

# REGULAR LANGUAGES

# **Definition**

Let  $\Sigma$  be an alphabet. A language  $L \subset \mathsf{str}(\Sigma)$  is called *regular* if there exists a finite automaton that recognizes it.

# Regular operations

Let  $A, B \subset \mathsf{str}(\Sigma)$  be languages in  $\Sigma$ .

## Union

The union (alternation) of A and B is, as usual, the set  $A \cup B$ .

#### Concatenation

The concatenation of A and B is the set  $\{xy \mid x \in A \text{ and } y \in B\}$ , where xy denotes length |x| + |y| string which is the concatenation of x and y.

#### Multi-concatenation

The star (Kleene star, multi-concatenation) of A is the set

$$\{x \in \mathsf{str}(\Sigma) \mid \exists k > 0, x = y_1 y_2 \cdots y_k, y_i \in A\}.$$

By this definition we do mean to include the empty string  $\varnothing$  in  $A^*$ , regardless of A.

## **Notation**

We denote the alternation of A and B by  $A \cup B$  as usual, but other notations include A + B,  $A \mid B$ , and  $A \vee B$ . We denote

the concatenation of A and B by AB, but other notations include  $A \circ B$ . We denote the star of A by  $A^*$ .

