Loss Functions

1 Why

We want to compare inductors by comparing their produced predictors. We compare predictors by judging predictions.

2 Definition

A *loss* function is a nonnegative real-valued function on pairs which is zero only on repeated pairs. It need not be symmetric. We interpret the first argument of the loss function as our prediction and the second as the recorded value.

The *loss of a predictor on a pair* is the result of the loss function on the pair. Similarly, the *loss of a predictor on a sequence* of pairs is the sum of the losses on the pairs. The *average loss of a predictor on a sequence* is the loss divided by the length of the sequence.

2.1 Notation

Let $(a, b) \in A \times B$ where A and B are non-empty sets. Let $\ell : B \times B \to \mathsf{R}$. Let $f : A \to B$. The loss of f on (a, b) is

$$\ell(f(a), b).$$

Let $s = ((a^1, b^1), \dots, (a^n, b^n))$ be a record sequence. The loss of f on s is

$$\sum_{k=1}^{n} \ell(f(a^k), b^k).$$

The average loss of f on s is

$$\frac{1}{n}\sum_{k=1}^{n} \ell(f(a^k), b^k).$$

2.2 Dual Record Prediction Evaluators

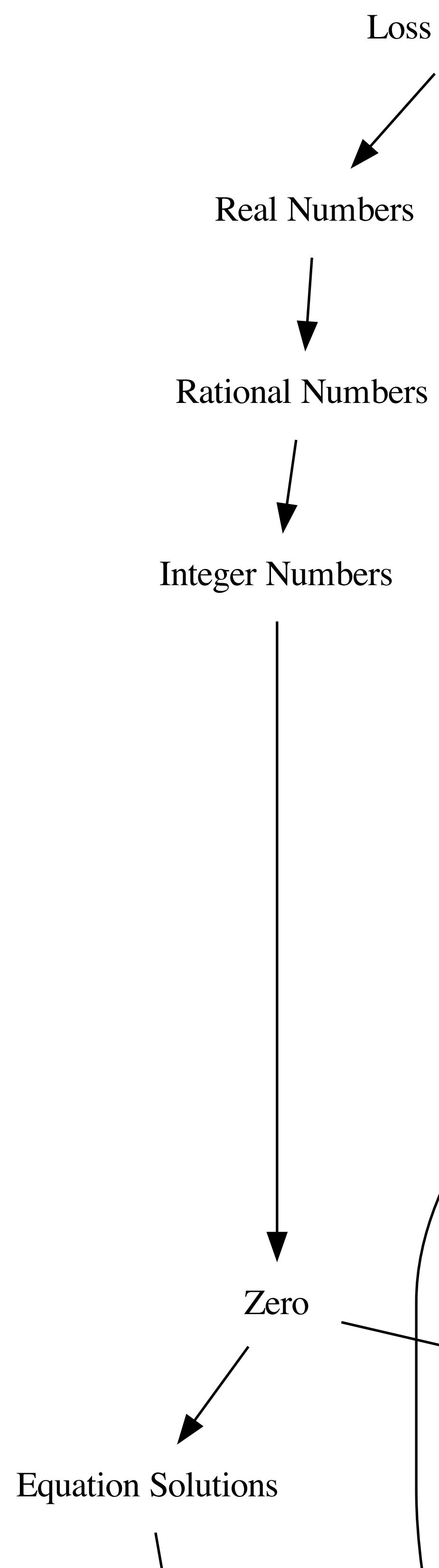
Let i be an inductor and let r and s be two record sequences. Denote the predictor $i(r)$ associated with r by f. Let g be a prediction evaluator. Let $s = ((u^1, v^n), \dots, (u^n, v^n))$. Then consider

$$\sum_{i=1}^{n} g(f(u^n), v^n)).$$

Consider an inductor, two record sequences, and a prediction evaluator. Consider Consider the predictor associated with first record sequences. Consider the evaluator which sums the prediction errors on the second record sequence of the the predictor induced on the first record sequence.

The natural evaluator associated with this inductor is the We consider the pred The first record sequencerecord sequence

Commonly evaluators have structure. We fix a record sequence and consider the predictor induced by it. We consider a second record sequence and compare the predictor's result for a precept with the postcept paired with it in the second record sequence.


Loss
Real Numbers
Rational Numbers
Integer Numbers
Zero
Equation Solutions