Why

The inverse of a function interacts nicely with family unions, family intersections and complements.

Results

Let $f : X \to Y$. Throughout this sheet, let $f^{-1} : \mathcal{P}(Y) \to \mathcal{P}(X)$. And take $\{B_i\}$ to be a family of subsets of Y.[1]

Proposition 1. $f^{-1}(\cup_i B_i) = \cup_i f^{-1}(B_i)$

Proposition 2. $f^{-1}(\cup_i B_i) = \cap_i f^{-1}(B_i)$

Proposition 3. $f^{-1}(Y - B) = X - f^{-1}(B)$

Properties for function image

Notice that $f(\cup_i A_i) = \cup_i f(A_i)$ but not for interesctions. Nor is there a similar correspondence for complements. There are some relations, which we list below.[2]

Proposition 4. $f(A \cap B) = f(A) \cap f(B)$ *if and only if f is one-to-one.*

Proposition 5. *For all $A \subset X$, $f(X - A) = Y - f(A)$ if and only if f is one-to-one.*

Proposition 6. *For all $A \subset X$, $Y - f(A) \subset f(X - A)$ if and only if f is onto.*

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[1]The proofs of the following will appear in future editions.
[2]Accounts of these facts will appear in future editions.

Inverses Unions Intersections and Complements

Family Unions and Intersections

Families

Functions

Relations

Ordered Pair Projections

Set Products

Set Powers

Generalized Set Dualities

Set Unions and Intersections

Set Dualities

Intersection of Empty Set

Unordered Triples

Set Intersections

Pair Unions

Ordered Pairs

Pair Intersections

Set Complements

Set Unions

Set Differences

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects

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