Why

We have seen that the integrable functions form a vector space. How about the square integrable functions? And so on.[1]

Definition

The *integrable function spaces* are a collection of function spaces, one for each real number $p \geq 1$, for which the pth power of the absolute value of the function is integrable.[2]

Notation

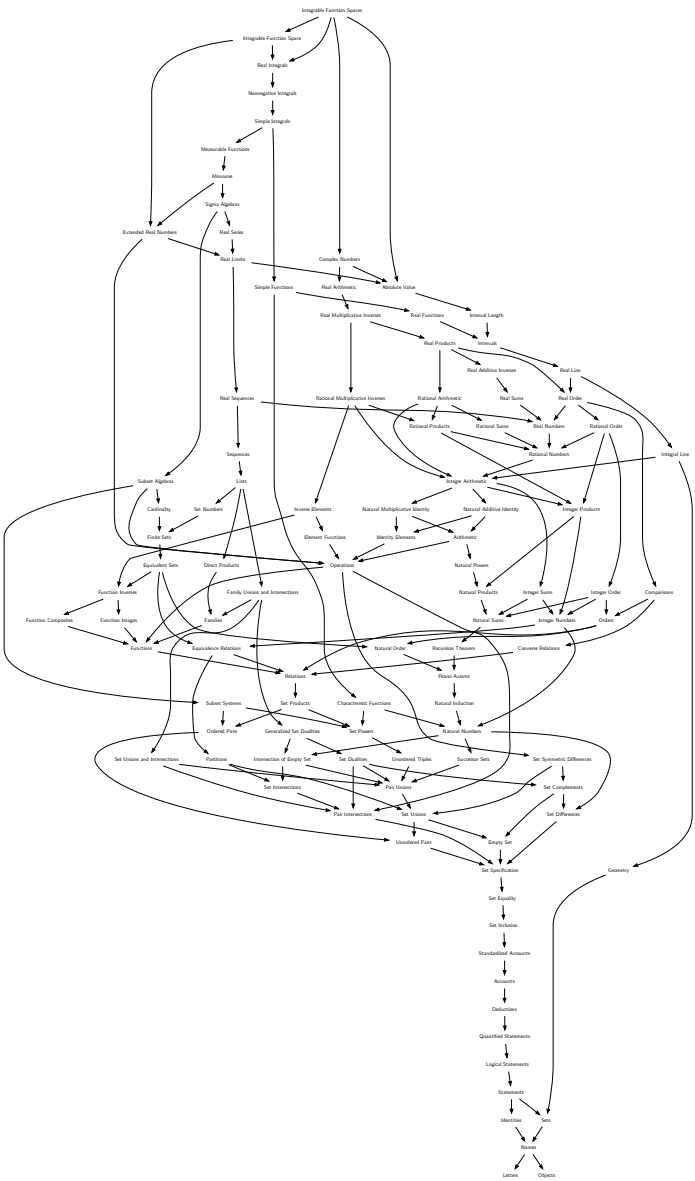
Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $p \geq 1$. We denote the integrable function space corresponding to p by $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{R})$. We have defined it by

$$\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{R}) = \left\{ \text{ measurable } f : X \to R \;\middle|\; \int |f|^p d\mu < \infty \right\}$$

Let $\mathbf{C}$ denote the set of complex numbers. Similarly for complex-valued functions, we denote the pth space by $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbf{C})$.

[1] Future sheets are likely to being with L^2.

[2] Future editions will include the case where $p = \infty$.


Integrable Function Spaces
Integrable Function Space
Real Integrals
Nonnegative Integrals
Simple Integrals
Measurable Functions
Measures
Sigma Algebras
Extended Real Numbers
Real Series
Real Limits
Complex Numbers
Absolute Value
Simple Functions
Real Arithmetic
Real Functions
Interval Length
Real Multiplicative Inverses
Real Products
Intervals
Real Additive Inverses
Real Line
Real Sequences
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Rational Products
Rational Sums
Real Numbers
Rational Order
Rational Numbers
Integral Line
Sequences
Subset Algebras
Lists
Integer Arithmetic
Cardinality
Set Numbers
Inverse Elements
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Finite Sets
Element Functions
Identity Elements
Arithmetic
Equivalent Sets
Direct Products
Operations
Natural Powers
Function Inverses
Family Unions and Intersections
Natural Products
Integer Sums
Integer Order
Comparisons
Function Composition
Function Images
Families
Natural Sums
Integer Numbers
Orders
Functions
Equivalence Relations
Natural Order
Recursion Theorem
Converse Relations
Relations
Peano Axioms
Subset Systems
Set Products
Characteristic Functions
Natural Induction
Ordered Pairs
Generalized Set Duality
Set Powers
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Duality
Unordered Triples
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Geometry
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects