Optimization

1 Why

Given a correspondence between objects in a set with objects in an totally ordered set, we are interested in the objects which correspond to minimal or maximal elements of the ordered set.

2 Definition

Consider a non-empty set and a chain, with a function associating to each element of a set an element of the chain (a chain is a set with a total order).

A *minimizer* of the function over the set is an element of the set whose result under the function is a minimal in the function's range. In other words, the result of the function on that element is as small as the result of the function on any other element in its domain. Similarly, a *maximizer* of the function over the set is an element whose result under the function is maximal in the function's range.

In the context of minimizers and maximizers, we call the function f the *criterion*. Often f is a restriction of a function defined on a superset. In this case, we call the

We call f the *ordering* function. We call A the *feasible set* and we call $a \in A$ a *feasible element*. An element $a \in A$ is a *minimizer* of f if $f(a)$ is minimal in $f(A)$. An element $a \in A$ is a *maximizer* of f if $f(a)$ is maximal in $f(A)$.

3 Notation

Let $(C, \prec)$ be a chain. We denote the minimization problem to find an element $a \in A$ to minimize $f : A \to C$ by

$$\begin{aligned} \textbf{find} \quad & a \in A \\ \textbf{to minimize} \quad & f(a) \end{aligned}$$

We denote the minimizers by

$$\textsf{\textbf{minimizers}}(f).$$