Why

We want to solve linear equations.

Example

Suppose we want to find $x_1, x_2 \in \mathbf{R}$ to satisfy the system of linear equations

$$3x_1 + 2x_2 = 10, \text{ and}$$

$$6x_1 + 5x_2 = 20,$$

with constants $((3, 2), 10)$ and $((6, 4), 20)$.

We can associate to the first equation an *equation for x_1 in terms of x_2*. We call this *solving the first equation for x_1*.

$$3x_1 + 2x_2 = 10 \longleftrightarrow 3x_1 = 10 - 2x_2 \longleftrightarrow x_1 = (1/3)(10 - 2x_2).$$

Define $f_1 : \mathbf{R} \to \mathbf{R}$ as $f_1(y) = (1/3)(10 - 2y)$. Then $3x_1 + 2x_2 = 10$ if and only if $x_1 = f_1(x_2)$. We have written x_1 *as a function* of x_2 and obtained a new equation. The equation is not linear, however, as f_1 is not linear.

Using the equation for x_1 in terms of x_2, we can substitute this equation into our second linear equation. The two linear equations hold if and only if

$$6f(x_2) + 5x_2 = 20 \longleftrightarrow 20 - 4x_2 + 5x_2 = 20 \longleftrightarrow x_2 = 0.$$

So the equations are satisifed if and only if x_2 is 0. If $x_2 = 0$, then $3x_1 = 10$ and $6x_1 = 20$. Both of these are equivalent to $x_1 = 10/3$. So we have that x_1 must be $10/3$ and x_2 must be 0.

Clearly this is *a solution.* Is it the only one?[1]

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[1] Future editions will expand.

Real Linear Equation Solutions

Real Linear Equations

Linear Functions

Arrays

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space

Real Plane

Real Line

Real Summation

Real Order

Integral Line

Natural Summation

Real Numbers

Rational Order

Geometry

Lists

Rational Numbers

Direct Products

Set Numbers

Integer Arithmetic

Natural Fractions

Integer Products

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Equivalent Sets

Identity Elements

Natural Arithmetic

Family Unions and Intersections

Family Operations

Function Inverses

Comparisons

Natural Equations

Integer Order

Integer Sums

Natural Powers

Families

Function Composites

Function Images

Operations

Equations

Orders

Integer Numbers

Natural Products

Functions

Converse Relations

Equivalence Relations

Natural Order

Natural Sums

Ordered Pair Projections

Relations

Recursion Theorem

Set Products

Peano Axioms

Generalized Set Dualities

Set Powers

Natural Induction

Set Unions and Intersections

Set Dualities

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Partitions

Set Symmetric Differences

Natural Numbers

Ordered Pairs

Pair Unions

Set Intersections

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects