Definition

A *decision problem* is a pair (I, Y) of *instances* I and *yes-instances* $Y \subset I$.

Examples

Subgraph isomorphism

Let $\mathcal{G}(n)$ be the set of graphs with n vertices. For $m \leq n$, define $I = \mathcal{G}(n) \times \mathcal{G}(m)$. A graph $G_1 = (V_1, E_1)$ is *subgraph-isomorphic* to a graph $G_2 = (V_2, E_2)$ if there exists $V' \subset V_1$, $E' \subset E_1$ with $|V'| = |V_2|$, $|E'| = |E_2|$, and bijection $f : V' \to V_1$ so that

$$\{u, v\} \in E_2 \longleftrightarrow \{f(u), f(v)\} \in E'.$$

Define

$$Y = \{(G_1, G_2) \in I \mid G_1 \text{ is subgraph-isomorphic to } G_2\}$$

We call (Y, I) the *subgraph-isomorphism problem*.

Traveling Salesman

Denote by $S^n \subset \mathbf{Z}^{n \times n}$ the set of symmetric integer-valued matrices. Define $I = S^n \times \mathbf{Z}$. A *n-tour* is a numbering $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ and has *cost* $C(\pi)$ with respect to D defined by

$$C(\pi) = D_{\pi(n), \pi(1)} + \sum_{j=1}^{n-1} D_{\pi(i), \pi(i+1)}$$

A tour is B-bounded if its cost is no greater than B. Define $Y = \{(D, B) \mid \text{ there is a } B - \text{bounded tour with respect to } D\}$.

Decision Problems

Undirected Graphs

Set Numbers

Finite Sets

Equivalent Sets

Natural Order   Function Inverses

Integer Numbers   Function Images   Function Composites

Equivalence Relations   Peano Axioms   Functions

Natural Induction   Relations

Set Products

Set Powers

Natural Numbers

Partitions   Intersection of Empty Set   Successor Sets   Unordered Triples

Set Intersections   Pair Unions   Ordered Pairs

Set Differences   Set Unions

Pair Intersections   Empty Set   Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects