Why

We consider a simple distribution on sequences.[1]

Definition

A *memory chain* (*markov chain*[2], *memory model*, *markov model*) *on* a set A of *length n* is a joint distribution $p : A^n \to [0,1]$ satisfying

$$p(a) = f(a_1) \prod_{i=2}^{n} g(a_i, a_{i-1})$$

for some distribution $f : A \to [0,1]$ and $g : A^2 \to [0,1]$ is a function for which $g(\cdot, \alpha) : A \to [0,1]$ is a distribution on A for every $\alpha \in A$.

Proposition 1. *p so defined is a distribution. The function g is the conditional distribition $p_{i|i-1}$ for $i = 2, \ldots, n$. The function f is the first marginal p_1.*

For this reason, we call g the *conditional distribution* of the Markov chain. We call f the *initial distribution*.

Terminology

Let $A = \prod_i A_i$ and $p : A \to [0,1]$ be a distribution. On one hand, p is said to be *memoryless* (or have *zero-order memory*) if $p = \prod_i p_i$. In particular, $p_{i|i-1}(\alpha, \beta) = p_i(\alpha)$ for every $i = 2, \ldots, n$. On the other hand, p is said to have *first-order memory* if $p = p_1 \prod_{i=2}^{n} p_{i|i-1}$

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[1]Future editions will modify and expand.

[2]This term is universal. We avoid it in these sheets because of the Bourbaki project's policy on naming. The skeptical reader will note, at least, out term and this term terms have the same intials.

If, in addition, $A_i = A_1$ for all $i = 1, \ldots, n$, then we say that p has *homogenous conditionals* if $p_{i|i-1} = p_{j|j-1}$ for $i \neq j = 2, \ldots, n$. In this language, a memory chain is a joint distribution with first-order memory and homogogenous conditionals.

Memory Chains

Conditional Distributions

Conditional Event Probabilities   Marginal Distributions

Event Probabilities   Joint Distributions

Probability Distributions

Intervals

Real Line

Real Summation   Real Order

Rational Order   Real Numbers

Rational Numbers

Integral Line

Integer Arithmetic   Lists

Integer Products   Natural Additive Identity   Natural Multiplicative Identity

Set Numbers

Arithmetic   Identity Elements

Finite Sets   Direct Products

Natural Powers   Natural Summation

Equivalent Sets

Integer Sums   Integer Order   Comparisons   Family Operations   Function Inverses   Family Unions and Intersections

Natural Products

Natural Sums   Integer Numbers   Orders   Operations   Converse Relations   Equivalence Relations   Function Images   Function Composites   Families   Functions

Recursion Theorem   Natural Order   Relations

Peano Axioms   Set Products

Uncertain Outcomes   Natural Induction   Set Powers   Generalized Set Dualities

Natural Numbers   Successor Sets   Unordered Triples   Set Dualities   Intersection of Empty Set   Partitions   Set Unions and Intersections

Set Symmetric Differences   Pair Unions   Set Intersections   Ordered Pairs

Set Complements   Set Unions   Pair Intersections

Set Differences   Empty Set   Unordered Pairs

Set Specification   Geometry

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects