

LINEAR TRANSFORMATION PRODUCTS

Why

We can consider function composition for linear maps?

Definition

Given vector spaces U, V, W over the same field \mathbf{F} and linear maps $T \in \mathcal{L}(U, V)$ and $S \in \mathcal{L}(V, W)$, the *product* of S and T is the linear map $R \in \mathcal{L}(U, W)$ defined by

$$R(u) = S(T(u))$$
 for all $u \in U$

(Prove that R so defined is linear). In other words, the product is $S \circ T$.

This definition only makes sense if T maps into the domain of S. We often say that the maps are *conforming* in this case.

Notation

Often the product is denoted ST (instead of $S \circ T$).

Algebraic properties

Proposition 1 (associativity). Suppose T_1, T_2, T_3 are three linear maps so that conforming for $T_1T_2T_3$. Then

$$(T_1T_2)T_3 = T_1(T_2T_3)$$

Not commutative

Image of zero

Proposition 2. Suppose T is a linear map from V to W. Then T(0) = 0.

