Definition

Consider two distributions on the same finite set. The *entropy* of the first distribution *relative* to the second distribution is the difference of the cross entropy of the first distribution relative to the second and the entropy of the second distribution. We call it the *relative entropy* of the first distribution with the second distribution. People also call the relative entropy the *Kullback-Leibler divergence* or *KL divergence*.

Notation

Let A be a non-empty finite set. Let $p : A \to \mathbf{R}$ and $q : A \to \mathbf{R}$ be distributions. Let $H(q, p)$ denote the cross entropy of p relative to q and let $H(q)$ denote the entropy of q. The entropy of p relative to q is

$$H(q, p) - H(q).$$

Herein, we denote the entropy of p relative to q by $d(q, p)$.

A similarity function

The relative entropy is a similarity function between distributions.

Proposition 1. *Let q and p be distributions on the same set. Then $d(q, p) \geq 0$ with equality if and only if $p = q$.*

So, d has a few of the properties of a metric. However, d is not a metric; for example, it is not symmetric.

Proposition 2. *There exist distributions $p : A \to \mathbf{R}$ and $q : A \to \mathbf{R}$ (with A a non-empty finite set) such that*

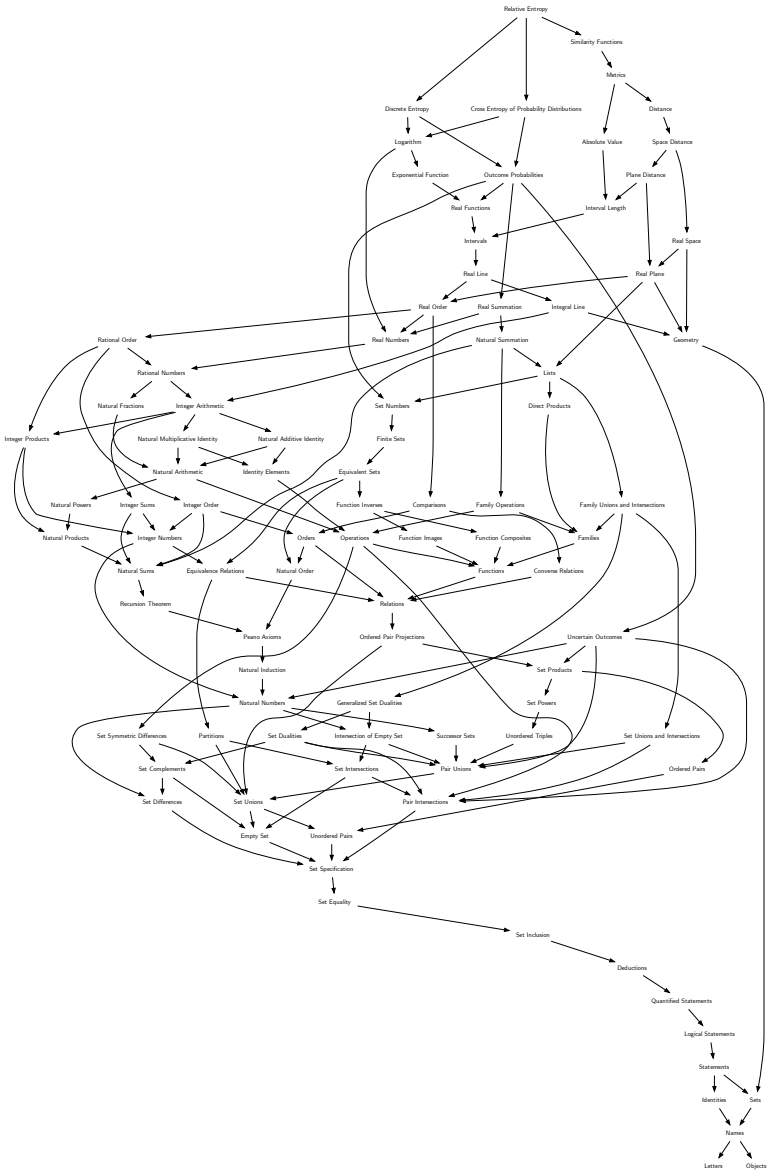
$$d(q, p) \neq d(p, q).$$

Optimization perspective

A solution to finding a distribution $p : A \to \mathbf{R}$ to

$$\text{minimize} \quad d(q, p),$$

is $p^\star = q$.


Relative Entropy
Similarity Functions
Metrics
Discrete Entropy
Cross Entropy of Probability Distributions
Distance
Logarithm
Absolute Value
Space Distance
Exponential Function
Outcome Probabilities
Plane Distance
Real Functions
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Order
Real Summation
Integral Line
Rational Order
Real Numbers
Natural Summation
Geometry
Rational Numbers
Lists
Natural Fractions
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Function Inverses
Comparisons
Family Operations
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Natural Order
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Partitions
Set Dualities
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects