



Definition

A conditional distribution (density) $q : Y \times X \rightarrow \mathbf{R}$ is *functionally parametrizable* if there exists a function $f : X \rightarrow \Theta$ and distribution (density) family $\{p^{(\theta)} : Y \rightarrow \mathbf{R}\}_{\theta \in \Theta}$ satisfying $q(y, x) \equiv p^{(f(x))}(\gamma)$ for all $x \in X$ and $y \in Y$.

In this case we call f the *parameterizer* and we call $\{p^{(\theta)}\}_{\theta \in \Theta}$ the *parameterized family*. A *parameterized conditional distribution* is an ordered pair whose first coordinate is a function from X to Θ and whose second coordinate is a family of distributions on X with parameter set Θ . For a particular choice of parameterizer and family, it induces a conditional distribution.

Since all conditional distributions are functionally parametrizable (consider $\{q(\cdot, \xi)\}_{\xi \in X}$ with parameters X and identity parameterizer), we are interested in parameterizers and parameterized families that are simple. Said differently, we are interested in approximating a conditional distribution by selecting an appropriate parameterizer and parameterized family.

If $\{f_\phi : X \rightarrow \Theta\}_{\phi \in \Phi}$ is a family of functions and $\{q^{(\theta)}\}$ is a family of distributions, then $\{p^{(\phi)} : X \times Z \rightarrow \mathbf{R}\}_{\phi}$ defined by $p^{(\phi)}(\cdot, \zeta) \equiv q^{f_\phi(\zeta)}$ is a conditional distribution family called a *functionally parameterized conditional distribution family*. In other words, by selecting some parameters ϕ , we induce a conditional distribution $p^{(\phi)} : X \times Z \rightarrow \mathbf{R}$.

We similarly define *parameterized conditional densities* and *functionally parameterized conditional density families*.

Basic example

Let $Z = \{1, 2\}$ and $X = \mathbf{R}$. Let $f : \{1, 2\} \rightarrow \mathbf{R} \times \mathbf{R}_+$ be defined by $f(1) = (\mu_1, \sigma_1)$ and $f(2) = (\mu_2, \sigma_2)$. Let $\{g^{(\theta)}\}_\theta$ be the normal family. Then $(f, \{g^{(\theta)}\})$ is a *functionally parameterized conditional density*.¹

¹Future editions will modify.

