Why

What is the natural generalization of a smooth function to functions defined on sets of $\mathbf{R}^k$.[1]

Definition

Let $U \subset \mathbf{R}^d$ be an open set (see Real Open Sets). A function $f : U \to \mathbf{R}$ is *smooth* if all its partial derivatives exists and are continuous.

More generally, let $X \subset \mathbf{R}^d$. A function $f : X \to R$ is *smooth* if there exists an open set $U \subset \mathbf{R}^d$ and a smooth $F : U \to \mathbf{R}$ so that $F(x) = f(x)$ for all $x \in U \cap X$.

Example

The identity map is smooth. In other words, let $f : \mathbf{R}^d \to \mathbf{R}$ be so that $X \subset \mathbf{R}^d$. Then $f : X \to \mathbf{R}$ s

Properties

Proposition 1. *The composition of two smooth functions is smooth.*

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[1]Future editions will expand.

Smooth Multivariate Functions

Smooth Real Functions

Higher Order Derivatives

Second Derivatives   Partial Derivatives

Real Open Sets   Differentiable Functions   Multivariate Functions

N-Dimensional Space

Real Functions   Distance

Space Distance

Plane Distance

Interval Length

Intervals   Real Space

Real Line   Real Plane

Integral Line   Real Order

Geometry   Rational Order   Real Numbers   Lists

Rational Number   Set Numbers   Direct Products

Integer Arithmetic   Finite Sets

Integer Products   Natural Additive Identity   Natural Multiplicative Identity   Equivalent Sets

Arithmetic   Identity Elements   Function Inverses   Family Unions and Intersections

Natural Powers   Integer Sums   Integer Order   Comparisons   Families

Natural Products   Integer Numbers   Orders   Operations   Function Images   Function Composites

Natural Sums   Natural Order   Converse Relations   Equivalence Relations   Functions

Recursion Theorem   Relations

Peano Axioms   Ordered Pair Projections

Natural Induction   Set Products

Natural Numbers   Set Powers   Generalized Set Dualities

Set Symmetric Differences   Successor Sets   Unordered Triples   Intersection of Empty Set   Set Dualities   Partitions   Set Unions and Intersections

Set Complements   Pair Unions   Set Intersections   Ordered Pairs

Set Differences   Set Unions   Pair Intersections

Empty Set   Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects

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