Why

Under which operations is the set of measurable functions closed?

Overview

Measurable functions are closed under composition and concatenation. Taking these facts together with the observation that continuous functions are measurable, we conclude that measurable functions are closed under addition, multiplication, and (with suitable nonzero assumptions) division.

Results

Proposition 1. *The composition of two measurable functions is measurable.*

Proof. Let $(X, \mathcal{A})$, $(Y, \mathcal{B})$, $(Z, \mathcal{C})$ be measurable spaces. Let $f : X \to Y$ and $g : Y \to Z$ be measurable functions. Define $h = g \circ f$.

Let $C \in \mathcal{C}$. Measurability of g implies $g^{-1}(C) \in \mathcal{B}$. This fact, together with measurability of f, implies $f^{-1}(g^{-1}(C)) \in \mathcal{A}$. Since $h^{-1} = f^{-1} \circ g^{-1}$, we conclude that h is measurable. $\square$

Proposition 2. *A continuous function between topological spaces is measurable with respect to the topological sigma algebras.*

Proposition 3. *The concatentation of two measurable functions is measurable.*

Proof. Let $(X, \mathcal{A})$, $(Y, \mathcal{B})$ and $(Z, \mathcal{C})$ be measurable spaces. Let $f : X \to Y$ and $g : X \to Z$ be measurable. Define $h : X \to Y \times Z$ by $h(x) = (f(x), g(x))$. $\square$

Proposition 4. *Let $(X, \mathcal{A})$ be a measurable space and let R denote the real numbers. Let $f, g : X \to R$ be measurable.*

Then, $f + g$ and fg are measurable.

Measurable Function Operations

Measurable Functions

Measures

Length Measure   Sigma Algebras

Real Series   Extended Real Numbers

Real Limits

Absolute Value

Interval Length

Intervals

Real Line

Real Order   Real Sequences

Subset Algebras   Rational Order   Real Numbers   Sequence

Cardinality   Integral Line   Lists

Set Numbers   Rational Numbers   Direct Products

Finite Sets   Integer Arithmetic

Natural Additive Identity   Natural Multiplicative Identity   Natural Fractions

Equivalent Sets   Integer Products   Identity Elements   Natural Arithmetic

Comparisons   Function Inverses   Integer Order   Integer Sums   Operations   Natural Powers   Family Unions and Intersections

Orders   Function Image   Function Composites   Integer Numbers   Natural Products   Families

Converse Relations   Natural Order   Functions   Equivalence Relations   Natural Sums

Relations   Recursion Theorem

Ordered Pair Projections   Peano Axioms

Subset Systems   Set Products   Natural Induction   Generalized Set Dualities

Ordered Pairs   Set Powers   Natural Numbers   Set Symmetric Differences   Set Dualities   Set Unions and Intersections

Partitions   Unordered Triples   Intersection of Empty Set   Successor Sets

Set Intersections   Pair Unions   Set Complements

Set Unions   Set Differences   Pair Intersections

Unordered Pairs   Empty Set

Geometry   Set Specification

Set Equality

Set Inclusion

Deductions

Quantified Statements

Logical Statements

Statements

Sets   Identities

Names

Letters   Objects

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