



# Rooted Tree Linear Cascades

## 1 Why

TODO

## 2 Definition

A *d-dimensional rooted tree linear cascade* is a sequence of four objects: a tree on  $\{1, \dots, d\}$ , a vertex of the tree, a family of real numbers indexed by the edges of the tree, and a  $d$ -dimensional random vector whose covariance matrix is the identity matrix. The cascade is called “ $d$ -dimensional” because we associate it with a random vector (defined as a function of that in the form of its definition) whose codomain is  $\mathbf{R}^d$ .

The tree together with the vertex form a rooted tree. The graph associated with the rooted tree and the family of real numbers together form a weighted graph.

The idea is to use the weights and the tree structure to recursively define a random vector in terms of elements of the given random vector. Let  $C = (T, i, w, e)$  be a  $d$ -dimensional rooted tree linear cascade. So  $T$  is a tree on  $\{1, \dots, d\}$ ,  $i \in \{1, \dots, d\}$  and  $w : T \rightarrow \mathbf{R}$ , and  $e : \Omega \rightarrow \mathbf{R}^d$  for some probability space  $(A, \mathcal{A}, \mathbf{P})$ . The random vector associated with  $C$  is the random variable  $x : \Omega \rightarrow \mathbf{R}^d$  defined by

$$x_i = e_i \text{ and } x_j = w_{\{\mathbf{pa}_j, j\}} x_{\mathbf{pa}_j} + e_j \text{ for } j \neq i.$$

In other words,

$$e = Ax$$

where  $A$  is lower triangle and extremely sparse.

## 2.1 Notation

Let  $(A, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $e : A \rightarrow \mathbf{R}^d$  be a random vector, let  $T$  be a tree on  $\{1, \dots, d\}$  with  $a_{ij} = a_{ji}$  the weight on edge  $\{i, j\} \in T$ . We