



## Why

The most important families are those indexed by (subsets of) the natural numbers.

## Definition

A *finite sequence* is a family whose index set is  $\{1, \dots, n\}$  for some  $n \in \mathbf{N}$ . The *length* of a finite sequence is the size of its index set. If the codomain of a sequence is  $A$ , we say the sequence is *in*  $A$ .

Let  $A$  be a set with  $|A| = n$ . In this case, another term for a finite sequence is a *string* (or *list*). A sequence  $a : \{1, \dots, n\} \rightarrow A$  is an *ordering* of  $A$  if  $a$  is invertible. In this case, we call the inverse a *numbering* of  $A$ . An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

## Notation

Since the natural numbers are ordered, we regularly denote finite sequences from left to right between parentheses. For example, we denote  $a : \{1, \dots, 4\} \rightarrow A$  by  $(a_1, a_2, a_3, a_4)$ .

## Relation to Direct Products

A *natural direct product* is a product of a sequence of sets. We denote the direct product of a sequence of sets  $A_1, \dots, A_n$  by  $\prod_{i=1}^n A_i$ . If each  $A_i$  is the same set  $A$ , then we denote the

product  $\prod_{i=1}^n A_i$  by  $A^n$ . In this case, we call an element (the sequence  $a = (a_1, a_2, \dots, a_n) \in A^n$ ) an *n-tuple* or *tuple*. The set of sequences in a set  $A$  is the direct product  $A^n$ .

## Infinite Sequences

An *infinite sequence* is a family whose index set is  $\mathbf{N}$  (the set of natural numbers without zero). The *nth term* or *coordinate* of a sequence is the result of the *nth* natural number,  $n \in \mathbf{N}$ .<sup>1</sup>

### Notation

Let  $A$  be a non-empty set and  $a : \mathbf{N} \rightarrow A$ . Then  $a$  is a (infinite) sequence in  $A$ .  $a(n)$  is the *nth* term. We also denote  $a$  by  $(a_n)_n$  and  $a(n)$  by  $a_n$ . If  $\{A_n\}_{n \in \mathbf{N}}$  is an infinite sequence of sets, then we denote the direct product of the sequence by  $\prod_{i=1}^{\infty} A_i$ .

### Natural unions and intersections

We denote the family union of the finite sequence of sets  $A_1, \dots, A_n$  by  $\cup_{i=1}^n A_i$ . We denote the family of the infinite sequence of sets  $(A_n)_n$  by  $\cup_{i=1}^{\infty} A_i$ . Similarly, we denote the intersections of a finite and infinite sequence of sets  $\{A_i\}$  by  $\cap_{i=1}^n A_i$  and  $\cap_{i=1}^{\infty} A_i$ , respectively.

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<sup>1</sup>Future editions may also comment that we are introducing language for the steps of an infinite process.

