



Why

We want to extend our notion of entropy (see Discrete Entropy) to real-valued (continuous) random variables.

Definition

The *differential entropy* of a probability density function is the integral of the density against the negative log of the density. This definition made to be similar to the case of discrete entropy. If a real-valued random variable has a density, then we call the differential entropy of its density the *differential entropy* of the random variable.

Notation

Let $f : \mathbf{R}^n \rightarrow \mathbf{R}$ be a probability density function. The differential entropy of f is

$$-\int f \log f$$

We denote the differential entropy of f by $h(f)$.

Example

Let $x : \Omega \rightarrow \mathbf{R}$ be uniform on $[0, 1/2]$. Then $h(x) = \log 1/2 < 0$.

Problems

We have $h(ax) = h(x) + \log |a|$. In generally $h(Ax) = h(x) + \log |A|$.

Differences still meaningful

Even though the value of the differential entropy is not necessarily a good analogy to discrete entropy, differences still are. In particular, the following holds

$$I(X; Y) = H(Y) - H(Y | X) = H(X) = H(X | Y)$$

