Subsets

1 Why

We want to speak of sets which contain all the elements of other sets.

2 Two Sets

A **subset** of a set A is any set B for which each element of the set B is an element of the set A. In this case, we say that B is a subset of A. Conversely, we say that A is a **superset** of B.

Every set is a subset of itself. So if the set A is the set B, then A is a subset of B and B is a subset of A. Conversely, if A is a subset of B and B is a subset of A, then A is B. To argue that A is B, we argue that membership in A implies membership in B and second, we argue that membership in B implies membership in A.

The **power set** of a set is the set of all subsets of that set. It includes the set itself and the empty set. We call these two sets **improper subsets** of the set. We call all other sets **proper subsets**.

2.1 Notation

Let A and B be sets. We denote that A is a subset of B by $A \subset B$. We read the notation $A \subset B$ aloud as “A subset B".

If $A \subset B$ and $B \subset A$, then $A = B$. The converse also holds.

We denote the power set of A by 2^A, read aloud as “two to the A." $A \in 2^A$ and $\varnothing \in 2^A$. However, $A \subset 2^A$ is false.

2.2 Examples

Let a, b, c be distinct objects. Let $A = \{a, b, c\}$ and $B = \{a, b\}$. Then $B \subset A$. In other notation, $B \in 2^A$. As always, $\varnothing \in 2^A$ and $A \in 2^A$ as well. In this case, we can list the elements (which are sets) of the power set:

$$2^A = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}.$$