Topological Space

1 Why

We want to generalize the notion of continuity.

2 Definition

A **topological space** is a subset algebra for which: (1) the empty set and the base set are distinguished, (2) the intersection of a finite family of distinguished subsets is distinguished, and (3) the union of a family of distinguished subsets is distinguished. We call the set of distinguished subsets the **topology**. We call the distinguished subsets the **open sets**.

2.1 Notation

Let A a non-empty set. For the set of distinguished sets, we use $\mathcal{T}$, a mnemonic for topology, read aloud as "script T". We denote elements of $\mathcal{T}$ by O, a mnemonic for open. We denote the topological space with base set A and topology $\mathcal{T}$ by $(A, \mathcal{T})$. We denote the properties satisfied by elements of $\mathcal{T}$:

1. $X, \varnothing \in \mathcal{T}$

2. $\{O_i\}_{i=1}^n \subset \mathcal{T} \implies \cap_{i=1}^n O_i \in \mathcal{T}$

3. $\{O_\alpha\}_{\alpha \in I} \subset \mathcal{T} \implies \cup_{\alpha \in I} \in \mathcal{T}$