Why

Every matrix $A \in \mathbf{R}^{m \times n}$ maps the unit ball in $\mathbf{R}^n$ to an ellipsoid in $\mathbf{R}^m$.

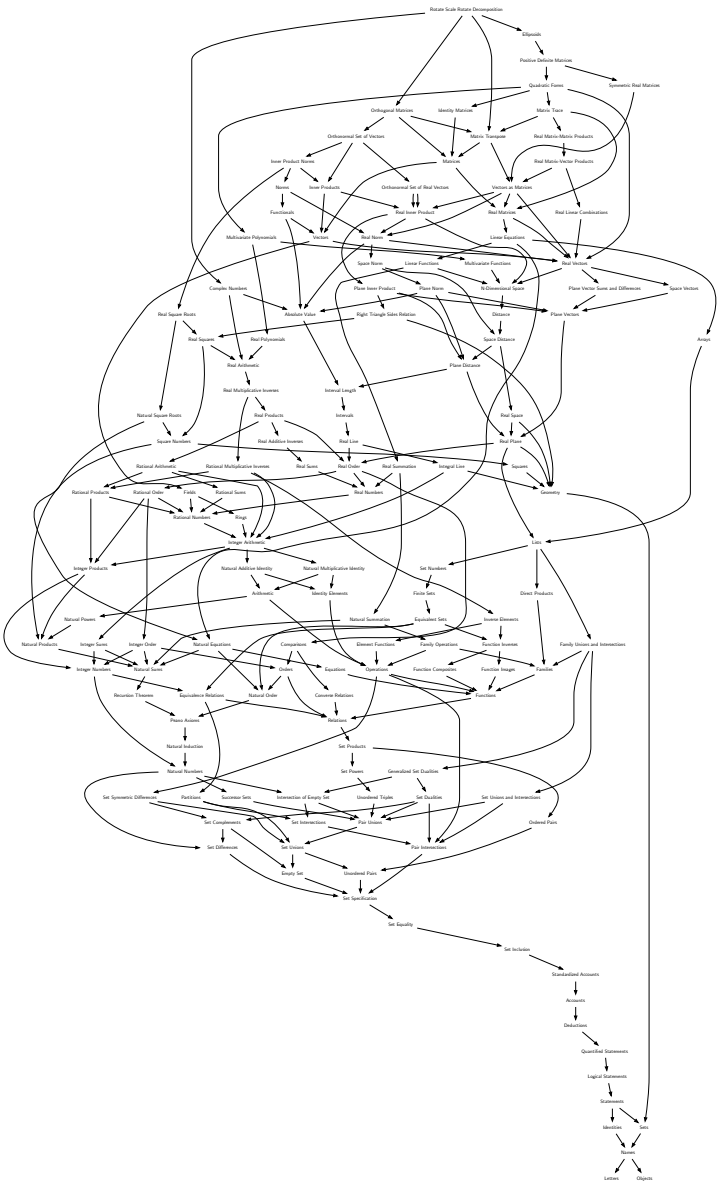
Definition

A *rotate scale rotate decomposition* (or *rotate scale rotate factorization*) of a matrix $A \in \mathbf{R}^{m \times n}$ is an ordered triple (U, S, V) where U and V are orthgonal and S is diagonal decreasing ($S_{11} \geq S_{22} \geq \cdots \geq S_{pp}$, where $p = \min\{m, n\}$) satisfying

$$A = USV^\top.$$

Other (universal) terminology includes the *singular value decomposition* or *SVD* of A. We call diagonal elements of S the *singular values* of A. We call the column vectors of U the *left singular vectors* or *output singular vectors*. We call the column vectors of V the *right singular vectors* or *input singular vectors*. We refer to them collectively as the *singular vectors*.

$$Av_i = \sigma_i u_i.$$


Rotate Scale Rotate Decomposition
Ellipsoids
Positive Definite Matrices
Quadratic Forms
Symmetric Real Matrices
Orthogonal Matrices
Identity Matrices
Matrix Trace
Orthonormal Set of Vectors
Matrix Transpose
Real Matrix-Matrix Products
Inner Product Norms
Matrices
Real Matrix-Vector Products
Norms
Inner Products
Orthonormal Set of Real Vectors
Vectors as Matrices
Functionals
Real Inner Products
Real Matrices
Real Linear Combinations
Multivariate Polynomials
Vectors
Real Norm
Linear Equations
Space Norm
Linear Functions
Multivariate Functions
Real Metrics
Complex Numbers
Plane Inner Product
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Real Square Roots
Absolute Value
Right Triangle Sides Relation
Distance
Plane Vectors
Real Squares
Real Polynomials
Space Distance
Arrays
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Intervals
Natural Square Roots
Real Products
Real Space
Square Numbers
Real Addition Inverses
Real Line
Real Plane
Rational Arithmetic
Rational Multiplication Inverses
Real Sums
Real Order
Real Summation
Integral Line
Squares
Rational Products
Rational Order
Fields
Rational Sums
Real Numbers
Geometry
Rational Numbers
Rings
Integer Arithmetic
Lists
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Arithmetic
Identity Elements
Finite Sets
Direct Products
Natural Powers
Natural Summation
Equivalent Sets
Inverse Elements
Natural Products
Integer Sums
Integer Order
Natural Equations
Comparisons
Element Functions
Family Operations
Exercise Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Orders
Equations
Operations
Function Composition
Function Images
Functions
Recursion Theorem
Equivalence Relations
Natural Order
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Unions
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Unions
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects