



Why

Since every affine set is a translate of a unique subspace, we can represent them by representing the vector and the subspace.

Definition

Recall that M is affine means $M = S + a$ for some subspace S and vector $a \in \mathbf{R}^n$. The dimension of M is the dimension of the subspace. Suppose $\dim S = k$, then there exists $Q \in \mathbf{R}^{n \times k}$ with $Q^\top Q = I$, so that for any $x \in S$, there exists unique $z \in \mathbf{R}^k$ with $x = Qz$. Since $M = S + a$, we have

$$M = \{y \in \mathbf{R}^n \mid (\exists z \in \mathbf{R}^k)(y = a + Qz)\}$$

We also denote this set $\{a + Qz \mid z \in \mathbf{R}^k\}$.

