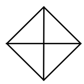


Why

We want to talk about the size of a set.

Definition

Two sets are *equivalent* if there exists a bijection between them. Let X be a set. Then set equivalence as a relation in X^* is an equivalence relation (see Equivalence Relations).

Notation

If A and B are sets and they are equivalent, then we write $A \sim B$, read aloud as "A is equivalent to B."

Basic Result

Every set is equivalent to itself, whether two sets are equivalent does not depend on the order in which we consider them, and if two sets are equivalent to the same set then they are equivalent to each other. These facts can be summarized by the following proposition.

Proposition 1. *Let X a set. Then $\sim$ is an equivalence relation on X^*.*[1]

For natural numbers

Proposition 2. *Every proper subset of a natural number is equivalent to some smaller natural number.*[2]

[1] The proof is direct and will appear in future editions.

[2] The proof, which uses induction, will appear in future editions.

Equivalence to subsets

It is unusual that a set can be equivalents to a proper subset of itself.

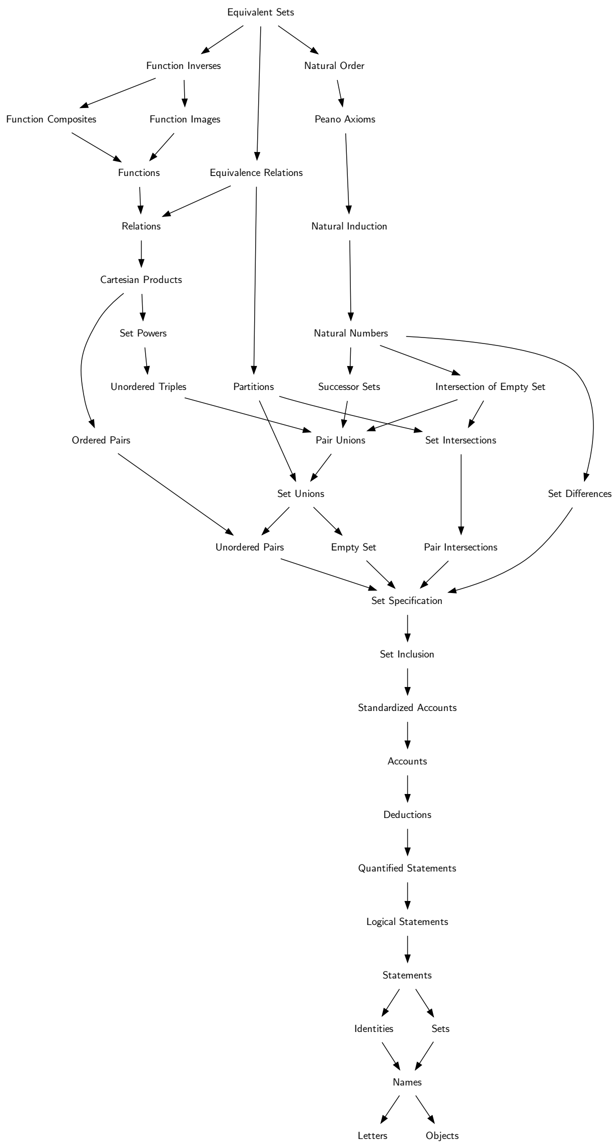
Proposition 3. *A set may be equivalent to a proper subset of itself.*

Proof. The example is the set of natural numbers and the function $f(n) = n^+$. It is a bijection from ω onto N.[3] $\square$

However, this never holds for natural numbers.

Proposition 4. *If $n \in \omega$ then $n \not\sim x$ for any $x \subset n$ and $x \neq n$.*

[3]The account will be expanded in future editions.



Equivalent Sets
Function Inverses
Natural Order
Function Composites
Function Images
Peano Axioms
Functions
Equivalence Relations
Relations
Natural Induction
Cartesian Products
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects