Why

A simple example of an embedding.[1]

Definition

Fix $d \in \mathbf{N}$. A *polynomial feature map* of degree d is a function $\phi : \mathbf{R} \to \mathbf{R}^d$ with

$$\phi(x) = \begin{pmatrix} 1 & x^2 & \cdots & x^d \end{pmatrix}^\top.$$

For $x \in \mathbf{R}$, we call $\phi(x)$ the *polynomial embedding* of x.

A *polynomial regressor* is a least squares linear predictor using a polynomial feature embedding (of any degree, but to be precise one must specify the degree). The task of consructing a linear predictor is often referred to as *polynomial regression*.

Given a dataset of paired records $(x^1, y^1), \ldots, (x^n, y^n) \in \mathbf{R}^2$, one can construct a predictor $g : \mathbf{R} \to \mathbf{R}$ for y by embedding the dataset $(\phi(x^1), \ldots, \phi(x^n))$ and finding the least squares linear regressor $f : \mathbf{R}^d \to \mathbf{R}$ for y. One defines the predictor $g : \mathbf{R} \to \mathbf{R}$ by $g(\phi(x))$.

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[1]Future editions will expand, or perhaps collapse this sheet.