



Why

What is the natural generalization of a smooth function to functions defined on sets of \mathbf{R}^k .¹

Definition

Let $U \subset \mathbf{R}^d$ be an open set (see Real Open Sets). A function $f : U \rightarrow \mathbf{R}$ is *smooth* if all its partial derivatives exists and are continuous. More

More generally, let $X \subset \mathbf{R}^d$. A function $f : X \rightarrow \mathbf{R}$ is *smooth* if there exists an open set $U \subset \mathbf{R}^d$ and a smooth $F : U \rightarrow \mathbf{R}$ so that $F(x) = f(x)$ for all $x \in U \cap X$.

Example

The identity map is smooth. In other words, let $f : \mathbf{R}^d \rightarrow \mathbf{R}$ be so that $X \subset \mathbf{R}^d$. Then $f : X \rightarrow \mathbf{R}$ is

Properties

Proposition 1. *The composition of two smooth functions is smooth.*

¹Future editions will expand.

