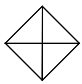


Why

We want to capture the useful properties of the standard basis vectors.

Definition

A set of vectors $\{v_1, \dots, v_k\}$ is *independent* if

$$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_k v_k = 0 \Rightarrow \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0.$$

Notice that independence is a property of a set of vectors, not of any vector in particular.

Unique representation

Suppose $v_1, \dots, v_k$ are independent and we have

$$x = \sum_{i=1}^{k} \alpha_i v_i \quad \text{and} \quad x = \sum_{i=1}^{k} \beta_i v_i.$$

Then

$$0 = x - x = \sum_{i=1}^{n} (\alpha_i - \beta_i) v_k.$$

Using the definition of independence, we conclude $\alpha_i - \beta_i = 0$ for $i = 1, \dots k$. Consequently, $\alpha_i = \beta_i$. In other words, if x can be represented as a linear combination of the vectors $v_1, \dots, v_k$, that representation is *unique*. We have shown that independence implies uniqueness? What of the converse?

We show that lack of independence gives a lack of uniqueness. Suppose there exists $\alpha_1, \dots, \alpha_k$, not all zero, so that

$$\alpha_1 v_1 + +\alpha_2 v_2 + \cdots + \alpha_k v_k = 0.$$

In particular, suppose $\alpha_i \neq 0$. Then we have

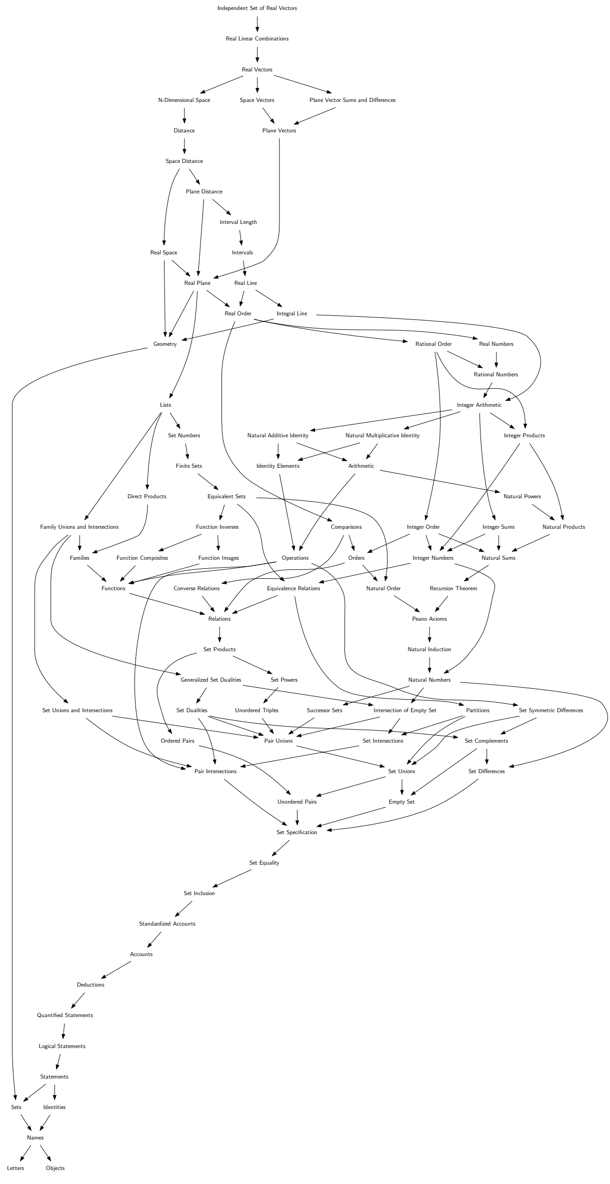
$$v_i = (1/\alpha_i) \sum_{j \neq i} \alpha_j v_j.$$

Suppose x can be written as a linear combination of $v_1, \dots, v_k$. In other words, there are $\beta_1, \dots, \beta_k$ so that

$$x = \beta_1 v_1 + \beta_2 v_2 + \cdots + \beta_k v_k$$

Then also,

$$x = \beta \sum_{j \neq i} \beta_i v_i$$


Independent Set of Real Vectors
Real Linear Combinations
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects