Why

We often want to discuss lists without regard to order. This is natural, for instance, in discussing number factorizations. There, the associativity and commutativity of natural multipliciation mean that the factorization $(2, 3)$ and $(3, 2)$ are equivalent, in the sense that they both factor 6.

Definition

Given a set A, a *multiset* (or *mset*, *bag*) of elements *of* or *in* A is a function $m : A \to \mathbf{N}$. In this case, the result $m(a)$ is called the *multiplicity* of $a \in A$. For this reason, another term for the object we have named a multiset is *multiplicity function*. An $a \in A$ with $m(a) \neq 0$ is called an *element* of the multiset.

The *size* (or *cardinality*) of a multiset m is the sum of the multiplicities of its elements. In summation notation, the size is defined to be

$$\sum_{a \in A} m(a)$$

The set

$$\{a \in A \mid m(a) > 0\}$$

is called the *support* of the multiset.

Examples

A natural use for multisets is in factorizations of natural numbers.

Notation

Notation for multisets is nonstandard—here are some common in use. If a and b are two distinct objects and $A = \{a, b\}$ is the unordered pair containing them, the multiset $m : A \to \mathbf{N}$ defined by $m(a) = 2$ and $m(b) = 3$ is sometimes denoted

$$\rbag a, a, b, b, b \lbag$$

or

$$\{\{a, a, b, b, b\}\}$$

or alternatively

$$[a, a, b, b, b]$$

or

$$\{a^2, b^3\}$$

As it appears to the present authors, none of these is desirable. This last notation seems to be a reference to the notation for natural powers (a sheet not required by this one) and the use of multisets in number factorizations .

Other terminology

Other authors refer to the ordered pair (A, m) as the multi set, and call A the *underlying set*. We avoid this terminology, for the reason that $\operatorname{dom} m = A$.

Multisets

Number Factorizations

Natural Summation

Prime Numbers

Lists

Natural Products

Set Numbers

Direct Products

Natural Sums

Finite Sets

Recursion Theorem

Equivalent Sets

Family Unions and Intersections

Natural Order

Function Inverses

Family Operations

Peano Axioms

Function Composites

Function Images

Families

Operations

Equivalence Relations

Functions

Natural Induction

Relations

Natural Numbers

Ordered Pair Projections

Successor Sets

Generalized Set Dualities

Set Products

Intersection of Empty Set

Set Powers

Set Partitions

Ordered Pairs

Unordered Triples

Set Unions and Intersections

Set Dualities

Set Intersections

Set Symmetric Differences

Pair Unions

Pair Intersections

Set Complements

Set Differences

Set Unions

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects