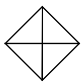


Why

If we interpret a list of two numbers as displacement in a plane, and a list of three numbers as displacement in a space, what of a list of n numbers as displacement in $\mathbf{R}^n$?

Definition

A *real vector* (*vector*, *n-dimensional vector*, *n-vector*) is a length-n list of real numbers.

Algebra

For $x, y \in \mathbf{R}^n$, we define the *real vector sum* (or *sum*) of x and y as the vector $z \in \mathbf{R}^n$ where $z_i = x_i + y_i$ for $i = 1, \dots, n$. As usual, we denote the sum by $x + y$, so

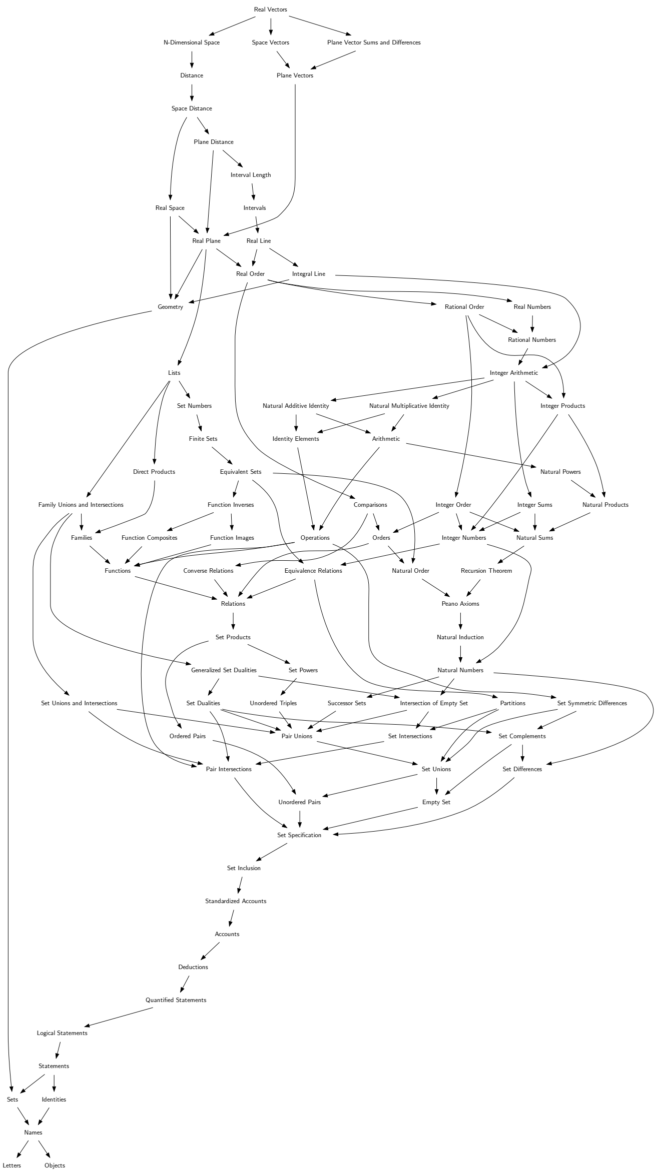
$$x + y = (x_1 + y_1, \dots, x_n + y_n).$$

For $\alpha \in \mathbf{R}$ and $x \in \mathbf{R}^n$, *real scalar-vector product* (or *scalar product*, *product*) $z \in \mathbf{R}^n$ is defined by $z_i = \alpha x_i$ for $i = 1, \dots, n$. As usual, we denote the product αx, and write

$$\alpha x = (\alpha x_1, \dots, \alpha x_n).$$

Our intuition for both of these *operations* comes from their special cases in $\mathbf{R}^2$ and $\mathbf{R}^3$. As usual, the *real-vector difference* (or *difference*) of x and y is the vector $z \in \mathbf{R}^n$ defined by $z_i = x_i - y_i$ for $i = 1, \dots, n$. As usual, we denote it by $x - y$, and note that $x - y = x + (-y)$.

The algebra given here for vectors is natural in view of their generalization as n-dimensional *displacements*. However, we keep in mind that this algebra is over lists of numbers, and that these sums and products can be defined on these lists of numbers regardless of interpretation.



Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects