Why

We integrate over a product space by integrating one coordinate at a time.

Result

Proposition 1. *Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{A}, \nu)$ be σ-finite measurable spaces. Let $f : X \times Y \to [-\infty, \infty]$ be $\mathcal{A} \times \mathcal{B}$-measurable and $\mu \times \nu$-integrable. Then*

1. *For μ-almost every x in X the section f_x is ν-integrable and for ν-almost every y in Y the section f^y is μ-integrable,*

2. *the functions I_f and J_f defined by*

$$I_f(x) = \begin{cases} \int_Y f_x d\nu & \text{if } f_x \text{ is } \nu\text{-integrable,} \\ 0 & \text{otherwise} \end{cases}$$

   and

$$J_f(y) = \begin{cases} \int_X f^y d\mu & \text{if } f^y \text{ is } \mu\text{-integrable,} \\ 0 & \text{otherwise} \end{cases}$$

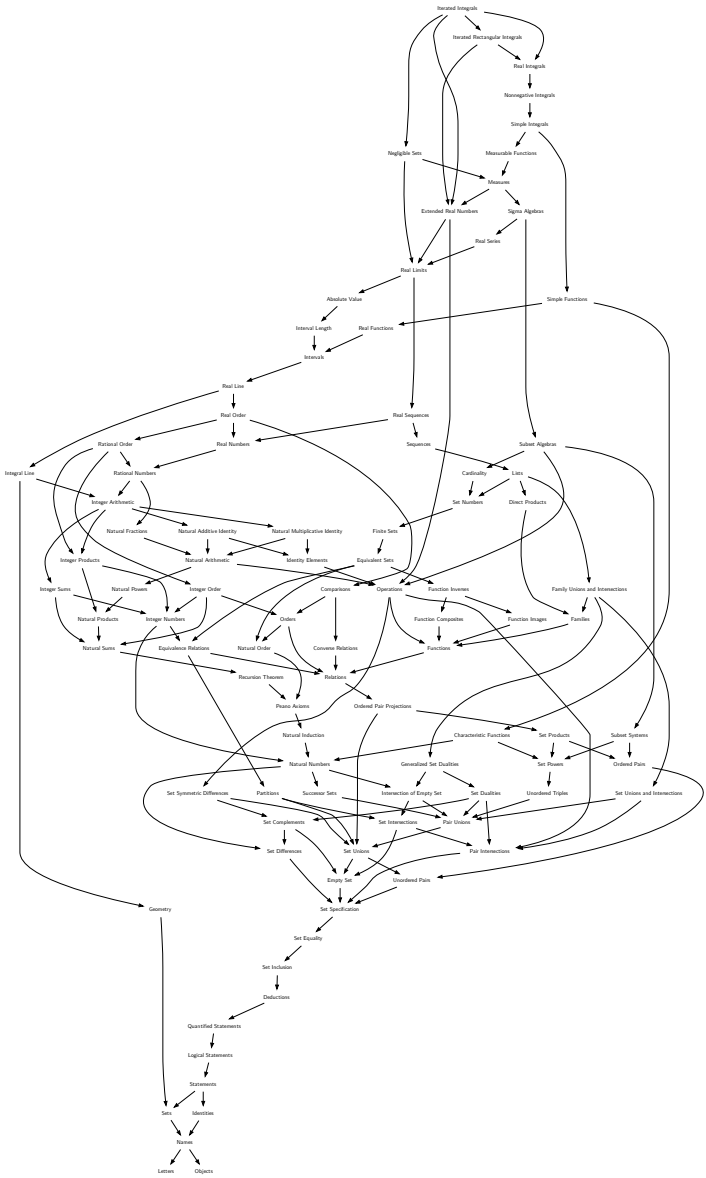
   belong to $\mathcal{L}(X, \mathcal{A}, \mu, \mathbf{R})$ and $\mathcal{L}(Y, \mathcal{A}, \nu, \mathbf{R})$ respectively, and

3. *the relation*

$$\int_{X \times Y} f d(\mu \times \nu) = \int_X I_f d\mu = \int_Y J_f d\nu$$

   holds.

The above is called *Fubini's Theorem*. Next: *Tonelli's theorem*.


Iterated Integrals
Iterated Rectangular Integrals
Real Integrals
Nonnegative Integrals
Simple Integrals
Negligible Sets
Measurable Functions
Measures
Extended Real Numbers
Sigma Algebras
Real Series
Real Limits
Absolute Value
Simple Functions
Interval Length
Real Functions
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Subset Algebras
Integral Line
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Fractions
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Integer Products
Natural Arithmetic
Identity Elements
Equivalent Sets
Integer Sums
Natural Powers
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Natural Order
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects