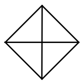


Why

TODO

Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\mathbf{cov}(f, g)$. We defined it:

$$\mathbf{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\,\mathbf{E}(g).$$

0.1 Properties

PROPOSITION **1.** *Covariance is symmetric and billinear.*

Proof. TODO □

PROPOSITION **2.** *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\mathbf{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\,\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \mathbf{var}(f).$$

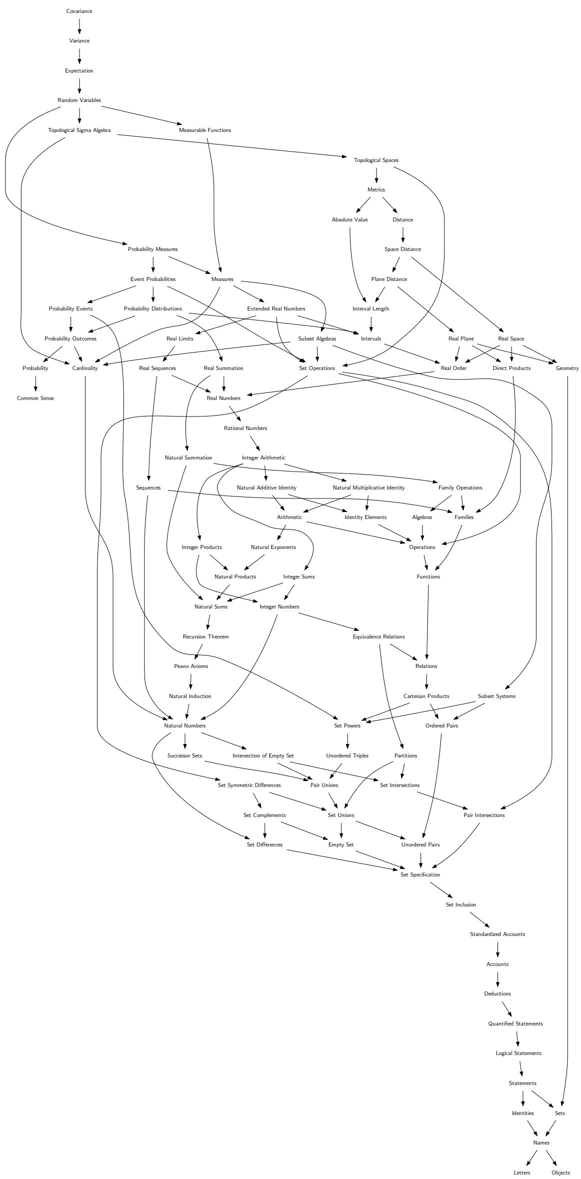
□

Proposition 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\begin{aligned}\mathsf{var}\left(\sum_{i=1}^{n} f_i\right) &= \mathsf{cov}\left(\sum_{i=1}^{n} f_i, \sum_{i=1}^{n} f_i\right) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} \mathsf{cov}(f_i, f_j)\end{aligned}$$

□


Covariance
Variance
Expectation
Random Variables
Topological Sigma Algebra
Measurable Functions
Topological Spaces
Metrics
Absolute Value
Distance
Probability Measures
Space Distance
Event Probabilities
Measures
Plane Distance
Probability Events
Probability Distributions
Extended Real Numbers
Interval Length
Probability Outcomes
Real Limits
Subset Algebras
Intervals
Real Plane
Real Space
Probability
Cardinality
Real Sequences
Real Summation
Set Operations
Real Order
Direct Products
Geometry
Common Sense
Real Numbers
Rational Numbers
Natural Summation
Integer Arithmetic
Sequences
Natural Additive Identity
Natural Multiplicative Identity
Family Operations
Arithmetic
Identity Elements
Algebras
Families
Integer Products
Natural Exponents
Operations
Natural Products
Integer Sums
Functions
Natural Sums
Integer Numbers
Recursion Theorem
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Cartesian Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Set Unions
Pair Intersections
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects