



## Monotone Classes

### 1 Why

### 2 Definition

The *limit* of an increasing sequence of sets is the family union of the sequence. The *limit* of a decreasing sequence of sets is the family intersection of the sequence.

A *monotone limit* of an sequence of sets is the limit of a monotone sequence.

A *monotone class* is a subset system in which monotone limits of monotone sequences of distinguished sets are distinguished. We call the distinguished sets a *monotone class*.

#### 2.1 Notation

Let  $A$  a non-empty set with partial order  $\preceq$ . Let  $(A, \mathcal{A})$  be a subset space on  $A$ .

Let  $(A_n)_n$  be an increasing or decreasing sequence in  $\mathcal{A}$ . We denote the limit of  $(A_n)_n$  by  $\lim_n A_n$ .

If  $(A_n)_n$  is increasing,  $\lim_n A_n = \cup_n A_n$ . If  $(A_n)_n$  is decreasing,  $\lim_n A_n = \cap_n A_n$ .

If  $(A, \mathcal{A})$  is a monotone space, then for all monotone  $(A_n)_n$  in  $\mathcal{A}$ ,  $\lim_n A_n \in \mathcal{A}$ . In this case,  $\mathcal{A}$  is a monotone class.