

MATRIX SPACE

Why

We can view the set of real-valued $n \times k$ matrices as a vector space over \mathbb{R} .

Definition

The matrix sum of two matrices $A, B \in \mathbb{R}^{n \times k}$ is the matrix $C \in \mathbb{R}^{n \times k}$ defined by $C_{ij} = A_{ij} + B_{ij}$. In other words, the matrix C is given by summing the entires of A and B "entry-wise". We denote the matrix sum by A + B.

For $\alpha \in \mathbb{R}$, the α -scaled version of $A \in \mathbb{R}^{n \times k}$ is the matrix $C \in \mathbb{R}^{n \times k}$ given by $C_{ij} = \alpha A_{ij}$. In other words, the matrix C is given by scaling the entries of A "entry-wise". We denote the α -scaled version of A by αA . These two definitions are justified by the following.

The $n \times k$ -matrix space is the vector space over $\mathbb{R}^{n \times k}$ in which addition is given by the matrix sumer and scalar multiplication by entry-wise scaling.¹

Subspace of symmetric matrices

The subset of symmetric n by n matrices is a subset of $\mathbb{R}^{n \times n}$.

¹Future editions will rework this sheet.

