



FINITE SETS

Why

We want to talk about the size of a set.¹

Definition

A *finite* set is one that is equivalent to some natural number; an infinite set is one which is not finite. From this we can show that ω is infinite. This justifies the language we used in *Natural Numbers* about the principle of infinity. The principle of infinity asserts the existence of a particular infinite set; namely ω .

Motivation for set number

It happens that if a set is equivalent to a natural number, it is equivalent to only one natural number.

Proposition 1. *A set can be equivalent to at most one natural number.*²

A consequence is that a finite set is never equivalent to a proper subset of itself. So long as we are considering finite sets, a piece (subset) is always less than than the whole (original set).

Proposition 2. *A finite set is never equivalent to a proper subset of itself.*

¹Will be expanded in future editions.

²Future edition will include proof, which uses comparability of numbers and the results of *Equivalent Sets*).

Subsets of finite sets

Every subset of a natural number is equivalent to a natural number.³ A consequence is:

Proposition 3. *Every subset of a finite set is finite.*⁴

Unions of finite sets

Proposition 4. *if A and B are finite, then $A \cup B$ is finite.*

Products of finite sets

Proposition 5. *If A and B are finite, then $A \times B$ is finite.*

Powers of finite sets

Proposition 6. *If A is finite then A^* is finite.*

Functions between finite sets

Proposition 7. *If A and B are finite, then A^B is finite.*

³This requires proof, and may become a proposition in future editions.

⁴An account will appear in future editions.

