



## Why

The inverse of a function interacts nicely with family unions, family intersections and complements.

## Results

Let  $f : X \rightarrow Y$ . Throughout this sheet, let  $f^{-1} : \mathcal{P}(Y) \rightarrow \mathcal{P}(X)$ . And take  $\{B_i\}$  to be a family of subsets of  $Y$ .<sup>1</sup>

**Proposition 1.**  $f^{-1}(\cup_i B_i) = \cup_i f^{-1}(B_i)$

**Proposition 2.**  $f^{-1}(\cap_i B_i) = \cap_i f^{-1}(B_i)$

**Proposition 3.**  $f^{-1}(Y - B) = X - f^{-1}(B)$

## Properties for function image

Notice that  $f(\cup_i A_i) = \cup_i f(A_i)$  but not for intersections. Nor is there a similar correspondence for complements. There are some relations, which we list below.<sup>2</sup>

**Proposition 4.**  $f(A \cap B) = f(A) \cap f(B)$  if and only if  $f$  is one-to-one.

**Proposition 5.** For all  $A \subset X$ ,  $f(X - A) = Y - f(A)$  if and only if  $f$  is one-to-one.

**Proposition 6.** For all  $A \subset X$ ,  $Y - f(A) \subset f(X - A)$  if and only if  $f$  is onto.

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<sup>1</sup>The proofs of the following will appear in future editions.

<sup>2</sup>Accounts of these facts will appear in future editions.



