



# Countable Probability Distributions

## 1 Why

We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

## 2 Why

Consider a set  $A$ . If  $A$  has  $n$  elements, then a probability distribution on  $A$  is  $p : A \rightarrow \mathbf{R}$  where  $p(a) = 1/n$ . There is a natural candidate.

What if  $A$  is the set of natural numbers  $\mathbf{N}$ . The principle difficulty is that not all sequences of real numbers  $a : \mathbf{N} \rightarrow \mathbf{R}$  are summable.

A *(countable) probability distribution* on  $\mathbf{N}$  is  $p : \mathbf{N} \rightarrow \mathbf{R}$  where  $p \geq 0$  and

$$\sum_{n=1}^{\infty} p(n) = 1.$$