Definition

The *cumulative distribution function* of a real-valued random variable is the function mapping a real number to the measure of the set of outcomes for which the random variable takes value less than or equal to the number. Notation (below) helps.

The range of the cumulative distribution function is the interval $[0, 1]$, since the measure of the base set is one and all measures are non-negative.

We often abbreviate the words "cumulative distribution function" by *c.d.f.* or *cdf*.

Notation

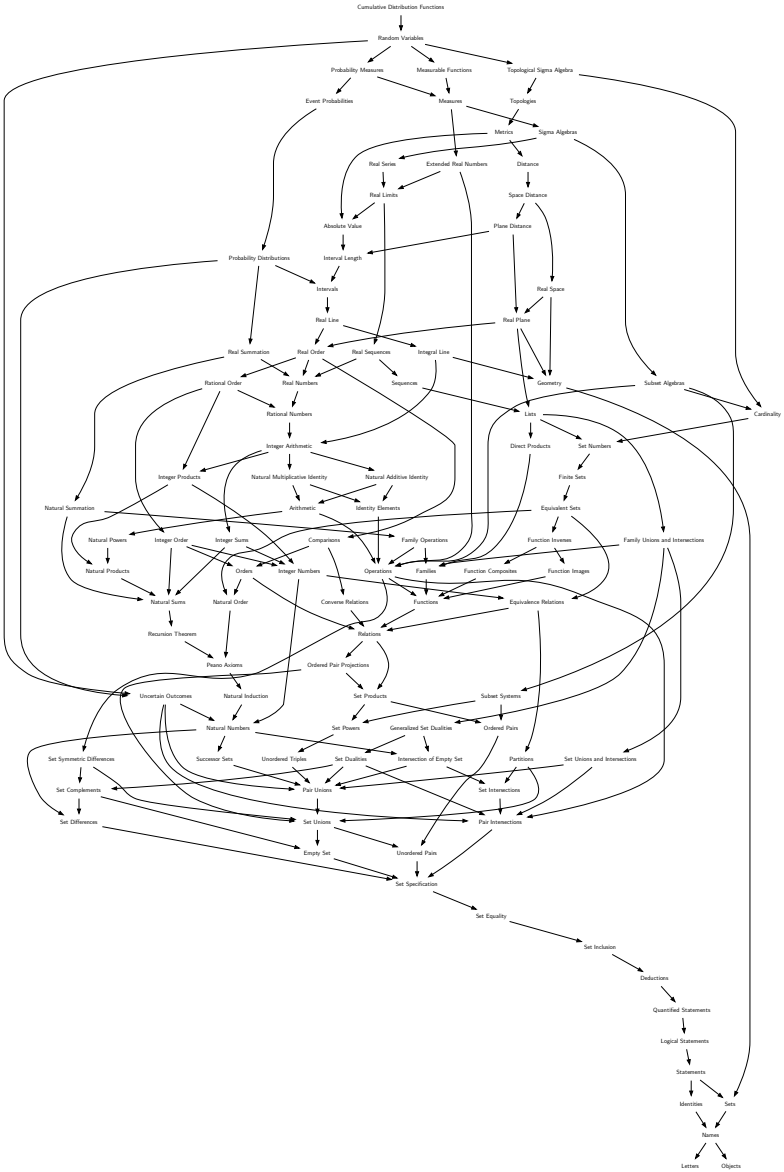
Let $(X, \mathcal{A}, \mu)$ be a probability space. Let $f : X \to \mathbf{R}$ be a measurable function (a real-valued random variable). We denote the cumulative distribution function of f by $F_f : \mathbf{R} \to [0, 1]$. We defined it by

$$F_f(t) = \mu(\{x \in X \mid f(x) \leq t\}).$$

Properties

Proposition 1. *The cumulative distribution function of any real-valued random variable*

1. *is non-decreasing,*
2. *is right-continuous,*
3. *tends to one as its argument tends toward infinity, and*
4. *tends to zero as its argument tends towards negative infinity.*


Cumulative Distribution Functions
Random Variables
Probability Measures
Measurable Functions
Topological Sigma Algebra
Event Probabilities
Measures
Topologies
Metrics
Sigma Algebras
Real Series
Extended Real Numbers
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Probability Distributions
Interval Length
Intervals
Real Space
Real Line
Real Plane
Real Summation
Real Order
Real Sequences
Integral Line
Subset Algebras
Rational Order
Real Numbers
Sequences
Geometry
Cardinality
Rational Numbers
Lists
Integer Arithmetic
Direct Products
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Summation
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Order
Integer Sums
Comparisons
Family Operations
Function Inverses
Family Unions and Intersections
Natural Products
Orders
Integer Numbers
Operations
Families
Function Composites
Function Images
Natural Sums
Natural Order
Converse Relations
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Uncertain Outcomes
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Generalized Set Dualities
Ordered Pairs
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects