Relation Properties

1   Why

Often relations are defined over a single set. These have a few simple properties.

2   Definitions

- A relation is *reflexive* if every element is related to itself.

- A relation is *symmetric* if two objects are related regardless of their order.

- A relation is *antisymmetric* if two different objects are related only in one order, and never both.

- A relation is *transitive* if a first element is related to a second element and the second element is related to the third element, then the first and third element are related

2.1   Notation

Let R be a relation on a non-empty set A. R is **reflexive** if

$$(a, a) \in R$$

for all $a \in A$. R is **transitive** if

$$(a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R$$

for all $a, b, c \in A$. R is **symmetric** if

$$(a, b) \in R \implies (b, a) \in R$$

for all $a, b \in A$. R is **anti-symmetric** if

$$(a, b) \in R \implies (b, a) \notin R$$

for all $a, b \in A$.