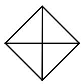


Why

Vectors can be identified with matrices of width 1.

Canonical identification

We identify $\mathbf{R}^n$ with $\mathbf{R}^{n\times 1}$ in the obvious way. For this reason, we call $x \in \mathbf{R}^{n\times 1}$ (meaning $x \in \mathbf{R}^n$) a *column vector.*

For the reasons that we identify $\mathbf{R}^n$ with $\mathbf{R}^{n\times 1}$, we write the vector $a = (a_1, a_2, a_3) \in \mathbf{R}^3$ as

$$\begin{bmatrix} a\_1 \\ a\_2 \\ a\_3 \end{bmatrix} \text{ or } \begin{pmatrix} a\_1 \\ a\_2 \\ a\_3 \end{pmatrix}.$$

We could as easily also identify $\mathbf{R}^n$ with $\mathbf{R}^{1\times n}$. We avoid this convention. However, by analogy with the language "column vector," we refer to the *matrix* $y \in \mathbf{R}^{1\times n}$ as a *row vector.*

Matrix transpose

We frequently move from $\mathbf{R}^{n\times 1}$ and $\mathbf{R}^{1\times n}$. If $a \in \mathbf{R}^{n\times 1}$, we denote $b \in \mathbf{R}^{1\times n}$ defined by $b_i = a_i$ by $a^\top$.

More generally, given a matrix $A \in \mathbf{R}^{m\times n}$, we denote the matrix $B \in \mathbf{R}^{m\times n}$ defined by $B_{ij} = A_{ji}$ by $A^\top$. Notice that the entries of i and j have swapped. We call the matrix B the *transpose* of A, and similarly call $a^\top$ the *transpose* of the vector a. Clearly, $(A^\top)^\top = A$, which includes $(a^\top)^\top = a$.

Reals as vectors

There is a similar, and similarly obvious, identification of scalars $a \in \mathbf{R}$ with the 1-vectors $\mathbf{R}^1$ (and so with the 1 by 1 matrices $\mathbf{R}^{1\times 1}$). Given our definition of matrix-vector products, if we identify $a \in \mathbf{R}$ with $A \in \mathbf{R}^{1\times 1}$ where $A_{11} = a$, then $Ax = ax$.

Familiar concepts, new notation

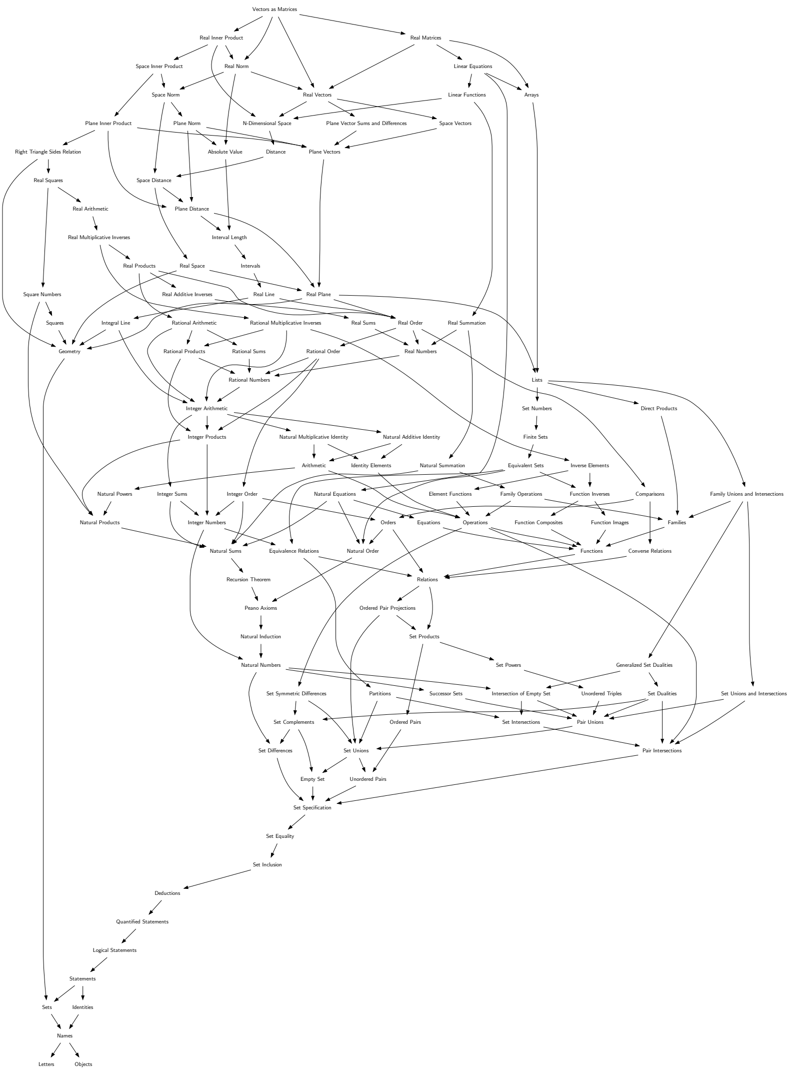
These identifications and the notation of transposition give allow us to write several familiar concepts in a compact notation. We write the norm as

$$\|x\| = \sqrt{x\_1^2 + x\_2^2 + \cdots + x\_n^2} = \sqrt{x^\top x}.$$

We write the inner product as

$$\langle x, y \rangle = x\_1y\_1 + x\_2y\_2 + \cdots + x\_ny\_n = x^\top y.$$

We express the symmetry of the inner product by $x^\top y = y^\top x$.



Vectors as Matrices
Real Inner Product
Real Matrices
Space Inner Product
Real Norm
Linear Equations
Space Norm
Real Vectors
Linear Functions
Arrays
Plane Inner Product
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Right Triangle Sides Relation
Absolute Value
Distance
Plane Vectors
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Square Numbers
Real Additive Inverses
Real Line
Real Plane
Squares
Integral Line
Rational Arithmetic
Rational Multiplicative Inverses
Real Sums
Real Order
Real Summation
Geometry
Rational Products
Rational Sums
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Natural Summation
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Integer Order
Natural Equations
Element Functions
Family Operations
Function Inverses
Comparisons
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Equations
Operations
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Natural Order
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Ordered Pairs
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects