



FINITE SIGNED MEASURES

Why

For the difference of two (signed) measures to be well-defined, we need one of the two to be finite. Otherwise, the measure of the difference on the base set involves subtracting ∞ from ∞ .

Definition

A *finite* signed measure is one for which the measure of every set is finite. This condition is equivalent to the base set having finite measure (see below).

Result

PROPOSITION 1. *A signed measure is finite if and only if it is finite on the base set.*

Proof. Let (X, \mathcal{A}) be a measurable space. Let $\mu : \mathcal{A} \rightarrow [-\infty, \infty]$ be a signed measure. (\Rightarrow) If μ is finite, then $\mu(X)$ is finite since $X \in \mathcal{A}$. (\Leftarrow) Next, suppose $\mu(X)$ is finite. Let $A \in \mathcal{A}$. Then $X = A \cup (X - A)$, with these sets disjoint, so by countable additivity of μ , $\mu(X) = \mu(A) + \mu(X - A)$. Since $\mu(X)$ finite, $\mu(A)$ and $\mu(X - A)$ are both finite. \square

