Why

[1]

Definition

Let $(X, \mathbf{R})$ be a vector space. A function $f : X \times X \to \mathbf{R}$ is an *inner product* on the vector space $(X, \mathbf{R})$ if

1. $f(x, x) \geq 0, = 0 \longleftrightarrow x = 0$,
2. $f(x + y, z) = f(x, z) + f(y, z)$,
3. $f(x, y) = f(y, x)$, and
4. $f(\alpha x, y) = \alpha f(x, y)$.

An *inner product space* is an ordered pair: a real vector space and an inner product.[2]

Examples

$\mathbf{R}^n$ with the usual inner product is an inner product space. Some authors call any finite-dimensional inner product space over the real numbers is a *Euclidean vector space*.

Examples

If $f : X \times X \to \mathbf{R}$ is an inner product we regularly denote $f(x, x)$ by $\langle x, x \rangle$.

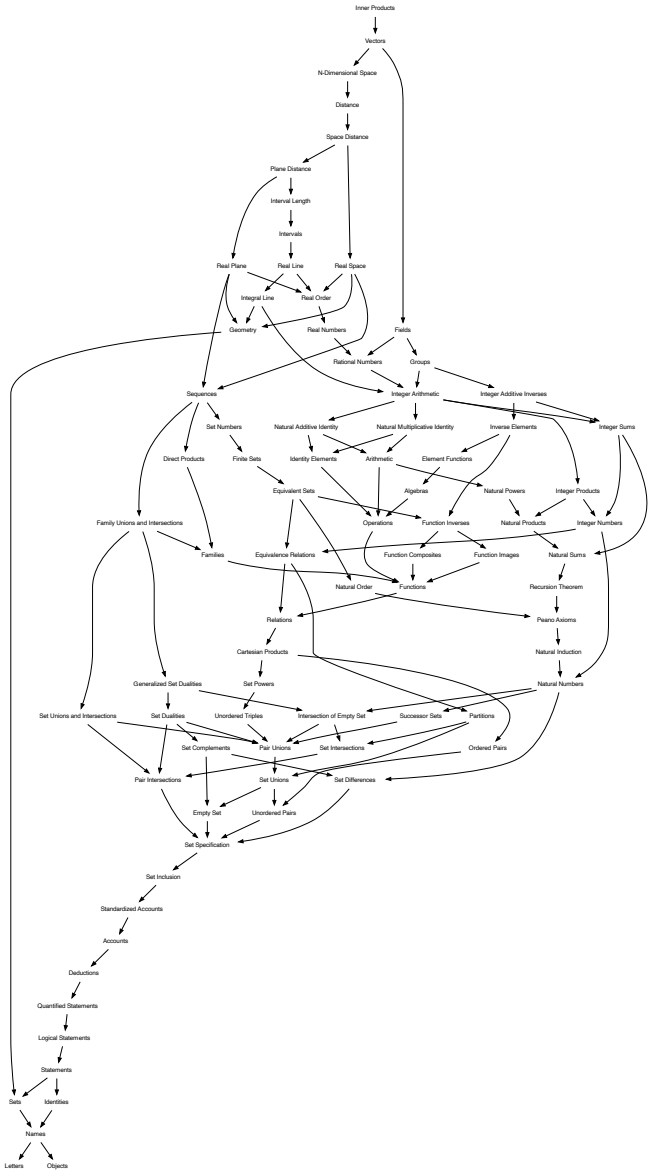
[1] Future editions will complete and rework this sheet.

[2] Future editions will discuss complex inner products.

Orthogonality

Two vectors in an inner product space are *orthogonal* if their inner product is zero. An *orthogonal family of vectors* in an inner product space is a family of vectors for which distinct family members are orthogonal.

A vector is *normalized* if its inner product with itself is one.


Inner Products
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Plane
Real Line
Real Space
Integral Line
Real Order
Geometry
Real Numbers
Fields
Rational Numbers
Groups
Sequences
Integer Arithmetic
Integer Additive Inverses
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Inverse Elements
Integer Sums
Direct Products
Finite Sets
Identity Elements
Arithmetic
Element Functions
Equivalent Sets
Algebras
Natural Powers
Integer Products
Operations
Function Inverses
Natural Products
Integer Numbers
Family Unions and Intersections
Families
Equivalence Relations
Function Composites
Function Images
Natural Sums
Natural Order
Functions
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects