Why

Let $X = \{a, b\}$ and $Y = \{0, 1\}$. The dataset $((a, 0))$ is consistent, but it is not functionally complete. On the other hand, the dataset $((a, 0), (b, 0), (a, 0), (a, 0), (a, 0), (a, 1))$ is complete but it is not *functionally* consistent.

In general, if $y_i \neq y_j$ for some i and j where $x_i = x_j$, then the dataset is not functionally consistent. In the preceding example, both $(a, 0)$ and $(a, 1)$ appear.

If we emphasize the "predictive" aspect of a functional inductor, we interpret the input as an object we "see before" the output. And so treat $y \in Y$ as an uncertain outcome which is the element associated to $x \in X$.

In this case, we may use the language of probability to discuss this uncertain outcome. If, for example, Y is finite, we can associate a distribution with each input $x \in X$.

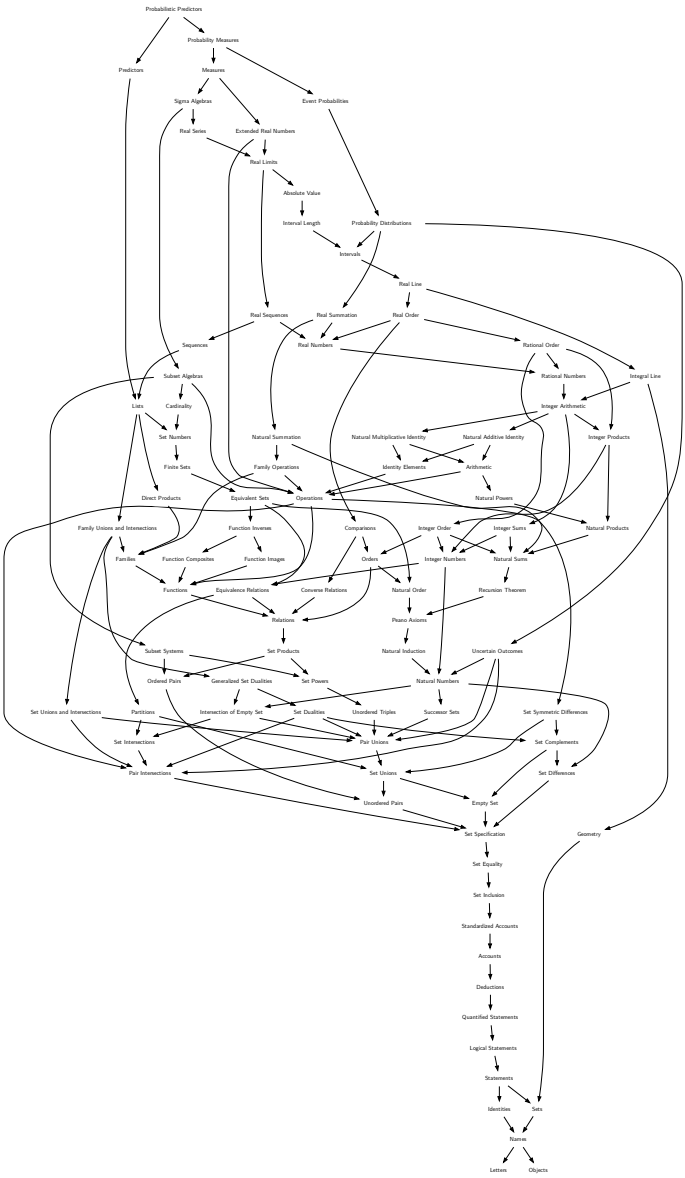
Definition

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable spaces.

A *probabilistic functional inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a family of measures on $(Y, \mathcal{Y})$, indexed by X. We call a function from inputs to output measures a *probabilistic predictor*. We call the distribution a *probabilistic prediction*.

Notation

Let $\mathcal{M}(Y, \mathcal{Y})$ be the set of measures on Y. Let D be a dataset in $(X \times Y)^n$. Let $g : X \to \mathcal{M}(Y, \mathcal{Y})$ a probabilistic predictor. Let $G_n(X \times Y)^n \to (X \to \mathcal{M}(Y, \mathcal{Y}))$ be a predictive probabilistic inductor. Then $G_n(D)$ is a family of measures $\{g_x : \mathcal{Y} \to [0, 1]\}_{x \in X}$.


Probabilistic Predictors
Probability Measures
Predictors
Measures
Sigma Algebras
Event Probabilities
Real Series
Extended Real Numbers
Real Limits
Absolute Value
Interval Length
Probability Distributions
Intervals
Real Line
Real Sequences
Real Summation
Real Order
Real Numbers
Rational Order
Sequences
Subset Algebras
Rational Numbers
Integral Line
Lists
Cardinality
Integer Arithmetic
Set Numbers
Natural Summation
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Finite Sets
Family Operations
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Operations
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Sums
Functions
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects