Why

We want language for all of the elements of a first set being the elements of a second set.

Definition

Denote a set by A and a set by B. If every element of the set denoted by A is an element of the set denoted by B, then we say that the set denoted by A is a *subset* of the set denoted by B.

We say that the set denoted by A is *included* in the set denoted by B. We say that the set denoted by B is a *superset* of the set denoted by A or that the set denoted by B *includes* the set denoted by A.

Every set is included in and includes itself.

Notation

Let A denote a set and B denote a set. We denote that the set A is included in the set B by $A \subset B$. In other words, $A \subset B$ means $(\forall x)((x \in A) \longrightarrow (x \in B))$. We read the notation $A \subset B$ aloud as "A is included in B" or "A subset B". Or we write $B \supset A$, and read it aloud "B includes A" or "B superset A". $B \supset A$ also means $(\forall x)((x \in A) \longrightarrow (x \in B))$.

Properties

There are some properties that our intuition suggests inclusion should have. First, every set should include itself. We describe this fact by saying that inclusion is *reflexive*.

Proposition 1 (Reflexive)**.** *Every set is included in itself.*

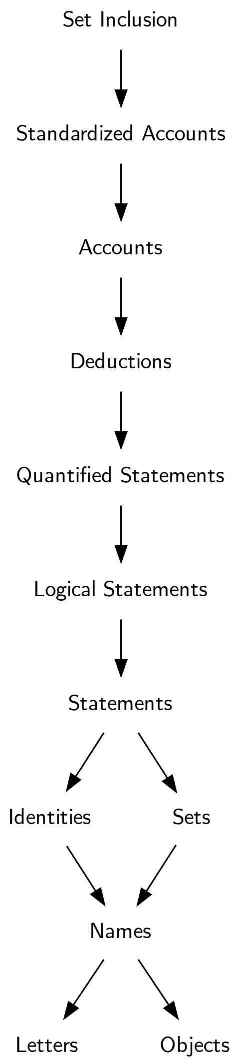
Proof. `(1) name` A; `(2) have` $(\forall x)(x \in A \longrightarrow x \in A)$; `(3) thus` $A \subset A$ `by SetInclusion:Definition`. □

Next, we expect that if one set is included in another, This fact is described by saying that inclusion is *transitive*

Proposition 2 (Transitive)**.** *If a set is included in another, and the latter in yet another, then the first is included in the last.*

Proof. `(1) name` A, B, C; `(2) have` $A \subset B$ `(3) have` $B \subset C$ `(4) thus` $A \subset C$ `by` modus ponens. □

Equality (=) shares these two properties. Let A denote an object. Then $A = A$. Let B and C also denote objects. If $A = B$ and $B = C$, then $A = C$. Of course, inclusion is not symmetric.. Belonging ($\in$) may be, but need not be reflexive and transitive.


Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects