



## Why

We want to speak of the pairwise conditional distributions of a particular joint distribution.<sup>1</sup>

## Definition

Suppose  $A_1, \dots, A_n$  is a list of finite sets and  $p : \prod_{i=1}^n A_i \rightarrow [0, 1]$  is a distribution on the (finite) product  $\prod_{i=1}^n A_i$ .

For  $i \neq j \in \{1, \dots, n\}$ , the *conditional distribution* of  $i$  on  $j$  is the function  $p_{i|j} : A_i \times A_j \rightarrow \mathbf{R}$  defined so that  $p_{i|j}(\cdot, b)$  is the conditional distribution induced by conditioning on  $\{a \in \prod_{i=1}^n A_i \mid a_j = b\}$ .

For  $i, j = 1, \dots, n$  and  $i \neq j$ ,  $p_i$ ,  $p_{ij}$  and  $p_{i|j}$  satisfy

$$p_{i|j}(b, c)p_j(c) = p_{ij}(b, c) \quad \text{for all } b \in A_i, c \in A_j$$

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<sup>1</sup>Future editions will rework this sheet.



