Why

Can permuting the rows or columns of a matrix be represented by matrix multiplication?

Definition

Let $\sigma : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ be a permutation of n. The *permutation matrix* of σ is the matrix P defined by $P_{ij} = 1$ if $\sigma(i) = j$ and 0 otherwise. This is sometimes called the *column representation* (in contrast to the row representation, in which $P_{ij} = 1$ if $\sigma(j) = i$.

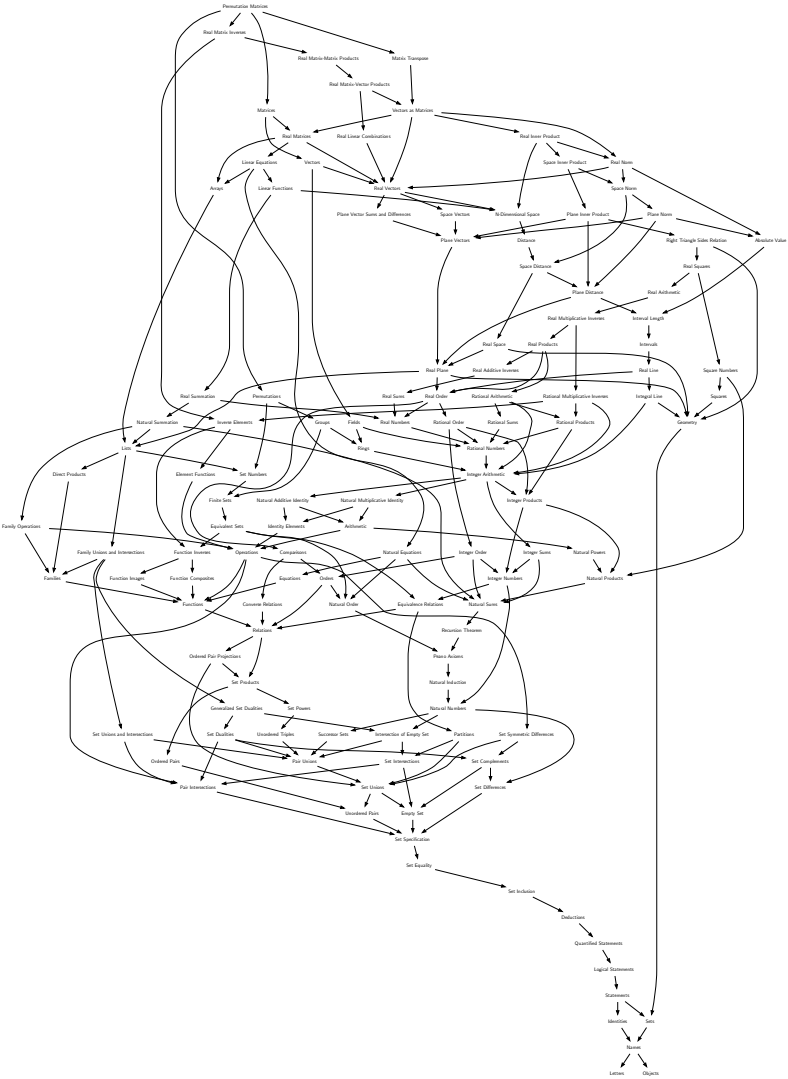
Let $A \in \mathbf{R}^{n \times n}$. Then pre-multipying A by P permutes the rows of A. In other words PA has the same rows as A but permuted according to σ. Similarly, post-multiplying by P permutes the columns of A. In other words, AP has the same columns as A but permuted according to σ. Clearly, we can also speak of permuting the components of a vector.

Composition

Let $\pi, \sigma \in S_n$ with corresponding permutation matrices P_σ and P_π. Then $P_\pi P_\sigma A$ has the same rows as A but permuted by $\pi\sigma$. Likewise, $AP_\pi P_\sigma$ has the same columns as A but permuted by $\pi\sigma$. Clearly, the identity permutation on $\{1, 2, \ldots, n\}$ is the identity $I \in \mathbf{R}^{n \times n}$.

Inverses

It is clear from the definition that $P_\sigma^{-1} = P_{\sigma^{-1}}$ and so if P is a permutation matrix then P^{-1} is $P^\top$.


Permutation Matrices
Real Matrix Inverses
Real Matrix-Matrix Products
Matrix Transpose
Real Matrix-Vector Products
Matrices
Vectors as Matrices
Real Matrices
Real Linear Combinations
Real Inner Product
Space Inner Product
Real Norm
Linear Equations
Vectors
Real Vectors
Space Norm
Arrays
Linear Functions
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Inner Product
Plane Norm
Plane Vectors
Distance
Right-Triangle Sides Relation
Absolute Value
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Real Multiplicative Inverses
Interval Length
Real Space
Real Products
Intervals
Real Plane
Real Additive Inverses
Real Line
Square Numbers
Real Summation
Permutations
Real Sums
Real Order
Rational Arithmetic
Rational Multiplication Inverses
Integral Line
Squares
Natural Summation
Inverse Elements
Groups
Fields
Real Numbers
Rational Order
Rational Sums
Rational Products
Counting
Lists
Rings
Natural Numbers
Direct Products
Element Functions
Set Numbers
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplication Identity
Integer Products
Equivalent Sets
Identity Elements
Arithmetic
Family Operations
Function Inverses
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Natural Equations
Integer Order
Integer Sums
Natural Powers
Family Unions and Intersections
Families
Function Images
Function Composition
Equations
Order
Integer Numbers
Natural Products
Functions
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Properties
Peano Axioms
Set Products
Natural Induction
Generalized Set Duality
Set Powers
Natural Numbers
Set Unions and Intersections
Set Duality
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects