



Definition

A nonempty set $S \subset \mathbf{R}^n$ is called a *subspace* (or *linear subspace*, *vector subspace*) if

1. $x + y \in S$ for all $x, y \in S$, and
2. $\alpha x \in S$ for all $\alpha \in \mathbf{R}$, $x \in S$.

We say that S is (1) *closed under vector addition* and (2) *closed under scalar multiplication*.

Examples

The set $S_1 = \mathbf{R}^n$ is a subspace. In other words, the entire set is a subspace of itself. The set $S_2 = \{0\}$, consisting of a single point, the origin, is a subspace. S_1 is the biggest subspace. In other words, if S' is another subspace of \mathbf{R}^n , then $S' \subset S_1$. If S is a subspace, it is nonempty, so there is $x \in S$, and it is closed under scalar multiplication, so $0 \cdot x = 0 \in S$. In other words, every subspace contains the origin. So S_2 is the smallest subspace, in the sense that if S' is another subspace $S_2 \subset S'$.

The span (see **Real Vectors Span**) of a set of vectors v_1, \dots, v_k is a subspace. For two subspaces $S, T \subset \mathbf{R}^n$, their sum

$$S + T = \{x + y \mid x \in S, y \in T\}$$

is a subspace.

Geometric intuition

Roughly speaking, a subspace S is a flat set which passes through the origin. In \mathbf{R}^2 , the subspaces are the lines. In \mathbf{R}^3 , the lines and the planes.

