Why

The Hilbert space is one of the most important generalizations of the past century.[1]

Definition

Let $(X, \mathbf{R})$ be a real vector space. An inner product $\langle \cdot, \cdot \rangle : X \times X \to \mathbf{R}$ induces a norm $\lVert \cdot \rVert : X \to \mathbf{R}$ defined by $\lVert x \rVert = \sqrt{\langle x, x \rangle}$ and metric $d : X \times X \to \mathbf{R}$ defined by $d(x, y) = \lVert x - y \rVert$.

If (X, d) is a complete metric space, we call $((X, \mathbf{R}), \langle \cdot, \cdot \rangle)$ a *complete inner product space* (or *Hilbert space*).[2]

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[1]Future editions will modify.

[2]The term Hilbert space is universal, but in accordance with the Bourbaki project's guidelines on naming, we will tend to use the term complete inner product space, even though this is longer.

Complete Inner Product Spaces

Complete Normed Spaces

Complete Metric Spaces — Norm Metrics — Inner Product Norms

Egyptic Sequences — Norms — Inner Products

Real Egyptic Sequences — Functionals — Real Inner Product

Metrics — Real Limits — Real Norm — Vectors

Absolute Value — Real Vectors

Plane Norm — Space Vector — Plane Vector Sums and Differences — N-Dimensional Space — Plane Inner Product

Distance — Plane Vector — Right Triangle Sides Relation — Real Square Roots

Space Distance — Real Square

Plane Distance — Real Arithmetic

Interval Length — Real Multiplicative Inverses

Intervals — Real Space — Real Products

Real Line — Real Plane — Real Additive Inverses

Real Sequences — Integral Line — Real Order — Real Sums — Rational Arithmetic — Rational Multiplicative Inverses

Geometry — Rational Order — Real Numbers — Fields — Rational Sums — Rational Products

Sequences — Rational Numbers — Rings

Lists — Integer Arithmetic

Direct Products — Set Numbers — Integer Products — Natural Multiplicative Identity — Natural Additive Identity

Finite Sets — Arithmetic — Identity Elements — Natural Square Roots

Equivalent Sets — Natural Powers — Inverse Elements — Square Numbers

Family Unions and Intersections — Comparisons — Integer Order — Integer Sums — Function Inverses — Natural Products — Element Functions — Squares — Natural Equations

Families — Orders — Integer Numbers — Function Images — Function Composites — Natural Sums — Operations — Equations

Equivalence Relations — Converse Relations — Natural Order — Functions — Recursion Theorem

Relations — Peano Axioms

Set Products — Natural Induction

Set Unions and Intersection — Generalized Set Dualities — Set Powers — Natural Numbers

Partitions — Intersection of Empty Set — Set Dualities — Unordered Triples — Successor Sets — Set Symmetric Differences

Set Intersections — Pair Unions — Set Complements — Ordered Pairs

Pair Intersections — Set Unions — Set Differences

Empty Set — Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects