

## Topological Spaces

## Why

We want to generalize the notion of continuity.

## **Definition**

A topological space is a subset system for which: (1) the empty set and the base set are distinguished, (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the set of distinguished subsets the topology. We call the distinguished subsets the open sets.

## **Notation**

Let A be a non-empty set. For the set of distinguished sets, we use  $\mathcal{T}$ , a mnemonic for topology, read aloud as "script T". We denote elements of  $\mathcal{T}$  by O, a mnemonic for open. We denote the topological space with base set A and topology  $\mathcal{T}$  by  $(A, \mathcal{T})$ . We denote the properties satisfied by elements of  $\mathcal{T}$ :

1. 
$$X, \emptyset \in \mathcal{T}$$

2. 
$$\{O_i\}_{i=1}^n \subset \mathcal{T} \longrightarrow \bigcap_{i=1}^n O_i \in \mathcal{T}$$

3. 
$$\{O_{\alpha}\}_{\alpha \in I} \subset \mathcal{T} \longrightarrow \bigcup_{\alpha \in I} \in \mathcal{T}$$

