Why

We want fractions.[1]

Rational equivalence

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We say that the elements (a, b) and (c, d) of this set are *rational equivalent* if $ad = bc$. Briefly, the intuition is that (a, b) represents a over b In the usual notation, (a, b) represents "a/b". So this equivalence relation says these two are the same if $a/b = c/d$ or else $ad = bc$.

Proposition 1. *Rational equivalence is an equivalence relation on* $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$.[2]

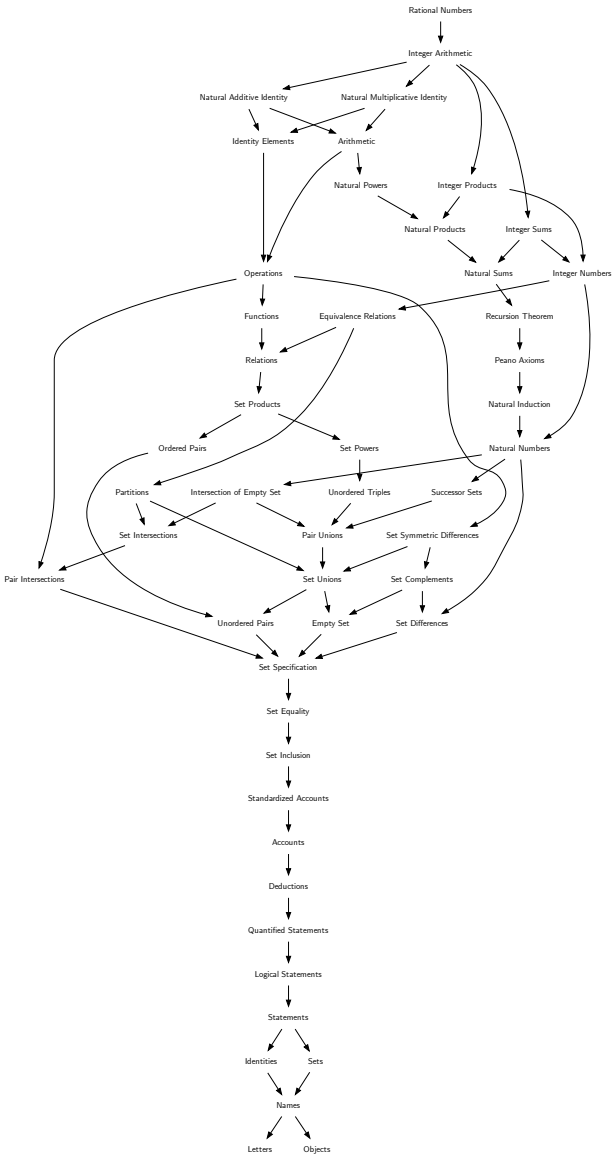
Definition

The *set of rational numbers* is the set of equivalence classes (see Equivalence Classes) of $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ under ratioanl equivalence. We call an element of the set of rational numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

Notation

We denote the set of rationals by $\mathbf{Q}$.[3] If we denote rational equivalence by $\sim$ then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$.

[1] This why will be expanded in future editions.
[2] Future editions will include an account.
[3] From what we can tell, $\mathbf{Q}$ is a mnemonic for "quantity," from the latin "quantitas."


Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Identity Elements
Arithmetic
Natural Powers
Integer Products
Natural Products
Integer Sums
Natural Sums
Integer Numbers
Operations
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Successor Sets
Set Intersections
Pair Unions
Set Symmetric Differences
Pair Intersections
Set Unions
Set Complements
Unordered Pairs
Empty Set
Set Differences
Set Specification
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