$$\diamondsuit$$

Halfspaces

1 Why

TODO

2 Definition

For a nonzero $b \in \mathbf{R}^n$ and $\beta \in \mathbf{R}$ the sets

$$\{x \in \mathbf{R}^n \mid \langle x, b \rangle \le \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x, b \rangle \ge \beta\},$$

are *closed halfspaces* and the sets

$$\{x \in \mathbf{R}^n \mid \langle x, b \rangle < \beta\}, \quad \{x \in \mathbf{R}^n \mid \langle x, b \rangle > \beta\},$$

are *open halfspaces.*

Each of these is nonempty and convex. As with hyperplanes, the same four sets appear if one uses $\lambda\beta$ and λb above, so the halfspaces depend only on the hyperplane $H = \{x \mid \langle x, b \rangle = \beta\}$.