



Matrix Trace

1 Why

TODO

2 Definition

The *trace* of a square real matrix is the sum of the elements on its diagonal.

2.1 Notation

We denote the function which associates a matrix with its trace by $\mathbf{tr} : \mathbf{R}^{n \times n} \rightarrow \mathbf{R}$. Let $A \in \mathbf{R}^{n \times n}$. Then

$$\mathbf{tr} A = \sum_{i=1}^n A_{ii}.$$

3 Properties

Proposition 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n \times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\mathbf{tr} C = \sum_{i=1}^n C_{ii} = \sum_{i=1}^n \alpha A_{ii} + \beta B_{ii} = \alpha \sum_{i=1}^n A_{ii} + \beta \sum_{i=1}^n B_{ii} = \alpha \mathbf{tr} A + \beta \mathbf{tr} B.$$

□

Proposition 2. *Let $A, B \in \mathbf{R}^{n \times n}$.*

$$\mathbf{tr}(AB) = \mathbf{tr}(BA)$$

Proof. TODO

□