



Extended Real Numbers

1 Why

That some limits grow without bound leads us to add two elements to the set of real numbers.

2 Definition

The set of **extended real numbers** is the union of the set of real numbers with a set containing two elements: one we call **positive infinity** and we call **negative infinity**.

2.1 Extended Arithmetic

We extend addition to all but one ordered pair of elements of the new set. The sum of any real number with a real number is defined as before. The sum any real number with positive infinity is positive infinity. The sum any real number with negative infinity is negative infinity. The sum of positive infinity with positive infinity is positive infinity. The sum of negative infinity with negative infinity is negative infinity. We do not define the sum of positive infinity and negative infinity.

2.2 Notation

Let R denote the set of real numbers. We denote the element positive infinity by $+\infty$ and we denote the element negative infinity by $-\infty$. The set of extended real numbers is the set $R \cup \{+\infty, -\infty\}$.

3 Intervals

We