Why

Consider bounding the measure where two functions differ. We would look at their absolute value, and the measure of where this is greater than zero. The absolute value of their difference is a non-negative measurable random variable.

Result

We bound the measure that a non-negative measurable real-valued function exceeds some value by its integral.

Proposition 1. *Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $g : X \to [0, \infty]$ be measurable. Then for all $t > 0$,*

$$\mu(\{x \in X \mid g(x) \geq t\}) \leq \frac{\int g d\mu}{t}.$$

Proof. Let $A = \{x \in X \mid g(x) \geq t\}$. Define $h : X \to R$ by

$$h(x) = \begin{cases} 1 & \text{if } g(x) \geq t \\ 0 & \text{otherwise.} \end{cases}$$

First, $\mu(A) = \int h d\mu$. Second, $h \leq g$. So:

$$\mu(\{x \in X \mid g(x) \geq t\}) = \int h d\mu \leq \int g/t d\mu = \frac{\int g d\mu}{t}.$$

□

This proposition is universally referred to as *Markov's inequality*.[1]

[1] We eschew this terminology in accordance with the project's policy on naming.

Tail Measure Upper Bound
Variance
Expectation
Random Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Outcome Variable Covariance
Measures
Topologies
Sigma Algebras
Metric
Outcome Variable Expectation
Extended Real Numbers
Real Sets
Distance
Induced Probability Distributions
Real Square Roots
Real Lists
Outcome Variable Events
Square Distance
Real Square
Event Probabilities
Phase Distance
Absolute Value
Real Arithmetic
Probability Distributions
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Space
Natural Square Roots
Square Numbers
Real Power
Real Lists
Real Addition Inverses
Real Sequences
Real Summation
Rational Multiplication Inverses
Real Order
Real Sums
Rational Arithmetic
Integral Lists
Squares
Sequences
Real Numbers
Rational Order
Rational Products
Rational Sums
Geometry
Subset Algebras
Rational Numbers
Cardinality
Lists
Set Numbers
Direct Products
Uncertain Outcomes
Integer Arithmetic
Finite Sets
Natural Addition Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Inverse Elements
Natural Summation
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Family Operations
Element Functions
Comparison
Integer Order
Integer Sums
Natural Powers
Natural Equations
Functions
Function Composition
Function Images
Operations
Order
Integer Numbers
Natural Products
Equation
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Subset Systems
Set Products
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Generalized Set Duality
Unordered Tuples
Natural Numbers
Set Unions and Intersections
Partitions
Insertion of Empty Set
Set Duality
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Axioms
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Axioms
Lexicon
Objects