



## Why

### Definition

A *normal random function* (or *normal process* or *gaussian process*)<sup>1</sup> is a real-valued random function with the property that any subset of results has a multivariate normal density.

Let  $(\Omega, \mathcal{A}, \mathbf{P})$  be a probability space. Let  $x : I \rightarrow (\Omega \rightarrow \mathbf{R})$ . Then  $x$  is a normal random function if there exists  $m : I \rightarrow \mathbf{R}$  and positive definite  $k : I \times I \rightarrow \mathbf{R}$  with the property that if  $J \subset I$ ,  $|J| = d$ , then  $x_J \sim \mathcal{N}(m(J), k(J \times J))$ . In other words,  $x_J : \Omega \rightarrow \mathbf{R}^d$  is a Gaussian random vector. We call  $m$  the *mean function* and  $k$  the *covariance function*.

### Random function interpretation

Many authorities discuss a normal random function as “putting a prior” on a “space” (see, for example, Real Function Space) of functions. One can draw a sample from this space by first selecting  $\omega \in \Omega$ , and then defining a sample  $f : I \rightarrow \mathbf{R}$  by  $f(i) = x(i, \omega)$ .

### Multivariate normal special case

If the index set is finite, and can be ordered, then the normal random function is a multivariate normal random vector.

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<sup>1</sup>The choice of “normal” is a result of the Bourbaki project’s convention to eschew historical names. Though here, as in **Multivariate Normals** the language of the project is nonstandard.



