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Probability Density Function

1 Why

TODO

2 Definition

A *probability density function* of a real-valued random variable is a measurable function from the real numbers to real numbers which is (a) non-negative and (b) the integral of the function over a cover-length-measurable set with respect to the cover measure to the cover length is the probability that the random variable takes value in that set. We often abbreviate probability density function as *p.d.f*, read aloud as "pdf".

2.1 Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space. Let f be a real-valued random variable on X. Let λ denote the cover length. A function $g : R \to R$ is a probability density of f if $g \geq 0$ and of f if

$$\mu(\{x \in X \mid f(x) \in A\}) = \int_A g d\lambda.$$

2.2 Uniqueness

Proposition 1. *If a probability density function exists, it is unique (up to almost everywhere).*

Proof. ☐