Why

We can cast various common probabilistic regression models into the probabilistic errors linear model by mentioning the input space and feature maps. This unifies our analysis.

Definition

A *line fit model* has input space R and output space R. We use a regression function $\phi : \mathsf{R} \to \mathsf{R}^2$ defined by $\phi(t) = (1, t)^\top$.

We think of $t \in T \subset \mathsf{R}$ as a "dose level" (T is an interval). Given dose levels $t_1, \ldots, t_\ell$ and repetitions $n_1, \ldots, n_\ell$ we obtain the design matrix. Here the regression function generates a line segment embedded in the plane R^2. We call the parameters the *intercept parameter* and *slope parameter*.

A *parabola fit model* has input space R and output space R. We use a regression function $\phi : \mathsf{R} \to \mathsf{R}^3$ defined by $\phi(t) = (1, t, t^2)^\top$. Here the regression space is a segment of a parabola embedded in space R^3 (since $t \in T$ an interval).

These two are instance of *polynomial fit models* of degree $d \geq 1$, in which the regression function becomes $\phi : \mathsf{R} \to \mathsf{R}^{d+1}$ defined by $\phi(t) = (1, t, t^2, \ldots, t^d)^\top$. In this case, the regression range $\phi(T)$ is a one-dimensional curve embedded in R^{d+1}. In cases in which it is clear that the input space is a single real variable t, a linear model for a line fit (parabola fit, polynomial fit of degree d) is called a *first-degree model* (*second-degree model*, *dth degree model*).

m-way models

We can generalize to *m-way dth degree polynomial fit models* in which the input space is $X \subset \mathbf{R}^m$ and the regression function $\phi : \mathbf{R}^m \to \mathbf{R}^k$ (k is $d+m$ choose d) is the vector of all monomials of degree d in m variables.

For example, a two-way third-degree model has a regression function

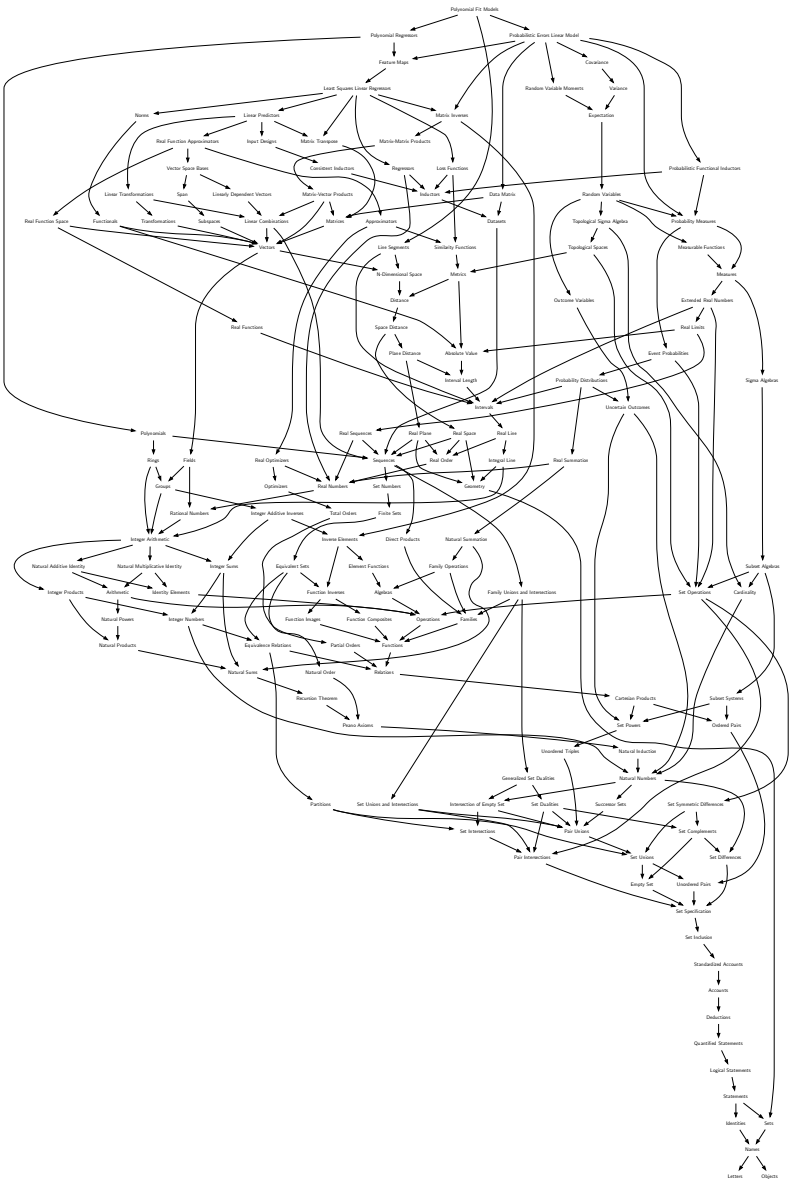
$$\phi(t_1, t_2) = \begin{bmatrix} 1 & t_1 & t_2 & t_1^2 & t_1 t_2 & t_2^2 & t_1^3 & t_1^2 t_2 & t_1 t_2^2 & t_2^3 \end{bmatrix}^\top.$$

Or consider a three way second-degree model with regression function

$$\phi(t_1, t_2, t_3) = \begin{bmatrix} 1 & t_1 & t_2 & t_3 & t_1^2 & t_1 t_2 & t_1 t_3 & t_2^2 & t_2 t_3 & t_3^2 \end{bmatrix}^\top.$$

Both models will result in parameter vectors of size ten. We call these models *saturated* because they have every possible dth degree power or cross product of variables. In generally, a m-way dth degree model has $d+m$ choose d mean parameters.

In contrast to saturated models we can talk about *nonsaturated* models. For example, a nonsaturated two-way second-degree model has $\phi : \mathbf{R}^2 \to \mathbf{R}^4$ where $\phi(t_1, t_2) = (1, t_1, t_2, t_1^2)^\top$.


Polynomial Fit Models
Polynomial Regression
Probabilistic Error Linear Model
Feature Maps
Covariance
Random Variable Moments
Variance
Least Squares Linear Regression
Expectation
Norms
Linear Prediction
Matrix Inverses
Real Function Approximation
Input Designs
Matrix Transpose
Matrix-Matrix Products
Probabilistic Functional Induction
Vector Space Bases
Constant Induction
Regression
Loss Functions
Induction
Data Matrix
Random Variables
Linear Transformations
Span
Linearly Dependent Vectors
Matrix-Vector Products
Real Function Space
Functionals
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Topological Sigma Algebra
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Similarity Functions
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Measurable Functions
N-Dimensional Space
Metrics
Measures
Distance
Outcome Variables
Extended Real Numbers
Real Function
Space Distance
Real Limits
Plane Distance
Absolute Value
Event Probabilities
Interval Length
Probability Distributions
Sigma Algebras
Intervals
Uncertain Outcomes
Polynomials
Real Sequences
Real Plane
Real Space
Real Line
Rings
Fields
Real Optimizers
Sequences
Real Order
Integral Line
Real Summation
Groups
Optimizers
Real Numbers
Set Numbers
Geometry
Rational Numbers
Integer Additive Inverses
Total Orders
Finite Sets
Integer Arithmetic
Inverse Elements
Direct Products
Natural Summation
Natural Additive Identity
Natural Multiplication Identity
Integer Sums
Equivalent Sets
Element Functions
Family Operations
Subset Algebras
Integer Products
Arithmetic
Identity Elements
Function Inverses
Algebras
Family Unions and Intersections
Set Operations
Cardinality
Natural Powers
Integer Numbers
Function Images
Function Composites
Operations
Families
Natural Products
Equivalence Relations
Partial Orders
Functions
Natural Sums
Natural Order
Relations
Cartesian Products
Subset Systems
Recursion Theorem
Set Powers
Ordered Pairs
Peano Axioms
Natural Induction
Unordered Triples
Natural Numbers
Generalized Set Duality
Partitions
Set Unions and Intersections
Intersection of Empty Set
Set Duality
Successor Sets
Set Symmetric Difference
Set Intersection
Pair Unions
Set Complements
Pair Intersection
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects