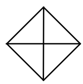


Definition

The *trace* of a square real matrix is the sum of its diagonal entries.

Notation

We denote the function which associates a matrix with its trace by $\mathrm{tr} : \mathbf{R}^{n\times n} \to \mathbf{R}$. The trace of $A \in \mathbf{R}^{n\times n}$ is

$$\mathrm{tr}\, A = \sum\_i = 1^n A\_ii.$$

Properties

Proposition 1. *The trace is a linear function on the vector space of $n\times n$ real matrices.*

Proof. Let $A, B \in \mathbf{R} Matnn$ and $\alpha, \beta \in \mathbf{R}$. Define $C = \alpha A + \beta B$. Then $C_{ii} = \alpha A_{ii} + \beta B_{ii}$. So

$$\begin{aligned} \mathrm{tr}\, C = \sum\_i = 1^n C\_ii = \sum\_i = 1^n \alpha A\_ii + \beta B\_ii \\ = \alpha \sum\_i = 1^n A\_ii + \beta \sum\_i = 1^n B\_ii \\ = \alpha\, \mathrm{tr}\, A + \beta\, \mathrm{tr}\, B. \end{aligned}$$

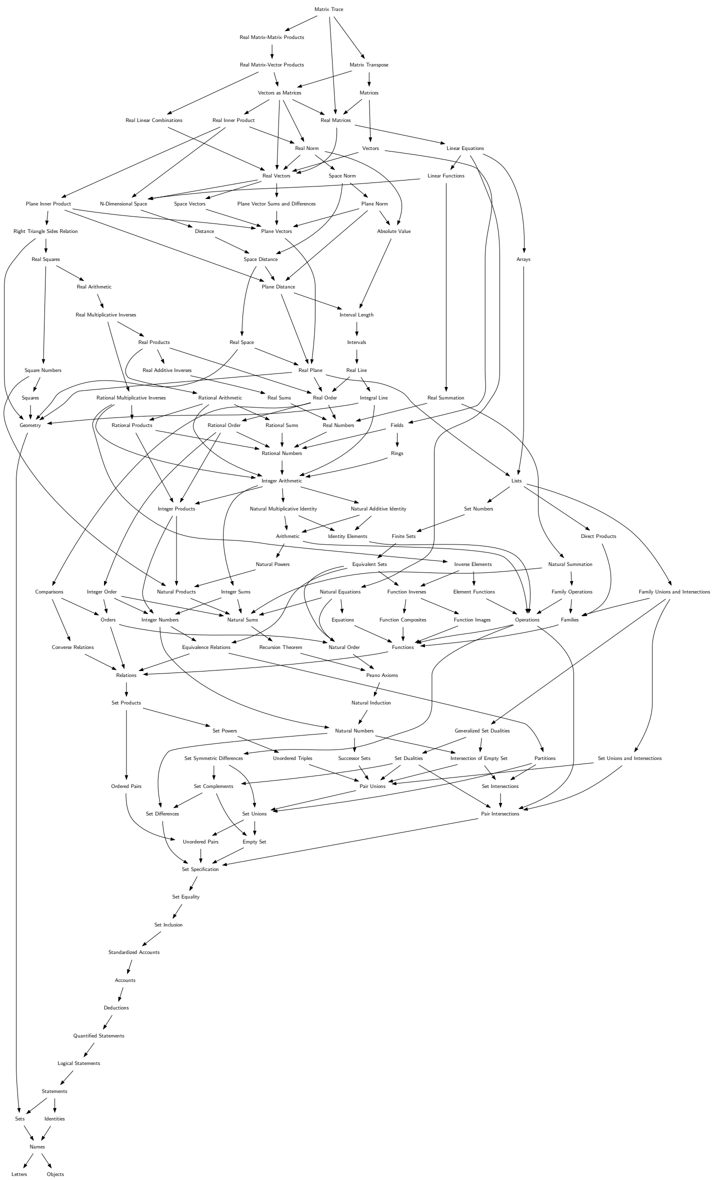
□

Proposition 2. *Let $A, B \in \mathbf{R}^{n\times n}$. Then*

$$\mathrm{tr}\,(AB) = \mathrm{tr}\,(BA)\,.$$

In other words, "matrices commute under the trace operator."

Proposition 3. *Let $A \in \mathbf{R}^{n\times n}$. Then $\mathrm{tr}\, A = \mathrm{tr}\, A^\top$.*


Matrix Trace
Real Matrix-Matrix Products
Real Matrix-Vector Products
Matrix Transpose
Vectors as Matrices
Matrices
Real Linear Combinations
Real Inner Product
Real Matrices
Real Norm
Vectors
Linear Equations
Real Vectors
Space Norm
Linear Functions
Plane Inner Product
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Plane Norm
Right Triangle Sides Relation
Distance
Plane Vectors
Absolute Value
Real Squares
Space Distance
Arrays
Real Arithmetic
Plane Distance
Real Multiplicative Inverses
Interval Length
Real Products
Real Space
Intervals
Square Numbers
Real Additive Inverses
Real Plane
Real Line
Squares
Rational Multiplicative Inverses
Rational Arithmetic
Real Sums
Real Order
Integral Line
Real Summation
Geometry
Rational Products
Rational Order
Rational Sums
Real Numbers
Fields
Rational Numbers
Rings
Integer Arithmetic
Lists
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Set Numbers
Arithmetic
Identity Elements
Finite Sets
Direct Products
Natural Powers
Equivalent Sets
Inverse Elements
Natural Summation
Comparisons
Integer Order
Natural Products
Integer Sums
Natural Equations
Function Inverses
Element Functions
Family Operations
Family Unions and Intersections
Orders
Integer Numbers
Natural Sums
Equations
Function Composites
Function Images
Operations
Families
Converse Relations
Equivalence Relations
Recursion Theorem
Natural Order
Functions
Relations
Peano Axioms
Set Products
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Symmetric Differences
Unordered Triples
Successor Sets
Set Dualities
Intersection of Empty Set
Partitions
Set Unions and Intersections
Ordered Pairs
Set Complements
Pair Unions
Set Intersections
Set Differences
Set Unions
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects