Why

We want numbers to count with.[1]

Definition

The *successor* of a set is the set which is the union of the set with the singleton of the set. In other words, the successor of a set A is $A \cup \{A\}$. This definition has sense for any set, but is of interest only for those particular sets introduced here.

These sets are the following (and their successors): We call the empty set *zero*.[2] We call the successor of the empty set *one*. In other words, one is $\varnothing \cup \{\varnothing\} = \{\varnothing\}$. We call the successor of one *two*. In other words, two is $\{\varnothing\} \cup \{\{\varnothing\}\} = \{\varnothing, \{\varnothing\}\}$. Likewise, the successor of two we call *three* and the successor of three we call *four*. And we continue as usual,[3] using the English language in the typical way.

A set is a *successor set* if it contains zero and if it contains the successor of each of its elements.

Notation

Let x be a set. We denote the successor of x by x^+. We defined it by

$$x^+ := x \cup \{x\}$$

We denote one by 1. We denote two by 2. We denote three by 3. We

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[1]Future editions will expand on this sheet with a more justified why.

[2]In future editions, zero may be a separate sheet.

[3]Future editions will assume less in the introduction of natural numbers.

denote four by 4. So

$$0 = \varnothing$$
$$1 = 0^+ = \{0\}$$
$$2 = 1^+ = \{0, 1\}$$
$$3 = 2^+ = \{0, 1, 2\}$$
$$4 = 3^+ = \{0, 1, 2, 3\}$$

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                    Successor Sets
                          |
                          v
                     Pair Unions
                          |
                          v
                     Set Unions
                       /     \
                      v       v
              Empty Set       Unordered Pairs
                      \       /
                       v     v
                   Set Specification
                          |
                          v
                     Set Equality
                          |
                          v
                     Set Inclusion
                          |
                          v
                 Standardized Accounts
                          |
                          v
                      Accounts
                          |
                          v
                     Deductions
                          |
                          v
                 Quantified Statements
                          |
                          v
                  Logical Statements
                          |
                          v
                     Statements
                       /     \
                      v       v
              Identities       Sets
                      \       /
                       v     v
                       Names
                       /     \
                      v       v
                Letters       Objects
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