

#### SMOOTH MULTIVARIATE FUNCTIONS

### Why

What is the natural generalization of a smooth function to functions defined on sets of  $\mathbb{R}^k$ .<sup>1</sup>

#### **Definition**

Let  $U \subset \mathbb{R}^d$  be an open set (see Real Open Sets). A function  $f: U \to \mathbb{R}$  is *smooth* if all its partial derivatives exists and are continuous.

More generally, let  $X \subset \mathbf{R}^d$ . A function  $f: X \to R$  is *smooth* if there exists an open set  $U \subset \mathbf{R}^d$  and a smooth  $F: U \to \mathbf{R}$  so that F(x) = f(x) for all  $x \in U \cap X$ .

## Example

The identity map is smooth. In other words, let  $f: \mathbf{R}^d \to \mathbf{R}$  be so that  $X \subset \mathbf{R}^d$ . Then  $f: X \to \mathbf{R}$  s

# **Properties**

 ${\bf Proposition} \ {\bf 1.} \ \ {\it The \ composition \ of \ two \ smooth \ functions \ is \ smooth.}$ 

<sup>&</sup>lt;sup>1</sup>Future editions will expand.

