Why

How can we construct distributions which factor according to a directed graph.[1]

Definition

Let $\bar{G} = (G, A)$ be a typed graph (see Function Graphs) with directed and acyclic G. For source vertices i, let $g_i : A_i \to [0, 1]$ be a distribution and otherwise let $g_i : A_{\mathrm{pa}_i} \to [0, 1]$ denote a conditional distribution so that g is a sequence of functions.

We call the ordered pair $(\bar{G}, g)$ a *distribution graph* (or *distribution network, conditional distribution network, conditional distribution graph, bayesian network[2], directed probabilistic graphical model*).

The *distribution* of $(\bar{G}, g)$ is the function $p : \prod_i A_i \to [0, 1]$ defined by

$$p(a) \prod_{\mathrm{pa}_i = \varnothing} g_i(a_i) \prod_{\mathrm{pa}_i \neq \varnothing} g_{i|\mathrm{pa}_i} g_i(a_i, a_{\mathrm{pa}_i}).$$

It is, of course, a distribution. And it factors according to the directed and acyclic graph G.

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[1]Future editions might flip the order of this sheet with that of directed graph distributions since, in the genetic approach, in may be more natural to think of constructing such distributions before analyzing them. This is partially motivated by the acyclic constraint here, which restricts the graphs according to which a distribution can factor.

[2]Indeed, this term is near universal in certain literatures. We avoid it in these sheets as a result of the Bourbaki project's policy on naming.

Distribution Graphs

Directed Graph Distribution

Hidden Memory Chains

Rooted Tree Distributions

Memory Chains

Conditional Distributions

Marginal Distributions

Conditional Event Probabilities

Joint Distributions

Event Probabilities

Probability Distribution

Intervals

Real Line

Function Graphs

Rooted Trees

Real Summation

Real Order

Trees

Real Numbers

Rational Order

Directed Graphs

Undirected Paths

Rational Numbers

Integral Line

Undirected Graphs

Lists

Integer Arithmetic

Set Numbers

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Finite Sets

Direct Products

Identity Elements

Arithmetic

Equivalent Sets

Natural Summation

Natural Powers

Family Unions and Intersections

Function Inverses

Family Operations

Comparisons

Integer Order

Integer Sums

Natural Products

Function Composite

Function Images

Families

Operations

Orders

Integer Numbers

Natural Sums

Equivalence Relations

Functions

Converse Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Set Products

Natural Induction

Uncertain Outcomes

Generalized Set Dualities

Set Powers

Natural Numbers

Set Unions and Intersections

Partitions

Intersection of Empty Set

Set Dualities

Unordered Triples

Successor Sets

Set Symmetric Differences

Set Intersections

Ordered Pairs

Pair Unions

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Geometry

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects