Why

We can interpret the codomain of a random variable as a new sample space, since the underlying probability distribution induces a new probability distribution.

Definition

Let $p : \Omega \to \mathbf{R}$ be a probability distribution and $x : \Omega \to V$ an outcome variable. Define $q : V \to \mathbf{R}$ by

$$q(a) = \mathbf{P}[x = a].$$

Since events $x^{-1}(a)$ for $a \in V$ partition Ω, $\sum_{a \in A} q(a) = 1$. We call q the *induced distribution* (or *induced probability mass function*) of the random variable x. Thus we can think of V as a set of outcomes, which we call the outcomes *induced* by x.

Notation

It is common to denote it by p_x.

If $x : \Omega \to V$ is a random variable and $f : V \to U$, then if we define $y : \Omega \to V$ so that $y \equiv f(x)$, y is a random variable with induced distribution $p_y : \Omega \to \mathbf{R}$ satisfying

$$p_y(b) = \sum_{a \in V \mid y(a) = b} p_x(a).$$

As a matter of practical computation, we can evaluate probabilities having to do with the outcome variable x using p_x instead of p. For example with x as in the example above, $\mathbf{P}(x = 4 \text{ or } x = 5) = p_x(4) + p_x(5)$, rather than $\sum_{\omega \in \Omega \mid x(\omega) = 4 \text{ or } x(\omega) = 5} p(\omega)$.

Induced Probability Distributions

Outcome Variable Events

Event Probabilities

Probability Distributions

Intervals

Real Line

Real Order　　Real Summation

Rational Order　　Real Numbers

Rational Numbers　　Integral Line

Integer Arithmetic

Integer Products　　Natural Additive Identity　　Natural Multiplicative Identity

Arithmetic　　Identity Elements

Natural Powers　　Natural Summation

Comparisons　　Integer Order　　Integer Sums　　Natural Products　　Family Operations

Orders　　Integer Numbers　　Families　　Operations

Converse Relations　　Natural Order　　Equivalence Relations　　Recursion Theorem　　Functions

Relations　　Peano Axioms　　Uncertain Outcomes

Set Products　　Natural Induction

Ordered Pairs　　Set Powers　　Natural Numbers

Unordered Triples

Successor Sets　　Intersection of Empty Set　　Partitions

Pair Unions　　Set Symmetric Differences　　Set Intersections

Set Unions　　Set Complements　　Pair Intersections

Unordered Pairs　　Set Differences　　Empty Set

Set Specification　　Geometry

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities　　Sets

Names

Letters　　Objects

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