Why

We want to talk rooting a tree at a given vertex.[1]

Definition

A *rooted tree* is an ordered pair $((V, T), r)$ where (V, T) is a tree and $r \in V$ is a distinguished vertex which we call the *root*. We visualize rooted trees with the root at the top (see Figure 1).

Parents and Children

Suppose w is the first vertex on the path from the root to a non-root vertex v. Since there is only one such path, w is unique and we call it *the parent* of v. Conversely, we call v *a child* of w. We denote the set of children of v by $\mathsf{ch}(v)$. A vertex may have no children or it may have many children. If it has no children we call it a *leaf*.

We define the *parent function* $\mathsf{pa} : V \to V$ with the convention that the *parent of the root* is the root. The *parent of degree* k where $k > 0$ is $\mathsf{pa}^k(x)$ where pa^k is the composite of pa with itself k times. So, in particular, $\mathsf{pa}^{k+1}(v) = \mathsf{pa}(\mathsf{pa}^k(v))$. We define the *parent of degree* 0 of v to be v, and denote it by $\mathsf{pa}^0(v) = v$. For the tree visualized in Figure 1, $\mathsf{pa}(i) = g$, $\mathsf{pa}^2(i) = d$, $\mathsf{pa}^3(i) = a$.

If $w = \mathsf{pa}^k(v)$ for some $k \geq 0$, then w is a *ancestor* of v and v is a *descendent* of w. We use the term *proper ancestor*

[1] Future editions will expand this intuition.

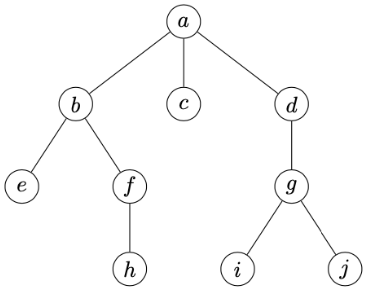
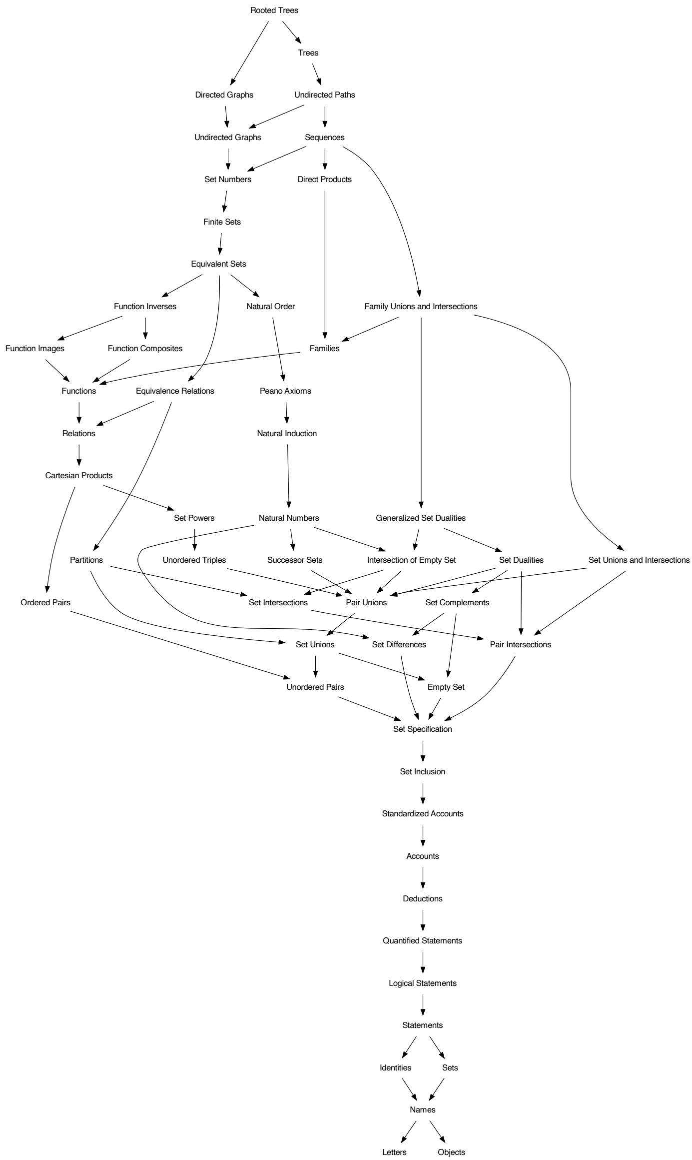


Figure 1: A rooted tree with root a.

and *proper descendent* if $k > 0$ (i.e., $w \neq v$).

The *depth* or *level* of a vertex v is its distance (see Trees) to the root. We denote the level of a vertex v by $\mathsf{\textbf{lev}}(v)$. The level of the root is 0. If $\mathsf{\textbf{lev}}(v) = k > 0$, then $\mathsf{\textbf{pa}}^k(v)$ is the root. The level function $\mathsf{\textbf{lev}}$ satisfies $\mathsf{\textbf{lev}}(v) = \mathsf{\textbf{lev}}(\mathsf{\textbf{pa}}(v)) + 1$.


Rooted Trees
Trees
Directed Graphs
Undirected Paths
Undirected Graphs
Sequences
Set Numbers
Direct Products
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Images
Function Composites
Families
Functions
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Cartesian Products
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects