



## RATIONAL NUMBERS

### Why

We want fractions.<sup>1</sup>

### Definition

Consider  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ . We say that the elements  $(a, b)$  and  $(c, d)$  of this set are *rational equivalent* if  $ad = bc$ . Briefly, the intuition is that  $(a, b)$  represents  $a$  over  $b$ , or in the usual notation “ $a/b$ ”. So this equivalence relation says these two are the same if  $a/b = c/d$  or else  $ad = bc$ .

**Proposition 1.** *Rational equivalence is an equivalence relation on  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ .*

We define the *set of rational numbers* to be the set of equivalence classes (see **Equivalence Classes**) under rational equivalence on  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ . We call an element of the set of rational numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

### Notation

We denote the set of rationals by  $\mathbf{Q}$ .<sup>2</sup> If we denote rational equivalence by  $\sim$  then  $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$ .

---

<sup>1</sup>This why will be expanded in future editions.

<sup>2</sup>From what we can tell so far,  $\mathbf{Q}$  is a mnemonic for “quantity”, from the latin “quantitas”.



