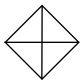


Why

We want numbers to count with.[1]

Definition

The *successor* of a set is the set which is the union of the set with the singleton of the set. In other words, the successor of a set A is $A \cup \{A\}$. This definition has sense for any set, but is of interest only for those particular sets introduced here.

These sets are the following (and their successors): We call the empty set *zero*.[2] We call the successor of the empty set *one*. In other words, one is $\varnothing \cup \{\varnothing\} = \{\varnothing\}$. We call the successor of one *two*. In other words, two is $\{\varnothing\} \cup \{\{\varnothing\}\} = \{\varnothing, \{\varnothing\}\}$. Likewise, the successor of two we call *three* and the successor of three we call *four*. And we continue as usual,[3] using the English language in the typical way.

A set is a *successor set* if it contains zero and if it contains the successor of each of its elements.

Notation

Let x be a set. We denote the successor of x by x^+. We defined it by

$$x^+ := x \cup \{x\}$$

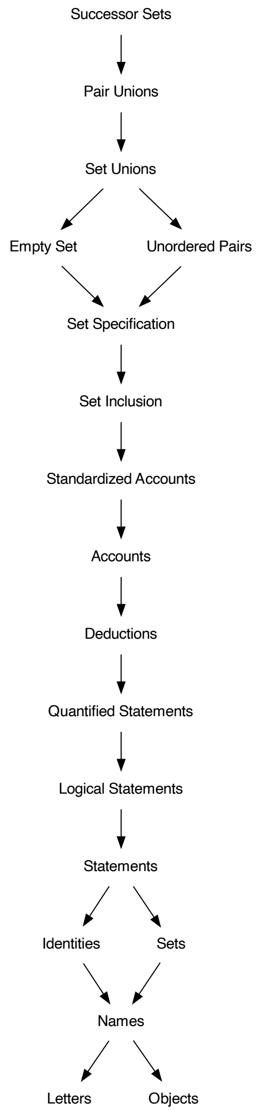
[1]Future editions will expand on this sheet with a more justified why.

[2]In future editions, zero may be a separate sheet.

[3]Future editions will assume less in the introduction of natural numbers.

We denote one by 1. We denote two by 2. We denote three by 3. We denote four by 4. So

$$
\begin{aligned}
0 &= \varnothing \\
1 &= 0^+ = \{0\} \\
2 &= 1^+ = \{0, 1\} \\
3 &= 2^+ = \{0, 1, 2\} \\
4 &= 3^+ = \{0, 1, 2, 3\}
\end{aligned}
$$


Successor Sets
Pair Unions
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects