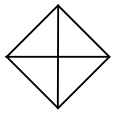

Tree Distribution Approximation Solution

1 Why

We can characterize the solutions of tree-distribution approximation.

2 Solution

Proposition 1. *Let q be a distribution on A. Let T be a tree on $\{1, \ldots, d\}$. Let p_j be the parent of vertex j for the T rooted at vertex i, $j = 1, \ldots, n$ and $j \neq i$. Then the distribution p on A defined by*

$$p = q_i \prod_{j \neq i} q_{j|p_j}$$

achieves minimum entropy relative to q among all distributions which factor according to T.

Proposition 2. *Let q be a distribution on A. A tree T is a solution to the problem above if and only if it is a maximal spanning tree of the mutual information graph of q.*