Why

We want to add and multiply complex numbers.[1]

Definition

Let $z_1, z_2 \in \mathbf{C}$ with $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$. The *complex product* of z_1 and z_2 is the complex number $(x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2)$.

Notation

Properties

Proposition 1 (Distributivity). *For all $z_1, z_2, z_3 \in \mathbf{C}$, we have* $z_1(z_2 + z_3)$ *and* $z_1 z_2 + z_1 z_3$

Relation to $\mathbf{R}^2$

Addition in $\mathbf{C}$ corresponds to the usual addition of the corresponding vectors in the plane $\mathbf{R}^2$. In other words, it corresponds to element-wise addition. However multiplication in $\mathbf{C}$ is not componentwise multiplication in $\mathbf{R}^2$.

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[1]Future editions will expand in the genetic account for introducing complex numbers.

Complex Arithmetic → Complex Products → Complex Sums → Complex Numbers → Absolute Value, Real Arithmetic

Absolute Value → Interval Length → Intervals → Real Line

Real Arithmetic → Real Multiplicative Inverses → Real Products → Real Additive Inverses → Real Sums

Rational Multiplicative Inverses · Rational Arithmetic · Real Order

Rational Products · Rational Sums · Real Numbers · Rational Order

Integral Line · Rational Numbers

Integer Arithmetic

Natural Multiplicative Identity · Natural Additive Identity · Integer Products

Identity Elements · Arithmetic

Inverse Elements · Natural Powers

Function Inverses · Element Functions · Operations · Natural Products · Integer Sums · Integer Order · Comparisons

Function Composites · Function Images · Natural Sums · Integer Numbers · Orders

Functions · Recursion Theorem · Equivalence Relations · Natural Order · Converse Relations

Peano Axioms · Relations

Natural Induction · Set Products

Natural Numbers · Set Powers · Ordered Pairs

Partitions · Intersection of Empty Set · Successor Sets · Unordered Triples

Set Symmetric Differences · Set Intersections · Pair Unions

Pair Intersections · Set Complements · Set Unions

Empty Set · Set Differences · Unordered Pairs

Geometry

Set Specification → Set Equality → Set Inclusion → Standardized Accounts → Accounts → Deductions → Quantified Statements → Logical Statements → Statements → Sets, Identities → Names → Letters, Objects

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