Why

A *chord* for a path in an undirected graph is an edge between two non-consecutive vertices of the path; similarly for a cycle. We think of a chord as a one-edge shortcut between two vertices of a path. If a path has a chord, it can be reduced to a shorter path. So a shortest path between two vertices is chordless. The converse, however, is not true: a chordless path is not necessarily a shortest path.

An undirected graph is *chordal* if every cycle of length greather than three has a chord. Using this property, to every cycle in a chordal graph there corresponds at least one cycle of length three. Chordal graphs are also called *rigid-circuit graphs*, *triangulated graphs*, *perfect elimination graphs*, *decomposable graphs*, and *acyclic graphs*. But we will only ever call them chordal graphs.

A *cactus graph* is an undirected graph with no cycles of length greater than three. Both trees and forests are cactus graphs. All cactus graphs are trivially chordal.

Notation

Let $G = (V, E)$ be an undirected graph. A chord in a path $(v_0, v_1, \ldots, v_k)$ of G is an edge $\{v_i, v_j\}$ with $|j - i| > 1$. A chord in a cycle $(v_0, v_1, \ldots, v_{k-1}, v_0)$ of G is an edge $\{v_i, v_j\}$ with $(j - i) \mod k > 1$.

Examples

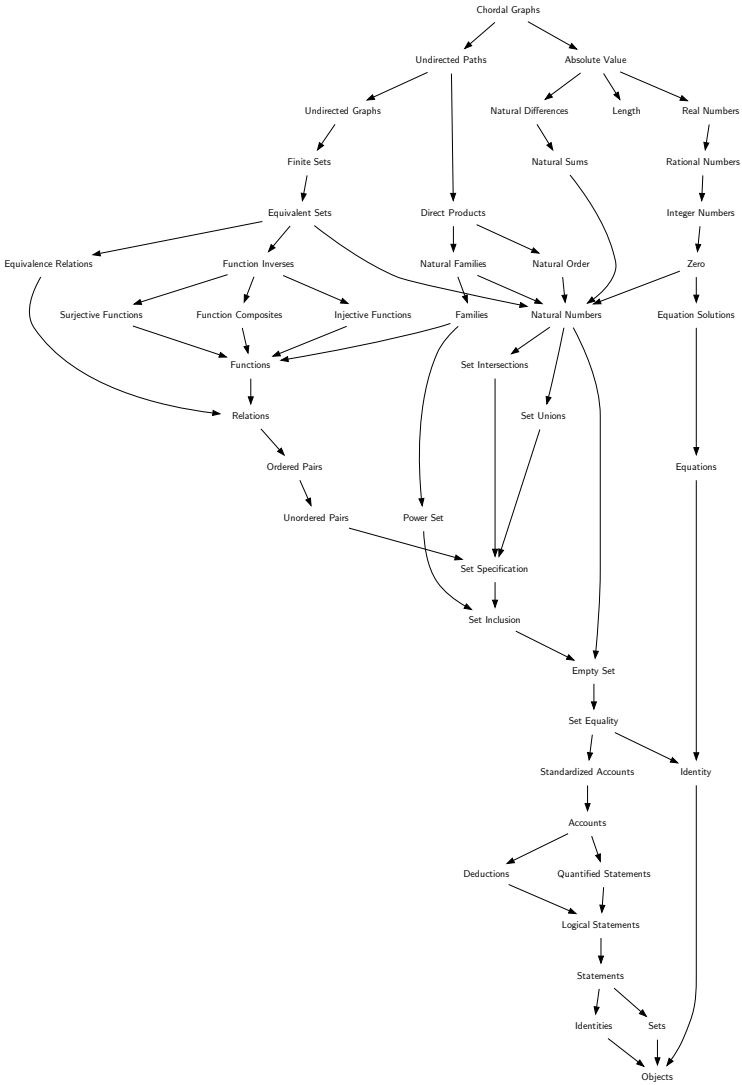
Trees and forests (and any cactus graph) are trivially chordal. Additionally:

Proposition **1.** *Complete graphs are chordal.*

Properties

Proposition **2.** *Any subgraph of a chordal graph is chordal.*

Proof. A consequence of the definition, since every cycle of the subgraph is a cycle of the original graph, and so also with the chords. □


Chordal Graphs
Undirected Paths
Absolute Value
Undirected Graphs
Natural Differences
Length
Real Numbers
Finite Sets
Natural Sums
Rational Numbers
Equivalent Sets
Direct Products
Integer Numbers
Equivalence Relations
Function Inverses
Natural Families
Natural Order
Zero
Surjective Functions
Function Composites
Injective Functions
Families
Natural Numbers
Equation Solutions
Functions
Set Intersections
Relations
Set Unions
Ordered Pairs
Equations
Unordered Pairs
Power Set
Set Specification
Set Inclusion
Empty Set
Set Equality
Standardized Accounts
Identity
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Objects