



## CHARACTERISTIC FUNCTIONS

### Why

We represent rectangles by functions.

### Definition

The *characteristic function* of a  $A$  of a set  $X$  is a function from  $X$  to 2 which is 1 if the argument is in  $A$  and 0 otherwise.

If the base set is the real numbers and the subset is an interval, then the characteristic function is a rectangle with height one and the width of the interval.

The function which assigns to each subset  $A$  of  $X$  to characteristic function of  $A$  is a one-to-one function from  $X^*$  onto  $2^X$ .

### Notation

Let  $A$  be a non-empty set and  $B \subset A$ . We denote the characteristic function of  $B$  in  $A$  by  $\chi_B : A \rightarrow R$ . The Greek letter  $\chi$  is a mnemonic for “characteristic”.

The subscript indicates the set on which the function is one. In other words, for all  $B \subset A$ ,  $\chi_B^{-1}(\{1\}) = B$ .

If  $B$  is an interval and  $\alpha$  is a real number then  $\alpha\chi_B$  is a rectangle with height  $\alpha$ .



