Why

We use a normal random function model to make a regressor.

Definition

Let $F : \Omega \to (A \to \mathbf{R})$ be a normal random function with mean function $m : A \to \mathbf{R}$ and covariance function $k : A \times A \to \mathbf{R}$ over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Let the family of random variables (or stochastic process) of F be $f : A \to (\Omega \to \mathbf{R})$.

Let e be a normal random vector with mean zero and covariance Σ_e. Let $a^1, \ldots, a^n \in A$. We sometimes call the sequence $a^1, \ldots, a^n$ the *design*. Define $y : \Omega \to \mathbf{R}^d$ by

$$y_i = f(a^i) + e_i$$

We call y the *observation vector* or *observation random vector*. We call e the *error vector* or *noise vector*. In this context, $f(a^i)$ is sometimes called the *signal*.

Let $b^1, \ldots, b^m \in A$. Define $z : \Omega \to \mathbf{R}^d$ by $z_i = f(b^i)$ for $i = 1, \ldots, n$. So z_i is the random variable corresponding to the family at index $b^i \in A$. Then (y, z) is normal. We call the conditional density of z given y the *predictive density* for b given a.

Proposition 1. *Define $m_a \in \mathbf{R}^n$ by $(m(a^1), \cdots, m(a^n))^\top$ and define m_b by $(m(b^1), \cdots, m(b^m))^\top$.*[1] *Define $\Sigma_a \in \mathbf{R}^{n \times n}$ by*

$$\begin{pmatrix} k(a^1, a^1) & \cdots & k(a^1, a^n) \\ \vdots & \ddots & \vdots \\ k(a^n, a^1) & \cdots & k(a^n, a^n) \end{pmatrix}$$

and define $\Sigma_{ba} \in \mathbf{R}^{m \times n}$ by

$$\begin{pmatrix} k(b^1, a^1) & \cdots & k(b^1, a^n) \\ \vdots & \ddots & \vdots \\ k(b^m, a^1) & \cdots & k(b^m, a^n) \end{pmatrix}.$$

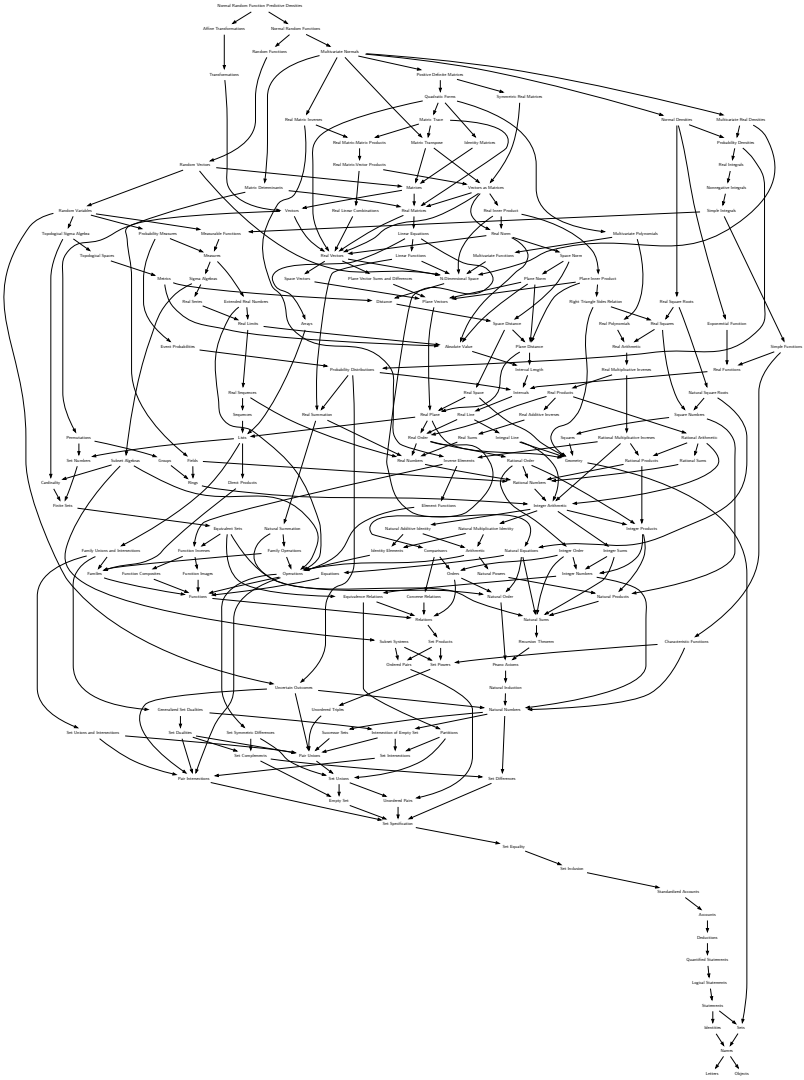
[1] Future editions will fix the re-use of the symbol m.

The predictive density $g_{z|y}(\cdot, \gamma) : \mathbf{R}^m \to \mathbf{R}$ *of* $b \in A$ *for design* $a^1, \ldots, a^n$ *is normal with mean.*

$$m_b + K_{ba} \left(K_a + \Sigma_e\right)^{-1} (\gamma - m_a)$$

and covariance

$$\Sigma_b - \Sigma_{ba} \left(\Sigma_a + \Sigma_e\right)^{-1} \Sigma_{ab}.$$


Normal Random Function Predictor Densities
Affine Transformations
Normal Random Functions
Random Functions
Multivariate Normals
Transformations
Positive Definite Matrices
Quadratic Forms
Symmetric Real Matrices
Real Matrix Inverses
Matrix Trace
Normal Densities
Multivariate Real Densities
Real Matrix-Matrix Products
Matrix Transpose
Identity Matrices
Probability Measures
Real Matrix-Vector Products
Real Integrals
Random Vectors
Matrix Determinants
Matrices
Square Matrices
Nonnegative Integrals
Random Variables
Vectors
Real Linear Transformations
Real Matrices
Real Inner Product
Simple Integrals
Topological Sigma Algebra
Probability Measures
Measurable Functions
Linear Equations
Real Norms
Multivariate Polynomials
Topological Spaces
Linear Functions
Multivariate Functions
Inner Sets
Sets
Sigma Algebra
Square Vectors
Plane Vector Sums and Differences
Euclidean Space
Plane Norms
Plane Inner Product
Metrics
Real Series
Extended Real Numbers
Distance
Plane Vectors
Right Triangle Sides Relation
Real Square Roots
Real Sums
Sums
Square Distance
Real Polynomials
Real Squares
Exponential Function
Event Probabilities
Absolute Value
Plane Distance
Real Arithmetic
Simple Functions
Probability Distributions
Interval Length
Real Multiplication Inverses
Real Functions
Real Sequences
Integers
Real Numbers
Natural Square Roots
Sequences
Real Inequalities
Real Order
Real Addition Inverses
Square Numbers
Real Order
Real Line
Natural Numbers
Rational Multiplication Inverses
Rational Arithmetic
Permutations
Real Subsets
Integer Elements
Rational Order
Integers
Set Numbers
Subset Algebra
Groups
Fields
Real Numbers
Rational Numbers
Rational Products
Rational Sums
Cardinality
Rings
Order Products
Rational Numbers
Finite Sets
Element Functions
Image Intervals
Equivalent Sets
Natural Enumeration
Natural Addition Identity
Natural Multiplication Identity
Image Products
Family Unions and Intersections
Function Inverses
Family Cartesians
Identity Elements
Integers
Natural Numbers
Image Order
Image Sums
Sets
Function Composition
Function Images
Equivalence
Equivalence
Order
Natural Powers
Image Numbers
Functions
Equivalence Relations
Converse Relations
Natural Order
Natural Products
Relations
Natural Sums
Characteristic Functions
Subset Systems
Set Products
Elementary Theorems
Ordered Pairs
Set Powers
Power Axioms
Natural Induction
Unordered Sets
Unordered Triples
Natural Numbers
Set Unions and Intersections
Set Symmetric Differences
Intersection of Empty Set
Partitions
Set Complements
Set Subsets
Pair Intersections
Set Unions
Set Differences
Singleton Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Axioms
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Symbols
Letters
Digraphs