Why

Suppose I have three stones in my hands. If I have two in my left hand, how many are in my right hand?

Discussion

Denote by a the number of stones in my left hand. Denote by b the number of stones in my right hand. Then the number of stones in both hands is $a + b$. Both a and b are natural numbers. In other words, a is 0 or 1 or 2 or... Likewise for b.

We express that I have two stones in my left hand by the equation $a = 2$. We express that I have have five stones in total by the equation that $a + b = 5$. Because we have identified the names a and b with tangible objects in my hands, these equations are statements about tangible objects in my hands. But the *numbers* involved are intangible objects. In any case, we have two equations in two unknowns.

We express the question "how many stones do I have in my right hand?" by asking for a solution (see Equations) to the equation $a + b = 5$. We can start by trying natural numbers in order. Is $(a, b) = (0, 0)$ a solution? Well, $0 + 0 = 0$, and $0 \neq 5$, so it is not. Is $(a, b) = (1, 0)$ a solution? Well, $l + 0 = 1$, and $1 \neq 5$, so it is not. Well, $(a, b) = (0, 1)$ a solution? Well, $l + 0 = 1$, and $1 \neq 5$, so it is not. Likewise for $(0, 2)$, $(1, 2)$ and so on. Some people call this the process of *guess and check*.

Continuing this way we find that $(3, 2)$ and $(2, 3)$ is a solu-

tion. Indeed, $3 + 2 = 2 + 3 = 5$. We are, however, interested in solutions to both equations

$$a = 3$$
$$a + b = 5$$

Both $(3, 2)$ and $(2, 3)$ satisfy the second equation, but only $(3, 2)$ satisfies both. Are there other solutions?[1]

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[1]Future editions will expand.

Natural Equations

Equations　　　　　Natural Sums

Functions　　Natural Order　　Recursion Theorem

Relations　　　　　　　　Peano Axioms

Cartesian Products　　　　　Natural Induction

Set Powers　　　　　　Natural Numbers

Unordered Triples　　Successor Sets　　Intersection of Empty Set

Ordered Pairs　　Pair Unions　　Set Intersections

Set Unions　　　　　　Set Differences

Unordered Pairs　　Empty Set　　Pair Intersections

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities　　Sets

Names

Letters　　Objects