

COMPLEX EXPONENTIAL

Definition¹

The complex exponential of a complex number $z \in \mathbf{C}$ is the complex number

$$1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \dots = \sum_{n=1}^{\infty} \frac{z^n}{n!}.$$

Notation

We denote the complex exponential function $\exp: \mathbf{C} \to \mathbf{C}$, and so denote the complex exponential of $z \in \mathbf{C}$ by $\exp(z)$, as usual. This overloaded notation is justified by the fact that the complex exponential agrees with the real exponential function on reals. It is also common to denote $\exp(z)$ by e^z .

Relation to power series of \sin and \cos

It can be shown² that for $x \in \mathbf{R}$,

$$\exp(ix) = \cos(x) + i\sin(x).$$

¹Future editions may modify this sheet, as there are several (equivalent) characterizations of the complex exponential function.

²And future editions will show it

