## Why

How many ways are there to arrange $n$ people at a table with $n$ seats? In other words, how many ways can we order $n$ objects?

## Discussion

There is one way to arrange a single person and there are two ways to arrange two people. What of three? First, we can pick a seat, and decide which person to put in that chair. We have three choices of people. Then, whomever we choose, there are only two ways to seat the other two people in the other two chairs. Since there were three ways of picking whom to sit in the firs seat, there are $3 \times 2 = 6$ ways to order the three people. We reason similarly for four people.

## Definition

Let $f : \mathbf{N} \to \mathbf{N}$ be defined recursively by $f(1) = 1$ and $f(2) = 2f(1)$, and $f(n) = nf(n - 1)$ for $n \in \mathbf{N}$. $f$ exists by the recursion theorem (see Recursion Theorem). We call the result $f(n)$ of $n$ the *factorial* of $n$. We call $f$ the *factorial function*.

## Notation

We denote the factorial of $n$ by $n!$, read aloud "n factorial".

Factorials

Sequences    Natural Products

Lists    Natural Sums

Set Numbers    Direct Products

Finite Sets    Recursion Theorem

Equivalent Sets

Function Inverses    Natural Order    Family Unions and Intersections

Function Composites    Function Images    Peano Axioms    Families

Equivalence Relations    Functions

Relations    Natural Induction

Set Products

Set Powers

Natural Numbers    Generalized Set Dualities

Partitions    Unordered Triples    Successor Sets    Intersection of Empty Set    Set Dualities    Set Unions and Intersections

Ordered Pairs    Set Intersections    Pair Unions    Set Complements

Set Unions    Set Differences    Pair Intersections

Unordered Pairs    Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities    Sets

Names

Letters    Objects