Categories

1   Why

We generalize the notion of sets and functions.

2   Definition

A **category** is a collection of objects together with a set of **maps** for each ordered pair of objects. The set of maps has a binary operation called **composition**, whose induced algebra is associative and contains identities.

As the fundamental example, consider the category whose objects are sets and whose maps are functions. The sets are the objects of the category. The functions are the maps. The rule of composition is ordinary function composition. The map identities are the identity functions. We call this category the **category of sets.**

2.1   Notation

Our notation for categories is guided by our generalizing the notions of set and functions.

We denote categories with upper-case latin letters in script; for example, $\mathcal{C}$. We read $\mathcal{C}$ aloud as "script C." Upper case latin letters remind that the category is a set of objects. The script form reminds that these objects may themselves be sets.

We denote the objects of a category by upper-case latin letters, for example A, B, C; an allusion to the idea that these generalize sets. We denote the maps for an ordered pair of objects (A, B) by $A \to B$; an

allusion to the function notation. We denote members of $A \to B$ using lower case latin letters f, g, h, in allusion function notation.