

INDEPENDENT RANDOM VARIABLES

Why

What does it mean for two random variables to be independent? What are the events associated with a random variable?

Definition

Two random variables are *independent* if the sigma algebras generated by the random variables are independent. In general, a family of random variables are *independent* if the sigma algebras generated by the random variables are independent.

Notation

Let (X, \mathcal{A}, μ) be a probability space and (Y, \mathcal{B}) be a measurable space. Let $f_1, f_2 : X \to Y$ be random variables. If the random variables are independent we write $f_1 \perp f_2$.

Results

Proposition 1. Let f_1, \ldots, f_n be independent real-valued random variables defined on a probability space (X, \mathcal{A}, μ) . Let B_1, \ldots, B_n be Borel sets of real numbers and let $A_i = f_i^{-1}(B_i)$. Let $A = \bigcap_{i=1}^n f_i^{-1}(B_i)$. Then

$$\mu(A) = \prod_{i=1}^{n} \mu(A_i)$$

Proof. Since f_i are independent, so are the sigma algebras they generate. A_i are in each of these sigma algebras, so by definition of independence the measure of the intersection is the product of the measures.

