Trees

1 Why

Tree branches split and do not recombine. We formalize this property in the language of graphs.

2 Definition

A *tree* is a set of two-element sets with the following property: if we take the union over all the sets and define a relation on this union such that two elements are related if they appear as a two element set, then the ordered pair of the union and this relation is a graph that is connected and acyclic.

Thus every tree corresponds to an undirected, connected and acyclic graph. We avoid defining a tree to be that, though, because we want to keep around only the most important object: the set of two-element sets. From that object we can get the vertex set and edge set of the graph. We need the concepts from graphs to talk about which sets of two-element sets are trees, but we want the notation from trees to be amenable to their nature. The notation we use will bear us out.

2.1 Notation

Let T be a tree, a mnemonic for “tree." Let V be the vertex set associated with T. In other words,

$$V = \cup_{e \in T} e$$

. Let E be so that $(u, v) \in E$ if and only if $\{u, v\} \in T$ for every $u, v \in V$. By construction, the graph (V, E) is undirected. T a tree means (V, E) is connected and acyclic. Let V be a non-empty finite set and $E \subset V \times V$ such that (V, E) is a tree.

A major motivator for our definition of trees is so that we can write $\{u, v\} \in T$. TODO

3 Properties

Proposition 1. *There is only one path between any two vertices in a tree.*

Proof. Suppose to the contrary that there were two paths from vertex u to vertex v. Then by combining these paths we obtain a cycle. But the tree has no cycles. So there must not be two paths between any two vertices. $\square$