Why

Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\mathbf{tr} : \mathbf{R}^{n \times n} \to \mathbf{R}$. Let $A \in \mathbf{R}^{n \times n}$. Then

$$\mathbf{tr}\, A = \sum_{i=1}^{n} A_{ii}.$$

Properties

Prop. 1. *The trace is a linear function on the vector space of $n \times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R}^{n \times n}$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\mathbf{tr}\, C = \sum_{i=1}^{n} C_{ii} = \sum_{i=1}^{n} \alpha A_{ii} + \beta B_{ii}$$

$$= \alpha \sum_{i=1}^{n} A_{ii} + \beta \sum_{i=1}^{n} B_{ii}$$

$$= \alpha\, \mathbf{tr}\, A + \beta\, \mathbf{tr}\, B.$$

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[1]Future editions will include, in the genetic tradition.

$\square$

Prop. 2. *Let $A, B \in \mathbf{R}^{n \times n}$.*

$$\mathbf{tr}\,(AB) = \mathbf{tr}\,(BA)$$

[2]

In other words, "matrices commute under the trace operator."

Prop. 3. *Let $A \in \mathbf{R}^{n \times n}$. Then $\mathbf{tr}\,A = \mathbf{tr}\,A^{\top}$.*

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[2]Future editions will include an account.

Matrix Trace

Matrix-Matrix Products — Real Matrices

Matrix-Vector Products — Matrix Transpose

Linear Combinations — Matrices

Vectors

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Intervals

Real Line — Real Space — Real Plane

Integral Line — Real Order — Sequences

Geometry — Real Numbers — Fields — Direct Products

Rational Numbers — Groups — Set Numbers

Integer Arithmetic — Integer Additive Inverse — Finite Sets

Integer Sums — Natural Additive Identity — Natural Multiplicative Identity — Inverse Elements

Arithmetic — Identity Elements — Element Functions — Equivalent Sets

Integer Products — Natural Powers — Algebras — Operations — Function Inverses — Family Unions and Intersections

Integer Numbers — Natural Products — Function Composites — Function Images — Families

Natural Sums — Equivalence Relations — Functions

Recursion Theorem — Natural Order — Relations

Peano Axioms

Natural Induction — Cartesian Products

Natural Numbers — Set Powers — Generalized Set Dualities

Successor Sets — Unordered Triples — Intersection of Empty Set — Partitions — Set Dualities — Set Unions and Intersections

Ordered Pairs — Pair Unions — Set Intersections — Set Complements

Set Differences — Set Unions — Pair Intersections

Unordered Pairs — Empty Set

Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets — Identities

Names

Letters — Objects

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