Why

We want to discuss making a single decision.[1] To discuss decisions, we first we speak of the choices to be made.

Definition

We have a set which includes all possible choices. The set is called the *actions*, *acts*, *decisions*, *choices* or *designs*.

We also have a set which includes all possible outcomes. Often the outcomes are uncertain, and associated with the future (see Uncertain Outcomes).

The intuition is that we will select an action prior to observing the outcome. We need some way to talk about which actions and outcomes are preferable to others. A *preference* is a total order on the product of the set of actions and outcomes.

A *simple decision problem* is a triple $(A, O, \preceq)$ in which A is a set of actions, O is a set of outcomes, and $\preceq$ is a preference.

Example: party

Consider deciding whether to host a party indoors or outdoors. We are unsure of the weather. We have a set of two actions $A = \{\textsc{In}, \textsc{Out}\}$ and a set of two outcomes $O = \{\textsc{Rain}, \textsc{Shine}\}$.

Although many orders on $A \times O$ exist, one such order is

$$(\textsc{Out}, \textsc{Shine}) \prec (\textsc{In}, \textsc{Rain}) \prec (\textsc{In}, \textsc{Shine}) \prec (\textsc{Out}, \textsc{Rain}).$$

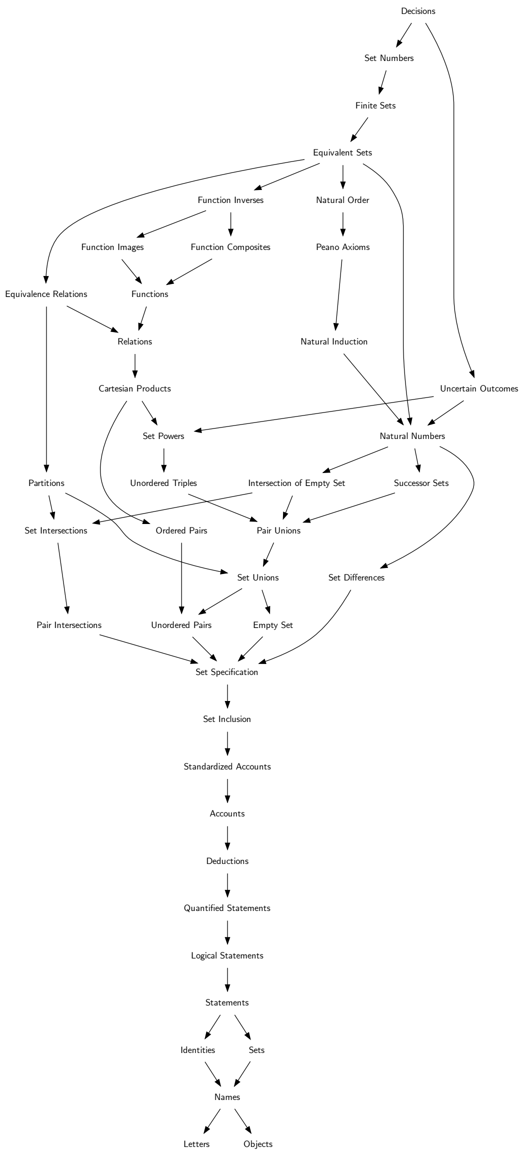
[1] Future editions will expand.

In other words, having the party outside in the sun is preferred to having it inside when it is raining, but both of these are preferred to having it inside when the sun is shining, and all of these are preferred to having it outdoors in the rain.

Best actions

Let $(A, O, \preceq)$ be a simple decision problem. Let $s \in S$. An action $a \in A$ is *best for outcome* o if $(a, o) \preceq (a', o)$ for all $a' \in A$. An action $a \in A$ is *best for all outcomes* (or *uniformly best*) if, for all $o \in O$, $(a, o) \preceq (a', o')$ for all $a' \in A, b' \in O$.

In the party example, the best action for $\textsc{Rain}$ is $\textsc{In}$ and the best action for $\textsc{Shine}$ is $\textsc{Out}$. On the other hand, there is no uniformly best action. For the action $\textsc{Out}$, there is an outcome $\textsc{Rain}$, and action-outcome pair $(\textsc{In}, \textsc{Rain})$ so that $(\textsc{In}, \textsc{Rain}) \prec (\textsc{Out}, \textsc{Rain})$. In other words, $\textsc{Out}$ is not uniformly best, because there is an outcome, $\textsc{Rain}$ for which the action $\textsc{In}$ is preferred. Similarly for $\textsc{In}$.


Decisions
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Function Images
Function Composites
Peano Axioms
Equivalence Relations
Functions
Relations
Natural Induction
Cartesian Products
Uncertain Outcomes
Set Powers
Natural Numbers
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Intersections
Ordered Pairs
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects