Why

We want to quantify the error of compressing a real-valued random variable.

Definition

Let $\mathcal{X}$ be a finite set and $q : \mathbf{R} \to \mathcal{X}$ a quantization (see Quantizations) of $\mathbf{R}$. Also, fix a probability space $(\Omega, \mathcal{A}, \mathbf{P})$ and a random variable $x : \Omega \to \mathbf{R}$.

The *compression* $\hat{x} : \Omega \to \mathcal{X}$ of x under q is $q \circ x$. A *distortion function* for x and $\hat{x}$ is a function

$$d : (\Omega \to \mathbf{R}) \times (\Omega \to \mathcal{X}) \to \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

Examples

The *expected mean-squared-error distortion* d_{mse} between x and $\hat{x}$ is

$$d_{\mathrm{mse}}(x, \hat{x}) = \mathbf{E}[(x - \hat{x})^2]$$

The *Kulback-Liebler distortion* d_{kld} defined by

$$d_{\mathrm{kld}}(x, \hat{x}) = \mathbf{E}[d_{\mathrm{kl}}(\mathbf{P}(y \in \cdot \mid x, \hat{x}) \mid \mathbf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.[1]

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[1] Future editions will clarify this sentence.