Why

We generalize convex sets to arbitrary vector spaces.

Definition

Suppose V is a vector space over the real field $\mathbf{R}$. A set $C \subset V$ is *convex* if it contains the closed line segment between every pair of distinct points.

In other words, a set $C \subset V$ is convex if

$$\lambda x + (1 - \lambda)y \in C \quad \text{for all } x, y \in C \text{ and } \lambda \in [0, 1].$$

Concrete Sets

Real Concrete Sets

Topological Interiors

Real Halfspaces

Real Hyperplanes

Subspace Orthogonal Complements

Subspace Dimension

Vector Space Dimension

Vector Space Bases

Affine Set Dimension

Affinely Independent Vectors — Orthogonal Complements — Span — Linearly Dependent Vectors

Real Affine Sets and Subspaces — Real Affine Hulls — Real Subspace Dimension — Vector Subspaces

Real Affine Sets — Real Vector Bases — Real Subspaces — Linear Combinations — Real Inner Products — Vectors as Matrices

N-Dimensional Line Segments — Real Affine Combinations — Independent Set of Real Vectors — Real Vector Span — Real Inner Product — Vectors — Real Matrices — Topological Closures

Real Translates — N-Dimensional Lines — Topological Neighborhoods — Real Linear Combination — Space Inner Product — Real Norm — Real Vectors — Real Linear Equations — Linear Functions — Arrays — Real Set Closure

N-Dimensional Space — Metrics — Plane Inner Product — Plane Norm — Space Vector — Plane Vector Sums and Differences — Real Limits

Topologies — Space Norm

Right Triangle Side Relation — Distance — Space Vector — Absolute Value — Plane Vector

Real Squares — Space Distance

Real Arithmetic — Plane Distance

Real Multiplication Inverse — Interval Length

Intervals — Real Products — Real Space

Square Numbers — Real Line — Real Addition Inverse — Real Plane

Square — Integral Line — Real Order — Rational Arithmetic — Rational Multiplication Inverse — Real Sums

Geometry — Rational Order — Rational Products — Rational Sums — Fields — Real Numbers — Real Summation — Real Sequences

Rational Numbers — Rings — Natural Summation — Sequences

Integer Arithmetic — Natural Fractions — Lists

Integer Products — Natural Additive Identity — Natural Multiplication Identity — Inverse Elements — Set Numbers — Direct Products

Identity Elements — Natural Arithmetic — Element Functions — Natural Equations — Family Operations — Function Inverses — Finite Sets — Equivalent Sets — Family Unions and Intersections

Comparisons — Integer Order — Integer Sums — Natural Powers — Operations — Equations — Function Images — Function Composition — Families

Orders — Integer Numbers — Natural Products — Natural Sums — Natural Order — Equivalence Relations — Functions

Converse Relations — Recursion Theorem — Relations

Peano Axioms — Ordered Pair Projections

Natural Induction — Set Products

Natural Numbers — Set Powers — Generalized Set Dualities

Unordered Triple — Set Duality — Intersection of Empty Set — Set Partitions — Set Unions and Intersections

Set Symmetric Differences — Successor Sets — Pair Union — Set Intersections

Ordered Pairs — Set Complements — Set Unions — Pair Intersections

Set Differences

Unordered Pair — Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects