Why

[1]

Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\mathsf{cov}(f, g)$. We defined it:

$$\mathsf{cov}(f, g) = \mathsf{E}(fg) - \mathsf{E}(f)\,\mathsf{E}(g).$$

Properties

Prop. 1. *Covariance is symmetric and billinear.*[2]

Prop. 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\mathsf{cov}(f, f) = \mathsf{E}(ff) - \mathsf{E}(f)\,\mathsf{E}(f) = \mathsf{E}(f^2) - (\mathsf{E}(f))^2 = \mathsf{var}(f).$$

□

[1] Future editions will include this.

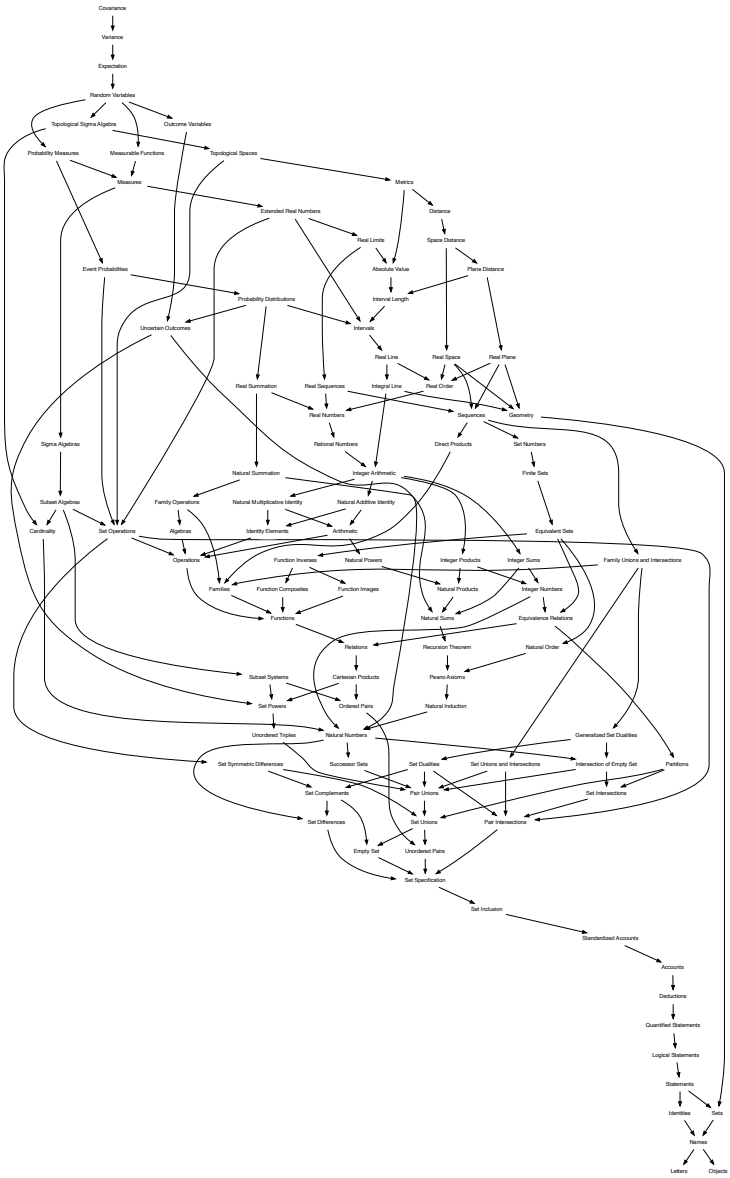
[2] Future editions will include an account.

Prop. 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\begin{aligned}\mathsf{var}\left(\sum_{i=1}^{n} f_i\right) &= \mathsf{cov}\left(\sum_{i=1}^{n} f_i, \sum_{i=1}^{n} f_i\right) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} \mathsf{cov}(f_i, f_j)\end{aligned}$$

$\square$


Covariance
Variance
Expectation
Random Variables
Outcome Variables
Topological Sigma Algebra
Probability Measures
Measurable Functions
Topological Spaces
Metrics
Measures
Extended Real Numbers
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Event Probabilities
Interval Length
Probability Distributions
Intervals
Uncertain Outcomes
Real Line
Real Space
Real Plane
Real Order
Integral Line
Real Summation
Real Sequences
Real Numbers
Sequences
Geometry
Rational Numbers
Direct Products
Set Numbers
Sigma Algebras
Integer Arithmetic
Natural Summation
Finite Sets
Natural Multiplicative Identity
Natural Additive Identity
Family Operations
Subset Algebras
Cardinality
Set Operations
Algebras
Identity Elements
Arithmetic
Equivalent Sets
Operations
Function Inverses
Natural Powers
Integer Products
Integer Sums
Family Unions and Intersections
Families
Function Composites
Function Images
Natural Products
Integer Numbers
Functions
Natural Sums
Equivalence Relations
Relations
Recursion Theorem
Natural Order
Subset Systems
Cartesian Products
Peano Axioms
Set Powers
Ordered Pairs
Natural Induction
Unordered Triples
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects