Why

Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\operatorname{tr} : \mathbf{R}^{n \times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n \times n}$. Then

$$\operatorname{tr} A = \sum_{i=1}^{n} A_{ii}.$$

Properties

Prop. 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n \times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\operatorname{tr} C = \sum_{i=1}^{n} C_{ii} = \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii}$$

$$= \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii}$$

$$= \alpha \operatorname{tr} A + \beta \operatorname{tr} B.$$

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[1]Future editions will include, in the genetic tradition.

$\square$

Prop. 2. *Let $A, B \in \mathbf{R}^{n \times n}$.*

$$\operatorname{tr}(AB) = \operatorname{tr}(BA)$$

[2]

In other words, "matrices commute under the trace operator."

Prop. 3. *Let $A \in \mathbf{R}^{n \times n}$. Then $\operatorname{tr} A = \operatorname{tr} A^{\top}$.*

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[2]Future editions will include an account.

Matrix Trace

Matrix-Matrix Products Real Matrices

Matrix-Vector Products Matrix Transpose

Linear Combinations Matrices

Vectors

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space Real Plane Real Line

Sequences Real Order Integral Line

Direct Products Geometry Real Numbers Fields

Set Numbers Rational Numbers Groups

Finite Sets Integer Arithmetic Integer Additive Inverse

Natural Additive Identity Natural Multiplicative Identity Inverse Elements Integer Sums

Equivalent Sets Identity Elements Arithmetic Element Functions

Family Unions and Intersections Algebras Function Inverses Natural Powers Integer Products

Families Comparisons Operations Function Composites Function Images Natural Products Integer Numbers

Orders Converse Relations Functions Equivalence Relations Natural Sums

Natural Order Relations Recursion Theorem

Peano Axioms

Set Products Natural Induction

Generalized Set Dualities Set Powers Natural Numbers

Set Unions and Intersections Set Dualities Unordered Triples Successor Sets Intersection of Empty Set Partitions

Set Complements Ordered Pairs Pair Unions Set Intersections

Pair Intersections Set Differences Set Unions

Empty Set Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects

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