



## Why

We define integrals using an infinite process; in order for each step of the process to make sense, we need functions to be measurable.<sup>1</sup>

## Definition

A function between the base sets of two measurable spaces is *measurable* with respect to the distinguished sets of the two spaces if the inverse image of every distinguished subset of the codomain is a distinguished subset of the domain.

## Notation

Let  $(X, \mathcal{A})$  and  $(Y, \mathcal{B})$  be measurable spaces. Then a function  $f : X \rightarrow Y$  is measurable if  $B \in \mathcal{B}$  implies  $f^{-1}(B) \in \mathcal{A}$ . We say that  $f$  is measurable with respect to  $\mathcal{A}$  and  $\mathcal{B}$ .

In this case, we sometimes say  $f$  is a measurable function from  $(X, \mathcal{A})$  to  $(Y, \mathcal{B})$ . We say,  $f : (X, \mathcal{A}) \rightarrow (Y, \mathcal{B})$  is measurable, read aloud as “ $f$  from  $X$ ,  $\mathcal{A}$  to  $Y$ ,  $\mathcal{B}$  is measurable.”

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<sup>1</sup>This statement contains a forward reference to **Real Integrals**, and so may be modified in future editions.



