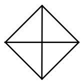


Why

If R corresponds to a line, and R^2 to a plane, and R^3 to space, does R^4 correspond to anything? What of R^5?

Definition

Let n be a natural number. We call the set R^n *n-dimensional space*. We call elements of R^n *points* and call the point associated with $x = (x_1, x_2, \dots, x_n) \in \mathsf{R}^n$ with $x_i = 0$ for $1 \leqq i \leqq n$ the *origin*. When it is clear from context, we denote the point x with $x_i = 0$ for all $i = 1, \dots, n$ by 0. Likewise, we denote teh point x with $x_i = 1$ for all $i = 1, \dots, n$ by 1 when it is clear from context.

Visualization

We can not visualize n-dimensional space. Thus, our intuition for it comes from real space (see Real Space).

Distance

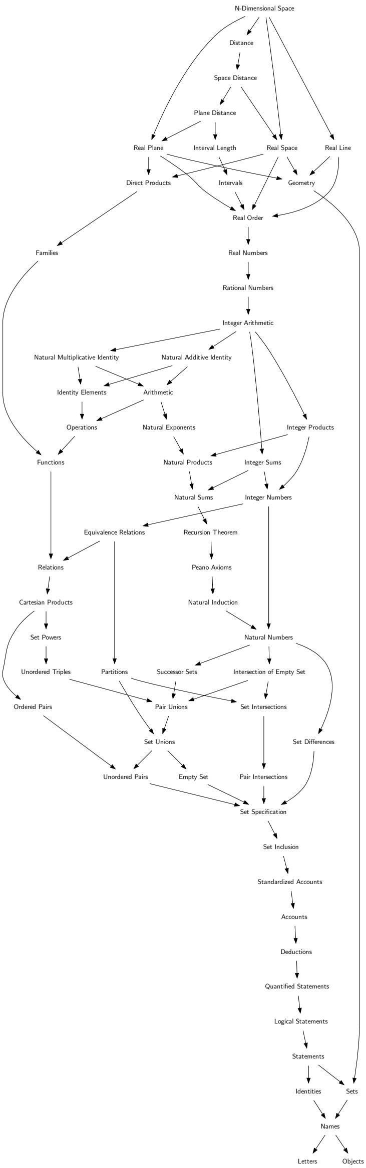
A natural notion of distance for R^n is the extension of the Euclidean distance. We define the distance between $(x_1, x_2, \dots, x_n)$, $(y_1, y_2, \dots, y_n) \in \mathsf{R}^n$ as

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2}$$

This is sometimes called the *Euclidean distance for n-dimensional space*. Does this have the properties that distance has in the plane and in space? We discussed these properties It does. Denote the function which associates to $x, y \in \mathsf{R}^n$ their distance $d : \mathsf{R}^n \times \mathsf{R}^n \to \mathsf{R}$. So $d(x, y)$ is the distance between the points corresponding to x and y.

Proposition 1. *d is non-negative, symmetric, and the distance between two points is no larger than the sum of the distances with any third object.*[1]

[1]Future editions will include an account.


N-Dimensional Space
Distance
Space Distance
Plane Distance
Real Plane
Interval Length
Real Space
Real Line
Direct Products
Intervals
Geometry
Real Order
Families
Real Numbers
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Identity Elements
Arithmetic
Operations
Natural Exponents
Integer Products
Functions
Natural Products
Integer Sums
Natural Sums
Integer Numbers
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Set Powers
Natural Numbers
Unordered Triples
Partitions
Successor Sets
Intersection of Empty Set
Ordered Pairs
Pair Unions
Set Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects