Definition

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex product* of z_1 and z_2 is the complex number

$$(x_1x_2 - y_1y_2, x_1y_2 + y_1x_2).$$

Notation

We denote the complex product of z_1 and z_2 by $z_1 \cdot z_2$ or z_1z_2.

The notation overloads that used for real numbers. This overloading is justified by the fact that the complex product of two purely real complex numbers z_1 and z_2 the purely real complex number whose real part is the product of the real parts of z_1 and z_2.

Recall that we denote $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. This notation is a mnemonic for the definition of a complex product if we treat $i^2 = -1$.

$$\begin{aligned} z_1z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2). \end{aligned}$$

Properties

Proposition 1 (Commutativity)**.** *For all $z_1, z_2 \in \mathbf{C}$, we have $z_1z_2 = z_2z_1$.*

Proposition 2 (Associativity)**.** *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have*

$$z_1(z_2z_3) = (z_1z_2)z_3.$$

Complex multiplication

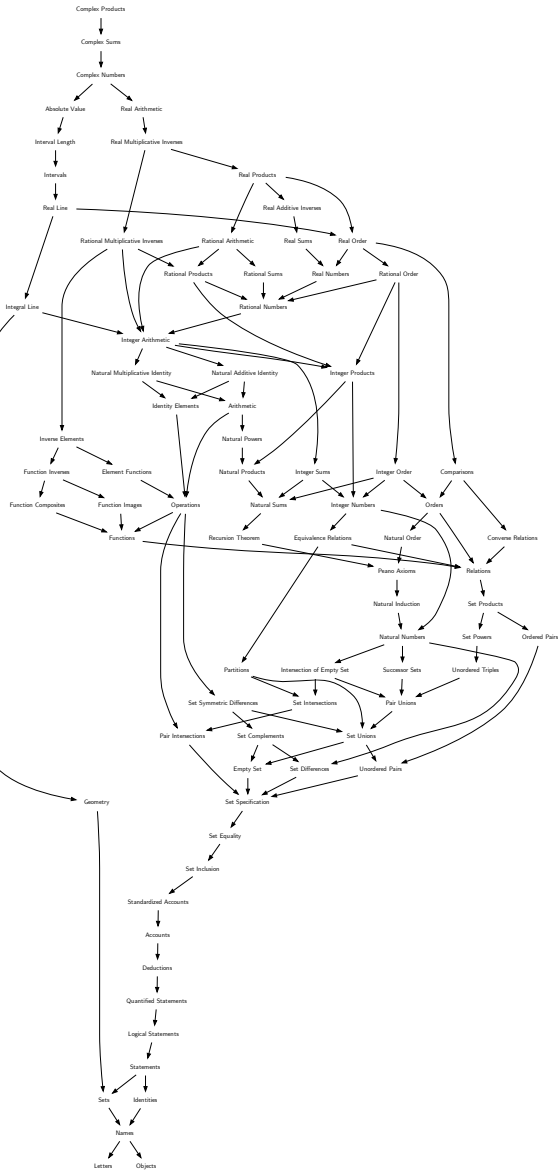
We call the operation that associates a pair of complex numbers with their product *complex multiplication*. The operation is symmetric (commutative).

Multiplicative identity and inverse

Notice that the complex number $(1, 0)$ is the multiplicative identity. It is unique,[1] and so we call it the *complex multiplicative identity.*

We call the operation $(z, w) \mapsto z/w$ *complex division* and we call z/w the *(complex) quotient* of z with w.

[1] Future editions will include an account


Complex Products
Complex Sums
Complex Numbers
Absolute Value
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Line
Real Additive Inverses
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Rational Products
Rational Sums
Real Numbers
Rational Order
Integral Line
Rational Numbers
Integer Arithmetic
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Identity Elements
Arithmetic
Natural Powers
Inverse Elements
Function Inverses
Element Functions
Natural Products
Integer Sums
Integer Order
Comparisons
Function Composites
Function Images
Operations
Natural Sums
Integer Numbers
Orders
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Recursion Theorem
Equivalence Relations
Natural Order
Converse Relations
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Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Pair Intersections
Set Complements
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Set Differences
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Geometry
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects