



## Why

What is the best linear regressor if we choose according to a weighted squared loss function.

## Definition

Suppose we have a paired dataset of  $n$  records with inputs in  $\mathbf{R}^d$  and outputs in  $\mathbf{R}$ . A *weighted least squares linear predictor* for nonnegative weights  $w \in \mathbf{R}^n$ ,  $w \geq 0$ , is a linear transformation  $f : \mathbf{R}^d \rightarrow \mathbf{R}$  (the field is  $\mathbf{R}$ ) which minimizes

$$\frac{1}{n} \sum_{i=1}^n w_i (y_i - x^\top a^i)^2.$$

Some authors refer to this process of selecting a linear predictor as the *weighted least-squares problem*.

Define  $W \in \mathbf{R}^{n \times n}$  so that  $W_{ii} = w_i$  and  $W_{ij} = 0$  when  $i \neq j$ . So, in particular,  $W$  is a diagonal matrix. We want to find  $x$  to minimize

$$\|W(Ax - y)\|$$

## Solution

**Proposition 1.** *There exists a unique weighted least squares linear predictor and its parameters are given by*

$$(A^\top W A^\top)^{-1} A^\top W y.$$



