Definition

Let Σ be an alphabet. A language $L \subset \mathsf{str}(\Sigma)$ is called *regular* if there exists a finite automaton that recognizes it.

Regular operations

Let $A, B \subset \mathsf{str}(\Sigma)$ be languages in Σ.

Union

The *union* (*alternation*) of A and B is, as usual, the set $A \cup B$.

Concatenation

The *concatenation* of A and B is the set $\{xy \mid x \in A \text{ and } y \in B\}$, where xy denotes length $|x| + |y|$ string which is the concatenation of x and y

Multi-concatenation

The *star* (*Kleene star*, *multi-concatenation*) of A is the set

$$\{x \in \mathsf{str}(\Sigma) \mid \exists k \geq 0, x = y_1 y_2 \cdots y_k, y_i \in A\}.$$

By this definition we do mean to include the empty string $\varnothing$ in A^*, regardless of A.

Notation

We denote the alternation of A and B by $A \cup B$ as usual, but other notations include $A + B$, $A \mid B$, and $A \vee B$. We denote the concatenation of A and B by AB, but other notations include $A \circ B$. We denote the star of A by A^*.

Regular Languages

Finite Automata — Formal Languages

Alphabets

Lists

Direct Products — Set Numbers

Finite Sets

Equivalent Sets

Function Inverses — Natural Order — Family Unions and Intersections

Function Images — Function Composites — Families

Functions — Equivalence Relations

Relations — Peano Axioms

Ordered Pair Projections — Natural Induction

Set Products — Set Powers — Natural Numbers — Generalized Set Dualities

Partitions — Unordered Triples — Successor Sets — Intersection of Empty Set — Set Dualities — Set Unions and Intersections

Ordered Pairs — Set Intersections — Pair Unions — Set Complements — Pair Intersections

Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects