



N-DIMENSIONAL SPACE

Why

If \mathbf{R} corresponds to a line, and \mathbf{R}^2 to a plane, and \mathbf{R}^3 to space, does \mathbf{R}^4 correspond to anything? What of \mathbf{R}^5 ?

Definition

Let n be a natural number. We call the set \mathbf{R}^n *n-dimensional space*. We call elements of \mathbf{R}^n *points*.

We call the point associated with $x = (x_1, x_2, \dots, x_n) \in \mathbf{R}^n$ with $x_i = 0$ for $1 \leq i \leq n$ the *origin*. When clear from context, we denote the origin by 0. Similarly, we denote the point x with $x_i = 1$ for all $i = 1, \dots, n$ by 1.

Visualization

We can not visualize n -dimensional space. Thus, our intuition for it comes from real space (see Real Space).

Distance

A natural notion of distance for \mathbf{R}^n is the extension of the Euclidean distance. We define the distance between $(x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \in \mathbf{R}^n$ as

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$$

This is sometimes called the *Euclidean distance for n-dimensional space*. Does this have the properties that distance has in the plane and in space? We discussed these properties It does. Denote the function which associates to $x, y \in \mathbf{R}^n$ their distance

$d : \mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}$. So $d(x, y)$ is the distance between the points corresponding to x and y .

Proposition 1. *d is non-negative, symmetric, and the distance between two points is no larger than the sum of the distances with any third object.*¹

Order

Let $x, y \in \mathbf{R}^n$. If $x_i < y_i$ for all $i = 1, \dots, n$ then we say x is *less than* y . Likewise, if $x_i \leq y_i$ for all $i = 1, \dots, n$ then we say $x \leq y$. Likewise for $>$ and \geq .

Notation

If $x \in \mathbf{R}^n$ is less than $y \in \mathbf{R}^n$ then we write $x < y$. Similarly for $x \leq y$, $x > y$ and $x \geq y$.

¹Future editions will include an account.

