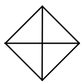


Definition

A *hyperplane* in n-dimensional space is an $(n-1)$-dimensional affine set.

Since the $n-1$-dimensional subspaces are the orthogonal complements of the one-dimensional subspaces, they are the sets which can be specified by

$$\{x \in \mathbf{R}^n \mid x \perp b\}$$

for $b \in \mathbf{R}^n$. The hyperplanes are translates of these,

$$\begin{aligned}\{x \in \mathbf{R}^n \mid x \perp b\} + a &= \{x + a \mid \langle x \rangle b = 0\} \\ &= \{y \mid \langle y - a \rangle b = 0\} = \{y \mid \langle y \rangle b = \beta\},\end{aligned}$$

where $\beta = \langle a \rangle b$.

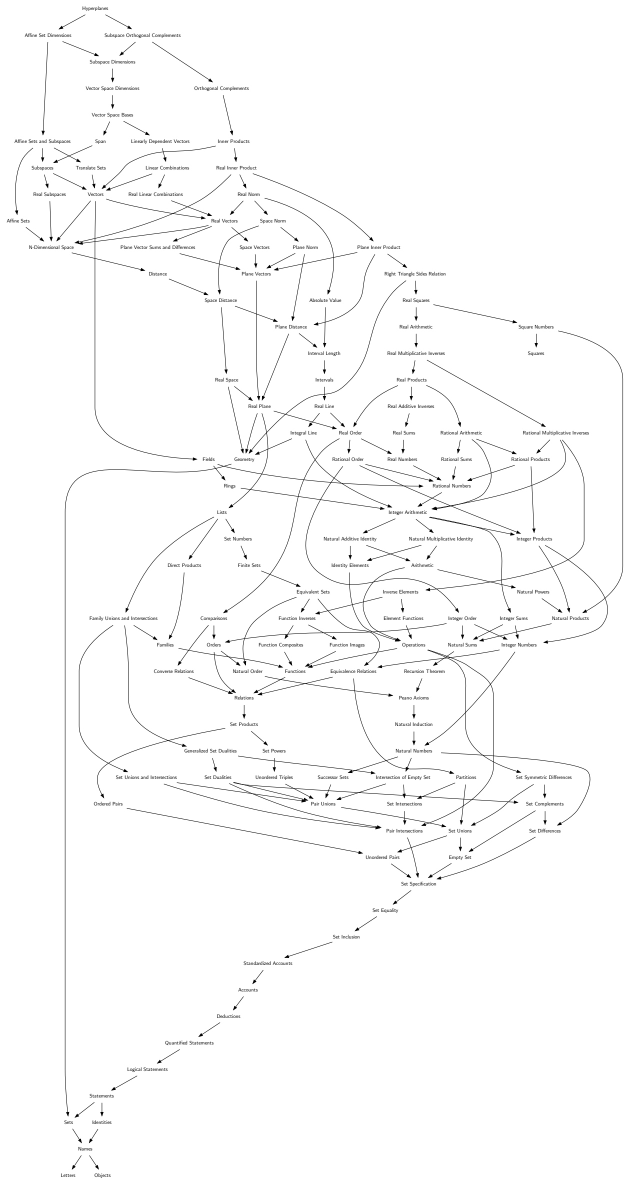
Characterization

Prop. 1. *$H \subset \mathbf{R}^n$ is a hyperplane if and only if there exists $\beta \in \mathbf{R}$ and nonzero $b \in \mathbf{R}^n$ so that*

$$H = \{x \in \mathbf{R}^n \mid \langle x \rangle b = \beta\}.$$

Remark 1. *b and β are unique up to a common nonzero multiple. For example, b, β and $2b, 2\beta$ give the same hyperplane.*

Remark 2. *The vector b is called a* normal *to the hyperplane.*


Hyperplanes
Affine Set Dimensions
Subspace Orthogonal Complements
Subspace Dimensions
Vector Space Dimensions
Orthogonal Complements
Vector Space Bases
Affine Sets and Subspaces
Span
Linearly Dependent Vectors
Inner Products
Subspaces
Translate Sets
Linear Combinations
Real Inner Product
Real Subspaces
Vectors
Real Linear Combinations
Real Norm
Affine Sets
Real Vectors
Space Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Plane Norm
Plane Inner Product
Distance
Plane Vectors
Right Triangle Sides Relation
Space Distance
Absolute Value
Real Squares
Plane Distance
Real Arithmetic
Square Numbers
Interval Length
Real Multiplicative Inverses
Squares
Real Space
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Fields
Geometry
Rational Order
Real Numbers
Rational Sums
Rational Products
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Inverse Elements
Natural Powers
Family Unions and Intersections
Comparisons
Function Inverses
Element Functions
Integer Order
Integer Sums
Natural Products
Families
Orders
Function Composites
Function Images
Operations
Natural Sums
Integer Numbers
Converse Relations
Natural Order
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects