

## MAXIMUM CONDITIONAL ESTIMATES

## Why

We want to estimate a random vector  $x: \Omega \to \mathbf{R}^d$  from a random vector  $y: \Omega \to \mathbf{R}^n$ .

## Definition

Denote by  $g: \mathbf{R}^d \times \mathbf{R}^n \to \mathbf{R}$  the joint density for (x,y).<sup>1</sup> Denote the conditional density for x given y by  $g_{x|y}: \mathbf{R}^d \times \mathbf{R}^n \to \mathbf{R}$ . In this setting,  $g_{x|y}$  is called the *posterior density*,  $g_x$  is called the *prior density*, and  $g_{y|x}$  is called the *likelihood density* and  $g_y$  is called the *marginal likelihood density*.

As usual (and assuming  $g_y > 0$ ), the posterior is related to the likelihood, prior and marginal likelihood by

$$g_{x|y} \equiv \frac{g_x g_{y|x}}{g_y}.$$

A maximum conditional estimate for  $x:\Omega\to \mathbf{R}^n$  given that y has taken the value  $\gamma\in \mathbf{R}^n$  is a maximizer  $\xi\in \mathbf{R}^d$  of  $g_{x|y}(\xi,\gamma)$ . It is also called the maximum a posteriori estimate or MAP estimate. The maximum conditional estimate is natural, in part, because it also maximizes the joint density, since  $g(\xi,\gamma)=g_y(\gamma)g_{x|y}(\xi,\gamma)$  for all  $\xi\in \mathbf{R}^d$  and  $\gamma\in \mathbf{R}^n$ .

 $<sup>^1\</sup>mathrm{Future}$  editions will comment on the existence of such a density.

