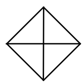


Why

We are frequently interested in finding minimizers of real functions.[1]

Definition

An *optimization problem* is a pair $(\mathcal{X}, f)$ in which $\mathcal{X}$ is a nonempty set called the *constraint set* and $f : \mathcal{X} \to \mathbf{R}$ is called the *objective* (or *cost function*).

If $\mathcal{X}$ is finite we call the optimization problem *discrete*. If $\mathcal{X} \subset \mathbf{R}^d$ we call the optimization problem *continuous*.

We refer to all elements of the constraint set as *feasible*. We refer to an element $x \in \mathcal{X}$ of the constraint set as *optimal* if $f(x) = \inf_{z \in \mathcal{X}} f(z)$. We also refer to optimal elements as *solutions* of the optimization problem.

It is common for f and $\mathcal{X}$ to depend on some other, known, given objects. In this case, these objects are often called *parameters* or *problem data*.

Notation

We often write optimization problems as

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathcal{X}. \end{array}$$

In this case we call x the *decision variable*.

Extended reals

It is common to let $f : \mathcal{X} \to \bar{\mathbf{R}}$, and allow there to exist $x \in \mathcal{X}$ for which $f(x) = \infty$. This technique can be used to embed further constraints in the objective. For example, we interpret $f(x) = +\infty$ to mean x is *infeasible*.

[1] Future editions will modify and expand.

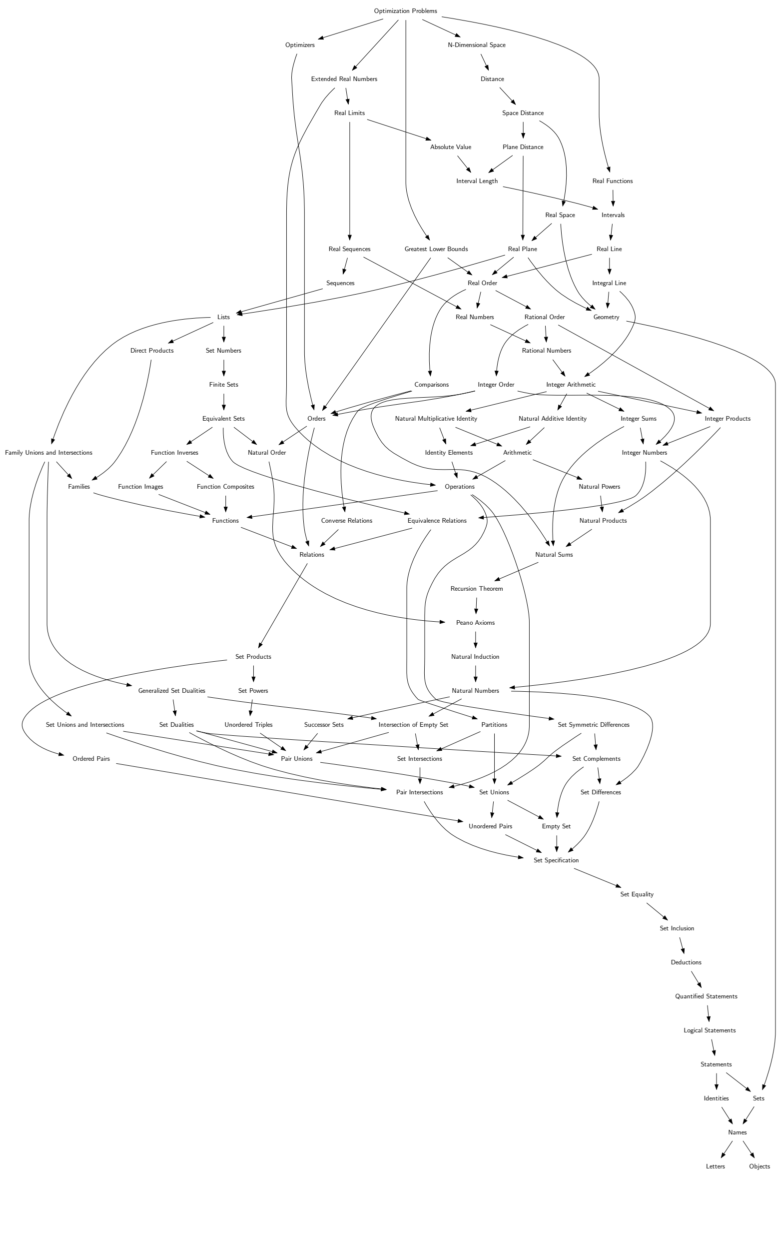
Maximization

If we have some function $g : \mathcal{X} \to \bar{\mathbf{R}}$ that we wish to maximize, we can always convert it to an optimization problem by defining $f : \mathcal{X} \to \bar{\mathbf{R}}$ by $f(x) = -g(x)$. In this case g is often called a *reward* (*utility*, *profit*).

Solvers

Let $\mathcal{P} = \{(X_a, f_a : X_a \to \bar{\mathbf{R}})\}_{a \in A}$ be a family of optimization problems. A *solver* (or *solution method*, *solution algorithm*) for $\mathcal{P}$ is a function $S : A \to \mathcal{S}$ such that S_a is a solution of the problem (X_a, f_a).

Loosely speaking, the difficulty of "solving" the optimization problem $(\mathcal{X}, f)$ depends on the properties of $\mathcal{X}$ and f and the problem "size". For example, when $\mathcal{X} \subset \mathbf{R}^d$ the difficulty is related to the "dimension" d of $x \in \mathcal{X}$.


Optimization Problems
Optimizers
N-Dimensional Space
Extended Real Numbers
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Interval Length
Real Functions
Real Space
Intervals
Real Sequences
Greatest Lower Bounds
Real Plane
Real Line
Sequences
Real Order
Integral Line
Lists
Real Numbers
Rational Order
Geometry
Direct Products
Set Numbers
Rational Numbers
Finite Sets
Comparisons
Integer Order
Integer Arithmetic
Equivalent Sets
Orders
Natural Multiplicative Identity
Natural Additive Identity
Integer Sums
Integer Products
Family Unions and Intersections
Function Inverses
Natural Order
Identity Elements
Arithmetic
Integer Numbers
Families
Function Images
Function Composites
Operations
Natural Powers
Functions
Converse Relations
Equivalence Relations
Natural Products
Relations
Natural Sums
Recursion Theorem
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
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Set Complements
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Set Unions
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Set Inclusion
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Statements
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