Why

We can use families to think about unions and intersections.

Family Unions

Let $A : I \to \mathcal{P}(X)$ be a family of subsets. We refer to the union (see Set Unions) of the range (see Relations) of the family the *family union*. We denote it $\cup_{i \in I} A_i$.

Proposition 1. $(x \in \cup_{i \in I} A_i) \longleftrightarrow (\exists i)(x \in A_i)$

If $I = \{a, b\}$ is a pair with $a \neq b$, then $\cup_{i \in I} = A_a \cup A_b$.

There is no loss of generality in considering family unions. Every set of sets is a family: consider the identity function from the set of sets to itself.

We can also show generalized associative and commutative law[1] for unions.

Proposition 2. *Let $\{I_j\}$ be a family of sets and define $K = \cup_j I_j$. Then* $\cup_{k \in K} A_k = \cup_{j \in J}(\cup_{i \in I_j} A_i)$.[2]

Family Intersection

If we have a nonempty family of subsets $A : I \to \mathcal{P}(X)$, we call the intersection (see Set Intersections) of the range of the family the *family intersection*. We denote it $\cap_{i \in I} A_i$.

Proposition 3. $x \in \cap_{i \in I} A_i \longleftrightarrow (\forall i)(x \in A_i)$

Similarly we can derive associative and commutative laws for intersection[3]. They can be derived as for unions, or from the facts of unions using generalized DeMorgan's laws (see Generalized SSet Dualities.

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[1] The commutative law will appear in future editions.

[2] An account will appear in future editions.

[3] Statements of these will be given in future editions.

ßConnections

The following are easy[4]

Let $\{A_i\}$ be a family of subsets of X and let $B \subset X$.

Proposition 4. $B \cap \bigcup_i A_i = \bigcup_i (B \cap A_i)$

Proposition 5. $B \cup \bigcap_i A_i = \bigcap_i (B \cup A_i)$

Let $\{A_i\}$ and $\{B_j\}$ be families of sets.[5]

Proposition 6. $(\bigcup_i A_i) \cap (\bigcup_j B_j) = \bigcup_{i,j} (A_i \cap B_j)$

Proposition 7. $(\bigcap_i A_u) \cup (\bigcap_j B_j) = \bigcap_{i,j} (A_i \cup B_j)$.

Proposition 8. $\cap_i X_i \subset X_j \subset \cup_i X_i$ *for each* j.

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[4]Accounts will appear in future editions.

[5]An account of the notation used and the proofs will appear in future editions.

Family Unions and Intersections

Families

Functions

Relations

Set Products

Generalized Set Dualities

Set Powers

Set Dualities

Intersection of Empty Set

Set Unions and Intersections

Unordered Triples

Set Intersections

Pair Unions

Ordered Pairs

Set Complements

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Set Differences

Empty Set

Unordered Pairs

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