

LINEAR EQUATIONS

Why

Linear equations are ubiquitous.

Definition

Given $a \in \mathbb{R}^n$ and $y \in \mathbb{R}$, suppose we want to find $x \in \mathbb{R}^n$ satisfying

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = y.$$

We refer to this expression as a real linear equation or linear equation. We treat each component $x_i \in \mathbf{R}$ as a variable and we treat each component $a_i \in \mathbf{R}$ and $y \in \mathbf{R}$ as constants. We call the pair (a, y) the real linear equation constants.

The source of the terminology "linear" is by viewing the left hand side as a function. Define $f: \mathbf{R}^n \to \mathbf{R}$ by $f(x) = \sum_i a_i x_i$. We want to find $x \in \mathbf{R}^n$ to satisfy f(x) = b. Notice that f is a linear real function.²

Moreover, to each linear function $f: \mathbf{R}^d \to \mathbf{R}$ there exists a vector $a \in \mathbf{R}^d$ so that $f(x) = \sum_i a_i x_i$. For this reason, if we are given several linear function f_1, \ldots, f_m , then we can think of these as several vectors a^1, \ldots, a^n . If we are also given $b_i \in \mathbf{R}$ for each $i = 1, \ldots, m$, then we have the vector $b \in \mathbf{R}^m$

We can define the two-dimensional array $A \in \mathbf{R}^{m \times n}$ by $A_{ij} = a_j^i$. For this reason, a *linear system of equations* is a pair (A, b). A solution of a linear system of equations is a vector $x \in \mathbf{R}^n$ satisfying the equations

$$A_{11}x_{1} + A_{12}x_{2} + \cdots + A_{1n}x_{n} = b_{1}$$

$$A_{21}x_{1} + A_{22}x_{2} + \cdots + A_{2n}x_{n} = b_{2}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$A_{m1}x_{1} + A_{m2}x_{2} + \cdots + A_{mn}x_{n} = b_{n}$$

Other terminology includes a system of linear equations or linear system or simultaneous linear equations

¹Future editions will clarify.

²Future editions may require a sheet here.

