Why

If a directed graph has no cycles, then it has a nice property.[1]

Definition

Directed and acyclic graphs (sometimes $DAGs$) have some useful properties. Clearly, every subgraph induced on a directed acyclic graph is a directed acyclic graph.

Proposition 1. *Let (V, E) be a directed acyclic graph. Then there exists a vertex $v \in V$ which is a source and a vertex $w \in V$ which is a sink.*

Proof. There exists a directed path of maximum length. It must start at a source and end at a sink.[2] □

A *topological numbering*, *topological sort* or *topological ordering* of a directed graph (V, E) is a numbering $\sigma : \{1, \ldots, \nu m V\} \to V$ satisfying

$$(v, w) \in E \longrightarrow \sigma^{-1}(v) < \sigma^{-1}(w).$$

3

Proposition 2. *There exists a topological sort for every acyclic graph.*

Proof. Let (V, F) be a directed acyclic graph. There exists a source vertex, v_1. Set $\sigma(1) = v_1$. Take the subgraph induced by $V - \{v_1\}$. It is directed acyclic, and so has a source vertex, v_2. Set $\sigma(2) = v_2$. Continue in this way.[4] □

---

[1]Future editions will expand this vague introduction.
[2]Future editions will expand.
[3]Future editions will further explain this concept.
[4]Future editions will clarify and expand.

Directed Acyclic Graphs

Directed Paths

Directed Graphs

Undirected Graphs

Set Numbers

Finite Sets

Equivalent Sets

Function Inverses   Natural Order

Function Composites   Function Images   Peano Axioms

Functions   Equivalence Relations

Relations

Set Products   Natural Induction

Set Powers   Natural Numbers

Unordered Triples   Partitions   Successor Sets   Intersection of Empty Set

Ordered Pairs   Pair Unions   Set Intersections

Set Unions   Set Differences

Unordered Pairs   Empty Set   Pair Intersections

Set Specification

Set Equality

Set Inclusion

Standardized Accounts

Accounts

Deductions

Quantified Statements

Logical Statements

Statements

Identities   Sets

Names

Letters   Objects

4