

## Cumulative Distribution Function From Density

## 1 Why

The density of a random variable determines the cumulative distribution function.

## 2 Result

**Proposition 1.** A probability density function of a random variable, if it exists, characterizes the cumulative distribution function.

*Proof.* Let  $(X, \mathcal{A}, \mu)$  be a probability space. Let f be a real-valued random variable on X. Let  $\lambda$  denote the cover length. Let g be a probability density of f: Then:

$$F_f(t) = \mu(\{x \in X \mid f(x) \in (-\infty, t])\})$$
$$= \int_{(-\infty, t]} g d\lambda.$$

