



## Why

We want language and notation involving order.<sup>1</sup>

## Comparisons

A *comparison* is a statement (see **Statements**) involving a partial (which may or may not be total) order.

## Notation

Let  $A$  be a set. We tend to denote an arbitrary partial order on  $A$  by  $\preceq$ . So  $(A, \preceq)$  is a partially ordered set.

As usual (see **Relations**), we write  $a \preceq b$  to mean  $(a, b) \in A$ . Alternatively, we write  $b \succeq a$  to mean  $a \preceq b$ . In other words,  $\succeq$  is the inverse relation (see **Converse Relations**) of  $\preceq$ .

## Predecessors and successors

If  $a \preceq b$  and  $a \neq b$ , we write  $a \prec b$  and say that  $a$  *precedes*  $b$ . In this case we call  $a$  the *predecessor* of  $b$ . Alternatively, under the same conditions, we write  $b \succ a$  and we say that  $b$  *succeeds*  $a$ . In this case we call  $b$  the *successor* of  $a$ .

## Induced partial orders

Of course, the object we have defined and denoted by  $\prec$  is a relation on  $A$ . It satisfies (i) for no elements  $x$  and  $y$  do  $x \prec y$  and  $y \prec x$  hold simultaneously and (ii) if  $x \prec y$  and  $y \prec z$ , then  $x \prec z$  (i.e.,  $\prec$  is transitive). It is worthwhile to observe that if  $S$  is a relation satisfying (i) and (ii), then the relation  $R$  defined to mean  $(a, b) \in S$  or  $a = b$  is a partial order on  $A$ .

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<sup>1</sup>In the present edition, this sheet can be thought of as an extended notation section for **Orders**.

### Strict and weak relations

This connection between  $\preceq$  and  $\prec$  can be generalized. The *strict relation* corresponding to a relation  $R$  on a set  $A$  is the relation  $S$  on  $A$  defined by  $(a, b) \in S$  if  $(a, b) \in R$  and  $a \neq b$ . The *weak relation* corresponding to a relation  $S'$  on a set  $A$  is the relation  $R'$  defined by  $(a, b) \in R'$  if  $(a, b) \in S'$  or  $a = b$ . For this reason, a relation is said to *partially order* a set if it is a partial order or if its corresponding weak relation is one.



