

## ROOTED TREES

## Why

We want to talk rooting a tree at a given vertex.<sup>1</sup>

## **Definition**

A rooted tree is an ordered pair ((V,T),r) where (V,T) is a tree and  $r \in V$  is a distinguished vertex which we call the root. We visualize rooted trees with the root at the top (see the figure below).

## Parents and children

Suppose w is the first vertex on the path from the root to a non-root vertex v. Since there is only one such path, w is unique and we call it the parent of v. Conversely, we call v a child of w. We denote the set of children of v by ch(v). A vertex may have no children or it may have many children. If it has no children we call it a leaf.

We define the parent function pa :  $V \to V$  with the convention that the parent of the root is the root. The parent of degree k where k > 0 is  $\operatorname{pa}^k(x)$  where  $\operatorname{pa}^k$  is the composite of pa with itself k times. So, in particular,  $\operatorname{pa}^{k+1}(v) = \operatorname{pa}(\operatorname{pa}^k(v))$ . We define the parent of degree 0 of v to be v, and denote it by  $\operatorname{pa}^0(v) = v$ . For the tree visualized in the figure below,  $\operatorname{pa}(i) = g$ ,  $\operatorname{pa}^2(i) = d$ ,  $\operatorname{pa}^3(i) = a$ .

If  $w = \operatorname{pa}^k(v)$  for some  $k \geq 0$ , then w is a ancestor of v and v is a descendent of w. We use the term proper ancestor and proper descendent if k > 0 (i.e.,  $w \neq v$ ).

The depth or level of a vertex v is its distance (see Trees) to the root. We denote the level of a vertex v by lev(v). The level of the root is 0. If lev(v) = k > 0, then  $pa^k(v)$  is the root. The level function lev satisfies lev(v) = lev(pa(v)) + 1.

<sup>&</sup>lt;sup>1</sup>Future editions will expand this intuition.



