Why

We want to find the roots of negative numbers.[1]

Definition

A *complex number* is an ordered pair of real numbers. The *real part* of a complex number is its first coordinate. The *imaginary part* of a complex number is its second coordinate.

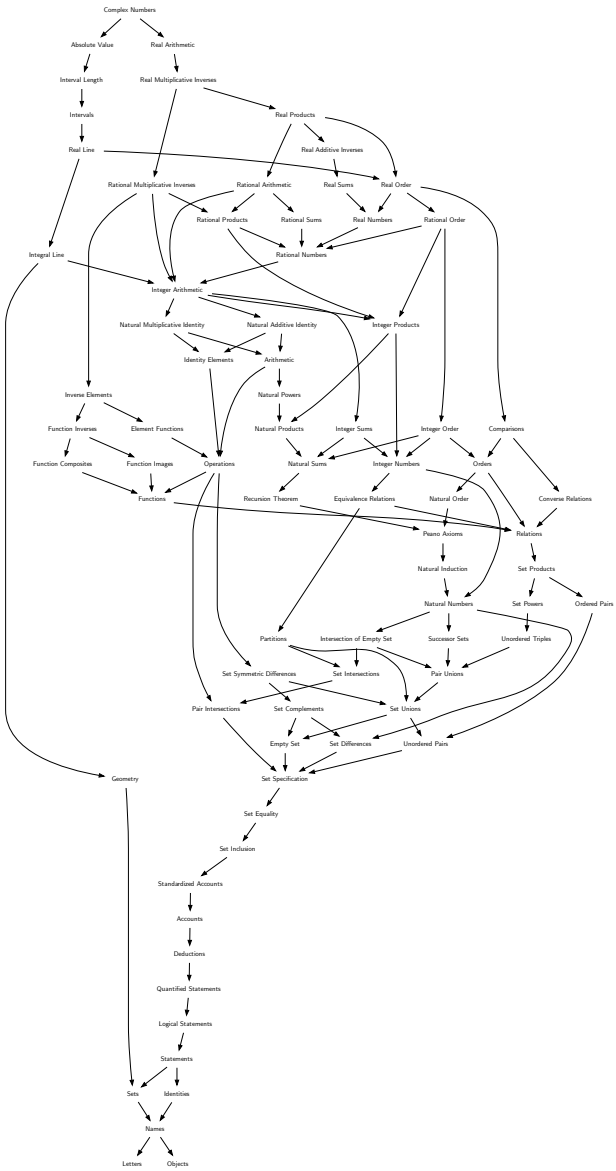
We can identify the imaginary numbers with no complex part (i.e., $\{(a,b) \in \mathbf{R}^2 \mid b = 0\}$ with $\mathbf{R}$ in the obvious way. For this reason, such a complex number is sometimes referred to as a *purely real number* Conversely, a complex number with zero imaginary part (i.e., an element of $\{(a,b) \in \mathbf{R}^2 \mid a = 0\}$) is said to be a *purely imaginary number*.

Notation

When treating $\mathbf{R}^2$ as the set of complex numbers, we denote it by $\mathbf{C}$. Let $z \in \mathbf{C}$ with $z = (a,b)$. The real part of z is a and its imaginary part is b. It is universal to denote z by $a + ib$, and to call i an (or the) *imaginary number*. Some authors use j, it is a matter of notation.

We denote the real part of z by $\mathrm{Re}(z)$, read "real of z," and the imaginary part by $\mathrm{Im}(z)$, read "imaginary of z." So, in particular, $\mathrm{Re}(z) = a$ and $\mathrm{Im}(z) = b$.

[1] Future editions will modify this, and will discuss the existence of solutions of algebraic equations.


Complex Numbers
Absolute Value
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Additive Inverses
Real Line
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Rational Products
Rational Sums
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Identity Elements
Arithmetic
Inverse Elements
Natural Powers
Function Inverses
Element Functions
Natural Products
Integer Sums
Integer Order
Comparisons
Function Composites
Function Images
Operations
Natural Sums
Integer Numbers
Orders
Functions
Recursion Theorem
Equivalence Relations
Natural Order
Converse Relations
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
Set Unions
Empty Set
Set Differences
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
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Identities
Names
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Objects