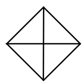


Why

We want to add repeatedly.

Defining Result

Proposition 1. *For each natural number m, there exists a function $p_m : \omega \to \omega$ which satisfies*

$$p_m(0) = 0 \quad \text{and} \quad p_m(n^+) = (p_m(n))^+ + m$$

for every natural number n.

Proof. The proof uses the recursion theorem (see Recursion Theorem).[1] □

Let m and n be natural numbers. The value $p_m(n)$ is the *product* of m with n.

Notation

We denote the product $p_m(n)$ by $m \cdot n$. We often drop the $\cdot$ and write $m \cdot n$ as mn.

Properties

The properties of products are direct applications of the principle of mathematical induction (see Natural Induction).[2]

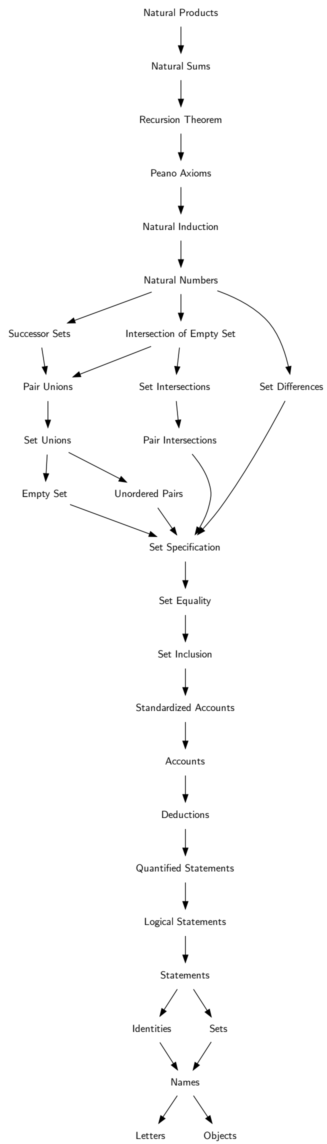
Proposition 2 (Associativity)**.** *Let k, m, and n be natural numbers. Then*

$$(k \cdot m) \cdot n = k \cdot (m \cdot n).$$

Proposition 3. *Let m and n be natural numbers. Then*

$$m \cdot n = n \cdot m.$$

[1] Future editions will give the entire account.
[2] Future editions will include the accounts.


Natural Products
Natural Sums
Recursion Theorem
Peano Axioms
Natural Induction
Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects