



Rooted Trees

1 Why

We want to talk about orienting the edges of a tree away from a given vertex.

2 Definition

A *rooted tree* is an ordered pair whose first object is a tree and whose second object is a vertex of the tree. We call the vertex the *root*.

We associate with a rooted tree the directed graph constructed by orienting all edges away from the root. For a particular rooted tree, which is to say a particular choice of root, we obtain a directed graph. There are many choices of root, and so naturally we wonder if each choice of root corresponds to a different directed graph. Since trees are connected, each choice of root does lead to a different graph. If they were disconnected, some of the graphs may turn out to be the same. So there is a one-to-one correspondence between rooted trees and their associated directed graphs, and so we could have defined a rooted tree as a directed graph with particular properties.

2.1 Notation

Let (V, T) be a tree. Let V be a non-empty set and $E \subset V \times V$. We denote the tree $T = V$ rooted at vertex i by (T, i) .

3 Properties

Proposition 1. *Let (T, i) be a rooted tree. In the directed graph corresponding to this rooted tree every vertex has one parent.*

We denote the parent of vertex i by \mathbf{pa}_i .