Why

We want to subtract numbers.[1]

Definition

Consider the set $\omega \times \omega$. This set is the set of ordered pairs of ω. In other words, the ordered pairs of natural numbers.

We call two such pairs (a, b) and (c, d) of $\omega \times \omega$ *integer equivalent* if

$$a + d = b + c$$

Briefly, the intuition is that (a, b) represents a less b, or in the usual notation "$a - b$".[2] So this equivalence relation says these two are the same if $a - b = c - d$. Rearranging gives $a + d = b + c$.

Proposition 1. *Integer equivalence is an equivalence relation.*[3]

The *set of integer numbers* is the set of equivalence classes (see Equivalence Relations) under integer equivalence on $\omega \times \omega$. We call an element an *integer number* (or *integer*).

Notation

We denote the set of integers by **Z**. If we denote integer equivalence by $\sim$ then $\mathbf{Z} = (\omega \times \omega)/\!\sim$.

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[1]Future editions will change this why. In particular, by referencing Inverse Elementsand the lack thereof in ω.

[2]This account will be expanded in future editions.

[3]The proof is straightforward. It will be included in future editions.

Integer Numbers

Equivalence Relations

Relations

Ordered Pair Projections

Set Products

Set Powers

Unordered Triples

Natural Numbers

Successor Sets

Intersection of Empty Set

Ordered Pairs

Partitions

Pair Unions

Set Unions

Set Intersections

Set Differences

Unordered Pairs

Empty Set

Pair Intersections

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects