



Tree Density Approximation Solution

1 Why

What can we say about tree density approximation? It turns out to be similar to what we can say about tree distribution approximation.

2 Results

Proposition 1. *Let $g : \mathbf{R}^n \rightarrow \mathbf{R}$ be a density. Let T be a tree on $\{1, \dots, n\}$. Let p_j be the parent of vertex j for the T rooted at vertex i , $j = 1, \dots, n$ and $j \neq i$. Then the density $f : \mathbf{R}^d \rightarrow \mathbf{R}$ defined by*

$$f = g_i \prod_{j \neq i} g_{j|p_j}$$

achieves minimum differential entropy relative to g among all densities which factor according to T .

Proposition 2. *Let $g : \mathbf{R}^n \rightarrow \mathbf{R}$ be a density. A tree T is a solution to the problem above if and only if it is a maximum spanning tree of the differential mutual information graph of g .*