Why

What is inverse under matrix multiplication.

Definition

Recall that if $A \in \mathbf{R}^{m \times n}$ then $x \mapsto Ax$ is a function from $\mathbf{R}^n$ to $\mathbf{R}^m$. Clearly, if $m \neq n$, then the inverse of f can not exist.[1]

Now suppose that $A \in \mathbf{R}^{n \times n}$. Of course, the inverse may not exist. Consider, for example if A was the n by n matrix of zeros. If there exists a matrix B so that $BA = I$ we call B the *left inverse* of A and likewise if $AC = I$ we call C the *right inverse* of A. In the case that A is square, the right inverse and left inverse coincide.

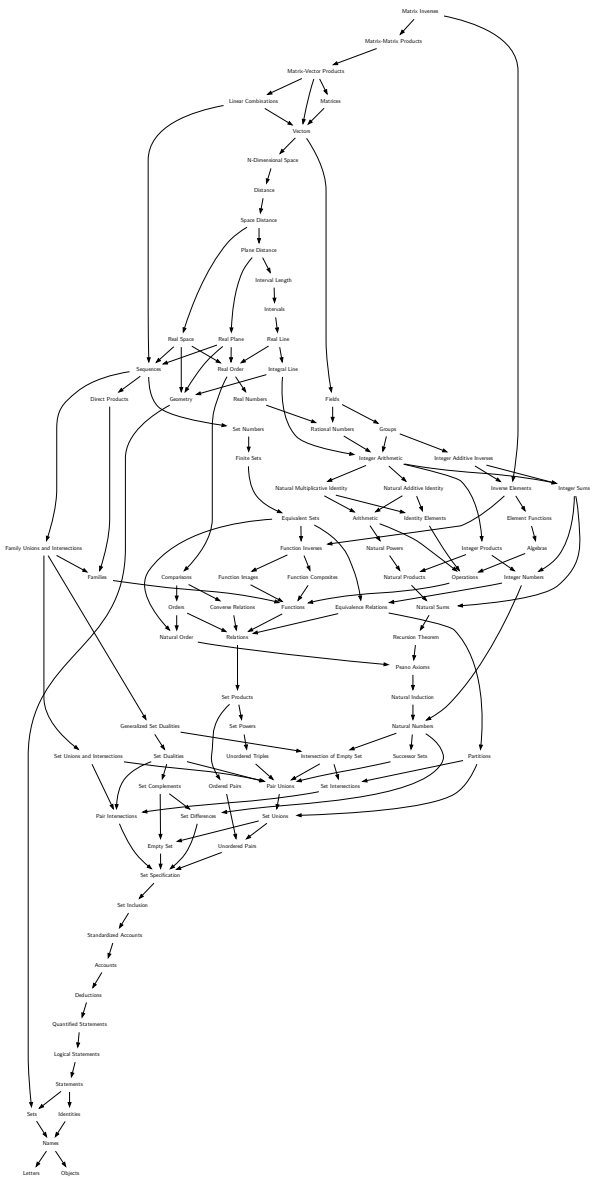
Proposition 1. *Let $A, B, C \in \mathbf{R}^{n \times n}$. Let $BA = I$ and $AC = I$. Then $B = C$.*

Proof. Since $BA = AC$ we have $BBA = BAC$ so $B = C$ since $BA = I$. □

Notation

Let F be a field. Let $A \in F^{n \times n}$ be invertible. We follow the notation of inverse elements and denote the inverse of A by A^{-1}.

[1] Future editions will expand.


Matrix Inverses
Matrix-Matrix Products
Matrix-Vector Products
Linear Combinations
Matrices
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Sequences
Real Order
Integral Line
Direct Products
Geometry
Real Numbers
Fields
Set Numbers
Rational Numbers
Groups
Finite Sets
Integer Arithmetic
Integer Additive Inverses
Natural Multiplicative Identity
Natural Additive Identity
Inverse Elements
Integer Sums
Equivalent Sets
Arithmetic
Identity Elements
Element Functions
Family Unions and Intersections
Function Inverses
Natural Powers
Integer Products
Algebra
Families
Comparisons
Function Images
Function Composites
Natural Products
Operations
Integer Numbers
Orders
Converse Relations
Functions
Equivalence Relations
Natural Sums
Natural Order
Relations
Recursion Theorem
Peano Axioms
Set Products
Natural Induction
Generalized Set Qualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Qualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Ordered Pairs
Pair Unions
Set Intersections
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects