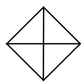


Why

How many ways are there to split n objects into nonoverlapping groups, when the objects are indistinguishable?

Definition

Suppose n is a nonzero natural number. A *partition* of n is a *nonincreasing* list of *nonzero* natural numbers whose sum is n. The requirement that the list of numbers be nonincreasing makes the representation unique. The terms of the list are called the *parts* of the partition. The number n is sometimes called the *weight* of the partition. The number of times a particular number appears in the list is called the *multiplicity* of that part.

Examples

What are the partitions of the number 5?

$$\begin{aligned}
5 &= 5 \\
&= 4 + 1 \\
&= 3 + 2 \\
&= 3 + 1 + 1 \\
&= 2 + 2 + 1 \\
&= 2 + 1 + 1 + 1 \\
&= 1 + 1 + 1 + 1 + 1
\end{aligned}$$

These seven identities correspond to the seven partitions of 5, namely $(5,)$, $(4, 1)$, $(3, 2)$, $(3, 1, 1)$, $(2, 2, 1)$, $(2, 1, 1, 1)$, $(1, 1, 1, 1, 1)$. The multiplicity of 1 in these partitions is 0, 1, 0, 2, 1, 3, 5, respectively.

Notation

Suppose λ is a list in $\mathbf{N}$ of length $r \geq 1$. Then $\lambda = (\lambda_1, \ldots, \lambda_r)$ is a *partition* of $n \in \mathbf{N}$ if

$$\lambda_1 + \lambda_2 + \cdots + \lambda_r = n$$

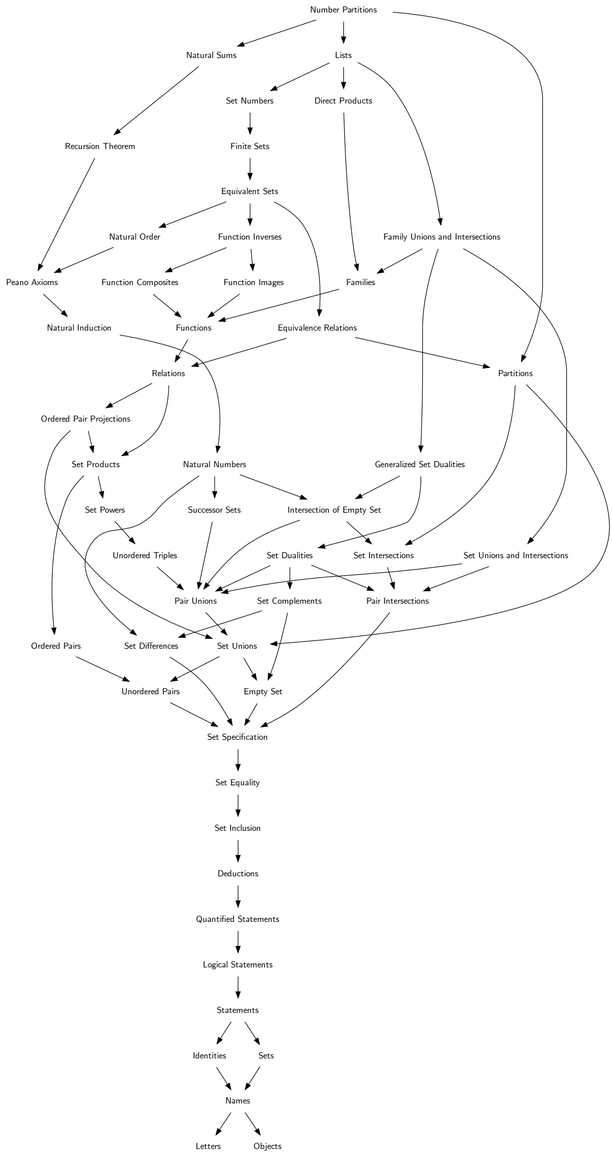
The terms of λ are called the *parts* of the partition. λ_i is the ith part, where $i = 1, \ldots, r$. λ is a nonincreasing list means

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r$$

Some authors denote the *weight* of λ by $|\lambda|$.

Partition function

How many partitions are there of the number n? We denote *this number* by $p(n)$. From the examples above, $p(5) = 7$.


Number Partitions
Natural Sums
Lists
Set Numbers
Direct Products
Recursion Theorem
Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Family Unions and Intersections
Peano Axioms
Function Composites
Function Images
Families
Natural Induction
Functions
Equivalence Relations
Relations
Partitions
Ordered Pair Projections
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Intersections
Set Unions and Intersections
Pair Unions
Set Complements
Pair Intersections
Ordered Pairs
Set Differences
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects