



SET INTERSECTIONS

Why

We can consider intersections of more than two sets.

Definition

Let \mathcal{A} denote a set of sets. In other words, every element of \mathcal{A} is a set. And suppose that \mathcal{A} has at least one set (i.e., $\mathcal{A} \neq \emptyset$). Let C denote a set such that $C \in \mathcal{A}$. Then consider the set,

$$\{x \in C \mid (\forall A)(A \in \mathcal{A} \longrightarrow x \in A)\}.$$

This set exists by the principle of specification (see [Set Specification](#)). Moreover, the set does not depend on which set we picked. So the dependence on C does not matter. It is unique by the axiom of extension (see [Set Equality](#)). This set is called the *intersection* of \mathcal{A} .

Notation

We denote the intersection of \mathcal{A} by $\bigcap \mathcal{A}$.

Equivalence with pair intersections

As desired, the the set denoted by \mathcal{A} is a pair (see [Unordered Pairs](#)) of sets, the pair intersection (see [Pair Intersections](#)) coincides with intersection as we have defined it in this sheet.¹

Proposition 1. $\bigcap\{A, B\} = A \cap B$

¹A full account of the proof will appear in future editions.

