



Signed Measure Examples

1 Why

Let us consider examples of signed measures. TODO

2 Examples

Consider an integrable function defined on some measurable space. The extended-real-valued function which assigns to each distinguished set the value of the integrating the function over that set is a signed measure. See Example 1.

Consider

Example 1. Let (X, \mathcal{A}, μ) a measure space. Let R denote the set of real numbers. Let $f : X \rightarrow R$ an integrable function. Define $\nu : \mathcal{A} \rightarrow R$ by

$$\nu(A) = \int_A f d\mu.$$

Then ν is a signed measure.

Proof. First,

$$\nu(\emptyset) = \int f \chi_{\emptyset} d\mu = \int 0 d\mu = 0.$$

Next, let $\{A_n\}_n$ disjoint. Notice that,

$$\chi_{\cup_{i=1}^n A_k} = \sum_{i=k}^n \chi_{A_k}$$

so for all n ,

□