

TREES

Definition

An undirected graph is a *tree* if it is connected and acyclic. An undirected graph is a *forest* if it is acyclic. Each connected component of a forest is a tree, motivating the definition.

Properties

Proposition 1. There is a unique path between any two vertices of a tree.

Proof. Such a path exists because the tree is connected. Such a path is unique because the existence of two separate paths would create a cycle. \Box

Distance

The distance between two vertices v and w in a tree is the length of the unique path connnecting v and w. Recall that the length of a path is the number of edges, or one fewer than the number of vertices. If v and w are adjacent in the tree, then there distance is 1.

