



# Probability Distributions

## 1 Why

We want to talk about probability over finite sets.

## 2 Definition

A *probability distribution* or *probability mass function* is a real-valued function from a set of outcomes which is non-negative and normalized. A real-valued function on a finite set is *normalized* if the sum of its results is 1. We will refer to these as *distributions*. The *probability of an outcome* is the result of the outcome under the distribution.

### 2.1 Notation

Let  $A$  be a set of outcomes and Let  $p : A \rightarrow \mathbf{R}$  be a distribution; then

- $p(a) > 0$  for each  $a \in A$  and
- $\sum_{a \in A} p(a) = 1$

**Proposition 1.** *If  $p : A \rightarrow \mathbf{R}$  is a distribution, then  $p(A) \subset [0, 1]$ .*

*Proof.* Let  $a \in A$ . First,  $p(a) \geq 0$  by definition. Second, since  $p$  is normalized,  $\sum_{b \in A} p(b) = 1$ . And  $p(a) \leq \sum_{b \in A} p(b)$ , so  $p(a) \leq 1$ .

□