Why

Definition

Suppose $U_1, \ldots, U_m$ are subsets of V The *sum* of $U_1, \ldots, U_m$ is the set

$$\{u_1 + \cdots + u_m \mid u_1 \in U_1, \ldots, u_m \in U_m\}$$

For subspaces

The sum of two subspaces is a subspace. Moreover, it is the smallest subspace containing both subspaces.

Proposition 1. *Suppose $U_1, \ldots, U_m$ are subspaces of a vector space V. The $U_1 + \cdots + U_m$ is the smallest subspace containing $U_1, \ldots, U_m$*

Subspace Sums

Vector Subspaces

Real Subspace

Real Vectors Span — Translate Sets

Vectors — Real Linear Combinations

Real Vectors — Real Translates

Plane Vector Sums and Differences — Space Vectors — N-Dimensional Space

Plane Vectors

Distance

Space Distance

Plane Distance

Interval Length — Real Space

Intervals

Real Line — Real Plane

Integral Line — Real Order

Fields — Real Numbers — Rational Order — Geometry

Rings — Rational Numbers — Lists

Integer Arithmetic — Natural Fractions — Set Numbers — Direct Products

Natural Additive Identity — Natural Multiplicative Identity — Integer Products — Finite Sets

Identity Elements — Natural Arithmetic — Equivalent Sets — Function Inverses — Family Unions and Intersections

Integer Sums — Natural Powers — Integer Order — Comparison — Function Composites — Function Images — Families

Operations — Natural Products — Integer Numbers — Orders — Converse Relations — Functions

Natural Sums — Natural Order — Equivalence Relations

Recursion Theorem — Relations

Peano Axioms — Ordered Pair Projections

Natural Induction — Set Products

Natural Numbers — Set Powers — Generalized Set Dualities

Set Symmetric Differences — Partitions — Successor Sets — Intersection of Empty Set — Unordered Triples — Set Dualities — Set Unions and Intersections

Set Complements — Set Intersections — Pair Unions — Ordered Pairs

Set Differences — Set Unions — Pair Intersections

Empty Set — Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects