



Nets



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1 Why

We generalize the notion of sequence to index sets beyond the naturals.

2 Definition

A sequence is a function on the natural numbers; this set has two important properties: (a) we can order the natural numbers and (b) we can always go “further out.”

To elaborate on property (b): if handed two natural numbers m and n , we can always find another, for example $\max\{m, n\} + 1$, larger than m and n . We might think of larger as “further out” from the first natural number: 1.

Combining these two observations, we define a directed set:

Definition 1. *A directed set is a set D with a partial order \preceq satisfying one additional property: for all $a, b \in D$, there exists $c \in D$ such that $a \preceq c$ and $b \preceq c$.*

Definition 2. *A net is a function on a directed set.*

A sequence, then, is a net. The directed set is the set of natural numbers and the partial order is $m \preceq n$ if $m \leq n$.

2.1 Notation

Directed sets involve a set and a partial order. We commonly assume the partial order, and just denote the set. We use the letter D as a mnemonic for directed.

For nets, we use function notation and generalize sequence notation. We denote the net $x : D \rightarrow A$ by $\{a_\alpha\}$, emulating notation for sequences. The use of α rather than n reminds us that D need not be the set of natural numbers.