Submodular Optimization: From Discrete to Continuous and Back

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Amin Karbasi

Penn

Yale

Slides + references: http://iid.yale.edu/icml/icml-20.md/

This Talk

$$S^* = \arg\max_{S \subseteq V, S \in \mathcal{I}} f(S)$$

Perfect Oracle

Discrete Methods

- Centralized
- Streaming
- Distributed

Imperfect Oracle

Continuous Methods

This Talk

$$S^* = \arg\max_{S \subseteq V, S \in \mathcal{I}} f(S)$$

Perfect Oracle

Discrete Methods

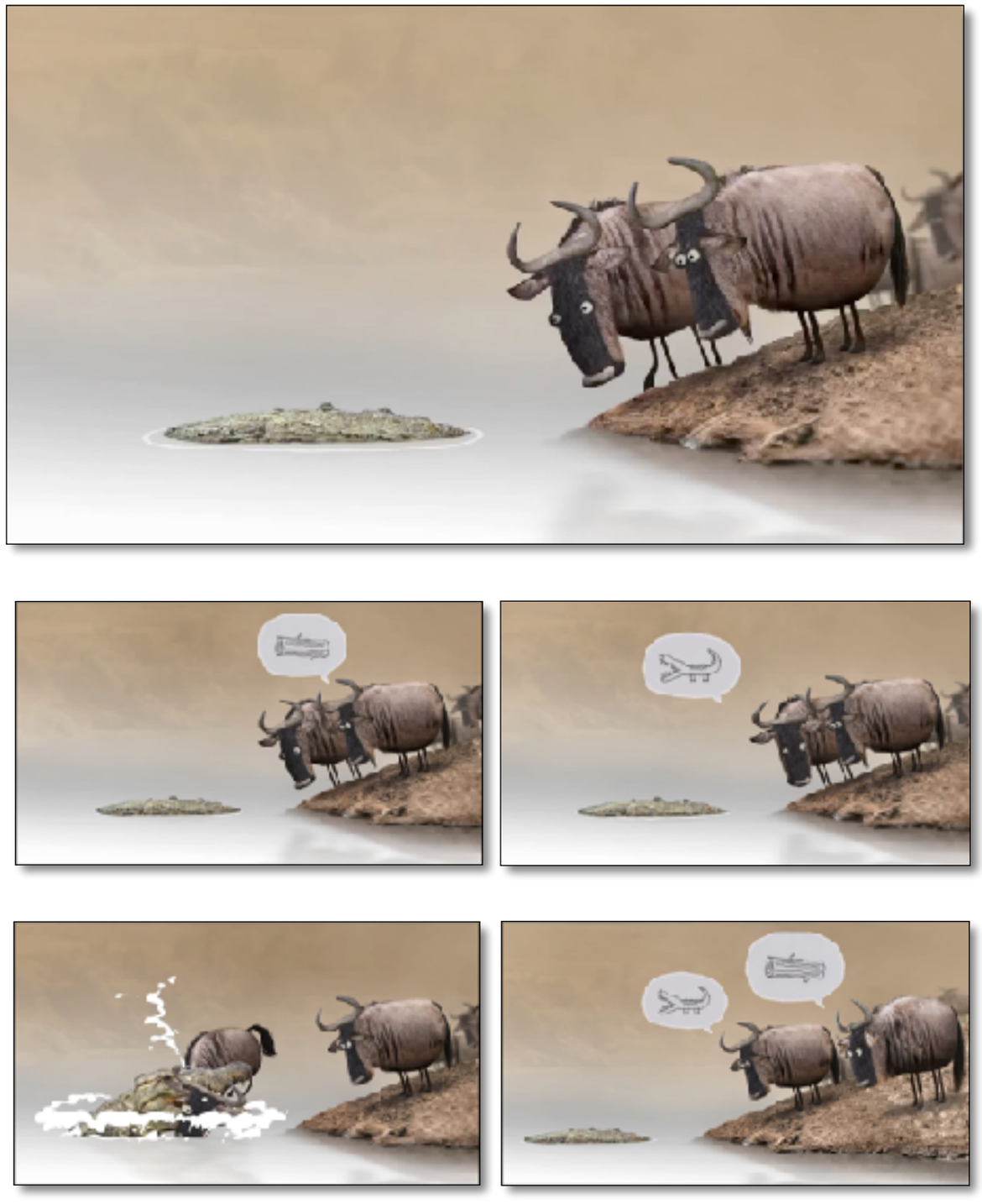
- **Centralized**
- **Streaming**
- **Distributed**

Imperfect Oracle

Continuous Methods

Data Stream

Data Stream



Problem: Greedy can be impractical

- Greedy approaches require random access to the complete data set; but for truly large-scale problems, where data is residing on disk, or arriving over time at a fast pace they are often impractical.

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Is it possible to summarize a massive data set "on the fly": at any point of time we have access only to a small fraction of data?

$$S^* = \arg\max f(S)$$
$$S \subseteq V, S \in \mathcal{I}$$

Setting

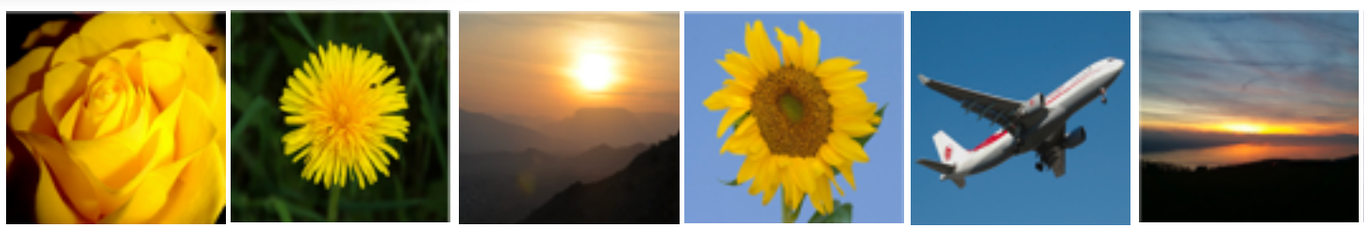
- Streaming

Stream Greedy

- Consider the following problem where f is **monotone** submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$

Data Stream e



$$f(S \cup \{e\} \setminus \{x\}) - f(S) > 0$$

$$f\left(\left\{ \quad x_1 \quad x_2 \quad x_3 \quad \right\}\right) \; S$$

"Budgeted Nonparametric Learning from Data Stream", R. Gomes, A. Krause, 2010

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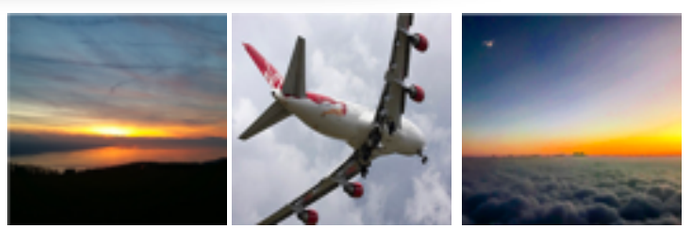

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- Efficient in practice but in the worst case $f(S_{\text{Stream-Greedy}}) \leq \frac{1}{k}\text{OPT}$

"Budgeted Nonparametric Learning from Data Stream", R. Gomes, A. Krause, 2010

Bad Example

- Consider the following problem where f is **monotone** submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$

Stream Good

$$f(S) = \sum_{x \in \cup_{v \in S} v} w(x)$$

$$w(1) = \cdots = w(k) = 1$$
$$w(k+1) = \cdots = w(2k) = 1 + \epsilon$$
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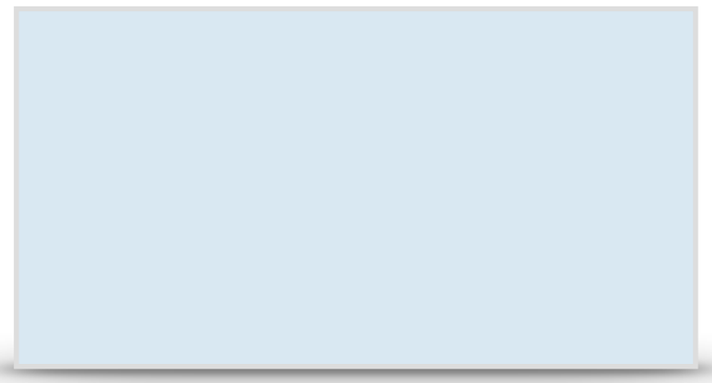
$$w(k^2+1) = \cdots = w(k^2+k) = 1 + k\epsilon$$
$$\text{and } \epsilon \ll 1$$



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Solution

Thresholding

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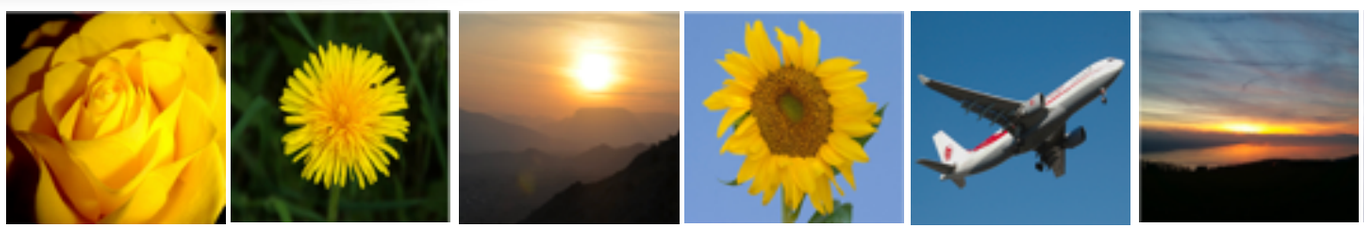

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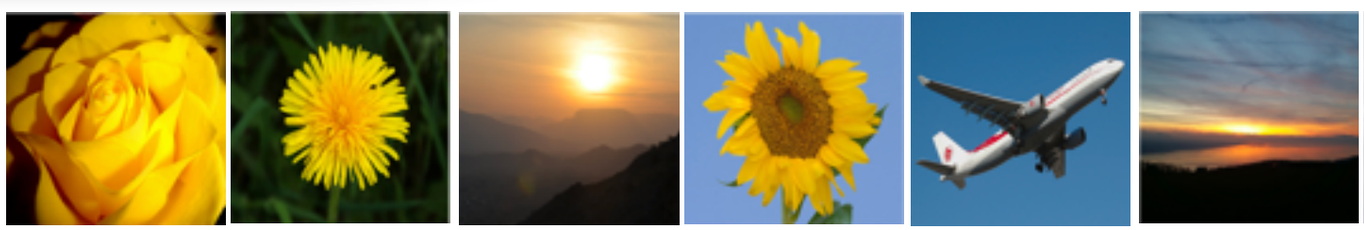

$$f(S \cup \{e\} \setminus \{x\}) - f(S) \geq \frac{f(S)}{k}$$

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[Buchbinder, Feldman, Schwartz]

For monotone submodular functions, Threshold-Greedy, with memory O(k) achieves

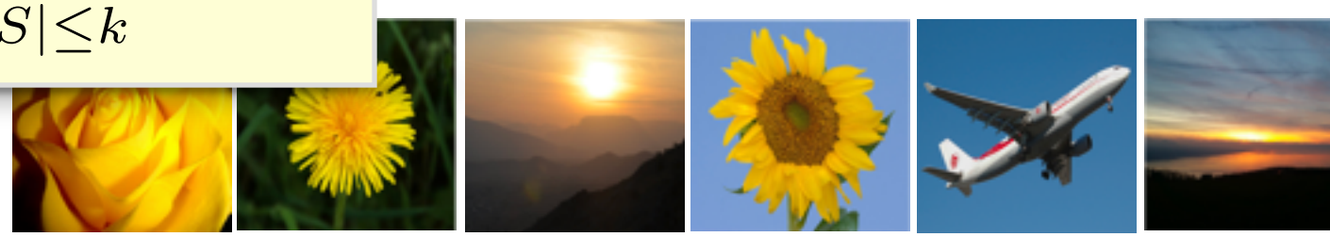
$$f(S_{\text{Threshold-Greedy}}) \geq \frac{1}{4}\text{OPT}$$

"Online Submodular Maximization with Preemption", 2015

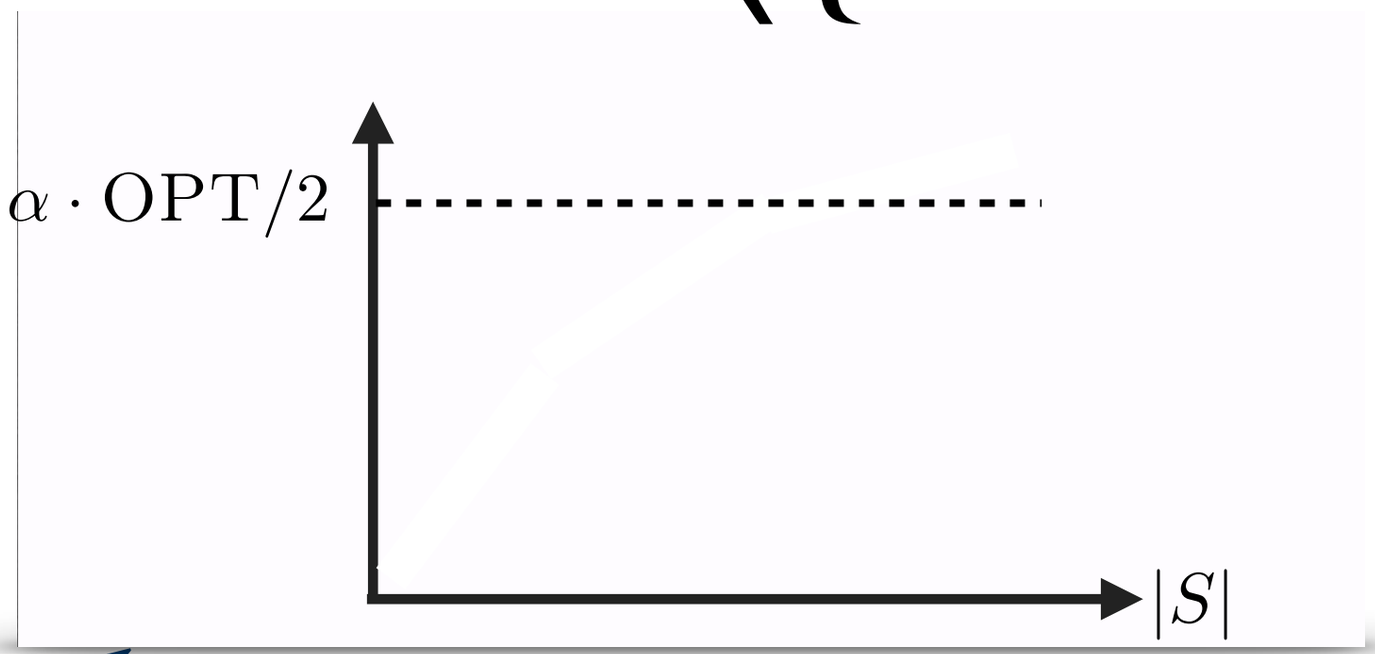
Thresholding + Guessing

- Consider the following problem where f is monotone submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$



$$f(\{\quad\}) \longrightarrow \max$$



Thresholding + Guessing

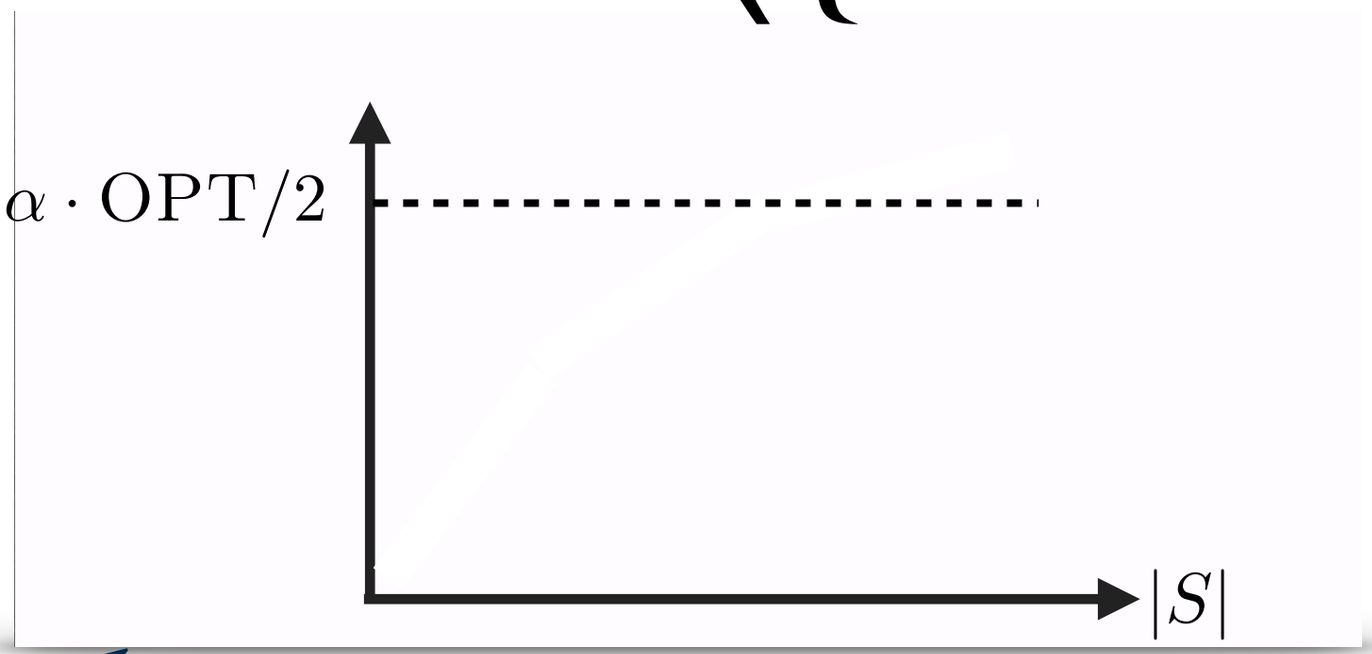
- Consider the following problem where f is monotone submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$

Data Stream

$$\mathrm{OPT} \geq \tau \geq \alpha \cdot \mathrm{OPT}$$
$$f(e_i \mid S) \geq \tau/(2k)$$

$f(\{\quad\}) \longrightarrow$ max



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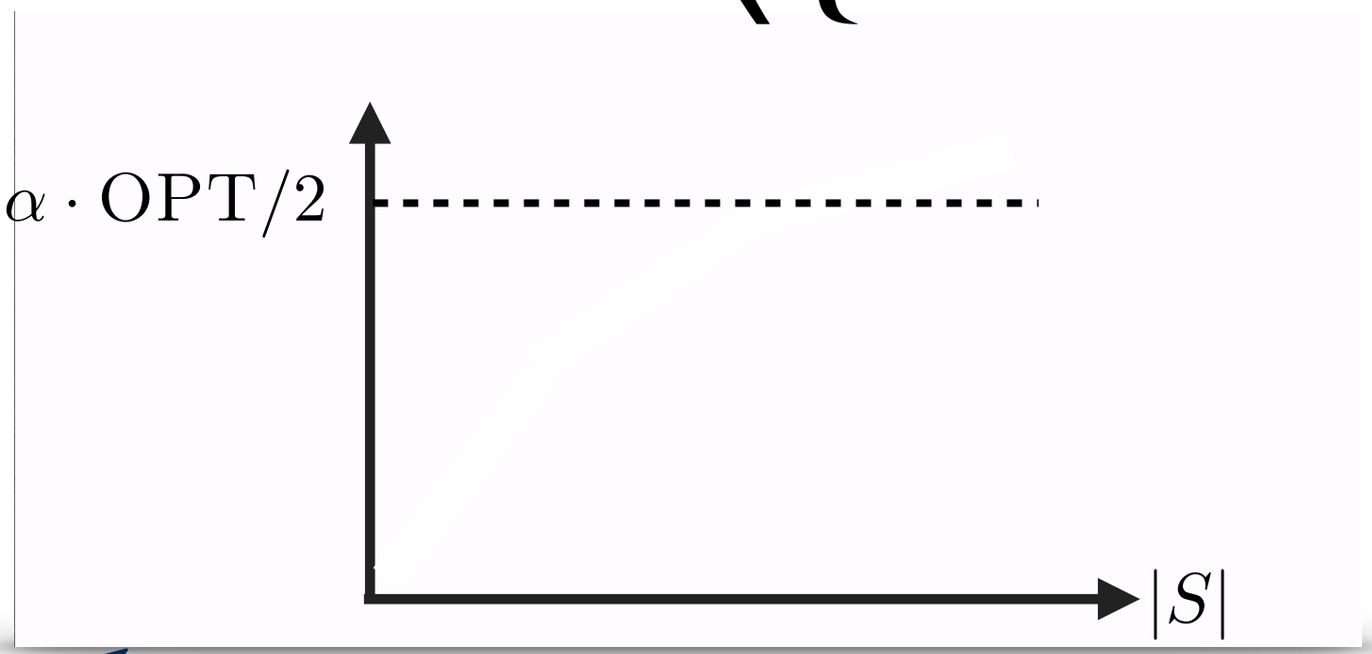

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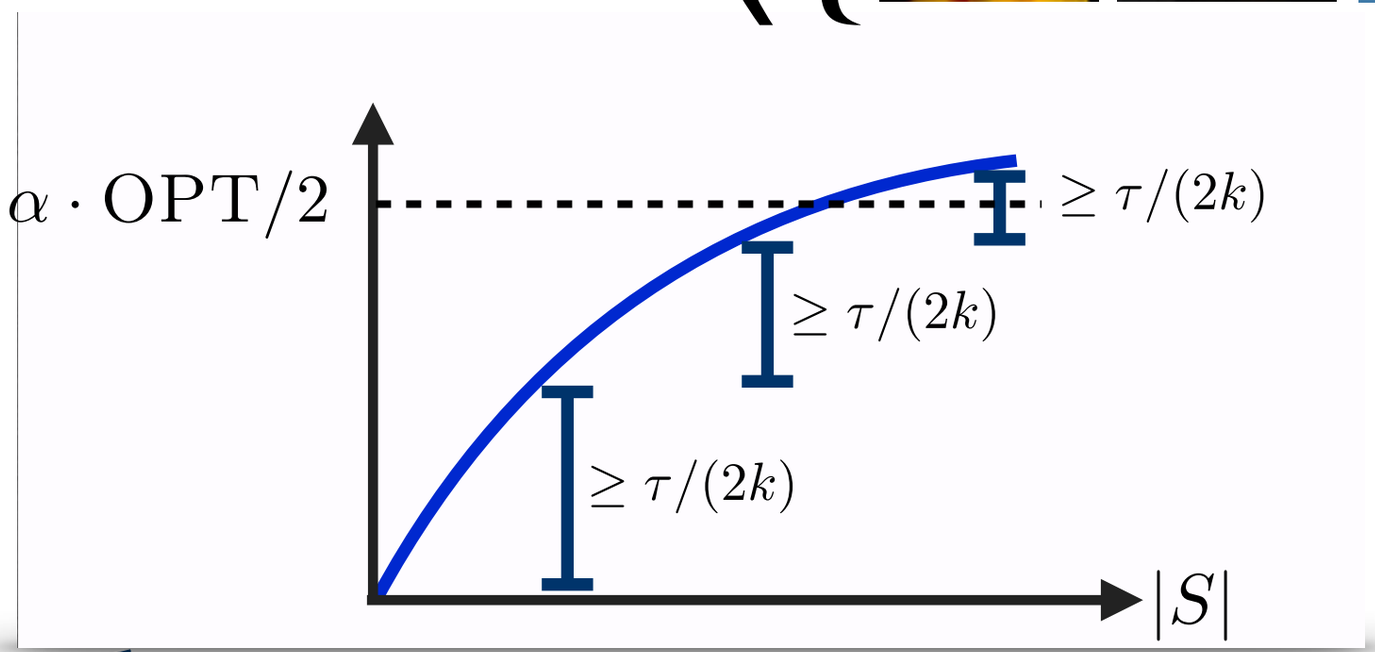

Properties:

- 1 pass
- $f(S) \geq \alpha \cdot \mathrm{OPT}/2$
- $O(k)$ memory
- $O(1)$ update time

