

Partial Orders

Why

We want to handle elements of a set in a particular order.

Definition

A relation R in set X is called *antisymmetric* if, for every $x, y \in X$, xRy and yRx implies that x = y. A partial order is a reflexive, transitive, and antisymmetric relation. If $(a, b) \in R$ we say that a precedes b and that b succeeds a.

A partially ordered set (X, R) is a set X and a partial order R on X. The language partial is meant to suggest that two elements need not be comparable. For example, suppose R is $\{(a,a) \mid a \in A\}$; we may justifiably call this no order at all and call A totally unordered, but it is a partial order by our definition.

Notation

We denote a partial order on a set A by \leq . We read \leq aloud as "precedes or equal to" and so read $a \leq b$ aloud as "a precedes or is equal to b." If $a \leq b$ but $a \neq b$, we write $a \prec b$, read aloud as "a precedes b."

