Result

This result is called sometimes called the *probability inverse transform.*

Proposition 1. *Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space and let $X : \Omega \to \mathbf{R}$ be a random variable with cumulative distribution function $F_X : \mathbf{R} \to [0,1]$. Suppose ${F_X}^{-1} : [0,1] \to \mathbf{R}$ exists, then $Y = {F_X}^{-1} \circ X$ is a random variable with cumulative distribution function $F_Y : [0,1] \to [0,1]$ satisfying $F_Y(y) = y$.*

Remark 1. *The conclusion is equivalent to the following: Y has a density and that density is the the standard unform density (see Uniform Densities).*

Proof. Express $F_Y(\gamma) = \mathbf{P}[Y \leq \gamma] = \mathbf{P}(Y^{-1}([0,\gamma]))$ Notice[1]

$$\begin{aligned} Y^{-1}([0,\gamma]]) &= \{\omega \in \Omega \mid Y(\omega) \leq \gamma\} \\ &= \{\omega \in \Omega \mid F_X(X(\omega)) \leq \gamma\} \\ &= \{\omega \in \Omega \mid X(\omega) \leq F_X^{-1}(\gamma)\}. \quad = X^{-1}(\cdots). \end{aligned}$$

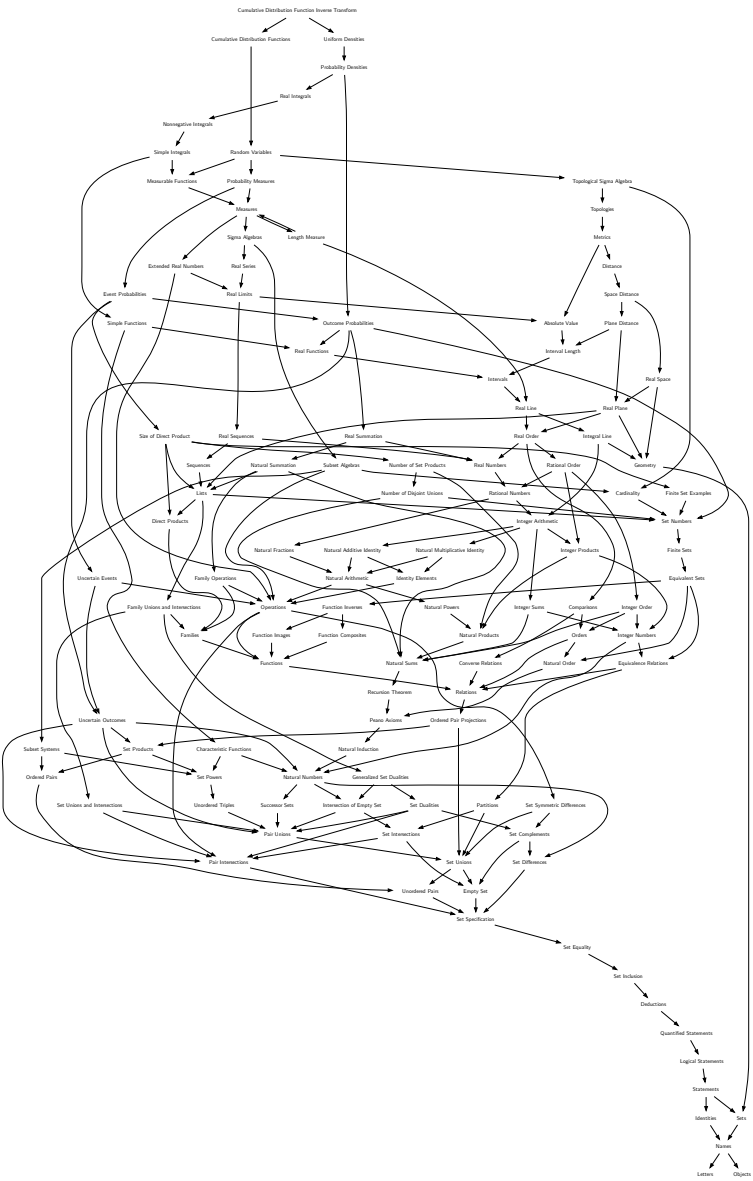
□

Remark 2. *Using different notation the above can be expressed succinctly as*

$$\begin{aligned} F_Y(\gamma) = \mathbf{P}[Y \leq \gamma] &= \mathbf{P}[F_X \circ X \leq \gamma] \\ &= \mathbf{P}[X \leq {F_X}^{-1}(\gamma)] = F_X({F_X}^{-1}(\gamma)) = \gamma. \end{aligned}$$

Future editions will discuss *inverse transform sampling.*

[1] Future editions will complete.


Cumulative Distribution Function Inverse Transform
Cumulative Distribution Functions
Uniform Densities
Probability Densities
Real Integrals
Nonnegative Integrals
Simple Integrals
Random Variables
Measurable Functions
Probability Measures
Topological Sigma Algebra
Topologies
Measures
Length Measure
Sigma Algebras
Metrics
Real Series
Distance
Extended Real Numbers
Real Limits
Space Distance
Event Probabilities
Absolute Value
Plane Distance
Simple Functions
Outcome Probabilities
Interval Length
Real Functions
Real Space
Intervals
Real Line
Real Plane
Real Order
Integral Line
Size of Direct Product
Real Sequences
Real Summation
Sequences
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Finite Set Examples
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Number of Disjoint Unions
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Integer Arithmetic
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Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Uncertain Events
Family Operations
Natural Arithmetic
Identity Elements
Equivalent Sets
Operations
Function Inverses
Natural Powers
Integer Sums
Comparisons
Integer Order
Family Unions and Intersections
Function Images
Function Composites
Natural Products
Orders
Integer Numbers
Families
Functions
Natural Sums
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Relations
Uncertain Outcomes
Peano Axioms
Ordered Pair Projections
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Set Unions and Intersections
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects