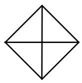


Why

The matrix of a linear system and its row reduction are related by a matrix multiplication.

Main observation

Proposition 1. *Let* $(A \in \mathsf{R}^{n \times n}, b \in \mathsf{R}^n)$ *be a linear system. Let* (C, d) *be the* ij*-reduction of* (A, b)*. There exists* $L \in \mathsf{R}^{n \times n}$ *so that*

$$C = LA \text{ and } d = Lb.$$

Proof. Define $L \in \mathsf{R}^{n \times n}$ by $L_{ii} = 0$ and

$$L_{ij} = \begin{cases} 1 & \text{if } i = j \\ -A_{kj}/A_{ij} & \text{if } k \neq i \text{ and } A_{ij} \neq 0 \\ 0 & \text{otherwise .} \end{cases}$$

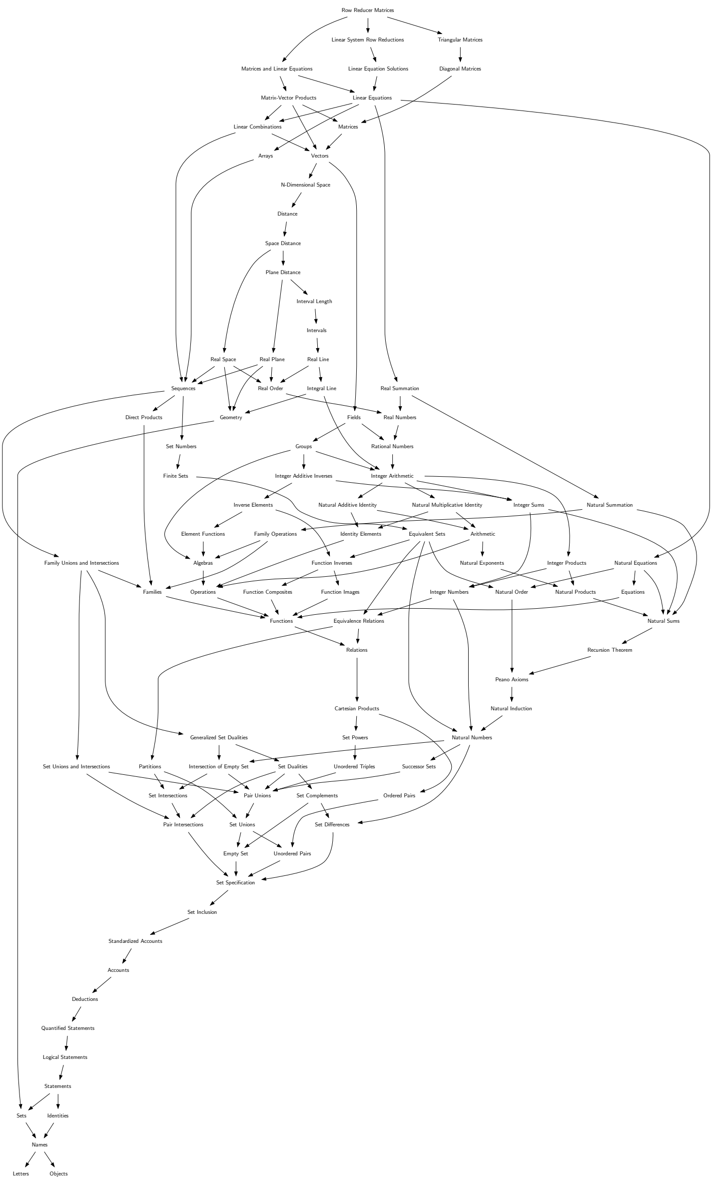
□

For this reason, we call L in Proposition 1 a *row reducer matrix* or *row reducing matrix* or *row reducer*.

A simple consequence is that there exists $(L_1, \dots, L_{n-1})$ in $\mathsf{R}^{n \times n}$ of lower triangular matrices so that the ordinary row reduction of (A, b) is $(L_{n-1} \cdots L_2 L_1 A, L_{n-1} \cdots L_2 L_1 b)$.

For example

$$L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -A_{21}/A_{11} & 1 & 0 & \cdots & 0 \\ -A_{31}/A_{11} & 0 & 1 & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & 0 \\ -A_{n1}/A_{11} & 0 & 1 & \ddots & 0 \end{bmatrix}$$



Row Reducer Matrices
Linear System Row Reductions
Triangular Matrices
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Natural Multiplicative Identity
Integer Sums
Natural Summation
Element Functions
Family Operations
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Equivalent Sets
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Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
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