$$\diamondplus$$

Set Specification

1 Why

Can we always construct subsets?

2 Definition

We will say that we can. We assert that to every set and every sentence predicated of elements of the set there exists a second set (a subset of the first) whose elements satisfy the sentence. It is an consequence of the axiom of extension that this set is unique.

This is sometimes called the *axiom of specification*. We call it the set obtained from the first set by *specifying* elements according to the sentence.

2.1 Notation

Let A be a set. Let $S(a)$ be a sentence. We use the notation

$$\{\, a \in A \mid \ S(a) \,\}$$

to denote the subset of A specified by S. We read the symbol $\mid$ aloud as "such that." We read the whole notation aloud as "a in A such that..."

We call the notation *set-builder notation*. Set-builder notation avoids enumerating elements. This notation is really indispensable for sets which have many members, too many to reasonably write down.

3 Example

For example, let a, b, c, d be distinct objects. Let $A = \{a, b, c, d\}$. Then $\{x \in A \mid x \neq a\}$ is the set $\{b, c, d\}$

Now let B be an arbitrary set. The set $\{b \in B \mid b \neq b\}$ specifies the empty set. Since the statement $b \neq b$ is false for all objects b.