Thresholding + Guessing

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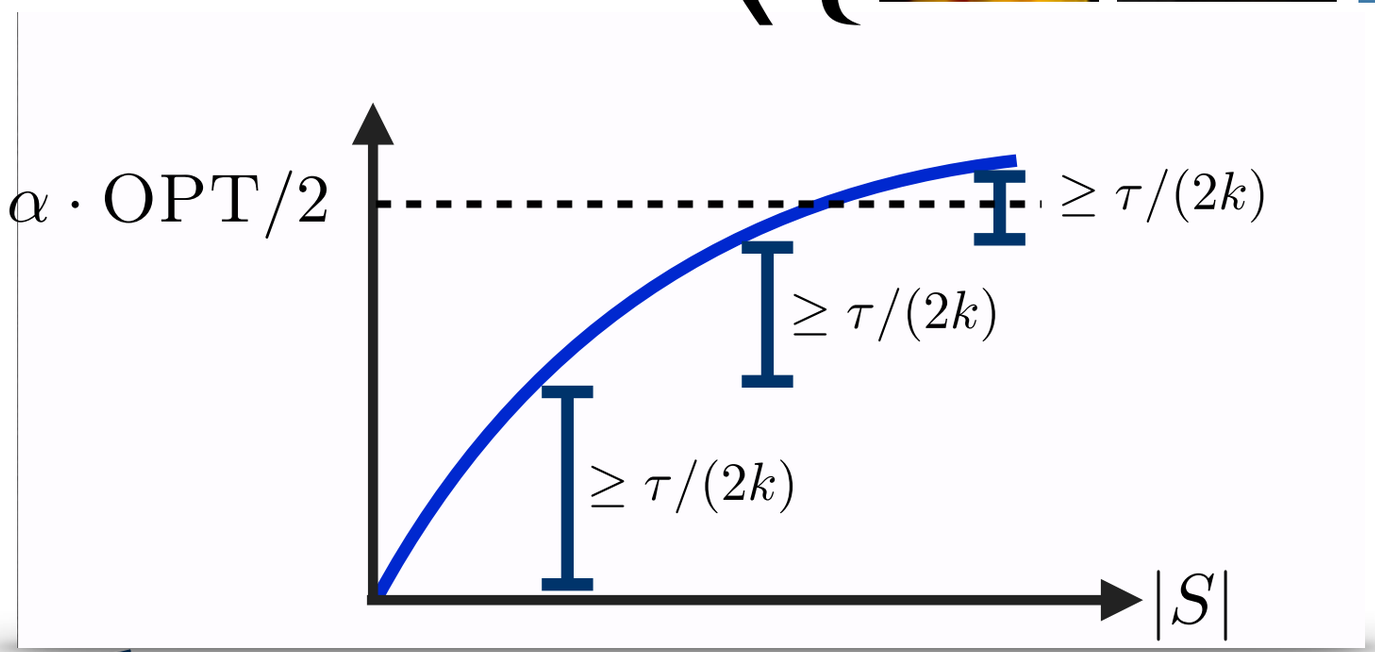

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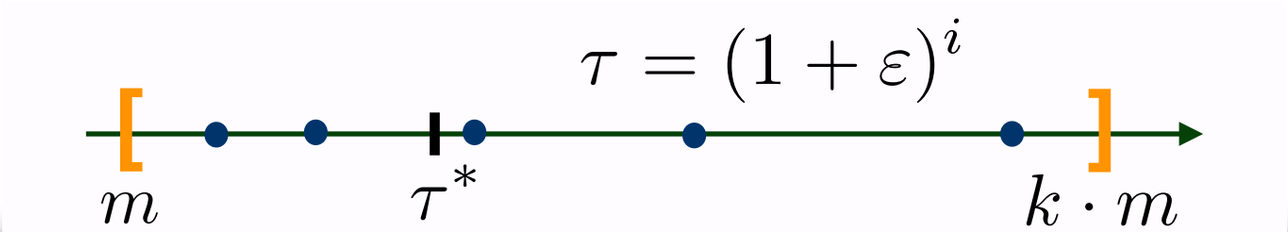

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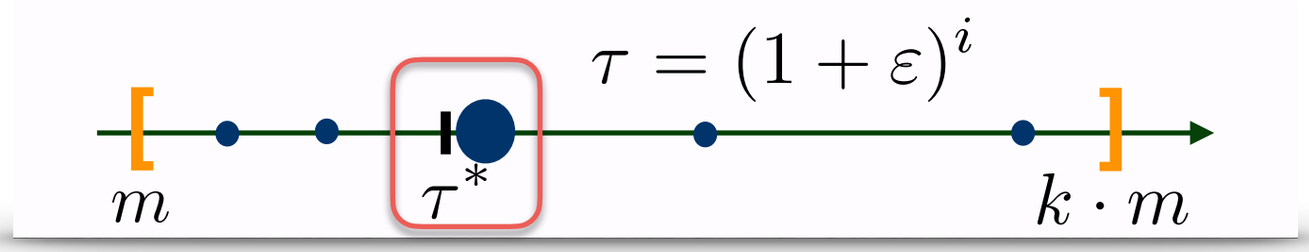

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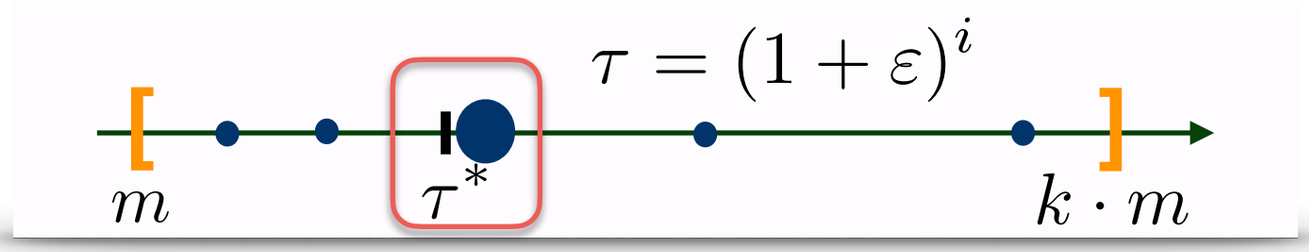
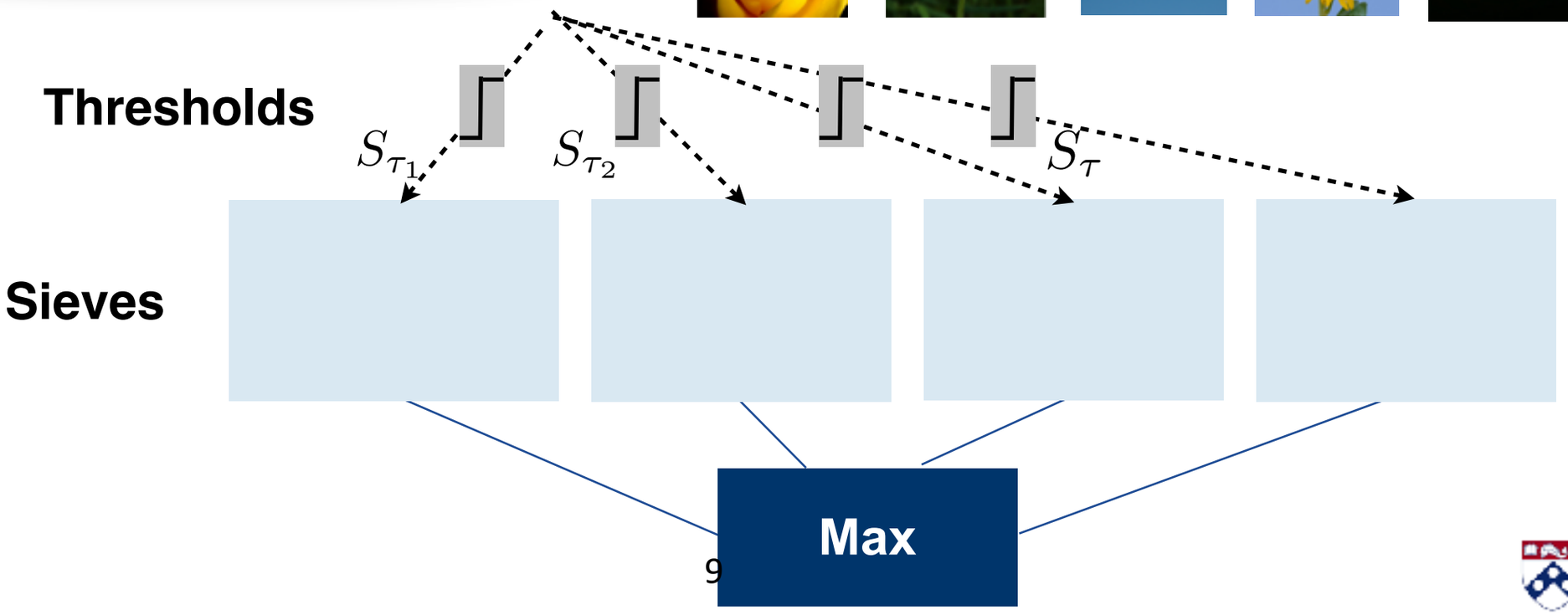


$f(e_i \mid S) \geq \tau/(2k)$



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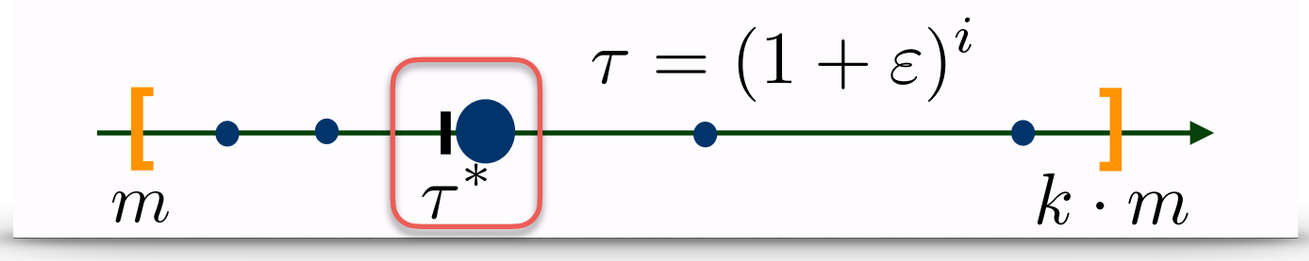

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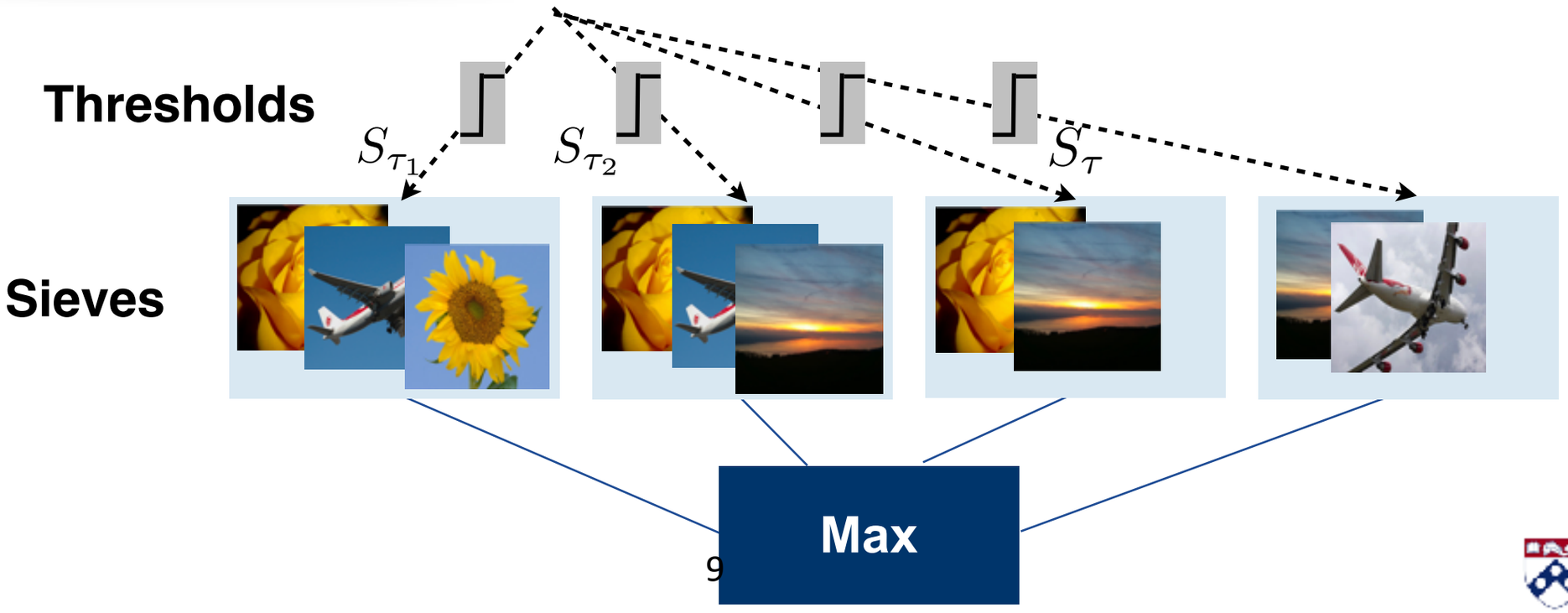

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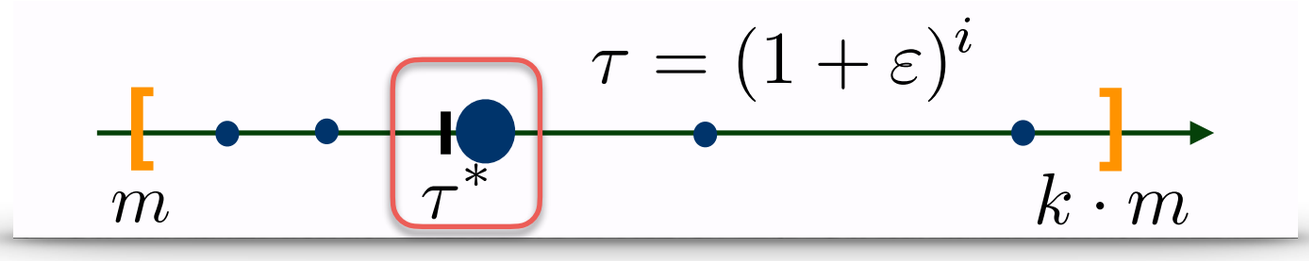
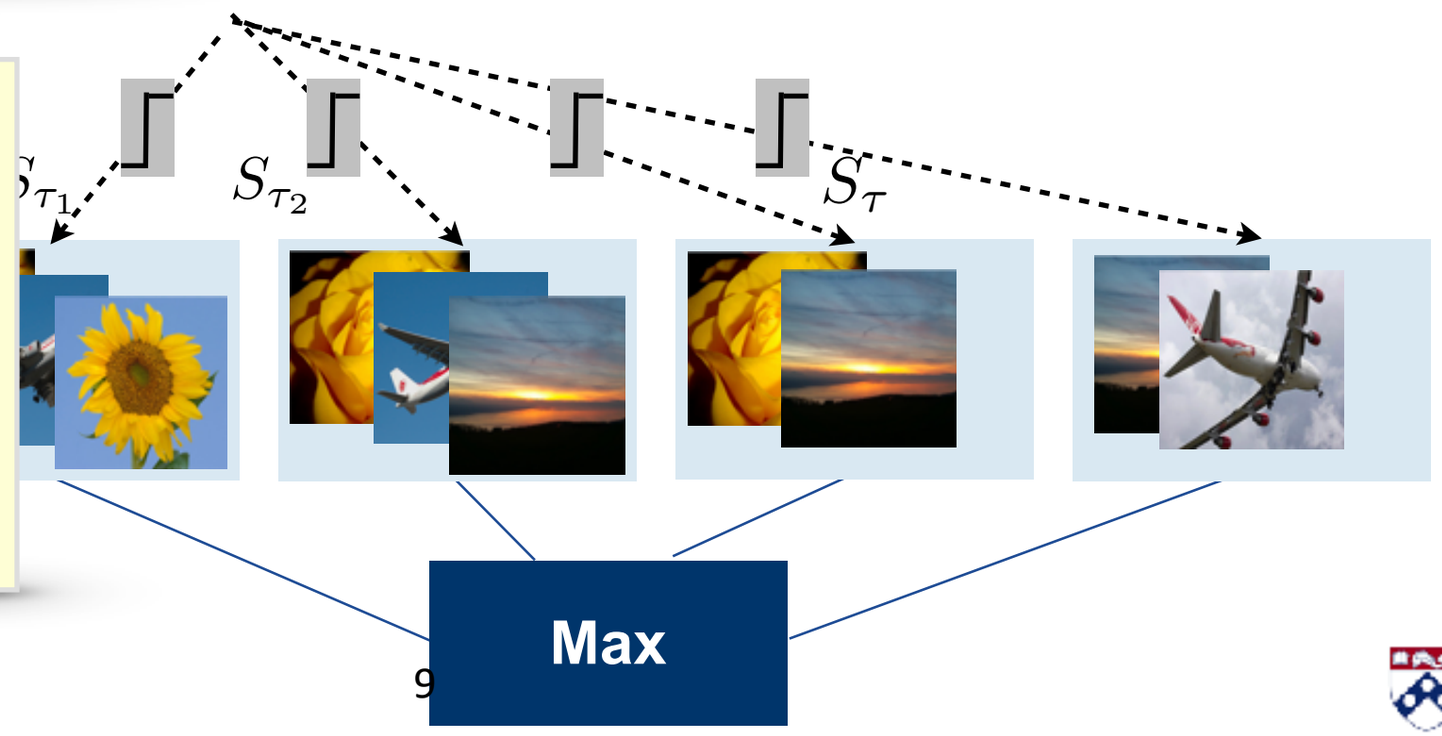


$f(e_i \mid S) \geq \tau/(2k)$

Properties:

- 1 pass
- $f(S) \geq (1/2 - \varepsilon)\mathrm{OPT}$
- $O(k \log k/\varepsilon)$ memory
- $O(\log k/\varepsilon)$ update time



SIEVE-STREAMING

[Badanidiyuru, Mirzasoleiman, Karbasi, Krause]

For monotone submodular functions, SIEVE-STREAMING with a $O(k\log(k)/\varepsilon)$ memory achieves

$$f(S_{\text{Sieve-Streaming}}) \geq \left(\frac{1}{2} - \epsilon\right) \text{OPT}$$

"Streaming Submodular Maximization: Massive Data Summarization on the Fly", 2014

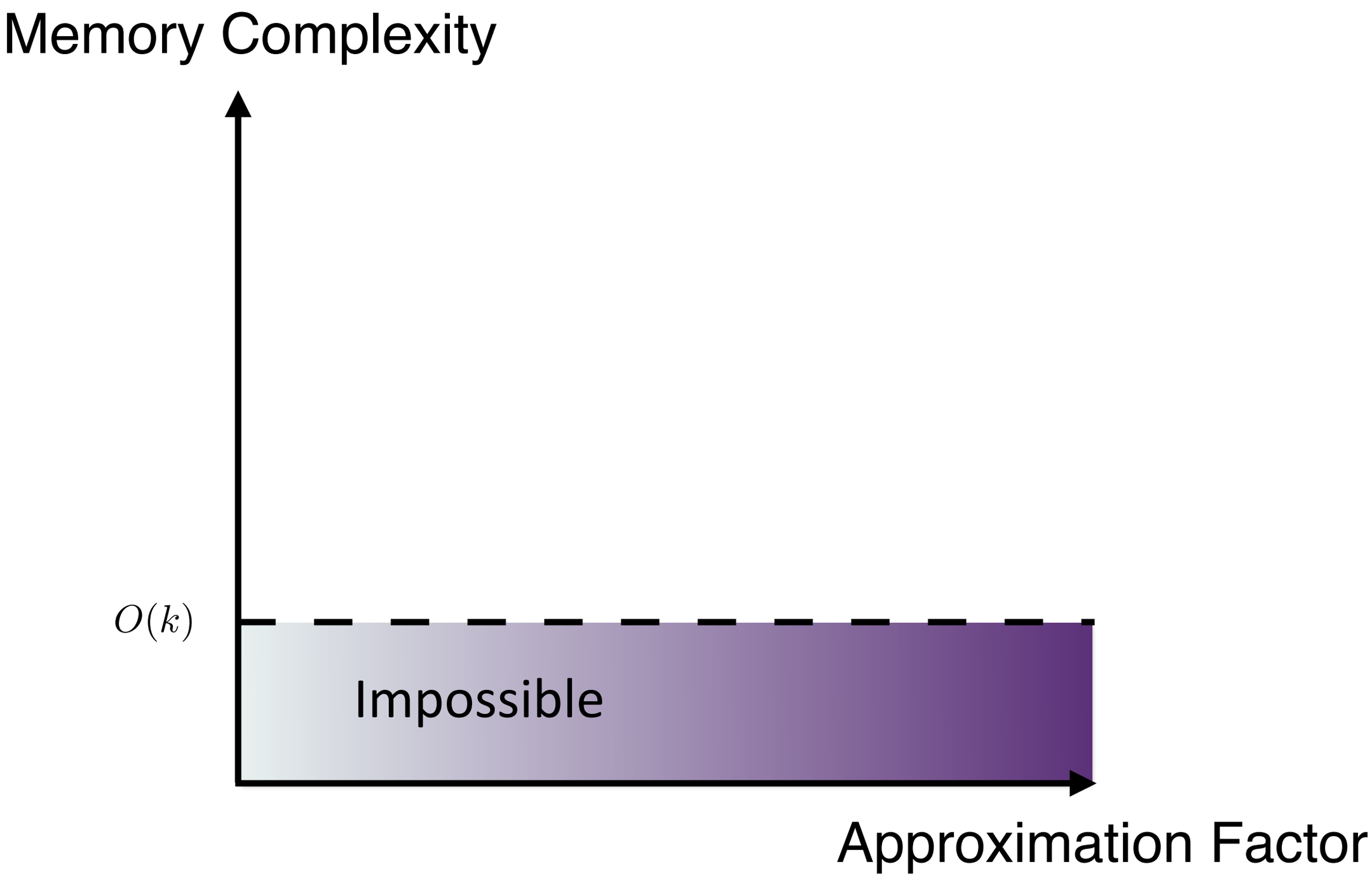
[Kazemi, Mitrovic, Zadimoghaddam, Lattanzi, Karbasi]

For monotone submodular functions, SIEVE-STREAMING++ with a $O(k)$ memory achieves

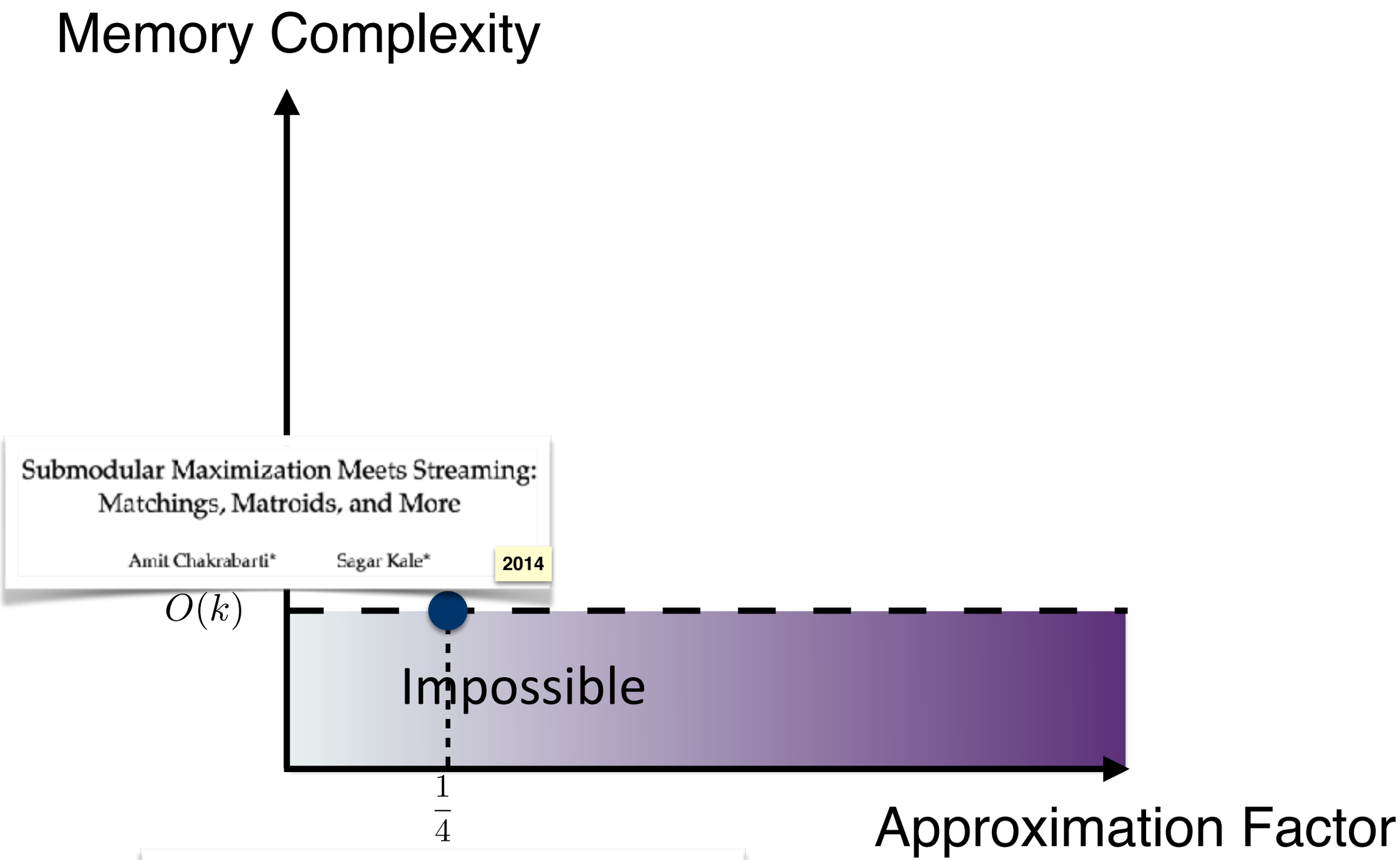
$$f(S_{\text{Sieve-Streaming++}}) \geq \left(\frac{1}{2} - \epsilon\right) \text{OPT}$$

