Why

It happens that if a distribution factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So the property of factoring according to a rooted tree is really property of factoring according to a tree.

Definition

0.1 Defining Result

Prop. 1. *Let $A_1, \ldots, A_n$ be finite non-empty sets and define $A = \prod_{i=1}^{n} A_n$. Let $p : A \to [0, 1]$ be a distribution and let T be a tree on $\{1, \ldots, n\}$.*

Prop. 2. *If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

Undirected Definition

A distribution p *factors according to the tree T* if it factors according to the T rooted at any vertex.

Existence and Uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

Existence

A distribution p need not factor according to a tree.

Uniqueness

A distribution p may factor according to multiple trees.

Tree Distributions

Rooted Tree Distributions

Conditional Distributions

Conditional Event Probability

Marginal Distributions

Event Probabilities

Joint Distributions

Probability Distributions

Intervals

Real Line

Real Order

Real Summation

Rooted Trees

Real Numbers

Trees

Directed Graphs

Integral Line

Rational Numbers

Undirected Paths

Integer Arithmetic

Sequences

Undirected Graphs

Natural Multiplicative Identity

Natural Additive Identity

Direct Products

Set Numbers

Arithmetic

Identity Elements

Natural Summation

Finite Sets

Integer Products

Natural Powers

Family Operations

Equivalent Sets

Integer Sums

Natural Products

Set Operations

Algebras

Family Unions and Intersections

Function Inverses

Integer Numbers

Natural Sums

Operations

Families

Function Composites

Function Images

Recursion Theorem

Natural Order

Functions

Equivalence Relations

Peano Axioms

Relations

Natural Induction

Uncertain Outcomes

Cartesian Products

Natural Numbers

Set Powers

Generalized Set Dualities

Set Unions and Intersections

Partitions

Set Symmetric Differences

Successor Sets

Unordered Triples

Set Dualities

Intersection of Empty Set

Set Intersections

Ordered Pairs

Set Complements

Pair Unions

Set Unions

Pair Intersections

Set Differences

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects