### Why

We can view the set of sequences as vector spaces and give them norms.

### Bounded Sequences

Let $\mathbf{C}^{\mathbf{N}}$ denote the set of complex-valued sequences. Define $\ell^\infty \subset \mathbf{C}^{\mathbf{N}}$ to be the set of all *bounded sequences*. That is,

$$\ell^\infty = \left\{ x \in \mathbf{C}^{\mathbf{N}} \;\middle|\; \exists M \in \mathbf{R} \text{ with } |x_i| < M \text{ for all } i \right\}.$$

Then $\ell^\infty$ with componentwise addition and scalar multiplication is a vector space.

**Exercise 1.** *Define* $\|\cdot\|_\infty : \ell^\infty \to \mathbf{R}$ *by*

$$\|x\|_\infty = \sup_{n \in \mathbf{N}} |x_n| .$$

*Then* $\|\cdot\|_\infty$ *is a norm on* $\ell^\infty$*.*

### Absolutely Summable Sequences

Let $\ell^1 \subset \mathbf{C}^{\mathbf{N}}$ denote the set of all *absolutely summable sequences*. In other words, for $x \in \mathbf{C}^{\mathbf{N}}$, $x \in \ell^1$ if

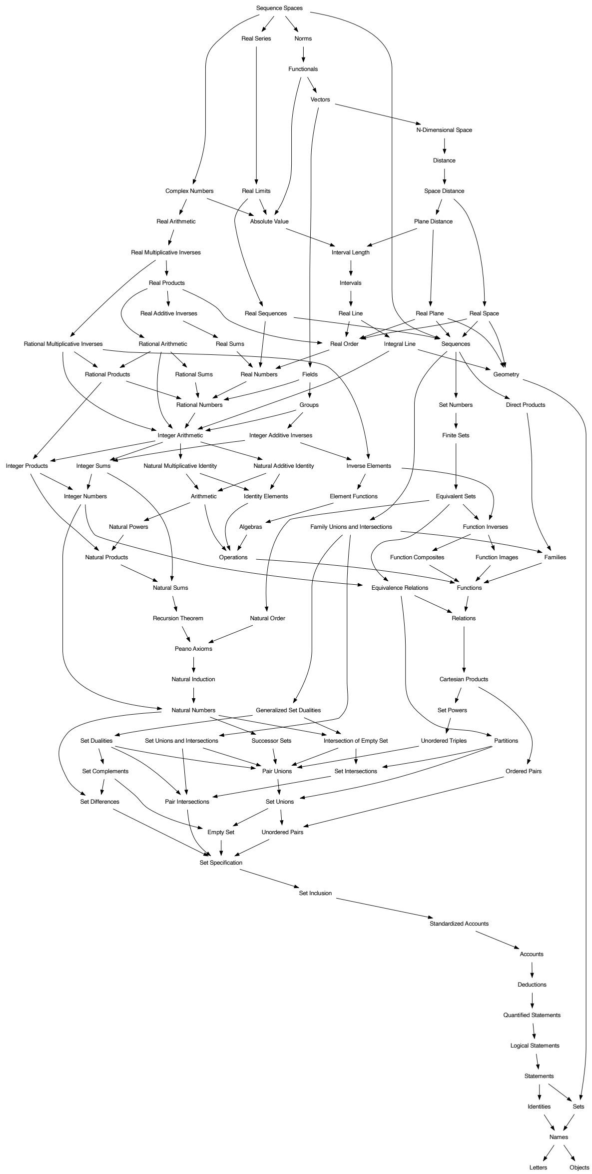
$$\sum_{n=1}^{\infty} |x_n| < \infty.$$

Then $\ell^1$ is a vector space with componentwise addition and scalar multiplication.

**Exercise 2.** *Define* $\|\cdot\|_1 : \ell^1 \to \mathsf{R}$ *by*

$$\|x\|_1 = \sum_{n=1}^{\infty} |x_n|$$

*Then* $\|\cdot\|_1$ *is a norm on* $\ell^1$.


Sequence Spaces
Real Series
Norms
Functionals
Vectors
N-Dimensional Space
Distance
Complex Numbers
Real Limits
Space Distance
Real Arithmetic
Absolute Value
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Intervals
Real Additive Inverses
Real Sequences
Real Line
Real Plane
Real Space
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Sequences
Rational Products
Rational Sums
Real Numbers
Fields
Geometry
Rational Numbers
Groups
Set Numbers
Direct Products
Integer Arithmetic
Integer Additive Inverses
Finite Sets
Integer Products
Integer Sums
Natural Multiplicative Identity
Natural Additive Identity
Inverse Elements
Integer Numbers
Arithmetic
Identity Elements
Element Functions
Equivalent Sets
Natural Powers
Algebras
Family Unions and Intersections
Function Inverses
Natural Products
Operations
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Natural Order
Relations
Peano Axioms
Natural Induction
Cartesian Products
Natural Numbers
Generalized Set Dualities
Set Powers
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects