

## Powers and Unions

## Why

How does the power set relate to a union?

**§Notation Preliminaries** 

Let E denote a set. Let  $\mathcal{A}$  denote a set of subsets of the set denoted by E. We define  $\bigcup_{A \in \mathcal{A}} A$  to mean  $\cap \mathcal{A}$ .

## **Basic Properties**

Here are some basic interactions between the powerset and unions.<sup>1</sup>

**Proposition 1.**  $E^* \cup F^* \subset (E \cup F)^*$ 

Proposition 2.  $\bigcup_{X \in \mathcal{C}} X^* \subset (\bigcup_{X \in \mathcal{C}} X)^*$ 

Proposition 3.  $E = \bigcup E^*$ 

**Proposition 4.**  $(\bigcup E)^* \supset E$ .

Typically  $E \neq (\bigcup E)^*$ , in which case E is a proper subset.

<sup>&</sup>lt;sup>1</sup>Future editions will expand on these propositions and provide accounts of them.

