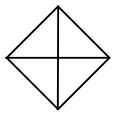

# Relations

## 1 Why

How can we relate the elements of two sets?

## 2 Definition

A **relation** between two non-empty sets $A$ and $B$ is a subset of $A \times B$. A relation on a single set $C$ is a subset of $C \times C$.

Let $a \in A$ and $b \in B$. The pair $(a, b)$ may or may not be in a relation on $A$ and $B$. If $A \neq B$, then $(b, a)$ is not a member of the product $A \times B$, and therefore not in any relation on $A$ and $B$. If $A = B$, however, it may be that $(b, a)$ is in the relation.

### 2.1 Notation

Let $A$ and $B$ be nonempty sets with $a \in A$ and $b \in B$. Since relations are sets, we can use upper case Latin letters. Let $R$ be a relation on $A$ and $B$. We denote that $(a, b) \in R$ by $aRb$, read aloud as "a in relation $R$ to b."

When $A = B$, we tend to use other symbols instead of letters. For example, $\sim$, $=$, $<$, $\leq$, $\prec$, and $\preceq$.