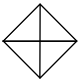


Why

In the statistical learning framework, since the algorithm only has access to the training set, it is natural to select a hypothesis which performs well on the training set.

Definition

Let $((\Omega, \mathcal{A}, \mathcal{P}), \{x_i : \Omega \to \mathcal{X}\}_{i=1}^n, f : \mathcal{X} \to \mathcal{Y})$ be *probabilistic data-generation model* so that $S : \Omega \to (\mathcal{X} \times \mathcal{Y})^n$ is the training dataset.

The *training error* of a classifier $h : \mathcal{X} \to \mathcal{Y}$ is

$$({}^1\!/\!{}_m) \left| \{i \in \{1, 2, \dots, m\} \mid h(x_i) \neq y_i\} \right|.$$

Other terminology includes *empirical error* and *empirical risk.* For these reasons, the learning paradigm of selecting a predictor h to minimizer the empirical risk is called *empirical risk minimization* or *ERM.*

Overfitting

Although selecting a classifier to minimize the empirical risk seems natural, it can be foolish. Let $A \subset \mathcal{X} \subset \mathsf{R}^2$ and $\mathcal{Y} = \{0, 1\}$. Suppose that the true classifier $f : \mathcal{X} \to \mathcal{Y}$ is $f(x) = 1$ if $x \in A$ and $f(x) = 0$ otherwise. Suppose that for the underlying distribution $(\mathcal{X}, \mathcal{A}, \mathbf{P})$ we have $A \in \mathcal{A}$ and $\mathbf{P}(A) = {}^1\!/\!{}_2$.

For the training set S in $\mathcal{X} \times \mathcal{Y}$, the hypothesis $h : \mathcal{X} \to \mathcal{Y}$

defined by

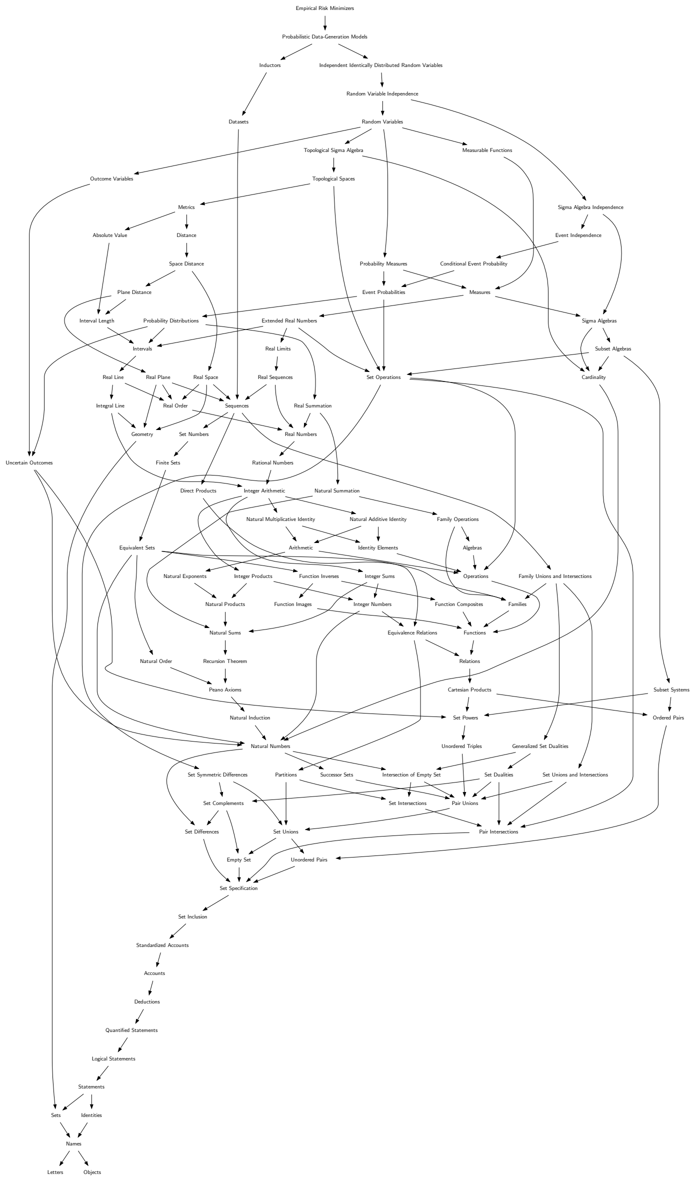
$$h_S(x) = \begin{cases} y_i & \text{if } x_i = x \\ 0 & \text{otherwise.} \end{cases}$$

achieves zero empirical risk but has error (w.r.t. $\mathbf{P}$) of $1/2$. Such a classifier is said to be *overfit* or to exhibit *overfitting*. It is said to fit the training dataset "too well."

Inductive Bias

In spite of the prior example, are there conditions under which ERM does not overfit? One such formulation is to ask whether there are situations in which good performance on empirical risk implies (or makes it highly likely to achieve) good performance on the underling distribution.

One simple approach is to constrain the set of predictors considered. A *hypothesis class* is a subset of predictors $\mathcal{H} \subset \mathcal{X} \to \mathcal{Y}$. The *restricted empirical risk minimizer* for $\mathcal{H}$ is a hypothesis $h \in \mathcal{H}$ minimizing the empirical risk on the data set. Many authorities call the hypothesis class a *inductive bias* and speak of "biasing" the "learning algorithm". Since one specifies the hypothesis class prior to the data it often said to "encode prior knowledge about the problem to be learned."


Empirical Risk Minimizers
Probabilistic Data-Generation Models
Inductors
Independent Identically Distributed Random Variables
Random Variable Independence
Datasets
Random Variables
Topological Sigma Algebra
Measurable Functions
Outcome Variables
Topological Spaces
Sigma Algebra Independence
Metrics
Absolute Value
Distance
Event Independence
Space Distance
Probability Measures
Conditional Event Probability
Plane Distance
Event Probabilities
Measures
Interval Length
Probability Distributions
Extended Real Numbers
Sigma Algebras
Intervals
Real Limits
Subset Algebras
Real Line
Real Plane
Real Space
Real Sequences
Set Operations
Cardinality
Integral Line
Real Order
Sequences
Real Summation
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Natural Multiplicative Identity
Natural Additive Identity
Family Operations
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Function Images
Integer Numbers
Function Composites
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Natural Induction
Set Powers
Ordered Pairs
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Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
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Statements
Sets
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Names
Letters
Objects