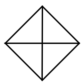


Why

We want to talk about several objects in order.

Definition

Suppose A is a set. A *list* (or *finite sequence*, *n-tuple*, *string*, *dataset*) *in* A (or *of* elements *from* or *of* A) is a function

$$a : \{1, \dots, n\} \to A.$$

In other words, a list is a *family* whose index set is $\{1, \dots, n\}$. The *length* (or *size*) of the list is the size of its domain. The *kth entry* (or *term*, *record*) of A is the result a_k of k; here $k \in \{1, \dots, n\}$.

Notation

Since the natural numbers are naturally ordered, we denote lists using this order, from left to right, between parentheses. For example, we denote the function $a : \{1, \dots, 4\} \to A$ by (a_1, a_2, a_3, a_4).

Orderings and numberings

Let A be a set with $|A| = n$. A sequence $a : \{1, \dots, n\} \to A$ is an *ordering* of A if a is invertible. In this case, we call the inverse a *numbering* of A. An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

Relation to Direct Products

A *natural direct product* is a product of a list of sets. We denote the direct product of a list of sets $A_1, \dots, A_n$ by $\prod_{i=1}^{n} A_i$. If each A_i is the same set A, then we denote the product $\prod_{i=1}^{n} A_i$ by A^n. The direct product A^n is the set of lists in A .

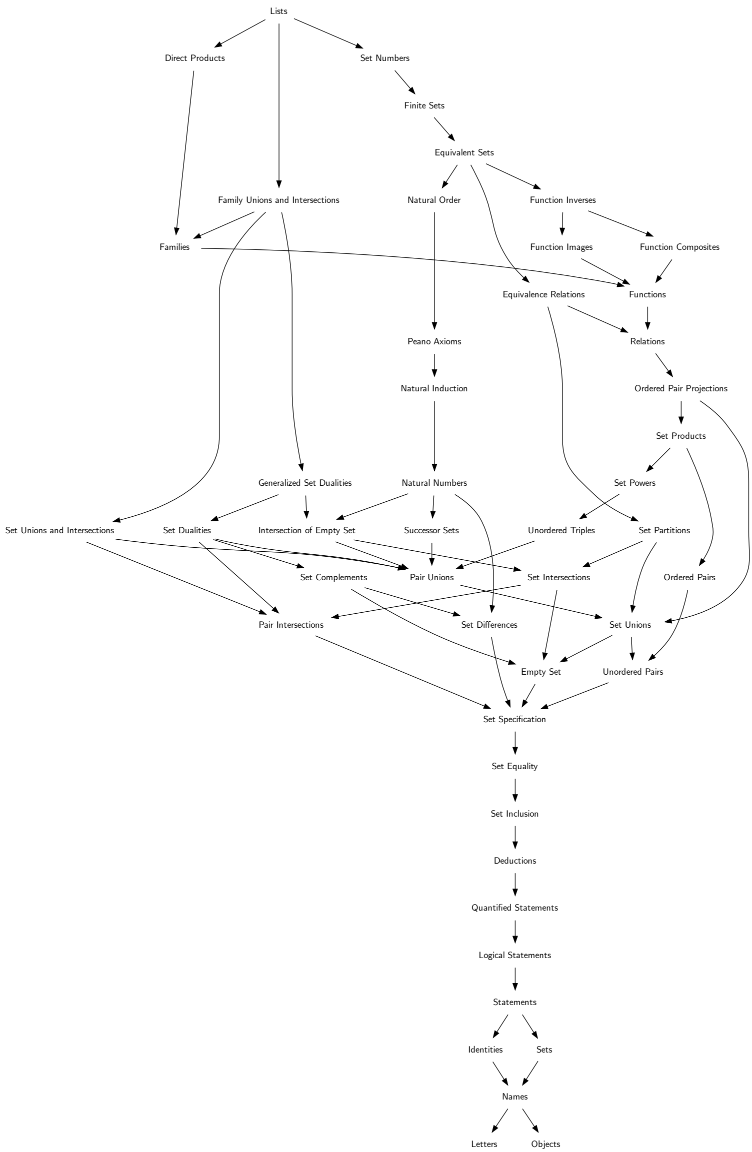
Natural unions and intersections

We denote the family union of the list of sets $A_1, \dots, A_n$ by $\cup_{i=1}^n A_i$. Similarly, we denote the intersection by $\cap_{i=1}^n A_i$.

Slices

An *index range* for a list s of length n is a pair (i, j) for which $1 \leq i < j \leq n$. The *slice* corresponding to (i, j) is the length $j - i$ list s' defined by $s'_1 = s_i$, $s'_2 = s_{i+1}, \dots, s'_j = s_{i+j-1}$.

We denote the (i, j)-slice of s by $s_{i:j}$. If $i = 1$ we use $s_{:j}$ and if $j = n$ we use $s_{i:}$ as shorthands for the slices $s_{1:j}$ and $s_{i:n}$.


Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Images
Function Composites
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects