Order Relations

1 Why

TODO

2 Definition

Let R a relation on the non-empty set A. We call R **anti-symmetric** if for two nonequal elements $a, b \in A$, $(a, b) \in R \implies (b, a) \notin R$. If R is reflexive, transitive, and anti-symmetric then we call R a **partial order** on A.

A **partially ordered set** is a set together with a partial order. The language partial is meant to suggest that two elements need not be comparable. For example, suppose R is $\{(a, a) \mid a \in A\}$; we may justifiably call this no order at all and call A totally unordered, but it is a partial order by our definition.

Often we want all elements of the set A to be comparable. We call R **connexive** if for all $a, b \in A$, $(a, b) \in R$ or $(b, a) \in R$. If R is a partial order and connexive, we call it a **total order**.

A **totally ordered set** is a set together with a total order. The language is a faithful guide: we can compare any two elements. Still, we prefer one word to three, and so we will use the shorter term **chain** for a totally ordered set; other terms include **simply ordered set** and **linearly ordered set**.

2.1 Notation

We denote total and partial orders on a set A by $\preceq$. We read $\preceq$ aloud as "precedes or equal to" and so read $a \preceq b$ aloud as "a precedes or is equal to b." If $a \preceq b$ but $a \neq b$, we write $a \prec b$, read aloud as "a precedes b."