Why

We want to find a low-dimensional affine set into which we can project some high-dimensional data.

Problem

For $a \in \mathbf{R}^n$ and $U \in \mathbf{R}^{n \times k}$, the set $W(a, U) = \{a + Uz\}z \in \mathbf{R}^n$ is an affine set. Denote the projection of $x \in \mathbf{R}^n$ onto $W(a, U)$ by $\text{proj}_{W(a,U)}(x)$.

Given $x^{(1)}, \ldots, x^{(m)} \in \mathbf{R}^n$, and a dimension k, we want to choose a and U to minimize

$$\sum_{i=1}^{m} \|x - \text{proj}_{W(a,U)}(x)\|^2,$$

the sum of squares of the distance from x to its projection in $W(a, U)$.

Minimum Residual Affine Subspaces

Real Affine Set Representations

Affine Set Dimensions

Real Subspace Representations

Subspace Dimensions

Vector Space Dimensions

Vector Space Bases

Real Matrices with Orthonormal Columns

Linearly Dependent Vectors

Span

Affine Sets and Subspaces

Real Matrix-Matrix Products

Real Translates

Real Matrix-Vector Products

Real Subspaces

Vector as Matrices

Orthonormal Set of Real Vectors

Real Vector Projections

Linear Combinations

Real Vector Span

Real Matrices

Real Inner Product

Real Norm

Optimization Problems

Vector

Affine Sets

Real Linear Combinations

Linear Equations

N-Dimensional Lines

Real Vectors

Linear Functions

Plane Inner Product

Right Triangle Sides Relation

Extended Real Numbers

Plane Vector Sums and Differences

Space Vectors

N-Dimensional Space

Distance

Space Norm

Plane Norm

Real Square

Real Limits

Space Distance

Plane Distance

Real Arithmetic

Interval Length

Real Functions

Plane Vectors

Arrays

Real Multiplication Inverses

Real Products

Real Line

Intervals

Greatest Lower Bounds

Real Plane

Real Addition Inverses

Real Sequences

Square Numbers

Real Summation

Sequences

Natural Multiplicative Inverses

Natural Arithmetic

Real Sums

Square

Integral Unit

Real Order

Optimizers

Fields

Natural Products

Rational Sums

Real Numbers

Geometry

Rational Order

Direct Products

Set Numbers

Rings

Integer Arithmetic

Rational Numbers

Finite Sets

Inverse Elements

Function Inverses

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Integer Order

Comparison

Orders

Family Unions and Intersections

Natural Summation

Natural Equation

Family Operations

Element Function

Identity Elements

Arithmetic

Integer Sums

Equivalent Sets

Integer Numbers

Concrete Relations

Equations

Function

Natural Order

Function Inverse

Function Image

Operations

Equivalence Relations

Natural Powers

Natural Products

Functions

Natural Sums

Relations

Recursion Theorem

Peano Axioms

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Successor Sets

Unordered Triples

Set Unions and Intersections

Set Dualities

Set Symmetric Differences

Partitions

Intersection of Empty Set

Pair Union

Ordered Pairs

Set Complements

Set Intersection

Set Difference

Pair Intersection

Set Union

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects