## Why

A simple example of an embedding.[1]

## Definition

Fix $d \in \mathbf{N}$. A *polynomial feature map* of degree $d$ is a function $\phi : \mathbf{R} \to \mathbf{R}^d$ with

$$\phi(x) = \begin{pmatrix} 1 & x^2 & \cdots & x^d \end{pmatrix}^\top.$$

For $x \in \mathbf{R}$, we call $\phi(x)$ the *polynomial embedding* of $x$.

A *polynomial regressor* is a least squares linear predictor using a polynomial feature embedding (of any degree, but to be precise one must specify the degree). The task of consructing a linear predictor is often referred to as *polynomial regression.*

Given a dataset of paired records $(x^1, y^1), \ldots, (x^2, y^2) \in \mathbf{R}^2$, one can construct a predictor $g : \mathbf{R} \to \mathbf{R}$ for $y$ by embedding the dataset $(\phi(x^1), \ldots, \phi(x^n))$ and finding the least squares linear regressor $f : \mathbf{R}^d \to \mathbf{R}$ for $y$. One defines the predictor $g : \mathbf{R} \to \mathbf{R}$ by $g(\phi(x))$.

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[1]Future editions will expand, or perhaps collapse this sheet.

Polynomial Regression

Feature Maps

Least Squares Linear Regression

Matrix Inverses

Matrix-Matrix Products     Linear Prediction

Real Matrix-Matrix Products   Input Designs   Matrix-Vector Products   Loss Functions   Matrix Transpose   Real Function Approximation

Regression   Consistent Induction   Real Matrix-Vector Products   Predictor Performance Metrics   Matrices   Vector Space Bases   Norms   Approximation   Real Function Space

Prediction   Linearly Dependent Vectors   Linear Transformation   Span   Vectors as Matrices   Real Inner Product   Functionals   Similarity Functions

Linear Combinations   Transformations   Subspaces   Real Matrices   Real Norm   Metrics   Real Function

Real Linear Combinations   Vectors   Linear Equations   Absolute Value

Real Vector   Linear Functions

Plane Vector Sums and Differences   Space Vectors   Plane Norm   Plane Inner Product   Bi-Dimensional Space

Arrays   Plane Vectors   Right Triangle Sides Relation   Distance

Real Square   Space Distance

Real Arithmetic   Square Numbers   Plane Distance   Interval Length

Real Multiplicative Inverses   Squares   Real Space   Intervals

Real Products   Real Additive Inverses   Real Plane   Real Line

Polynomials   Fields   Real Sums   Real Order   Real Optimizers   Integral Line

Lists   Rational Multiplicative Inverses   Real Summation   Rings   Rational Arithmetic   Real Numbers   Rational Order   Geometry

Inverse Elements   Rational Products   Rational Sum   Rational Numbers

Element Functions   Direct Products   Set Numbers   Integer Arithmetic   Optimizers

Finite Sets   Natural Multiplicative Identity   Natural Additive Identity   Equivalent Sets   Natural Summation   Integer Sums   Integer Products   Integer Order   Comparisons

Function Inverses   Identity Elements   Arithmetic   Family Operations   Family Union and Intersections   Integer Numbers   Orders   Natural Equations

Function Composites   Function Images   Operations   Families   Natural Powers   Equations   Natural Order

Equivalence Relations   Converse Relations   Functions   Natural Products

Relations   Natural Sums

Recursion Theorem

Peano Axioms

Natural Induction

Set Products   Set Powers   Generalized Set Dualities   Natural Numbers

Set Symmetric Differences   Partitions   Set Dualities   Unordered Triples   Intersection of Empty Set   Successor Sets   Set Unions and Intersections

Ordered Pairs   Set Complement   Set Intersections   Pair Unions

Set Unions   Set Differences   Pair Intersections

Unordered Pairs   Empty Set

Set Specification

Set Inclusion   Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects