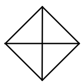


Why

How can we construct distributions which factor according to a directed graph.[1]

Definition

Let $\bar{G} = (G, A)$ be a typed graph (see Typed Graphs) with directed and acyclic G. For source vertices i, let $g_i : A_i \to [0, 1]$ be a distribution and otherwise let $g_i : A \times A_{\mathrm{pa}_i} \to [0, 1]$ denote a function satisfying $g_i(\cdot, x)$ is a distribution for every $x \in X_{\mathrm{pa}_i}$.

We call the ordered pair $(\bar{G}, g)$ a *distribution graph* (or *distribution network*, *conditional distribution network*, *conditional distribution graph*, *bayesian network*,[2] *directed probabilistic graphical model*, *directed graphical model*).

The *distribution* of $(\bar{G}, g)$ is the function $p : \prod_i A_i \to [0, 1]$ defined by

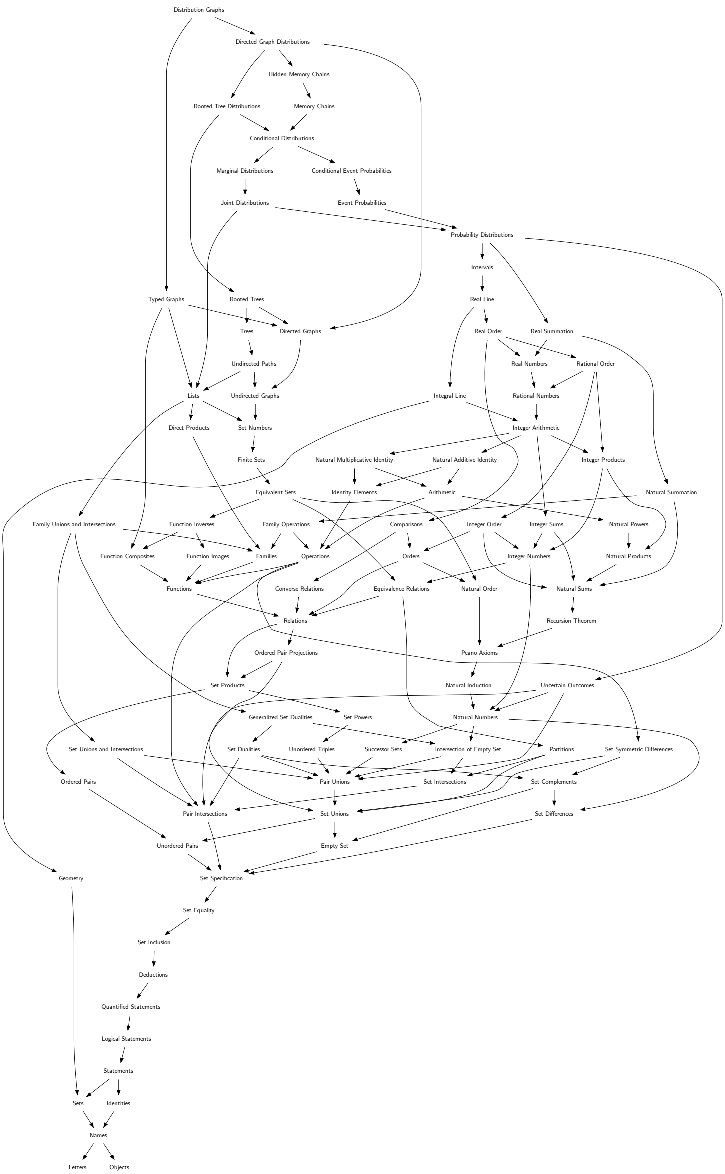
$$p(a) = \prod_{\mathrm{pa}_i = \varnothing} g_i(a_i) \prod_{\mathrm{pa}_i \neq \varnothing} g_i(a_i, a_{\mathrm{pa}_i}).$$

It is, of course, a distribution. And it factors according to the directed and acyclic graph G. Also, the g_i, $i = 1, \dots, n$, are the conditionals.[3]

[1] Future editions might flip the order of this sheet with that of directed graph distributions since, in the genetic approach, in may be more natural to think of constructing such distributions before analyzing them. This is partially motivated by the acyclic constraint here, which restricts the graphs according to which a distribution can factor.

[2] Indeed, this term is near universal in certain literatures. We avoid it in these sheets as a result of the Bourbaki project's policy on naming.

[3] Future editions will elaborate and give a proof.


Distribution Graphs
Directed Graph Distributions
Hidden Memory Chains
Rooted Tree Distributions
Memory Chains
Conditional Distributions
Marginal Distributions
Conditional Event Probabilities
Joint Distributions
Event Probabilities
Probability Distributions
Intervals
Typed Graphs
Rooted Trees
Real Line
Trees
Directed Graphs
Real Order
Real Summation
Undirected Paths
Real Numbers
Rational Order
Lists
Undirected Graphs
Integral Line
Rational Numbers
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Equivalent Sets
Identity Elements
Arithmetic
Natural Summation
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Powers
Function Composites
Function Images
Families
Operations
Orders
Integer Numbers
Natural Products
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Uncertain Outcomes
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects