



NATURAL ORDER

Why

We count in order.¹

Defining Result

We say that two natural numbers m and n are *comparable* if $m \in n$ or $m = n$ or $n \in m$.

Proposition 1. *Any two natural numbers are comparable.*²

In fact, more is true.

Proposition 2. *For any two natural numbers, exactly one of $m \in n$, $m = n$ and $n \in m$ is true.*³

Proposition 3. $m \in n \longleftrightarrow m \subset n$.

If $m \in n$, then we say that m is *less than* n . We also say in this case that m is *smaller than* n . If we know that $m = n$ or m is less than n , we say that m is *less than or equal to* n .

Notation

If m is less than n we write $m < n$, read aloud “ m less than n .” If m is less than or equal to n , we write $m \leq n$, read aloud “ m less than or equal to n .”

¹Future editions will expand.

²Future editions will include an account.

³Use the fact that no natural number is a subset of itself. Future editions will expand this account. See *Peano Axioms*).

Properties

Notice that $<$ and \leq are relations on ω (see *Relations*).⁴

Proposition 4 (Reflexivity). \leq is reflexive, but $<$ is not.

Proposition 5 (Symmetry). Both \leq and $<$ are not symmetric.

Proposition 6 (Transitivity). Both \leq and $<$ are transitive.

Proposition 7 (Antisymmetry). If $m \leq n$ and $n \leq m$, then $m = n$.

⁴Proofs of the following propositions will appear in future editions.

