



## Why

How many ways are there to split  $n$  objects into nonoverlapping groups, when the objects are indistinguishable?

## Definition

Suppose  $n$  is a nonzero natural number. A *partition* of  $n$  is a *nonincreasing* list of *nonzero* natural numbers whose sum is  $n$ . The requirement that the list of numbers be nonincreasing makes the representation unique. The terms of the list are called the *parts* of the partition. The number  $n$  is sometimes called the *weight* of the partition. The number of times a particular number appears in the list is called the *multiplicity* of that part.

## Examples

What are the partitions of the number 5?

$$\begin{aligned}
 5 &= 5 \\
 &= 4 + 1 \\
 &= 3 + 2 \\
 &= 3 + 1 + 1 \\
 &= 2 + 2 + 1 \\
 &= 2 + 1 + 1 + 1 \\
 &= 1 + 1 + 1 + 1 + 1
 \end{aligned}$$

These seven identities correspond to the seven partitions of 5, namely  $(5, )$ ,  $(4, 1)$ ,  $(3, 2)$ ,  $(3, 1, 1)$ ,  $(2, 2, 1)$ ,  $(2, 1, 1, 1)$ ,  $(1, 1, 1, 1, 1)$ . The multiplicity of 1 in these partitions is 0, 1, 0, 2, 1, 3, 5, respectively.

## Notation

Suppose  $\lambda$  is a list in  $\mathbf{N}$  of length  $r \geq 1$ . Then  $\lambda = (\lambda_1, \dots, \lambda_r)$  is a *partition* of  $n \in \mathbf{N}$  if

$$\lambda_1 + \lambda_2 + \dots + \lambda_r = n$$

The terms of  $\lambda$  are called the *parts* of the partition.  $\lambda_i$  is the  $i$ th part, where  $i = 1, \dots, r$ .  $\lambda$  is a nonincreasing list means

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_r$$

Some authors denote the *weight* of  $\lambda$  by  $|\lambda|$ .

## Partition function

How many partitions are there of the number  $n$ ? We denote *this number* by  $p(n)$ . From the examples above,  $p(5) = 7$ .



