Equivalence Relations

1 Why

$\boxed{\textit{TODO}}$

2 Definition

Let R a relation on the non-empty set A. If aRa, then we call R **reflexive**. If aRb if and only if bRa then we call R **symmetric**. If aRb and bRc together imply aRc, then we call R **transitive**. If R is reflexive, symmetric, and transitive we call it an **equivalence relation**.

For an element $a \in A$, we call the set of elements in relation R to a the **equivalence class** of a. The key observation, recorded and proven below, is that the equivalence classes partition the set A. A frequent technique is to define an appropriate equivalence relation on a large set A and then to work with the set of equivalence classes of A.

We call the set of equivalence classes the **quotient set** of A under R. An equally good name is the divided set of A under R, but this terminology is not standard. The language in both cases reminds us that $\sim$ partitions the set A into equivalence classes.

2.1 Notation

If R is an equivalence relation on a set A, we use the symbol $\sim$. When alone, $\sim$ is read aloud as "sim," but we still read $a \sim b$ aloud as "a equivalent to b." We denote the quotient set of A under $\sim$ by $A/\sim$, read aloud as "A quotient sim".