



Optimization

1 Why

Given a correspondence between objects in a set with objects in an totally ordered set, we are interested in the objects which correspond to minimal or maximal elements of the ordered set.

2 Definition

Let A be a non-empty set and let (C, \leq) be chain. Let $f : A \rightarrow C$.

The **minimization problem over** A associated with f is to find an element $a \in A$ so that $f(a)$ is minimal in $f(A)$. The **maximization problem over** A associated with f is to find an element $a \in A$ so $f(a)$ is maximal in $f(A)$. We call either of these an **optimization problem**.

We call f the **ordering** function. We call A the **feasible set** and we call $a \in A$ a **feasible element**. An element $a \in A$ is a **minimizer** of f if $f(a)$ is minimal in $f(A)$. An element $a \in A$ is a **maximizer** of f if $f(a)$ is maximal in $f(A)$.

3 Notation

Let (C, \prec) be a chain. We denote the minimization problem to find an element $a \in A$ to minimize $f : A \rightarrow C$ by

$$\begin{array}{ll} \text{find} & a \in A \\ \text{to minimize} & f(a) \end{array}$$

We denote the minimizers by

$$\mathbf{minimizers}(f).$$