



# Sequences

## 1 Why

We introduce language for the steps of an infinite process.

## 2 Definition

Let  $A$  be a non-empty set. A *sequence in  $A$*  is a function from the natural numbers to the set. The  $n$ th term of a sequence is the result of the  $n$ th natural number; it is an element of the set.

### 2.1 Notation

Let  $A$  be a non-empty set.  $a : \mathbf{N} \rightarrow A$  is a sequence in  $A$ .  $a(n)$  is the  $n$ th term. We also denote  $a$  by  $(a_n)_n$  and  $a(n)$  by  $a_n$ .

## 3 Subsequences

A *subindex* is a monotonically increasing function from and to the natural numbers. Roughly, it selects some ordered infinite subset of natural numbers. A *subsequence* of a first sequence is any second sequence which is the composition of the first sequence with a subindex.

## 4 Notation

Let  $i : N \rightarrow N$  such that  $n < m \implies i(n) < i(m)$ . Then  $i$  is a subindex. Let  $b = a \circ i$ . Then  $b$  is a subsequence of  $a$ . We denote it by  $\{b_{i(n)}\}_n$  and the  $n$ th term by  $b_{i(n)}$ .

TODO: integrate, from direct products

If  $I$  is the set of natural numbers we denote the direct product by

$$\prod_{i=1}^{\infty} A_i.$$

We denote an element of  $\prod_{i=1}^{\infty} A_i$  by  $(a_i)$  with the understanding that  $a_i \in A_i$  for all  $i = 1, 2, 3, \dots$ . If  $A_i = A$  for all  $i = 1, 2, 3, \dots$ , then  $(a_i)$  is a sequence in  $A$ .