

CONVEX SETS

Why

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Definition

The closed line segment between two points in n-dimensional space is the set of points which can be expressed as the sum of the first point and a scalar multiple of the difference between the second point and the first; where the scalar is in the interval [0,1]. Thus, the closed line segment between two points is a subset of the line though the two points. The open line segment between x and y is the closed line segment with the points x and y.

A convex set contains every closed line segement between any two points. Every affine set is convex. Thus, convex sets are more general.

Notation

Let x and y in \mathbb{R}^n . We can express the closed line segment between x and y as

$${x + a(y - x) \mid 0 \le a \le 1, x, y \in \mathbb{R}^n}.$$

Notice that x + a(y - x) = (1 - a)x + ay.

Proposition 1. Every affine set is convex.

¹Future editions will clarify.

PROPOSITION 2. The intersection of a family of convex sets is convex.

Proposition 3. The translate of a convex set is convex. The scalar multiple of a convex set si convex.

