Why

What does it mean for two random variables to be independent? What are the events associated with a random variable? TODO

Definition

Defining Result

PROPOSITION **1.** *The set of inverse images the distinguished sets of a measurable space under a function from a set to that space is a sigma algebra.*

If the first set and the function are measurable, the sigma algebra is a sub sigma algebra of the domain sigma algebra.

Proof. Let $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ be measurable spaces. Let $f : X \to Y$ be a measurable function. Define

$$\mathcal{C} := \{f^{-1}(B) \mid B \in \mathcal{B}\}.$$

First, since $f^{-1}(Y) = X$, $X \in \mathcal{C}$.

Second, let $C \in \mathcal{C}$. Then there is B such that $C = f^{-1}(B)$. Then $X - C = X - f^{-1}(B) = f^{-1}(Y - B)$. Since $\mathcal{B}$ is a sigma algebra, $B \in \mathcal{B}, Y - B \in \mathcal{B}$ and so $X - C \in \mathcal{C}$.

Finally, let $(C_n)_n \subset \mathcal{C}$. Then for every n there exists a $B_n \in \mathcal{B}$ so that $C_n = f^{-1}(B_n)$. Then:

$$\cup_n C_n = \cup_n f^{-1}(B_n) = f^{-1}(\cup B_n).$$

Since $\mathcal{B}$ is a sigma algebra, $\cup_n B_n \in \mathcal{B}$ and so $\cup_n C_n \in \mathcal{C}$.

Since f is measurable, $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}$, and so $\mathcal{C} \subset \mathcal{A}$. $\qquad\square$

The sigma algebra *generated by a random variable* is the sigma algebra consisting of the inverse images of every measurable set of the codomain.

The sigma algebra generated by a family of random variables is the sigma algebra generated by the union of the sigma algebras generated individually by each of the random variables.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f : X \to Y$ be a random variable. Denote by $\sigma(f)$ the sigma algebra generated by f.

Results

PROPOSITION **2.** *The sigma algebra generated by a family of random variables is the smallest sigma algebra for with respect to which each random variable is measurably.*

Random Variable Sigma Algebra

Random Variables

Probability Measures   Measurable Functions

Measures

Extended Real Numbers   Event Probabilities   Topological Sigma Algebra

Real Limits   Probability Distributions   Topological Spaces

Real Sequences   Intervals   Real Summation   Probability Events   Metrics

Sequences   Real Numbers   Natural Summation   Probability Outcomes   Subset Algebras   Distance

Rational Numbers   Family Operations   Direct Products   Probability   Cardinality   Set Operations   Length Common Notions

Integer Numbers   Natural Families   Injective Functions   Algebra   Natural Order   Common Sense

Zero   Subset Systems   Function Composites   Families   Operations   Natural Sums

Equation Solutions   Power Set   Functions   Natural Numbers   Set Symmetric Differences

Relations   Set Intersections   Set Unions

Ordered Pairs   Set Complements

Equations   Unordered Pairs   Empty Set

Set Specification

Set Inclusion   Sentences

Set Equality

Identity   Sets

Objects

4