Why

We abstract the properties of the natural numbers under natural addition and multiplication.

Definition

A *semiring* $(S, +, \cdot)$ satisfies all the properties of a ring (see Rings) *except* that addition $+$ need not have additive inverses.

Examples

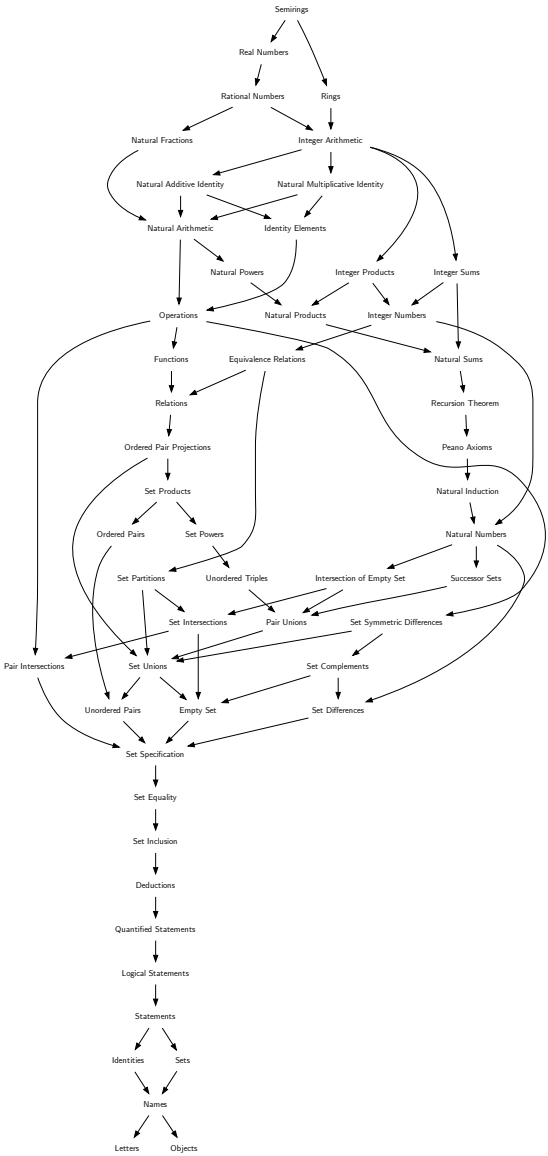
Set of natural numbers. The set $(\mathbf{N}, +, \cdot)$ where $+$ and $\cdot$ denote natural addition and multiplication respectively is a semiring.

Nonnegative real numbers with max and multiplication Notice that

$$\max(a, b) = \max(b, a) \quad \text{for all } a, b \in \mathbf{R}$$

$$\max(a, \max(b, c)) = \max(\max(a, b), c) \quad \text{for all } a, b, c \in \mathbf{R}$$

So $\max : \mathbf{R}^2 \to \mathbf{R}$ is a commutative and associative operation. The identity is 0, $\max(a, 0) = a$ for all $a \in \mathbf{R}_+$. Notice that there is no inverse element. Of course, $\cdot$ is associative and has identities.


Semirings
Real Numbers
Rational Numbers
Rings
Natural Fractions
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Natural Arithmetic
Identity Elements
Natural Powers
Integer Products
Integer Sums
Operations
Natural Products
Integer Numbers
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Set Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects