Why

We want to visualize relations.

Definition

A *directed graph* (or *digraph*) is a pair (V, E) in which V is a nonempty set and E is a subset of $V \times V$. In other words, E is a relation on V. We call the elements of V *vertices* and the elements of E *edges*.

Example

For example, define V and E by

$$V = \{1, 2, 3, 4\} \quad \text{and} \quad E = \{(1, 2), (1, 3), (2, 4), (3, 4)\}$$

It is worth drawing this graph.

Edge and vertex terminology

Let $(v, w) \in E$. We say that (v, w) is an edge *from v to w*, and that it is an *outgoing edge* of v and an *incoming edge* of w. We call v a *parent* of w and we call w a *child* of v. We say that the edge (v, w) is *incident* to v and w.

The *child set* of a vertex is the set of its child vertices and similarly for the *parent set*; we refer to these sets as the *children* and *parents* of the vertex, respectively. The *indegree* of a vertex is number parents it has and the *outdegree* is the number of children it has.

A vertex is a *source* vertex if it only has outgoing edges (i.e., is the child of no vertex its parent set is empty) and a vertex is a *sink* if it only has incoming edges (i.e., is the parent of no vertex).

A directed graph is *complete* if every vertex is both a child and parent of every other vertex.

Notation

We denote by $\mathrm{pa} : V \to \mathcal{P}(V)$ and $\mathrm{ch} : V \to \mathcal{P}(V)$ the functions associating to each vertex its set of parents and set of children, respectively. As usual, we denote the parents of vertex v by pa_v and the children by ch_v.

Self-loops

If x is a vertex, and (x, x) is an edge, we call such an edge a *self-loop* (or just *loop*). Many authorities exclude self-loops in their definition of directed graphs, but we allow them. To make the distinction, we call a graph with no *loops simple* (a *simple graph*).

Directed Graphs

Set Numbers

Finite Sets

Equivalent Sets

Natural Order

Function Inverses

Function Composites

Function Images

Equivalence Relations

Functions

Peano Axioms

Relations

Natural Induction

Ordered Pair Projections

Set Products

Natural Numbers

Set Powers

Intersection of Empty Set

Successor Sets

Unordered Triples

Pair Unions

Partitions

Ordered Pairs

Set Differences

Set Intersections

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects