Why

We can embed the undirected graphs as certain garphs directed graphs.

Definition

Suppose that $G = (V, E)$ is a directed graph. If $(v, w) \in E$ *and* $(w, v) \in E$ we call (v, w) (and (w, v)) and *bidirected edge* (or *undirected edge*). In other words, an edge $(v, w) \in E$ is undirected if (v, w) is also in E.

If every edge of a directed graph is undirected, then we call the graph *undirected*. If some edges are an some are not, we call G a *partially directed graph*.

Notation

Suppose (V, E) is a directed graph. It is common to write $v \to w$ for

$$(v, w) \in E \text{ and } (w, v) \notin E.$$

It is common to write $a \sim \beta$ if

$$(v, w) \in E \text{ and } (w, v) \in E$$

Similarly, we write $v \not\to w$ if $(v, w) \notin E$ and $v \not\sim w$ if

$$(v, w) \notin E \text{ and } (w, v) \notin E$$

Undirected version

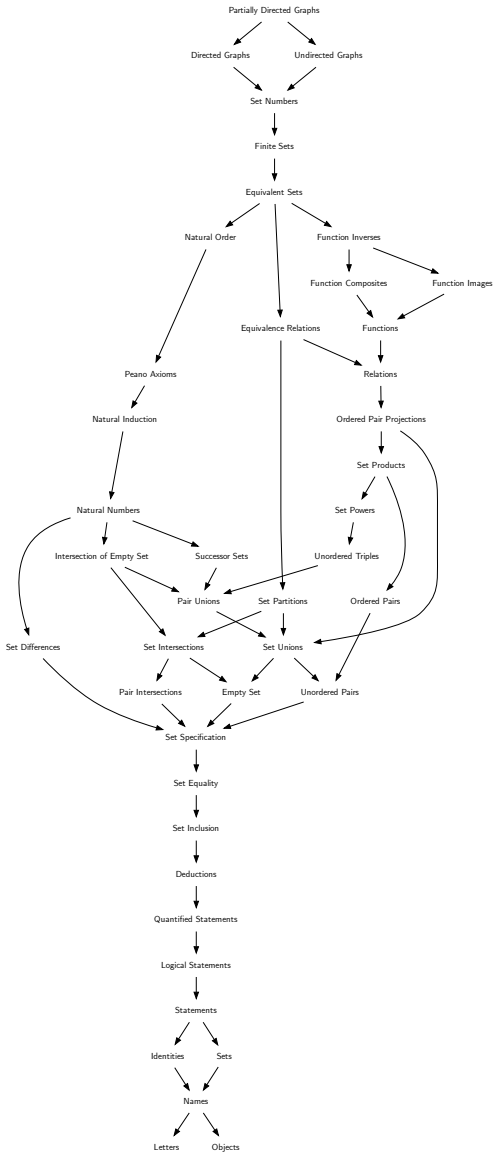
As before, the *undirected version* (or *skeleton*) of G is the *undirected* partially directed graph defined satisfying $u \sim v$ if $u \to v$ or $u \sim v$.

Subgraphs

Given a graph $G = (V, E)$ and a set $A \subset V$, the *subgraph* of $G = (V, E)$ corresponding to A is the graph denoted G_A defined by $(A, E \cap (A \times A)$.

Completeness

Suppose (V, E) is a partially directed graph. In the context of partially directed graphs, the graph is *complete* if $(u, v) \in E$ or $(v, u) \in E$ for any two vertices u and v. A subset A is *complete* if the subgraph G_A is complete. A complete subset that is maximal (w.r.t. $\subseteq$) is called a *clique*.


Partially Directed Graphs
Directed Graphs
Undirected Graphs
Set Numbers
Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Function Composites
Function Images
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Pair Unions
Set Partitions
Ordered Pairs
Set Differences
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects