Why

We want to define area under a real function. We begin with functions whose area under the curve is self-evident.

Definition

Consider a measure space. The characteristic function of any measurable set is measurable. A simple function is measurable if and only if each element of its simple partition is measurable.

The *integral* of a measurable non-negative simple function is the sum of the products of the measure of each piece with the value of the function on that piece. For example, the integral of a measurable characteristic function of a subset is the measure of that subset.

The *integral operator* is the real-valued function which associates each measurable non-negative simple function with its integral. The simple integral is non-negative, so the integral operator is a non-negative function.

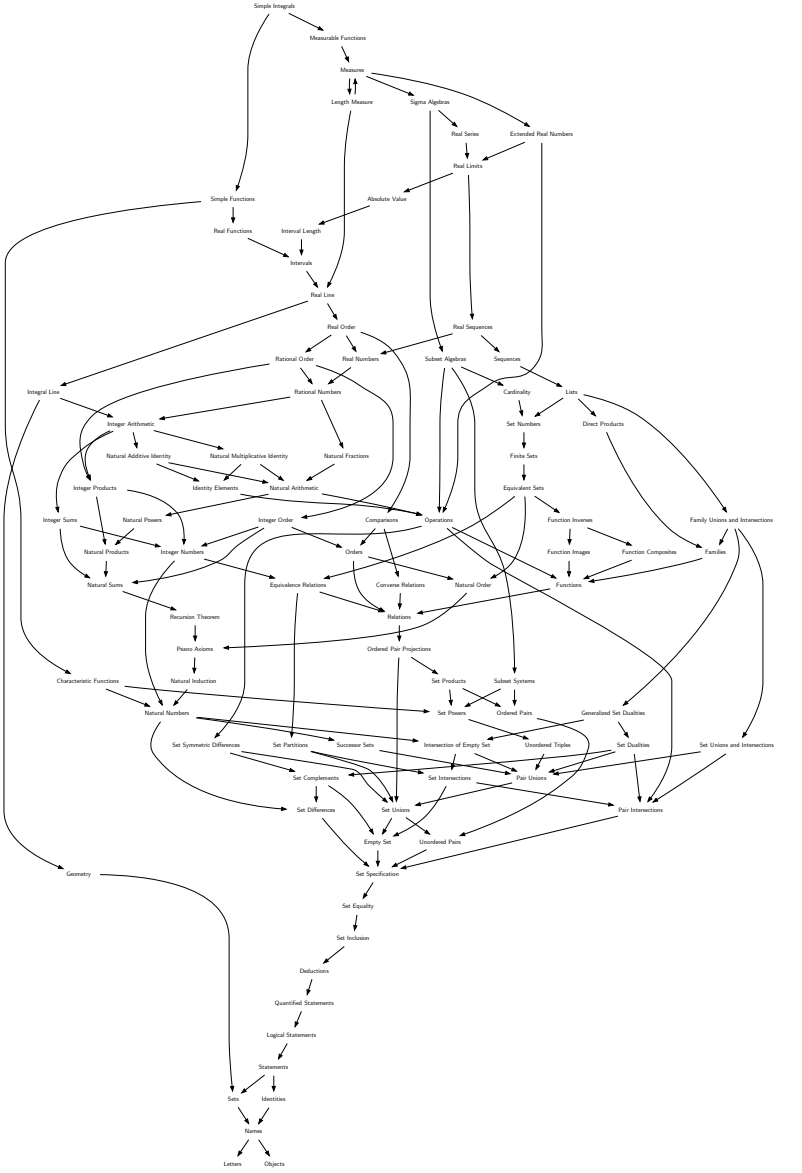
Notation

Suppose $(X, \mathcal{A}, \mu)$ is a measure space and $f : X \to \mathbf{R}$ is a measurable simple function; i.e., there are $A_1, \ldots, A_n \in \mathcal{A}$ and $a_1, \ldots, a_n \in \mathbf{R}$ with:

$$f = \sum_{i=1}^{n} a_i \chi_{A_i}.$$

We denote the integral of f with respect to measure μ by $\int f d\mu$, which we defined as

$$\int f d\mu := \sum_{i=1}^{n} a_i \mu(A_i).$$


Simple Integrals
Measurable Functions
Measures
Length Measure
Sigma Algebras
Real Series
Extended Real Numbers
Real Limits
Simple Functions
Absolute Value
Real Functions
Interval Length
Intervals
Real Line
Real Order
Real Sequences
Rational Order
Real Numbers
Subset Algebras
Sequences
Integral Line
Rational Numbers
Cardinality
Lists
Integer Arithmetic
Set Numbers
Direct Products
Natural Additive Identity
Natural Multiplicative Identity
Natural Fractions
Finite Sets
Integer Products
Identity Elements
Natural Arithmetic
Equivalent Sets
Integer Sums
Natural Powers
Integer Order
Comparisons
Operations
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Order
Function Images
Function Composites
Families
Natural Sums
Equivalence Relations
Converse Relations
Natural Order
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Characteristic Functions
Natural Induction
Set Products
Subset Systems
Natural Numbers
Set Powers
Ordered Pairs
Generalized Set Dualities
Set Symmetric Differences
Set Partitions
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Dualities
Set Unions and Intersections
Set Complements
Set Intersections
Pair Unions
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects