Why

What of the generalization to a multivariate normal.

Result

Prop. 1. *Let* $(x^1, \ldots, x^n)$ *be a dataset in* $\mathbf{R}^d$*. Let* f *be a multivariate normal density with mean*

$$\frac{1}{n}\sum_{k=1}^{d} x^k$$

and covariance

$$\frac{1}{n}\sum_{k=1}^{n}\left(x^k - \frac{1}{n}\sum_{k=1}^{n} x^k\right)\left(x^k - \frac{1}{n}\sum_{k=1}^{n} x^k\right)^\top.$$

Then f *is a maximum likelihood multivariate normal density.*

Proof. We express the log likelihood

$$\sum_{k=1}^{n} -\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu) - \frac{1}{2}\log(2\pi)^d - \frac{1}{2}\log \mathbf{det}\,\Sigma$$

Let $P = \Sigma^{-1}$. The $\log \mathbf{det}\,\Sigma$ is $\log \mathbf{det}\,P^{-1}$ is $\log\left(\mathbf{det}\,P\right)^{-1}$ is $-\log \mathbf{det}\,P$. Use matrix calculus to get

$$\frac{\partial \ell}{\partial P} = \sum_{k=1}^{n}(x^k-\mu)(x^k-\mu)^\top - P^{-1}.$$

□

We call these two objects the *maximum likelihood mean* or *empirical mean* and *maximum likelihood covariance* or *empirical covariance* of the dataset. We call the normal density with the empirical mean and empirical covariance the *empirical normal* of the dataset.

Multivariate Normal Maximum Likelihood
Multivariate Normals
Positive Definite Matrices
Symmetric Matrices
Quadratic Forms
Matrix Inverses
Matrix Trace
Matrix Determinants
Normal Maximum Likelihood
Matrix-Matrix Products
Matrix Transpose
Identity Matrices
Real Matrices
Density Maximum Likelihood
Matrix-Vector Products
Multivariate Real Densities
Density Selection
Normal Densities
Linear Combinations
Matrices
Real Vectors
Probability Densities
Exponential Function
Vectors
Real Integrals
N-Dimensional Space
Real Functions
Nonnegative Integrals
Distance
Simple Integrals
Real Square Roots
Square Distance
Maximum Likelihood
Measurable Functions
Real Squares
Plane Distance
Distributor Selection
Measures
Optimizers
Real Arithmetic
Extended Real Numbers
Interval Length
Real Multiplicative Inverses
Probability Distributions
Total Orders
Intervals
Real Products
Signed Algebras
Datums
Real Limits
Real Spans
Real Planes
Real Sequences
Real Lists
Real Addition Inverses
Sequences
Permutations
Integral Lists
Fields
Real Order
Rational Multiplicative Inverses
Real Sums
Rational Arithmetic
Real Summation
Geometry
Set Numbers
Groups
Real Numbers
Rational Sums
Rational Products
Finite Sets
Integer Additive Inverses
Rational Numbers
Partial Orders
Direct Products
Inverse Elements
Integer Arithmetic
Natural Summation
Equivalent Sets
Family Operations
Element Functions
Natural Multiplication Identity
Natural Additive Identity
Integer Sums
Integer Products
Subset Algebras
Natural Square Roots
Family Unions and Intersections
Function Inverses
Algebras
Identity Elements
Arithmetic
Set Operations
Integer Numbers
Natural Equations
Square Numbers
Families
Function Image
Function Composition
Operations
Natural Exponents
Equations
Natural Order
Square
Equivalence Relations
Functions
Natural Products
Relations
Natural Sums
Recursion Theorem
Cardinality
Peano Axioms
Simple Functions
Cartesian Products
Subset Systems
Characteristic Functions
Uncertain Outcomes
Natural Induction
Ordered Pairs
Generalized Set Duality
Partitions
Set Powers
Natural Numbers
Set Unions and Intersections
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Nouns
Letters
Objects