



Complex Measures

1 Why

We allow measures to take complex values. TODO

2 Definition

A complex-valued function on a sigma algebra is *countably additive* if the result of the function applied to the union of a disjoint countable family of distinguished sets is the limit of the partial sums of the results of the function applied to each of the sets individually. The limit of the partial sums must exist irregardless of the summand order.

A *complex measure* is an complex-valued function on a sigma algebra that is (1) zero on the empty set and (2) countably additive. We call the result of the function applied to a set in the sigma algebra the *complex measure* (or when no ambiguity arises, the *measure*) of the set.

Since the codomain of a complex measures is the complex numbers, the sum corresponding to every countable union must be absolutely convergent (?) (TODO: define).

2.1 Notation

We denote complex measures by μ a mnemonic for “measure.” Let C denote the set of complex numbers. Let (X, \mathcal{A}) be a measurable space and let $\mu : \mathcal{A} \rightarrow C$. Then μ is a complex measure if

1. $\mu(\emptyset) = 0$ and
2. $\mu(\cup_i A_i) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \mu(A_k)$ for all disjoint families $(A_n)_n$.