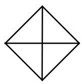


Definition

Two subspaces $S, T \subset \mathbf{R}^n$ are *orthogonal* if

$$x^\top y = 0 \text{ for all } x \in S, y \in T.$$

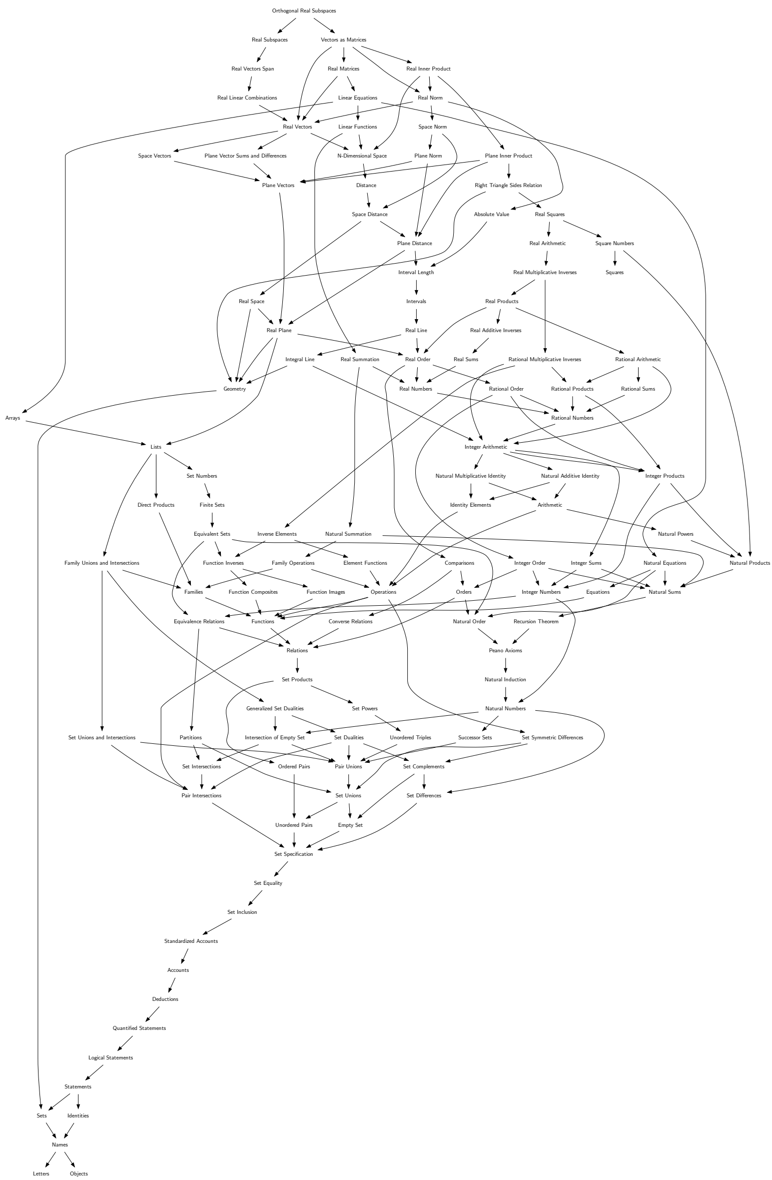
For any set $S \subset \mathbf{R}^n$ (not necessarily a subspace), the *orthogonal complement* of S is the set

$$S^\perp = \{x \in \mathbf{R}^n \mid x^\top y = 0 \text{ for all } y \in S\}.$$

$S^\perp$ is the set of all vectors which are orthogonal to every vector in S.

Orthgonal complement is a subspace

Notice that $S^\perp$ is always a subspace. If $x \in S^\perp$, then $x^\top y = 0$ for all $y \in S$. So then $(\alpha x)^\top y = \alpha(x^\top y) = 0$ for all $\alpha \in \mathbf{R}$ and $y \in S$. We conclude $\alpha x \in S^\perp$ for all $\alpha \in \mathbf{R}$. In other words, $S^\perp$ is closed under scalar multiplication. If $x, z \in S^\perp$, then $(x + z)^\top y = x^\top y + z^\top y = 0 + 0 = 0$. We conclude that $x + z \in S^\perp$ for all $x, z \in S^\perp$. In other words, $S^\perp$ is closed under vector addition. Consequently, $S^\perp$ is a subspace.



Orthogonal Real Subspaces
Real Subspaces
Vectors as Matrices
Real Vectors Span
Real Matrices
Real Inner Product
Real Linear Combinations
Linear Equations
Real Norm
Real Vectors
Linear Functions
Space Norm
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Norm
Plane Inner Product
Plane Vectors
Distance
Right Triangle Sides Relation
Space Distance
Absolute Value
Real Squares
Plane Distance
Real Arithmetic
Square Numbers
Interval Length
Real Multiplicative Inverses
Squares
Real Space
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Summation
Real Order
Real Sums
Rational Multiplicative Inverses
Rational Arithmetic
Geometry
Real Numbers
Rational Order
Rational Products
Rational Sums
Arrays
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Inverse Elements
Natural Summation
Natural Powers
Family Unions and Intersections
Function Inverses
Family Operations
Element Functions
Comparisons
Integer Order
Integer Sums
Natural Equations
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Equations
Natural Sums
Equivalence Relations
Functions
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects