Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

Notation

Let $(X, \mathcal{A}, \mathbf{P})$ be a probability space. Let $(Y, \mathcal{B})$ a measurable space. A random variable is a measurable function $f : X \to Y$.

Some authors tend to denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. They reserve lower case letters x, y, z for the elements of the codomains of these functions. In such cases, they often denote the set of outcomes as Ω, which we have mentionedis a mnemonic for outcomes.

Special notation for common cases

Some authors use notation for the probability of particular, common sets. Suppose $(\Omega, \mathcal{A}, \mathbf{P})$ is a probability space with $X : \Omega \to \mathbf{R}$ a real-valued random variable. Many authors use $\mathbf{P}(X \in A)$ (or $P(X \in A)$, no bold) to denote

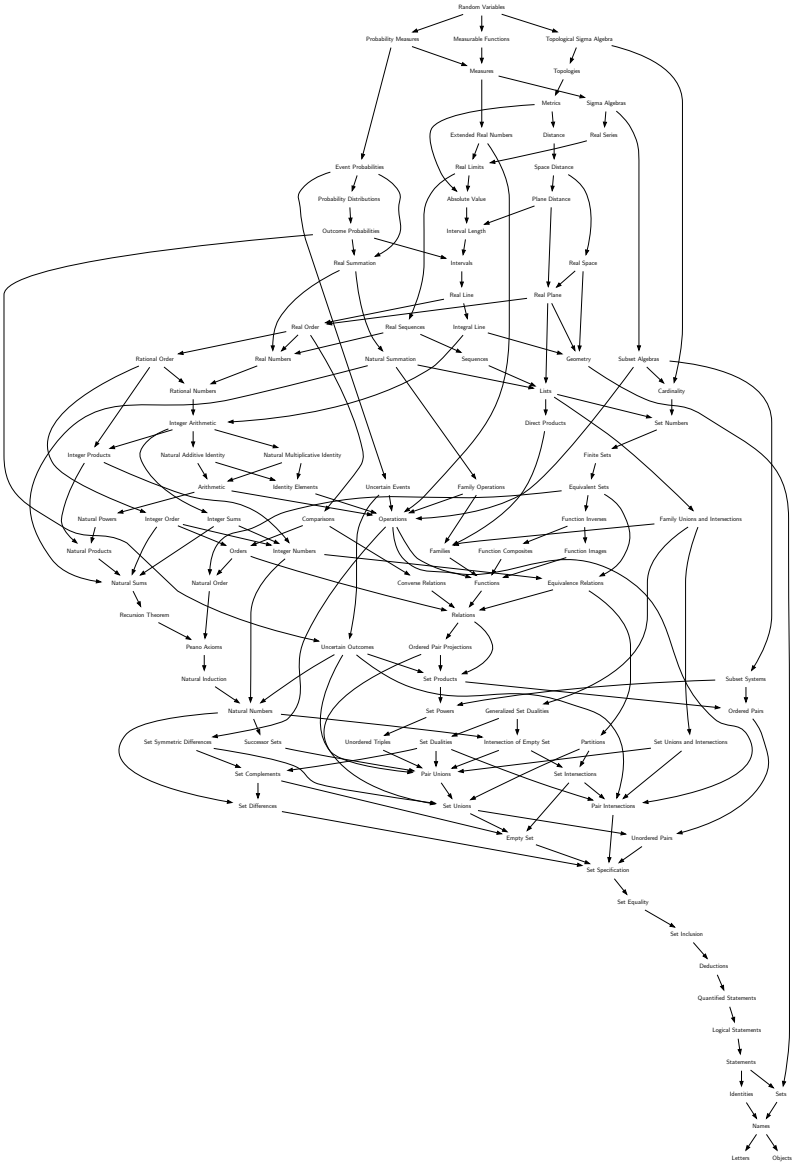
$$\mathbf{P}(X^{-1}(A)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\})$$

where $A \in \mathcal{B}(\mathbf{R})$, the Borel sigma algebra on $\mathbf{R}$. We will tend to use brackets in place of parentheses for clarity. So we will write $\mathbf{P}[X \in A]$.

Similar to the above, suppose $Y : \Omega \to \mathbf{R}$ is a random variable and $B \in \mathcal{B}(\mathbf{R})$. Then we will use $\mathbf{P}[X \in A, Y \in B]$ to denote

$$\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B)) = \mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

Similarly for n random variables $X_1, \ldots, X_n : \Omega \to \mathbf{R}$, and Borel sets $A_1, \ldots, A_n$, we will use $\mathbf{P}[X_1 \in A_1, \ldots, X_n \in A_n]$ to denote $\mathbf{P}(\cap_{i=1}^n X_i^{-1}(A_i))$.


Random Variables
Probability Measures
Measurable Functions
Topological Sigma Algebra
Measures
Topologies
Metrics
Sigma Algebras
Extended Real Numbers
Distance
Real Series
Event Probabilities
Real Limits
Space Distance
Probability Distributions
Absolute Value
Plane Distance
Outcome Probabilities
Interval Length
Real Summation
Intervals
Real Space
Real Line
Real Plane
Real Order
Real Sequences
Integral Line
Rational Order
Real Numbers
Natural Summation
Sequences
Geometry
Subset Algebras
Rational Numbers
Lists
Cardinality
Direct Products
Set Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Uncertain Events
Family Operations
Equivalent Sets
Natural Powers
Integer Order
Integer Sums
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Orders
Integer Numbers
Families
Function Composites
Function Images
Natural Sums
Natural Order
Converse Relations
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Uncertain Outcomes
Ordered Pair Projections
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Generalized Set Dualities
Ordered Pairs
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects