



## Bounded Linear Norm

### 1 Why

We can give the set of bounded linear functions between two norm spaces a norm.

### 2 Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

#### 2.1 Notation

Let  $((V_1, F_1), |\cdot|_1)$  and  $((V_2, F_2), |\cdot|_2)$  be two norm spaces. Let  $f : V_1 \rightarrow V_2$  be linear and bounded. The norm of  $f$  is the smallest  $C$  so that

$$|f(v)|_2 \leq C|v|_1.$$

### 3 Equivalent Formulation

**Proposition 1.** *Let  $((V_1, F_1), |\cdot|_1)$  and  $((V_2, F_2), |\cdot|_2)$  be two norm spaces. Let  $f : V_1 \rightarrow V_2$  be bounded and linear. The norm of  $f$  is*

$$\sup_{|x|_1=1} |f(x)|_2.$$