



Why

Do the rational numbers correspond (in the sense Homomorphisms) to elements of the reals.

Main Result

Indeed, roughly speaking the rationals correspond to elements of the reals which are bounded above by that rational. Denote by $\tilde{\mathbf{R}}$ the set $\{q \in \mathbf{R} \mid \exists s \in \mathbf{Q}, q = \{t \in \mathbf{Q} \mid t < s\}\}$.

Proposition 1. *The fields $(\tilde{\mathbf{R}}, +_{\mathbf{R}} \mid \tilde{\mathbf{R}}, \cdot_{\mathbf{R}} \mid \tilde{\mathbf{R}})$ and $(\mathbf{Q}, +_{\mathbf{Q}}, \cdot_{\mathbf{Q}})$ are homomorphic.¹*

Proof. The function is $f : \mathbf{Q} \rightarrow \tilde{\mathbf{R}}$ with $f(q) = \{r \in \mathbf{Q} \mid r < q\}$ \square

¹Indeed, more is true and will be included in future editions. There is an *order perserving* field homomorphism.

