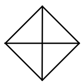


## Why

There is a natural orientation of an ordered undirected graph.

## Motivating result

An ordered undirected graph can be converted into a directed graph by orienting the edges from lower to higher index. The *orientation* of an ordered undirected graph $((V, E), \sigma)$ is the directed graph $(V, F)$ where

$$\{v, w\} \in V \longrightarrow (v, w) \in F \text{ and } \sigma^{-1}(v) < \sigma^{-1}(w).$$

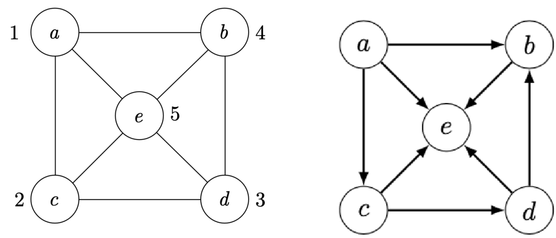
In other words, we can "convert" the ordered undirected graph by "orienting" the edges from lower to higher index.

**Proposition 1.** *Let $G = ((V, E), \sigma)$ be an ordered undirected graph. The orientation of $G$ is acyclic.*

*Proof.* Contradiction on the existence of a cycle.[1] □

Conversely, let $(V, F)$ be directed acyclic. To each topological numbering $\sigma$ of $(V, F)$ (see Directed Paths) there exists an ordered undirected graph $((V, E), \sigma)$ where $(V, E)$ is the skeleton of $(V, F)$.

## Example




[1] Future editions will expand.

Let $G = ((V, E), \sigma)$ be an undirected graph with

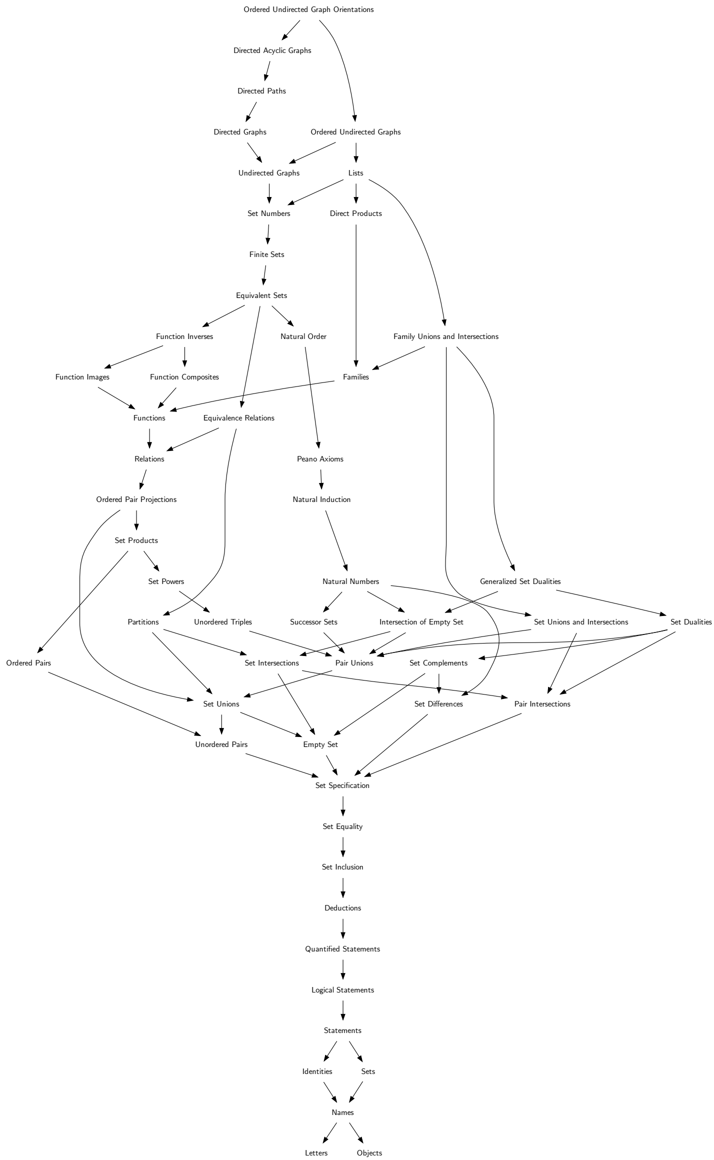
$$V = \{a, b, c, d, e\},$$

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\},$$

and

$$\sigma(1) = a \quad \sigma(2) = c \quad \sigma(3) = d \quad \sigma(4) = b \quad \sigma(e) = 5.$$

We visualize $G$ and its (directed acyclic) orientation above.


Ordered Undirected Graph Orientations
Directed Acyclic Graphs
Directed Paths
Directed Graphs
Ordered Undirected Graphs
Undirected Graphs
Lists
Set Numbers
Direct Products
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Images
Function Composites
Families
Functions
Equivalence Relations
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Unions and Intersections
Set Dualities
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects