Why

We want to split up a measure and the pieces should not—roughly speaking—overlap.[1]

Definition

A measure is *concentrated* on a set if the measure on the complement of the set is zero. A signed or complex measure is *concentrated* on a set if its variation is concentrated on the set.[2]

Two measures (or signed or complex measures) are *mutually singular* if there exists a set on which one is concentrated and on whose complement the other is concentrated. In other words, there exists a decomposition of the space into two sets such that one measure if concentrated on one piece and the other measure is concentrated on the other piece.

Notation

Let $(X, \mathcal{A})$ be a measurable space and let μ be a measure. Then μ is concentrated on a set $C \in \mathcal{A}$ if $\mu(X - C) = 0$. If ν is a signed or complex measure, then ν is concentrated on $C \in \mathcal{A}$

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[1]Future editions will improve upon this justification. It may be that the intuition from linear algebra need be more explicitly used. This would seem right, since dealing with signed measures and such is partly to make the space of measures a vector space.

[2]Future editions may remove the dependence on variation of a measure. The benefit is observed in the next paragraph.

if $|\nu|$ is concentrated on C; in which case $|\nu|(X - C) = 0$.

Let μ and ν be measures on $(X, \mathcal{A})$. Then μ and ν are mutually singular if there exists a set $A \in \mathcal{A}$ so that μ is concentrated on A and ν is concentrated on $X - A$. We denote that two measures are singular by $\mu \perp \nu$, read aloud as "mu perp nu".

Exercises

Exercise 1. *Let μ and ν be measures on a measurable space $(X, \mathcal{A})$ such that for any $\varepsilon > 0$, there is a set $A \in \mathcal{A}$ such that $\mu(A) < \varepsilon$ and $\nu(C(A)) < \varepsilon$. Show that $\mu \perp \nu$.*

Singular Measure

Variation Measure

Measure

Sigma Algebras

Real Series

Extended Real Numbers

Real Limits

Absolute Value

Interval Length

Intervals

Real Line

Real Order

Real Numbers

Real Sequences

Sequences

Rational Order

Integral Line

Rational Numbers

Integer Arithmetic

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Arithmetic

Identity Elements

Lists

Subset Algebra

Set Numbers

Cardinality

Set Operations

Finite Sets

Natural Powers

Operations

Equivalent Sets

Direct Products

Family Unions and Intersections

Integer Sums

Integer Order

Comparisons

Function Inverses

Natural Products

Natural Sums

Orders

Function Composite

Function Images

Families

Recursion Theorem

Natural Order

Equivalence Relations

Converse Relations

Functions

Peano Axioms

Relations

Natural Induction

Set Products

Subset Systems

Natural Numbers

Generalized Set Dualities

Set Powers

Ordered Pairs

Set Symmetric Differences

Partitions

Intersection of Empty Set

Successor Sets

Set Dualities

Unordered Triples

Set Unions and Intersections

Set Complements

Set Intersections

Pair Unions

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects