



## Expectation Deviation Upper Bound

### 1 Why

We bound the probability that a random variance deviates from its mean using its variance.

### 2 Result

**Proposition 1.** *Let  $f$  be a square-integrable real-valued random variable on the probability space  $(X, \mathcal{A}, \mu)$ . Then for  $t > 0$ ,*

$$\mu(|f - \mathbf{E}(f)| \geq t) \leq \frac{\mathbf{var} f}{t^2}.$$

*Proof.* The set  $|f - \mathbf{E}(f)| \geq t$  is  $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$ . This set is  $\{x \in X \mid (f(x) - \mathbf{E}(f))^2 \geq t^2\}$ . By using the non-negative inequality

$$\mu(\{x \in X \mid (f(x) - \mathbf{E}(f))^2 \geq t^2\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))^2}{t^2}.$$

We recognize the numerator of the right hand side as the variance. □

The above is also called *Chebychev's Inequality*