



## Why

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### Definition

A *deep conditional family* is family of probabilistic generating pairs which are parameterized by a neural network (see **Parameterized Distributions and Neural Networks**). A *deep generative family* (or *deep latent variable model*, *DLVM*) is a neural network parameterized generative conditional family.

A *variational autoencoder* (or *VAE*) on *observations*  $X$  and *latents*  $Z$  is an ordered pair  $(\{(p_z^{(\theta)}, p_{x|z}^{(\theta)})\}_{\theta \in \Theta}, \{q_{z|x}^{(\phi)}\}_{\phi \in \Phi})$  whose first coordinate is a deep generative family from  $Z$  to  $X$  and whose second coordinate is deep conditional family from  $X$  to  $Z$ .

We call

with *latent distribution* (density)  $p_z : Z \rightarrow \mathbf{R}$  and *observation distribution* (density)  $p_x : X \rightarrow \mathbf{R}$  is a ordered pair  $()$  discrete (continuous) latent set  $Z$  and discrete (continuous) observation set  $X$  is a tuple

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<sup>1</sup>Future editions will include. Future editions may also change the name of this sheet. It is also likely that there will be added prerequisite sheets on variational inference.

## Parameterizing distributions

### Definition

where (a)  $\nu$  is an autoencoder (which need not be regular, see *Autoencoders*), (b)  $q_{z|x} : Z \times X \rightarrow \mathbf{R}$  is a conditional distribution (density) called the *recognition distribution* (*recognition density*), (c)  $p_z : Z \rightarrow \mathbf{R}$  is a distribution (density) called the *latent prior model*, and (d)  $p_{x|z} : X \times Z \rightarrow \mathbf{R}$  is a conditional distribution (density) called the *generating model*.

In other words, for (a)  $q_{z|x}(\cdot, \xi) : Z \rightarrow \mathbf{R}$  is a distribution (density) for each  $\xi \in X$  and for (d)  $p_{x|z}(\cdot, \zeta) : X \rightarrow \mathbf{R}$  is a distribution (density) on  $X$  for each  $\zeta \in Z$ .

If the model has discrete latent set and discrete observation set (or continuous latent set and continuous observation set), the *joint distribution* (*joint density*)  $p_{zx} : Z \times X \rightarrow \mathbf{R}$  is defined by  $p_{zx} = p_z p_{x|z}$ . The *observation distribution*

A *continuous-continuous variational autoencoder family* (*discrete-discrete*, *discrete-continuous*, *continuous-discrete*) is a tuple

$$(\nu, \{(q^{(\theta)}, p_z^{(\theta)}, p_{x|z}^{(\theta)})\}_{\theta \in \Theta}),$$

where:

- $\nu$  is an autoencoder with encoder  $f : \mathbf{R}^d \rightarrow \mathbf{R}^k$  and decoder  $g : \mathbf{R}^k \rightarrow \mathbf{R}^d$ . The autoencoder need not be regular, see *Autoencoders*.
- $\Theta \subset \mathbf{R}^p$ . The *parameter set* (or *parameter space*).

- $q^{(\theta)} : \mathbf{R}^h \rightarrow \mathbf{R}$  is a density (distribution, density, distribution), for each  $\theta \in \Theta$ . We call  $\{q^{(\theta)}\}_{\theta \in \Theta}$  the *recognition model family*.
- $p_z^\theta : \mathbf{R}^h \rightarrow \mathbf{R}$  is a density (distribution, distribution, density), for each  $\theta \in \Theta$ . We call  $\{p_z^{(\theta)}\}_\theta$  the *latent prior model family*.
- $p_{x|z}^{(\theta)} : \mathbf{R}^d \times \mathbf{R}^h \rightarrow \mathbf{R}$  is a conditional density (distribution, density, distribution). In other words,  $p_{x|z}^{(\theta)}(\cdot, \zeta) : \mathbf{R}^d \rightarrow \mathbf{R}$  is a density (distribution, density, distribution) for every  $\zeta \in \mathbf{R}^d$ . We call  $\{p_{x|z}^{(\theta)}\}_{\theta \in \Theta}$  the *observation model family*.

A *variational autoencoder* (or *VAE*) may refer to any of the above. The convention we have adopted is “latent type”-“observation type”.



