Why

[1]

Definition

Define $S \in \mathbf{R}^{d \times d}$ by

$$S = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

For a vector $x \in \mathbf{R}^d$ the *down shift* of x is Sx.

Let $A \in \mathbf{R}^{d \times d}$ be a matrix with columsn $a_1, \dots, a_d$. A is a *circulant matrix* if $a_1 = Sa_d$, $a2 = Sa_2$, and $a_i = A_{i-1}$ for $i = 2, \dots, d$.

Example

For example, the matrix

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

is a circulant matrix.

Characterization

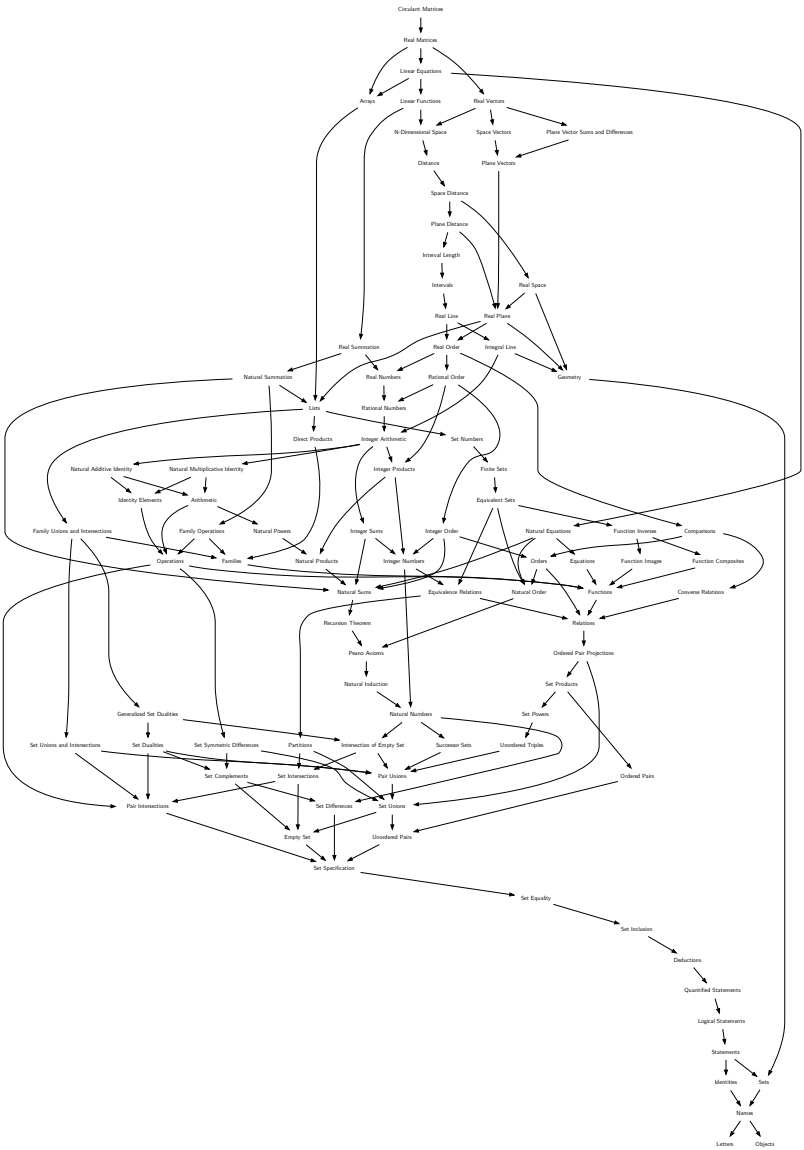
A matrix $C \in \mathbf{R}^{d \times d}$ is circulant if and only if there exists $c_0, \dots, c_{d-1}$ so that

$$C = c_0 I + c_1 S + c_2 S^2 + \cdot s + c_{n-1} S^{n-1}.$$

[1] Future sheets will include. These matrices arise in practice and each has the same eigenvectors.

Properties

The sum and product of any two circulant matrix is circulant. In other words, the circulant matrices with the usual matrix addition and multiplication form a commutative ring.


Circulant Matrices
Real Matrices
Linear Equations
Arrays
Linear Functions
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Summation
Real Order
Integral Line
Natural Summation
Real Numbers
Rational Order
Geometry
Lists
Rational Numbers
Direct Products
Integer Arithmetic
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Family Unions and Intersections
Family Operations
Natural Powers
Integer Sums
Integer Order
Natural Equations
Function Inverses
Comparisons
Operations
Families
Natural Products
Integer Numbers
Orders
Equations
Function Images
Function Composites
Natural Sums
Equivalence Relations
Natural Order
Functions
Converse Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Set Symmetric Differences
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Complements
Set Intersections
Pair Unions
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects