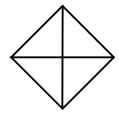

Graphs

1 Why

We want to visualize relations.

2 Definition

A *graph* is a set and a relation on the set. The graph is *undirected* if the relation is symmetric; otherwise the graph is *directed*.

A *vertex* of the graph is an element of the set. The set is called the *vertex set*. An *edge* of the graph is an element of the relation. The relation is called the *edge set*.

If the graph is directed, we call the first element of an edge the *parent* of the second element. We call the second element of an edge the *child* of the first element. So we can discuss the set of parents or set of children of a particular vertex (these sets may be empty).

2.1 Notation

We denote the vertex set by V, a mnemonic for vertex. We denote the edge set by E, a mnemonic for edge. We denote a graph by (V, E). If the vertex set is assumed, or if every vertex appears in E we can unambiguously refer to the graph by E.

Let $\mathrm{pa} : V \to V^*$ and $\mathrm{ch} : V \to V^*$ be the functions associating to each vertex its set of parents and set of children, respectively. As usual, we denote the parents of vertex v by pa_v and the children by ch_v.

2.2 Visualization

We visualize a graph by drawing a point for each vertex. If two vertices u and v are in relation, we draw a line from the point corresponding to u to the point corresponding to v with an arrow at the point corresponding to v. If the graph is undirected, we omit arrows. Here are all undirected graphs on three vertices.

3 Paths

A path in a graph is a sequence of vertices with the property that consecutive vertices are related. A path *cycles* if a vertex appears more than once. A path is *finite* if the sequence is finite. A *loop* is a finite path that cycles once. A finite path from vertex u to vertex v is a path starting with u and ending with v. The *length* of a finite path is the length of the sequence.

4 Properties

A graph is *connected* if there is a path between every pair of vertices. A graph is *acyclic* if none of its paths cycle.