



## COMPLEX PRODUCTS

### Definition

Let  $z_1, z_2 \in \mathbf{C}$  with  $z_1 = (x_1, y_1)$  and  $z_2 = (x_2, y_2)$ . The *complex product* of  $z_1$  and  $z_2$  is the complex number

$$(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2).$$

### Notation

We denote the complex product of  $z_1$  and  $z_2$  by  $z_1 \cdot z_2$  or  $z_1z_2$ . The notation overloads that used for real numbers. This overloading is justified by the fact that the complex product of two purely real complex numbers  $z_1$  and  $z_2$  is the purely real complex number whose real part is the product of the real parts of  $z_1$  and  $z_2$ .

Recall that we denote  $z_1 = x_1 + iy_1$  and  $z_2 = x_2 + iy_2$ . This notation is a mnemonic for the definition of a complex product if we treat  $i^2 = -1$ .

$$\begin{aligned} z_1z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2). \end{aligned}$$

### Properties

**Proposition 1** (Commutativity). *For all  $z_1, z_2 \in \mathbf{C}$ , we have  $z_1z_2 = z_2z_1$ .*

**Proposition 2** (Associativity). *For all  $z_1, z_2, z_3 \in \mathbf{C}$ , we have and  $z_1(z_2z_3) = (z_1z_2)z_3$ .*

## Complex multiplication

We call the operation that associates a pair of complex numbers with their product *complex multiplication*. The operation is symmetric (commutative).

## Multiplicative identity and inverse

Notice that the complex number  $(1, 0)$  is the multiplicative identity. It is unique,<sup>1</sup> and so we call it the *complex multiplicative identity*.

We call the operation  $(z, w) \mapsto z/w$  *complex division* and we call  $z/w$  the *(complex) quotient* of  $z$  with  $w$ .

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<sup>1</sup>Future editions will include an account

