Why

We want to talk rooting a tree at a given vertex.[1]

Definition

A *rooted tree* is an ordered pair $((V, T), r)$ where (V, T) is a tree and $r \in V$ is a distinguished vertex which we call the *root*. We visualize rooted trees with the root at the top (see Figure 1).

Parents and Children

Suppose w is the first vertex on the path from the root to a non-root vertex v. Since there is only one such path, w is unique and we call it *the parent* of v. Conversely, we call v *a child* of w. We denote the set of children of v by $\text{ch}(v)$. A vertex may have no children or it may have many children. If it has no children we call it a *leaf*.

We define the *parent function* $\text{pa} : V \to V$ with the convention that the *parent of the root* is the root. The *parent of degree k* where $k > 0$ is $\text{pa}^k(x)$ where pa^k is the composite of pa with itself k times. So, in particular, $\text{pa}^{k+1}(v) = \text{pa}(\text{pa}^k(v))$. We define the *parent of degree 0* of v to be v, and denote it by $\text{pa}^0(v) = v$. For the tree visualized in Figure 1, $\text{pa}(i) = g$, $\text{pa}^2(i) = d$, $\text{pa}^3(i) = a$.

If $w = \text{pa}^k(v)$ for some $k \geq 0$, then w is a *ancestor* of v and v is a *descendent* of w. We use the term *proper ancestor*

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[1]Future editions will expand this intuition.

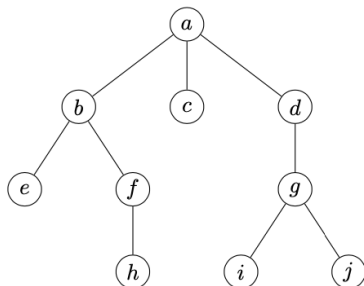Figure 1: A rooted tree with root a.

and *proper descendent* if $k > 0$ (i.e., $w \neq v$).

The *depth* or *level* of a vertex v is its distance (see Trees) to the root. We denote the level of a vertex v by $\mathrm{lev}(v)$. The level of the root is 0. If $\mathrm{lev}(v) = k > 0$, then $\mathrm{pa}^k(v)$ is the root. The level function lev satisfies $\mathrm{lev}(v) = \mathrm{lev}(\mathrm{pa}(v)) + 1$.

Rooted Trees

Trees

Directed Graphs

Undirected Paths

Undirected Graphs

Sequences

Set Numbers

Direct Products

Finite Sets

Equivalent Sets

Function Inverses

Natural Order

Family Unions and Intersections

Function Images

Function Composites

Families

Functions

Equivalence Relations

Peano Axioms

Relations

Natural Induction

Set Products

Set Powers

Natural Numbers

Generalized Set Dualities

Partitions

Unordered Triples

Successor Sets

Intersection of Empty Set

Set Dualities

Set Unions and Intersections

Ordered Pairs

Set Intersections

Pair Unions

Set Complements

Set Unions

Set Differences

Pair Intersections

Unordered Pairs

Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects