Why

Linear equations are ubiquitous.

Definition

Given $a \in \mathbf{R}^n$ and $y \in \mathbf{R}$, suppose we want to find $x \in \mathbf{R}^n$ satisfying

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = y.$$

We refer to this expression as a *real linear equation* or *linear equation*. We treat each component $x_i \in \mathbf{R}$ as a variable and we treat each component $a_i \in \mathbf{R}$ and $y \in \mathbf{R}$ as constants. We call the pair (a, y) the *real linear equation constants*.[1]

The source of the terminology "linear" is by viewing the left hand side as a function. Define $f : \mathbf{R}^n \to \mathbf{R}$ by $f(x) = \sum_i a_i x_i$. We want to find $x \in \mathbf{R}^n$ to satisfy $f(x) = b$. Notice that f is a *linear* real function.[2]

Moreover, to each linear function $f : \mathbf{R}^d \to \mathbf{R}$ there exists a vector $a \in \mathbf{R}^d$ so that $f(x) = \sum_i a_i x_i$. For this reason, if we are given several linear function $f_1, \ldots, f_m$, then we can think of these as several vectors $a^1, \ldots, a^n$. If we are also given $b_i \in \mathbf{R}$ for each $i = 1, \ldots, m$, then we have the vector $b \in \mathbf{R}^m$

We can define the two-dimensional array $A \in \mathbf{R}^{m \times n}$ by $A_{ij} = a_j^i$. For this reason, a *linear system of equations* is a

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[1]Future editions will clarify.

[2]Future editions may require a sheet here.

pair (A, b). A solution of a linear system of equations is a vector $x \in \mathbf{R}^n$ satisfying the equations

$$
\begin{aligned}
A_{11}x_1 + \quad A_{12}x_2 + \quad \cdots + \quad A_{1n}x_n &= \quad b_1 \\
A_{21}x_1 + \quad A_{22}x_2 + \quad \cdots + \quad A_{2n}x_n &= \quad b_2 \\
\vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad\; &\quad\; \vdots \\
A_{m1}x_1 + \quad A_{m2}x_2 + \quad \cdots + \quad A_{mn}x_n &= \quad b_n
\end{aligned}
$$

Other terminology includes a *system of linear equations* or *linear system* or *simultaneous linear equations*

Linear Equations

Linear Functions

Arrays

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space

Real Plane

Real Line

Integral Line

Real Order

Real Summation

Geometry

Real Numbers

Rational Order

Rational Numbers

Lists

Integer Arithmetic

Set Numbers

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Direct Products

Finite Sets

Identity Elements

Arithmetic

Equivalent Sets

Natural Summation

Natural Powers

Family Unions and Intersections

Function Inverses

Family Operations

Comparisons

Integer Order

Integer Sums

Natural Equations

Natural Products

Families

Function Images

Function Composites

Operations

Orders

Integer Numbers

Equations

Natural Sums

Functions

Converse Relations

Equivalence Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Set Products

Generalized Set Dualities

Set Powers

Natural Numbers

Natural Induction

Set Unions and Intersections

Set Dualities

Unordered Triples

Intersection of Empty Set

Successor Sets

Partitions

Set Symmetric Differences

Ordered Pairs

Pair Unions

Set Intersections

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects