Definition

Let $x : \Omega \to \mathbf{R}$ a random variable and $p : \Omega \to \mathbf{R}$ a distribution. The *mean square* of x is $\mathbf{E}(x^2)$. Define $\mu = \mathbf{E}(x)$. We can express $\mathbf{E}(x^2) = \mathbf{E}(x)^2 + \mathrm{cov}(x)$ since

$$\begin{aligned}\mathrm{cov}(x) = \mathbf{E}((x-\mu)^2) &= \mathbf{E}(x^2 - 2\mu x + \mu^2) \\ &= \mathbf{E}(x^2) - 2\mu\mathbf{E}(x) + \mu^2 \\ &= \mathbf{E}(x^2) - \mu^2.\end{aligned}$$

We refer to this relation as the *mean-variance decomposition* of x.[1]

The *n'th moment* of x is $\mathbf{E}(x^n)$. The mean is the first moment. The covariance is the second moment minus the square of the first moment.

[1] Future editions will modify this sheet, and likely motivate this decomposition in terms of minimum square error estimation.

Outcome Variable Moments
Outcome Variable Covariance
Outcome Variable Expectation
Induced Probability Distributions
Outcome Variable Events
Real Square Roots
Event Probabilities
Real Squares
Probability Distributions
Real Arithmetic
Outcome Probabilities
Real Multiplicative Inverses
Real Summation
Intervals
Real Products
Real Line
Real Additive Inverses
Rational Multiplicative Inverses
Real Sums
Rational Arithmetic
Real Order
Natural Square Roots
Natural Summation
Rational Products
Real Numbers
Rational Sums
Rational Order
Square Numbers
Rational Numbers
Lists
Integral Line
Squares
Direct Products
Set Numbers
Integer Arithmetic
Geometry
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Inverse Elements
Identity Elements
Arithmetic
Family Unions and Intersections
Uncertain Events
Family Operations
Function Inverses
Element Functions
Integer Sums
Natural Powers
Integer Order
Natural Equations
Comparisons
Families
Function Image
Function Composition
Operations
Integer Numbers
Natural Products
Equations
Orders
Equivalence Relations
Functions
Natural Sums
Converse Relations
Natural Order
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Uncertain Outcomes
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Duality
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersection
Set Complements
Pair Intersections
Set Union
Set Difference
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deduction
Quantified Statements
Logical Statements
Statements
Identities
Sets
Nouns
Letters
Objects