



## Nonnegative Integrals

### 1 Why

We want to define area under an extended real function. We use the infinite process to approximate the area under a nonnegative extended real function using simple functions.

### 2 Definition

Consider a measure space.

The *integral* of a measurable nonnegative function is the supremum of integrals over nonnegative simple functions pointwise less than or equal to the function.

#### 2.1 Notation

Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $f : X \rightarrow [0, \infty]$  be measurable. We denote the integral of  $f$  with respect to the measure  $\mu$  by  $\int f d\mu$ . We defined:

$$\int f d\mu = \sup \left\{ \int g d\mu \mid g \in \mathcal{SF}_+(X) \text{ and } g \leq f \right\}.$$