

### PROBABILITY VECTORS

# Why

We can identify probability distributions with vectors.

## **Definition**

Let  $p: \Omega \to \mathbf{R}$  be a probability distribution on a finite set  $\Omega = \{\omega_1, \ldots, \omega_n\}$ . We can associate p with the vector  $x \in \mathbf{R}^n$  defined by  $x_i = p(\omega_i)$  for  $i = 1, \ldots, n$ . We call this vector y the probability vector associated with p. The conditions on p mean that (1)  $\langle 1, y \rangle = 1$  and (2)  $y \geq 0$  (i.e.,  $y_i \geq 0$  for all  $i = 1, \ldots, n$ .).

Conversely, suppose  $z \in \mathbf{R}$  satisfies (1) and (2). Then  $q: \Omega \to \mathbf{R}$  defined by  $q(\omega_i) = z_i$  for  $i = 1, \ldots, n$  is a probability distribution. For this reason, we call z satisfying the conditions a *distribution vector*. Notice that implicit in this correspondence is a numbering  $\omega: \{1, \ldots, n\} \to \Omega$  of the set of outcomes  $\Omega$ .

### Expectation

Suppose  $\rho \in \mathbf{R}^n$  is a distribution vector corresponding to  $p: \Omega \to \mathbf{R}$  its corresponding distribution. Let  $x: \Omega \to \mathbf{R}$  and (similar to  $\rho$ ) define  $\xi \in \mathbf{R}^n$  by  $\xi_i = x(\omega_i)$  for  $i = 1, \ldots, n$ . Then  $\mathbf{E} x = \langle \rho, \xi \rangle$ .

### Example

For 
$$\rho = (-1, -1, 1, 1, 2)$$
 and  $\xi = (0.1, 0.15, 0.1, 0.25, 0.4)$ 

$$\mathbf{E}x = \langle \rho, \xi \rangle - 1 - 0.15 + 0.1 + 0.25 + 2(0.4) = 0.9.$$

