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Definition

Let $(A, +, \cdot)$ be a ring.

A *polynomial* in A of *degree* d is a function $p : A \rightarrow A$ for which there exists a finite sequence $c = (c_0, c_1, \dots, c_{d-1}, c_d) \in A^{d+1}$ satisfying

$$p(a) = c_0 + c_1 a^1 + c_2 a^2 + \dots + c_d a^d,$$

for all $a \in A$. We call the sequence c the *polynomial coefficients*, and call the c_i the *coefficients* of p . We call $d + 1$ the *order* of the polynomial.

Clearly, to every polynomial in A of degree d there corresponds a sequence in A of length $d + 1$, and vice versa. For this reason, we can identify polynomials by their coefficients.

Examples

The function $f : A \rightarrow A$ is a polynomial of degree 0 and order 1 if there exists c_0 so that

$$f(a) = c_0$$

for all $a \in A$.

The function $g : A \rightarrow A$ is a polynomial of degree 1 and order 2 if there exists c_0 and c_1 so that

$$g(a) = c_0 + c_1 a$$

The function $h : A \rightarrow A$ is a polynomial of degree 2 and order 3 if there exists c_0 and c_1 so that

$$h(a) = c_0 + c_1 a + c_2 a^2.$$

In other words, a second degree polynomial is a quadratic.

¹Future editions will include, and most likely will build on quadratics and an appeal to the simplicity of the “natural” algebraic operations.

