

FUNCTION RESTRICTIONS AND EXTENSIONS

Why

The relationship between the inclusion map and the identity map is characteristic of making small functions out of large ones.¹

Definition

Let $X \subset Y$ and $f: Y \to Z$. There is a natural function $g: X \to Z$, namely the one defined by g(x) = f(x) for all $x \in X$. We call g the restriction of f to X. We call f an extension of g to Y. Clearly, there may be more than one extension of a function

Notation

We denote the restriction of $f: Y \to Z$ to the set $X \subset Y$ by $f \mid X$.

Example

A simple example is the that the inclusion mapping from X to Y with $X \subset Y$ is a restriction of the identity map on X

An extension order

Here is a natural order involving set extensions and restrictions. Fix two sets A and B. Let F be the set of all functions $f: X \to Y$ with $X \subset A$ and $Y \subset B$. Define a relation R in F by $(f,g) \in R$ if $\mathrm{dom}\, f \subset \mathrm{dom}\, g$ and f(x) = g(x) for all x in $\mathrm{dom}\, f$. In other words, $(f,g) \in R$ if f is a restriction of g (or, equivalently, g is an extension of f. We recognize that R is a special case of the inclusion partial order by recognizing the elements of F as subsets $A \times B$.

 $^{^{1}\}mathrm{Future}$ editions will modify this language.

