

Inverses of Composite Relations

Why

How do inverse and converse relations interact.

Results

Let R be a relation between X and Y and let S be a relation between Y and Z.

Proposition 1. $(RS)^{-1} = S^{-1}R^{-1}$

Identity Relations

Recall that I is the identity relation on X if x I y if and only if x = y.

Proposition 2. Let R be a relation on X. Let I be the identity relation on X. Then RI = IR = R.

One would like $RR^{-1} \supset I$, $R^{-1}R \supset I$. The father of the son is the father and the son of the father is the son. But the empty relation violates these claims.

Relation Properties

Proposition 3. R is symmetric if and only if $R \subset R^{-1}$

Proposition 4. R is reflextive if and only if $I \subset R$

Proposition 5. R is transitive if and only if $RR \subset R$.

