Convergence in Probability

1 Why

laws of large numbers

2 Definition

A sequence of random variables convergences in probability if it converges in measure.

2.1 Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $\{f_n\}_n$ a sequence of real-valued measurable functions on X. Let $f : X \to R$ be measurable function. If f_n converges in measure to f we write: $f_n \longrightarrow f$ in probability, read aloud as "f n goes to f in probability."

Suppose $f_n \longrightarrow f$ in probability. Then for every $\epsilon > 0$,

$$\lim_{n\to\infty} \mu(\{x \in X \mid |f_n(x) - f(x)| > \epsilon\}) = 0.$$