



Equivalent Sets

1 Why

We want to talk about the size of a set.

2 Definition

Two sets are *equivalent* if there exists a bijection between them.

Proposition 1. *Set equivalence in the sense defined above is an equivalence relation in the power set of a set.*

Proposition 2. *Every proper subset of a natural number is equivalent to some smaller natural number.*

Proof. TODO induction

□

TODO: smaller defined?

Proposition 3. *A set can be equivalent to a proper subset of itself.*

Halmos' example here is not a bijection, though...

Proposition 4. *If n is a natural number, then n is not equivalent to a proper subset of itself.*

Proposition 5. *A set can be equivalent to at most one natural number.*

Proposition 6. *The set of natural numbers is infinite.*

Proposition 7. *A finite set is never equivalent to a proper subset of itself.*

Proposition 8. *Every subset of a finite set is finite.*

Proposition 9. *Every subset of a natural number is equivalent to a natural number.*