Nets

Why

We generalize the notion of sequence to index sets beyond the naturals.

Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go "further out."

To elaborate on property (b): if handed two natural numbers m and n, we can always find another, for example $\max\{m, n\}+1$, larger than m and n. We might think of larger as "further out" from the first natural number: 1.

Combining these to observations, we define a directed set:

Definition 1. *A directed set is a set D with a partial order $\preceq$ satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.*

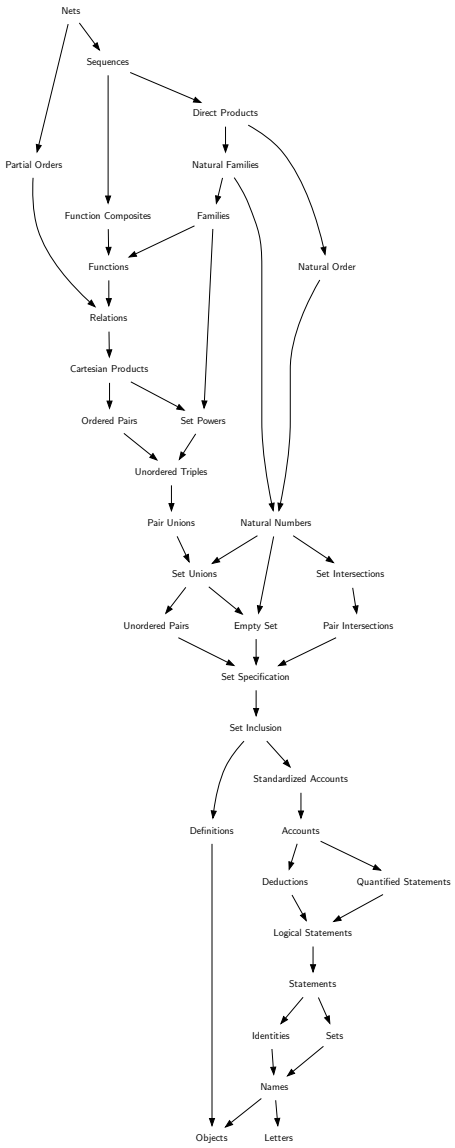
Definition 2. *A net is a function on a directed set.*

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \to A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.


Nets
Sequences
Direct Products
Partial Orders
Natural Families
Function Composites
Families
Functions
Natural Order
Relations
Cartesian Products
Ordered Pairs
Set Powers
Unordered Triples
Pair Unions
Natural Numbers
Set Unions
Set Intersections
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters