



## Multivariate Normals

### 1 Why

We generalize the normal density to  $d$ -dimensional space.

### 2 Definition

Let  $f : \mathbf{R}^d \rightarrow \mathbf{R}$  be a density such that

$$f(x) = \frac{1}{\sqrt{(2\pi)^d \mathbf{det} \Sigma}} \exp \left( -\frac{1}{2} (x - \mu)^\top \Sigma^{-1} (x - \mu) \right)$$

where  $\mu \in \mathbf{R}^d$ ,  $\Sigma \in \mathbf{S}^d$ , and  $\Sigma \succ 0$ . We call  $f$  a *multivariate normal density*. A multivariate normal density with  $d = 1$  is a normal density, so we refer to multivariate normal densities as *normal densities* without ambiguity. We frequently use the word normal as a substantive, and refer to *normals* when we mean multivariate normal densities. Some people call a multivariate normal distribution a *multivariate gaussian distribution* and speak of *gaussians* instead of normals.

We call  $\mu$  the *mean* and  $\Sigma$  the *covariance matrix*. We call  $\Sigma^{-1}$  the *precision matrix*.