



Why

If we interpret a list of two numbers as displacement in a plane, and a list of three numbers as displacement in a space, what of a list of n numbers as displacement in \mathbf{R}^n ?

Definition

A *real vector* (*vector*, *n -dimensional vector*, *n -vector*) is a length- n list of real numbers.

Algebra

For $x, y \in \mathbf{R}^n$, we define the *real vector sum* (or *sum*) of x and y as the vector $z \in \mathbf{R}^n$ where $z_i = x_i + y_i$ for $i = 1, \dots, n$. As usual, we denote the sum by $x + y$, so

$$x + y = (x_1 + y_1, \dots, x_n + y_n).$$

For $\alpha \in \mathbf{R}$ and $x \in \mathbf{R}^n$, *real scalar-vector product* (or *scalar product*, *product*) $z \in \mathbf{R}^n$ is defined by $z_i = \alpha x_i$ for $i = 1, \dots, n$. As usual, we denote the product αx , and write

$$\alpha x = (\alpha x_1, \dots, \alpha x_n).$$

Our intuition for both of these *operations* comes from their special cases in \mathbf{R}^2 and \mathbf{R}^3 . As usual, the *real-vector difference* (or *difference*) of x and y is the vector $z \in \mathbf{R}^n$ defined by $z_i = x_i - y_i$ for $i = 1, \dots, n$. As usual, we denote it by $x - y$, and note that $x - y = x + (-y)$.

The algebra given here for vectors is natural in view of their generalization as n -dimensional *displacements*. However, we keep in mind that this algebra is over lists of numbers, and that these sums and products can be defined on these lists of numbers regardless of interpretation.

