Why

We model a real-valued output as corrupted by small random errors. Thus, we can talk about a dataset which is "close" to being consistent with a linear predictor.

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $x \in \mathbf{R}^d$ and $e : \Omega \to \mathbf{R}^n$. For $A \in \mathbf{R}^{n \times d}$, define $y : \Omega \to \mathbf{R}^n$ by $y = Ax + e$. We call (x, A, e) a *probabilistic errors linear model*. We call y the *response vector*, A the *model matrix* and e the *error vector*.

Moment assumptions

The most basic distributional assumption for a probabilistic errors linear model pertain to the expectation and variance. Since $\mathrm{E}(y) = Ax + \mathrm{E}(e)$ and $\mathrm{var}(y) = \mathrm{var}(e)$, these assumptions can be given for e or for y.

If $\mathrm{E}(x) = 0$ and $\mathrm{var}(y) = \sigma^2 I$ then we call (x, A, e) a *classical linear model with moment assumptions*. Notice that the components of e are assumed uncorrelated. We have $d + 1$ unknowns (the $d \times 1$ entires of θ and scalar parameter σ^2.

In this case $\mathrm{E}(y_i) = a^{i\top}\theta$ and so θ is called the *mean parameter vector* and σ^2 is called the *model variance*. The model variance indicates the variability inherent in the observations. Neither the mean nor variance of the error depends on the regression vector x nor on the parameter vector θ.

Examples

Consider the *two-sample problem* in which we have two populations with (unknown) mean responses $\alpha_1, \alpha_2 \in \mathbf{R}$. We observe these responses with (perhaps unknown) common variance σ^2, and assume that errors are uncorrelated.

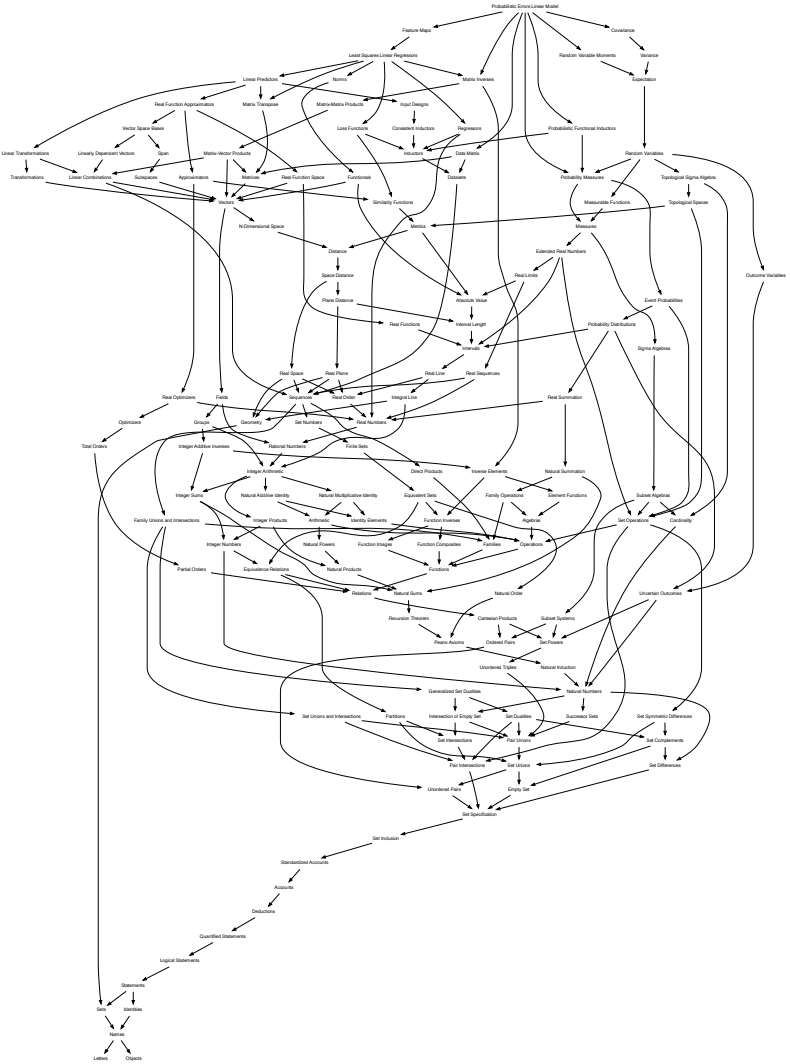
We define $y^1 = \alpha_1 \mathbf{1} + e^1$ and $y^2 = \alpha_2 \mathbf{1} + e^2$ so that we can stack these and obtain

$$y = \begin{bmatrix} y^1 \\ y^2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \mathbf{1} \\ \alpha_2 \mathbf{1} \end{bmatrix} + \begin{bmatrix} e^1 \\ e^2 \end{bmatrix}.$$

To cast this in our standard form we define

$$A = \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \cdots & \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \end{bmatrix} & \cdots & \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix}^\top, \quad x = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}.$$

with regression vectors $a_1 = (1, 0)$ and $x_2 = (0, 1)$ repeated n_1 and n_2 times, respectively. An input design for this model involves specifying a sequence of these two vectors, which (with the uncorrelated assumption) reduces to dictating how many responses should be collected from each population. The inputs here is really the set $\mathcal{X} = \{1, 2\}$. The feature function is $\phi : \mathcal{X} \to \mathbf{R}^2$ defined by $\phi(1) = (1, 0)$ and $\phi(2) = (0, 1)$. And so the regression range is $\{(1, 0), (0, 1)\}$.


Probabilistic Errors Linear Model
Feature Maps
Covariance
Least Squares Linear Regressors
Random Variable Moments
Variance
Linear Predictors
Norms
Matrix Inverses
Summation
Real Function Approximators
Matrix Transpose
Matrix-Matrix Products
Input Designs
Vector Space Bases
Loss Functions
Consistent Inductors
Regressors
Probabilistic Functional Inductors
Linear Transformations
Linearly Dependent Vectors
Span
Matrix-Vector Products
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Random Variables
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Real Function Space
Functionals
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Probability Measures
Topological Sigma Algebras
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Similarity Functions
Measurable Functions
Topological Spaces
N-Dimensional Space
Metrics
Measures
Distance
Extended Real Numbers
Space Distance
Real Limits
Outcome Variables
Absolute Value
Plane Distance
Event Probabilities
Real Functions
Interval Length
Probability Distributions
Intervals
Sigma Algebras
Real Space
Real Plane
Real Line
Real Sequences
Real Optimizers
Fields
Sequences
Real Order
Integral Line
Real Summation
Optimizers
Groups
Geometry
Set Numbers
Real Numbers
Total Orders
Integer Additive Inverses
Rational Numbers
Finite Sets
Integer Arithmetic
Direct Products
Inverse Elements
Natural Summation
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Family Operations
Element Functions
Subset Algebras
Family Unions and Intersections
Integer Products
Arithmetic
Identity Elements
Function Inverses
Algebras
Set Operations
Cardinality
Integer Numbers
Natural Powers
Function Images
Function Composites
Families
Operations
Partial Orders
Equivalence Relations
Natural Products
Functions
Relations
Natural Sums
Natural Order
Uncertain Outcomes
Recursion Theorem
Cartesian Products
Subset Systems
Peano Axioms
Ordered Pairs
Set Powers
Unordered Triples
Natural Induction
Generalized Set Duplies
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Duplies
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
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Statements
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Identities
Norms
Lattices
Objects