**Why**

In some sense, $\mathbf{R}^n$ is the only finite-dimensional vector space. In what sense?

**Definition**

An *isomorphism* is an invertible linear transformation between two vector spaces. Two vector spaces are *isomorphic* if there exists and isomorphism between them.

**Key Result**

**Prop. 1.** *Two finite-dimensional vector spaces are isomorphic if and only if they have the same dimension.*

**Corollary 1.** *Two finite-dimensional vector spaces are isomorphic if and only if they have the same dimension.*

Vector Space Isomorphisms

Vector Space Dimensions

Vector Space Bases

Linear Transformations — Linearly Dependent Vector — Span

Transformations — Linear Combinations — Subspaces

Vectors

Real Vectors

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals — Real Space

Real Line — Real Plane

Real Order — Integral Line

Fields — Real Numbers — Rational Order — Geometry

Groups — Rational Numbers

Integer Additive Inverses — Integer Arithmetic

Inverse Elements — Natural Multiplicative Identity — Natural Additive Identity — Integer Products — Lists

Element Functions — Identity Elements — Arithmetic — Set Numbers

Integer Sums — Algebras — Natural Powers — Finite Sets — Direct Products

Operations — Natural Products — Function Inverses — Integer Order — Comparisons — Equivalent Sets — Family Unions and Intersections

Natural Sums — Function Composites — Function Images — Integer Numbers — Orders — Families

Recursion Theorem — Functions — Natural Order — Converse Relations — Equivalence Relations

Peano Axioms — Relations

Natural Induction — Set Products — Generalized Set Dualities

Natural Numbers — Set Powers — Intersection of Empty Set — Partitions — Set Dualities — Set Unions and Intersections

Successor Sets — Unordered Triples

Ordered Pairs — Pair Unions — Set Intersections — Set Complements

Set Unions — Set Differences — Pair Intersections

Unordered Pairs — Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects