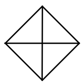


Why

We bound the probability that a random variance deviates from its mean using its variance.

Result

Proposition 1. *Let f be a square-integrable real-valued random variable on the probability space $(X, \mathcal{A}, \mu)$. Then for $t > 0$,*

$$\mu[|f - \mathbf{E}(f)| \geq t] \leq \frac{\operatorname{var} f}{t^2}.$$

Proof. The symbols $|f - \mathbf{E}(f)| \geq t$ denote the set $\{x \in X \mid |f(x) - \mathbf{E}(f)| \geq t\}$. This set is the same as the set

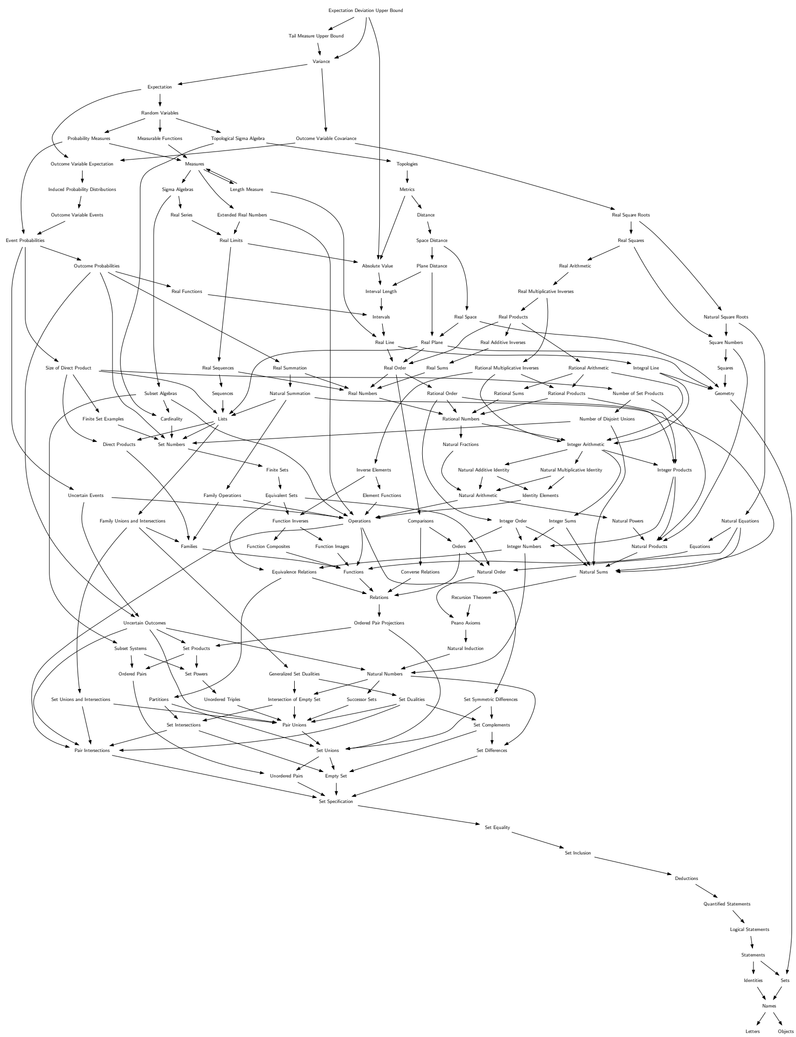
$$\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}.$$

By using the tail measure upper bound,

$$\mu(\left\{x \in X \;\middle|\; (f(x) - \mathbf{E}(f))^2 \geq t^2\right\}) \leq \frac{\mathbf{E}(f - \mathbf{E}(f))^2}{t^2}.$$

We recognize the numerator of the right hand side as the varianceof f. □

The above is also called *Chebychev's Inequality.*



Expectation Deviation Upper Bound
Tail Measure Upper Bound
Variance
Expectation
Random Variables
Probability Measures
Measurable Functions
Topological Sigma Algebra
Outcome Variable Covariance
Outcome Variable Expectation
Measures
Topologies
Induced Probability Distributions
Sigma Algebras
Length Measure
Metrics
Outcome Variable Events
Real Series
Extended Real Numbers
Distance
Real Square Roots
Event Probabilities
Real Limits
Space Distance
Real Squares
Outcome Probabilities
Absolute Value
Plane Distance
Real Arithmetic
Real Functions
Interval Length
Real Multiplicative Inverses
Intervals
Real Space
Real Products
Natural Square Roots
Real Line
Real Plane
Real Additive Inverses
Square Numbers
Size of Direct Product
Real Sequences
Real Summation
Real Order
Real Sums
Rational Multiplicative Inverses
Rational Arithmetic
Integral Line
Squares
Subset Algebras
Sequences
Natural Summation
Real Numbers
Rational Order
Rational Sums
Rational Products
Number of Set Products
Geometry
Finite Set Examples
Cardinality
Lists
Rational Numbers
Number of Disjoint Unions
Direct Products
Set Numbers
Natural Fractions
Integer Arithmetic
Finite Sets
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Uncertain Events
Family Operations
Equivalent Sets
Element Functions
Natural Arithmetic
Identity Elements
Family Unions and Intersections
Function Inverses
Operations
Comparisons
Integer Order
Integer Sums
Natural Powers
Natural Equations
Families
Function Composites
Function Images
Orders
Integer Numbers
Natural Products
Equations
Equivalence Relations
Functions
Converse Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Uncertain Outcomes
Subset Systems
Set Products
Natural Induction
Ordered Pairs
Set Powers
Generalized Set Dualities
Natural Numbers
Set Unions and Intersections
Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Dualities
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects