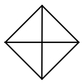

Why

[1]

Definition

Let (V, E) be a directed graph. A *directed path* between vertex v and vertex $w \neq v$ is a finite sequence of distinct vertices, whose first coordinate is v and whose last coordinate is w, and whose consecutive coordinates (as ordered pairs) are edges in the graph. We say that a path between v and w is from v to w. The *length* of the path is one less than the number of vertices: namely, the number of edges.

Two vertices are *connected* in a graph if there exists at least one path between them. A directed graphis *connected* if there is a path between every pair of vertices. A graph is *acyclic* if none of its paths cycle.

Other Terminology

Some authors allow paths to contain repeated vertices, and call a path with distinct vertices a *simple path*. Similarly, some authors allow a cycle to contain repeated vertices, and call a path with distinct vertices a *simeple cycle* or *circuit*. Some authors use the term *loop* instead of *cycle*.

[1]Future editions will include.

Directed Acyclic Graph

Directed and acyclic graphs (sometimes $DAGs$) have some useful properties. Clearly, every subgraph induced on a directed acyclic graph is a directed acyclic graph.

Proposition 1. *Let (V, E) be a directed acyclic graph. Then there exists a vertex $v \in V$ which is a source and a vertex $w \in V$ which is a sink.*

Proof. There exists a directed path of maximum length. It must start at a source and end at a sink.[2] □

A *topological numbering*, *topological sort* or *topological order* of a directed graph (V, E) is a numbering $\sigma : \{1, \ldots, |V|\} \to V$ satisfying

$$(v, w) \in E \longrightarrow \sigma^{-1}(v) < \sigma^{-1}(w).^3$$

Proposition 2. *There exists a topological sort for every acyclic graph.*

Proof. Let (V, F) be a directed acyclic graph. There exists a source vertex, v_1. Set $\sigma(1) = v_1$. Take the subgraph induced by $V - \{v_1\}$. It is directed acyclic, and so has a source vertex, v_2. Set $\sigma(2) = v_2$. Continue in this way.[4] □

[2]Future editions will expand.

[3]Future editions will further explain this concept.

[4]Future editions will clarify and expand.

Directed Paths
Directed Graphs
Undirected Graphs
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Function Images
Function Composites
Peano Axioms
Functions
Equivalence Relations
Relations
Natural Induction
Cartesian Products
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects