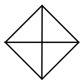


Why

In the case of several variables, what do the row reducer matrices correspond to in ordinary row reduction?

Definition

Let $(A \in \mathsf{R}^{m \times m}, b \in \mathsf{R}^{m})$ be an ordinarily reducible linear system. The *ordinary reducer sequence* (or *ordinary row reducer sequence*) is a sequence of row reducer matrices $L_1, \dots L_{m-1}$ so that

$$A_1 = L_1 A \text{ and } A_i = L_i A_{i-1} \text{ for } 2 \leq i \leq m-1.$$

In other words, $U \in \mathsf{R}^{m \times m}$ defined by

$$U = L_{m-1} \cdots L_2 L_1 A$$

is the ordinary row reduction of A.

Formulae

Let x_1 be the first column of A and let x_k be the kth column of A_{k-1} for $k = 2, \dots, m-1$. The the transformation L_k is chosen so that

$$x_k = \begin{bmatrix} x_{1k} \\ \vdots \\ x_{kk} \\ x_{k+1,k} \\ \vdots \\ x_{mk} \end{bmatrix} \quad \xrightarrow{L_k} \quad L_k x_k = \begin{bmatrix} x_{1k} \\ \vdots \\ x_{kk} \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

we subtract $\ell_{jk} = {}^{x_{jk}}/_{x_{kk}}$ for $k \le j \le m$ times row k from row j. We call l_{jk} the *row multiplier*. The matrix L_k has the form

$$L_k = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & -\ell_{k+1,k} & 1 & & \\ & & \vdots & & \ddots & \\ & & -\ell_{mk} & & & 1 \end{bmatrix}$$

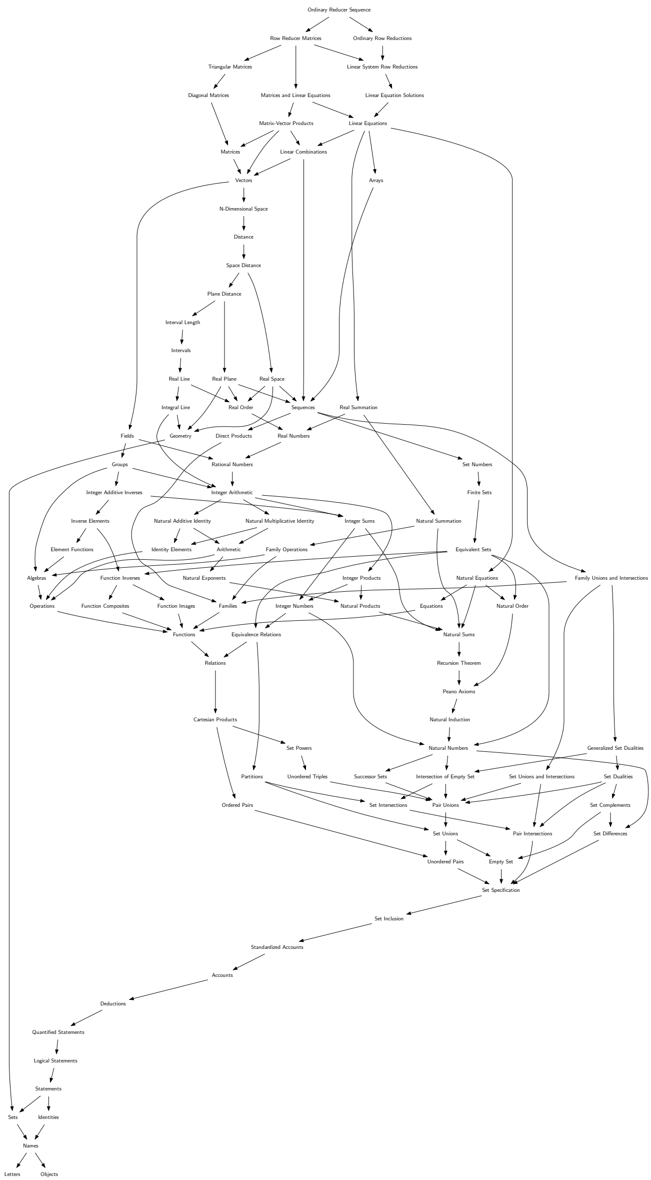
Properties

As an immediate consequence of the formula for L_k given above, we have

Proposition 1. *Every reducer in an ordinary reducer sequence is unit lower triangular.*

The goal of row reduction is to introduce zeros below the diagonal so to allow for back-substitution. The following proposition states that ordinary row reduction succeeds in creating an upper triangular matrix..

Proposition 2. *The ordinary row reduction of a matrix is upper triangular.*


Ordinary Reducer Sequence
Row Reducer Matrices
Ordinary Row Reductions
Triangular Matrices
Linear System Row Reductions
Diagonal Matrices
Matrices and Linear Equations
Linear Equation Solutions
Matrix-Vector Products
Linear Equations
Matrices
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Vectors
Arrays
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Distance
Space Distance
Plane Distance
Interval Length
Intervals
Real Line
Real Plane
Real Space
Integral Line
Real Order
Sequences
Real Summation
Fields
Geometry
Direct Products
Real Numbers
Groups
Rational Numbers
Set Numbers
Integer Additive Inverses
Integer Arithmetic
Finite Sets
Inverse Elements
Natural Additive Identity
Natural Multiplicative Identity
Integer Sums
Natural Summation
Element Functions
Identity Elements
Arithmetic
Family Operations
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Algebras
Function Inverses
Natural Exponents
Integer Products
Natural Equations
Family Unions and Intersections
Operations
Function Composites
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