Why

If the base set of a sequence has a partial order, then we can discuss its relation to the order of sequence.

Definition

A sequence on a partially ordered set is *non-decreasing* if whenever a first index precedes a second index the element associated with the first index precedes the element associated with the second element. A sequence on a partially ordered set is *increasing* if it is non-decreasing and no two elements are the same.

A sequence on a partially ordered set is *non-increasing* if whenever a first index precedes a second index the element associated with the first index succedes the element associated with the second element. A sequence on a partially ordered set is *decreasing* if it is non-increasing and no two elements are the same.

A sequence on a partially ordered set is *monotone* if it is non-decreasing, or non-increasing. An increasing sequence is non-decreasing. A decreasing sequences is non-increasing. A sequence on a partially ordered set is *strictly monotone* if it is decreasing, or increasing.

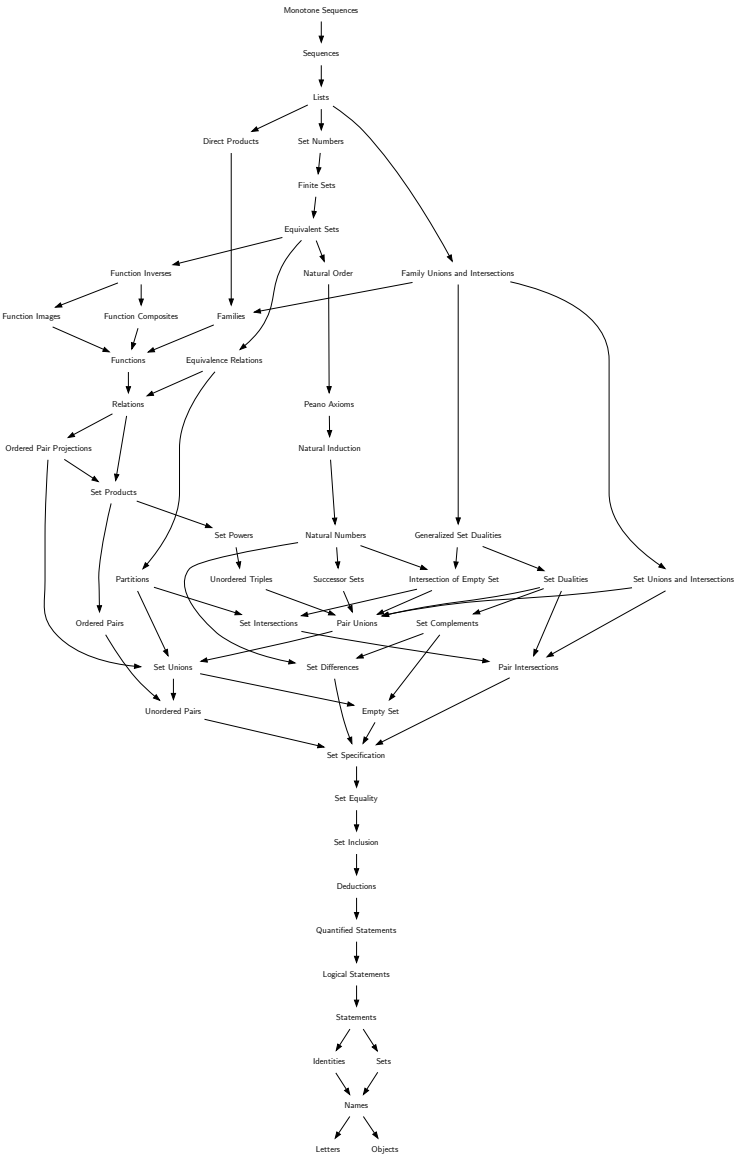
Notation

Let A a non-empty set with partial order $\preceq$. Let $(a_n)_n$ a sequence in A.

The sequence is non-decreasing if $n \leq m \longrightarrow a_n \preceq a_m$, and increasing if $n < m \longrightarrow a_n \prec a_m$. The sequence is non-increasing if $n \leq m \longrightarrow a_n \succeq a_m$, and decreasing if $n < m \longrightarrow a_n > a_m$.

Examples

Example 1. *Let A a non-empty set and $(A_n)_n$ a sequence of sets in $\mathcal{P}(A)$. Partially order elements of $\mathcal{P}(A)$ by the relation contained in.*


Monotone Sequences
Sequences
Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Function Inverses
Natural Order
Family Unions and Intersections
Function Images
Function Composites
Families
Functions
Equivalence Relations
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Set Powers
Natural Numbers
Generalized Set Dualities
Partitions
Unordered Triples
Successor Sets
Intersection of Empty Set
Set Dualities
Set Unions and Intersections
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects