

Inner Product Norms

1 Why

An inner product gives rise to a norm naturally.

2 Definition

Proposition 1. Let (V, F) be a vector space. Let $f: V \times V \to \mathsf{F}$ be an inner product such that $f(x, x) \in \mathsf{R}$. Let $g: V \to \mathsf{R}$ such that

$$g(x) = \sqrt{f(x, x)}.$$

Then g is a norm.

The *norm* of a vector in an inner product space is the square root of the inner product of the vector with itself.

2.1 Notation