



## POWER SET

### Why

We want to consider the subsets of a given set. Does a set exist which contains all the subsets.

### Definition

We say yes.

**Principle 1** (Powers). *For every set, there exists a set containing all of the subsets.*

We call the existence of this set the *principles of powers* and we call the set the *power set*. As usual, the principle of extension gives uniqueness (see *Set Equality*). The power set of a set includes the set itself and the empty set.

### Notation

We denote the power set of  $A$  by  $A^*$ , read aloud as “powerset of  $A$ .”  $A \in A^*$  and  $\emptyset \in A^*$ . However,  $A \subset A^*$  is false.

### Example

Let  $a, b, c$  be distinct objects. Let  $A = \{a, b, c\}$  and  $B = \{a, b\}$ . Then  $B \subset A$ . In other notation,  $B \in A^*$ . As always,  $\emptyset \in A^*$  and  $A \in A^*$  as well. In this case, we can list the elements (which are sets) of the power set:

$$A^* = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}.$$



