

REAL FUNCTION EPIGRAPHS

Definition

Let $f: D \to \mathbf{R}$ be a multivariate real-valued function where $D \subset \mathbf{R}^d$. The graph of f is the set in \mathbf{R}^{d+1} defined by

$$\{(x, f(x)) \in \mathbf{R}^d \times \mathbf{R} \mid x \in D\}.$$

The epigraph of f is the set in \mathbb{R}^{d+1} defined by

$$\{(x,t) \in \mathbf{R}^d \mid x \in D \text{ and } f(x) \le t\}.$$

Notation

We denote the epigraph of a function f by epi f.

Connecting convex sets and convex functions

A function $f: \mathbf{R}^d \to \mathbf{R}$ is convex if and only if epi f is a convex set.

Note on terminology

The prefix "epi" is Greek, meaning "above".

