



Why

Lots of things are (approximately) linear.¹

Definition

A transformation is *linear* if the result of a linear combination of the two vectors is the linear combination of the results of the vectors (using the same coefficients). The transformation is linear *with respect to* the field of the two vector spaces.

We use the term transformation (Transformations) for emphasis and reminder that the function is defined on a vector space. Of course, \mathbf{R} is a vector space and so a function $f : \mathbf{R} \rightarrow \mathbf{R}$ may be linear. It is, therefore, common to speak of *linear functions*.

Often authors will use the word *operator* for linear functions. It seems, generally, that this term is commonly reserved for the case in which the vector space discussed is a function space (or, at least, infinite dimensional).

Notation

Let (V_1, F) and (V_2, F) be two vector spaces over the same field. Let $f : V_1 \rightarrow V_2$. f is linear means

$$f(au + bv) = af(u) + bf(v)$$

for all $a, b \in F$ and $u, v \in V_1$.

¹Future editions will expand on this why. In particular, the intuition of proportionality.

