

ORDERED UNDIRECTED GRAPH ORIENTATIONS

Why

There is a natural orientation of an ordered undirected graph.

Motivating result

An ordered undirected graph can be converted into a directed graph by orienting the edges from lower to higher index. The *orientation* of an ordered undirected graph $((V, E), \sigma)$ is the directed graph (V, F) where

$$\{v,w\} \in V \longrightarrow (v,w) \in F \text{ and } \sigma^{-1}(v) < \sigma^{-1}(w).$$

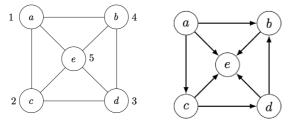
In other words, we can "convert" the ordered undirected graph by "orienting" the edges from lower to higher index.

Proposition 1. Let $G = ((V, E), \sigma)$ be an ordered undirected graph. The orientation of G is acyclic.

Proof. Contradiction on the existence of a cycle.
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Conversely, let (V, F) be directed acyclic. To each topological numbering σ of (V, F) (see Directed Paths) there exists an ordered undirected graph $((V, E), \sigma)$ where (V, E) is the skeleton of (V, F).

Example



¹Future editions will expand.

Let $G=((V,E),\sigma)$ be an undirected graph with

$$V = \{a, b, c, d, e\},\$$

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\},\$$

and

$$\sigma(1) = a$$
 $\sigma(2) = c$ $\sigma(3) = d$ $\sigma(4) = b$ $\sigma(e) = 5$.

We visualize G and its (directed acyclic) orientation above.

