Why

We generalize the normal density to d-dimensional space.

Definition

Let $f : \mathbf{R}^d \to \mathbf{R}$ be a density such that

$$f(x) = \frac{1}{\sqrt{(2\pi)^d \det \Sigma}} \exp\left(-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\right)$$

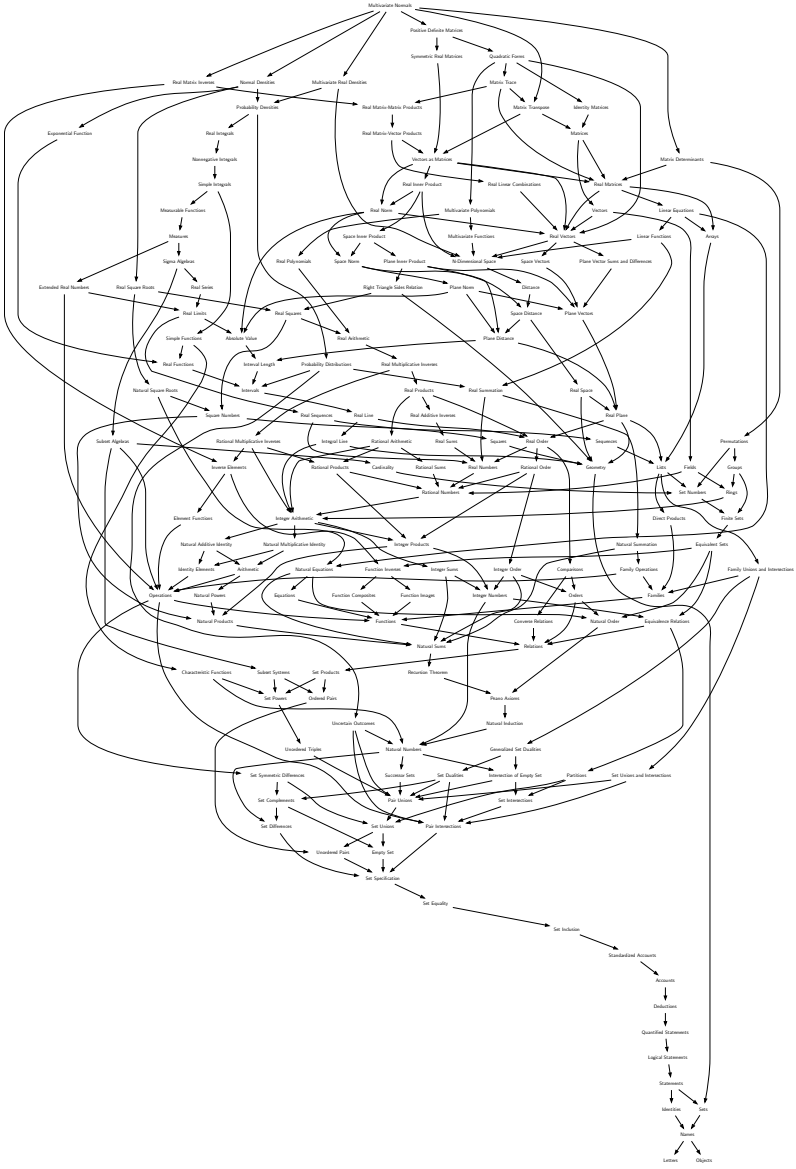
where $\mu \in \mathbf{R}^d$, $\Sigma \in \mathbf{S}^d$, and $\Sigma \succ 0$. We call f a *multivariate normal density*. A multivariate normal density with $d = 1$ is a normal density, so we refer to multivariate normal densities as *normal densities* without ambiguity. We frequently use the word normal as a substantive, and refer to *normals* when we mean multivariate normal densities. Many people call a multivariate normal distribution a *multivariate gaussian distribution* and speak of *gaussians* instead of normals.[1]

We call μ the *mean* and Σ the *covariance matrix*. We call Σ^{-1} the *precision matrix*.

Maximum

The maximum of a normal density is its mean, $\mu \in \mathbf{R}^d$.

[1] We avoid this usage in accordance with the project's policy on historical names.


Multivariate Nonzero
Positive Definite Matrices
Symmetric Real Matrices
Quadratic Forms
Real Matrix Inverses
Normal Densities
Multivariate Real Densities
Matrix Trace
Probability Densities
Real Matrix-Matrix Products
Matrix Transpose
Identity Matrices
Exponential Function
Real Integrals
Real Matrix-Vector Products
Matrices
Nonnegative Integrals
Vectors as Matrices
Matrix Determinants
Simple Integrals
Real Inner Product
Real Linear Combinations
Real Matrices
Measurable Functions
Real Norm
Multivariate Polynomials
Vectors
Linear Equations
Measures
Space Inner Product
Multivariate Functions
Real Vectors
Linear Functions
Angle
Sigma Algebras
Real Polynomials
Space Norm
Plane Inner Product
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Extended Real Numbers
Real Square Roots
Real Series
Right Triangle Sides Relation
Plane Norm
Distance
Real Limits
Real Squares
Space Distance
Plane Vectors
Simple Functions
Absolute Value
Real Arithmetic
Plane Distance
Real Functions
Interval Length
Probability Distributions
Real Multiplication Inverses
Natural Square Roots
Intervals
Real Products
Real Summation
Real Space
Square Numbers
Real Sequences
Real Line
Real Additive Inverses
Real Plane
Subset Algebras
Rational Multiplicative Inverses
Integral Line
Rational Arithmetic
Real Sums
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Real Order
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Element Functions
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Direct Products
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Natural Additive Identity
Natural Multiplication Identity
Integer Products
Natural Summation
Equivalent Sets
Identity Elements
Arithmetic
Natural Equations
Function Inverses
Integer Sums
Integer Order
Comparisons
Family Operations
Family Union and Intersections
Operations
Natural Powers
Equations
Function Composition
Function Images
Integer Numbers
Orders
Families
Natural Products
Functions
Cartesian Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Characteristic Functions
Subset Systems
Set Products
Recursion Theorem
Set Powers
Ordered Pairs
Peano Axioms
Natural Induction
Uncertain Outcomes
Natural Numbers
Unordered Triple
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects