Function Composition

1 Why

We want a notion for applying two functions one after the other. We apply a first function then a second function.

2 Definition

Consider two functions for which the codomain of the first function is the domain of the second function.

The *composition* of the second function with the first function is the function which associates each element in the first's domain with the element in the second's codomain that the second function associates with the result of the first function.

The idea is that we take an element in the first domain. We apply the first function to it. We obtain an element in the first's codomain. This result is an element of the second's domain. We apply the second function to this result. We obtain an element in the second's codomain. The composition of the second function with the first is the function so constructed.

2.1 Notation

Let A, B, C be non-empty sets. Let $f : A \to B$ and $g : B \to C$. We denote the composition of g with f by $g \circ f$ read aloud as “g composed with f." To make clear the domain and comdomain, we denote the composition $g \circ f : A \to C$.

In previously introduced notation, $g \circ f$ satisfies

$$(g \circ f)(a) = g(f(a))$$

for all $a \in A$.