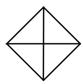


Why

We often want to discuss lists without regard to order. This is natural, for instance, in discussing number factorizations. There, the associativity and commutativity of natural multipliciation mean that the factorization $(2, 3)$ and $(3, 2)$ are equivalent, in the sense that they both factor 6.

Definition

Given a set A, a *multiset* (or *mset*) of elements *of* or *in* A is a function $m : A \to \mathbf{N}$. In this case, the result $m(a)$ is called the *multiplicity* of $a \in A$. For this reason, another term for the object we have named a multiset is *multiplicity function.* An $a \in A$ with $m(a) \neq 0$ is called an *element* of the multiset.

The *size* (or *cardinality*) of a multiset m is the sum of the multiplicities of its elements. In summation notation, the size is defined to be

$$\sum_{a \in A} m(a)$$

The set

$$\{a \in A \mid m(a) > 0\}$$

is called the *support* of the multiset.

Examples

A natural use for multisets is in factorizations of natural numbers.

Notation

Notation for multisets is nonstandard—here are some common in use. If a and b are two distinct objects and $A = \{a, b\}$ is the unordered pair containing them, the multiset $m : A \to \mathbf{N}$ defined by $m(a) = 2$ and $m(b) = 3$ is sometimes denoted

$$\{\{a, a, b, b, b\}\}$$

or alternatively

$$[a, a, b, b, b]$$

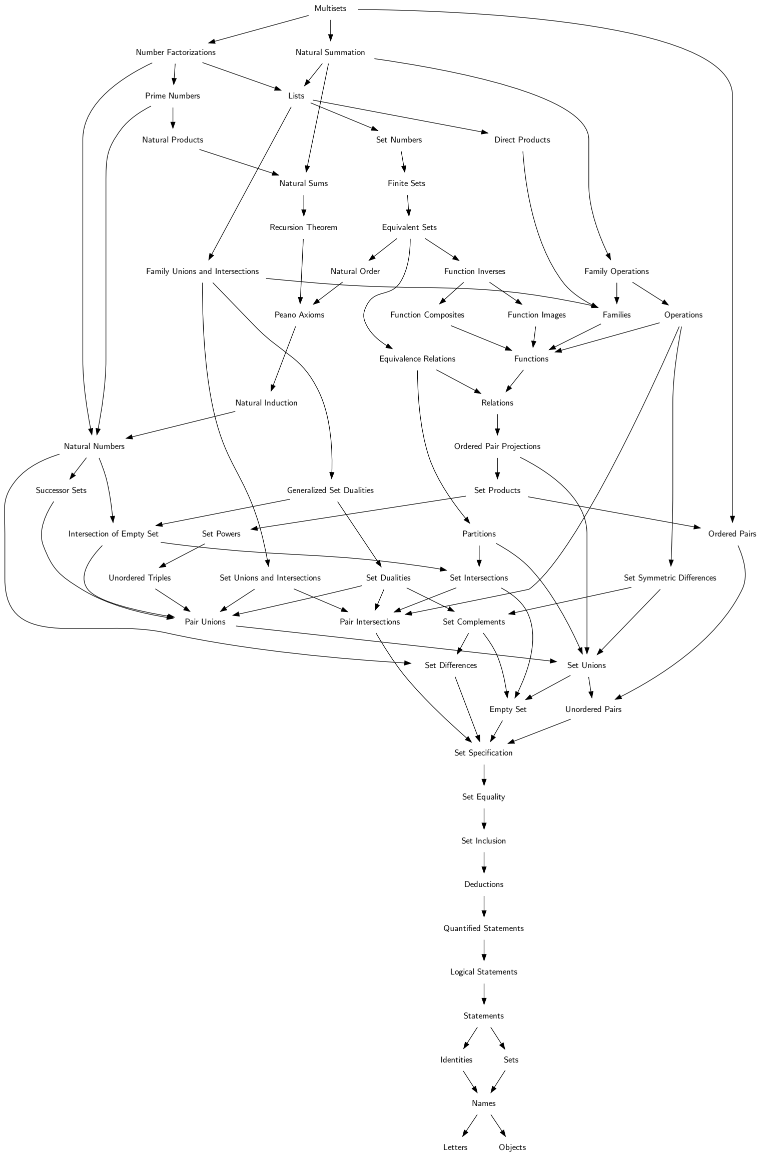
or

$$\{a^2, b^3\}$$

As it appears to the present authors, none of these is desirable. This last notation seems to be a reference to the notation for natural powers (a sheet not required by this one) and the use of multisets in number factorizations .

Other terminology

Other authors refer to the ordered pair (A, m) as the multi set, and call A the *underlying set*. We avoid this terminology, for the reason that $\operatorname{dom} m = A$.


Multisets
Number Factorizations
Natural Summation
Prime Numbers
Lists
Natural Products
Set Numbers
Direct Products
Natural Sums
Finite Sets
Recursion Theorem
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Family Operations
Peano Axioms
Function Composites
Function Images
Families
Operations
Equivalence Relations
Functions
Natural Induction
Relations
Natural Numbers
Ordered Pair Projections
Successor Sets
Generalized Set Dualities
Set Products
Intersection of Empty Set
Set Powers
Partitions
Ordered Pairs
Unordered Triples
Set Unions and Intersections
Set Dualities
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects