## Why

We want to talk about an object in some set—and the properties of this object—without knowing the precise identity of the object. Such language is useful in discussing the future, degrees of belief, and arbitrary objects from large populations.

## Definition

We have a set $\Omega$ which includes all possible objects. We call it the *sample space* (or *set of outcomes*). We call an element of the set of outcomes an *outcome* (or *possibility*, *sample*, *elementary event*).

An *event* (*compound event*, *random event*) is a subset of outcomes. For events $A, B \subset \Omega$, we interpret $A \cup B$ as the event that either $A$ *or* $B$ occurs. Similarly we interpret $A \cap B$ as the event that *both $A$ and $B$* occur. We interpret $\Omega - A$, the *complement* of $A$ in $\Omega$, as the event that $A$ *does not* occur.

An *outcome variable* (or *random variable*) is a function from $\Omega$ to $V$, where $V$ is a set of *values*. If the set is named _, we call the function a _-valued outcome variable on $\Omega$.

## Example: coin

We want to talk about the result of flipping a coin. The coin has two sides. When we flip the coin, it lands heads or tails. We model these outcomes with the set $\{0, 1\}$. If the coin lands tails, we say that outcome 0 has occurred. If the coin lands heads, we say that outcome 1 has occurred.

## Example: die

We want to talk about the result of rolling a die. The die has six sides. When we roll the die, one of the six sides is facing up. We model this uncertain outcome with $\{1, 2, 3, 4, 5, 6\}$, whose elements represent the number of pips facing up.

Define $O = \{1, 3, 5\}$ and $E = \{2, 4, 6\}$. We interpret $O$ as the event that the number of pips is odd, and $E$ as the event that the number of pips is even.

### Example: two dice

We want to talk about the sum of the pips shown facing up after rolling two dice. We may take as our set of outcome $\{1, \ldots, 12\}$, whose elements correspond to the sum. We interpret $\{x \in \Omega \mid x \geq 10\}$ as the event that the sum of the two dice is greater than or equal to 10.

Alternatively, we may take the outcomes $\{1, \ldots, 6\}^2$ and define an outcome variable $s : \{1, \ldots, 6\}^2 \to \{1, \ldots, 12\}$ by

$$s((d_1, d_2)) = d_1 + d_2.$$

We interpret this natural-number-valued outcome variable $s$ as sum of the two dice. The event that the sum of the two dice is greater than or equal to to 10 corresponds to the set $\{(d_1, d_2) \in \{1, \ldots, 6\} \mid s((d_1, d_2)) \geq 10\}$.

Uncertain Outcomes

Natural Numbers

Intersection of Empty Set

Successor Sets

Set Intersections

Set Differences

Pair Unions

Pair Intersections

Set Unions

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects