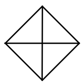


Why

We generalize our notion of *size* to n-dimensional space.

Definition

The *norm* (or *Euclidean norm*) of $x \in \mathbf{R}^n$ is

$$\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

A vector $u \in \mathbf{R}^n$ with $\|u\| = 1$ is called a *unit vector*.

Notation

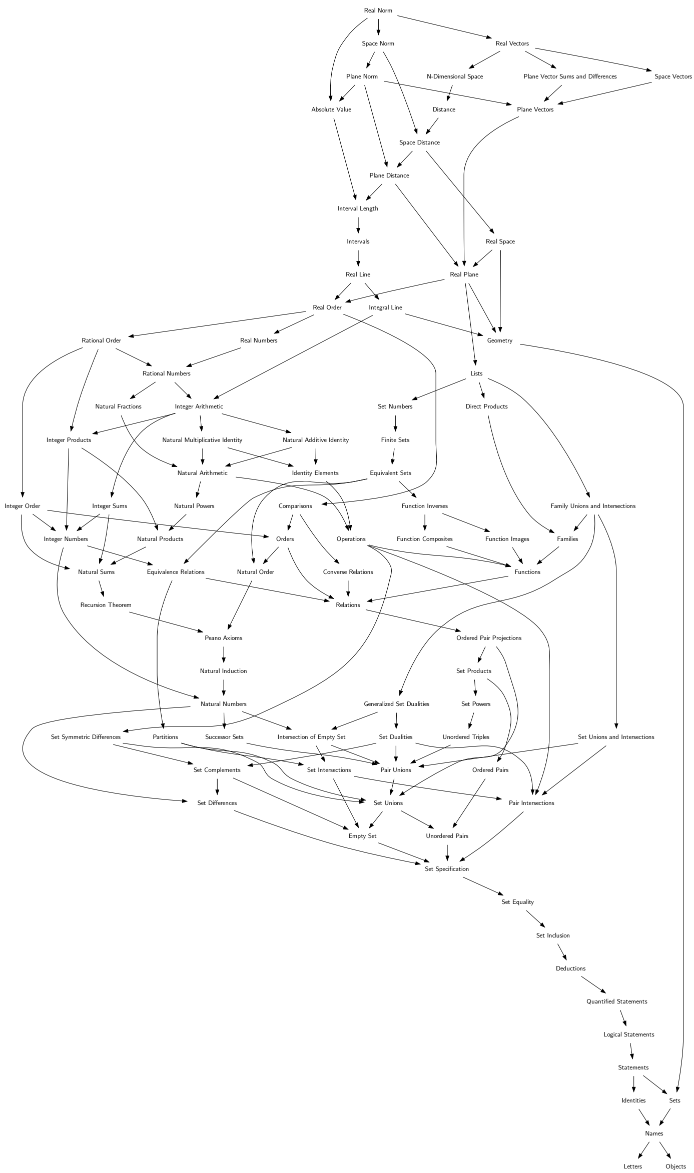
We denote the norm of x by $\|x\|$. In other words, $\|\cdot\| : \mathbf{R}^n \to \mathbf{R}$ is a function from vectors to real numbers. The notation follows the notation of absolute value, the *magnitude* of a real number, and the double verticals remind us that x is a vector. A warning: some authors write $|x|$ for the norm of x when it is understood that $x \in \mathbf{R}^n$.

We understand the norm of x by comparison with the distance function $d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$. On one hand, the norm of x is $d(x, 0)$. So $\|x\|$ measures the length of the vector x from the origin 0. On the other hand, $d(x, y) = \|x - y\|$. So $\|x - y\|$ measures the distance between x and y.

Properties

The norm has several important properties:

1. $\|\alpha x\| = |\alpha|\|x\|$, called *(absolute) homogeneity*,
2. $\|x + y\| \leq \|x\| + \|y\|$, called the *triangle inequality*,
3. $\|x\| \geq 0$, called *non-negativity*, and
4. $\|x\| = 0 \longleftrightarrow x = 0$, called *definiteness*.


Real Norm
Space Norm
Real Vectors
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Absolute Value
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Order
Integral Line
Rational Order
Real Numbers
Geometry
Rational Numbers
Lists
Natural Fractions
Integer Arithmetic
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Arithmetic
Identity Elements
Equivalent Sets
Integer Order
Integer Sums
Natural Powers
Comparisons
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Orders
Operations
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects