Why

We want to discuss rearranging the order of some set.

Definition

Let X be a nonempty set. A *permutation* is a bijection from X to X.

Let $\pi : X \to X$ and $\sigma : X \to X$ be two permutations. Then $\pi \circ \sigma$ is a permutation. Also, the identity function $\mathsf{id}_X : X \to X$ is a permutation. In particular $\pi \, \mathsf{id} = \mathsf{id} \, \pi$ for all permutations. Since each permutation is invertible, inverse elements exist. And the associative law is valid for permutations since function composition is associative. Therefore the set of permutations with the operation of composition is a group (see Groups). We call it the *symmetric group on X*.

If X is finite and $|X| = n$, then we can associate each element of X with a number $\{1, 2, \ldots, n\}$ and in so doing consider permutations of the set $\{1, \ldots, n\}$. This special symmetric group is called the *symmetric group of degree n*.

Notation

We denote the symmetric group on X by $\mathsf{Sym}(X)$. It is common to denote the symmetric group of degree n by S_n.

Permutations

Set Numbers   Groups

Finite Sets   Integer Arithmetic

Integer Additive Inverses   Natural Multiplicative Identity   Natural Additive Identity

Inverse Elements   Integer Sums   Arithmetic   Identity Elements

Equivalent Sets   Element Functions   Integer Products   Natural Exponents

Function Inverses   Integer Numbers   Algebras   Natural Products

Equivalence Relations   Function Images   Function Composites   Natural Sums   Operations

Natural Order   Recursion Theorem   Functions

Peano Axioms   Relations

Natural Induction   Cartesian Products

Natural Numbers   Set Powers

Partitions   Intersection of Empty Set   Successor Sets   Unordered Triples

Set Intersections   Pair Unions   Ordered Pairs

Set Differences   Set Unions

Pair Intersections   Empty Set   Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects