Why

The most important families are those indexed by (subsets of) the natural numbers.

Definition

A *finite sequence* is a family whose index set is a nonzero natural number. In other words, a finite sequence is a function whose domain is a nonzero natural number.

The *length* of a finite sequence is the size of its index set (which is the same as the domain of the sequence, see Set Numbers). Some authors call a finite sequence, in certain contexts, a *numbering* or, in other contexts, a *string.* A sequence whose codomain is some nonempty A is said to be a *sequence in A*.

Notation

Since the index set of a finite sequence is ordered (see Natural Order), we write sequences in order from left to right. For example, let A be a set and let $a : 4 \to A$ a sequence. We denote a by listing its elements in between parentheses, as in (a_1, a_2, a_3, a_4).

Relation to Direct Products

A *natural direct product* is a product of a sequence of sets. We denote the direct product of a sequence of sets A_1, ..., A_n by $\prod_{i=1}^{n} A_i$. If each A_i is the same set A, then we denote the

product $\prod_{i=1}^n A_i$ by A^n. In this case, we call an element (the sequence $a = (a_1, a_2, \ldots, a_n) \in A^n$) an *n-tuple* or *tuple*. The set of sequences in a set A is the direct product A^n.

Infinite Sequences

An *infinite sequence* is a family whose index set is N (the set of natural numbers without zero). The *nth term* of a sequence is the result of the nth natural number, $n \in \mathsf{N}$.[1]

Notation

Let A be a non-empty set and $a : \mathsf{N} \to A$. Then a is a (infinite) sequence in A. $a(n)$ is the nth term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n. If $\{A_n\}_{n\in\mathsf{N}}$ is an infinite sequence of sets, then we denote the direct product of the sequence by $\prod_{i=1}^\infty A_i$.

Natural unions and intersections

We denote the family union of the finite sequence of sets A_1, $\ldots$, A_n by $\cup_{i=1}^n A_i$. We denote the family of the infinite sequence of sets $(A_n)_n$ by $\cup_{i=1}^\infty A_i$. Similarly, we denote the intersections of a finite and infinite sequence of sets $\{A_i\}$ by $\cap_{i=1}^n A_i$ and $\cap_{i=1}^\infty A_i$, respectively.

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[1]Future editions may also comment that we are introducing language for the steps of an infinite process.

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