



## REAL SUMS

### Why

We want to add real numbers.<sup>1</sup>

### Definition

The *real sum* of two real numbers  $R$  and  $S$  is the set

$$\{t \in \mathbf{Q} \mid \exists r \in R, s \in S \text{ with } t = r + s\}.$$

### Notation

We denote the sum of two real numbers  $x$  and  $y$  by  $x + y$ .

### Properties

We can show the following.<sup>2</sup>

**Proposition 1** (Associative).  $x + (y + z) = (x + y) + z$

**Proposition 2** (Commutative).  $x + y = y + x$

**Proposition 3** (Identity). *The set of negative rational numbers is the additive identity.*

We denote the additive identity of  $\mathbf{R}$  under  $+$  by  $0_{\mathbf{R}}$ . When it is clear from context, we call  $0_{\mathbf{R}}$  “zero”.

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<sup>1</sup>Future editions will expand.

<sup>2</sup>Accounts will appear in future editions.



