



Tree Distributions

1 Why

It happens that if a distribution factors according to a tree rooted at a particular vertex, it factors according to a tree rooted at any vertex. So the property of factoring according to a rooted tree is really property of factoring according to a tree.

2 Definition

2.1 Defining Result

Proposition 1. *Let A_1, \dots, A_n be finite non-empty sets and define $A = \prod_{i=1}^n A_i$. Let $p : A \rightarrow [0, 1]$ be a distribution and let T be a tree on $\{1, \dots, n\}$.*

Proposition 2. *If a distribution factors according to a tree rooted at a vertex it factors according to that tree rooted at any vertex.*

2.2 Undirected Definition

A distribution p *factors according to the tree T* if it factors according to the T rooted at any vertex.

3 Existence and Uniqueness

Trees are not a property of distributions, since there is no one-to-one correspondence, as demonstrated by the following propositions.

3.1 Existence

A distribution p need not factor according to a tree.

3.2 Uniqueness

A distribution p may factor according to multiple trees.