



## Definition

A *quadratic form* is a multivariate polynomial each term of which has degree two.

## Representation

Let  $f : \mathbf{R}^n \rightarrow \mathbf{R}$  be a quadratic form. There exists a matrix  $A \in \mathbf{R}^{n \times n}$  so that

$$x^\top Ax = \sum_{i,j} A_{ij} x_i x_j.$$

Suppose  $A$  is not symmetric. Then

$$f(x) = f(x)/2 + f(x)/2 = \frac{1}{2}x^\top Ax + \frac{1}{2}x^\top A^\top x = x^\top \left(\frac{1}{2}(A + A^\top)\right)x.$$

Define  $B = 1/2(A^\top + A)$ . We call  $B$  the *symmetric part* of  $A$ . Since every matrix  $A$  has a symmetric part, we can always assume that the matrix for a quadratic form is symmetric. If it is not, replace it with its symmetric part, obtaining the same function.

## Under trace

Observe that  $\text{tr } x^\top Ax = \text{tr } Axx^\top = \text{tr } xx^\top A$ .



