Integrable Function Space

1 Why

The integrable functions are a vector space.

2 Definition

The *integrable function space* corresponding to a measure space is the set of real-valued functions which are integrable with respect to the measure. The term space is appropriate because this set is a real vector space. If we scale an integrable function, it remains integrable. If we add two integrable functions, the sum is integrable. Thus, a linear combination of integrable functions is integrable. The zero function is the zero element of the vector space.

TODO The open question is: what elements of our geometric intuition can we bring to a space of functions. Do functions have a size? Are certain functions near each other?

2.1 Notation

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let R denote the set of real numbers and let C denote the set of complex numbers.

We denote set the real-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, R)$, read aloud as "the real integrable functions on the measure space X script A mu." We denote set the complex-valued integrable functions on X by $\mathcal{I}(X, \mathcal{A}, \mu, C)$, read aloud as "the complex integrable functions on the measure space X script

A mu." When the field is irrelevant, we denote them by $\mathcal{I}(X, \mathcal{A}, \mu)$, read aloud as "integrable functions on the measure space X script A mu." The $\mathcal{I}$ is a mnemonic for "integrable."