Why

Definition

A *hyperplane* in n-dimensional space is an $(n-1)$-dimensional affine set.

Since the $n-1$-dimensional subspaces are the orthogonal complements of the one-dimensional subspaces, they are the sets which can be specified by

$$\{x \in \mathbf{R}^n \mid x \perp b\}$$

for $b \in \mathbf{R}^n$. The hyperplanes are translates of these,

$$\{x \in \mathbf{R}^n \mid x \perp b\} + a = \{x + a \mid \langle x, b \rangle = 0\}$$
$$= \{y \mid \langle y - a, b \rangle = 0\} = \{y \mid \langle y, b \rangle = \beta\},$$

where $\beta = \langle a, b \rangle$.

Characterization

PROPOSITION **1.** *$H \subset \mathbf{R}^n$ is a hyperplane if and only if there exists $\beta \in \mathbf{R}$ and nonzero $b \in \mathbf{R}^n$ so that*

$$H = \{x \in \mathbf{R}^n \mid \langle x, b \rangle = \beta\}.$$

Remark 2. *b and β are unique up to a common nonzero multiple. For example, b, β and $2b, 2\beta$ give the same hyperplane.*

Remark 3. *The vector b is called a* normal *to the hyperplane.*

Hyperplane

Affine Set Dimension Subspace Orthogonal Complements

Subspace Dimensions Orthogonal Complements

Vector Space Dimensions

Vector Space Bases Inner Products

Affine Sets and Subspaces Span Linearly Dependent Vectors

Translate Sets Subspaces Linear Combinations

Affine Sets Vectors

N-Dimensional Space Numberings

Distance Finite Sets

Space Distance

Plane Distance

Interval Length

Real Plane Real Line Real Space Intervals

Geometry Direct Products Real Order

Families

Real Numbers Fields

Rational Numbers Groups

Integer Arithmetic Integer Additive Inverses

Natural Additive Identity Natural Multiplicative Identity Inverse Elements Integer Sums

Identity Elements Element Functions Arithmetic

Algebras Function Inverses Natural Exponents Integer Products Equivalent Sets

Operations Function Composites Function Images Natural Products Integer Numbers

Functions Natural Sums Equivalence Relations Natural Order

Relations Recursion Theorem

Peano Axioms

Natural Induction

Cartesian Products

Set Powers Natural Numbers

Unordered Triples Successor Sets Intersection of Empty Set Partitions

Ordered Pairs Pair Unions Set Intersections Set Differences

Set Unions

Unordered Pairs Empty Set Pair Intersections

Set Specification

Set Inclusion

Standardized Accounts

Accounts Definitions

Deductions Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects

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