Why

We can cast various common probabilistic regression models into the probabilistic errors linear model by mentioning the input space and feature maps. This unifies our analysis.

Definition

A *line fit model* has input space R and output space R. We use a regression function $\phi : \mathsf{R} \to \mathsf{R}^2$ defined by $\phi(t) = (1, t)^\top$.

We think of $t \in T \subset \mathsf{R}$ as a "dose level" (T is an interval). Given dose levels $t_1, \ldots, t_\ell$ and repetitions $n_1, \ldots, n_\ell$ we obtain the design matrix. Here the regression function generates a line segment embedded in the plane R^2. We call the parameters the *intercept parameter* and *slope parameter*.

A *parabola fit model* has input space R and output space R. We use a regression function $\phi : \mathsf{R} \to \mathsf{R}^3$ defined by $\phi(t) = (1, t, t^2)^\top$. Here the regression space is a segment of a parabola embedded in space R^3 (since $t \in T$ an interval).

These two are instance of *polynomial fit models* of degree $d \geq 1$, in which the regression function becomes $\phi : \mathsf{R} \to \mathsf{R}^{d+1}$ defined by $\phi(t) = (1, t, t^2, \ldots, t^d)^\top$. In this case, the regression range $\phi(T)$ is a one-dimensional curve embedded in R^{d+1}. In cases in which it is clear that the input space is a single real variable t, a linear model for a line fit (parabola fit, polynomial fit of degree d) is called a *first-degree model* (*second-degree model*, *dth degree model*).

m-way models

We can generalize to *m-way dth degree polynomial fit models* in which the input space is $X \subset \mathbf{R}^m$ and the regression function $\phi : \mathbf{R}^m \to \mathbf{R}^k$ (k is $d+m$ choose d) is the vector of all monomials of degree d in m variables.

For example, a two-way third-degree model has a regression function

$$\phi(t_1, t_2) = \begin{bmatrix} 1 & t_1 & t_2 & t_1^2 & t_1 t_2 & t_2^2 & t_1^3 & t_1^2 t_2 & t_1 t_2^2 & t_2^3 \end{bmatrix}^\top.$$

Or consider a three way second-degree model with regression function

$$\phi(t_1, t_2, t_3) = \begin{bmatrix} 1 & t_1 & t_2 & t_3 & t_1^2 & t_1 t_2 & t_1 t_3 & t_2^2 & t_2 t_3 & t_3^2 \end{bmatrix}^\top.$$

Both models will result in parameter vectors of size ten. We call these models *saturated* because they have every possible dth degree power or cross product of variables. In generally, a m-way dth degree model has $d+m$ choose d mean parameters.

In contrast to saturated models we can talk about *nonsaturated* models. For example, a nonsaturated two-way second-degree model has $\phi : \mathbf{R}^2 \to \mathbf{R}^4$ where $\phi(t_1, t_2) = (1, t_1, t_2, t_1^2)^\top$.

Polynomial Fit Models
Polynomial Regression
Probabilistic Errors Linear Model
Parameter Magic
Covariance
Variance
Random Variable Moments
Least Squares Linear Regression
Expectation
Linear Predictors
Matrix Inverses
Real Function Approximations
Input Matrix
Matrix Transpose
Matrix-Matrix Products
Vector Space Basis
Hyperplanes
Consistent Inductors
Loss Functions
Data Matrix
Probabilistic Functional Inductors
Span
Linear Transformations
Linearly Dependent Vectors
Basis
Matrix-Vector
Matrices
Real Function Space
Isomorphisms
Transformations
Linear Combinations
Determinants
Factors
Real Matrices
Random Variables
Approximators
Vectors
Topological Signed Algebra
Measurable Functions
Probability Measures
Real Vectors
Real Errors
Linear Inequalities
Similarity Functions
Linear Functions
Topological Spaces
Measures
Mathematical Space
Metrics
Norms
Sigma Algebras
Distance
Real Series
Accumulated Real Numbers
Space-Distance
Real Limits
Outcomes Variables
Absolute Value
Place-Distance
Event Probabilities
Real Functions
Interval Length
Probability Distributions
Intervals
Real Space
Real Sequences
Real Intervals
Sequences
Polynomials
Real Line
Real Plane
Ratios
Fields
Real Numbers
Real Numbers
Subset Algebras
Groups
Rational Order
Geometry
Set Numbers
Set Operations
Integer Addition Inverses
Rational Numbers
Optimizers
Cardinality
Finite Sets
Integer Multiplication
Inverse Elements
Direct Products
Natural Summation
Natural Addition Identity
Natural Multiplication Identity
Integer Products
Integer Order
Commutators
Element Functions
Equivalent Sets
Family Operations
Integer Sums
Integer Sums
Identity Elements
Associativity
Algebra
Functions Inverse
Natural Sequences
Family Unions and Intersections
Natural Powers
Function Inverse
Function Composition
Natural Sums
Cardinal
Equations
Natural Products
Equivalence Relations
Converse Relations
Functions
Natural Series
Abstract
Uncertain Outcomes
Recursion Theorem
Set Products
Subset Systems
Ordered Pairs
Set Pairs
Peano Axioms
Natural Induction
Ordered Triples
Natural Successor
Generalized Set Duplicates
Functions
Successor Sets
Intersection of Empty Set
Set Unions and Intersections
Set Partitions
Set Symmetric Differences
Set Intersections
Set Unions
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Induction
Standardized Abstracts
Axioms
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Atoms
Letters
Objects