Prop. 1. *Let (V, F) be a complete inner product space. Let M be a closed subspace. then*

$$H = M \oplus M^{\perp}.$$

That is for all $x \in V$, there exists unique $z \in M$ and $q \in M^{\perp}$ such that $x = z + w$.

Complete Inner Product Decomposition

Orthogonal Complements

Inner Products

Vectors

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Plane · Real Line · Real Space

Integral Line · Real Order

Geometry · Real Numbers · Fields

Rational Numbers · Groups

Integer Arithmetic · Integer Additive Inverse

Sequences · Set Numbers · Natural Additive Identity · Natural Multiplicative Identity · Inverse Elements · Integer Sums

Direct Products · Finite Sets · Identity Elements · Arithmetic · Element Functions

Equivalent Sets · Algebras · Natural Powers · Integer Products

Family Unions and Intersections · Operations · Function Inverses · Natural Products · Integer Numbers

Families · Equivalence Relations · Function Composites · Function Images · Natural Sums

Natural Order · Functions · Recursion Theorem

Relations · Peano Axioms

Cartesian Products · Natural Induction

Generalized Set Dualities · Set Powers · Natural Numbers

Set Unions and Intersections · Set Dualities · Unordered Triples · Intersection of Empty Set · Successor Sets · Partitions

Set Complements · Pair Unions · Set Intersections · Ordered Pairs

Pair Intersections · Set Unions · Set Differences

Empty Set · Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets · Identities

Names

Letters · Objects