



## Why

We want to talk rooting a tree at a given vertex.<sup>1</sup>

## Definition

A *rooted tree* is an ordered pair  $((V, T), r)$  where  $(V, T)$  is a tree and  $r \in V$  is a distinguished vertex which we call the *root*. We visualize rooted trees with the root at the top (see Figure 1).

## Parents and Children

Suppose  $w$  is the first vertex on the path from the root to a non-root vertex  $v$ . Since there is only one such path,  $w$  is unique and we call it *the parent* of  $v$ . Conversely, we call  $v$  a *child* of  $w$ . We denote the set of children of  $v$  by  $\mathbf{ch}(v)$ . A vertex may have no children or it may have many children. If it has no children we call it a *leaf*.

We define the *parent function*  $\mathbf{pa} : V \rightarrow V$  with the convention that the *parent of the root* is the root. The *parent of degree  $k$*  where  $k > 0$  is  $\mathbf{pa}^k(x)$  where  $\mathbf{pa}^k$  is the composite of  $\mathbf{pa}$  with itself  $k$  times. So, in particular,  $\mathbf{pa}^{k+1}(v) = \mathbf{pa}(\mathbf{pa}^k(v))$ . We define the *parent of degree 0* of  $v$  to be  $v$ , and denote it by  $\mathbf{pa}^0(v) = v$ . For the tree visualized in Figure 1,  $\mathbf{pa}(i) = g$ ,  $\mathbf{pa}^2(i) = d$ ,  $\mathbf{pa}^3(i) = a$ .

If  $w = \mathbf{pa}^k(v)$  for some  $k \geq 0$ , then  $w$  is a *ancestor* of  $v$  and  $v$  is a *descendent* of  $w$ . We use the term *proper ancestor*

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<sup>1</sup>Future editions will expand this intuition.

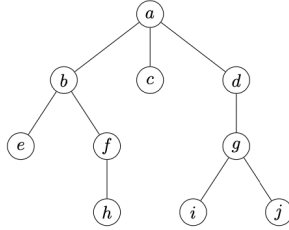


Figure 1: A rooted tree with root  $a$ .

and *proper descendent* if  $k > 0$  (i.e.,  $w \neq v$ ).

The *depth* or *level* of a vertex  $v$  is its distance (see **Trees**) to the root. We denote the level of a vertex  $v$  by  $\mathbf{lev}(v)$ . The level of the root is 0. If  $\mathbf{lev}(v) = k > 0$ , then  $\mathbf{pa}^k(v)$  is the root. The level function  $\mathbf{lev}$  satisfies  $\mathbf{lev}(v) = \mathbf{lev}(\mathbf{pa}(v)) + 1$ .

## Orderings

A *topological ordering* of a rooted tree is an ordering  $\sigma$  for which  $v \prec_{\sigma} \mathbf{pa}(v)$ . If there are  $n$  vertices, the root has index  $n$  and every other vertex has an index which is lower than its parent. The *first descendent* of  $v$  (which we denote  $\mathbf{fdesc}(v)$ ) is the descendent with the lowest index.

A *postordering* is a topological ordering in which descendents of a vertex are given consecutive numbers. Postorderings reduce common questions to checking precedence. For example, to check if a vertex  $w$  is a proper descendent of  $v$  one checks if  $\mathbf{fdesc}(v) \preceq w \prec v$ .

