

REAL MULTIPLICATIVE INVERSES

Why

What is the multiplicative inverse in the reals?

Result

We can show the following.¹

Proposition 1. The multiplicative inverse of $R \in \mathbb{R}$, $R \neq 0_{\mathbb{R}}$,

1. if $0_{\mathbf{Q}} \in R$, then

$$S = \{ q \in \mathbf{Q} \mid q \le 0_{\mathbf{Q}} \} \cup \left\{ r^{-1} \mid \exists s < r, (r \notin R) \right\}$$

is a multiplicative inverse of R.

2. if $0_{\mathbf{Q}} \notin R$, then case (1) applies to -R. Let S be the multiplicative inverse of -R. Then the additive inverse of S, i.e., -S is a multiplicative inverse of R.

Notation

We denote the multiplicative inverse of $r \in \mathbf{R}$ by r^{-1} . We denote $q \cdot (r^{-1})$ by q/r.

Division

We call the operation $(a, b) \mapsto a/b$ real division. We call the product of a and the multiplicative inverse of b the (real) quotient of a and b.

 $^{^{1}\}mathrm{The}$ account will appear in future editions.

