Why

What is the generalisation of smooth functions between Euclidean spaces.

Definition

Let $U \subset \mathbf{R}^n$ be open. A function $f : U \to \mathbf{R}^m$ is *smooth* if each of its components is smooth.

More generally, let $X \subset \mathbf{R}^n$ (not necessarily open). We call $g : X \to \mathbf{R}^m$ smooth if for each $x \in X$ there exists an open set $V \subset \mathbf{R}^n$ with $x \in V$ and smooth function $G : U \to \mathbf{R}^n$ so that $G(y) = g(y)$ for all $y \in U \cap X$. In this case we say that g can be *locally extended* to a smooth map on open sets.

Smooth Functions

Smooth Multivariate Functions

Smooth Real Functions

Higher Order Derivatives

Second Derivatives   Partial Derivatives

Real Open Sets   Multivariate Functions   Differentiable Functions

N-Dimensional Space

Distance   Real Functions

Space Distance

Plane Distance

Interval Length

Real Space   Intervals

Real Plane   Real Line

Real Order   Integral Line

Geometry   Rational Order   Real Numbers

Rational Numbers

Lists   Integer Arithmetic

Set Numbers   Natural Multiplicative Identity   Natural Additive Identity   Integer Products

Finite Sets   Identity Elements   Arithmetic   Natural Powers

Direct Products   Equivalent Sets   Integer Order   Integer Sums   Natural Products

Family Unions and Intersections   Function Inverses   Comparisons   Integer Numbers   Natural Sums

Families   Function Composite   Function Images   Operations   Orders   Natural Order   Recursion Theorem

Functions   Converse Relations   Equivalence Relations   Peano Axioms

Relations   Natural Induction

Set Products

Generalized Set Dualities   Set Powers   Natural Numbers

Set Unions and Intersections   Set Dualities   Unordered Triples   Successor Sets   Intersection of Empty Set   Partitions   Set Symmetric Differences

Ordered Pairs   Pair Union   Set Intersections   Set Complements

Pair Intersections   Unordered Pairs   Set Union   Set Differences

Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects