Why

Does a set exist which contains zero, and on and two, and three, and all the rest?

Definition

In *Successor Sets*, we said "and we continue as usual using the English language..." in our definition of zero, and one and two and three. Can this really be carried on and on? We will say yes. We will say that there exists a set which contains zero and contains the successor of each of its elements.

Principle 1 (Natural Numbers). *A set which contains 0 and contains the successor each of its elements exists.*

This principle is sometimes also called the *principle of infinity.*

We want this set to be unique. The principle says one successor set exists, but not that it is unique. To see that it is unique, notice that the intersection of a nonempty family of successor sets is a successor set.[1] Consider the intersection of the family of all successor sets. The intersection is nonempty by the principle of infinity (see *Intersection of Empty Set* for this subtlety). The axiom of extension guarantees that this intersection, which is a successor set contained in every other successor set, is unique. We summarize:

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[1] This account will be expanded in future editions.

Proposition 1. *There exists a unique successor smallest successor set.*

A *natural number* or *number* or *natural* is an element of this minimal successor set. The *set of natural numbers* or *natural numbers* or *naturals* or *numbers* is the minimal successor set.

Notation

We denote the set which exists by Proposition 1 by ω. [2] We denote the set of natural numbers without 0 by N, a mnemonic for natural. In other words $\mathsf{N} = \omega - \{0\}$. We often denote elements of ω or N by n, a mnemonic for number, or m, a letter close to n.

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[2]We use this notation to follow many authorities on the subject, and to meet the exigencies of time in producing this first edition. Future editions are likely to rework the treatment.

Natural Numbers

Set Intersections → Successor Sets

Pair Intersections → Pair Unions

Set Unions

Empty Set → Unordered Pairs

Set Specification

Set Inclusion

Standardized Accounts

Definitions → Accounts

Deductions → Quantified Statements

Logical Statements

Statements

Identities → Sets

Names

Objects → Letters