

COMBINED ORDERS

Why

Given multiple orders, can we combine them?

Discussion

Suppose we have two orders \prec_1 and \prec_2 on A. Define \prec by $a \prec b$ if and only if $a \prec_1 b$ and $b \prec_2 a$. Notice that \prec is reflexive, transitive, and antisymmetric, and so it is a partial order. Call it the *combined* order.

Here's the rub. Even if \prec_1 and \prec_2 are total, \prec may not be total.¹ Consider the basic case $A = \{a, b\}$ and $\prec_1 = \{(a, a), (a, b), (b, b)\}$ and $\prec_2 = \{(a, a), (b, a), (b, b)\}$ Then $\prec = \{(a, a), (b, b)\}$, a partial order, to be sure, but not really any order at all.

There is not anything to be done about it, it is a fact. Total orders do not (necessarily) induce total combined orders.

¹Future editions will include and expand.

