

PROBABILISTIC LINEAR MODEL

Why

If we treat the parameters of a linear function as a random variable, an inductor for the predictor is equivalent to an estimator for the parameters.¹

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $x : \Omega \to \mathbf{R}^d$. Define $g : \Omega \to (\mathbf{R}^d \to \mathbf{R})$ by $g(\omega)(a) = a^{\top}x(\omega)$, for $a \in \mathbf{R}^d$. In other words, for each outcome $\omega \in \Omega$, $g_{\omega} : \mathbf{R}^d \to \mathbf{R}$ is a linear function with parameters $x(\omega)$. g_{ω} is the function of interest.

Let $a^1, \ldots, a^n \in \mathbb{R}^d$ a dataset with data matrix $A \in \mathbb{R}^{n \times d}$. Let $e : \Omega \to \mathbb{R}^n$ independent of x, and define $y : \Omega \to \mathbb{R}^n$ by

$$y = Ax + e$$
.

In other words, $y_i = x^{\top} a^i + e_i$.

We call (x, A, e) a probabilistic linear model. Other terms include linear model, statistical linear model, linear regression model, bayesian linear regression, and bayesian analysis of the linear model.² We call x the parameters, A a design, e the error or noise vector, and y the observation vector.

One may want an estimator for the parameters x in terms of y or one may be modeling the function g and want to predict g(a) for $a \in A$ not in the dataset.

¹Future editions will offer further discussion.

²The word bayesian is in reference to treating the object of interest—x—as a random variable.

Inconsistency

In this model, the dataset is assumed to be inconsistent as a result of the random errors. In these cases, the error vector e may model a variety of sources of error ranging from inaccuracies in the measurements (or measurement devices) to systematic errors from the "inapproriateness" of the use of a linear predictor.³ In this case the linear part is sometimes called the deterministic effect of the response on the input $a \in A$.

Moment assumptions

One route to be more specific about the underlying distribution of the random vector is give its mean and variance. It is common to give the mean of E(w)

Mean and variance

Proposition 1.
$$E(y) = A E(x) + E(w)^4$$

Proposition 2.
$$cov((x,y)) = A cov(x)A^{T} + cov e^{5}$$

A simple consequence is that, if x and

³Future editions will clarify and may excise this sentence.

⁴By linearity. Full account in future editions.

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