Why

We want to subtract numbers.[1]

Definition

Consider the set $\omega \times \omega$. This set is the set of ordered pairs of ω. In other words, the ordered pairs of natural numbers.

We call two such pairs (a, b) and (c, d) *equivalent* $a + d = b + c$. Briefly, the intuition is that (a, b) represents a less b, or in the usual notation "$a - b$".[2] So this equivalence relation says these two are the same if $a - b = c - d$. Rearranging gives $a + d = b + c$.

Proposition 1. *Integer equivalence is an equivalence relation.*[3]

We define the *set of integer numbers* to be the set of equivalence classes (see Equivalence Relations) under integer equivalence on $\omega \times \omega$. We call an element of the set of integer numbers an *integer number* or an *integer*. We call the set of integer numbers the *set of integers* or *integers* for short.

Notation

We denote the set of integers by **Z**. If we denote integer equivalence by $\sim$ then $\mathbf{Z} = (\omega \times \omega)/\sim$.

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[1]Future editions will change this why. In particular, by referencing Inverse Elementsand the lack thereof in ω.

[2]This account will be expanded in future editions.

[3]The proof is straightforward. It will be included in future editions.

Integer Numbers

Equivalence Relations

Relations

Set Products

Set Powers

Natural Numbers

Unordered Triples   Partitions   Successor Sets   Intersection of Empty Set

Ordered Pairs   Pair Unions   Set Intersections   Set Differences

Set Unions

Unordered Pairs   Empty Set   Pair Intersections

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects