Dependent Events

Why

We want to talk about how knowledge of one aspect of an outcome can give us knowledge about another aspect.

Definition

Let $\mathbf{P} : \mathcal{P}(\Omega) \to \mathbf{R}$ be a probability measure.

Two events $A, B \subset \Omega$ are *independent* if

$$\mathbf{P}(A \cap B) = \mathbf{P}(A)\mathbf{P}(B).$$

In other words, they are independent if the probability of their intersection is the product of their respective probabilities. Otherwise, we call A and B *dependent*.

In the case that $\mathbf{P}(B) \neq 0$, then $\mathbf{P}(A \cap B) = \mathbf{P}(A)\mathbf{P}(B)$ is equivalent to $\mathbf{P}(A \mid B) = \mathbf{P}(A)$. We interpret this second expression as encoding the fact that the occurence of event B does not change the "credibility" of the event A.

Example: two dice

Define $\Omega = \{(\omega_1, \omega_2) \mid \omega_i \in \{1, \ldots, 6\}\}$, and interpret $\omega \in \Omega$ as corresponding to pips face up after rolling two dice. Define $p : \Omega \to \mathbf{R}$ by $p(\omega) = 1/36$.

Two events are $A = \{\omega \in \Omega \mid \omega_1 + \omega_2 > 5\}$, "the sum is greater than 5", and $B = \{\omega \in \Omega \mid \omega_1 > 3\}$, "the number of pips on the first die in greater than 3". Then $\mathbf{P}(A) = 26/36$. Also, $\mathbf{P}(A \mid B) = 17/16$. So, these events are dependent.

Roughly speaking, we say that knowing B tells us something about A. In this case, we say that it "makes A more probable."

Dependent Events

Conditional Event Probabilities

Event Probabilities

Probability Distributions

Intervals

Real Line

Real Order

Real Summation

Rational Order

Real Numbers

Integral Line

Rational Numbers

Integer Arithmetic

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Arithmetic

Identity Elements

Natural Summation

Natural Powers

Integer Sums

Natural Products

Integer Order

Comparisons

Family Operations

Algebras

Uncertain Outcomes

Set Operations

Integer Numbers

Natural Sums

Orders

Operations

Families

Equivalence Relations

Recursion Theorem

Natural Order

Converse Relations

Functions

Relations

Peano Axioms

Partitions

Natural Induction

Set Products

Natural Numbers

Set Powers

Ordered Pairs

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Intersections

Pair Unions

Set Symmetric Differences

Pair Intersections

Set Unions

Set Complements

Geometry

Unordered Pairs

Empty Set

Set Differences

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects