Singular Measures

1 Why

TODO

2 Definition

A measure is *concentrated* on a set if the measure on the complement of the set is zero. A signed or complex measure is *concentrated* on a set if its variation is concentrated on the set.

Two measures (or signed or complex measures) are *mutually singular* if there exists a set on which one is concentrated and on whose complement the other is concentrated.

2.1 Notation

Let $(X, \mathcal{A})$ be a measurable space and let μ be a measure. Then μ is concentrated on a set $C \in \mathcal{A}$ if $\mu(X - C) = 0$. If ν is a signed or complex measure, then ν is concentrated on $C \in \mathcal{A}$ if $|\nu|$ is concentrated on C; in which case $|\nu|(X - C) = 0$.

Let μ and ν be measures on $(X, \mathcal{A})$. Then μ and ν are mutually singular if there exists a set $A \in \mathcal{A}$ so that μ is concentrated on A and ν is concentrated on $X - A$. We denote that two measures are singular by $\mu \perp \nu$, read aloud as “mu perp nu".