Why

[1]

Definition

A *distribution family* (*density family*) on X is a family of distributions (densities) $\{p^{(\theta)}\}_{\theta \in \Theta}$ on X. We call the index set Θ (see Families) the *parameters*. Frequently $\Theta \subset \mathsf{R}^p$.

Similarly, a *conditional distribution family* (*conditional density family*) on Z from X is a family $\{q^{(\theta)}\}_{\theta \in \Theta}$ whose terms $q^{(\theta)} : Z \times X \to \mathsf{R}$ are such that $q^{(\theta)}(\cdot, \xi) : Z \to \mathsf{R}$ is a distribution (density) for every $\xi \in X$.

Examples

For example, let $\Theta = [0, 1]$ and consider the family of distributions $\{p^{(\theta)} : \{0, 1\} \to [0, 1]\}_{\theta \in [0,1]}$ defined by, for each $\theta \in [0, 1]$,

$$p^{(\theta)}(1) = \theta \text{ and } p^{(\theta)}(0) = 1 - \theta.$$

This family is called the *Bernoulli family* and $p^{(\theta)}$ is called a *Bernoulli distribution* with parameter θ.

For a second example, let $\Theta = \mathsf{R} \times \mathsf{R}_+$ and consider the family of densities $\{f^{(\theta)} : \mathsf{R} \to \mathsf{R}\}_{\theta \in \Theta}$ defined by, for each $\theta = (\mu, \sigma) \in \Theta$,

$$f^{(\theta)}(x) = (1/\sqrt{2\pi}\sigma) \exp((x-\mu)/\sigma^2).$$

[1] Future editions will include.

This family is called the *normal family* and $f^{(\theta)}$ with $\theta = (\mu, \sigma)$ is called a *normal density* with mean μ and variance σ^2.

Functional parameterization

A conditional distribution $q : X \times Z \to \mathbf{R}$ is *functionally parametrizable* if there exists a function $f : X \to \Theta$ and distribution family $\{p^{(\theta)}\}_{\theta \in \Theta}$ on Z satisfying $q(\cdot, \zeta) \equiv p^{(f(\zeta))}$. In this case we call f the *parametrization function* and we call $\{p^{(\theta)}\}_{\theta \in \Theta}$ the *parameterized family.* We call $(f, \{p^{(\theta)}\}_\theta)$ a *functionally parameterized conditional distribution* on Z from X, since for each $\zeta \in Z$, $p^{f(\zeta)} : X \to [0, 1]$ is a distribution

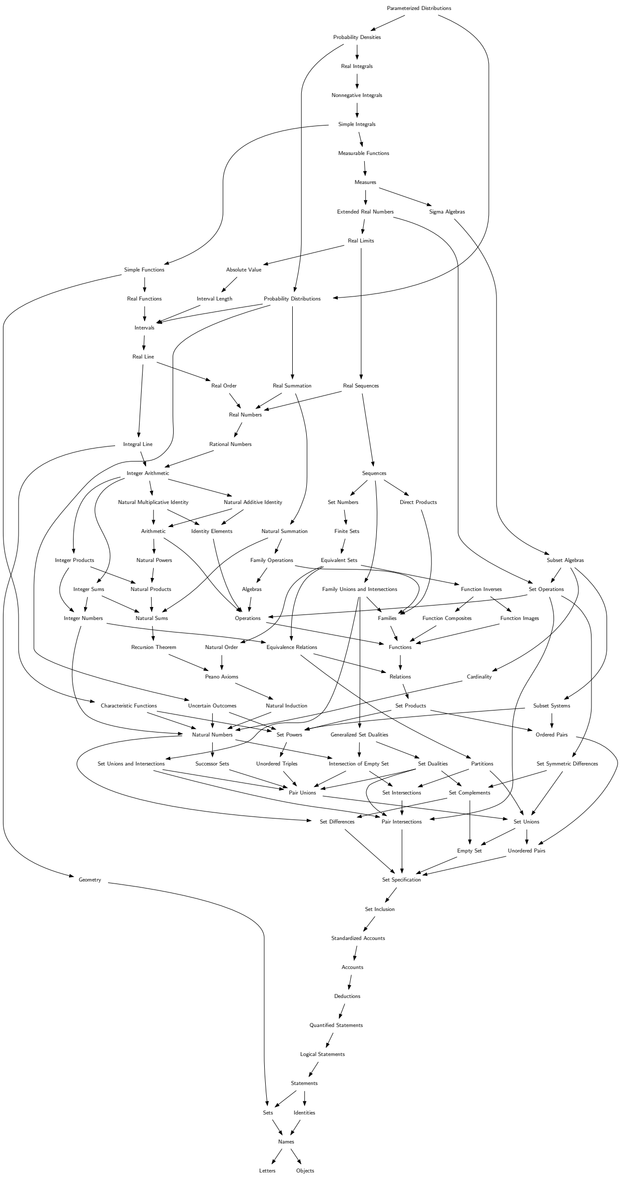
All conditional distributions $q : X \times Z \to \mathbf{R}$ are functionally parametrizable (consider $\{q(\cdot, \xi)\}_{\xi \in X}$ with parameters X and identity parameterization). Restricted families interest us.

Example

Let $Z = \{1, 2\}$ and $X = \mathbf{R}$. Let $f : \{1, 2\} \to \mathbf{R} \times \mathbf{R}_+$ be defined by $f(1) = (\mu_1, \sigma_1)$ and $f(2) = (\mu_2, \sigma_2)$. Let $\{g^{(\theta)}\}_\theta$ be the normal family. Then $(f, \{g^{(\theta)}\})$ is a *functionally parameterized conditional density.*

Parametrizing the function

If $\{f_\phi : X \to \Theta\}_{\phi \in \Phi}$ is a family of functions and $\{q^{(\theta)}\}$ is a family of distributions, then $\{p^{(\phi)} : X \times Z \to [0, 1]\}_\phi$ defined by $p^{(\phi)}(\cdot, \zeta) \equiv q^{f_\phi(\zeta)}$ is a conditional distribution family called a *functionally parameterized conditional distribution family.*


Parameterized Distributions
Probability Densities
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Extended Real Numbers
Sigma Algebras
Real Limits
Simple Functions
Absolute Value
Real Functions
Interval Length
Probability Distributions
Intervals
Real Line
Real Order
Real Summation
Real Sequences
Real Numbers
Integral Line
Rational Numbers
Integer Arithmetic
Sequences
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Natural Summation
Finite Sets
Integer Products
Natural Powers
Family Operations
Equivalent Sets
Subset Algebras
Integer Sums
Natural Products
Algebras
Family Unions and Intersections
Function Inverses
Set Operations
Integer Numbers
Natural Sums
Operations
Families
Function Composites
Function Images
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Cardinality
Characteristic Functions
Uncertain Outcomes
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Generalized Set Dualities
Ordered Pairs
Set Unions and Intersections
Successor Sets
Unordered Triples
Intersection of Empty Set
Set Dualities
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects