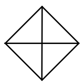

Why

[1]

1 Definition

Define $S \in \mathbf{R}^{d\times d}$ by

$$S = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

For a vector $x \in \mathbf{R}^d$ the *down shift* of x is Sx.

Let $A \in \mathbf{R}^{d\times d}$ be a matrix with columsn $a_1, \dots, a_d$. A is a *circulant matrix* if $a_1 = Sa_d$, $a2 = Sa_2$, and $a_i = A_{i-1}$ for $i = 2, \dots, d$.

Example

For example, the matrix

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

is a circulant matrix.

[1] Future sheets will include. These matrices arise in practice and each has the same eigenvectors.

Characterization

A matrix $C \in \mathsf{R}^{d\times d}$ is circulant if and only if there exists $c_0, \ldots, c_{d-1}$ so that

$$C = c_0 I + c_1 P + c_2 P^2 + \cdots + c_{n-1} P^{n-1}.$$

Properties

The sum and product of any two circulant matrix is circulant. In other words, the circulant matrices with the usual matrix addition and multiplication form a commutative ring.


Circulant Matrices
Real Matrices
Matrices
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Plane
Real Line
Real Space
Integral Line
Real Order
Geometry
Real Numbers
Fields
Rational Numbers
Groups
Sequences
Integer Arithmetic
Integer Additive Inverses
Set Numbers
Natural Additive Identity
Natural Multiplicative Identity
Inverse Elements
Integer Sums
Direct Products
Finite Sets
Identity Elements
Arithmetic
Element Functions
Equivalent Sets
Algebras
Natural Powers
Integer Products
Family Unions and Intersections
Operations
Function Inverses
Natural Products
Integer Numbers
Families
Equivalence Relations
Function Composites
Function Images
Natural Sums
Natural Order
Functions
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects