



## Definition

A set of vectors  $\{v_1, \dots, v_k\}$  is a *basis* for a subspace  $S \subset \mathbf{R}^n$  if

$$S = \text{span}\{v_1, \dots, v_k\} \quad \text{and} \quad \{v_1, \dots, v_k\} \text{ is independent.}$$

This definition captures two competing properties. The first is that the set is large, in the sense that any vector in  $S$  can be represented as a linear combination of vectors in  $\{v_1, \dots, v_k\}$ . Simultaneously, the set is small, in the sense that no vector in the set is a linear combination of the others. In other words, there is no extra vector in the set.

Linear independence is equivalent to uniqueness of representation of the vectors representable as a linear combination of  $v_1, \dots, v_k$ . In other words,  $\{v_1, \dots, v_k\}$  is a basis for  $S$  if each vector  $x \in S$  can be uniquely expressed as

$$x = \alpha_1 v_1 + \dots + \alpha_k v_k.$$



