



Real Intervals

1 Why

We use frequently subsets of the real numbers which correspond to segments of the line.

2 Definition

Take two real numbers, with the first less than the second.

An **interval** is one of four sets.

1. the set of real numbers larger than the first number and smaller than the second; we say the interval is **open**.
2. adf

(2) the union of (1) with the set containing only the first number
(3) the union of (1) with the set containing only the second number
(4) the union of (1) with the set containing only the first and second number. The **endpoints** of the interval are the first and second number.

The **open interval** from the first to second is the set of all real numbers larger than first and smaller than the second; set

(1) above. We identify this set with the line segment from the first number to the second number, excluding the endpoints.

The **closed interval** from the first to the second is the union of the set of all real numbers larger than the first and smaller than the second and the set containing the two numbers. The open interval from the first number to the second is a subset of the closed interval from the first to the second.

2.1 Notation

Denote the set of real numbers by R . Let $a, b \in R$ with $a < b$.

We denote the open interval from a to b by (a, b) . This notation, although standard, is the same as that for ordered pairs; no confusion arises with adequate context.

We denote the closed interval from a to b by $[a, b]$. We record the fact $(a, b) \subset [a, b]$ in our new notation.

3 Half-Closed

A **half-closed interval** from a first to second real number is one of two sets: either (1) the union of the open interval from the first to the second with the set containing only the first number, or (2) the union of the open interval from the first to the second with the set containing only the second number.

In the first case, we say the half-closed interval is **closed on the left** and **open on the right**.

In the second case, we say the half-closed interval is **open on the left** and **closed on the right**. A **half-open interval** is a half-closed interval.

3.1 Notation

Denote the set of real numbers by R . Let $a, b \in R$ with $a < b$.

We denote the half-closed interval which is closed on the left by $[a, b)$. We denote the half-closed interval which is closed on the right by $(a, b]$.

3.2 Intervals