Why

Convex functions satisfy an interpolation property.

Discussion

By definition, given $S \subset \mathbf{R}^n$, $f : S \to \mathbf{R}$ is convex means that the point

$$(1 - \lambda)(x\mu) + \lambda(x, \nu) = ((1 - \lambda)x + \lambda y, (1 - \lambda)\mu + \lambda\nu)$$

belongs to epi f whenever (x, μ) and (y, ν) belong to epi f and $0 \leq \lambda \leq 1$. Said differently, we have $(1 - \lambda)x + \lambda y \in S$, and

$$f((1 - \lambda)x + \lambda y) \leq (1 - \lambda)\mu + \lambda\nu$$

whenever $x \in S$, $y \in S$, $f(x) \leq \mu \in \mathbf{R}$, and $f(y) \leq \nu \in \mathbf{R}$.

Concave functions have a similar property, with the inequalities flipped, and affine functions satisfy with qualities. This shows that the functions $f : \mathbf{R}^n \to \mathbf{R}$ we are calling affine coincide exactly with the affine transformations from $\mathbf{R}^n$ to $\mathbf{R}$.

Visualization

Characterizations of Convex Functions

Convex Multivariate Functions

Real Convex Functions · Real Function Epigraphs · Affine Transformations of Real Convex Sets

Real Function Graphs · Real Convex Sets

Real Functions · Real Halfspaces

Extended Real Numbers · Real Hyperplanes · Multivariate Functions

Subspace Orthogonal Complement

Topological Interiors · Subspace Dimension

Vector Space Dimension

Vector Space Bases · Affine Set Dimension · Affinely Independent Vectors

Orthogonal Complements · Linearly Dependent Vectors · Span · Real Subspace Dimension · Real Affine Sets and Subspace · Real Affine Hulls

Vector Subspace

Real Inner Products · Vectors as Matrices · N-Dimensional Line Segments · Real Subspaces · Real Vector Bases · Real Translates · Real Affine Sets

Topological Closures · Real Inner Product · Linear Combinations · Real Vector Span · Independent Set of Real Vectors · Real Affine Combinations · Real Matrices

Topological Neighbourhoods · Vectors · Space Inner Product · Real Norm · Real Vectors · Real Linear Combinations · N-Dimensional Lines · Real Linear Equations

Real Set Closure · Topologies · Space Norm · Linear Functions · Maps

Real Limits · Metrics · Plane Norm · Plane Inner Product · Space Vectors · Plane Vector Sums and Differences · N-Dimensional Space

Absolute Value · Right Triangle Sides Relation · Plane Vectors · Distance

Real Square · Plane Distance · Space Distance

Real Arithmetic · Interval Length · Real Space

Real Multiplication Inverses · Intervals · Square Numbers

Real Products · Real Line · Real Plane · Squares

Real Addition Inverses · Real Sums · Real Order · Rational Arithmetic · Real Sequences · Integral Line · Real Summation · Identity

National Multiplication Inverses · Fields · Real Numbers · Natural Sums · Rational Order · Rational Products · Sequences · Natural Summation

Rings · Rational Numbers

Integer Arithmetic · Natural Functions · Direct Products · Set Number

Inverse Elements · Natural Multiplication Identity · Natural Additive Identity · Integer Products · Finite Sets

Element Function · Family Operations · Identity Elements · Natural Arithmetic · Equivalent Sets

Compositions · Operations · Function Inverses · Function Restrictions and Extensions · Integer Order · Integer Sums · Natural Powers · Natural Equations · Family Unions and Intersections

Converse Relations · Function Composition · Function Images · Orders · Integer Numbers · Families · Natural Products · Equations

Functions · Equivalence Relations · Natural Sums · Natural Order

Relations · Recursion Theorem

Ordered Pair Projections · Peano Axioms

Set Products · Natural Induction

Ordered Pairs · Set Powers · Natural Numbers · Generalized Set Dualities

Set Symmetric Differences · Partitions · Unordered Triples · Intersection of Empty Set · Successor Sets · Set Dualities · Set Unions and Intersections

Ordered Pairs · Set Complements · Set Intersection · Pair Unions

Set Differences · Set Union · Pair Intersections

Unordered Pairs · Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities · Sets

Names

Letters · Objects