Why

We can identify any linear functional $F : \mathbf{R}^n \to \mathbf{R}$ with a vector $y \in \mathbf{R}^n$ so that $F(x) = \langle x, y \rangle$. We generalize this result to complete inner product spaces.

Motivating result

The following is known as the *Riesz representation theorem* (or *Riesz-Fréchet representation theorem*, or *Riesz theorem*, or *Riesz-Fréchet theorem*).

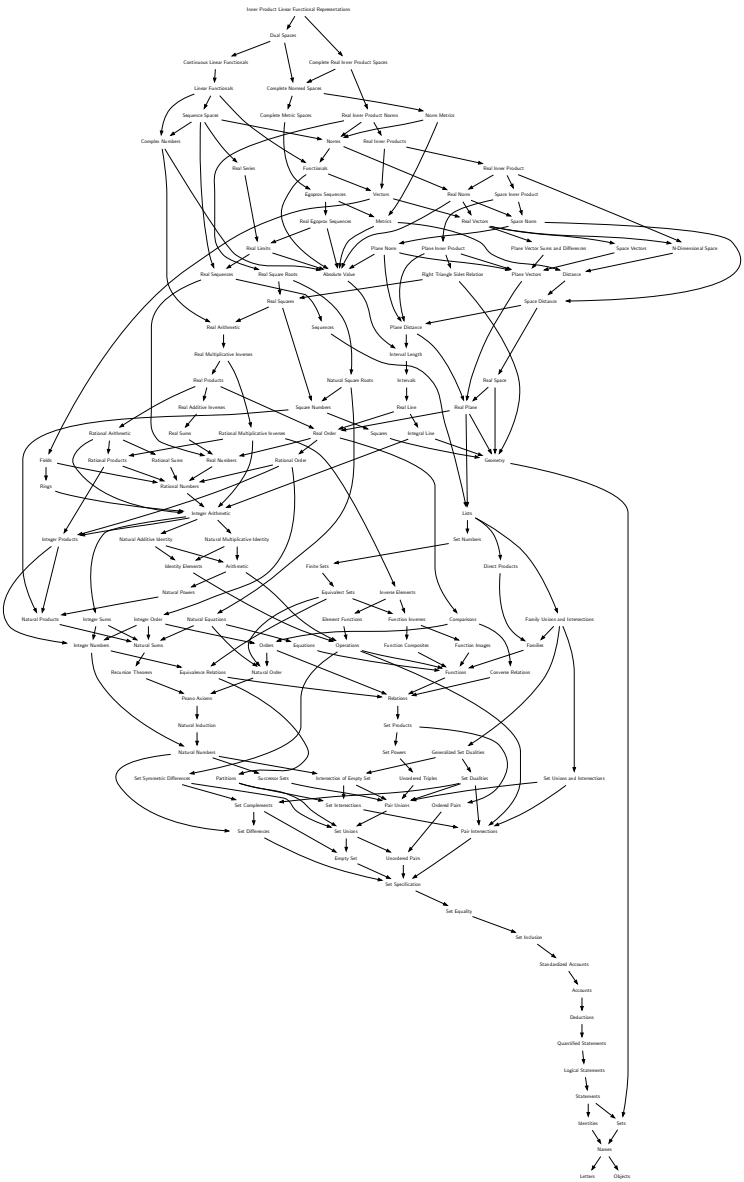
Proposition 1. *Let $((V, k), \langle \cdot, \cdot \rangle)$ be a complete inner product space and let $F : V \to k$ be a continuous linear functional on V. There exists a unique $y \in V$ so that*

$$F(x) = \langle x, y \rangle$$

for all $x \in V$. Moreover $\|y\| = \|F\|_*$.

Clearly $\mathbf{R}^n$ is a complete inner product space, and so this this theorem says the expected. We can identify linear functionals on $\mathbf{R}^n$ with elements (vectors) in $\mathbf{R}^n$.[1]

[1] Future editions will expand further.


Inner Product Linear Functional Representation
Dual Spaces
Continuous Linear Functionals
Complete Real Inner Product Spaces
Linear Functionals
Complete Normed Spaces
Sequence Spaces
Complete Metric Spaces
Real Inner Product Norm
Norm Metrics
Complex Numbers
Norm
Real Inner Products
Real Series
Functionals
Real Inner Product
Bounded Sequences
Vectors
Real Norm
Space Inner Product
Real Bounded Sequences
Metrics
Real Vectors
Space Norm
Real Limits
Plane Norm
Plane Inner Product
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Real Sequences
Real Square Roots
Absolute Value
Right Triangle Sides Relation
Plane Vectors
Distance
Real Squares
Space Distance
Real Arithmetic
Sequences
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Natural Square Roots
Intervals
Real Space
Real Addition Inverses
Square Numbers
Real Line
Real Plane
Rational Arithmetic
Real Sums
Rational Multiplicative Inverses
Real Order
Squares
Integral Line
Geometry
Fields
Rational Products
Rational Sums
Real Numbers
Rational Order
Rings
Rational Numbers
Integer Arithmetic
Lists
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Identity Elements
Arithmetic
Finite Sets
Direct Products
Natural Powers
Equivalent Sets
Inverse Elements
Natural Products
Integer Sums
Integer Order
Natural Equations
Element Functions
Function Inverses
Comparisons
Family Unions and Intersections
Integer Numbers
Natural Sums
Order
Equations
Operations
Function Composition
Function Images
Families
Recursive Theorem
Equivalence Relations
Natural Order
Functions
Converse Relations
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects