

### RATIONAL NUMBERS

## Why

We want fractions.<sup>1</sup>

# Rational equivalence

Consider  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ . We say that the elements (a, b) and (c, d) of this set are rational equivalent if ad = bc. Briefly, the intuition is that (a, b) represents a over b In the usual notation, (a, b) represents "a/b". So this equivalence relation says these two are the same if a/b = c/d or else ad = bc.

**Proposition 1.** Rational equivalence is an equivalence relation on  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ .

## **Definition**

The set of rational numbers is the set of equivalence classes (see Equivalence Classes) of  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$  under rational equivalence. We call an element of the set of rational numbers a rational number or rational. We call the set of rational numbers the set of rationals or rationals for short.

#### Notation

We denote the set of rationals by  $\mathbf{Q}$ .<sup>3</sup> If we denote rational equivalence by  $\sim$  then  $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$ .

 $<sup>^{1}\</sup>mathrm{This}$  why will be expanded in future editions.

<sup>&</sup>lt;sup>2</sup>Future editions will include an account.

 $<sup>^3</sup>$ From what we can tell so far, **Q** is a mnemonic for "quantity", from the latin "quantitas".

