Why

We consider the probabilistic linear model in which all random variables are normal.

Definition

A *normal linear model* is a probabilistic linear model in which the parameter and noise vectors have normal (Gaussian) densities. The model is also called the *Gaussian linear model* or the *linear model with Gaussian noise*.

Let $(x : \Omega \to \mathbf{R}^d, A \in \mathbf{R}^{n\times d}, e : \Omega \to \mathbf{R}^n)$ be a probabilistic linear model over the probability space $(\Omega, \mathcal{A}, \mathbf{P})$ in which x and e have normal densities. Recall that a probabilistic linear model has observation vector $y : \Omega \to \mathbf{R}^n$ defined by

$$y = Ax + e.$$

Conditional density of x on y

Since x and e are normal and independent, y is normal.[1] Moreover, the random vector (x, y) is normal with covariance

$$\begin{pmatrix} \Sigma_x & \Sigma_x A^\top \\ A\Sigma_x & A\Sigma_x A^\top + \Sigma_e \end{pmatrix}.$$

So the conditional density (see Normal Conditionals) of $g_{x|y}(\cdot, \gamma)$ is normal with mean

$$\Sigma_x A^\top (A\Sigma_x A^\top + \Sigma_e)^{-1}\gamma$$

and covariance

$$\Sigma_x - \Sigma_x A^\top (A\Sigma_x A^\top + \Sigma_e)^{-1} A\Sigma_x.$$

This density is sometimes called the posterior for the parameters given the observations. So the parameter posterior of the normal linear model is normal.

[1] Future editions will include an account.

We can write the conditional mean as

$$(\Sigma_x^{-1} + A^\top \Sigma_e^{-1} A)^{-1} A^\top \Sigma_e^{-1}$$

and the conditional covariance as[2]

$$(\Sigma_x^{-1} + A^\top \Sigma_e^{-1} A)^{-1}.$$

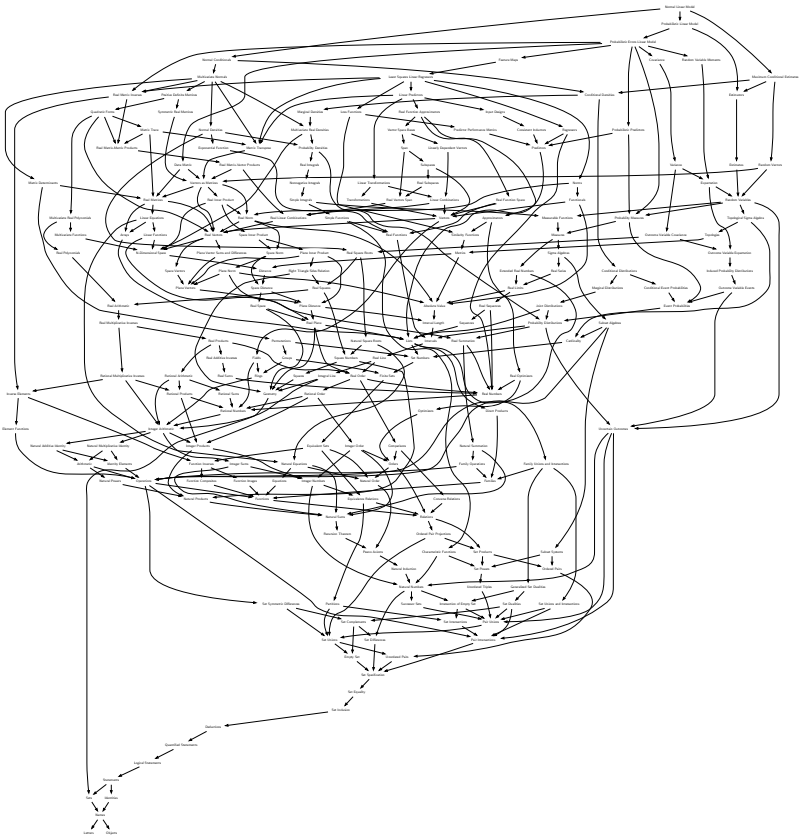
Very frequently we use these forms when $d < n$. In other words, in the case that we have fewer unknowns than measurements. In that case Σ_x is smaller than $A\Sigma_x A^\top$.

Maximum conditional estimate of x

Proposition 1. *The maximum conditional estimate of $x : \Omega \to \mathbf{R}^d$ given observed value $\gamma \in \mathbf{R}^n$ of $y : \Omega \to \mathbf{R}^n$ is the conditional mean*

$$\Sigma_x A^\top (A\Sigma_x A^\top + \Sigma_e)^{-1} \gamma.$$

[2] A proof will appear in future editions. Use the matrix inversion lemma or facts about inverses.


Natural Linear Model
Probabilistic Linear Model
Probabilistic Principal Components Analysis
Factor Analysis
Gaussian
Random Variable Measures
Measurable Functional Estimator
Factor Map
Bernoulli Distribution
Multivariate Bernoulli
Bernoulli Process
Bernoulli Trial
Binomial Distribution
Multinomial Distribution
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