



# Graphs

## 1 Why

We want to visualize relations.

## 2 Definition

A **graph** is a set and a relation on the set. The graph is **undirected** if the relation is symmetric; otherwise the graph is **directed**.

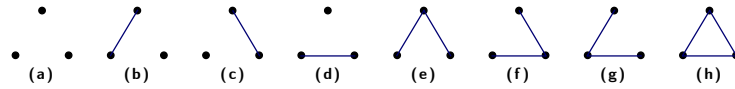
A **vertex** of the graph is an element of the set. The set is called the **vertex set**. An **edge** of the graph is an element of the relation. The relation is called the **edge set**.

### 2.1 Notation

We denote the vertex set by  $V$ , a mnemonic for vertex. We denote the edge set by  $E$ , a mnemonic for edge. We denote a graph by  $(V, E)$ . If the vertex set is assumed we can unambiguously refer to the graph by  $E$ .

## 2.2 Visualization

We visualize a graph by drawing a point for each vertex. If two vertices  $u$  and  $v$  are in relation, we draw a line from the point corresponding to  $u$  to the point corresponding to  $v$  with an arrow at the point corresponding to  $v$ . If the graph is undirected, we omit arrows. Here are all undirected graphs on three vertices.



## 3 Paths

A path in a graph is a sequence of vertices with the property that consecutive vertices are related. A path **cycles** if a vertex appears more than once. A path is **finite** if the sequence is finite. A **loop** is a finite path that cycles once. A finite path from vertex  $u$  to vertex  $v$  is a path starting with  $u$  and ending with  $v$ . The **length** of a finite path is the length of the sequence.

## 4 Properties

A graph is **connected** if there is a path between every pair of vertices. A graph is **acyclic** if none of its paths cycle.