



Why

We want to repeatedly multiply.

Defining result

Proposition 1. *For each natural number m , there exists a function $e_m : \omega \rightarrow \omega$ which satisfies*

$$e_m(0) = 1 \quad \text{and} \quad e_m(n^+) = (e_m(n))^+ \cdot m$$

for every natural number n .

Proof. The proof uses the recursion theorem (see Recursion Theorem).¹

□

Let m and n be natural numbers. The value $p_m(n)$ is the *power* of m with n . Or the *n th power* of m

Notation

We denote the n th power of m by m^n .

Properties

Here are some basic properties of powers.

Proposition 2. *Let k , m , and n be natural numbers. Then*

$$m^n m^k = m^{k+n}.$$

Proposition 3. *Let k , m , and n be natural numbers. Then*

$$(m^n)^k = m^{nk}.$$

¹Future editions will give the entire account.

