Rational Numbers

Why

We want fractions.[1]

Definition

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We say that the elements (a, b) and (c, d) of this set are *rational equivalent* if $ad = bc$. Briefly, the intuition is that (a, b) represents a over b, or in the usual notation "a/b". So this equivalence relation says these two are the same if $a/b = c/d$ or else $ad = bc$.

Proposition 1. *Rational equivalence is an equivalence relation on* $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$.

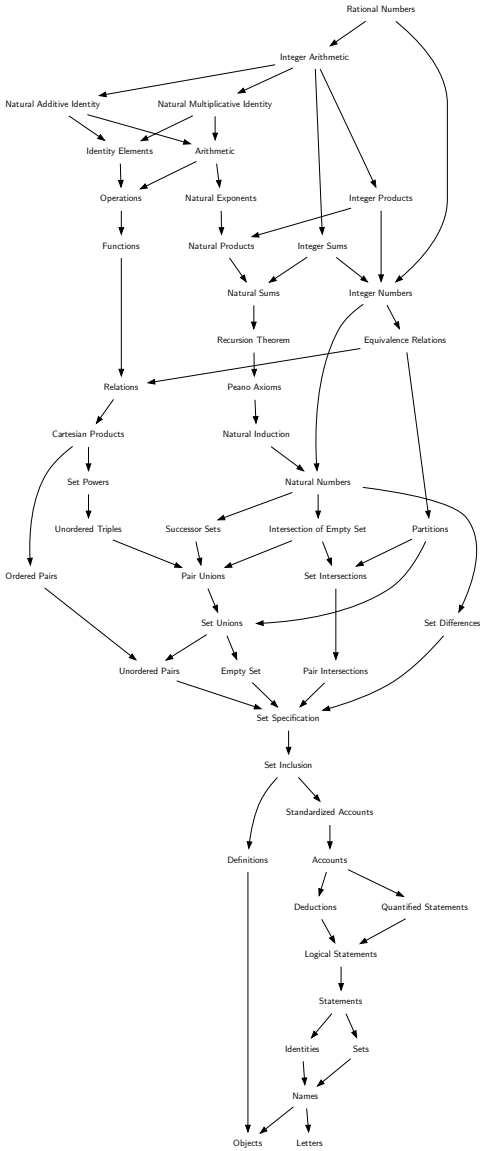
We define the *set of rational numbers* to be the set of equivalence classes (see *Equivalence Classes*) under ratioanl equivalence on $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We call an element of the set of ratioanl numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

Notation

We denote the set of rationals by $\mathbf{Q}$.[2] If we denote rational equivalence by $\sim$ then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$.

[1] This why will be expanded in future editions.

[2] From what we can tell so far, $\mathbf{Q}$ is a mnemonic for "quantity", from the latin "quantitas".


Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Identity Elements
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Natural Sums
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