Why

If $\mathbf{R}$ corresponds to a line, and $\mathbf{R}^2$ to a plane, and $\mathbf{R}^3$ to space, does $\mathbf{R}^4$ correspond to anything? What of $\mathbf{R}^5$?

Definition

Let n be a natural number. We call the set $\mathbf{R}^n$ *n-dimensional space* (or *Euclidean n-space*, *real coordinate space*, *real Euclidean space*). We call elements of $\mathbf{R}^n$ *points*. We identify $\mathbf{R}^1$ with $\mathbf{R}$ in the obvious way.

We call the point associated with $x = (x_1, x_2, \ldots, x_n) \in \mathbf{R}^n$ with $x_i = 0$ for $1 \leqq i \leqq n$ the *origin*. We denote the origin by 0. Similarly, we denote the point x with $x_i = 1$ for all $i = 1, \ldots, n$ by 1.

Visualization

We can not visualize n-dimensional space. Thus, our intuition for it comes from real space (see Real Space).

Distance

A natural notion of distance for $\mathbf{R}^n$ generalizes that in $\mathbf{R}^2$ and $\mathbf{R}^3$. We define the *distance* (or *Euclidean distance*) between $(x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots, y_n) \in \mathbf{R}^n$ as

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2}.$$

Does this have the properties that distance has in the plane and in space? We discussed these properties It does. Denote the function which associates to $x, y \in \mathbf{R}^n$ their distance $d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$. So $d(x, y)$ is the distance between the points corresponding to x and y.

Proposition 1. *d is non-negative, symmetric, and the distance between two points is no larger than the sum of the distances with any third object.*[1]

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[1] Future editions will include an account.

Order

Let $x, y \in \mathbf{R}^n$. If $x_i < y_i$ for all $i = 1, \ldots, n$ then we say x is *less than* y. Likewise, if $x_i \leq y_i$ for all $i = 1, \ldots, n$ then we say $x \leq y$. Likewise for $>$ and $\geq$.

Notation

If $x \in \mathbf{R}^n$ is less than $y \in \mathbf{R}^n$ then we write $x < y$. Similarly for $x \leq y$, $x > y$ and $x \geq y$. Other notation in the literature for $\mathbf{R}^n$ includes E^n, which is a mnemonic for "euclidean."

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space

Real Line

Real Plane

Integral Line

Real Order

Geometry

Real Numbers

Rational Order

Lists

Rational Numbers

Set Numbers

Direct Products

Integer Arithmetic

Natural Fractions

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Equivalent Sets

Identity Elements

Natural Arithmetic

Family Unions and Intersections

Natural Powers

Integer Sums

Integer Order

Comparisons

Function Inverses

Operations

Natural Products

Integer Numbers

Orders

Function Images

Function Composites

Families

Natural Sums

Equivalence Relations

Converse Relations

Functions

Natural Order

Recursion Theorem

Relations

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Set Symmetric Differences

Partitions

Successor Sets

Intersection of Empty Set

Unordered Triples

Set Dualities

Set Unions and Intersections

Set Complements

Set Intersections

Pair Unions

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects