



## MINIMUM MEAN SQUARED ERROR ESTIMATOR

### Why

What is the best estimator for a real-value random variable if we consider the squared loss.

### Definition

We want to estimate a random variable  $x : \Omega \rightarrow \mathbf{R}^n$  from a random variable  $y : \Omega \rightarrow \mathbf{R}^n$  using an estimator  $\phi : \mathbf{R}^n \rightarrow \mathbf{R}^n$ .

**Proposition 1.** *The mmse estimator is the conditional mean.*

Let  $x : \Omega \rightarrow \mathbf{R}^n$  and  $y : \Omega \rightarrow \mathbf{R}^m$ . A *minimum mean squared error estimator* or *MMSE estimator* or *least square estimator* for  $x$  given  $y$  is an estimator  $f : \mathbf{R}^m \rightarrow \mathbf{R}^n$  which minimizes  $\mathbb{E}\|f(y) - x\|^2$ .



