

QUADRATIC FORMS

Definition

A quadratic form is a multivariate polynomial each term of which has degree two.

Representation

Let $f: \mathbf{R}^n \to \mathbf{R}$ be a quadratic form. There exists a matrix $A \in \mathbf{R}^{n \times n}$ so that

$$x^{\top} A x = \sum_{i,j} A_{ij} x_i x_j.$$

Suppose A is not symmetric. Then

$$f(x) = f(x)/2 + f(x)/2 = \frac{1}{2}x^{\top}Ax + \frac{1}{2}x^{\top}A^{\top}x = x^{\top}(\frac{1}{2}(A + A^{\top}))x.$$

Define $B = 1/2(A^{\top} + A)$. We call B the symmetric part of A. Since every matrix A has a symmetric part, we can always assume that the matrix for a quadratic form is symmetric. If it is not, replace it with its symmetric part, obtaining the same function.

Under trace

Observe that $\operatorname{tr} x^{\top} A x = \operatorname{tr} A x x^{\top} = \operatorname{tr} x x^{\top} A$.

