Undirected Graphs

1 Why

We want to visualize symmetric relations.

2 Definition

An *undirected graph* is a finite nonempty set and a set of (unordered) pairs of its elements. We call the elements of the first set the *vertices* of the graph and the elements of the second set the *edges*.

Two vertices are *adjacent* if their pair is an edge in the set. We say that the corresponding edge is *incident* to those vertices. The *adjacency set* of a vertex is the set of vertices adjacent to it. The *degree* of a vertex is the number of vertices adjacent to it; in other words, the size of its adjacency set. A graph is *complete* if each pair of two distinct vertices is adjacent. A *clique* is a maximal complete subgraph.

2.1 Other Terminology

Some authors define a clique as any set of vertices whose corresponding subgraph is complete; we prefer the term *complete subgraph* here. Some authors call the adjacency set the *neighborhood* of the vertex and call the union of the adjacency set of a vertex with the singleton of that same vertex the *closed neighborhood* of the vertex.

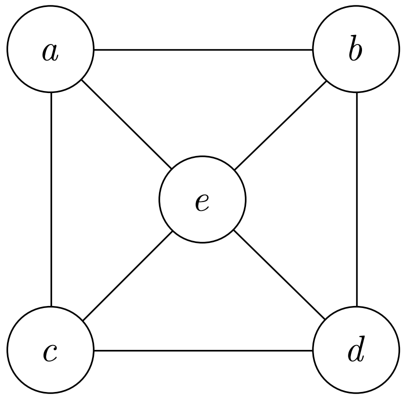

Figure 1: Undirected graph.

2.2 Notation

Let V be a non-empty set and let E be a subset of $\{\{u, v\} \mid u, v \in V\}$. As usual, we denote the ordered pair consisting of V and E by (V, E). We say "Let $G = (V, E)$" be a graph, implying the relevant properties of V and E.

3 Example

Let $V = \{a, b, c, d, e\}$ and

$$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}\}.$$