Why

We can identify probability distributions with vectors.

Definition

Let $p : \Omega \to \mathbf{R}$ be a probability distribution on a finite set $\Omega = \{\omega_1, \ldots, \omega_n\}$. We can associate p with the vector $x \in \mathbf{R}^n$ defined by $x_i = p(\omega_i)$ for $i = 1, \ldots, n$. We call this vector y the *probability vector* associated with p. The conditions on p mean that (1) $\langle 1, y \rangle = 1$ and (2) $y \geq 0$ (i.e., $y_i \geq 0$ for all $i = 1, \ldots, n$.).

Conversely, suppose $z \in \mathbf{R}$ satifies (1) and (2). Then $q : \Omega \to \mathbf{R}$ defined by $q(\omega_i) = z_i$ for $i = 1, \ldots, n$ is a probability distribution. For this reason, we call z satisfying the conditions a *distribution vector*. Notice that implicit in this correspondence is a numbering $\omega : \{1, \ldots, n\} \to \Omega$ of the set of outcomes Ω.

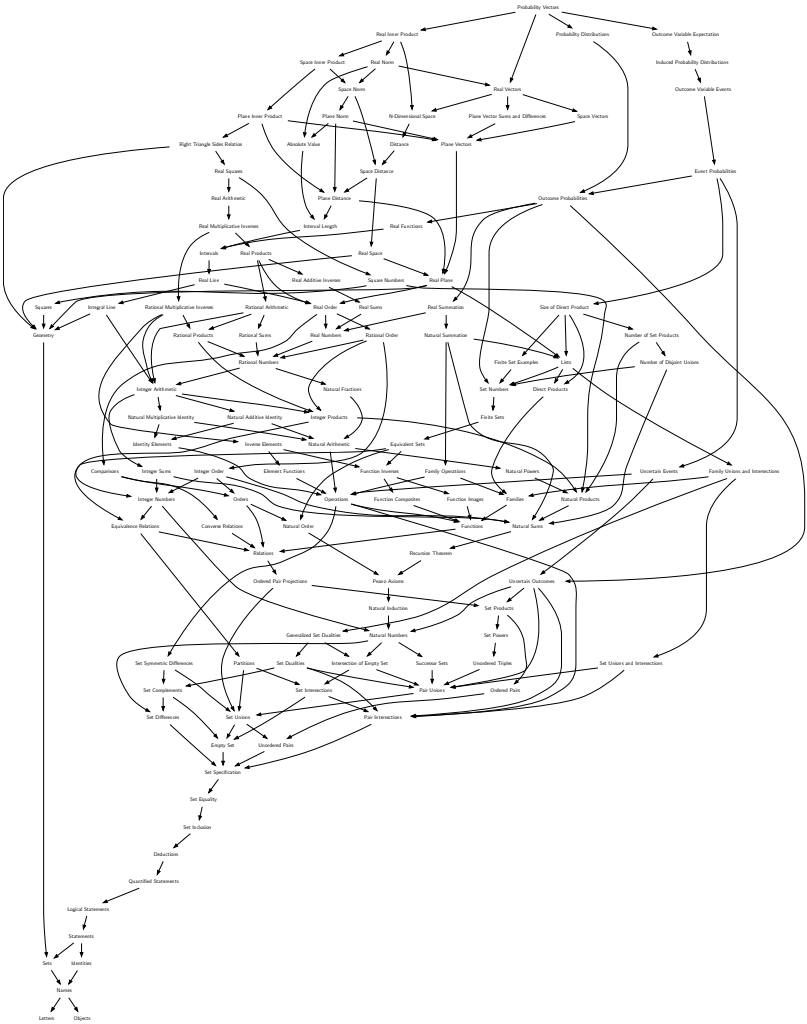
Expectation

Suppose $\rho \in \mathbf{R}^n$ is a distribution vector corresponding to $p : \Omega \to \mathbf{R}$ its corresponding distribution. Let $x : \Omega \to \mathbf{R}$ and (similar to ρ) define $\xi \in \mathbf{R}^n$ by $\xi_i = x(\omega_i)$ for $i = 1, \ldots, n$. Then $\mathbf{E}x = \langle \rho, \xi \rangle$.

Example

For $\rho = (-1, -1, 1, 1, 2)$ and $\xi = (0.1, 0.15, 0.1, 0.25, 0.4)$

$$\mathbf{E}x = \langle \rho, \xi \rangle - 1 - 0.15 + 0.1 + 0.25 + 2(0.4) = 0.9.$$


Probability Vectors
Real Inner Product
Probability Distributions
Outcome Variable Expectation
Induced Probability Distributions
Space Inner Product
Real Norm
Space Norm
Real Vectors
Outcome Variable Events
Plane Inner Product
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Right Triangle Sides Relation
Absolute Value
Distance
Plane Vectors
Real Squares
Space Distance
Event Probabilities
Real Arithmetic
Plane Distance
Outcome Probabilities
Real Multiplication Inverses
Interval Length
Real Functions
Intervals
Real Products
Real Space
Real Line
Real Additive Inverses
Square Numbers
Real Plane
Squares
Integral Line
Rational Multiplicative Inverses
Rational Arithmetic
Real Order
Real Sums
Real Summations
Size of Direct Product
Geometry
Rational Products
Rational Sums
Real Numbers
Rational Order
Natural Summation
Number of Set Products
Rational Numbers
Finite Set Examples
Lists
Number of Disjoint Unions
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Natural Multiplicative Identity
Natural Addition Identity
Integer Products
Finite Sets
Identity Elements
Inverse Elements
Natural Arithmetic
Equivalent Sets
Comparisons
Integer Sums
Integer Order
Element Functions
Function Inverses
Family Operations
Natural Powers
Uncertain Events
Family Unions and Intersections
Integer Numbers
Orders
Operations
Function Compositions
Function Images
Families
Natural Products
Equivalence Relations
Converse Relations
Natural Order
Functions
Natural Sums
Relations
Recursive Theorems
Ordered Pair Projections
Peano Axioms
Uncertain Outcomes
Natural Induction
Set Products
Generalized Set Duality
Natural Numbers
Set Powers
Set Symmetric Differences
Partitions
Set Duality
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specifications
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects