Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? TODO: perhaps do L^2 first then generalize.

Definition

The *integrable function spaces* are a collection of function spaces, one for each real number $p \geq 1$, for which the pth power of the absolute value of the function is integrable.

TODO: case ∞

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $p \geq 1$. Let R denote the set of real numbers. We denote the integrable function space corresponding to p by $\mathcal{L}^p(X, \mathcal{A}, \mu, R)$. We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, R) = \left\{ \text{ measurable } f : X \to R \,\middle|\, \int |f|^p d\mu < \infty \right\}$$

Let C denote the set of complex numbers. Similarly for complex-valued functions, we denote the pth space by $\mathcal{L}^p(X, \mathcal{A}, \mu, C)$.

Integrable Function Spaces

Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions   Simple Functions

Characteristic Functions   Measures

Real Functions   Extended Real Numbers   Partitions

Intervals   Real Limits

Absolute Value   Real Sequences

Length   Natural Differences   Real Numbers   Sequences   Subset Algebras

Rational Numbers   Direct Products   Function Composites   Cardinality   Set Operations

Integer Numbers   Natural Order   Natural Families   Injective Functions   Algebras

Zero   Natural Sums   Families   Subset Systems   Operations

Equation Solutions   Natural Numbers   Power Set   Functions   Set Symmetric Differences

Set Intersections   Relations   Set Unions

Ordered Pairs   Set Complements

Empty Set   Unordered Pairs

Equations   Set Specification

Set Inclusion

Set Equality   Sentences

Identity   Sets

Objects