Why

[1]

Definition

Let $a \in \mathbf{R}^n$ and $b \in \mathbf{R}$. Suppose we want to find $x \in \mathbf{R}^n$ to satisfy

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b.$$

We refer to this expression as a *real linear equation* or *linear equation*. We treat x_is as a variables and we treat the a_is and b as constants. We call the pair (a, b) the *real linear equation constants*.[2]

The source of the terminology "linear" is by viewing the left hand side as a function. Define $f : \mathbf{R}^n \to \mathbf{R}$ by $f(x) = \sum_i a_ix_i$. We want to find $x \in \mathbf{R}^n$ to satisfy $f(x) = b$. Notice that f is a *linear* real function.[3]

Moreover, to each linear function $f : \mathbf{R}^d \to \mathbf{R}$ there exists a vector $a \in \mathbf{R}^d$ so that $f(x) = \sum_i a_ix_i$. For this reason, if we are given several linear function $f_1, \ldots, f_m$, then we can think of these as several vectors $a^1, \ldots, a^n$. If we are also given $b_i \in \mathbf{R}$ for each $i = 1, \ldots, m$, then we have the vector $b \in \mathbf{R}^m$

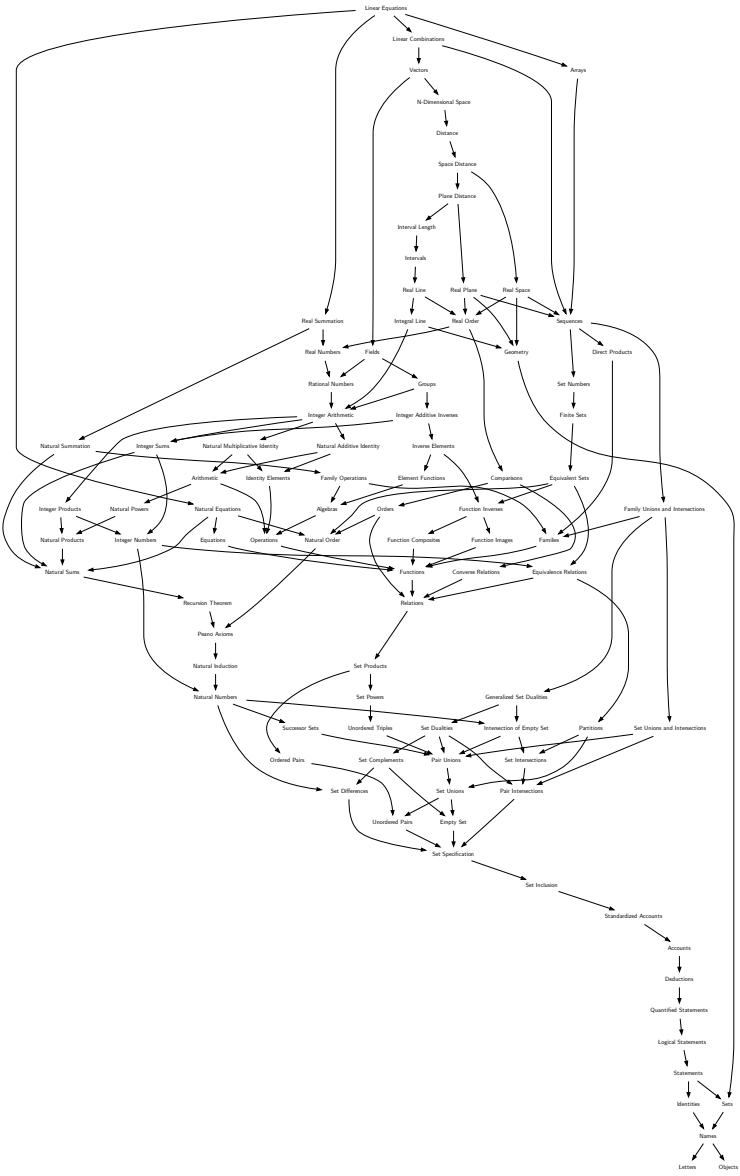
We can define the two-dimensional array $A \in \mathbf{R}^{m\times n}$ by $A_{ij} = a^i_j$. For this reason, a *linear system of equations* is a

[1] Future editions will include.
[2] Future editions will clarify.
[3] Future editions may require a sheet here.

pair (A, b). A solution of a linear system of equations is a vector $x \in \mathbf{R}^n$ satisfying the equations

$$\begin{array}{rrcrcr}
A_{11}x_1 + & A_{12}x_2 + & \cdots + & A_{1n}x_n = & b_1 \\
A_{21}x_1 + & A_{22}x_2 + & \cdots + & A_{2n}x_n = & b_2 \\
\vdots & \vdots & & \vdots & \vdots \\
A_{m1}x_1 + & A_{m2}x_2 + & \cdots + & A_{mn}x_n = & b_n
\end{array}$$

Other terminology includes a *system of linear equations* or *linear system* or *simultaneous linear equations*


Linear Equations
Linear Combinations
Arrays
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Real Summation
Integral Line
Real Order
Sequences
Real Numbers
Fields
Geometry
Direct Products
Rational Numbers
Groups
Set Numbers
Integer Arithmetic
Integer Additive Inverses
Finite Sets
Natural Summation
Integer Sums
Natural Multiplicative Identity
Natural Additive Identity
Inverse Elements
Arithmetic
Identity Elements
Family Operations
Element Functions
Comparisons
Equivalent Sets
Integer Products
Natural Powers
Natural Equations
Algebras
Orders
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Equations
Operations
Natural Order
Function Composition
Function Images
Families
Natural Sums
Functions
Converse Relations
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects