Set Intersections

Why

Surely we can do a similar with pair intersections for several sets.

Definition

Let $\mathcal{A}$ denote a set of sets. In other words, every element of $\mathcal{A}$ is a set. And suppose that $\mathcal{A}$ has at least one set (i.e., $\mathcal{A} \neq \varnothing$). Let C denote a set such that $C \in \mathcal{A}$. Then consider the set,

$$\{x \in C \mid (\forall A)(A \in \mathcal{A} \longrightarrow x \in A)\}.$$

This set exists by the principle of specification (see *Set Specification*). Moreover, the set does not depend on which set we picked. So the dependence on C does not matter. It is unique by the axiom of extension (see *Set Equality*). This set is called the *intersection* of $\mathcal{A}$.

ßNotation

We denote the intersection of $\mathcal{A}$ by $\bigcap \mathcal{A}$.

ßEquivalence with pair intersections

As desired, the if the set $\mathcal{A}$ is a pair of sets, the pair intersection coincides with the intersection.[1]

Proposition 1. $\bigcap\{A, B\} = A \cap B$

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[1] A full account of the proof will appear in future editions.

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Pair Intersections

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Set Specification

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