Why

If $\mathbf{R}$ corresponds to a line, and $\mathbf{R}^2$ to a plane, and $\mathbf{R}^3$ to space, does $\mathbf{R}^4$ correspond to anything? What of $\mathbf{R}^5$?

Definition

Let n be a natural number. We call the set $\mathbf{R}^n$ *n-dimensional space*. We call elements of $\mathbf{R}^n$ *points*.

We call the point associated with $x = (x_1, x_2, \ldots, x_n) \in \mathbf{R}^n$ with $x_i = 0$ for $1 \leqq i \leqq n$ the *origin*. When clear from context, we denote the origin by 0. Similarly, we denote the point x with $x_i = 1$ for all $i = 1, \ldots, n$ by 1.

Visualization

We can not visualize n-dimensional space. Thus, our intuition for it comes from real space (see Real Space).

Distance

A natural notion of distance for $\mathbf{R}^n$ is the extension of the Euclidean distance. We define the distance between $(x_1, x_2, \ldots, x_n)$, $(y_1, y_2, \ldots, y_n) \in \mathbf{R}^n$ as

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2}$$

This is sometimes called the *Euclidean distance for n-dimensional space*. Does this have the properties that distance has in the plane and in space? We discussed these properties It does. Denote the function which associates to $x, y \in \mathbf{R}^n$ their distance

$d : \mathsf{R}^n \times \mathsf{R}^n \to \mathsf{R}$. So $d(x, y)$ is the distance between the points corresponding to x and y.

Proposition 1. *d is non-negative, symmetric, and the distance between two points is no larger than the sum of the distances with any third object.*[1]

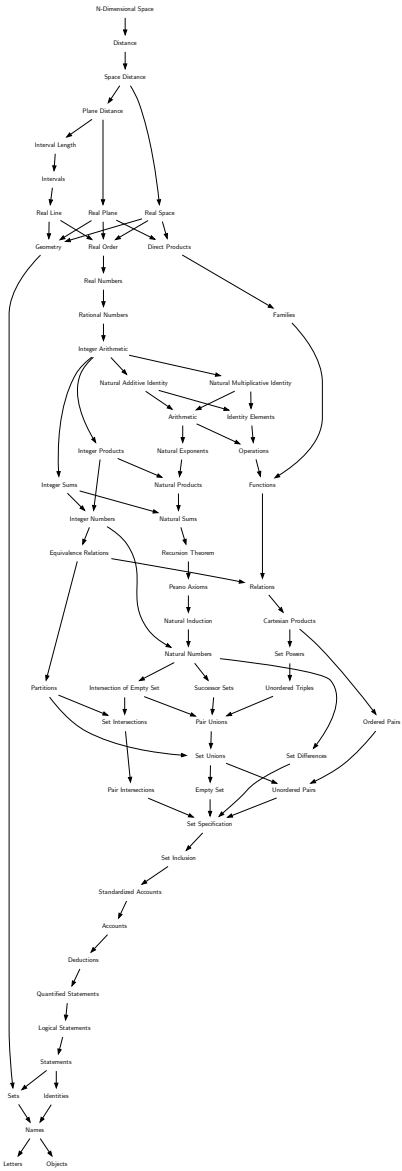
Order

Let $x, y \in \mathsf{R}^n$. If $x_i < y_i$ for all $i = 1, \ldots, n$ then we say x is *less than* y. Likewise, if $x_i \leq y_i$ for all $i = 1, \ldots, n$ then we say $x \leq y$. Likewise for $>$ and $\geq$.

Notation

If $x \in \mathsf{R}^n$ is less than $y \in \mathsf{R}^n$ then we write $x < y$. Similarly for $x \leq y$, $x > y$ and $x \geq y$.

[1] Future editions will include an account.


N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Geometry
Real Order
Direct Products
Real Numbers
Rational Numbers
Families
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Integer Products
Natural Exponents
Operations
Integer Sums
Natural Products
Functions
Integer Numbers
Natural Sums
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects