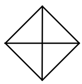


Why

[1]

Result

This result is called sometimes called the *probability inverse transform.*

Proposition 1. *Let* $(\Omega, \mathcal{F}, \mathsf{P})$ *be a probability space and let* $X : \Omega \to \mathsf{R}$ *be a random variable with cumulative distribution function* $F_X : \mathsf{R} \to [0, 1]$. *Suppose* ${F_X}^{-1} : [0, 1] \to \mathsf{R}$ *exists, then* $Y = {F_X}^{-1} \circ X$ *is a random variable with cumulative distribution function* $F_Y : [0, 1] \to [0, 1]$ *satisfying* $F_Y(y) = y$.

Remark 1. *The conclusion is equivalent to the following:* Y *has a density and that density is the the standard unform density (see Uniform Densities).*

Proof. Express $F_Y(\gamma) = \mathsf{P}[Y \leq \gamma] = \mathsf{P}(Y^{-1}([0, \gamma]))$ Notice

$$
\begin{aligned}
Y^{-1}([0, \gamma]]) &= \{\omega \in \Omega \mid Y(\omega) \leq \gamma\} \\
&= \{\omega \in \Omega \mid F_X(X(\omega)) \leq \gamma\} \\
&= \{\omega \in \Omega \mid X(\omega) \leq F_X^{-1}(\gamma)\}. \quad = X^{-1}(\cdots).^2
\end{aligned}
$$

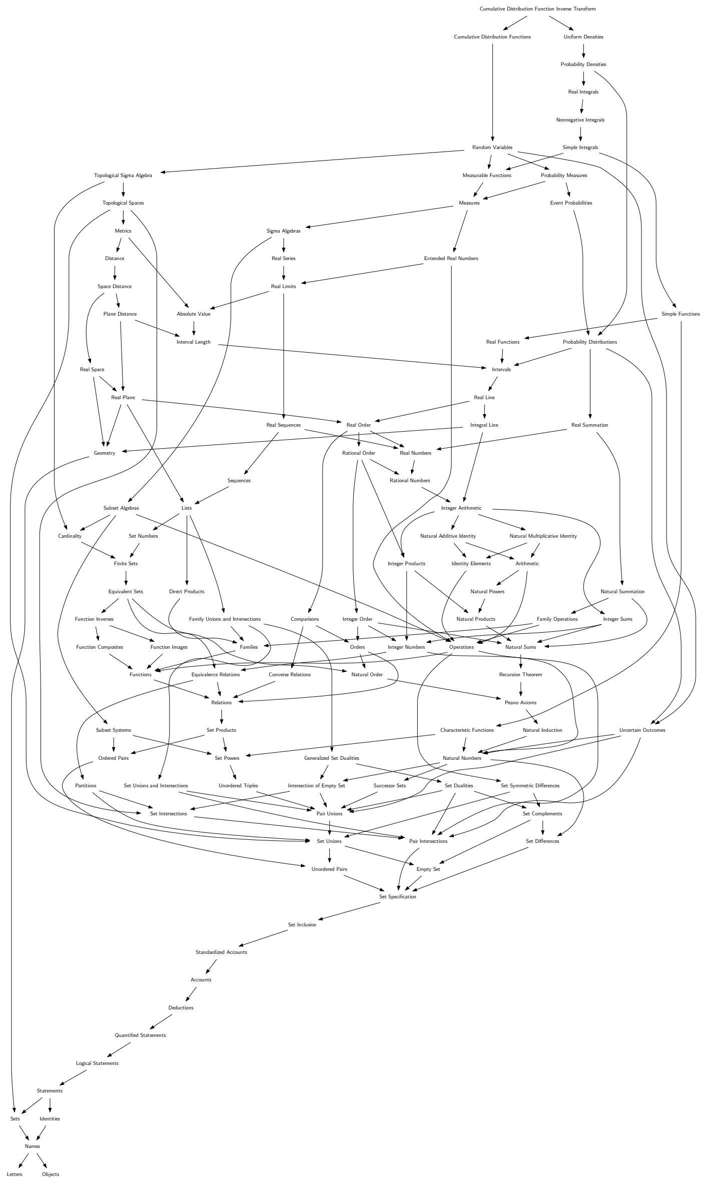
□

Remark 2. *Using different notation the above can be expressed succinctly as*

$$
\begin{aligned}
F_Y(\gamma) = \mathsf{P}[Y \leq \gamma] &= \mathsf{P}[F_X \circ X \leq \gamma] \\
&= \mathsf{P}[X \leq {F_X}^{-1}(\gamma)] = F_X({F_X}^{-1}(\gamma)) = \gamma.
\end{aligned}
$$

[1]Future editions will include.

Future editions will discuss *inverse transform sampling*.


Cumulative Distribution Function Inverse Transform
Cumulative Distribution Functions
Uniform Densities
Probability Densities
Real Integrals
Nonnegative Integrals
Simple Integrals
Random Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Topological Spaces
Measures
Event Probabilities
Metrics
Sigma Algebras
Distance
Real Series
Extended Real Numbers
Space Distance
Real Limits
Plane Distance
Absolute Value
Simple Functions
Interval Length
Real Functions
Probability Distributions
Real Space
Intervals
Real Plane
Real Line
Real Sequences
Real Order
Integral Line
Real Summation
Geometry
Rational Order
Real Numbers
Sequences
Rational Numbers
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Integer Products
Identity Elements
Arithmetic
Equivalent Sets
Direct Products
Natural Powers
Natural Summation
Function Inverses
Family Unions and Intersections
Comparisons
Integer Order
Natural Products
Family Operations
Integer Sums
Function Composites
Function Images
Families
Orders
Integer Numbers
Operations
Natural Sums
Functions
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Partitions
Set Unions and Intersections
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Set Unions
Pair Intersections
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects