Why

Let $X = \{a, b\}$ and $Y = \{0, 1\}$. The dataset $((a, 0))$ is consistent, but it is not functionally complete. On the other hand, the dataset $((a, 0), (b, 0), (a, 0), (a, 0), (a, 0), (a, 1))$ is complete but it is not *functionally* consistent.

In general, if $y_i \neq y_j$ for some i and j where $x_i = x_j$, then the dataset is not functionally consistent. In the preceding example, both $(a, 0)$ and $(a, 1)$ appear.

If we emphasize the "predictive" aspect of a functional inductor, we interpret the input as an object we "see before" the output. And so treat $y \in Y$ as an uncertain outcome which is the element associated to $x \in X$.

In this case, we may use the language of probability to discuss this uncertain outcome. If, for example, Y is finite, we can associate a distribution with each input $x \in X$.

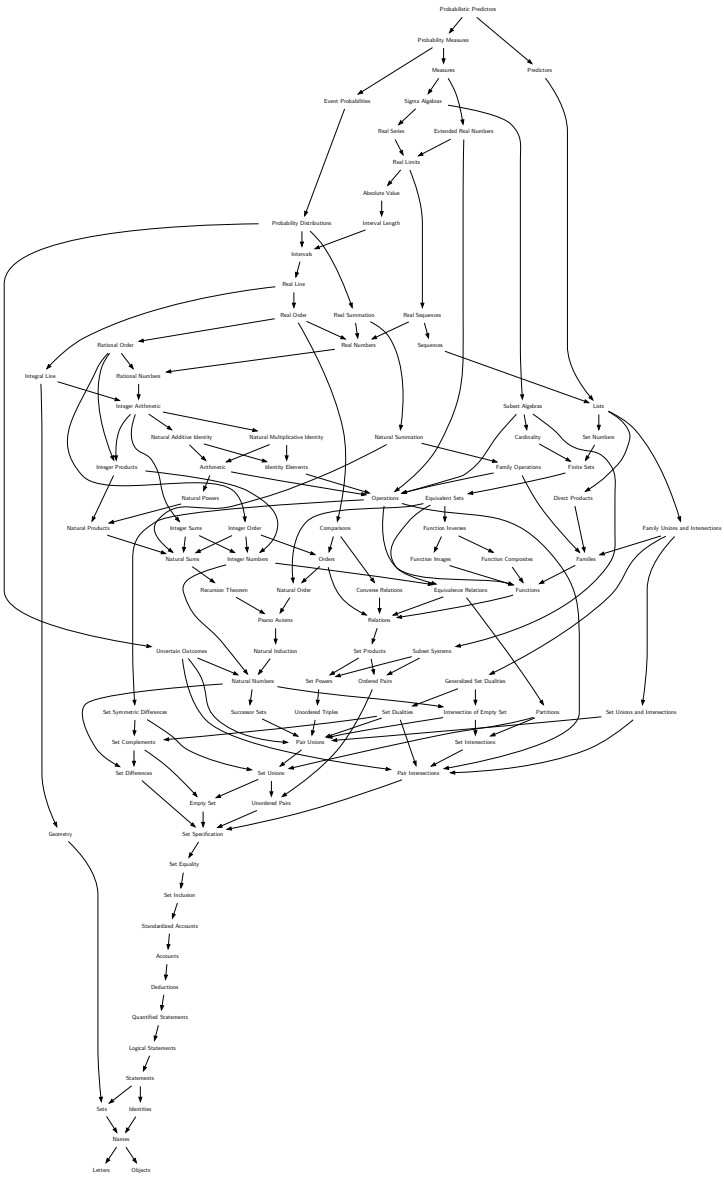
Definition

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable spaces.

A *probabilistic functional inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a family of measures on $(Y, \mathcal{Y})$, indexed by X. We call a function from inputs to output measures a *probabilistic predictor*. We call the distribution a *probabilistic prediction*.

Notation

Let $\mathcal{M}(Y, \mathcal{Y})$ be the set of measures on Y. Let D be a dataset in $(X \times Y)^n$. Let $g : X \to \mathcal{M}(Y, \mathcal{Y})$ a probabilistic predictor. Let $G_n(X \times Y)^n \to (X \to \mathcal{M}(Y, \mathcal{Y}))$ be a predictive probabilistic inductor. Then $G_n(D)$ is a family of measures $\{g_x : \mathcal{Y} \to [0, 1]\}_{x \in X}$.


Probabilistic Predictors
Probability Measures
Measures
Predictors
Event Probabilities
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Absolute Value
Probability Distributions
Interval Length
Intervals
Real Line
Real Order
Real Summation
Real Sequences
Real Numbers
Sequences
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Subset Algebras
Lists
Cardinality
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Natural Summation
Integer Products
Arithmetic
Identity Elements
Family Operations
Finite Sets
Natural Powers
Operations
Equivalent Sets
Direct Products
Natural Products
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Orders
Function Image
Function Composites
Families
Recursion Theorem
Natural Order
Convene Relations
Equivalence Relations
Functions
Peano Axioms
Relations
Uncertain Outcomes
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects