MATHEMATICS FOR ECONOMICS

PROBLEM SET 9

The problems came from Simon and Blume (1994).

Chapter 19: 10, 11, 13, 18

Consider Example 17.3 in Simon and Blume (1994), about a price-discriminating monopolist. Since the demand and the cost functions are linear, there are six possible parameters we could vary in this problem: the slope and the intersect of the inverse demand curve in market 1, the slope and intercept of the inverse demand curve in market 2, and the slope and intercept of the cost function.

a) As a benchmark, compute how much profit the firm makes at the optimal solution in this worked-out example.

b) Now suppose that the intercept of the inverse demand curve in market 2 changes. Replace the 100 with a parameter $a$, and rewrite the profit function. Use the Envelope Theorem to compute how much the monopolist's profits would change with a marginal increase in $a$ from 100 to something slightly larger than 100. In other words, compute the derivative of the optimal value of F with respect to $a$.

c) To see how convenient it was to use the Envelope Theorem, redo the calculation in (b) by doing things more explicitly. Solve for the optimal Q1 and Q2 as a function of $a$, and then plug these values into the profit function to find out the optimal profit as a function of $a$. Finally, take the derivative of the optimal profit with respect to $a$. (Notice how many more calculations are required by this method than in b), but see that you get the same answer both ways.)

d) Now consider a different parameter. This time suppose we change the slope of the cost function from 20 to some number $c$ slightly greater than 20. What is the marginal change in the monopolist's profit as the parameter $c$ increases? Use the Envelope Theorem to obtain the result.

e) Check your answer in d) by doing things in the long way: compute optimal profit as a function of the parameter $c$, then take the derivative directly, as in part c).

In addition, you are asked to answer the following open question in Note 9

1. Complete Example 3
2. Complete Example 4