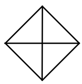


Why

We want a notion of reversing functions.

Definition

Reversing functions does not make sense if the function is not one-to-one. Let $f : X \to Y$. If x_1 goes to y and x_2 goes to y (i.e., $f(x_1) = f(x_2) = y$), then what should y go to. One answer is that we should have a function which gives all the domain values which could lead to y. This is the inverse image (see Function Images) $f^{-1}(\{y\})$. Nor does reversing functions make sense if f is not onto. If there does not exist $x \in X$ so that $y = f(x)$, then $f^{-1}(\{y\}) = \varnothing$.

In the case, however, that the function is one-to-one and onto, then each element of the domain corresponds to one and only one element of the codomain and vice versa. In this case, for all $y \in Y$, $f^{-1}(\{y\})$ is a singleton $\{x\}$ where $f(x) = y$. In this case, we define a fucntion $g : Y \to X$ so that $g(y) = x$ if and only if $f(x) = y$.

Proposition 1 (Uniqueness). *Let $f : A \to B$, $g : B \to A$, and $h : B \to A$. If g and h are both inverse functions of f, then $g = h$.*

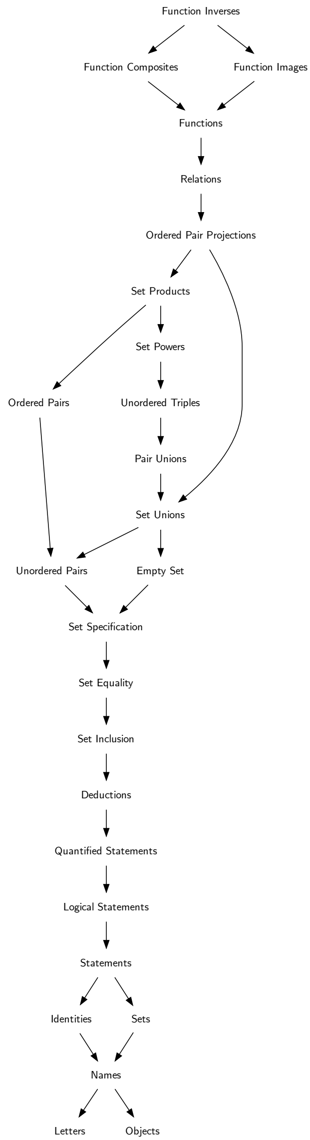
Proposition 2 (Existence). *If a function is one-to-one and onto, it has an inverse; and conversely.*[1]

Composites and inverses

Let $f : X \to Y$ and $g : Y \to Z$. Then g^{-1} maps $\mathcal{P}(Z)$ to $\mathcal{P}(Y)$ and f^{-1} maps $\mathcal{P}(Y)$ to $\mathcal{P}(X)$. Then the following is immediate

Proposition 3. $(gf)^{-1} = f^{-1}g^{-1}$

[1] A proof will appear in future editions.


Function Inverses
Function Composites
Function Images
Functions
Relations
Ordered Pair Projections
Set Products
Set Powers
Ordered Pairs
Unordered Triples
Pair Unions
Set Unions
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects