



## Why

We use a normal random function model to make a regressor.

## Definition

Let  $F : \Omega \rightarrow (A \rightarrow \mathbf{R})$  be a normal random function with mean function  $m : A \rightarrow \mathbf{R}$  and covariance function  $k : A \times A \rightarrow \mathbf{R}$  over the probability space  $(\Omega, \mathcal{A}, \mathbf{P})$ . Let the family of random variables (or stochastic process) of  $F$  be  $f : A \rightarrow (\Omega \rightarrow \mathbf{R})$ .

Let  $e$  be a normal random vector with mean zero and covariance  $\Sigma_e$ . Let  $a^1, \dots, a^n \in A$ . We sometimes call the sequence  $a^1, \dots, a^n$  the *design*. Define  $y : \Omega \rightarrow \mathbf{R}^d$  by

$$y_i = f(a^i) + e_i$$

We call  $y$  the *observation vector* or *observation random vector*. We call  $e$  the *error vector* or *noise vector*. In this context,  $f(a^i)$  is sometimes called the *signal*.

Let  $b^1, \dots, b^m \in A$ . Define  $z : \Omega \rightarrow \mathbf{R}^d$  by  $z_i = f(b^i)$  for  $i = 1, \dots, m$ . So  $z_i$  is the random variable corresponding to the family at index  $b^i \in A$ . Then  $(y, z)$  is normal. We call the conditional density of  $z$  given  $y$  the *predictive density* for  $b$  given  $a$ .

**Proposition 1.** Define  $m_a \in \mathbf{R}^n$  by  $(m(a^1), \dots, m(a^n))^\top$  and

define  $m_b$  by  $(m(b^1), \dots, m(b^m))^\top$ .<sup>1</sup> Define  $\Sigma_a \in \mathbf{R}^{n \times n}$  by

$$\begin{pmatrix} k(a^1, a^1) & \cdots & k(a^1, a^n) \\ \vdots & \ddots & \vdots \\ k(a^n, a^1) & \cdots & k(a^n, a^n) \end{pmatrix}$$

and define  $\Sigma_{ba} \in \mathbf{R}^{m \times n}$  by

$$\begin{pmatrix} k(b^1, a^1) & \cdots & k(b^1, a^n) \\ \vdots & \ddots & \vdots \\ k(b^m, a^1) & \cdots & k(b^m, a^n) \end{pmatrix}.$$

The predictive density  $g_{z|y}(\cdot, \gamma) : \mathbf{R}^m \rightarrow \mathbf{R}$  of  $b \in A$  for design  $a^1, \dots, a^n$  is normal with mean.

$$m_b + K_{ba} (K_a + \Sigma_e)^{-1} (\gamma - m_a)$$

and covariance

$$\Sigma_b - \Sigma_{ba} (\Sigma_a + \Sigma_e)^{-1} \Sigma_{ab}.$$

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<sup>1</sup>Future editions will fix the re-use of the symbol  $m$ .

