Tree-Structured Distribution Approximation

1 Why

2 Problem

Tree-structured distribution approximation is a mathematical optimization problem in which we find a tree-structured distribution which minimizes its entropy relative to a given distribution over the set of all distributions which factor according to trees.

2.1 Notation

Let $A_1, \ldots, A_n$ be non-empty sets. Let q be a distribution on $\prod_{i=1}^{n} A_i$. Let d denote the relative entropy.

We want to find a distribution p on A and tree T on $\{1, \ldots, n\}$ to

$$\begin{aligned} \text{minimize} \quad & d(q, p) \\ \text{subject to} \quad & p \text{ factors according to the tree } T \end{aligned}$$

3 Solution

Proposition 1. *Let q be a distribution on A. Let T be a tree on $\{1, \ldots, d\}$. Let p_j be the parent of vertex j for the T rooted at vertex i, $j = 1, \ldots, n$ and $j \neq i$. Then the distribution p on A defined by*

$$p = q_i \prod_{j \neq i} q_{j|p_j}$$

achieves minimum entropy relative to q among all distributions which factor according to T.

Proposition 2. *Let q be a distribution on A. Let T be a tree on $\{1, \ldots, d\}$. Let p_j be the parent of vertex j for the T rooted at vertex i, $j = 1, \ldots, n$ and $j \neq i$. Then the distribution p on A defined by*

$$p = q_i \prod_{j \neq i} q_{j|p_j}$$

achieves minimum entropy relative to q among all distributions which factor according to T.

Proposition 3. *Let q be a distribution on A. A tree T is a solution to the problem above if and only if it is a minimum spanning tree of the mutual information graph of q.*