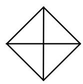


Why

We want a set which corresponds to our notion of points on a line.[1]

Rational cuts

We call a subset R of $\mathbf{Q}$ a *rational cut* if (a) $R \neq \varnothing$, (b) $R \neq \mathbf{Q}$, (c) for all $q \in R$, $r \leq q \longrightarrow r \in R$, and (d) R has no greatest element. Briefly, the intuition is that the point is the set of all rationals to less than (or, potentially, equal to) some particular rational number.[2]

Definition

The *set of real numbers* is the set of all rational cuts. This set exists by an application of the principle of selection (see Set Selection) to the power set (see Set Powers) of $\mathbf{Q}$. We call an element of the set of real numbers a *real number* or a *real*. We call the set of real numbers the *set of reals* or *reals* for short.

Notation

We follow tradition and denote the set of real numbers by $\mathbf{R}$, likely a mnemonic for "real."

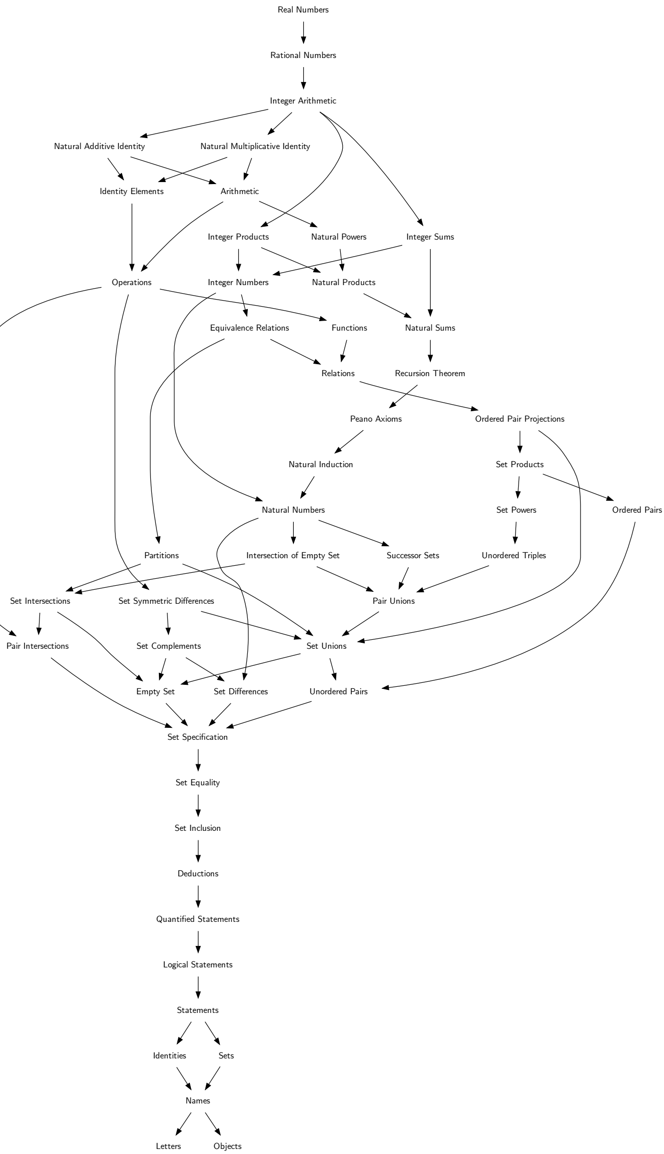
Other terminology

Some authors call a real number a *quantity* or a *continuous quantity*. The real numbers, then, are said to be *continuous*. When contrasting (using this terminology) a finite set with the real numbers, one refers to the finite set as *discrete*.[3]

[1] Future editions will modify and expand this justification.

[2] This brief intuition will be expanded upon in future sheets.

[3] Future editions may move this discussion later, to the discussion of the cardinality of the reals.


Real Numbers
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Identity Elements
Arithmetic
Integer Products
Natural Powers
Integer Sums
Operations
Integer Numbers
Natural Products
Equivalence Relations
Functions
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects