Matrix notation

Let $u_1 \ldots, u_k \in \mathbf{R}^n$. Define $U \in \mathbf{R}^{n \times k}$ so that

$$U = \begin{bmatrix} u_1 & u_2 & \cdots & u_k \end{bmatrix}.$$

Then $\{u_1, \ldots, u_k\}$ is an orthonormal set mean

$$U^\top U = I_k$$

Notice that if $k < n$, $UU^\top \neq I$, since its rank is at most k.

Real Matrices with Orthonormal Columns

Real Matrix-Matrix Products

Real Matrix-Vector Products

Orthonormal Set of Real Vectors · Vectors as Matrices · Real Linear Combinations · Real Matrices

Real Inner Product · Real Norm · Linear Equations

Space Inner Product · Real Vectors · Linear Function · Arrays

Space Norm

Plane Inner Product · Plane Norm · N-Dimensional Space · Plane Vector Sums and Differences · Space Vectors

Right Triangle Sides Relation · Absolute Value · Distance · Plane Vectors

Real Squares · Space Distance

Real Arithmetic · Plane Distance

Real Multiplication Inverses · Interval Length

Square Numbers · Real Products · Real Space · Intervals

Squares · Integral Line · Real Addition Inverses · Real Line · Real Plane

Rational Arithmetic · Rational Multiplication Inverses · Real Sums · Real Order · Real Summation

Geometry · Rational Sums · Rational Products · Real Numbers · Rational Order · Natural Summation

Integer Arithmetic · Rational Numbers · Lists

Set Numbers · Direct Products

Integer Products · Natural Addition Identity · Natural Multiplication Identity · Finite Sets

Arithmetic · Identity Elements · Equivalent Sets · Inverse Elements · Family Operation · Family Union and Intersection

Natural Powers · Integer Sums · Integer Order · Natural Equations · Element Function · Function Inverses · Comparison · Families

Natural Products · Integer Numbers · Orders · Equations · Operations · Function Composites · Function Images

Natural Sums · Equivalence Relation · Natural Order · Functions · Converse Relations

Recursion Theorem · Relations

Peano Axioms · Ordered Pair Projections

Natural Induction · Set Products · Generalized Set Duality

Natural Numbers · Set Powers · Set Union and Intersection

Set Symmetric Differences · Partition · Successor Sets · Intersection of Empty Set · Unordered Triples · Set Duality

Set Complements · Set Intersection · Pair Union · Ordered Pairs

Set Differences · Set Union · Pair Intersection

Empty Set · Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deduction

Quantified Statements

Logical Statements

Statements · Identities

Sets · Names

Letters · Objects