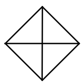


Why

Can we represent the function associating a linear system with its row reduction by matrix multiplication?

Main observation

The following proposition affirmatively answers the question.

Proposition 1. *Let $(A \in \mathbf{R}^{m \times m}, b \in \mathbf{R}^m)$ be a linear system with $A_{kk} \neq 0$ and (C, d) the kth reduction of (A, b). Then there exists a matrix $L \in \mathbf{R}^{m \times m}$ so that $C = LA$ and $d = Lb$.*

Proof. Define $L \in \mathbf{R}^{m \times m}$ by $L_{st} = 1$ if $s = t$, $-A_{sj}/A_{ij}$ if $k < s \leq m$ and zero otherwise. □

For this reason, we call L in Proposition 1 a *row reducer matrix* or *row reducing matrix* or *row reducer*. The row reducer matrix for the kth reduction of (A, b) has the form

$$L_k = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & {}^{A_{ik}}\!/\!{}_{A_{kk}} & 1 & & \\ & & \vdots & & \ddots & \\ & & {}^{A_{mk}}\!/\!{}_{A_{kk}} & & & 1 \end{bmatrix}$$

So the following is immediate

Prop. 2. *Row reducing matrices are unit lower triangular.*

Example

For example, the $(1,1)$-reduction of (A,b) in which

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 6 & 7 & 9 & 5 \\ 8 & 7 & 9 & 8 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

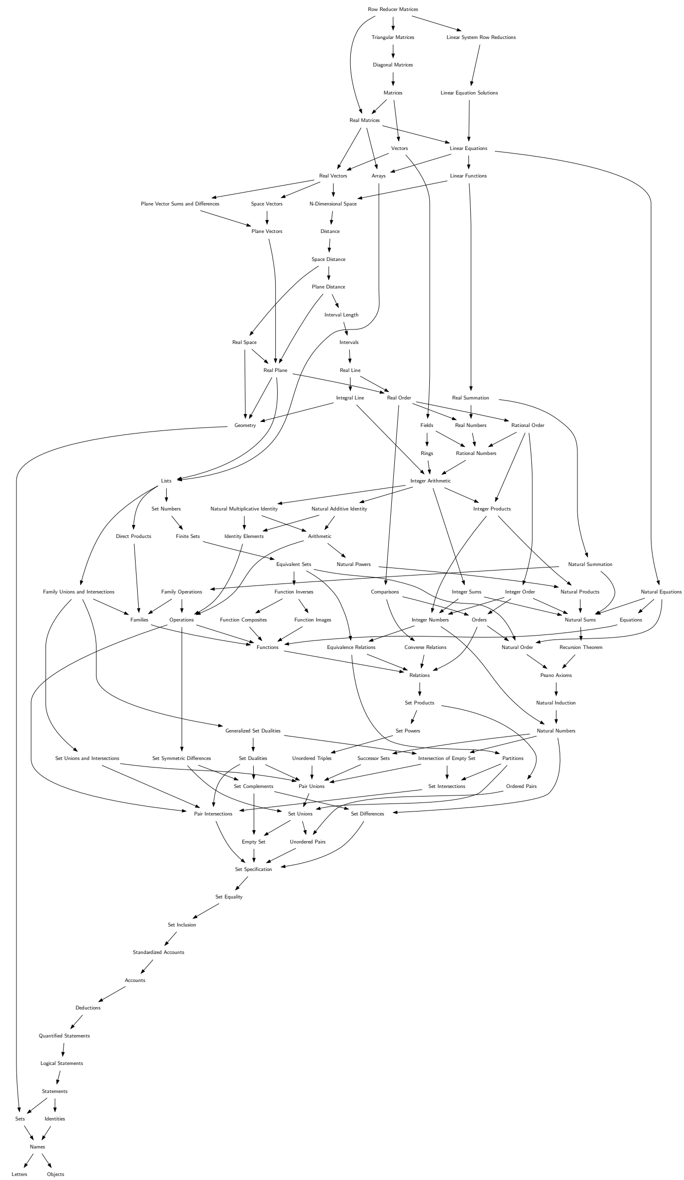
is the linear system

$$A' = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix} \text{ and } b' = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

The row reducer is $L \in R^{4\times4}$ defined by

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix}.$$

One can check that $A' = LA$ and $b' = Lb$, and clearly L is unit lower triangular.


Row Reducer Matrices
Triangular Matrices
Linear System Row Reductions
Diagonal Matrices
Matrices
Linear Equation Solutions
Real Matrices
Vectors
Linear Equations
Real Vectors
Arrays
Linear Functions
Plane Vector Sums and Differences
Space Vectors
N-Dimensional Space
Plane Vectors
Distance
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Plane
Real Line
Integral Line
Real Order
Real Summation
Geometry
Fields
Real Numbers
Rational Order
Rings
Rational Numbers
Lists
Integer Arithmetic
Set Numbers
Natural Multiplicative Identity
Natural Additive Identity
Integer Products
Direct Products
Finite Sets
Identity Elements
Arithmetic
Equivalent Sets
Natural Powers
Natural Summation
Family Unions and Intersections
Family Operations
Function Inverses
Comparisons
Integer Sums
Integer Order
Natural Products
Natural Equations
Families
Operations
Function Composites
Function Images
Integer Numbers
Orders
Natural Sums
Equations
Functions
Equivalence Relations
Converse Relations
Natural Order
Recursion Theorem
Relations
Peano Axioms
Set Products
Natural Induction
Generalized Set Dualities
Set Powers
Natural Numbers
Set Unions and Intersections
Set Symmetric Differences
Set Dualities
Unordered Triples
Successor Sets
Intersection of Empty Set
Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Standardized Accounts
Accounts
Deductions
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Logical Statements
Statements
Sets
Identities
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Letters
Objects