



## Definition

A polynomial  $p : \mathbf{R} \rightarrow \mathbf{R}$  is *nonnegative* (a *nonnegative polynomial*, *non-negative real polynomial*) if

$$p(x) \geq 0 \quad \text{for all } x \in \mathbf{R}$$

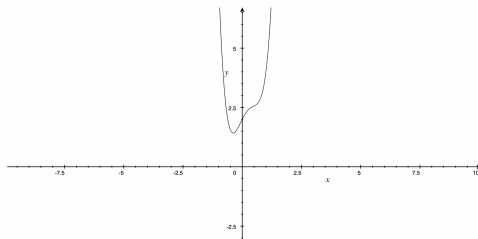
In this case, we call  $p$  *positive semidefinite* or *PSD*.

## Testing nonnegativity

Given polynomial  $p$ , how do we know if  $p$  is (globally) nonnegative? Consider  $p : \mathbf{R} \rightarrow \mathbf{R}$  defined by

$$p(x) = 5x^4 - 4x^3 - x^2 + 2x + 2$$

We visualize the graph of  $p$  below.



Given the coefficients of  $p$ , namely the list  $(2, 2, -1, -4, 5) \in \mathbf{R}^5$ , how can we tell? It is not so obvious, but if we write

$$p(x) = (x^2 + 1)^2 + (2x^2 - x - 1)^2,$$

then it is readily apparent that  $p \geq 0$  since all squares are nonnegative.

We can ask two questions:

1. If  $p$  is a nonnegative polynomial, can it be written as a sum of squares?
2. If  $p$  is a sum of squares, how do we find them?



