

FINITE AUTOMATA

Definition

A finite automaton (or machine, deterministic finite automaton) $M = (Q, \Sigma, \delta, q_0, F)$ is a list where Q and Σ is are finite sets (alphabets), $\delta : Q \times \Sigma \to Q$, $q_0 \in Q$ and $F \subset Q$.

We call Q the states, Σ the alphabet, δ the transition function, q_0 the start state (initial state), and F the accept states (or final states). An input $u \in \text{str}(\Sigma)$ results in a state sequence $x \in \text{str}(Q)$ with $x_1 = q_0$ and $x_{i+1} = \delta(x_i, u_i)$ for $i = 1, \ldots, |u|$. M accepts x if $x_{|x|+1} \in F$. The set of all strings that M accepts is the language of the machine M. We say that M recognizes or accepts this set. Although a language may accept many different strings, it only ever accepts one language. For example, if the machine accepts no strings, then it accepts the language \varnothing .

A $L \subset \mathsf{str}(\Sigma)$ is called regular if there exists a finite automaton that recognizes it.

Example

Define
$$Q = \{q_1, q_2, q_3\}, \Sigma = \{0, 1\},$$
 define $\delta: Q \times \Sigma \to Q$ by $\delta(q)$

