Definition

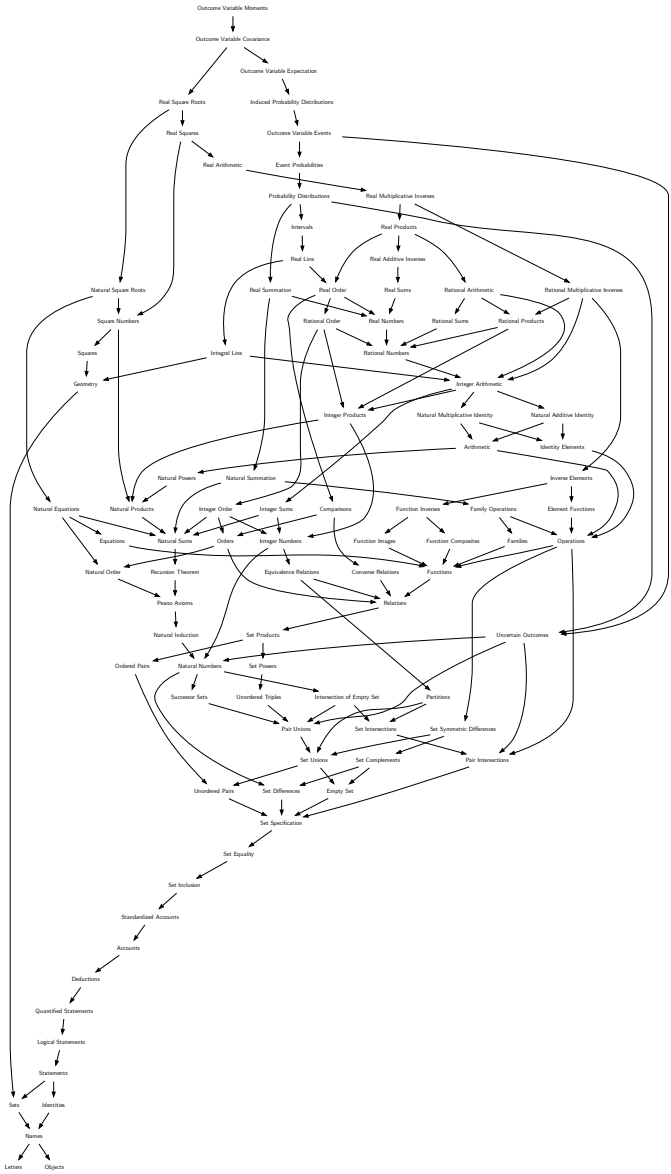
Let $x : \Omega \to \mathbf{R}$ a random variable and $p : \Omega \to \mathbf{R}$ a distribution. The *mean square* of x is $\mathbf{E}(x^2)$. Define $\mu = \mathbf{E}(x)$. We can express $\mathbf{E}(x^2) = \mathbf{E}(x)^2 + \mathrm{cov}(x)$ since

$$\begin{aligned}\mathrm{cov}(x) = \mathbf{E}((x-\mu)^2) &= \mathbf{E}(x^2 - 2\mu x + \mu^2) \\ &= \mathbf{E}(x^2) - 2\mu\mathbf{E}(x) + \mu^2 \\ &= \mathbf{E}(x^2) - \mu^2.\end{aligned}$$

We refer to this relation as the *mean-variance decomposition* of x.[1]

The *n'th moment* of x is $\mathbf{E}(x^n)$. The mean is the first moment. The covariance is the second moment minus the square of the first moment.

[1] Future editions will modify this sheet, and likely motivate this decomposition in terms of minimum square error estimation.


Outcome Variable Moments
Outcome Variable Covariance
Outcome Variable Expectation
Real Square Roots
Induced Probability Distributions
Real Squares
Outcome Variable Events
Real Arithmetic
Event Probabilities
Probability Distributions
Real Multiplicative Inverses
Intervals
Real Products
Real Line
Real Additive Inverses
Natural Square Roots
Real Summation
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Square Numbers
Rational Order
Real Numbers
Rational Sums
Rational Products
Squares
Integral Line
Rational Numbers
Geometry
Integer Arithmetic
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Arithmetic
Identity Elements
Natural Powers
Natural Summation
Inverse Elements
Natural Equations
Natural Products
Integer Order
Integer Sums
Comparisons
Function Inverses
Family Operations
Element Functions
Equations
Natural Sums
Orders
Integer Numbers
Function Images
Function Composites
Families
Operations
Natural Order
Recursion Theorem
Equivalence Relations
Converse Relations
Functions
Peano Axioms
Relations
Natural Induction
Set Products
Uncertain Outcomes
Ordered Pairs
Natural Numbers
Set Powers
Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions
Pair Unions
Set Intersections
Set Symmetric Differences
Set Unions
Set Complements
Pair Intersections
Unordered Pairs
Set Differences
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects