



# Measurable Functions

## 1 Why

We define integrals using an infinite process; in order for each step of the process to make sense, need functions to be measurable. Maybe: point to simple functions so that the why is clear.

## 2 Definition

A function between the base sets of two measurable spaces is **measurable** with respect to the distinguished sets of the two spaces if the inverse image of every distinguished subset of the codomain is a distinguished subset of the domain.

### 2.1 Notation

Let  $(X, \mathcal{A})$  and  $(Y, \mathcal{B})$  be measurable spaces. Then a function  $f : X \rightarrow Y$  is measurable if  $B \in \mathcal{B}$  implies  $f^{-1}(B) \in \mathcal{A}$ . We say that  $f$  is measurable with respect to  $\mathcal{A}$  and  $\mathcal{B}$ .

In this case, we sometimes say  $f$  is a measurable function from  $(X, \mathcal{A})$  to  $(Y, \mathcal{B})$ . We say,  $f : (X, \mathcal{A}) \rightarrow (Y, \mathcal{B})$  is measur-

able, read aloud as “f from X, A to Y, B is measurable.”