



## Definition

Let  $x : \Omega \rightarrow \mathbf{R}$  a random variable and  $p : \Omega \rightarrow \mathbf{R}$  a distribution. The *mean square* of  $x$  is  $\mathbf{E}(x^2)$ . Define  $\mu = \mathbf{E}(x)$ . We can express  $\mathbf{E}(x^2) = \mathbf{E}(x)^2 + \mathbf{cov}(x)$  since

$$\begin{aligned}\mathbf{cov}(x) &= \mathbf{E}((x - \mu)^2) = \mathbf{E}(x^2 - 2\mu x + \mu^2) \\ &= \mathbf{E}(x^2) - 2\mu\mathbf{E}(x) + \mu^2 \\ &= \mathbf{E}(x^2) - \mu^2.\end{aligned}$$

We refer to this relation as the *mean-variance decomposition* of  $x$ .<sup>1</sup>

The *n'th moment* of  $x$  is  $\mathbf{E}(x^n)$ . The mean is the first moment. The covariance is the second moment minus the square of the first moment.

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<sup>1</sup>Future editions will modify this sheet, and likely motivate this decomposition in terms of minimum square error estimation.



