



TOPOLOGICAL SPACES

Why

We want to generalize the notion of continuity.

Definition

A *topological space* is a base set and a set distinguished subsets of this set for which: (1) the empty set and the base set are distinguished, (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the set of distinguished subsets the *topology*. We call the distinguished subsets the *open sets*.

Notation

Let A be a non-empty set. For the set of distinguished sets, we use \mathcal{T} , a mnemonic for topology, read aloud as “script T”. We denote elements of \mathcal{T} by O , a mnemonic for open. We denote the topological space with base set A and topology \mathcal{T} by (A, \mathcal{T}) . We denote the properties satisfied by elements of \mathcal{T} :

1. $X, \emptyset \in \mathcal{T}$
2. $\{O_i\}_{i=1}^n \subset \mathcal{T} \longrightarrow \bigcap_{i=1}^n O_i \in \mathcal{T}$
3. $\{O_\alpha\}_{\alpha \in I} \subset \mathcal{T} \longrightarrow \bigcup_{\alpha \in I} O_\alpha \in \mathcal{T}$

Examples

\mathbf{R} with the open intervals as the open sets is a topological space.

