Why

Sums of non-negative functions are increasing, and workable with the monotone convergence theorem.

Result

Prop. 1. *The integral of the limit of the partial sums of a sequence of measurable, nonnegative, extended-real-valued functions is the limit of the partial sums of the integrals.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space, and let $f_n :\to [0, \infty]$ a $\mathcal{A}$-measurable function for every natural number n. We want to show that:

$$\int \sum_{k=1}^{\infty} f_k d\mu = \sum_{k=1}^{\infty} \int f_k d\mu.$$

We apply the monotone convergence theorem to the sequence $\{\sum_{i=1}^{n} f_i\}_n$. This sequence is nondecreasing because $f_n \geq 0$ for all n. $\qquad\square$

Real Integral Series Convergence

Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions

Measure

Sigma Algebras

Extended Real Numbers

Simple Functions

Real Limits

Absolute Value

Real Functions

Interval Length

Intervals

Real Line

Real Order

Real Sequences

Real Numbers

Rational Numbers

Integral Line

Integer Arithmetic

Sequences

Set Numbers

Direct Products

Natural Additive Identity

Natural Multiplicative Identity

Identity Elements

Arithmetic

Finite Sets

Equivalent Sets

Natural Powers

Integer Products

Subset Algebras

Set Operations

Comparisons

Natural Products

Integer Sums

Function Inverses

Function Composites

Family Unions and Intersections

Order

Operations

Natural Sums

Integer Numbers

Natural Order

Functions

Families

Converse Relations

Recursion Theorem

Equivalence Relations

Cardinality

Relations

Peano Axioms

Subset Systems

Set Products

Characteristic Functions

Natural Induction

Generalized Set Dualities

Ordered Pairs

Set Powers

Natural Numbers

Partitions

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Dualities

Set Unions and Intersections

Set Symmetric Differences

Set Complements

Set Intersections

Pair Unions

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Geometry

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects