Set Equality

1 Why

When are two sets the same?

2 Definition

Consider the sets A and B. If A is B, then every element of A is an element of B and every element of B is an element of A.

What of the converse? If every element of A is an element of B and vice versa is A the same as B? We axiomatically answer this question in the affirmative.

Thus: Two sets are the same if and only if they have the same elements. We call the elements of a set its *extension* and this above statement is sometimes called the *axiom of extension*.

The importance is that we have given ourselves a way to argue two sets are equivalent. Argue the consequence of the first paragraph, and the use the axiom of extension to conclude that the sets are the same.

2.1 Notation

Let A and B be two sets. As with any objects, we denote that A and B are equal by $A = B$. The axiom of extension is

$$A = B \Leftrightarrow (a \in A \implies a \in B) \wedge (b \in B \implies b \in A).$$