Why

We want to combine two groups.[1]

Defining result

Proposition 1. *For each natural number m, there exists a function $s_m : \omega \to \omega$ which satisfies*

$$s_m(0) = m \quad and \quad s_m(n^+) = (s_m(n))^+$$

for every natural number n.

Proof. The proof uses the recursion theorem (see Recursion Theorem).[2]

$\square$

Let m and n be natural numbers. The value $s_m(n)$ is the *sum* of m with n.

Notation

We denote the sum $s_m(n)$ by $m + n$.

Properties

The properties of sums are direct applications of the principle of mathematical induction (see Natural Induction).[3]

Proposition 2 (Associative)**.** *Let k, m, and n be natural numbers. Then*

$$(k + m) + n = k + (m + n).$$

Proposition 3 (Commutative)**.** *Let m and n be natural numbers. Then*

$$m + n = n + m.$$

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[1]Future editions will change this section.
[2]Future editions will give the entire account.
[3]Future editions will include the accounts.

Relation to addition

Proposition 4 (Distributive)**.** *Let k, m, and n be natural numbers. Then*

$$k \cdot (m + n) = (k \cdot m) + (k \cdot n).$$

Natural Sums

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Recursion Theorem

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Peano Axioms

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Natural Induction

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Natural Numbers

Successor Sets — Intersection of Empty Set — Set Differences

Pair Unions

Set Unions — Set Intersections

Unordered Pairs — Empty Set — Pair Intersections

Set Specification

↓

Set Equality

↓

Set Inclusion

↓

Deductions

↓

Quantified Statements

↓

Logical Statements

↓

Statements

Identities — Sets

Names

Letters — Objects