Why

[1]

Definition

Let $(X, \mathbf{R})$ be a vector space. A function $f : X \times X \to \mathbf{R}$ is an *inner product* on the vector space $(X, \mathbf{R})$ if

1. $f(x, x) \geq 0, = 0 \longleftrightarrow x = 0$,
2. $f(x + y, z) = f(x, z) + f(y, z)$,
3. $f(x, y) = f(y, x)$, and
4. $f(\alpha x, y) = \alpha f(x, y)$.

An *inner product space* is an ordered pair: a real vector space and an inner product.[2]

Examples

$\mathbf{R}^n$ with the usual inner product is an inner product space. Some authors call any finite-dimensional inner product space over the real numbers is a *Euclidean vector space*.

Examples

If $f : X \times X \to \mathbf{R}$ is an inner product we regularly denote $f(x, x)$ by $\langle x, x \rangle$.

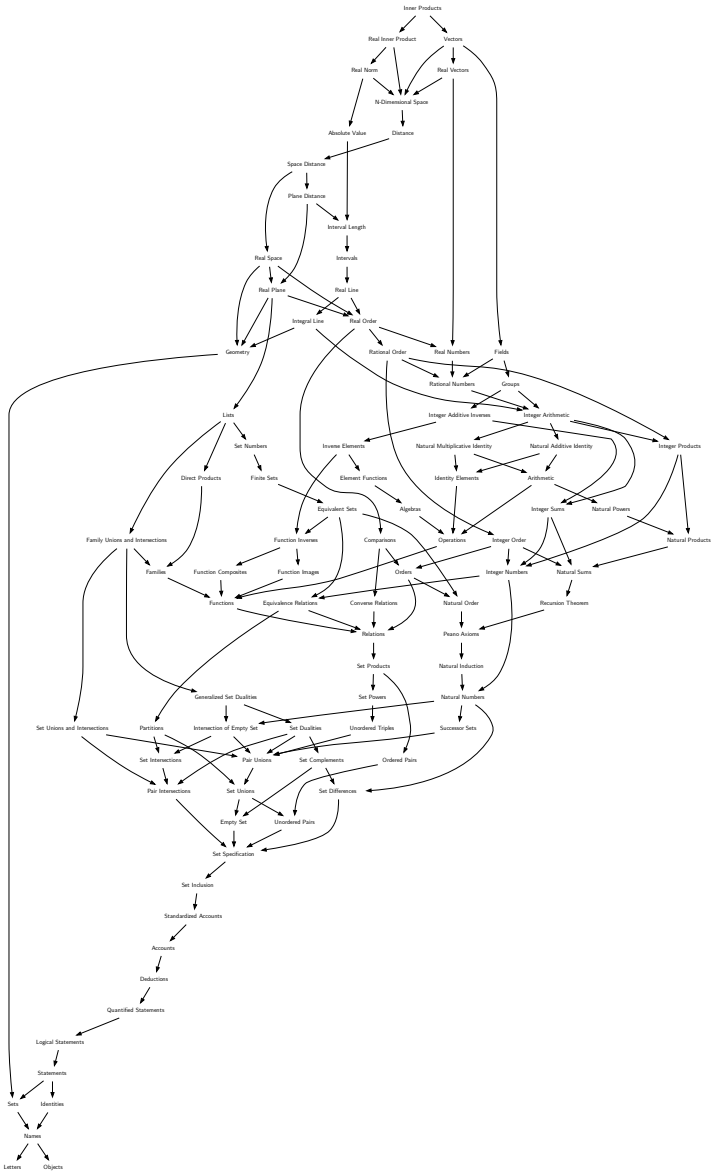
[1] Future editions will complete and rework this sheet.

[2] Future editions will discuss complex inner products.

Orthogonality

Two vectors in an inner product space are *orthogonal* if their inner product is zero. An *orthogonal family of vectors* in an inner product space is a family of vectors for which distinct family members are orthogonal.

A vector is *normalized* if its inner product with itself is one.


Inner Products
Real Inner Product
Vectors
Real Norm
Real Vectors
N-Dimensional Space
Absolute Value
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Fields
Rational Numbers
Groups
Integer Arithmetic
Lists
Integer Additive Inverses
Set Numbers
Inverse Elements
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Element Functions
Identity Elements
Arithmetic
Equivalent Sets
Algebras
Integer Sums
Natural Powers
Family Unions and Intersections
Function Inverses
Comparisons
Operations
Integer Order
Natural Products
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Sums
Functions
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Unordered Triples
Successor Sets
Set Intersections
Pair Unions
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects