Monotone Class

1 Why

2 Definition

The *limit* of an increasing sequence of sets is the family union of the sequence. The *limit* of a decreasing sequence of sets is the family intersection of the sequence.

A *monotone limit* of an sequence of sets is the limit of a monotone sequence.

A *monotone space* is a subset space in which monotone limits of monotone sequences of distinguished sets are distinguished. We call the distinguished sets a *monotone class*.

2.1 Notation

Let A a non-empty set with partial order $\preceq$. Let $(A, \mathcal{A})$ be a subset space on A.

Let $(A_n)_n$ be an increasing or decreasing sequence in $\mathcal{A}$. We denote the limit of $(A_n)_n$ by $\lim_n A_n$.

If $(A_n)_n$ is increasing, $\lim_n A_n = \cup_n A_n$. If $(A_n)_n$ is decreasing, $\lim_n A_n = \cap_n A_n$.

If $(A, \mathcal{A})$ is a monotone space, then for all monotone $(A_n)_n$ in $\mathcal{A}$, $\lim_n A_n \in \mathcal{A}$. In this case, $\mathcal{A}$ is a montone class.