



## Why

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### Definition

Let  $(A, +, \cdot)$  be a ring.

A *polynomial* in  $A$  of *degree*  $d$  is a function  $p : A \rightarrow A$  for which there exists a finite sequence  $c = (c_0, c_1, \dots, c_{d-1}, c_d) \in A^{d+1}$  satisfying

$$p(a) = c_0 + c_1a^1 + c_2a^2 + \cdots + c_da^d,$$

for all  $a \in A$ . We call the sequence  $c$  the *polynomial coefficients*, and call the  $c_i$  the *coefficients* of  $p$ . We call  $d + 1$  the *order* of the polynomial.

Clearly, to every polynomial in  $A$  of degree  $d$  there corresponds a sequence in  $A$  of length  $d + 1$ , and vice versa. For this reason, we can identify polynomials by their coefficients.

### Examples

The function  $f : A \rightarrow A$  is a polynomial of degree 0 and order 1 if there exists  $c_0$  so that

$$f(a) = c_0$$

for all  $a \in A$ .

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<sup>1</sup>Future editions will include, and most likely will build on quadratics and an appeal to the simplicity of the “natural” algebraic operations.

The function  $g : A \rightarrow A$  is a polynomial of degree 1 and order 2 if there exists  $c_0$  and  $c_1$  so that

$$g(a) = c_0 + c_1a$$

The function  $h : A \rightarrow A$  is a polynomial of degree 2 and order 3 if there exists  $c_0$  and  $c_1$  so that

$$h(a) = c_0 + c_1a + c_2a^2.$$

In other words, a second degree polynomial is a quadratic.

The function  $p : A \rightarrow A$  is a *polynomial* of degree  $d$  and order  $d+1$  if there exists a  $d+1$  length sequence  $(c_0, c_1, \dots, c_d)$  in  $A$  so that

$$p(a) = c_0 + c_1a + \dots + c_da^d.$$

