



## Why

Do the integer numbers correspond (in the sense of Homomorphisms) to elements of the rationals.

## Main result

Indeed, roughly speaking the integers correspond to rationals whose denominator is 1. Define

$$\tilde{Q} := \{[(a, b)] \in \mathbf{Q} \mid b = 1_{\mathbf{Z}}\}.$$

**Proposition 1.** *The rings  $(\tilde{Q}, +_{\mathbf{Q}} \mid \tilde{Q}, \cdot_{\mathbf{Q}} \mid \tilde{Q})$  and  $(Z, +_{\mathbf{Z}}, \cdot_{\mathbf{Z}})$  are homomorphic.<sup>1</sup>*

*Proof.* The function is  $f : \mathbf{Z} \rightarrow \mathbf{Q}$  with  $f(z) = [(z, 1)]$ .<sup>2</sup> □

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<sup>1</sup>Indeed, more is true and will be included in future editions. There is an *order preserving* ring homomorphism.

<sup>2</sup>The full account will appear in future editions.



