Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\text{cov}(f, g)$. We defined it:

$$\text{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\mathbf{E}(g).$$

Properties

Proposition 1. *Covariance is symmetric and billinear.*

Proposition 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\text{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \text{var}(f).$$

$\square$

Proposition 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \ldots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \ldots, n$. Using the billinearity,

$$\text{var} \sum_{i=1}^n f_i = \text{cov}\left(\sum_{i=1}^n f_i, \sum_{i=1}^n f_i\right)$$

$$= \sum_{i=1}^n \sum_{j=1}^n \text{cov}(f_i, f_j)$$

$\square$

Covariance

Variance

Expectation

Random Variable

Topological Sigma Algebra — Measurable Function — Probability Measure

Measure — Topological Space — Outcome Variable Covariance

Sigma Algebras — Metrics — Outcome Variable Expectation

Extended Real Numbers — Real Series — Distance — Induced Probability Distribution — Real Square Roots

Real Limits — Outcome Variable Events — Space Distance — Real Square

Event Probability — Plane Distance — Absolute Value — Real Arithmetic — Natural Square Roots

Probability Distribution — Interval Length — Real Multiplicative Inverse

Intervals — Real Products — Real Space — Square Numbers

Real Plane — Real Line — Real Addition Inverse

Real Sequences — Real Summation — Rational Multiplicative Inverse — Real Order — Real Sums — Rational Arithmetic — Integral Line — Squares

Subset Algebras — Sequences — Rational Order — Rational Products — Rational Sums — Geometry

Cardinality — Lists — Rational Numbers

Set Numbers — Direct Products — Uncertain Outcome — Integer Arithmetic — Integer Products

Finite Sets — Natural Additive Identity — Natural Multiplicative Identity

Equivalent Sets — Inverse Elements — Natural Summation — Identity Elements — Arithmetic

Family Unions and Intersections — Function Inverses — Family Operations — Element Functions — Comparison — Integer Sums — Natural Powers

Families — Function Composition — Function Images — Operations — Integer Order — Natural Products — Equations

Functions — Converse Relations — Equivalence Relations — Order — Integer Numbers — Natural Sums — Natural Equations

Relations — Natural Order

Subset Systems — Set Products — Recursion Theorem

Ordered Pairs — Set Powers — Peano Axioms

Set Unions and Intersections — Generalized Set Dualities — Unordered Triples — Natural Numbers — Natural Induction

Partitions — Intersection of Empty Set — Set Dualities — Successor Sets — Set Symmetric Differences

Set Intersections — Set Complements

Pair Intersections — Pair Unions — Set Union — Set Differences

Unordered Pairs — Empty Set

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities — Sets

Names

Letters — Objects