Tree-Structured Distributions

1 Why

2 Definition

2.1 Rooted Definition

Let p be a distribution on a d-dimensional space. p **factors according to the rooted tree** on $\{1, \ldots, d\}$ rooted at a vertex k if it can be written as a product of p_k and the conditionals of $p_{i|j}$ for $i, j = 1, \ldots, d$ and $i \neq j$ and $i \neq k$ where j is the parent of i in the rooted tree.

2.2 Defining Result

Proposition 1. *If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

2.3 Undirected Definition

A distribution p **factors according to the tree** T if it factors according to the T rooted at any vertex.

3 Existence and Uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

3.1 Existence

A distribution p need not factor according to a tree.

3.2 Uniqueness

A distribution p may factor according to multiple trees.