



## Random Variables

### 1 Why

TODO

### 2 Definition

A *random variable* is a measurable map from a probability space to a measurable space.

A *real-valued random variable* is a measurable map between the probability space and the set of real numbers with its topological sigma algebra.

#### 2.1 Notation

Let  $(X, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $(Y, \mathcal{B})$  a measurable space. Then a random variable is a measurable function  $f : X \rightarrow Y$ .

Some authors denote real-valued random variables by upper case Latin letters: for example,  $X, Y, Z$ . In this case, the base probability space is denoted by  $\Omega$ , a mnemonic for “outcomes.”. Let  $(\Omega, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $X : \Omega \rightarrow \mathbf{R}$  be measurable. Then  $X$  is a real-valued random variable.

Some authors use notation for the probability of certain common sets. Let  $A \in \mathcal{B}(\mathbf{R})$ . Let  $\mathbf{P}(X \in A)$  denote  $\mathbf{P}(X^{-1}(A))$ . These are

equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A\}).$$

Next, let  $Y : \Omega \rightarrow R$  a measurable function and let  $B \in \mathcal{B}(R)$ . Similar to the above, let  $\mathbf{P}(X \in A, Y \in B)$  denote  $\mathbf{P}(X^{-1}(A) \cap Y^{-1}(B))$ . These are equivalent to

$$\mathbf{P}(\{\omega \in \Omega \mid X(\omega) \in A \text{ and } Y(\omega) \in B\}).$$

