



Definition

A function $f : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is *linear* if

1. $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbf{R}^n$, and
2. $f(\alpha x) = \alpha f(x)$ for all $\alpha \in \mathbf{R}$ and $x \in \mathbf{R}^n$.

Equivalently, $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$ for all $\alpha, \beta \in \mathbf{R}$ and $x, y \in \mathbf{R}^n$.

In this case, we say that *superposition* holds.

