



## Why

We want to visualize symmetric relations.

## Definition

An *undirected graph* (or *graph*, *network*) is a pair  $(V, E)$  in which  $V$  is a nonempty set and  $E$  is a subset of *unordered* pairs of elements in  $V$ . We call the elements of  $V$  the *vertices* (or *points*) of the graph and the elements of  $E$  the *edges* (or *arcs*, *lines*). We call  $(V, E)$  an undirected graph on  $V$ .

Two vertices are *adjacent* (or *neighboring*) if their pair is in the edge set. We say that the corresponding edge is *incident* to those vertices. The *adjacency set* of a vertex is the set of vertices adjacent to it. The *degree* of a vertex is the number of vertices adjacent to it; in other words, the size of its adjacency set. A graph is *complete* if each pair of two distinct vertices is adjacent.

The *complement* of  $(V, E)$  is the graph  $(V, F)$  where  $F$  is the complement of  $E$  in the set of pairs from  $V$ .

## Other terminology

Some authors call the adjacency set the *neighborhood* of the vertex. They call the union of the adjacency set of the vertex  $v \in V$  with the singleton  $\{v\}$  the *closed neighborhood* of  $v$ .

When  $\{x, y\} \in E$ , some authors say that the edge “joins” the vertices, and call  $x$  and  $y$  the *end vertices* (or *ends*) of the edge.

Some authors call two *edges adjacent* if they have exactly one common end vertex.

## Self-loops

Some authors include a concept of a self-loop, which is meant to be an edge from of a “self-loop”—an edge from one node to itself. However, we

do not allow this as the set  $\{u, u\}$  is *not* a pair.

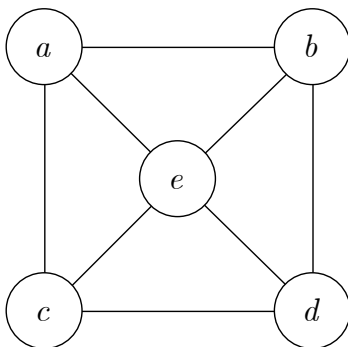
### Notation

Let  $V$  be a nonempty set. Let  $E \subset \{\{v, w\} \mid v, w \in V\}$ . Then the pair  $(V, E)$  is an undirected graph. We regularly say “Let  $G = (V, E)$ ” be a graph, in which the relevant properties of  $V$  and  $E$  are implicit.

The notation  $\{v, w\} \in E$  for an edge between vertices  $v, w \in V$  reminds us that the edges are unordered pairs of distinct vertices. We denote the adjacency set of  $v$  by  $\text{adj}(v)$  and the degree of  $v$  by  $\deg(v)$ .

Some authors will denote the vertex set of a graph they are denoting by  $G$  by  $V(G)$  and the edges set by  $E(G)$ .

### Example



Suppose  $V = \{a, b, c, d, e\}$ . Define

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\}.$$

In visualizations of graphs, the vertices are frequently represented as circles or rectangles in the plane and edges are shown as lines connecting the vertices.

