Definition

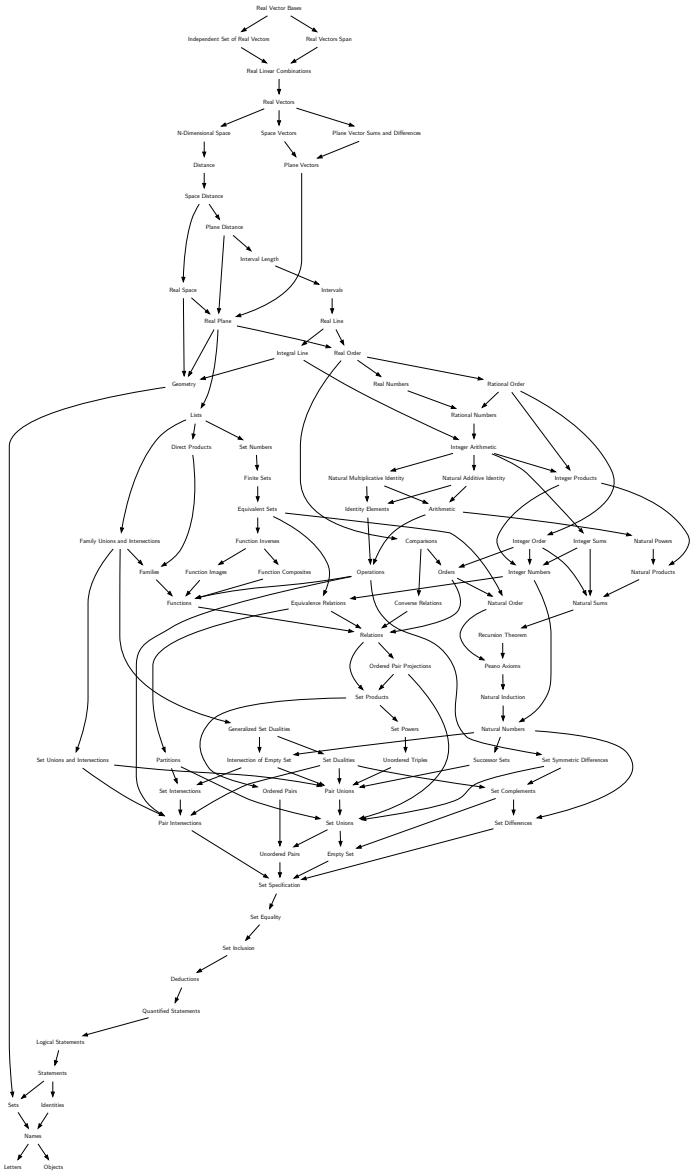
A set of vectors $\{v_1, \ldots, v_k\}$ is a *basis* for a subspace $S \subset \mathbf{R}^n$ if

$$S = \operatorname{span}\{v_1, \ldots, v_k\} \quad \text{and} \quad \{v_1, \ldots, v_k\} \text{ is independent.}$$

This definition captures two competing properties. The first is that the set is large, in the sense that any vector in S can be represented as a linear combination of vectors in $\{v_1, \ldots, v_k\}$. Simultaneously, the set is small, in the sense that no vector in the set is a linear combination of the others. In other words, there is no extra vector in the set.

Linear independence is equivalent to uniqueness of representation of the vectors representable as a linear combination of $v_1, \ldots, v_k$. In other words, $\{v_1, \ldots, v_k\}$ is a basis for S if each vector $x \in S$ can be uniquely expressed as

$$x = \alpha_1 v_1 + \cdots + \alpha_k v_k.$$


Real Vector Bases
Independent Set of Real Vectors
Real Vectors Span
Real Linear Combinations
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Geometry
Real Numbers
Rational Order
Rational Numbers
Lists
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Equivalent Sets
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Powers
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Natural Products
Functions
Equivalence Relations
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects