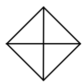

Why

We want to visualize relations.

Definition

A *directed graph* (or *digraph*, *graph*) is a pair (V, E) in which V is a nonempty set and E is a subset of $V \times V$. In other words, E is a relation on V. We call the elements of V *vertices* and the elements of E *edges*.

Example

For example, define V and E by

$$V = \{1, 2, 3, 4\} \quad \text{and} \quad E = \{(1, 2), (1, 3), (2, 4), (3, 4)\}$$

It is worth drawing this graph.

Edge and vertex terminology

Let $(v, w) \in E$. We say that (v, w) is an edge *from v to w*, and that it is an *outgoing edge* of v and an *incoming edge* of w. We call v a *parent* of w and we call w a *child* of v. We say that the edge (v, w) is *incident* to v and w.

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. The *indegree* of a vertex is number parents it has and the *outdegree* is the number of children it has.

The *parents*, *children*, and *neighbors* of a *set* A of vertices each defined to be the set of vertices which *are not* in the set but *are* the parents, children or neighbors of some vertex in the set defined.

A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

Notation

We denote by $\mathrm{pa} : V \to \mathcal{P}(V)$ and $\mathrm{ch} : V \to \mathcal{P}(V)$ the functions associating to each vertex its set of parents and set of children, respectively. As usual, we denote the parents of vertex v by pa_v and the children by ch_v.

Self-loops

If x is a vertex, and (x, x) is an edge, we call such an edge a *self-loop* (or just *loop*). Many authorities exclude self-loops in their definition of directed graphs, but we allow them. To make the distinction, we call a graph with no *loops simple* (a *simple graph*).


Directed Graphs
Set Numbers
Finite Sets
Equivalent Sets
Natural Order
Function Inverses
Function Composites
Function Images
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Pair Unions
Partitions
Ordered Pairs
Set Differences
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects