

Closest Point Property

Suppose H is a Hilbert space $A\subset H$ closed and convex. then if $x\in H$, there exists a unique $z\in A$ closest to x. There exists a unique $z\in A$ closest to x such that

$$d(z,x) = \inf_{y \in A} d(y,x).$$

Proof. Take any sequence $\{y_n\}_n$ such that

$$d(y_n, x) \to d = \inf_{y \in A} d(y, x)$$