1 Why

We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

2 Why

Consider a set A. If A has n elements, then a probability distribution on A is $p : A \to \mathbf{R}$ where $p(a) = 1/n$. There is a natural candidate.

What if A is the set of natural numbers $\mathbf{N}$. The principle difficulty is that not all sequences of real numbers $a : \mathbf{N} \to \mathbf{R}$ are summable.