

DIRECT PRODUCTS

Why

We generalize the idea of a produce of two sets to a product of n sets.

Direct Products

The direct product of a natural family is the set of ordered sequences of elements from each set in the family. We call the elements of the direct product n-tuples. We call the ith element in an n-tuple the ith coordinate. This language is meant to follow that used in defining ordered pairs. Two coordinates in a sequence are consecutive if their natural difference is 1.

Notation

Let A_1, \ldots, A_n be a natural family of sets. We denote its direct product by

$$\prod_{i=1}^n A_i.$$

We read this notation as "product over alpha in I of A subalpha." We denote an element of $\prod_{i=1}^{n} A_i$ by (a_1, a_2, \ldots, a_n) with the understanding that $a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n$.

If
$$A_i = A$$
 for $i = 1, ..., n$, then we denote $\prod_{i=1}^n A_i$ by A^n .