



**Definition**

A *finite automaton* (or *machine*, *deterministic finite automaton*)  $M = (Q, \Sigma, \delta, q_0, F)$  is a list where  $Q$  and  $\Sigma$  are finite sets (alphabets),  $\delta : Q \times \Sigma \rightarrow Q$ ,  $q_0 \in Q$  and  $F \subset Q$ .

We call  $Q$  the *states*,  $\Sigma$  the *alphabet*,  $\delta$  the *transition function*,  $q_0$  the *start state* (*initial state*), and  $F$  the *accept states* (or *final states*). An input  $u \in \text{str}(\Sigma)$  results in a state sequence  $x \in \text{str}(Q)$  with  $x_1 = q_0$  and  $x_{i+1} = \delta(x_i, u_i)$  for  $i = 1, \dots, |u|$ .  $M$  *accepts*  $x$  if  $x_{|x|+1} \in F$ . The set of all strings that  $M$  accepts is the *language* of the machine  $M$ . We say that  $M$  *recognizes* or *accepts* this set. Although a language may accept many different strings, it only ever accepts one language. For example, if the machine accepts no strings, then it accepts the language  $\emptyset$ .

A  $L \subset \text{str}(\Sigma)$  is called *regular* if there exists a finite automaton that recognizes it.

**Example**

Define  $Q = \{q_1, q_2, q_3\}$ ,  $\Sigma = \{0, 1\}$ , define  $\delta : Q \times \Sigma \rightarrow Q$  by  $\delta(q$



