



### Why

We want to quantify the error of compressing a real-valued random variable.

### Definition

Let  $\mathcal{X}$  be a finite set and  $q : \mathbf{R} \rightarrow \mathcal{X}$  a quantization (see Quantizations) of  $\mathbf{R}$ . Also, fix a probability space  $(\Omega, \mathcal{A}, \mathbf{P})$  and a random variable  $x : \Omega \rightarrow \mathbf{R}$ .

The *compression*  $\hat{x} : \Omega \rightarrow \mathcal{X}$  of  $x$  under  $q$  is  $q \circ x$ . A *distortion function* for  $x$  and  $\hat{x}$  is a function

$$d : (\Omega \rightarrow \mathbf{R}) \times (\Omega \rightarrow \mathcal{X}) \rightarrow \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

### Examples

The *expected mean-squared-error distortion*  $d_{\text{mse}}$  between  $x$  and  $\hat{x}$  is

$$d_{\text{mse}}(x, \hat{x}) = \mathbf{E}[(x - \hat{x})^2]$$

The *Kulback-Liebler distortion*  $d_{\text{kld}}$  defined by

$$d_{\text{kld}}(x, \hat{x}) = \mathbf{E}[d_{\text{kl}}(\mathbf{P}(y \in \cdot \mid x, \hat{x}) \mid \mathbf{P}(y \in \cdot \mid \hat{x}))]$$

where  $y$  is some random variable that depends on  $x$ .<sup>1</sup>

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<sup>1</sup>Future editions will clarify this sentence.



