



## Why

Linear equations are ubiquitous.

## Definition

Given  $a \in \mathbf{R}^n$  and  $y \in \mathbf{R}$ , suppose we want to find  $x \in \mathbf{R}^n$  satisfying

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = y.$$

We refer to this expression as a *real linear equation* or *linear equation*. We treat each component  $x_i \in \mathbf{R}$  as a variable and we treat each component  $a_i \in \mathbf{R}$  and  $y \in \mathbf{R}$  as constants. We call the pair  $(a, y)$  the *real linear equation constants*.<sup>1</sup>

The source of the terminology “linear” is by viewing the left hand side as a function. Define  $f : \mathbf{R}^n \rightarrow \mathbf{R}$  by  $f(x) = \sum_i a_i x_i$ . We want to find  $x \in \mathbf{R}^n$  to satisfy  $f(x) = b$ . Notice that  $f$  is a *linear* real function.<sup>2</sup>

Moreover, to each linear function  $f : \mathbf{R}^d \rightarrow \mathbf{R}$  there exists a vector  $a \in \mathbf{R}^d$  so that  $f(x) = \sum_i a_i x_i$ . For this reason, if we are given several linear function  $f_1, \dots, f_m$ , then we can think of these as several vectors  $a^1, \dots, a^n$ . If we are also given  $b_i \in \mathbf{R}$  for each  $i = 1, \dots, m$ , then we have the vector  $b \in \mathbf{R}^m$

We can define the two-dimensional array  $A \in \mathbf{R}^{m \times n}$  by  $A_{ij} = a_j^i$ . For this reason, a *linear system of equations* is a pair  $(A, b)$ . A solution of a linear system of equations is a vector  $x \in \mathbf{R}^n$  satisfying the equations

$$\begin{array}{ccccccc} A_{11}x_1 + & A_{12}x_2 + & \cdots + & A_{1n}x_n = & b_1 \\ A_{21}x_1 + & A_{22}x_2 + & \cdots + & A_{2n}x_n = & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ A_{m1}x_1 + & A_{m2}x_2 + & \cdots + & A_{mn}x_n = & b_n \end{array}$$

Other terminology includes a *system of linear equations* or *linear system* or *simultaneous linear equations*

<sup>1</sup>Future editions will clarify.

<sup>2</sup>Future editions may require a sheet here.

