

## RANDOM VARIABLES JOINT LAW

## Why

We name the image measure of a collection of real-valued random variables.

## **Definition**

The *joint law* of a sequence of n real-valued random variables is the image measure of the tuple-valued function whose components are the individual random variables.

## Notation

Let  $(X, \mathcal{A}, \mu)$  be a probability space and  $(Y, \mathcal{B})$  be a measurable space. Let  $f_1, \ldots, f_n : X \to Y$  be random variables. Define  $f : X \to Y^n$  by  $(f(x))_i = f_i(x)$ . The joint law is the image measure of f.

We denote the joint law of  $\{f_i\}$  by  $\mu_{f_1,\dots,f_n}: \mathcal{A} \to [0,\infty]$ . We defined it by

$$\mu_{f_1, \dots, f_n}(A) = \mu(\{x \in X \mid f(x) \in A\}).$$

for all A in the product sigma algebra on  $Y^n$ .

