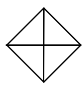


Why

If we treat the parameters of a linear function as a random variable, an inductor for the predictor is equivalent to an estimator for the parameters.[1]

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $x : \Omega \to \mathsf{R}^d$. Define $g : \Omega \to (\mathsf{R}^d \to \mathsf{R})$ by $g(\omega)(a) = a^\top x(\omega)$, for $a \in \mathsf{R}^d$. In other words, for each outcome $\omega \in \Omega$, $g_\omega : \mathsf{R}^d \to \mathsf{R}$ is a linear function with parameters $x(\omega)$. g_ω is the function of interest.

Let $a^1, \dots, a^n \in \mathsf{R}^d$ a dataset with data matrix $A \in \mathsf{R}^{n \times d}$. Let $e : \Omega \to \mathsf{R}^n$ independent of x, and define $y : \Omega \to \mathsf{R}^n$ by

$$y = Ax + e.$$

In other words, $y_i = x^\top a^i + e_i$.

We call (x, A, e) a *probabilistic linear model*. Other terms include *linear model*, *statistical linear model*, *linear regression model*, *bayesian linear regression*, and *bayesian analysis of the linear model*.[2] We call x the parameters, A a *design*, e the *error* or *noise* vector, and y the *observation* vector.

One may want an estimator for the parameters x in terms of y or one may be modeling the function g and want to predict $g(a)$ for $a \in A$ not in the dataset.

[1] Future editions will offer further discussion.

[2] The word bayesian is in reference to treating the object of interest—x—as a random variable.

Inconsistency

In this model, the dataset is assumed to be inconsistent as a result of the random errors. In these cases, the error vector e may model a variety of sources of error ranging from inaccuracies in the measurements (or measurement devices) to systematic errors from the "inapproriateness" of the use of a linear predictor.[3] In this case the linear part is sometimes called the *deterministic effect* of the response on the input $a \in A$.

Moment assumptions

One route to be more specific about the underlying distribution of the random vector is give its mean and variance. It is common to give the mean of $\mathrm{E}(w)$

Mean and variance

Proposition 1. $\mathrm{E}(y) = A\,\mathrm{E}(x) + \mathrm{E}(w)$[4]

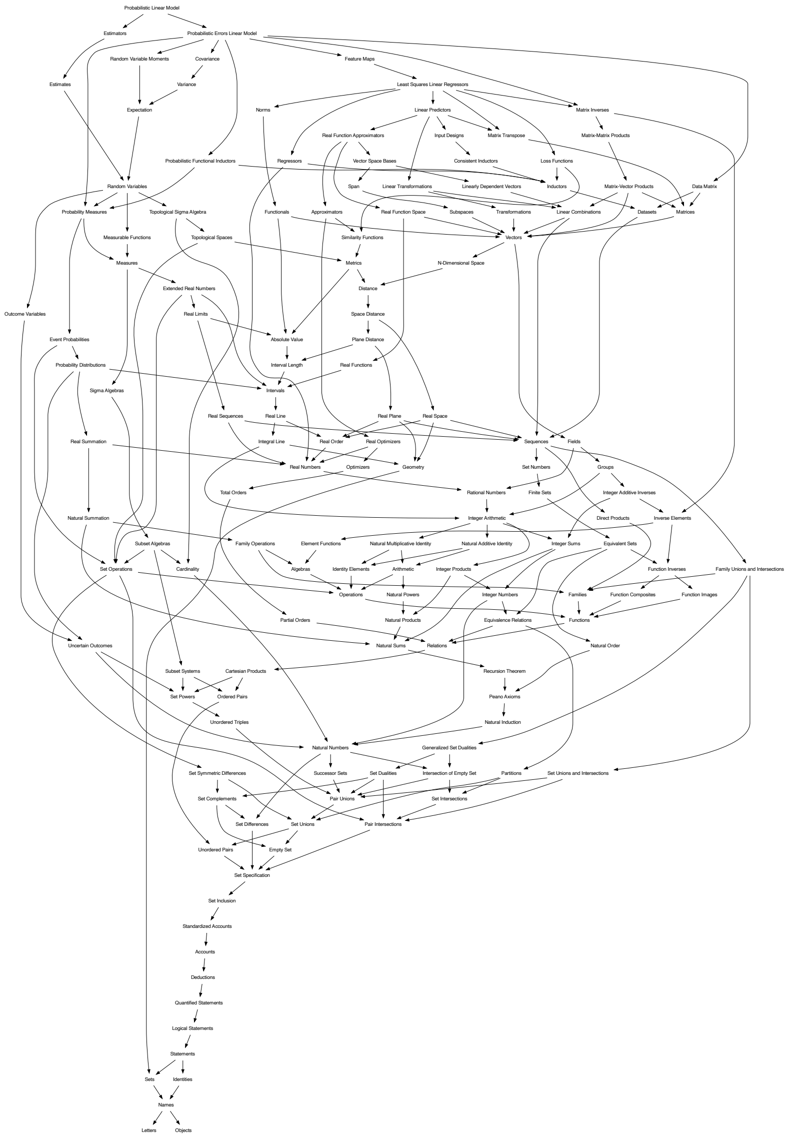
Proposition 2. $\mathrm{cov}((x, y)) = A\,\mathrm{cov}(x)A^\top + \mathrm{cov}\,e$[5]

A simple consequence is that, if x and

[3] Future editions will clarify and may excise this sentence.

[4] By linearity. Full account in future editions.

[5] Full account in future editions.


Probabilistic Linear Model
Estimators
Probabilistic Errors Linear Model
Random Variable Moments
Covariance
Feature Maps
Estimates
Variance
Least Squares Linear Regressors
Expectation
Norms
Linear Predictors
Matrix Inverses
Real Function Approximators
Input Designs
Matrix Transpose
Matrix-Matrix Products
Probabilistic Functional Inductors
Regressors
Vector Space Bases
Consistent Inductors
Loss Functions
Random Variables
Span
Linear Transformations
Linearly Dependent Vectors
Inductors
Matrix-Vector Products
Data Matrix
Probability Measures
Topological Sigma Algebra
Functionals
Approximators
Real Function Space
Subspaces
Transformations
Linear Combinations
Datasets
Matrices
Measurable Functions
Topological Spaces
Similarity Functions
Vectors
Measures
Metrics
N-Dimensional Space
Extended Real Numbers
Distance
Outcome Variables
Real Limits
Space Distance
Event Probabilities
Absolute Value
Plane Distance
Probability Distributions
Interval Length
Real Functions
Sigma Algebras
Intervals
Real Sequences
Real Line
Real Plane
Real Space
Real Summation
Integral Line
Real Order
Real Optimizers
Sequences
Fields
Real Numbers
Optimizers
Geometry
Set Numbers
Groups
Total Orders
Rational Numbers
Finite Sets
Integer Additive Inverses
Natural Summation
Integer Arithmetic
Direct Products
Inverse Elements
Subset Algebras
Family Operations
Element Functions
Natural Multiplicative Identity
Natural Additive Identity
Integer Sums
Equivalent Sets
Set Operations
Cardinality
Algebras
Identity Elements
Arithmetic
Integer Products
Function Inverses
Family Unions and Intersections
Operations
Natural Powers
Integer Numbers
Families
Function Composites
Function Images
Partial Orders
Natural Products
Equivalence Relations
Functions
Uncertain Outcomes
Natural Sums
Relations
Natural Order
Subset Systems
Cartesian Products
Recursion Theorem
Set Powers
Ordered Pairs
Peano Axioms
Unordered Triples
Natural Induction
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects