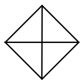


Why

We want to model uncertain outcomes in dynamical systems.[1]

Definition

Let $\mathcal{X}_0, \mathcal{X}_1, \dots, \mathcal{X}_T$ and $\mathcal{U}_0, \mathcal{U}_1, \dots, \mathcal{U}_{T-1}$ be sets. Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. Let $\mathcal{W}_0, \dots, \mathcal{W}_{T-1}$. Let $w_t : \Omega \to \mathcal{W}_t$ for $t = 0, \dots, T-1$ be random variablesk. For $t = 0, \dots, T-1$, let $f_t : \mathcal{X}_t \times \mathcal{U}_t \times \mathcal{W}_t \to \mathcal{X}_{t+1}$.

We call the sequence

$$((\mathcal{X}_t)_{t=0}^{T}), (\mathcal{U}_t)_{t=0}^{T-1}, (w_t)_{t=0}^{T-1}, (f_t)_{t=1}^{T-1})$$

a *stochastic discrete-time dynamical system*. We call w_t the *noise* variables.

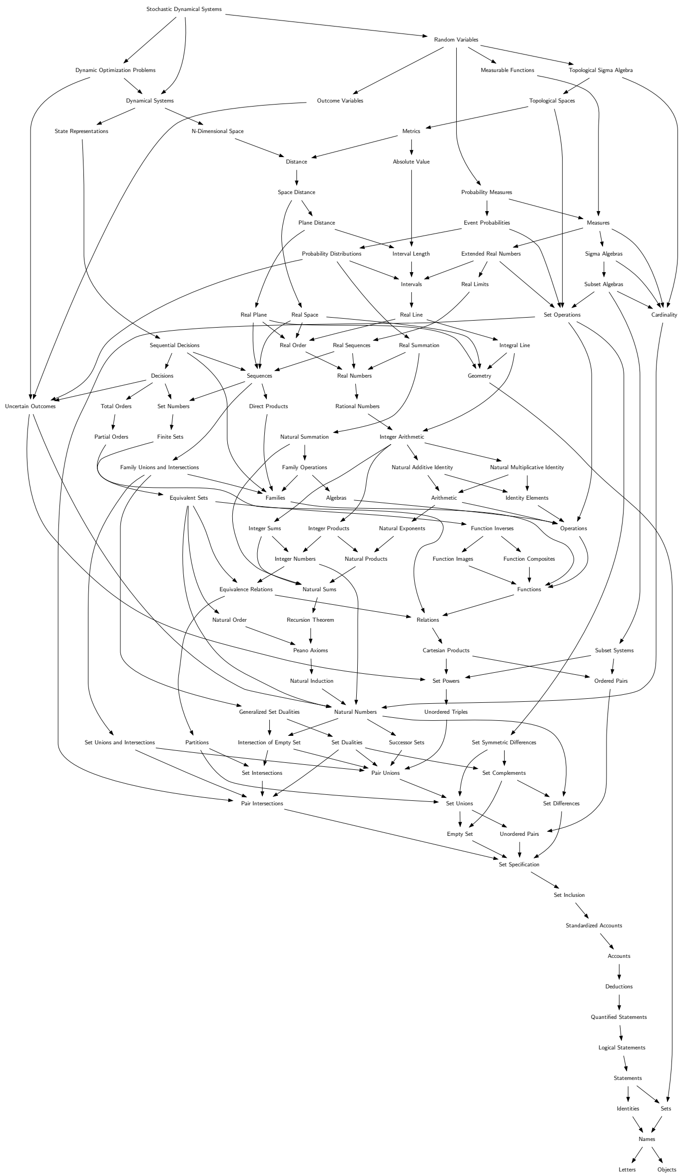
1 Problem

Let $x_0 : \Omega \to \mathcal{X}_0$ be a random variable. Define $x_1, \dots, x_T$ by

$$x_{t+1} = f_t(x_t, u_t, w_t)$$

for $t = 0, \dots, T-1$. Here we see that we are effectively modeling the state transition functions as nondeterministic. In other words, it is uncertain which state we will arrive in given our current state and action. In fact, the choice u_t only determines the distribution of x_{t+1}. Here x_0 is (still) called the *initial state* and is a random variable, usually assumed independent of the w_t.

[1] Future editions will expand.


Stochastic Dynamical Systems
Random Variables
Dynamic Optimization Problems
Measurable Functions
Topological Sigma Algebra
Dynamical Systems
Outcome Variables
Topological Spaces
State Representations
N-Dimensional Space
Metrics
Distance
Absolute Value
Space Distance
Probability Measures
Plane Distance
Event Probabilities
Measures
Probability Distributions
Interval Length
Extended Real Numbers
Sigma Algebras
Intervals
Real Limits
Subset Algebras
Real Plane
Real Space
Real Line
Set Operations
Cardinality
Sequential Decisions
Real Order
Real Sequences
Real Summation
Integral Line
Decisions
Sequences
Real Numbers
Geometry
Uncertain Outcomes
Total Orders
Set Numbers
Direct Products
Rational Numbers
Partial Orders
Finite Sets
Natural Summation
Integer Arithmetic
Family Unions and Intersections
Family Operations
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Families
Algebras
Arithmetic
Identity Elements
Integer Sums
Integer Products
Natural Exponents
Function Inverses
Operations
Integer Numbers
Natural Products
Function Images
Function Composites
Equivalence Relations
Natural Sums
Functions
Natural Order
Recursion Theorem
Relations
Peano Axioms
Cartesian Products
Subset Systems
Natural Induction
Set Powers
Ordered Pairs
Generalized Set Dualities
Natural Numbers
Unordered Triples
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects