$$\diamondsuit$$

Power Set

1   Why

We want to consider the subsets of a given set. Does a set exist which contains all the subsets.

2   Definition

We say yes.

We call this set the *power set*. It includes the set itself and the empty set.

2.1   Notation

We denote the power set of A by A^*, read aloud as "powerset of A." $A \in A^*$ and $\varnothing \in A^*$. However, $A \subset A^*$ is false.

2.2   Example

Let a, b, c be distinct objects. Let $A = \{a, b, c\}$ and $B = \{a, b\}$. Then $B \subset A$. In other notation, $B \in A^*$. As always, $\varnothing \in A^*$ and $A \in A^*$ as well. In this case, we can list the elements (which are sets) of the power set:

$$A^* = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}.$$