Why[1]

Definition

An *ordering* of an undirected graph is an ordering (see Lists) of its vertices. An *ordered undirected graph* is an ordered pair $((V, E), \sigma : \{1, 2, \ldots, |V|\} \to V)$ where (V, E) is an undirected graph (see Undirected Graphs) and σ is an ordering of the vertex set V.

Notation

Let $((V, E), \sigma)$ be an ordered undirected graph. We commonly associate it with (V, E, σ) and call this ordered triple an undirected graph as well. But, throughout these sheets, an ordered undirected graph is an ordered pair.

We denote that $\sigma^{-1}(v) < \sigma^{-1}(w)$ by $v \prec_\sigma w$ and $v \succeq_\sigma w$ by $\sigma^{-1}(v) \leq \sigma^{-1}(w)$. We occasionally omit the subscripts in $\prec_\sigma$ and $\succeq_\sigma$ when clear from context.

Visualization

We visualize an ordered undirected graph by labeling its nodes with the indices of each vertex. Let (V, E) be an undirected graph with $V = \{a, b, c, d, e\}$ and

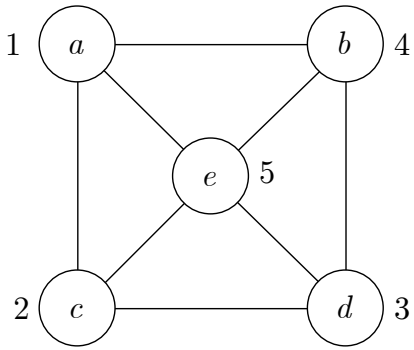
$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\}.$$

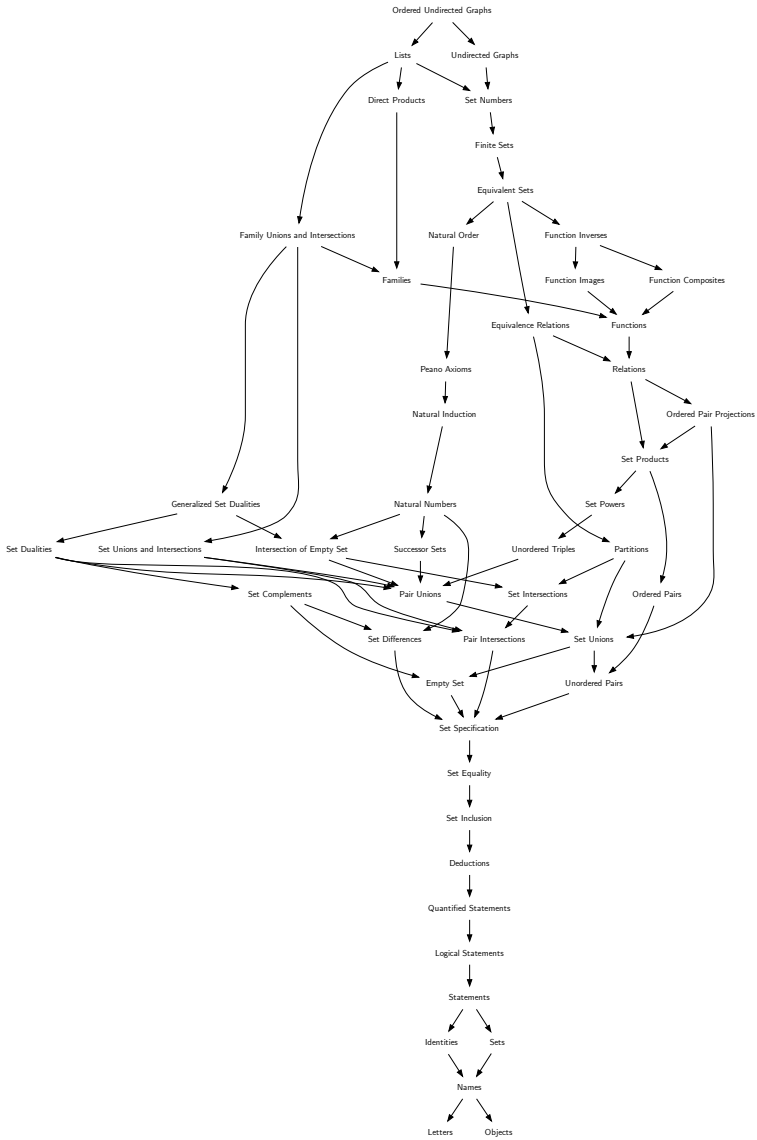
Let $\sigma : \{1, \ldots, 5\} \to V$ be an ordering with

$$\sigma(1) = a \quad \sigma(2) = c \quad \sigma(3) = d \quad \sigma(4) = b \quad \sigma(e) = 5.$$

We visualize the ordered graph in the figure.

[1] Future editions will include. This sheet is needed, for example, in discussing perfect elimination orderings.


1
a
b
4
e
5
2
c
d
3


Ordered Undirected Graphs
Lists
Undirected Graphs
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Images
Function Composites
Equivalence Relations
Functions
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Dualities
Set Unions and Intersections
Intersection of Empty Set
Successor Sets
Unordered Triples
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects