Definition

A *real linear transformation* is a function $f : \mathbf{R}^n \to \mathbf{R}^m$ satisfying

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \quad \text{for all } x, y \in \mathbf{R}^n \text{ and } \alpha, \beta \in \mathbf{R}$$

Equivalently, f is (a) *homogenous* $f(\alpha x) = \alpha f(x)$ and (b) *additive* $f(x + y) = f(x) + f(y)$.

Real Linear Transformations

Multivariate Vector Linear Functions

Vector-Valued Multivariate Functions

Linear Functions — Real Vectors — Multivariate Functions

Plane Vector Sums and Differences — Space Vectors — N-Dimensional Space

Plane Vector — Distance

Space Distance

Plane Distance

Interval Length

Intervals — Real Space

Real Plane — Real Line

Real Summation — Real Order — Integral Line

Natural Summation — Real Numbers — Rational Order — Geometry

Rational Numbers

Lists — Set Numbers — Natural Fractions — Integer Arithmetic

Direct Products

Finite Sets — Natural Additive Identity — Natural Multiplicative Identity — Integer Products

Equivalent Sets — Natural Arithmetic — Identity Elements

Family Unions and Intersections — Family Operations — Function Inverses — Comparisons — Integer Order — Integer Sums — Natural Powers

Families — Function Images — Function Composites — Operations — Orders — Integer Numbers — Natural Products

Functions — Converse Relations — Natural Order — Equivalence Relations — Natural Sums

Relations — Recursion Theorem

Ordered Pair Projections — Peano Axioms

Set Products — Natural Induction

Set Powers — Generalized Set Dualities — Natural Numbers

Set Unions and Intersections — Set Dualities — Successor Sets — Intersection of Empty Set — Set Partitions — Set Symmetric Differences

Unordered Triples

Ordered Pairs — Pair Unions — Set Complements

Pair Intersections — Set Intersections — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects