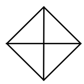


Why

We want to count, forever.

Definition

We assert two additonal self-evident and indispensable properties of these natural numbers. First, one is the successor of no other element. Second, if we have a subset of the naturals containing one with the property that it contains successors of its elements, then that set is equal to the natural numbers. We call this second property the **principle of mathematical induction.**

These two properties, along with the existence and uniqueness of successors are together called *Peano's axioms* for the natural numbers. When in familiar company, we freely assume Peano's axioms.

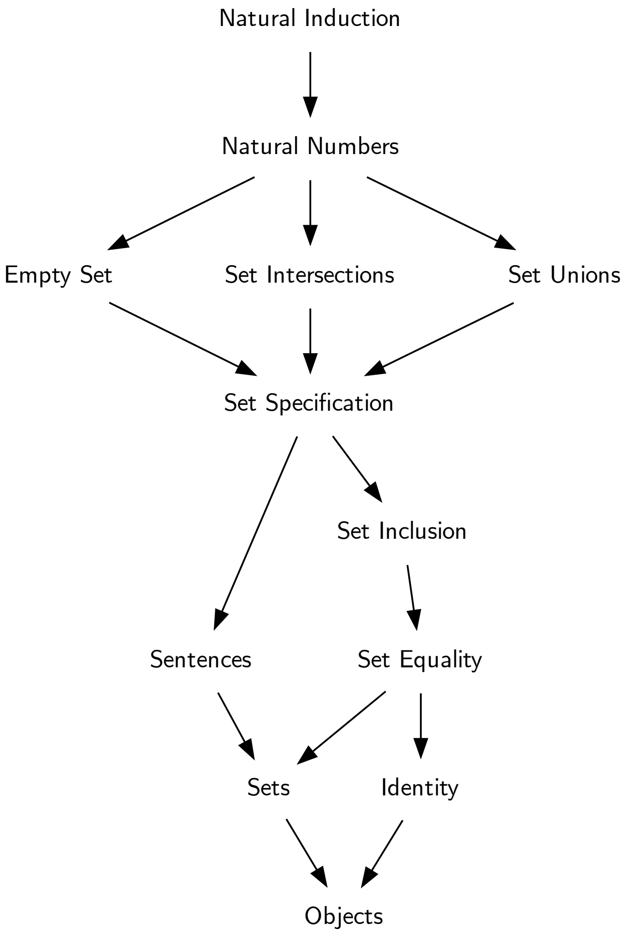
Notation

As an exercise in the notation assumed so far, we can write Peano's axioms: N is a set along with a function $s : N \to N$ such that

1. $s(n)$ is the successor of n for all $n \in N$.
2. s is one-to-one; $s(n) = s(m) \implies m = n$ for all $m, n \in N$.

3. There does not exist $n \in N$ such that $s(n) = 1$.

4. If $T \subset N$, $1 \in T$, and $s(n) \in T$ for all $n \in T$, then $T = N$.


Natural Induction
Natural Numbers
Empty Set
Set Intersections
Set Unions
Set Specification
Set Inclusion
Sentences
Set Equality
Sets
Identity
Objects