Why

Is a sufficient condition for mutual independence of a set of events the fact that each pair of events of the set are independent? The surprising answer is no.

Definition

Suppose P is a event probability function on a finite sample space Ω. The events $A_1, \ldots, A_n$ are *independent* (or *mutually independent*), if for all indices $i \neq j$.

$$P(A_i \cap A_j) = P(A_i)P(A_j)$$

Clearly, if $A_1, \ldots, A_n$ are mutually independent, then they are pairwise independent. The converse, however, is not true.

Counterexample: two tosses

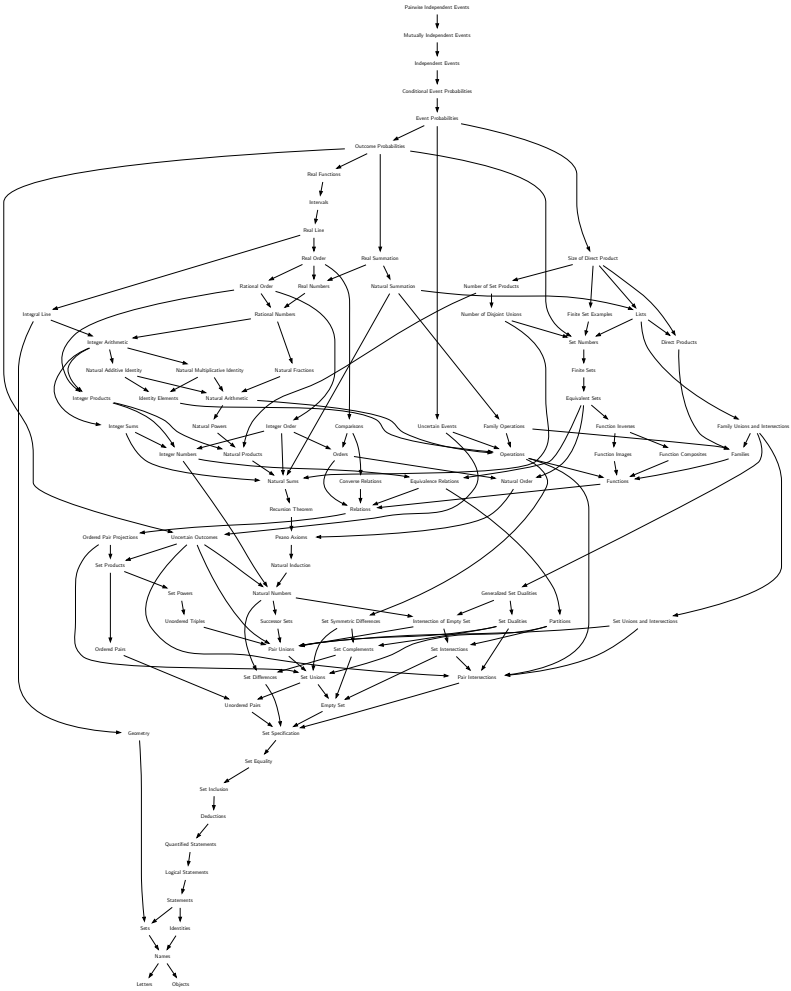
As usual, model tossing a coin twice with the sample space $\Omega = \{0,1\}^2$. Put a distribution $p : \Omega \to [0,1]$ so that $p(\omega) = 1/4$ for all $\omega \in \Omega$. Define $A = \{(1,0),(1,1)\}$ (the first toss is heads) , $B = \{(0,1),(1,1)\}$ (the second toss is heads), and $C = \{(0,0),(1,1)\}$. Then $P(A) = P(B) = P(C) = 1/2$, and

$$P(A \cap B) = P(B \cap C) = P(A \cap C) = 1/4.$$

Hence A, B, C are pairwise independent. However,

$$P(A \cap B \cap C) = 1/4 \neq P(A)P(B)P(C).$$

So A, B, C are *not* mutually independent.


Pairwise Independent Events
Mutually Independent Events
Independent Events
Conditional Event Probabilities
Event Probabilities
Outcome Probabilities
Real Functions
Intervals
Real Line
Real Order
Real Summation
Size of Direct Product
Rational Order
Real Numbers
Natural Summation
Number of Set Products
Integral Line
Rational Numbers
Number of Disjoint Unions
Finite Set Examples
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Additive Identity
Natural Multiplication Identity
Natural Fractions
Finite Sets
Integer Products
Identity Elements
Natural Arithmetic
Equivalent Sets
Integer Sums
Natural Powers
Integer Order
Comparisons
Uncertain Events
Family Operations
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Converse Relations
Equivalence Relations
Natural Order
Functions
Recursion Theorem
Relations
Ordered Pair Projections
Uncertain Outcomes
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Intersection of Empty Set
Set Dualities
Partitions
Set Unions and Intersections
Ordered Pairs
Pair Unions
Set Complements
Set Intersections
Pair Intersections
Set Differences
Set Unions
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects