



## Why

Can a set have no elements?

## Definition

Sure. A set exists by the principle of existence (see **Sets**); denote it by  $A$ . Specify elements (see **Set Specification**) of any set that exists using the universally false statement  $x \neq x$ . We denote that set by  $\{x \in A \mid x \neq x\}$ . It has no elements. In other words,  $(\forall x)(x \notin A)$ . The principle of extension (see **Set Equality**) says that the set obtained is unique (contradiction).<sup>1</sup> We call the unique set with no elements the *empty set*.

## Notation

We denote the empty set by  $\emptyset$ . In other words, in all future accounts (see **Accounts**), there are two implicit lines. First, “**name**  $\emptyset$ ” and second “**have**  $(\forall x)(x \notin \emptyset)$ ”.

## Properties

It is immediate from our definition of the empty set and of the definition of inclusion (see **Set Inclusion**) that the empty set is included in every set (including itself).

**Proposition 1.**  $(\forall A)(\emptyset \subset A)$

*Proof.* Suppose toward contradiction that  $\emptyset \not\subset A$ . Then there exists  $y \in \emptyset$  such that  $y \notin A$ . But this is impossible, since  $(\forall x)(x \notin \emptyset)$ .  $\square$

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<sup>1</sup>This account will be expanded in the next edition.

