Why

We want to add repeatedly.

Defining Result

Proposition 1. *For each natural number m, there exists a function $p_m : \omega \to \omega$ which satisfies*

$$p_m(0) = 0 \quad and \quad p_m(n^+) = (p_m(n))^+ + m$$

for every natural number n.

Proof. The proof uses the recursion theorem (see *Recursion Theorem*).[1]   □

Let m and n be natural numbers. The value $p_m(n)$ is the *product* of m with n.

Notation

We denote the product $p_m(n)$ by $m \cdot n$. We often drop the $\cdot$ and write $m \cdot n$ as mn.

Properties

The properties of products are direct applications of the principle of mathematical induction (see *Natural Induction*).[2]

---

[1]Future editions will give the entire account.

[2]Future editions will include the accounts.

Proposition 2 (Associativity)**.** *Let k, m, and n be natural numbers. Then*

$$(k \cdot m) \cdot n = k \cdot (m \cdot n).$$

Proposition 3. *Let m and n be natural numbers. Then*

$$m \cdot n = n \cdot m.$$

Natural Products

↓

Natural Sums

↓

Recursion Theorem

↓

Peano Axioms

↓

Natural Induction

↓

Natural Numbers

Successor Sets   Intersection of Empty Set

Pair Unions   Set Intersections   Set Differences

Set Unions   Pair Intersections

Empty Set   Unordered Pairs

Set Specification

↓

Set Inclusion

Standardized Accounts

Definitions   Accounts

Deductions   Quantified Statements

Logical Statements

↓

Statements

Identities   Sets

Names

Objects   Letters