



## Why

We want fractions.<sup>1</sup>

## Rational equivalence

Consider  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ . We say that the elements  $(a, b)$  and  $(c, d)$  of this set are *rational equivalent* if  $ad = bc$ . Briefly, the intuition is that  $(a, b)$  represents  $a$  over  $b$ . In the usual notation,  $(a, b)$  represents “ $a/b$ ”. So this equivalence relation says these two are the same if  $a/b = c/d$  or else  $ad = bc$ .

**Proposition 1.** *Rational equivalence is an equivalence relation on  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ .*<sup>2</sup>

## Definition

The *set of rational numbers* is the set of equivalence classes (see Equivalence Classes) of  $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$  under rational equivalence. We call an element of the set of rational numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

## Notation

We denote the set of rationals by  $\mathbf{Q}$ .<sup>3</sup> If we denote rational equivalence by  $\sim$  then  $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$ .

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<sup>1</sup>This why will be expanded in future editions.

<sup>2</sup>Future editions will include an account.

<sup>3</sup>From what we can tell,  $\mathbf{Q}$  is a mnemonic for “quantity”, from the latin “quantitas.”



