Measure Space

1 Why

We want to generalize the notions of length, area, and volume.

2 Definition

A *measurable space* is a sigma algebra. We call the distinguished subsets the *measurable sets*.

A *measure* on a measurable space is a function from the sigma algebra to the positive extended reals. A *measure space* is a measurable space and a measure.

2.1 Notation

2.2 Properties

Proposition 1. *Let* $(A, \mathcal{A})$ *be a measurable space and* $m : \mathcal{A} \to [0, \infty]$ *be a measure.*

If $B \subset C \subset A$*, then* $m(B) \leq m(C)$*. We call this property the of measures monotonicity of measure.*

Proposition 2. *For a measure space $(A, \mathcal{A}, m)$.*

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Proposition 3. *For a measure space $(A, \mathcal{A}, m)$.*

If $\{A_n\} \subset \mathcal{A}$ a countable family, then $m(\cup A_n) \leq \sum_i m(A_i)$.

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Proposition 5. *For a measure space $(A, \mathcal{A}, m)$.*

$$m(\cup_{n=1}^{\infty} A_i) = \lim_{n \to \infty} m(A_i)$$

Proposition 6. *For a measure space $(A, \mathcal{A}, m)$.*

$$m(\cap_{n=1}^{\infty} A_i) = \lim_{n \to \infty} m(A_i)$$

2.3 Examples

Example 7. *counting measure*