Ordered Pairs

Why

We speak of an ordered pair of objects: one selected from a first set and one selected from a second set.

Definition

Let A and B be nonempty sets. Let $a \in A$ and $b \in B$. The *ordered pair* of a and b is the set $\{\{a\}, \{a, b\}\}$. The *first coordinate* of $\{\{a\}, \{a, b\}\}$ is a and the *second coordinate* is b.

The *product* of A and B is the set of all ordered pairs. This set is also called the *cartesian product*. If $A \neq B$, the ordering causes the product of A and B to differ from the product of B with A. If $A = B$, however, the symmetry holds.

Notation

We denote the ordered pair $\{\{a\}, \{a, b\}\}$ by (a, b). We denote the product of A with B by $A \times B$, read aloud as "A cross B." In this notation, if $A \neq B$, then $A \times B \neq B \times A$.

Taste

Notice that $a \notin (a, b)$ and similarly $b \notin (a, b)$. These facts led us to use the terms first and second "coordinate" above rather than element. Neither a nor b is an element of the ordered pair (a, b). On the other hand, it is true that $\{a\} \in (a, b)$ and $\{a, b\} \in (a, b)$. These facts are odd. Should they bother us?

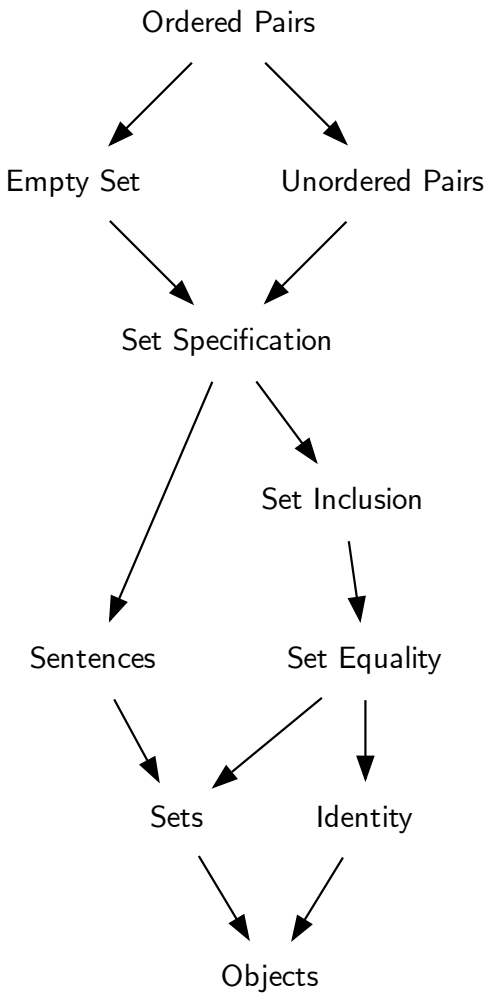
We chose to define ordered pairs in terms of sets so that we could reuse notions about a particular type of object (sets) that we had already developed. We chose what we may call conceptual simplicty (reusing notions from sets) over defining a new type of object (the ordered pair) with its own primitive properties. Taking the former path, rather than the latter is a matter of taste, really, and not a logical consequence of the nature of things.

The argument for our taste is as follows. We already know about sets, so let's use them, and let's forget cases like $\{a, b\} \in (a, b)$ (called by some authors "pathologies"). It does not bother us that our construction admits many true (but irrelevant) statements. Such is the case in life. Plus, suppose we did choose to make the object (a, b) primitive. Sure, we would avoid oddities like $\{a\} \in (a, b)$. And we might even get statements like $a \in (a, b)$ to be true. But to do so we would have to define the meaning of $\in$ for the case in which the right hand object is an "ordered pair". Our current route avoids introducing any new concepts, and simply names a construction in our current concepts.

Equality

PROPOSITION **1.** *$(a, b) = (c, d)$ if and only if $a = b$ and $c = d$.*

Proof. TODO □


Ordered Pairs
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Sentences
Set Equality
Sets
Identity
Objects