



# Total Orders

## 1 Why

Often we want all elements of the set  $A$  to be comparable.

## 2 Definition

We call  $R$  *connexive* if for all  $a, b \in A$ ,  $(a, b) \in R$  or  $(b, a) \in R$ . If  $R$  is a partial order and connexive, we call it a *total order*.

A *totally ordered set* is a set together with a total order. The language is a faithful guide: we can compare any two elements. Still, we prefer one word to three, and so we will use the shorter term *chain* for a totally ordered set; other terms include *simply ordered set* and *linearly ordered set*.

Let  $C = (A, R)$  be a chain. A *minimal element* of  $C$  is an element which precedes all other elements. A *maximal element* of  $C$  is an element which is preceded by all other elements.