Why

What is the best linear regressor if we choose according to a weighted squared loss function.

Definition

Suppose we have a paired dataset of n records with inputs in $\mathbf{R}^d$ and outputs in $\mathbf{R}$. A *weighted least squares linear predictor* for nonnegative weights $w \in \mathbf{R}^n$, $w \geq 0$, is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$\frac{1}{n} \sum_{i=1}^{n} w_i (y_i - x^\top a^i)^2.$$

Some authors refer to this process of selecting a linear predictor as the *weighted least-squares problem*.

Define $W \in \mathbf{R}^{n \times n}$ so that $W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix. We want to find x to minimize

$$\|W(Ax - y)\|$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$\left(A^\top W A^\top\right)^{-1} A^\top W y.$$

Weighted Least Square Linear Regressors

Inner Products Least Square Linear Regressors

Regressors Loss Functions Linear Predictors Matrix Inverse

Norms Induction Matrix Transpose Matrix-Matrix Products Linear Transformations

Functionals Matrices Transformations Linear Combinations

Vectors

N-Dimensional Space Datasets

Absolute Value Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Line Real Plane Real Space Sequences

Integral Line Real Order Direct Products

Real Numbers Geometry Fields Set Numbers

Rational Numbers Groups

Integer Arithmetic Integer Additive Inverses Finite Sets

Integer Sums Natural Additive Identity Natural Multiplicative Identity Inverse Elements

Arithmetic Identity Elements Element Functions Equivalent Sets

Natural Exponents Integer Products Algebras Family Unions and Intersections Function Inverses

Natural Products Integer Numbers Operations Families Function Images Function Composites

Natural Sums Equivalence Relations Functions

Recursion Theorem Natural Order Relations

Peano Axioms

Natural Induction Cartesian Products

Generalized Set Dualities Natural Numbers Set Powers

Set Dualities Successor Sets Set Unions and Intersections Intersection of Empty Set Unordered Triples Partitions

Set Complements Pair Unions Set Intersections Ordered Pairs

Set Differences Pair Intersections Set Unions

Empty Set Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects

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