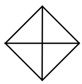


Why

We can consider function composition for linear maps?

Definition

Given vector spaces U, V, W over the same field $\mathbf{F}$ and linear maps $T \in \mathcal{L}(U, V)$ and $S \in \mathcal{L}(V, W)$, the *product* of S and T is the linear map $R \in \mathcal{L}(U, W)$ defined by

$$R(u) = S(T(u)) \quad \text{for all } u \in U$$

(Prove that R so defined is linear). In other words, the product is $S \circ T$.

This definition only makes sense if T maps into the domain of S. We often say that the maps are *conforming* in this case.

Notation

Often the product is denoted ST (instead of $S \circ T$).

Algebraic properties

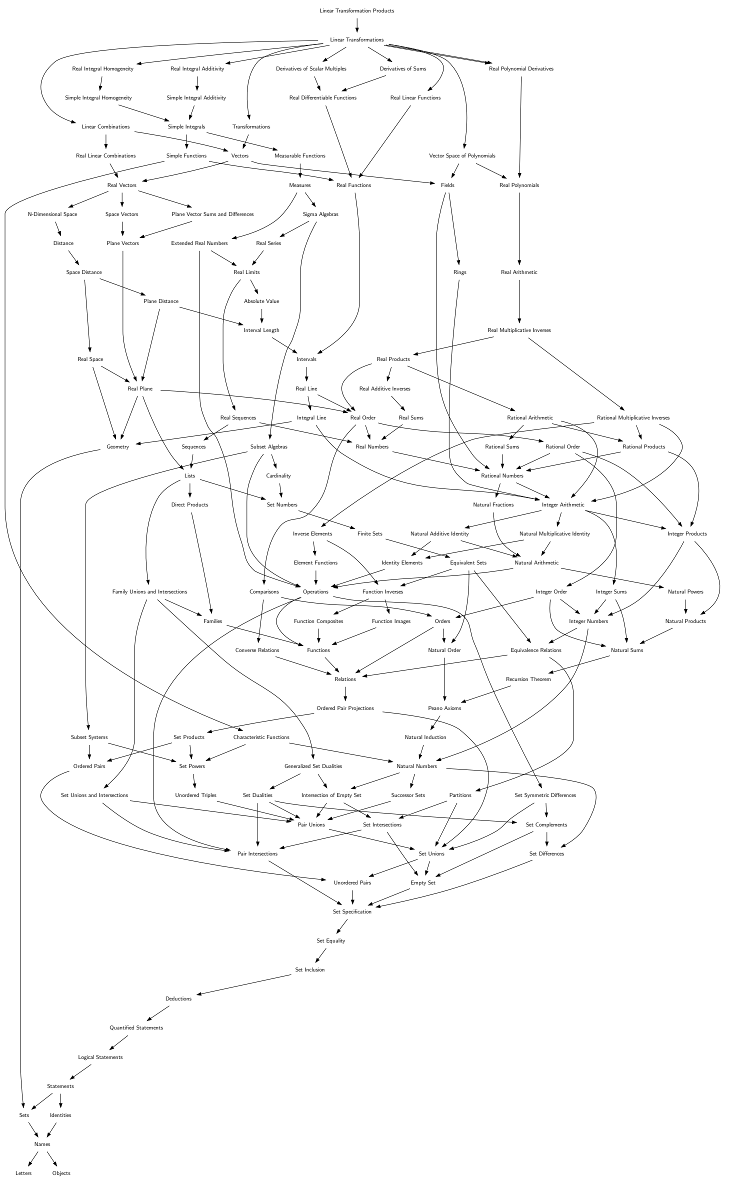
Proposition 1 (associativity). *Suppose T_1, T_2, T_3 are three linear maps so that conforming for $T_1T_2T_3$. Then*

$$(T_1T_2)T_3 = T_1(T_2T_3)$$

Not commutative

Image of zero

Proposition 2. *Suppose T is a linear map from V to W. Then $T(0) = 0$.*


Linear Transformation Products
Linear Transformations
Real Integral Homogeneity
Real Integral Additivity
Derivatives of Scalar Multiples
Derivatives of Sums
Real Polynomial Derivatives
Simple Integral Homogeneity
Simple Integral Additivity
Real Differentiable Functions
Real Linear Functions
Linear Combinations
Simple Integrals
Transformations
Real Linear Combinations
Simple Functions
Vectors
Measurable Functions
Vector Space of Polynomials
Real Vectors
Measures
Real Functions
Fields
Real Polynomials
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Sigma Algebras
Distance
Plane Vectors
Extended Real Numbers
Real Series
Space Distance
Real Limits
Rings
Real Arithmetic
Plane Distance
Absolute Value
Interval Length
Real Multiplicative Inverses
Real Space
Intervals
Real Products
Real Plane
Real Line
Real Additive Inverses
Real Sequences
Integral Line
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Geometry
Sequences
Subset Algebras
Real Numbers
Rational Sums
Rational Order
Rational Products
Lists
Cardinality
Rational Numbers
Direct Products
Set Numbers
Natural Fractions
Integer Arithmetic
Inverse Elements
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Element Functions
Identity Elements
Equivalent Sets
Natural Arithmetic
Family Unions and Intersections
Comparisons
Operations
Function Inverses
Integer Order
Integer Sums
Natural Powers
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Products
Converse Relations
Functions
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Unordered Triples
Set Dualities
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects