Why

We want to generalize the notion of continuity.

Definition

Given a set X, a *topology* on X is a set of subsets of X for which (1) the empty set base set are distinguished (2) the intersection of a *finite* family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the elements of the topology the *open sets*.

A *topological space* is an ordered pair: a base set and a set distinguished subsets of the base set which are a topology.

Notation

Let X be a non-empty set. For the set of distinguished sets, we tend to use $\mathcal{T}$, a mnemonic for topology, read aloud as "script T". We tend to denote elements of $\mathcal{T}$ by O, a mnemonic for open. We denote the topological space with base set X and topology $\mathcal{T}$ by $(X, \mathcal{T})$. We denote the properties satisfied by elements of $\mathcal{T}$:

1. $X, \varnothing \in \mathcal{T}$

2. if $O_1, \ldots, O_n \in \mathcal{T}$, then $\bigcap_{i=1}^{n} O_i \in \mathcal{T}$

3. if $O_\alpha \in \mathcal{T}$ for all $\alpha \in I$, then $\bigcup_{\alpha \in I} \in \mathcal{T}$

Examples

R with the open intervals as the open sets is a topological space.

Topologies

Metrics

Distance

Absolute Value

Space Distance

Plane Distance

Interval Length

Real Space

Intervals

Real Line

Real Plane

Real Order

Integral Line

Geometry

Rational Order

Real Numbers

Lists

Rational Numbers

Direct Products

Natural Fractions

Integer Arithmetic

Set Numbers

Integer Products

Natural Multiplicative Identity

Natural Additive Identity

Finite Sets

Family Unions and Intersections

Integer Order

Integer Sums

Natural Arithmetic

Identity Elements

Equivalent Sets

Function Inverses

Integer Numbers

Natural Products

Natural Powers

Comparisons

Operations

Function Composites

Function Images

Families

Natural Sums

Equivalence Relations

Natural Order

Converse Relations

Functions

Recursion Theorem

Relations

Orders

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Generalized Set Dualities

Set Powers

Unordered Triples

Set Unions and Intersections

Set Symmetric Differences

Partitions

Successor Sets

Intersection of Empty Set

Set Dualities

Set Complements

Set Intersections

Pair Unions

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects