

COMPLEX CONJUGATES

Why

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Definition

The complex conjugate (or conjugate) of a complex number z is the complex number whose real part matches z and whose imaginary part is the additive inverse of z. The complex conjugate of a purely real number is the same purely real number. In other words, the complex conjugate of a complex number with no imaginary part is the same complex number.

Notation

We denote the complex conjugate of the complex number $z \in \mathbf{C}$ by z^* . Other common notation includes \bar{z} , read "z bar". If there exists $a, b \in \mathbf{R}$ so that z = (a, b), then $z^* = (a, -b)$.

Geometric interpretation

Taking the conjugate of a complex numbers corresponds to a reflection across the real axis in the plane.

Properties

A complex number z is real if and only if $z = z^*$ and it is imaginar if and only if $z = -z^*$.

Proposition 1. For $z \in \mathbb{C}$, we have

$$\operatorname{Re}(z) = \frac{z + z^*}{2}$$
 and $\operatorname{Im}(z) = \frac{z - \bar{z}}{2i}$.

¹Future editions will include.

