Definition

The *translate* of $S \subset \mathbf{R}^n$ *by* the vector $a \in \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid \exists x \in S \text{ such that } z = x + a\}.$$

Notation

We often use the abbreviated notation $S + a$ for the translate of S by a. It is sometimes also convenient to extend set-builder notation and write

$$S + a = \{x + a \mid x \in M\}.$$

The right hand side is slick notation for the definition given above.

Sums and differences

The *sum* (or *Minkowski sum*) of two sets $S, T \subset \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x + y)\}.$$

Likewise, the *difference* (or *Minkowski difference*) of two sets $S, T \subset \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x - y)\}.$$

Notation

We denote the sum of S and T by $S + T$, and the difference by $S - T$.[1] We often use the slick notation

$$\{x + y \mid x \in S, y \in T\} \text{ and } \{x - y \mid x \in S, y \in T\},$$

for these two sets. Notice that in this notation

$$\{a\} + B = a + B$$

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[1]This second notation unfortunately conflicts with our notation for set differences. Future editions will correct.

Scaled sets

Given a set $A \subset \mathbf{R}^n$ and a $\lambda \in \mathbf{R}$, the set which is A *scaled by* (or *scaled set*, *scaling*) is

$$\{z \in \mathbf{R}^n \mid (\exists x \in A)(z = \lambda x)\}$$

We often denote this set by λA. As before, we often use the slick notation

$$\lambda A = \{\lambda a \mid a \in A\}$$

The set $(-1)A$ is denoted $-A$

Homothetic sets

A set A is *homothetic* to a set B if there is $x \in \mathbf{R}^n$ and $\lambda \neq 0$ so taht

$$A = x + \lambda B$$

If $\lambda > 0$, A is *positively homothetic* to B.

Real Translates

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Real Space

Intervals

Real Line

Real Plane

Integral Line

Real Order

Geometry

Real Numbers

Rational Order

Lists

Rational Numbers

Set Numbers

Direct Products

Integer Arithmetic

Natural Fractions

Finite Sets

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Equivalent Sets

Family Unions and Intersections

Identity Elements

Natural Arithmetic

Natural Powers

Integer Sums

Integer Order

Comparisons

Function Inverses

Families

Operations

Natural Products

Integer Numbers

Orders

Function Images

Function Composites

Natural Sums

Equivalence Relations

Converse Relations

Functions

Natural Order

Recursion Theorem

Relations

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Set Symmetric Differences

Partitions

Successor Sets

Intersection of Empty Set

Unordered Triples

Set Dualities

Set Unions and Intersections

Set Complements

Set Intersections

Pair Unions

Ordered Pairs

Set Differences

Set Unions

Pair Intersections

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects