Why

[1]

Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

Notation

Let $(X, \mathcal{A}, \mathsf{P})$ be a probability space. Let $(Y, \mathcal{B})$ a measurable space. Then a random variable is a measurable function $f : X \to Y$.

Some authors denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. In this case, the base probability space is denoted by Ω, a mnemonic for "outcomes." Let $(\Omega, \mathcal{A}, \mathsf{P})$ be a probability space. Let $X : \Omega \to \mathsf{R}$ be measurable. Then X is a real-valued random variable.

Some authors use notation for the probability of particular, common sets. Although the authors often use regular parentheses, we will use [,] for precision. Let $A \in \mathcal{B}(R)$. Let

[1] Future editions will include this.

$\mathbf{P}[X \in A]$ denote $\mathbf{P}(X^{-1}(A))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

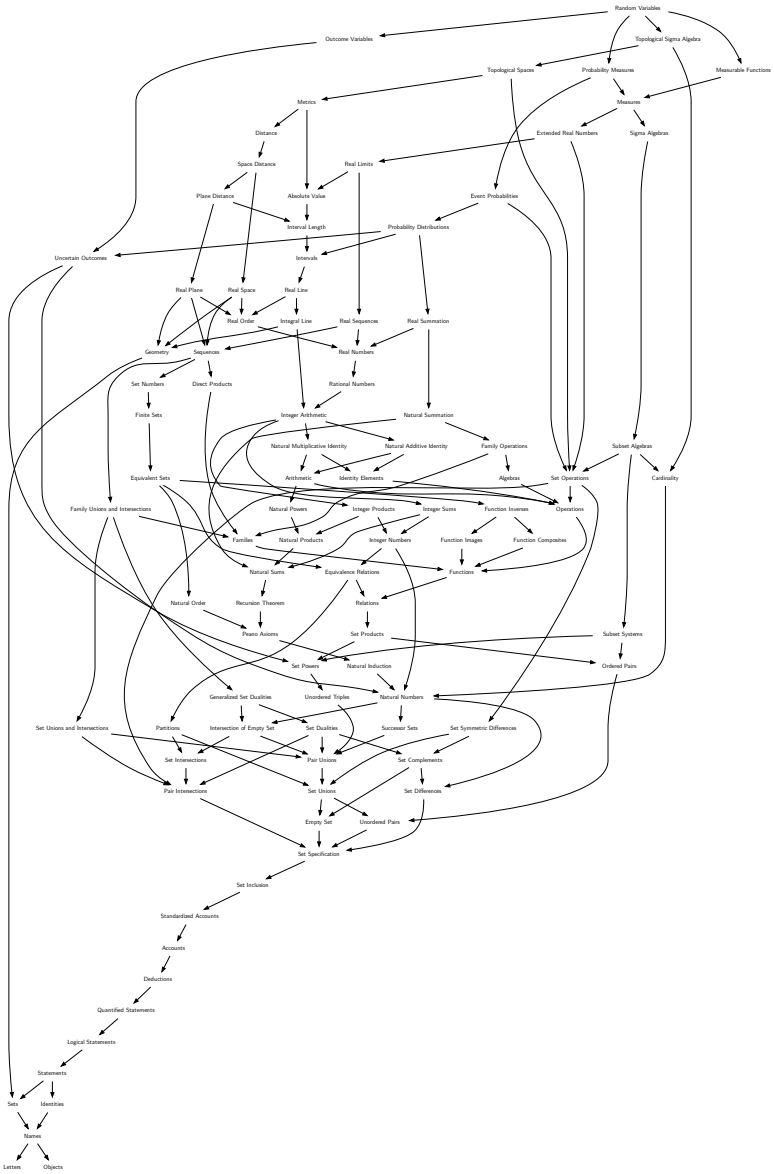
As we mentioned, some authors use $P(X \in A)$ for $P[X \in A]$, which is mostly harmless.

Next, let $Y : \Omega \to R$ a measurable function and let $B \in \mathcal{B}(R)$. Similar to the above, let $\mathbf{P}[X \in A, Y \in B]$ denote $\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B))$. These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \ldots, X_n : \Omega \to \mathbf{R}$,

$$\mathbf{P}[X_1 \in A_1, \ldots, X_n \in A_n] = \mathbf{P}(\cap_{i=1}^n X_i^{-1}(A_i)).$$


Random Variables
Outcome Variables
Topological Sigma Algebra
Topological Spaces
Probability Measures
Measurable Functions
Metrics
Measures
Distances
Extended Real Numbers
Sigma Algebras
Space Distance
Real Limits
Plane Distance
Absolute Value
Event Probabilities
Interval Length
Probability Distributions
Uncertain Outcomes
Intervals
Real Plane
Real Space
Real Line
Real Order
Integral Line
Real Sequences
Real Summation
Geometry
Sequences
Real Numbers
Set Numbers
Direct Products
Rational Numbers
Finite Sets
Integer Arithmetic
Natural Summation
Natural Multiplicative Identity
Natural Additive Identity
Family Operations
Subset Algebras
Arithmetic
Identity Elements
Algebras
Set Operations
Cardinality
Equivalent Sets
Natural Powers
Integer Products
Integer Sums
Function Inverses
Operations
Family Unions and Intersections
Families
Natural Products
Integer Numbers
Function Images
Function Composites
Natural Sums
Equivalence Relations
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Subset Systems
Set Powers
Natural Induction
Ordered Pairs
Generalized Set Dualities
Unordered Triples
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects