Why

The inverse of a function interacts nicely with family unions, family intersections and complements.

Results

Let $f : X \to Y$. Throughout this sheet, let $f^{-1} : Y^* \to X^*$. And take $\{B_i\}$ to be a family of subsets of Y.[1]

Proposition 1. $f^{-1}(\cup_i B_i) = \cup_i f^{-1}(B_i)$

Proposition 2. $f^{-1}(\cup_i B_i) = \cap_i f^{-1}(B_i)$

Proposition 3. $f^{-1}(Y - B) = X - f^{-1}(B)$

Properties for Function Image

Notice that $f(\cup_i A_i) = \cup_i f(A_i)$ but not for interesctions. Nor is there a similar correspondence for complements. There are some relations, which we list below.[2]

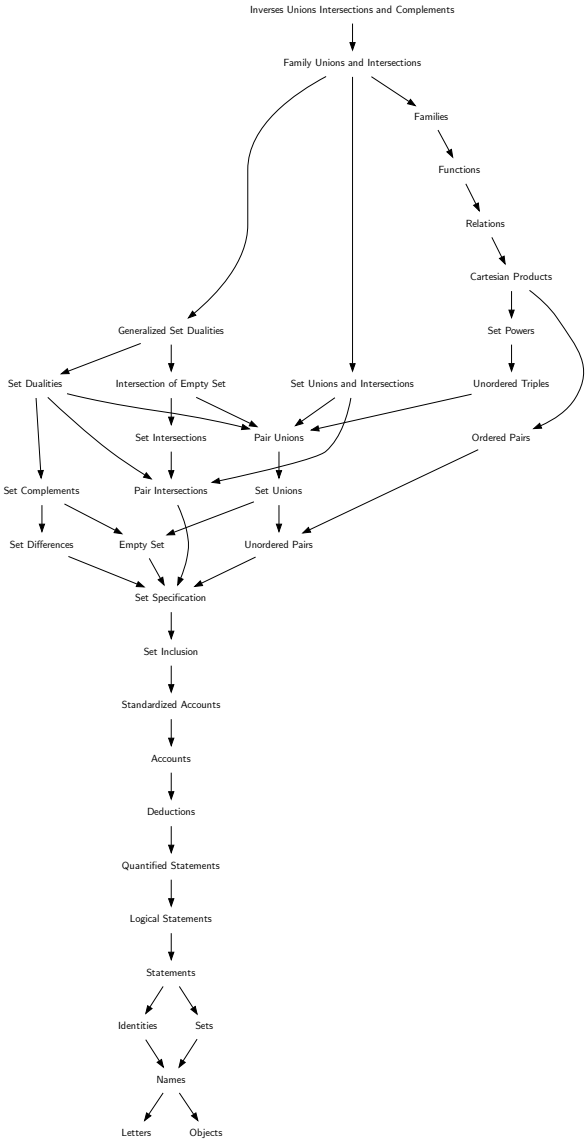
Proposition 4. *$f(A \cap B) = f(A) \cap f(B)$ if and only if f is one-to-one.*

Proposition 5. *For all $A \subset X$, $f(X - A) = Y - f(A)$ if and only if f is one-to-one.*

Proposition 6. *For all $A \subset X$, $Y - f(A) \subset f(X - A)$ if and only if f is onto.*

[1] The proofs of the following will appear in future editions.

[2] Accounts of these facts will appear in future editions.


Inverses Unions Intersections and Complements
Family Unions and Intersections
Families
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