



## Why

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## Definition

A sequence of functions  $(g_1, \dots, g_\ell)$  is *composable* if  $g_i$  is composable with  $g_{i-1}$  for  $i = 2, \dots, \ell$ . In this case we write  $g_\ell \circ g_{\ell-1} \circ \dots \circ g_2 \circ g_1$ . For example, we write  $g_3 \circ g_2 \circ g_1$  for  $(g_1, g_2, g_3)$ .

A *neural network* (or *feedforward neural network*) from  $\mathbf{R}^n$  to  $\mathbf{R}^m$  is a sequence of composable functions  $(g_1, \dots, g_\ell)$ ,  $\text{dom } g_1 = \mathbf{R}^n$ ,  $\text{ran } g_\ell \subset \mathbf{R}^m$ , satisfying

$$g_i(\xi) = h_i(A_i \xi + b_i)$$

for some conforming matrices  $A_i$ , vectors  $b_i$  and functions  $h_i$ .

The *i*th *layer* of the neural network is the *i*th function  $g_i$ . The *i*th *activation* of the neural network is the function  $h_i$ . A *neural network* is called *deep* if its number of layers is larger than 3.

We call the composition of the layers of the neural network the *network predictor* (or just *predictor*). We also call it *the function* of the network.<sup>2</sup>

A *multi-layer perceptron* (*MLP*) is a neural network with 2 layers (1 *hidden layer*) and for which  $A_i$  and  $b_i$  have unrestricted nonzero entries.

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<sup>1</sup>Future editions will include. Future editions may change the name of this sheet to *computation networks*, or may add prerequisite sheet on computation graphs.

<sup>2</sup>Many authorities refer to a neural network as a function. Strictly speaking that is true for us, as well, since a sequence is a function. But the meaning of the common use is in reference to the *network predictor*.



