



## Why

Does a set exist containing the elements shared between two sets? How might we construct such a set?

## Definition

Let  $A$  and  $B$  denote sets. Consider the set  $\{x \in A \mid x \in B\}$ . This set exists by the principle of specification (see **Set Specification**). Moreover  $(y \in \{x \in A \mid x \in B\}) \longleftrightarrow (y \in A \wedge y \in B)$ . In other words,  $\{x \in A \mid x \in B\}$  contains all the elements of  $A$  that are also elements of  $B$ .

We can also consider  $\{x \in B \mid x \in A\}$ , in which we have swapped the positions of  $A$  and  $B$ . Similarly, the set exists by the principle of specification (see **Set Specification**) and again  $y \in \{x \in B \mid x \in A\} \longleftrightarrow (y \in B \wedge y \in A)$ . Of course,  $y \in A \wedge y \in B$  means the same as<sup>1</sup>  $y \in B \wedge y \in A$  and so by the principle of extension (see **Set Equality**)

$$\{x \in A \mid x \in B\} = \{x \in B \mid x \in A\}.$$

We call this set the *pair intersection* of the set denoted by  $A$  with the set denoted by  $B$ .

## Notation

We denote the intersection of the set denoted by  $A$  with the set denoted by  $B$  by  $A \cap B$ . We read this notation aloud as “A intersect B”.

## Basic properties

All the following results are immediate.<sup>2</sup>

**Proposition 1.**  $A \cap \emptyset = \emptyset$

**Proposition 2** (Commutativity).  $A \cap B = B \cap A$

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<sup>1</sup>Future editions will name and cite this rule.

<sup>2</sup>Proofs of these results will appear in the next edition.

**Proposition 3** (Associativity).  $(A \cap B) \cap C = A \cap (B \cap C)$

**Proposition 4.**  $A \cap A = A$

**Proposition 5.**  $(A \subset B) \longleftrightarrow (A \cap B = A).$

