

## MATRIX SPACE

## Why

We can view the set of real-valued  $n \times k$  matrices as a vector space over  $\mathbb{R}$ .

## Definition

The matrix sum of two matrices  $A, B \in \mathbb{R}^{n \times k}$  is the matrix  $C \in \mathbb{R}^{n \times k}$  defined by  $C_{ij} = A_{ij} + B_{ij}$ . In other words, the matrix C is given by summing the entires of A and B "entrywise". We denote the matrix sum by A + B.

For  $\alpha \in \mathbb{R}$ , the  $\alpha$ -scaled version of  $A \in \mathbb{R}^{n \times k}$  is the matrix  $C \in \mathbb{R}^{n \times k}$  given by  $C_{ij} = \alpha A_{ij}$ . In other words, the matrix C is given by scaling the entries of A "entry-wise". We denote the  $\alpha$ -scaled version of A by  $\alpha A$ . These two definitions are justified by the following.

The  $n \times k$ -matrix space is the vector space over  $\mathbb{R}^{n \times k}$  in which addition is given by the matrix sumer and scalar multiplication by entry-wise scaling.<sup>1</sup>

## Subspace of symmetric matrices

The subset of symmetric n by n matrices is a subset of  $\mathbb{R}^{n \times n}$ .

<sup>&</sup>lt;sup>1</sup>Future editions will rework this sheet.

