



### Definition

A *decision problem* is a pair  $(I, Y)$  of *instances*  $I$  and *yes-instances*  $Y \subset I$ .

### Examples

#### Subgraph isomorphism

Let  $\mathcal{G}(n)$  be the set of graphs with  $n$  vertices. For  $m \leq n$ , define  $I = \mathcal{G}(n) \times \mathcal{G}(m)$ . A graph  $G_1 = (V_1, E_1)$  is *subgraph-isomorphic* to a graph  $G_2 = (V_2, E_2)$  if there exists  $V' \subset V_1$ ,  $E' \subset E_1$  with  $|V'| = |V_2|$ ,  $|E'| = |E_2|$ , and bijection  $f : V' \rightarrow V_2$  so that

$$\{u, v\} \in E_2 \iff \{f(u), f(v)\} \in E'.$$

Define

$$Y = \{(G_1, G_2) \in I \mid G_1 \text{ is subgraph-isomorphic to } G_2\}$$

We call  $(Y, I)$  the *subgraph-isomorphism problem*.

#### Traveling Salesman

Denote by  $S^n \subset \mathbf{Z}^{n \times n}$  the set of symmetric integer-valued matrices. Define  $I = S^n \times \mathbf{Z}$ . A *n-tour* is a numbering  $\pi : \{1, \dots, n\} \rightarrow \{1, \dots, n\}$  and has *cost*  $C(\pi)$  with respect to  $D$  defined by

$$C(\pi) = D_{\pi(n), \pi(1)} + \sum_{j=1}^{n-1} D_{\pi(j), \pi(j+1)}$$

A tour is *B-bounded* if its cost is no greater than  $B$ . Define  $Y = \{(D, B) \mid \text{there is a } B\text{-bounded tour with respect to } D\}$ .

