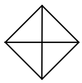


Why

We want to combine two groups.[1]

Defining Result

Proposition 1. *For each natural number m, there exists a function $s_m : \omega \to \omega$ which satisfies*

$$s_m(0) = m \quad \textit{and} \quad s_m(n^+) = (s_m(n))^+$$

for every natural number n.

Proof. The proof uses the recursion theorem (see *Recursion Theorem*).[2] □

Let m and n be natural numbers. The value $s_m(n)$ is the *sum* of m with n.

Notation

We denote the sum $s_m(n)$ by $m + n$.

Properties

The properties of sums are direct applications of the principle of mathematical induction (see *Natural Induction*).[3]

---

[1] Future editions will change this section.
[2] Future editions will give the entire account.
[3] Future editions will include the accounts.

Proposition 2 (Associative)**.** *Let k, m, and n be natural numbers. Then*

$$(k + m) + n = k + (m + n).$$

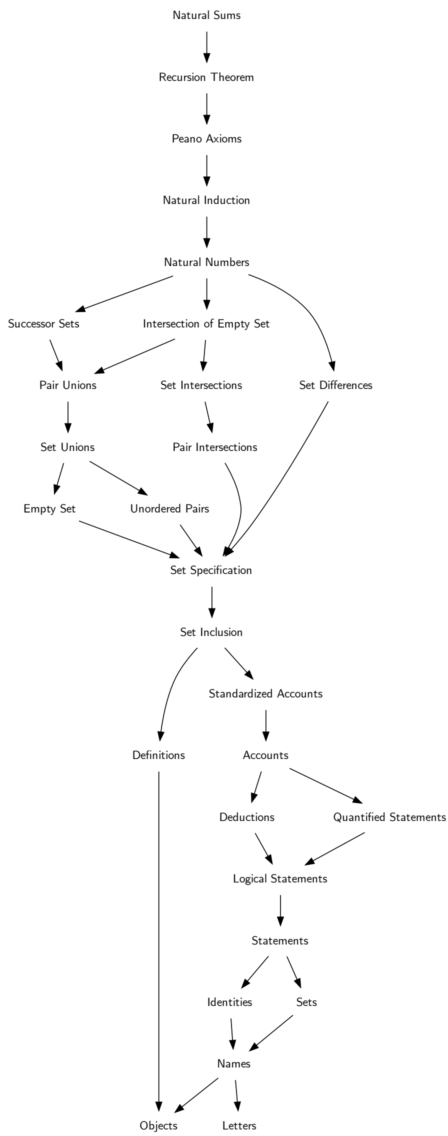
Proposition 3 (Commutative)**.** *Let m and n be natural numbers. Then*

$$m + n = n + m.$$

Relation to Addition

Proposition 4 (Distributive)**.** *Let k, m, and n be natural numbers. Then*

$$k \cdot (m + n) = (k \cdot m) + (k \cdot n).$$


Natural Sums
Recursion Theorem
Peano Axioms
Natural Induction
Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Definitions
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters