

FIELDS

Why

We generalize the algebraic structure of addition and multiplication over the rationals.

Definition

A field is a ring $(R, +, \cdot)$ for which \cdot is commutative (i.e., ab = ba for all $a, b \in R$) and \cdot has inverses for all elements except 0. In this case, we refer to field addition and field multiplication.

We also give names to the objects which have one of these additional properties or the other. A ring which for which multipliation is commutative is called a *commutative ring*. Note that a ring is *always* commutative with respect to addition, here the term commutative refers to multiplication. A ring for which there are inverse elements, excepting 0, is called a *division ring*).

Notation

Since our guiding example is the set of rationals \mathbf{Q} with addition and multiplication defined in the usual manner, and we use a bold font for \mathbf{Q} , we tend to denote an arbitrary field by \mathbf{F} , a mnemonic for "field."

Field operations

Along with field addition and field multiplication, we call the function which takes an element of a field to its additive inverse and the function which takes an element of a field to its multiplicative inverse the *field operations*.

