Why

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Definition

Let $(A, +, \cdot)$ be a ring.

A *polynomial* in A of *degree* d is a function $p : A \to A$ for which there exists a finite sequence $c = (c_0, c_1, \dots, c_{d-1}, c_d) \in A^{d+1}$ satisfying

$$p(a) = c_0 + c_1 a^1 + c_2 a^2 + \cdots + c_d a^d,$$

for all $a \in A$. We call the sequence c the *polynomial coefficients*, and call the c_i the *coefficients* of p. We call $d + 1$ the *order* of the polynomial.

Clearly, to every polynomial in A of degree d there corresponds a sequence in A of length $d + 1$, and vice versa. For this reason, we can identify polynomials by their coefficients.

Examples

The function $f : A \to A$ is a polynomial of degree 0 and order 1 if there exists c_0 so that

$$f(a) = c_0$$

for all $a \in A$.

The function $g : A \to A$ is a polynomial of degree 1 and order 2 if there exists c_0 and c_1 so that

$$g(a) = c_0 + c_1 a$$

The function $h : A \to A$ is a polynomial of degree 2 and order 3 if there exists c_0 and c_1 so that

$$h(a) = c_0 + c_1 a + c_2 a^2.$$

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[1]Future editions will include, and most likely will build on quadratics and an appeal to the simplicity of the "natural" algebraic operations.

In other words, a second degree polynomial is a quadratic.

The function $p : A \to A$ is a *polynomial* of degree d and order $d+1$ if there exists a $d+1$ length sequence $(c_0, c_1, \ldots, c_d)$ in A so that

$$p(a) = c_0 + c_1 a + \cdots + c_d a^d.$$

Polynomials

Real Polynomials

Rings

Real Arithmetic

Real Multiplicative Inverses

Real Products

Real Additive Inverses

Rational Multiplicative Inverses

Rational Arithmetic

Real Sums

Real Order

Rational Products

Rational Sums

Real Numbers

Rational Order

Rational Numbers

Lists

Direct Products

Set Numbers

Integer Arithmetic

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Equivalent Sets

Inverse Elements

Identity Elements

Arithmetic

Family Unions and Intersections

Function Inverses

Element Functions

Integer Sums

Natural Powers

Integer Order

Comparisons

Families

Function Images

Function Composites

Operations

Natural Products

Integer Numbers

Orders

Functions

Equivalence Relations

Natural Order

Natural Sums

Converse Relations

Recursion Theorem

Relations

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Generalized Set Dualities

Natural Numbers

Set Powers

Set Unions and Intersections

Partitions

Set Dualities

Intersection of Empty Set

Set Symmetric Differences

Successor Sets

Unordered Triples

Set Intersections

Set Complements

Pair Unions

Ordered Pairs

Pair Intersections

Set Differences

Set Unions

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects