



Metric Space Examples

1 Why

We give examples of metric spaces

2 Example

Example 1. Let n be a natural number. Let A be \mathbb{R}^n and define $d : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ by

$$d(a, b) = \sqrt{(a_1 - b_1)^2 + \cdots + (a_n - b_n)^2}.$$

(A, d) is a metric space.

Example 2. Let A be the unit circle in \mathbb{R}^2 . So $A = \{x \in \mathbb{R}^2 \mid x_1^2 + x_2^2 = 1\}$. Let $d_1 : A \times A \rightarrow \mathbb{R}$ defined by

$$d(a, b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}.$$

Let $d_2 : A \times A \rightarrow \mathbb{R}$ defined as the arc length between the two points. Both (A, d_1) and (A, d_2) are metric spaces.

Example 3. Let $A = C([0, 1], \mathbb{R})$. Let $d_1 : A \times A \rightarrow \mathbb{R}$ be such that

$$d_1(a, b) = \max_{x \in [0, 1]} |a(x) - b(x)|.$$

Let λ be the outer cover measure. Let $d_2 : A \times A \rightarrow \mathbb{R}$ be such that

$$d_2(a, b) = \int_{[0, 1]} |f - g| d\lambda.$$

Both (A, d_1) and (A, d_2) metric spaces.

