



**Definition**

Let  $X$  and  $Y$  be sets. A *hidden memory chain* (*hidden markov chain*, *hidden memory model*, *hidden markov model*,<sup>1</sup> *HMM*) with *hiddens* (or *latents*)  $X$  and *observations*  $Y$  of length  $n$  is a joint distribution  $p : X^n \times Y^n \rightarrow [0, 1]$  satisfying

$$p(x, y) = f(x)g(y_1, x_1) \prod_{i=1}^n h(x_i, x_{i-1})g(y_i, x_i),$$

where  $f : X \rightarrow [0, 1]$  is a distribution, and  $g$  and  $h$  are functions satisfying  $g(\cdot, \xi)$  and  $h(\cdot, \xi)$  are distributions on  $Y$  and  $X$ , respectively, for all  $\xi \in X$ .

**Proposition 1.**  *$p$  so defined is a distribution. The function  $f$  is the distribution  $p_1$ . For all  $i = 1, \dots, n$ ,  $p_{n+1|i} \equiv gg$ . For all  $i = 2, \dots, n$ ,  $p_{i|i-1} \equiv h$ .*<sup>2</sup>

Clearly,  $p_{1,\dots,n}$  is a memory chain (see **Memory Chains**). For this reason, we continue to refer to  $h$  as the *conditional distribution*. We continue to refer to  $f$  as the *initial distribution*. We refer to  $g$  as the *observation distribution*.

The word “hidden” refers to the situation in which we observe outcomes  $y$ , and we hypothesize that they were “generated by” unobserved outcomes  $x$ .

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<sup>1</sup>This term is universal. We avoid it because of the Bourbaki project’s policy on naming. The skeptical reader will note (as in **Memory Chains**) that our term and this term have the same initials.

<sup>2</sup>Future editions will define everything needed in this proposition in the proposition statement, as opposed to saying “ $p$  so defined”



