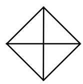


Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\mathbf{cov}(f, g)$. We defined it:

$$\mathbf{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\mathbf{E}(g).$$

Properties

Prop. 1. *Covariance is symmetric and billinear.*[1]

Prop. 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\mathbf{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \mathbf{var}(f).$$

□

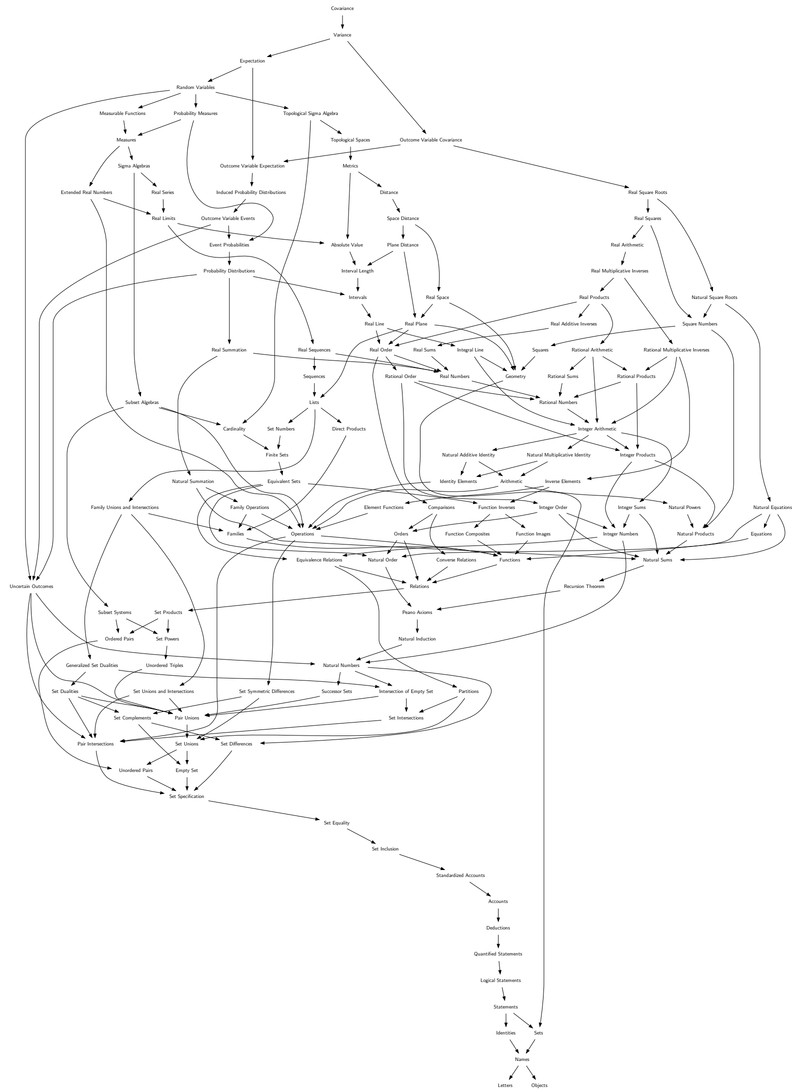
Prop. 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \dots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \dots, n$. Using the billinearity,

$$\begin{aligned}\mathbf{var}\left(\sum_{i=1}^n f_i\right) &= \mathbf{cov}\left(\sum_{i=1}^n f_i, \sum_{i=1}^n f_i\right) \\ &= \sum_{i=1}^n \sum_{j=1}^n \mathbf{cov}(f_i, f_j)\end{aligned}$$

□

[1] Future editions will include an account.


Covariance
Variance
Expectation
Random Variables
Measurable Functions
Probability Measures
Topological Sigma Algebra
Measures
Topological Spaces
Outcome Variable Covariance
Sigma Algebras
Outcome Variable Expectation
Metrics
Extended Real Numbers
Real Series
Induced Probability Distributions
Distance
Real Square Roots
Real Limits
Outcome Variable Events
Space Distance
Real Squares
Event Probabilities
Absolute Value
Plane Distance
Real Arithmetic
Probability Distributions
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Natural Square Roots
Real Line
Real Plane
Real Additive Inverses
Square Numbers
Real Summation
Real Sequences
Real Order
Real Sums
Integral Line
Squares
Rational Arithmetic
Rational Multiplicative Inverses
Sequences
Rational Order
Real Numbers
Geometry
Rational Sums
Rational Products
Subset Algebras
Lists
Rational Numbers
Cardinality
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Natural Summation
Equivalent Sets
Identity Elements
Arithmetic
Inverse Elements
Family Unions and Intersections
Family Operations
Element Functions
Comparisons
Function Inverses
Integer Order
Integer Sums
Natural Powers
Natural Equations
Families
Operations
Orders
Function Composites
Function Images
Integer Numbers
Natural Products
Equations
Equivalence Relations
Natural Order
Converse Relations
Functions
Natural Sums
Uncertain Outcomes
Relations
Recursion Theorem
Subset Systems
Set Products
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Generalized Set Dualities
Unordered Triples
Natural Numbers
Set Dualities
Set Unions and Intersections
Set Symmetric Differences
Successor Sets
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects