Why

We want to discuss interactive decision making.

Discussion

We are interested in talking about situations in which there are several *decision makers*, *agents* or *players*, each of which are making decisions that will affect the outcome for all involved.

Example: rock paper scissors

Consider the game "rock-paper-scissors" in which there are two players A and B. Each player may choose one of the three actions ROCK, PAPER, SCISSORS. To play, each player simultaneously selects an action, and these are compared. Here, both agents have the same actions.

Definition

There is a finite set of *players*, or *agents*, or *controllers*. Let $\mathcal{I}$ be a finite set with $|\mathcal{I}| = n$, the players.

In rock-paper-scissors, for example, $\mathcal{I} = \{A, B\}$. There, each player could pick one of the three actions. Define $\mathcal{A}_A = \mathcal{A}_B = \{\text{ROCK}, \text{PAPER}, \text{SCISSORS}\}$. We call $\mathcal{A}_A$ the actions of A and $\mathcal{A}_B$ the actions of B.

We have a set of outcomes $\mathcal{O} = \{\text{A WINS}, \text{B WINS}, \text{TIE}\}$. Let $f : \mathcal{A}_A \times C_B \to \mathcal{O}$ defined by $f(\text{ROCK}, \text{SCISSORS}) = \text{A WINS}$

The set of *players*. Let S be a finite set, the set of *states*. For $i = 1, \ldots, n$, let $\{A_s^p\}_{s \in S}$ be a family of sets, the *action sets by state*. Define $\mathcal{A}^i = \cup_s A_s^p$ the set of *actions* for player $i = 1, \ldots, n$.

Let $f : S \times \prod_i \mathcal{A}^i \to S$, the *game dynamics* or *transition function*.

Games

Arrays

Lists　　　Decisions

Set Numbers　　　Direct Products

Finite Sets

Equivalent Sets　　　Orders

Family Unions and Intersections　　Natural Order　　Function Inverses

Families　　Function Composites　　Function Images

Equivalence Relations　　Peano Axioms　　Functions

Natural Induction　　　Relations

Uncertain Outcomes　　　Ordered Pair Projections

Generalized Set Dualities　　Natural Numbers　　Set Products

Partitions　　Intersection of Empty Set　　Successor Sets　　Set Powers

Set Unions and Intersections　　Set Dualities　　Set Intersections　　Unordered Triples

Pair Intersections　　Set Complements　　Pair Unions

Set Differences　　Set Unions　　Ordered Pairs

Empty Set　　Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities　　Sets

Names

Letters　　Objects

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