Why

How does the power set relate to an intersection?

Notation preliminaries

First, if we have a set of sets—denote it $\mathcal{C}$—and all members are subsets of a fixed set—denote it E—then the set of sets is a subset of $\mathcal{P}(E)$. In this case, we can write

$$\bigcap\{X \in \mathcal{P}(E) \mid x \in \mathcal{C}\}$$

Which is a sort of justification for the notation

$$\bigcap_{X \in \mathcal{C}} X.$$

Basic properties

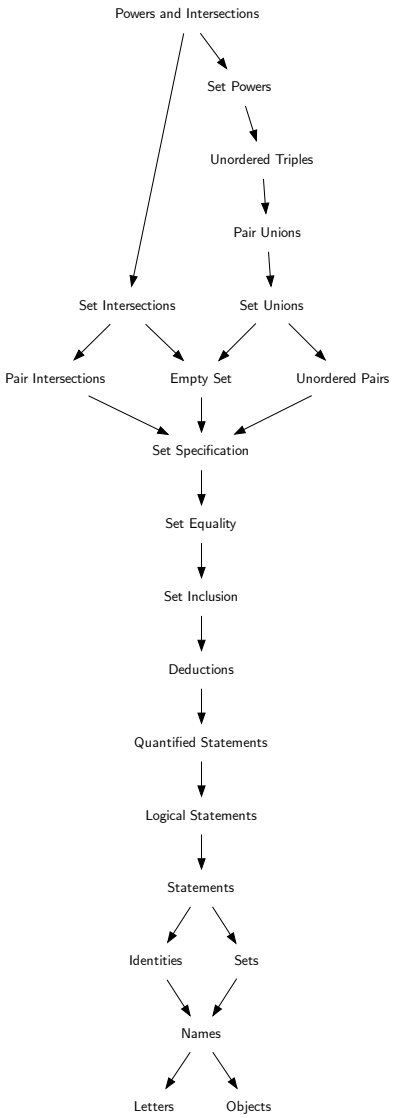
Here are some basic interactions between the powerset and intersections.[1]

Proposition 1. $\mathcal{P}(A) \cap \mathcal{P}(F) = \mathcal{P}((A \cap F))$

Proposition 2. $\bigcap_{X \in \mathcal{A}} \mathcal{P}(A) = \mathcal{P}((\bigcap_{X \in \mathcal{A}} A))$

Proposition 3. $\bigcap_{X \in \mathcal{P}(E)} X = \varnothing$

[1] Future editions will expand on these propositions and provide accounts of them.


Powers and Intersections
Set Powers
Unordered Triples
Pair Unions
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects