



## SET INTERSECTIONS

### Why

We can consider intersections of more than two sets.

### Definition

Let  $\mathcal{A}$  denote a set of sets. In other words, every element of  $\mathcal{A}$  is a set. And suppose that  $\mathcal{A}$  has at least one set (i.e.,  $\mathcal{A} \neq \emptyset$ ). Let  $C$  denote a set such that  $C \in \mathcal{A}$ . Then consider the set,

$$\{x \in C \mid (\forall A)(A \in \mathcal{A} \longrightarrow x \in A)\}.$$

This set exists by the principle of specification (see *Set Specification*). Moreover, the set does not depend on which set we picked. So the dependence on  $C$  does not matter. It is unique by the axiom of extension (see *Set Equality*). This set is called the *intersection* of  $\mathcal{A}$ .

### Notation

We denote the intersection of  $\mathcal{A}$  by  $\bigcap \mathcal{A}$ .

### Equivalence with pair intersections

As desired, the the set denoted by  $\mathcal{A}$  is a pair (see *Unordered Pairs*) of sets, the pair intersection (see *Pair Intersections*) coincides with intersection as we have defined it in this sheet.<sup>1</sup>

**Proposition 1.**  $\bigcap\{A, B\} = A \cap B$

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<sup>1</sup>A full account of the proof will appear in future editions.



