Why

[1]

Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\operatorname{tr} : \mathbf{R}^{n\times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n\times n}$. Then

$$\operatorname{tr} A = \sum_{i=1}^{n} A_{ii}.$$

Properties

Prop. 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n\times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\begin{aligned}\operatorname{tr} C = \sum_{i=1}^{n} C_{ii} &= \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii} \\ &= \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii} \\ &= \alpha \operatorname{tr} A + \beta \operatorname{tr} B.\end{aligned}$$

[1] Future editions will include, in the genetic tradition.

$\square$

Prop. 2. *Let* $A, B \in \mathsf{R}^{n \times n}$.

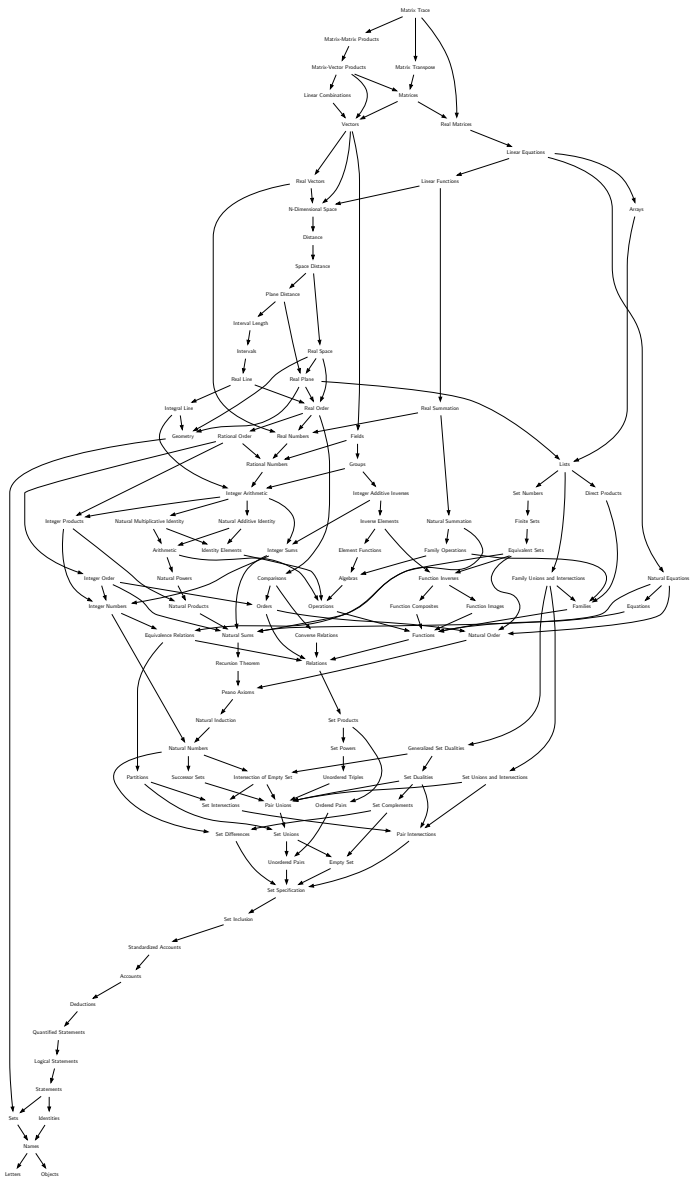
$$\operatorname{tr}(AB) = \operatorname{tr}(BA)$$

[2]

In other words, "matrices commute under the trace operator."

Prop. 3. *Let* $A \in \mathsf{R}^{n \times n}$. *Then* $\operatorname{tr} A = \operatorname{tr} A^{\top}$.

[2] Future editions will include an account.


Matrix Trace
Matrix-Matrix Products
Matrix-Vector Products
Matrix Transpose
Linear Combinations
Matrices
Vectors
Real Matrices
Linear Equations
Real Vectors
Linear Functions
Axioms
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Integral Line
Real Order
Real Summation
Geometry
Rational Order
Real Numbers
Fields
Rational Numbers
Groups
Lists
Integer Arithmetic
Integer Additive Inverses
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Inverse Elements
Natural Summation
Finite Sets
Arithmetic
Identity Elements
Integer Sums
Element Functions
Family Operations
Equivalent Sets
Natural Powers
Comparisons
Algebras
Function Inverses
Family Unions and Intersections
Natural Equations
Integer Order
Natural Products
Orders
Operations
Function Compositions
Function Images
Families
Equations
Integer Numbers
Equivalence Relations
Natural Sums
Converse Relations
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
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Logical Statements
Statements
Sets
Identities
Names
Lemmas
Objects