Families

1 Why

It is useful to have some language and notation for talking about a set of sets.

2 Definition

A *family* of sets is a set of sets. Experience shows that it is useful to have these associated with the elements of a well-known second set.

An *indexed family of sets* is a function from one set to the power set of a second set. We call the first set the *index set*. We call the second set the *base set*. The range of the indexed family of sets is, of course, a family.

2.1 Notation

Let A and I be be a non-empty sets. We use I as a mnemonic for “index" set. Let $a : I \to A^*$ be a family. For $i \in I$, we follow the function notation and denote the result of applying a to i by a_i.

We denote the range of the family by family of a_α indexed with I by $\{a_\alpha\}_{\alpha \in I}$, which is short-hand for set-builder notation. We read this notation “a sub-alpha, alpha in I."