



## Why

We compress the notation for linear equations.

## Definition

A *real matrix* (*matrix*, *rectangular array*) is a two-dimensional array of real numbers. We denote the elements of a matrix in a grid between two rectangular braces, as in

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}.$$

We call  $n$  and  $m$  the *dimensions* of the matrix. We call  $n$  the *height* and  $m$  the *width*. If the height of the matrix is the same as the width of the matrix then we call the matrix *square*. If the height is larger than the width, we call the matrix *tall*. If the width is larger than the height, we call the matrix *wide*.

## Linear equations

Recall that we are interested in solutions of the linear equations

$$\begin{aligned} y_1 &= A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n, \\ y_2 &= A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n, \\ &\vdots \\ y_n &= A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n. \end{aligned}$$

We have suggestively used the notation  $A_{ij}$  for the coefficients of the equations, so they are the entries of  $A \in \mathbf{R}^{m \times n}$ .

### **A primer on matrix-vector products**

Using the notation  $A \in \mathbf{R}^{m \times n}$  and  $x \in \mathbf{R}^n$  we want a compressed way to write the above system of linear equations. Define the *real matrix-vector product*  $z \in \mathbf{R}^m$  of  $A$  with  $x$  by

$$z_i = \sum_{j=1}^n A_{ij}x_j, \quad i = 1, \dots, m.$$

We denote the matrix vector product  $z$  by  $Ax$ .

### **Notation**

We express the above system of linear equations as

$$y = Ax,$$

where  $y = (y_1, \dots, y_m) \in \mathbf{R}^m$  and  $x = (x_1, \dots, x_n) \in \mathbf{R}^n$ . The compact notation  $y = Ax$  is sometimes called the *matrix form* of the  $m$  linear equations and  $A$  the *coefficient matrix*.

### **Note on terminology**

The word “matrix” is from the Latin “mater,” meaning mother, and has an old sense in English similar “womb.” The matrix is source of many determinants (discussed elsewhere).

