

## MONOTONE CLASSES

## Definition

The *limit* of an increasing sequence of sets is the family union of the sequence. The *limit* of a decreasing sequence of sets is the family intersection of the sequence.

A monotone limit of an sequence of sets is the limit of a monotone sequence.

A monotone class is a subset system in which monotone limits of monotone sequences of distinguished sets are distinguished. We call the distinguished sets a monotone class.

## **Notation**

Let A a non-empty set with partial order  $\leq$ . Let  $(A, \mathcal{A})$  be a subset space on A.

Let  $(A_n)_n$  be an increasing or decreasing sequence in  $\mathcal{A}$ . We denote the limit of  $(A_n)_n$  by  $\lim_n A_n$ .

If  $(A_n)_n$  is increasing,  $\lim_n A_n = \bigcup_n A_n$ . If  $(A_n)_n$  is decreasing,  $\lim_n A_n = \bigcap_n A_n$ .

If  $(A, \mathcal{A})$  is a monotone space, then for all monotone  $(A_n)_n$  in  $\mathcal{A}$ ,  $\lim_n A_n \in \mathcal{A}$ . In this case,  $\mathcal{A}$  is a monotone class.

