Why

Certain sparse matrices are easier to work with, especially those with chordal sparsity patterns.[1]

Definition

A *sparsity pattern* E of *order* n is a set of (unordered) pairs of $V = \{1, \ldots, n\}$. A sparsity pattern is *chordal* if the undirected graph (V, E) is chordal.

A symmetric matrix is said to *have a sparsity pattern* if its ijth entry is zero whenever $\{i, j\}$ is not in the sparsity pattern. The diagonal entries and off-diagonal entries for pairs appearing in the sparsity pattern may or may not be zero.

The graph whose vertices are one through n and whose edge set is the sparsity patter is called the *sparsity graph*

A sparsity pattern is not a property of a matrix because it is not unique (unless all off-diagonal entries are non-zero). If a matrix has a particular sparsity pattern it has every sparsity pattern which is a superset of it. In other words, every matrix has the sparsity pattern which is the set of all pairs of integers.

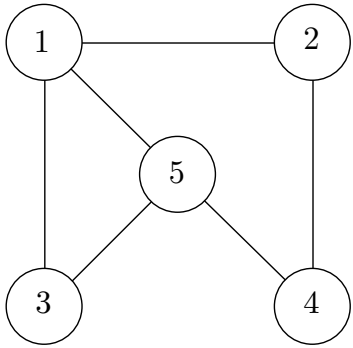
Notation

Let $E \subset \{\{i, j\} \mid i, j \in \{1, 2, \ldots, n\}\}$. A symmetric matrix $A \in \mathbf{S}^n$ is said to have sparsity pattern E if $A_{ij} = A_{ji} = 0$ wheneer $i \neq j$ and $\{i, j\} \notin E$. The graph $G = (V, E)$ where $V = \{1, 2, \ldots, n\}$ is the sparsity graph associated with E.

We will denote the symmetric matrices of order n with sparsity pattern E by $\mathbf{S}^n_E$.

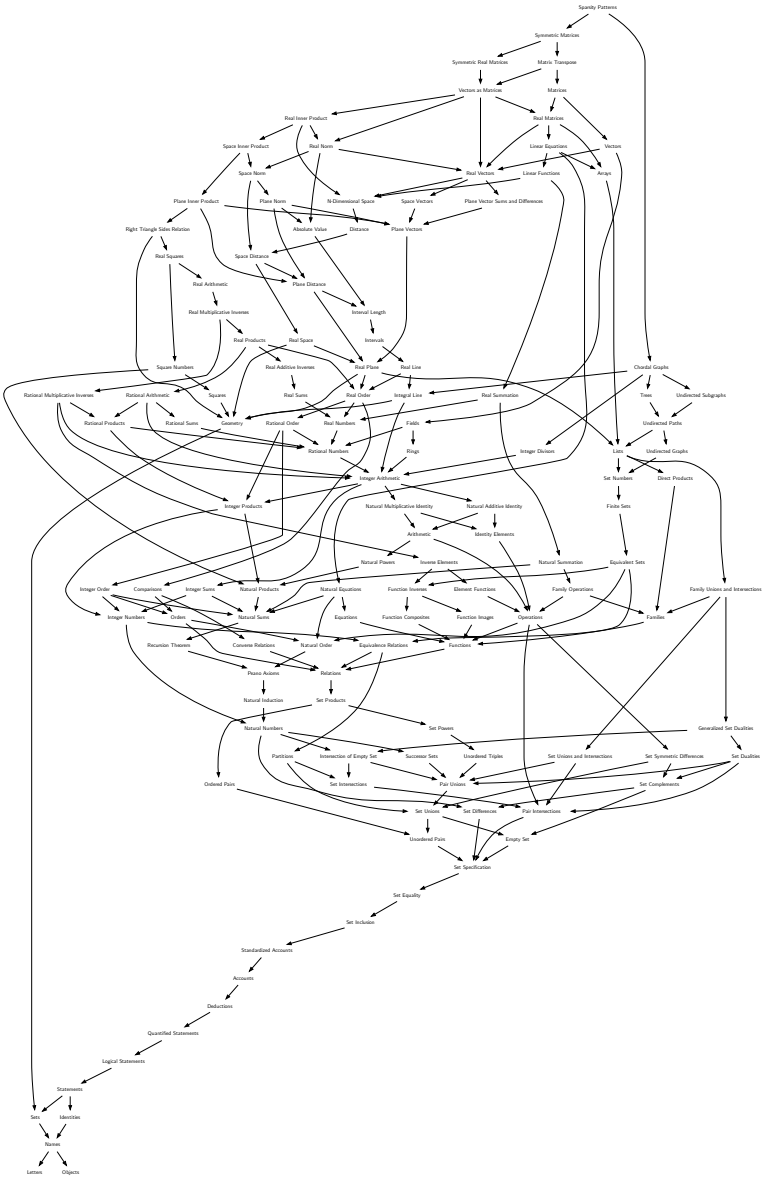
[1] Future editions will expand.

Example



The figure above shows a sparsity graph for the matrix

$$A := \begin{bmatrix} A_{11} & A_{21} & A_{31} & 0 & A_{51} \\ A_{21} & A_{22} & 0 & A_{42} & 0 \\ A_{31} & 0 & A_{33} & 0 & A_{54} \\ 0 & A_{42} & 0 & A_{44} & A_{45} \\ A_{51} & 0 & A_{53} & A_{54} & A_{55} \end{bmatrix}.$$


Sparsity Patterns
Symmetric Matrices
Symmetric Real Matrices
Matrix Transpose
Vectors as Matrices
Matrices
Real Inner Product
Real Matrices
Space Inner Product
Real Norm
Linear Equations
Vectors
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Interval Length
Intervals
Real Products
Real Space
Square Numbers
Real Additive Inverses
Real Plane
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Integer Products
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Family Unions and Intersections
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Set Powers
Generalized Set Dualities
Partitions
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Unions and Intersections
Set Symmetric Differences
Set Dualities
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
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Names
Lemas
Objects