

## Real Diffeomorphisms

## Why

We want to think about two abstract spaces as being equivalent. <sup>1</sup>

## Definition

Let  $X \subset \mathbb{R}^n$  and  $Y \subset \mathbb{R}^m$ . A smooth, invertible function  $f: X \to Y$  is a diffeomorphism  $f^{-1}$  is smooth. X and Y are diffeomorphic if such a function exists.

The key is the relation diffeomorphic is an equivalence relation. It is reflexive because the identity map is smooth and invertible. It is symmetric since if f is a diffeomorphism from X to Y then  $f^{-1}$  is a diffeomorphism from Y to X. It is transitive because the composition of two smooth functions is smooth.

## **Differential Topology**

Differential topology studies properties of  $X \subset \mathbb{R}^n$  which do not change under diffeomorphism.

<sup>&</sup>lt;sup>1</sup>Future editions will include.

