Why

We consider the set of results of a set of domain elements.

Definition

The *image* of a set of domain elements under a function is the set of their results. Though the set of domain elements may include several distinct elements, the image may still be a singleton, since the function may map all of elements to the same result.

Using this language, the range (see Functions) of a function is the image of its domain. The range includes all possible results of the function. If the range does not include some element of the codomain, then the function maps no domain elements to that codomain element.

Notation

Let $f : A \to B$. We denote the image of $C \subset A$ by $f(C)$, read aloud as "f of C." This notation is overloaded: for every $c \in C$, $f(c) \in A$, whereas $f(C) \subset A$. Read aloud, the two are indistinguishable, so we must be careful to specify whether we mean an element c or a set C. Following this notation for function images, we denote the range of f by $f(A)$. In this notation, we can record that f maps X onto Y by $f(X) = Y$.

Notational ambiguity

The notation $f(A)$ is can be ambiguous in the case that A is both an element and a set of elements of the domain of f. For example, consider $f : \{\{a\}, \{b\}, \{a, b\}\} \to X$. Then $f(\{a, b\})$ is ambiguous. We will avoid this ambiguity by making clear which we mean in particular cases.

Inverse Images

Similarly to how we can define $f : \mathcal{P}(X) \to \mathcal{P}(Y)$ for $A \subset X$

$$f(A) = \{y \in Y \mid (\exists x)(x \in a \land y = f(x))\},$$

we can define $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$ for $B \subset X$

$$f^{-1}(B) = \{x \in X \mid (\exists y)(y \in B \wedge y = f(x))\}.$$

In other words, $f^{-1}(B)$ is the set of all elements of the domain which give the elements in B of the range. We call $f^{-1}(B)$ the *inverse image* of B. Another name less commonly used is *counter image* or *counterimage*.

Connections

Here are some connections.[1]

Proposition 1. *Let $f : X \to Y$ and $B \subset Y$. $f(f^{-1}(B)) \subset B$. If f is onto, then $f(f^{-1}(B)) \subset B$.*

Proposition 2. *Let $f : X \to Y$ and $A \subset X$. $A \subset f^{-1}(f(A))$. If f is one-to-one, then $A = f^{-1}(f(A))$.*

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[1]The proofs are straightfoward, and will appear in future editions.

Function Images

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Functions

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Relations

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Set Products

Set Powers

Ordered Pairs   Unordered Triples

Pair Unions

Set Unions

Unordered Pairs   Empty Set

Set Specification

↓

Set Equality

↓

Set Inclusion

↓

Standardized Accounts

↓

Accounts

↓

Deductions

↓

Quantified Statements

↓

Logical Statements

↓

Statements

Identities   Sets

Names

Letters   Objects