Why

Given a subset of the real line, what is its length?

Background

Let $a, b \in R$ with $a \leq b$. The *length* of the closed interval of the real numbers $[a, b]$ is $b - a$. The length is non-negative.

A family $\{A_\alpha\}_{\alpha \in I}$ is *disjoint* if for $\alpha, \beta \in I$, $\alpha \neq \beta$, then $A_\alpha \cap A_\beta = \varnothing$. A set A can be *partioned* into a family if there exists a disjoint family whose union is A. A set $A \subset R$ is *simple* if it can be partitioned into a countable family whose members are closed intervals. The above discussion suggests that we should define the length of a simple set as the sum of the lengths of sets which parition it.

The above discussion suggests that if we wish to define a function length : $2^R \to R \cup \{-\infty, \infty\}$, we should ask that (1) $\text{length}(A) \geq 0$, (2) $\text{length}([a, b]) = b - a$, (3) for disjoint closed intervals $\{A_n\}_{n \in N}$, $\text{length}(A_i) = \sum_i \text{length}(A_i)$, and (4) for all $A \subset R$ and $a \in R$, $\text{length}(A + x) = \text{length}(A)$.

Converse

Define the equivalence relation $\sim$ on R by by $x \sim y$ if $x \sim y \in Q$

Notation

Let A be a set and $\mathcal{A} \subset \mathcal{P}(A)$. We denote the subset algebra of A and $\mathcal{A}$ by $(A, \mathcal{A})$, read aloud as "A, script A."

Properties

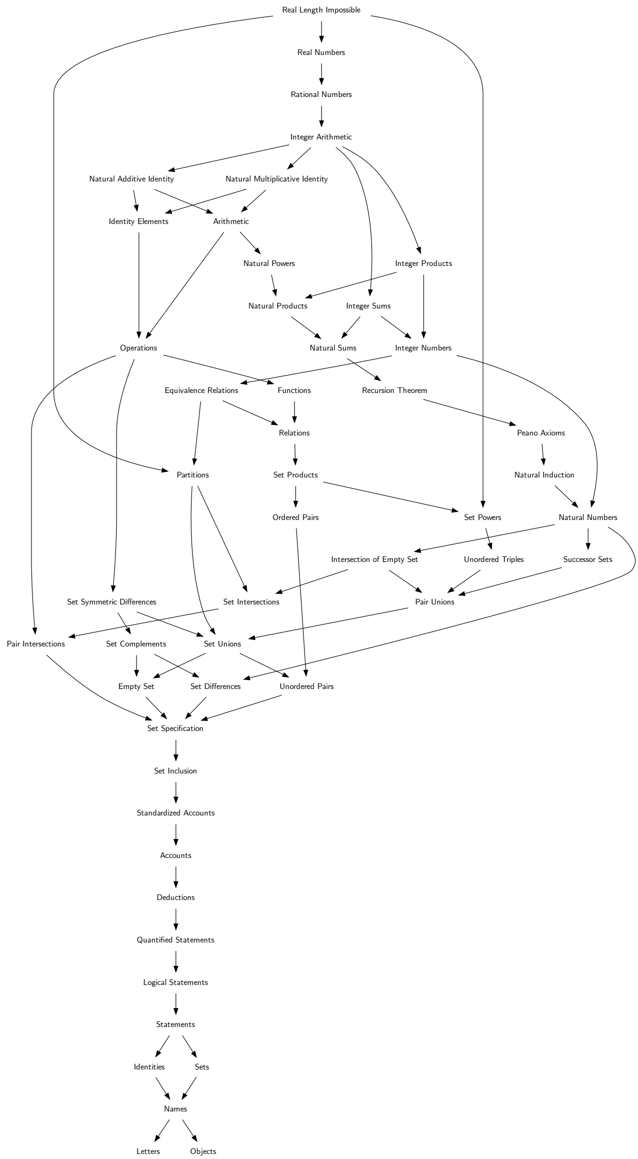
Prop. 1. *For any set A, 2^A is a sigma algebra.*

Prop. 2. *The intersection of a family of sigma algebras is a sigma algebra.*

Generation

Prop. 3. *Let A a set and $\mathcal{B}$ a set of subsets. There is a unique smallest sigma algebra $(A, \mathcal{A})$ with $\mathcal{B} \subset \mathcal{A}$.*

We call the unique smallest sigma algebra containing B the *generated sigma algebra* of B.



Real Length Impossible
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Identity Elements
Arithmetic
Natural Powers
Integer Products
Natural Products
Integer Sums
Operations
Natural Sums
Integer Numbers
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Partitions
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Intersection of Empty Set
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects