Definition

Let A be a set and let $\leq$ be an order[1] on A.

An *upper bound* for $B \subset A$ is an element $a \in A$ so that $b \leq a$ for all $b \in B$. A set is *bounded from above* if it has a least upper bound. A *least upper bound* for B is an element $c \in A$ so that c is an upper bound and $c < a$ for all other upper bounds a.

Proposition 1. *If there is a least upper bound it is unique.*[2]

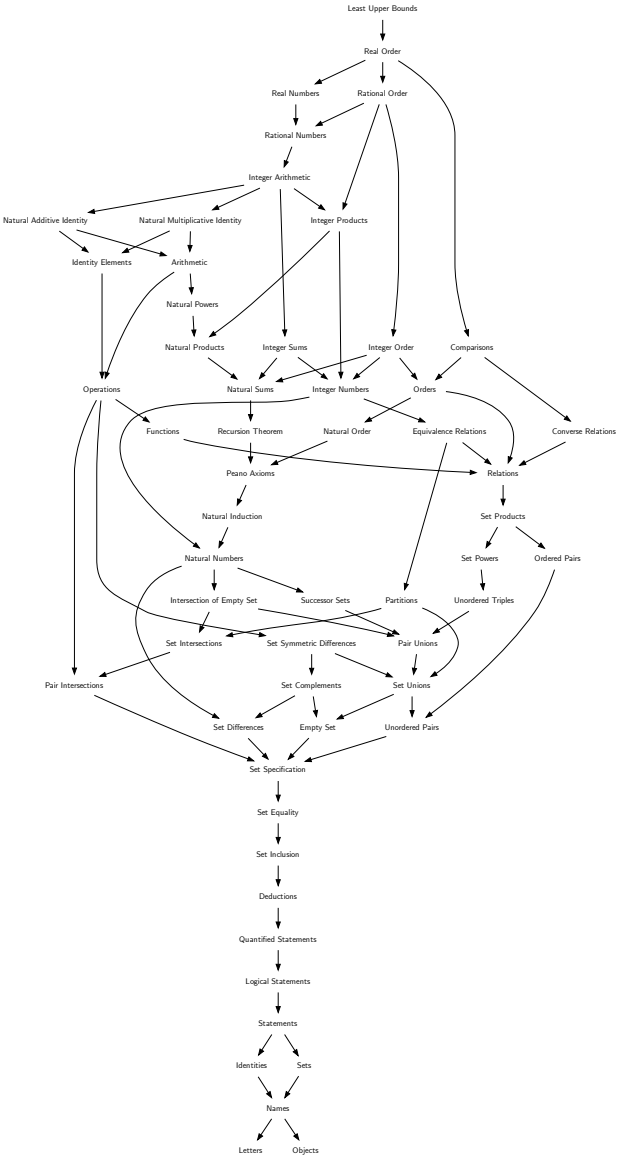
We call the unique least upper bound of a set (if it exists) the *supremum*.

Notation

We denote the supremum of a set $B \subset A$ by $\sup A$.

[1] To be defined in future editions, but understood in the usual way. See Natural Orderor Integer Orderor Rational Orderetc.

[2] Proof in future editions.


Least Upper Bounds
Real Order
Real Numbers
Rational Order
Rational Numbers
Integer Arithmetic
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Identity Elements
Arithmetic
Natural Powers
Natural Products
Integer Sums
Integer Order
Comparisons
Operations
Natural Sums
Integer Numbers
Orders
Functions
Recursion Theorem
Natural Order
Equivalence Relations
Converse Relations
Peano Axioms
Relations
Natural Induction
Set Products
Natural Numbers
Set Powers
Ordered Pairs
Intersection of Empty Set
Successor Sets
Partitions
Unordered Triples
Set Intersections
Set Symmetric Differences
Pair Unions
Pair Intersections
Set Complements
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects