

## **SET INCLUSION**

# Why

We want language for all of the elements of a first set being the elements of a second set.

## Definition

Denote a set by A and a set by B. If every element of the set denoted by A is an element of the set denoted by B, then we say that the set denoted by A is a *subset* of the set denoted by B.

We say that the set denoted by A is *included* in the set denoted by B. We say that the set denoted by B is a *superset* of the set denoted by A or that the set denoted by B includes the set denoted by A.

Every set is included in and includes itself.

#### Notation

Let A denote a set and B denote a set. We denote that the set A is included in the set B by  $A \subset B$ . In other words,  $A \subset B$  means  $(\forall x)((x \in A) \longrightarrow (x \in B))$ . We read the notation  $A \subset B$  aloud as "A is included in B" or "A subset B". Or we write  $B \supset A$ , and read it aloud "B includes A" or "B superset A".  $B \supset A$  also means  $(\forall x)((x \in A) \longrightarrow (x \in B))$ .

#### **Properties**

There are some properties that our intuition suggests inclusion should have. First, every set should include itself. We describe this fact by saying that inclusion is *reflexive*.

Proposition 1 (Reflexive). Every set is included in itself

*Proof.* (1) name 
$$A$$
; (2) have  $(\forall x)(x \in A \longrightarrow x \in A)$ ; (3) thus  $A \subset A$  by SetInclusion:Definition.

Next, we expect that if one set is included in another, This fact is described by saying that inclusion is *transitive* 

**Proposition 2** (Transitive). If a set is included in another, and the latter in yet another, then the first is included in the last.

*Proof.* (1) name 
$$A, B, C$$
; (2) have  $A \subset B$  (3) have  $B \subset C$  (4) thus  $A \subset C$  by modus ponens.

Equality (=) shares these two properties. Let A denote an object. Then A = A. Let B and C also denote objects. If A = B and B = C, then A = C. Of course, inclusion is not symmetric. Belonging ( $\in$ ) may be, but need not be reflexive and transitive.

