



# Optimization

## 1 Why

Given a correspondence between objects in a set with objects in an totally ordered set, we are interested in the objects which correspond to minimal or maximal elements of the ordered set.

## 2 Definition

Let  $A$  be a non-empty set and let  $(C, \leq)$  be chain. Let  $f : A \rightarrow C$ .

The *minimization problem over  $A$*  associated with  $f$  is to find an element  $a \in A$  so that  $f(a)$  is minimal in  $f(A)$ . The *maximization problem over  $A$*  associated with  $f$  is to find an element  $a \in A$  so  $f(a)$  is maximal in  $f(A)$ . We call either of these an *optimization problem*.

We call  $f$  the *ordering* function. We call  $A$  the *feasible set* and we call  $a \in A$  a *feasible element*. An element  $a \in A$  is a *minimizer* of  $f$  if  $f(a)$  is minimal in  $f(A)$ . An element  $a \in A$  is a *maximizer* of  $f$  if  $f(a)$  is maximal in  $f(A)$ .

## 3 Notation

Let  $(C, \prec)$  be a chain. We denote the minimization problem to find an element  $a \in A$  to minimize  $f : A \rightarrow C$  by

**find**    $a \in A$   
**to minimize**    $f(a)$

We denote the minimizers by

**minimizers**( $f$ ).