

CODES

Why

Can we generalize the idea of flash codes.¹

Definition

Let X be a set and a A be an alphabet set. We denote the set of all finite sequences (strings) in A by S(A). We read S(A) aloud as "the strings in A." The length zero string is \emptyset .

A code for X in A is a function from X to S(A). In this context, we refer to the finite set A as an alphabet and we call c(x) the codeword of x. The length of $x \in X$, with respect to a code $c: X \to S(A)$, is the length of the sequence c(x) (its codeword). We call a code nonsingular if it is injective.

Examples

Define
$$c : \{\alpha, \beta\} \to \{0, 1\}$$
 by $c(\alpha) = (0, 1)$ and $c(\beta) = (1, 1)$.

Code extensions

Let $s, t \in \mathcal{S}(A)$ of length m and n respectively. The concatenation of s with t is the length m + n string $u \in \mathcal{S}(A)$ defined by $u_1 = s_1, \ldots, u_m = s_m$ and $u_{m+1} = t_1, \ldots, u_{m+n} = t_n$. We denote the concatenation of s and t by st. Note, however, that $st \neq ts$, although s(tr) = (st)r.

Given a code $c: X \to \mathcal{S}(A)$, we can produce a code for $\mathcal{S}(X)$ in a natural way. The *extension* of c is the function $C: \mathcal{S}(X) \to \mathcal{S}(A)$

¹The reliance of this sheet on Flash Codes and Dot-Dash Codes is for this justification, and not for any of the terms presented.

²Future editions will include additional examples.

defined, for $\xi = (\xi_1, \dots, \xi_n) \in \mathcal{S}(X)$, by

$$C(\xi) = c(\xi_1) \cdots c(\xi_n).$$

We call an code uniquely decodable if its extension is injective. In other words, given the code $C(\xi)$ for a sequence $\xi \in \mathcal{S}(X)$, we can recover ξ . We call $C(\xi)$ the encoding of ξ . We call ξ the decoding of $C(\xi)$.

