Metrics

Why

We want to talk about a set with a prescribed quantitative degree of closeness (or distance) between its elements.

Definition

The correspondences which serve as a degree of closeness, or measure of distance, must satisify our notions of distances previously developed.

A function on ordered pairs which does not depend on the order of the elements so considered is *symmetric*. A function into the real numbers which takes only non-negative values is *non-negative*. A repeated pair is an ordered pair of the same element twice. A function which satisfies a triangle inequality for any three elements is *triangularly transitive*.

A *metric* (or *distance function*) is a function on ordered pairs of elements of a set which is symmetric, non-negative, zero only on repeated pairs, and triangularly transitive. A *metric space* is an ordered pair: a nonempty set with a metric on the set.

In a metric space, we say that one pair of objects is *closer* together if the metric of the first pair is smaller than the metric value of the second pair.

Notice that a set can be made into different metric spaces by using different metrics.

Notation

Let A be a set and let R be the set of real numbers. We commonly denote a metric by the letter d, as a mnemonic for "distance." Let $d : A \times A \to R$. Then d is a metric if:

1. it is non-negative, which we tend to denote by

$$d(a, b) \geq 0, \quad \forall a, b \in A.$$

2. it is 0 only on repeated pairs, which we tend to denote by

$$d(a, b) = 0 \Leftrightarrow a = b, \quad \forall a, b \in A.$$

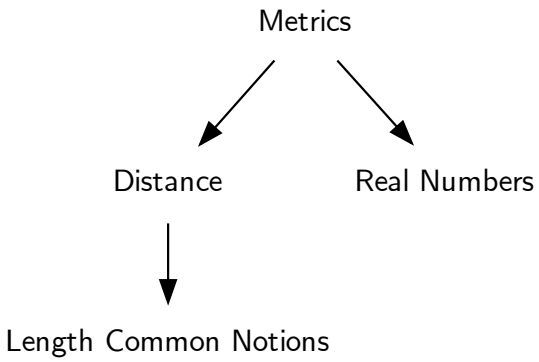
3. it is symmetric, which we tend to denote by:

$$d(a, b) = d(b, a), \quad \forall a, b \in A.$$

4. it is triangularly transitive, which we tend to denote by

$$d(a, b) \leq d(a, c) + d(c, b), \quad \forall a, b, c \in A.$$

As usual, we denote the metric space of A with d by (A, d).


Metrics
Distance
Real Numbers
Length Common Notions