



# Measurable Sections

## 1 Why

Toward a theory of iterated integrals, we need to know that set and function sections are measurable.

## 2 Results

**Proposition 1.** *Let  $(X, \mathcal{A})$  and  $(Y, \mathcal{B})$  be measurable spaces. For any  $E \in \mathcal{A} \times \mathcal{B}$ , the sections  $E_x$  and  $E^y$  are measurable for any  $x \in X$  and  $y \in Y$ .*

*Proof.* TODO □

**Proposition 2.** *Let  $(X, \mathcal{A})$  and  $(Y, \mathcal{B})$  be measurable spaces. Let  $f : X \times Y \rightarrow F$ , where  $F$  is the extended real numbers or the complex numbers, and  $f$  is measurable (using the appropriate sigma algebra of the codomain). The sections  $f_x : Y \rightarrow F$  and  $f^y : X \rightarrow F$  are measurable for each  $x \in X$  and  $y \in Y$ .*

*Proof.* TODO □