Why

We are regularly referring to a few common growth classes.

Definitions

Let $c \in \mathbf{R}$. Then we name the following growth classes

growth class	name
$O(1)$	*constant growth class*
$O(\log(x))$	*logarithmic growth class*
$O(\log(x)^c)$	*polylogarithmic growth class*
$O(x)$	*linear growth class*
$O(x^2)$	*quadratic growth class*
$O(x^c)$	*polynomial growth class*
$O(c^x)$	*exponential growth class*

We have written these in order:

$$O(1) \subset O(\log(x)) \subset O((\log(x))^c) \subset \cdots \subset O(x^c) \subset O(c^x).$$

A function that grows faster (is in the upper growth class) of a power of x is called *superpolynomial*. One that grows slower than c^n for some $c \in \mathbf{R}$ is called *subexponential*. The class $O(\log(x^c))) = O(\log(x))$ since $\log(x^c) = c \log x$. Similarly, for all $c_1, c_2 > 0$, $O(\log_{c_1}(x)) = O(\log_{c_2}(x))$.

This list is useful because of the following

Proposition 1. *Let $f, g : \mathbf{R} \to \mathbf{R}$ and defined $h : \mathbf{R} \to \mathbf{R}$ by $h = f + g$. If $O(f) \subset O(g)$, then $h \in O(g)$.*

In other words, if a function h is the sum of f and g and g is growing faster, then g (the one growing faster) determines the order of h.

Common Growth Classes

Function Growth Classes — Logarithm — Polynomials

Absolute Value — Exponential Function — Real Polynomials

Interval Length — Real Functions — Real Arithmetic

Intervals — Real Multiplicative Inverses

Real Products — Rings

Real Line — Real Additive Inverses

Real Order — Real Sums — Rational Arithmetic — Rational Multiplicative Inverses

Rational Order — Real Numbers — Rational Sums — Rational Products

Integral Line — Rational Numbers — Lists

Integer Arithmetic — Set Numbers — Direct Products

Integer Products — Natural Multiplicative Identity — Natural Additive Identity — Finite Sets

Arithmetic — Identity Elements — Equivalent Sets — Inverse Elements

Integer Order — Comparisons — Integer Sums — Natural Powers — Element Functions — Function Inverses — Family Unions and Intersections

Orders — Integer Numbers — Natural Products — Operations — Function Composites — Function Images — Functions

Natural Sums — Natural Order — Converse Relations — Equivalence Relations — Families

Recursion Theorem — Relations

Peano Axioms — Ordered Pair Projections

Natural Induction — Set Products

Natural Numbers — Set Powers — Generalized Set Dualities — Set Unions and Intersections

Successor Sets — Unordered Triples — Set Symmetric Differences — Intersection of Empty Set — Set Dualities — Partitions

Ordered Pairs — Pair Unions — Set Complements — Set Intersections

Set Differences — Set Unions — Pair Intersections

Unordered Pairs — Empty Set

Geometry — Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects