Why

Let $X = \{a, b\}$ and $Y = \{0, 1\}$. The dataset $((a, 0))$ is consistent, but it is not functionally complete. On the other hand, the dataset $((a, 0), (b, 0), (a, 0), (a, 0), (a, 0), (a, 1))$ is complete but it is not *functionally* consistent.

In general, if $y_i \neq y_j$ for some i and j where $x_i = x_j$, then the dataset is not functionally consistent. In the preceding example, both $(a, 0)$ and $(a, 1)$ appear.

If we emphasize the "predictive" aspect of a functional inductor, we interpret the input as an object we "see before" the output. And so treat $y \in Y$ as an uncertain outcome which is the element associated to $x \in X$.

In this case, we may use the language of probability to discuss this uncertain outcome. If, for example, Y is finite, we can associate a distribution with each input $x \in X$.

Definition

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable spaces.

A *probabilistic functional inductor* (for a dataset of size n in $X \times Y$) is a function mapping a dataset in $(X \times Y)^n$ to a family of measures on $(Y, \mathcal{Y})$, indexed by X. We call a function from inputs to output measures a *probabilistic predictor*. We call the distribution a *probabilistic prediction*.

Notation

Let $\mathcal{M}(Y, \mathcal{Y})$ be the set of measures on Y. Let D be a dataset in $(X \times Y)^n$. Let $g : X \to \mathcal{M}(Y, \mathcal{Y})$ a probabilistic predictor. Let $G_n(X \times Y)^n \to (X \to \mathcal{M}(Y, \mathcal{Y}))$ be a predictive probabilistic inductor. Then $G_n(D)$ is a family of measures $\{g_x : \mathcal{Y} \to [0, 1]\}_{x \in X}$.

Probabilistic Inductors

Probabilistic Dataset Models

Inductors — Probability Measures

Event Probabilities — Measures

Extended Real Numbers — Probability Distributions

Real Limits — Intervals

Datasets

Real Line

Real Sequences — Real Summation — Real Order

Real Numbers

Rational Numbers — Integral Line

Sequences — Set Numbers — Integer Arithmetic

Direct Products — Natural Additive Identity — Natural Multiplicative Identity

Finite Sets — Natural Summation — Identity Elements — Arithmetic — Sigma Algebras

Family Operations — Equivalent Sets — Natural Exponents — Subset Algebras — Integer Products

Family Unions and Intersections — Function Inverses — Algebras — Set Operations — Natural Products — Integer Sums — Cardinality

Families — Function Composites — Function Images — Operations — Natural Sums — Integer Numbers

Functions — Equivalence Relations — Natural Order — Recursion Theorem

Relations — Peano Axioms

Cartesian Products — Subset Systems — Uncertain Outcomes — Natural Induction

Ordered Pairs — Generalized Set Dualities — Set Powers — Natural Numbers

Set Unions and Intersections — Partitions — Intersection of Empty Set — Set Dualities — Unordered Triples — Successor Sets — Set Symmetric Differences

Set Intersections — Pair Unions — Set Complements

Pair Intersections — Set Unions — Set Differences

Unordered Pairs — Empty Set

Set Specification — Geometry

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects