



## Why

We further drop conditions on the structure of the binary operations, and study only the algebraic structure of addition over the integers.

## Definition

A *group* is an *algebra*  $(G, \circ)$  for which  $\circ : G \times G \rightarrow G$  is associative, has an identity element in  $G$ , and has inverse elements. It is a *commutative group* (or *abelian group*) if  $\circ$  is commutative.

## Additive groups

Suppose that  $(R, +, \cdot)$  is ring. Then  $(R, +)$  is a commutative group. Conversely, suppose  $(G, +)$  is a commutative group. Define multiplication on  $S$  by  $a \cdot b = 0$  for all  $a, b \in R$ . Then  $(S, +, \cdot)$  is a ring, called the *zero ring* of  $(G, +)$ . For this reason, it is customary to write  $+$  for the operations  $\circ$  when handling commutative groups.



