



## Why

What is the best estimate for a random variable if we consider the square error?

## Definition

Let  $(\Omega, \mathcal{A}, \mathbf{P})$  be a probability space and  $x : \Omega \rightarrow \mathbf{R}$  a random variable. A *minimum mean squared error estimate* or *MMSE estimate* or *least square estimate* is a value  $\xi \in \mathbf{R}$  which minimizes  $\mathbf{E}(x - \xi)^2$ .

**Proposition 1.** *There is a unique MMSE estimate and it is given by  $\mathbf{E}(x)$ .*

## Vector Case

Let  $(\Omega, \mathcal{A}, \mathbf{P})$  be a probability space and  $y : \Omega \rightarrow \mathbf{R}^n$  a random variable.<sup>1</sup>

A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* is a value  $\xi \in \mathbf{R}$  which minimizes  $\mathbf{E}\|x - \xi\|^2$ .

**Proposition 2.** *There is a unique MMSE estimator and it is given by  $\mathbf{E}(y)$ .*

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<sup>1</sup>Future editions might collapse this into the previous case.



