$$\diamondplus$$

Iterated Integrals

# 1 Why

We integrate over a product space by integrating one coordinate at a time. TODO: WHY?

# 2 Result

**Proposition 1.** *Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be $\sigma$-finite measurable spaces. Let $f : X \times Y \to [-\infty, \infty]$ be $\mathcal{A} \times \mathcal{B}$-measurable and $\mu \times \nu$-integrable. Then*

1. *For $\mu$-almost every $x$ in $X$ the section $f_x$ is $\nu$-integrable and for $\nu$-almost every $y$ in $Y$ the section $f^y$ is $\mu$-integrable,*

2. *the functions $I_f$ and $J_f$ defined by*

$$I_f(x) = \begin{cases} \int_Y f_x d\nu & \text{if } f_x \text{ is } \nu\text{-integrable,} \\ 0 & \text{otherwise} \end{cases}$$

   *and*

$$J_f(y) = \begin{cases} \int_X f^y d\mu & \text{if } f^y \text{ is } \mu\text{-integrable,} \\ 0 & \text{otherwise} \end{cases}$$

   *belong to $\mathcal{L}(X, \mathcal{A}, \mu, R)$ and $\mathcal{L}(Y, \mathcal{B}, \nu, R)$ respectively, and*

3. *the relation*

$$\int_{X \times Y} f d(\mu \times \nu) = \int_X I_f d\mu = \int_Y J_f d\nu$$

   *holds.*

Meas

Characteristic F

Real Funct

Interval

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