Why

We want to extend our notion of entropy (see Discrete Entropy) to real-valued (continuous) random variables.

Definition

The *differential entropy* of a probability density function is the integral of the density against the negative log of the density. This definition made to be similar to the case of discrete entropy. If a real-valued random variable has a density, then we call the differential entropy of its density the *differential entropy* of the random variable.

Notation

Let $f : \mathbf{R}^n \to \mathbf{R}$ be a probability density function. The differential entropy of f is

$$- \int f \log f$$

We denote the differential entropy of f by $h(f)$.

Example

Let $x : \Omega \to \mathbf{R}$ be uniform on $[0, 1/2]$. Then $h(x) = \log 1/2 < 0$.

Problems

We have $h(ax) = h(x) + \log|a|$. In general $h(Ax) = h(x) + \log|A|$.

Differences still meaningful

Even though the value of the differential entropy is not necessarily a good analogy to discrete entropy, differences still are. In particular, the following holds

$$I(X;Y) = H(Y) - H(Y \mid X) = H(X) = H(X \mid Y)$$

Differential Entropy

Multivariate Real Densities

Probability Densities

N-Dimensional Space

Distance

Real Integrals

Nonnegative Integrals

Simple Integrals

Measurable Functions

Measures

Sigma Algebras

Extended Real Numbers

Real Series

Space Distance

Plane Distance

Real Limits

Absolute Value

Simple Functions

Discrete Entropy

Interval Length

Real Functions

Probability Distributions

Real Space

Real Plane

Intervals

Logarithm

Real Line

Geometry

Sequences

Integral Line

Real Sequences

Real Order

Real Summation

Real Numbers

Rational Order

Subset Algebras

Cardinality

Lists

Rational Numbers

Set Numbers

Integer Arithmetic

Finite Sets

Direct Products

Natural Additive Identity

Natural Multiplicative Identity

Integer Products

Equivalent Sets

Identity Elements

Arithmetic

Natural Summation

Natural Power

Function Inverses

Family Unions and Intersections

Family Operations

Comparisons

Integer Order

Integer Sums

Natural Products

Function Composites

Function Images

Families

Operations

Orders

Integer Numbers

Natural Sums

Equivalence Relations

Functions

Converse Relations

Natural Order

Recursion Theorem

Relations

Peano Axioms

Subset Systems

Ordered Pairs

Set Products

Natural Induction

Characteristic Functions

Uncertain Outcomes

Set Powers

Generalized Set Dualities

Natural Numbers

Partitions

Unordered Triples

Intersection of Empty Set

Successor Sets

Set Dualities

Set Symmetric Differences

Set Unions and Intersections

Set Intersection

Pair Unions

Set Complements

Pair Intersection

Set Unions

Set Differences

Unordered Pairs

Empty Set

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets

Identities

Names

Letters

Objects