Definition

A polynomial $p : \mathbf{R} \to \mathbf{R}$ is *nonnegative* (a *nonnegative polynomial*, *nonnegative real polynomial*) if

$$p(x) \geq 0 \quad \text{for all } x \in \mathbf{R}$$

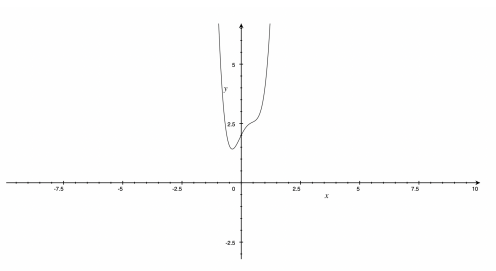
In this case, we call p *positive semidefinite* or *PSD*.

Testing nonnegativity

Given polynomial p, how do we know if p is (globally) nonnegative? Consider $p : \mathbf{R} \to \mathbf{R}$ defined by

$$p(x) = 5x^4 - 4x^3 - x^2 + 2x + 2$$

We visualize the graph of p below.



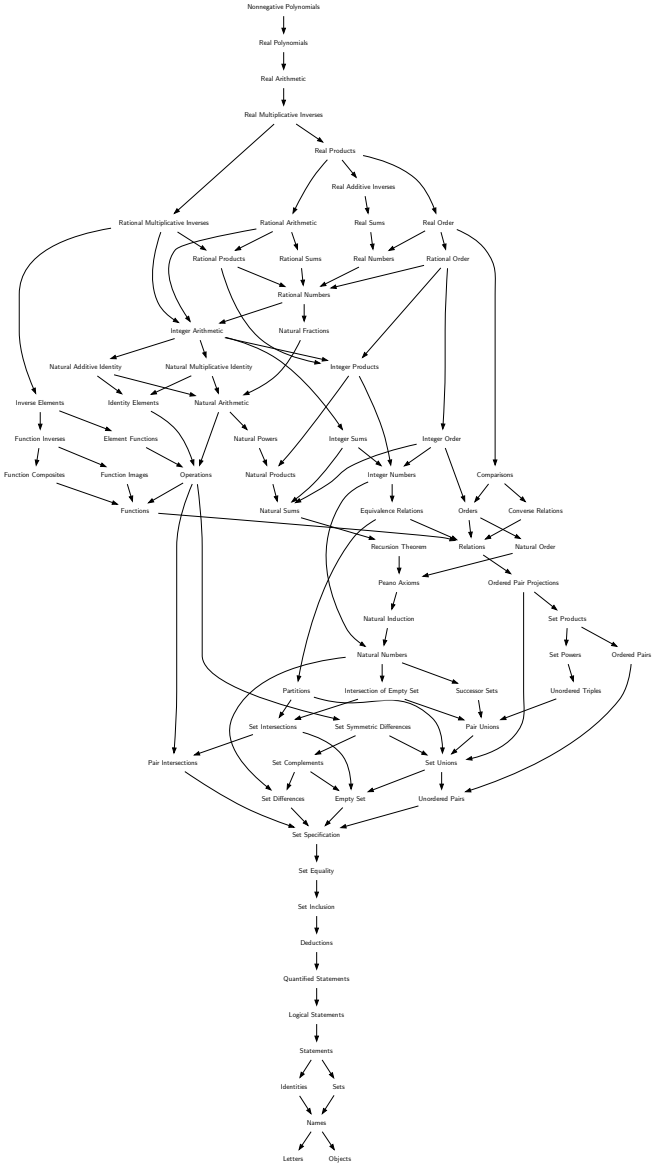
Given the coefficients of p, namely the list $(2, 2, -1, -4, 5) \in \mathbf{R}^5$, how can we tell? It is not so obvious, but if we write

$$p(x) = (x^2 + 1)^2 + (2x^2 - x - 1)^2,$$

then it is readily apparent that $p \geq 0$ since all squares are nonnegative.

We can ask two questions:

1. If p is a nonnegative polynomial, can it be written as a sum of squares?
2. If p is a sum of squares, how do we find them?


Nonnegative Polynomials
Real Polynomials
Real Arithmetic
Real Multiplicative Inverses
Real Products
Real Additive Inverses
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Rational Products
Rational Sums
Real Numbers
Rational Order
Rational Numbers
Integer Arithmetic
Natural Fractions
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Inverse Elements
Identity Elements
Natural Arithmetic
Function Inverses
Element Functions
Natural Powers
Integer Sums
Integer Order
Function Composites
Function Images
Operations
Natural Products
Integer Numbers
Comparisons
Functions
Natural Sums
Equivalence Relations
Orders
Converse Relations
Recursion Theorem
Relations
Natural Order
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects