Why

What is the best linear predictor if we choose according to a particular norm.

Definition

Suppose we have a paired dataset of n records with inputs in $\mathbf{R}^d$ and outputs in $\mathbf{R}$. A *norm weighted least squares linear predictor* for a norm $g : \mathbf{R}^n \to \mathbf{R}$ is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$g(y - Ax).$$

Weight matrix

Let $\|\cdot\|_W$ be the weighted norm for some positive semidefinite weight matrix W. We want to find x to minimize

$$\|y - AX\|_W.$$

This problem is referred to by many authors as *weighted least squares* or the *weighted least squares problem*.

Diagonal weight matrix

A special case of norm weighted least squares with a weighted norm is the usual weighted least squares problem (see Weighted Least Squares Linear Predictors). Consider weighted least squares with weights $w \in \mathbf{R}^n$, $w \geq 0$. Define $W \in \mathbf{R}^{n \times n}$ so that

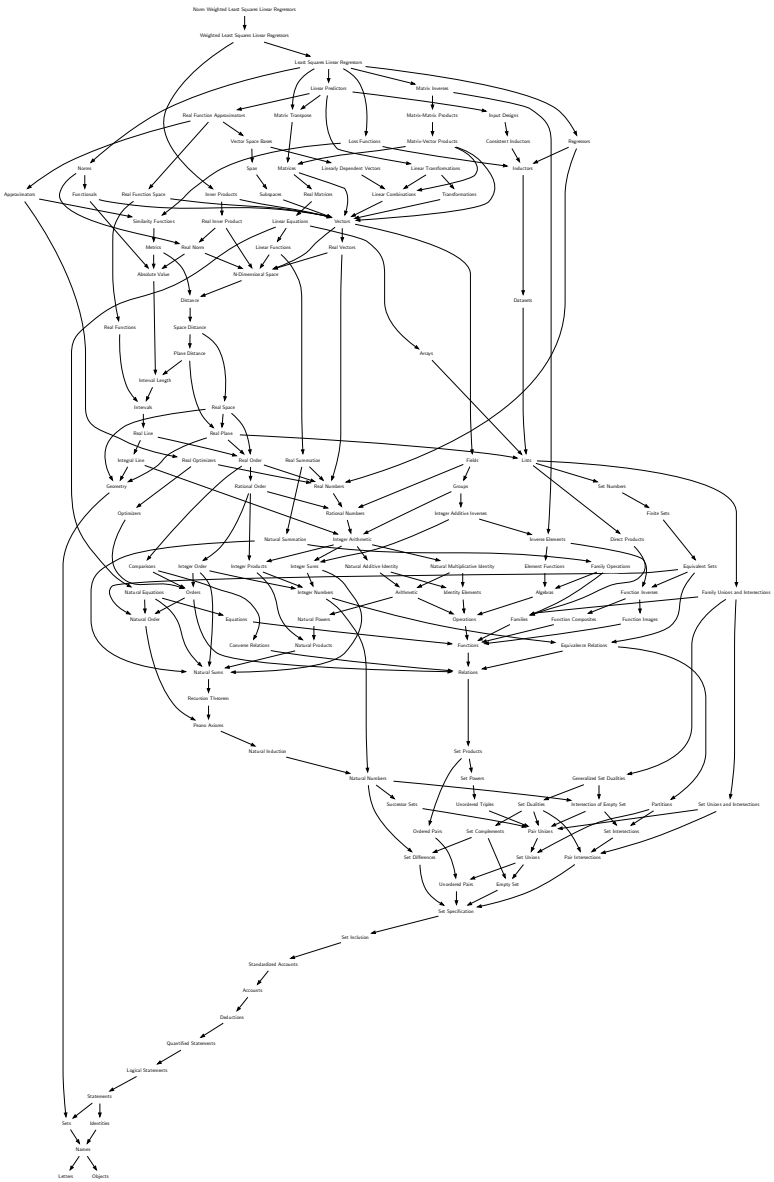
$W_{ii} = w_i$ and $W_{ij} = 0$ when $i \neq j$. So, in particular, W is a diagonal matrix and

$$\|y - Ax\|_W = \sum_{i=1}^{n} w_i (y_i - x^\top a_i)^2.$$

Solution

Proposition 1. *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$\left(A^\top W A\right)^{-1} A^\top W y.$$


Non-Weighted Least Squares Linear Regression
Weighted Least Squares Linear Regression
Least Squares Linear Regression
Linear Prediction
Matrix Inverses
Real Function Approximation
Matrix Transpose
Matrix-Matrix Products
Input Designs
Vector Space Bases
Loss Functions
Matrix-Vector Products
Consistent Inductors
Regressors
Norms
Span
Matrices
Linearly Dependent Vectors
Linear Transformations
Inductors
Approximations
Functionals
Real Function Spaces
Inner Products
Subspaces
Real Matrices
Linear Combinations
Transformations
Similarity Functions
Real Inner Product
Linear Equations
Vectors
Metrics
Real Norms
Linear Functions
Real Vectors
Absolute Value
N-Dimensional Space
Datasets
Distance
Real Functions
Space Distance
Arrays
Plane Distance
Interval Length
Intervals
Dual Space
Real Line
Real Plane
Integral Line
Real Optimizers
Real Order
Real Summation
Fields
Lists
Geometry
Rational Order
Real Numbers
Groups
Set Numbers
Optimizers
Rational Numbers
Integer Additive Inverses
Finite Sets
Natural Summation
Integer Arithmetic
Inverse Elements
Direct Products
Comparisons
Integer Order
Integer Products
Integer Sums
Natural Additive Identity
Natural Multiplication Identity
Element Functions
Family Operations
Equivalent Sets
Natural Equations
Orders
Integer Numbers
Arithmetic
Identity Elements
Algebras
Function Inverses
Family Unions and Intersections
Natural Order
Equations
Natural Powers
Operations
Families
Function Compositions
Function Images
Converse Relations
Natural Products
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Duality
Successor Sets
Unordered Triples
Set Duality
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Definitions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Atoms
Letters
Objects