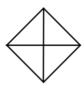


Why

If a directed graph has no cycles, then it has a nice property.[1]

Definition

Directed and acyclic graphs (sometimes *DAGs*) have some useful properties. Clearly, every subgraph induced on a directed acyclic graph is a directed acyclic graph.

Proposition 1. *Let (V, E) be a directed acyclic graph. Then there exists a vertex $v \in V$ which is a source and a vertex $w \in V$ which is a sink.*

Proof. There exists a directed path of maximum length. It must start at a source and end at a sink.[2] □

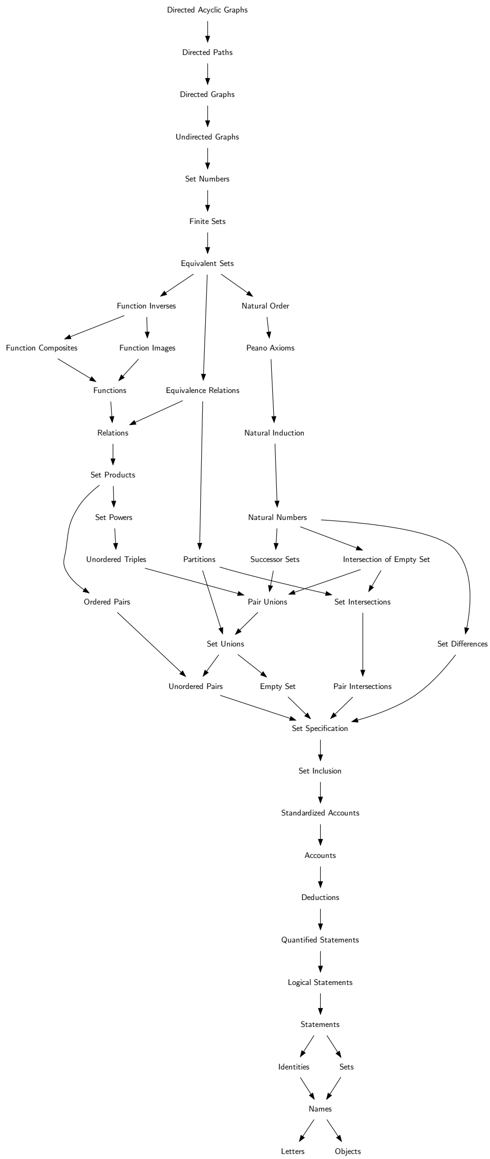
A *topological numbering*, *topological sort* or *topological ordering* of a directed graph (V, E) is a numbering $\sigma : \{1, \dots, |V|\} \to V$ satisfying

$$(v, w) \in E \longrightarrow \sigma^{-1}(v) < \sigma^{-1}(w).^3$$

Proposition 2. *There exists a topological sort for every acyclic graph.*

Proof. Let (V, F) be a directed acyclic graph. There exists a source vertex, v_1. Set $\sigma(1) = v_1$. Take the subgraph induced by $V - \{v_1\}$. It is directed acyclic, and so has a source vertex, v_2. Set $\sigma(2) = v_2$. Continue in this way.[4] □

[1] Future editions will expand this vague introduction.
[2] Future editions will expand.
[3] Future editions will further explain this concept.
[4] Future editions will clarify and expand.


Directed Acyclic Graphs
Directed Paths
Directed Graphs
Undirected Graphs
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Relations
Natural Induction
Set Products
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects