Why

If we stack a rectangle on top of itself we have a rectangle twice the height. The additivity principle says that the area of the so-formed rectangle is the sum of the areas of the stacked rectangles. Our definition of integral for simple functions has this property.

Result

Proposition 1. *The simple non-negative integral operator is homogenous over non-negative real values.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $\text{SF}_+(X)$ denote the non-negative real-valued simple functions on X. Define $s : \text{SF}_+(X) \to [0, \infty]$ by $s(f) = \int f d\mu$ for $f \in \text{SF}_+(X)$. In this notation, we want to show that $s(\alpha f) = \alpha s(f)$ for all $\alpha \in [0, \infty)$ and $f \in \text{SF}_+(X)$. Toward this end, let $f \in \text{SF}_+(X)$ with the simple partition $\{A_n\} \subset \mathcal{A}$ and $\{a_n\} \subset [0, \infty]$.

First, let $\alpha \in (0, \infty)$. Then $\alpha f \in \text{SF}_+(X)$, with the simple partition $\{A_n\} \subset \mathcal{A}$ and $\{\alpha a_n\} \subset [0, \infty]$.

$$s(\alpha f) = \sum_{i=1}^{n} \alpha a_n \mu(A_i) = \alpha \sum_{i=1}^{n} a_n \mu(A_i) = \alpha s(f).$$

If $\alpha = 0$, then αf is uniformly zero; it is the non-negative simple with partition $\{X\}$ and $\{0\}$. Regardless of the measure of X, this non-negative simple function is zero. Recall that we define $0 \cdot \infty = \infty \cdot 0 = 0$. $\qquad \square$

Simple Integral Homogeneity

Simple Integrals

Measurable Functions

Measure

Sigma Algebras

Real Series — Extended Real Numbers

Real Limits

Absolute Value

Simple Functions

Real Functions

Interval Length

Intervals

Real Line

Real Order — Real Sequences

Subset Algebras

Rational Order — Real Numbers — Sequences

Cardinality

Rational Numbers — Integral Line — Lists

Set Numbers — Integer Arithmetic — Direct Products

Finite Sets

Integer Products — Natural Multiplicative Identity — Natural Additive Identity

Equivalent Sets

Arithmetic — Identity Elements

Comparisons — Function Inverses — Integer Order — Integer Sums — Natural Powers — Operations — Family Unions and Intersections

Orders — Function Images — Function Composites — Natural Products — Families

Converse Relations — Natural Order — Functions — Integer Numbers — Equivalence Relations — Natural Sums

Relations

Recursion Theorem

Ordered Pair Projections

Peano Axioms

Subset Systems — Set Products — Characteristic Functions

Natural Induction

Ordered Pairs — Set Powers

Natural Numbers — Generalized Set Dualities

Partitions — Unordered Triples — Successor Sets — Intersection of Empty Set — Set Symmetric Differences — Set Dualities — Set Unions and Intersections

Set Intersections — Pair Unions — Set Complements

Set Unions — Set Differences — Pair Intersections

Unordered Pairs — Empty Set

Set Specification — Geometry

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects