

## LINEAR EQUATIONS

Why

1

## Definition

Let  $a \in \mathbb{R}^n$  and  $b \in \mathbb{R}$ . Suppose we want to find  $x \in \mathbb{R}^n$  to satisfy

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b.$$

We refer to this expression as a real linear equation or linear equation. We treat  $x_i$ s as a variables and we treat the  $a_i$ s and b as constants. We call the pair (a, b) the real linear equation constants.<sup>2</sup>

The source of the terminology "linear" is by viewing the left hand side as a function. Define  $f: \mathbb{R}^n \to \mathbb{R}$  by  $f(x) = \sum_i a_i x_i$ . We want to find  $x \in \mathbb{R}^n$  to satisfy f(x) = b. Notice that f is a linear real function.<sup>3</sup>

Moreover, to each linear function  $f: \mathbb{R}^d \to \mathbb{R}$  there exists a vector  $a \in \mathbb{R}^d$  so that  $f(x) = \sum_i a_i x_i$ . For this reason, if we are given several linear function  $f_1, \ldots, f_m$ , then we can think of these as several vectors  $a^1, \ldots, a^n$ . If we are also given  $b_i \in \mathbb{R}$  for each  $i = 1, \ldots, m$ , then we have the vector  $b \in \mathbb{R}^m$ 

We can define the two-dimensional array  $A \in \mathbb{R}^{m \times n}$  by  $A_{ij} = a_j^i$ . For this reason, a linear system of equations is a

<sup>&</sup>lt;sup>1</sup>Future editions will include.

<sup>&</sup>lt;sup>2</sup>Future editions will clarify.

<sup>&</sup>lt;sup>3</sup>Future editions may require a sheet here.

pair (A, b). A solution of a linear system of equations is a vector  $x \in \mathbb{R}^n$  satisfying the equations

$$A_{11}x_{1} + A_{12}x_{2} + \cdots + A_{1n}x_{n} = b_{1}$$

$$A_{21}x_{1} + A_{22}x_{2} + \cdots + A_{2n}x_{n} = b_{2}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$A_{m1}x_{1} + A_{m2}x_{2} + \cdots + A_{mn}x_{n} = b_{n}$$

Other terminology includes a system of linear equations or linear system or simultaneous linear equations

