



## Why

We want to visualize the probabilistic relations between components of outcomes in probabilistic models over large (e.g., product) outcome sets.<sup>1</sup>

## Definition

Suppose  $X_1, \dots, X_k$  are sets and define  $X = \prod_{i=1}^k X_i$ . For  $x \in X$  and  $S \subset \{1, \dots, n\}$ , denote the subvector of  $x$  indexed (in order) by  $S$  by  $x_S$ .<sup>2</sup>

A distribution  $p : X \rightarrow [0, 1]$  *factors according to a directed graph* on  $\{1, \dots, n\}$  with parent function  $\text{pa} : \{1, \dots, n\} \rightarrow \mathcal{P}(\{1, \dots, n\})$  if

$$p(x) = \prod_{\text{pa}_i = \emptyset} g_i(x_i) \prod_{\text{pa}_i \neq \emptyset} g_i(x_i, x_{\text{pa}_i}),$$

where  $g_i$  is a distribution for all  $i$  which  $\text{pa}_i = \emptyset$  and  $g_i(\cdot, \xi)$  is a distribution for all  $\xi \in \prod_{j \in \text{pa}_i} A_j$ ,  $i$  for which  $\text{pa}_i \neq \emptyset$ .

**Proposition 1.**  *$p$  so defined is a distribution, and the  $g_i$  are the marginals and conditionals.*<sup>3</sup>

## Examples

Consider a rooted tree distribution (see [Rooted Tree Distributions](#)), or a memory chain (see [Memory Chains](#)), or a hidden memory chain (see [Hidden Memory Chains](#)).<sup>4</sup>

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<sup>1</sup>Future editions will modify and expand. The title of the sheet may change, since another interpretation for the words “directed graph distribution” is a distribution on directed graphs.

<sup>2</sup>Future editions will rework this treatment, perhaps combining it with the sheet [Index Matrices](#), which will possibly be split up.

<sup>3</sup>Future editions will be precise and give an account.

<sup>4</sup>Future editions will expand.