"Submodular Streaming in All Its Glory: Tight Approximation, Minimum Memory and Low Adaptive Complexity", 2019

Streaming Algorithms: Cardinality Constraint



Streaming Algorithms: Cardinality Constraint



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Penn

Streaming Algorithms: Cardinality Constraint

Memory Complexity

Streaming Submodular Maximization: Massive Data Summarization on the Fly

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Andreas Krause
ETH Zurich
krausea@ethz.ch

2014

$O(k \log k)$

Submodular Maximization Meets Streaming: Matchings, Matroids, and More

Amit Chakrabarti* Sagar Kale*

2014

$O(k)$

Impossible

$\frac{1}{4}$ $\frac{1}{2} - \epsilon$

Approximation Factor

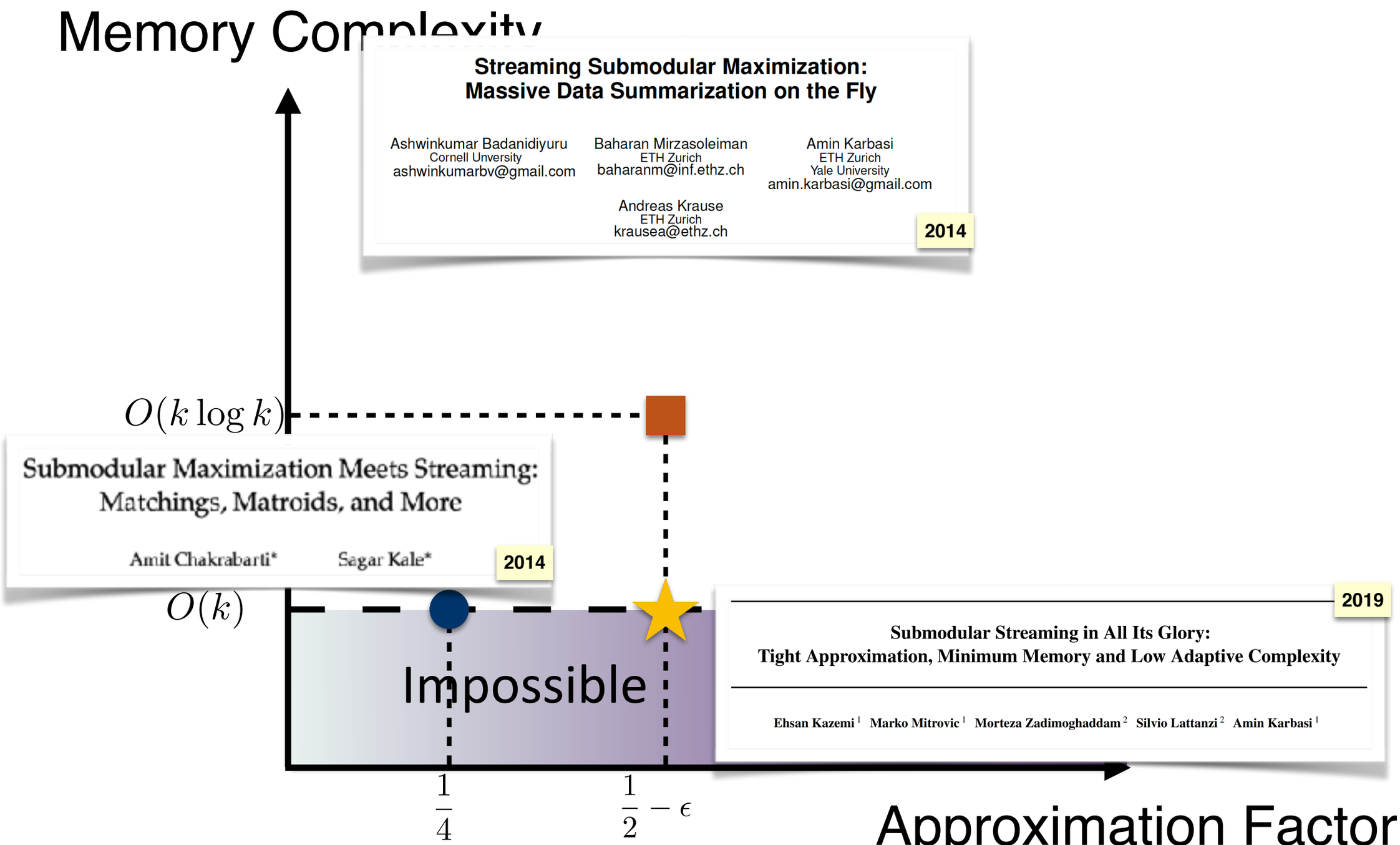
Online Submodular Maximization with Preemption

