



Graphs

1 Why

We want to visualize relations.

2 Definition

A **graph** is a set and a relation on the set. The graph is **undirected** if the relation is symmetric; otherwise the graph is **directed**.

A **vertex** of the graph is an element of the set. The set is called the **vertex set**. An **edge** of the graph is an element of the relation. The relation is called the **edge set**.

2.1 Notation

We denote the vertex set by V , a mnemonic for vertex. We denote the edge set by E , a mnemonic for edge. We denote a graph by (V, E) . If the vertex set is assumed we can unambiguously refer to the graph by E .

2.2 Visualization

We visualize a graph by drawing a point for each vertex. If two vertices u and v are in relation, we draw a line from the point corresponding to u to the point corresponding to v with an arrow at the point corresponding to v . If the graph is undirected, we omit arrows. Here are all undirected graphs on three vertices.

3 Paths

A path in a graph is a sequence of vertices with the property that consecutive vertices are related. A path **cycles** if a vertex appears more than once. A path is **finite** if the sequence is finite. A **loop** is a finite path that cycles once. A finite path from vertex u to vertex v is a path starting with u and ending with v . The **length** of a finite path is the length of the sequence.

4 Properties

A graph is **connected** if there is a path between every pair of vertices. A graph is **acyclic** if none of its paths cycle.