



## Why

We discuss inferring (or learning) functions from examples.

## Definitions

Suppose  $\mathcal{U}$  and  $\mathcal{V}$  are two sets. A *predictor* from  $\mathcal{U}$  to  $\mathcal{V}$  is a function  $f : \mathcal{U} \rightarrow \mathcal{V}$ . We call  $\mathcal{U}$  the *inputs*,  $\mathcal{V}$  the *outputs*, and  $f(u)$  the *prediction* of  $f$  on  $u \in \mathcal{U}$ .

An *inductor* is a function from datasets in  $\mathcal{U} \times \mathcal{V}$  to predictors from  $\mathcal{U}$  to  $\mathcal{V}$ . A *learner* (or *learning algorithm*) is a family of inductors whose index set is  $\mathbf{N}$ , and whose  $n$ th term is an inductor for a dataset of size  $n$ .

## Notation

Let  $D$  be a dataset of size  $n$  in  $\mathcal{U} \times \mathcal{V}$ . Let  $g : \mathcal{U} \rightarrow \mathcal{V}$ , a predictor, which makes prediction  $g(u)$  on input  $u \in \mathcal{U}$ . Let  $G_n : (\mathcal{U} \times \mathcal{V})^n \rightarrow (\mathcal{U} \times \mathcal{V})$  be an inductor, so that  $G_n(D)$  is the predictor which the inductor associates with dataset  $D$ . Then  $\{G_n : (\mathcal{U} \times \mathcal{V})^n \rightarrow \mathcal{V}^{\mathcal{U}}\}_{n \in \mathbf{N}}$  is a learner.

## Relations

Functions are relations, so we might ask if *inferring* relations may be a more general and difficult problem than inferring functions. The following consideration shows that this is *not* the case.

A *relation inductor* is a function from finite datasets in  $\mathcal{U} \times \mathcal{V}$  to *relations* on  $\mathcal{U} \times \mathcal{V}$ . Suppose  $R$  is a relation between  $\mathcal{U}$  and  $\mathcal{V}$ . Suppose the function  $f : \mathcal{U} \times \mathcal{V} \rightarrow \{0, 1\}$  is such that

$$f(u, v) = 1 \quad \text{if and only if} \quad (u, v) \in R$$

Given  $f$  we can find  $R$ , and given  $R$  we can find  $f$ . Thus, instead of learning the *relation*  $R$  we can think of learning the *function*  $f$ . In other words, if we have an inductor for  $f$ , we have a *relation* inductor for  $R$ .

## Consistent and complete datasets

What can a dataset tell us?

Suppose  $D = ((u_i, v_i))_{i=1}^n$  be a dataset and  $R \subset X \times Y$  a relation.  $D$  is *consistent with  $R$*  if  $(u_i, v_i) \in R$  for all  $i = 1, \dots, n$ .  $D$  is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take  $R = \mathcal{U} \times \mathcal{V}$ ).  $D$  is *functionally consistent* if it is consistent with a function; in this case,  $x_i = x_j \Rightarrow y_i = y_j$ .  $D$  is *functionally complete* if  $\cup_i \{x_i\} = X$ . In this case, the dataset includes every element of the relation.

### Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent variables*, *explained variables*, *postcepts*, *targets*, *outcomes*, *labels* or *observational outcomes*. An input-output pair is sometimes called a *record pair*.

Other terms for a learner include *supervised learning algorithm*. Other terms for a predictor include *input-output mapping*, *prediction rule*, *hypothesis*, *concept*, and *classifier*.

