Why

Can we think of linear maps as vectors?

Definitions

Suppose V and W are some vector spaces over a field **F**. Denote the linear maps from V to W by $\mathcal{L}(V, W)$ as usual.

Addition. Given $S, T \in \mathcal{L}(V, W)$ the *sum* of S and T is the linear map $R \in \mathcal{L}(V, W)$ defined by

$$Rv = Sv + Tv \quad \text{for all } v \in V$$

Scalar multiplication. Given $S \in \mathcal{L}(V, W)$ the *(scalar) product* of λ and T is the linear map $Q \in \mathcal{L}(V, W)$ defined by

$$Qv = \lambda Tv \quad \text{for all } v \in V$$

Proposition 1. *Suppose V and W are two vector spaces over the same field **F**. Then $\mathcal{L}(V, W)$ is a vector space over the field **F** with respect to the operations of addition and scalar multiplication just defined.*

The additive identity of the vector space $\mathcal{L}(V, W)$ is the zero map $0 \in \mathcal{L}(V, w)$.

Notation

Given $S, T \in \mathcal{L}(V, W)$ the *sum* of S and T and $\lambda \in$ **F**, we denote the sum of S and T by $S + T$. Hence,

$$(S + T)(v) = Sv + Tv \quad \text{for all } v \in V$$

We denote the product of λ and T by λT. Hence,

$$(\lambda T)(v) = \lambda(Tv) \quad \text{for all } v \in V$$

Vector Space of Linear Transformations

Linear Transformations

Real Polynomial Derivatives · Derivatives of Scalar Multiples · Derivatives of Sums · Real Integral Additivity · Real Integral Homogeneity

Real Linear Functions · Real Differentiable Functions · Simple Integral Additivity · Simple Integral Homogeneity

Transformations · Linear Combinations · Simple Integrals

Vector Space of Polynomials

Vectors · Real Linear Combinations · Simple Functions · Measurable Functions

Real Polynomials · Fields · Real Functions · Measures · Real Vector

Length Measure · Sigma Algebras · N-Dimensional Space · Plane Vector Sums and Differences · Space Vectors

Real Arithmetic · Rings · Real Series · Extended Real Numbers · Distance · Plane Vectors

Real Limits · Space Distance · Plane Distance

Real Multiplicative Inverse · Absolute Value · Interval Length

Real Products · Intervals · Real Space

Real Additive Inverse · Real Line · Real Plane

Rational Multiplicative Inverse · Rational Arithmetic · Real Sums · Real Order · Integral Line · Real Sequences

Rational Products · Rational Order · Rational Sums · Real Numbers · Subset Algebras · Sequences · Geometry

Rational Numbers · Cardinality · Lists

Integer Arithmetic · Natural Fractions · Set Numbers · Direct Products

Integer Products · Natural Additive Identity · Natural Multiplicative Identity · Finite Sets · Inverse Elements

Natural Arithmetic · Identity Elements · Equivalent Sets · Element Functions

Integer Sums · Integer Order · Natural Power · Operations · Function Inverses · Comparisons · Family Unions and Intersections

Integer Numbers · Natural Products · Orders · Function Image · Function Composite · Families

Natural Sum · Equivalence Relations · Natural Order · Functions · Converse Relations

Recursion Theorem · Relations

Peano Axioms · Ordered Pair Projections

Natural Induction · Characteristic Functions · Set Products · Subset Systems

Natural Numbers · Generalized Set Dualities · Set Powers · Ordered Pairs

Set Symmetric Differences · Set Partitions · Intersection of Empty Set · Successor Sets · Set Dualities · Unordered Triples · Set Unions and Intersections

Set Complements · Set Intersections · Pair Unions

Set Differences · Set Unions · Pair Intersections

Empty Set · Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deduction

Quantified Statements

Logical Statements

Statements

Identities · Sets

Names

Letters · Objects