Why

Do the rational numbers correspond (in the sense of Homomorphisms) to elements of the reals.

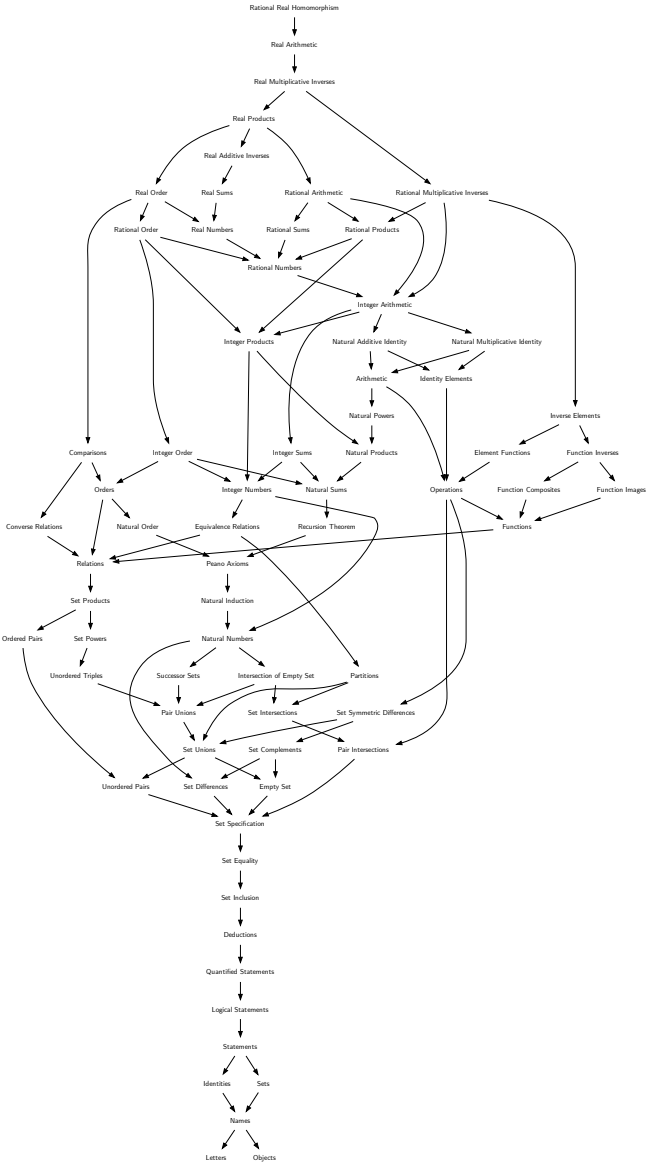
Main result

Indeed, roughly speaking the rationals correspond to elements of the reals which are bounded above by that rational. Denote by $\tilde{\mathbf{R}}$ the set $\{q \in \mathbf{R} \mid \exists s \in \mathbf{Q}, q = \{t \in \mathbf{Q} \mid t < s\}\}$.

Proposition 1. *The fields* $(\tilde{\mathbf{R}}, +_{\mathbf{R}} \mid \tilde{\mathbf{R}}, \cdot_{\mathbf{R}} \mid \tilde{\mathbf{R}})$ *and* $(Q, +_{\mathbf{Q}}, \cdot_Q)$ *are homomorphic.*[1]

Proof. The function is $f : \mathbf{Q} \to \mathbf{R}$ with $f(q) = \{(r \in \mathbf{Q} \mid r < q\}$ □

[1] Indeed, more is true and will be included in future editions. There is an *order perserving* field homomorphism.


Rational Real Homomorphism
Real Arithmetic
Real Multiplicative Inverses
Real Products
Real Additive Inverses
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Rational Order
Real Numbers
Rational Sums
Rational Products
Rational Numbers
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Inverse Elements
Comparisons
Integer Order
Integer Sums
Natural Products
Element Functions
Function Inverses
Orders
Integer Numbers
Natural Sums
Operations
Function Composites
Function Images
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Functions
Relations
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects