Chordal Graphs

1 Why

A *chord* for a path in an undirected graph is an edge between two non-consecutive vertices of the path; similarly for a cycle. We think of a chord as a one-edge shortcut between two vertices of a path. If a path has a chord, it can be reduced to a shorter path. So a shortest path between two vertices is chordless. The converse, however, is not true: a chordless path is not necessarily a shortest path.

An undirected graph is *chordal* if every cycle of length greather than three has a chord. Using this property, to every cycle in a chordal graph there corresponds at least one cycle of length three. Chordal graphs are also called *rigid-circuit graphs*, *triangulated graphs*, *perfect elimination graphs*, *decomposable graphs*, and *acyclic graphs*. But we will only ever call them chordal graphs.

A *cactus graph* is an undirected graph with no cycles of length greater than three. Both trees and forests are cactus graphs. All cactus graphs are trivially chordal.

1.1 Notation

Let $G = (V, E)$ be an undirected graph. A chord in a path $(v_0, v_1, \ldots, v_k)$ of G is an edge $\{v_i, v_j\}$ with $|j-i| > 1$. A chord in a cycle $(v_0, v_1, \ldots, v_{k-1}, v_0)$ of G is an edge $\{v_i, v_j\}$ with $(j - i) \mod k > 1$.

2 Examples

Trees and forests (and any cactus graph) are trivially chordal. Additionally:

Proposition 1. *Complete graphs are chordal.*

3 Properties

Proposition 2. *Any subgraph of a chordal graph is chordal.*

Proof. A consequence of the definition, since every cycle of the subgraph is a cycle of the original graph, and so also with the chords. □