Why

Let $X = \{a, b\}$ and $Y = \{0, 1\}$. The dataset $((a, 0))$ is consistent, but it is not functionally complete. On the other hand, the dataset $((a, 0), (b, 0), (a, 0), (a, 0), (a, 0), (a, 1))$ is complete but it is not *functionally* consistent.

In general, if $y_i \neq y_j$ for some i and j where $x_i = x_j$, then the dataset is not functionally consistent. In the preceding example, both $(a, 0)$ and $(a, 1)$ appear.

If we emphasize the "predictive" aspect of a functional inductor, we interpret the input as an object we "see before" the output. And so treat $y \in Y$ as an uncertain outcome which is the element associated to $x \in X$.

In this case, we may use the language of probability to discuss this uncertain outcome. If, for example, Y is finite, we can associate a distribution with each input $x \in X$.

Definition

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable spaces.

A *probabilistic functional inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a family of measures on $(Y, \mathcal{Y})$, indexed by X. We call a function from inputs to output measures a *probabilistic predictor*. We call the distribution a *probabilistic prediction*.

Notation

Let $\mathcal{M}(Y, \mathcal{Y})$ be the set of measures on Y. Let D be a dataset in $(X \times Y)^n$. Let $g : X \to \mathcal{M}(Y, \mathcal{Y})$ a probabilistic predictor. Let $G_n(X \times Y)^n \to (X \to \mathcal{M}(Y, \mathcal{Y}))$ be a predictive probabilistic inductor. Then $G_n(D)$ is a family of measures $\{g_x : \mathcal{Y} \to [0, 1]\}_{x \in X}$.

Probabilistic Predictive Inductors

Inductors   Probability Measures

Event Probabilities   Measures

Extended Real Numbers   Probability Distributions

Real Limits   Intervals

Real Line

Real Sequences   Real Summation   Real Order

Datasets

Real Numbers

Rational Numbers   Integral Line

Sequences   Integer Arithmetic

Direct Products   Set Numbers

Natural Additive Identity   Natural Multiplicative Identity

Finite Sets

Natural Summation   Identity Elements   Arithmetic   Sigma Algebras

Family Operations   Equivalent Sets

Natural Exponents   Subset Algebras   Integer Products

Family Unions and Intersections   Function Inverses   Algebras   Set Operations   Natural Products   Integer Sums   Cardinality

Function Composites   Function Images   Operations   Natural Sums   Integer Numbers

Families   Functions   Equivalence Relations   Natural Order   Recursion Theorem

Relations   Peano Axioms

Cartesian Products   Subset Systems   Uncertain Outcomes   Natural Induction

Ordered Pairs   Generalized Set Dualities   Set Powers   Natural Numbers

Set Unions and Intersections   Partitions   Intersection of Empty Set   Set Dualities   Unordered Triples   Successor Sets   Set Symmetric Differences

Set Intersections   Pair Unions   Set Complements

Pair Intersections   Set Unions   Set Differences

Unordered Pairs   Empty Set

Set Specification   Geometry

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects