Why

We define the area under an extended real function.

Definition

The *positive part* of an extended-real-valued function is the function mapping each element to the maximum of the function's result and zero. The *negative part* of an extended-real-valued function is the function mapping each element to the maximum of the additive inverse of function's result and zero.

We decompose an extended-real-valued function as the difference of its positive part and its negative part. Both the positive and negative parts are non-negative extended-real-valued functions.

Consider a measure space. An *integrable* function is a measurable extended-real-valued function for which the non-negative integral of the posititve part and the non-negative integral of the negative part of the function are finite. The *integral* of an integrable function is the difference of the non-negative integral of the posititive part and and the non-negative integral of the negative part.

If one but not both of the parts of the function are finite, we say that the integral *exists* and again define it as before. In this way we avoid arithmetic between two infinities.

Notation

Suppose A nonempty and $g : A \to \bar{\mathbf{R}}$. We denote the positive part of g by g^+ and the negative part of g by g^- so that

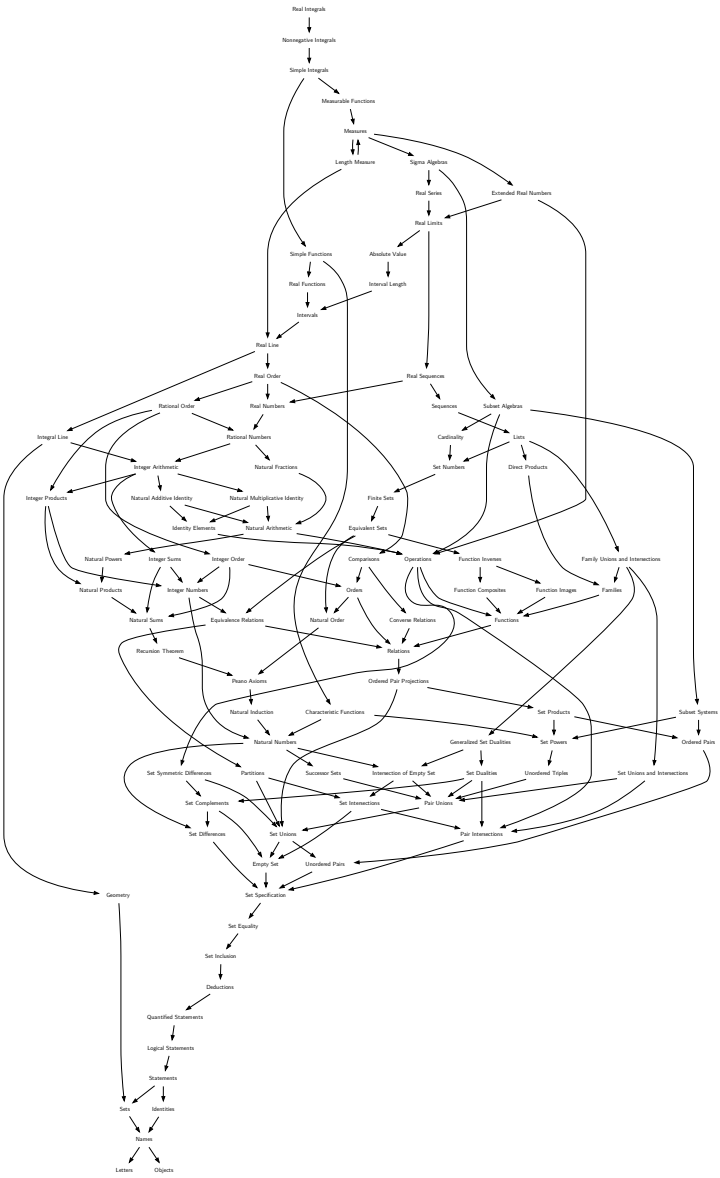
$$g^+(x) = \max\{g(x), 0\} \quad \text{and} \quad g^-(x) = \max\{-g(x), 0\} \quad \text{for all } x \in A$$

and $g = g^+ - g^-$. Of course, $g^+ \geq 0$ and $g^- \geq 0$.

Suppose $(X, \mathcal{A}, \mu)$ is a measure space and $f : X \to \bar{\mathbf{R}}$ is measurable and one of $\int f^+ d\mu$ or $\int f^- d\mu$ is finite (if both are finite, f is integrable).

We denote the integral of f with respect to the measure μ by $\int f d\mu$ so that

$$\int f d\mu = \int f^+ d\mu - \int f^- d\mu$$


Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Length Measure
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Simple Functions
Absolute Value
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Subset Algebras
Integral Line
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
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Objects