



Marginal Distributions

1 Why

2 Definition

Consider a joint distribution with n components. We associate with this joint n *marginal distributions*.

For $i = 1, \dots, n$, the i th *marginal distribution* of the joint is the distribution over the i th set in the product which assigns to each element of that set the sum of probabilities of outcomes whose i th component matches that element.

For $i, j = 1, \dots, n$ and $i \neq j$, the i, j th *marginal distribution* of the joint is the distribution over the product of the i th and j th sets in the original product which assigns to each element in the product the sum of probabilities of outcomes whose i component matches the first component of the product and whose j th component matches the j th component of the product.

2.1 Notation

Let A_1, \dots, A_n be non-empty finite sets. Define $A = \prod_{i=1}^n A_i$ and let $p : A \rightarrow \mathbf{R}$ be a joint distribution.

For $i = 1, \dots, n$, define $p_i : A_i \rightarrow \mathbf{R}$ by

$$p_i(b) = \sum_{a_i=b} p(a).$$

for each $b \in A_i$. p_i is the i th marginal of p .

Similarly, for $i, j = 1, \dots, n$ and $i \neq j$ define $p_{ij} : A_i \times A_j \rightarrow \mathbf{R}$ by

$$p_{ij}(b, c) = \sum_{a_i=b, a_j=c} p(a)$$

for every $b \in A_i$ and $c \in A_j$. Then p_{ij} is the i, j th marginal of p .