



## EQUATIONS

### Why

We name a statement which involves an identity.<sup>1</sup>

### Definition

An *equation* is statement (see **Statements**) relating two terms by the relation of identity (see **Identities**). Some authors also call an equation an *equality*. The symbol “=” is called the (or an) *equals sign* or *equals symbol*.

### Variables

Let  $X$  and  $Y$  be sets and let  $f, g : X \rightarrow Y$ . In the statement  $(\forall x)(f(x) = g(x))$ ,  $f$  and  $g$  are *free* names and  $x$  is a *bound* name (see **Quantified Statements**).

For convenience, we often refer to the equation  $f(x) = g(x)$  without the quantifier  $\forall x$ . In this case,  $x$  appears free, but is not. In this context, the statement  $f(x) = g(x)$  has  $x$  implicitly bound. There are two senses here, though. The first is that  $x$  is bound because it is “subordinate” to the quantifier  $\forall$ . The particular symbol is irrelevant; the symbol  $y$  works just as well. In a second sense, though, the name is “free,” as it is a placeholder and the choice of the symbol  $x$  does not matter.

We use different terminology for this common case. In discussing  $f(x) = g(x)$ , we call the placeholder name  $x$  a *variable* and we call the names  $f$  and  $g$  *constants*. The language is

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<sup>1</sup>Future editions will modify this statement and sheet.

meant to convey  $f$  and  $g$  are *fixed* in the present discussion, as indicated by the usual language “Let  $f$  and  $g$  ...”.

## Solutions

We are often interested in finding objects in some set to satisfy an equation. For example, we are interested in finding an object  $\xi \in X$  to satisfy  $f(\xi) = g(\xi)$ . In this setting we call the variable  $\xi$  in the equation an *unknown*.

We call an element  $\xi \in X$  a *solution* of the equation if  $f(\xi) = g(\xi)$ . We call the set  $\{\xi \in X \mid f(\xi) = g(\xi)\}$  the *solution set*. If the solution set is non-empty, we say that a solution *exists*. If the solution set is a singleton, we say that the solution is *unique*.

We are often interested in solutions which satisfy several equations at once. For example, we have the equations  $f_1(x) = g_1(x)$  and  $h(x) = i(x)$  and so on. We want  $x$  to satisfy these. Here it is *set of equations*, *simultaneous equations*, or a *system of equations*.

## Finding solutions

We often talk about *finding* or *searching* for solutions or *solving equations*. We say: “We want to *find*  $x \in X$  to satisfy  $f(x) = g(x)$ .” In addition to  $f(x) = g(x)$ , we may include other statements about  $x$ . The language is meant to convey that we are searching for an object which we will name, as a variable,  $x$ , and we want this object to satisfy the statements.

