



## Why

If a directed graph has no cycles, then it has a nice property.<sup>1</sup>

## Definition

Directed and acyclic graphs (sometimes *DAGs*) have some useful properties. Clearly, every subgraph induced on a directed acyclic graph is a directed acyclic graph.

**Proposition 1.** *Let  $(V, E)$  be a directed acyclic graph. Then there exists a vertex  $v \in V$  which is a source and a vertex  $w \in V$  which is a sink.*

*Proof.* There exists a directed path of maximum length. It must start at a source and end at a sink.<sup>2</sup> □

A *topological numbering*, *topological sort* or *topological ordering* of a directed graph  $(V, E)$  is a numbering  $\sigma : \{1, \dots, |V|\} \rightarrow V$  satisfying

$$(v, w) \in E \longrightarrow \sigma^{-1}(v) < \sigma^{-1}(w).^3$$

**Proposition 2.** *There exists a topological sort for every acyclic graph.*

*Proof.* Let  $(V, E)$  be a directed acyclic graph. There exists a source vertex,  $v_1$ . Set  $\sigma(1) = v_1$ . Take the subgraph induced by  $V - \{v_1\}$ . It is directed acyclic, and so has a source vertex,  $v_2$ . Set  $\sigma(2) = v_2$ . Continue in this way.<sup>4</sup> □

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<sup>1</sup>Future editions will expand this vague introduction.

<sup>2</sup>Future editions will expand.

<sup>3</sup>Future editions will further explain this concept.

<sup>4</sup>Future editions will clarify and expand.



