Why

We can give the set of bounded linear functions between two norm spaces a norm.

Definition

The *norm* of a bounded linear function is the smallest real number by which we can bound the result on a vector times the norm of that vector.

Notation

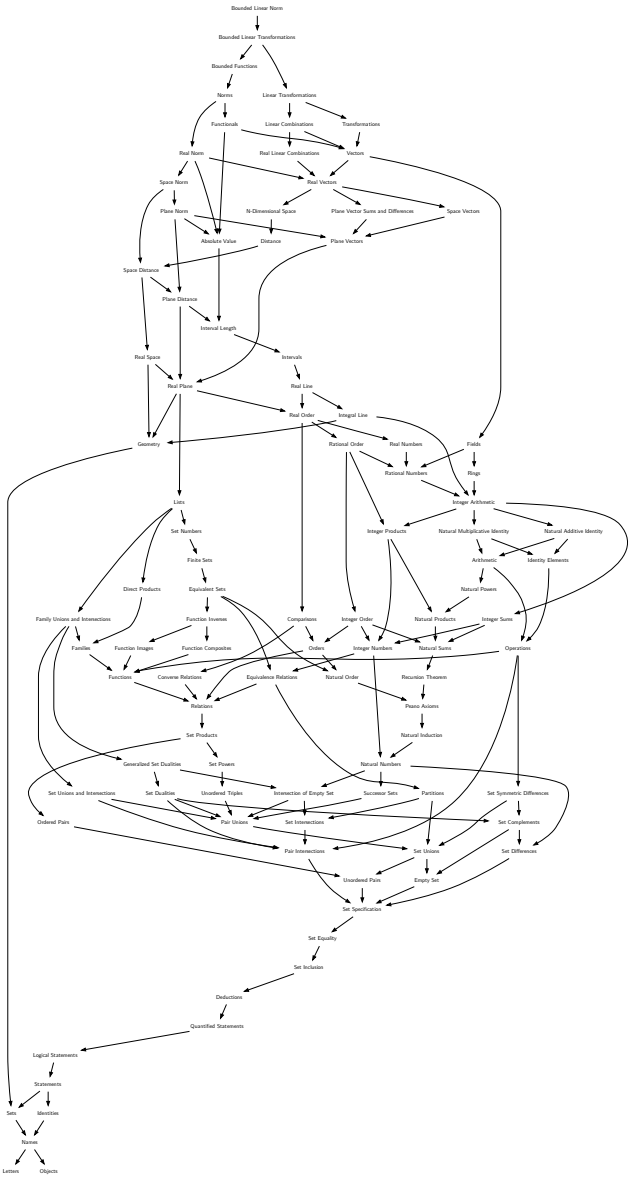
Let $((V_1, F_1), \|\cdot\|_1)$ and $((V_2, F_2), \|\cdot\|_2)$ be two norm spaces. Let $f : V_1 \to V_2$ be linear and bounded. The norm of f is the smallest C so that

$$\|f(v)\|_2 \leq C\|v\|_1.$$

Equivalent formulation

Proposition 1. *Let* $((V_1, F_1), \|\cdot\|_1)$ *and* $((V_2, F_2), \|\cdot\|_2)$ *be two norm spaces. Let* $f : V_1 \to V_2$ *bounded and linear. The norm of* f *is*

$$\sup_{\|x\|_1=1} \|f(x)\|_2.$$


Bounded Linear Norm
Bounded Linear Transformations
Bounded Functions
Norms
Linear Transformations
Functionals
Linear Combinations
Transformations
Real Norm
Real Linear Combinations
Vectors
Space Norm
Real Vectors
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Absolute Value
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Fields
Rational Numbers
Rings
Integer Arithmetic
Lists
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Arithmetic
Identity Elements
Direct Products
Equivalent Sets
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Natural Products
Integer Sums
Families
Function Images
Function Composites
Orders
Integer Numbers
Natural Sums
Operations
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Duality
Set Power
Natural Numbers
Set Unions and Intersections
Set Duality
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects