



LDU Factorizations

1 Why

An *LDU factorization* of a $n \times n$ positive definite matrix permuted by $\{1, 2, \dots, n\}$ is an ordered pair whose first coordinate is a unit lower triangular matrix and whose second coordinate is a positive diagonal matrix, with the property that the given matrix permuted according to the permutation is equal to the product of the unit lower triangular matrix, the diagonal matrix, and the transpose of the unit lower triangular matrix.

2 Notation

Let (L, D) be an LDU factorization of $A \in \mathbf{S}_{++}^n$ permuted by the permutation $\sigma : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$. Then

$$P_\sigma A P_\sigma^\top = LDL^\top.$$

Alternatively,

$$A = P_\sigma^\top LDL^\top P_\sigma.$$

3 Existence

Proposition 1. *For every positive definite matrix $A \in \mathbf{S}_{++}^n$ and permutation $\sigma : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$ there exists a unique LDU factorization of A permuted by σ .*