

## **DISTORTION FUNCTIONS**

## Why

We want to quantify the error of compressing a real-valued random variable.

## **Definition**

Let  $\mathcal{X}$  be a finite set and  $q: \mathbf{R} \to \mathcal{X}$  a quantization (see Quantizations) of  $\mathbf{R}$ . Also, fix a probability space  $(\Omega, \mathcal{A}, \mathbf{P})$  and a random variable  $x: \Omega \to \mathbf{R}$ .

The compression  $\hat{x}: \Omega \to \mathcal{X}$  of x under q is  $q \circ x$ . A distortion function for x and  $\hat{x}$  is a function

$$d: (\Omega \to \mathbf{R}) \times (\Omega \to \mathcal{X}) \to \mathbf{R}.$$

Roughly speaking, a distortion function is meant to quantify the error in using this compression.

## **Examples**

The expected mean-squared-error distortion  $d_{\text{mse}}$  between x and  $\hat{x}$  is

$$d_{\text{mse}}(x, \hat{x}) = \mathbf{E}[(x - \hat{x})^2]$$

The Kulback-Liebler distortion  $d_{kld}$  defined by

$$d_{\mathrm{kld}}(x,\hat{x}) = \mathsf{E}[d_{\mathrm{kl}}(\mathsf{P}(y \in \cdot \mid x,\hat{x}) \mid \mathsf{P}(y \in \cdot \mid \hat{x}))]$$

where y is some random variable that depends on x.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Future editions will clarify this sentence.

