



## Why

We can embed the undirected graphs as certain garphs directed graphs.

## Definition

Suppose that  $G = (V, E)$  is a directed graph. If  $(v, w) \in E$  and  $(w, v) \in E$  we call  $(v, w)$  (and  $(w, v)$ ) and *bidirected edge* (or *undirected edge*). In other words, an edge  $(v, w) \in E$  is undirected if  $(v, w)$  is also in  $E$ .

If every edge of a directed graph is undirected, then we call the graph *undirected*. If some edges are an some are not, we call  $G$  a *partially directed graph*.

## Notation

Suppose  $(V, E)$  is a directed graph. It is common to write  $v \rightarrow w$  for

$$(v, w) \in E \text{ and } (w, v) \notin E.$$

It is common to write  $a \sim \beta$  if

$$(v, w) \in E \text{ and } (w, v) \in E$$

Similarly, we write  $v \nrightarrow w$  if  $(v, w) \notin E$  and  $v \nleftarrow w$  if

$$(v, w) \notin E \text{ and } (w, v) \notin E$$

## Undirected version

As before, the *undirected version* (or *skeleton*) of  $G$  is the *undirected* partially directed graph defined satisfying  $u \sim v$  if  $u \rightarrow v$  or  $u \leftarrow v$ .

## Subgraphs

Given a graph  $G = (V, E)$  and a set  $A \subset V$ , the *subgraph* of  $G = (V, E)$  corresponding to  $A$  is the graph denoted  $G_A$  defined by  $(A, E \cap (A \times A))$ .

## Completeness

Suppose  $(V, E)$  is a partially directed graph. In the context of partially directed graphs, the graph is *complete* if  $(u, v) \in E$  or  $(v, u) \in E$  for any two vertices  $u$  and  $v$ . A subset  $A$  is *complete* if the subgraph  $G_A$  is complete. A complete subset that is maximal (w.r.t.  $\subseteq$ ) is called a *clique*.



