



## Why

We can add and scale matrices, so the  $m \times n$  matrices are a vector space over  $\mathbf{R}$ .

## Definition

The *matrix sum* of two matrices  $A, B \in \mathbf{R}^{m \times n}$  is the matrix  $C \in \mathbf{R}^{m \times n}$  defined by  $C_{ij} = A_{ij} + B_{ij}$ . In other words, the matrix  $C$  is given by summing the entries of  $A$  and  $B$  “entry-wise”. We denote the matrix sum by  $A + B$ .

For  $\alpha \in \mathbf{R}$ , the  $\alpha$ -*scaling* of  $A \in \mathbf{R}^{m \times n}$  is the matrix  $C \in \mathbf{R}^{m \times n}$  defined by  $C_{ij} = \alpha A_{ij}$ . In other words, the matrix  $C$  is given by scaling the entries of  $A$  “entry-wise”. We denote the  $\alpha$ -scaled version of  $A$  by  $\alpha A$ . These two definitions are justified by the following.

The *matrix space* (or *matrix vector space*) is the vector space  $\mathbf{R}^{m \times n}$  in which addition is given by the matrix sums and scalar multiplication by entry-wise scaling. Sometimes we reference explicitly the  $m \times n$  *matrix space*.

## Subspaces of symmetric matrices

Consider the space of square matrices  $\mathbf{R}^{n \times n}$ . It is obvious that the subset of symmetric matrices is a subspace. Adding two symmetric matrices gives a symmetric matrix. Scaling a symmetric matrix by a real number gives a symmetric matrix. The matrix all of whose entries are 0 is symmetric.



