



Why

We want to visualize symmetric relations.

Definition

An *undirected graph* (or *graph*) is a pair (V, E) in which V is a nonempty set and E is a subset of *unordered* pairs of elements in V . We call the elements of V the *vertices* of the graph and the elements of E the *edges*. We call (V, E) an undirected graph on V .

Two vertices are *adjacent* (or *neighboring*) if their pair is in the edge set. We say that the corresponding edge is *incident* to those vertices. The *adjacency set* of a vertex is the set of vertices adjacent to it. The *degree* of a vertex is the number of vertices adjacent to it; in other words, the size of its adjacency set. A graph is *complete* if each pair of two distinct vertices is adjacent.

The *complement* of (V, E) is the graph (V, F) where F is the complement of E in the set of pairs from V .

Other terminology

Some authors call the adjacency set the *neighborhood* of the vertex. They call the union of the adjacency set of the vertex $v \in V$ with the singleton $\{v\}$ the *closed neighborhood* of v .

When $\{x, y\} \in E$, some authors say that the edge “joins” the vertices, and call x and y the *end vertices* of the edge.

Some authors call two *edges adjacent* if they have exactly one common end vertex.

Self-loops

Some authors include a concept of a self-loop, which is meant to be an edge from of a “self-loop”—an edge from one node to itself. However, we do not allow this as the set $\{u, u\}$ is *not* a pair.

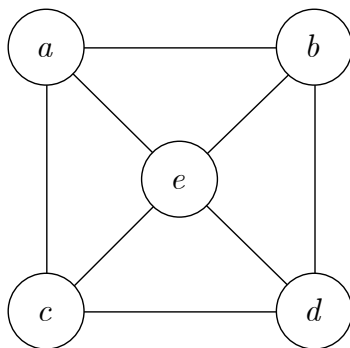
Notation

Let V be a nonempty set. Let $E \subset \{\{v, w\} \mid v, w \in V\}$. Then the pair (V, E) is an undirected graph. We regularly say “Let $G = (V, E)$ ” be a graph, in which the relevant properties of V and E are implicit.

The notation $\{v, w\} \in E$ for an edge between vertices $v, w \in V$ reminds us that the edges are unordered pairs of distinct vertices. We denote the adjacency set of v by $\text{adj}(v)$ and the degree of v by $\deg(v)$.

Some authors will denote the vertex set of a graph they are denoting by G by $V(G)$ and the edges set by $E(G)$.

Example



Suppose $V = \{a, b, c, d, e\}$. Define

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\}.$$

In visualizations of graphs, the vertices are frequently represented as circles or rectangles in the plane and edges are shown as lines connecting the vertices.

