Set Operations

Why

We want to consider the elements of two sets together at once, and other sets created from two sets.

Definitions

Let A and B be two sets.

The *union* of A with B is the set whose elements are in either A *or* B *or* both. The key word in the definition is *or*.

The *intersection* of A with B is the set whose elements are in both A *and* B. The keyword in the definition is *and*.

Viewed as operations, both union and intersection commute; this property justifies the language "with." The intersection is a subset of A, of B, and of the union of A with B.

The *symmetric difference* of A and B is the set whose elements are in the union but not in the intersection. The symmetric difference commutes because both union and intersection commute; this property justifies the language "and." The symmetric difference is a subset of the union.

Let C be a set containing A. The *complement* of A in C is the symmetric difference of A and C. Since $A \subset C$, the union is C and the intersection is A. So the complement is the "left-over" elements of B after removing the elements of A.

We call these four operations *set-algebraic operations*.

Notation

Let A, B be sets. We denote the union of A with B by $A \cup B$, read aloud as "A union B." $\cup$ is a stylized U. We denote the intersection of A with B by $A \cap B$, read aloud as "A intersect B." We denote the symmetric difference of A and B by $A + B$, read aloud as "A symdiff B." "Delta" is a mnemonic for difference.

Let C be a set containing A. We denote the complement of A in C by $C - A$, read aloud as "C minus A."

Results

Proposition **1.** *For all sets A and B the operations $\cup$, $\cap$, and $+$ commute.*

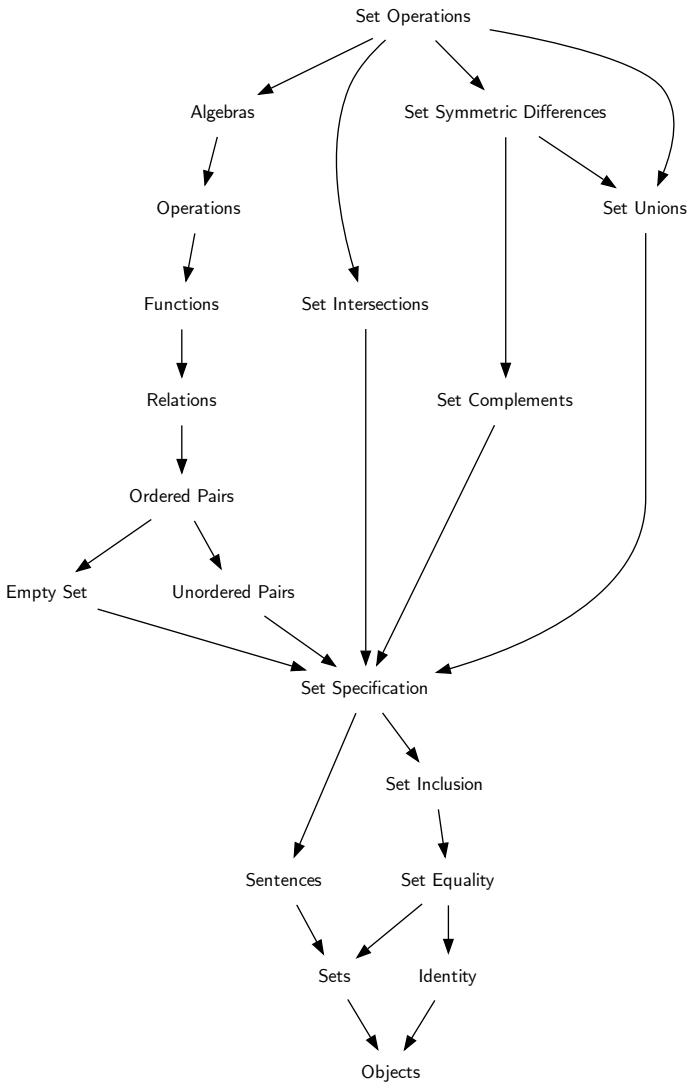
Proposition **2.** *Let S a set. For all sets $A, B \subset S$,*

$$\begin{aligned} &(1) \quad S - (A \cup B) = (S - A) \cap (S - B) \\ &(2) \quad S - (A \cap B) = (S - A) \cup (S - B). \end{aligned}$$

Proposition **3.** *Let S a set. For all sets $A, B \subset S$,*

$$A + B = (A \cup B) \cap C_S(A \cap B)$$

$\boxed{TODO : notation}$


Set Operations
Algebras
Set Symmetric Differences
Operations
Set Unions
Functions
Set Intersections
Relations
Set Complements
Ordered Pairs
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Sentences
Set Equality
Sets
Identity
Objects