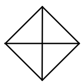


Why

We can interpret the codomain of a random variable as a new sample space, since the underlying probability distribution induces a new probability distribution.

Definition

Let $p : \Omega \to \mathsf{R}$ be a probability distribution and $x : \Omega \to V$ an outcome variable. Define $q : V \to \mathsf{R}$ by

$$q(a) = \mathsf{P}[x = a].$$

Since events $x^{-1}(a)$ for $a \in V$ partition Ω, $\sum_{a \in A} q(a) = 1$. We call q the *induced distribution* (or *induced probability mass function*) of the random variable x. Thus we can think of V as a set of outcomes, which we call the outcomes *induced* by x.

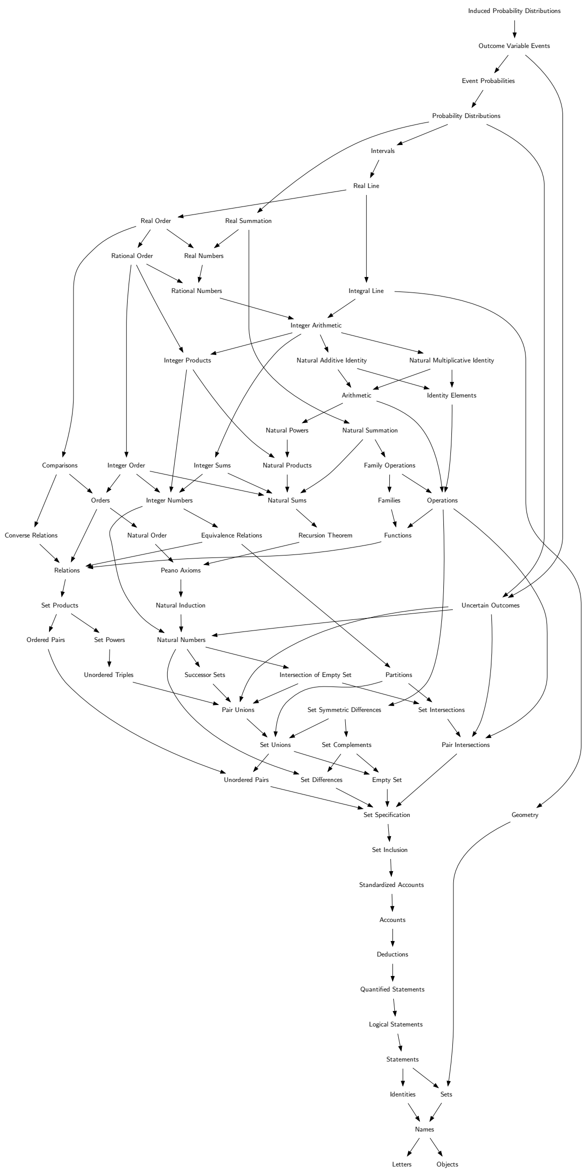
Notation

It is common to denote it by p_x.

If $x : \Omega \to V$ is a random variable and $f : V \to U$, then if we define $y : \Omega \to V$ so that $y \equiv f(x)$, y is a random variable with induced distribution $p_y : \Omega \to \mathsf{R}$ satisfying

$$p_y(b) = \textstyle\sum_{a \in V | y(a) = b} p_x(a).$$

As a matter of practical computation, we can evaluate probabilities having to do with the outcome variable x using p_x instead of p. For example with x as in the example above, $\mathsf{P}(x = 4 \text{ or } x = 5) = p_x(4) + p_x(5)$, rather than $\sum_{\omega \in \Omega | x(\omega) = 4 \text{ or } x(\omega) = 5} p(\omega)$.


Induced Probability Distributions
Outcome Variable Events
Event Probabilities
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Rational Order
Real Numbers
Rational Numbers
Integral Line
Integer Arithmetic
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Arithmetic
Identity Elements
Natural Powers
Natural Summation
Comparisons
Integer Order
Integer Sums
Natural Products
Family Operations
Orders
Integer Numbers
Natural Sums
Families
Operations
Converse Relations
Natural Order
Equivalence Relations
Recursion Theorem
Functions
Relations
Peano Axioms
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Pair Unions
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects