



PEANO AXIOMS

Why

Historically considered a fountainhead for all of mathematics.

Discussion

Our discussion of sets has succeeded in making Peano's axioms provable propositions.

1. $0 \in \omega$.
2. $n \in \omega \longrightarrow n^+ \in \omega$.
3. Suppose $S \subset \omega$, $0 \in S$, and $(n \in S \longrightarrow n^+ \in S)$. Then $S = \omega$.

Proposition 1 (Peano's First Axiom). $0 \in \omega$.

Proposition 2 (Peano's Second Axiom). $n \in \omega \longrightarrow n^+ \in \omega$.

Proposition 3 (Peano's Third Axiom). *Suppose $S \subset \omega$, $0 \in S$, and $(n \in S \longrightarrow n^+ \in S)$. Then $S = \omega$.*

Peano's third axiom is called the *principle of mathematical induction*.

Proposition 4 (Peano's Fourth Axiom). $n^+ \neq 0$ for all $n \in \omega$.

Proposition 5 (Peano's Fifth Axiom). *Suppose $n, m \in \omega$ with $n^+ = m^+$. Then $n = m$.*

