Monotone Sequences

1 Why

If the base set of a sequence has a partial order, then we can discuss its relation to the order of sequence.

2 Definition

A sequence on a partially ordered set is *non-decreasing* if whenever a first index precedes a second index the element associated with the first index precedes the element associated with the second element. A sequence on a partially ordered set is *increasing* if it is non-decreasing and no two elements are the same.

A sequence on a partially ordered set is *non-increasing* if whenever a first index precedes a second index the element associated with the first index succedes the element associated with the second element. A sequence on a partially ordered set is *decreasing* if it is non-increasing and no two elements are the same.

A sequence on a partially ordered set is *monotone* if it is non-decreasing, or non-increasing. An increasing sequence is non-decreasing. A decreasing sequences is non-increasing. A sequence on a partially ordered set is *strictly monotone* if it is decreasing, or increasing.

2.1 Notation

Let A a non-empty set with partial order $\preceq$. Let $(a_n)_n$ a sequence in A.

The sequence is non-decreasing if $n \leq m \implies a_n \preceq a_m$, and increasing if $n < m \implies a_n \prec a_m$. The sequence is non-increasing if $n \leq m \implies a_n \succeq a_m$, and decreasing if $n < m \implies a_n > a_m$.

3 Examples

Example 1. *Let A a non-empty set and $(A_n)_n$ a sequence of sets in A^*. Partially order elements of A^* by the relation contained in.*