



# Finite Signed Measures

## 1 Why

For the difference of two (signed) measures to be well-defined, we need one of the two to be finite. Otherwise, the measure of the difference on the base set involves subtracting  $\infty$  from  $\infty$ .

## 2 Definition

A *finite* signed measure is one for which the measure of every set is finite. This condition is equivalent to the base set having finite measure (see below).

## 3 Result

**Proposition 1.** *A signed measure is finite if and only if it is finite on the base set.*

*Proof.* Let  $(X, \mathcal{A})$  be a measurable space. Let  $\mu : \mathcal{A} \rightarrow [-\infty, \infty]$  be a signed measure.  $(\Rightarrow)$  If  $\mu$  is finite, then  $\mu(X)$  is finite since  $X \in \mathcal{A}$ .  $(\Leftarrow)$  Next, suppose  $\mu(X)$  is finite. Let  $A \in \mathcal{A}$ . Then  $X = A \cup (X - A)$ , with these sets disjoint, so by countable

additivity of  $\mu$ ,  $\mu(X) = \mu(A) + \mu(X - A)$ . Since  $\mu(X)$  finite,  $\mu(A)$  and  $\mu(X - A)$  are both finite.  $\square$