



INTERSECTION OF EMPTY SET

Why

We only define set intersections for nonempty sets of sets. Why?

Discussion

Which objects are specified by the sentence $(\forall x \in \emptyset)(x \in X)$? Well, since no objects fail to satisfy the statement,¹ the sentence specifies all objects. So in other words, the condition we used to define set intersections (**Set Intersections**) specifies the “set of everything”. In order to maintain other more desirable set principles like selection, we have said that such a set does not exist (see **Set Specification**).

If, however, all sets under consideration are subsets of one particular set—denote it E —then we can define intersections as follows. Let \mathcal{C} be a possibly nonempty collection of sets

$$\bigcap \mathcal{C} = \{X \in E \mid (\forall X \in \mathcal{C})(x \in X)\}.$$

This definition agrees with that given in **Set Intersections**. In particular, it is the intersection of the set $\mathcal{C} \cup \{E\}$

Another definition

This begs the following question. Why not define intersections by selecting from the union. Let \mathcal{A} be a possibly nonempty

¹Future editions will offer an account of this.

set of sets. Then define:

$$\bigcap \mathcal{A} = \{x \in \bigcup \mathcal{A} \mid (\forall A \in \mathcal{A})(x \in A)\}.$$

If \mathcal{A} is empty, so is $\bigcup \mathcal{A}$ and then there are no elements in the set to select from so $\bigcap \mathcal{A}$ is empty. This does not agree with the previous definitions for the empty set, but does for all other sets of sets.

For these reasons, the intersection of the empty set is a delicate thing.²

²Future editions will expand on the preference for the former definition.

