

UNDIRECTED GRAPHS

1 Why

We want to visualize symmetric relations.

2 Definition

An *undirected graph* is a finite nonempty set and a set of (unordered) pairs of its elements. We call the elements of the first set the *vertices* of the graph and the elements of the second set the *edges*.

Two vertices are adjacent if their pair is an edge in the set. We say that the corresponding edge is incident to those vertices. The adjacency set of a vertex is the set of vertices adjacent to it. The degree of a vertex is the number of vertices adjacent to it; in other words, the size of its adjacency set. A graph is complete if each pair of two distinct vertices is adjacent. A clique is a maximal complete subgraph.

Other Terminology

Some authors define a clique as any set of vertices whose corresponding subgraph is complete; we prefer the term *complete* subgraph here. Some authors call the adjacency set the neighborhood of the vertex and call the union of the adjacency set of a vertex with the singleton of that same vertex the closed neighborhood of the vertex.

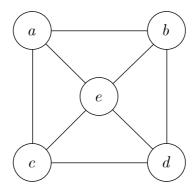


Figure 1: Undirected graph.

Notation

Let V be a non-empty set and let E be a subset of $\{\{u,v\} \mid u,v \in V\}$. As usual, we denote the ordered pair consisting of V and E by (V,E). We say "Let G=(V,E)" be a graph, implying the relevant properties of V and E.

Example

Let
$$V = \{a, b, c, d, e\}$$
 and

$$E = \{\{a,b\}, \{a,c\}, \{a,e\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,e\}, \{d,e\}\}.$$

