Why

We want to subtract numbers.[1]

Definition

Consider the set $\omega \times \omega$. This set is the set of ordered pairs of ω. In other words, the ordered pairs of natural numbers.

We call two such pairs (a, b) and (c, d) of $\omega \times \omega$ *integer equivalent* if

$$a + d = b + c$$

Briefly, the intuition is that (a, b) represents a less b, or in the usual notation "$a - b$".[2] So this equivalence relation says these two are the same if $a - b = c - d$. Rearranging gives $a + d = b + c$.

Proposition 1. *Integer equivalence is an equivalence relation.*[3]

The *set of integer numbers* is the set of equivalence classes (see Equivalence Relations) under integer equivalence on $\omega \times \omega$. We call an element an *integer number* (or *integer*).

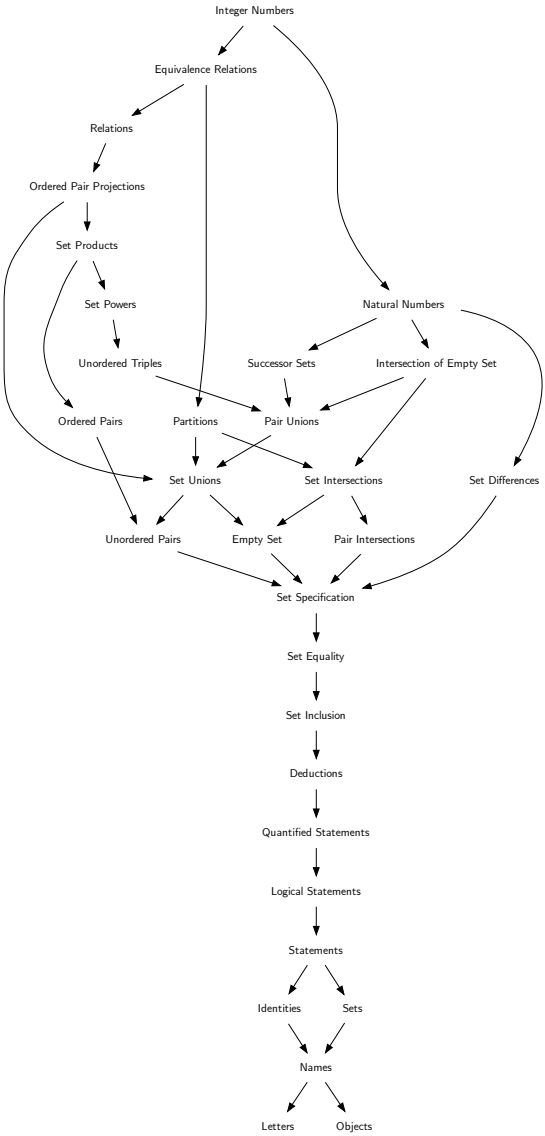
Notation

We denote the set of integers by $\mathbf{Z}$. If we denote integer equivalence by $\sim$ then $\mathbf{Z} = (\omega \times \omega)/\sim$.

[1] Future editions will change this why. In particular, by referencing Inverse Elementsand the lack thereof in ω.

[2] This account will be expanded in future editions.

[3] The proof is straightforward. It will be included in future editions.


Integer Numbers
Equivalence Relations
Relations
Ordered Pair Projections
Set Products
Set Powers
Unordered Triples
Natural Numbers
Successor Sets
Intersection of Empty Set
Ordered Pairs
Partitions
Pair Unions
Set Unions
Set Intersections
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects