



# Relation Properties

## 1 Why

Often relations are defined over a single set. These have a few simple properties.

## 2 Definitions

- A relation is *reflexive* if every element is related to itself.
- A relation is *symmetric* if two objects are related regardless of their order.
- A relation is *antisymmetric* if two different objects are related only in one order, and never both.
- A relation is *transitive* if a first element is related to a second element and the second element is related to the third element, then the first and third element are related

### 2.1 Notation

Let  $R$  be a relation on a non-empty set  $A$ .  $R$  is reflexive if

$$(a, a) \in R$$

for all  $a \in A$ .  $R$  is transitive if

$$(a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R$$

for all  $a, b, c \in A$ .  $R$  is symmetric if

$$(a, b) \in R \implies (b, a) \in R$$

for all  $a, b \in A$ .  $R$  is anti-symmetric if

$$(a, b) \in R \implies (b, a) \notin R$$

for all  $a, b \in A$ .