Why

Definition

A *hyperplane* in n-dimensional space is an $(n-1)$-dimensional affine set.

Since the $n-1$-dimensional subspaces are the orthogonal complements of the one-dimensional subspaces, they are the sets which can be specified by

$$\{x \in \mathbf{R}^n \mid x \perp b\}$$

for $b \in \mathbf{R}^n$. The hyperplanes are translates of these,

$$\begin{aligned}\{x \in \mathbf{R}^n \mid x \perp b\} + a &= \{x + a \mid \langle x \rangle b = 0\} \\ &= \{y \mid \langle y - a \rangle b = 0\} = \{y \mid \langle y \rangle b = \beta\},\end{aligned}$$

where $\beta = \langle a \rangle b$.

Characterization

Prop. 1. *$H \subset \mathbf{R}^n$ is a hyperplane if and only if there exists $\beta \in \mathbf{R}$ and nonzero $b \in \mathbf{R}^n$ so that*

$$H = \{x \in \mathbf{R}^n \mid \langle x \rangle b = \beta\}.$$

Remark 1. *b and β are unique up to a common nonzero multiple. For example, b, β and $2b, 2\beta$ give the same hyperplane.*

Remark 2. *The vector b is called a* normal *to the hyperplane.*


Hyperplanes
Affine Set Dimensions
Subspace Orthogonal Complements
Subspace Dimensions
Vector Space Dimensions
Orthogonal Complements
Vector Space Bases
Affine Sets and Subspaces
Span
Linearly Dependent Vectors
Inner Products
Translate Sets
Subspaces
Linear Combinations
Real Inner Product
Vectors
Real Linear Combinations
Real Norm
Affine Sets
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Plane Norm
Plane Inner Product
Distance
Plane Vectors
Right Triangle Sides Relation
Space Distance
Absolute Value
Real Squares
Plane Distance
Real Arithmetic
Square Numbers
Interval Length
Real Multiplicative Inverses
Squares
Real Space
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Order
Rational Multiplicative Inverses
Real Sums
Rational Arithmetic
Geometry
Rational Order
Fields
Real Numbers
Rational Sums
Rational Products
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Inverse Elements
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Element Functions
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Ordered Pairs
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects