



## Why

Conditioning defines a new distribution on the set of outcomes.

## Definition

The conditional probability of any two events defines a new probability mass function over the set of outcomes which is in the second event. The conditional probability of an outcome is the probability of the second event divided into the original probability of the outcome.

## Notation

Let  $p$  be a probability mass function on the set of outcomes  $A$  with the corresponding event probability function  $\mathbf{P}$ . Then the distribution conditioned on  $B$  is  $q : A \rightarrow \mathbf{R}$ , defined by

$$q(a) = \begin{cases} \frac{p(a)}{\mathbf{P}(B)} & \text{if } a \in B \\ 0 & \text{otherwise .} \end{cases}$$

In other words,

$$\mathbf{P}_{A,p}(C \mid B) = \mathbf{P}_{A,q}(C)$$

## Generally

1

Let  $A_1, \dots, A_n$  be a sequence of non-empty finite sets. Let  $A = \prod_{i=1}^n A_i$ . Let  $p : A \rightarrow \mathbf{R}$  be a distribution on  $A$ . We

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<sup>1</sup>Future editions will rework this sheet.

denote the conditional distribution of  $i$  on  $j$  of  $p$  by  $p_{i|j} : A_i \times A_j \rightarrow \mathbf{R}$  For  $i, j = 1, \dots, n$  and  $i \neq j$ ,  $p_i$  satisfies

$$p_{i|j}(b, c)p_j(c) = p_{ij}(b, c)$$

for every  $b \in A_i$  and  $c \in A_j$ .

