Why

We further drop conditions on the structure of the binary operations, and study only the algebraic structure of addition over the integers.

Definition

A *group* is an algebra$(G, \circ)$ for which $\circ : G \times G \to G$ is associative, has an identity element in G, and has inverse elements. A group is a *commutative group* (or *abelian group*) if $\circ$ is commutative. A group is a *finite group* if G is a finite set.

Additive groups

Suppose that $(R, +, \cdot)$ is ring. Then $(R, +)$ is a commutative group. Conversely, suppose $(G, +)$ is a commutative group. Define multiplication on S by $a \cdot b = 0$ for all $a, b \in R$. Then $(S, +, \cdot)$ is a ring, called the *zero ring* of $(G, +)$. For this reason, it is customary to write $+$ for the operations $\circ$ when handling commutative groups.

Group Operations

Along with the group operation, we call the function which maps an element to its inverse element the *group operations*.

Groups

Finite Sets   Rings

Integer Arithmetic

Natural Additive Identity   Natural Multiplicative Identity

Equivalent Sets

Function Inverses

Integer Sums   Integer Products   Arithmetic   Identity Elements

Function Composites   Function Images   Integer Numbers   Natural Products   Operations

Natural Powers

Functions   Equivalence Relations   Natural Sums

Relations   Natural Order   Recursion Theorem

Ordered Pair Projections   Peano Axioms

Set Products   Natural Induction

Ordered Pairs   Set Powers   Natural Numbers

Unordered Triples   Partitions   Successor Sets   Intersection of Empty Set

Pair Unions   Set Intersections   Set Symmetric Differences

Set Unions   Set Complements   Pair Intersections

Unordered Pairs   Empty Set   Set Differences

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects