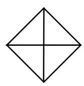


Why

We want a notion of reversing functions.

Definition

An *identity function* is a relation on a set which is functional and reflexive. It associates each element in the set with itself. There is only one identity function associated to each set.

Consider two functions for which the codomain of the first function is the domain of the second function and the codomain of the second function is the domain of the first function. These functions are *inverse functions* if the composition of the second with the first is the identity function on the first's domain and the composition of the first with the second is the identity function on the second's domain.

In this case we say that the second function is an *inverse* of the second, and vice versa. When an inverse exists, it is unique, so we refer to the *inverse* of a function. We call the first function *invertible*. Other names for an invertible function include *bijection*.

Notation

Let A a non-empty set. We denote the identity function on A by id_A, read aloud as "identity on A." id_A maps A onto A.

Let A, B be non-empty sets. Let $f : A \to B$ and $g : B \to A$

be functions. f and g are inverse functions if $g \circ f = \mathrm{id}_A$ and $f \circ g = \mathrm{id}_B$.

The Inverse

We discuss existence and uniqueness of an inverse.

Proposition **1.** *Let $f : A \to B$, $g : B \to A$, and $h : B \to A$.*

If g and h are both inverse functions of f, then $g = h$.

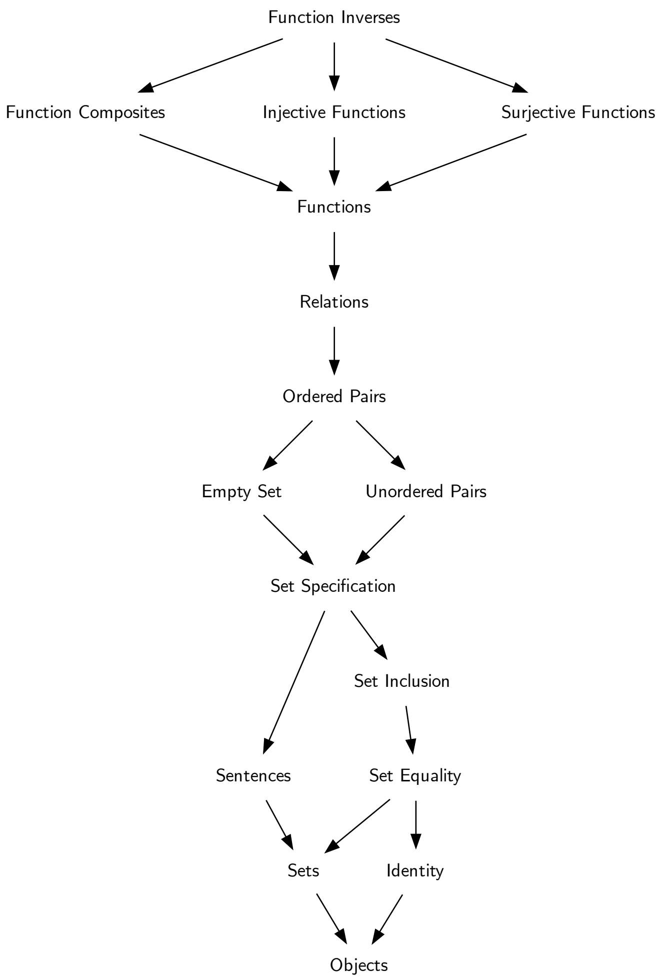
Proof. $\square$

Proposition **2.** *If a function is one-to-one and onto, it has an inverse.*

Proof. $\square$

Inverse Images

The *inverse image* of a codomain set under a function is the set of elements which map to elements of the codomain set under the function. We denote the inverse image of $D \subset B$ by $f^{-1}(D)$, read aloud as "f inverse D."


Function Inverses
Function Composites
Injective Functions
Surjective Functions
Functions
Relations
Ordered Pairs
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Sentences
Set Equality
Sets
Identity
Objects