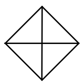


Why

We want to measure the size of vectors in $\mathbf{R}^n$.[1]

Definition

The *norm* (*Euclidean norm*) of $x \in \mathbf{R}^n$ is

$$\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

Notation

We denote the norm of x by $\|x\|$. In other words, $\|\cdot\| : \mathbf{R}^n \to \mathbf{R}$ is a function from vectors to real numbers. The notation follows the notation of absolute value, the *magnitude* of a real number, and the double verticals remind us that x is a vector. A warning: some authors write $|x|$ for the norm of x when it is understood that $x \in \mathbf{R}^n$.

We understand the norm of x by comparison with the distance function $d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$. On one hand, the norm of x is $d(x, 0)$. So $\|x\|$ measures the length of the vector x from the origin 0. On the other hand, $d(x, y) = \|x - y\|$. So $\|x - y\|$ measures the distance between x and y.

Properties

The norm has several important properties

[1] Future editions may expand. They will certainly include images of the plane.

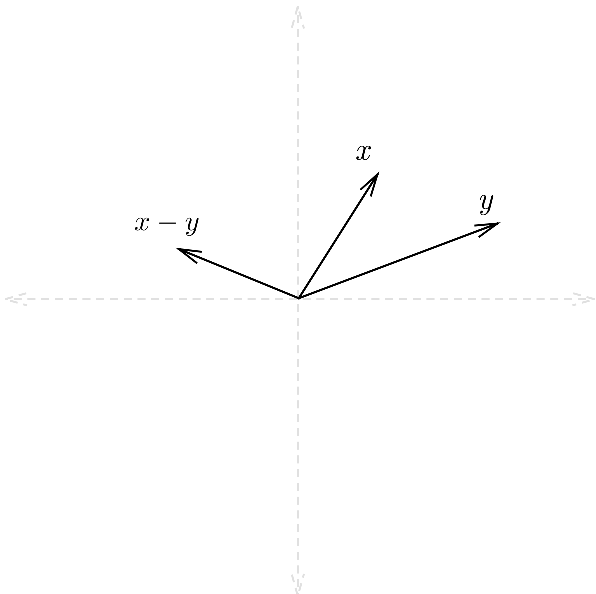
1. $\|\alpha x\| = |\alpha|\|x\|$, called *(absolute) homogeneity*,

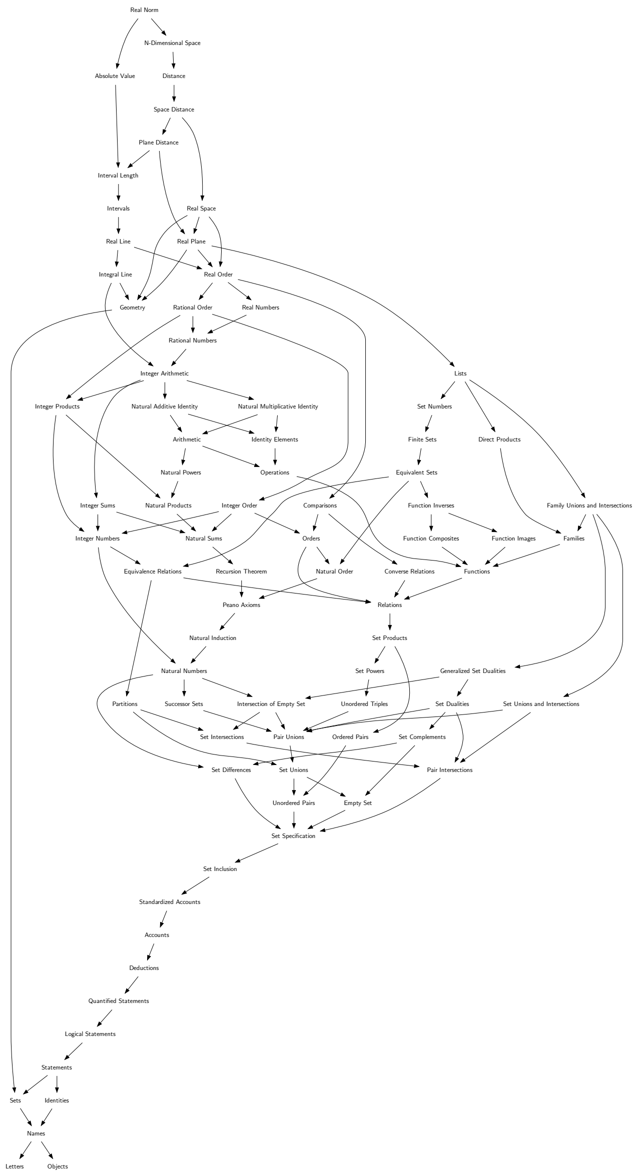
2. $\|x + y\| \leq \|x\| + \|y\|$, called the *triangle inequality*,

3. $\|x\| \geq 0$, called *non-negativity*, and

4. $\|x\| = 0 \longleftrightarrow x = 0$, called *definiteness*.

Visualization




Real Norm
N-Dimensional Space
Absolute Value
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Integer Arithmetic
Lists
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Arithmetic
Identity Elements
Finite Sets
Direct Products
Natural Powers
Operations
Equivalent Sets
Integer Sums
Natural Products
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Sums
Orders
Function Composites
Function Images
Families
Equivalence Relations
Recursion Theorem
Natural Order
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Ordered Pairs
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects