Filled Graphs

1 Why

TODO Needed for perfect elimination orderings.

2 Definition

An ordered graph is *filled* or *monotone transitive* if all higher neighborhoods induce complete subgraphs.

3 Notation

Let $G_\sigma = (V, E, \sigma)$ be an ordered graph. G_σ is filled if

$$u, v \in \overset{+}{\mathsf{adj}}(v) \implies \{u, v\} \in E.$$

In other word, if $i < j < k$ so that $\{\sigma(i), \sigma(j)\} \in E$ and $\{\sigma(i), \sigma(k)\} \in E$ then $\{\sigma(j), \sigma(k)\} \in E$.

4 Chordality

Proposition 1. *If (V, E, σ) is a filled graph, then (V, E) is chordal.*

Proof. Take the vertex with the lowest index on a cycle of length greater than three. Take $\square$