

RATIONAL NUMBERS

Why

We want fractions.¹

Definition

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We say that the elements (a, b) and (c, d) of this set are rational equivalent if ad = bc. Briefly, the intuition is that (a, b) represents a over b, or in the usual notation "a/b". So this equivalence relation says these two are the same if a/b = c/d or else ad = bc.

Proposition 1. Rational equivalence is an equivalence relation on $Z \times (Z - \{0_Z\})$.

We define the set of rational numbers to be the set of equivalence classes (see Equivalence Classes) under ratioanl equivalence on $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We call an element of the set of ratioanl numbers a rational number or rational. We call the set of rational numbers the set of rationals for short.

Notation

We denote the set of rationals by \mathbf{Q}^2 . If we denote rational equivalence by \sim then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$.

¹This why will be expanded in future editions.

 $^{^2}$ From what we can tell so far, **Q** is a mnemonic for "quantity", from the latin "quantitas".

