## Why

We are constantly thinking of the integers as the endpoints of equal length segments of a line.

Discussion

We commonly associate elements of the integers with the endpoints of equal-length segments of a real line. Take segment S_0 of L with endpoints p and q. Associate the point p with 0. Associate the point q with 1. Take a segment S_1 of equal length, non-overlapping with S_0, who shares the endpoint q. Associate the second endpoint of this segment 2. Continue with the rest. We call the line so formed the *integral line* of unit S_0.

$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$$
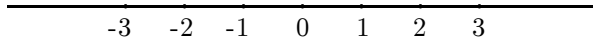
Integral Distance

Let $f : \mathbf{Z} \to \mathbf{Z}$ be defined by $f(a,b) = a - b$ if $a > b$ and $f(a,b) = b - a$ if $b > a$. Notice that f is symmetric: $f(a,b) = f(b,a)$. The (geometric) interpretation of f is the distance between the points associated with the two integers $a, b \in \mathbf{Z}$ in some integral line. We call f the *integral distance*. Notice that $f(a,b) > 0$ for all $a, b \in \mathbf{Z}$.

Notation

We denote the distance between $a, b \in \mathbf{Z}$ by $|a - b|$.

Integral Line

Integer Arithmetic

Natural Multiplicative Identity

Natural Additive Identity

Identity Elements

Arithmetic

Natural Powers

Integer Products

Natural Products

Integer Sums

Operations

Natural Sums

Integer Numbers

Functions

Recursion Theorem

Equivalence Relations

Relations

Peano Axioms

Set Products

Natural Induction

Ordered Pairs

Set Powers

Natural Numbers

Unordered Triples

Successor Sets

Intersection of Empty Set

Partitions

Set Symmetric Differences

Pair Unions

Set Intersections

Pair Intersections

Set Complements

Set Unions

Set Differences

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects