



LOSS FUNCTIONS

Why

We compare inductors by comparing the predictors they produce. We compare predictors by judging predictions.

Definition

A *loss* function is a nonnegative real-valued function on pairs which is zero only on repeated pairs. It need not be symmetric. We interpret the first argument of the loss function as our prediction and the second as the recorded value.

The *loss of a predictor on a pair* is the result of the loss function on the pair. Similarly, the *loss of a predictor on a sequence* of pairs is the sum of the losses on the pairs. The *average loss of a predictor on a sequence* is the loss divided by the length of the sequence. The average loss is also known as the *empirical risk* of the predictor on the dataset.

Notation

Let $(a, b) \in A \times B$ where A and B are non-empty sets. Let $\ell : B \times B \rightarrow \mathbf{R}$. Let $f : A \rightarrow B$. The loss of f on (a, b) is

$$\ell(f(a), b).$$

Let $s = ((a^1, b^1), \dots, (a^n, b^n))$ be a record sequence. The loss of f on s is

$$\sum_{k=1}^n \ell(f(a^k), b^k).$$

The average loss of f on s is

$$\frac{1}{n} \sum_{k=1}^n \ell(f(a^k), b^k).$$

