



Definition

An optimization problem is (X, f) is an *inequality constrained space optimization problem* if $X \subset \mathbf{R}^n$, $f : \mathbf{R}^n \rightarrow \mathbf{R}$, and there exists $g : \mathbf{R}^n \rightarrow \mathbf{R}^m$ so that

$$X = \{x \in \mathbf{R}^n \mid g(x) \leq 0\}$$

For this reason, (f, g) is sometimes called the *problem data* (*abstract problem data*) of the problem.

Notation

We often write such problems as: given $f : \mathbf{R}^n \rightarrow \mathbf{R}$ and $g : \mathbf{R}^n \rightarrow \mathbf{R}^m$, find $x \in \mathbf{R}^n$ to

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & g(x) \leq 0 \end{array}$$

Some authors abbreviate inequality constrained space optimization problem as ICP.

Handles equality constraints

Suppose $f : \mathbf{R}^n \rightarrow \mathbf{R}$ and $h : \mathbf{R}^n \rightarrow \mathbf{R}^m$ are the (abstract) problem data for an *equality* constrained space optimization problem. Define $g : \mathbf{R}^n \rightarrow \mathbf{R}^{2m}$ so that

$$g(x) = (h(x), -h(x)) \quad \text{for all } x \in \mathbf{R}^n$$

Then the ECP (f, h) and ICP (f, g) have the same feasible set and optimal solutions. In other words, given an equality constrained problem we can always write it as an inequality constrained problem.

