



Index Matrices

1 Why

TODO

2 Definition

An *index sequence* of order n is a finite sequence of distinct elements of $\{1, 2, \dots, n\}$ whose length is less than or equal to n . We call the i th coordinate of an index sequence the *i -index* of the sequence. The *index matrix* associated with an index sequence is the $r \times n$ matrix whose i, j th entry is 1 if the index sequence's i th coordinate is j , and 0 otherwise. If $r = n$ then the index matrix is a permutation matrix.

Multiplying a vector by an index matrix produces a permuted subvector. The *subvector* of an n -vector associated with a length- r index sequence is the product of the $r \times n$ index matrix with the n -dimensional vector. Its i th entry is the i -index entry of the vector.

2.1 Other Terminology

Some authors use the term *index set* for index sequences; but since these are sequences (which are functions, and so relations, and so sets), they are not sets of indices, so we avoid this usage.

2.2 Notation

Let $r \leq n$ be natural numbers. Let $\alpha : \{1, 2, \dots, r\} \rightarrow \{1, 2, \dots, n\}$ be an index sequence. We denote the index matrix associated with α by P_α . This matrix P_α is an element of $\mathbf{N}^{r \times n}$ and is defined by

$$(P_\alpha)_{ij} = \begin{cases} 1 & j = \alpha(i) \\ 0 & \text{otherwise.} \end{cases}$$

Let A be a nonempty set and let $x \in A^n$. then the subvector of x associated with P_α (and so with α) is

$$P_\alpha x = (x_{\alpha(1)}, \dots, x_{\alpha(r)})$$

We denote the product $P_\alpha x$ by x_α .

We denote the product $P_\alpha X P_\alpha^\top$ by $X_{\alpha\alpha}$.

3 Multiplication

The product of the $n \times r$ transpose of an index matrix with an r vector is

