Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\operatorname{tr} : \mathbf{R}^{n\times n} \to \mathbf{R}$. The trace of $A \in \mathbf{R}^{n\times n}$ is

$$\operatorname{tr} A = \sum _i = 1^n A_ii.$$

Properties

Proposition 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}Matnn$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\begin{aligned}\operatorname{tr} C = \sum _i = 1^n C_ii &= \sum _i = 1^n \alpha A_ii + \beta B_ii \\ &= \alpha \sum _i = 1^n A_ii + \beta \sum _i = 1^n B_ii \\ &= \alpha \operatorname{tr} A + \beta \operatorname{tr} B.\end{aligned}$$

□

Proposition 2. *Let $A, B \in \mathbf{R}^{n\times n}$. Then*

$$\operatorname{tr}(AB) = \operatorname{tr}(BA).$$

In other words, "matrices commute under the trace operator."

Proposition 3. *Let $A \in \mathbf{R}^{n\times n}$. Then* $\operatorname{tr} A = \operatorname{tr} A^\top$.

Matrix Trace
Real Matrix-Matrix Products
Matrix Transpose
Real Matrix-Vector Products
Vectors as Matrices
Real Linear Combinations
Real Inner Product
Real Matrices
Space Inner Product
Real Norm
Linear Equations
Space Norm
Real Vectors
Linear Functions
Arrays
Plane Inner Product
Plane Norm
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Right Triangle Sides Relation
Absolute Value
Distance
Plane Vectors
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Square Numbers
Real Additive Inverses
Real Line
Real Plane
Square
Integral Line
Rational Arithmetic
Rational Multiplicative Inverses
Real Sums
Real Order
Real Summation
Geometry
Rational Sums
Rational Products
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Natural Summation
Equivalent Sets
Inverse Elements
Natural Powers
Integer Sums
Natural Equations
Integer Order
Comparisons
Family Operations
Element Functions
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Equations
Orders
Operations
Function Composites
Function Images
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Ordered Pairs
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects