Definition

A subset $M \subset \mathbf{R}^n$ is a *smooth manifold* of dimension d if for every $x \in M$, there exists a neighborhood V of x in X that is diffeomorphic to an open subset U of $\mathbf{R}^d$. In this case we say that the set is *locally diffeomorphic* to $\mathbf{R}^d$.

A diffeomorphism $\phi : U \to V$ is called a *parameterization* of the neighborhood of V.

Its inverse diffeomorphism ϕ^{-1} is called a *coordinate system* (or system of *coordinates*) on V.

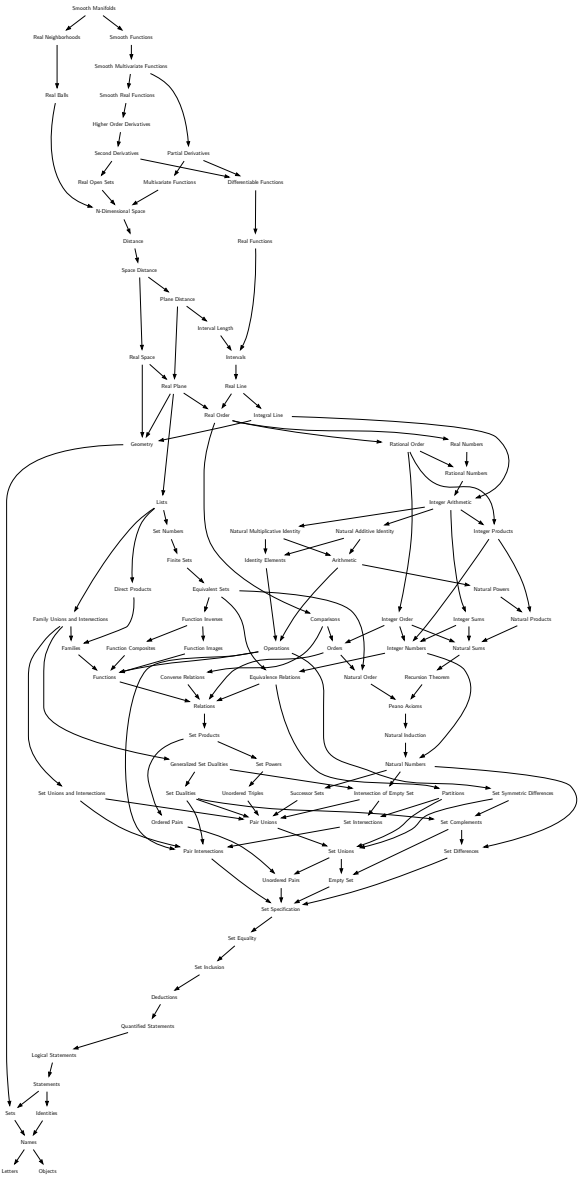
Notation

We denote the dimension of a manifold M by $\dim M$.

Submanifolds

If X and Z are both manifolds in R^n and $Z \subset X$, then we call Z a *submanifold* of X. In particular, X is a submanifold of R^n. Any open set of a manifold X is a submanifold X.[1]

[1] Future editions will expand.


Smooth Manifolds
Real Neighborhoods
Smooth Functions
Smooth Multivariate Functions
Real Balls
Smooth Real Functions
Higher Order Derivatives
Second Derivatives
Partial Derivatives
Real Open Sets
Multivariate Functions
Differentiable Functions
N-Dimensional Space
Distance
Real Functions
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Real Order
Integral Line
Geometry
Rational Order
Real Numbers
Rational Numbers
Integer Arithmetic
Lists
Set Numbers
Natural Multiplication Identity
Natural Additive Identity
Integer Products
Finite Sets
Identity Elements
Arithmetic
Natural Powers
Direct Products
Equivalent Sets
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Products
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects