



Why

We are frequently interested in finding minimizers of real functions.¹

Definition

An *optimization problem* is a pair (\mathcal{X}, f) in which \mathcal{X} is a nonempty set called the *constraint set* and $f : \mathcal{X} \rightarrow \mathbf{R}$ is called the *objective* (or *cost function*).

If \mathcal{X} is finite we call the optimization problem *discrete*. If $\mathcal{X} \subset \mathbf{R}^d$ we call the optimization problem *continuous*.

We refer to all elements of the constraint set as *feasible*. We refer to an element $x \in \mathcal{X}$ of the constraint set as *optimal* if $f(x) = \inf_{z \in \mathcal{X}} f(z)$. We also refer to optimal elements as *solutions* of the optimization problem.

It is common for f and \mathcal{X} to depend on some other, known, given objects. In this case, these objects are often called *parameters* or *problem data*.

Notation

We often write optimization problems as

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathcal{X}. \end{array}$$

In this case we call x the *decision variable*.

¹Future editions will modify and expand.

Extended reals

It is common to let $f : \mathcal{X} \rightarrow \bar{\mathbf{R}}$, and allow there to exist $x \in \mathcal{X}$ for which $f(x) = \infty$. This technique can be used to embed further constraints in the objective. For example, we interpret $f(x) = +\infty$ to mean x is *infeasible*.

Maximization

If we have some function $g : \mathcal{X} \rightarrow \bar{\mathbf{R}}$ that we wish to maximize, we can always convert it to an optimization problem by defining $f : \mathcal{X} \rightarrow \bar{\mathbf{R}}$ by $f(x) = -g(x)$. In this case g is often called a *reward* (*utility*, *profit*).

Solvers

Let $\mathcal{P} = \{(X_a, f_a : X_a \rightarrow \bar{\mathbf{R}})\}_{a \in A}$ be a family of optimization problems. A *solver* (or *solution method*, *solution algorithm*) for \mathcal{P} is a function $S : A \rightarrow \mathcal{S}$ such that S_a is a solution of the problem (X_a, f_a) .

Loosely speaking, the difficulty of “solving” the optimization problem (\mathcal{X}, f) depends on the properties of \mathcal{X} and f and the problem “size”. For example, when $\mathcal{X} \subset \mathbf{R}^d$ the difficulty is related to the “dimension” d of $x \in \mathcal{X}$.

