

GREATEST LOWER BOUNDS

Definition

Suppose (A, \leq) is a partially ordered set. A lower bound for $B \subset A$ is an element $a \in A$ satisfying

$$a \le b$$
 for all $b \in B$

In words, a is a predecessor of every element of B. A set is bounded from below if it has a lower bound. A greatest lower bound for B is an element $c \in A$ so that c is a lower bound and c < a for all other lower bounds a.

Proposition 1. If there is a greatest lower bound it is unique.¹

We call the unique greatest lower bound of a set (if it exists) the *infimum*.

Notation

We denote the infimum of a set $B \subset A$ by inf A.

¹Proof in future editions.

