Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?[1]

Definition

Two random variables are *independent* if the sigma algebras generated by the random variables are independent. In general, a family of random variables are *independent* if the sigma algebras generated by the random variables are independent.

Notation

Let $(X, \mathcal{A}, \mu)$ be a probability space and $(Y, \mathcal{B})$ be a measurable space. Let $f_1, f_2 : X \to Y$ be random variables. If the random variables are independent we write $f_1 \perp f_2$.

Results

Prop. 1. *Let $f_1, \ldots, f_n$ be independent real-valued random variables defined on a probability space $(X, \mathcal{A}, \mu)$.*

Let $B_1, \ldots, B_n$ be Borel sets of real numbers and let $A_i = f_i^{-1}(B_i)$. Let $A = \cap_{i=1}^n f_i^{-1}(B_i)$. Then

$$\mu(A) = \prod_{i=1}^n \mu(A_i)$$

Proof. Since f_i are independent, so are the sigma algebras they generate. A_i are in each of these sigma algebras, so by definition of independence the measure of the intersection is the product of the measures. $\square$

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[1]Future editions will modify this.

Independent Random Variables

Independent Sigma Algebras — Random Variables

Dependent Events — Probability Measures — Measurable Functions — Topological Sigma Algebra

Conditional Event Probabilities — Measure

Sigma Algebras — Event Probabilities — Topological Spaces

Real Series — Extended Real Numbers — Metrics

Real Limits — Distance

Space Distance

Absolute Value — Plane Distance

Probability Distributions — Interval Length

Intervals — Real Space

Real Sequences — Real Summation — Real Line — Real Plane

Sequences — Real Order — Integral Line

Real Numbers — Rational Order — Geometry

Subset Algebras — Lists — Cardinality — Rational Numbers

Set Numbers — Integer Arithmetic

Direct Products — Finite Sets — Natural Additive Identity — Natural Multiplicative Identity — Integer Products

Natural Summation — Identity Elements — Equivalent Sets — Arithmetic

Family Unions and Intersections — Family Operations — Function Inverses — Comparisons — Integer Order — Integer Sums — Natural Powers

Families — Operations — Function Composites — Function Images — Orders — Integer Numbers — Natural Products

Functions — Converse Relations — Equivalence Relations — Natural Order — Natural Sums

Subset Systems — Set Products — Relations — Recursion Theorem

Ordered Pairs — Set Powers — Peano Axioms

Uncertain Outcomes — Natural Induction

Generalized Set Dualities — Unordered Triples — Natural Numbers

Set Unions and Intersections — Set Dualities — Intersection of Empty Set — Successor Sets — Partitions — Set Symmetric Differences

Pair Unions — Set Intersections — Set Complements

Pair Intersections — Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects

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