

## Numbered Partitions

## Definition

Given a set S, a numbered partition of S (or labeled partition) is a numbering of some partition of a S. In other words, a numbered partition  $(\pi_1, \ldots, \pi_p)$  of S is such that

$$\{\pi_1, \dots, \pi_p\}$$
 partitions  $S$ 

As before,  $\pi_i$  is called a *part* of the partition, for i = 1, ..., n. In this case p is the *size* and  $(|\pi_1|, ..., |\pi_p|)$  is the *shape* of the numbered partition.

When we speak simultaneously of partitions (as defined in Partitions) and numbered partitions, we sometimes call a partition an unnumbered partition (or unlabeled partition, allocation). In this case, the size of a partition P of S is the multiset  $m: \mathbf{N} \to \mathbf{N}$  defined so that m(k) is the number of parts of size k in P.

