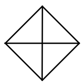

Why

We have seen that the matrices are a vector space. Are they an inner product space?

Definition

The *matrix scalar product* of $A \in \mathbf{R}^{n \times k}$ and $B \in \mathbf{R}^{n \times k}$ is the following product

$$\sum_{i=1}^{n} \sum_{j=1}^{k} a_{ij} b_{ij}.$$

Using the matrix trace, we can denote this as $\operatorname{tr} A^\top B$. Some authors call this the *Euclidean matrix scalar product*, *matrix inner product* or *Frobenius inner product*.

Proposition 1. *The matrix scalar product is an inner product.*

For example, symmetry of the product is a consequence of the fact that a square matrix and its transpose have identical traces. Commutativity of the trace yields $\operatorname{tr} A^\top B = \operatorname{tr} B A^\top$, where the LHS is the scalar product of $B^\top$ and $A^\top$. In other words, transposition "preserves" the matrix scalar product.

With this inner product, $\mathbf{R}^{n \times k}$ is a Euclidean vector space (see Inner procuts) of dimension nk. For the case of $k = 1$, we recover a model[1] for the usual space $\mathbf{R}^n$.

Notation

We commonly denote the matrix inner product by $\langle A, B \rangle$.

Induced norm

The matrix inner product induces a norm in the usual way. This norm is sometimes called the *matrix-vector norm* (or *Frobenius norm*) and is

[1] Future editions will define this term.

often denoted for a matrix $A \in \mathbf{R}^{m \times n}$ by $\|A\|_F$.