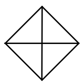


Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

Result

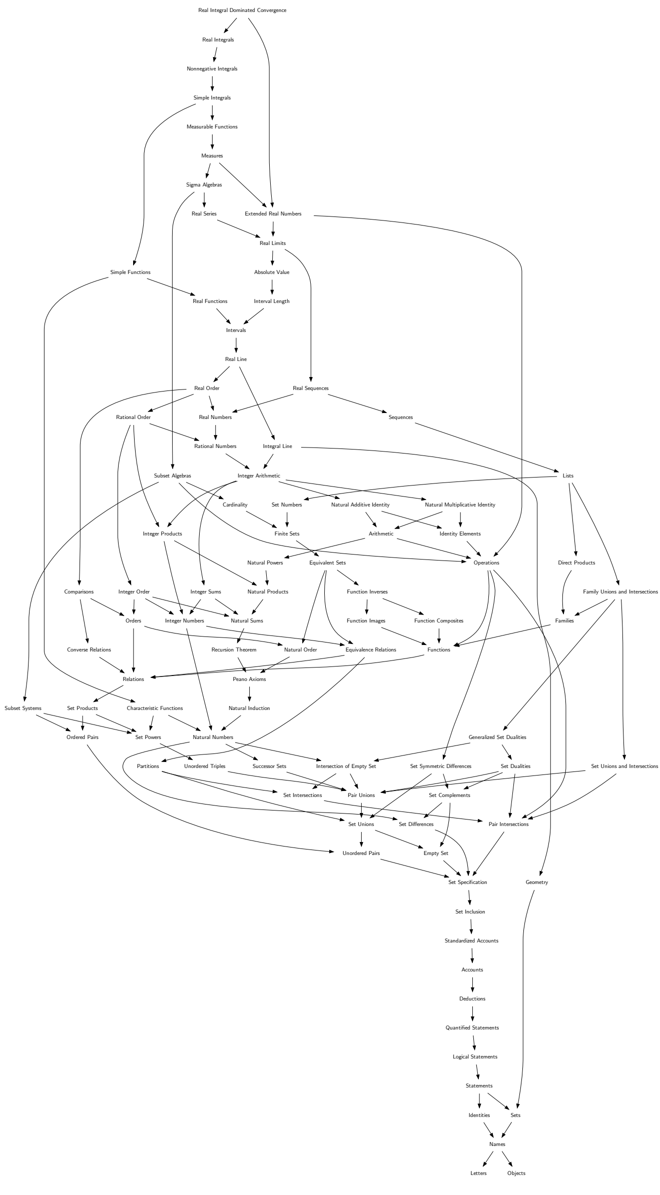
When context is clear, we refer to the following proposition as the *dominated convergence theorem*.

Prop. 1. *The integral of the almost everywhere limit of a sequence of measurable, extended-real-valued, almost-everywhere bounded functions is the limit of the sequence of integrals of the functions.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f : X \to [-\infty, \infty]$ be a $\mathcal{A}$-measurable function. Let $f_n :\to [-\infty, \infty]$ a $\mathcal{A}$-measurable function for every natural number n so that $(f_n)_n$ converges almost everywhere to f. Let $g : X \to [0, \infty]$ be an integrable function which dominates f_n almost everywhere for each n. We want to show that:

$$\int f d\mu = \lim_n \int f_n d\mu.$$

□


Real Integral Dominated Convergence
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Simple Functions
Absolute Value
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Rational Numbers
Integral Line
Subset Algebras
Integer Arithmetic
Lists
Cardinality
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Arithmetic
Identity Elements
Natural Powers
Equivalent Sets
Operations
Direct Products
Comparisons
Integer Order
Integer Sums
Natural Products
Function Inverses
Family Unions and Intersections
Orders
Integer Numbers
Natural Sums
Function Images
Function Composites
Families
Converse Relations
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Relations
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects