

#### INDEPENDENT RANDOM VARIABLES

# Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?

# **Definition**

Two random variables are *independent* if the sigma algebras generated by the random variables are independent. In general, a family of random variables are *independent* if the sigma algebras generated by the random variables are independent.

### Notation

Let  $(X, \mathcal{A}, \mu)$  be a probability space and  $(Y, \mathcal{B})$  be a measurable space. Let  $f_1, f_2 : X \to Y$  be random variables. If the random variables are independent we write  $f_1 \perp f_2$ .

#### Results

**Proposition 1.** Let  $f_1, \ldots, f_n$  be independent real-valued random variables defined on a probability space  $(X, \mathcal{A}, \mu)$ . Let  $B_1, \ldots, B_n$  be Borel sets of real numbers and let  $A_i = f_i^{-1}(B_i)$ . Let  $A = \bigcap_{i=1}^n f_i^{-1}(B_i)$ . Then

$$\mu(A) = \prod_{i=1}^{n} \mu(A_i)$$

*Proof.* Since  $f_i$  are independent, so are the sigma algebras they generate.  $A_i$  are in each of these sigma algebras, so by definition of independence the measure of the intersection is the product of the measures.

