



## Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?<sup>1</sup>

## Definition

Two random variables are *independent* if the sigma algebras generated by the random variables are independent. In general, a family of random variables are *independent* if the sigma algebras generated by the random variables are independent.

## Notation

Let  $(X, \mathcal{A}, \mu)$  be a probability space and  $(Y, \mathcal{B})$  be a measurable space. Let  $f_1, f_2 : X \rightarrow Y$  be random variables. If the random variables are independent we write  $f_1 \perp f_2$ .

## Results

**PROPOSITION 1.** *Let  $f_1, \dots, f_n$  be independent real-valued random variables defined on a probability space  $(X, \mathcal{A}, \mu)$ .*

*Let  $B_1, \dots, B_n$  be Borel sets of real numbers and let  $A_i = f_i^{-1}(B_i)$ . Let  $A = \cap_{i=1}^n f_i^{-1}(B_i)$ . Then*

$$\mu(A) = \prod_{i=1}^n \mu(A_i)$$

*Proof.* Since  $f_i$  are independent, so are the sigma algebras they

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<sup>1</sup>Future editions will modify this.

generate.  $A_i$  are in each of these sigma algebras, so by definition of independence the measure of the intersection is the product of the measures.  $\square$

