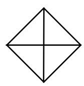


Definition

In the case of several variables, we want to do several row reductions before back-substitution.[1]

Definition

A linear system $S = (A \in \mathbf{R}^{m \times n}, b \in \mathbf{R}^m)$ is *ordinarily reducible* (or *simply reducible*) if there exists a sequence $S_1, \ldots, S_{n-1}$ of systems such that S_1 is the 11-reduction of S and S_i is the ii-reduction of S_{i-1} for $i = 1, \ldots, n-1$. In this case, we call S_{n-1} the *ordinary reduction* (or *simple reduction*) of S

Proposition 1. *The solution set of the ordinary reduction of an ordinarily reducible system is the same as that of the system.*

Proof. Let S be the first system and $S_1, \ldots, S_{n-1}$ be a sequence of reductions. Then all S_i have same solution set and S_1 has the same solution set as S. $\square$

Example

Suppose we want to find $x_1, x_2, x_3, x_4 \in \mathbf{R}$ to satisfy

$$
\begin{aligned}
2x_1 + x_2 \quad & +x_3 & &= 1, \\
4x_1 + 3x_2 \quad & +3x_3 + x_4 & &= 2, \\
8x_1 + 7x_2 \quad & +9x_3 + 5x_4 & &= 3, \text{ and} \\
6x_1 + 7x_2 \quad & +9x_3 + 8x_4 & &= 4.
\end{aligned}
$$

[1] Future editions will expand.

First we reduce by subtracting twice row 1 from row 2, four times row 1 from row 3, and three times row 1 from row 4.

$$S_1 = (\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}).$$

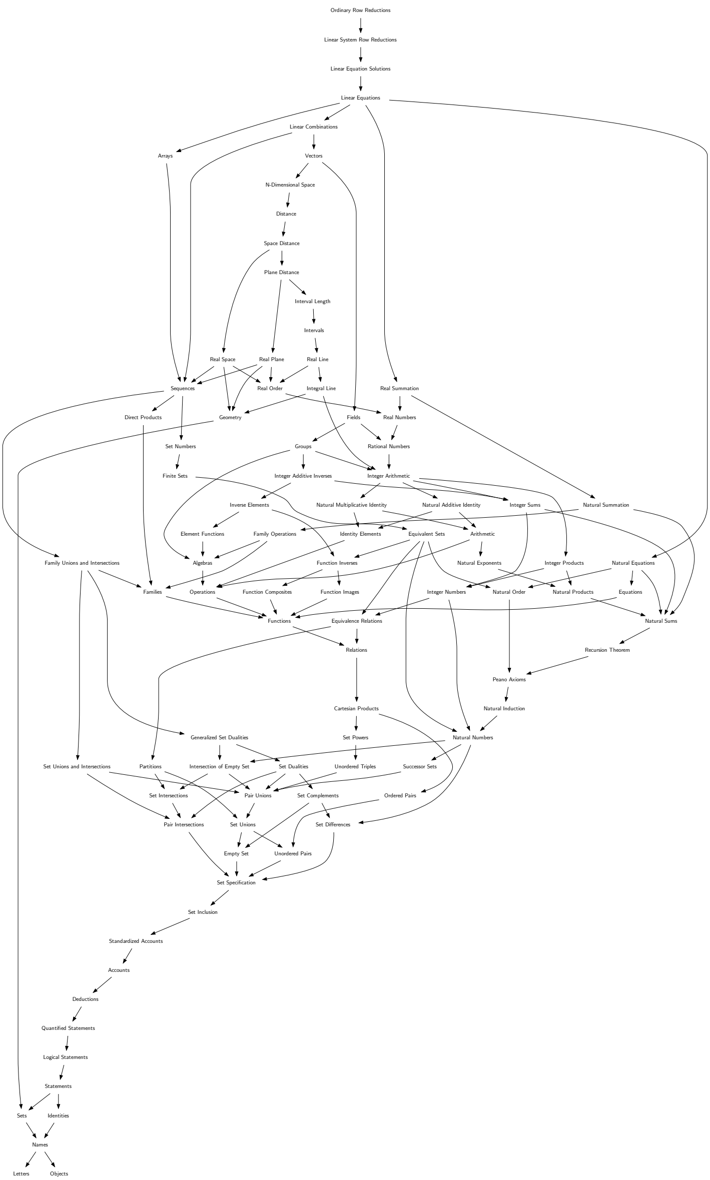
We then subtract three times row 2 from row 3 and four times row 2 from row 4 to obtain

$$S_2 = (\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}).$$

Finally, we subtract two times row 3 from row 4 to obtain S_4, which we can write as

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 1, \\ x_2 + x_3 + x_4 &= 0, \\ 2x_3 + 2x_4 &= -1, \quad \text{and} \\ 2x_4 &= 3. \end{aligned}$$

We can now back-substitute to find $x_4 = 3/2$, $x_3 = -2$, $x_2 = 1/2$ and $x_1 = 5/4$. Proposition 1 says that this is the only solution of S, as well.


Ordinary Row Reductions
Linear System Row Reductions
Linear Equation Solutions
Linear Equations
Linear Combinations
Arrays
Vectors
N-Dimensional Space
Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Space
Real Plane
Real Line
Sequences
Real Order
Integral Line
Real Summation
Direct Products
Geometry
Fields
Real Numbers
Set Numbers
Groups
Rational Numbers
Finite Sets
Integer Additive Inverses
Integer Arithmetic
Inverse Elements
Natural Multiplicative Identity
Natural Additive Identity
Integer Sums
Natural Summation
Element Functions
Family Operations
Identity Elements
Equivalent Sets
Arithmetic
Family Unions and Intersections
Algebras
Function Inverses
Natural Exponents
Integer Products
Natural Equations
Families
Operations
Function Composites
Function Images
Integer Numbers
Natural Order
Natural Products
Equations
Functions
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Peano Axioms
Cartesian Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects