



Why

We want to speak of infinite processes, and to do so we define sequences indexed by \mathbf{N} . In other words, important families are those indexed by the natural numbers.

Definition

A *sequence* (or *infinite sequence*) is a family whose index set is \mathbf{N} (the set of natural numbers without zero). The *n th term* or *coordinate* of a sequence is the result of the n th natural number, $n \in \mathbf{N}$.¹

Notation

Let A be a non-empty set and $a : \mathbf{N} \rightarrow A$. Then a is a (infinite) sequence in A . $a(n)$ is the n th term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n . If $\{A_n\}_{n \in \mathbf{N}}$ is an infinite sequence of sets, then we denote the direct product of the sequence by $\prod_{i=1}^{\infty} A_i$.

Sometimes the set of infinite sequences in A are denoted $A^{\mathbf{N}}$ or A^{∞} .

Natural unions and intersections

We denote the family of the infinite sequence of sets $(A_n)_n$ by $\cup_{i=1}^{\infty} A_i$. Similarly, we denote the intersection of an infinite sequence of sets by $\cap_{i=1}^{\infty} A_i$, respectively.

¹Future editions may also comment that we are introducing language for the steps of an infinite process.

