Result

This result is called sometimes called the *probability inverse transform.*

Proposition 1. *Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space and let $X : \Omega \to \mathbf{R}$ be a random variable with cumulative distribution function $F_X : \mathbf{R} \to [0,1]$. Suppose ${F_X}^{-1} : [0,1] \to \mathbf{R}$ exists, then $Y = {F_X}^{-1} \circ X$ is a random variable with cumulative distribution function $F_Y : [0,1] \to [0,1]$ satisfying $F_Y(y) = y$.*

Remark 1. *The conclusion is equivalent to the following: Y has a density and that density is the the standard unform density (see Uniform Densities).*

Proof. Express $F_Y(\gamma) = \mathbf{P}[Y \leq \gamma] = \mathbf{P}(Y^{-1}([0,\gamma]))$ Notice[1]

$$\begin{aligned} Y^{-1}([0,\gamma]]) &= \{\omega \in \Omega \mid Y(\omega) \leq \gamma\} \\ &= \{\omega \in \Omega \mid F_X(X(\omega)) \leq \gamma\} \\ &= \{\omega \in \Omega \mid X(\omega) \leq F_X^{-1}(\gamma)\}. \quad = X^{-1}(\cdots). \end{aligned}$$

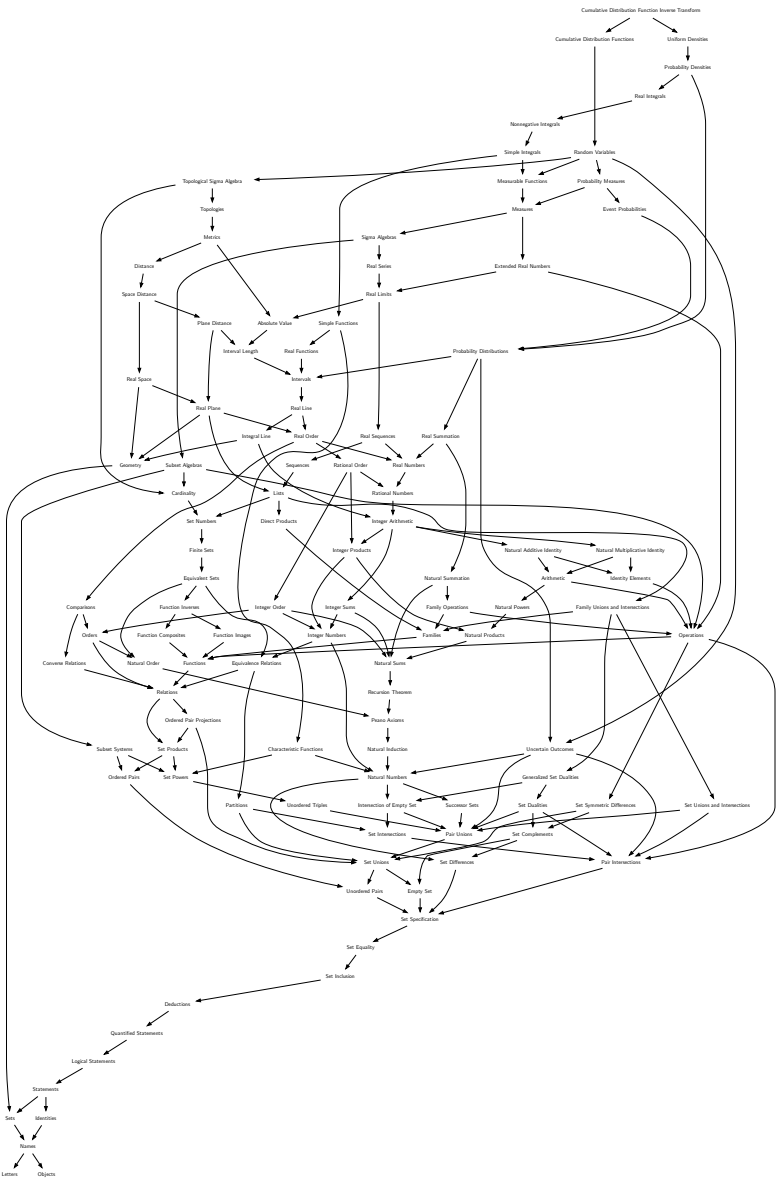
□

Remark 2. *Using different notation the above can be expressed succinctly as*

$$\begin{aligned} F_Y(\gamma) = \mathbf{P}[Y \leq \gamma] &= \mathbf{P}[F_X \circ X \leq \gamma] \\ &= \mathbf{P}[X \leq {F_X}^{-1}(\gamma)] = F_X({F_X}^{-1}(\gamma)) = \gamma. \end{aligned}$$

Future editions will discuss *inverse transform sampling.*

[1] Future editions will complete.


Cumulative Distribution Function Inverse Transform
Cumulative Distribution Functions
Uniform Densities
Probability Densities
Real Integrals
Nonnegative Integrals
Simple Integrals
Random Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Topologies
Measures
Event Probabilities
Metrics
Sigma Algebras
Distances
Real Series
Extended Real Numbers
Space Distance
Real Limits
Plane Distance
Absolute Value
Simple Functions
Interval Length
Real Functions
Probability Distributions
Intervals
Real Space
Real Line
Real Plane
Integral Line
Real Order
Real Sequences
Real Summation
Geometry
Subset Algebras
Sequences
Rational Order
Real Numbers
Cardinality
Lists
Rational Numbers
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplication Identity
Arithmetic
Identity Elements
Equivalent Sets
Natural Summation
Comparisons
Function Inverses
Integer Order
Integer Sums
Family Operations
Natural Powers
Family Unions and Intersections
Order
Function Composites
Function Images
Integer Numbers
Families
Natural Products
Operations
Converse Relations
Natural Order
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Unions and Intersections
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects