



Why

We discuss learning (or inferring) relations from examples.

Definition

Let X and Y be sets. A *relation inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a relation between X and Y . We frequently use the term inductor to refer to a family of inductors, indexed by $n \in \mathbf{N}$.

An inductor is *functional* if it produces functions. In this case, we call the elements of X the *inputs* and the elements of Y the *outputs*. We call a function from inputs to outputs a *predictor* and call the result of an input under a predictor a *prediction*. Using this language, a functional inductor maps datasets to predictors. A predictor maps inputs to outputs.

To every relation between X and Y corresponds a characteristic function on $X \times Y$ and vice versa. For this reason, henceforth by *inductor* we mean a functional inductor. A relational inductor on a dataset in $(X \times Y)^n$ can be modeled by a functional inductor on a dataset in $((X \times Y) \times \{0, 1\})^n$.

Notation

Let D be a dataset of size n in $X \times Y$. Let $g : X \rightarrow Y$, a predictor, which makes prediction $g(x)$ on input $x \in X$. Let $G_n : (X \times Y)^n \rightarrow (X \times Y)$ be an inductor. Then $G_n(D)$ is the predictor which the inductor associates with dataset D . And

$\{G_n : (X \times Y)^n \rightarrow \mathcal{P}((X \times Y))\}_{n \in \mathbf{N}}$ is a family of inductors.

Consistent and complete datasets

Let $D = ((x_i, y_i))_{i=1}^n$ be a dataset and $R \subset X \times Y$ a relation. D is *consistent with* R if each $(x_i, y_i) \in R$. D is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take $R = X \times Y$). D is *functionally consistent* if it is consistent with a function; in this case, $x_i = x_j \longrightarrow y_i = y_j$. D is *functionally complete* if $\cup_i \{x_i\} = X$. If a dataset is complete, then it includes every element of the relation.

Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent variables*, *explained variables*, *postcepts*, *targets*, *outcomes*, *labels* or *observational outcomes*.

Other terms for a predictor include *input-output mapping*, *prediction rule*, *hypothesis*, *concept*, or *classifier*. Since a predictor can be used to *guess* the output of an input, some authors call an inductor (or family of inductors) a *learner* or *learning algorithm* or *supervised learning algorithm* and refer to the argument as the *training dataset*. Often the word “supervised” is included, as in *supervised learning*. The language intends to indicate that inputs are given along with outputs, and these outputs “provide supervision to the algorithm.”

