Why

We discuss linear systems of equations using the algebra of matrices.[1]

Discussion

Let $(A \in \mathbf{R}^{m \times n}, b \in \mathbf{R}^m)$ be a linear system of equations. The two-dimensional array A is a matrix. Recall that we want to find $x \in \mathbf{R}^n$ to satisfy the simultaneous equations

$$A_{11}x_1 \cdots \quad A_{1n}x_n = b_1$$
$$\vdots$$
$$A_{m1}x_1 \cdots \quad A_{mn}x_n = b_m$$

Using the notation for a matrix-vector product, we can compactly write the above as

$$Ax = b.$$

This short statement encodes all m linear equations. For this reason A is often called the *coefficient matrix.*

Moreover it provides an algebraic and geometric interpretation of solving systems of linear equations. The algebraic interpretation is that we are interested in the invertibility of the map $x \mapsto Ax$. In other words, we are interested in the existence of an inverse element of A. The geometric interpretation is that A transforms the vector x.

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[1]Future sheets may invert this ordering, and motivate matrices by linear equations.

Matrices and Linear Equations

Matrix-Vector Products

Linear Equations

Matrices

Linear Combinations

Vectors

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Line

Real Plane

Real Space

Integral Line

Real Order

Real Summation

Sequences

Fields

Geometry

Real Numbers

Direct Products

Groups

Rational Numbers

Set Numbers

Integer Additive Inverses

Integer Arithmetic

Finite Sets

Inverse Elements

Natural Multiplicative Identity

Natural Additive Identity

Integer Sums

Element Functions

Identity Elements

Arithmetic

Family Operations

Equivalent Sets

Algebra

Function Inverses

Integer Products

Natural Exponents

Families

Natural Equations

Family Unions and Intersections

Operations

Integer Numbers

Function Composites

Function Images

Natural Products

Equations

Natural Order

Equivalence Relations

Functions

Natural Sums

Relations

Recursion Theorem

Peano Axioms

Cartesian Products

Natural Induction

Set Powers

Natural Numbers

Generalized Set Dualities

Partitions

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Unions and Intersections

Set Dualities

Ordered Pairs

Set Intersection

Pair Unions

Set Complements

Set Unions

Pair Intersections

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects