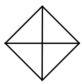


Why

A simple (albeit indirect) way to obtain a sigma algebra, is to start with some sets, and then to add all the sets needed to make the starting set closed under the various operations.

Definition

The *generated sigma algebra* for a set of subsets is the smallest sigma algbera containing the set of subsets. We must prove the existence and uniqueness of this sigma algebra.

Proposition 1. *The intersection of a non-empty set of sigma algebras over the same set is a sigma algebra.*

Proof. Given a family of sigma algebras $\{(A, \mathcal{A}_\alpha\}_{\alpha \in I}$ over some set, define $\mathcal{A} = \cap_{\alpha \in I} \mathcal{A}_\alpha$.

1. For all $\alpha \in I$, $A \in \mathcal{A}_\alpha$, thus $A \in \mathcal{A}$; condition (a).
2. For all $B \in \mathcal{A}$, for all $\alpha \in I$, $B \in \mathcal{A}_\alpha$. Thus, for all $\alpha \in I$, $C_A(B) \in \mathcal{A}_\alpha$. And so $C_A(B) \in \mathcal{A}$; condition (b).
3. For all sequences $\{B_n\} \subset \mathcal{A}$, $\{B_n\} \subset \mathcal{A}_\alpha$ for all α. Thus $\cup_n B_n \in \mathcal{A}_\alpha$ for all α and so $\cup_n B_n \in \mathcal{A}$; condition (c).

□

On the other hand, the union of a set of sigma algebras can fail to be a sigma algebra.

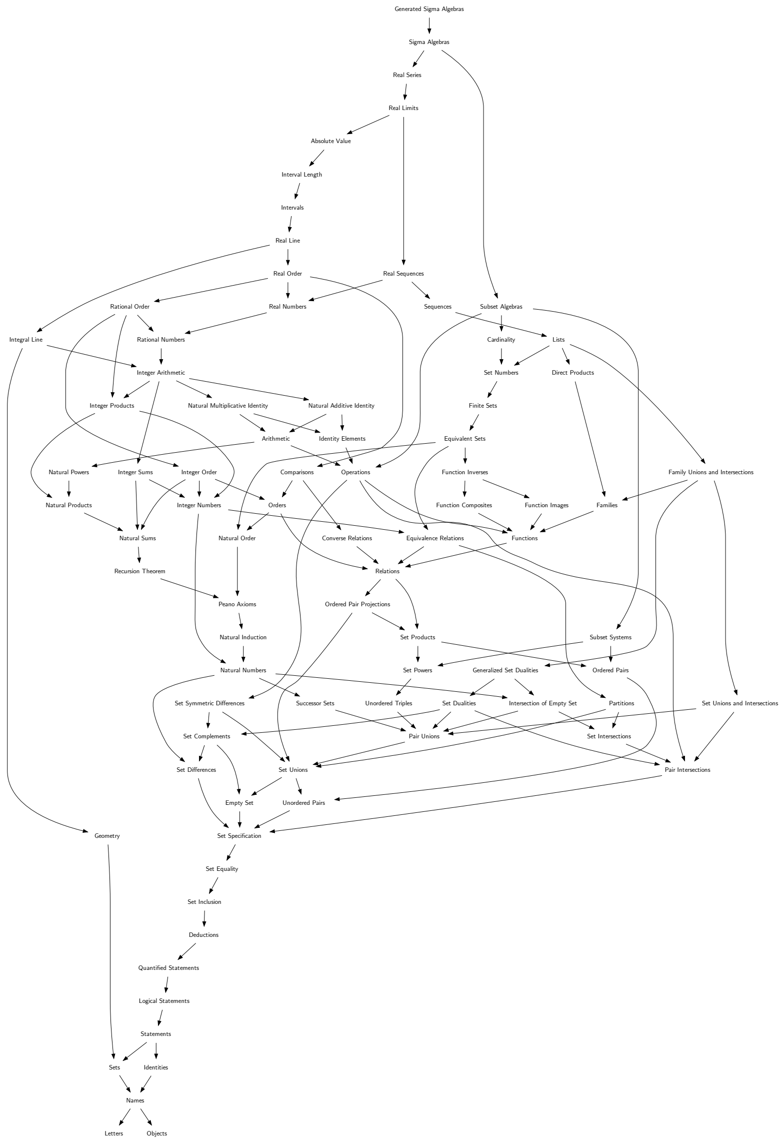
Proposition 2. *If A is a set and $\mathcal{A} \subset 2^A$, then there is a unique a smallest sigma algebra containing $\mathcal{A}$.*

Proof. We know of one sigma algebra containing $\mathcal{A}$: the power set of A. Thus, the set of sigma algebras containing $\mathcal{A}$ is not empty. Proposition **??** implies the intersection of all such sigma algebras (containing

$\mathcal{A}$) is a sigma algebra. The intersection contains $\mathcal{A}$, and is contained in all other sigma algebras with this property, so is a smallest sigma algebra containing $\mathcal{A}$. If $\mathcal{B}, \mathcal{C}$ were two smallest sigma algebras, then $\mathcal{B} \subset \mathcal{C}$ and $\mathcal{C} \subset \mathcal{B}$, but then $\mathcal{B} = \mathcal{C}$; thus the smallest sigma algebra is unique. $\square$

Notation

Let A be a set and $\mathcal{A} \subset \mathcal{P}(A)$. We denote the sigma algebra generated by $\mathcal{A}$ by $\sigma(\mathcal{A})$.


Generated Sigma Algebras
Sigma Algebras
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Subset Algebras
Integral Line
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Function Composites
Function Images
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Generalized Set Dualities
Ordered Pairs
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects