



## Event Probabilities

### 1 Why

If we have some outcomes and a probability mass function, we can construct a function which assigns probabilities to events.

### 2 Definition

The *probability* of an event is the sum of the probabilities of the outcomes in the event. The *event probability function* is the correspondence assigning events to their probabilities.

### 3 Notation

Let  $A$  be a set of outcomes and  $p$  a probability mass function on  $A$ . Let  $B \subset A$  be an event. Let  $\mathbf{P} : 2^A \rightarrow R$  be the event probability function:

$$\mathbf{P}(B) = \sum_{b \in B} p(b).$$

Notice that  $\mathbf{P}$  depends on the set of outcomes  $A$  and the probability mass function  $p$ . Sometimes we will include this when denoting  $\mathbf{P}$  and denote it by  $\mathbf{P}_{A,p}$ .

### 4 Properties

**Proposition 1.** *Let  $\mathbf{P}$  be the event probability function of a probability mass function  $p$  on a set of outcomes  $A$ . then*

1.  $\mathbf{P}(B) \geq 0$  for all  $B \subset A$
2.  $\mathbf{P}(A) = 1$
3.  $\mathbf{P}(B \cup C) = \mathbf{P}(B) + \mathbf{P}(C)$  for  $B, C \subset A$  and  $B \cap C = \emptyset$