

Matrices and Linear Transformations

1 Why

All linear transformations are equivalent to multiplication by a matrix.

2 Main Result

Proposition 1. Let $(V_1, \mathbf{F}), (V_2, \mathbf{F})$ be two vector spaces and $f: V_1 \to V_2$ be a transformation between them.

If f is linear, then there exists bases for the spaces and a matrix A in \mathbf{F} for which matrix multiplication is equivalent to applying f.

If A is a matrix in F f is linear if and only if there exists a matrix in \mathbf{F} whose coordinates in V_1 to coordinates in V_2 .



