

NORMAL DENSITIES

Why

We want a density that is symmetric about some center value with some spread.

Definition

Let $f : \mathbb{R} \to \mathbb{R}$ be a density. If there exists $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}$ with $\sigma > 0$ so that for each $x \in \mathbb{R}$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

then f is a normal density. A normal density is often called a Gaussian density.¹ We often drop the word density and use refer to these as normals or Gaussians, using these words as substantives.

We call the special case when $\mu = 0$ and $\sigma = 1$ the standard normal density or standard gaussian density.

Maximum

The maximum of a normal density is μ .

¹We do not use this term in accordance with the Bourbaki project's policy on historical names.

