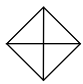


Why

We would like to speak about an object, which is a member of some set, and some attributes of this objects—without knowing the precise identity of the object. Such language would be especially useful in discussing games of chance.

Definition

We have a set Ω which includes all possible objects. We call it the *set of outcomes* (or *sample space*). We call an element of the set of outcomes an *outcome* (or *possibility*, *sample*, *elementary event*, *simple event*, *sample point*).

An *event* (*compound event*, *random event*) is a subset of outcomes. For events $A, B \subset \Omega$, we interpret $A \cup B$ as the event that either A *or* B occurs. Similarly we interpret $A \cap B$ as the event that *both* A *and* B occur. We interpret $\Omega - A$, the *complement* of A in Ω, as the event that A *does not* occur.

An *outcome variable* (or *random variable*) is a function from Ω to V, where V is a set. In this context, V is called the set of *values* of the random variable.

Example: coin

We want to talk about the result of flipping a coin. The coin has two sides. When we flip the coin, it lands heads or tails. We model these outcomes with the set $\{0, 1\}$. If the coin lands tails, we say that outcome 0 has occurred. If the coin lands heads, we say that outcome 1 has occurred.

Example: die

We want to talk about the result of rolling a die. The die has six sides. When we roll the die, one of the six sides is facing up. We model this uncertain outcome with $\{1, 2, 3, 4, 5, 6\}$, whose elements represent the number of pips facing up.

Define $O = \{1, 3, 5\}$ and $E = \{2, 4, 6\}$. We interpret O as the event that the number of pips is odd, and E as the event that the number of pips is even.

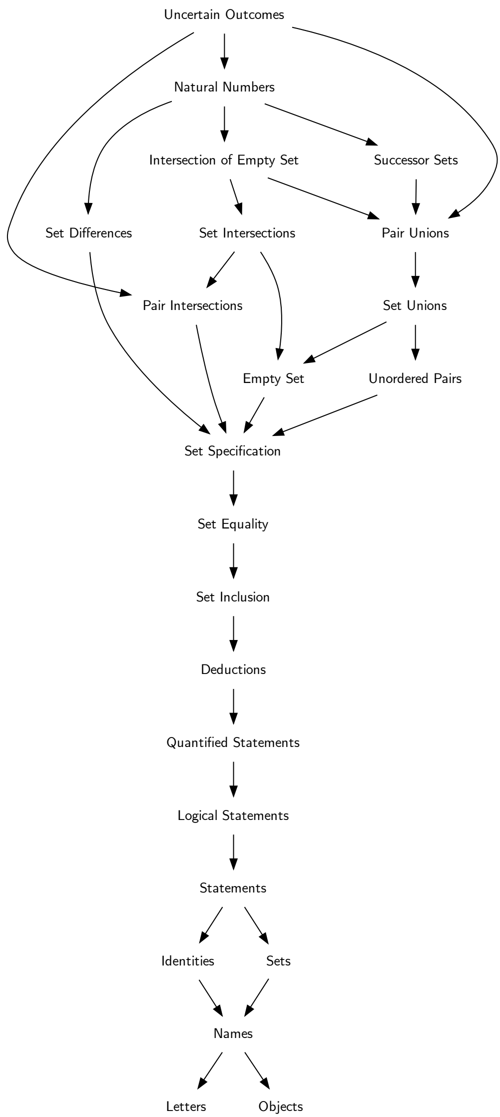
Example: two dice

We want to talk about the sum of the pips shown facing up after rolling two dice. We may take as our set of outcome $\{1, \ldots, 12\}$, whose elements correspond to the sum. We interpret $\{x \in \Omega \mid x \geq 10\}$ as the event that the sum of the two dice is greater than or equal to 10.

Alternatively, we may take the outcomes $\{1, \ldots, 6\}^2$ and define an outcome variable $s : \{1, \ldots, 6\}^2 \to \{1, \ldots, 12\}$ by

$$s((d_1, d_2)) = d_1 + d_2.$$

We interpret this natural-number-valued outcome variable s as sum of the two dice. The event that the sum of the two dice is greater than or equal to to 10 corresponds to the set $\{(d_1, d_2) \in \{1, \ldots, 6\} \mid s((d_1, d_2)) \geq 10\}$.


Uncertain Outcomes
Natural Numbers
Intersection of Empty Set
Successor Sets
Set Differences
Set Intersections
Pair Unions
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects