Why

Linear predictors are simple and we know how to select the parameters. The main downside is that there may not be a linear relationship between inputs and outputs.

Definition

A *feature map* (or *regression function*) for outputs A is a mapping $\phi : A \to \mathbf{R}^d$. In this setting, we call $a \in A$ the *raw input record* and we call $\phi(a)$ an *embedding*, *feature embedding* or *feature vector*. We call the components of a feature vector the *features*. We call $\phi(A)$ the *regression range*.

A feature map is *faithful* if, whenever records a_i and a_j are in some sense "similar" in the set A, the embeddings $\phi(a_i)$ and $\phi(a_j)$ are close in the vector space $\mathbf{R}^d$.

Since it is common for raw input records $a \in A$ to consist of many fields, it is regular to have several feature maps ϕ_i which operate component-wise on the fields of a. These are sometimes called *basis functions*, by analogy with real function approximators (see Real Function Approximators). We concatenate these field feature maps and commonly add a constant feature 1. Since $\mathbf{R}^d$ is a vector space, it is common to refer to it in this case as the *feature space*.

Given a dataset $a = (a^1, \ldots, a^n)$ in A and a feature map $\phi : A \to \mathbf{R}^d$, the *embedded dataset* of a with respect to ϕ is the dataset $(\phi(a^1), \ldots, \phi(a^n)$ in $\mathbf{R}^d$.

Featurized consistency: a route around $X \neq \mathbf{R}^d$

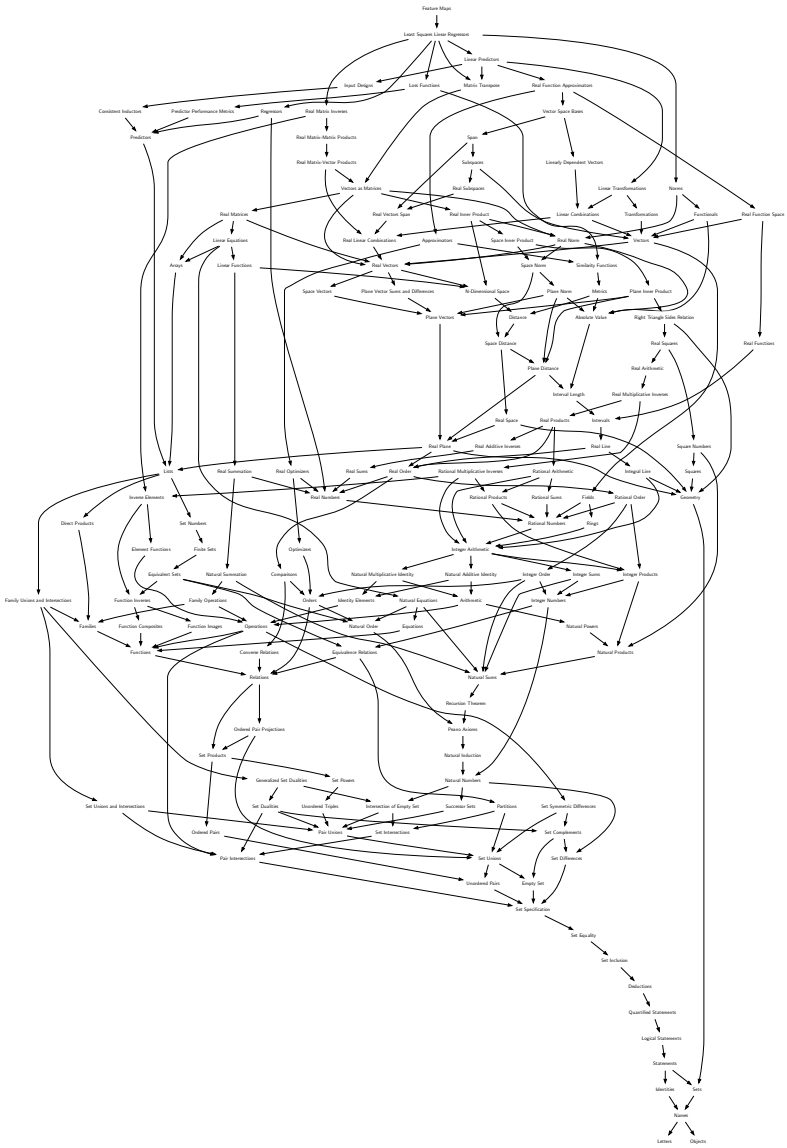
Recall that a dataset is parametrically consistent with the family $\{h_\theta : X \to Y\}_\theta$ if there exists $\theta^\star$ so that the dataset is consistent with $\theta^\star$. We saw how to pick θ if we use a linear model with a squared loss (see Least Squares Linear Regressors).

Let $\mathcal{G} = \{g_\theta : \mathbf{R}^d \to \mathbf{R}\}_\theta$. A dataset is *featurized parametrically consistent* with respect to the family $\mathcal{G}$ and the feature map $\phi : X \to \mathbf{R}^d$ if it is parametrically consistent with respect to $\mathcal{G} \circ \phi = \{g \circ \phi \mid g \in \mathcal{G}\}$.

The interpretation is that we have transformed the problem of selecting a predictor on an arbitrary space X to the problem of selecting a predictor on the space $\mathbf{R}^d$. In so doing, we can continue to use simple predictors, such as those that are linear and minimize the squared error on the dataset.[1]

In other words, we have "shifted emphasis" from the model function $h : X \to \mathbf{R}$ to the *regression function* from $\mathbf{R}^d \to \mathbf{R}$. If we know the features and the input x, then we know the *regression vector* $\phi(x)$. The *regression range* is the set $\{\phi(x) \mid x \in X\}$. In this case linearity pertains to the parameters $\theta \in \mathbf{R}^d$ instead of the inputs (or experimental conditions) $x \in X$.

[1] Future editions are likely to modify this section.


Feature Maps
Least Squares Linear Regression
Linear Prediction
Input Designs
Loss Functions
Matrix Transpose
Real Function Approximation
Consistent Induction
Predictive Performance Metrics
Regressors
Real Matrix Inverses
Vector Space Bases
Prediction
Real Matrix-Matrix Products
Span
Subspaces
Linearly Dependent Vectors
Real Matrix-Vector Products
Vectors as Matrices
Real Subspaces
Linear Transformations
Norms
Real Matrices
Real Vector Spaces
Real Inner Product
Linear Combinations
Transformations
Functionals
Real Function Space
Linear Equations
Real Linear Combinations
Approximation
Space Inner Product
Real Norms
Vectors
Arrays
Linear Functions
Real Vectors
Space Norms
Similarity Functions
Space Vectors
Plane Vector Sums and Differences
N-Dimensional Space
Plane Norms
Matrices
Plane Inner Product
Plane Vectors
Distance
Absolute Value
Right Triangle Side Relation
Space Distance
Real Squares
Real Functions
Plane Distance
Real Arithmetic
Interval Length
Real Multiplication Inverses
Real Products
Intervals
Real Space
Real Plane
Real Additive Inverses
Real List
Square Numbers
Lists
Real Summation
Real Optimizers
Real Sums
Real Order
Rational Multiplication Inverses
Rational Arithmetic
Integral List
Squares
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Real Numbers
Rational Products
Rational Sums
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Element Functions
Finite Sets
Optimizers
Equivalent Sets
Natural Summation
Comparisons
Natural Multiplication Identity
Natural Additive Identity
Integer Order
Integer Sums
Integer Products
Family Unions and Intersections
Function Inverses
Family Operations
Order
Identity Elements
Natural Exponents
Arithmetic
Integer Numbers
Functions
Function Composition
Function Images
Operations
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Exponents
Natural Powers
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Equivalence Relations
Natural Products
Relations
Natural Sums
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Natural Induction
Set Products
Generalized Set Duaities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Duaities
Unordered Triples
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deduction
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects