Why

We have a space X and a family of probability measures $\mathcal{P}$ on this space. Assume $x \in X$ drawn from a fixed, unknown measure $P \in \mathcal{P}$. Given x, how should we guess P?

Definition

A *probabilistic model* (or *statistical model*, *parametric statistical model*, *statistical experiment*) is a family of probability measures over the same measurable space $(X, \mathcal{F})$. Call the index set the *parameter set* or *set of parameters*. The set X is called the *sample space*. A *statistic* is any function on the sample space.

Notation

Let $(X, \mathcal{F})$ denote a measurable space. We usually denote the parameter by Θ, and denote the family

$$\mathcal{P} = \{\mathbf{P}_\theta : \mathcal{F} \to [0,1] \mid \mathbf{P}_\theta \text{ a measure}, \theta \in \Theta\}.$$

Often $\Theta \subset \mathbf{R}^d$.

Example: coin flips

The usual model for n flips of a coin takes $X = \{0,1\}^n$, the set of binary n-tuples. For $\theta \in [0,1]$, a distribution $p_\theta(x) = \theta^t(1-\theta)^{n-t}$ where $t = t(x) = x_1 + \cdots + x_n$ is defined on X. A probability measure $\mathbf{P}_\theta$ is defined on $\mathcal{P}(X)$ in the the usual way. Thus, the probabilistic model is $\{\mathbf{P}_\theta \mid \theta \in [0,1]\}$. Given x, we are asked to guess θ.

Decisions

A *decision procedure* (*estimator*, *statistical procedure*) is a measurable function $A : \mathcal{X} \to \mathcal{A}$ where $\mathcal{A}$ is a a set, called the *actions* or *decisions*. Often $\mathcal{A} = \Theta$, in which case $A(x)$ givens an *estimate* of θ, which we denote $\hat{\theta}(x)$.

Judging decisions

Given a *loss function* $L : \mathcal{A} \times \Theta \to \bar{\mathbf{R}}$, the *risk* of A is

$$R(A, \theta) = \mathbf{E} L(A(x), \theta).$$

Probabilistic Models
Expectation
Outcome Variable Expectation
Random Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Topologies
Induced Probability Distributions
Measures
Metrics
Outcome Variable Events
Sigma Algebras
Distance
Real Series
Extended Real Numbers
Space Distance
Event Probabilities
Real Limits
Plane Distance
Absolute Value
Outcome Probabilities
Interval Length
Real Functions
Integrals
Real Space
Real Plane
Real Line
Integral Line
Real Order
Real Summation
Real Sequences
Size of Direct Product
Geometry
Rational Order
Real Numbers
Natural Summation
Number of Set Products
Sequences
Subset Algebras
Rational Numbers
Number of Disjoint Unions
Lists
Finite Set Examples
Cardinality
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Natural Fractions
Finite Sets
Identity Elements
Natural Arithmetic
Uncertain Events
Family Operations
Equivalent Sets
Comparisons
Integer Order
Integer Sums
Natural Power
Operations
Function Inverses
Family Unions and Intersections
Orders
Integer Numbers
Natural Products
Function Images
Function Composites
Families
Natural Sums
Natural Order
Equivalence Relations
Functions
Converse Relations
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Uncertain Outcomes
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Power
Ordered Pairs
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects