

## **HALFSPACES**

## Why

TODO

## **Definition**

For a nonzero  $b \in \mathbb{R}^n$  and  $\beta \in \mathbb{R}$  the sets

$$\{x \in \mathbf{R}^n \mid \langle x \rangle b \le \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x \rangle b \ge \beta\},$$

are closed halfspaces and the sets

$$\{x \in \mathbf{R}^n \mid \langle x \rangle b \le \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x \rangle b \ge \beta\},$$

are open halfspaces.

Each of these is nonempty and convex. As with hyperplanes, the same four sets appear if one uses  $\lambda\beta$  and  $\lambda b$  above, so the halfspaces depend only on the hyperplane  $H = \{x \mid \langle x \rangle b = \beta\}$ .

