



## SUCCESSOR SETS

### Why

We want numbers to count with.<sup>1</sup>

### Definition

The *successor* of a set is the set which is the union of the set with the singleton of the set. In other words, the successor of a set  $A$  is  $A \cup \{A\}$ . This definition has sense for any set, but is of interest only for those particular sets introduced here.

These sets are the following (and their successors): We call the empty set *zero*.<sup>2</sup> We call the successor of the empty set *one*. In other words, one is  $\emptyset \cup \{\emptyset\} = \{\emptyset\}$ . We call the successor of one *two*. In other words, two is  $\{\emptyset\} \cup \{\{\emptyset\}\} = \{\emptyset, \{\emptyset\}\}$ . Likewise, the successor of two we call *three* and the successor of three we call *four*. And we continue as usual,<sup>3</sup> using the English language in the typical way.

A set is a *successor set* if it contains zero and if it contains the successor of each of its elements.

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<sup>1</sup>Future editions will expand on this sheet with a more justified why.

<sup>2</sup>In future editions, zero may be a separate sheet.

<sup>3</sup>Future editions will assume less in the introduction of natural numbers.

## Notation

Let  $x$  be a set. We denote the successor of  $x$  by  $x^+$ . We defined it by

$$x^+ := x \cup \{x\}$$

We denote one by 1. We denote two by 2. We denote three by 3. We denote four by 4. So

$$0 = \emptyset$$

$$1 = 0^+ = \{0\}$$

$$2 = 1^+ = \{0, 1\}$$

$$3 = 2^+ = \{0, 1, 2\}$$

$$4 = 3^+ = \{0, 1, 2, 3\}$$

