Why

What of lower upper triangular factorizations when the system is symmetric?

Definition

A *symmetric lower upper triangular factorization* of A is a pair of matrices $(L \in \mathbf{R}^{m \times m}, U \in \mathbf{R}^{m \times m})$ where L is unit lower triangular, U is upper triangular and $A = LU$.

$$A = LL^\top.$$

Other terminology includes *lower upper triangular factorization, LU decomposition, LU factorization.* Define $R = L^\top$, then

$$A = R^\top R.$$

Let $A \in \mathbf{R}^{n \times n}$. Then the ordinary row reduction of A is a matrix U which is upper triangular.be symmetric. A *lower upper triangular decomposition* A is a pair of matrices $(L, L^\top)$ where $L \in \mathbf{R}^{n \times n}$ is lower triangular, has nonnegative real diagonal entries, and satisfies

$$A = LL^\top.$$

Other terminology includes *lower upper triangular factorization, LU decomposition, LU factorization.* Define $R = L^\top$, then

$$A = R^\top R.$$

Basic properties

Proposition 1. *Let $A \in \mathbf{R}^{m \times m}$ be positive definite. Then there exists unique lower triangular matrix $L \in \mathbf{R}^{n \times n}$ so that*

$$A = LL^\top.$$

So, in the case that A is positive definite, a lower upper triangular decomposition exists and is unique. Therefore we refer to it as *the upper*

lower triangular decomposition of A. It is also known (universally) as the *Cholesky decomposition* or *Cholesky factorization* of A.

Proposition 2. *If A is positive semidefinite, there exists a permutation matrix P for which there is a unique L so that*

$$P^\top A P = L L^\top.$$

This second proposition says that there is a unique Cholesky decomposition for a particular permutation (or pivoting) of A.

Unitriangular form

A *lower diagonal upper decomposition* (or *lower diagonal upper factorization*) of a matrix A a sequence $(L, D, L^\top)$ where $L \in \mathbf{R}^{n \times n}$ is unit lower triangular, $D \in \mathbf{R}^{n \times n}$ is diagonal with real nonnegative entries and

$$A = L D L^\top.$$

Other terminology includes *LDL decomposition, LDL factorization, LDU factorization, LDU decomposition*.

If $(L \in \mathbf{R}^{n \times n}, D \in \mathbf{R}^{n \times n}, L^\top)$ is a LDU decomposition of $A \in \mathbf{R}^{n \times n}$, then $(L D^{1/2} A = L D L^\top$ then $(\tilde{L} D^{1/2}, D^{1/2} L^\top)$ is a LU decomposition. Conversely, if $(B, B^\top)$ is a LU decomposition and S is the diagonal matrix satisfying $S_{ii} = B_{ii}$ for $i = 1, \ldots, n$, then $(B S^{-1}, S^2, S^{-1} B^\top)$ is a LDU decomposition of A.

Symmetric Lower Upper Triangular Factorizations

Lower Upper Triangular Factorizations

Index Matrices   Symmetric Matrices   Positive Definite Matrices

Ordinary Reduced Factorization   Permutation Matrices   Quadratic Forms

Ordinary Row Reductions   Row Reducer Matrices   Matrix Square

Matrix Trace

Linear System Row Reductions   Triangular Matrices   Real Matrix Inverses   Identity Matrices   Real Matrix-Matrix Products

Matrix Transpose

Diagonal Matrices   Real Matrix-Vector Products   Symmetric Real Matrices

Matrices   Real Linear Combination   Vector as Matrix

Linear Equation Solutions   Real Matrices   Real Inner Product

Linear Equation   Vector   Real Norm   Multivariate Polynomials

Linear Functions   Real Vectors   Space Norm   Multivariate Functions

Space Vectors   Plane Vector Sums and Differences   Plane Norm   Plane Inner Product   N-Dimensional Space

Plane Vectors   Absolute Value   Distance   Real Square Roots   Right Triangle Sides Relation

Space Distance   Real Polynomials   Real Squares

Arrays   Plane Distance   Real Arithmetic

Interval Length   Real Multiplication Inverses   Natural Square Roots

Permutations   Real Plane   Real Line   Real Addition Inverses   Real Space   Real Products   Square Numbers

Real Summation   Real Order   Real Sums   Rational Multiplicative Inverses   Rational Arithmetic   Integral Line   Geometry

Inverse Elements   Lists   Groups   Fields   Real Numbers   Rational Sums   Rational Order   Rational Products

Element Functions   Direct Products   Set Numbers   Rings   Rational Numbers

Finite Sets   Integer Arithmetic   Integer Products

Equivalent Sets   Natural Summation   Natural Multiplication Identity   Natural Additive Identity   Integer Order   Integer Sums

Family Union and Intersection   Function Inverses   Family Operations   Comparisons   Identity Elements   Arithmetic   Natural Equations   Integer Numbers

Function Composition   Function Images   Families   Operations   Equations   Orders   Natural Power

Functions   Converse Relations   Equivalence Relations   Natural Order   Natural Products

Relations   Natural Sums

Recursion Theorem

Peano Axioms

Set Products   Natural Induction

Generalized Set Dualities   Set Powers   Natural Numbers

Set Unions and Intersections   Set Dualities   Unordered Triples   Successor Sets   Intersection of Empty Set   Partitions   Set Symmetric Differences

Ordered Pairs   Pair Unions   Set Intersections   Set Complements   Set Differences

Pair Intersections   Unordered Pairs   Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements   Identities   Sets

Names

Letters   Objects

5