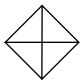


Why

We want language for all of the elements of a first set being the elements of a second set.

Definition

Denote a set by A and a set by B. If every element of the set denoted by A is an element of the set denoted by B, then we say that the set denoted by A is a *subset* of the set denoted by B. We say that the set denoted by A is *included* in the set denoted by B. We say that the set denoted by B is a *superset* of the set denoted by A or that the set denoted by B *includes* the set denoted by A. A set includes and is included in itself.

If the sets denoted by A and B are identical, then each contains the other. If $A = B$, then the set denoted by A includes the set denoted by B and the set denoted by B includes the set denoted by A. The axiom of extension asserts the converse also holds. If the set denoted by A includes the set denoted by B and the set denoted by$B$ includes the set denoted by A, then A and B denote the same set. In other words, if the set denoted by A is a subset of the set denoted by B and the set denoted by B a subset of the set denoted by A, then $A = B$.

The empty set is a subset of every other set.

Account 1. Empty Set Inclusion

1-2	`name`	$A, \varnothing$	
3	`have`	$\neg((\exists x)(x \in \varnothing))$	
4	`thus`	$(\forall x)((x \in \varnothing) \implies (x \in A))$	`by 3`
5	`i.e.`	$\varnothing \subset A$	`by 4`

Suppose toward contradiction that A were a set which did not include the empty set. Then there would exist an element in the empty set which is not in A. But then the empty set would not be empty. We call the empty set and A *improper subsets* of A. All other subsets we call *proper subsets.* In other words, B is an improper subset of A if and only if A includes B, $B \neq A$ and $B \neq \varnothing$.

Notation

Given two sets A and B, we denote that A is included in B by $A \subset B$. We read the notation $A \subset B$ aloud as "A is included in B" or "A subset B". Or we write $B \supset A$, and read it aloud "B includes A" or "B superset A".

In this notation, we express the axiom of extension

$$A = B \Leftrightarrow (A \supset B) \land (A \subset B).$$

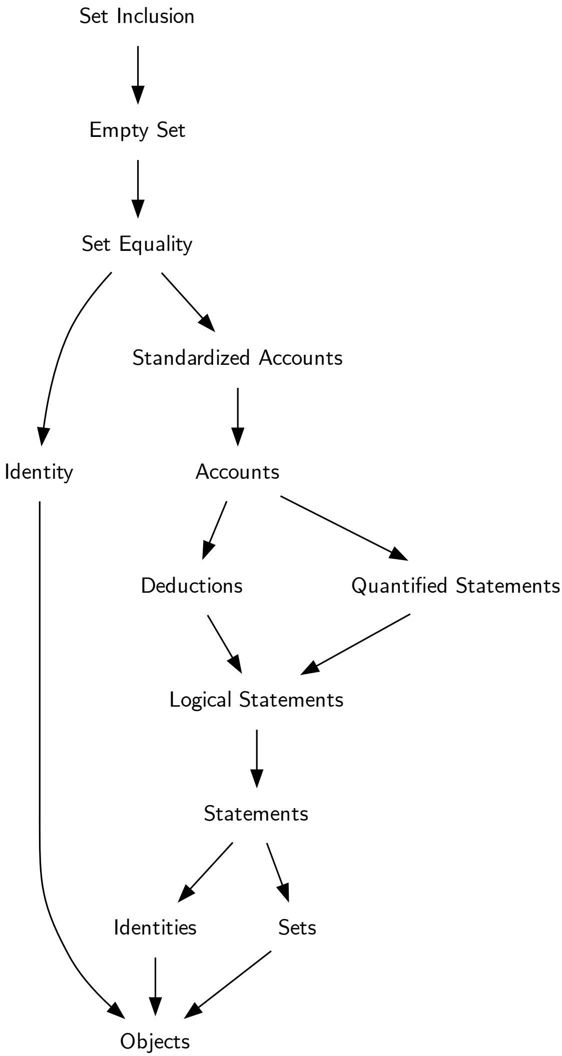
The notation $A \subset B$ is a concise symbolism for the sentence "every element of A is an element of B." Or for the alternative notation $a \in A \implies a \in B$.

Properties

Given a set A, $A \subset A$. Like equality, we say that inclusion is *reflexive*. Given sets A and B, if $A \subset B$ and $B \subset C$ then $A \subset C$. Like equality, we say that inclusion is *transitive*. If $A \subset B$ and $B \subset A$, then $A = B$ (by the axiom of extension). Unlike equality, which is symmetric, we say that inclusion is *antisymmetric*.

Comparison with belonging

Given a set A inclusion is reflexive. $A \subset A$ is always true. Is $A \in A$ ever true? Also, inclusion is transitive. Whereas belonging is not.


Set Inclusion
Empty Set
Set Equality
Standardized Accounts
Identity
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Objects