



## SEQUENCES

### Why

We introduce language for the steps of an infinite process.

### Definition

A *finite sequence* is a family whose index set is a natural number (excluding zero). An *infinite sequence* is a family whose index set is the set of natural numbers (without zero). The *nth term* of a sequence (finite or infinite) is the result of the *nth* natural number. Let  $A$  be a non-empty set. A sequence in  $A$  is a function from the natural numbers to the set.

### Notation

Let  $A$  be a non-empty set. Let  $a : \mathbf{N} \rightarrow A$ . Then  $a$  is a sequence in  $A$ .  $a(n)$  is the *nth* term. We also denote  $a$  by  $(a_n)_n$  and  $a(n)$  by  $a_n$ .

### Natural Unions and intersections

If  $\{A_i\}$  is a finite sequence of sets indexed by  $\{1, 2, \dots, n\}$ , then we denote the union of the family by

$$\cup_{i=1}^n A_i$$

If  $\{A_i\}$  is an infinite sequence of sets, then we denote the union of the family by

$$\cup_{i=1}^{\infty} A_i.$$

Similarly, we denote the intersections of a finite and infinite sequence of sets  $\{A_i\}$  by

$$\cap_{i=1}^n A_i \quad \text{and} \quad \cap_{i=1}^{\infty} A_i.$$

respectively.

