



Why

Suppose we flip a coin until it lands heads. What is the probability that we see a head after one flip? Two flips? n flips? We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

Discussion

Consider a set A .¹ If A has n elements, then a probability distribution on A is $p : A \rightarrow \mathbf{R}$ where $p(a) = 1/n$. There is a natural candidate.

What if A is the set of natural numbers \mathbf{N} . The principle difficulty is that not all sequences of real numbers $a : \mathbf{N} \rightarrow \mathbf{R}$ are summable.

Definition

A (*countable*) *probability distribution* on \mathbf{N} is $p : \mathbf{N} \rightarrow \mathbf{R}$ where $p \geq 0$ and

$$\sum_{n=1}^{\infty} p(n) = 1.$$

¹Future editions will rework this sheet.

