



ORDER AND ARITHMETIC

Why

How does arithmetic preserve order?

Results

The following are standard useful results.¹

Proposition 1. *If $m < n$, then $m + k < n + k$ for all k .*

Proposition 2. *If $m < n$ and $k \neq 0$, then $m \cdot k < n \cdot k$.*

Proposition 3 (Least Element). *If E is a nonempty set of natural numbers, there exists $k \in E$ such that $k \leq m$ for all $m \in E$.*

Proposition 4 (Greatest Element). *If E is a nonempty set of natural numbers, there exists $k \in E$ such that $m \leq k$ for all $m \in E$.*

¹The accounts of which will appear in future editions.

