Why

If a system (A, b) is ordinarily reducible, then there exists L unit lower triangular and U upper triangular so that $A = LU$. When does such a factorization exist?

Definition

Let $A \in \mathbf{R}^{m \times m}$. A *lower upper triangular factorization* of A is a pair of matrices $(L \in \mathbf{R}^{m \times m}, U \in \mathbf{R}^{m \times m})$ where L is unit lower triangular, U is upper triangular and $A = LU$. Other terminology includes *lower upper triangular decomposition*, *LU factorization*, and *LU decomposition*.

Proposition 1. *If (A, b) is ordinarily reducible, a LU factorization exists.*

What about an LU-factorization when (A, b) is not ordinarily reducible? The main issue is that we may encounter a diagonal entry of some reduction of A which is zero.

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