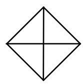


## Why

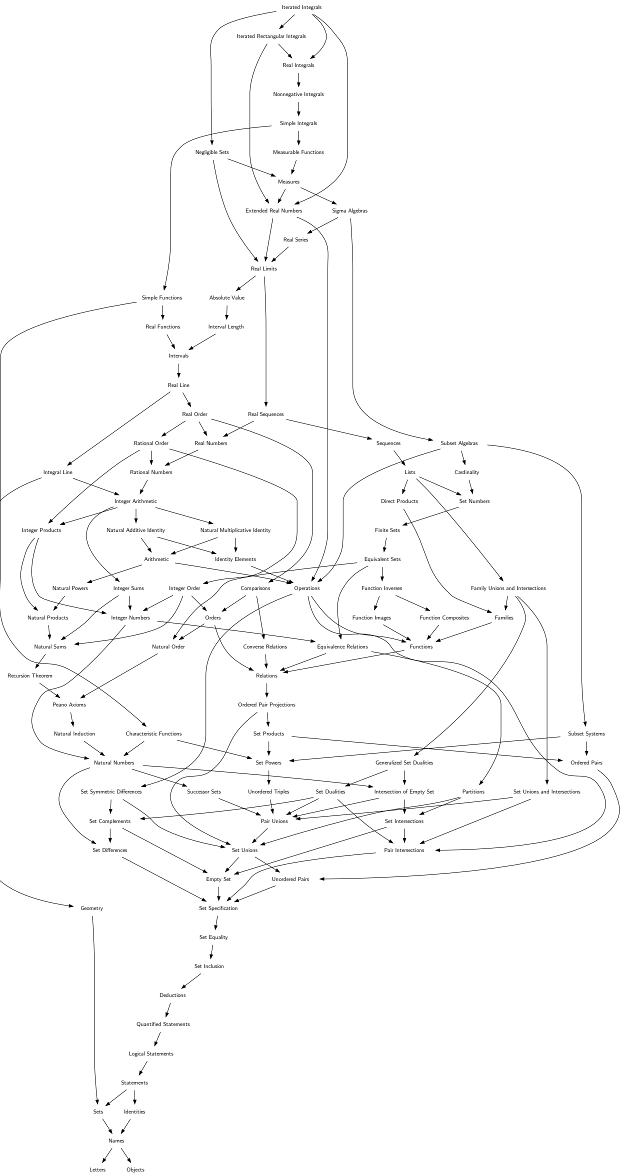
We integrate over a product space by integrating one coordinate at a time.

## Result

**Proposition 1.** *Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{A}, \nu)$ be $\sigma$-finite measurable spaces. Let $f : X \times Y \to [-\infty, \infty]$ be $\mathcal{A} \times \mathcal{B}$-measurable and $\mu \times \nu$-integrable. Then*

1. *For $\mu$-almost every $x$ in $X$ the section $f_x$ is $\nu$-integrable and for $\nu$-almost every $y$ in $Y$ the section $f^y$ is $\mu$-integrable,*
2. *the functions $I_f$ and $J_f$ defined by*
$$I_f(x) = \begin{cases} \int_Y f_x d\nu & \text{if } f_x \text{ is } \nu\text{-integrable,} \\ 0 & \text{otherwise} \end{cases}$$
*and*
$$J_f(y) = \begin{cases} \int_X f^y d\mu & \text{if } f^y \text{ is } \mu\text{-integrable,} \\ 0 & \text{otherwise} \end{cases}$$
*belong to $\mathcal{L}(X, \mathcal{A}, \mu, \mathbf{R})$ and $\mathcal{L}(Y, \mathcal{A}, \nu, \mathbf{R})$ respectively, and*
3. *the relation*
$$\int_{X \times Y} f d(\mu \times \nu) = \int_X I_f d\mu = \int_Y J_f d\nu$$
*holds.*

The above is called *Fubini's Theorem.* Next: *Tonelli's theorem.*


Iterated Integrals
Iterated Rectangular Integrals
Real Integrals
Nonnegative Integrals
Simple Integrals
Negligible Sets
Measurable Functions
Measures
Extended Real Numbers
Sigma Algebras
Real Series
Real Limits
Simple Functions
Absolute Value
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Sequences
Subset Algebras
Integral Line
Rational Numbers
Lists
Cardinality
Integer Arithmetic
Direct Products
Set Numbers
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Finite Sets
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Function Images
Function Composites
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Natural Numbers
Set Powers
Generalized Set Dualities
Ordered Pairs
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects