Why

How should we represent a subspace computationally?

Definition

Given a subspace $S \in \mathbf{R}^n$, since it is finite dimensional, there exists a finite basis for the space. This basis can be made orthonormal. Therefore every subspace S has an orthonormal basis $q_1, \ldots, q_k$, where k is the dimension of the subspace. We can stack these as a matrix. Define $Q \in \mathbf{R}^{n \times k}$ by

$$Q = \begin{bmatrix} q_1 & q_2 & \cdots & q_k \end{bmatrix}.$$

For every $x \in S$, there exists unique coefficients $z \in \mathbf{R}^k$ so that

$$x = Qz.$$

Therefore we have a one-to-one correspondence between vectors $x \in S$ and their coordinates $z \in \mathbf{R}^k$.

Real Subspace Representations

Real Matrices with Orthonormal Columns

Real Matrix-Matrix Products

Real Matrix-Vector Products

Orthonormal Set of Real Vectors · Vector as Matrices

Real Inner Product · Real Linear Combination · Real Matrices

Space Inner Product · Real Norm · Linear Equation

Space Norm · Real Vectors · Linear Function · Arrays

Plane Inner Product · Plane Norm · N-Dimensional Space · Space Vector · Plane Vector Sums and Differences

Right Triangle Sides Relation · Absolute Value · Distance · Plane Vectors

Real Square · Space Distance

Real Arithmetic · Plane Distance

Real Multiplicative Inverse · Interval Length

Real Products · Real Space · Intervals

Square Numbers · Real Additive Inverse · Real Line · Real Plane

Square · Integral Line · Rational Arithmetic · Rational Multiplicative Inverse · Real Sums · Real Order · Real Summation

Geometry · Rational Sum · Rational Products · Rational Order · Real Numbers · Lists

Rational Numbers · Set Numbers · Direct Products

Integer Arithmetic · Finite Sets

Integer Products · Natural Multiplicative Identity · Natural Additive Identity

Arithmetic · Identity Elements · Natural Summation · Equivalent Sets · Inverse Elements

Natural Powers · Integer Sums · Integer Order · Natural Equations · Comparisons · Element Functions · Family Operations · Function Inverses · Family Unions and Intersections

Natural Products · Integer Numbers · Order · Equations · Operations · Function Composition · Function Images · Families

Natural Sums · Natural Order · Equivalence Relations · Converse Relations · Functions

Recursion Theorem · Relations

Peano Axioms · Ordered Pair Projections

Natural Induction · Set Products

Natural Numbers · Generalized Set Dualities

Set Symmetric Differences · Partition · Successor Sets · Intersection of Empty Set · Unordered Triples · Set Dualities · Set Unions and Intersections

Set Complements · Ordered Pairs · Set Intersection · Pair Unions

Set Differences · Set Union · Unordered Pairs · Pair Intersections

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets · Identities

Names

Letters · Objects