Why

We want to build vectors out of other vectors using scalar multiplication and vector addition.

Definition

A *linear combination from* a $\mathbf{R}^n$ is an ordered pair: the first coordinate is a sequence of vectors and the second is a sequence in $\mathbf{R}$ (which we call the *scalars*). The *result* of a linear combination is the sum of the results of scaling each vector by the corresponding scalar in the sequence; itself a vector in the space.

A *trivial linear combination* is one whose sequence of scalars is zero at each coordinate. The result of any trivial linear combination is the zero vector. A *nontrivial linear combination* is one which is not trivial. In other words, to be nontrivial, there must exist at least one index of the scalar sequence whose corresponding value is nonzero.

We say that a given vector *can be written as a linear combination of* a sequence of vectors if there exists a sequence of scalars such that the result of the linear combination of that sequence of vectors and scalars is the given vector. In other words, a vector can be written as a linear combination of some other vectors if there exists scalars for those other vectors such that scaling them and adding the results gives the vector.

Notation

Let $v = (v_1, \ldots, v_n)$ be a sequence of vectors in $\mathbf{R}^n$ and $a = (a_1, \ldots, a_n)$ be a sequence of scalars in $\mathbf{R}$. Then (v, a) is a linear

combination and we can express its result by

$$a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

If (v, a) is trivial, then $a_i = 0$ for $i \in \{1, 2, \ldots, n\}$ and the result of (v, a) is 0 (the zero vector). Otherwise, there exists $i \in \{1, 2, \ldots, n\}$ such that $a_i \neq 0$; of course, the result of (v, a) may still be 0. A vector u can be written as a linear combination of the vectors $v_1, v_2, \ldots, v_n$ if there exists scalars $a_1, a_2, \ldots, a_n$ so that

$$u = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n.$$

We often number (see Lists) a finite set of vectors. In this case, we say "Let $\{u_1, \ldots, u_n\}$ be a (finite) set of vectors."

Real Linear Combinations

Real Vectors

N-Dimensional Space Space Vectors Plane Vector Sums and Differences

Distance Plane Vectors

Space Distance

Plane Distance

Interval Length

Real Space Intervals

Real Plane Real Line

Real Order Integral Line

Geometry Rational Order Real Numbers

Rational Numbers

Lists Integer Arithmetic

Set Numbers Natural Additive Identity Natural Multiplicative Identity Integer Products

Finite Sets Identity Elements Arithmetic

Direct Products Equivalent Sets Natural Powers

Family Unions and Intersections Function Inverses Comparisons Integer Order Integer Sums Natural Products

Families Function Composites Function Images Operations Orders Integer Numbers Natural Sums

Functions Converse Relations Equivalence Relations Natural Order Recursion Theorem

Relations Peano Axioms

Set Products Natural Induction

Generalized Set Dualities Set Powers Natural Numbers

Set Unions and Intersections Set Dualities Unordered Triples Successor Sets Intersection of Empty Set Partitions Set Symmetric Differences

Ordered Pairs Pair Unions Set Intersections Set Complements

Pair Intersections Set Unions Set Differences

Unordered Pairs Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects