



## Why

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### Definition

A neural network  $\nu$  commutes with a neural network  $\mu$  if their associated predictors commute as functions.

An *autoencoder* (or *feedforward autoencoder*) is a pair of neural networks  $((\phi_1, \dots, \phi_k), (\psi_1, \dots, \psi_\ell))$ . If the networks commute and  $\text{dom } \phi_1 = \text{dom } \psi_\ell$ , we call the autoencoder *regular*. We call the predictor of the first network the *encoder* and the predictor the second network the *decoder*. We call the image of an input to the encoder an *embedding* (or *feature vector*, *representation*, *code*).

### Compressive autoencoders

Let  $(\phi, \psi)$  be regular and let  $f : \mathbf{R}^d \rightarrow \mathbf{R}^k$  be the encoder and  $g : \mathbf{R}^k \rightarrow \mathbf{R}^d$  be the decoder. If  $k < d$ , we call the autoencoder *compressive*. Otherwise, we call the autoencoder *noncompressive*. An autoencoder is *perfect* if  $g \circ f$  is the identity function. Clearly, a compressive autoencoder can not be perfect.

Let us relax our notion of perfect by introducing a similarity function  $\ell : \mathbf{R}^d \times \mathbf{R}^d \rightarrow \mathbf{R}$  (see **Similarity Functions**). An autoencoder is optimal with respect to  $\ell$  if it minimizes

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<sup>1</sup>Future editions will include. Future editions may also change the name of this sheet.

$\int_{\mathbf{R}^d} \ell(g(f(z)), z) dz$ . This integral may diverge. Even if it converges for some autoencoders, there may not be an optimal autoencoder, or a unique one.

If we parameterize a family of autoencoders  $\{x_\theta\}_{\theta \in \Theta}$  by a compact set  $\Theta$ , ...<sup>2</sup>

It is natural to be interested in compressive autoencoders.

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<sup>2</sup>Future editions will continue.



