



# Matroids

## 1 Why

We generalize the notion of linear dependence.

## 2 Definition

A *matroid* is a finite subset algebra satisfying:

1. The subset of a distinguished set is distinguished.
2. For two distinguished subsets of nonequal cardinality, there is an element of the base set in the complement of the smaller set in the bigger set whose singleton union with the smaller set is a distinguished set.

An *independent subset* of a matroid is a distinguished subset. A *dependent subset* of a matroid is an undistinguished subset.

### 2.1 Notation

We follow the notation of subset algebras, but use  $M$  for the base set, a mnemonic for matroid, and  $\mathcal{I}$  for the distinguished sets, a mnemonic for independent.

Let  $(M, \mathcal{I})$  a matroid. We denote the properties by

1.  $A \in \mathcal{I} \wedge B \subset A \implies B \in \mathcal{I}$ .

$$2. \ A, B \in \mathcal{I} \wedge |A| < |B| \implies \exists x \in M : (A \cup \{x\}) \in \mathcal{I}$$