



## Definition

A nonempty set  $S \subset \mathbf{R}^n$  is called a *subspace* (*linear subspace*) if

1.  $x + y \in S$  for all  $x, y \in S$ , and
2.  $\alpha x \in S$  for all  $\alpha \in \mathbf{R}$ ,  $x \in S$ .

We say that that  $S$  is (1) *closed under vector addition* and (2) *closed under scalar multiplication*.

## Examples

The set  $S_1 = \mathbf{R}^n$  is a subspace. In other words, the entire set is a subspace of itself. The set  $S_2 = \{0\}$ , consisting of a single point, the origin, is a subspace.  $S_1$  is the biggest subspace. In other words, if  $S'$  is another subspace of  $\mathbf{R}^n$ , then  $S' \subset S_1$ . If  $S$  is a subspace, it is nonempty, so there is  $x \in S$ , and it is closed under scalar multiplication, so  $0 \cdot x = 0 \in S$ . In other words, every subspace contains the origin. So  $S_2$  is the smallest subspace, in the sense that if  $S'$  is another subspace  $S_2 \subset S'$ .

The span (see **Real Vectors Span**) of a set of vectors  $v_1, \dots, v_k$  is a subspace. For two subspaces  $S, T \subset \mathbf{R}^n$ , their sum

$$S + T = \{x + y \mid x \in S, y \in T\}$$

is a subspace.

## Geometric intuition

Roughly speaking, a subspace  $S$  is a flat set which passes through the origin. In  $\mathbf{R}^2$ , the subspaces are the lines. In  $\mathbf{R}^3$ , the lines and the planes.



