Why

What is the inverse element under matrix multiplication.

Definition

Recall that if $A \in \mathbf{R}^{m \times n}$ then $x \mapsto Ax$ is a function from $\mathbf{R}^n$ to $\mathbf{R}^m$. Clearly, if $m \neq n$, then the inverse of f can not exist.[1]

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that $BA = I$ we call B the *left inverse* of A and likewise if $AC = I$ we call C the *right inverse* of A. In the case that A is square, the right inverse and left inverse coincide.

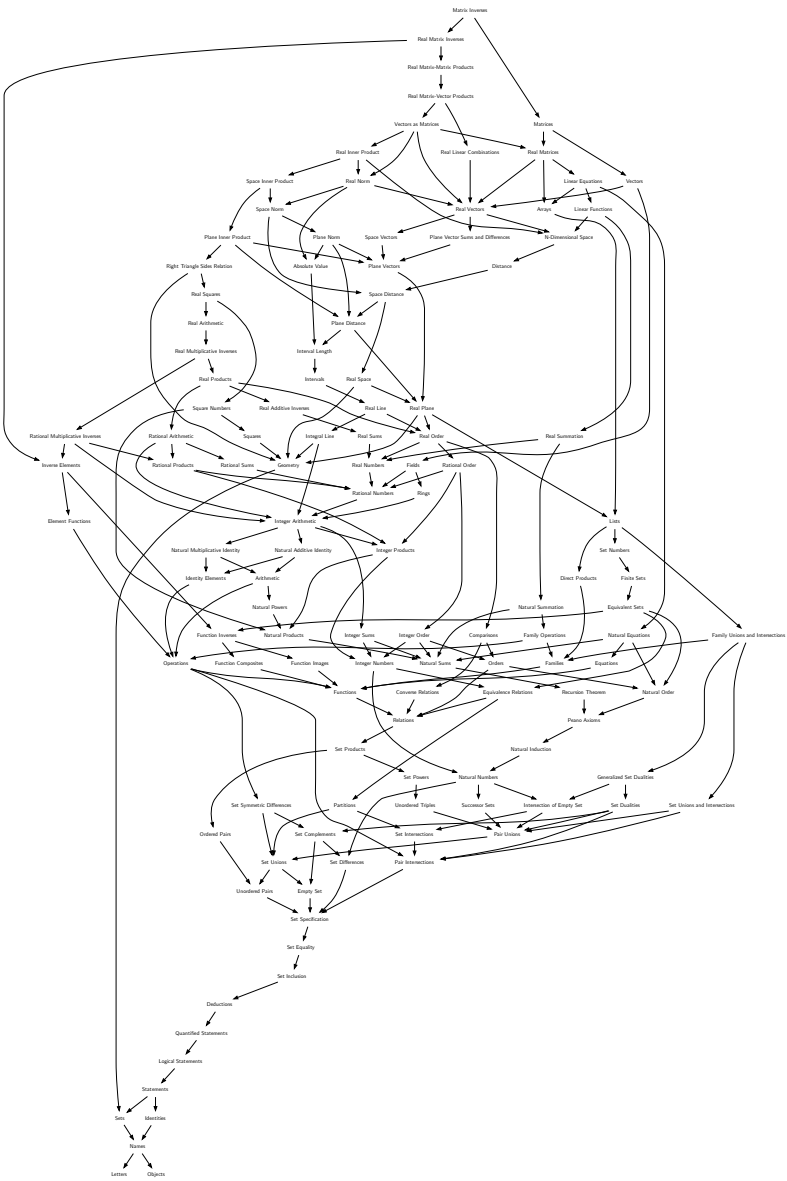
Proposition 1. *Let $A, B, C \in \mathbf{R}^{n \times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

Proof. Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. □

Notation

Let $\mathbf{F}$ be a field. Let $A \in \mathbf{F}^{n \times n}$ be invertible. We follow the notation of inverse elements and denote the inverse of A by A^{-1}.

[1] Future editions will expand.


Matrix Inverses
Real Matrix Inverses
Real Matrix-Matrix Products
Real Matrix-Vector Products
Vectors as Matrices
Matrices
Real Inner Product
Real Linear Combinations
Real Matrices
Space Inner Product
Real Norm
Linear Equations
Vectors
Space Norm
Real Vectors
Arrays
Linear Functions
Plane Inner Product
Plane Norm
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Right Triangle Sides Relation
Absolute Value
Plane Vectors
Distance
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Square Numbers
Real Additive Inverses
Real Line
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Squares
Integral Line
Real Sums
Real Order
Real Summation
Inverse Elements
Rational Products
Rational Sums
Geometry
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Rings
Element Functions
Integer Arithmetic
Lists
Integer Products
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Direct Products
Finite Sets
Identity Elements
Arithmetic
Natural Powers
Natural Summation
Equivalent Sets
Function Inverses
Natural Products
Integer Sums
Integer Order
Comparisons
Family Operations
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Ordered Pairs
Set Complements
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects