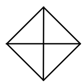


Why

The product of two sets is a (sub)set of ordered pairs. Is every set of ordered pairs a subset of a product of two sets?

Result

The answer is easily seen to be yes. Let R denote a set of ordered pairs. So for $x \in R$, $x = \{\{a\}, \{a, b\}\}$. First consider $\bigcup R$. Then $\{a\} \in \bigcup R$ and $\{a, b\} \in \bigcup R$. Next consder $\bigcup \bigcup R$. Then $a, b \in \bigcup \bigcup R$. So if we want two sets—denote them by A and B—so that $R \subset A \times B$, we can take both A and B to be the set $\bigcup \bigcup R$.

Projections

We often want to shrink the sets A and B to include only the *relevant* members. In other words, to include only those members which appear as either the first coordinate (for A) or second coordinate (for B) in an element of R. We can do this by specifying the elements of $\bigcup \bigcup R$ which are actually a first coordinate or second coordinate for some ordered pair in the set R.

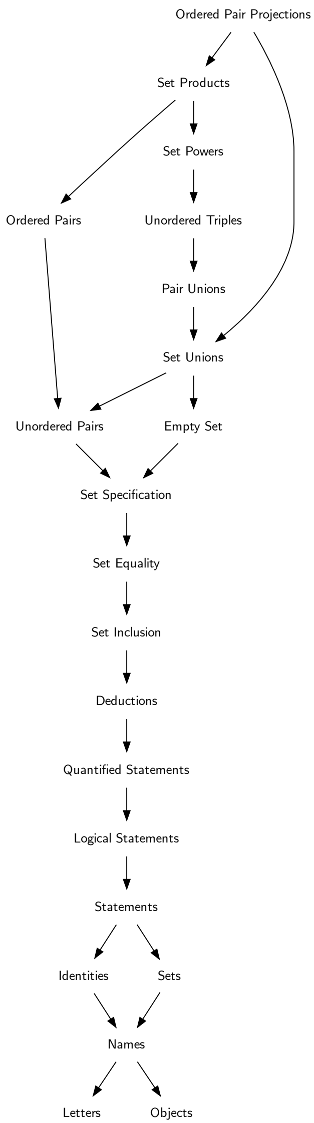
Define

$$A' = \{a \in A \mid (\exists b)((a, b) \in R)\},$$

and likewise

$$B' = \{b \in B \mid (\exists a)((a, b) \in R)\}.$$

We call A' the *projection onto the first coordinate* and B' the *projection onto the second coordinate.*


Ordered Pair Projections
Set Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects