Why

Do the integer numbers correspond (in the sense of Homomorphisms) to elements of the rationals.

Main result

Indeed, roughly speaking the integers correspond to rationals whose denominator is 1. Define

$$\tilde{Q} := \{[(a, b)] \in \mathbf{Q} \mid b = 1_{\mathsf{Z}}\}.$$

Proposition 1. *The rings* $(\tilde{\mathbf{Q}}, +_{\mathbf{Q}} \mid \tilde{\mathbf{Q}}, \cdot_{\mathbf{Q}} \mid \tilde{\mathbf{Q}})$ *and* $(Z, +_{\mathsf{Z}}, \cdot_{\mathsf{Z}})$ *are homomorphic.*[1]

Proof. The function is $f : \mathbf{Z} \to \mathbf{Q}$ with $f(z) = [(z, 1)]$.[2] □

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[1]Indeed, more is true and will be included in future editions. There is an *order perserving* ring homomorphism.

[2]The full account will appear in future editions.

Integer Rational Homomorphism

Rational Arithmetic
Rational Multiplicative Inverses
Rational Additive Inverses

Rational Products
Rational Sums

Rational Numbers

Integer Arithmetic

Integer Products
Natural Additive Identity
Natural Multiplicative Identity

Arithmetic
Identity Elements
Integer Additive Inverses

Natural Powers
Integer Sums
Inverse Elements

Natural Products
Integer Numbers
Element Functions
Function Inverses

Natural Sums
Operations
Function Composites
Function Images

Recursion Theorem
Equivalence Relations
Functions

Peano Axioms
Relations

Natural Induction
Set Products

Natural Numbers
Set Powers
Ordered Pairs

Successor Sets
Unordered Triples
Intersection of Empty Set
Partitions

Set Symmetric Differences
Pair Unions
Set Intersections

Set Complements
Set Unions
Pair Intersections

Set Differences
Empty Set
Unordered Pairs

Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Identities
Sets

Names

Letters
Objects