Why

Can permuting the rows or columns of a matrix be represented by matrix multiplication?

Definition

Let $\sigma : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ be a permutation of n. The *permutation matrix* of σ is the matrix P defined by $P_{ij} = 1$ if $\sigma(i) = j$ and 0 otherwise. This is sometimes called the *column representation* (in contrast to the *row representation*, in which $P_{ij} = 1$ if $\sigma(j) = i$.

Let $A \in \mathbf{R}^{n \times n}$. Then pre-multipying A by P permutes the rows of A. In other words PA has the same rows as A but permuted according to σ. Similarly, post-multiplying by P permutes the columns of A. In other words, AP has the same columns as A but permuted according to σ. Clearly, we can also speak of permuting the components of a vector.

Composition

Let $\pi, \sigma \in S_n$ with corresponding permutation matrices P_σ and P_π. Then $P_\pi P_\sigma A$ has the same rows as A but permuted by $\pi\sigma$. Likewise, $AP_\pi P_\sigma$ has the same columns as A but permuted by $\pi\sigma$. Clearly, the identity permutation on $\{1, 2, \ldots, n\}$ is the identity $I \in \mathbf{R}^{n \times n}$.

Inverses

It is clear from the definition that $P_\sigma^{-1} = P_{\sigma^{-1}}$ and so if P is a permutation matrix then P^{-1} is $P^\top$.