

MINIMUM MEAN SQUARED ERROR ESTIMATES

Why

What is the best estimate for a random variable if we consider the square error?

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $x : \Omega \to \mathbf{R}$ a random variable. A minimum mean squared error estimate or MMSE estimate or least square estimate is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E}(x - \xi)^2$.

Proposition 1. There is a unique MMSE estimate and it is given by $\mathbf{E}(x)$.

Vector case

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $y : \Omega \to \mathbf{R}^n$ a random variable.

A minimum mean squared error estimator or MMSE estimator or least square estimator is a value $\xi \in \mathbf{R}$ which minimizes $\mathbf{E} ||x - \xi||^2$.

Proposition 2. There is a unique MMSE estimator and it is given by $\mathbf{E}(y)$.

¹Future editions might collapse this into the previous case.

