



## Why

We want to consider all the subsets of a given set.

## Definition

We do not yet have a principle stating that such a set exists, but our intuition suggests that it does.

**Principle 1** (powers). *For every set, there exists a set of its subsets.*

We call the existence of this set the *principle of powers* and we call the set the *power set*.<sup>1</sup> As usual, the principle of extension gives uniqueness (see **Set Equality**). The power set of a set includes the set itself and the empty set.

## Notation

Let  $A$  denote a set. We denote the power set of  $A$  by  $\mathcal{P}(A)$ , read aloud as “powerset of  $A$ .”  $A \in \mathcal{P}(A)$  and  $\emptyset \in \mathcal{P}(A)$ . However,  $A \subset \mathcal{P}(A)$  is false.

## Examples

Let  $a, b, c$  denote distinct objects. Let  $A = \{a, b, c\}$  and  $B = \{a, b\}$ . Then  $B \subset A$ . In other notation,  $B \in \mathcal{P}(A)$ . Showing each of the following is straightforward.

1. The empty set:  $\mathcal{P}(\emptyset) = \{\emptyset\}$
2. Singletons:  $\mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$
3. Pairs:  $\mathcal{P}(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
4. Triples:

$$\mathcal{P}(\{a, b, c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$

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<sup>1</sup>This terminology is standard, but unfortunate. Future editions may change these terms.

## Properties

We can guess the following easy properties.<sup>2</sup>

**Proposition 1.**  $\emptyset \in \mathcal{P}(A)$

**Proposition 2.**  $A \in \mathcal{P}(A)$

We call  $A$  and  $\emptyset$  the *improper* subsets of  $A$ . All other subsets we call *proper*.

## Basic fact

**Proposition 3.**  $E \subset F \longrightarrow \mathcal{P}(E) \subset \mathcal{P}(F)$

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<sup>2</sup>Future editions will expand this account.

