Why

We want to talk about probability over measurable (not necessarily finite) sets. We will use our intuition from probability distributions.

Definition

The principal difficulty is assigning nonzero numbers to infinitely many elements of a set.

Consider the set of natural numbers. suppo Suppose the set that we can not assign nonzero probability to each element of the set. There are infinitely many, and so We can not assign nonzero probability to each individual element of the set, since there are infinitely many it would never normalize. Instead, we assign probability to subsets of the set. In other words, we associate a measure with the set. The measure is normalized, meaning that the measure of the whole set, the event corresponding to any outcome occuring is 1.

A *probability density* (or *probability density function*, *pdf*) is a real-valued function from a set of outcomes which is nonnegative and normalized. A real-valued function on measurable space is *normalized* if the measure if its integral over the space is 1.