

## Why

Can we generalize the idea of flash codes.<sup>1</sup>

## **Definition**

Let X be a set and A be an alphabet set. We denote the set of all finite sequences (strings) in A by  $\mathsf{str}(A)$ . We read  $\mathsf{str}(A)$  aloud as "the strings in A." The length zero string is  $\varnothing$ .

A code for X in A is a function from X to  $\mathsf{str}(A)$ . In this context, we refer to the finite set A as an alphabet and we call c(x) the codeword of x. The length of  $x \in X$ , with respect to a code  $c: X \to \mathsf{str}(A)$ , is the length of the sequence c(x) (its codeword). We call a code nonsingular if it is injective.

## **Examples**

Define 
$$c: \{\alpha, \beta\} \to \{0, 1\}$$
 by  $c(\alpha) = (0, )$  and  $c(\beta) = (1, ).^2$ 

## Code extensions

Let  $s, t \in \text{str}(A)$  of length m and n respectively. The concatenation of s with t is the length m+n string  $u \in \text{str}(A)$  defined by  $u_1 = s_1, \ldots, u_m = s_m$  and  $u_{m+1} = t_1, \ldots, u_{m+n} = t_n$ . We denote the concatenation of s and t by st. Note, however, that  $st \neq ts$ , although s(tr) = (st)r.

Given a code  $c: X \to \mathsf{str}(A)$ , we can produce a code for  $\mathsf{str}(X)$  in a natural way. The *extension* of c is the function  $C: \mathsf{str}(X) \to \mathsf{str}(A)$  defined, for  $\xi = (\xi_1, \dots, \xi_n) \in \mathsf{str}(X)$ , by

$$C(\xi) = c(\xi_1) \cdots c(\xi_n).$$

We call an code uniquely decodable if its extension is injective. In other

<sup>&</sup>lt;sup>1</sup>The reliance of this sheet on Flash Codes and Dot-Dash Codes is for this justification, and not for any of the terms presented.

<sup>&</sup>lt;sup>2</sup>Future editions will include additional examples.

words, given the code  $C(\xi)$  for a sequence  $\xi \in \mathsf{str}(X)$ , we can recover  $\xi$ . We call  $C(\xi)$  the *encoding* of  $\xi$ . We call  $\xi$  the *decoding* of  $C(\xi)$ .

