



Why

What is the inverse element under matrix multiplication.

Definition

Recall that if $A \in \mathbf{R}^{m \times n}$ then $x \mapsto Ax$ is a function from \mathbf{R}^n to \mathbf{R}^m . Clearly, if $m \neq n$, then the inverse of f can not exist.¹

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that $BA = I$ we call B the *left inverse* of A and likewise if $AC = I$ we call C the *right inverse* of A . In the case that A is square, the right inverse and left inverse coincide.

Proposition 1. *Let $A, B, C \in \mathbf{R}^{n \times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

Proof. Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. □

Notation

Let F be a field. Let $A \in F^{n \times n}$ be invertible. We follow the notation of inverse elements and denote the inverse of A by A^{-1} .

¹Future editions will expand.

