Why

The most important families are those indexed by (subsets of) the natural numbers.

Definition

A *finite sequence* (or *list*) is a family whose index set is $\{1, \ldots, n\}$ for some $n \in \mathbb{N}$. The *length* of a finite sequence is the size of its index set. If the codomain of a sequence is A, we say the sequence is *in A*.

Let A be a set with $|A| = n$. In this case, another term for a finite sequence is a *string* (or *list*). A sequence $a : \{1, \ldots, n\} \to A$ is an *ordering* of A if a is invertible. In this case, we call the inverse a *numbering* of A. An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

Notation

Since the natural numbers are ordered, we regularly denote finite sequences from left to right between parentheses. For example, we denote $a : \{1, \ldots, 4\} \to A$ by (a_1, a_2, a_3, a_4).

Relation to Direct Products

A *natural direct product* is a product of a sequence of sets. We denote the direct product of a sequence of sets A_1, …, A_n by $\prod_{i=1}^{n} A_i$. If each A_i is the same set A, then we denote the

product $\prod_{i=1}^{n} A_i$ by A^n. In this case, we call an element (the sequence $a = (a_1, a_2, \ldots, a_n) \in A^n$) an *n-tuple* or *tuple*. The set of sequences in a set A is the direct product A^n.

Infinite Sequences

An *infinite sequence* is a family whose index set is N (the set of natural numbers without zero). The *nth term* or *coordinate* of a sequence is the result of the nth natural number, $n \in \mathsf{N}$.[1]

Notation

Let A be a non-empty set and $a : \mathsf{N} \to A$. Then a is a (infinite) sequence in A. $a(n)$ is the nth term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n. If $\{A_n\}_{n \in \mathsf{N}}$ is an infinite sequence of sets, then we denote the direct product of the sequence by $\prod_{i=1}^{\infty} A_i$.

Natural unions and intersections

We denote the family union of the finite sequence of sets A_1, ..., A_n by $\cup_{i=1}^{n} A_i$. We denote the family of the infinite sequence of sets $(A_n)_n$ by $\cup_{i=1}^{\infty} A_i$. Similarly, we denote the intersections of a finite and infinite sequence of sets $\{A_i\}$ by $\cap_{i=1}^{n} A_i$ and $\cap_{i=1}^{\infty} A_i$, respectively.

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[1]Future editions may also comment that we are introducing language for the steps of an infinite process.

Slices[2]

An *index range* for a length-n sequence s is a pair (i, j) for which $1 \leq i < j \leq n$. The *slice* corresponding to the index range (i, j) is the length $j - i$ sequence s' defined by $s'_1 = s_i$, $s'_2 = s_{i+1}, \ldots, s'_j = s_{i+j-1}$. We denote the (i, j)-slice of s by $s_{i:j}$. If $i = 1$ we use $s_{:j}$ and if $j = n$ we use $s_{i:}$ as shorthands.

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[2]Future editions may break sequences in finite sequences and infinite sequences. This would simplify the sheet and remove the dependence on the principle of infinity. Future editions may also use the term "list" instead of finite sequence.

Sequences

Direct Products   Set Numbers

Finite Sets

Equivalent Sets

Family Unions and Intersections   Natural Order   Function Inverses

Families   Function Composites   Function Images

Peano Axioms   Functions   Equivalence Relations

Natural Induction   Relations

Cartesian Products

Generalized Set Dualities   Natural Numbers   Set Powers

Set Dualities   Set Unions and Intersections   Intersection of Empty Set   Successor Sets   Unordered Triples   Partitions

Set Complements   Pair Unions   Set Intersections   Ordered Pairs

Set Differences   Pair Intersections   Set Unions

Empty Set   Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects