Why

What is the natural generalization of a smooth function to functions defined on sets of $\mathbf{R}^k$.[1]

Definition

Let $U \subset \mathbf{R}^d$ be an open set (see Real Open Sets). A function $f : U \to \mathbf{R}$ is *smooth* if all its partial derivatives exists and are continuous.

More generally, let $X \subset \mathbf{R}^d$. A function $f : X \to R$ is *smooth* if there exists an open set $U \subset \mathbf{R}^d$ and a smooth $F : U \to \mathbf{R}$ so that $F(x) = f(x)$ for all $x \in U \cap X$.

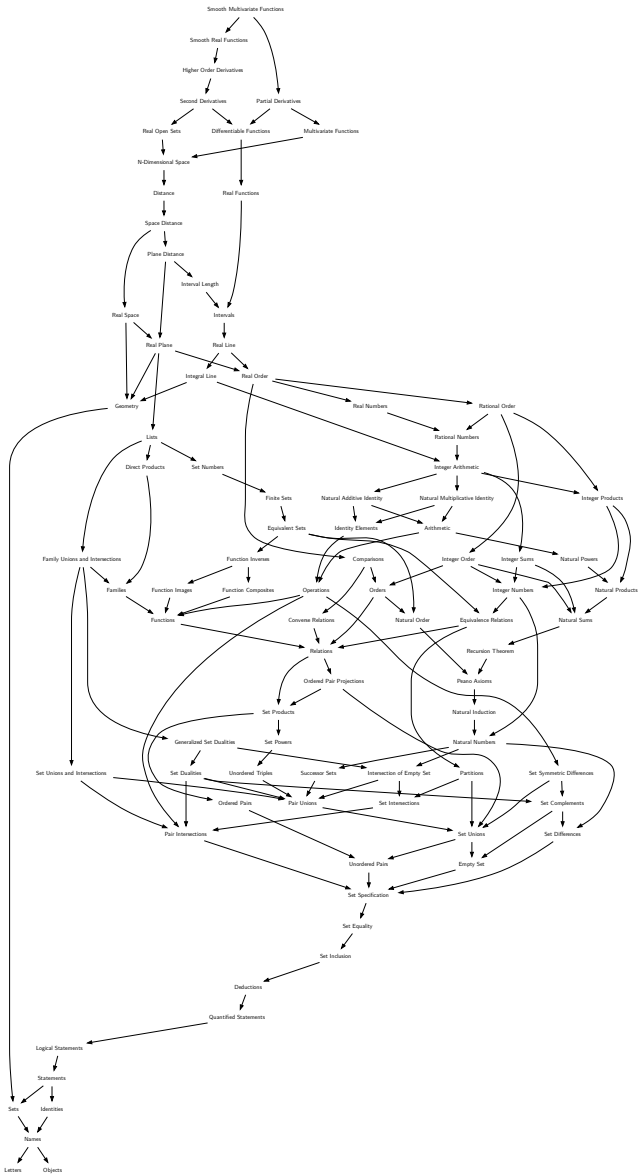
Example

The identity map is smooth. In other words, let $f : \mathbf{R}^d \to \mathbf{R}$ be so that $X \subset \mathbf{R}^d$. Then $f : X \to \mathbf{R}$ s

Properties

Proposition 1. *The composition of two smooth functions is smooth.*

[1] Future editions will expand.


Smooth Multivariate Functions
Smooth Real Functions
Higher Order Derivatives
Second Derivatives
Partial Derivatives
Real Open Sets
Differentiable Functions
Multivariate Functions
N-Dimensional Space
Distance
Real Functions
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Geometry
Real Numbers
Rational Order
Rational Numbers
Lists
Integer Arithmetic
Direct Products
Set Numbers
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Comparisons
Integer Order
Integer Sums
Natural Powers
Families
Function Images
Function Composites
Operations
Orders
Integer Numbers
Natural Products
Functions
Converse Relations
Natural Order
Equivalence Relations
Natural Sums
Relations
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Symmetric Differences
Ordered Pairs
Pair Unions
Set Intersections
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects