



Topological Spaces

1 Why

We want to generalize the notion of continuity.

2 Definition

A *topological space* is a subset system for which: (1) the empty set and the base set are distinguished, (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the set of distinguished subsets the *topology*. We call the distinguished subsets the *open sets*.

2.1 Notation

Let A be a non-empty set. For the set of distinguished sets, we use \mathcal{T} , a mnemonic for topology, read aloud as “script T”. We denote elements of \mathcal{T} by O , a mnemonic for open. We denote the topological space with base set A and topology \mathcal{T} by (A, \mathcal{T}) . We denote the properties satisfied by elements of \mathcal{T} :

1. $X, \emptyset \in \mathcal{T}$
2. $\{O_i\}_{i=1}^n \subset \mathcal{T} \implies \bigcap_{i=1}^n O_i \in \mathcal{T}$
3. $\{O_\alpha\}_{\alpha \in I} \subset \mathcal{T} \implies \bigcup_{\alpha \in I} O_\alpha \in \mathcal{T}$