Why

How does arithmetic preserve order?

Results

The following are standard useful results.[1]

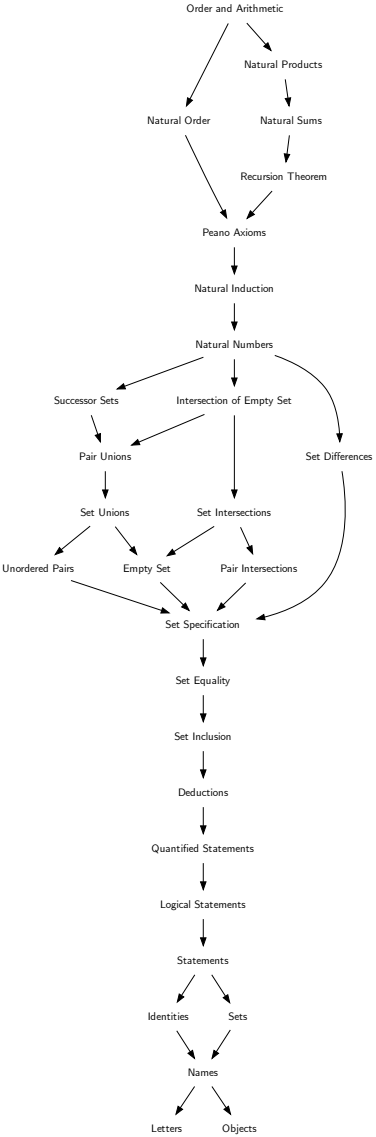
Proposition 1. *If $m < n$, then $m + k < n + k$ for all k.*

Proposition 2. *If $m < n$ and $k \neq 0$, then $m \cdot k < n \cdot k$.*

Proposition 3 (Least Element)**.** *If E is a nonempty set of natural numbers, there exists $k \in E$ such that $k \leq m$ for all $m \in E$.*

Proposition 4 (Greatest Element)**.** *If E is a nonempty set of natural numbers, there exists $k \in E$ such that $m \leq k$ for all $m \in E$.*

[1] The accounts of which will appear in future editions.


Order and Arithmetic
Natural Products
Natural Order
Natural Sums
Recursion Theorem
Peano Axioms
Natural Induction
Natural Numbers
Successor Sets
Intersection of Empty Set
Pair Unions
Set Differences
Set Unions
Set Intersections
Unordered Pairs
Empty Set
Pair Intersections
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects