



## SET COMPLEMENTS

### Why

It is often the case in considering set differences that all sets considered are subsets of one set.

### Definition

Let  $A$  and  $B$  denote sets. In many cases, we take the difference between a set and one contained in it. In other words, we assume that  $B \subset A$ . In this case, we often take complements relative to the same set  $A$ . So we do not refer to it, and instead refer to the relative complement of  $B$  in  $A$  as the *complement* of  $B$ .

### Notation

Let  $A$  denote a set, and let  $B$  denote a set for which  $B \subset A$ . We denote the relative complement of  $B$  in  $A$  by  $C_A(B)$ . When we need not mention the set  $A$ , and instead speak of the complement of  $B$  without qualification, we denote this complement by  $C(B)$ .

### Complement of a complement

One nice property of a complement when  $B \subset A$  is:

**Proposition 1.**  $(B \subset A) \longleftrightarrow (C_A(C_A(B)) = B)$

### Basic Facts

Let  $E$  denote a set and let  $A$  and  $B$  denote sets satisfying  $A, B \subset E$ . Then take all complements with respect to  $E$ . Here

are some immediate consequences of the definition of complements.<sup>1</sup>

**Proposition 2.**  $C(C(A)) = A$

**Proposition 3.**  $C(\emptyset) = E$

**Proposition 4.**  $C(E) = \emptyset$

**Proposition 5.**  $A \subset B \longleftrightarrow C(B) \subset C(A)$

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<sup>1</sup>Proofs will appear in future editions.

