Functions: Maths JEE

G V V Sharma*

1. The values of $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lie in the interval.......
2. For the function f(x) = $\frac{x}{1+e^{1/x}}$, x $\neq$ 0 and f(x) = 0, x = 0 the derivative from the right, $f^{'}(0+)$= ...., and the derivative from the left, $f^{'}(0-)$= ....
3. The domain of the funtion f(x)=$\sin^{-1}\left(log_2\frac{x^2}{2}\right)$ is given by ....
4. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is.......and out of these.......are onto functions.
5. If
$$f(x) = \sin ln\Big(\frac{\sqrt{4-x^2}}{1-x}\Big),$$
then domain of f(x) is.... and its range is......
6. There are exactly two distinct linear functions.......,and......which map [-1,1] onto [0,2].
7. If f is an even function defined on the interval (-5,5), then four real values of x satisfying the equation f(x)=$f(\frac{x+1}{x+2})$ are.........., ..., ..., and......
8. If
$$f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + cosxcos\Big(x + \frac{\pi}{3}\Big)$$
and g($\frac{5}{4}$) = 1, then (gof)(x)= ....
9. If f(x)=$(a - x^n)^{1/n}$ where $a > 0$ and n is a positive integer, then f[f(x)]=x.
10. The function f(x)=$\frac{x^2+4x+30}{x^2-8x+18}$ is not one-to-one.
11. If $f_1(x)$ and $f_2(x)$ are defined on domains D_1 and D_2 respectively, then $f_1(x) + f_2(x)$ is defined on $D_1 \cup D_2$.
12. Let R be the set of real numbers. If f:R $\rightarrow$ R is a function defined by f(x)=x^2, then f is :
    a) Injective but not surjective
    b) Surjective but not injective
    c) Bijective
    d) None of these.
13. The entire graphs of the equation y = $x^2 + kx - x + 9$ is stirctly above the x-axis if and only if
    a) $k < 7$
    b) $-5 < k < 7$
    c) $k > -5$
    d) None of these.
14. Let f(x)= $\left|x - 1\right|$. Then
    a) $f(x^2) = (f(x))^2$
    b) f(x+y)=f(x)+f(y)
    c) $f(\left|x\right|) = \left|f(x)\right|$
    d) None of these
15. If x satisfies $\left|x-1\right| + \left|x-2\right| + \left|x-3\right| \geq 6$,then

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

a) $0 \leq x \leq 4$
b) $x \leq -2$ or $x \geq 4$
c) $x \leq 0$ or $x \geq 4$
d) None of these

16. If f(x)=cos(lnx), then f(x)f(y) - $\frac{1}{2}\left[f(\frac{x}{y}) + f(xy)\right]$ has the value
a) -1
b) $\frac{1}{2}$
c) -2
d) none of these

17. The domain of definition of the function y=$\frac{1}{log_{10}(1-x)}$ + $\sqrt{x+2}$ is
a) (-3,2) excluding -2.5
b) $[0, 1]$ excluding 0.5
c) $[-2, 1)$ excluding 0
d) none of these

18. Which of the following functions is periodic?
a) f(x)=x-[x] where [x] denotes the largest integer less than or equal to the real number x
b) f(x)=sin $\frac{1}{x}$ for $x \neq 0$, f(0)=0
c) f(x)=xcos x
d) none of these

19. Let f(x)=sin x and g(x) = ln $\left|x\right|$. If the ranges of the composition functions fog and gof are R_1 and R_2 respectively, then
a) $R_1 = \{u : -1 \leq u < 1\}, R_2 = \{v : -\infty < v < 0\}$
b) $R_1 = \{u : -\infty < u < 0\}, R_2 = \{v : -1 \leq v \leq 0\}$
c) $R_1 = \{u : -1 < u < 1\}, R_2 = \{v : -\infty < v < 0\}$
d) $R_1 = \{u : -1 \leq u \leq 1\}, R_2 = \{v : -\infty < v \leq 0\}$

20. Let $f(x) = (x+1)^2 - 1$, x ≥ -1. Then the set $\{x : f(x) = f^{-1}(x)\}$ is
a) $\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\}$
b) $\{0, 1, -1\}$
c) $\{0, -1\}$
d) empty

21. The function f(x) = $\left|px - q\right|$ + r$\left|x\right|$, $x \in (-\infty, \infty)$ where p > 0, q > 0, r > 0 assumes its minimum value only on one point if
a) p $\neq$ q
b) r $\neq$ q
c) r $\neq$ p
d) p = q = r

22. Let $f(x)$ be defined for all $x > 0$ and be continuos. Let $f(x)$ satisfy $f(\frac{x}{y}) = f(x) - f(y)$ for all x,y and $f(e) = 1$. Then
a) $f(x)$ is bounded
b) $f(\frac{1}{x}) \to 0$ as $x \to 0$
c) $xf(x) \to 1$ as $x \to 0$
d) $f(x) = lnx$

23. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is
a) $\frac{1}{2}^{x(x-1)}$
b) $\frac{1}{2}(1 + \sqrt{1 + 4log_2 x})$
c) $\frac{1}{2}(1 - \sqrt{1 + 4log_2 x})$
d) not defined

24. Let $f : R \to$ R be any function. Define g: R $\to$ R by $g(x) = |f(x)|$ for all x. Then g is
   a) onto if f is onto
   b) one-one if f is one-one
   c) continuos if f is continuos
   d) differentiable if f is differentiable
25. The domain of definition of the function $f(x)$ given by the equation $2^x + 2^y = 2$ is
   a) $0 < x \leq 1$
   b) $0 \leq x \leq 1$
   c) $-\infty < x \leq 0$
   d) $-\infty < x < 1$
26. Let $g(x) = 1 + x - [x]$ and

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0. \\ 1, & x > 0 \end{cases}$$

   then for all x, $f(g(x))$ is equal to
   a) x
   b) 1
   c) $f(x)$
   d) $g(x)$
27. If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals
   a) $(x + \sqrt{x^2 - 4})/2$
   b) $x/(1 + x^2)$
   c) $(x - \sqrt{x^2 - 4})/2$
   d) $1 + \sqrt{x^2 - 4}$
28. The domain of definition of $f(x) = \frac{log_2(x+3)}{x^2+3x+2}$ is
   a) $R\backslash\{-1, -2\}$
   b) $(-2, \infty)$
   c) R$\backslash\{-1, -2, -3\}$
   d) $(-3, \infty)\backslash\{-1, -2\}$
29. Let E=$\{1, 2, 3, 4\}$ and F=$\{1, 2\}$. Then the number of onto functions from E to F is
   a) 14
   b) 16
   c) 12
   d) 8
30. Let $f(x) = \frac{\alpha x}{x+1}$,x $\neq$ -1. Then, for what value of α is $f(f(x)) = x$?
   a) $\sqrt{2}$
   b) $-\sqrt{2}$
   c) 1
   d) -1
31. Suppose $f(x) = (x + 1)^2$ for x $\geq$ -1. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line y=x then $g(x)$ equals
   a) $-\sqrt{x} - 1, x \geq 0$
   b) $\frac{1}{(x+1)^2}, x > -1$
   c) $\sqrt{x + 1}, x \geq -1$
   d) $\sqrt{x} - 1, x \geq 0$
32. Let function $f : R \to$ R be defined by $f(x) = 2x + \sin x$ for x $\in$ R, then f is

a) one-to-one and onto
b) one-to-one but NOT onto
c) onto but NOT one-to-one
d) neither one-to-one nor onto

33. If $f : [0, \infty) \to [0, \infty)$, and $f(x) = \frac{x}{1+x}$ then f is
a) one-one and onto
b) one-one but not onto
c) onto but not one-one
d) neither one-one nor onto

34. Domain of the definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued x ,is
a) $[-\frac{1}{4}, \frac{1}{2}]$
b) $[-\frac{1}{2}, \frac{1}{2}]$
c) $[-\frac{1}{2}, \frac{1}{9}]$
d) $[-\frac{1}{4}, \frac{1}{4}]$

35. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}; x \epsilon \mathrm{R}$ is
a) $(1, \infty)$
b) $(1, \frac{11}{7}]$
c) $(1, \frac{7}{3}]$
d) $(1, \frac{7}{5}]$

36. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that min $f(x) > maxg(x)$, then the relation between b and c, is
a) no real value of b & c
b) $0 < c < b\sqrt{2}$
c) $\left|c\right| < \left|b\right| \sqrt{2}$
d) $\left|c\right| > \left|b\right| \sqrt{2}$

37. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain
a) $[0, \frac{\pi}{2}]$
b) $[-\frac{\pi}{4}, \frac{\pi}{4}]$
c) $[-\frac{\pi}{2}, \frac{\pi}{2}]$
d) $[0, \pi]$

38. If the functions $f(x)$ and $g(x)$ are defined on R→R such that

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

;

$$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

then $(f - g)(x)$ is
a) one-one & onto
b) neither one-one nor onto
c) one-one but not onto
d) onto but not one-one

39. X and Y are two sets and $f : X \to Y$. If $\{f(c) = y; c \subset X, y \subset Y\}$ and $\{f^{-1}(d) = x; d \subset Y, x \subset X\}$, then the true statement is
a) $f(f^{-1}(b))$ = b
b) $f^{-1}(f(a))$ = a

c) $f(f^{-1}(b))$ = b,b $\subset$ y
d) $f(f^{-1}(a))$ = a,a $\subset$ x

40. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ where $f''(x) = -f(x)$ and $g(x) = f'(x)$ and given that F(5)=5, then F(10) is equal to
a) 5
b) 10
c) 0
d) 15

41. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for n $\geq$ 2 and $g(x) = \underbrace{(fofo......of)}_{\text{f occurs n times}}(x)$. Then $\int x^{n-2}g(x)dx$ equals.
a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$
b) $\frac{1}{(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$
c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$
d) $\frac{1}{(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$

42. Let f, g and h be real-valued functions defined on the interval $[0,1]$ by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f,g and h on $[0,1]$, then
a) a = b and c $\neq$ b
b) a = c and a $\neq$ b
c) a $\neq$ b and c $\neq$ b
d) a = b = c

43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in$ R. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is
a) $\pm\sqrt{n\pi}$, n $\in \{0,1,2.....\}$
b) $\pm\sqrt{n\pi}$, n $\in \{1,2.....\}$
c) $\frac{\pi}{2} + 2n\pi$, n $\in \{.....-2,-1,0,1,2.....\}$
d) $2n\pi$, n $\in \{.....-2,-1,0,1,2.....\}$

44. The function $f : [0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is
a) one-one and onto
b) onto but not one-one
c) one-one but not onto
d) neither one-one nor onto

45. If y=$f(x) = \frac{x+2}{x-1}$ then
a) $x = f(y)$
b) $f(1)$=3
c) y increases with x for $x < 1$
d) f is a rational function of x

46. Let $g(x)$ be a function defined on $[-1,1]$. If the area of the equilateral triangle with two of its vertices at (0,0) and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is
a) $g(x) = \pm\sqrt{1-x^2}$
b) $g(x) = \sqrt{1-x^2}$
c) $g(x) = -\sqrt{1-x^2}$
d) $g(x) = \sqrt{1+x^2}$

47. If f(x)=cos$[\pi^2]x + \cos[-\pi^2]x$, where [x] stands for the greatest integer function, then
a) $f(\frac{\pi}{2}) = -1$
b) $f(\pi) = 1$

c) $f(-\pi) = 0$
d) $f(\frac{\pi}{4}) = 1$

48. If $f(x)$=3x-5, then $f^{-1}(x)$
a) is given by $\frac{1}{3x-5}$
b) is given by $\frac{x+5}{3}$
c) does not exist because f is not one-one
d) does not exist because f is not onto.

49. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then
a) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
b) $f(x) = \sin x, g(x) = |x|$
c) $f(x) = x^2,\ g(x) = \sin\sqrt{x}$
d) f and g cannot be determined.

50. Let f:(0,1) $\to$ R be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then
a) f is not invertible on (0,1)
b) $f \neq f^{-1}$ on (0,1) and $f^{'}(b) = \frac{1}{f^{'}(0)}$
c) $f = f^{-1}$ on (0,1) and $f^{'}(b) = \frac{1}{f^{'}(0)}$
d) f^{-1} is differentiable (0,1)

51. Let f:(-1,1) $\to$ IR be such that $f(\cos 4\Theta) = \frac{2}{2-\sec^2\Theta}$ for $\Theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the values of $f(\frac{1}{3})$ is
a) 1-$\sqrt{\frac{3}{2}}$
b) 1+$\sqrt{\frac{3}{2}}$
c) 1-$\sqrt{\frac{2}{3}}$
d) 1+$\sqrt{\frac{2}{3}}$

52. The funciton $f(x) = 2|x| + |x+2|$ - $||x+2| - 2|x||$ has a local minimum or a local maximum at x=
a) -2
b) $\frac{-2}{3}$
c) 2
d) $\frac{2}{3}$

53. Let $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \to R$ be given by $f(x) = (log(\sec x + \tan x))^3$. Then
a) $f(x)$ is an odd function
b) $f(x)$ is one-one function
c) $f(x)$ is an onto function
d) $f(x)$ is an even function

54. Let a∈R and let f: R $\to$ R be given by $f(x) = x^5 - 5x + a$. Then
a) $f(x)$ has three real roots if a>4
b) $f(x)$ has only real root if a>4
c) $f(x)$ has three real roots if a<-4
d) $f(x)$ has three real roots if -4<a<4

55. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in$R and $g(x) = \frac{\pi}{2}\sin x$ for all x∈R. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then which of the following is true?
a) Range of f is $[-\frac{1}{2}, \frac{1}{2}]$
b) Range of fog is $[-\frac{1}{2}, \frac{1}{2}]$
c) $\lim_{x\to 0}\frac{f(x)}{g(x)} = \frac{\pi}{6}$
d) There is an $x \in$R such that $(gof)(x) = 1$

56. Find the domain and range of the function $f(x) = \frac{x^2}{1+x^2}$. Is the function one-to-one?

57. Draw the graph of $y = \left|x\right|^{1/2}$ for $-1 \le x \le 1$.
58. If $f(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$, find $f(6)$.
59. Consider the following relations in the set of real numbers R. $R = \{(x, y); x \in R, y \in R, x^2 + y^2 \le 25\}$ $R' = \{(x, y) : x \in R, y \in R, y \ge \frac{4}{9}x^2\}$. Find the domain and the range of $R \cap R'$. Is the relation $R \cap R'$ a function?
60. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B.
61. Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1$, $f(y) \neq 1$,$f(z) \neq 2$ determine $f^{-1}(1)$.
62. Let R be the set of real numbers and f: R → R be such that for all x and y in R$\left|f(x) - f(y)\right| \le \left|x - y\right|^3$. Prove that $f(x)$ is a constant.
63. Find the natural number $'a'$ for which $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, where the function $'f'$ satisfies the relation $f(x + y) = f(x)f(y)$ for all natural numbers x,y and further $f(1) = 2$.
64. Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$.
65. A function $f : IR \to IR$, where IR is the set of real numbers, defined by $f(x) = \frac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$. Find the interval of values of α for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer.
66. Let $f(x) = Ax^2 + Bx + c$ where A,B,C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers 2A,A+B and C are all integers. Conversely, prove that if the numbers 2A,A+B and C are all integers then $f(x)$ is an integer whenever x is an integer.
67. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is
68. The value of $((log_2 9)^2)^{\frac{1}{log_2(log_2 9)}} \times (\sqrt{7})^{\frac{1}{log_4 7}}$ is ......
69. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is ......
70. The domain of $\sin^{-1}[log_3(x/3)]$ is
    a) $[1, 9]$
    b) $[-1, 9]$
    c) $[-9, 1]$
    d) $[-9, -1]$
71. The function $f(x) = log(x + \sqrt{x^2 + 1})$, is
    a) neither an even nor an odd function
    b) an even function
    c) an odd function
    d) a periodic function.
72. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + log_{10}(x^3 - x)$, is
    a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
    b) (a,2)
    c) $(-1, 0) \cup (a, 2)$
    d) $(1, 2) \cup (2, \infty)$.
73. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all x, y ∈ R and f(1)=7, then $\sum_{r=1}^{n} f(r)$ is
    a) $\frac{7n(n+1)}{2}$
    b) $\frac{7n}{2}$
    c) $\frac{7(n+1)}{2}$
    d) $7n + (n + 1)$

74. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$

is

a) neither one-one nor onto
b) one-one but not onto
c) onto but not one-one
d) one-one and onto both.

75. The range of the function $f(x) =^{7-x} P_{x-3}$ is

a) $\{1, 2, 3, 4, 5\}$
b) $\{1, 2, 3, 4, 5, 6\}$
c) $\{1, 2, 3, 4\}$
d) $\{1, 2, 3, \}$

76. Let $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$, is onto, then the interval of S is

a) $[-1, 3]$
b) $[-1, 1]$
c) $[0, 1]$
d) $[0, 3]$

77. The graph of the function $y = f(x)$ is symmetrical about the line x=2, then

a) $f(x) = -f(-x)$
b) $f(2 + x) = f(2 - x)$
c) $f(x) = f(-x)$
d) $f(x + 2) = f(x - 2)$

78. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

a) $[1, 2]$
b) $[2, 3)$
c) $[1, 2]$
d) $[2, 3]$

79. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = tan^{-1} \frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

a) $(0, \frac{\pi}{2})$
b) $[0, \frac{\pi}{2})$
c) $[-\frac{\pi}{2}, \frac{\pi}{2})$
d) $(-\frac{\pi}{2}, \frac{\pi}{2})$

80. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	Function
(a). $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(b). $[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(c). $(-\infty, \frac{1}{3}]$	$3x^2 - 2x + 1$
(d). $(-\infty, -4)$	$x^3 + 6x^2 + 6$

81. A real valued function $f(x)$ satisfies the functional equation

$$f(x - y) = f(x)f(y) - f(a - x)f(a + y)$$

where a is a given constant and $f(0) = 1, f(2a - x)$ is equal to

a) $-f(x)$
b) $f(x)$
c) $f(a) + f(a - x)$
d) $f(-x)$

82. The Largest interval lying in $(\frac{-\pi}{2}, \frac{\pi}{2})$ for which the function,

$$f(x) = 4^{-x^2} + cos^{-1}(\frac{x}{2} - 1) + log(\cos x))$$

, is defined, is

a) $[-\frac{\pi}{4}, \frac{\pi}{2})$
b) $[0, \frac{\pi}{2})$
c) $[0, \pi]$
d) $(-\frac{\pi}{2}, \frac{\pi}{2})$

83. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where

$$Y = \{y \in N : y = 4x + 3 for some x \in N\}$$

a) $g(y) = \frac{3y+4}{3}$
b) $g(y) = 4 + \frac{y+3}{4}$
c) $g(y) = \frac{y+3}{4}$
d) $g(y) = \frac{y-3}{4}$

84. Let $f(x) = (x + 1)^2 - 1$,x $\geq$ -1
Statement-1 : The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}\}$
Statement-2 : f is a bijection

a) Statement-1 is true,Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

85. For real x, let $f(x) = x^3 + 5x + 1$, then

a) f is onto R but not one-one
b) f is one-one and onto R
c) f is neither one-one nor onto R
d) f is one-one but not onto R

86. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is

a) $(0, \infty)$
b) $(-\infty, 0)$
c) $(-\infty, \infty) - \{0\}$
d) $(-\infty, \infty)$

87. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to:

a) $f_3(x)$
b) $f_3(x)$
c) $f_2(x)$
d) $f_1(x)$

88. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:

a) 6

b) 8
c) 4
d) 14

89. If the function $f : R-\{1,-1\}$. A defined by f(x)=$\frac{x^2}{1-x^2}$, is surjective, then A is equal to:
a) R-$\{-1\}$
b) $[0,\infty)$
c) R-$[-1,0)$
d) R-$(-1,0)$

90. Let

$$\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1),$$

where the function f satisifies f(x+y)=f(x)f(y) for all natural numbers x,y and f(a) is = 2. Then the natural number $'a'$ is:
a) 2
b) 16
c) 4
d) 3

Match the following

91. Let the function defined in Colum 1 have domain $(-\frac{\pi}{2},\frac{\pi}{2})$ and range $(-\infty,\infty)$

Column I	Column II
(A) 1+2x	(p) onto but not one-one
(B) $\tan x$	(q) one-one but not onto
	(r) one-one and onto
	(s) neither one-one nor onto

92. Let $f(x)=\frac{x^2-6x+5}{x^2-5x+6}$
Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	Column II
(A) If $-1<x<1$, then $f(x)$ satisfies	(p) $0<f(x)<1$
(B) If $1<x<2$, then $f(x)$ satisfies	(q) $f(x)<0$
(C) If $3<x<5$, then $f(x)$ satisfies	(r) $f(x)>0$
(D) If $x>5$, then $f(x)$ satisfies	(s) $f(x)<1$

93. Let $E_1=\{x\in R : x\neq 1 and \frac{x}{x-1}>0\}$ and $E_2=\{x\in E_1 : \sin^{-1}\left(log_e(\frac{x}{x-1})\right) is a real number\}$. (Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $[-\frac{\pi}{2},\frac{\pi}{2}]$).
Let $f : E_1\to R$ be the function defined by $f(x)=log_e(\frac{x}{x-1})$ and g: $E_2\to$ R be the function defined by $g(x)=sin^{-1}(log_e(\frac{x}{x-1}))$.
The correct option is:
a) $P\to 4; Q\to 2; R\to 1; S\to 1$
b) $P\to 4; Q\to 2; R\to 1; S\to 6$
c) $P\to 3; Q\to 3; R\to 6; S\to 5$
d) $P\to 4; Q\to 3; R\to 6; S\to 5$

LIST-I	LIST-II
P. The range of f is	1. $(-\infty, \frac{1}{1-e}] \cup [\frac{e}{e-1}, \infty)$
Q. The range of g contains	2. (0,1)
R. The domain of f contains	3. $[-\frac{1}{2}, \frac{1}{2}]$
S. The domain of g is	4. $(-\infty, 0) \cup (0, \infty)$
	5. $(-\infty, \frac{e}{e-1}]$
	6. $(-\infty, 0) \cup (\frac{1}{2}, \frac{e}{e-1}]$