



# Complex Inner Products

## 1 Why

What is an inner product if we take a vector space over the complex numbers.

## 2 Definition

An inner product over a complex vector space is positive definite, Hermitian, and linear in the first argument.

### 2.1 Alternate Conventions

### 2.2 Notation

Let  $C$  be the set of complex numbers. Let  $(V, C)$  be a complex vector space. Let  $f : V \times V \rightarrow C$  be a function such that

1.  $f(x, x) \geq 0$ ,  $f(x, x) = 0 \Leftrightarrow x = 0$ ;
2.  $f(x, y) = \overline{f(y, x)}$
3.  $f(ax + by, z) = a f(x, z) + b f(y, z)$