Definition

Suppose $(A, \leq)$ is a partially ordered set.

An *upper bound* for $B \subset A$ is an element $a \in A$ so that $b \leq a$ for all $b \in B$. A set is *bounded from above* if it has a least upper bound. A *least upper bound* for B is an element $c \in A$ so that c is an upper bound and $c < a$ for all other upper bounds a.

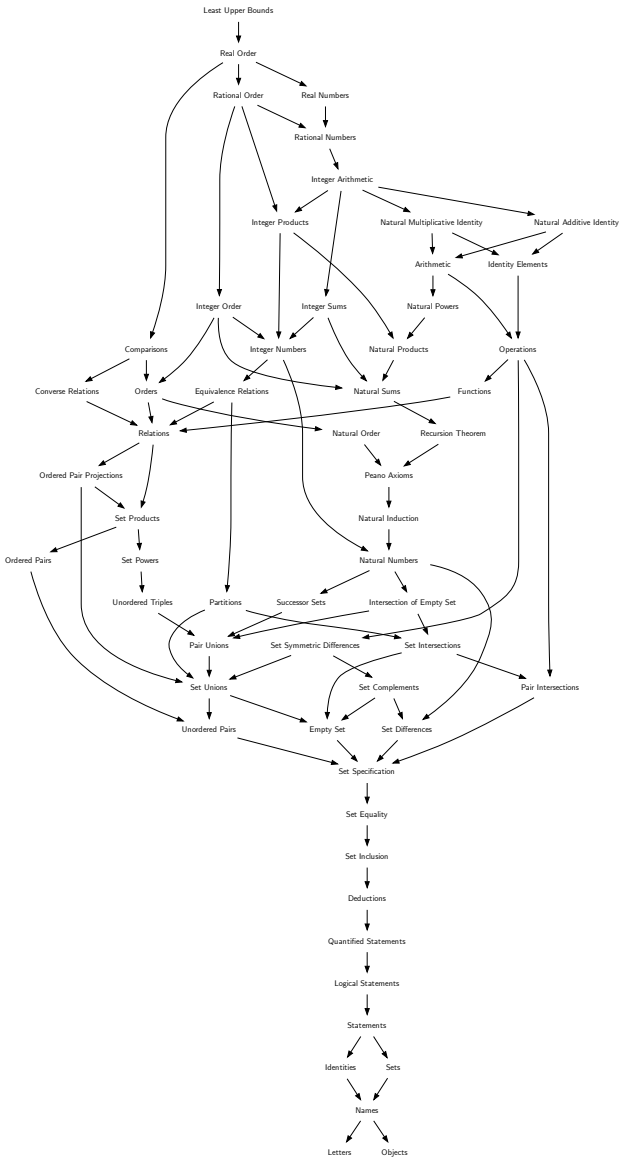
Proposition 1. *If there is a least upper bound it is unique.*[1]

We call the unique least upper bound of a set (if it exists) the *supremum*.

Notation

We denote the supremum of a set $B \subset A$ by $\sup A$.

[1] Proof in future editions.


Least Upper Bounds
Real Order
Rational Order
Real Numbers
Rational Numbers
Integer Arithmetic
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Arithmetic
Identity Elements
Integer Order
Integer Sums
Natural Powers
Comparisons
Integer Numbers
Natural Products
Operations
Converse Relations
Orders
Equivalence Relations
Natural Sums
Functions
Relations
Natural Order
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Pair Unions
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects