



## Why

The matrix of a linear system and its row reduction are related by a matrix multiplication.

## Main observation

**Proposition 1.** *Let  $(A \in \mathbf{R}^{n \times n}, b \in \mathbf{R}^n)$  be a linear system. Let  $(C, d)$  be the  $ij$ -reduction of  $(A, b)$ . There exists  $L \in \mathbf{R}^{n \times n}$  so that*

$$C = LA \text{ and } d = Lb.$$

*Proof.* Define  $L \in \mathbf{R}^{n \times n}$  by  $L_{ii} = 0$  and

$$L_{ij} = \begin{cases} 1 & \text{if } i = j \\ -A_{kj}/A_{ij} & \text{if } k \neq i \text{ and } A_{kj} \neq 0 \\ 0 & \text{otherwise .} \end{cases}$$

□

A simple consequence is that there exists  $(L_1, \dots, L_{n-1})$  in  $\mathbf{R}^{n \times n}$  of lower triangular matrices so that the ordinary row reduction of  $(A, b)$  is  $(L_{n-1} \cdots L_2 L_1 A, L_{n-1} \cdots L_2 L_1 b)$ .

**For example**

$$L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -A_{21}/A_{11} & 1 & 0 & \cdots & 0 \\ -A_{31}/A_{11} & 0 & 1 & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & 0 \\ -A_{n1}/A_{11} & 0 & 1 & \ddots & 0 \end{bmatrix}$$



