Algebras

1 Why

We name a set together with an operation.

2 Basics

An **algebra** is an ordered pair whose first element is a non-empty set and whose second element is an operation on that set. The **ground set** of the algebra is the set on which the operation is defined.

2.1 Notation

Let A be a non-empty set and let $+ : A \times A \to A$ be an operation on A. As usual, we denote the ordered pair by $(A, +)$.

3 Identity Elements

We call $e \in A$ an **identity element** if (1) $e + a = e$ and (2) $a + e = e$ for all $a \in A$. If only (1) holds, we call e a **left**

identity. If only (2) holds, we call e a **right identity**.

4 Inverse Elements

We call $b \in A$ an **inverse element** of $a \in A$ if (1) $b + a = e$ and (2) $a + b = e$. If only (1) holds, we call e a **left inverse**. If only (2) holds, we call e a **right inverse**.