



## Why

We generalize the algebraic structure of addition and multiplication over the rationals.

## Definition

A *field* is a ring  $(R, +, \cdot)$  for which  $\cdot$  is commutative (i.e.,  $ab = ba$  for all  $a, b \in R$ ) and  $\cdot$  has inverses for all elements except 0. In this case, we refer to *field addition* and *field multiplication*.

We also give names to the objects which have one of these additional properties or the other. A ring for which multiplication is commutative is called a *commutative ring*. Note that a ring is *always* commutative with respect to addition, here the term commutative refers to multiplication. A ring for which there are inverse elements, excepting 0, is called a *division ring*.

## Notation

Since our guiding example is the set of rationals  $\mathbf{Q}$  with addition and multiplication defined in the usual manner, and we use a bold font for  $\mathbf{Q}$ , we tend to denote an arbitrary field by  $\mathbf{F}$ , a mnemonic for “field.”



