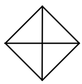


Why

What is the best linear regressor if we choose according to a squared loss function.

Definition

Let $X \in \mathbf{R}^{n \times d}$ and $y \in \mathbf{R}^d$. In other words, we have a paired dataset of records with inputs in $\mathbf{R}^d$ (the rows of X) and outputs in $\mathbf{R}$ (the elements of y).

A *least squares linear predictor* or *linear least squares predictor* is a linear transformation $f : \mathbf{R}^d \to \mathbf{R}$ (the field is $\mathbf{R}$) which minimizes

$$\frac{1}{n} \sum_{i=1}^{n} (f(x^i) - y_i)^2.$$

over the dataset of pairs $(x^1, y_1), \dots, (x^n, y_n) \in \mathbf{R}^d \times \mathbf{R}$ where $\left(x^i\right)^\top$ is the ith row of X for $i = 1, \dots, n$.

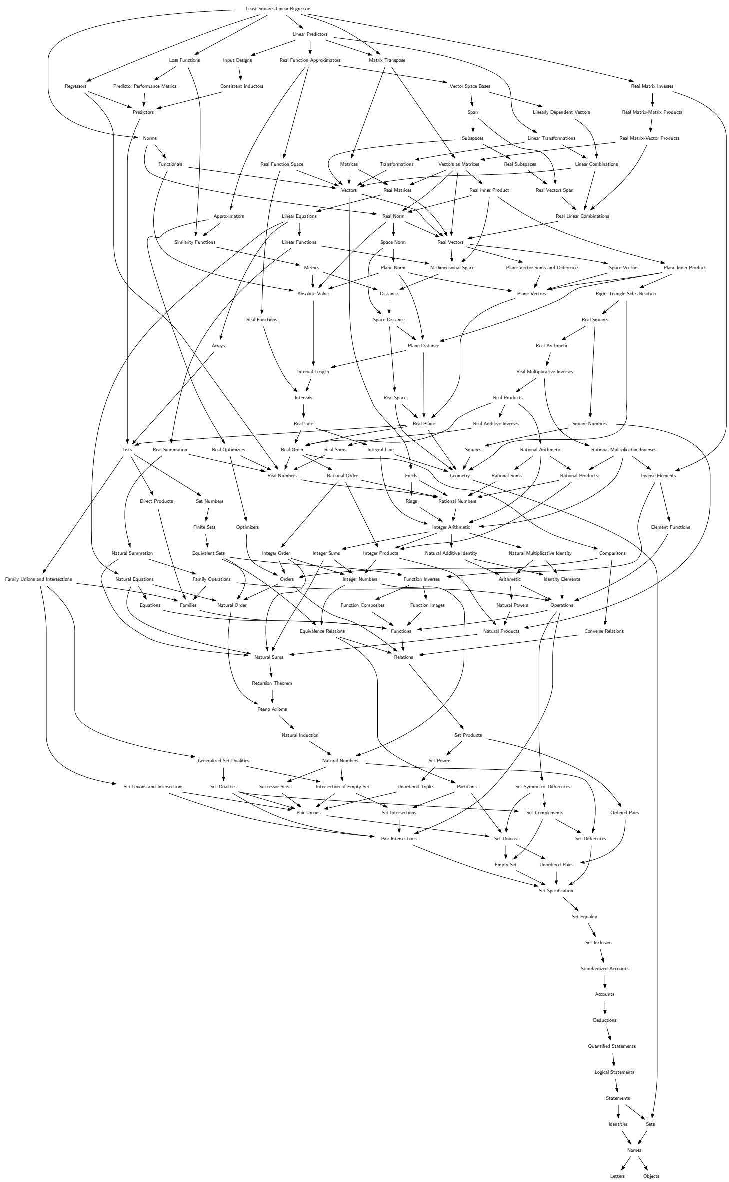
The set of linear functions from $\mathbf{R}^d$ to $\mathbf{R}$ is in one-to-one correspondence with $\mathbf{R}^d$. So we want to find $\theta \in \mathbf{R}^d$ to minimize

$$\frac{1}{n} \|X\theta - y\|^2.$$

Solution

Proposition 1. *There exists a unique linear least squares predictor and its parameters are given by* $\left(X^\top X\right)^{-1} X^\top y$.[1]

[1] Future editions will include an account.


Least Squares Linear Regressors
Linear Predictors
Loss Functions
Input Designs
Real Function Approximators
Matrix Transpose
Regressors
Predictor Performance Metrics
Consistent Inductors
Vector Space Bases
Real Matrix Inverses
Predictors
Span
Linearly Dependent Vectors
Real Matrix-Matrix Products
Norms
Subspaces
Linear Transformations
Real Matrix-Vector Products
Functionals
Real Function Space
Matrices
Transformations
Vectors as Matrices
Real Subspaces
Linear Combinations
Vectors
Real Matrices
Real Inner Product
Real Vectors Span
Approximators
Linear Equations
Real Norm
Real Linear Combinations
Similarity Functions
Linear Functions
Space Norm
Real Vectors
Metrics
Plane Norm
N-Dimensional Space
Plane Vector Sums and Differences
Space Vectors
Plane Inner Product
Absolute Value
Distance
Plane Vectors
Right Triangle Sides Relation
Real Functions
Space Distance
Real Squares
Arrays
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Real Line
Real Plane
Real Additive Inverses
Square Numbers
Lists
Real Summation
Real Optimizers
Real Order
Real Sums
Integral Line
Squares
Rational Arithmetic
Rational Multiplicative Inverses
Real Numbers
Rational Order
Fields
Geometry
Rational Sums
Rational Products
Inverse Elements
Direct Products
Set Numbers
Rings
Rational Numbers
Finite Sets
Optimizers
Integer Arithmetic
Element Functions
Natural Summation
Equivalent Sets
Integer Order
Integer Sums
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Comparisons
Family Unions and Intersections
Natural Equations
Family Operations
Orders
Integer Numbers
Function Inverses
Arithmetic
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Equations
Families
Natural Order
Function Composites
Function Images
Natural Powers
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Converse Relations
Natural Sums
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Peano Axioms
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Set Powers
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Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects