Why

We have a space X and a family of probability measures $\mathcal{P}$ on this space. Assume $x \in X$ drawn from a fixed, unknown measure $P \in \mathcal{P}$. Given x, how should we guess P?

Definition

A *probabilistic model* (or *statistical model*, *parametric statistical model*, *statistical experiment*) is a family of probability measures over the same measurable space $(X, \mathcal{F})$. Call the index set the *parameter set* or *set of parameters*. The set X is called the *sample space*. A *statistic* is any function on the sample space.

Notation

Let $(X, \mathcal{F})$ denote a measurable space. We usually denote the parameter by Θ, and denote the family

$$\mathcal{P} = \{\mathbf{P}_\theta : \mathcal{F} \to [0,1] \mid \mathbf{P}_\theta \text{ a measure}, \theta \in \Theta\}.$$

Often $\Theta \subset \mathbf{R}^d$.

Example: coin flips

The usual model for n flips of a coin takes $X = \{0,1\}^n$, the set of binary n-tuples. For $\theta \in [0,1]$, a distribution $p_\theta(x) = \theta^t(1-\theta)^{n-t}$ where $t = t(x) = x_1 + \cdots + x_n$ is defined on X. A probability measure $\mathbf{P}_\theta$ is defined on $\mathcal{P}(X)$ in the the usual way. Thus, the probabilistic model is $\{\mathbf{P}_\theta \mid \theta \in [0,1]\}$. Given x, we are asked to guess θ.

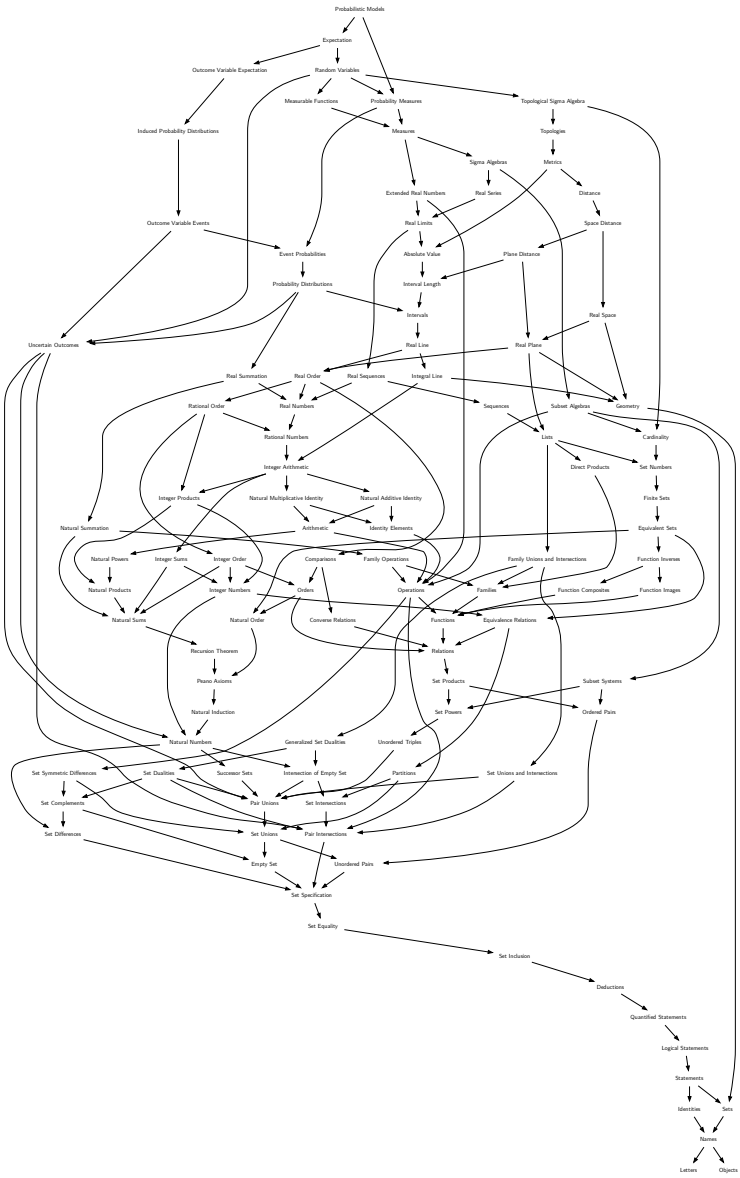
Decisions

A *decision procedure* (*estimator*, *statistical procedure*) is a measurable function $A : \mathcal{X} \to \mathcal{A}$ where $\mathcal{A}$ is a a set, called the *actions* or *decisions*. Often $\mathcal{A} = \Theta$, in which case $A(x)$ givens an *estimate* of θ, which we denote $\hat{\theta}(x)$.

Judging decisions

Given a *loss function* $L : \mathcal{A} \times \Theta \to \bar{\mathbf{R}}$, the *risk* of A is

$$R(A, \theta) = \mathbf{E} L(A(x), \theta).$$


Probabilistic Models
Expectation
Outcome Variable Expectation
Random Variables
Measurable Functions
Probability Measures
Topological Sigma Algebra
Induced Probability Distributions
Measures
Topologies
Sigma Algebras
Metrics
Extended Real Numbers
Real Series
Distance
Real Limits
Space Distance
Outcome Variable Events
Absolute Value
Event Probabilities
Plane Distance
Interval Length
Probability Distributions
Intervals
Real Space
Uncertain Outcomes
Real Line
Real Plane
Real Summation
Real Order
Real Sequences
Integral Line
Rational Order
Real Numbers
Sequences
Subset Algebras
Geometry
Rational Numbers
Lists
Cardinality
Integer Arithmetic
Direct Products
Set Numbers
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Natural Summation
Arithmetic
Identity Elements
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Family Operations
Family Unions and Intersections
Function Inverses
Natural Products
Integer Numbers
Orders
Operations
Families
Function Compositions
Function Images
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Natural Order
Converse Relations
Functions
Equivalence Relations
Recursion Theorem
Relations
Peano Axioms
Set Products
Subset Systems
Natural Induction
Set Powers
Ordered Pairs
Natural Numbers
Generalized Set Dualities
Unordered Triples
Set Symmetric Differences
Set Dualities
Successor Sets
Intersection of Empty Set
Partitions
Set Unions and Intersections
Pair Unions
Set Intersections
Set Complements
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects