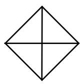


Why

In the case that it is not possible to easily identify (or guess) the limit of a sequence, we are naturally interested in a simple condition on the sequence which is equivalent to convergence.

Definition

A sequence $(x_n)_{n \in \mathbf{N}}$ in $\mathbf{R}$ is said to be *egopox* (or *Cauchy* or a *Cauchy sequence*) if for every $\varepsilon > 0$, there exists $N \in \mathbf{N}$ so that for all $m, n > N$, $|x_m - x_n| < \varepsilon$. We call this property of the sequence *(eventual) egoproximity.*

Notation

We sometimes denote this property as

$$|x_n - x_m| \to 0 \quad \text{as} \quad m, n \to \infty.$$

Example

For example, consider $\lim_{N \to \infty} \sum_{n=1}^{N} 1/n^3$.

Sufficiency in R

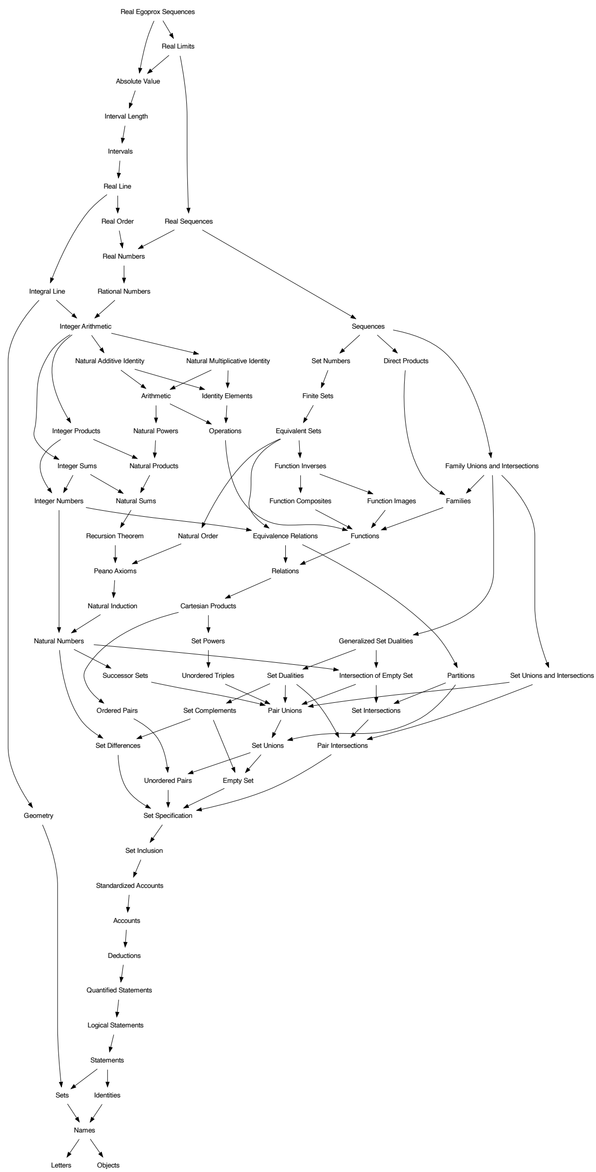
Clearly a convergent sequence is egoprox.[1] What of the converse? Recall that we think of egoprox sequences as "bunching up." For the reals, if a sequence is bunching up, then our intuition is that it should be converging. In other words, an

[1] Future editions may elaborate here.

egoprox real sequence always converges. The egoprox condition is sufficient. Bunching up is sufficient.

Proposition 1. *If* $(x_n)_{n \in \mathsf{N}}$ *is egoprox, then there exists* $x_0 \in \mathsf{R}$ *so that* $\lim_{n \to \infty} x_n = x_0$.

In other words, in R egoproximity is equivalent to convergence. The above is sometimes called the *Bolzano-Weierstrass theorem.*


Real Egoprox Sequences
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Sequences
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Finite Sets
Integer Products
Natural Powers
Operations
Equivalent Sets
Integer Sums
Natural Products
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects