Why

We want a notation for expressing the sum of several natural numbers.

Definition

Let $s = (m_1, \ldots, m_n)$ be a sequence of natural numbers. The *sequence sum* of s is the result of first summing the first two numbers, then summing the result with the third number, and so on, until we have summed all the numbers

Notation

We denote the sum of a sequence using by using the $\sum$ symbol. $\sum$ is the capital greek letter "sigma" and is a mnemonic for "sum."

Let $(m_1, \ldots, m_n)$ be a sequence of natural numbers. Let us denote by m_i an element of the sequence, where $i = 1, \ldots, n$.

We denote that the sum ranges over an (ordered) index set $\{1, \ldots, n\}$ by writing $\sum_{i=1}^{n}$. We denote the sequence sum

$$\sum_{i=1}^{n} m_i.$$

Summing over finite sets

Suppose A is a finite set and $f : A \to \mathbf{N}$ is a function. Then we the notation

$$\sum_{a \in A} f(a)$$

is notation for $\sum_{i=1}^{n} f(\sigma(i))$ where $\sigma : \{1, \ldots, |A|\} \to A$ is any numbering of A. The numbering is inconsequential because addition is commutative.

Natural Summation

Lists

Natural Sums

Direct Products

Set Numbers

Finite Sets

Recursion Theorem

Equivalent Sets

Family Operations

Family Unions and Intersections

Function Inverses

Natural Order

Operations

Families

Function Images

Function Composites

Peano Axioms

Functions

Equivalence Relations

Relations

Natural Induction

Ordered Pair Projections

Set Products

Generalized Set Dualities

Set Powers

Natural Numbers

Set Symmetric Differences

Set Unions and Intersections

Set Dualities

Unordered Triples

Intersection of Empty Set

Successor Sets

Partitions

Ordered Pairs

Set Complements

Pair Unions

Set Intersections

Pair Intersections

Set Differences

Set Unions

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects