



# Characteristic Functions

## 1 Why

We represent rectangles by functions.

## 2 Definition

The *characteristic function* of a subset of some base set is the function from the base set to the real numbers which maps elements contained in the subset to value one and maps all other elements to zero. The range of the function is the set consisting of the real numbers one and zero.

If the base set is the real numbers and the subset is an interval, then the characteristic function is a rectangle with height one and the width of the interval.

### 2.1 Notation

Let  $A$  be a non-empty set and  $B \subset A$ . We denote the characteristic function of  $B$  in  $A$  by  $\chi_B : A \rightarrow R$ . The Greek letter  $\chi$  is a mnemonic for "characteristic".

The subscript indicates the set on which the function is one.  
In other words, for all  $B \subset A$ ,  $\chi_B^{-1}(\{1\}) = B$ .

If  $B$  is an interval and  $\alpha$  is a real number then  $\alpha\chi_B$  is a rectangle with height  $\alpha$ .