Why

We can view the set of sequences as vector spaces and give them norms.

Bounded sequences

Let $\mathbf{C}^{\mathbf{N}}$ denote the set of complex-valued sequences. Define $\ell^\infty \subset \mathbf{C}^{\mathbf{N}}$ to be the set of all *bounded sequences*. That is,

$$\ell^\infty = \left\{ x \in \mathbf{C}^{\mathbf{N}} \;\middle|\; \exists M \in \mathbf{R} \text{ with } |x_i| < M \text{ for all } i \right\}.$$

Then ℓ^∞ with componentwise addition and scalar multiplication is a vector space.

Exercise 1. *Define* $\|\cdot\|_\infty : \ell^\infty \to \mathbf{R}$ *by*

$$\|x\|_\infty = \sup_{n \in \mathbf{N}} |x_n|.$$

Then $\|\cdot\|_\infty$ *is a norm on* ℓ^∞.

Absolutely summable sequences

Let $\ell^1 \subset \mathbf{C}^{\mathbf{N}}$ denote the set of all *absolutely summable sequences*. In other words, for $x \in \mathbf{C}^{\mathbf{N}}$, $x \in \ell^1$ if

$$\sum_{n=1}^{\infty} |x_n| < \infty.$$

Then ℓ^1 is a vector space with componentwise addition and scalar multiplication.

Exercise 2. *Define* $\|\cdot\|_1 : \ell^1 \to \mathbf{R}$ *by*

$$\|x\|_1 = \sum_{n=1}^{\infty} |x_n|$$

Then $\|\cdot\|_1$ *is a norm on* ℓ^1.

Sequence Spaces
Norms
Real Series
Real Sequence Space
Functionals
Real Norm
Vectors
Space Norm
Real Vectors
N-Dimensional Space
Plane Norm
Space Vectors
Plane Vector Sums and Differences
Complex Numbers
Real Limits
Distance
Plane Vectors
Absolute Value
Real Sequences
Space Distance
Real Arithmetic
Sequences
Plane Distance
Interval Length
Real Multiplicative Inverses
Real Space
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Integral Line
Real Order
Rational Arithmetic
Real Sums
Rational Multiplicative Inverses
Geometry
Rational Order
Fields
Rational Sums
Real Numbers
Rational Products
Rings
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Inverse Elements
Comparisons
Integer Order
Integer Sums
Natural Powers
Element Functions
Function Inverses
Family Unions and Intersections
Orders
Integer Numbers
Natural Products
Operations
Function Composites
Function Images
Families
Converse Relations
Natural Order
Natural Sums
Equivalence Relations
Functions
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Natural Numbers
Set Powers
Generalized Set Dualities
Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects