



## Why

We want to talk about several objects in order.

## Definition

Suppose  $A$  is a set. A *list* (or *finite sequence*,  *$n$ -tuple*, *string*, *dataset*) in  $A$  (or *of elements from* or *of*  $A$ ) is a function

$$a : \{1, \dots, n\} \rightarrow A.$$

In other words, a list is a *family* whose index set is  $\{1, \dots, n\}$ . The *length* (or *size*) of the list is the size of its domain. The  $k$ th *entry* (or *term*, *record*) of  $A$  is the result  $a_k$  of  $k$ ; here  $k \in \{1, \dots, n\}$ .

## Notation

Since the natural numbers are **naturally ordered**, we denote lists using this order, from left to right, between parentheses. For example, we denote the function  $a : \{1, \dots, 4\} \rightarrow A$  by  $(a_1, a_2, a_3, a_4)$ .

## Orderings and numberings

Let  $A$  be a set with  $|A| = n$ . A sequence  $a : \{1, \dots, n\} \rightarrow A$  is an *ordering* of  $A$  if  $a$  is invertible. In this case, we call the inverse a *numbering* of  $A$ . An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

## Relation to Direct Products

A *natural direct product* is a product of a list of sets. We denote the direct product of a list of sets  $A_1, \dots, A_n$  by  $\prod_{i=1}^n A_i$ . If each  $A_i$  is the same set  $A$ , then we denote the product  $\prod_{i=1}^n A_i$  by  $A^n$ . The direct product  $A^n$  is the set of lists in  $A$ .

## Natural unions and intersections

We denote the family union of the list of sets  $A_1, \dots, A_n$  by  $\cup_{i=1}^n A_i$ . Similarly, we denote the intersection by  $\cap_{i=1}^n A_i$ .

## Slices

An *index range* for a list  $s$  of length  $n$  is a pair  $(i, j)$  for which  $1 \leq i < j \leq n$ . The *slice* corresponding to  $(i, j)$  is the length  $j - i$  list  $s'$  defined by  $s'_1 = s_i, s'_2 = s_{i+1}, \dots, s'_j = s_{i+j-1}$ .

We denote the  $(i, j)$ -slice of  $s$  by  $s_{i:j}$ . If  $i = 1$  we use  $s_{:j}$  and if  $j = n$  we use  $s_{i:}$  as shorthands for the slices  $s_{1:j}$  and  $s_{i:n}$ .

