Measure Space

1 Why

We want to generalize the notions of length, area, and volume.

2 Definition

A **measurable space** is a sigma algebra. We call the distinguished subsets the **measurable sets**.

A **measure** on a measurable space is a function from the sigma algebra to the positive extended reals. A **measure space** is a measurable space and a measure.

2.1 Notation

2.2 Properties

Proposition 1. *Let $(A, \mathcal{A})$ be a measurable space and $m : \mathcal{A} \to [0, \infty]$ be a measure.*

*If $B \subset C \subset A$, then $m(B) \leq m(C)$. We call this property the of measures **monotonicity of measure**.*

Proposition 2. *For a measure space* $(A, \mathcal{A}, m)$.

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Proposition 3. *For a measure space* $(A, \mathcal{A}, m)$.

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We this property the ***sub-additivty of measure***.

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2.3 Examples

Example 7. ***counting measure***