Why

Do the integer numbers correspond (in the sense of Homomorphisms) to elements of the rationals.

Main Result

Indeed, roughly speaking the integers correspond to rationals whose denominator is 1. Define

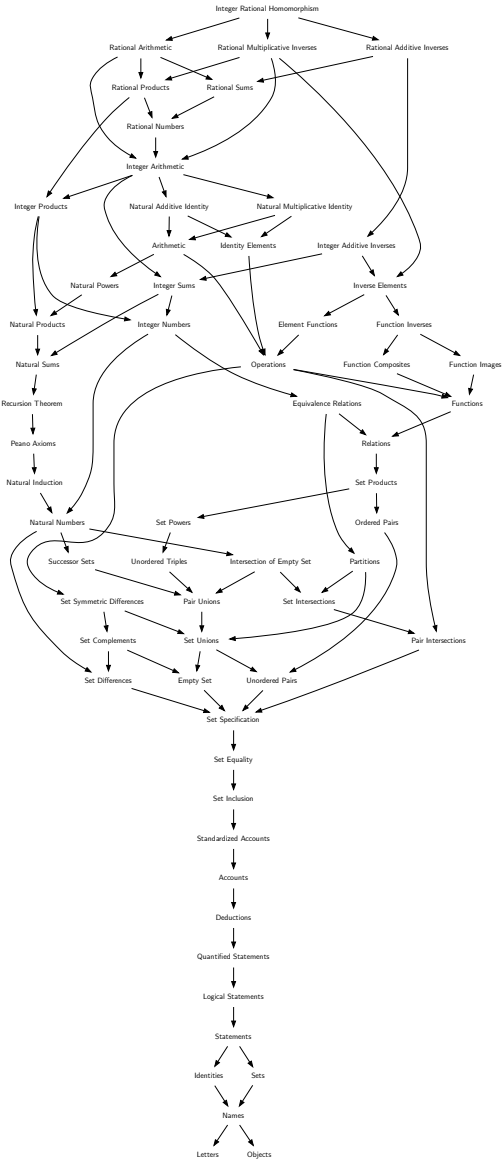
$$\tilde{Q} := \{[(a,b)] \in \mathbf{Q} \mid b = 1_{\mathbf{Z}}\}.$$

Proposition 1. *The rings* $(\tilde{\mathbf{Q}}, +_{\mathbf{Q}} \mid \tilde{\mathbf{Q}}, \cdot_{\mathbf{Q}} \mid \tilde{\mathbf{Q}})$ *and* $(Z, +_{\mathbf{Z}}, \cdot_{\mathbf{Z}})$ *are homomorphic.*[1]

Proof. The function is $f : \mathbf{Z} \to \mathbf{Q}$ with $f(z) = [(z,1)]$.[2] □

[1] Indeed, more is true and will be included in future editions. There is an *order perserving* ring homomorphism.

[2] The full account will appear in future editions.


Integer Rational Homomorphism
Rational Arithmetic
Rational Multiplicative Inverses
Rational Additive Inverses
Rational Products
Rational Sums
Rational Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Integer Additive Inverses
Natural Powers
Integer Sums
Inverse Elements
Natural Products
Integer Numbers
Element Functions
Function Inverses
Natural Sums
Operations
Function Composites
Function Images
Recursion Theorem
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Set Unions
Pair Intersections
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects