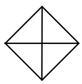


Why

The most important families are those indexed by (subsets of) the natural numbers.

Definition

A *finite sequence* (or *list*) is a family whose index set is $\{1, \dots, n\}$ for some $n \in \mathbf{N}$. The *length* of a finite sequence is the size of its index set. If the codomain of a sequence is A, we say the sequence is *in* A.

Let A be a set with $|A| = n$. In this case, another term for a finite sequence is a *string* (or *list*). A sequence $a : \{1, \dots, n\} \to A$ is an *ordering* of A if a is invertible. In this case, we call the inverse a *numbering* of A. An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

Notation

Since the natural numbers are ordered, we regularly denote finite sequences from left to right between parentheses. For example, we denote $a : \{1, \dots, 4\} \to A$ by (a_1, a_2, a_3, a_4).

Relation to Direct Products

A *natural direct product* is a product of a sequence of sets. We denote the direct product of a sequence of sets $A_1, \dots, A_n$ by $\prod_{i=1}^{n} A_i$. If each A_i is the same set A, then we denote the product $\prod_{i=1}^{n} A_i$ by A^n. In this case, we call an element (the sequence $a = (a_1, a_2, \ldots, a_n) \in A^n$) an *n-tuple* or *tuple*. The set of sequences in a set A is the direct product A^n.

Infinite Sequences

An *infinite sequence* is a family whose index set is N (the set of natural numbers without zero). The *nth term* or *coordinate* of a sequence is the result of the nth natural number, $n \in \mathsf{N}$.[1]

Notation

Let A be a non-empty set and $a : \mathsf{N} \to A$. Then a is a (infinite) sequence in A. $a(n)$ is the nth term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n. If $\{A_n\}_{n \in \mathsf{N}}$ is an infinite sequence of sets, then we denote the direct product of the sequence by $\prod_{i=1}^{\infty} A_i$.

Natural unions and intersections

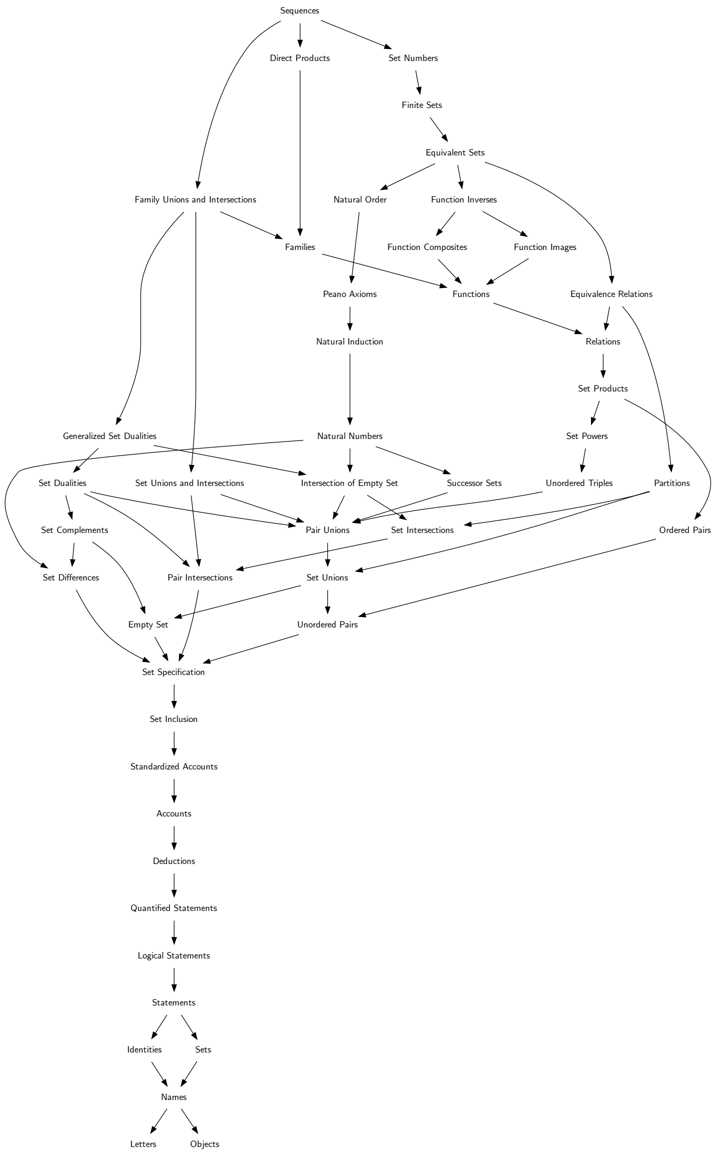
We denote the family union of the finite sequence of sets A_1, $\ldots$, A_n by $\cup_{i=1}^{n} A_i$. We denote the family of the infinite sequence of sets $(A_n)_n$ by $\cup_{i=1}^{\infty} A_i$. Similarly, we denote the intersections of a finite and infinite sequence of sets $\{A_i\}$ by $\cap_{i=1}^{n} A_i$ and $\cap_{i=1}^{\infty} A_i$, respectively.

[1] Future editions may also comment that we are introducing language for the steps of an infinite process.

Slices[2]

An *index range* for a length-n sequence s is a pair (i,j) for which $1 \leq i < j \leq n$. The *slice* corresponding to the index range (i,j) is the length $j-i$ sequence s' defined by $s'_1 = s_i$, $s'_2 = s_{i+1}, \dots, s'_j = s_{i+j-1}$. We denote the (i,j)-slice of s by $s_{i:j}$. If $i = 1$ we use $s_{:j}$ and if $j = n$ we use $s_{i:}$ as shorthands.

[2]Future editions may break sequences in finite sequences and infinite sequences. This would simplify the sheet and remove the dependence on the principle of infinity. Future editions may also use the term “list” instead of finite sequence.


Sequences
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Natural Induction
Relations
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
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Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects