Definition

A *subspace* of a vector space is a subset of vectors that is a vector space when restriction vector addition and scalar multiplication to the set. In other words, a subspace is a subset of a vector space which is closed under vector addition and scalar multiplication.

For example, the entire set of vectors is a subspace. As a second example, the set consisting only of the zero vector is a subspace; we call this the *zero subspace*. These two subspaces are the *trivial subspaces*. A *nontrivial subspace* is a subspace that is not trivial.

Notation

Let $(V, \mathbf{F})$ be a vector space. Let $U \subset V$ with

$$\alpha u + \beta v \in U$$

for all $\alpha, \beta \in \mathbf{F}$ and $u, v \in U$. Then U is a subspace of $(V, \mathbf{F})$.

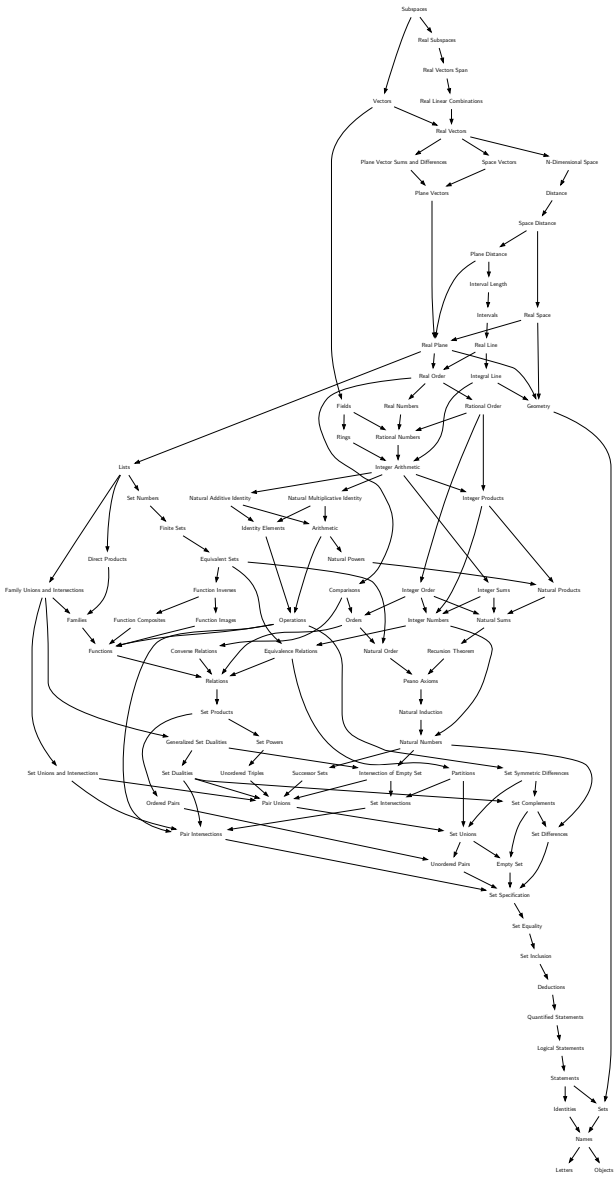
Properties

Proposition 1. *The intersection of a family of subspaces is a subspace.*

Proposition 2. *There exists a family of subspaces whose union is not a subspace;*

Remark 1. *In other words: the union of a family subspaces need not be a subspace.*

Proposition 3. *A subspace must contain the zero vector; in other words, every subspace is nonempty.*


Subspaces
Real Subspaces
Real Vectors Span
Vectors
Real Linear Combinations
Real Vectors
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Real Order
Integral Line
Fields
Real Numbers
Rational Order
Geometry
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects