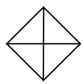

Definition

The *covariance* between two random variables which are each integrable and whose product is integrable is the expectation of their product less the product of their expectation.

Notation

Let f and g be two integrable random variables with fg integrable. Denote the covariance of f with g by $\operatorname{cov}(f, g)$. We defined it:

$$\operatorname{cov}(f, g) = \mathbf{E}(fg) - \mathbf{E}(f)\mathbf{E}(g).$$

Properties

Proposition 1. *Covariance is symmetric and billinear.*

Proposition 2. *The covariance of a random variable with itself is its variance.*

Proof. Let f be a square-integrable real-valued random variable, then

$$\operatorname{cov}(f, f) = \mathbf{E}(ff) - \mathbf{E}(f)\mathbf{E}(f) = \mathbf{E}(f^2) - (\mathbf{E}(f))^2 = \operatorname{var}(f).$$

□

Proposition 3. *The variance of a sum of integrable real-valued random variables whose pairwise products are integrable is the double sum of the pairwise covariances.*

Proof. Let $f_1, \dots, f_n$ be integrable random variables with $f_i f_j$ integrable for all $i, j = 1, \dots, n$. Using the billinearity,

$$\begin{aligned} \operatorname{var} \textstyle\sum_{i=1}^n f_i &= \operatorname{cov}(\sum_{i=1}^n f_i, \sum_{i=1}^n f_i) \\ &= \sum_{i=1}^n \sum_{j=1}^n \operatorname{cov}(f_i, f_j) \end{aligned}$$

□


Covariance
Variance
Expectation
Random Variables
Topological Sigma Algebra
Measurable Functions
Probability Measures
Measures
Topologies
Outcome Variable Covariance
Sigma Algebras
Metrics
Outcome Variable Expectation
Extended Real Numbers
Real Series
Distance
Induced Probability Distributions
Real Square Roots
Real Limits
Outcome Variable Events
Space Distance
Real Squares
Event Probabilities
Plane Distance
Absolute Value
Real Arithmetic
Probability Distributions
Interval Length
Real Multiplicative Inverses
Intervals
Real Products
Real Space
Natural Square Roots
Real Plane
Real Line
Real Additive Inverses
Square Numbers
Real Sequences
Real Summation
Rational Multiplicative Inverses
Real Order
Real Sums
Rational Arithmetic
Integral Line
Squares
Subset Algebras
Sequences
Real Numbers
Rational Order
Rational Products
Rational Sums
Geometry
Cardinality
Lists
Rational Numbers
Set Numbers
Direct Products
Uncertain Outcomes
Integer Arithmetic
Finite Sets
Natural Additive Identity
Natural Multiplicative Identity
Integer Products
Equivalent Sets
Inverse Elements
Natural Summation
Identity Elements
Arithmetic
Family Unions and Intersections
Function Inverses
Family Operations
Element Functions
Comparisons
Integer Order
Integer Sums
Natural Powers
Natural Equations
Families
Function Composites
Function Images
Operations
Orders
Integer Numbers
Natural Products
Equations
Functions
Converse Relations
Equivalence Relations
Natural Order
Natural Sums
Relations
Recursion Theorem
Subset Systems
Set Products
Peano Axioms
Ordered Pairs
Set Powers
Natural Induction
Generalized Set Dualities
Unordered Triples
Natural Numbers
Set Unions and Intersections
Partitions
Intersection of Empty Set
Set Dualities
Successor Sets
Set Symmetric Differences
Set Intersections
Pair Unions
Set Complements
Pair Intersections
Set Unions
Set Differences
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects