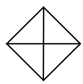


Why

We abstract the notion of inner product to an arbitrary vector space.

Definition

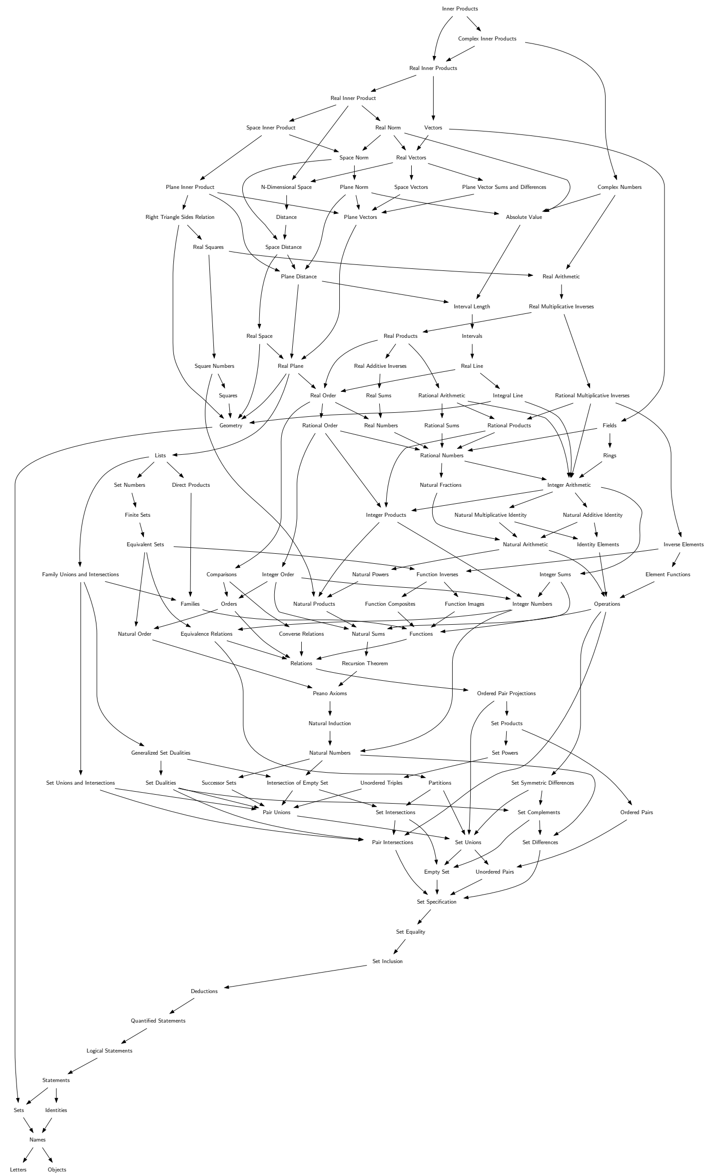
Suppose F is a field which is either R or $\mathbf{C}$. Let (V, F) be a vector space. Then a function $f : V \times V \to \mathsf{F}$ is an *inner product* on V if

1. $f(x, x) \geq 0$, $f(x, x) = 0 \Leftrightarrow x = 0$;
2. $f(x, y) = \overline{f(y, x)}$
3. $f(ax + by, z) = a(x, z) + b(y, z)$

A *inner product space* (or *pre-Hilbert space*) is a tuple (V, f) where V is an inner product space over F and $f : V^2 \to \mathsf{F}$ is an inner product.

Notation

Suppose V is a vector space over the field F. We regularly denote an arbitrary inner product for V by $\langle \cdot, \cdot \rangle : V^2 \to \mathsf{F}$. So we would denote the inner product of the vector x with the vector y by $\langle x, y \rangle$.


Inner Products
Complex Inner Products
Real Inner Products
Real Inner Product
Space Inner Product
Real Norm
Vectors
Space Norm
Real Vectors
Plane Inner Product
N-Dimensional Space
Plane Norm
Space Vectors
Plane Vector Sums and Differences
Complex Numbers
Right Triangle Sides Relation
Distance
Plane Vectors
Absolute Value
Real Squares
Space Distance
Plane Distance
Real Arithmetic
Interval Length
Real Multiplicative Inverses
Real Space
Real Products
Intervals
Square Numbers
Real Plane
Real Additive Inverses
Real Line
Squares
Real Order
Real Sums
Rational Arithmetic
Integral Line
Rational Multiplicative Inverses
Geometry
Rational Order
Real Numbers
Rational Sums
Rational Products
Fields
Lists
Rational Numbers
Rings
Set Numbers
Direct Products
Natural Fractions
Integer Arithmetic
Finite Sets
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Equivalent Sets
Natural Arithmetic
Identity Elements
Inverse Elements
Family Unions and Intersections
Comparisons
Integer Order
Natural Powers
Function Inverses
Integer Sums
Element Functions
Families
Orders
Natural Products
Function Composites
Function Images
Integer Numbers
Operations
Natural Order
Equivalence Relations
Converse Relations
Natural Sums
Functions
Relations
Recursion Theorem
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Partitions
Set Symmetric Differences
Pair Unions
Set Intersections
Set Complements
Ordered Pairs
Pair Intersections
Set Unions
Set Differences
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Letters
Objects