

## MULTIVARIATE VECTOR LINEAR FUNCTIONS

## **Definition**

A function  $f: \mathbf{R}^n \to \mathbf{R}^m$  is linear if

- 1. f(x+y) = f(x) + f(y) for all  $x, y \in \mathbb{R}^n$ , and
- 2.  $f(\alpha x) = \alpha f(x)$  for all  $\alpha \in \mathbf{R}$  and  $x \in \mathbf{R}^n$ .

Equivalently,  $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$  for all  $\alpha, \beta \in \mathbf{R}$  and  $x, y \in \mathbf{R}^n$ . In this case, we say that *superposition* holds.

