



## Why

We want to model natural phenomena.<sup>1</sup>

## Definition

Let  $X_0, X_1, \dots, X_T$  be a sequence of sets and let  $f_t : X_t \rightarrow X_{t+1}$  for  $t = 0, \dots, T - 1$ . We call  $((X_0, \dots, X_T), (f_1, \dots, f_{T-1}))$  a *deterministic discrete-time dynamical system*.

We call the index  $t$  the *epoch*, the *stage* or the *period*. We call  $X_t$  the *state space* at period  $t$ . We call  $f_t$  the *transition function* or *dynamics function*.

Let  $x_0 \in \mathcal{X}_0$ . Define a state sequence  $x_1 \in \mathcal{X}_1, \dots, x_T \in \mathcal{X}_T$  by

$$x_{t+1} = f_t(x_t, u_t).$$

In this case we call  $x_0$  the *initial state*. We call the  $x_t$  the *trajectory* associated with initial state  $x_0$ .

We call  $T$  the *horizon*. In the case that we have an infinite sequence of state sets, input sets, and dynamics, then we refer to a *infinite-horizon* dynamical system. To use language in contrast with this case, we refer to the dynamical system when  $T$  is finite as a *finite-horizon* dynamical system.

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<sup>1</sup>Future editions will modify, and may develop dynamic systems via the genetic approach by appealing to their classical use in Newtonian physics for modeling celestial mechanics.



