Metric Continuity

1 Why

We define continuity for functions between metric spaces.

2 Definition

Our inspiration is continuity of functions from the set of real numbers to the set of real numbers. There we decided on a definition which codified our intuition that numbers which are sufficiently close to each other are mapped to numbers that are close to each other.

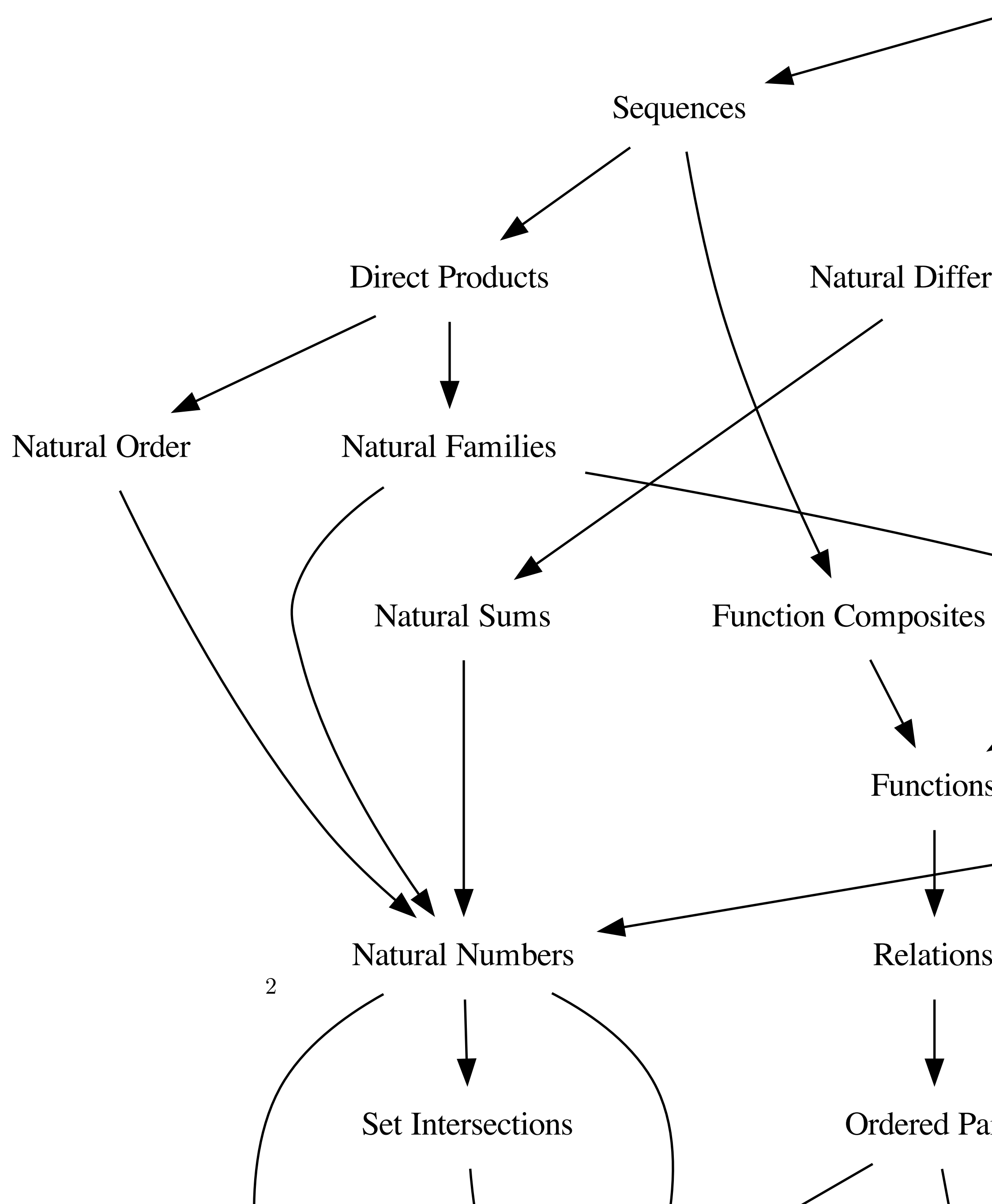
A function from a first metric space to a second metric space is *continuous at* an object of its domain if, for every positive real number (no matter how small), there is a second positive real number (possibly, though not necessarily, smaller) so that every element in the domain whose distance to the fixed object is less than the second positive number has a result under the function whose distance to the result of the fixed object is less than the first positive number.

A function between metric spaces is continuous if it is *continuous* at every object of its domain.

2.1 Notation

Let (A, d) and (B, d') be metric spaces. Let $f : (A, d) \to (B, d')$. Then f is continuous at $\bar{a} \in A$, if for all real numbers $\varepsilon > 0$, there exists a real number $\delta > 0$ such that for all $a \in A$,

$$d(\bar{a}, a) < \delta \implies d'(f(\bar{a}), f(a)) < \varepsilon.$$



Sequences
Direct Products
Natural Differ
Natural Order
Natural Families
Natural Sums
Function Composites
Functions
Natural Numbers
Relations
Set Intersections
Ordered Pa

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