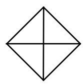


Why

If $\mathbf{R}$ corresponds to a line, and $\mathbf{R}^2$ to a plane, and $\mathbf{R}^3$ to space, does $\mathbf{R}^4$ correspond to anything? What of $\mathbf{R}^5$?

Definition

Let n be a natural number. We call the set $\mathbf{R}^n$ *n-dimensional space* (or *Euclidean n-space*). We call elements of $\mathbf{R}^n$ *points*. We identify $\mathbf{R}^1$ with $\mathbf{R}$ in the obvious way.

We call the point associated with $x = (x_1, x_2, \dots, x_n) \in \mathbf{R}^n$ with $x_i = 0$ for $1 \leqq i \leqq n$ the *origin*. We denote the origin by 0. Similarly, we denote the point x with $x_i = 1$ for all $i = 1, \dots, n$ by 1.

Visualization

We can not visualize n-dimensional space. Thus, our intuition for it comes from real space (see Real Space).

Distance

A natural notion of distance for $\mathbf{R}^n$ generalizes that in $\mathbf{R}^2$ and $\mathbf{R}^3$. We define the *distance* (*Euclidean distance*) between $(x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \in \mathbf{R}^n$ as

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2}.$$

Does this have the properties that distance has in the plane and in space? We discussed these properties It does. Denote the function which associates to $x, y \in \mathbf{R}^n$ their distance $d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$. So $d(x, y)$ is the distance between the points corresponding to x and y.

Proposition 1. *d is non-negative, symmetric, and the distance between two points is no larger than the sum of the distances with any third object.*[1]

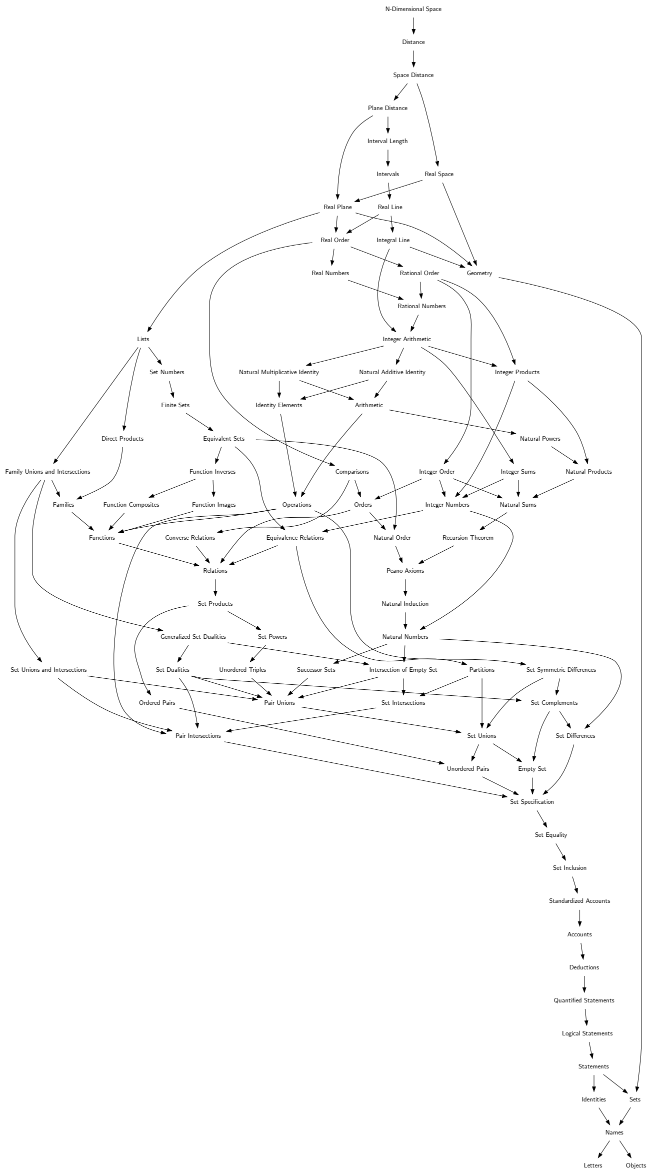
[1] Future editions will include an account.

Order

Let $x, y \in \mathbf{R}^n$. If $x_i < y_i$ for all $i = 1, \ldots, n$ then we say x is *less than* y. Likewise, if $x_i \leq y_i$ for all $i = 1, \ldots, n$ then we say $x \leq y$. Likewise for $>$ and $\geq$.

Notation

If $x \in \mathbf{R}^n$ is less than $y \in \mathbf{R}^n$ then we write $x < y$. Similarly for $x \leq y$, $x > y$ and $x \geq y$. Other notation in the literature for $\mathbf{R}^n$ includes E^n, which is a mnemonic for "euclidean."


N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Real Order
Integral Line
Real Numbers
Rational Order
Geometry
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects