Why

We want a notion of distance between elements of the real line.

Definition

The *absolute value* of a real number is the greater of itself and its additive inverse. In other words, if x is positive, then the absolute value of x is x. If x is negative, then the absolute value of x is $-x$ (which would be a positive real number).

Notation

We denote the absolute value of a real number $x \in \mathbf{R}$ by $|x|$.

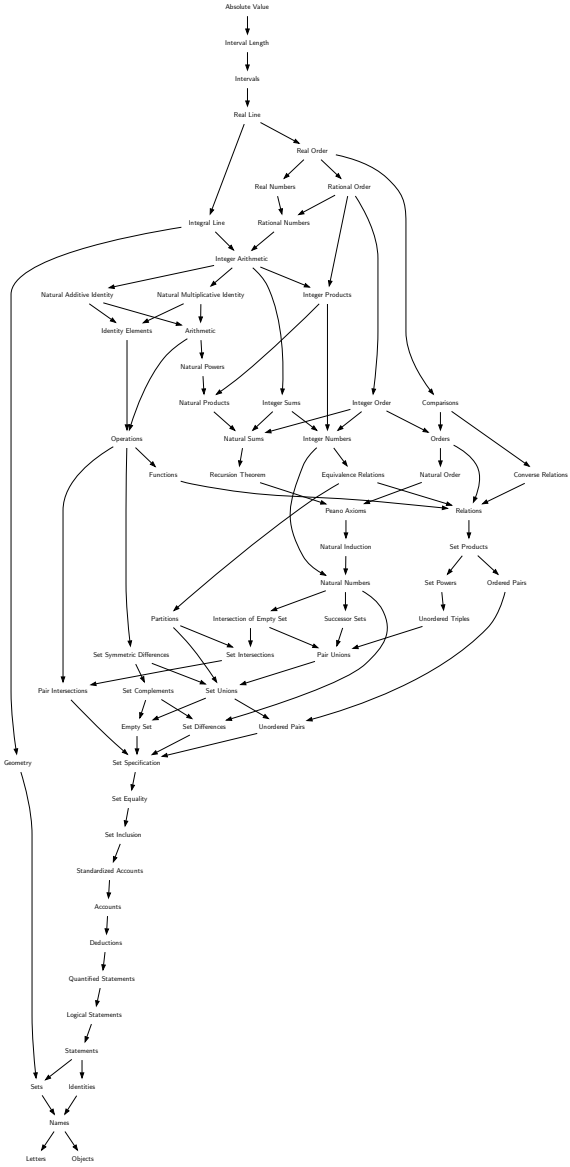
Distance

The absolute value can be interpreted as the distance between the point corresponding to the real number and the point corresponding to 0. We can generalize this idea. Consider $x, y \in \mathbf{R}$. If $x > y$, then $x - y > 0$ and so the distance between the corresponding points is $x - y$. If $x < y$ then $y - x > 0$, and so the distance is $y - x$.

The observation is that $|-x| = |x|$. So

$$|y - x| = |-(x - y)| = |x - y|.$$

So if we just care about the distance between the points corresponding to y and x, we can consider $|x - y|$, without regard for their order. In other words, the function $(x, y) \quad |x - y|$ is symmetric in x and y.


Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Identity Elements
Arithmetic
Natural Powers
Natural Products
Integer Sums
Integer Order
Comparisons
Operations
Natural Sums
Integer Numbers
Orders
Functions
Recursion Theorem
Equivalence Relations
Natural Order
Converse Relations
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects