Definition

Proposition 1. *Let W be a positive semidefinite n by n matrix. Then $g : \mathbf{R}^n \to \mathbf{R}$ defined by $g(x) = \sqrt{x^\top W x}$ is a norm on $\mathbf{R}^n$.*

Notation

Let $W \in \mathbf{R}^{n \times n}$, positive semidefinite. Then we denote the norm corresponding to W by $\|\cdot\|_W$. So then, the norm of a vector $x \in \mathbf{R}^n$ is $\|x\|_w$. Notice that $\|x\|_W = \|W^{1/2} x\|_2$.

Visualization

[1]

We can compare the Euclidean norm on $\mathbf{R}^2$ with the weighted norm given by

$$W = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$

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[1]Future editions will visualize these norms as function on $\mathbf{R}^2$, via contour plots.

Weighted Norms

Matrix Transpose

Vectors as Matrices

Norms

Real Matrices

Real Inner Product

Functionals

Linear Equations

Space Inner Product

Real Norm

Vectors

Arrays

Linear Functions

Real Vectors

Space Norm

Plane Vector Sums and Differences

Space Vectors

N-Dimensional Space

Plane Norm

Plane Inner Product

Right Triangle Sides Relation

Plane Vector

Distance

Absolute Value

Space Distance

Real Square

Real Arithmetic

Plane Distance

Interval Length

Real Multiplicative Inverse

Intervals

Real Products

Real Space

Square Numbers

Real Plane

Real Line

Real Additive Inverse

Square

Real Summation

Rational Multiplicative Inverse

Real Order

Real Sums

Integral Line

Rational Arithmetic

Geometry

Real Numbers

Rational Products

Rational Sums

Fields

Rational Order

Rational Numbers

Rings

Integer Arithmetic

Integer Products

Lists

Set Numbers

Natural Multiplicative Identity

Natural Additive Identity

Direct Products

Finite Sets

Identity Elements

Arithmetic

Inverse Elements

Equivalent Sets

Natural Summation

Natural Powers

Family Unions and Intersections

Function Inverses

Element Functions

Family Operations

Comparisons

Natural Equations

Integer Order

Integer Sums

Natural Products

Families

Function Image

Function Composite

Operations

Equations

Orders

Natural Sums

Integer Numbers

Functions

Converse Relations

Natural Order

Equivalence Relation

Recursion Theorem

Relations

Peano Axioms

Set Products

Natural Induction

Generalized Set Dualities

Set Powers

Natural Numbers

Set Unions and Intersections

Set Dualities

Unordered Triples

Intersection of Empty Set

Successor Sets

Partitions

Set Symmetric Differences

Ordered Pairs

Pair Unions

Set Intersections

Set Complements

Pair Intersections

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects