Why

We further drop conditions on the structure of the binary operations, and study only the algebraic structure of addition over the integers.

Definition

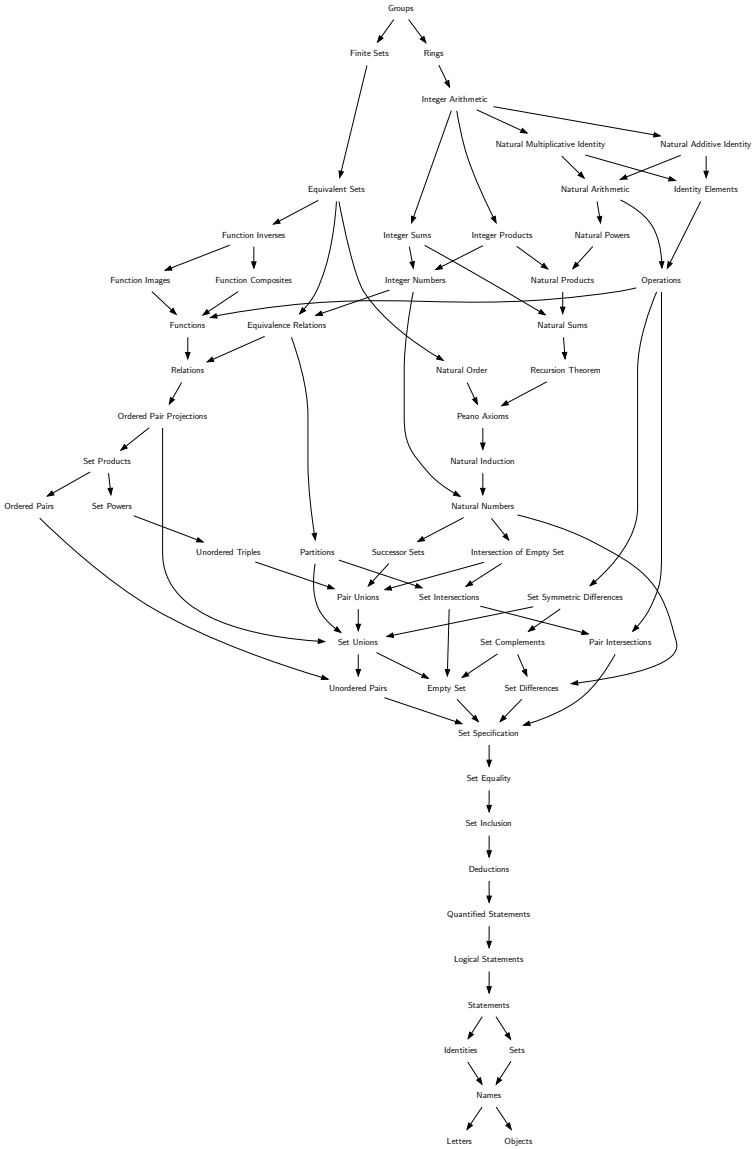
A *group* is an $\mathsf{algebra}(G, \circ)$ for which $\circ : G \times G \to G$ is associative, has an identity element in G, and has inverse elements. A group is a *commutative group* (or *abelian group*) if $\circ$ is commutative. A group is a *finite group* if G is a finite set.

Additive groups

Suppose that $(R, +, \cdot)$ is ring. Then $(R, +)$ is a commutative group. Conversely, suppose $(G, +)$ is a commutative group. Define multiplication on S by $a \cdot b = 0$ for all $a, b \in R$. Then $(S, +, \cdot)$ is a ring, called the *zero ring* of $(G, +)$. For this reason, it is customary to write $+$ for the operation $\circ$ when handling commutative groups.

Group Operations

Along with the group operation, we call the function which maps an element to its inverse element the *group operations*.


Groups
Finite Sets
Rings
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Natural Arithmetic
Identity Elements
Function Inverses
Integer Sums
Integer Products
Natural Powers
Function Images
Function Composites
Integer Numbers
Natural Products
Operations
Functions
Equivalence Relations
Natural Sums
Relations
Natural Order
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Ordered Pairs
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Empty Set
Set Differences
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects