Why

We have seen that the matrices are a vector space. Are they an inner product space?

Definition

The *matrix scalar product* of $A \in \mathbf{R}^{n \times k}$ and $B \in \mathbf{R}^{n \times k}$ is the following product

$$\sum_{i=1}^{n} \sum_{j=1}^{k} a_{ij} b_{ij}.$$

Using the matrix trace, we can denote this as $\operatorname{tr} A^{\top} B$. Some authors call this the *Euclidean matrix scalar product*, *matrix inner product* or *Frobenius inner product*.

Proposition 1. *The matrix scalar product is an inner product.*

For example, symmetry of the product is a consequence of the fact that a square matrix and its tranpose have identical traces. Commutativity of the trace yields $\operatorname{tr} A^{\top} B = \operatorname{tr} B A^{\top}$, where the LHS is the scalar product of $B^{\top}$ and $A^{\top}$. In other words, transposition "preserves" the matrix scalar product.

With this inner product, $\mathbf{R}^{n \times k}$ is a Euclidean vector space (see Inner procuts) of dimension nk. For the case of $k = 1$, we recover a model[1] for the usual space $\mathbf{R}^{n}$.

Notation

We commonly denote the matrix inner product by $\langle A, B \rangle$.

Induced norm

The matrix inner product induces a norm in the usual way. This norm is sometimes called the *matrix-vector norm* (or *Frobenius norm*) and is

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[1]Future editions will define this term.

often denoted for a matrix $A \in \mathbf{R}^{m \times n}$ by $\|A\|_F$.

Matrix Scalar Product

Matrix Space

Matrix Trace

Subspaces

Symmetric Matrices

Real Matrix-Matrix Products

Real Inner Product Norms

Matrix Transpose

Symmetric Real Matrices

Real Subspaces

Real Matrix-Vector Products

Norms

Real Inner Products

Matrices

Vectors as Matrices

Functionals

Real Inner Product

Real Matrices

Real Vector Span

Vectors

Space Inner Product

Real Norm

Real Linear Combinations

Linear Equations

Plane Inner Product

Space Norm

N-Dimensional Space

Plane Vector Sums and Differences

Space Vectors

Linear Functions

Arrays

Right Triangle Sides Relation

Real Square Roots

Plane Norm

Distance

Plane Vectors

Real Squares

Absolute Value

Space Distance

Real Arithmetic

Plane Distance

Real Multiplication Inverses

Interval Length

Real Products

Natural Square Roots

Real Space

Intervals

Real Plane

Square Numbers

Real Additive Inverses

Real Line

Lists

Square

Rational Arithmetic

Integral Line

Real Sums

Real Order

Rational Multiplicative Inverse

Real Summation

Set Numbers

Fields

Geometry

Rational Products

Rational Sums

Real Numbers

Rational Order

Finite Sets

Direct Products

Rings

Rational Numbers

Integer Arithmetic

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Natural Summation

Equivalent Sets

Inverse Elements

Family Unions and Intersections

Integer Sums

Arithmetic

Integer Order

Identity Elements

Natural Equations

Comparisons

Element Function

Family Operations

Function Inverses

Families

Natural Powers

Integer Numbers

Order

Operations

Equations

Function Composites

Function Images

Natural Products

Natural Order

Converse Relations

Equivalence Relations

Functions

Natural Sums

Relations

Recursion Theorem

Peano Axioms

Set Products

Natural Induction

Set Power

Generalized Set Dualities

Natural Numbers

Intersection of Empty Set

Partitions

Set Unions and Intersections

Set Symmetric Differences

Successor Sets

Unordered Triples

Set Dualities

Ordered Pairs

Set Intersections

Set Complements

Pair Unions

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects

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