Why

Can we always construct subsets?

Definition

We will say that we can. Let A denote a set. Let s denote a statement in which A appears unbound. We assert that there is a set, denote it by A' for which belonging is equivalent to the statement denoted by s. It is a consequence of the axiom of extension that this set is unique. This assertion is sometimes called the *axiom of specification* is this assertion. We call the second set (obtained from the first) the set obtained by *specifying* elements according to the sentence.

For example:

Account 1. Example Specification

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name   A,y
thus   (∃A')((x ∈ A') ⟺ (x ≠ y)))   by   Axiom:Specifica
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Notation

Let A be a set. Let $S(a)$ be a sentence. We use the notation

$$\{a \in A \mid \; S(a) \;\}$$

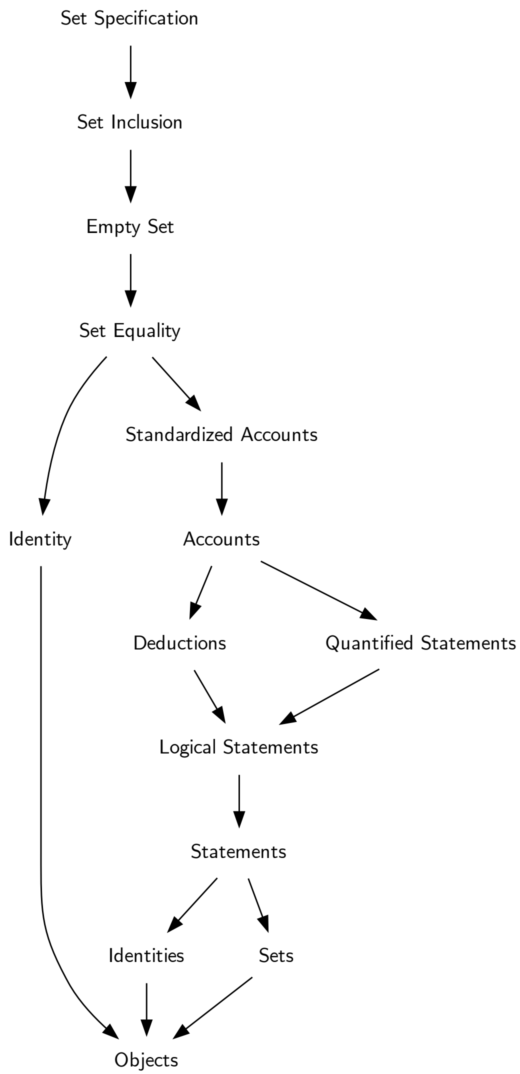
to denote the subset of A specified by S. We read the symbol $\mid$ aloud as "such that." We read the whole notation aloud as "a in A such that..."

We call the notation *set-builder notation*. Set-builder notation avoids enumerating elements. This notation is really indispensable for sets which have many members, too many to reasonably write down.

Example

For example, let a, b, c, d be distinct objects. Let $A = \{a, b, c, d\}$. Then $\{x \in A \mid x \neq a\}$ is the set $\{b, c, d\}$

Now let B be an arbitrary set. The set $\{b \in B \mid b \neq b\}$ specifies the empty set. Since the statement $b \neq b$ is false for all objects b.


Set Specification
Set Inclusion
Empty Set
Set Equality
Standardized Accounts
Identity
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Objects