Real Integral Limit Inferior Bound

1 Why

TODO

2 Result

Proposition 1. *The integral of the limit inferior of a sequence of measurable, nonnegative, extended-real-valued functions is no larger than the limit inferior of the sequence of integrals.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space, and let $f_n :\to [0, \infty]$ a $\mathcal{A}$-measurable function for every natural number n. We want to show that if

$$\int \liminf_n f_n d\mu \leq \liminf_n \int f_n d\mu.$$

$\square$