Definition

A *distribution family* (*density family*) on X is a family of distributions (densities) $\{p^{(\theta)}\}_{\theta \in \Theta}$ on X.

We call the index set Θ (see Families) the *parameters*. Frequently $\Theta \subset \mathbf{R}^p$ where $p \in \mathbf{N}$.

Similarly, a *conditional distribution family* (*conditional density family*) on Z from X is a family $\{q^{(\theta)}\}_{\theta \in \Theta}$ whose terms $q^{(\theta)} : Z \times X \to \mathbf{R}$ are such that $q^{(\theta)}(\cdot, \xi) : Z \to \mathbf{R}$ is a distribution (density) for every $\xi \in X$.

Examples

For example, let $\Theta = [0, 1]$ and consider the family of distributions $\{p^{(\theta)} : \{0, 1\} \to [0, 1]\}_{\theta \in [0,1]}$ defined by, for each $\theta \in [0, 1]$,

$$p^{(\theta)}(1) = \theta \text{ and } p^{(\theta)}(0) = 1 - \theta.$$

This family is called the *Bernoulli family* and $p^{(\theta)}$ is called a *Bernoulli distribution* with parameter θ.

For a second example, let $\Theta = \mathbf{R} \times \mathbf{R}_+$ and consider the family of densities $\{f^{(\theta)} : \mathbf{R} \to \mathbf{R}\}_{\theta \in \Theta}$ defined by, for each $\theta = (\mu, \sigma) \in \Theta$,

$$f^{(\theta)}(x) = (1/\sqrt{2\pi}\sigma) \exp((x - \mu)/\sigma^2).$$

This family is called the *normal family* and $f^{(\theta)}$ with $\theta = (\mu, \sigma)$ is called a *normal density* with mean μ and variance σ^2.

Distribution Families

Probability Densities

Real Integrals   Probability Distributions

Nonnegative Integrals

Simple Integrals

Measurable Function

Measures

Sigma Algebras

Extended Real Numbers   Real Series

Real Limits

Absolute Value

Interval Length   Simple Functions   Outcome Probabilities

Real Functions

Intervals

Real Line

Subset Algebras   Real Order   Real Sequences   Real Summation

Rational Order   Real Numbers   Sequence   Natural Summation

Integral Line   Cardinality   Rational Numbers   Lists

Integer Arithmetic   Set Numbers   Direct Products

Natural Multiplicative Identity   Natural Additive Identity   Integer Products   Finite Sets   Family Operations

Identity Elements   Arithmetic   Equivalent Sets

Comparisons   Operations   Integer Order   Integer Sums   Natural Powers   Function Inverse   Family Unions and Intersections

Orders   Integer Numbers   Natural Products   Function Image   Function Composites   Families

Converse Relations   Equivalence Relations   Natural Order   Natural Sums   Functions

Relations   Recursion Theorem

Ordered Pair Projections   Peano Axioms   Uncertain Outcomes

Subset Systems   Set Products   Characteristic Functions   Natural Induction

Ordered Pairs   Set Powers   Natural Numbers   Generalized Set Dualities

Set Symmetric Differences   Partitions   Unordered Triples   Intersection of Empty Set   Successor Sets   Set Dualities   Set Unions and Intersections

Set Complements   Set Intersections   Pair Unions

Set Unions   Set Differences   Pair Intersections

Unordered Pairs   Empty Set

Geometry   Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects

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