

TOPOLOGIES

Why

We want to generalize the notion of continuity.

Definition

Let X be a set. A topology is a set of subsets of X for which (1) the empty set base set are distinguished (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the elements of the topology the $open \ sets$.

A topological space is an ordered pair: a base set and a set distinguished subsets of the base set which are a topology.

Notation

Let X be a non-empty set. For the set of distinguished sets, we tend to use \mathcal{T} , a mnemonic for topology, read aloud as "script T". We tend to denote elements of \mathcal{T} by O, a mnemonic for open. We denote the topological space with base set X and topology \mathcal{T} by (X, \mathcal{T}) . We denote the properties satisfied by elements of \mathcal{T} :

- 1. $X, \emptyset \in \mathcal{T}$
- 2. if $O_1, \ldots, O_n \in \mathcal{T}$, then $\bigcap_{i=1}^n O_i \in \mathcal{T}$
- 3. if $O_{\alpha} \in \mathcal{T}$ for all $\alpha \in I$, then $\bigcup_{\alpha \in I} \in \mathcal{T}$

Examples

 $\boldsymbol{\mathsf{R}}$ with the open intervals as the open sets is a topological space.

