Why

We want language and notation involving order.[1]

Comparisons

A *comparison* is a statement (see Statements) involving a partial (which may or may not be total) order.

Notation

Let A be a set. We tend to denote an arbitrary partial order on A by $\preceq$. So $(A, \preceq)$ is a partially ordered set.

As usual (see Relations), we write $a \preceq b$ to mean $(a, b) \in A$. Alternatively, we write $b \succeq a$ to mean $a \preceq b$. In other words, $\succeq$ is the inverse relation (see Converse Relations) of $\preceq$.

Predecessors and successors

If $a \preceq b$ and $a \neq b$, we write $a \prec b$ and say that a *precedes* b. In this case we call a the *predecessor* of b. Alternatively, under the same conditions, we write $b \succ a$ and we say that b *succeeds* a. In this case we call b the *successor* of a.

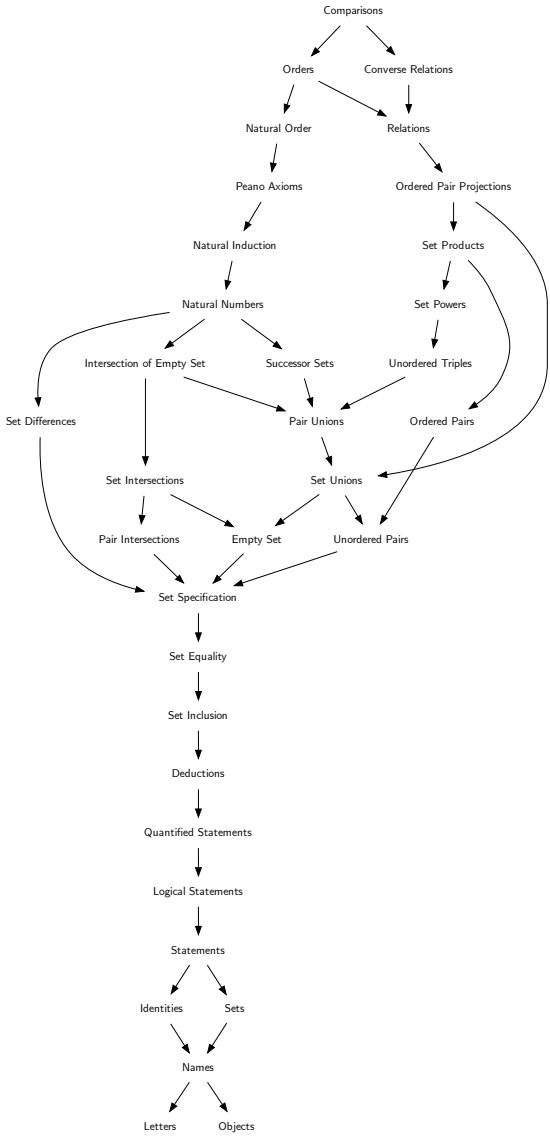
Induced partial orders

Of course, the object we have defined and denoted by $\prec$ is a relation on A. It satisfies (i) for no elements x and y do $x \prec y$ and $y \prec x$ hold simultaneously and (ii) if $x \prec y$ and $y \prec z$, then $x \prec z$ (i.e., $\prec$ is transitive). It is worthwhile to observe that if S is a relation satisfying (i) and (ii), then the relation R defined to mean $(a, b) \in S$ or $a = b$ is a partial order on A.

[1] In the present edition, this sheet can be thought of as an extended notation section for Orders.

Strict and weak relations

This connection between $\preceq$ and $\prec$ can be generalized. The *strict relation* corresponding to a relation R on a set A is the relation S on A defined by $(a, b) \in S$ if $(a, b) \in R$ and $a \neq b$. The *weak relation* corresponding to a relation S' on a set A is the relation R' defined by $(a, b) \in R'$ if $(a, b) \in S'$ or $a = b$. For this reason, a relation is said to *partially order* a set if it is a partial order or if its corresponding weak relation is one.


Comparisons
Orders
Converse Relations
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Differences
Pair Unions
Ordered Pairs
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects