Relations

# 1 Why

How can we relate the elements of two sets?

# 2 Definition

A *relation* between two nonempty sets is a subset of their cross product. A relation on a single set is a subset of the cross product of it with itself.

The *domain* of a relation is the set of all elements which appear as the first coordinate of some ordered pair of the relation. The *range* of a relation is the set of all elements which appear as the second coordinate of some ordered pair of the relation.

## 2.1 Notation

Let $A$ and $B$ be two nonempty sets. A relation on $A$ and $B$ is a subset of $A \times B$. Let $C$ be a nonempty set. A relation on a $C$ is a subset of $C \times C$.

Let $a \in A$ and $b \in B$. The ordered pair $(a, b)$ may or may not be in a relation on $A$ and $B$. Also notice that if $A \neq B$, then $(b, a)$ is not a member of the product $A \times B$, and therefore not in any relation on $A$ and $B$. If $A = B$, however, it may be that $(b, a)$ is in the relation.

### 2.2 Notation

Let $A$ and $B$ be nonempty sets with $a \in A$ and $b \in B$. Since relations are sets, we can use upper case Latin letters. Let $R$ be a relation on $A$ and $B$. We denote that $(a, b) \in R$ by $aRb$, read aloud as "a in relation $R$ to b."

When $A = B$, we tend to use other symbols instead of letters. For example, $\sim$, $=$, $<$, $\leq$, $\prec$, and $\preceq$.

## 3 Properties

Often relations are defined over a single set, and there are a few useful properties to distinguish.

A relation is *reflexive* if every element is related to itself. A relation is *symmetric* if two objects are related regardless of their order. A relation is *antisymmetric*antisymmetric if two different objects are related only in one order, and never both. A relation is *transitive* if a first element is related to a second element and the second element is related to the third element, then the first and third element are related.

### 3.1 Notation

Let $R$ be a relation on a non-empty set $A$. $R$ is reflexive if

$$(a, a) \in R$$

for all $a \in A$. $R$ is transitive if

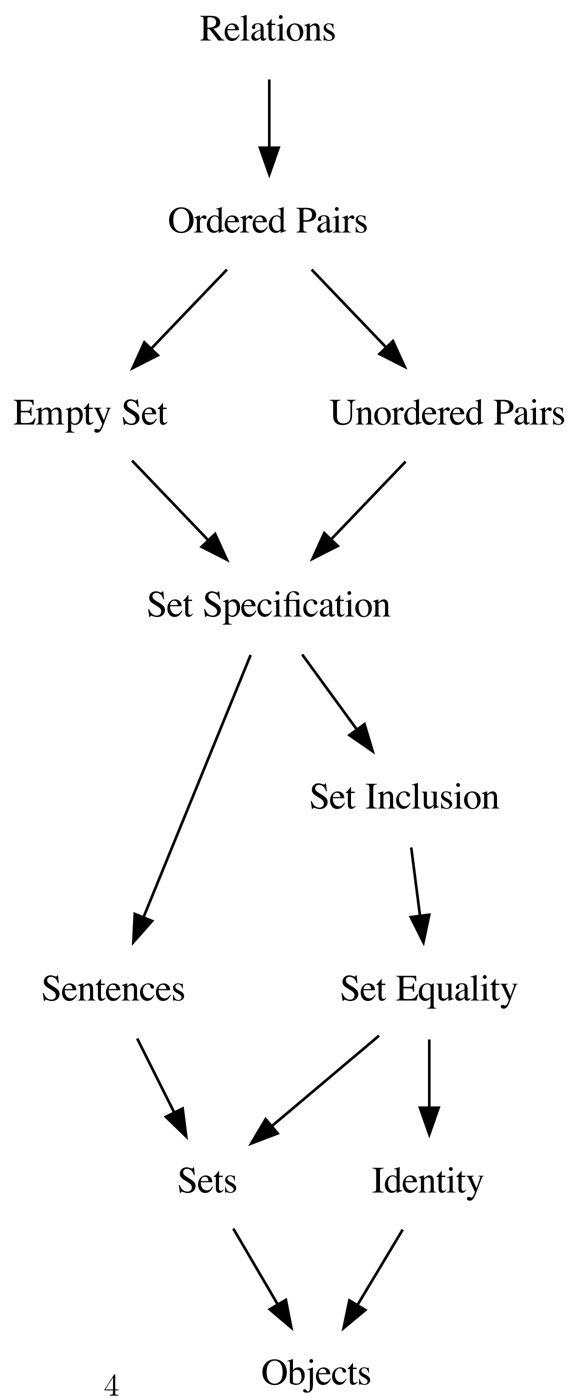
$$(a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R$$

for all $a, b, c \in A$. $R$ is symmetric if

$$(a, b) \in R \implies (b, a) \in R$$

for all $a, b \in A$. $R$ is anti-symmetric if

$$(a, b) \in R \implies (b, a) \notin R$$




for all $a, b \in A$.