



Why

Many problems simplify if the graph involved is chordal.¹

Paths

Let G be an undirected graph. A *chord* in a path p of G is an edge between two non-consecutive vertices of p . So a chord of the path (v_1, v_1, \dots, v_k) is an edge $\{v_i, v_j\}$ with $|j - i| > 1$.

We interpret a chord as a “one-edge shortcut” between two vertices of a path. If a path p has a chord, it can be reduced to a shorter path p' by “skipping” vertices. In other words, the shortest path between two vertices is chordless. However, a chordless path need not be a shortest path. See Figure 1.

Graphs

A chord of a cycle $(v_1, v_2, \dots, v_{k-1}, v_1)$ is an edge $\{v_i, v_j\}$ with $(j - i) \bmod k > 1$. An undirected graph G is *chordal* if every cycle with more than three edges has a chord.

If G is chordal, every cycle in G can be reduced to a cycle of length three. We sometimes call a cycle of length three a *triangle*. For this reason, chordal graphs are also sometimes called *triangulated graphs*. Other terminology includes *rigid-circuit graphs*, *triangulated graphs*, *perfect elimination graphs*, *decomposable graphs*.²

¹Future editions will expand.

²See Vanenberghe and Anderson, 2014.

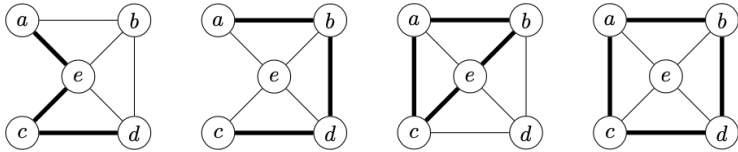


Figure 1: The edge $\{e, d\}$ is a chord in the path (a, e, c, d) of the first graph. The path (a, v, d, c) is chordless. The edge $\{a, e\}$ is a chord in the cycle (a, v, e, c, a) of the second graph. The cycle (a, b, d, c, a) is chordless.

The last graph in Figure 1 is not chordal because the cycle (a, b, d, c, a) has length four and no chord. Adding the edge $\{b, c\}$ or $\{a, d\}$ would make the graph chordal. An immediate consequence of the definition that G be chordal is that any subgraph of G is chordal.

Simple Examples

Since trees and forests have no cycles, they are chordal. Similarly, any graph with no cycles longer than three edges are trivially chordal. Such graphs are sometimes called *cactus graphs*. The complete graphs are also trivially chordal.

