



# Product Metrics

## 1 Why

Given  $n$  sets each with metrics, there is a standard way of turning the direct product of the sets into a metric space. In other words, defining a distance on the tuples of elements from the sets.

## 2 Motivating Result

**Proposition 1.** *Let  $(A_1, d_1), (A_2, d_2), \dots, (A_n, d_n)$  be metric spaces.*

*Let  $A$  be  $\prod_{i=1}^n A_i$  and let  $R$  be the set of real numbers. Define*

*$d : A \times A \rightarrow R$  by*

$$d(a, b) = \max\{d_1(a_1, b_1), \dots, d_n(a_n, b_n)\}.$$

*Then  $(A, d)$  is a metric space.*