



## Rooted Trees

### 1 Why

We want to talk about orienting the edges of a tree away from a given vertex.

### 2 Definition

A *rooted tree* is an ordered pair whose first object is a tree and whose second object is a vertex of the tree. We call the vertex the *root*.

We associate with a rooted tree the directed graph constructed by orienting all edges away from the root. For a particular rooted tree, which is to say a particular choice of root, we obtain a directed graph. There are many choices of root, and so naturally we wonder if each choice of root corresponds to a different directed graph. Since trees are connected, each choice of root does lead to a different graph. If they were disconnected, some of the graphs may turn out to be the same. So there is a one-to-one correspondence between rooted trees and their associated directed graphs, and so we could have defined a rooted tree as a directed graph with particular properties.

#### 2.1 Notation

Let  $(V, T)$  be a tree. Let  $V$  be a non-empty set and  $E \subset V \times V$ . We denote the tree  $T = V$  rooted at vertex  $i$  by  $(T, i)$ .

### 3 Properties

**Proposition 1.** *Let  $(T, i)$  be a rooted tree. In the directed graph corresponding to this rooted tree every vertex has one parent.*

We denote the parent of vertex  $i$  by  $\mathbf{pa}_i$ .