



Why

We want to define area under a real function. We start with defining functions for which this notion is obvious.

Definition

A *simple function* (or *step function*) is a real-valued function whose range is a finite set. We can write simple function as the sum of the characteristic functions for the inverse image elements.

We can partition the range of the function into a finite family of one-element sets. Then the family whose members are the inverse images of these sets partitions the domain. We call this the *simple partition* of the function.

Notation

We denote the set of simple functions on A by $\text{SF}(A)$. We denote subset of non-negative simple real functions with domain A by $\text{SF}_+(A)$.

Let $f \in \text{SF}(A)$. Let $\{a_1, \dots, a_n\} = f(A)$. Define $A_i = f^{-1}(\{a_i\})$. Then $f = \sum_{i=1}^n a_i \chi_{A_i}$.

