Why

We have considered the number of ways we can arrange 52 cards into a deck. What of n cards? A moment's reflection indicates this will also be the number of ways to arrange n objects in order (where the objects need not be cards).

Definition

By the fundamental principle of counting, there are n ways to select the first card, $n-1$ ways to select the second, and so on. Thus, the number of ways of stacking n cards in a deck is

$$n(n-1)(n-2)\cdots 1$$

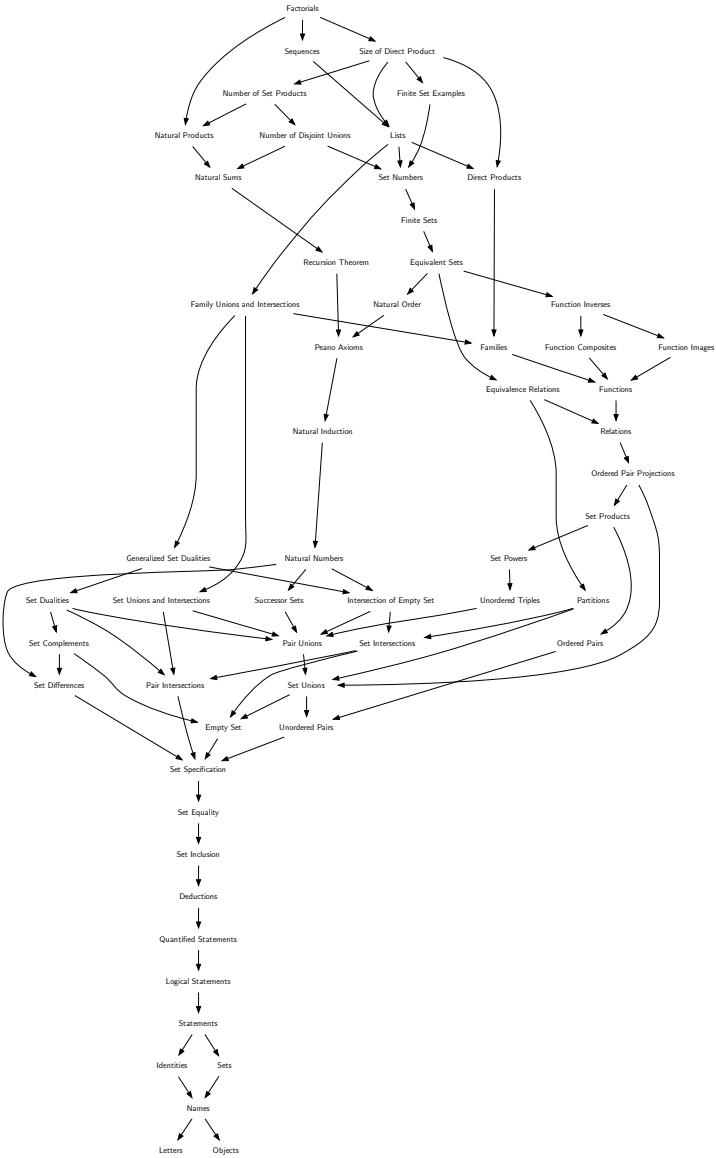
We call this number the *factorial* of n, or *n-factorial*.

Factorial function. Define $f : \mathbf{N} \to \mathbf{N}$ recursively by $f(1) = 1$ and $f(2) = 2f(1)$, and $f(n) = nf(n-1)$ for $n \in \mathbf{N}$ (f exists by the the recursion theorem—see Recursion Theorem). f is defined such that $f(n)$ is n factorial, for which reason we call f the *factorial function*. For convenience, we extend f to ω[1] by defining $f(0) = 1$.

Notation

We denote the factorial of n by $n!$, read aloud "n factorial". So for example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ and $0! = 1$.

[1] See Natural Numbers.


Factorials
Sequences
Size of Direct Product
Number of Set Products
Finite Set Examples
Natural Products
Number of Disjoint Unions
Lists
Natural Sums
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Direct Products
Finite Sets
Recursion Theorem
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Set Dualities
Set Unions and Intersections
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
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Pair Unions
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Ordered Pairs
Set Differences
Pair Intersections
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Unordered Pairs
Set Specification
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