Relations

1 Why

We want to relate elements of two sets.

2 Definition

A **relation** between two non-empty sets A and B is a subset of $A \times B$. So then, naturally, a relation on a single set C is a subset of $C \times C$.

2.1 Notation

As relations are sets, our de facto protocol is to denote them by upper case capital letters, for example, the letter R. Let R a relation on A and B. If $(a, b) \in R$, we often write aRb, read aloud as "a in relation R to b."

In many cases, though, we forego the set notation and use particular symbols. Often the symbols we use are meant to be suggestive of the relation. Some examples include $\sim$, $=$, $<$, $\leq$, $\prec$, and $\preceq$.

3 Properties

Let R a relation on a non-empty set A. R is **reflexive** if $(a, a) \in R$ for all $a \in A$. R is **transitive** if $(a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R$ for all $a, b, c \in A$. R is **symmetric** if $(a, b) \in R \implies (b, a) \in R$ for all $a, b \in A$. R is **anti-symmetric** if $(a, b) \in R \implies (b, a) \notin R$ for all $a, b \in A$.