Why

We are given an element of some set, and want to find an element (in some subset) which is most similar to it.

Definition

Consider a non-empty set, one of its subsets, and a similarity function on it. An *approximator* of an element of the set is any element of the subset. So we call the subset the set of *approximators*. One approximator may be more similar than another. An *optimal* approximator is a minimizer of the similarity function over the set of approximators.

Notation

Let B be a non-empty set. Let $A \subset B$. Let $d : B \times B \to \mathsf{R}$ be a similarity function. For $b \in B$, every $a \in A$ is an approximator of b. An optimal b is a solution of

$$\begin{aligned} \text{minimize} \quad & d(b, a) \\ \text{subject to} \quad & a \in A. \end{aligned}$$

ε approximations

Let $a, b \in B$. For $\varepsilon > 0$, we say that an element b *ε-approximates* $a \in A$ if $d(a, b) \le \varepsilon$.

Approximators

Similarity Functions

Metrics

Distance

Absolute Value

Space Distance

Plane Distance

Interval Length

Intervals

Real Line

Real Space

Real Plane

Real Optimizers

Integral Line

Real Order

Real Numbers

Geometry

Optimizers

Rational Numbers

Integer Arithmetic

Sequences

Natural Additive Identity

Natural Multiplicative Identity

Set Numbers

Direct Products

Arithmetic

Identity Elements

Comparisons

Finite Sets

Integer Products

Natural Powers

Orders

Operations

Equivalent Sets

Family Unions and Intersections

Integer Sums

Natural Products

Natural Order

Function Inverses

Integer Numbers

Natural Sums

Function Images

Function Composites

Families

Recursion Theorem

Converse Relations

Equivalence Relations

Functions

Peano Axioms

Relations

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Partitions

Successor Sets

Intersection of Empty Set

Unordered Triples

Set Dualities

Set Unions and Intersections

Ordered Pairs

Set Intersections

Pair Unions

Set Complements

Set Differences

Set Unions

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects

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