



## REAL FUNCTIONS

### Why

We name functions whose codomain is the real numbers.

### Definition

A *real function* is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

### Notation

Let  $A$  be a set. Let  $f : A \rightarrow \mathbf{R}$ .  $f$  is a real function. If  $A = \mathbf{R}$ , then  $f \in \mathbf{R} \rightarrow \mathbf{R}$ . To speak of functions defined on intervals, let  $a, b \in \mathbf{R}$ . Let  $g : [a, b] \rightarrow \mathbf{R}$ . Then  $g$  is a real function defined on a closed interval. Let  $h : (a, b) \rightarrow \mathbf{R}$ . Then  $h$  is a real function defined on an open interval.

We regularly declare the interval and the function at once. For example, “let  $f : [a, b] \rightarrow \mathbf{R}$ ” is understood to mean “let  $a$  and  $b$  be real numbers with  $a < b$ , let  $[a, b]$  be the closed interval with them as endpoints, and let  $f$  be a real-valued function whose domain is this interval”. We read the notation  $f : [a, b] \rightarrow \mathbf{R}$  aloud as “ $f$  from closed  $a$   $b$  to  $\mathbf{R}$ .” We use  $f : (a, b) \rightarrow \mathbf{R}$  similarly (read aloud “ $f$  from open  $a$   $b$  to  $\mathbf{R}$ ”).

### Examples

**Example 1.** Let  $c \in \mathbf{R}$ . Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be such that  $f(x) = c$  for every  $x \in \mathbf{R}$ .  $f$  is a real function.

**Example 2.** Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  with  $f(x) = 2x^2 + 1$  for all  $x \in \mathbf{R}$ .

*f* is a real function.

**Example 3.** Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  with

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{otherwise.} \end{cases}$$

*f* is a real function.

