



Why

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1 Definition

Define $S \in \mathbf{R}^{d \times d}$ by

$$S = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

For a vector $x \in \mathbf{R}^d$ the *down shift* of x is Sx .

Let $A \in \mathbf{R}^{d \times d}$ be a matrix with columns a_1, \dots, a_d . A is a *circulant matrix* if $a_1 = Sa_d$, $a_2 = Sa_1$, and $a_i = Sa_{i-1}$ for $i = 2, \dots, d$.

Example

For example, the matrix

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

is a circulant matrix.

¹Future sheets will include. These matrices arise in practice and each has the same eigenvectors.

