Why

We are frequently interested in finding minimizers of real functions.[1]

Definition

An *optimization problem* is a pair $(\mathcal{X}, f : \mathcal{X} \to \mathbf{R})$ in which $\mathcal{X}$ is a nonempty set called the *constraint set* and f is called the *objective* (or *cost function*).

If $\mathcal{X}$ is finite we call the optimization problem *discrete*. If $\mathcal{X} \subset \mathbf{R}^d$ we call the optimization problem *continuous*.

We refer to all elements of the constraint set as *feasible*. We refer to an element $x \in \mathcal{X}$ of the constraint set as *optimal* if $f(x) = \inf_{z \in \mathcal{X}} f(z)$. We also refer to optimal elements as *solutions* of the optimization problem.

It is common for f and $\mathcal{X}$ to depend on some other, known, given objects. In this case, these objects are often called *parameters* or *problem data*.

Notation

We often write optimization problems as

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \mathcal{X}. \end{aligned}$$

In this case we call x the *decision variable*.

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[1]Future editions will modify and expand.

Extended reals

It is common to let $f : \mathcal{X} \to \bar{\mathsf{R}}$, and allow there to exist $x \in \mathcal{X}$ for which $f(x) = \infty$. This is a trick to embed further constraints in the objective.

Maximization

If we have some function $g : \mathcal{X} \to \bar{\mathsf{R}}$ that we wish to maximize, we can always convert it to an optimization problem by defining $f : \mathcal{X} \to \bar{\mathsf{R}}$ by $f(x) = -g(x)$. In this case g is often called a *reward* (*utility*, *profit*).

Solvers

Let $\mathcal{P} = \{(X_a, f_a : X_a \to \bar{\mathsf{R}})\}_{a \in A}$ be a family of optimization problems. A *solver* (or *solution method*, *solution algorithm*) for $\mathcal{P}$ is a function $S : A \to \mathcal{S}$ such that S_a is a solution of the problem (X_a, f_a).

Loosely speaking, the difficulty of "solving" the optimization problem $(\mathcal{X}, f)$ depends on the properties of $\mathcal{X}$ and f and the problem "size". For example, when $\mathcal{X} \subset \mathsf{R}^d$ the difficulty is related to the "dimension" d of $x \in \mathcal{X}$.

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