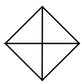


Why

For general measure theory, we need an algebra of sets closed under countable unions; we define such an object.[1]

Definition

A *countably summable subset algebra* is a subset algebra for which (1) the base set is distinguished (2) the complement of a distinguished set is distinguished (3) the union of a sequence of distinguished sets is distinguished.

The name is justified, as each countably summable subset algebra is a subset algebra, because the union of $A_1, \dots, A_n$ coincides with the union of $A_1, \cdots, A_n, A_n, A_n \cdots$.

We call the set of distinguished sets a *sigma algebra* (or *sigma field*) *on* the base set. This language is justified (as for a regular subset algebra) by the closure properties of the sigma algebra under the usual set operations. We sometimes write are σ-algebra and σ-field.

A *sub-σ-algebra* (*sub-sigma-algebra*) is a subset of a sigma algebra which is itself a sigma algebra.

Notation

Let $(A, \mathcal{A})$ be a countably summable subset algebra. We often say "let $\mathcal{A}$ be a sigma algebra on A." Since the largest element of the sigma algebra is the base set, we can also say (without ambiguity): "let $\mathcal{A}$ be a sigma algebra." In this last case, the base set is $\cup\mathcal{A}$.

[1] Future editions will make no reference to measure theory. The entire development will be for follow the historical development for handling integration.

Examples

Example 1. *For any set A, 2^A is a sigma algebra.*

Example 2. *For any set A, $\{A, \varnothing\}$ is a sigma algebra.*

Example 3. *Let A be an infinite set. Let $\mathcal{A}$ the collection of finite subsets of A. $\mathcal{A}$ is not a sigma algebra.*

Example 4. *Let A be an infinite set. Let $\mathcal{A}$ be the collection subsets of A such that the set or its complement is finite. $\mathcal{A}$ is not a sigma algebra.*

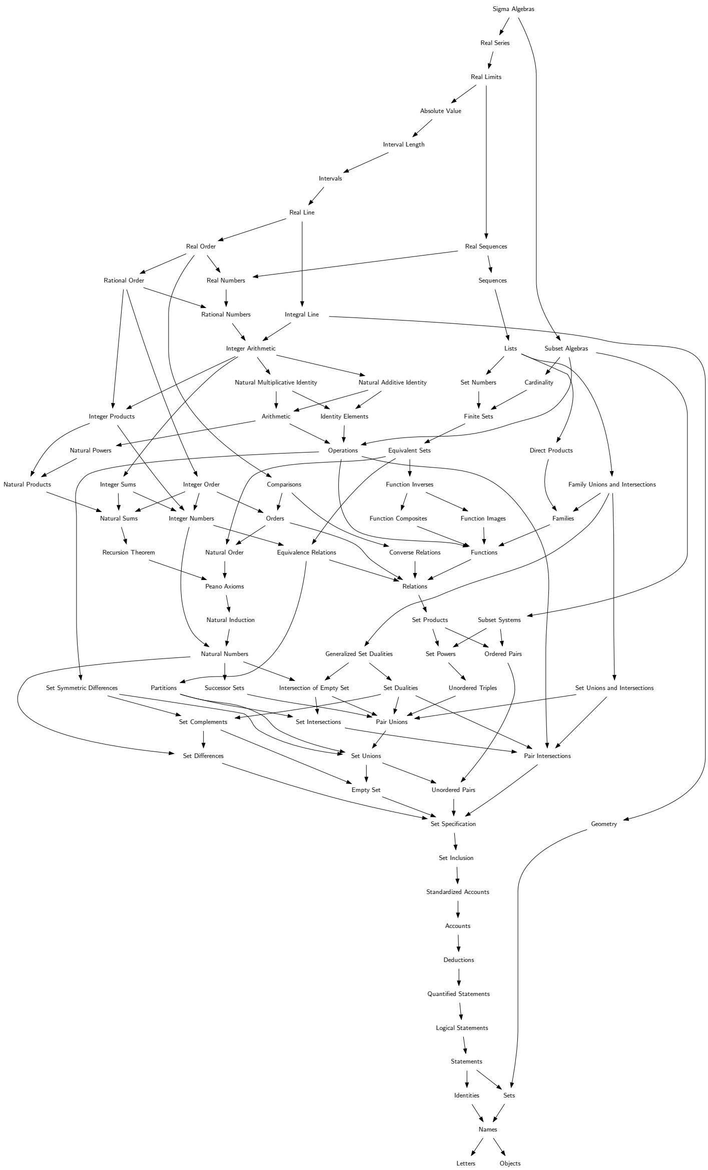
Proposition 1. *The intersection of a family of sigma algebras is a sigma algebra.*

Example 5. *For any infinite set A, let $\mathcal{A}$ be the set*

$$\{B \subset A \mid |B| \leq \aleph_0 \vee |C_A(B)| \leq \aleph_0\}.$$

$\mathcal{A}$ is an algebra; the countable/co-countable algebra.[2]

[2] Future editions will clean up and modify these examples.



Sigma Algebras
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Rational Numbers
Integral Line
Integer Arithmetic
Lists
Subset Algebras
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Cardinality
Integer Products
Arithmetic
Identity Elements
Finite Sets
Natural Powers
Operations
Equivalent Sets
Direct Products
Natural Products
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Sums
Integer Numbers
Orders
Function Composites
Function Images
Families
Recursion Theorem
Natural Order
Equivalence Relations
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Subset Systems
Natural Numbers
Generalized Set Dualities
Set Powers
Ordered Pairs
Set Symmetric Differences
Partitions
Successor Sets
Intersection of Empty Set
Set Dualities
Unordered Triples
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Geometry
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects