



Definition

A square real matrix is *symmetric* (we call it a *symmetric matrix*) if its values do not depend on the order of the indices. In other words, a matrix is symmetric if the values above and below the diagonal are a mirror image.

Examples

The matrix

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

is symmetric. The matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

is not.

Notation

Suppose $A \in \mathbf{R}^{n \times n}$ with $A^\top = A$ (i.e., symmetric). We denote the set of all such symmetric $n \times n$ real matrices by \mathbf{S}^n . So

$$\mathbf{S}^n = \{A \in \mathbf{R}^{n \times n} \mid A^\top = A\}.$$

In particular, $\mathbf{S}^n \subset \mathbf{R}^{n \times n}$.

