



## Why

Can flashlights speak?

Speech can be made to correspond to words. And words are sequences of letters. So, roughly, can we communicate letters with a flashlight? No sounds allowed.

## Discussion

Well, what can we do with a flashlight? Turn it on and off. So we can show blinks of light. Here's a thought: One blink means  $a$ . Two blinks means  $b$ . Three blinks means  $c$ . Four blinks means  $d$ . And so on.

Here's the rub: how do we know when one letter stops, and another begins. Put another way, does 7 blinks mean  $(g)$ , or  $(b, a, d)$ , since  $2 + 1 + 4 = 7$ , or  $(c, d)$ , since  $3 + 4 = 7$ , or  $(d, c)$  and so on. If we change the game, maybe we can change the color of the flashlight. Or give a half-flash. Is a half-flash a "quick" flash, or is it a flash in which we have covered up half of the end of the flashlight?

The first question is a clue. The duration of the blink. How about the duration between blinks? A one second pause in between blinks for a letter. A three second pause for blinks in between letters. A five second pause for a space, to indicate a change of a word? Or an additional symbol, a "space", in our alphabet, communicated with 27 (!) blinks?

## Definition

Let  $\star$  represent a blink. The *flash code*<sup>1</sup> for the alphabet  $A$  (the latin lower case letters) is the function  $f : A \rightarrow \mathcal{S}(\{\star\})$  defined by  $f(a) = \star$ ,  $f(b) = \star\star$ ,  $f(c) = \star\star\star$ ,  $f(d) = \star\star\star\star$  and so on. We call  $f(a)$  the *code word* of  $a \in A$ . It is a sequence of blinks.

The idea is that if we give someone the *code word*  $f(a)$  for a letter of our alphabet  $a \in A$ , they will be able to tell us  $a$ . In the language of functions, we want  $f$  to have an inverse. Clearly,  $f$  is invertible.

It is natural to extend the flash code for  $A$  to strings of  $A$ . Let  $\square$  represent a long pause. Then we encode a sequence  $x \in \mathcal{S}(A)$  by applying  $f$  to each element of  $x$  in turn, and including a  $\square$  in between. The *flash code* for the strings  $\mathcal{S}(A)$  is the function  $g : \mathcal{S}(A) \rightarrow \mathcal{S}(\{\star, \square\})$  defined recursively, as  $g(x) = f(x'_1)$  for length one strings  $x'$ ,  $g(x'') = f(x'_1)\square f(x_2)$  and for length  $n$  strings  $x$  by  $f(x_1)\square g(x_{2:n})$ . For example, we encode the word (string)  $(b, a, d)$  as

$$\star\star\square\star\square\star\star\star\star \tag{1}$$

Now suppose we are given a sequence as shown in (1). Well, so long as  $f$  is invertible, we look can decode the codewords (the blinks, in this case) in between the squares and recover the word.

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<sup>1</sup>The etymology of code is from the Latin codex, in the legal tradition of a list of statutes. We may well use the term correspondence, or function, but code is standard.

