Definition[1]

The *complex exponential* of a complex number $z \in \mathbf{C}$ is the complex number

$$1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \cdots = \sum_{n=1}^{\infty} \frac{z^n}{n!}.$$

Notation

We denote the complex exponential funciton $\exp : \mathbf{C} \to \mathbf{C}$, and so denote the complex exponential of $z \in \mathbf{C}$ by $\exp(z)$, as usual. This overloaded notation is justified by the fact that the complex exponential agrees with the real exponential function on reals. It is also common to denote $\exp(z)$ by e^z.

Relation to power series of $\sin$ and $\cos$

It can be shown[2] that for $x \in \mathbf{R}$,

$$\exp(ix) = \cos(x) + i\sin(x).$$

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[1]Future editions may modify this sheet, as there are several (equivalent) characterizations of the complex exponential function.

[2]And future editions will show it

Complex Exponential

Exponential Function

Complex Arithmetic

Complex Products

Real Functions

Complex Sums

Complex Numbers

Absolute Value   Real Arithmetic

Interval Length   Real Multiplicative Inverses

Intervals   Real Products

Real Line   Real Additive Inverses

Rational Arithmetic   Real Sums   Real Order   Rational Multiplicative Inverses

Rational Sums   Real Numbers   Rational Products   Rational Order

Integral Line   Rational Numbers

Integer Arithmetic

Natural Additive Identity   Natural Multiplicative Identity   Integer Products

Identity Elements   Arithmetic

Natural Powers   Inverse Elements

Element Functions   Integer Sums   Natural Products   Integer Order   Comparisons   Function Inverses

Operations   Integer Numbers   Natural Sums   Orders   Function Images   Function Composites

Equivalence Relations   Recursion Theorem   Natural Order   Converse Relations   Functions

Peano Axioms   Relations

Natural Induction   Set Products

Natural Numbers   Set Powers   Ordered Pairs

Partitions   Intersection of Empty Set   Successor Sets   Unordered Triples

Set Intersections   Set Symmetric Differences   Pair Unions

Pair Intersections   Set Complements   Set Unions

Empty Set   Set Differences   Unordered Pairs

Geometry   Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects

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