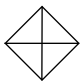


Why

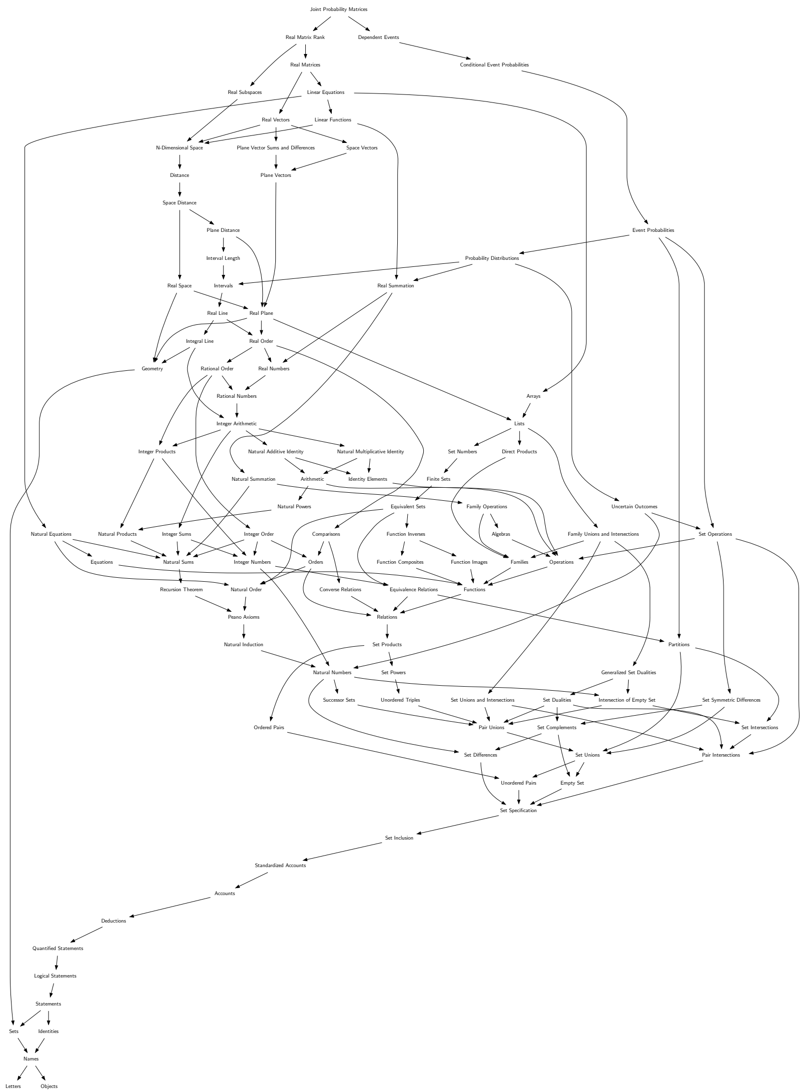
We can characterize the dependence of two events in terms of the rank of a particular matrix.

Definition

Given a probability measure $\mathsf{P} : \mathcal{P}(\Omega) \to \mathsf{R}$ on the finite set Ω and two events $A, B \subset \Omega$, the *joint probability matrix* of A and B is the matrix

$$M = \begin{bmatrix} \mathsf{P}(A \cap B) & \mathsf{P}(A \cap C_\Omega(B)) \\ \mathsf{P}(C_\Omega(A) \cap B) & \mathsf{P}(C_\Omega(A) \cap C_\Omega(B)) \end{bmatrix}.$$

Since B and $C_\Omega(B)$ partition Ω, use the the total law of probability to express is $\mathsf{P}(A) = \mathsf{P}(A \cap B) + \mathsf{P}(A \cap C_\Omega(B))$. Likewise, $\mathsf{P}(C_\Omega(A)) = \mathsf{P}(C_\Omega(A) \cap B) + \mathsf{P}(C_\Omega(A) \cap C_\Omega(B))$. For these reasons, the sum of the entries of the first row of M is $\mathsf{P}(A)$ and the sum of the entries of the second row of M is $\mathsf{P}(C_\Omega(A))$. The events A and B are independent if and only if $\mathsf{rank}(M) = 1$


Joint Probability Matrices
Real Matrix Rank
Dependent Events
Real Matrices
Conditional Event Probabilities
Real Subspaces
Linear Equations
Real Vectors
Linear Functions
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Distance
Plane Vectors
Space Distance
Plane Distance
Event Probabilities
Interval Length
Probability Distributions
Real Space
Intervals
Real Summation
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Arrays
Integer Arithmetic
Lists
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Set Numbers
Direct Products
Natural Summation
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Equivalent Sets
Family Operations
Uncertain Outcomes
Natural Equations
Natural Products
Integer Sums
Integer Order
Comparisons
Function Inverses
Algebras
Family Unions and Intersections
Set Operations
Equations
Natural Sums
Integer Numbers
Orders
Function Composites
Function Images
Families
Operations
Recursion Theorem
Natural Order
Converse Relations
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Partitions
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Dualities
Intersection of Empty Set
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Complements
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects