



# Convergence In Probability

## 1 Why

laws of large numbers

## 2 Definition

A sequence of random variables converges in probability if it converges in measure.

### 2.1 Notation

Let  $(X, \mathcal{A}, \mu)$  be a measure space. Let  $(f_n)_n$  a sequence of real-valued measurable functions on  $X$ . Let  $f : X \rightarrow \mathbb{R}$  be measurable function. If  $f_n$  converges in measure to  $f$  we write:  $f_n \rightarrow f$  in probability, read aloud as “f n goes to f in probability.”

Suppose  $f_n \rightarrow f$  in probability. Then for every  $\varepsilon > 0$ ,

$$\lim_{n \rightarrow \infty} \mu(\{x \in X \mid |f_n(x) - f(x)| > \varepsilon\}) = 0.$$