Why

We want to define area under a real function. We start with defining functions for which this notion is obvious.

Definition

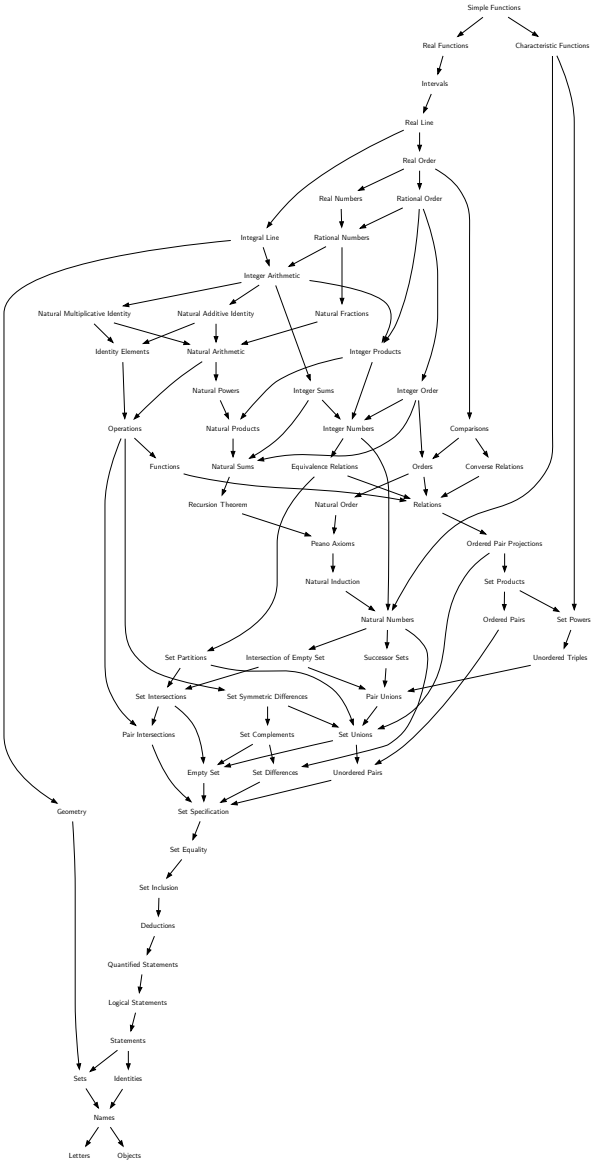
A *simple function* (or *step function*) is a real-valued function whose range is a finite set. We can write simple function as the sum of the characteristic functions fo the inverse image elements.

We can partition the range of the function into a finite family of one-elements sets. Then the family whose members are the inverse images of these sets partitions the domain. We call this the *simple partition* of the function.

Notation

We denote the set of simple functions on A by $\mathrm{SF}(A)$. We denote subset of non-negative simple real functions with domain A by by $\mathrm{SF}_+(A)$.

Let $f \in \mathrm{SF}(A)$. Let $\{a_1, \ldots, a_n\} = f(A)$. Define $A_i = f^{-1}(\{a_i\})$. Then $f = \sum_{i=1}^{n} a_i \chi_{A_i}$.


Simple Functions
Real Functions
Characteristic Functions
Intervals
Real Line
Real Order
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Natural Fractions
Identity Elements
Natural Arithmetic
Integer Products
Natural Powers
Integer Sums
Integer Order
Operations
Natural Products
Integer Numbers
Comparisons
Functions
Natural Sums
Equivalence Relations
Orders
Converse Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Ordered Pairs
Set Powers
Set Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects