



Why

The product of two sets is a (sub)set of ordered pairs. Is every set of ordered pairs a subset of a product of two sets?

Result

The answer is easily seen to be yes. Let R denote a set of ordered pairs. So for $x \in R$, $x = \{\{a\}, \{a, b\}\}$. First consider $\bigcup R$. Then $\{a\} \in \bigcup R$ and $\{a, b\} \in \bigcup R$. Next consider $\bigcup\bigcup R$. Then $a, b \in \bigcup\bigcup R$. So if we want two sets—denote them by A and B —so that $R \subset A \times B$, we can take both A and B to be the set $\bigcup\bigcup R$.

Projections

We often want to shrink the sets A and B to include only the *relevant* members. In other words, to include only those members which appear as either the first coordinate (for A) or second coordinate (for B) in an element of R . We can do this by specifying the elements of $\bigcup\bigcup R$ which are actually a first coordinate or second coordinate for some ordered pair in the set R .

Define

$$A' = \{a \in A \mid (\exists b)((a, b) \in R)\},$$

and likewise

$$B' = \{b \in B \mid (\exists a)((a, b) \in R)\}.$$

We call A' the *projection onto the first coordinate* and B' the *projection onto the second coordinate*.

