Why

We want a notion for applying two functions one after the other. We apply a first function then a second function.

Definition

Consider two functions. And suppose the range of the first is a subset of the domain of the second. In other words, every value of the first is in the domain (and so can be used as an argument) for the second.

The *composite* or *composition* of the second function with the first function is the function which associates each element in the first's domain with the element in the second's codomain that the second function associates with the result of the first function.

The idea is that we take an element in the first domain. We apply the first function to it. We obtain an element in the first's codomain. This result is an element of the second's domain. We apply the second function to this result. We obtain an element in the second's codomain. The composition of the second function with the first is the function so constructed. Of course the order of composition is important.

Notation

Let A, B, C be non-empty sets. Let $f : A \to B$ and $g : B \to C$. We denote the composition of g with f by $g \circ f$ read aloud as "g composed with f." To make clear the domain and comdomain, we denote the composition $g \circ f : A \to C$. $g \circ f$ is defined by

$$(g \circ f)(a) = g(f(a)).$$

for all $a \in A$. Sometimes the notation gf is used for $g \circ f$.

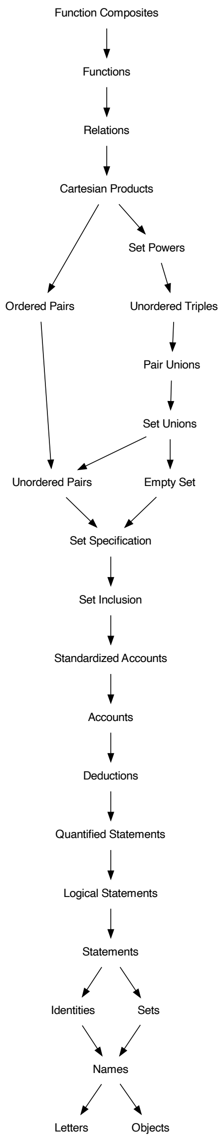
Basic Properties

Function composiiton is associative but not commutative.[1] Indeed, even if $f \circ g$ is defined, $g \circ f$ may not be.

Proposition 1 (Associative)**.** *Let $f : X \to Y$, $g : Y \to Z$ and $h : Z \to U$ Then $(f \circ g) \circ h = f \circ (g \circ h)$*[2]

[1] Future editions will include a counterexample.

[2] The proof is straightforward. Future editions will include it.


Function Composites
Functions
Relations
Cartesian Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects