Why

1

Definition

A subset $M \subset \mathbf{R}^n$ is a *smooth manifold* of dimension d if for every $x \in M$, there exists a neighborhood V of x in X that is diffeomorphic to an open subset U of $\mathbf{R}^d$. In this case we say that the set is *locally diffeomorphic* to $\mathbf{R}^d$.

A diffeomorphism $\phi : U \to V$ is called a *parameterization* of the neighborhood of V. Its inverse diffeomorphism ϕ^{-1} is called a *coordinate system* (or system of *coordinates*) on V.

Notation

We denote the dimension of a manifold M by $\dim M$.

Submanifolds

If X and Z are both manifolds in R^n and $Z \subset X$, then we call Z a *submanifold* of X. In particular, X is a submanifold of R^n. Any open set of a manifold X is a submanifold X.[2]

---

[1]Future editions will include.

[2]Future editions will expand.

Smooth Manifolds

Smooth Functions · Real Neighborhoods

Smooth Multivariate Functions

Smooth Real Functions · Real Balls

Higher Order Derivatives

Second Derivatives · Partial Derivatives

Differentiable Functions · Real Open Sets · Multivariate Functions

N-Dimensional Space

Real Functions

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Line · Real Space · Real Plane

Integral Line · Real Order · Geometry

Real Numbers

Rational Numbers

Integer Arithmetic · Sequences

Natural Multiplicative Identity · Natural Additive Identity · Set Number · Direct Products

Arithmetic · Identity Elements · Finite Sets

Natural Powers · Integer Products · Operations · Equivalent Sets

Natural Products · Integer Sums · Function Inverses · Family Unions and Intersections

Natural Sums · Integer Numbers · Function Composite · Function Image · Families

Recursion Theorem · Natural Order · Equivalence Relations · Functions

Peano Axioms · Relations

Natural Induction · Cartesian Products

Natural Numbers · Set Power · Generalized Set Dualities

Partitions · Intersection of Empty Set · Successor Sets · Unordered Triples · Set Dualities · Set Unions and Intersections

Ordered Pairs · Set Intersections · Pair Unions · Set Complements

Set Unions · Set Differences · Pair Intersections

Unordered Pairs · Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities · Sets

Names

Letters · Objects