LINEAR COMBINATIONS

Why

We want to build vectors out of other vectors using scalar multiplication and vector addition.

Definition

A *linear combination from* a vector space is an ordered pair: the first coordinate is a sequence of vectors and the second is a sequence of scalars. The *result* of a linear combination is the sum of the results of scaling each vector by the corresponding scalar in the sequence; itself a vector in the space.

A *trivial linear combination* is one whose sequence of scalars is zero at each coordinate. The result of any trivial linear combination is the zero vector. A *nontrivial linear combination* is one which is not trivial. In other words, to be nontrivial, there must exist at least one index of the scalar sequence whose corresponding value is nonzero.

We say that a given vector *can be written as a linear combination of* a sequence of vectors if there exists a sequence of scalars such that the result of the linear combination of that sequence of vectors and scalars is the given vector. In other words, a vector can be written as a linear combination of some other vectors if there exists scalars for those other vectors such that scaling them and adding the results gives the vector.

Notation

Let $(V, \mathbf{F})$ be a vector space. Let $v = (v_1, \ldots, v_n)$ be a sequence of vectors in V and $a = (a_1, \ldots, a_n)$ be a sequence of scalars in $\mathbf{F}$. Then (v, a) is a linear combination and we can express its result by

$$a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

If (v, a) is trivial, then $a_i = 0$ for $i \in \{1, 2, \ldots, n\}$ and the result of (v, a) is 0 (the zero vector). Otherwise, there exists $i \in \{1, 2, \ldots, n\}$ such that $a_i \neq 0$; of course, the result of (v, a) may still be 0. A vector u can be

written as a linear combination of the vectors $v_1, v_2, \ldots, v_n$ if there exists scalars $a_1, a_2, \ldots, a_n$ so that

$$u = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

We often number (see Lists) a set of finite vectors. In this case, we say "Let $\{u_1, \ldots, u_n\}$ be a (finite) set of vectors."

Linear Combinations

Vectors

Real Linear Combinations

Real Vectors

Plane Vector Sums and Differences

Space Vectors

N-Dimensional Space

Plane Vectors

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Space

Real Plane

Real Line

Real Order

Integral Line

Fields

Real Numbers

Rational Order

Geometry

Rings

Rational Numbers

Lists

Integer Arithmetic

Set Numbers

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Finite Sets

Identity Elements

Arithmetic

Direct Products

Equivalent Sets

Natural Powers

Family Unions and Intersections

Function Inverse

Comparisons

Integer Order

Integer Sum

Natural Products

Families

Function Composites

Function Images

Operations

Orders

Integer Numbers

Natural Sums

Functions

Converse Relations

Equivalence Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Set Products

Generalized Set Dualities

Set Powers

Natural Induction

Set Unions and Intersections

Set Dualities

Unordered Triples

Successor Sets

Intersection of Empty Set

Partitions

Set Symmetric Differences

Natural Numbers

Ordered Pair

Pair Unions

Set Intersections

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects