

REAL CONVEX FUNCTIONS

Why

We speak of functions which always bends up.¹

Definition

A convex real-valued function is a function defined on a convex set of real numbers for which the result of the function on a convex combination of any two points in the domain is smaller than the convex combination of the same length of the value of the function on the endpoints.² A function $f: \mathbb{R} \to \mathbb{R}$ is concave if the function -f is convex.

Notation

Let $A \subset \mathbb{R}$ be a convex set. The function $f: A \to \mathbb{R}$ is convex if, for any $a, b \in A$ and $t \in [0, 1]$,

$$f(ta + (1 - t)b) \le tf(a) + (1 - t)f(b).$$

¹Future editions may expand.

²Future editions will include figures

