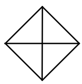


Why

When does the technique of row reductions prevail?

Multivariable row reductions

Let $S = (A \in \mathsf{R}^{m \times m}, b \in \mathsf{R}^m)$ be a linear system with $A_{kk} \neq 0$. The *kth row reduction* of S is the linear system (C, d) with $C_{st} = A_{st} - (A_{sk}/A_{kk})A_{kt}$ if $i < s \leq m$ and $C_{st} = A_{st}$ otherwise.

The idea, as in the example in Linear System Row Reductions, is to eliminate variable k from equations $k+1, \dots, m$. We are taking the kth column of A from

$$\begin{bmatrix} A_{1k} \\ \vdots \\ A_{kk} \\ A_{k+1,k} \\ \vdots \\ A_{mk} \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} A_{1k} \\ \vdots \\ A_{kk} \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

We interpret the ith row reduction as *subtracting equations* of the system or *reducing rows* of the array A. If $a^i, c^i \in \mathsf{R}^n$ denote the ith rows of A and C, $c^i = a^i - (A_{ik}/A_{kk})a^k$ for $k < i \leq m$, In other words, we obtain the ith row of matrix C by subtracting a multiple of the kth row of matrix A from the ith row of matrix A, for $k < i \leq m$. The following is an immediate consequence of real arithmetic.

Proposition 1. *Let $(A \in \mathsf{R}^{m \times n}, b \in \mathsf{R}^n)$ be a linear system which row reduces to (C, d). Then $x \in \mathsf{R}^n$ is a solution of (A, b) if and only if it is a solution of (C, d).*

Ordinary reductions

We call the system S *ordinarily reducible* if there exists a sequence of systems $S_1, \ldots, S_{m-1}$ so that S_1 is the 11-reduction of S and S_i is the ii-reduction of S_{i-1} for $i = 1, \ldots, n-1$. In this case, we call S_{n-1} the *final ordinary reduction* (or just *ordinary reduction*) of S. The following is an immediate consequence of Proposition 1.

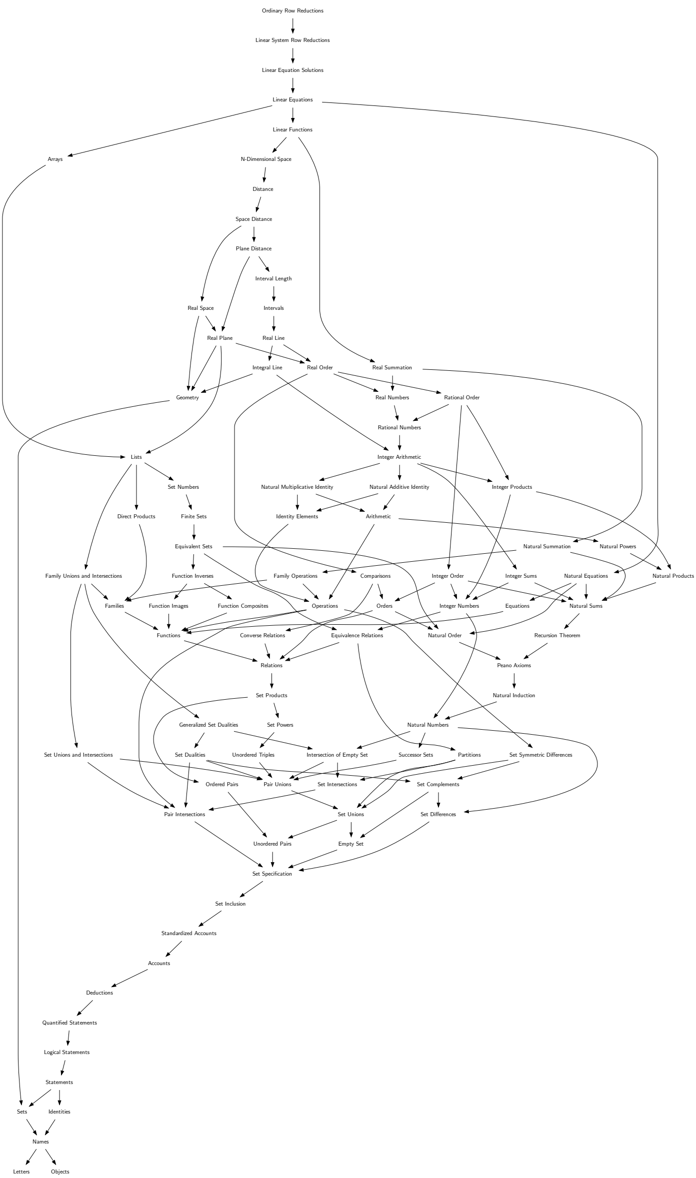
Proposition 2. *Let S' be the (final) ordinary reduction of S. Then S and S' have equivalent solution sets.*

This process of constructing the ordinary reduction is called *Gauss elimination* or *Gaussian elimination.* We call the kkth entry of system S_{k-1} the *pivot.* In an ordinarily reducible system, the pivots are nonzero.

The idea is that a system is ordinarily reducible if we can take row reductions in sequence an end up with a system that is easy to back-substitute and solve. The difficulty is that this need not be the case. For example, consider the following obvious difficulty. The system (A, b) in which

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

is not ordinarily reducible, but clearly solvable.



Ordinary Row Reductions
Linear System Row Reductions
Linear Equation Solutions
Linear Equations
Linear Functions
Arrays
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Real Summation
Geometry
Real Numbers
Rational Order
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Natural Summation
Natural Powers
Family Unions and Intersections
Function Inverses
Family Operations
Comparisons
Integer Order
Integer Sums
Natural Equations
Natural Products
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Equations
Natural Sums
Functions
Converse Relations
Equivalence Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects