R}^{n \times n}$. Then the ordinary row reduction of A is a matrix U which is upper triangular.be symmetric. A *lower upper triangular decomposition* A is a pair of matrices $(L, L^\top)$ where $L \in \mathbf{R}^{n \times n}$ is lower triangular, has nonnegative real diagonal entries, and satisfies

$$A = LL^\top.$$

Other terminology includes *lower upper triangular factorization*, *LU decomposition*, *LU factorization.* Define $R = L^\top$, then

$$A = R^\top R.$$

Basic properties

Proposition 1. *Let $A \in \mathbf{R}^{m \times m}$ be positive definite. Then there exists unique lower triangular matrix $L \in \mathbf{R}^{n \times n}$ so that*

$$A = LL^\top.$$

So, in the case that A is positive definite, a lower upper triangular decomposition exists and is unique. Therefore we refer to it as *the upper* *lower triangular decomposition* of A. It is also known (universally) as the *Cholesky decomposition* or *Cholesky factorization* of A.

Proposition 2. *If A is positive semidefinite, there exists a permutation matrix P for which there is a unique L so that*

$$P^\top AP = LL^\top.$$

This second proposition says that there is a unique Cholesky decomposition for a particular permutation (or pivoting) of A.

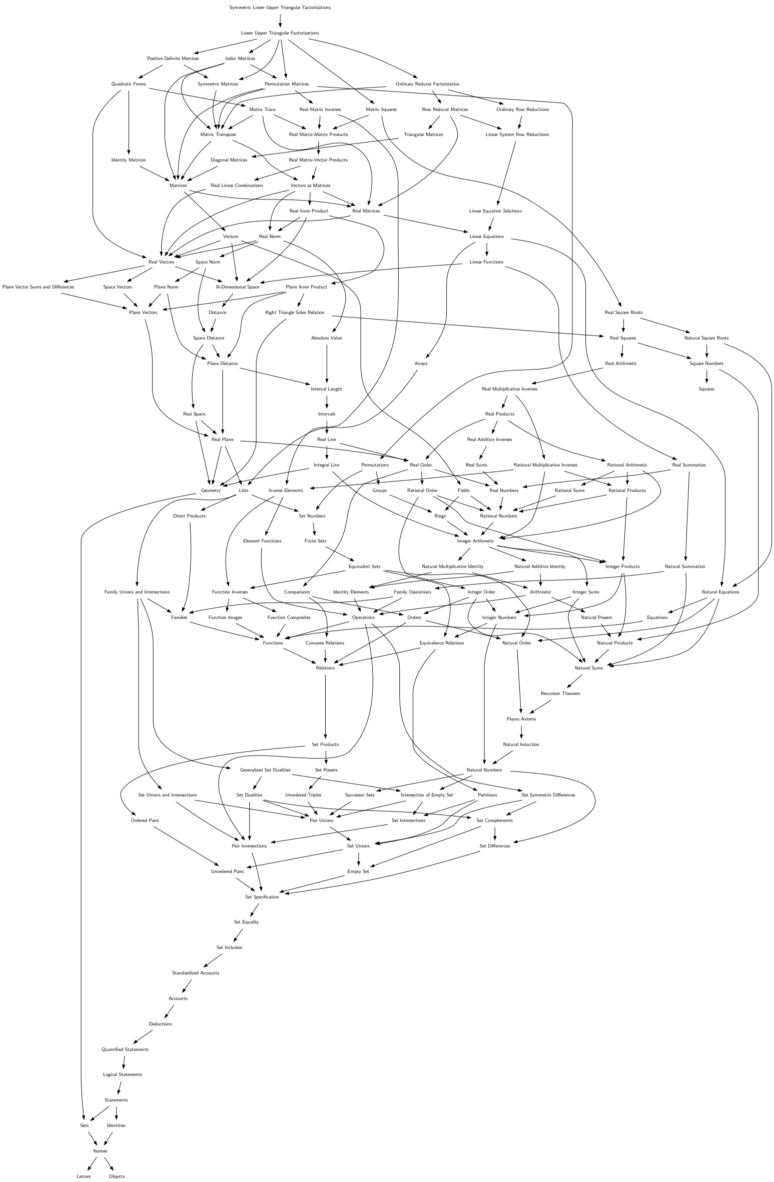
Unitriangular form

A *lower diagonal upper decomposition* (or *lower diagonal upper factorization*) of a matrix A a sequence $(L, D, L^\top)$ where $L \in \mathbf{R}^{n\times n}$ is unit lower triangular, $D \in \mathbf{R}^{n\times n}$ is diagonal with real nonnegative entries and

$$A = LDL^\top.$$

Other terminology includes *LDL decomposition*, *LDL factorization*, *LDU factorization*, *LDU decomposition*.

If $(L \in \mathbf{R}^{n\times n}, D \in \mathbf{R}^{n\times n}, L^\top)$ is a LDU decomposition of $A \in \mathbf{R}^{n\times n}$, then $(LD^{1/2}A = LDL^\top$ then $(\tilde{L}D^{1/2}, D^{1/2}L^\top)$ is a LU decomposition. Conversely, if $(B, B^\top)$ is a LU decomposition and S is the diagonal matrix satisfying $S_{ii} = B_{ii}$ for $i = 1, \dots, n$, then $(BS^{-1}, S^2, S^{-1}B^\top)$ is a LDU decomposition of A.


Symmetric Lower Upper Triangular Factorizations
Lower Upper Triangular Factorizations
Positive Definite Matrices
Index Matrices
Quadratic Forms
Symmetric Matrices
Permutation Matrices
Ordinary Reducer Factorization
Matrix Trace
Real Matrix Inverses
Matrix Squares
Row Reducer Matrices
Ordinary Row Reductions
Matrix Transpose
Real Matrix-Matrix Products
Triangular Matrices
Linear System Row Reductions
Identity Matrices
Diagonal Matrices
Real Matrix-Vector Products
Matrices
Real Linear Combinations
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Recursion Theorem
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects