



## Why

Lots of things are (approximately) linear.<sup>1</sup>

## Definition

A transformation is *linear* if the result of a linear combination of the two vectors is the linear combination of the results of the vectors (using the same coefficients). The transformation is linear *with respect to* the field of the two vector spaces.

We use the term transformation (Transformations) for emphasis and reminder that the function is defined on a vector space. Of course,  $\mathbf{R}$  is a vector space and so a function  $f : \mathbf{R} \rightarrow \mathbf{R}$  may be linear. It is, therefore, common to speak of *linear functions*.

Often authors will use the word *operator* for linear functions. It seems, generally, that this term is commonly reserved for the case in which the vector space discussed is a function space (or, at least, infinite dimensional).

## Notation

Let  $(V_1, F)$  and  $(V_2, F)$  be two vector spaces over the same field. Let  $f : V_1 \rightarrow V_2$ .  $f$  is linear means

$$f(au + bv) = af(u) + bf(v)$$

for all  $a, b \in F$  and  $u, v \in V_1$ .

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<sup>1</sup>Future editions will expand on this why. In particular, the intuition of proportionality.



