Why

It is natural to embed a dataset.

Definition

Let $(x : \Omega \to \mathbf{R}^d, A \in \mathbf{R}^{n \times d}, e : \Omega \to \mathbf{R}^n)$ be a probabilistic linear model over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Let $\phi : \mathbf{R}^d \to \mathbf{R}^{d'}$ be a feature map.

We call the sequence (x, A, e, ϕ) a *featurized probabilistic linear model* (also *embedded probabilistic linear model*). We interpret the model as a random field $h : \Omega \to (\mathbf{R}^d \to \mathbf{R})$ which is a linear function of the features

$$h_\omega(a) = \phi(a)^\top x(\omega).$$

Denote the data matrix of the embedded feature vectors by $\phi(A)$. In other words, $\phi(A) \in \mathbf{R}^{n \times d'}$ is a matrix whose rows are feature vectors. Then (x, A, e, ϕ) corresponds to the probabilistic linear model $(x, \phi(A), e)$.

Normal case

In the normal (Gaussian) case, the parameter posterior $g_{x|y}(\cdot, \gamma)$ is a normal density with mean

$$\Sigma_x \phi(A)^\top \left(\phi(A) \Sigma_x \phi(A)^\top + \Sigma_e \right)^{-1} \gamma$$

and covariance

$$\left({\Sigma_x}^{-1} + \phi(A)^\top {\Sigma_e}^{-1} \phi(A) \right)^{-1}.$$

The predictive density for $a \in \mathbf{R}^d$ is normal with mean

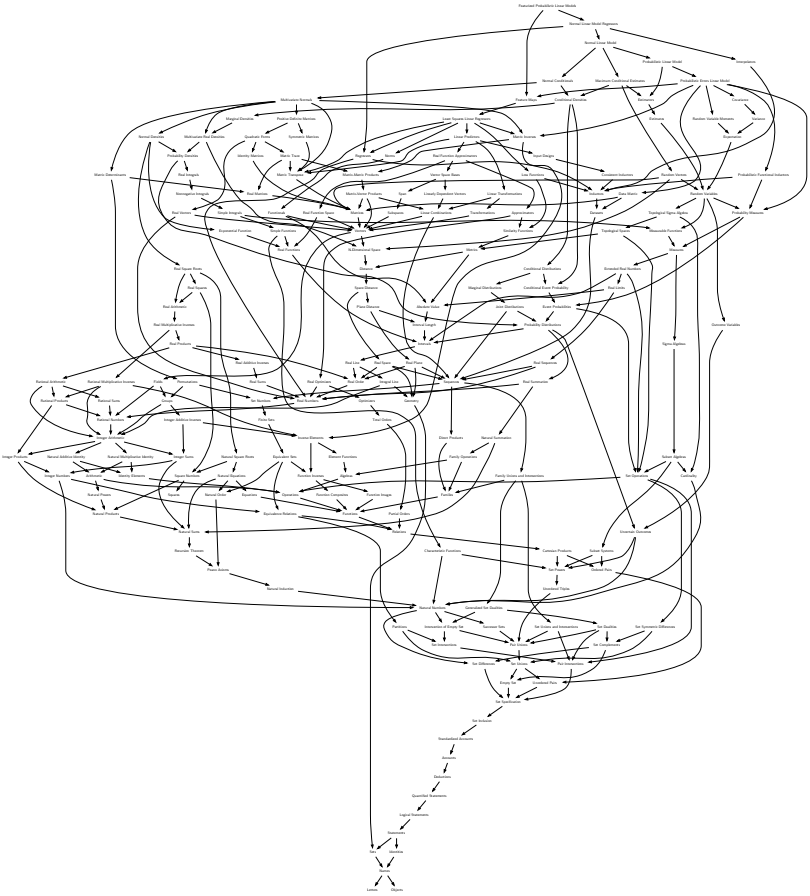
$$\phi(a)^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x\phi(A)^\top + \Sigma_e\right)^{-1} \gamma.$$

and covariance

$$\phi_a{}^\top \Sigma_x \phi_a - \phi_a{}^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x\phi(A)^\top + \Sigma_e\right)^{-1} \phi(A)\Sigma_x\phi_a.$$

So the *featurized linear regressor* is the predictor $h : \mathbf{R}^d \to \mathbf{R}$ defined by

$$h(a) = \phi(a)^\top \Sigma_x \phi(A)^\top \left(\phi(A)\Sigma_x\phi(A)^\top + \Sigma_e\right)^{-1} \gamma.$$


Generalized Probabilistic Linear Models
Barrier Gross Model Regression
Barrier Model
Probabilistic Linear Model
Regulators
General Coordinates
Gross Domestic Product
Gross Domestic Product Growth
Probabilistic Gross Domestic Product
Gross Domestic Product Growth Model
Gross Domestic Product Growth Model
Multivariate Regression
Regular Gross Domestic Product
Regular Gross Domestic Product Growth
Regular Variable Moments
Regular Variable Moments
Regular Variable Moments