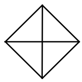


Why

1

Definition

The *event sigma algebra* of $A \in \mathcal{F}$ where $(\Omega, \mathcal{F})$ is a measurable space is the set sub-σ-algebra $\{\varnothing, A, A^c, \Omega\}$.

A family of events events are *independent* if the event sigma algebras are independent.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space. Let $A \in \mathcal{A}$ be an event. The sigma algebra generated by A is $\{\varnothing, A, X - A, X\}$. We denote it by $\sigma(A)$.

Let $B \in \mathcal{A}$. If A is independent of B we write $A \perp B$.

Equivalent Condition

Proposition 1. *Two events are independent if and only if the measure of their intersection is the product of their measures.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a probability space. Let $A, B \in \mathcal{A}$.

($\Rightarrow$) If $A \perp B$, then by definition $A \in \sigma(A)$ and $B \in \sigma(B)$ and so:

$$\mu(A \cap B) = \mu(A)\mu(B).$$

($\Leftarrow$) Conversely, let $a \in \sigma(A)$ and $b \in \sigma(B)$. If $a = \varnothing$ or $b = \varnothing$ then $a \cap b = \varnothing$. So

$$\mu(a \cap b) = \mu(\varnothing) = \mu(a)\mu(b),$$

since one of the two measures on the right hand side is zero. On the other hand, if $a = X$, then $a \cap b = b$ and so

$$\mu(a \cap b) = \mu(b) = \mu(a)\mu(b),$$

[1] Future editions will include

since $\mu(a) = \mu(X) = 1$. Likewise if $b = X$.

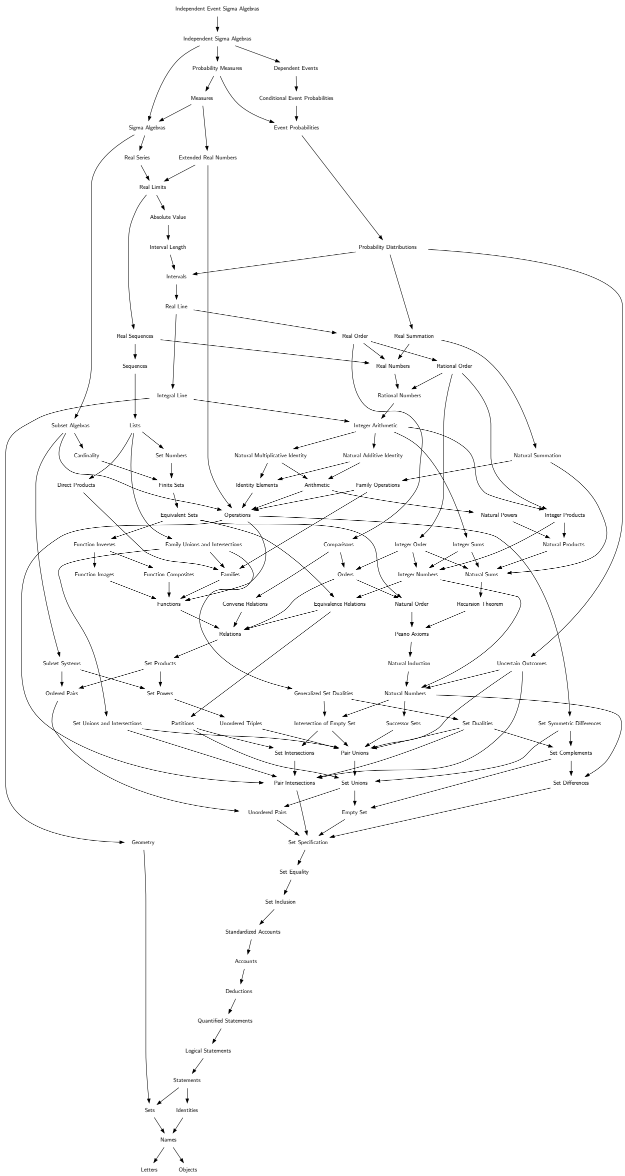
So it remains to verify $\mu(a \cap b) = \mu(a)\mu(b)$ for the cases $a \in \{A, X - A\}$ and $b \in \{B, X - B\}$. If $a = A$, and $b = B$, then the identity follows by hypothesis. Next, observe that $A \cap (X - B) = A - (A \cap B)$ and $(A \cap B) \subset A$ so $\mu(X) < \infty$ allows us to deduce:

$$\begin{aligned}
\mu(A \cap (X - B)) &= \mu(A - (A \cap B)) \\
&= \mu(A) - \mu(A \cap B) \\
&= \mu(A)(1 - \mu(B)) \\
&= \mu(A)\mu(X - A).
\end{aligned}$$

Similar for $X - A$ and B. Finally, recall that $\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B)$. So then,

$$\begin{aligned}
\mu((X - A) \cap (X - B)) &= 1 - \mu(A \cup B) \\
&= 1 - \mu(A) - \mu(B) + \mu(A \cap B) \\
&= 1 - \mu(A) - \mu(B) + \mu(A)\mu(B) \\
&= (1 - \mu(A))(1 - \mu(B)) \\
&= \mu(X - A)\mu(X - B).
\end{aligned}$$

□


Independent Event Sigma Algebras
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Probability Measures
Dependent Events
Measures
Conditional Event Probabilities
Sigma Algebras
Event Probabilities
Real Series
Extended Real Numbers
Real Limits
Absolute Value
Interval Length
Probability Distributions
Intervals
Real Line
Real Sequences
Real Order
Real Summation
Sequences
Real Numbers
Rational Order
Rational Numbers
Integral Line
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Natural Summation
Direct Products
Finite Sets
Identity Elements
Arithmetic
Family Operations
Equivalent Sets
Operations
Natural Powers
Integer Products
Function Inverses
Family Unions and Intersections
Comparisons
Integer Order
Integer Sums
Natural Products
Function Images
Function Composites
Families
Orders
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Natural Sums
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Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
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Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
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Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects