



## Why

We are frequently interested in finding minimizers of real functions.<sup>1</sup>

## Definition

An *optimization problem* is a pair  $(\mathcal{X}, f)$  in which  $\mathcal{X}$  is a nonempty set called the *constraint set* and  $f : \mathcal{X} \rightarrow \mathbf{R}$  is called the *objective* (or *cost function*).

If  $\mathcal{X}$  is finite we call the optimization problem *discrete*. If  $\mathcal{X} \subset \mathbf{R}^d$  we call the optimization problem *continuous*.

We refer to all elements of the constraint set as *feasible*. We refer to an element  $x \in \mathcal{X}$  of the constraint set as *optimal* if  $f(x) = \inf_{z \in \mathcal{X}} f(z)$ . We also refer to optimal elements as *solutions* of the optimization problem.

It is common for  $f$  and  $\mathcal{X}$  to depend on some other, known, given objects. In this case, these objects are often called *parameters* or *problem data*.

## Notation

We often write optimization problems as

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathcal{X}. \end{array}$$

In this case we call  $x$  the *decision variable*.

## Extended reals

It is common to let  $f : \mathcal{X} \rightarrow \bar{\mathbf{R}}$ , and allow there to exist  $x \in \mathcal{X}$  for which  $f(x) = \infty$ . This technique can be used to embed further constraints in the objective. For example, we interpret  $f(x) = +\infty$  to mean  $x$  is *infeasible*.

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<sup>1</sup>Future editions will modify and expand.

## Maximization

If we have some function  $g : \mathcal{X} \rightarrow \bar{\mathbf{R}}$  that we wish to maximize, we can always convert it to an optimization problem by defining  $f : \mathcal{X} \rightarrow \bar{\mathbf{R}}$  by  $f(x) = -g(x)$ . In this case  $g$  is often called a *reward* (*utility*, *profit*).

## Solvers

Let  $\mathcal{P} = \{(X_a, f_a : X_a \rightarrow \bar{\mathbf{R}})\}_{a \in A}$  be a family of optimization problems. A *solver* (or *solution method*, *solution algorithm*) for  $\mathcal{P}$  is a function  $S : A \rightarrow \mathcal{S}$  such that  $S_a$  is a solution of the problem  $(X_a, f_a)$ .

Loosely speaking, the difficulty of “solving” the optimization problem  $(\mathcal{X}, f)$  depends on the properties of  $\mathcal{X}$  and  $f$  and the problem “size”. For example, when  $\mathcal{X} \subset \mathbf{R}^d$  the difficulty is related to the “dimension”  $d$  of  $x \in \mathcal{X}$ .

