



Why

A probability event function is a measure on the measurable space consisting of the set of outcomes and the set of events.

Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The word “space” is natural, since the notion of a measure generalized the notion of volume in real space (see **Real Space** and **N-Dimensional Space**). The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra. The measure in a probability space corresponds to the event probability function.

Notation

Let (X, \mathcal{A}) be a measurable space.¹ We denote the sigma-algebra by \mathcal{A} , as usual. We denote a probability measure by \mathbf{P} , a mnemonic for “probability,” and intended to remind of the event probability function.²

Thus, we often say “Let $(X, \mathcal{A}, \mathbf{P})$ be a probability space.” Many authors associate an event $A \in \mathcal{A}$ with a function $\pi : X \rightarrow \{0, 1\}$ so that $A = \{x \in X \mid \pi(x) = 1\}$. In this context, it is common to write $\mu[\pi(x)]$ for $\mu(A)$.

¹Often, other authors will denote the set of outcomes (here denoted by X) by Ω , an apparent mnemonic for “outcomes”.

²Often, other authors will use notation including P , Pr , Prob , and pr .

