Sequences

1 Why

We introduce language for the steps of an infinite process.

2 Definition

Let A be a non-empty set. A *sequence in* A is a function from the natural numbers to the set. The *nth term* of a sequence is the result of the nth natural number; it is an element of the set.

2.1 Notation

Let A be a non-empty set. $a : \mathsf{N} \to A$. is a sequence in A. $a(n)$ is the nth term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n.

3 Subsequences

A *subindex* is a monotonically increasing function from and to the natural numbers. Roughly, it selects some ordered infinite subset of natural numbers. A *subsequence* of a first sequence is any second sequence which is the composition of the first sequence with a subindex.

4 Notation

Let $i : N \to N$ such that $n < m \implies i(n) < i(m)$. Then i is a subindex. Let $b = a \circ i$. Then b is a subsequence of a. We denote it by $\{b_{i(n)}\}_n$ and the nth term by $b_{i(n)}$.

TODO: integrate, from direct products

If I is the set of natural numbers we denote the direct product by

$$\prod_{i=1}^{\infty} A_i.$$

We denote an element of $\prod_{i=1}^{\infty} A_i$ by (a_i) with the understanding that $a_i \in A_i$ for all $i = 1, 2, 3, \ldots$. If $A_i = A$ for all $i = 1, 2, 3, \ldots$, then (a_i) is a sequence in A.


Natural