



EMPTY SET

Why

Can a set have no elements?

Definition

Sure. Take a set. At least one exists by the principle of existence (see *Sets*). Now, take the universally false statement $x \neq x$ and apply the principle of specification (see *Set Specification*) to that set. The set we get has no elements, since no object satisfies $x \neq x$. The principle of extension says that the set obtained is unique (contradiction).¹ We call this set with no elements *the empty set*.

Notation

We denote the empty set by \emptyset . In other words, in all future accounts (see *Accounts*), there are two implicit lines. First, “name \emptyset ” and second “have $(\forall x)(x \notin \emptyset)$ ”.

¹This account will be expanded in the next edition.

