

MATRIX RINGS

Why

Matrices with elements in a ring form a ring.

Example

Let $(R, +, \cdot)$ be a ring. Define C = A + B by $C_{ij} = A_{ij} + B_{ij}$ and define C = A + B by $C_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$, as with real matrices, for $A, B \in \mathbb{R}^{n \times n}$. Then $(\mathbb{R}^{n \times n}, +, -)$ is a ring. In other words, the set of $n \times n$ matrices with elements in R is a ring, with the usual addition and multiplication of matrices.

The additive identity of the ring is the matrix $0 \in R^{n \times n}$ for which $0_{ij} = 0 \in R$. The multiplicative identity the matrix I for which $I_{ii} = 1 \in R$ for i = 1, ..., n and $I_{ij} = 0 \in R$ for $i \neq j = 1, ..., n$. As seen with real-valued matrices, multiplication on $R^{n \times n}$ need not be commutative even if R is.

Exercise 1. Show that $R^{n \times n}$ is not a division ring when n > 1.

