Relations

1 Why

We want to relate elements of two sets.

2 Definition

A **relation** between two non-empty sets A and B is a subset of $A \times B$. A relation on a single set C is a subset of $C \times C$.

2.1 Notation

We denote relations with upper case capital latin letters because they are sets. Let R be a relation on A and B. We denote that $(a, b) \in R$ by aRb, read aloud as "a in relation R to b."

Often, instead of latin letters we use other symbols. For example, $\sim$, $=$, $<$, $\leq$, $\prec$, and $\preceq$.

3 Properties

Let R be a relation on a non-empty set A. R is **reflexive** if

$$(a, a) \in R$$

for all $a \in A$. R is **transitive** if

$$(a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R$$

for all $a, b, c \in A$. R is **symmetric** if

$$(a, b) \in R \implies (b, a) \in R$$

for all $a, b \in A$. R is **anti-symmetric** if

$$(a, b) \in R \implies (b, a) \notin R$$

for all $a, b \in A$.