



EQUATIONS

Why

We name a statement which involves an identity.¹

Definition

An *equation* is any statement (see **Statements**) relating two terms by the relation of identity (see **Identities**). Some authors also call an equation an *equality*. The symbol “=” is called the (or an) *equals sign* or *equals symbol*.

Variables

It is regularly the case that we are interested in equations relating all objects of one set to another. For example: Let X and Y be sets and let $f : X \rightarrow Y$ and $g : X \rightarrow Y$. We may write the logical assertion $(\forall x)(f(x) = g(x))$. In this case it is understood that f and g are *free* names and x is a *bound* name (see **Quantified Statements**).

We will regularly, however, refer to the equation $f(x) = g(x)$ without the quantifier $\forall x$. In this case, x appears free, but is not. In other words, in the statement $f(x) = g(x)$, depending on context, x is an implicitly bound name. This usage is in slight offense to normal English usage. The name is *bound* (see **Quantified Statements**) because the use of the particular symbol x is irrelevant. We may as well have used the symbol y , and so the name is bound to the quantifier \forall .

¹Future editions will expand on this statement.

In a second sense, however, the choice of name is “free”, and the name is meant as a placeholder (see **Names**). For these reasons, we introduce terminology for this common case. We are discussing the equation

$$f(x) = g(x).$$

The name x we call a *variable*. It is a placeholder name, which is bound in the quantified statement. But the particular choice of name is irrelevant. In contrast to the variable x , the names f and g we call *constant*. We refer to them as the *constants* in the equation. The meaning is meant to convey that these names are *fixed* in the present discussion. That we are considering a particular function f and function g .

