Why

The inverse of a function interacts nicely with family unions, family intersections and complements.

Results

Let $f : X \to Y$. Throughout this sheet, let $f^{-1} : Y^* \to X^*$. And take $\{B_i\}$ to be a family of subsets of Y.[1]

Proposition 1. $f^{-1}(\cup_i B_i) = \cup_i f^{-1}(B_i)$

Proposition 2. $f^{-1}(\cup_i B_i) = \cap_i f^{-1}(B_i)$

Proposition 3. $f^{-1}(Y - B) = X - f^{-1}(B)$

Properties for Function Image

Notice that $f(\cup_i A_i) = \cup_i f(A_i)$ but not for interesctions. Nor is there a similar correspondence for complements. There are some relations, which we list below.[2]

Proposition 4. $f(A \cap B) = f(A) \cap f(B)$ *if and only if f is one-to-one.*

Proposition 5. *For all $A \subset X$, $f(X - A) = Y - f(A)$ if and only if f is one-to-one.*

Proposition 6. *For all $A \subset X$, $Y - f(A) \subset f(X - A)$ if and only if f is onto.*

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[1]The proofs of the following will appear in future editions.

[2]Accounts of these facts will appear in future editions.

Inverses Unions Intersections and Complements

Family Unions and Intersections

Families

Functions

Relations

Cartesian Products

Set Powers

Generalized Set Dualities

Set Dualities Intersection of Empty Set Set Unions and Intersections Unordered Triples

Set Intersections Pair Unions Ordered Pairs

Set Complements Pair Intersections Set Unions

Set Differences Empty Set Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Definitions Accounts

Deductions Quantified Statements

Logical Statements

Statements

Identities Sets

Names

Objects Letters