Sequences

1 Why

We introduce language for the steps of an infinite process.

2 Definition

Let A be a non-empty set. A **sequence in** A is a function from the natural numbers to the set. The n**th term** of a sequence is the result of nth natural number; it is an element of the set.

A **subindex** is a monotonically increasing function from and to the natural numbers. Roughly, it selects some ordered infinite subset of natural numbers. A **subsequence** of a first sequence is any second sequence which is the composition of the first sequence with a subindex.

Another way of describing a sequence is as an element of the direct product of a family of indentical sets indexed by the natural numbers.

2.1 Notation

Keep A as a non-empty set. Denote the natural numbers by N. Let $a : N \to A$. Then a is a sequence and $a(n)$ is the nth term. We denote a by $\{a_n\}_n$ and $a(n)$ by a_n.

Let $i : N \to N$ such that $n < m \implies i(n) < i(m)$. Then i is a subindex. Let $b = a \circ i$. Then b is a subsequence of a. We denote it by $\{b_{i(n)}\}_n$ and the nth term by $b_{i(n)}$.