



## Definition

Let  $(V, E)$  be an undirected graph. An (undirected) *path* between vertex  $v \in V$  and vertex  $w \neq v$  is a finite sequence of distinct vertices, whose first coordinate is  $v$  and whose last coordinate is  $w$ , and whose consecutive coordinates are adjacent in the graph. We call the first and last coordinate the *endpoints* of the path. We say the path is *between* its endpoints.

The *length* of a path is one less than the number of vertices: namely, the number of edges. Notice that this definition disagrees with the definition of the “length” of the sequence of vertices (namely, the number of vertices). The length of a path is always at least one: there exists a path of length one between any two adjacent vertices. If a path has length two or greater, we call a vertex which is not the first or last vertex an *interior vertex*.

Two vertices are *connected* in a graph if there exists at least one path between them. A graph is *connected* if each pair of vertices is connected. Recall that two vertices are **adjacent** if they are connected by a path of length one. In contrast, two vertices are connected if they are connected by a path of any length. In other words, all adjacent vertices are connected.

A *cycle* is a sequence whose first and last coordinate are identical, all other coordinates are distinct, and consecutive coordinates are adjacent. An undirected graph is *acyclic* if there are no cycles of its vertices.

## Other terminology

Some authors allow paths to contain repeated vertices, and call a path with distinct vertices a *simple path*. Similarly, some authors allow a cycle to contain repeated vertices, and call a path with distinct vertices a *simple cycle* or *circuit*. Some authors use the term *loop* instead of *cycle*.

## Notation

Let  $G = (V, E)$  be a graph. A path between  $v$  and  $w$  (with  $v \neq w$ ) in  $G$  is a sequence  $(v_0, v_1, \dots, v_k)$  where  $v_0 = v$  and  $v_k = w$  and  $\{v_i, v_{i+1}\} \in E$  for  $i = 0, \dots, k - 1$ .

