



# Tree-Structured Densities

## 1 Why

## 2 Definition

### 2.1 Rooted Definition

Let  $f : \mathbf{R}^d \rightarrow \mathbf{R}$  be a density.  $f$  **factors according to the rooted tree** on  $\{1, \dots, d\}$  rooted at a vertex  $k$  if it can be written as a product of  $f_k$  and the conditionals of  $f_{i|j}$  for  $i, j = 1, \dots, d$  and  $i \neq j$  and  $i \neq k$  where  $j$  is the parent of  $i$  in the rooted tree.

### 2.2 Defining Result

**Proposition 1.** *If a density factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

### 2.3 Undirected Definition

A density  $f$  **factors according to the tree**  $T$  if it factors according to the  $T$  rooted at any vertex.

### 3 Existence and Uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

#### 3.1 Existence

A distribution  $f$  need not factor according to a tree.

#### 3.2 Uniqueness

A distribution  $f$  may factor according to multiple trees.