

COUNTABLE PROBABILITY DISTRIBUTIONS

Why

Suppose we flip a coin until it lands heads. What is the probability that we see a head after one flip? Two flips? n flips? We want to extend our notion of probability distribution to a set with infinite elements, but only countably many.

Why

Consider a set A.¹ If A has n elements, then a probability distribution on A is $p:A\to \mathbf{R}$ where p(a)=1/n. There is a natural candidate.

What if A is the set of natural numbers \mathbb{N} . The principle difficulty is that not all sequences of real numbers $a: \mathbb{N} \to \mathbb{R}$ are summable.

A (countable) probability distribution on \mathbb{N} is $p: \mathbb{N} \to \mathbb{R}$ where $p \geq 0$ and

$$\sum_{n=1}^{\infty} p(n) = 1.$$

¹Future editions will rework this sheet.

