Why

We name functions whose codomain is the real numbers.

Definition

A *real function* is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

Notation

Let A be a set. Let $f : A \to \mathbf{R}$. f is a real function. If $A = \mathbf{R}$, then $f \in \mathbf{R} \to \mathbf{R}$. To speak of functions defined on intervals, let $a, b \in \mathbf{R}$. Let $g : [a, b] \to \mathbf{R}$. Then g is a real function defined on a closed interval. Let $h : (a, b) \to \mathbf{R}$. Then h is a real function defined on an open interval.

We regularly declare the interval and the function at once. For example, "let $f : [a, b] \to \mathbf{R}$" is understood to mean "let a and b be real numbers with $a < b$, let $[a, b]$ be the closed interval with them as endpoints, and let f bea real-valued function whose domain is this interval". We read the notation $f : [a, b] \to \mathbf{R}$ aloud as "f from closed a b to $\mathbf{R}$." We use $f : (a, b) \to \mathbf{R}$ similarly (read aloud "f from open a b to $\mathbf{R}$").

Examples

Example 1. *Let $c \in \mathbf{R}$. Let $f : \mathbf{R} \to \mathbf{R}$ be such that $f(x) = c$ for every $x \in \mathbf{R}$. f is a real function.*

Example 2. *Let $f : \mathbf{R} \to \mathbf{R}$ with $f(x) = 2x^2 + 1$ for all $x \in R$. f is a real function.*

Example 3. *Let $f : \mathbf{R} \to \mathbf{R}$ with*

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{otherwise.} \end{cases}$$

f is a real function.

Real Functions

Intervals

Real Line

Real Order

Real Numbers

Rational Order

Integral Line

Rational Numbers

Integer Arithmetic

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Identity Elements

Arithmetic

Natural Powers

Natural Products

Integer Sums

Integer Order

Comparisons

Operations

Natural Sums

Integer Numbers

Orders

Functions

Recursion Theorem

Equivalence Relations

Natural Order

Converse Relations

Peano Axioms

Relations

Natural Induction

Set Products

Natural Numbers

Set Powers

Ordered Pairs

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Symmetric Differences

Set Intersections

Pair Unions

Pair Intersections

Set Complements

Set Unions

Empty Set

Set Differences

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects