Why

We want to model the progressive fill-in during Gauss elimination with graphs.

Definition

Let $G = ((V, E), \sigma)$ be an ordered undirected graph with $|V| = n$. Define $E_0 = E$. The *elimination edge sequence* of G is a sequence $(E_1, \ldots, E_{n-1})$ defined by

$$E_i = E_{i-1} \cup$$
$$\{\{v, w\} \mid v \succ \sigma(i), w \succ \sigma(i), \text{ and } \{\sigma(i), w\} \in E_{i-1}\}$$

for $i = 1, \ldots, n-1$. The difference between E_{i-1} and E can be described by saying that the higher neighborhood of the intermediate graph $((V, E_{i-1}), \sigma)$ is made complete by the addition of edges between all non-adjacent vertices.

The *elimination graph* of G is the graph $((V, E_{n-1}), \sigma)$.

Elimination Graphs

Pivoted Row Reductions

Ordinary Row Reductions

Linear System Row Reductions

Linear Equation Solutions

Linear Equations

Linear Combinations

Vectors

Arrays

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Line

Real Plane

Real Space

Ordered Undirected Graphs

Real Summation

Integral Line

Real Order

Geometry

Fields

Sequences

Undirected Graphs

Real Numbers

Rational Numbers

Groups

Finite Sets

Set Numbers

Integer Arithmetic

Integer Additive Inverses

Direct Products

Natural Summation

Natural Multiplication Identity

Natural Additive Identity

Integer Sums

Inverse Elements

Equivalent Sets

Arithmetic

Identity Elements

Family Operations

Element Functions

Natural Equations

Natural Exponents

Integer Products

Algebra

Function Inverses

Family Unions and Intersections

Equations

Natural Products

Natural Order

Integer Numbers

Operations

Function Images

Function Composites

Families

Natural Sums

Functions

Equivalence Relations

Recursion Theorem

Relations

Peano Axioms

Cartesian Products

Natural Induction

Natural Numbers

Set Powers

Generalized Set Dualities

Successor Sets

Unordered Triples

Intersection of Empty Set

Partitions

Set Dualities

Set Unions and Intersections

Ordered Pairs

Pair Unions

Set Intersections

Set Complements

Set Differences

Set Unions

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects