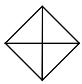


Why

Does convergence almost everywhere of a sequence of measurable functions guarantee measurability of the limit function? It does on complete measure spaces, and we can use this result to "weaken" the hypotheses of many theorems.

Results

A measure is *complete* if every subset of a measurable set of measure zero is measurable. If the measure is complete, then every negligible set must be measurable.

We begin with a transitivity property: almost everywhere equality of two functions allows us to infer measurability of one from the other.

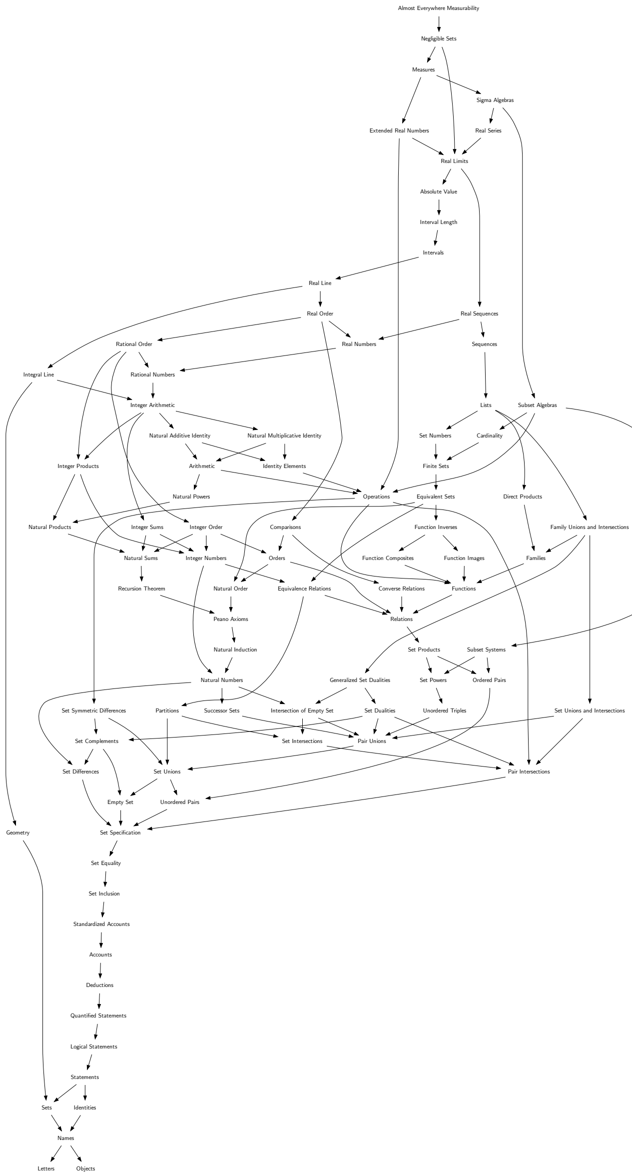
Prop. 1. *Let $(X, \mathcal{A}, \mu)$ be a measure space Let $f, g : X \to [-\infty, \infty]$ with $f = g$ almost everywhere. If μ is complete and f is $\mathcal{A}$-measurable, then g is $\mathcal{A}$-measurable.*

Proof. □

Prop. 2. *Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f_n : X \to [-\infty, \infty]$ for all natural numbers n and $f : X \to [-\infty, \infty]$ with $(f_n)_n$ converging to f almost everywhere. If μ is complete and and f_n is measurable for each n, then f is $\mathcal{A}$-measurable.*

Proof. [1] □

[1] Future editions will include.


Almost Everywhere Measurability
Negligible Sets
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Integral Line
Rational Numbers
Integer Arithmetic
Lists
Subset Algebras
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Cardinality
Integer Products
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Operations
Equivalent Sets
Direct Products
Natural Products
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Orders
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects