



**Why**

Given multiple orders, can we combine them?

**Discussion**

Suppose we have two orders  $\prec_1$  and  $\prec_2$  on  $A$ . Define  $\prec$  by  $a \prec b$  if and only if  $a \prec_1 b$  and  $b \prec_2 a$ . Notice that  $\prec$  is reflexive, transitive, and antisymmetric, and so it is a partial order. Call it the *combined order*.

Here's the rub. Even if  $\prec_1$  and  $\prec_2$  are total,  $\prec$  may not be total.<sup>1</sup> Consider the basic case  $A = \{a, b\}$  and  $\prec_1 = \{(a, a), (a, b), (b, b)\}$  and  $\prec_2 = \{(a, a), (b, a), (b, b)\}$ . Then  $\prec = \{(a, a), (b, b)\}$ , a partial order, to be sure, but not really any order at all.

There is not anything to be done about it, it is a fact. Total orders do not (necessarily) induce total combined orders.

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<sup>1</sup>Future editions will include and expand.



