

Why

We use a normal random function model to make a regressor.

Definition

Let $F: \Omega \to (A \to \mathbf{R})$ be a normal random function with mean function $m: A \to \mathbf{R}$ and covariance function $k: A \times A \to \mathbf{R}$ over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Let the family of random variables (or stochastic process) of F be $f: A \to (\Omega \to \mathbf{R})$.

Let e be a normal random vector with mean zero and covariance Σ_e . Let $a^1, \ldots, a^n \in A$. We sometimes call the sequence a^1, \ldots, a^n the design. Define $y: \Omega \to \mathbb{R}^d$ by

$$y_i = f(a^i) + e_i$$

We call y the observation vector or observation random vector. We call e the error vector or noise vector. In this context, $f(a^i)$ is sometimes called the signal.

Let $b^1, \ldots, b^m \in A$. Define $z : \Omega \to \mathbf{R}^d$ by $z_i = f(b^i)$ for $i = 1, \ldots, n$. So z_i is the random variable corresponding to the family at index $b^i \in A$. Then (y, z) is normal. We call the conditional density of z given y the predictive density for b given a.

Proposition 1. Define $m_a \in \mathbf{R}^n$ by $(m(a^1), \dots, m(a^n))^{\top}$ and define m_b by $(m(b^1), \dots, m(b^m))^{\top}$. Define $\Sigma_a \in \mathbf{R}^{n \times n}$ by

$$\begin{pmatrix} k(a^1, a^1) & \cdots & k(a^1, a^n) \\ \vdots & \ddots & \vdots \\ k(a^n, a^1) & \cdots & k(a^n, a^n) \end{pmatrix}$$

and define $\Sigma_{ba} \in \mathbf{R}^{m \times n}$ by

$$\begin{pmatrix} k(b^1, a^1) & \cdots & k(b^1, a^n) \\ \vdots & \ddots & \vdots \\ k(b^m, a^1) & \cdots & k(b^m, a^n) \end{pmatrix}.$$

 $^{^{1}}$ Future editions will fix the re-use of the symbol m.

The predictive density $g_{z|y}(\cdot,\gamma): \mathbf{R}^m \to \mathbf{R}$ of $b \in A$ for design a^1,\ldots,a^n is normal with mean.

$$m_b + K_{ba}(K_a + \Sigma_e)^{-1}(\gamma - m_a)$$

and covariance

$$\Sigma_b - \Sigma_{ba}(\Sigma_a + \Sigma_e)^{-1}\Sigma_{ab}.$$

