



## Definition

Given a set  $S$ , a *numbered partition* of  $S$  (or *labeled partition*) is a numbering of some partition of a  $S$ . In other words, a numbered partition  $(\pi_1, \dots, \pi_p)$  of  $S$  is such that

$$\{\pi_1, \dots, \pi_p\} \quad \text{partitions} \quad S$$

As before,  $\pi_i$  is called a *part* of the partition, for  $i = 1, \dots, p$ . In this case  $p$  is the *size* and  $(|\pi_1|, \dots, |\pi_p|)$  is the *shape* of the numbered partition.

When we speak simultaneously of partitions (as defined in **Partitions**) and numbered partitions, we sometimes call a *partition* an *unnumbered partition* (or *unlabeled partition*, *allocation*). In this case, the *size* of a partition  $P$  of  $S$  is the multiset  $m : \mathbf{N} \rightarrow \mathbf{N}$  defined so that  $m(k)$  is the number of parts of size  $k$  in  $P$ .



