



# Conditional Distributions

## 1 Why

Conditioning defines a new probability mass function on the set of outcomes.

## 2 Definition

The conditional probability of any two events defines a new probability mass function over the set of outcomes which is in the second event. The conditional probability of an outcome is the probability of the second event divided into the original probability of the outcome.

## 3 Notation

Let  $p$  be a probability mass function on the set of outcomes  $A$  with the corresponding event probability function  $\mathbf{P}$ . Let  $R$  be the set of real numbers. Then the conditional probability mass

function of  $C$  conditioned on  $B$  is  $q : C \rightarrow R$ , defined so that

$$q(a) = \begin{cases} \frac{p(a)}{\mathbf{P}(B)} & \text{if } a \in B \\ 0 & \text{otherwise .} \end{cases}$$

## 4 Definition

### 4.1 Notation

Let  $R$  denote the set of real numbers. Let  $A_1, \dots, A_n$  be a sequence of non-empty finite sets. Let  $A = \prod_{i=1}^n A_i$  Let  $p : A \rightarrow R$  be a distribution on  $A$ . We denote the conditional distribution of  $i$  on  $j$  of  $p$  by  $p_{i|j} : A_i \times A_j \rightarrow R$  For  $i, j = 1, \dots, n$  and  $i \neq j$   $p_i$  satisfies

$$p_{i|j}(b, c)p_j(c) = p_{ij}(b, c)$$

for every  $b \in A_i$  and  $c \in A_j$ .