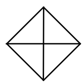


Why

Certain sparse matrices are easier to work with, especially those with chordal sparsity patterns.[1]

Definition

A *sparsity pattern* E of *order* n is a set of (unordered) pairs of $V = \{1, \dots, n\}$. A sparsity pattern is *chordal* if the undirected graph (V, E) is chordal.

A symmetric matrix is said to *have a sparsity pattern* if its ijth entry is zero whenever $\{i, j\}$ is not in the sparsity pattern. The diagonal entries and off-diagonal entries for pairs appearing in the sparsity pattern may or may not be zero.

The graph whose vertices are one through n and whose edge set is the sparsity patter is called the *sparsity graph*

A sparsity pattern is not a property of a matrix because it is not unique (unless all off-diagonal entries are non-zero). If a matrix has a particular sparsity pattern it has every sparsity pattern which is a superset of it. In other words, every matrix has the sparsity pattern which is the set of all pairs of integers.

Notation

Let $E \subset \{\{i, j\} \mid i, j \in \{1, 2, \dots, n\}\}$. A symmetric matrix $A \in \mathbf{S}^n$ is said to have sparsity pattern E if $A_{ij} = A_{ji} = 0$ wheneer $i \neq j$ and $\{i, j\} \notin E$. The graph $G = (V, E)$ where $V = \{1, 2, \dots, n\}$ is the sparsity graph associated with E.

We will denote the symmetric matrices of order n with sparsity pattern E by $\mathbf{S}^n_E$.

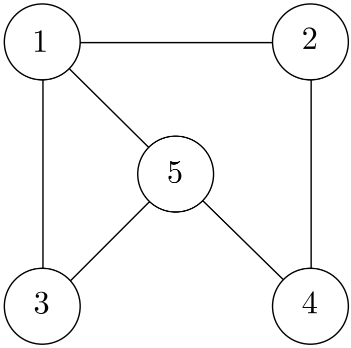


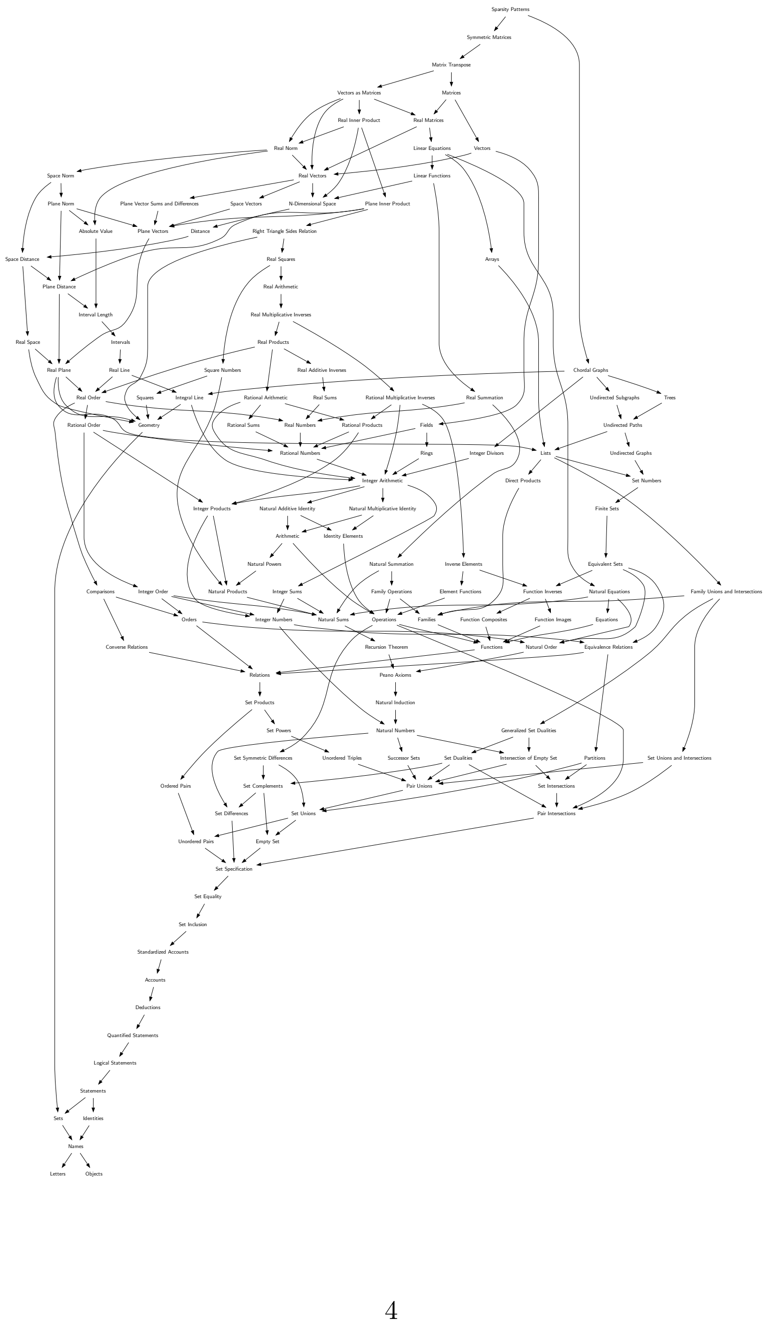
Figure 1: Sparsity graph for the matrix A.

Example

Figure 1 shows a sparsity graph for the matrix

$$A := \begin{bmatrix} A_{11} & A_{21} & A_{31} & 0 & A_{51} \\ A_{21} & A_{22} & 0 & A_{42} & 0 \\ A_{31} & 0 & A_{33} & 0 & A_{54} \\ 0 & A_{42} & 0 & A_{44} & A_{45} \\ A_{51} & 0 & A_{53} & A_{54} & A_{55} \end{bmatrix}.$$

[1]Future editions will expand.


Sparsity Patterns
Symmetric Matrices
Matrix Transpose
Vectors as Matrices
Matrices
Real Inner Product
Real Matrices
Real Norm
Linear Equations
Vectors
Space Norm
Real Vectors
Linear Functions
Plane Norm
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Inner Product
Absolute Value
Plane Vectors
Distance
Right Triangle Sides Relation
Space Distance
Real Squares
Arrays
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Real Space
Intervals
Real Products
Real Plane
Real Line
Square Numbers
Real Additive Inverses
Chordal Graphs
Real Order
Squares
Integral Line
Rational Arithmetic
Real Sums
Rational Multiplicative Inverses
Real Summation
Undirected Subgraphs
Trees
Rational Order
Geometry
Rational Sums
Real Numbers
Rational Products
Fields
Undirected Paths
Rational Numbers
Rings
Integer Divisors
Lists
Undirected Graphs
Integer Arithmetic
Direct Products
Set Numbers
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Natural Powers
Natural Summation
Inverse Elements
Equivalent Sets
Comparisons
Integer Order
Natural Products
Integer Sums
Family Operations
Element Functions
Function Inverses
Natural Equations
Family Unions and Intersections
Orders
Integer Numbers
Natural Sums
Operations
Families
Function Composites
Function Images
Equations
Converse Relations
Recursion Theorem
Functions
Natural Order
Equivalence Relations
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Unordered Triples
Successor Sets
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects