



## Tree Distributions

### 1 Why

It happens that if a distribution factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So the property of factoring according to a rooted tree is really property of factoring according to a tree.

### 2 Definition

#### 2.1 Defining Result

**Proposition 1.** *Let  $A_1, \dots, A_n$  be finite non-empty sets and define  $A = \prod_{i=1}^n A_i$ . Let  $p : A \rightarrow [0, 1]$  be a distribution and let  $T$  be a tree on  $\{1, \dots, n\}$ .*

**Proposition 2.** *If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

#### 2.2 Undirected Definition

A distribution  $p$  factors according to the tree  $T$  if it factors according to the  $T$  rooted at any vertex.

### **3 Existence and Uniqueness**

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

#### **3.1 Existence**

A distribution  $p$  need not factor according to a tree.

#### **3.2 Uniqueness**

A distribution  $p$  may factor according to multiple trees.