

## WEIGHTED LEAST SQUARES LINEAR REGRESSORS

## Why

What is the best linear regressor if we choose according to a weighted squared loss function.

## Definition

Suppose we have a paired dataset of n records with inputs in  $\mathbb{R}^d$  and outputs in  $\mathbb{R}$ . A weighted least squares linear predictor for nonnegative weights  $w \in \mathbb{R}^n$ ,  $w \geq 0$ , is a linear transformation  $f: \mathbb{R}^d \to \mathbb{R}$  (the field is  $\mathbb{R}$ ) which minimizes

$$\frac{1}{n}\sum_{i=1}^{n}w_{i}(y_{i}-x^{\top}a^{i})^{2}.$$

Some authors refer to this process of selecting a linear predictor as the weighted least-squares problem.

Define  $W \in \mathbf{R}^{n \times n}$  so that  $W_{ii} = w_i$  and  $W_{ij} = 0$  when  $i \neq j$ . So, in particular, W is a diagonal matrix. We want to find x to minimize

$$||W(Ax-y)||$$

## Solution

**Proposition 1.** There exists a unique weighted least squares linear predictor and its parameters are given by

$$\left(A^\top W A^\top\right)^{-1} A^\top W y.$$

