



Why

We abstract the notion of inner product to an arbitrary vector space.

Definition

Suppose \mathbf{F} is a field which is either \mathbf{R} or \mathbf{C} . Let (V, \mathbf{F}) be a vector space. Then a function $f : V \times V \rightarrow \mathbf{F}$ is an *inner product* on V if

1. $f(x, x) \geq 0$, $f(x, x) = 0 \Leftrightarrow x = 0$;
2. $f(x, y) = \overline{f(y, x)}$
3. $f(ax + by, z) = a f(x, z) + b f(y, z)$

A *inner product space* (or *pre-Hilbert space*) is a tuple: a vector space over \mathbf{R} or \mathbf{C} and an inner product.

