

REAL FUNCTIONS

Why

We name functions whose codomain is the real numbers.

Definition

A real function is a real-valued function. The domain is often an interval of real numbers, but may be any non-empty set.

Notation

Given any set $A, f: A \to \mathbf{R}$ is a real function. If $A = \mathbf{R}$, then $f \in \mathbf{R} \to \mathbf{R}$.

We often speak of functions defined on intervals. Given $a,b \in \mathbf{R}$, then $g:[a,b] \to \mathbf{R}$ is a real function defined on a closed interval. The function $h:(a,b) \to \mathbf{R}$ is a real function defined on an open interval.

We regularly declare the interval and the function at once. For example, "let $f:[a,b]\to \mathbf{R}$ " is understood to mean "let a and b be real numbers with a< b, let [a,b] be the closed interval with them as endpoints, and let f be a real-valued function whose domain is this interval". We read the notation $f:[a,b]\to \mathbf{R}$ aloud as "f from closed a b to \mathbf{R} ." We use $f:(a,b)\to \mathbf{R}$ similarly (read aloud "f from open a b to \mathbf{R} ").

Examples

Example 1. Given $c \in \mathbb{R}$, define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = c \quad for \ all \ x \in \mathbf{R}$$

Example 2. Define $f : \mathbf{R} \to \mathbf{R}$ by

$$f(x) = 2x^2 + 1 \quad for \ all \ x \in \mathbf{R}$$

Example 3. Define $f : \mathbf{R} \to \mathbf{R}$ by

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{otherwise.} \end{cases}$$

