



## LINEAR EQUATION SOLUTIONS

### Why

We want to solve linear equations.

### Example

Suppose we want to find  $x_1, x_2 \in \mathbf{R}$  to satisfy the system of linear equations

$$3x_1 + 2x_2 = 10, \text{ and}$$

$$6x_1 + 5x_2 = 20,$$

with constants  $((3, 2), 10)$  and  $((6, 4), 20)$ .

We can associate to the first equation an *equation for  $x_1$  in terms of  $x_2$* . We call this *solving the first equation for  $x_1$* .

$$3x_1 + 2x_2 = 10 \iff 3x_1 = 10 - 2x_2 \iff x_1 = \frac{1}{3}(10 - 2x_2).$$

Define  $f_1 : \mathbf{R} \rightarrow \mathbf{R}$  as  $f_1(y) = \frac{1}{3}(10 - 2y)$ . Then  $3x_1 + 2x_2 = 10$  if and only if  $x_1 = f_1(x_2)$ . We have written  $x_1$  as a *function* of  $x_2$  and obtained a new equation. The equation is not linear, however, as  $f_1$  is not linear.

Using the equation for  $x_1$  in terms of  $x_2$ , we can substitute this equation into our second linear equation. The two linear equations hold if and only if

$$6f(x_2) + 5x_2 = 20 \iff 20 - 4x_2 + 5x_2 = 20 \iff x_2 = 0.$$

So the equations are satisfied if and only if  $x_2$  is 0. If  $x_2 = 0$ , then  $3x_1 = 10$  and  $6x_1 = 20$ . Both of these are equivalent to  $x_1 = \frac{10}{3}$ . So we have that  $x_1$  must be  $\frac{10}{3}$  and  $x_2$  must be 0.

Clearly this is *a solution*. Is it the only one?<sup>1</sup>

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<sup>1</sup>Future editions will expand.

