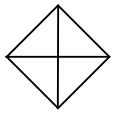

Tree Density Approximation

1 Why

2 Problem

Tree-structured density approximation is an optimization problem in which we select a tree-structured density minimizing differential entropy relative to a given density over.

2.1 Notation

Let $g : \mathbf{R}^n \to \mathbf{R}$ be a density. Let d denote the differential relative entropy. We want to

$$\begin{aligned} \textbf{find} \quad & \text{density } f \text{ and tree } T \text{ on } \{1, \ldots, n\} \\ \textbf{minimizing} \quad & d(g, f) \\ \textbf{subject to} \quad & f \text{ factors according to } T. \end{aligned}$$

3 Solution

Proposition 1. *Let $g : \mathbf{R}^n \to \mathbf{R}$ be a density. Let T be a tree on $\{1, \ldots, n\}$. Let p_j be the parent of vertex j for the T rooted at vertex i, $j = 1, \ldots, n$ and $j \neq i$. Then the density $f : \mathbf{R}^d \to \mathbf{R}$ defined by*

$$f = g_i \prod_{j \neq i} g_{j|p_j}$$

achieves minimum differential entropy relative to g among all densities which factor according to T.

Proposition 2. *Let $g : \mathbf{R}^n \to \mathbf{R}$ be a density. A tree T is a solution to the problem above if and only if it is a maximum spanning tree of the differential mutual information graph of g.*