Why

We want to visualize the probabilistic relations between components of outcomes in probabilistic models over large (e.g., product) outcome sets.[1]

Definition

Suppose $X_1, \ldots, X_k$ are sets. Define $X = \prod_{i=1}^k X_i$. For $x \in X$ and $S \subset \{1, \ldots, n\}$, denote the subvector of x indexed (in order) by S by x_S.[2]

A distribution $p : X \to [0, 1]$ *factors according to a directed graph* on $\{1, \ldots, n\}$ with parent function $\mathrm{pa} : \{1, \ldots, n\} \to \mathcal{P}(\{1, \ldots, n\})$ if

$$p(x) = \prod_{\mathrm{pa}_i = \varnothing} g_i(x_i) \prod_{\mathrm{pa}_i \neq \varnothing} g_i(x_i, x_{\mathrm{pa}_i}),$$

where g_i is a distribution for all i which $\mathrm{pa}_i = \varnothing$ and $g_i(\cdot, \xi)$ is a distribution for all $\xi \in \prod_{j \in \mathrm{pa}_i} A_j$, i for which $\mathrm{pa}_i \neq \varnothing$.

Proposition 1. *p so defined is a distribution, and the g_i are the marginals and conditionals.*[3]

Examples

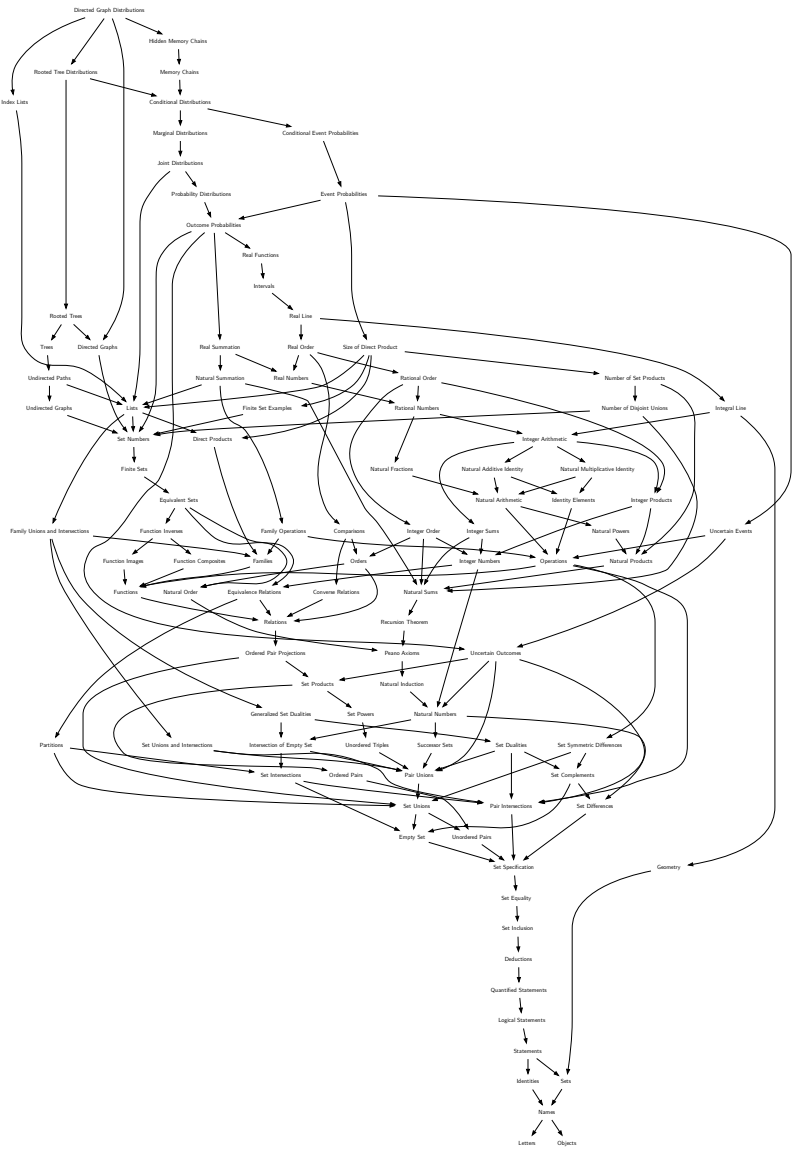
Consider a rooted tree distribution (see Rooted Tree Distributions), or a memory chain (see Memory Chains), or a hidden memory chain (see Hidden Memory Chains).[4]

[1] Future editions will modify and expand. The title of the sheet may change, since another interpretation for the words "directed graph distribution" is a distribution on directed graphs.

[2] Future editions will rework this treatment, perhaps combining it with the sheet Index Matrices, which will possibly be split up.

[3] Future editions will be precise and give an account.

[4] Future editions will expand.


Directed Graph Distributions
Hidden Memory Chains
Rooted Tree Distributions
Memory Chains
Index Lists
Conditional Distributions
Conditional Event Probabilities
Marginal Distributions
Joint Distributions
Probability Distributions
Event Probabilities
Outcome Probabilities
Real Functions
Intervals
Real Line
Rooted Trees
Trees
Directed Graphs
Real Summation
Real Order
Size of Direct Product
Undirected Paths
Natural Summation
Real Numbers
Rational Order
Number of Set Products
Undirected Graphs
Lists
Finite Set Examples
Rational Numbers
Number of Disjoint Unions
Integral Line
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Natural Fractions
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Natural Arithmetic
Identity Elements
Integer Products
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Powers
Uncertain Events
Function Images
Function Composites
Families
Orders
Integer Numbers
Operations
Natural Products
Functions
Natural Order
Equivalence Relations
Converse Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Uncertain Outcomes
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Partitions
Set Unions and Intersections
Intersection of Empty Set
Unordered Triples
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Set Unions
Pair Intersections
Set Differences
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects