REAL PRODUCTS

Why

We want to multiply real numbers.[1]

Definition

The *real product* of two real numbers R and S is defined

1. if R or S is $\{q \in \mathbf{Q} \mid q < 0_\mathbf{Q}\}$, then the $\{q \in \mathbf{Q} \mid q < 0_\mathbf{Q}\}$

2. otherwise,

   (a) if R or S is $0_\mathbf{R}$, then $0_\mathbf{R}$.

   (b) if $R, S \neq 0_\mathbf{R}$ and $0_\mathbf{R} \in R, S$, let T be

   $$\{t \in \mathbf{Q} \mid r \in R, s \in S, r, s \geq 0_\mathbf{Q}, t = r \cdot s\}$$

   then $T \cup \{q \in \mathbf{Q} \mid q \leq 0_\mathbf{Q}\}$[2]

   (c) If $R, S \neq 0_\mathbf{R}$, $0_\mathbf{R} \in R$ and $0_\mathbf{R} \notin S$, then the additive inverse of the product of $-R$ with S.

   (d) If $R, S \neq 0_\mathbf{R}$, $0_\mathbf{R} \notin R$ and $0_\mathbf{R} \in S$, then the additive inverse of the product of R with $-S$.

   (e) If $R, S \neq 0_\mathbf{R}$, and $0_\mathbf{R} \notin R, S$, then the product of $-R$ with $-S$.

Notation

We denote the product of two real numbers x and y by $x \cdot y$.

Properties

Proposition 1 (Associative). $x + (y + z) = (x + y) + z$

Proposition 2 (Commutative). $x + y = y + x$

Proposition 3 (Identity). *The set of all rationals less than $1_\mathbf{Q}$ is the multiplicative identity.*

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[1]Future editions will expand.

[2]We use $\geq$ in the usual way, it will be defined earlier in future editions.

We denote the the multiplicative identity by $1_{\mathbf{R}}$. When it is clear from context, we call $1_{\mathbf{R}}$ "one".

Real Products

Real Additive Inverses

Rational Arithmetic

Real Sums

Real Order

Rational Products

Rational Sums

Real Numbers

Rational Order

Rational Numbers

Integer Arithmetic

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Identity Elements

Arithmetic

Natural Powers

Integer Sums

Integer Order

Operations

Natural Products

Integer Numbers

Comparisons

Functions

Natural Sums

Equivalence Relations

Orders

Converse Relations

Recursion Theorem

Natural Order

Relations

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Set Powers

Ordered Pairs

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Intersections

Set Symmetric Differences

Pair Unions

Pair Intersections

Set Complements

Set Unions

Empty Set

Set Differences

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects