Why

What are some simple functions? Here's one answer: those that only involve addition and multiplication.[1]

Definition

A *real polynomial* (or *polynomial*) of degree d is a function $p : \mathbf{R} \to \mathbf{R}$ for which there exists a finite sequence $a = (a_0, a_1, \ldots, a_d)$ so that

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^d.$$

In particular, $q(x) = ax + b$ for $a, b \in \mathbf{R}$ is a polynomial of the first degree and $r(x) = ax^2 + bx + c$ for $a, b, c \in \mathbf{R}$ is a polynomial of the second degree.

In a sense, these are "simple" functions. We require addition (and substraction) and multiplication; but no division.

Properties

Proposition 1. *Let $p : \mathbf{R} \to \mathbf{R}$ be a polynomial of degree d. Then p is continuous.*

Proposition 2. *Let $p : \mathbf{R} \to \mathbf{R}$ be a polynomial of degree d. Then p has derivatives of all orders. Every derivative of p is a polynomial.*

Proposition 3. *Let $p : \mathbf{R} \to \mathbf{R}$ be a polynomial of degree d. The every derivative of order $d + 1$ or greater is the constant 0 function.*

---

[1]Future editions will modify and expand.

Real Polynomials

Real Arithmetic

Real Multiplicative Inverses

Real Products

Real Additive Inverses

Real Order   Real Sums   Rational Arithmetic   Rational Multiplicative Inverses

Rational Order   Real Numbers   Rational Sums   Rational Products

Rational Numbers

Integer Arithmetic

Integer Products   Natural Additive Identity   Natural Multiplicative Identity

Arithmetic   Identity Elements

Natural Powers

Inverse Elements

Comparisons   Integer Order   Integer Sums   Natural Products   Element Functions   Function Inverses

Orders   Integer Numbers   Natural Sums   Operations   Function Composites   Function Images

Converse Relations   Natural Order   Equivalence Relations   Recursion Theorem   Functions

Relations   Peano Axioms

Set Products   Natural Induction

Ordered Pairs   Set Powers   Natural Numbers

Unordered Triples   Successor Sets   Intersection of Empty Set   Partitions

Pair Unions   Set Intersections   Set Symmetric Differences

Set Unions   Set Complements   Pair Intersections

Unordered Pairs   Set Differences   Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects