Integrable Function Spaces

Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? TODO: perhaps do L^2 first then generalize.

Definition

The *integrable function spaces* are a collection of function spaces, one for each real number $p \geq 1$, for which the pth power of the absolute value of the function is integrable.

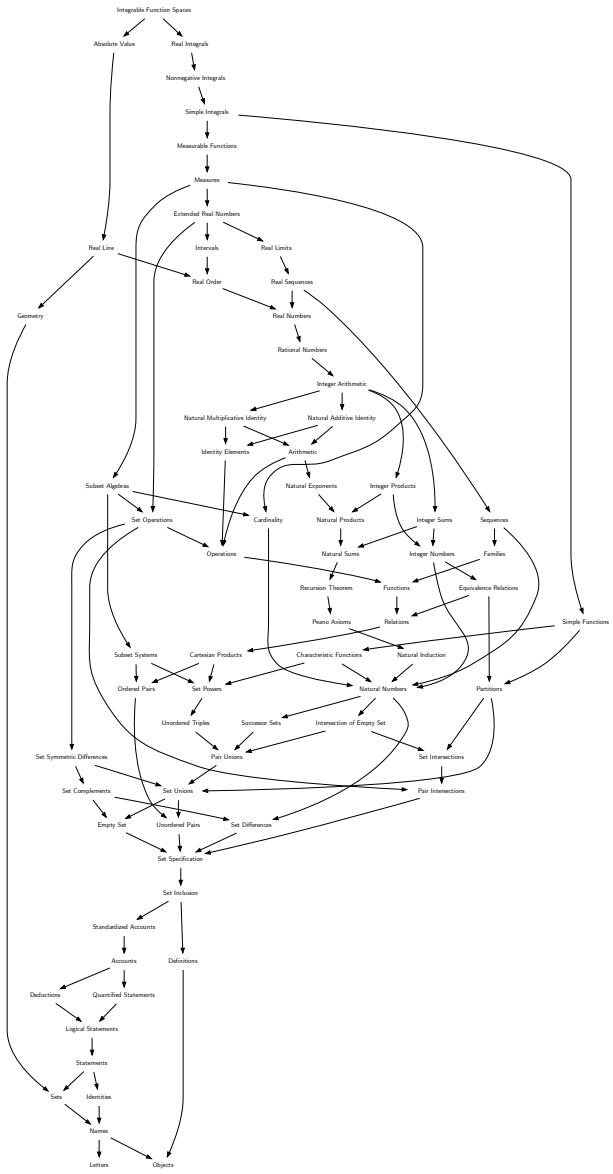
TODO: case ∞

Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $p \geq 1$. Let R denote the set of real numbers. We denote the integrable function space corresponding to p by $\mathcal{L}^p(X, \mathcal{A}, \mu, R)$. We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, R) = \left\{ \text{ measurable } f : X \to R \;\middle|\; \int |f|^p \, d\mu < \infty \right\}$$

Let C denote the set of complex numbers. Similarly for complex-valued functions, we denote the pth space by $\mathcal{L}^p(X, \mathcal{A}, \mu, C)$.


Integrable Function Spaces
Absolute Value
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Extended Real Numbers
Real Line
Intervals
Real Limits
Real Order
Real Sequences
Geometry
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Identity Elements
Arithmetic
Subset Algebras
Natural Exponents
Integer Products
Set Operations
Cardinality
Natural Products
Integer Sums
Sequences
Operations
Natural Sums
Integer Numbers
Families
Recursion Theorem
Functions
Equivalence Relations
Peano Axioms
Relations
Simple Functions
Subset Systems
Cartesian Products
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Differences
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Definitions
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects