Why

We want to discuss how quickly a function grows.[1]

Definition

Let $f : \mathsf{R} \to \mathsf{R}$. The *lower growth class* of f is the set of all functions $g : \mathsf{R} \to \mathsf{R}$ for which there exists $C, M > 0$ so that $\lvert g(x) \rvert \leq C \lvert f(x) \rvert$ for all $x > M$. The intuition is that if h is in the lower growth class of f, h does not grow faster than f.

The *upper growth class* of f is the set of all functions $g : \mathsf{R} \to \mathsf{R}$ for which there exists $C, M > 0$ so that $\lvert g(x) \rvert \geq C \lvert f(x) \rvert$ for all $x > M$. The intuition is that if h is in the upper growth class of f, h grows at least as fast as f.

The *exact growth class* (or *growth class*) of f is the set of all functions $g : \mathsf{R} \to \mathsf{R}$ for which there exists C_1, C_2, M so that $C_1 \lvert f(x) \rvert \leq \lvert g(x) \rvert \leq C_2 \lvert f(x) \rvert$ for all $x > M$. The intuition is that if h is in the growth class of f, then h and f grow at the same rate.

If a function $h : \mathsf{R} \to \mathsf{R}$ is in the upper growth class of f we say that h grows at order f.

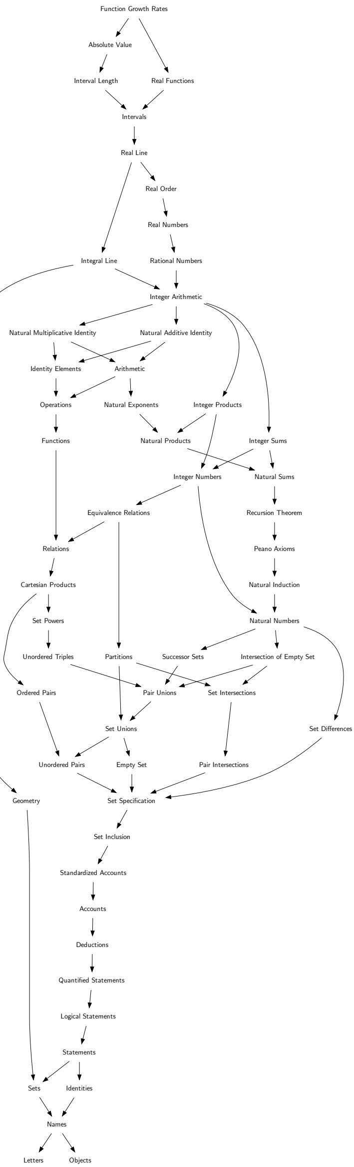
Notation

We denote the upper, lower and exact growth classes of a function $f : \mathsf{R} \to \mathsf{R}$ by of f by $O(f)$. We read the notation $O(f)$

[1]Future editions will expand.

as “order at most f,” we read $\Omega(f)$ as “order at least f,” and $\Theta(f)$ as *order exactly f*.

The letter O, therefore, is a nice mnemonic for order. zfrom use in Taylor approximations near zero. In this case of $|x| < 1$, $|x^p| < |x^q|$ if $q < p$. So higher order terms are “smaller” and “negligible.” This notation is sometimes called *Big O notation* or *Laundau’s symbol.*


Function Growth Rates
Absolute Value
Interval Length
Real Functions
Intervals
Real Line
Real Order
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Identity Elements
Arithmetic
Operations
Natural Exponents
Integer Products
Functions
Natural Products
Integer Sums
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects