



## Why

We want to talk about optimally eliminating variables in a system of linear equations.<sup>1</sup>

## Definition

An *ordered* undirected graph is *filled* or *monotone transitive* if all higher neighborhoods induce complete subgraphs. An ordering  $\sigma$  of an undirected graph  $(V, E)$  is a *perfect elimination ordering* if the ordered undirected graph  $((V, E), \sigma)$  is filled.

Let  $G = ((V, E), \sigma)$  be an ordered undirected graph.  $G$  is filled if, for all  $v \in V$ ,  $w, z \in \text{adj}^+(v) \longrightarrow \{w, z\} \in E$ . Equivalently stated,  $G$  is filled if, for all  $i < j < k$ ,  $\{\sigma(i), \sigma(j)\} \in E$  and  $\{\sigma(i), \sigma(k)\} \in E$  imply  $\{\sigma(j), \sigma(k)\} \in E$ .

## Chordality

**Prop. 1.** *If  $(V, E, \sigma)$  is a filled graph, then  $(V, E)$  is chordal.*

*Proof.* Take the vertex with the lowest index on a cycle of length greater than three. Take □

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<sup>1</sup>Future editions will expand. For example, this sheet is needed for perfect elimination orderings.



