Measurable Sections

1 Why

Toward a theory of iterated integrals, we need to know that set and function sections are measurable.

2 Results

Proposition 1. *Let* $(X, \mathcal{A})$ *and* $(Y, \mathcal{B})$ *be measurable spaces. For any* $E \in \mathcal{A} \times \mathcal{B}$*, the sections* E_x *and* E^y *are measurable for any* $x \in X$ *and* $y \in Y$*.*

Proof. TODO ☐

Proposition 2. *Let* $(X, \mathcal{A})$ *and* $(Y, \mathcal{B})$ *be measurable spaces. Let* $f : X \times Y \to F$*, where* F *is the extended real numnbers or the complex numbers, and* f *is measurable (using the appropriate sigma algebra of the codomain). The sections* $f_x : Y \to F$ *and* $f^y : X \to F$ *are measurable for each* $x \in X$ *and* $y \in Y$*.*

Proof. TODO ☐

Real F