



## Why

We have seen several concepts that consist of associating with a pair of sets a third set. For example, set unions and set intersections

## Definition

An *operation* (or *binary operation*, *law of composition*) on a set  $A$  is a function from  $A \times A$  to  $A$ .

Roughly speaking, operations *combine* (or *compose*) elements. We *operate* on ordered pairs.

## Example: set operations

Let  $X$  be a set. Define  $g : \mathcal{P}(X) \times \mathcal{P}(X) \rightarrow \mathcal{P}(X)$  by  $g(A, B) = A \cup B$ . Then  $g$ , the function which associates with two sets their union (see Pair Unions) is an operation on  $\mathcal{P}(X)$ . Likewise, define  $h : \mathcal{P}(X) \times \mathcal{P}(X) \rightarrow \mathcal{P}(X)$  by  $h(A, B) = A \cap B$ .

## Naming their properties

$\cup$  has several nice properties. For one  $A \cup B = B \cup A$  and  $(A \cup B) \cup C = A \cup (B \cup C)$ .

An operation with the first property, that the ordered pair  $(A, B)$  and  $(B, A)$  have the same result is called *commutative*. An operation with the second property, that when given three objects the order in which we operate does not matter is called *associative*.  $\cap$  shares these properties with  $\cup$ .

We call the operation of *forming unions* the function  $(A, B) \mapsto A \cup B$ . We call the operation of *forming intersections* the function  $(A, B) \mapsto A \cap B$ . We call the operation of *forming symmetric differences* the function  $(A, B) \mapsto A + B$ . Since forming unions commutes and is associative and likewise with forming intersections, forming symmetric differences also commutes.

## Algebras

Of course, any operation is defined on some set. For this reason, we define an *algebra* (or *algebraic structure*) as an ordered pair whose first element is a non-empty set and whose second element is an operation on that set. The *ground set* (or *underlying set*, *carrier set*, *domain*) of the algebra is the set on which the operation is defined.

