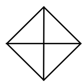


Why

What does it mean for an algorithm to be able to learn a concept?

Definition

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable input and output spaces.

A hypothesis class $\mathcal{H}$ of measurable functions from X to Y is *probably approximately correct learnable* (or *PAC learnable*) if (a) there exists an inductor $\mathcal{A} : (X \times Y)^n \to \mathcal{H}$, so that (b) for every underlying measure μ and labeling function $f : X \to Y$ (c) for every $\varepsilon, \delta \in (0, 1)$ (d) there exists $m_0 \in \mathbf{N}$ so that (e) for all $m \geq m_0$

$$\mu^m \left[\mu \left[\mathcal{A}((x_i, f(x_i))_{i=1}^n)(\xi) \neq f(\xi)\right] \leq \varepsilon\right] \geq 1 - \delta \tag{1}$$

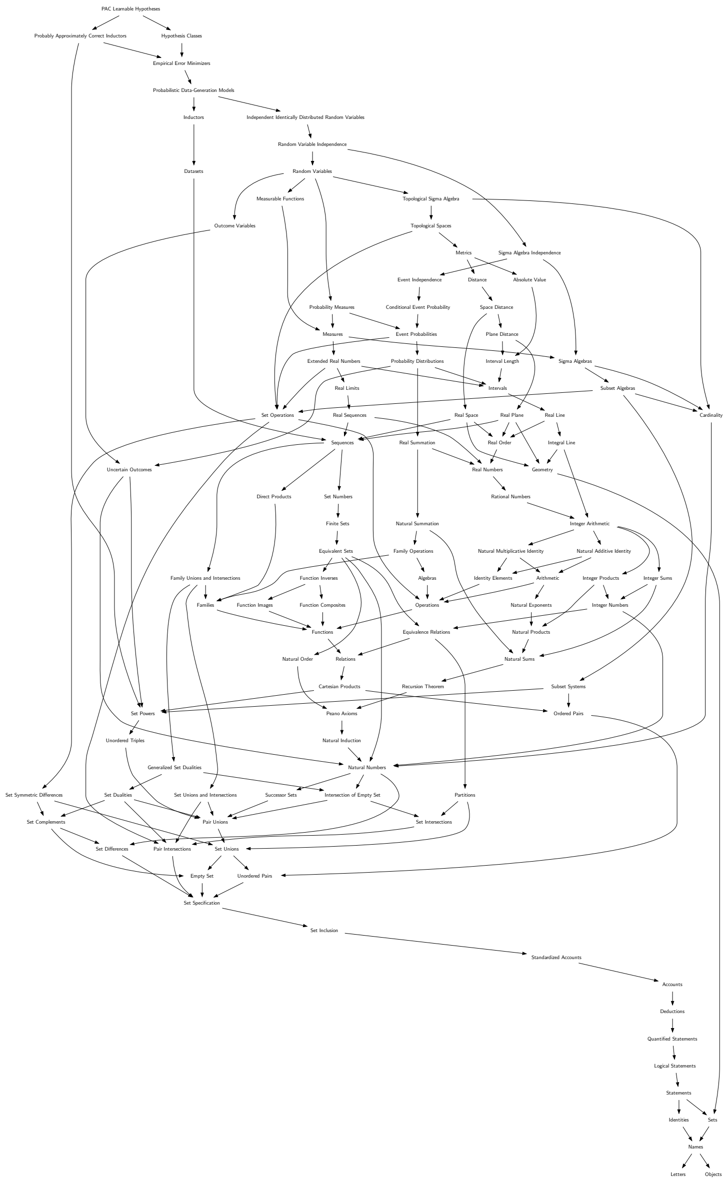
where $x \in X^m$ and $\xi \in X$. In this case we say that the inductor (or learning algorithm) $\mathcal{A}$ *PAC learns* $\mathcal{H}$.

Many authors offer much interpretation for this condition. The usual one is: "no matter the underlying distribution and correct labeling function, if someone specifies an accuracy and confidence we can tell them the number of samples they need so that the inductor outputs a hypothesis which is probably approximately correct."

Some authors require that the hypothesis class be realizable with respect to the underlying distribution and correct labeling function. In this case they refer to the above definition as the *agnostic PAC model*. We emphasize here that there this definition includes the notion of realizability. In other words. We emphasize again that this definition contains to "approximation parameters." The accuracy parameter ε corresponds to the "approximately correct" piece and the confidence parameters δ corresponds to the "probably" piece.

Sample complexity

Note that the existence of an m_0 above for each ε and δ is equivalent to requiring that there exists $m_0 : (0,1)^2 \to \mathsf{N}$ so that for all $m \geq m_0(\varepsilon, \delta)$ the condition in Equation 1 holds. There may exist multiple such m_0, so we define $\tilde{m} : (0,1)^2 \to \mathsf{N}$ so that $\tilde{m}(\varepsilon, \delta)$ is the smallest integer so that Equation (1) holds. We call $\tilde{m}$ the *sample complexity*. Clearly it is a function of ε and δ. It also depends on the hypothesis class $\mathcal{H}$ and the learning algorithm $\mathcal{A}$.


PAC Learnable Hypotheses
Probably Approximately Correct Inductors
Hypothesis Classes
Empirical Error Minimizers
Probabilistic Data-Generation Models
Inductors
Independent Identically Distributed Random Variables
Random Variable Independence
Datasets
Random Variables
Measurable Functions
Topological Sigma Algebra
Outcome Variables
Topological Spaces
Metrics
Sigma Algebra Independence
Event Independence
Distance
Absolute Value
Probability Measures
Conditional Event Probability
Space Distance
Measures
Event Probabilities
Plane Distance
Extended Real Numbers
Probability Distributions
Interval Length
Sigma Algebras
Real Limits
Intervals
Subset Algebras
Set Operations
Real Sequences
Real Space
Real Plane
Real Line
Cardinality
Sequences
Real Summation
Real Order
Integral Line
Uncertain Outcomes
Real Numbers
Geometry
Direct Products
Set Numbers
Rational Numbers
Finite Sets
Natural Summation
Integer Arithmetic
Equivalent Sets
Family Operations
Natural Multiplicative Identity
Natural Additive Identity
Family Unions and Intersections
Function Inverses
Algebras
Identity Elements
Arithmetic
Integer Products
Integer Sums
Families
Function Images
Function Composites
Operations
Natural Exponents
Integer Numbers
Functions
Equivalence Relations
Natural Products
Natural Order
Relations
Natural Sums
Cartesian Products
Recursion Theorem
Subset Systems
Set Powers
Peano Axioms
Ordered Pairs
Unordered Triples
Natural Induction
Generalized Set Dualities
Natural Numbers
Set Symmetric Differences
Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects