

## ROOTED TREES

## Why

We want to talk rooting a tree at a given vertex.<sup>1</sup>

## Definition

A rooted tree is an ordered pair ((V, T), r) where (V, T) is a tree and  $r \in V$  is a distinguished vertex which we call the root. We visualize rooted trees with the root at the top (see Figure 1).

## Parents and Children

Suppose w is the first vertex on the path from the root to a non-root vertex v. Since there is only one such path, w is unique and we call it the parent of v. Conversely, we call v a child of w. We denote the set of children of v by  $\mathbf{ch}(v)$ . A vertex may have no children or it may have many children. If it has no children we call it a leaf.

We define the parent function  $\mathbf{pa}: V \to V$  with the convention that the parent of the root is the root. The parent of degree k where k > 0 is  $\mathbf{pa}^k(x)$  where  $\mathbf{pa}^k$  is the composite of  $\mathbf{pa}$  with itself k times. So, in particular,  $\mathbf{pa}^{k+1}(v) = \mathbf{pa}(\mathbf{pa}^k(v))$ . We define the parent of degree 0 of v to be v, and denote it by  $\mathbf{pa}^0(v) = v$ . For the tree visualized in Figure 1,  $\mathbf{pa}(i) = g$ ,  $\mathbf{pa}^2(i) = d$ ,  $\mathbf{pa}^3(i) = a$ .

If  $w = \mathbf{pa}^k(v)$  for some  $k \geq 0$ , then w is a ancestor of v and v is a descendent of w. We use the term proper ancestor

<sup>&</sup>lt;sup>1</sup>Future editions will expand this intuition.

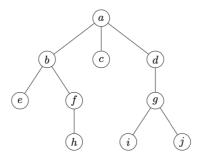


Figure 1: A rooted tree with root a.

and proper descendent if k > 0 (i.e.,  $w \neq v$ ).

The *depth* or *level* of a vertex v is its distance (see Trees) to the root. We denote the level of a vertex v by  $\mathbf{lev}(v)$ . The level of the root is 0. If  $\mathbf{lev}(v) = k > 0$ , then  $\mathbf{pa}^k(v)$  is the root. The level function  $\mathbf{lev}$  satisfies  $\mathbf{lev}(v) = \mathbf{lev}(\mathbf{pa}(v)) + 1$ .

