



Why

We want to discuss an inductor’s performance on consistent (but possibly incomplete) datasets.

We take two steps. First, put a measure on the set of training sets and only consider high-measure subsets. Second, consider predictors performing well in some tolerance.

Definition

Let (X, \mathcal{X}, μ) be a probability space and (Y, \mathcal{Y}) a measurable space. Let $f : X \rightarrow Y$ measurable. We call the pair $((X, \mathcal{X}, \mu), f)$ a *(supervised) probabilistic data model*.

We interpret μ as the *data-generating distribution* or *underlying distribution* and f as the *correct labeling function*. Many authors refer to a supervised probabilistic data model as the *statistical learning (theory) framework*.

Probable datasets

We put a measure on the set of datasets by using the product measure $(X^n, \mathcal{X}^n, \mu^n)$. We interpret this as a model for a training set of independent and identically distributed inputs.

For $\delta \in (0, 1)$, $\mathcal{S} \subset X^n$ is $1 - \delta$ -*representative* if $\mu^n(\mathcal{S}) \geq 1 - \delta$. If \mathcal{S} is $1 - \delta$ -representative for small δ , we think of \mathcal{S} as a set of “probable” or “reasonable” datasets. We call δ the *confidence parameter*.

Prediction error

The *error* of (measurable) $h : X \rightarrow Y$ (under μ and f) is

$$\text{error}_{\mu, f}(h) = \mu(\{x \in \mathcal{X} \mid h(x) \neq f(x)\}).$$

We interpret this as the probability that the predictor mislabels a point. The *accuracy* of h is $1 - \text{error}_{\mu, f}(h)$.

Since $(f, g) \mapsto \mu[f(x) \neq g(x)]$ is a metric on $L^2(X, \mathcal{X}, \mu, Y)$ we can talk about the error as the “distance” from the correct label classifier. Thus we will say that $\varepsilon \in (0, 1)$, (measurable) $h : \mathcal{X} \rightarrow \mathcal{Y}$ ε -*approximates* the correct labeling function f if $\text{error}(h) \leq \varepsilon$. Roughly speaking, if $\varepsilon \ll 1$, the the error of the hypothesis is “fairly small.” We call ε the *accuracy parameter*, since the accuracy of such a predictor is $1 - \varepsilon$.

Other terminology

A *hypothesis class* is a subset of the measurable functions from $X \rightarrow Y$. Other names for the error of a classifier include the *generalization error*, the *risk* or the *true error* or *loss*.

