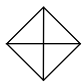


## Why

We want language and notation for selecting some of the entries (possibly, with reordering—i.e. permuting) from a list.

## Definition

An *index list* of *order* $n$ and *length* $r \leq n$ is a list of *distinct* elements of $\{1, 2, \dots, n\}$. Its *$i$-index* is the $i$th coordinate, where $i = 1, \dots, r$.

### Examples

Here are some index lists of order 5: $(1, 2, 3)$, $(3, 2, 1)$, $(4, 5, 1)$, $(5, 4, 3, 2, 1)$, $(3, )$. These have lengths 3, 3, 3, 7 and 1, respectively. The 3-index of the first is 3, and of the second is 1.

## Induced sublist

The *sublist* of an length-$n$ list $x$ *induced* by a length-$r$ index list $\alpha$ is the length-$r$ list $y$ whose *$i$th* entry is the value $x_{\alpha_i}$. In other words,

$$y_i = x_{\alpha_i}$$

For example, define $x = (6, 4, 5, 3, 9)$. The sublists associated with the example index lists above are $(6, 4, 5)$, $(5, 4, 6)$, $(3, 9, 6)$, $(9, 3, 5, 4, 6)$ and $(5, )$.

For a particular case, the third holds because

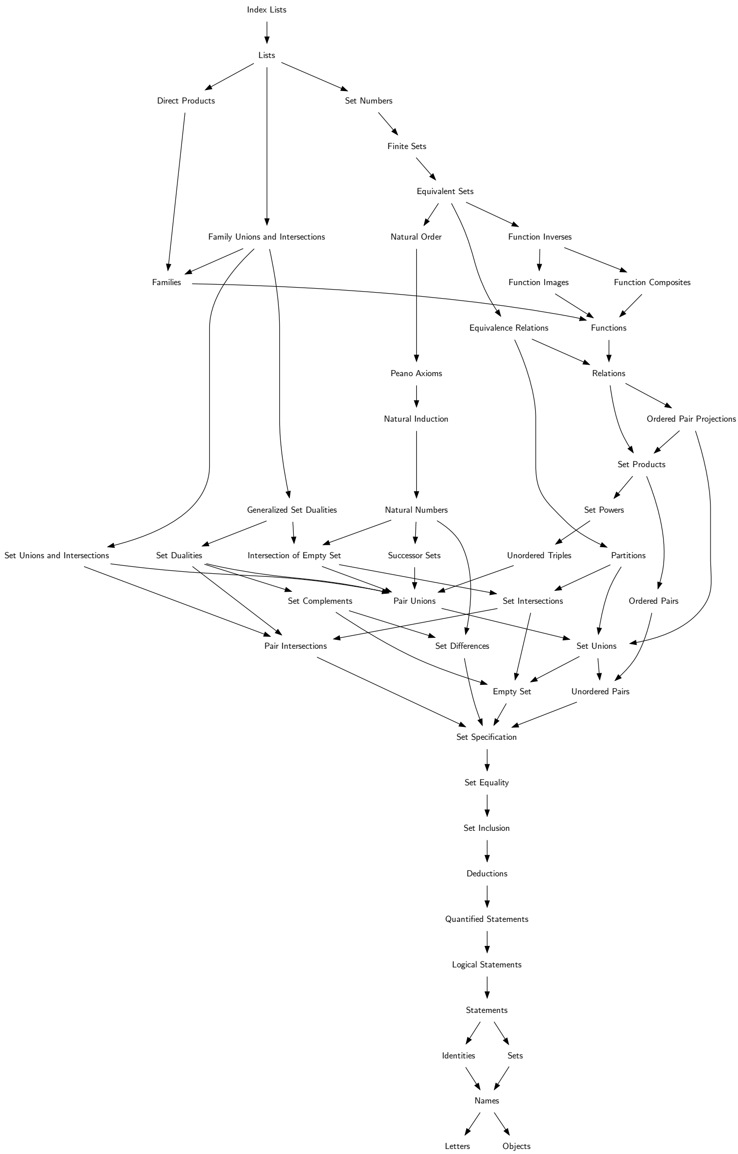
$$(3, 9, 6) = (x_4, x_5, x_1) = (x_{\alpha_1}, x_{\alpha_2}, x_{\alpha_3})$$

## Notation

We denote the induced sublist of list $x$ induced by index list $\alpha$ by $x_\alpha$. This is a slight abuse of notation, since we have so far defined a list with a subscript symbol mean the subscript-symbol term of that list. This ambiguity is avoided in our discussion if we keep in mind the types of the objects.

### Index sets

An *index set* $S \subset 1, \ldots, n$ can be associated with an index list in a natural way. It corresponds to the length-$|S|$ index list which has the elements of $S$ in their natural order. We denote the induced sublist by $x_S$.


Index Lists
Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Images
Function Composites
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects