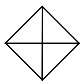


Why

[1]

Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\mathrm{tr} : \mathbf{R}^{n \times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n \times n}$. Then

$$\mathrm{tr}\, A = \sum_{i=1}^{n} A_{ii}.$$

Properties

Prop. 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n \times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\begin{aligned} \mathrm{tr}\, C = \sum_{i=1}^{n} C_{ii} &= \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii} \\ &= \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii} \\ &= \alpha\, \mathrm{tr}\, A + \beta\, \mathrm{tr}\, B. \end{aligned}$$

□

[1] Future editions will include, in the genetic tradition.

Prop. 2. *Let* $A, B \in \mathsf{R}^{n\times n}$.

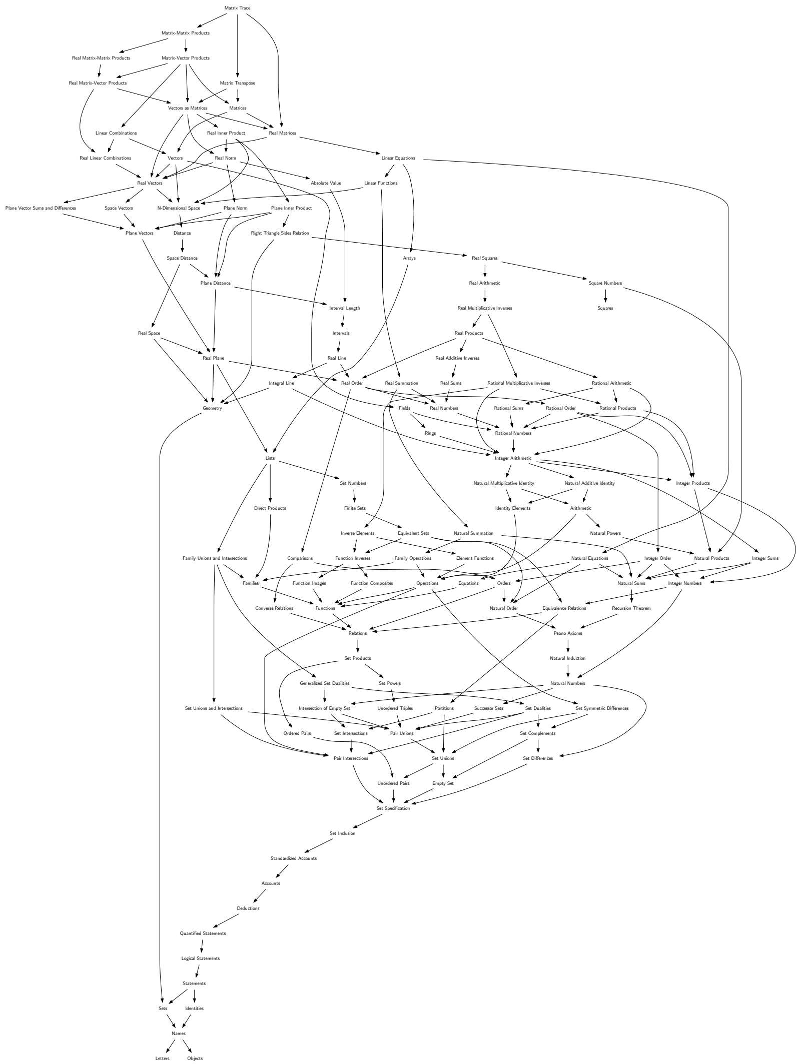
$$\operatorname{tr}(AB) = \operatorname{tr}(BA)$$

[2]

In other words, "matrices commute under the trace operator."

Prop. 3. *Let* $A \in \mathsf{R}^{n\times n}$. *Then* $\operatorname{tr} A = \operatorname{tr} A^\top$.

[2] Future editions will include an account.


Matrix Trace
Matrix-Matrix Products
Real Matrix-Matrix Products
Matrix-Vector Products
Real Matrix-Vector Products
Matrix Transpose
Vectors as Matrices
Matrices
Linear Combinations
Real Inner Product
Real Matrices
Real Linear Combinations
Vectors
Real Norm
Linear Equations
Real Vectors
Absolute Value
Linear Functions
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Norm
Plane Inner Product
Plane Vectors
Distance
Right Triangle Sides Relation
Space Distance
Arrays
Real Squares
Plane Distance
Real Arithmetic
Square Numbers
Interval Length
Real Multiplicative Inverses
Squares
Intervals
Real Space
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Order
Real Summation
Real Sums
Rational Multiplicative Inverses
Rational Arithmetic
Geometry
Fields
Real Numbers
Rational Sums
Rational Order
Rational Products
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Inverse Elements
Equivalent Sets
Natural Summation
Natural Powers
Family Unions and Intersections
Comparisons
Function Inverses
Family Operations
Element Functions
Natural Equations
Integer Order
Natural Products
Integer Sums
Families
Function Images
Function Composites
Operations
Equations
Orders
Natural Sums
Integer Numbers
Converse Relations
Functions
Natural Order
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Intersection of Empty Set
Unordered Triples
Partitions
Successor Sets
Set Dualities
Set Symmetric Differences
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects