Why

We want to solve linear equations.

Example

Suppose we want to find $x_1, x_2 \in \mathbf{R}$ to satisfy the system of linear equations

$$3x_1 + 2x_2 \& = 10, \text{ and}$$
$$6x_1 + 5x_2 \& = 20,$$

with constants $((3, 2), 10)$ and $((6, 4), 20)$.

We can associate to the first equation an *equation for x_1 in terms of x_2*. We call this *solving the first equation for x_1*.

$$3x_1 + 2x_2 = 10 \longleftrightarrow 3x_1 = 10 - 2x_2 \longleftrightarrow x_1 = \frac{1}{3}(10 - 2x_2).$$

Define $f_1 : \mathbf{R} \to \mathbf{R}$ as $f_1(y) = \frac{1}{3}(10 - 2y)$. Then $3x_1 + 2x_2 = 10$ if and only if $x_1 = f_1(x_2)$. We have written x_1 *as a function* of x_2 and obtained a new equation. The equation is not linear, however, as f_1 is not linear.

Using the equation for x_1 in terms of x_2, we can substitute this equation into our second linear equation. The two linear equations hold if and only if

$$6f(x_2) + 5x_2 = 20 \longleftrightarrow 20 - 4x_2 + 5x_2 = 20 \longleftrightarrow x_2 = 0.$$

So the equations are satisfed if and only if x_2 is 0. If $x_2 = 0$, then $3x_1 = 10$ and $6x_1 = 20$. Both of these are equivalent to $x_1 = \frac{10}{3}$. So we have that x_1 must be $\frac{10}{3}$ and x_2 must be 0.

Clearly this is *a solution.* Is it the only one?[1]

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[1]Future editions will expand.

Linear Equation Solutions

Linear Equations

Linear Functions

Arrays

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space   Real Plane   Real Line

Sequences   Integral Line   Real Order   Real Summation

Geometry

Real Numbers   Rational Order

Set Numbers   Direct Products

Rational Number

Finite Sets

Integer Arithmetic

Natural Summation

Equivalent Sets   Family Operations

Natural Multiplicative Identity   Natural Additive Identity   Integer Products

Family Unions and Intersections

Function Inverses   Function Composites   Families   Algebras   Companions   Identity Elements   Arithmetic   Integer Sums   Integer Order

Natural Equations

Function Images   Operations   Integer Numbers   Orders   Natural Powers   Equations

Equivalence Relations   Functions   Converse Relations   Natural Order   Natural Products

Natural Sums

Relations

Recursion Theorem

Peano Axioms

Set Products   Natural Numbers

Natural Induction

Generalized Set Dualities

Set Powers

Set Unions and Intersections   Partitions   Intersection of Empty Set   Set Dualities   Unordered Triples   Successor Sets

Set Intersections   Pair Unions   Set Complements   Ordered Pairs

Pair Intersections   Set Unions   Set Differences

Empty Set   Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects