Real Series

1 Why

We want to sum infinitely many real numbers.

2 Definition

Let n be a natural number. The nth *partial sum* of a sequence of real numbers is the sum of first n elements of the sequence. The first partial sum is the first term of the sequence. The third partial sum is the sum of first three elements of the sequence.

The *series* of the sequence is the sequence of partial sums. The sequence is *summable* if the series converges.

Since there exist sequences which do not converge, there exist sequences which are not summable. Consider the sequence which alternates between $+1$ and -1, and starts with $+1$. Its series alternates between $+1$ and 0, and so does not converge.

2.1 Notation

Let $(a_n)_n$ be a sequence of real numnbers. For natural number n, define:

$$s_n = \sum_{k=1}^{n} a_k.$$

Then $(s_n)_n$ is the series of $(a_n)_n$. If the series converges, then there exists a real number s, the limit, and we write:

$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

We read these relations aloud as "s is the limit as n goes to infinity of s n" and "s is the limit as n goes to infinity of the sum of a k from k equals 1 to n."

To avoid referencing s_n, we write:

$$\sum_{k=1}^{\infty} a_k = s,$$

read aloud as the "the sum from 1 to infinity of a k is s." The notation is subtle, and requires justification by the algebra of series. TODO