

## ORDERED UNDIRECTED GRAPH ORIENTATIONS

## Why

There is a natural orientation of an ordered undirected graph.

## Motivating Result

An ordered undirected graph can be converted into a directed graph by orienting the eges from lower to higher index. The *orientation* of an ordered undirected graph  $((V, E), \sigma)$  is the directed graph (V, F) where

$$\{v,w\} \in V \longrightarrow (v,w) \in F \text{ and } \sigma^{-1}(v) < \sigma^{-1}(w).$$

In other words, we can "convert" the ordered undirected graph by "orienting" the edges from lower to higher index.

**Proposition 1.** Let  $G = ((V, E), \sigma)$  be an ordered undirected graph. The orientation of G is acyclic.

*Proof.* Contradiction on the existence of a cycle. 
$$\Box$$

Conversely, let (V, F) be directed acyclic. To each topological numbering  $\sigma$  of (V, F) (see Directed Paths) there exists an ordered undirected graph  $((V, E), \sigma)$  where (V, E) is the skeleton of (V, F).

## Example

Let  $G = ((V, E), \sigma)$  be an undirected graph with

$$V = \{a, b, c, d, e\},\$$

$$E = \{\{a,b\},\{a,c\},\{a,e\},\{b,d\},\{b,e\},\{c,d\},\{c,e\},\{d,e\}\},$$

and

$$\sigma(1) = a$$
  $\sigma(2) = c$   $\sigma(3) = d$   $\sigma(4) = b$   $\sigma(e) = 5$ .

We visualize the ordered graph in Figure 1.

<sup>&</sup>lt;sup>1</sup>Future editions will expand.

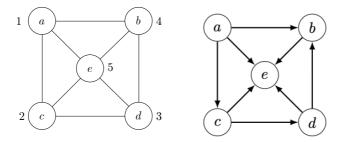


Figure 1: G and its (directed acyclic) orientation.

