Sequences

Why

We introduce language for the steps of an infinite process.

Definition

Let A be a non-empty set. A *sequence in A* is a function from the natural numbers to the set. The *nth term* of a sequence is the result of the nth natural number; it is an element of the set.

Notation

Let A be a non-empty set. $a : \mathsf{N} \to A$. is a sequence in A. $a(n)$ is the nth term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n.

Subsequences

A *subindex* is a monotonically increasing function from and to the natural numbers. Roughly, it selects some ordered infinite subset of natural numbers. A *subsequence* of a first sequence is any second sequence which is the composition of the first sequence with a subindex.

Notation

Let $i : N \to N$ such that $n < m \implies i(n) < i(m)$. Then i is a subindex. Let $b = a \circ i$. Then b is a subsequence of a. We denote it by $\{b_{i(n)}\}_n$ and the nth term by $b_{i(n)}$.

TODO: integrate, from direct products

If I is the set of natural numbers we denote the direct product by

$$\prod_{i=1}^{\infty} A_i.$$

We denote an element of $\prod_{i=1}^{\infty} A_i$ by (a_i) with the understanding that $a_i \in A_i$ for all $i = 1, 2, 3, \ldots$. If $A_i = A$ for all $i = 1, 2, 3, \ldots$, then (a_i) is a sequence in A.

Sequences
Direct Products
Natural Families
Function Composites
Natural Order
Families
Functions
Relations
Natural Numbers
Ordered Pairs
Set Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Power Set
Set Inclusion
Sentences
Set Equality
Sets
Identity
Objects