



Why

We further drop conditions on the structure of the binary operations, and study only the algebraic structure of addition over the integers.

Definition

A *group* is an *algebra* (G, \circ) for which $\circ : G \times G \rightarrow G$ is associative, has an identity element in G , and has inverse elements. It is a *commutative group* (or *abelian group*) if \circ is commutative.

Additive groups

Suppose that $(R, +, \cdot)$ is ring. Then $(R, +)$ is a commutative group. Conversely, suppose $(G, +)$ is a commutative group. Define multiplication on S by $a \cdot b = 0$ for all $a, b \in R$. Then $(S, +, \cdot)$ is a ring, called the *zero ring* of $(G, +)$. For this reason, it is customary to write $+$ for the operations \circ when handling commutative groups.

