Families

1 Why

TODO

2 Definition

We often refer to a collection of subsets of some set via some index which is an element of another set. We call the indexing set the **index set** and set of indexed sets an **indexed family**. We define the set whose elements are the objects which are contained in at least one family member the **family union**. We define the set whose elements are the obejcts which are contained in all of the family members the **family intersection**.

2.1 Notation

Let S be a set. We often denote the index set by I, though this is not required. For each $\alpha \in I$, let $A_\alpha \subset S$. We denote the family of A_α indexed with I by $\{A_\alpha\}_{\alpha \in I}$. We read this notation "A sub-alpha, alpha in I."

We denote the family union by $\cup_{\alpha \in I} A_\alpha$. We read this notation as "union over alpha in I of A sub-alpha." We denote family intersection by $\cap_{\alpha \in I} A_\alpha$. We read this notation as "intersection over alpha in I of A sub-alpha."

2.2 Results

Proposition 1 *For an indexed family $\{A_\alpha\}_{\alpha \in I}$ in S, if $I = \{i, j\}$ then*

$$\cup_{\alpha \in I} A_\alpha = A_i \cup A_j$$

and

$$\cap_{\alpha \in I} A_\alpha = A_i \cap A_j.$$

Proposition 2 *For an indexed family $\{A_\alpha\}_{\alpha \in I}$ in S, if $I = \emptyset$, then*

$$\cup_{\alpha \in I} A_\alpha = \emptyset$$

and

$$\cap_{\alpha \in I} A_\alpha = S.$$

Proposition 3 *For an indexed family $\{A_\alpha\}_{\alpha \in I}$ in S.*

$$C_S(\cup_{\alpha \in I} A_\alpha) = \cap_{\alpha \in I} C_S(A_\alpha)$$

and

$$C_S(\cap_{\alpha \in I} A_\alpha) = \cup_{\alpha \in I} C_S(A_\alpha).$$