Convex Functions

1   Why

We speak of a function which always bends up.

2   Definition

A *convex* real-valued function is a function defined on a convex set of real numbers for which the result of the function on a convex combination of any two points in the domain is smaller than the convex combination of the same length of the value of the function on the endpoints.

2.1   Notation

Let R denote the set of real numbers. Let $A \subset R$ be a convex set. The function $f : A \to R$ is convex if, for any $a, b \in A$ and $t \in [0, 1]$,

$$f(ta + (1 - t)b) \leq tf(a) + (1 - t)f(b).$$