



## Why

If a system  $(A, b)$  is ordinarily reducible, then there exists  $L$  unit lower triangular and  $U$  upper triangular so that  $A = LU$ . When does such a factorization exist?

## Definition

Let  $A \in \mathbf{R}^{m \times m}$ . A *lower upper triangular factorization* of  $A$  is a pair of matrices  $(L \in \mathbf{R}^{m \times m}, U \in \mathbf{R}^{m \times m})$  where  $L$  is unit lower triangular,  $U$  is upper triangular and  $A = LU$ . Other terminology includes *lower upper triangular decomposition*, *LU factorization*, and *LU decomposition*.

**Proposition 1.** *If  $(A, b)$  is ordinarily reducible, a LU factorization exists.*

What about an  $LU$ -factorization when  $(A, b)$  is not ordinarily reducible? The main issue is that we may encounter a diagonal entry of some reduction of  $A$  which is zero. Proposition 1

