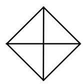


Definition

An operator $T \in \mathcal{L}(V)$ is called *self-adjoint* (or *Hermitian*) if the adjoint of T is itself. In symbols, T is self-adjoint if $T = T^*$. In other words, T is self-adjoint if and only if

$$\langle Tv, w \rangle = \langle v, Tw \rangle \quad \text{for all } v, w \in V$$

Properties

Proposition 1. *Suppose $S, T \in \mathcal{L}(V)$ are self-adjoint. The $S + T$ are self-adjoint. Also λT is adjoint for all real λ.*

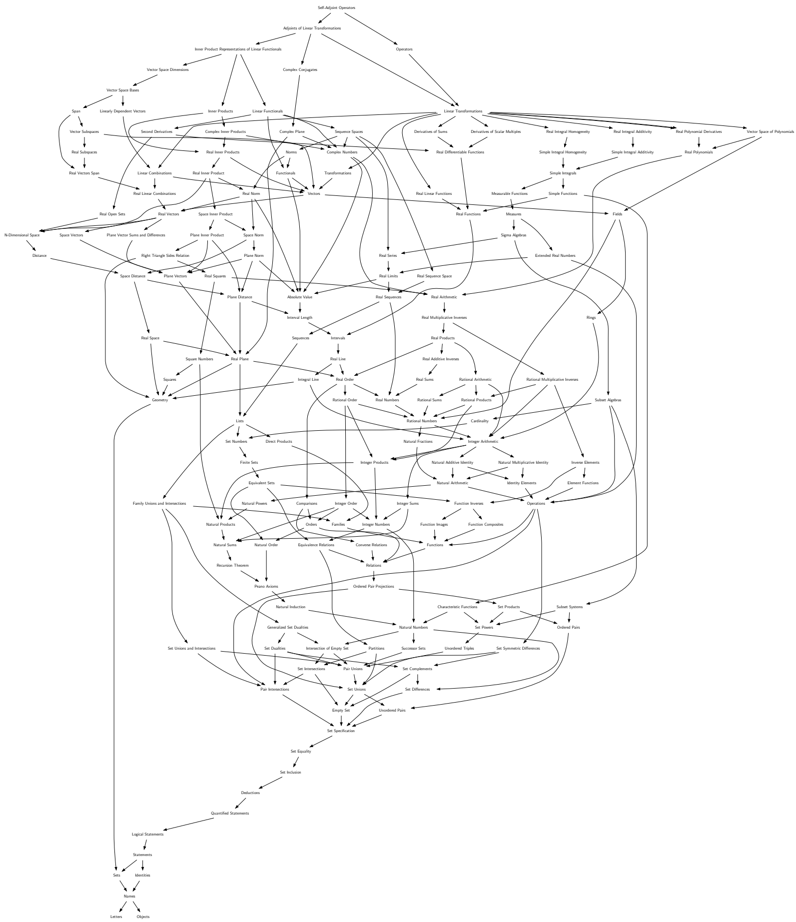
Notation

We will see that the adjoint on $\mathcal{L}(V)$ plays a role similar to complex conjugation on $\mathbf{C}$. The self-adjoint operators will seen to be analogous to the real numbers. A complex number is real if and only if $z = z^*$. Similarly, an operator is self-adjoint if and only if $T = T^*$.

Characterization for complex space

Proposition 2. *Suppose V is a complex inner product space and let $T \in \mathcal{L}(V)$. Then*

$$T = T^* \quad \longleftrightarrow \quad \langle Tv, v \rangle \in \mathbf{R} \textit{ for all } v \in V$$


Self-Adjoint Operators
Adjoints of Linear Transformations
Inner Product Representations of Linear Functionals
Operators
Vector Space Dimensions
Complex Conjugates
Vector Space Bases
Span
Linearly Dependent Vectors
Inner Products
Linear Functionals
Linear Transformations
Vector Subspaces
Second Derivatives
Complex Inner Products
Complex Plane
Sequence Spaces
Derivatives of Sums
Derivatives of Scalar Multiples
Real Integral Homogeneity
Real Integral Additivity
Real Polynomial Derivatives
Vector Space of Polynomials
Real Subspaces
Real Inner Products
Norms
Complex Numbers
Real Differentiable Functions
Simple Integral Homogeneity
Simple Integral Additivity
Real Polynomials
Real Vectors Span
Linear Combinations
Real Inner Product
Functionals
Transformations
Simple Integrals
Real Linear Combinations
Real Norm
Vectors
Real Linear Functions
Measurable Functions
Simple Functions
Real Open Sets
Real Vectors
Space Inner Product
Real Functions
Measures
Fields
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Plane Inner Product
Space Norm
Sigma Algebras
Distance
Right Triangle Sides Relation
Plane Norm
Real Series
Extended Real Numbers
Space Distance
Plane Vectors
Real Squares
Real Limits
Real Sequence Space
Plane Distance
Absolute Value
Real Sequences
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Rings
Real Space
Sequences
Intervals
Real Products
Square Numbers
Real Plane
Real Line
Real Additive Inverses
Squares
Integral Line
Real Order
Real Sums
Rational Arithmetic
Rational Multiplicative Inverses
Geometry
Rational Order
Real Numbers
Rational Sums
Rational Products
Subset Algebras
Rational Numbers
Cardinality
Lists
Set Numbers
Direct Products
Natural Fractions
Integer Arithmetic
Finite Sets
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Inverse Elements
Equivalent Sets
Natural Arithmetic
Identity Elements
Element Functions
Family Unions and Intersections
Natural Powers
Comparisons
Integer Order
Integer Sums
Function Inverses
Operations
Natural Products
Orders
Families
Integer Numbers
Function Images
Function Composites
Natural Sums
Natural Order
Equivalence Relations
Converse Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Characteristic Functions
Set Products
Subset Systems
Generalized Set Dualities
Natural Numbers
Set Powers
Ordered Pairs
Set Unions and Intersections
Set Dualities
Intersection of Empty Set
Partitions
Successor Sets
Unordered Triples
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects