

## **EQUATIONS**

## Why

We name a statement which involves an identity.<sup>1</sup>

## **Definition**

An equation is any statement (see Statements) relating two terms by the relation of identity (see Identities). Some authors also call an equation an equality. The symbol "=" is called the (or an) equals sign or equals symbol.

## **Variables**

It is regularly the case that we are interested in equations relating all objects of one set to another. For example: Let X and Y be sets and let  $f: X \to Y$  and  $g: X \to Y$ . We may write the logical assertion  $(\forall x)(f(x) = g(x))$ . In this case it is understood that f and g are free names and x is a bound name (see Quantified Statements).

We will regularly, however, refer to the equation f(x) = g(x) without the quantifier  $\forall x$ . In this case, x appears free, but is not. In other words, in the statement f(x) = g(x), depending on context, x is an implicitly bound name. This usage is in slight offense to normal English usage. The name is bound (see Quantified Statements) because the use of the particular symbol x is irrelevant. We may as well have used the symbol y, and so the name is bound to the quantifier  $\forall$ .

 $<sup>^{1}\</sup>mathrm{Future}$  editions will expand on this statement.

In a second sense, however, the choice of name is "free", and the name is meant as a placeholder (see Names). For these reasons, we introduce terminology for this common case. We are discussing the equation

$$f(x) = g(x)$$
.

The name x we call a variable. It is a placeholder name, which is bound in the quantified statement. But the particular choice of name is irrelevant. In contrast to the variable x, the names f and g we call constant. We refer to them as the contstants in the equation. They are fixed, and not meant to vary.

