Random Variables

1 Why

TODO

2 Definition

A **random variable** is a measurable map from a probability space to a measurable space.

A **real-valued random variable** is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

2.1 Notation

Let $(X, \mathcal{A}, p)$ be a probability space. Let R denote the set of real numbers. Let $(Y, \mathcal{B})$ a measurable space. Then a random variable is a measurable function $f : X \to Y$.

Some authors denote real-valued random variables by upper case Latin letters: for example, X, Y, Z. In this case, the base probability space is denoted by Ω, a mnemonic for "outcomes.".

Let $(\Omega, \mathcal{A}, p)$ be a probability space. Let R denote the set of real numbers, Let $X : \Omega \to R$ be measurable.

Some authors use notation for the probability of certain common sets. Let $A \in \mathcal{B}(R)$. Let $p(X \in A)$ denote $p(X^{-1}(A))$. These are equivalent to

$$p(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

Next, let $Y : \Omega \to R$ a measurable function and let $B \in \mathcal{B}(R)$. Similar to the above, let $p(X \in A, Y \in B)$ denote $p(X^{-1}(A) \cap Y^{-1}(B))$. These are equivalent to

$$p(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$