Why

We want to talk about how a function of multiple real-valued arguments changes with respect to changes in its arguments.

Definition

Consider a real-valued function on d-dimensional space. For $i = 1, \ldots, d$, Fix a point x. consider the limit of a sequence of quotients of the difference of the result of that function at a point the consider the limit of a sequence of quotients of the value changed at component The *partial derivative* of the function with respect to the ith the function which maps d-dimensional vectors of real numbers to the limit of a seq of all of the quotient between the point to argument is the limit of the rate with a The partial derivative of a

Let $f : \mathbf{R}^d \to \mathbf{R}$ For $i = 1, \ldots, d$, define Let $g_i : \mathbf{R}^d \to \mathbf{R}$ by

$$g_i(x) = \lim_{h \to 0} \frac{f(x + he_i) - f(x)}{h}$$

for each x

Notation

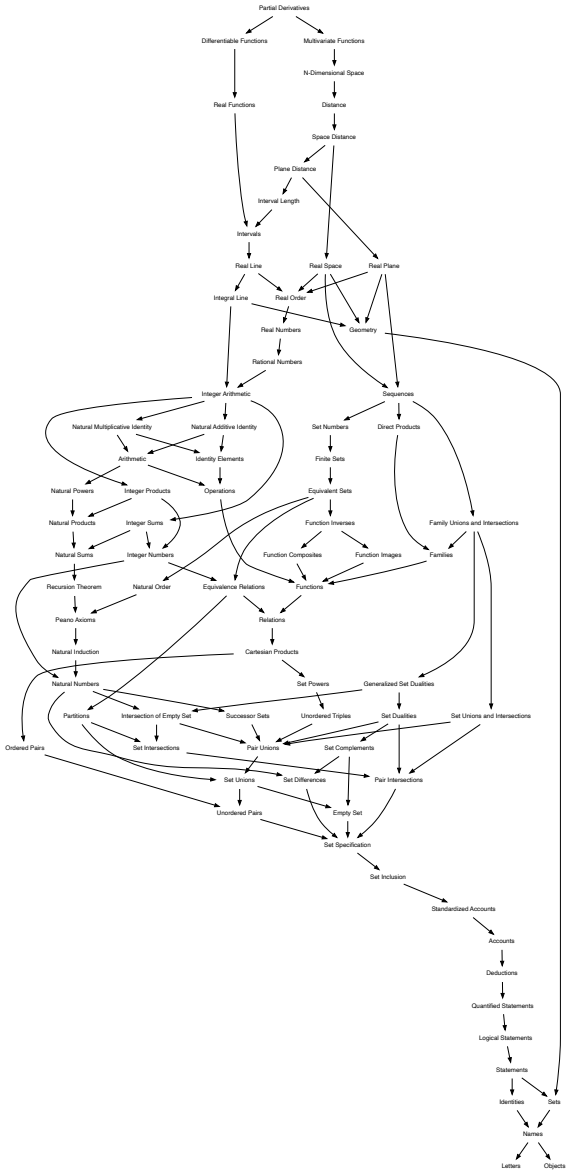
Gradient

The *gradient* of a multivariate function is the vector-valued function whose ith component is the the partial derivative of the function with respect to its ith argument.

Notation

Let $f : \mathsf{R}^n \to \mathsf{R}$. The gradient of f is frequently denoted ∇f. It is understood that $(\nabla f) \in \mathsf{R}^d \to \mathsf{R}^d$. An alternative notation is to use that similar for single derivatives and to denote the gradient (sometimes called derivative) of f by f' (assuming it exists). It is important to here note that although when $g : \mathsf{R} \to \mathsf{R}$, $g' \in (\mathsf{R} \to \mathsf{R})$, (and so is another function from and to reals) when $f : \mathsf{R}^d \to \mathsf{R}$, $f' \in \mathsf{R}^d \to \mathsf{R}^d$, and so is a vector-valued (not a real-valued) function.

There is (unfortunately) much notation for the individual partial derivatives; most of which we shall not (fortunately) have occasion to use in these sheets. One popular usage is the use of the ∂ symbol, read aloud as "partial." For example, if $f : \mathsf{R}^2 \to \mathsf{R}$ is a function of two arguments, each being referred to as x and y, then $\partial_x f$ denotes the partial derivative of f with respect to x and $\partial_y f$ denotes the partial derivative of f with respect to y. It is understood that $(\partial_x f) \in \mathsf{R}^d \to \mathsf{R}$. and likewise for $\partial_y f$. Another popular usage is $\partial f / \partial x$ for $\partial_x f$ and $\partial f / \partial y$ for $\partial_y f$. We will almost exclusively prefer the gradient notation.


Partial Derivatives
Differentiable Functions
Multivariate Functions
N-Dimensional Space
Real Functions
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Space
Real Plane
Integral Line
Real Order
Real Numbers
Geometry
Rational Numbers
Integer Arithmetic
Sequences
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Direct Products
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Integer Products
Operations
Equivalent Sets
Natural Products
Integer Sums
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects