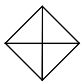


Why

A simple (albeit indirect) way to obtain a sigma algebra, is to start with some sets, and then to add all the sets needed to make the starting set closed under the various operations.

Definition

The *generated sigma algebra* for a set of subsets is the smallest sigma algbera containing the set of subsets. We must prove the existence and uniqueness of this sigma algebra.

Proposition 1. *The intersection of a non-empty set of sigma algebras over the same set is a sigma algebra.*

Proof. Given a family of sigma algebras $\{(A, \mathcal{A}_\alpha\}_{\alpha \in I}$ over some set, define $\mathcal{A} = \cap_{\alpha \in I} \mathcal{A}_\alpha$.

1. For all $\alpha \in I$, $A \in \mathcal{A}_\alpha$, thus $A \in \mathcal{A}$; condition (a).
2. For all $B \in \mathcal{A}$, for all $\alpha \in I$, $B \in \mathcal{A}_\alpha$. Thus, for all $\alpha \in I$, $C_A(B) \in \mathcal{A}_\alpha$. And so $C_A(B) \in \mathcal{A}$; condition (b).
3. For all sequences $\{B_n\} \subset \mathcal{A}$, $\{B_n\} \subset \mathcal{A}_\alpha$ for all α. Thus $\cup_n B_n \in \mathcal{A}_\alpha$ for all α and so $\cup_n B_n \in \mathcal{A}$; condition (c).

□

On the other hand, the union of a set of sigma algebras can fail to be a sigma algebra.

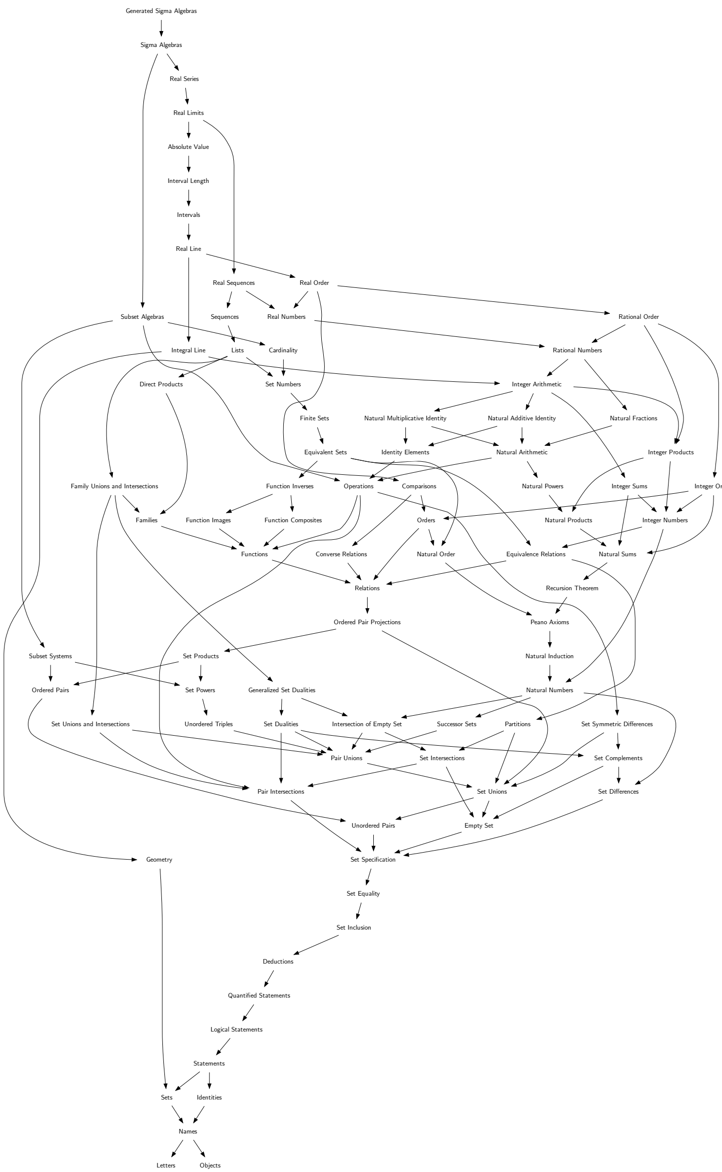
Proposition 2. *If A is a set and $\mathcal{A} \subset 2^A$, then there is a unique a smallest sigma algebra containing $\mathcal{A}$.*

Proof. We know of one sigma algebra containing $\mathcal{A}$: the power set of A. Thus, the set of sigma algebras containing $\mathcal{A}$ is not empty. Proposition **??** implies the intersection of all such sigma algebras (containing

$\mathcal{A}$) is a sigma algebra. The intersection contains $\mathcal{A}$, and is contained in all other sigma algebras with this property, so is a smallest sigma algebra containing $\mathcal{A}$. If $\mathcal{B}, \mathcal{C}$ were two smallest sigma algebras, then $\mathcal{B} \subset \mathcal{C}$ and $\mathcal{C} \subset \mathcal{B}$, but then $\mathcal{B} = \mathcal{C}$; thus the smallest sigma algebra is unique. $\square$

Notation

Let A be a set and $\mathcal{A} \subset \mathcal{P}(A)$. We denote the sigma algebra generated by $\mathcal{A}$ by $\sigma(\mathcal{A})$.



Generated Sigma Algebras
Sigma Algebras
Real Series
Real Limits
Absolute Value
Interval Length
Intervals
Real Line
Real Sequences
Real Order
Subset Algebras
Sequences
Real Numbers
Rational Order
Integral Line
Lists
Cardinality
Rational Numbers
Direct Products
Set Numbers
Integer Arithmetic
Finite Sets
Natural Multiplicative Identity
Natural Additive Identity
Natural Fractions
Equivalent Sets
Identity Elements
Natural Arithmetic
Integer Products
Family Unions and Intersections
Function Inverses
Operations
Comparisons
Natural Powers
Integer Sums
Integer Order
Families
Function Images
Function Composites
Orders
Natural Products
Integer Numbers
Functions
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Unordered Triples
Set Dualities
Intersection of Empty Set
Successor Sets
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Geometry
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects