

SMOOTH MANIFOLDS

Why

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Definition

A subset $M \subset \mathbb{R}^n$ is a *smooth manifold* of dimension d if for every $x \in M$, there exists a neighborhood V of x in X that is diffeomorphic to an open subset U of \mathbb{R}^d . In this case we say that the set is locally diffeomorphic to \mathbb{R}^d .

A diffeomorphism $\phi: U \to V$ is called a parameterization of the neighborhood of V. Its inverse diffeomorphism ϕ^{-1} is called a coordinate system (or system of coordinates) on V.

Notation

We denote the dimension of a manifold M by dim M.

Submanifolds

If X and Z are both manifolds in \mathbb{R}^n and $Z \subset X$, then we call Z a submanifold of X. In particular, X is a submanifold of \mathbb{R}^n . Any open set of a manifold X is a submanifold X.

¹Future editions will include.

²Future editions will expand.

