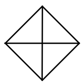


Definition

Let $(X, \mathbf{R})$ be a vector space. A function $f : X \times X \to \mathbf{R}$ is an *inner product* on the vector space $(X, \mathbf{R})$ if

1. $f(x, x) \geq 0, = 0 \longleftrightarrow x = 0,$
2. $f(x + y, z) = f(x, z) + f(y, z),$
3. $f(x, y) = f(y, x)$, and
4. $f(\alpha x, y) = \alpha f(x, y)$.

An *inner product space* is an ordered pair: a real vector space and an inner product.[1]

Examples

$\mathbf{R}^n$ with the usual inner product is an inner product space. Some authors call any finite-dimensional inner product space over the real numbers is a *Euclidean vector space.*

Notation

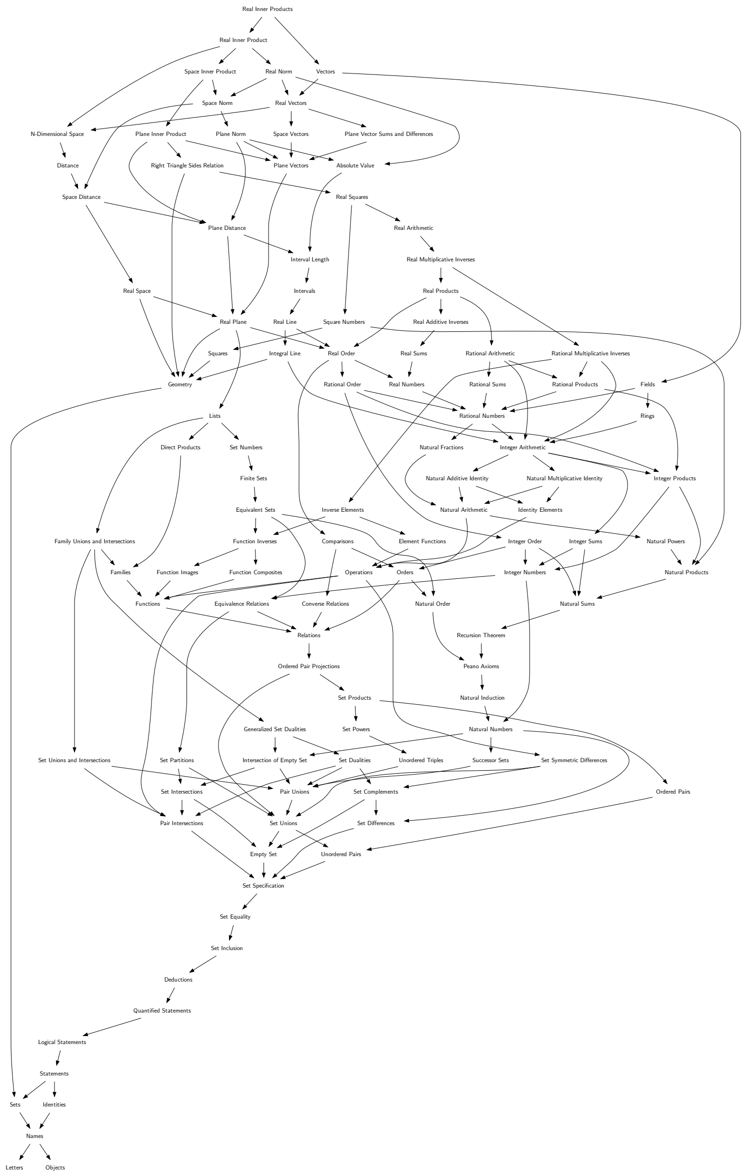
If $f : X \times X \to \mathbf{R}$ is an inner product we regularly denote $f(x, x)$ by $\langle x, x \rangle$.

Orthogonality

Two vectors in an inner product space are *orthogonal* if their inner product is zero. An *orthogonal family of vectors* in an inner product space is a family of vectors for which distinct family members are orthogonal.

A vector is *normalized* if its inner product with itself is one.

[1] Future editions will discuss complex inner products.


Real Inner Products
Real Inner Product
Space Inner Product
Real Norm
Vectors
Space Norm
Real Vectors
N-Dimensional Space
Plane Inner Product
Plane Norm
Space Vectors
Plane Vector Sums and Differences
Distance
Right Triangle Sides Relation
Plane Vectors
Absolute Value
Space Distance
Real Squares
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Real Space
Intervals
Real Products
Real Plane
Real Line
Square Numbers
Real Additive Inverses
Squares
Integral Line
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Geometry
Rational Order
Real Numbers
Rational Sums
Rational Products
Fields
Lists
Rational Numbers
Rings
Direct Products
Set Numbers
Natural Fractions
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Inverse Elements
Natural Arithmetic
Identity Elements
Family Unions and Intersections
Function Inverses
Comparisons
Element Functions
Integer Order
Integer Sums
Natural Powers
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Natural Products
Functions
Equivalence Relations
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects