Why

We want to think about two abstract spaces as being equivalent. [1]

Definition

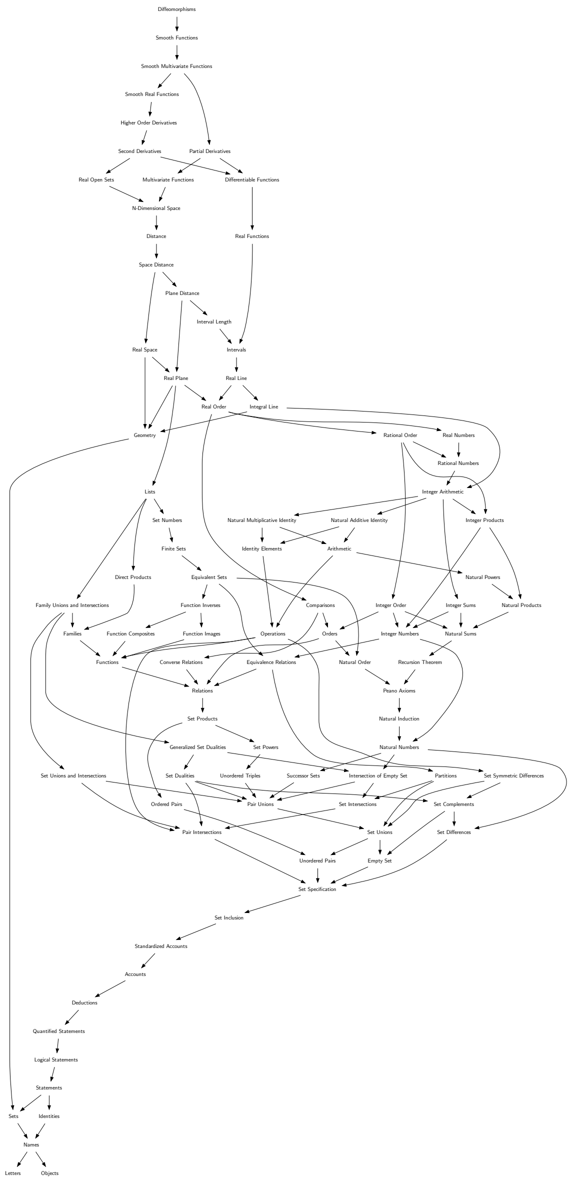
Let $X \subset \mathsf{R}^n$ and $Y \subset \mathsf{R}^m$. A smooth, invertible function $f : X \to Y$ is a *diffeomorphism* f^{-1} is smooth. X and Y are *diffeomorphic* if such a function exists.

The key is the relation diffeomorphic is an equivalence relation. It is reflexive because the identity map is smooth and invertible. It is symmetric since if f is a diffeomorphism from X to Y then f^{-1} is a diffeomorphism from Y to X. It is transitive because the composition of two smooth functions is smooth.

Differential Topology

Differential topology studies properties of $X \subset \mathsf{R}^n$ which do not change under diffeomorphism.

[1] Future editions will include.


Diffeomorphisms
Smooth Functions
Smooth Multivariate Functions
Smooth Real Functions
Higher Order Derivatives
Second Derivatives
Partial Derivatives
Real Open Sets
Multivariate Functions
Differentiable Functions
N-Dimensional Space
Distance
Real Functions
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects