Why

Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\text{cov}(f, g)$. We defined it:

$$\text{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\mathbf{E}(g).$$

Properties

Prop. 1. *Covariance is symmetric and billinear.*[2]

Prop. 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\text{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \text{var}(f).$$

□

---

[1]Future editions will include this.

[2]Future editions will include an account.

Prop. 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\mathrm{var}\left(\sum_{i=1}^{n} f_i\right) = \mathrm{cov}\left(\sum_{i=1}^{n} f_i, \sum_{i=1}^{n} f_i\right)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{cov}(f_i, f_j)$$

$\square$

Covariance

Variance

Expectation

Random Variables

Topological Sigma Algebra   Outcome Variables

Probability Measure   Measurable Functions   Topological Spaces

Measure   Metrics

Extended Real Numbers   Distance

Real Limits   Space Distance

Absolute Value   Plane Distance

Event Probabilities   Interval Length

Probability Distributions   Intervals

Uncertain Outcomes   Real Line   Real Space   Real Plane

Real Summation   Real Sequences   Integral Line   Real Order

Sigma Algebras   Real Numbers   Sequences   Geometry

Rational Numbers   Direct Products   Set Numbers

Subset Algebras   Natural Summation   Integer Arithmetic   Finite Sets

Cardinality   Family Operations   Natural Multiplicative Identity   Natural Additive Identity   Equivalent Sets

Set Operations   Algebras   Identity Elements   Arithmetic   Family Unions and Intersections

Operations   Function Inverses   Natural Powers   Integer Products   Integer Sums

Families   Function Composite   Function Image   Natural Products   Integer Numbers

Functions   Natural Sums   Equivalence Relations

Relations   Recursion Theorem   Natural Order

Subset Systems   Cartesian Products   Peano Axioms

Set Powers   Ordered Pairs   Natural Induction

Unordered Triples   Natural Numbers   Generalized Set Dualities

Set Symmetric Differences   Successor Sets   Set Dualities   Set Unions and Intersections   Intersection of Empty Set   Partitions

Set Complements   Pair Unions   Set Intersections

Set Differences   Set Unions   Pair Intersections

Empty Set   Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects