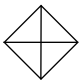


Why

We want to build vectors out of other vectors using scalar multiplication and vector addition.

Definition

A *linear combination from* a vector space is an ordered pair: the first coordinate is a sequence of vectors and the second is a sequence of scalars. The *result* of a linear combination is the sum of the results of scaling each vector by the corresponding scalar in the sequence; itself a vector in the space.

A *trivial linear combination* is one whose sequence of scalars is zero at each coordinate. The result of any trivial linear combination is the zero vector. A *nontrivial linear combination* is one which is not trivial. In other words, to be nontrivial, there must exist at least one index of the scalar sequence whose corresponding value is nonzero.

We say that a given vector *can be written as a linear combination of* a sequence of vectors if there exists a sequence of scalars such that the result of the linear combination of that sequence of vectors and scalars is the given vector. In other words, a vector can be written as a linear combination of some other vectors if there exists scalars for those other vectors such that scaling them and adding the results gives the vector.

Notation

Let (V, F) be a vector space. Let $v = (v_1, \dots, v_n)$ be a sequence of vectors in V and $a = (a_1, \dots, a_n)$ be a sequence of scalars in F. Then (v, a) is a linear combination and we can express its result by

$$a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

If (v, a) is trivial, then $a_i = 0$ for $i \in \{1, 2, \dots, n\}$ and the result of (v, a) is 0 (the zero vector). Otherwise, there exists $i \in \{1, 2, \dots, n\}$ such that $a_i \neq 0$; of course, the result of (v, a) may still be 0.

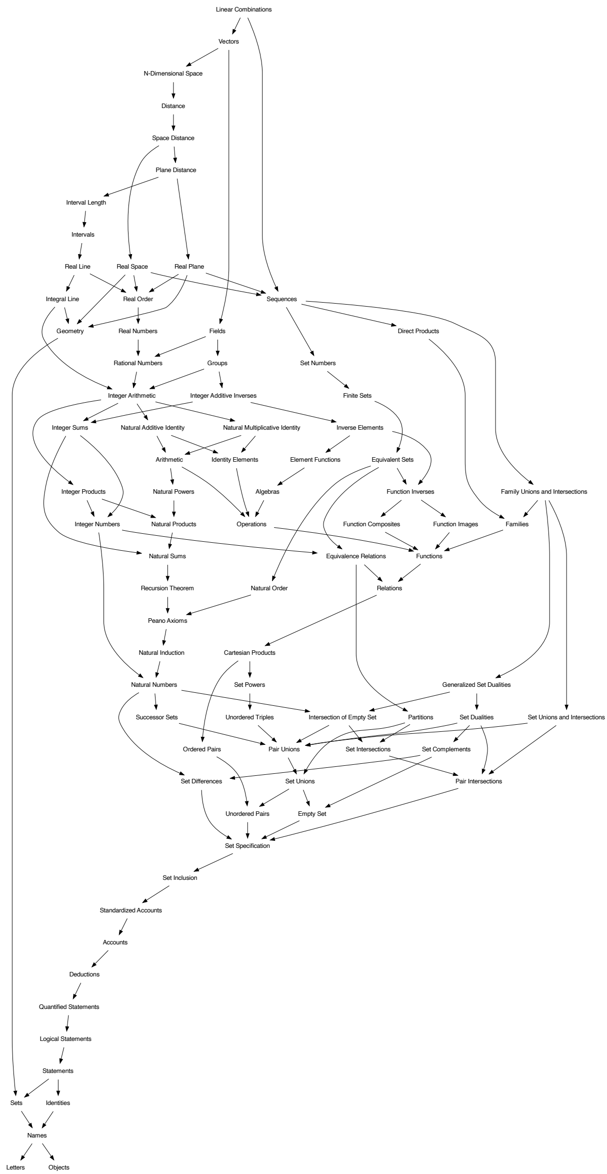
A vector u can be written as a linear combination of the vectors $v_1, v_2, \dots, v_n$ if there exists scalars $a_1, a_2, \dots, a_n$ so that

$$u = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

We often number (see Sequences) a set of finite vectors. In this case, we say "Let $\{u_1, \dots, u_n\}$ be a (finite) set of vectors."

Relationships

TODO span equivalence


Linear Combinations
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Space
Real Plane
Integral Line
Real Order
Sequences
Geometry
Real Numbers
Fields
Direct Products
Rational Numbers
Groups
Set Numbers
Integer Arithmetic
Integer Additive Inverses
Finite Sets
Integer Sums
Natural Additive Identity
Natural Multiplicative Identity
Inverse Elements
Arithmetic
Identity Elements
Element Functions
Equivalent Sets
Integer Products
Natural Powers
Algebras
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Products
Operations
Function Composites
Function Images
Families
Natural Sums
Equivalence Relations
Functions
Recursion Theorem
Natural Order
Relations
Peano Axioms
Natural Induction
Cartesian Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Set Dualities
Set Unions and Intersections
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects