**Why**

We generalize random variables and random vectors to a map from the outcome space to a space of functions.

**Definition**

A *random function* is a map from the set of outcomes of a probability space to a set of functions.[1]

**Notation**

Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $A$ and $B$ be sets and let $x : \Omega \to (A \to B)$ be a random function. For each outcome $\omega \in \Omega$, $x_\omega : A \to B$ is a function from $A$ to $B$.

**A family of random variables**

A *stochastic process* (or *random process* or *random field*) is a family of random variables. The term process is often used when the index set is associated with time and the term field is often used when the index set is associated with space.

We can associate to the random function $x : \Omega \to (A \to B)$ a family of random variables $y : A \to (\Omega \to B)$ defined by

$$y(a)(\omega) = x(\omega)(a).$$

The index set of the family is the domain of the set of functions. The codomain of each random variable is the codomain of the set of functions.

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[1]Future editions will make measurability precise.

The converse is also naturally true. Any family of random variables can be associated with a random function. In this way, a random function corresponds to a family of random variables, and any family of random variables corresponds to a random function.

For this reason many authors use the term random function or stochastic function and random process or stochastic process interchangeably. In these sheets, we will be specific.

Random Functions

Random Vector

Matrices

Real Matrices

Random Variables

Probability Measures — Measurable Functions — Linear Equations — Topological Sigma Algebra — Vectors

Measures — Linear Functions — Topological Space — Real Vector — Space Vector — Plane Vector Sums and Differences

Sigma Algebra — Metrics — N-Dimensional Space

Real Series — Extended Real Numbers — Distance — Plane Vectors

Event Probabilities — Real Limits — Space Distance — Arrays

Absolute Value — Plane Distance

Probability Distributions — Interval Length

Real Summation — Real Space

Intervals — Real Plane

Real Line

Real Order — Real Sequences — Integral Line — Sequences — Geometry

Rational Order — Fields — Real Numbers — Subset Algebra — Lists

Rings — Rational Numbers — Cardinality — Set Numbers — Direct Products

Integer Arithmetic — Finite Sets

Integer Products — Natural Additive Identity — Natural Multiplicative Identity

Natural Summation — Arithmetic — Identity Elements — Equivalent Sets

Natural Powers — Natural Equations — Integer Sums — Integer Order — Comparisons — Family Operations — Function Inverse — Family Unions and Intersections

Natural Products — Equation — Integer Numbers — Orders — Operations — Function Composites — Function Images — Families

Natural Sums — Natural Order — Converse Relations — Functions — Equivalence Relations

Recursion Theorem — Relations

Peano Axioms — Subset Systems — Set Products

Uncertain Outcomes — Natural Induction — Set Powers — Ordered Pairs

Natural Numbers — Unordered Triples — Generalized Set Dualities

Set Symmetric Difference — Successor Sets — Set Dualities — Intersection of Empty Set — Partitions — Set Unions and Intersections

Set Complements — Pair Unions — Set Intersections

Set Differences — Set Unions — Pair Intersections

Empty Set — Unordered Pairs

Set Specification

Set Inclusion — Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects