Probability Measures

1 Why

We use the language of measure theory to give a mathematical model for uncertain outcomes. TODO: probability intuition sheet.

2 Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The word *space* is natural, since we developed measure theory partly as a generalization of volume in three-dimensional space. The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra.

The measure in a probability space corresponds to the even probability function.

2.1 Notation

Let $(A, \mathcal{A})$ be a measurable space. We denote the set of outcomes by Ω, a mnemonic for "outcomes." We denote the sigma-algebra by $\mathcal{A}$, as usual. We denote a probability measure by $\mathbf{P}$, a mnemonic for

“probability," and intended to remind of the event probabilty function. Thus, we often say “Let $(A, \mathcal{A}, \mathbf{P})$ be a probability space."

2.2 Properties