Definition

Let X and Y be sets. A *hidden memory chain* (*hidden markov chain*, *hidden memory model*, *hidden markov model*,[1] *HMM*) with *hiddens* (or *latents*) Y and *observations* Y of *length* n is a joint distribution $p : X^n \times Y^n \to [0,1]$ satisfying

$$p(x,y) = f(x)g(y_1,x_1)\prod_{i=1}^{n} h(x_i,x_{i-1})g(y_i,x_i),$$

where $f : X \to [0,1]$ is a distribution, and g and h are functions satisfying $g(\cdot,\xi)$ and $h(\cdot,\xi)$ are distributions on Y and X, respectively, for all $\xi \in X$.

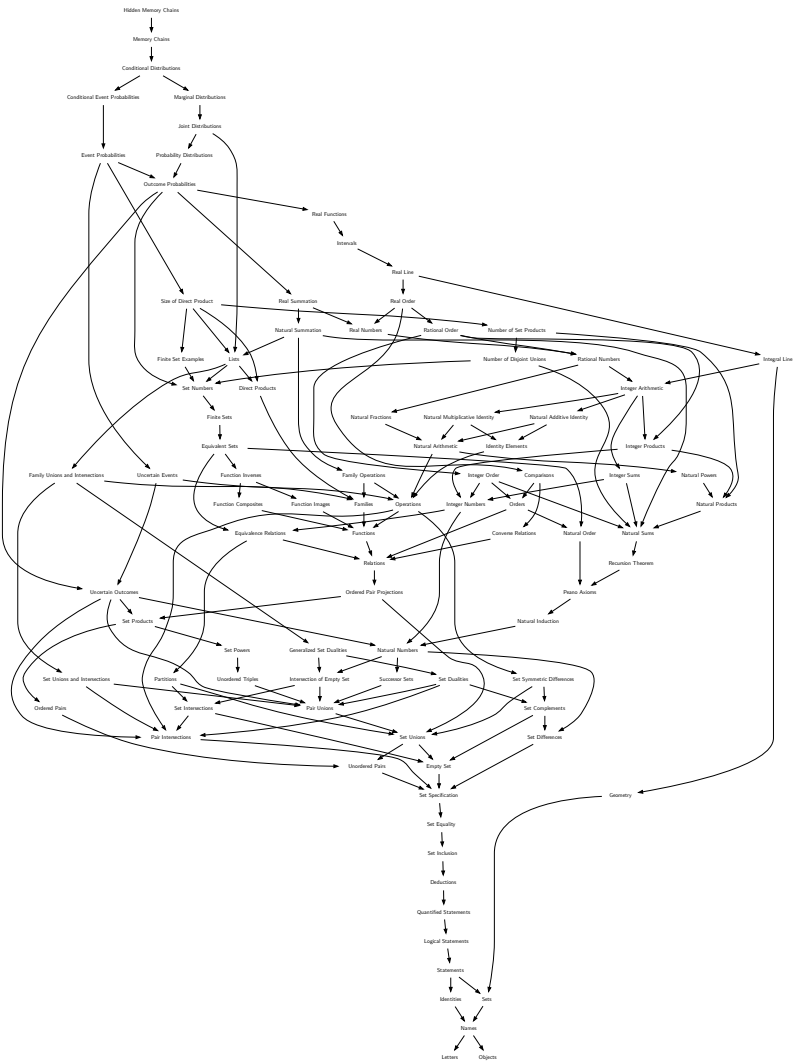
Proposition 1. *p so defined is a distribution. The function f is the distribition p_1. For all $i = 1,\ldots,n$, $p_{n+1|i} \equiv gg$. For all $i = 2,\ldots,n$, $p_{i|i-1} \equiv h$.*[2]

Clearly, $p_{1,\ldots,n}$ is a memory chain (see Memory Chains). For this reason, we continue to refer to h as the *conditional distribution.* We continute to refer to f as the *initial distribution.* We refer to h as the *observation distribution.*

The word "hidden" refers to the situation in which we observe outcomes y, and we hypothesize that they were "generated by" unobserved outcomes x.

[1] This term is universal. We avoid it because of the Bourbaki project's policy on naming. The skeptical reader will note (as in Memory Chains) that our term and this term have the same initials.

[2] Future editions will define everything needed in this proposition in the proposition statement, as ooposed to saying "p so defined"


Hidden Memory Chains
Memory Chains
Conditional Distributions
Conditional Event Probabilities
Marginal Distributions
Joint Distributions
Event Probabilities
Probability Distributions
Outcome Probabilities
Real Functions
Intervals
Real Line
Size of Direct Product
Real Summation
Real Order
Natural Summation
Real Numbers
Rational Order
Number of Set Products
Finite Set Examples
Lists
Number of Disjoint Unions
Rational Numbers
Integral Line
Set Numbers
Direct Products
Integer Arithmetic
Finite Sets
Natural Fractions
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Natural Arithmetic
Identity Elements
Integer Products
Family Unions and Intersections
Uncertain Events
Function Inverses
Family Operations
Integer Order
Comparisons
Integer Sums
Natural Powers
Function Composition
Function Images
Families
Operations
Integer Numbers
Orders
Natural Products
Equivalence Relations
Functions
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Uncertain Outcomes
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Difference
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Difference
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects