Why

We are regularly referring to a few common growth classes.

Definitions

Let $c \in \mathbf{R}$. Then we name the following growth classes

growth class	name
$O(1)$	*constant growth class*
$O(\log(x))$	*logarithmic growth class*
$O((\log(x))^c)$	*polylogarithmic growth class*
$O(x)$	*linear growth class*
$O(x^2)$	*quadratic growth class*
$O(x^c)$	*polynomial growth class*
$O(c^x)$	*exponential growth class*

We have written these in order:

$$O(1) \subset O(\log(x)) \subset O((\log(x))^c) \subset \cdots \subset O(x^c) \subset O(c^x).$$

A function that grows faster (is in the upper growth class) of a power of x is called *superpolynomial*. One that grows slower than c^n for some $c \in \mathbf{R}$ is called *subexponential*. The class $O(\log(x^c))) = O(\log(x))$ since $\log(x^c) = c \log x$. Similarly, for all $c_1, c_2 > 0$, $O(\log_{c_1}(x)) = O(\log_{c_2}(x))$.

This list is useful because of the following

Proposition 1. *Let* $f, g : \mathbb{R} \to \mathbb{R}$ *and defined* $h : \mathbb{R} \to \mathbb{R}$ *by* $h = f + g$. *If* $O(f) \subset O(g)$, *then* $h \in O(g)$.

In other words, if a function h is the sum of f and g and g is growing faster, then g (the one growing faster) determines the order of h.

Common Growth Classes

Exponential Function Function Growth Classes

Real Functions Absolute Value

Interval Length Logarithm

Intervals Polynomials

Real Line

Real Order

Real Numbers Rings

Integral Line Rational Numbers Groups

Integer Arithmetic Integer Additive Inverses

Integer Sums Natural Additive Identity Natural Multiplicative Identity Inverse Elements

Arithmetic Identity Elements Element Functions

Integer Products Natural Exponents Algebras Function Inverse

Natural Products Operations Function Composites Function Images

Integer Numbers Natural Sums Functions

Equivalence Relations Recursion Theorem

Relations Peano Axioms

Cartesian Products Natural Induction

Set Powers Natural Numbers

Partitions Unordered Triples Intersection of Empty Set Successor Sets

Set Intersections Ordered Pairs Pair Unions

Set Unions Set Differences

Pair Intersections Empty Set Unordered Pairs

Geometry Set Specification

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Sets Identities

Names

Letters Objects