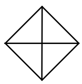


Why

We want symbols for “and”, “or”, “not”, and “implies”.

Definition

In *Statements* we discussed that nouns are names and that we will only use the present tense of the verbs “is” and “belongs”. We had statements like $a = b$ (identity) and $a \in A$ (belonging).

We call $=$ and $\in$ *relational symbols*. They say how the objects denoted by a pair of placeholder names relate to each other in the sense of being or belonging. We call $\_ = \_$ and $\_ \in \_$ *simple statements*. They denote simple sentences “the object denoted by _ is the object denoted by _” and “the object denoted by _ belongs to the set denoted by _”. We want want to assert that one make more complicated statements.

Conjunction

Consider the symbol $\land$. We will agree that it means “and”. If we want to make two simple statements like $a = b$ and $a \in A$ at once, we write write $(a = b) \land (a \in A)$. The symbol $\land$ is symmetric, reflecting the fact that a statement like $(a \in A) \land (a = b)$ means the same as $(a = b) \land (a \in A)$.

Disjunction

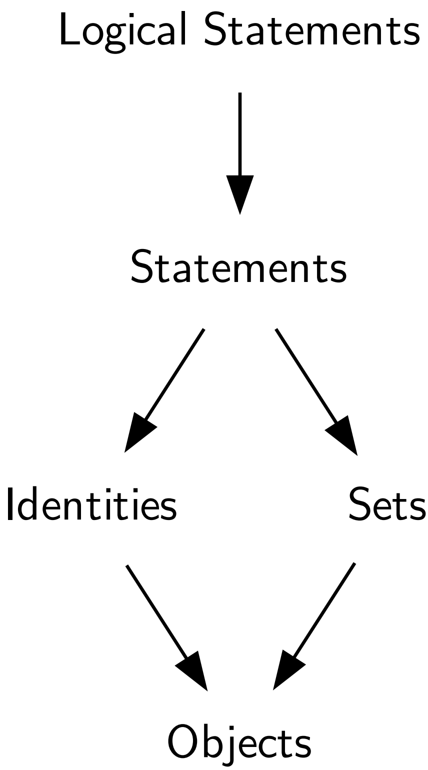
Consider the symbol $\lor$. We will agree that it means “or” in the sense of either one, the other, or both. If we want to say that possibly only one, but at least one of the simple statements like $a = b$ and $a \in A$, we write write $(a = b) \vee (a \in A)$. The symbol $\vee$ is symmetric, reflecting the fact that a statement like $(a \in A) \vee (a = b)$ means the same as $(a = b) \vee (a \in A)$.

Negation

Consider the simple $\neg$. We will agree that it means "not", in the sense of negating whatever it follows. If we want to say the opposite of a simple statement like $a = b$ we will write $\neg(a = b)$. We read it aloud as "not a is b". Of course, the more desirable english expression is "a is not b". Similarly, $\neg(a \in A)$ we read as "not, the object denoted by a belongs to the set denoted by A". Again, the more desirable english expression is something like "the object denoted by a does not belong to the set A" For these reasons, we introduce two new symbols $\neq$ and $\notin$. $a \neq b$ means $\neg(a = b)$ and $a \notin A$ means $\neg(a \in A)$.

Implication

Consider the symbol $\implies$. We will agree that it means "implies". For example $(a \in A) \implies (a \in B)$ means "the object denoted by a belongs to the object denoted by A implies the obejcted denoted by a belongs to the set denoted by B" It is the same as $(\neg(a \in A)) \vee (a \in B)$. In other words, if $a \in A$, then always $a \in B$. The symbol $\implies$ is not symmetric, since implication is not symmetric.


Logical Statements
Statements
Identities
Sets
Objects