

## Finite Measures

## 1 Why

Sometimes we want finite measures. TODO: which times?

## 2 Definition

A measurable set is *finite* if its measure is a real number. The measure space itself is *finite* if the base set is finite.

A measurable set is *sigma-finite* if there exists a sequence of finite measurable sets whose union is the set. The measure space itself is *sigma-finite* if the base set is sigma finite.

## 2.1 Notation

We denote that a measure space is finite by saying "Let  $(A, \mathcal{A}, \mu)$  and  $\mu(A) < +\infty$ ."

**Example 1.** Let (A, A) be a measurable space.

The counting measure on (A, A) is finite if and only if the base set is finite. It is sigma finite if and only if the base set is a union of a sequence of finite sets.

If  $A = 2^A$ , then the counting measure is sigma finite if and only if A is countable.

**Example 2.** A point mass measure is finite.

**Example 3.** Let R be the set of real numbers. The Lebesgue measure on  $(R, \mathcal{B}(R))$  is sigma finite.