

## METRIC SPACE EXAMPLES

## Why

We give examples of metric spaces

## Example

**Example 1.** Let n be a natural number. Let A be  $\mathbb{R}^n$  and define  $d: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$  by

$$d(a,b) = \sqrt{(a_1 - b_1)^2 + \dots + (x_n - y_n)^2}.$$

(A, d) is a metric space.

**Example 2.** Let A be the unit circle in  $R^2$ . So  $A = \{x \in R^2 \mid x_1^2 + b^2 = 1\}$ . Let  $d_1 : A \times A \to R$  defined by

$$d(a,b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}.$$

Let  $d_2: A \times A \to R$  defined as the arc length between the two points. Both  $(A, d_1)$  and  $(A, d_2)$  are metric spaces.

**Example 3.** Let A = C([0,1], R). Let  $d_1 : A \times A \to R$  be such that

$$d_1(a,b) = \max_{x \in [0,1]} |a(x) - b(x)|.$$

Let  $\lambda$  be the outer cover measure. Let  $d_2: A \times A \to R$  be such that

$$d_2(a,b) = \int_{[0,1]} |f - g| d\lambda.$$

Both  $(A, d_1)$  and  $(A, d_2)$  metric spaces.

