Why

Linear predictors are simple and we know how to select the parameters. The main downside is that there may not be a linear relationship between inputs and outputs.

Definition

A *feature map* (or *regression function*) for outputs A is a mapping $\phi : A \to \mathbf{R}^d$. In this setting, we call $a \in A$ the *raw input record* and we call $\phi(a)$ an *embedding*, *feature embedding* or *feature vector*. We call the components of a feature vector the *features*. We call $\phi(A)$ the *regression range*.

A feature map is *faithful* if, whenever records a_i and a_j are in some sense "similar" in the set A, the embeddings $\phi(a_i)$ and $\phi(a_j)$ are close in the vector space $\mathbf{R}^d$.

Since it is common for raw input records $a \in A$ to consist of many fields, it is regular to have several feature maps ϕ_i which operate component-wise on the fields of a. These are sometimes called *basis functions*, by analogy with real function approximators (see Real Function Apppoximators). We concatenate these field feature maps and commonly add a constant feature 1. Since $\mathbf{R}^d$ is a vector space, it is common to refer to it in this case as the *feature space*.

Given a dataset $a = (a^1, \ldots, a^n)$ in A and a feature map $\phi : A \to \mathbf{R}^d$, the *embedded dataset* of a with respect to ϕ is the dataset $(\phi(a^1), \ldots, \phi(a^n)$ in $\mathbf{R}^d$.

Featurized consistency: a route around $X \neq \mathbf{R}^d$

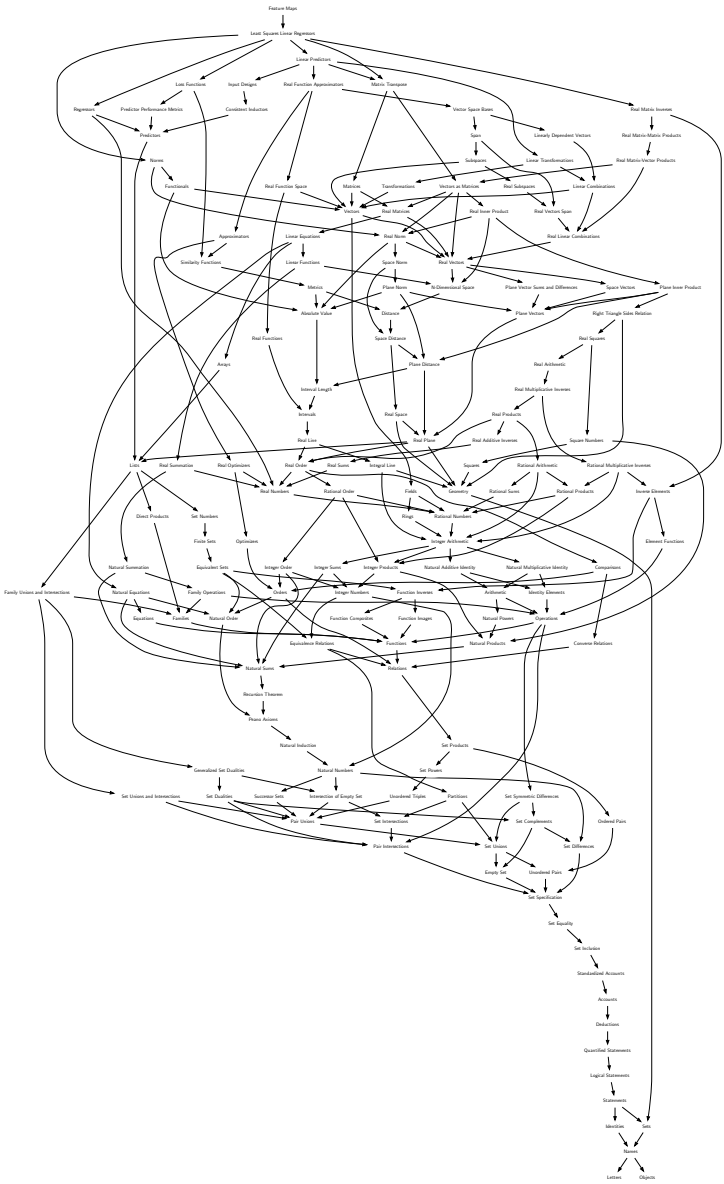
Recall that a dataset is parametrically consistent with the family $\{h_\theta : X \to Y\}_\theta$ if there exists $\theta^\star$ so that the dataset is consistent with $\theta^\star$. We saw how to pick θ if we use a linear model with a squared loss (see Least Squares Linear Regressors).

Let $\mathcal{G} = \{g_\theta : \mathbf{R}^d \to \mathbf{R}\}_\theta$. A dataset is *featurized parametrically consistent* with respect to the family $\mathcal{G}$ and the feature map $\phi : X \to \mathbf{R}^d$ if it is parametrically consistent with respect to $\mathcal{G} \circ \phi = \{g \circ \phi \mid g \in \mathcal{G}\}$.

The interpretation is that we have transformed the problem of selecting a predictor on an arbitrary space X to the problem of selecting a predictor on the space $\mathbf{R}^d$. In so doing, we can continue to use simple predictors, such as those that are linear and minimize the squared error on the dataset.[1]

In other words, we have "shifted emphasis" from the model function $h : X \to \mathbf{R}$ to the *regression function* from $\mathbf{R}^d \to \mathbf{R}$. If we know the features and the input x, then we know the *regression vector* $\phi(x)$. The *regression range* is the set $\{\phi(x) \mid x \in X\}$. In this case linearity pertains to the parameters $\theta \in \mathbf{R}^d$ instead of the inputs (or experimental conditions) $x \in X$.

[1] Future editions are likely to modify this section.


Feature Maps
Least Squares Linear Regression
Linear Predictors
Loss Functions
Input Designs
Real Function Representation
Matrix Transpose
Regression
Predictive Performance Metrics
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Vector Space Bases
Real Matrix Inverses
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Metrics
Plane Norm
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Plane Vector Sums and Differences
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Plane Inner Product
Absolute Value
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Plane Vectors
Right Triangle Sides Relation
Real Functions
Space Distance
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Arrays
Plane Distance
Real Arithmetic
Interval Length
Real Multiplication Inverses
Intervals
Real Space
Real Products
Real Line
Real Plane
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Generalized Set Dualities
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Set Unions and Intersections
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Set Unions
Set Differences
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects