

INTEGRABLE FUNCTION SPACES

Why

We have seen that the integrable functions form a vector space.

How about the square integrable functions? TODO: perhaps

do L^2 first then generalize.

Definition

The integrable function spaces are a collection of function spaces,

one for each real number $p \geq 1$, for which the pth power of the

absolute value of the function is integrable.

TODO: case ∞

Notation

Let (X, \mathcal{A}, μ) be a measure space. Let $p \geq 1$. Let R denote the

set of real numbers. We denote the integrable function space

corresponding to p by $\mathcal{L}^p(X, \mathcal{A}, \mu, R)$. We have defined it by

 $\mathcal{L}^p(X, \mathcal{A}, \mu, R) = \left\{ \text{ measurable } f: X \to R \mid \int |f|^p d\mu < \infty \right\}$

Let C denote the set of complex numbers. Similarly for

complex-valued functions, we denote the pth space by $\mathcal{L}^p(X, \mathcal{A}, \mu, C)$.

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