



## RELATIONS

### Why

How can we relate the elements of two sets?

### Definition

A *relation* between two nonempty sets is a subset of their cross product. A relation on a single set is a subset of the cross product of it with itself.

The *domain* of a relation is the set of all elements which appear as the first coordinate of some ordered pair of the relation. The *range* of a relation is the set of all elements which appear as the second coordinate of some ordered pair of the relation.

### Notation

Let  $A$  and  $B$  be two nonempty sets. A relation on  $A$  and  $B$  is a subset of  $A \times B$ . Let  $C$  be a nonempty set. A relation on a  $C$  is a subset of  $C \times C$ .

Let  $a \in A$  and  $b \in B$ . The ordered pair  $(a, b)$  may or may not be in a relation on  $A$  and  $B$ . Also notice that if  $A \neq B$ , then  $(b, a)$  is not a member of the product  $A \times B$ , and therefore not in any relation on  $A$  and  $B$ . If  $A = B$ , however, it may be that  $(b, a)$  is in the relation.

## Notation

Let  $A$  and  $B$  be nonempty sets with  $a \in A$  and  $b \in B$ . Since relations are sets, we can use upper case Latin letters. Let  $R$  be a relation on  $A$  and  $B$ . We denote that  $(a, b) \in R$  by  $aRb$ , read aloud as “a in relation  $R$  to b.”

When  $A = B$ , we tend to use other symbols instead of letters. For example,  $\sim$ ,  $=$ ,  $<$ ,  $\leq$ ,  $\prec$ , and  $\preceq$ .

## Properties

Often relations are defined over a single set, and there are a few useful properties to distinguish.

A relation is *reflexive* if every element is related to itself. A relation is *symmetric* if two objects are related regardless of their order. A relation is *antisymmetric* if two different objects are related only in one order, and never both. A relation is *transitive* if a first element is related to a second element and the second element is related to the third element, then the first and third element are related.

## Notation

Let  $R$  be a relation on a non-empty set  $A$ .  $R$  is reflexive if

$$(a, a) \in R$$

for all  $a \in A$ .  $R$  is transitive if

$$(a, b) \in R \wedge (b, c) \in R \longrightarrow (a, c) \in R$$

for all  $a, b, c \in A$ .  $R$  is symmetric if

$$(a, b) \in R \longrightarrow (b, a) \in R$$

for all  $a, b \in A$ .  $R$  is anti-symmetric if

$$(a, b) \in R \longrightarrow (b, a) \notin R$$

for all  $a, b \in A$ .



