



SET INCLUSION

Why

We want language for all of the elements of a first set being the elements of a second set.

Definition

Given two sets A and B , if every element of A is an element of B then we call that A is a *subset* of the B . We say that A is *included* in B . We say that B is a *superset* of A or that B *includes* A . A set A includes and is included in itself.

If $A = B$, then A includes B and B includes A . The axiom of extension asserts the converse also holds. If A includes B and B includes A , then $A = B$. In other words, if A is a subset of B and B a subset of A , then $A = B$.

The empty set is a subset of every other set. Suppose toward contradiction that A were a set which did not include the empty set. Then there would exist an element in the empty set which is not in A . But then the empty set would not be empty. We call the empty set and A *improper subsets* of A . All other subsets we call *proper subsets*. In other words, B is an improper subset of A if and only if A includes B , $B \neq A$ and $B \neq \emptyset$.

Notation

Given two sets A and B , we denote that A is included in B by $A \subset B$. We read the notation $A \subset B$ aloud as “ A is included

in B " or "A subset B ". Or we write $B \supset A$, and read it aloud "B includes A" or "B superset A".

In this notation, we express the axiom of extension

$$A = B \Leftrightarrow (A \supset B) \wedge (A \subset B).$$

The notation $A \subset B$ is a concise symbolism for the sentence "every element of A is an element of B ." Or for the alternative notation $a \in A \implies a \in B$.

Properties

Given a set A , $A \subset A$. Like equality, we say that inclusion is *reflexive*. Given sets A and B , if $A \subset B$ and $B \subset C$ then $A \subset C$. Like equality, we say that inclusion is *transitive*. If $A \subset B$ and $B \subset A$, then $A = B$ (by the axiom of extension). Unlike equality, which is symmetric, we say that inclusion is *antisymmetric*.

Comparison with belonging

Given a set A inclusion is reflexive. $A \subset A$ is always true. Is $A \in A$ ever true? Also, inclusion is transitive. Whereas belonging is not.

