

### INDEX MATRICES

## Why

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#### Definition

An *index sequence* of *order* n is a finite sequence of distinct elements of  $\{1, 2, ..., n\}$  whose length is less than or equal to n. We call the ith coordinate of an index sequence the i-index of the sequence. The index matrix associated with an index sequence is the  $r \times n$  matrix whose i, jth entry is 1 if the index sequences's ith coordinate is j, and 0 otherwise. If r = n then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an n-vector associated with a length-r index sequence is the product of the  $r \times n$  index matrix with the n-dimensional vector. Its ith entry is the i-index entry of the vector.

# Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions), they are not sets of indices, so we avoid this usage.

<sup>&</sup>lt;sup>1</sup>Future editions will include.

### **Notation**

Let  $r \leq n$  be natural numbers. Let  $\alpha : \{1, 2, ..., r\} \rightarrow \{1, 2, ..., n\}$  be an index sequence. We denote the index matrix associated with  $\alpha$  by  $P_{\alpha}$ . This matrix  $P_{\alpha}$  is an element of  $\mathbb{N}^{r \times n}$  and is defined by

$$(P_{\alpha})_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let A be a nonempty set and let  $x \in A^n$ . then the subvector of x associated with  $P_{\alpha}$  (and so with  $\alpha$ ) is

$$P_{\alpha}x = \left(x_{\alpha(1)}, \dots, x_{\alpha(r)}\right)$$

We denote the product  $P_{\alpha}x$  by  $x_{\alpha}$ .

We denote the product  $P_{\alpha}XP_{\alpha}^{\top}$  by  $X_{\alpha\alpha}$ .

## Multiplication

The product of the  $n \times r$  transpose of an index matrix with an r vector is the n vector with The

