

# **DIRECT PRODUCTS**

# Why

We generalize the product of two sets to a product of a family of sets. To do so we discuss sets of families.

## Discussion for pairs

Let A and B be sets. There is a natural correspondence between the product set  $A \times B$  (see Cartesian Products) and the set of families

$$Z = \{z : \{i, j\} \to (A \cup B) \mid z_i \in A \text{ and } z_b \in B\}.$$

The family  $z \in Z$  corresponds with the pair  $(z_i, z_i)$ . The pair (a, b) corresponds to the family  $z \in Z$  defined by z(i) = a and z(j) = b. So, ordered pairs can be put in one-to-one correspondence with families. The generalization of Cartesian products to more than two sets generalizes the notion for families.

#### Definition

Let X be a set. Let  $A: I \to X$  be a family of subsets of X. The *direct product* or *family Cartesian product* of A is the set of all families  $a: I \to X$  which satisfy  $a_i \in A_i$  for every  $i \in I$ .

A function on a product is called a function of several variables and, in particular, a function on the product  $X \times Y$  is called a function of two variables.

#### Notation

We denote the product of the family  $\{A_i\}$  by

$$\prod_{i \in I} A_i$$

We read this notation as "product over i in I of A sub-i."

### **Projections**

The word "projection" is used in two senses with families. Let I be a set, and let  $\{A_i\}$  be a family of sets. Define  $A = \prod_{i \in I} A_i$ .

First, let  $J \subset I$ . There is a natural correspondence between the elements of A and those of  $\prod_{j \in J} A_j$ . To each element  $a \in A$ , we restrict a to J and this is restriction is an element of  $\prod_{j \in J} A_j$ . The correspondence is called the *projection* of Aonto  $\prod_{i \in J} A_i$ . The projection in this sense is a set of families.

Second, consider the value of a family  $a \in A$  at j. We call  $a_j$  the projection of a onto index j or the j-coordinate of a. This word coordinate is meant to follow the language used in defining ordered pairs. The projection in this sense is an element of  $A_j$ .

