



Definition

A *basis* for a vector space is a set of linearly independent vectors whose span is the set of vectors of the space. For any vector in the space, there exists a linear combination of the basis vectors whose result is that vector. In this case, it is common to say that any vector in the space “can be written as a linear combination of the basis vectors.”

Since the basis is a linearly independent set, the linear combination of basis vectors is unique.

If we have a basis of n vectors for (V, \mathbf{F}) then each vector $v \in V$ can be written uniquely as a linear combination of the vectors in the basis. If we take the vector in the field which is these coefficients, then this is an isomorphism with the vector space $(\mathbf{F}^n, \mathbf{F})$. We call this the *coordinate vector*.

Characterizations

Proposition 1. *A set of vectors is a basis if and only if no proper superset of it is linearly independent.*

Proposition 2. *A set of vectors that spans the space is a basis if and only if no proper subset of it spans the space.*

