Why

The integrable functions are a vector space.

Definition

The *integrable function space* corresponding to a measure space is the set of real-valued functions which are integrable with respect to the measure. The term space is appropriate because this set is a real vector space. If we scale an integrable function, it remains integrable. If we add two integrable functions, the sum is integrable. Thus, a linear combination of integrable functions is integrable. The zero function is the zero element of the vector space.

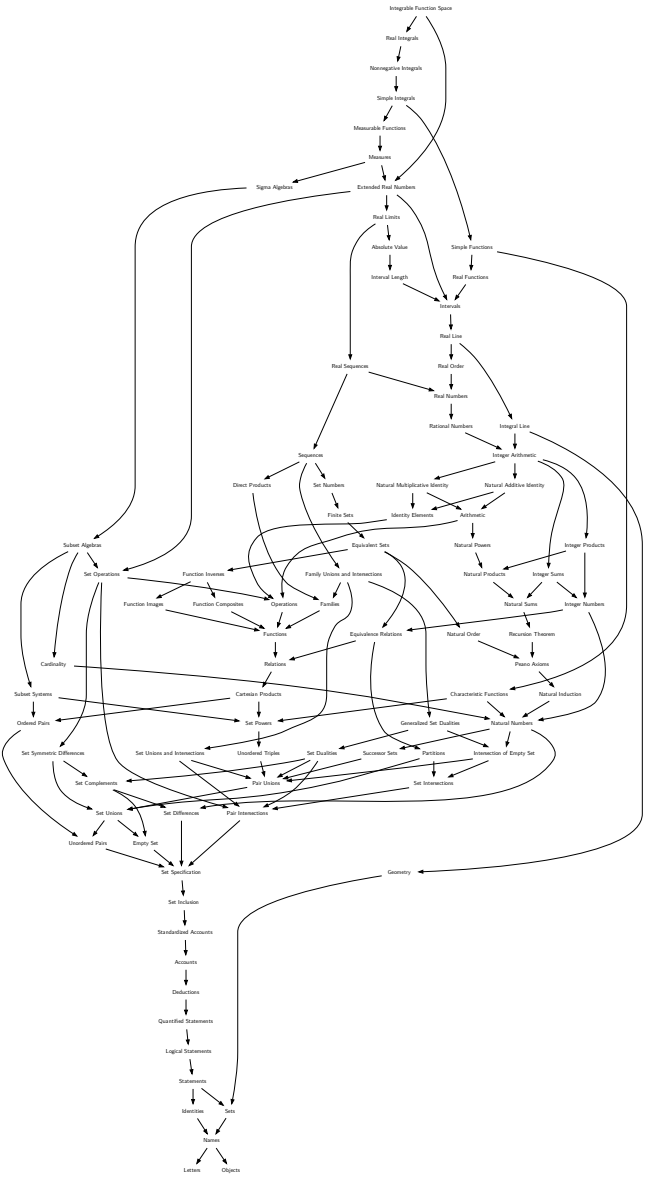
TODO The open question is: what elements of our geometric intuition can we bring to a space of functions. Do functions have a size? Are certain functions near each other?

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let R denote the set of real numbers and let C denote the set of complex numbers.

We denote set the real-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, R)$, read aloud as "the real integrable functions on the measure space X script A mu." We denote set the complex-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, C)$, read aloud as "the complex integrable functions on the measure space X script A mu." When the field is irrelevant, we denote them by

$\mathcal{I}(X, \mathcal{A}, \mu)$, read aloud as "integrable functions on the measure space X script A mu." The $\mathcal{I}$ is a mnemonic for "integrable."


Integrable Function Space
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Limits
Absolute Value
Simple Functions
Interval Length
Real Functions
Intervals
Real Line
Real Sequences
Real Order
Real Numbers
Rational Numbers
Integral Line
Sequences
Integer Arithmetic
Direct Products
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Identity Elements
Arithmetic
Subset Algebras
Equivalent Sets
Natural Powers
Integer Products
Set Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Sums
Function Images
Function Compositions
Operations
Families
Natural Sums
Integer Numbers
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Cardinality
Relations
Peano Axioms
Subset Systems
Cartesian Products
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Symmetric Differences
Set Unions and Intersections
Unordered Triples
Set Dualities
Successor Sets
Partitions
Intersection of Empty Set
Set Complements
Pair Unions
Set Intersections
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects