



Why

We want to extend addition to \mathbf{C} .

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex sum* of z_1 and z_2 is the complex number $(x_1 + x_2, y_1 + y_2)$.

Notation

For $z_1, z_2 \in \mathbf{C}$, we denote the complex sum of z_1 and z_2 by $z_1 + z_2$. The notation is justified because the complex sum of two purely real complex numbers corresponds to the purely real complex numbers whose real part is the real sum of the real parts of the first two numbers.

Recall that we denote $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. For example, we can express the definition of addition as

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2).$$

Properties

Proposition 1 (Commutativity). *For all $z_1, z_2 \in \mathbf{C}$, we have $z_1 + z_2 = z_2 + z_1$.*

Proposition 2 (Associativity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have and $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$.*

Complex addition

We call the operation that associates a pair of complex numbers with their sum *complex addition*. The operation is symmetric (commutative).

Additive identity and inverse

Notice that the complex number $(0,0)$ is the additive identity. It is unique,¹ and so we call it the *complex multiplicative identity*. Likewise, notice that additive inverse for (x, y) is $(-x, -y)$.

¹Future editions will include an account

