Why

Suppose we flip a coin n times. How should we define a probability distribution on $\{0,1\}^n$. Suppose instead we flip n different coins, each once. How now?

Discussion

If we model the outcome that coin i lands heads as 1 and that it lands tails by 0, then we are asking to treat of uncertain outcomes from the set $\{0,1\}^n$ of length-n sequences.

Here $x \in \{0,1\}^n$ could correspond to the outcome of flipping one coin n times or the outcome of flipping n different coins. This is a simple example of a frequent and obvious phenomenon in mathematics wherein the same mathematical model (here the outcome set) can be used to model different situations in the real world (here flipping the same coin several times, or flipping several different coins at once). Compare with numbers, etc.

Definition

Consider a distribution over a product of n sets, where the product is indexed by the first n natural numbers. We call the distribution a *joint distribution* with n components.

Joint Distributions

Probability Distributions

Intervals

Real Line

Real Summation — Real Order

Rational Order — Real Numbers

Rational Numbers

Integral Line — Lists

Integer Arithmetic

Direct Products — Set Numbers

Integer Products — Natural Additive Identity — Natural Multiplicative Identity

Finite Sets

Natural Summation — Arithmetic — Identity Elements — Equivalent Sets

Natural Powers — Integer Sums — Integer Order — Comparisons — Family Operations — Function Inverses — Family Unions and Intersections

Natural Products — Integer Numbers — Orders — Operations — Families — Function Composites — Function Images

Natural Sums — Natural Order — Converse Relations — Functions — Equivalence Relations

Recursion Theorem — Relations

Peano Axioms — Ordered Pair Projections

Uncertain Outcomes — Natural Induction — Set Products

Natural Numbers — Set Powers — Generalized Set Dualities

Set Symmetric Differences — Successor Sets — Unordered Triples — Set Dualities — Intersection of Empty Set — Partitions — Set Unions and Intersections

Set Complements — Pair Unions — Ordered Pairs — Set Intersections

Set Differences — Set Unions — Pair Intersections

Empty Set — Unordered Pairs

Set Specification — Geometry

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects