Definition

The *cumulative distribution function* of a real-valued random variable is the function mapping a real number to the measure of the set of outcomes for which the random variable takes value less than or equal to the number. Notation (below) helps.

The range of the cumulative distribution function is the interval $[0, 1]$, since the measure of the base set is one and all measures are non-negative.

We often abbreviate the words "cumulative distribution function" by *c.d.f.* or *cdf.*

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space. Let $f : X \to \mathbf{R}$ be a measurable function (a real-valued random variable). We denote the cumulative distribution function of f by $F_f : \mathbf{R} \to [0, 1]$. We defined it by

$$F_f(t) = \mu(\{x \in X \mid f(x) \leq t\}).$$

Properties

Proposition 1. *The cumulative distribution function of any real-valued random variable*

1. *is non-decreasing,*
2. *is right-continuous,*
3. *tends to one as its argument tends toward infinity, and*
4. *tends to zero as its argument tends towards negative infinity.*

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Successor Sets
Intersection of Empty Set
Set Dualities
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
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Statements
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