Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by tr : $\mathbf{R}^{n \times n} \to \mathbf{R}$. The trace of $A \in \mathbf{R}^{n \times n}$ is

$$\operatorname{tr} A = \sum\_i = 1^n A\_ii.$$

Properties

Proposition 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}Matnn$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\operatorname{tr} C = \sum\_i = 1^n C\_ii = \sum\_i = 1^n \alpha A\_ii + \beta B\_ii$$
$$= \alpha \sum\_i = 1^n A\_ii + \beta \sum\_i = 1^n B\_ii$$
$$= \alpha \operatorname{tr} A + \beta \operatorname{tr} B.$$

$\square$

Proposition 2. *Let $A, B \in \mathbf{R}^{n \times n}$. Then*

$$\operatorname{tr}(AB) = \operatorname{tr}(BA).$$

In other words, "matrices commute under the trace operator."

Proposition 3. *Let $A \in \mathbf{R}^{n \times n}$. Then $\operatorname{tr} A = \operatorname{tr} A^\top$.*

Matrix Trace

Real Matrix-Matrix Products

Matrix Transpose Real Matrix-Vector Products

Vectors as Matrices Real Linear Combinations

Real Inner Product Real Matrices

Space Inner Product Real Norm Linear Equations

Space Norm Real Vectors Linear Functions Arrays

Plane Inner Product Plane Norm N-Dimensional Space Space Vectors Plane Vector Sums and Differences

Right Triangle Sides Relation Absolute Value Distance Plane Vectors

Real Squares Space Distance

Real Arithmetic Plane Distance

Real Multiplicative Inverse

Real Products Real Space Interval Length

Square Numbers Real Additive Inverse Real Line Real Plane Intervals

Square Integral Line Rational Arithmetic Rational Multiplicative Inverse Real Sums Real Order Real Summation

Geometry Rational Sums Rational Products Rational Order Real Numbers

Rational Numbers Lists

Integer Arithmetic Set Numbers Direct Products

Integer Products Natural Additive Identity Natural Multiplicative Identity Finite Sets

Arithmetic Identity Elements Natural Summation Equivalent Sets Inverse Elements

Natural Power Integer Sum Natural Equations Integer Order Comparisons Family Operations Element Functions Function Inverse Families

Natural Products Integer Number Equations Orders Operations Function Composites Function Images Family Unions and Intersections

Natural Sums Converse Relation Equivalence Relations Functions

Natural Order Relations

Recursion Theorem Ordered Pair Projections

Peano Axioms Set Products

Natural Induction Set Powers Generalized Set Dualities

Natural Numbers Successor Sets Unordered Triples Set Dualities Intersection of Empty Set Partitions Set Unions and Intersections

Set Symmetric Difference

Set Complements Pair Unions Ordered Pairs Set Intersections

Set Differences Set Unions Pair Intersections

Empty Set Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deduction

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects