



## Why

A probability event function is a measure on the measurable space consisting of the set of outcomes and the set of events.

## Definition

A *probability measure* is a finite measure on a measurable space which assigns the value one to the base set. A finite measure can always be scaled to a probability measure, so these measures are standard examples of finite measures.

A *probability space* is a measure space whose measure is a probability measure. The word “space” is natural, since the notion of a measure generalized the notion of volume in real space (see **Real Space** and **N-Dimensional Space**). The *outcomes* of a probability space are the elements of the base set. The *set of outcomes* is the base set. The *events* are the elements of the sigma algebra. The measure in a probability space corresponds to the event probability function.

## Notation

Let  $(X, \mathcal{A})$  be a measurable space.<sup>1</sup> We denote the sigma-algebra by  $\mathcal{A}$ , as usual. We denote a probability measure by  $\mathbf{P}$ , a mnemonic for “probability,” and intended to remind of the event probability function.<sup>2</sup>

Thus, we often say “Let  $(X, \mathcal{A}, \mathbf{P})$  be a probability space.” Many authors associate an event  $A \in \mathcal{A}$  with a function  $\pi : X \rightarrow \{0, 1\}$  so that  $A = \{x \in X \mid \pi(x) = 1\}$ . In this context, it is common to write  $\mu[\pi(x)]$  for  $\mu(A)$ .

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<sup>1</sup>Often, other authors will denote the set of outcomes (here denoted by  $X$ ) by  $\Omega$ , an apparent mnemonic for “outcomes”.

<sup>2</sup>Often, other authors will use notation including  $P$ ,  $Pr$ ,  $\text{Prob}$ , and  $\text{pr}$ .



