



## COMPLEX NUMBERS

### Why

We want to find the roots of negative numbers.<sup>1</sup>

### Definition

A *complex number* is an ordered pair of real numbers. The *real part* of a complex number is its first coordinate. The *imaginary part* of a complex number is its second coordinate.

We can identify the imaginary numbers with no complex part (i.e.,  $\{(a, b) \in \mathbf{R}^2 \mid b = 0\}$  with  $\mathbf{R}$  in the obvious way. For this reason, such a complex number is sometimes referred to as a *purely real number*. Conversely, a complex number with zero imaginary part (i.e., an element of  $\{(a, b) \in \mathbf{R}^2 \mid a = 0\}$ ) is said to be a *purely imaginary number*.

### Notation

When treating  $\mathbf{R}^2$  as the set of complex numbers, we denote it by  $\mathbf{C}$ . Let  $z \in \mathbf{C}$  with  $z = (a, b)$ . The real part of  $z$  is  $a$  and its imaginary part is  $b$ . It is universal to denote  $z$  by  $a + ib$ , and to call  $i$  an (or the) *imaginary number*. Some authors use  $j$ , it is a matter of notation.

We denote the real part of  $z$  by  $\operatorname{Re}(z)$ , read “real of  $z$ ,” and the imaginary part by  $\operatorname{Im}(z)$ , read “imaginary of  $z$ .” So, in particular,  $\operatorname{Re}(z) = a$  and  $\operatorname{Im}(z) = b$ .

---

<sup>1</sup>Future editions will modify this, and will discuss the existence of solutions of algebraic equations.



