**Why**

Suppose we flip a coin until it lands heads. What is the probability that we see a head after one flip? Two flips? $n$ flips? We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

**Why**

Consider a set $A$.[1] If $A$ has $n$ elements, then a probability distribution on $A$ is $p : A \to \mathbf{R}$ where $p(a) = 1/n$. There is a natural candidate.

What if $A$ is the set of natural numbers $\mathbf{N}$. The principle difficulty is that not all sequences of real numbers $a : \mathbf{N} \to \mathbf{R}$ are summable.

A *(countable) probability distribution* on $\mathbf{N}$ is $p : \mathbf{N} \to \mathbf{R}$ where $p \geq 0$ and

$$\sum_{n=1}^{\infty} p(n) = 1.$$

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[1] Future editions will rework this sheet.

Countable Probability Distributions

Real Series

Real Limits

Cardinality

Absolute Value

Probability Distributions

Interval Length

Intervals

Real Line

Real Summation

Real Order

Real Sequences

Sequences

Real Numbers

Rational Order

Integral Line

Rational Numbers

Lists

Integer Arithmetic

Set Numbers

Natural Multiplicative Identity

Natural Additive Identity

Integer Products

Finite Sets

Direct Products

Identity Elements

Arithmetic

Equivalent Sets

Function Inverses

Natural Summation

Natural Powers

Family Operations

Natural Products

Integer Sums

Integer Order

Comparisons

Family Unions and Intersections

Function Images

Function Composites

Operations

Natural Sums

Integer Numbers

Orders

Families

Functions

Recursion Theorem

Natural Order

Converse Relations

Equivalence Relations

Peano Axioms

Relations

Natural Induction

Uncertain Outcomes

Set Products

Natural Numbers

Generalized Set Dualities

Set Powers

Set Symmetric Differences

Set Dualities

Successor Sets

Intersection of Empty Set

Unordered Triples

Set Unions and Intersections

Partitions

Set Complements

Pair Unions

Set Intersections

Ordered Pairs

Set Differences

Pair Intersections

Set Unions

Empty Set

Unordered Pairs

Geometry

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects

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