



Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? And so on.¹

Definition

The *integrable function spaces* are a collection of function spaces, one for each real number $p \geq 1$, for which the p th power of the absolute value of the function is integrable.²

Notation

Let (X, \mathcal{A}, μ) be a measure space. Let $p \geq 1$. We denote the integrable function space corresponding to p by $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{R})$. We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{R}) = \left\{ \text{measurable } f : X \rightarrow \mathbf{R} \mid \int |f|^p d\mu < \infty \right\}$$

Let \mathbf{C} denote the set of complex numbers. Similarly for complex-valued functions, we denote the p th space by $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{C})$.

¹Future sheets are likely to begin with L^2 .

²Future editions will include the case where $p = \infty$.

