



Why

We want to solve linear equations.

Example

Suppose we want to find $x_1, x_2 \in \mathbf{R}$ to satisfy the system of linear equations

$$3x_1 + 2x_2 = 10, \text{ and}$$

$$6x_1 + 5x_2 = 20,$$

with constants $((3, 2), 10)$ and $((6, 4), 20)$.

We can associate to the first equation an *equation for x_1 in terms of x_2* . We call this *solving the first equation for x_1* .

$$3x_1 + 2x_2 = 10 \iff 3x_1 = 10 - 2x_2 \iff x_1 = \frac{1}{3}(10 - 2x_2).$$

Define $f_1 : \mathbf{R} \rightarrow \mathbf{R}$ as $f_1(y) = \frac{1}{3}(10 - 2y)$. Then $3x_1 + 2x_2 = 10$ if and only if $x_1 = f_1(x_2)$. We have written x_1 as a *function* of x_2 and obtained a new equation. The equation is not linear, however, as f_1 is not linear.

Using the equation for x_1 in terms of x_2 , we can substitute this equation into our second linear equation. The two linear equations hold if and only if

$$6f(x_2) + 5x_2 = 20 \iff 20 - 4x_2 + 5x_2 = 20 \iff x_2 = 0.$$

So the equations are satisfied if and only if x_2 is 0. If $x_2 = 0$, then $3x_1 = 10$ and $6x_1 = 20$. Both of these are equivalent to $x_1 = \frac{10}{3}$. So we have that x_1 must be $\frac{10}{3}$ and x_2 must be 0.

Clearly this is a *solution*. Is it the only one?¹

¹Future editions will expand.

