Definition

Let $f : D \to \mathbf{R}$ be a multivariate real-valued function where $D \subset \mathbf{R}^d$. The *graph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}.$$

The *epigraph* of f is the set in $\mathbf{R}^{d+1}$ defined by

$$\{(x, t) \in \mathbf{R}^d \mid x \in D \text{ and } f(x) \leq t\}.$$

Notation

We denote the epigraph of a function f by epi f.

Connecting convex sets and convex functions

A function $f : \mathbf{R}^d \to \mathbf{R}$ is convex if and only if epi f is a convex set.

Note on terminology

The prefix "epi" is Greek, meaning "upon" or "above".

Real Function Epigraphs

Multivariate Functions — Convex Multivariate Functions

Real Convex Functions — Real Function Graphs

Real Convex Sets — Real Functions

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals — Real Space

Real Line — Real Plane

Real Order — Integral Line — Geometry

Rational Order — Real Numbers

Rational Numbers — Lists

Integer Arithmetic — Set Numbers — Direct Products

Integer Products — Natural Additive Identity — Natural Multiplicative Identity — Finite Sets

Arithmetic — Identity Elements — Equivalent Sets

Integer Order — Integer Sums — Natural Powers — Comparisons — Function Inverses — Family Unions and Intersections

Integer Numbers — Natural Products — Orders — Operations — Function Images — Function Composites — Families

Natural Sums — Equivalence Relations — Natural Order — Converse Relations — Functions

Recursion Theorem — Relations

Peano Axioms — Ordered Pair Projections

Natural Induction — Set Products

Natural Numbers — Generalized Set Dualities — Set Powers

Set Symmetric Differences — Partitions — Successor Sets — Intersection of Empty Set — Set Dualities — Unordered Triples — Set Unions and Intersections

Set Complements — Set Intersections — Pair Unions — Ordered Pairs

Set Differences — Set Unions — Pair Intersections

Empty Set — Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects