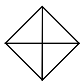


Why

What of lower upper triangular factorizations when the system is symmetric?

Definition

A *symmetric lower upper triangular factorization* of A is a pair of matrices $(L \in \mathbf{R}^{m \times m}, U \in \mathbf{R}^{m \times m})$ where L is unit lower triangular, U is upper triangular and $A = LU$.

$$A = LL^\top.$$

Other terminology includes *lower upper triangular factorization*, *LU decomposition*, *LU factorization.* Define $R = L^\top$, then

$$A = R^\top R.$$

Let $A \in \mathbf{R}^{n \times n}$. Then the ordinary row reduction of A is a matrix U which is upper triangular.be symmetric. A *lower upper triangular decomposition* A is a pair of matrices $(L, L^\top)$ where $L \in \mathbf{R}^{n \times n}$ is lower triangular, has nonnegative real diagonal entries, and satisfies

$$A = LL^\top.$$

Other terminology includes *lower upper triangular factorization*, *LU decomposition*, *LU factorization.* Define $R = L^\top$, then

$$A = R^\top R.$$

Basic properties

Proposition 1. *Let $A \in \mathbf{R}^{m \times m}$ be positive definite. Then there exists unique lower triangular matrix $L \in \mathbf{R}^{n \times n}$ so that*

$$A = LL^\top.$$

So, in the case that A is positive definite, a lower upper triangular decomposition exists and is unique. Therefore we refer to it as *the upper* *lower triangular decomposition* of A. It is also known (universally) as the *Cholesky decomposition* or *Cholesky factorization* of A.

Proposition 2. *If A is positive semidefinite, there exists a permutation matrix P for which there is a unique L so that*

$$P^\top AP = LL^\top.$$

This second proposition says that there is a unique Cholesky decomposition for a particular permutation (or pivoting) of A.

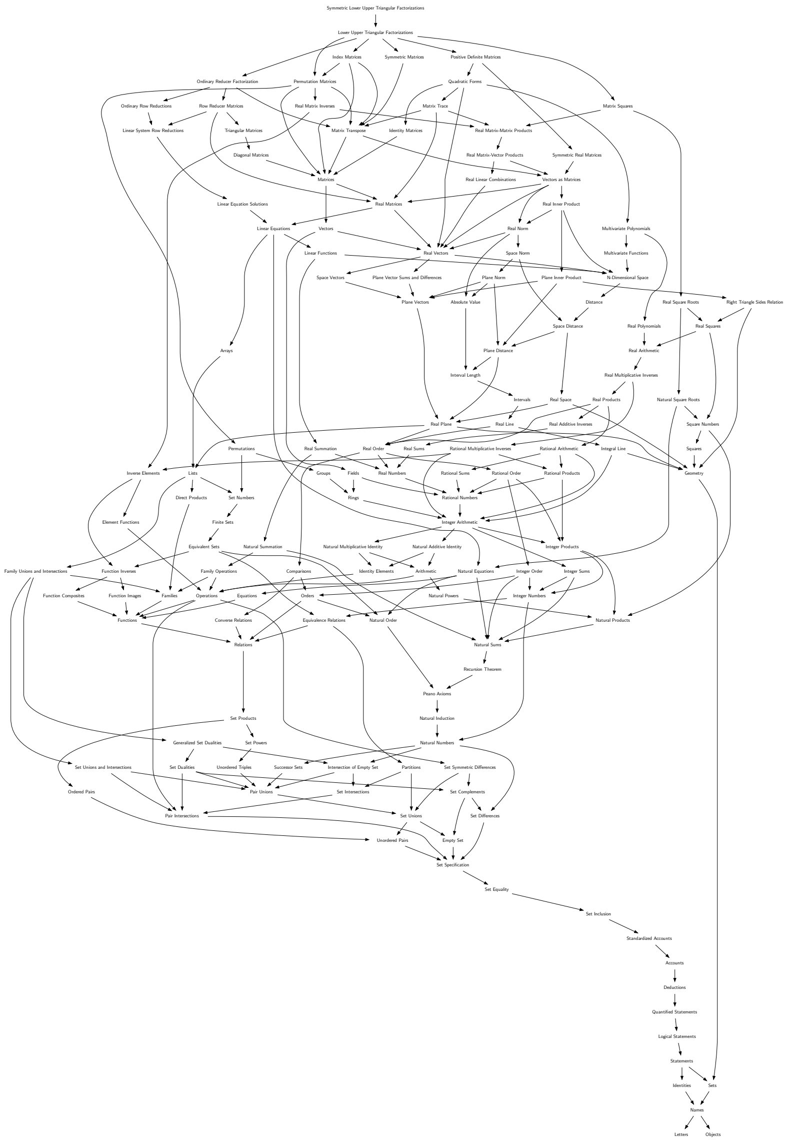
Unitriangular form

A *lower diagonal upper decomposition* (or *lower diagonal upper factorization*) of a matrix A a sequence $(L, D, L^\top)$ where $L \in \mathbf{R}^{n\times n}$ is unit lower triangular, $D \in \mathbf{R}^{n\times n}$ is diagonal with real nonnegative entries and

$$A = LDL^\top.$$

Other terminology includes *LDL decomposition*, *LDL factorization*, *LDU factorization*, *LDU decomposition*.

If $(L \in \mathbf{R}^{n\times n}, D \in \mathbf{R}^{n\times n}, L^\top)$ is a LDU decomposition of $A \in \mathbf{R}^{n\times n}$, then $(LD^{1/2}A = LDL^\top$ then $(\tilde{L}D^{1/2}, D^{1/2}L^\top)$ is a LU decomposition. Conversely, if $(B, B^\top)$ is a LU decomposition and S is the diagonal matrix satisfying $S_{ii} = B_{ii}$ for $i = 1, \dots, n$, then $(BS^{-1}, S^2, S^{-1}B^\top)$ is a LDU decomposition of A.


Symmetric Lower Upper Triangular Factorizations
Lower Upper Triangular Factorizations
Index Matrices
Symmetric Matrices
Positive Definite Matrices
Ordinary Reducer Factorization
Permutation Matrices
Quadratic Forms
Ordinary Row Reductions
Row Reducer Matrices
Real Matrix Inverses
Matrix Trace
Matrix Squares
Linear System Row Reductions
Triangular Matrices
Matrix Transpose
Identity Matrices
Real Matrix-Matrix Products
Diagonal Matrices
Real Matrix-Vector Products
Symmetric Real Matrices
Matrices
Real Linear Combinations
Vectors as Matrices
Linear Equation Solutions
Real Matrices
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Multivariate Functions
Space Vectors
Plane Vector Sums and Differences
Plane Norm
Plane Inner Product
N-Dimensional Space
Plane Vectors
Absolute Value
Distance
Real Square Roots
Right Triangle Sides Relation
Space Distance
Real Polynomials
Real Squares
Arrays
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Natural Square Roots
Real Plane
Real Line
Real Additive Inverses
Square Numbers
Permutations
Real Summation
Real Order
Real Sums
Rational Multiplicative Inverses
Rational Arithmetic
Integral Line
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Real Numbers
Rational Sums
Rational Order
Rational Products
Geometry
Direct Products
Set Numbers
Rings
Rational Numbers
Element Functions
Finite Sets
Integer Arithmetic
Equivalent Sets
Natural Summation
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Identity Elements
Arithmetic
Natural Equations
Integer Order
Integer Sums
Function Composites
Function Images
Families
Operations
Equations
Orders
Natural Powers
Integer Numbers
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Products
Relations
Natural Sums
Recursion Theorem
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects