**Definition**

The *translate* of $S \subset \mathbf{R}^n$ *by* $a \in \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid \exists x \in S \text{ such that } z = x + a\}.$$

**Notation**

We often use the abbreviated notation $S + a$ for the translate of $S$ by $a$. It is sometimes also convenient to extend set-builder notation and write

$$S + a = \{x + a \mid x \in M\}.$$

The right hand side is slick notation for the definition given above.

**Sums and differences**

The *sum* of two sets $S, T \subset \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x + y)\}.$$

Likewise, the *difference* of two sets $S, T \subset \mathbf{R}^n$ is the set

$$\{z \in \mathbf{R}^n \mid (\exists x \in S)(\exists y \in T)(z = x - y)\}.$$

**Notation**

We denote the sum of $S$ and $T$ by $S + T$, and the difference by $S - T$. We often use the slick notation

$$\{x + y \mid x \in S, y \in T\} \text{ and } \{x - y \mid x \in S, y \in T\},$$

for these two sets.

Real Translates

N-Dimensional Space

Distance

Space Distance

Plane Distance

Interval Length

Real Space

Intervals

Real Line

Real Plane

Real Order

Integral Line

Geometry

Rational Order

Real Numbers

Lists

Rational Numbers

Set Numbers

Direct Products

Integer Arithmetic

Integer Products

Natural Additive Identity

Natural Multiplicative Identity

Finite Sets

Arithmetic

Identity Elements

Equivalent Sets

Integer Order

Integer Sums

Natural Powers

Comparisons

Function Inverses

Family Unions and Intersections

Integer Numbers

Natural Products

Orders

Operations

Function Images

Function Composites

Families

Natural Sums

Equivalence Relations

Natural Order

Converse Relations

Functions

Recursion Theorem

Relations

Peano Axioms

Ordered Pair Projections

Natural Induction

Set Products

Natural Numbers

Set Powers

Generalized Set Dualities

Set Symmetric Differences

Partitions

Intersection of Empty Set

Successor Sets

Unordered Triples

Set Dualities

Set Unions and Intersections

Set Complements

Set Intersections

Pair Unions

Ordered Pairs

Pair Intersections

Set Differences

Set Unions

Empty Set

Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities

Sets

Names

Letters

Objects