



### Definition

Let  $(X, \mathbf{R})$  be a vector space. A function  $f : X \times X \rightarrow \mathbf{R}$  is an *inner product* on the vector space  $(X, \mathbf{R})$  if

1.  $f(x, x) \geq 0, = 0 \iff x = 0,$
2.  $f(x + y, z) = f(x, z) + f(y, z),$
3.  $f(x, y) = f(y, x),$  and
4.  $f(\alpha x, y) = \alpha f(x, y).$

An *inner product space* is an ordered pair: a real vector space and an inner product.<sup>1</sup>

### Examples

$\mathbf{R}^n$  with the usual inner product is an inner product space. Some authors call any finite-dimensional inner product space over the real numbers is a *Euclidean vector space*.

### Notation

If  $f : X \times X \rightarrow \mathbf{R}$  is an inner product we regularly denote  $f(x, x)$  by  $\langle x, x \rangle$ .

### Orthogonality

Two vectors in an inner product space are *orthogonal* if their inner product is zero. An *orthogonal family of vectors* in an inner product space is a family of vectors for which distinct family members are orthogonal.

A vector is *normalized* if its inner product with itself is one.

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<sup>1</sup>Future editions will discuss complex inner products.



