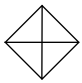


Why

We compress the notation for linear equations.

Definition

A *real matrix* (*real-valued matrix*, *matrix of real numbers*, *matrix*) is a two-dimensional array of real numbers. Recall that we are interested in solutions of the linear equations

$$\begin{aligned} y_1 &= A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n, \\ y_2 &= A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n, \\ &\vdots \\ y_n &= A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n. \end{aligned}$$

We have suggestively used the notation A_{ij} for the coefficients of the equations, so they are the entries of a matrix $A \in \mathbf{R}^{m \times n}$.

We call n and m the *dimensions* of the matrix. We call n the *height* and m the *width.* If the height of the matrix is the same as the width of the matrix then we call the matrix *square.* If the height is larger than the width, we call the matrix *tall.* If the width is larger than the height, we call the matrix *wide.*

Matrix-vector products

Using the notation $A \in \mathbf{R}^{m \times n}$ and $x \in \mathbf{R}^n$ we want a compressed way to write the above system of linear equations. Define the *real matrix-vector product* z of A with x by $z_i = \sum_{j=1}^{n} A_{ij}x_j$ We denote the matrix vector product z by Ax.

Notation

We express the above system of linear equations as

$$y = Ax,$$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}, \; A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}, \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}.$$

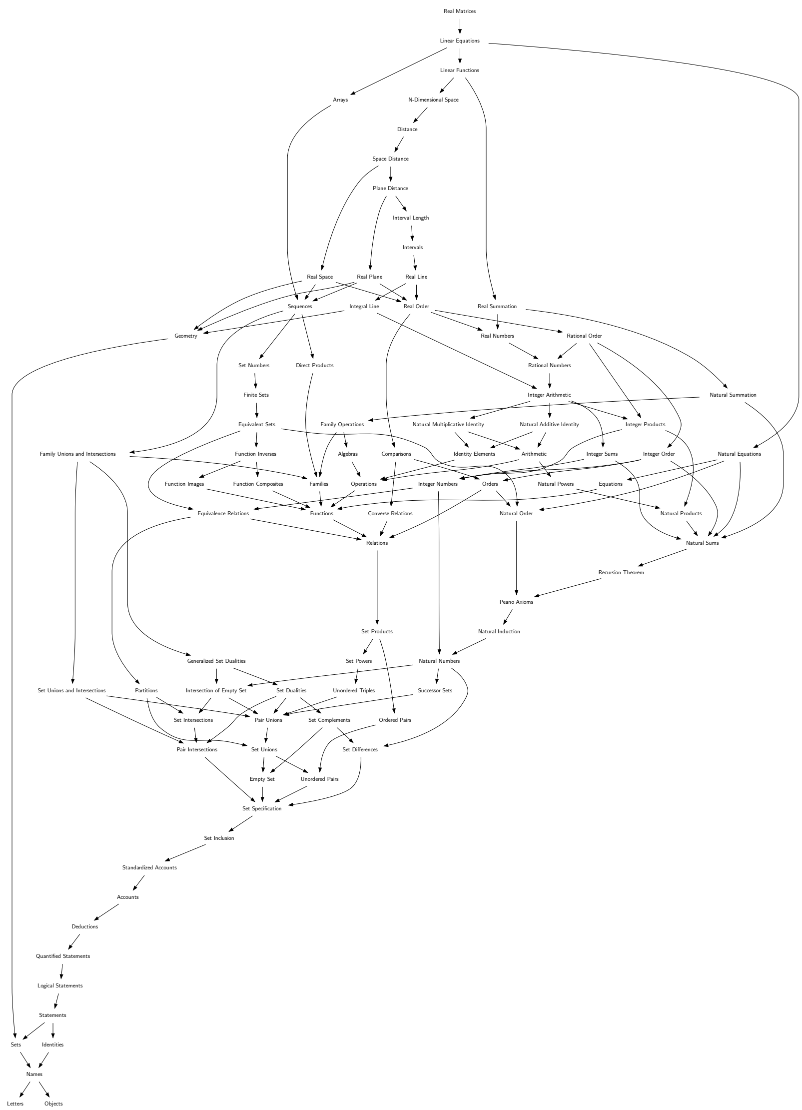
The compact notation $y = Ax$ is sometimes called the *matrix form* of the m linear equations and A the *coefficient matrix.*

This notation suggests both algebraic and geometric interpretations of solving systems of linear equations. The algebraic interpretation is that we are interested in the invertibility of the function $x \mapsto Ax$. In other words, we are interested in the existence of an inverse element of A. The geometric interpretation is that A transforms the vector x.

Note on terminology

The etymology of the word "matrix" is from the Latin "mater," meaning mother, and has an old sense in the English language similar to the sense of the English word "womb." The matrix is source of many determinants (discussed later).[1]

[1] Future editions may elide this discussion.


Real Matrices
Linear Equations
Linear Functions
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Sequences
Integral Line
Real Order
Real Summation
Geometry
Real Numbers
Rational Order
Set Numbers
Direct Products
Rational Numbers
Finite Sets
Integer Arithmetic
Natural Summation
Equivalent Sets
Family Operations
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Family Unions and Intersections
Function Inverses
Algebras
Comparisons
Identity Elements
Arithmetic
Integer Sums
Integer Order
Natural Equations
Function Images
Function Composites
Families
Operations
Integer Numbers
Orders
Natural Powers
Equations
Equivalence Relations
Functions
Converse Relations
Natural Order
Natural Products
Relations
Natural Sums
Recursion Theorem
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects