

Marginal Distributions

Why

Definition

Consider a joint distribution with n components. We associate with this joint n marginal distributions.

For i = 1, ..., n, the *ith marginal distribution* of the joint is the distribution over the *i*th set in the product which assigns to each element of that set the sum of probabilities of outcomes whose *i*th component matches that element.

For i, j = 1, ..., n and $i \neq j$, the i, jth marginal distribution of the joint is the distribution over the product of the ith and jth sets in the original product which assigns to each element in the product the sum of probabilities of outcomes whose i component matches the first component of the product and whose jth component matches the jth component of the product.

Notation

Let A_1, \ldots, A_n be non-empty finite sets. Define $A = \prod_{i=1}^n A_i$ and let $p: A \to \mathbf{R}$ be a joint distribution.

For i = 1, ..., n, define $p_i : A_i \to \mathbf{R}$ by

$$p_i(b) = \sum_{a_i = b} p(a).$$

for each $b \in A_i$. p_i is the *i*th marginal of p.

Similarly, for i, j = 1, ..., n and $i \neq j$ define $p_{ij} : A_i \times A_j \rightarrow$

R by

$$p_{ij}(b,c) = \sum_{a_i = b, a_j = c} p(a)$$

for every $b \in A_i$ and $c \in A_j$. Then p_{ij} is the i, jth marginal of p.

