Why

We want to visualize (nonsymmetric) relations.

Definition

A *directed graph* is a pair (V, E) in which V is a finite nonempty set and E is a subset of $V \times V$. In other words, E is a relation on V. We call the elements of the first set the *vertices* of the graph and the elements of the second set the *edges.*

Let $(v, w) \in E$. We say that (v, w) is an edge *from v to w*, and that it is an *outgoing edge* of v and an *incoming edge* of w. We call v a *parent* of w and we call w a child of v. We say that the edge (v, w) is *incident* to v and w.

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

Notation

Let $\mathrm{pa} : V \to \mathcal{P}(V)$ and $\mathrm{ch} : V \to \mathcal{P}(V)$ be the functions associating to each vertex its set of parents and set of children, respectively. As usual, we denote the parents of vertex v by pa_v and the children by ch_v.

Skeletons

The *skeleton* of the directed graph (V, E) is the undirected graph (V, F) where

$$F = \{\{v, w\} \subset V \mid (v, w) \in E \text{ or } (w, v) \in E\}.$$

In other words, the skeleton is an undirected graph whose vertex set is V and whose edges are all (unordered) pairs which appear as an ordered pair in the directed graph. If the (V, E) is a directed graph and E is a symmetric relation, then we the skeleton of (V, E) is a natural undirected graph to associate with (V, E). An *orientation* of an undirected graph G is a directed graph whose skeleton is G.

Directed Graphs

Undirected Graphs

Set Numbers

Finite Sets

Equivalent Sets

Function Inverses

Natural Order

Function Composites

Function Images

Peano Axioms

Functions

Equivalence Relations

Relations

Natural Induction

Set Products

Set Powers

Natural Numbers

Unordered Triples

Partitions

Successor Sets

Intersection of Empty Set

Ordered Pairs

Pair Unions

Set Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Pair Intersections

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects