



LINEAR COMBINATIONS

Why

We want to build vectors out of other vectors using scalar multiplication and vector addition.

Definition

A *linear combination* from a vector space is an ordered pair: the first coordinate is a sequence of vectors and the second is a sequence of scalars. The *result* of a linear combination is the sum of the results of scaling each vector by the corresponding scalar in the sequence; itself a vector in the space.

A *trivial linear combination* is one whose sequence of scalars is zero at each coordinate. The result of any trivial linear combination is the zero vector. A *nontrivial linear combination* is one which is not trivial. In other words, to be nontrivial, there must exist at least one index of the scalar sequence whose corresponding value is nonzero.

We say that a given vector *can be written as a linear combination of* a sequence of vectors if there exists a sequence of scalars such that the result of the linear combination of that sequence of vectors and scalars is the given vector. In other words, a vector can be written as a linear combination of some other vectors if there exists scalars for those other vectors such that scaling them and adding the results gives the vector.

Notation

Let (V, \mathbf{F}) be a vector space. Let $v = (v_1, \dots, v_n)$ be a sequence of vectors in V and $a = (a_1, \dots, a_n)$ be a sequence of scalars in \mathbf{F} . Then (v, a) is a linear combination and we can express its result by

$$a_1v_1 + a_2v_2 + \cdots + a_nv_n.$$

If (v, a) is trivial, then $a_i = 0$ for $i \in \{1, 2, \dots, n\}$ and the result of (v, a) is 0 (the zero vector). Otherwise, there exists $i \in \{1, 2, \dots, n\}$ such that $a_i \neq 0$; of course, the result of (v, a) may still be 0. A vector u can be written as a linear combination of the vectors v_1, v_2, \dots, v_n if there exists scalars a_1, a_2, \dots, a_n so that

$$u = a_1v_1 + a_2v_2 + \cdots + a_nv_n.$$

We often number (see **Lists**) a set of finite vectors. In this case, we say “Let $\{u_1, \dots, u_n\}$ be a (finite) set of vectors.”

