Complex Measures

Why

We allow measures to take complex values.[1]

Definition

A complex-valued function on a sigma algebra is *countably additive* if the result of the function applied to the union of a disjoint countable family of distinguished sets is the limit of the partial sums of the results of the function applied to each of the sets individually. The limit of the partial sums must exist irregardless of the summand order.

A *complex measure* is an complex-valued function on a sigma algebra that is (1) zero on the empty set and (2) countably additive. We call the result of the function applied to a set in the sigma algebra the *complex measure* (or when no ambiguity arises, the *measure*) of the set.

Since the codomain of a complex measures is the complex numbers, the sum corresponding to every countable union must be absolutely convergent (?) .[2]

Notation

We denote complex measures by μ a mnemonic for "measure." Let C denote the set of complex numbers. Let $(X, \mathcal{A})$ be a measurable space and let $\mu : \mathcal{A} \to C$. Then μ is a complex

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[1]Future editions will expand.
[2]Future editions will define.

measure if

1. $\mu(\varnothing) = 0$ and

2. $\mu(\cup_i A_i) = \lim_{n \to \infty} \sum_{k=1}^n \mu(A_k)$ for all disjoint families $(A_n)_n$.

Complex Measures — Measures — Sigma Algebras — Extended Real Numbers — Real Limits — Absolute Value — Interval Length — Intervals — Real Line — Real Sequences — Real Order — Real Numbers — Rational Numbers — Integral Line — Integer Arithmetic — Sequences — Direct Products — Set Numbers — Natural Additive Identity — Natural Multiplicative Identity — Finite Sets — Identity Elements — Arithmetic — Equivalent Sets — Natural Powers — Integer Products — Subset Algebras — Set Operations — Family Unions and Intersections — Function Inverses — Natural Products — Integer Sums — Families — Function Composites — Function Images — Operations — Natural Sums — Integer Numbers — Functions — Natural Order — Recursion Theorem — Equivalence Relations — Cardinality — Relations — Peano Axioms — Subset Systems — Cartesian Products — Generalized Set Dualities — Natural Induction — Ordered Pairs — Set Powers — Natural Numbers — Set Symmetric Differences — Set Dualities — Unordered Triples — Successor Sets — Set Unions and Intersections — Intersection of Empty Set — Partitions — Set Complements — Pair Unions — Set Intersections — Set Unions — Set Differences — Pair Intersections — Unordered Pairs — Empty Set — Set Specification — Geometry — Set Inclusion — Standardized Accounts — Accounts — Deductions — Quantified Statements — Logical Statements — Statements — Identities — Sets — Names — Letters — Objects