Definition

Let $(X, \mathbf{R})$ be a vector space. A function $f : X \times X \to \mathbf{R}$ is an *inner product* on the vector space $(X, \mathbf{R})$ if

1. $f(x, x) \geq 0, = 0 \longleftrightarrow x = 0$,
2. $f(x + y, z) = f(x, z) + f(y, z)$,
3. $f(x, y) = f(y, x)$, and
4. $f(\alpha x, y) = \alpha f(x, y)$.

An *inner product space* is an ordered pair: a real vector space and an inner product.[1]

Examples

$\mathbf{R}^n$ with the usual inner product is an inner product space. Some authors call any finite-dimensional inner product space over the real numbers is a *Euclidean vector space*.

Notation

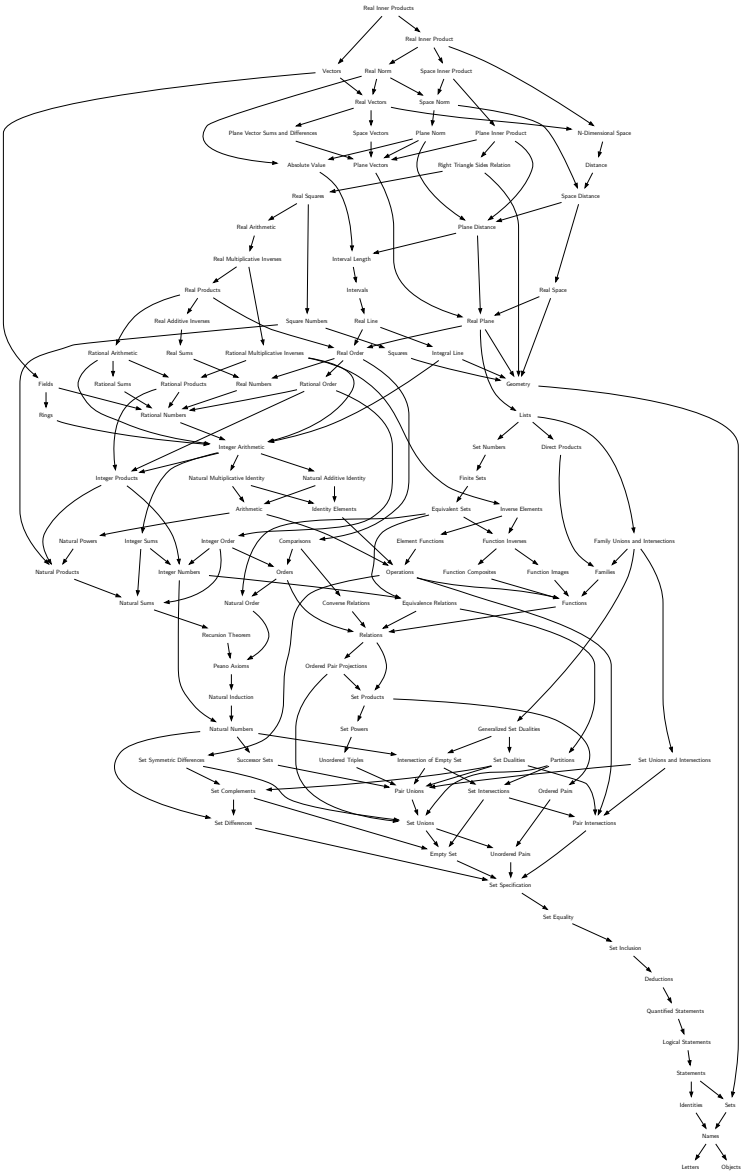
If $f : X \times X \to \mathbf{R}$ is an inner product we regularly denote $f(x, x)$ by $\langle x, x \rangle$.

Orthogonality

Two vectors in an inner product space are *orthogonal* if their inner product is zero. An *orthogonal family of vectors* in an inner product space is a family of vectors for which distinct family members are orthogonal.

A vector is *normalized* if its inner product with itself is one.

[1] Future editions will discuss complex inner products.


Real Inner Products
Real Inner Product
Vectors
Real Norm
Space Inner Product
Real Vectors
Space Norm
Plane Vector Sums and Differences
Space Vectors
Plane Norm
Plane Inner Product
N-Dimensional Space
Absolute Value
Plane Vectors
Right Triangle Sides Relation
Distance
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Square Numbers
Real Line
Real Plane
Rational Arithmetic
Real Sums
Rational Multiplicative Inverses
Real Order
Squares
Integral Line
Fields
Rational Sums
Rational Products
Real Numbers
Rational Order
Geometry
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Integer Order
Comparisons
Element Functions
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Composites
Function Images
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Intersection of Empty Set
Set Dualities
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects