Why

In the case of several variables, what do the row reducer matrices correspond to in ordinary row reduction?

Definition

Let $(A \in \mathsf{R}^{m \times m}, b \in \mathsf{R}^m)$ be an ordinarily reducible linear system. The *ordinary reducer sequence* (or *ordinary row reducer sequence*) is a sequence of row reducer matrices $L_1, \dots L_{m-1}$ so that

$$A_1 = L_1 A \text{ and } A_i = L_i A_{i-1} \text{ for } 2 \leq i \leq m-1.$$

In other words, $U \in \mathsf{R}^{m \times m}$ defined by

$$U = L_{m-1} \cdots L_2 L_1 A$$

is the ordinary row reduction of A.

Formulae

Let x_1 be the first column of A and let x_k be the kth column of A_{k-1} for $k = 2, \dots, m-1$. The the transformation L_k is chosen so that

$$x_k = \begin{bmatrix} x_{1k} \\ \vdots \\ x_{kk} \\ x_{k+1,k} \\ \vdots \\ x_{mk} \end{bmatrix} \quad \xrightarrow{L_k} \quad L_k x_k = \begin{bmatrix} x_{1k} \\ \vdots \\ x_{kk} \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

we subtract $l_{jk} = {}^{x_{jk}}/_{x_{kk}}$ for $k \le j \le m$ times row k from row j. We call l_{jk} the *row multiplier*. The matrix L_k has the form

$$L_k = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & -\ell_{k+1,k} & 1 & & \\ & & \vdots & & \ddots & \\ & & -\ell_{mk} & & & 1 \end{bmatrix}$$

Properties

Proposition 1. *Every reducer in the ordinary reducer sequence is unit lower triangular.*[1]

Proposition 2. *The ordinary row reduction of a matrix is upper triangular.*

Factorization perspective.

If the product $L_{m-1} \cdots L_2 L_1$ is invertible, then

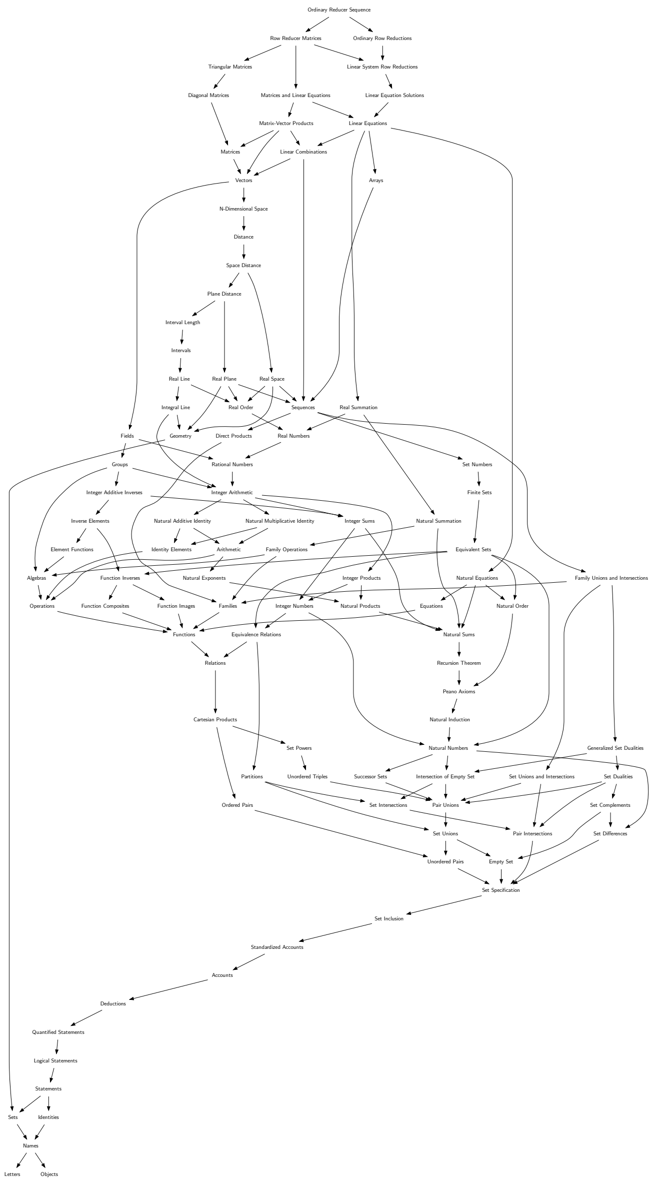
$$A = (L_{m-1} \cdots L_2 L_1)^{-1} U.$$

Of course,

$$(L_{m-1} \cdots L_2 L_1)^{-1} = L_1^{-1} L_2^{-1} \cdots L_{m-1}^{-1}.$$

So we are interested in the inverse of L_i for $i \le m-1$. The key insight is that $L_i{}^{-1}$ is L_i with the subdiagonal entries negated.

[1] Use result from row-reducer matrix.



Ordinary Reducer Sequence
Row Reducer Matrices
Ordinary Row Reductions
Triangular Matrices
Linear System Row Reductions
Diagonal Matrices
Matrices and Linear Equations
Linear Equation Solutions
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Integral Line
Real Order
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Integer Additive Inverses
Integer Arithmetic
Finite Sets
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Natural Summation
Element Functions
Identity Elements
Arithmetic
Family Operations
Equivalent Sets
Algebras
Function Inverses
Natural Exponents
Integer Products
Natural Equations
Family Unions and Intersections
Operations
Function Composites
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Generalized Set Dualities
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Intersection of Empty Set
Set Unions and Intersections
Set Dualities
Ordered Pairs
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Pair Unions
Set Complements
Set Unions
Pair Intersections
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