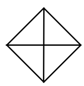


Why

We want an estimator for the parameters of a linear function, given observations of the function with additive noise.

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $x : \Omega \to \mathsf{R}^d$. Define $g : \Omega \to (\mathsf{R}^d \to \mathsf{R})$ by $g(\omega)(a) = a^\top x(\omega)$, for $a \in \mathsf{R}^d$. In other words, for each outcome $\omega \in \Omega$, $g_\omega : \mathsf{R}^d \to \mathsf{R}$ is a linear function with parameters $x(\omega)$. g_ω is the function of interest.

Let $a^1, \dots, a^n \in \mathsf{R}^d$ a dataset with data matrix $A \in \mathsf{R}^{n \times d}$. Let $e : \Omega \to \mathsf{R}^n$ independent of x, and define $y : \Omega \to \mathsf{R}^n$ by

$$y = Ax + e.$$

In other words, $y_i = x^\top a^i + e_i$.

We call (x, A, e) a *probabilistic linear model*. Other terms include *linear model*, *statistical linear model*, *linear regression model*, *bayesian linear regression*, and *bayesian analysis of the linear model*.[1] We call x the parameters, A a *design*, e the *error* or *noise* vector, and y the *observation* vector.

One may want an estimator for the parameters x in terms of y or one may be modeling the function g and want to predict $g(a)$ for $a \in A$ not in the dataset.

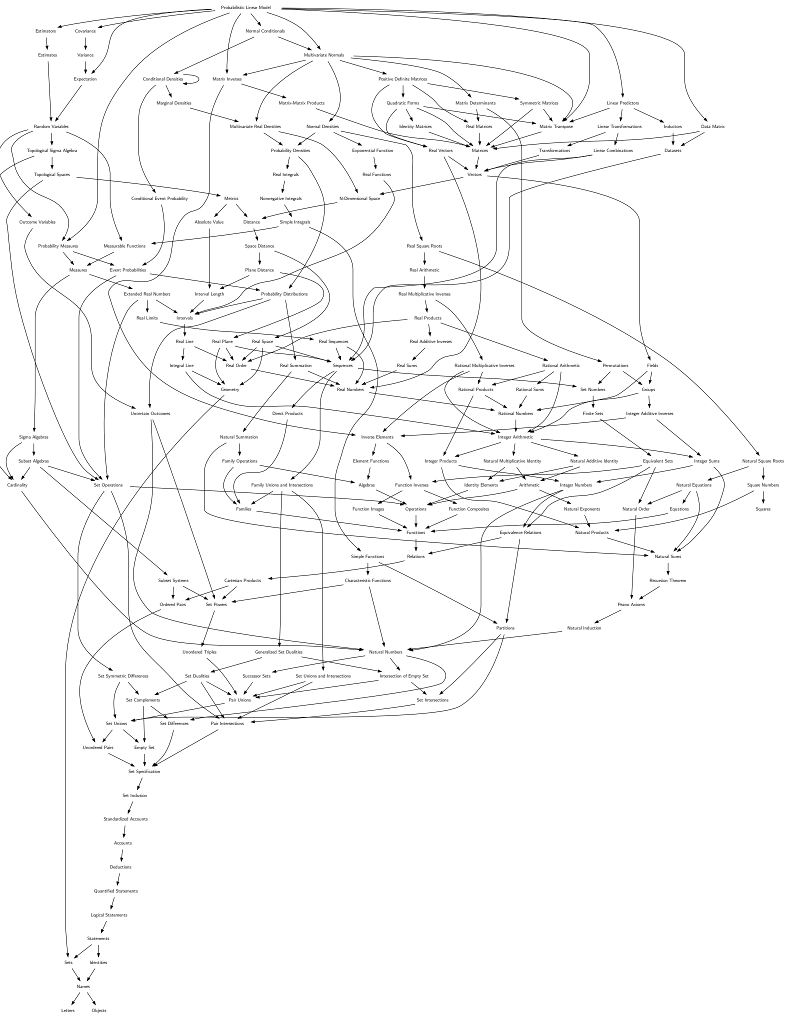
[1] The word bayesian is in reference to treating the object of interest—x—as a random variable.

Mean and variance

Proposition 1. $\mathsf{E}(y) = A\,\mathsf{E}(x) + \mathsf{E}(w)$[2]

Proposition 2. $\mathbf{cov}((x, y)) = A\,\mathbf{cov}(x)A^\top + \mathbf{cov}\,e$[3]

[2]By linearity. Full account in future editions.
[3]Full account in future editions.


Probabilistic Linear Model
Estimators
Covariance
Normal Conditionals
Estimates
Variance
Multivariate Normals
Expectation
Conditional Densities
Matrix Inverses
Positive Definite Matrices
Marginal Densities
Matrix-Matrix Products
Quadratic Forms
Matrix Determinants
Symmetric Matrices
Linear Predictors
Random Variables
Multivariate Real Densities
Normal Densities
Identity Matrices
Real Matrices
Matrix Transpose
Linear Transformations
Inductors
Data Matrix
Topological Sigma Algebra
Probability Densities
Exponential Function
Real Vectors
Matrices
Transformations
Linear Combinations
Datasets
Topological Spaces
Real Integrals
Real Functions
Vectors
Conditional Event Probability
Metrics
Nonnegative Integrals
N-Dimensional Space
Outcome Variables
Absolute Value
Distance
Simple Integrals
Probability Measures
Measurable Functions
Space Distance
Real Square Roots
Measures
Event Probabilities
Plane Distance
Real Arithmetic
Extended Real Numbers
Interval Length
Probability Distributions
Real Multiplicative Inverses
Real Limits
Intervals
Real Products
Real Line
Real Plane
Real Space
Real Sequences
Real Additive Inverses
Integral Line
Real Order
Real Summation
Sequences
Real Sums
Rational Multiplicative Inverses
Rational Arithmetic
Permutations
Fields
Geometry
Real Numbers
Rational Products
Rational Sums
Set Numbers
Groups
Uncertain Outcomes
Direct Products
Rational Numbers
Finite Sets
Integer Additive Inverses
Sigma Algebras
Natural Summation
Inverse Elements
Integer Arithmetic
Subset Algebras
Family Operations
Element Functions
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Integer Sums
Natural Square Roots
Cardinality
Set Operations
Family Unions and Intersections
Algebras
Function Inverses
Identity Elements
Arithmetic
Integer Numbers
Natural Equations
Square Numbers
Families
Function Images
Operations
Function Composites
Natural Exponents
Natural Order
Equations
Squares
Functions
Equivalence Relations
Natural Products
Simple Functions
Relations
Natural Sums
Subset Systems
Cartesian Products
Characteristic Functions
Recursion Theorem
Ordered Pairs
Set Powers
Peano Axioms
Partitions
Natural Induction
Unordered Triples
Generalized Set Dualities
Natural Numbers
Set Symmetric Differences
Set Dualities
Successor Sets
Set Unions and Intersections
Intersection of Empty Set
Set Complements
Pair Unions
Set Intersections
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects