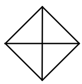

Result

We bound below the measure that a non-negative measurable real-valued function exceeds some value by its integral.

Proposition 1. *Let* $(X, \mathcal{A}, \mu)$ *be a measure space. Let* $g : X \to [0, \infty]$ *be measurable and square-integrable. Then for all* t *such that* $\int t d\mu \in [0, \int g d\mu)$*,*

$$\mu(\{x \in X \mid g(x) > t\}) \geq \frac{(\int (g-t)d\mu)^2}{\int g^2 d\mu}.$$

Proof. Let t such that $\int t d\mu \in [0, \int g)$. We have selected t so that $\int (g - t)d\mu \geq 0$. Define $h = (g-t)^+$ and $A = \{x \in X \mid h(x) > 0\}$. Then

$$\int (g-t)d\mu \leq \int h d\mu = \int h \chi_A d\mu \leq \sqrt{\int h^2 d\mu \int \chi_A^2 \, d\mu}$$

□

Now $g^2 > h^2$, so $\int g^2 d\mu \geq \int h^2 d\mu$. Also $\chi_A^2 = \chi_A$ so $\int \chi_A^2 = \mu(A)$. $h(x) > 0$ if and only if $g(x) \geq t$ for all x. So $A = \{x \in X \mid g(x) \geq t\}$. Combining we have:

$$\int (g-t)d\mu \leq \sqrt{(\int g^2 d\mu)\mu(A)}.$$

Proposition 2. *Let* X *be a random variable with* $\mathbf{E}(X^2) \leq \infty$*. Then for all* $t \in [0, \mathbf{E}(X))$*, we have*

$$P(X > t) \geq \frac{(\mathbf{E}(X) - t)^2}{\mathbf{E}X^2}.$$

The above is also called the *Paley-Zygmund Inequality*.

Tail Measure Lower Bound
Variance
Expectation
Random Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Topologies
Measures
Outcome Variable Covariance
Metrics
Sigma Algebras
Outcome Variable Expectation
Distance
Extended Real Numbers
Real Series
Induced Probability Distributions
Real Square Roots
Space Distance
Real Limits
Outcome Variable Events
Real Squares
Plane Distance
Absolute Value
Event Probabilities
Real Arithmetic
Interval Length
Probability Distributions
Real Multiplicative Inverses
Real Space
Intervals
Natural Square Roots
Real Products
Real Plane
Real Line
Square Numbers
Real Additive Inverses
Squares
Integral Line
Real Sequences
Real Summation
Real Order
Real Sums
Rational Multiplicative Inverses
Rational Arithmetic
Geometry
Sequences
Subset Algebras
Real Numbers
Rational Order
Rational Sums
Rational Products
Lists
Cardinality
Rational Numbers
Direct Products
Set Numbers
Uncertain Outcomes
Integer Arithmetic
Finite Sets
Inverse Elements
Natural Summation
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Family Operations
Element Functions
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Comparisons
Operations
Integer Order
Integer Sums
Natural Powers
Natural Equations
Families
Function Images
Function Composites
Orders
Integer Numbers
Natural Products
Equations
Equivalence Relations
Functions
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects