# Matrix Trace

## Why

TODO

## Definition

The *trace* of a square real matrix is the sum of the elements on its diagonal.

### Notation

We denote the function which associates a matrix with its trace by $\mathbf{tr} : \mathbf{R}^{n\times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n\times n}$. Then

$$\mathbf{tr}\, A = \sum_{i=1}^{n} A_{ii}.$$

## Properties

Proposition **1.** *The trace is a linear function on the vector space of $n \times n$ real matrices.*

*Proof.* Let $A, B \in \mathbf{R}^{n\times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\mathbf{tr}\, C = \sum_{i=1}^{n} C_{ii} = \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii} = \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii} = \alpha\, \mathbf{tr}\, A + \beta\, \mathbf{tr}\, B.$$

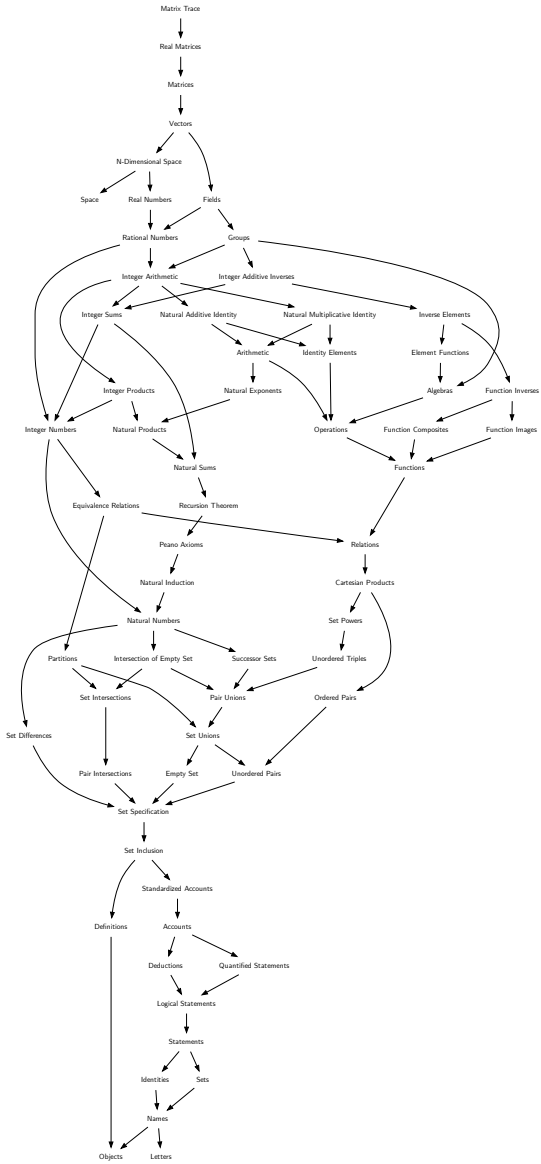
$\square$

Proposition **2.** *Let* $A, B \in \mathbf{R}^{n \times n}$.

$$\mathbf{tr}\,(AB) = \mathbf{tr}\,(BA)$$

*Proof.* TODO

$\square$


Matrix Trace
Real Matrices
Matrices
Vectors
N-Dimensional Space
Space
Real Numbers
Fields
Rational Numbers
Groups
Integer Arithmetic
Integer Additive Inverses
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Inverse Elements
Arithmetic
Identity Elements
Element Functions
Integer Products
Natural Exponents
Algebras
Function Inverses
Integer Numbers
Natural Products
Operations
Function Composites
Function Images
Natural Sums
Functions
Equivalence Relations
Recursion Theorem
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Pair Unions
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters