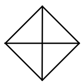


Why

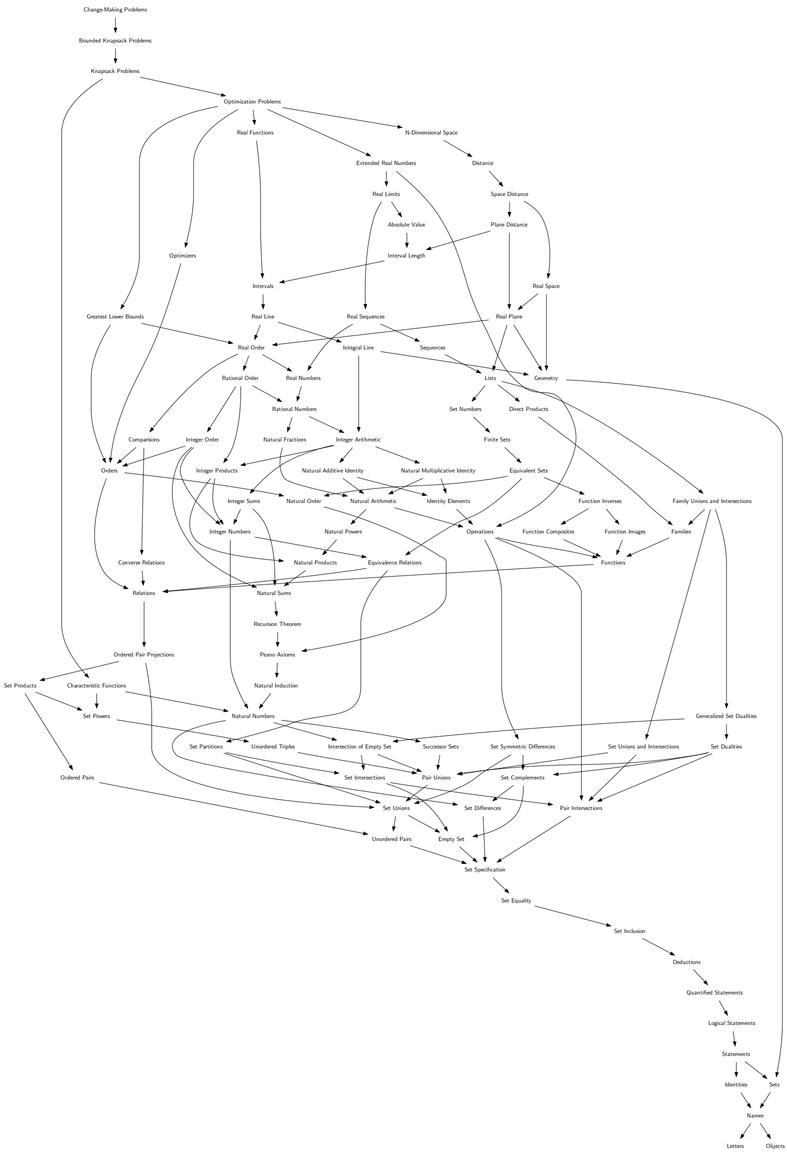
Suppose a cashier needs to provide $c \in \mathbf{Z}_+$ cents in change, and wants to do so using the using the fewest (or most) number of coins, each worth a different number of cents. We can model this as a problem similar to the bounded knapsack problem, in which we have an equality constraint instead of an inequality one.

Definition

Given $w : \{1, \dots, n\} \to \mathbf{R}_+$, $b \in \mathbf{Z}_+^n$, find $x \in \mathbf{Z}_+^n$ to

$$\begin{aligned} \text{minimize} \quad & \sum_i x_i \\ \text{subject to} \quad & \sum_{j=1}^n w_j x_j = c \\ & 0 \leq x \leq b, x \in \mathbf{Z}_n^+ \end{aligned}$$

This problem is often called a *change-making problem.* Without the budget constraints, it is called an *unbounded change-making problem.*


Change-Making Problems
Bounded Knapsack Problems
Knapsack Problems
Optimization Problems
Real Functions
N-Dimensional Space
Extended Real Numbers
Distance
Real Limits
Space Distance
Absolute Value
Plane Distance
Optimizers
Interval Length
Intervals
Real Space
Greatest Lower Bounds
Real Line
Real Sequences
Real Plane
Real Order
Integral Line
Sequences
Rational Order
Real Numbers
Lists
Geometry
Rational Numbers
Set Numbers
Direct Products
Comparisons
Integer Order
Natural Fractions
Integer Arithmetic
Finite Sets
Orders
Integer Products
Natural Additive Identity
Natural Multiplicative Identity
Equivalent Sets
Integer Sums
Natural Order
Natural Arithmetic
Identity Elements
Function Inverses
Family Unions and Intersections
Integer Numbers
Natural Powers
Operations
Function Composites
Function Images
Families
Converse Relations
Natural Products
Equivalence Relations
Functions
Relations
Natural Sums
Recursion Theorem
Ordered Pair Projections
Peano Axioms
Set Products
Characteristic Functions
Natural Induction
Set Powers
Natural Numbers
Generalized Set Dualities
Set Partitions
Unordered Triples
Intersection of Empty Set
Successor Sets
Set Symmetric Differences
Set Unions and Intersections
Set Dualities
Ordered Pairs
Set Intersections
Pair Unions
Set Complements
Set Unions
Set Differences
Pair Intersections
Unordered Pairs
Empty Set
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Letters
Objects