



SET INCLUSION

Why

We want language for all of the elements of a first set being the elements of a second set.

Definition

Denote a set by A and a set by B .

Definition 1. Subsets

If every element of the set denoted by A is an element of the set denoted by B .

We say that the set denoted by A is *included* in the set denoted by B . We say that the set denoted by B is a *superset* of the set denoted by A or that the set denoted by B *includes* the set denoted by A .

Every set is included in and includes itself.

Account 1.

1	name	A	
2	have	$(\forall x)(x \in A \longrightarrow x \in B)$	
3	thus	$A \subset B$	by Def 1

Notation

Let A denote a set and B denote a set. We denote that A is included in B by $A \subset B$. In other words, $A \subset B$ means

$(\forall x)((x \in A) \longrightarrow (x \in B))$. We read the notation $A \subset B$ aloud as “A is included in B” or “A subset B”. Or we write $B \supset A$, and read it aloud “B includes A” or “B superset A”. $B \supset A$ also means $(\forall x)((x \in A) \longrightarrow (x \in B))$.

Properties

Given a set A , $A \subset A$. Like equality, we say that inclusion is *reflexive*. Given sets A and B , if $A \subset B$ and $B \subset C$ then $A \subset C$. Like equality, we say that inclusion is *transitive*. If $A \subset B$ and $B \subset A$, then $A = B$ (by the axiom of extension). Unlike equality, which is symmetric, we say that inclusion is *antisymmetric*.

Comparison with belonging

Given a set A inclusion is reflexive. $A \subset A$ is always true. $A \in A$ may be true. Inclusion is transitive, whereas belonging is not.

