Why

The Hilbert space is one of the most important generalizations of the past century.[1]

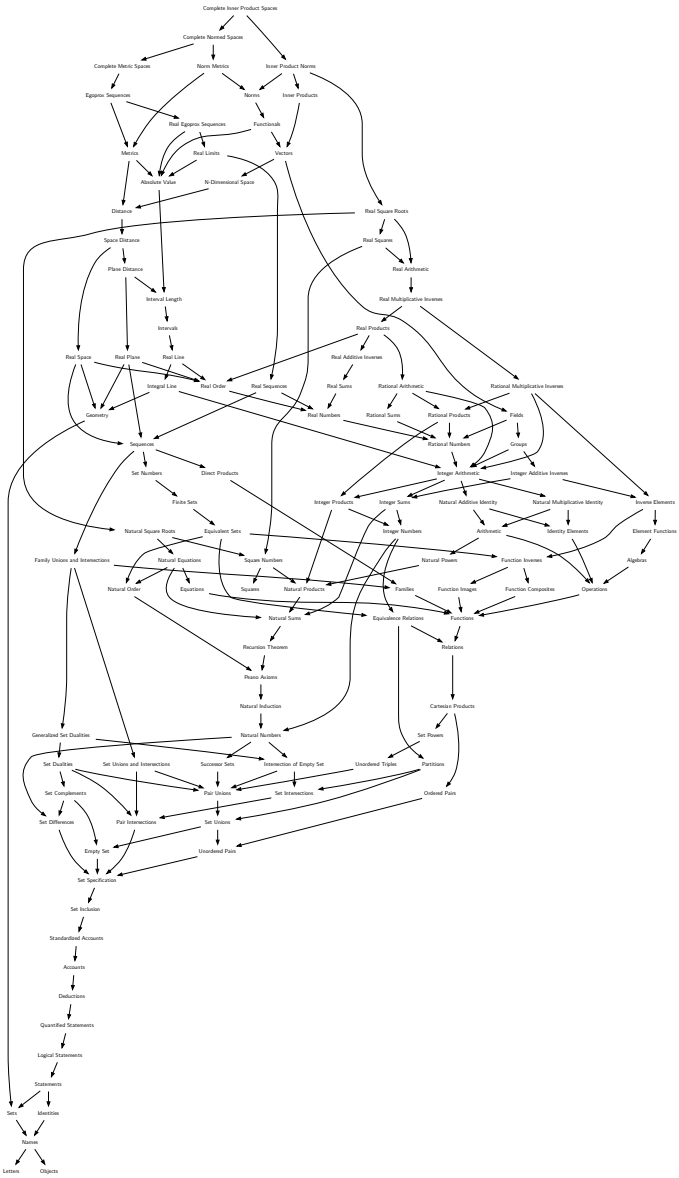
Definition

Let (X, R) be a real vector space. An inner product $\langle \cdot, \cdot \rangle : X \times X \to \mathsf{R}$ induces a norm $\|\cdot\| : X \to \mathsf{R}$ defined by $\|x\| = \sqrt{\langle x, x \rangle}$ and metric $d : X \times X \to \mathsf{R}$ defined by $d(x, y) = \|x - y\|$.

If (X, d) is a complete metric space, we call $((X, \mathsf{R}), \langle \cdot, \cdot \rangle)$ a *complete inner product space* (or *Hilbert space*).[2]

[1] Future editions will modify.

[2] The term Hilbert space is universal, but in accordance with the Bourbaki project's guidelines on naming, we will tend to use the term complete inner product space, even though this is longer.


Complete Inner Product Spaces
Complete Normed Spaces
Complete Metric Spaces
Norm Metrics
Inner Product Norms
Ergonic Sequences
Norms
Inner Products
Real Ergonic Sequences
Functionals
Metrics
Real Limits
Vectors
Absolute Value
N-Dimensional Space
Distance
Real Square Roots
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Space
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Order
Real Sequences
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Geometry
Real Numbers
Rational Sums
Rational Products
Fields
Rational Numbers
Groups
Sequences
Integer Arithmetic
Integer Additive Inverses
Set Numbers
Direct Products
Finite Sets
Integer Products
Integer Sums
Natural Addition Identity
Natural Multiplicative Identity
Inverse Elements
Natural Square Roots
Equivalent Sets
Integer Numbers
Arithmetic
Identity Elements
Element Functions
Family Unions and Intersections
Natural Equations
Square Numbers
Natural Powers
Function Inverses
Algebras
Natural Order
Equations
Squares
Natural Products
Families
Function Images
Function Composites
Operations
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Natural Induction
Cartesian Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects