Why

If A and B are two events in some finite sample space Ω and P is the event probability function of some distribution, we have seen that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

What if we have three events A, B, C? What of a list of n events $A_1, \ldots, A_n$?

Case of $n = 3$.

Proposition 1. *Suppose A, B, C are events in a finite sample space and P is any event probability function. Then*

$$\begin{aligned} P(A \cup B \cup C) = P(A) &+ P(B) + P(C) \\ &- P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &+ P(A \cap B \cap C) \end{aligned}$$

Proof. Express

$$\begin{aligned} P(A \cup B \cup C) &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\ &= P(A) + P(B) + P(C) - P(A \cap B) - P((A \cup B) \cap C) \end{aligned}$$

by using the inclusion-exclusion formula. Recall[1] that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

Now use the inclusion-exclusion formula again, and properties of set pair intersections to get

$$\begin{aligned} P((A \cup B) \cap C) &= P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C)) \\ &= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \end{aligned}$$

□

[1] See Set Unions and Intersections.

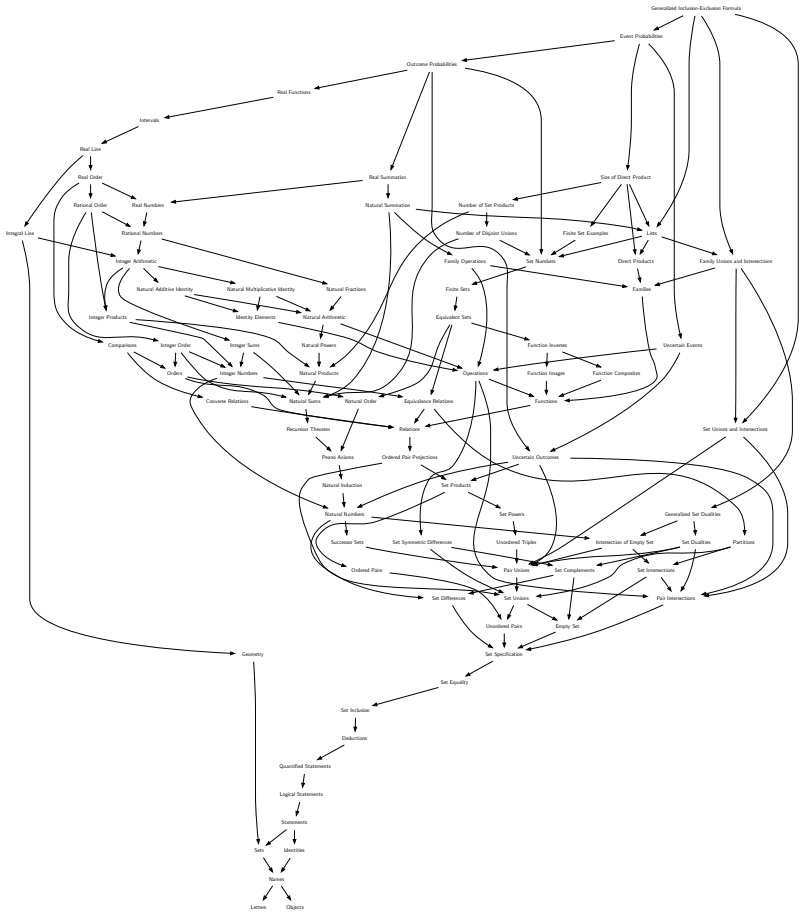
Case of n.

Proposition 2. *Suppose $A_1, \ldots, A_n$, are events in a finite sample space and P is any event probability function. Then*

$$\begin{aligned} P(A_1 \cup \cdots \cup A_n) = \sum P(A_i) - \sum P(A_i \cap A_j) \\ \sum P(A_i \cap A_j \cap A_k) - \cdots \\ (-1)^{n-1} P(A_1 \cap \cdots A_n) \end{aligned}$$

Proof. This can be shown by induction on the number of events n. Future editions will include. $\square$

Here there are $\binom{n}{r}$ terms in the rth sum, $r = 1, \ldots, n$.


Generalized Inclusion-Exclusion Formula
Event Probabilities
Outcome Probabilities
Real Functions
Intervals
Real Line
Real Order
Real Summation
Size of Direct Product
Rational Order
Real Numbers
Natural Summation
Number of Set Products
Integral Line
Rational Numbers
Number of Disjoint Unions
Finite Set Examples
Lists
Integer Arithmetic
Family Operations
Set Numbers
Direct Products
Family Unions and Intersections
Natural Additive Identity
Natural Multiplicative Identity
Natural Fractions
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Families
Integer Products
Identity Elements
Natural Arithmetic
Equivalent Sets
Comparisons
Integer Order
Integer Sums
Natural Powers
Function Inverses
Uncertain Events
Orders
Integer Numbers
Natural Products
Operations
Function Images
Function Compositions
Converse Relations
Natural Sums
Natural Order
Equivalence Relations
Functions
Recursion Theorem
Relations
Set Unions and Intersections
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Set Symmetric Differences
Unordered Triples
Intersection of Empty Set
Set Dualities
Partitions
Ordered Pairs
Pair Unions
Set Complements
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects