⬖ Product Sigma Algebras

1 Why

We want to generalize the construction of cover area as generated as a product of two cover lengths, and more generally for arbitrary measure spaces. TODO

2 Definition

Consider two measurable spaces. The *product base set* is the cartesian product of the first base set with the second base set. A first distinguished set is a distinguished set of the first measurable space, and likewise for a second distinguished set.

A *rectangle with measurable sides* is a set in product base set which is a product of a first distinguished set with a second distinguished set. The *product sigma algebra* is the sigma algebra generated by the rectangles with measurable sides.

The *product measurable space* is the measurable space whose base set is the product base set and whose sigma algebra is the product sigma algebra.

2.1 Notation

Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. The product base set is $X \times Y$. A set $R \in X \times Y$ is a rectangle with measurable sides if $R = A \times B$ for $A \in \mathcal{A}$ and $B \in \mathcal{B}$. We denote the product sigma algebra of $\mathcal{A}$ and $\mathcal{B}$ by $\mathcal{A} \times \mathcal{B}$. $(X \times Y, \mathcal{A} \times \mathcal{B})$ is the product measurable space.

Set Operations

Set Symmetric Differences

Alg

Set Unions

Set Interse