## Why

Let $A \in \mathbf{R}^{m\times n}$ and define $f : \mathbf{R}^n \to \mathbf{R}^m$ by $f(x) = Ax$. Then $f$ is a linear function from $\mathbf{R}^n$ to $\mathbf{R}^m$. Conversely, suppose $g : \mathbf{R}^n \to \mathbf{R}^m$ is a linear function. Then there exists a matrix $B \in \mathbf{R}^{m\times n}$ so that $g(z) = Bz$. Does this function have an inverse?

## Derivation

If $A \in \mathbf{R}^{m\times n}$, with $m \neq n$, then the inverse of $f$ can not exist. For a *square* matrix $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times n}$ is a *left inverse* if $BA = I$. In other words, $B$ is a left inverse element of $A$ in the algebra of matrices with the operation of multiplication. $C \in \mathbf{R}^{n\times n}$ is a *right inverse* if $AC = I$.

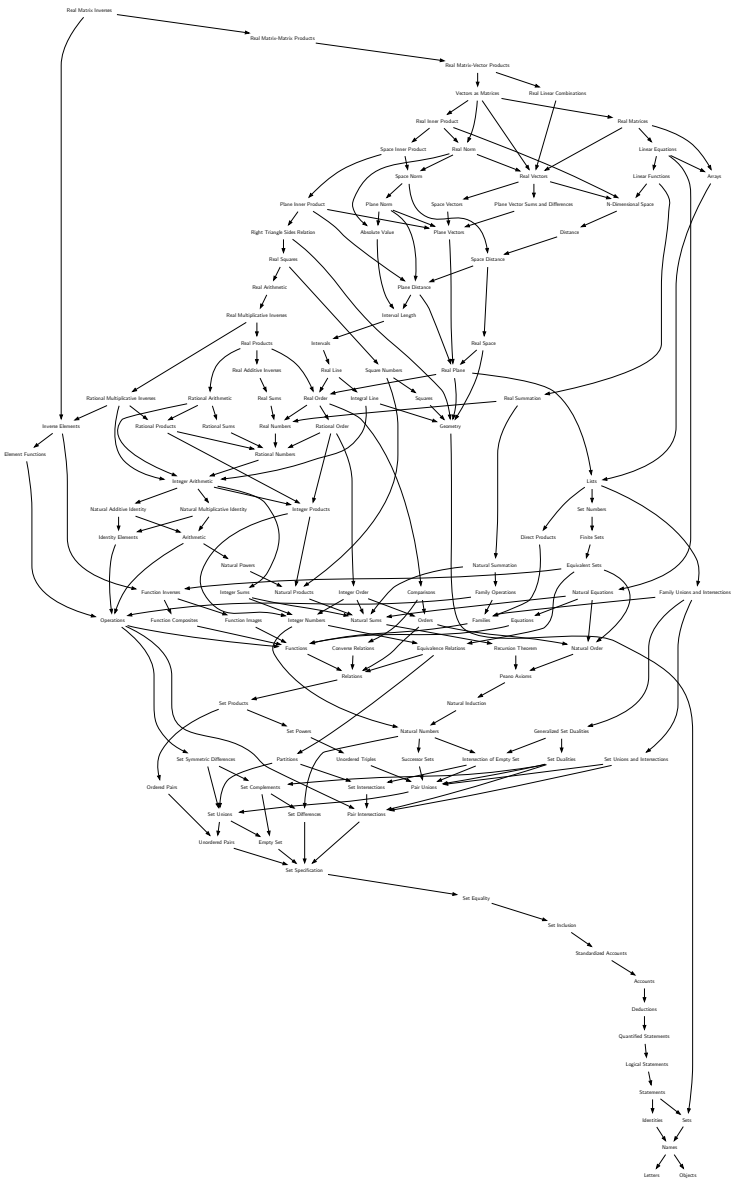
## Definition

We call a square matrix $A$ *invertible* if there is $B \in \mathbf{R}^{n\times n}$ so that $BA = I$.

Now suppose that $A \in \mathbf{R}^{n\times n}$. Of course, the inverse may not exist. Consider, for example if $A$ was the $n$ by $n$ matrix of zeros. If there exists a matrix $B$ so that $BA = I$ we call $B$ the *left inverse* of $A$ and likewise if $AC = I$ we call $C$ the *right inverse* of $A$. In the case that $A$ is square, the right inverse and left inverse coincide.

**Proposition 1.** *Let $A, B, C \in \mathbf{R}^{n\times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

*Proof.* Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. $\square$


Real Matrix Inverses
Real Matrix-Matrix Products
Real Matrix-Vector Products
Vectors as Matrices
Real Linear Combinations
Real Inner Product
Real Matrices
Space Inner Product
Real Norm
Linear Equations
Space Norm
Real Vectors
Linear Functions
Arrays
Plane Inner Product
Plane Norm
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Right Triangle Sides Relation
Absolute Value
Plane Vectors
Distance
Real Squares
Space Distance
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Space
Real Additive Inverses
Real Line
Square Numbers
Real Plane
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Squares
Real Summation
Inverse Elements
Rational Products
Rational Sums
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Geometry
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Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Set Numbers
Identity Elements
Arithmetic
Direct Products
Finite Sets
Natural Powers
Natural Summation
Equivalent Sets
Function Inverses
Integer Sums
Natural Products
Integer Order
Comparisons
Family Operations
Natural Equations
Family Unions and Intersections
Operations
Function Composites
Function Images
Integer Numbers
Natural Sums
Orders
Families
Equations
Functions
Converse Relations
Equivalence Relations
Recursion Theorem
Natural Order
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
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Set Symmetric Differences
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Complements
Set Intersections
Pair Unions
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Notes
Letters
Objects