



## COMPLEX DISTANCE

### Why

The identification of  $\mathbf{C}$  with a plane leads  $\mathbf{C}$  to naturally inherit  $\mathbf{R}^2$ 's notion of distance.

### Definition

The *absolute value* or *modulus* of  $z = (x, y) \in \mathbf{C}$  is the distance of  $z$  to the origin. If  $z \in \mathbf{C}$ , then the modulus of  $z$  is

$$\sqrt{x^2 + y^2}.$$

In other words, the modulus of  $z$  is the distance (in  $\mathbf{R}^2$  of  $z = (x, y)$  from the origin  $(0, 0)$ .

### Notation

We denote the modulus of  $z$  by  $|z|$ .

### Properties

**Proposition 1** (Triangle Inequality). *For all  $z, w \in \mathbf{C}$ ,*

$$|z + w| \leq |z| + |w|.$$

Also, for all  $z \in \mathbf{C}$ , we have  $|\operatorname{Re}(z)| \leq |z|$  and  $|\operatorname{Im}(z)| \leq |z|$ , and for all  $z, w \in \mathbf{C}$ ,

$$||z| - |w|| \leq |z - w|. <sup>1</sup>$$

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<sup>1</sup>This follows from the triangle inequality. Future editions will include an account.



