Why

In the case that it is not possible to easily identify (or guess) the limit of a sequence, we are naturally interested in a simple condition on the sequence which is equivalent to convergence.

Definition

A sequence $(x_n)_{n \in \mathbf{N}}$ in $\mathbf{R}$ is said to be *egopox* (or *Cauchy* or a *Cauchy sequence*) if for every $\varepsilon > 0$, there exists $N \in \mathbf{N}$ so that for all $m, n > N$, $|x_m - x_n| < \varepsilon$. We call this property of the sequence *(eventual) egoproximity.*

Notation

We sometimes denote this property as

$$|x_n - x_m| \to 0 \quad \text{as} \quad m, n \to \infty.$$

Example

For example, consider $\lim_{N \to \infty} \sum_{n=1}^{N} 1/n^3$.

Sufficiency in R

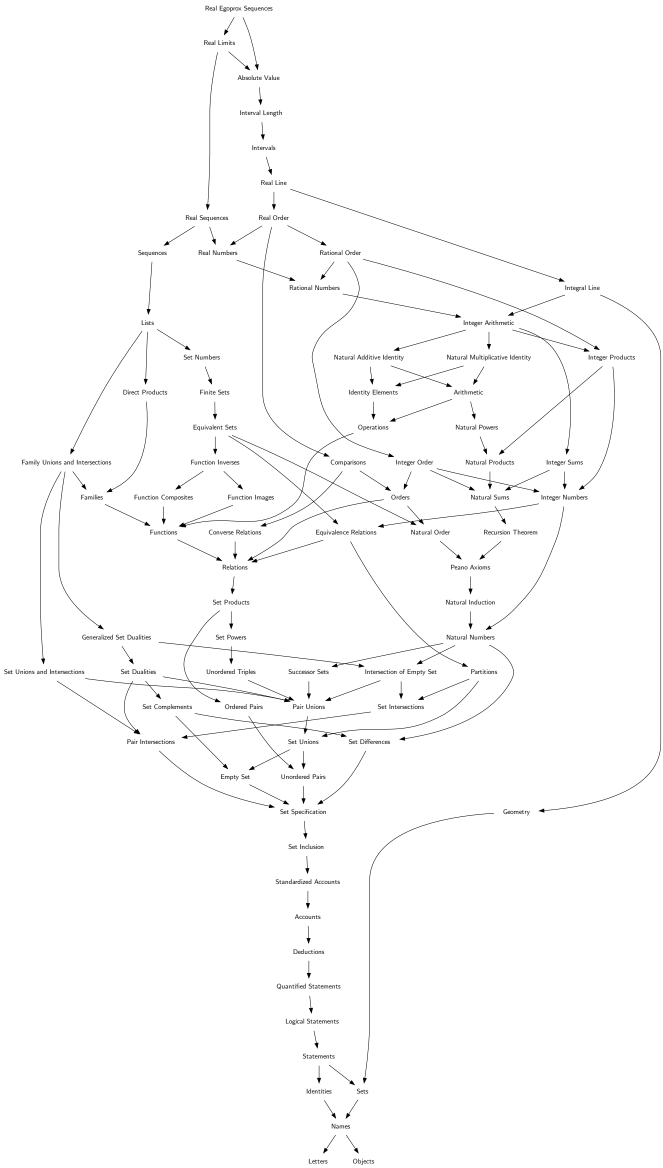
Clearly a convergent sequence is egoprox.[1] What of the converse? Recall that we think of egoprox sequences as "bunching up." For the reals, if a sequence is bunching up, then our intuition is that it should be converging. In other words, an

[1] Future editions may elaborate here.

egoprox real sequence always converges. The egoprox condition is sufficient. Bunching up is sufficient.

Proposition 1. *If* $(x_n)_{n\in\mathsf{N}}$ *is egoprox, then there exists* $x_0 \in \mathsf{R}$ *so that* $\lim_{n\to\infty} x_n = x_0$.

In other words, in R egoproximity is equivalent to convergence. The above is sometimes called the *Bolzano-Weierstrass theorem.*


Real Egoprox Sequences
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Sequences
Real Order
Sequences
Real Numbers
Rational Order
Rational Numbers
Integral Line
Lists
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Operations
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Natural Products
Integer Sums
Families
Function Composites
Function Images
Orders
Natural Sums
Integer Numbers
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Complements
Ordered Pairs
Pair Unions
Set Intersections
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects