



# Sparsity Patterns

## 1 Why

TODO

## 2 Definition

A *sparsity pattern* of order  $n$  is a set of pairs of natural numbers from one to  $n$ .

A symmetric matrix is said to *have a sparsity pattern* if its  $ij$ th entry is zero whenever  $\{i, j\}$  is not in the sparsity pattern. The diagonal entries and off-diagonal entries for pairs appearing in the sparsity pattern may or may not be zero.

The graph whose vertices are one through  $n$  and whose edge set is the sparsity pattern is called the *sparsity graph*.

A sparsity pattern is not a property of a matrix because it is not unique (unless all off-diagonal entries are non-zero). If a matrix has a particular sparsity pattern it has every sparsity pattern which is a superset of it. In other words, every matrix has the sparsity pattern which is the set of all pairs of integers.

### 2.1 Notation

Let  $E \subset \{\{i, j\} \mid i, j \in \{1, 2, \dots, n\}\}$ . A symmetric matrix  $A \in \mathbf{S}^n$  is said to have sparsity pattern  $E$  if  $A_{ij} = A_{ji} = 0$  whenever  $i \neq j$  and

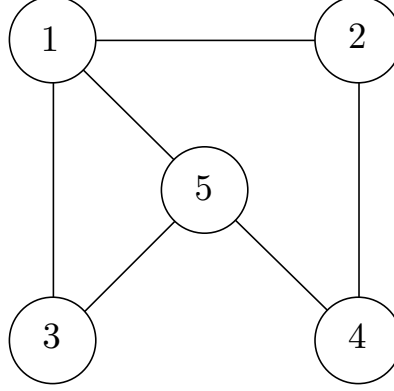


Figure 1: Sparsity graph for the matrix  $()$ .

$\{i, j\} \notin E$ . The graph  $G = (V, E)$  where  $V = \{1, 2, \dots, n\}$  is the sparsity graph associated with  $E$ .

We will denote the symmetric matrices of order  $n$  with sparsity pattern  $E$  by  $\mathbf{S}_E^n$ .

### 3 Example

Figure 1 shows a sparsity graph for the matrix

$$A := \begin{bmatrix} A_{11} & A_{21} & A_{31} & 0 & A_{51} \\ A_{21} & A_{22} & 0 & A_{42} & 0 \\ A_{31} & 0 & A_{33} & 0 & A_{54} \\ 0 & A_{42} & 0 & A_{44} & A_{45} \\ A_{51} & 0 & A_{53} & A_{54} & A_{55} \end{bmatrix}.$$