

Multivariate Normal Conditionals

1 Why

What of the conditional densities of a multivariate normal density.

2 Result

Proposition 1. Let

$$\mu = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}$$
 and $\Sigma = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$

Then the conditional density $f_{x|y}(\xi,\gamma)$ is is $\mathcal{N}(\bar{\mu}(\gamma),\bar{\Sigma})$ with

$$\bar{\mu}(\gamma) = \mu_1 + \Sigma_{xy} \Sigma_{yy}^{-1} (\gamma - \mu_y) \text{ and } \bar{\Sigma} = \Sigma_{xx} - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx}.$$