



## Why

We want to speak of infinite processes, and to do so we define sequences indexed by  $\mathbf{N}$ . In other words, important families are those indexed by the natural numbers.

## Definition

A *sequence* (*infinite sequence*) is a family whose index set is  $\mathbf{N}$  (the set of natural numbers without zero). The  *$n$ th term* or *coordinate* of a sequence is the result of the  *$n$ th* natural number,  $n \in \mathbf{N}$ .<sup>1</sup>

## Notation

Let  $A$  be a non-empty set and  $a : \mathbf{N} \rightarrow A$ . Then  $a$  is a (infinite) sequence in  $A$ .  $a(n)$  is the  *$n$ th term*. We also denote  $a$  by  $(a_n)_n$  and  $a(n)$  by  $a_n$ . If  $\{A_n\}_{n \in \mathbf{N}}$  is an infinite sequence of sets, then we denote the direct product of the sequence by  $\prod_{i=1}^{\infty} A_i$ .

## Natural unions and intersections

We denote the family of the infinite sequence of sets  $(A_n)_n$  by  $\cup_{i=1}^{\infty} A_i$ . Similarly, we denote the intersection of an infinite sequence of sets by  $\cap_{i=1}^{\infty} A_i$ , respectively.

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<sup>1</sup>Future editions may also comment that we are introducing language for the steps of an infinite process.



