



Why

We want fractions.¹

Rational equivalence

Consider $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$. We say that the elements (a, b) and (c, d) of this set are *rational equivalent* if $ad = bc$. Briefly, the intuition is that (a, b) represents a over b . In the usual notation, (a, b) represents “ a/b ”. So this equivalence relation says these two are the same if $a/b = c/d$ or else $ad = bc$.

Proposition 1. *Rational equivalence is an equivalence relation on $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$.*²

Definition

The *set of rational numbers* is the set of equivalence classes (see Equivalence Classes) of $\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\})$ under rational equivalence. We call an element of the set of rational numbers a *rational number* or *rational*. We call the set of rational numbers the *set of rationals* or *rationals* for short.

Notation

We denote the set of rationals by \mathbf{Q} .³ If we denote rational equivalence by \sim then $\mathbf{Q} = (\mathbf{Z} \times (\mathbf{Z} - \{0_{\mathbf{Z}}\}))/\sim$.

¹This why will be expanded in future editions.

²Future editions will include an account.

³From what we can tell, \mathbf{Q} is a mnemonic for “quantity”, from the latin “quantitas.”

