Why

An integral is a limit. When can we exchange this limit with another? We give a first result in the search for sufficient conditions to do so.

Result

When context is clear, we refer to the following proposition as the *dominated convergence theorem*.

Prop. 1. *The integral of the almost everywhere limit of a sequence of measurable, extended-real-valued, almost-everywhere bounded functions is the limit of the sequence of integrals of the functions.*

Proof. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $f : X \to [-\infty, \infty]$ be a $\mathcal{A}$-measurable function. Let $f_n :\to [-\infty, \infty]$ a $\mathcal{A}$-measurable function for every natural number n so that $(f_n)_n$ converges almost everywhere to f. Let $g : X \to [0, \infty]$ be an integrable function which dominates f_n almost everywhere for each n. We want to show that:

$$\int f d\mu = \lim_n \int f_n d\mu.$$

$\square$

Real Integral Dominated Convergence

Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions

Measures

Sigma Algebras

Real Series

Extended Real Numbers

Real Limits

Simple Functions

Absolute Value

Real Functions

Interval Length

Intervals

Real Line

Real Order

Real Sequences

Rational Order

Real Numbers

Sequences

Rational Numbers

Integral Line

Lists

Subset Algebras

Integer Arithmetic

Cardinality

Set Numbers

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Finite Sets

Arithmetic

Identity Elements

Natural Powers

Equivalent Sets

Operations

Direct Products

Comparisons

Integer Order

Integer Sums

Natural Products

Function Inverses

Family Unions and Intersections

Orders

Integer Numbers

Natural Sums

Function Images

Function Composites

Families

Converse Relations

Recursion Theorem

Natural Order

Equivalence Relations

Functions

Relations

Peano Axioms

Subset Systems

Set Products

Characteristic Functions

Natural Induction

Ordered Pairs

Set Powers

Natural Numbers

Generalized Set Dualities

Partitions

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Symmetric Differences

Set Dualities

Set Unions and Intersections

Set Intersections

Pair Unions

Set Complements

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Geometry

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects