Why

The identification of $\mathbf{C}$ with a plane leads $\mathbf{C}$ to naturally inherit R^2's notion of distance.

Definition

The *absolute value* or *modulus* of $z = (x, y) \in \mathbf{C}$ is the distance of z to the origin. If $z \in \mathbf{C}$, then the modulus of z is

$$\sqrt{x^2 + y^2}.$$

In other words, the modulus of z is the distance (in R^2 of $z = (x, y)$ from the origin $(0, 0)$.

Notation

We denote the modulus of z by $|z|$.

Properties

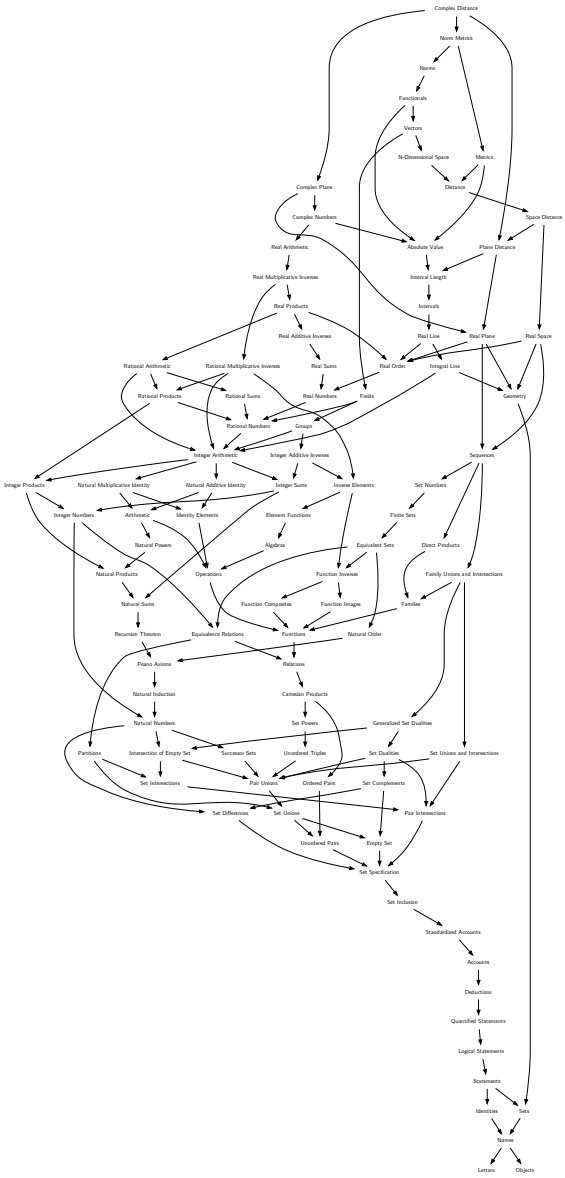
Proposition 1 (Triangle Inequality)**.** *For all $z, w \in \mathbf{C}$,*

$$|z + w| \leq |z| + |w|.$$

Also, for all $z \in \mathbf{C}$, we have $|\mathrm{Re}(z)| \leq |z|$ and $|\mathrm{Im}(z)| \leq |z|$, and for all $z, w \in \mathbf{C}$,

$$||z| - |w|| \leq |z - w|.$$ [1]

[1] This follows from the triangle inequality. Future editions will include an account.


Complex Distance
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Space Distance
Complex Plane
Complex Numbers
Absolute Value
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Line
Real Plane
Real Space
Real Additive Inverses
Real Order
Integral Line
Rational Arithmetic
Rational Multiplicative Inverses
Real Sums
Geometry
Rational Products
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Integer Arithmetic
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Integer Products
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Unordered Triples
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Set Unions and Intersections
Set Intersections
Pair Unions
Ordered Pairs
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Set Differences
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Unordered Pairs
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