Real Convergence

Why

We want to talk about sequences of real numbers which, as we go further and further in the sequence, get closer and closer to some fixed real number. Not all sequences have this property, but many do.

Definition

Consider a sequence of real numbers. If the sequence is going to end up somewhere in the real numbers, we need to know where. Consider a real number. Suppose that for any positive real number, no matter how small, there exists a natural number so that the any term corresponding to a natural number larger than the first natural number is no more different than that positive number from the real number considered.

In this case, we call the real number a *limit* of the sequence. We say that the sequence *converges* to that real number. Not all sequences *converge*.

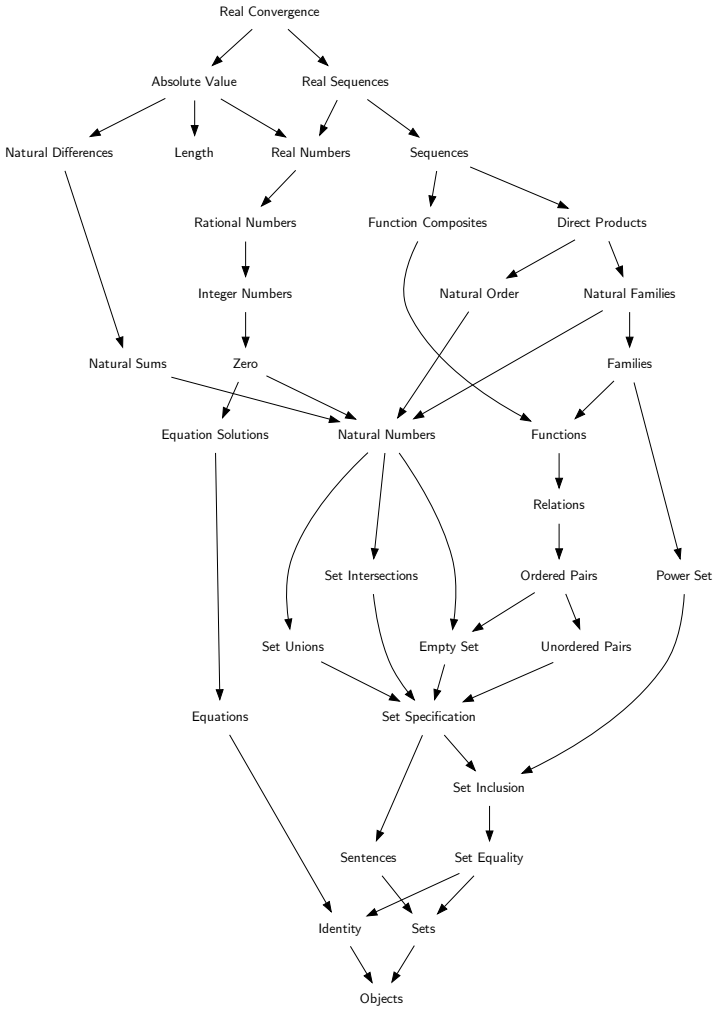
Notation

Let $x : \mathsf{N} \to \mathsf{R}$ be a sequence of real numbers. Let $y \in \mathsf{R}$.

First, suppose that for each $\varepsilon > 0$ there exists an $n_\varepsilon \in \mathsf{N}$ so that $|x_n - y| < \varepsilon$ for each $n \geq n_\varepsilon$. In this case, x converges to y. The real number y is the limit of the sequence x.

Second, suppose that there exists an $\varepsilon > 0$ so that for each

$n \in N$, there exists $m_n \geq n$ so that $|x_{m_n} - y| > \varepsilon$ for each $y \in \mathbf{R}$. In this case x does not converge.


Real Convergence
Absolute Value
Real Sequences
Natural Differences
Length
Real Numbers
Sequences
Rational Numbers
Function Composites
Direct Products
Integer Numbers
Natural Order
Natural Families
Natural Sums
Zero
Families
Equation Solutions
Natural Numbers
Functions
Relations
Set Intersections
Ordered Pairs
Power Set
Set Unions
Empty Set
Unordered Pairs
Equations
Set Specification
Set Inclusion
Sentences
Set Equality
Identity
Sets
Objects