Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\operatorname{cov}(f, g)$. We defined it:

$$\operatorname{cov}(f, g) = \mathsf{E}(fg) - \mathsf{E}(f)\mathsf{E}(g).$$

Properties

Prop. 1. *Covariance is symmetric and billinear.*[1]

Prop. 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\operatorname{cov}(f, f) = \mathsf{E}(ff) - \mathsf{E}(f)\mathsf{E}(f) = \mathsf{E}(f^2) - (\mathsf{E}(f))^2 = \operatorname{var}(f).$$

$\square$

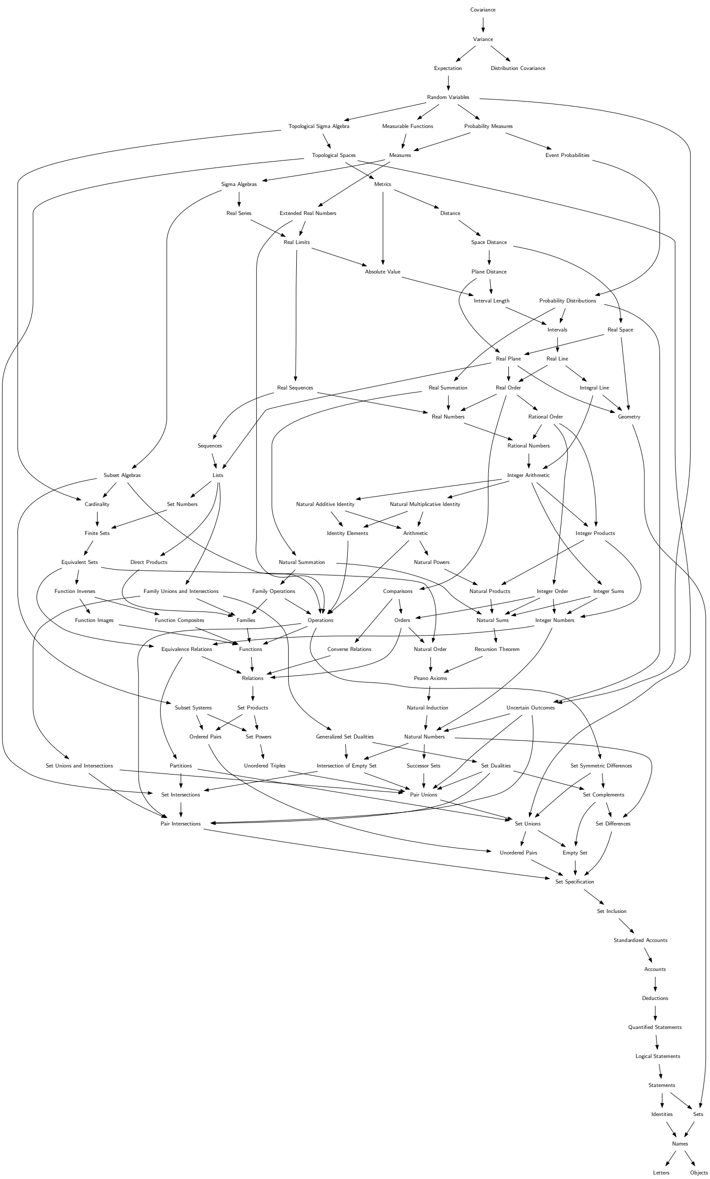
Prop. 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

[1] Future editions will include an account.

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\begin{aligned} \operatorname{var}\left(\sum_{i=1}^{n} f_i\right) &= \operatorname{cov}\left(\sum_{i=1}^{n} f_i, \sum_{i=1}^{n} f_i\right) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \operatorname{cov}(f_i, f_j) \end{aligned}$$

$\square$


Covariance
Variance
Expectation
Distribution Covariance
Random Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Topological Spaces
Measures
Event Probabilities
Sigma Algebras
Metrics
Real Series
Extended Real Numbers
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Interval Length
Probability Distributions
Intervals
Real Space
Real Plane
Real Line
Real Sequences
Real Summation
Real Order
Integral Line
Real Numbers
Rational Order
Geometry
Sequences
Rational Numbers
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Identity Elements
Arithmetic
Integer Products
Equivalent Sets
Direct Products
Natural Summation
Natural Powers
Function Inverses
Family Unions and Intersections
Family Operations
Comparisons
Natural Products
Integer Order
Integer Sums
Function Images
Function Composites
Families
Operations
Orders
Natural Sums
Integer Numbers
Equivalence Relations
Functions
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Subset Systems
Set Products
Natural Induction
Uncertain Outcomes
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects