



# Integrable Function Spaces

## 1 Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? TODO: perhaps do  $L^2$  first then generalize.

## 2 Definition

The *integrable function spaces* are a collection of function spaces, one for each real number  $p \geq 1$ , for which the  $p$ th power of the absolute value of the function is integrable.

TODO: case  $\infty$

### 2.1 Notation

Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $p \geq 1$ . Let  $R$  denote the set of real numbers. We denote the integrable function space corresponding to  $p$  by  $\mathcal{L}^p(X, \mathcal{A}, \mu, R)$ . We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, R) = \left\{ \text{measurable } f : X \rightarrow R \mid \int |f|^p d\mu < \infty \right\}$$

Let  $C$  denote the set of complex numbers. Similarly for complex-valued functions, we denote the  $p$ th space by  $\mathcal{L}^p(X, \mathcal{A}, \mu, C)$ .