Why

We generalize the notion of sequence to index sets beyond the naturals.

Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go "further out."

To elaborate on property (b): if handed two natural numbers m and n, we can always find another, for example $\max\{m, n\} + 1$, larger than m and n. We might think of larger as "further out" from the first natural number: 1.

Now combine these two observations. A *directed set* is a set D with a partial order $\preceq$ satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.

A *net* is a function on a directed set.

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \to A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.

Nets

Sequences

Lists

Direct Products

Set Numbers

Finite Sets

Equivalent Sets

Orders

Natural Order

Family Unions and Intersections

Function Inverses

Families

Function Images

Function Composites

Peano Axioms

Functions

Equivalence Relations

Natural Induction

Relations

Set Products

Natural Numbers

Generalized Set Dualities

Set Powers

Partitions

Set Dualities

Successor Sets

Intersection of Empty Set

Unordered Triples

Set Unions and Intersections

Set Complements

Pair Unions

Set Intersections

Ordered Pairs

Set Differences

Pair Intersections

Set Unions

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects