Why

Do the rational numbers correspond (in the sense *Homomorphisms*) to elements of the reals.

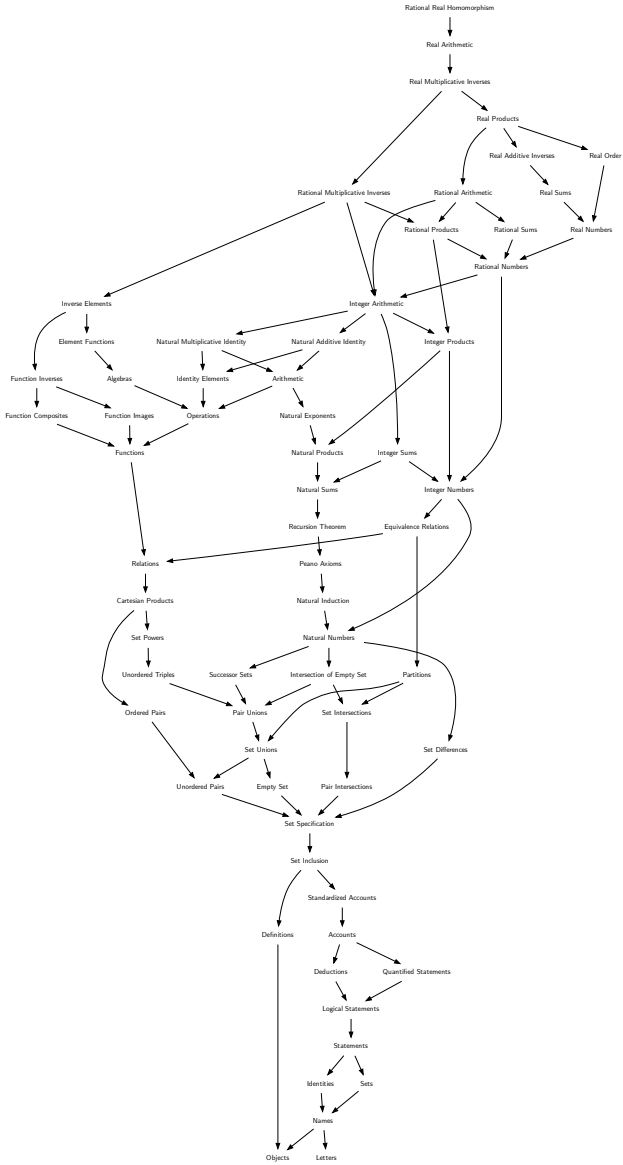
Main Result

Indeed, roughly speaking the rationals correspond to elements of the reals which are bounded above by that rational. Denote by $\tilde{\mathsf{R}}$ the set $\{q \in \mathsf{R} \mid \exists s \in \mathsf{Q}, q = \{t \in \mathsf{Q} \mid t < s\}\}$.

Proposition 1. *The fields* $(\tilde{\mathsf{R}}, +_{\mathsf{R}} \mid \tilde{\mathsf{R}}, \cdot_{\mathsf{R}} \mid \tilde{\mathsf{R}})$ *and* $(Q, +_{\mathsf{Q}}, \cdot_Q)$ *are homomorphic.*[1]

Proof. The function is $f : \mathsf{Q} \to \mathsf{R}$ with $f(q) = \{(r \in \mathsf{Q} \mid r < q\}$ $\square$

[1]Indeed, more is true and will be included in future editions. There is an *order perserving* field homomorphism.


Rational Real Homomorphism
Real Arithmetic
Real Multiplicative Inverses
Real Products
Real Additive Inverses
Real Order
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Rational Products
Rational Sums
Real Numbers
Rational Numbers
Inverse Elements
Integer Arithmetic
Element Functions
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Function Inverses
Algebras
Identity Elements
Arithmetic
Function Composites
Function Images
Operations
Natural Exponents
Functions
Natural Products
Integer Sums
Natural Sums
Integer Numbers
Recursion Theorem
Equivalence Relations
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters