



Why

We generalize the notion of sequence to index sets beyond the naturals.

Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go “further out.”

To elaborate on property (b): if handed two natural numbers m and n , we can always find another, for example $\max\{m, n\} + 1$, larger than m and n . We might think of larger as “further out” from the first natural number: 1.

Now combine these two observations. A *directed set* is a set D with a partial order \preceq satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.

A *net* is a function on a directed set.

Examples

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

Consider $\mathbf{N} \times \mathbf{N}$, and write $(a, b) \preceq (c, d)$ if $a \leq c$ and $b \leq d$. Clearly, (\mathbf{N}^2, \preceq) so defined is a partially ordered set. Notice that given $a = (a_1, a_2)$ and $b = (b_1, b_2)$ the point $(\max(a_1, b_1), \max(a_2, b_2))$ succeeds or is equal to both a and b . Thus (\mathbf{N}^2, \preceq) is a directed set on which we can define a net.

Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \rightarrow A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.

