



Why

We want to talk about particular attributes of an outcome, instead of the details of the outcomes themselves. These may be useful to specify events.

Definition

An *outcome variable* (or *random variable*) is a function from Ω to V , where V is a set. In this context, V is called the set of *values* of the random variable.

Examples

Sum of two dice. Suppose we model rolling two dice. We are interested in the sum of the pips shown facing up. Suppose we take as the set of outcome $\{1, \dots, 12\}$, whose elements correspond to the sum. We interpret $\{x \in \Omega \mid x \geq 10\}$ as the event that the sum of the two dice is greater than or equal to 10.

Alternatively, we may take the usual set of outcomes $\{1, \dots, 6\}^2$ and define an outcome variable $s : \{1, \dots, 6\}^2 \rightarrow \{1, \dots, 12\}$ by

$$s(d_1, d_2) = d_1 + d_2.$$

We interpret this natural-number-valued outcome variable s as sum of the two dice. The event that the sum of the two dice is greater than or equal to 10 corresponds to the set $\{(d_1, d_2) \in \{1, \dots, 6\}^2 \mid s(d_1, d_2) \geq 10\}$.

As a third alternative, again take the usual set of outcomes $\Omega = \{1, \dots, 6\}^2$. Define the outcome variable $X : \Omega \rightarrow \mathbf{N}$ to be the number of pips showing on the first die, $Y : \Omega \rightarrow \mathbf{N}$ to be the number of pips showing on the second die, and define $Z : \Omega \rightarrow \mathbf{N}$ by

$$Z(\omega) = X(\omega) + Y(\omega) \quad \text{for all } \omega \in \Omega$$

A standard notation for this relation between Z and X, Y by $Z = X + Y$. For example, if $\omega = (2, 5)$, $X(\omega) = 2$, $Y(\omega) = 5$, and $Z(\omega) = 7$.

The preceding three paragraphs highlight that there are several ways of denoting the same situation.

Tossing a fair coin n times. As before, we model n tosses of a fair coin with the sample space $\Omega = \{0, 1\}^n$. Define $X_i : \Omega \rightarrow \{0, 1\}$ by $X_i(\omega) = \omega_i$. We interpret $\{\omega \in \Omega \mid X_i(\omega) = 1\}$ as the event that toss i turns up heads, and likewise $\{\omega \in \Omega \mid X_i(\omega) = 0\}$ as the event that toss i turns up tails, for $i = 1, \dots, n$. We can define the function $X : \Omega \rightarrow 0, 1^n$ by $X(\omega) = \omega$ or by saying $X = (X_1, \dots, X_n)$. Suppose we define $H : \Omega \rightarrow \mathbf{N}$ by

$$H(\omega) = \sum_{i=1}^n X_i(\omega)$$

Or, alternatively, $H = \sum_{i=1}^n X_i$. In this case, we interpret H as the number of heads observed in the n tosses.

Notation

We often denote outcome variables by upper case letters $X : \Omega \rightarrow V$. The event $\{\omega \in \Omega \mid X(\omega) = x\}$, where $x \in V$, is often abbreviated $\{X = x\}$. The probability of this even is often abbreviated $P(X = x)$.

Similarly, for a subset $U \subset V$, the event $\{\omega \in \Omega \mid X(\omega) \in U\}$ is often abbreviated $\{X \in A\}$ and its probability abbreviated $P(X \in A)$.

Suppose $X : \Omega \rightarrow V_1$ and $Y : \Omega \rightarrow V_2$, and $A \subset V_1$ and $B \subset V_2$, then the event

$$\{X \in A\} \cap \{Y \in B\}$$

is often written $\{X \in A, Y \in B\}$.

