



**Definition**

**Proposition 1.** *Let  $W$  be a positive semidefinite  $n$  by  $n$  matrix. Then  $g : \mathbf{R}^n \rightarrow \mathbf{R}$  defined by  $g(x) = \sqrt{x^\top W x}$  is a norm on  $\mathbf{R}^n$ .*

**Notation**

Let  $W \in \mathbf{R}^{n \times n}$ , positive semidefinite. Then we denote the norm corresponding to  $W$  by  $\|\cdot\|_W$ . So then, the norm of a vector  $x \in \mathbf{R}^n$  is  $\|x\|_W$ . Notice that  $\|x\|_W = \|W^{1/2}x\|_2$ .

**Visualization**

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We can compare the Euclidean norm on  $\mathbf{R}^2$  with the weighted norm given by

$$W = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$

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<sup>1</sup>Future editions will visualize these norms as function on  $\mathbf{R}^2$ , via contour plots.



