



# Random Variables Joint Law

## 1 Why

We name the image measure of a collection of real-valued random variables.

## 2 Definition

The *joint law* of an ordered finite family of  $n$  real-valued random variables is the image measure of the tuple-valued function whose components are the individual random variables.

### 2.1 Notation

Let  $R$  denote the set of real numbers and let  $R^n$  denote set of  $n$ -tuples of real numbers. Let  $f_1, \dots, f_n$  be random variables defined on a probability space  $(X, \mathcal{A}, \mu)$ . Define  $f : X \rightarrow R^n$  by  $(f(x))_i = f_i(x)$ . The joint law is the image measure of  $f$ .

We denote the joint law of  $\{f_i\}$  by  $\mu_{f_1, \dots, f_n} : \mathcal{A} \rightarrow [0, \infty]$ . We defined it by

$$\mu_{f_1, \dots, f_n}(A) = \mu(\{x \in X \mid f(x) \in A\}).$$

for all  $A$  in the topological sigma algebra on  $R^n$ .