

CONFUSION MATRICES

Why

We can summarize the (label, prediction) pairs for a particular classifier on a particular dataset in a matrix.

Boolean case

Let A be a nonempty set and $B = \{-1, 1\}$. For a dataset $(a^1, b^i), \ldots, (a^n, b^n)$ in $A \times B$, and classifier $G : A \to B$, the confusion matrix C is defined

$$C = \begin{bmatrix} \# \text{ true negatives} & \# \text{ false negatives} \\ \# \text{ false positives} & \# \text{ true positives} \end{bmatrix} = \begin{bmatrix} C_{\text{tn}} & C_{\text{fn}} \\ C_{\text{fp}} & C_{\text{tp}} \end{bmatrix}.$$

Using this notation, $C_{\rm tn} + C_{\rm fn} + C_{\rm fp} + C_{\rm tp} = n$. $N_{\rm n} := C_{\rm tn} + C_{\rm fp}$ is the number of negative examples. $N_{\rm p} := C_{\rm fn} + C_{\rm tp}$ is the number of positive examples.

The diagonal elements of the confusion matrix give the numbers of correct predictions. The off-diagonal entries give the numbers of incorrect predictions for the two types of errors (see Classifier Errors).

In this notation, the false positive rate is $C_{\rm fp}/n$, the false negative rate is $C_{\rm fn}/n$ and the error rate is the sum of these, $(C_{\rm fn}+C_{\rm fp})/n$.

The true positive rate is $C_{\rm tp}/(C_{\rm fn}+C_{\rm tp})$. The true negative rate is $C_{\rm tn}/(C_{\rm tn}+C_{\rm fp})$. The false alarm rate is $C_{\rm fp}/(C_{\rm tn}+C_{\rm fp})$. The precision is $C_{\rm tp}/C_{\rm tp}+C_{\rm fp}$

