Why

Do solutions exist to a linear optimization problem which is feasible and bounded? Yes.

Result

Proposition 1. *Suppose* $A \in \mathbf{R}^{m \times n}$, $b \in \mathbf{R}^n$, *and* $c \in \mathbf{R}^n$ *so that*

$$P = \{x \in \mathbf{R}^n \mid Ax \leq b\} \neq \varnothing$$

and

$$\delta = \inf\{c^\top x \mid x \in P\} > -\infty$$

Then there exists $x^\star \in \mathbf{R}^n$ *with* $c^\top x^\star = \delta$.

For this reason, a linear program is sometimes abbreviated $\min\{cx \mid Ax \leq b\}$ instead of $\inf\{c^\top x\} Ax \leq b$.