Rational equivalence

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We say that the elements (a, b) and (c, d) of this set are *rational equivalent* if $ad = bc$. Briefly, the intuition is that (a, b) represents a over b In the usual notation, (a, b) represents "a/b". So this equivalence relation says these two are the same if $a/b = c/d$ or else $ad = bc$.

Proposition 1. *Rational equivalence is an equivalence relation on* $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$.[1]

Definition

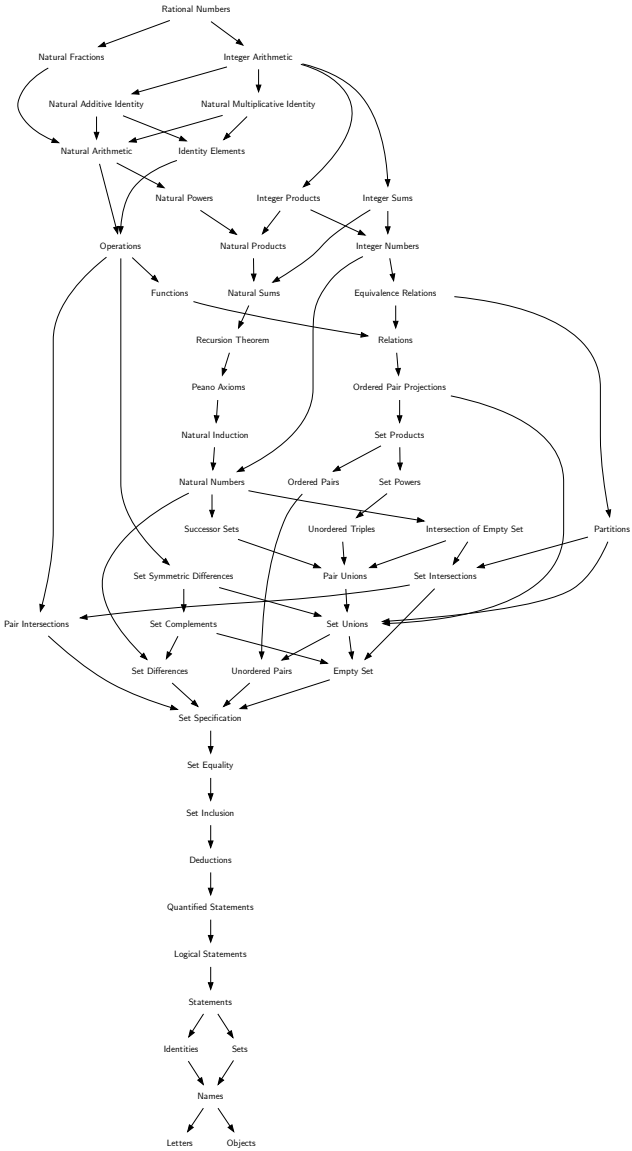
The *set of rational numbers* is the set of equivalence classes (see Equivalence Classes) of $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ under ratioanl equivalence. We call an element of the set of rational numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

Notation

We denote the set of rationals by $\mathbf{Q}$.[2] If we denote rational equivalence by $\sim$ then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$.

[1] Future editions will include an account.

[2] From what we can tell, $\mathbf{Q}$ is a mnemonic for "quantity", from the latin "quantitas." It may also be a mnemonic for quotient.


Rational Numbers
Natural Fractions
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Natural Arithmetic
Identity Elements
Natural Powers
Integer Products
Integer Sums
Operations
Natural Products
Integer Numbers
Functions
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Equivalence Relations
Recursion Theorem
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Set Products
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Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Pair Intersections
Set Complements
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
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Deductions
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Statements
Identities
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Objects