



# Families

## 1 Why

It is useful to have some language and notation for talking about a set of sets.

## 2 Definition

A *family* of sets is a set of sets. Experience shows that it is useful to have these associated with the elements of a well-known second set.

An *indexed family of sets* is a function from one set to the power set of a second set. We call the first set the *index set*. We call the second set the *base set*. The range of the indexed family of sets is, of course, a family.

### 2.1 Notation

Let  $A$  and  $I$  be non-empty sets. We use  $I$  as a mnemonic for “index” set. Let  $a : I \rightarrow A^*$  be a family. For  $i \in I$ , we follow the function notation and denote the result of applying  $a$  to  $i$  by  $a_i$ .

We denote the range of the family by family of  $a_\alpha$  indexed with  $I$  by  $\{a_\alpha\}_{\alpha \in I}$ , which is short-hand for set-builder notation. We read this notation “a sub-alpha, alpha in I.”