

ORTHOGONAL COMPLEMENTS

Definition

The *orthogonal complement* of a subset of an inner product space is the set of vectors which are orthogonal to every vector in the subset.

Here is the definition in symbols. Given an inner product space V and a subset $U \subset V$, the orthogonal complement of V is

$$\{v \in V \mid \langle v, u \rangle = 0 \text{ for every } u \in U\}$$

Notation

We denote the orthogonal complement of U by U^{\perp} .

Examples

Complements of lines and planes in \mathbb{R}^3 . If U is a line in \mathbb{R}^3 , then U^{\perp} is the plane containing the origin that is perpendicular to U. Conversely, if U is a plane in \mathbb{R}^3 containing the origin, then U^{\perp} is the line containing the origin that is perpendicular to U.

Properties

Proposition 1. Suppose V is an inner product space. Given any subset $U \subset V$,

- 1. U^{\perp} is a subspace of V.
- 2. $\{0\}^{\perp} = V$
- 3. $V^{\perp} = \{0\}$
- 4. $U\cap U^{\perp}\subset\{0\}$
- 5. if $W \subset U$, then $W^{\perp} \subset U^{\perp}$

