



## Why

We have considered the number of ways we can arrange 52 cards into a deck. What of  $n$  cards? A moment's reflection indicates this will also be the number of ways to arrange  $n$  objects in order (where the objects need not be cards).

## Definition

By the fundamental principle of counting, there are  $n$  ways to select the first card,  $n - 1$  ways to select the second, and so on. Thus, the number of ways of stacking  $n$  cards in a deck is

$$n(n - 1)(n - 2) \cdots 1$$

We call this number the *factorial* of  $n$ , or *n-factorial*.

*Factorial function.* Define  $f : \mathbf{N} \rightarrow \mathbf{N}$  recursively by  $f(1) = 1$  and  $f(2) = 2f(1)$ , and  $f(n) = nf(n - 1)$  for  $n \in \mathbf{N}$  ( $f$  exists by the recursion theorem—see **Recursion Theorem**).  $f$  is defined such that  $f(n)$  is  $n$  factorial, for which reason we call  $f$  the *factorial function*. For convenience, we extend  $f$  to  $\omega^1$  by defining  $f(0) = 1$ .

## Notation

We denote the factorial of  $n$  by  $n!$ , read aloud “ $n$  factorial”. So for example,  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$  and  $0! = 1$ .

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<sup>1</sup>See **Natural Numbers**.



