

## Product Metrics

## 1 Why

Given n sets each with metrics, there is a standard way of turning the direct product of the sets into a metric space. In other words, defining a distance on the tuples of elements from the sets.

## 2 Motivating Result

**Proposition 1.** Let  $(A_1, d_1), (A_2, d_2), \ldots, (A_n, d_n)$  be metric spaces. Let A be  $\prod_{i=1}^n A_n$  and let R be the set of real numbers. Define  $d: A \times A \to R$  by

$$d(a,b) = \max\{d_1(a_1,b_1),\ldots,d_n(a_n,b_n)\}.$$

Then (A, d) is a metric space.

