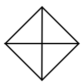


Why

We want to consider the elements of two sets together at once, and other sets created from two sets.

Definitions

Let A and B be two sets.

The *union* of A with B is the set whose elements are in either A *or* B *or* both. The key word in the definition is *or*.

The *intersection* of A with B is the set whose elements are in both A *and* B. The keyword in the definition is *and*.

Viewed as operations, both union and intersection commute; this property justifies the language "with." The intersection is a subset of A, of B, and of the union of A with B.

The *symmetric difference* of A and B is the set whose elements are in the union but not in the intersection. The symmetric difference commutes because both union and intersection commute; this property justifies the language "and." The symmetric difference is a subset of the union.

Let C be a set containing A. The *complement* of A in C is the symmetric difference of A and C. Since $A \subset C$, the union is C and the intersection is A. So the complement is the "left-over" elements of B after removing the elements of A.

We call these four operations *set-algebraic operations*.

Notation

Let A, B be sets. We denote the union of A with B by $A \cup B$, read aloud as "A union B." $\cup$ is a stylized U. We denote the intersection of A with B by $A \cap B$, read aloud as "A intersect B." We denote the symmetric difference of A and B by $A+B$, read aloud as "A symdiff B." "Delta" is a mnemonic for difference.

Let C be a set containing A. We denote the complement of A in C by $C - A$, read aloud as "C minus A."

Results

Proposition **1.** *For all sets A and B the operations $\cup$, $\cap$, and $+$ commute.*

Proposition **2.** *Let S a set. For all sets $A, B \subset S$,*

$$\text{(1)} \quad S - (A \cup B) = (S - A) \cap (S - B)$$

$$\text{(2)} \quad S - (A \cap B) = (S - A) \cup (S - B).$$

Proposition **3.** *Let S a set. For all sets $A, B \subset S$,*

$$A + B = (A \cup B) \cap C_S(A \cap B)$$

$\boxed{TODO : notation}$

Exercises

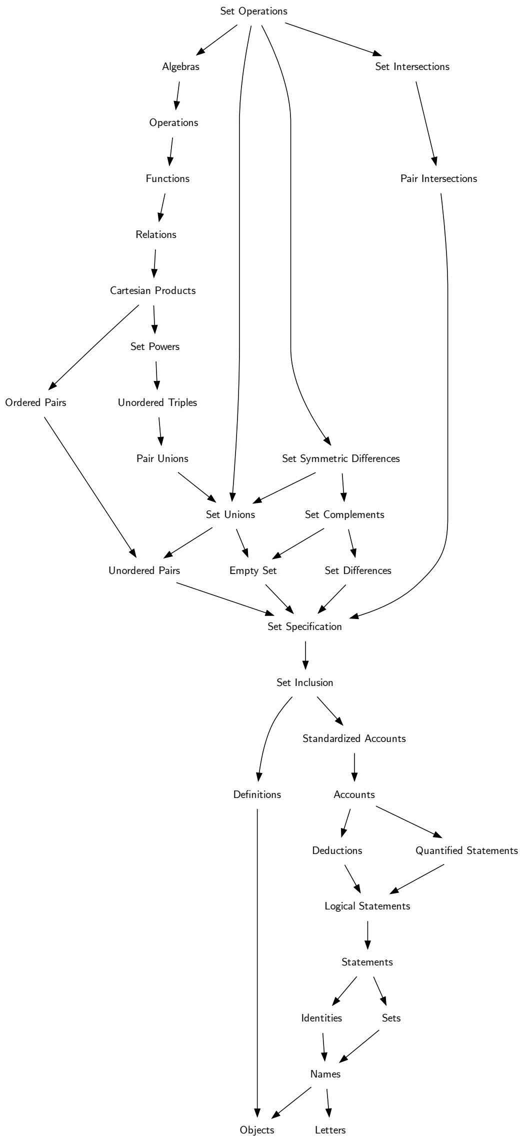
If A, B, X, and Y are sets, then

1. $(A \cup B) \times X) = (A \times X) \cup (B \times X)$,

2. $(A \cap B) \times (A \cap Y) = (A \times X) \cap (B \times Y)$,

3. $(A - B) \times X = (A \times X) - (B \times X)$.

If either $A = \varnothing$ or $B = \varnothing$, then $A \times B = \varnothing$, and conversely. If $A \subset X$ and $B \subset Y$, then $A \times B \subset X \times Y$, and (provided $A \times B \neq \varnothing$) conversely.


Set Operations
Algebras
Set Intersections
Operations
Functions
Pair Intersections
Relations
Cartesian Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Symmetric Differences
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters