**Why**

Once we have a notion of distance, we can define a more general notion of convergence.

**Definition**

Let $(X, d)$ be a metric space and let $(x_n)_{n \in \mathbf{N}}$ be a sequence in $X$. A *limit* of $(x_n)_{n \in \mathbf{N}}$ is an element $x \in X$ for which $d(x, x_n) \to 0$.

Metric Limits

Metrics

Distance

Space Distance    Real Limits

Absolute Value    Plane Distance

Interval Length

Intervals    Real Space

Real Line    Real Plane

Integral Line    Real Order    Real Sequences

Geometry    Rational Order    Real Numbers    Sequences

Rational Numbers    Lists

Integer Arithmetic    Set Numbers    Direct Products

Integer Products    Natural Multiplicative Identity    Natural Additive Identity    Finite Sets

Arithmetic    Identity Elements    Equivalent Sets

Natural Powers    Integer Sums    Integer Order    Comparisons    Function Inverses    Family Unions and Intersections

Natural Products    Integer Numbers    Orders    Operations    Function Images    Function Composites    Families

Natural Sums    Natural Order    Equivalence Relations    Converse Relations    Functions

Recursion Theorem    Relations

Peano Axioms    Ordered Pair Projections

Natural Induction    Set Products

Natural Numbers    Set Powers    Generalized Set Dualities

Set Symmetric Differences    Partitions    Intersection of Empty Set    Successor Sets    Unordered Triples    Set Dualities    Set Unions and Intersections

Set Complements    Set Intersections    Pair Unions    Ordered Pairs

Set Differences    Set Unions    Pair Intersections

Empty Set    Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets    Identities

Names

Letters    Objects