Why

We talk about an uncertain outcome by assigning credibility to various outcomes. We use our intuition of proportion to do so. We start with finite sets.

Definition

A *probability distribution* or *probability mass function* is a real-valued function from a set of outcomes which is non-negative and normalized. A real-valued function on a finite set is *normalized* if the sum of its results is 1. We will refer to these as *distributions*. The *probability of an outcome* is the result of the outcome under the distribution.

The probability of an outcome is meant to indicate the credibility of the outcome. The word probabiliy has its roots in the English word probable, which has the Middle English sense “worthy of belief”. The probability of an outcome then is how worthy of belief it is. In the case of flipping a coin, or rolling a die, all outcomes are equally worthy of belief.

When an outcome has a higher probability we say that is is *more probable* than another outcome. Remember that the intuition is that we are saying the outcome is more “credible”, or more “worthy of belief.”

Notation

Let A be a set of outcomes and let $p : A \to \mathsf{R}$ be a distribution. Then

$$p(a) \geq 0 \text{ for all } a \in A \text{ and } \sum_{a \in A} p(a) = 1.$$

Proposition 1. *If $p : A \to \mathsf{R}$ is a distribution, then $p(A) \subset [0, 1]$.*

Proof. Let $a \in A$. First, $p(a) \geq 0$ by definition. Second, since p is normalized, $\sum_{b \in A} p(b) = 1$. And $p(a) \leq \sum_{b \in A} p(b)$, so $p(a) \leq 1$.

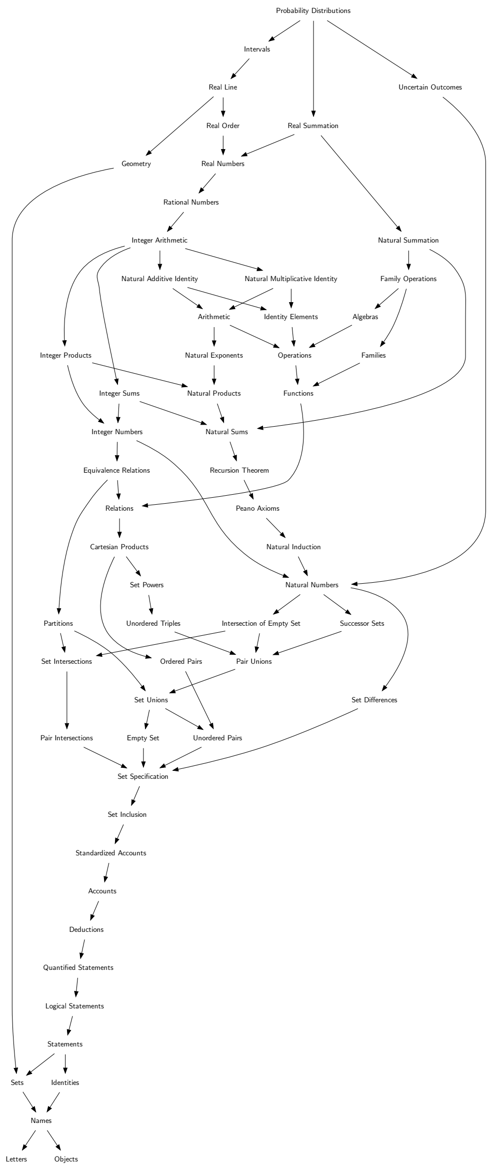
$\square$

Example: coin

When flipping a coin, both a heads and a tails are worthy of belief. Thus we say that the outcome 0 (tails) has probability $1/2$ and likewise for the outcome 1 (heads) In this case we say that we are modeling a *fair coin.*

Example: die

When rolling a die, all six sides are worthy of belief. Thus we say that the outcomes $1, 2, 3, 4, 5, 6$ each have probability $1/6$. Prior to roll, each is equally credible. In this case we say that we are modeing a *fair die.*


Probability Distributions
Intervals
Real Line
Uncertain Outcomes
Real Order
Real Summation
Geometry
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Summation
Natural Additive Identity
Natural Multiplicative Identity
Family Operations
Arithmetic
Identity Elements
Algebras
Integer Products
Natural Exponents
Operations
Families
Integer Sums
Natural Products
Functions
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Intersections
Ordered Pairs
Pair Unions
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects