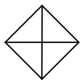


Why

What if the best estimator for a real-value random variable if we consider the squared loss.

Definition

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $x : \Omega \to \mathsf{R}$ a random variable.

A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value $\xi \in \mathsf{R}$ which minimizes $\mathsf{E}(x - \xi)^2$.

Proposition 1. *There is a unique MMSE estimator and it is given by* $\mathsf{E}(x)$.

Vector Case

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space and $y : \Omega \to \mathsf{R}^n$ a random variable.[1]

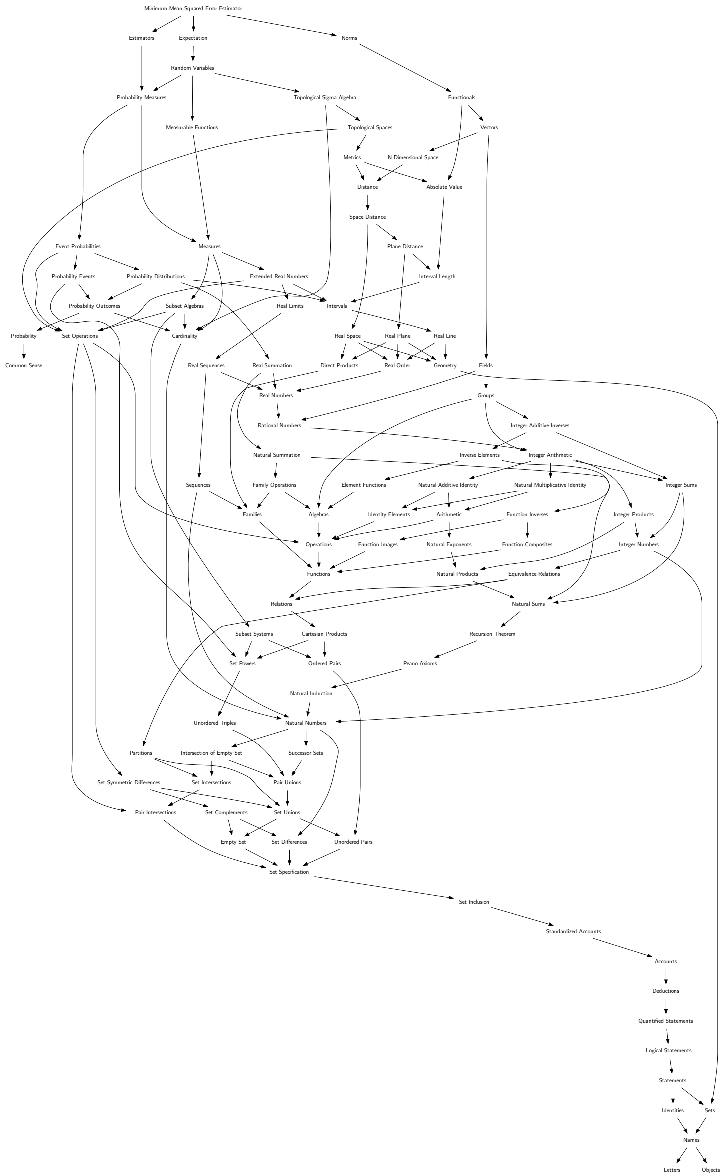
A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value $\xi \in \mathsf{R}$ which minimizes $\mathsf{E}|x - \xi|^2$.

Proposition 2. *There is a unique MMSE estimator and it is given by* $\mathsf{E}(y)$.

[1] Future editions might collapse this into the previous case.

Case with observation

Let $x : \Omega \to \mathsf{R}^n$ and $y : \Omega \to \mathsf{R}^m$. A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* for x given y is an estimator $f : \mathsf{R}^m \to \mathsf{R}^n$ which minimizes $\mathsf{E}|f(x) - y|^2$.



Minimum Mean Squared Error Estimator
Estimators
Expectation
Norms
Random Variables
Probability Measures
Topological Sigma Algebra
Functionals
Measurable Functions
Topological Spaces
Vectors
Metrics
N-Dimensional Space
Distance
Absolute Value
Space Distance
Event Probabilities
Measures
Plane Distance
Probability Events
Probability Distributions
Extended Real Numbers
Interval Length
Probability Outcomes
Subset Algebras
Real Limits
Intervals
Probability
Set Operations
Cardinality
Real Space
Real Plane
Real Line
Common Sense
Real Sequences
Real Summation
Direct Products
Real Order
Geometry
Fields
Real Numbers
Groups
Rational Numbers
Integer Additive Inverses
Natural Summation
Inverse Elements
Integer Arithmetic
Sequences
Family Operations
Element Functions
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Families
Algebras
Identity Elements
Arithmetic
Function Inverses
Integer Products
Operations
Function Images
Natural Exponents
Function Composites
Integer Numbers
Functions
Natural Products
Equivalence Relations
Relations
Natural Sums
Subset Systems
Cartesian Products
Recursion Theorem
Set Powers
Ordered Pairs
Peano Axioms
Natural Induction
Unordered Triples
Natural Numbers
Partitions
Intersection of Empty Set
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects