



Why

The most important families are those indexed by (subsets of) the natural numbers.

Definition

A *finite sequence* is a family whose index set is $\{1, \dots, n\}$ for some $n \in \mathbf{N}$. The *length* of a finite sequence is the size of its index set. If the codomain of a sequence is A , we say the sequence is *in* A .

Let A be a set with $|A| = n$. In this case, another term for a finite sequence is a *string* (or *list*). A sequence $a : \{1, \dots, n\} \rightarrow A$ is an *ordering* of A if a is invertible. In this case, we call the inverse a *numbering* of A . An ordering associates with each number a unique object and a numbering associates with each object a unique number (the object's *index*).

Notation

Since the natural numbers are ordered, we regularly denote finite sequences from left to right between parentheses. For example, we denote $a : \{1, \dots, 4\} \rightarrow A$ by (a_1, a_2, a_3, a_4) .

Relation to Direct Products

A *natural direct product* is a product of a sequence of sets. We denote the direct product of a sequence of sets A_1, \dots, A_n by $\prod_{i=1}^n A_i$. If each A_i is the same set A , then we denote the

product $\prod_{i=1}^n A_i$ by A^n . In this case, we call an element (the sequence $a = (a_1, a_2, \dots, a_n) \in A^n$) an *n-tuple* or *tuple*. The set of sequences in a set A is the direct product A^n .

Infinite Sequences

An *infinite sequence* is a family whose index set is \mathbf{N} (the set of natural numbers without zero). The *nth term* or *coordinate* of a sequence is the result of the *nth* natural number, $n \in \mathbf{N}$.¹

Notation

Let A be a non-empty set and $a : \mathbf{N} \rightarrow A$. Then a is a (infinite) sequence in A . $a(n)$ is the *nth* term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n . If $\{A_n\}_{n \in \mathbf{N}}$ is an infinite sequence of sets, then we denote the direct product of the sequence by $\prod_{i=1}^{\infty} A_i$.

Natural unions and intersections

We denote the family union of the finite sequence of sets A_1, \dots, A_n by $\cup_{i=1}^n A_i$. We denote the family of the infinite sequence of sets $(A_n)_n$ by $\cup_{i=1}^{\infty} A_i$. Similarly, we denote the intersections of a finite and infinite sequence of sets $\{A_i\}$ by $\cap_{i=1}^n A_i$ and $\cap_{i=1}^{\infty} A_i$, respectively.

¹Future editions may also comment that we are introducing language for the steps of an infinite process.

