Why

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Why

A conditional distribution (density) $q : Y \times X \to \mathbf{R}$ is *functionally parametrizable* if there exists a function $f : X \to \Theta$ and distribution (density) family $\{p^{(\theta)} : Y \to \mathbf{R}\}_{\theta \in \Theta}$ satisfying $q(y, x) \equiv p^{(f(x))}(\gamma)$ for all $x \in X$ and $y \in Y$.

In this case we call f the *parameterizer* and we call $\{p^{(\theta)}\}_{\theta \in \Theta}$ the *parameterized family*. A *parameterized conditional distribution* is an ordered pair whose first coordinate is a function from X to Θ and whose second coordinate is a family of distributions on X with parameter set Θ. For a particular choice of parameterizer and family, it induces a conditional distribution.

Since all conditional distributions are functionally parametrizable (consider $\{q(\cdot, \xi)\}_{\xi \in X}$ with parameters X and identity parameterizer), we are interested in parameterizers and parameterized families that are simple. Said differently, we are interested in approximating a conditional distribution by selecting an appropriate parameterizer and parameterized family.

If $\{f_\phi : X \to \Theta\}_{\phi \in \Phi}$ is a family of functions and $\{q^{(\theta)}\}$ is a family of distributions, then $\{p^{(\phi)} : X \times Z \to \mathbf{R}\}_\phi$ defined by $p^{(\phi)}(\cdot, \zeta) \equiv q^{f_\phi(\zeta)}$ is a conditional distribution family called a *functionally parameterized conditional distribution family*. In

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[1]Future editions will include.

other words, by selecting some parameters ϕ, we induce a conditional distribution $p^{(\phi)} : X \times Z \to \mathsf{R}$

We similarly define *parameterized conditional densities* and *functionally parameterized conditional density families*.

Basic example

Let $Z = \{1, 2\}$ and $X = \mathsf{R}$. Let $f : \{1, 2\} \to \mathsf{R} \times \mathsf{R}_+$ be defined by $f(1) = (\mu_1, \sigma_1)$ and $f(2) = (\mu_2, \sigma_2)$. Let $\{g^{(\theta)}\}_\theta$ be the normal family. Then $(f, \{g^{(\theta)}\})$ is a *functionally parameterized conditional density*.[2]

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[2] Future editions will modify.

Parameterized Distributions

Distribution Families

Probability Densities

Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions

Measures

Sigma Algebras Extended Real Numbers

Real Limits

Simple Functions

Absolute Value

Real Functions Interval Length Probability Distributions

Intervals

Real Line

Real Order Real Sequences Real Summation

Real Numbers

Integral Line Rational Numbers

Integer Arithmetic Set Numbers Sequences Direct Products Natural Summation

Natural Multiplicative Identity Natural Additive Identity

Subset Algebra Integer Products Arithmetic Identity Elements Finite Sets Equivalent Sets Family Operations

Integer Sums Set Operations Natural Powers Algebras Comparisons Family Unions and Intersections Function Inverses Families

Integer Numbers Natural Sums Operations Orders Function Images Function Composites

Natural Products Recursion Theorem Converse Relations Natural Order Equivalence Relations Functions

Cardinality Peano Axioms Relations

Natural Induction Characteristic Functions Uncertain Outcomes Subset Systems Set Products

Natural Numbers Set Powers Ordered Pairs Generalized Set Dualities

Set Symmetric Differences Successor Sets Unordered Triples Set Dualities Intersection of Empty Set Partitions Set Unions and Intersections

Set Complements Pair Unions Set Intersections

Set Differences Set Unions Pair Intersections

Empty Set Unordered Pair

Geometry Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects