



# Function Properties

## 1 Why

TODO

## 2 Definition

Let  $f : A \rightarrow B$ . The **image** of a set  $C \subset A$  is the set  $\{f(c) \in B \mid c \in C\}$ . The **range** of  $f$  is the image of the domain. The **inverse image** of a set  $D \subset B$  is the set  $\{a \in A \mid f(a) \in D\}$ .

The range need not equal the codomain; though it, like every other image, is a subset of the codomain. The function maps to domain *on* to the codomain if the range and codomain are equal; in this case we call the function **onto**. This language suggests that every element of the codomain is used by  $f$ . It means that for each element  $b$  of the codomain, we can find an element  $a$  of the domain so that  $f(a) = b$ .

An element of the codomain may be the result of several elements of the domain. This overlapping, using an element of the codomain more than once, is a regular occurrence. If a function is a unique correspondence in that every domain element has a different result, we call it *one-to-one*. This language is meant

to suggest that each element of the domain corresponds to one and exactly one element of the codomain, and vice versa.

## 2.1 Notation

Let  $f : A \rightarrow B$ . We denote the image of  $C \subset A$  by  $f(C)$ , read aloud as “f of C.” This notation is overloaded: for  $c \in C$ ,  $f(c) \in B$ , whereas  $f(C) \subset B$ . Read aloud, the two are indistinguishable, so we must be careful to specify whether we mean an element  $c$  or a set  $C$ . The property that  $f$  is onto can be written succinctly as  $f(A) = B$ . We denote the inverse image of  $D \subset B$  by  $f^{-1}(D)$ , read aloud as “f inverse D.”