



## Why

We want to subtract numbers.<sup>1</sup>

## Definition

Consider the set  $\omega \times \omega$ . This set is the set of ordered pairs of  $\omega$ . In other words, the ordered pairs of natural numbers.

We call two such pairs  $(a, b)$  and  $(c, d)$  of  $\omega \times \omega$  *integer equivalent* if

$$a + d = b + c$$

Briefly, the intuition is that  $(a, b)$  represents  $a$  less  $b$ , or in the usual notation “ $a - b$ ”.<sup>2</sup> So this equivalence relation says these two are the same if  $a - b = c - d$ . Rearranging gives  $a + d = b + c$ .

**Proposition 1.** *Integer equivalence is an equivalence relation.*<sup>3</sup>

The *set of integer numbers* is the set of equivalence classes (see **Equivalence Relations**) under integer equivalence on  $\omega \times \omega$ . We call an element an *integer number* (or *integer*).

## Notation

We denote the set of integers by **Z**. If we denote integer equivalence by  $\sim$  then  $\mathbf{Z} = (\omega \times \omega) / \sim$ .

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<sup>1</sup>Future editions will change this why. In particular, by referencing **Inverse Elements** and the lack thereof in  $\omega$ .

<sup>2</sup>This account will be expanded in future editions.

<sup>3</sup>The proof is straightforward. It will be included in future editions.