Niv Buchbinder* Moran Feldman† Roy Schwartz‡ 2015

Yale

Penn

Streaming Algorithms: Cardinality Constraint



Yale

Penn

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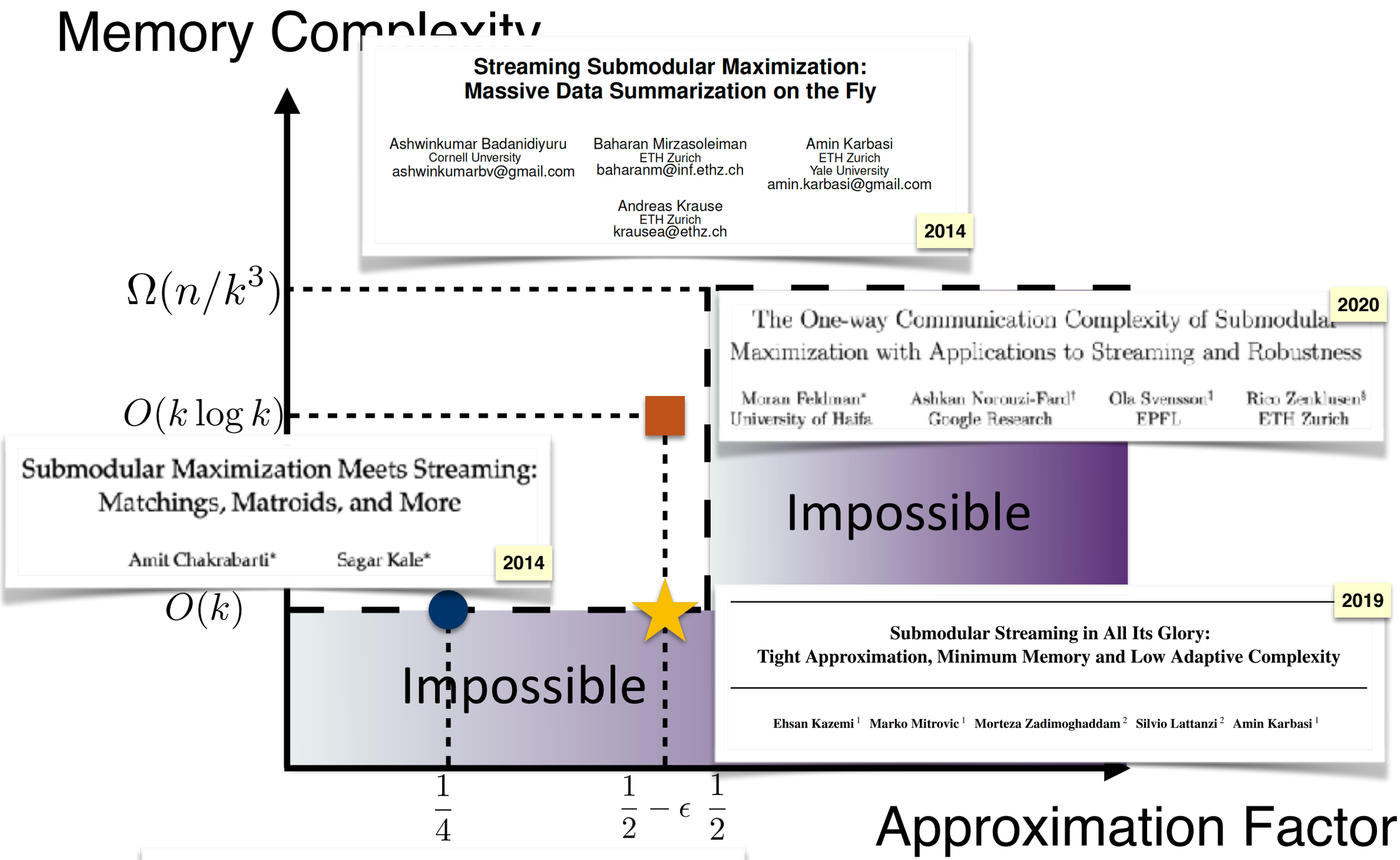
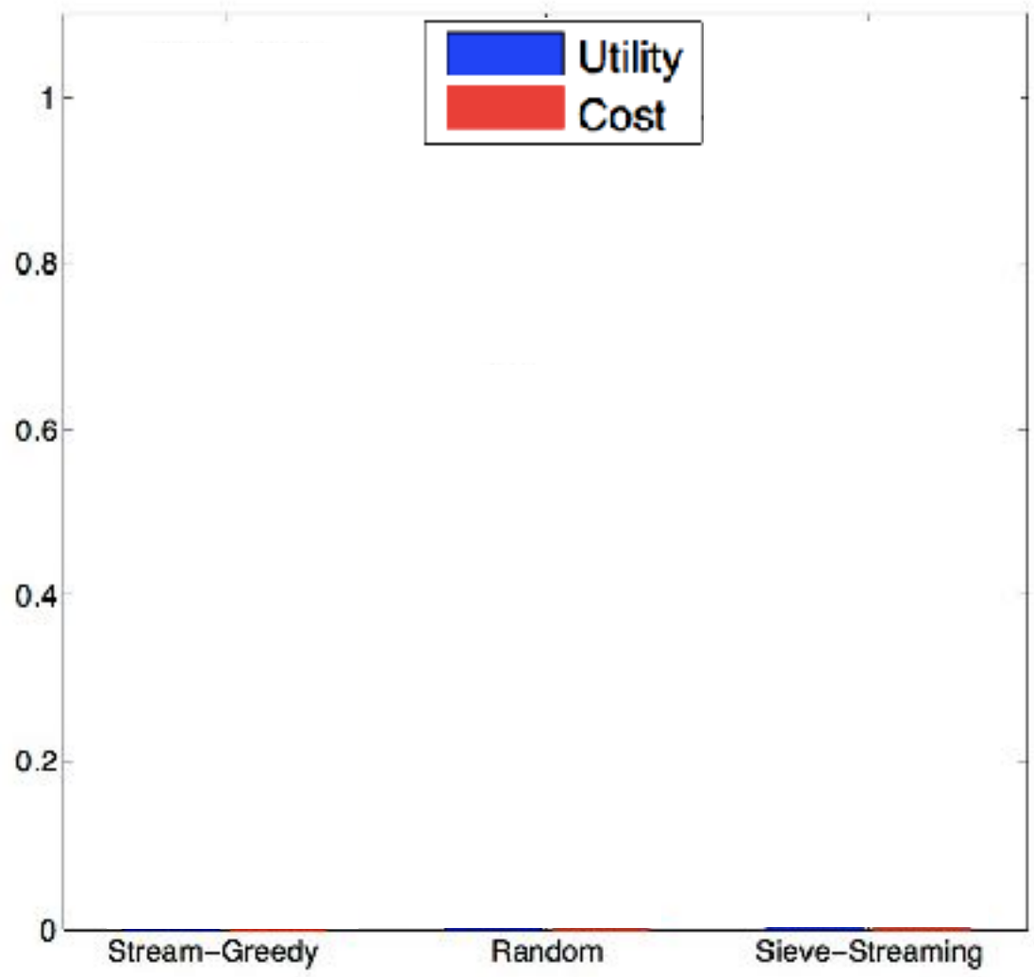


Yale

Penn

Nonparametric Regression

- **Census** dataset consists of 2,458,285 data points with 68 attributes



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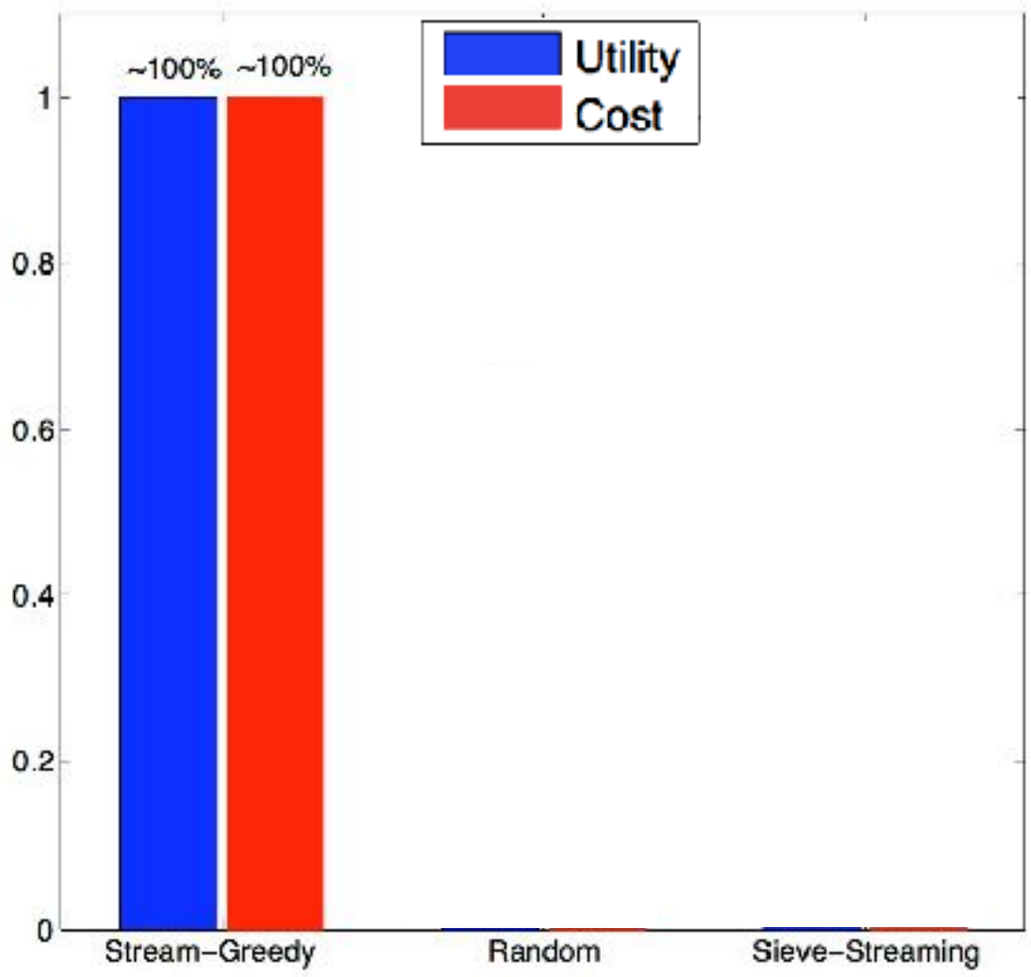

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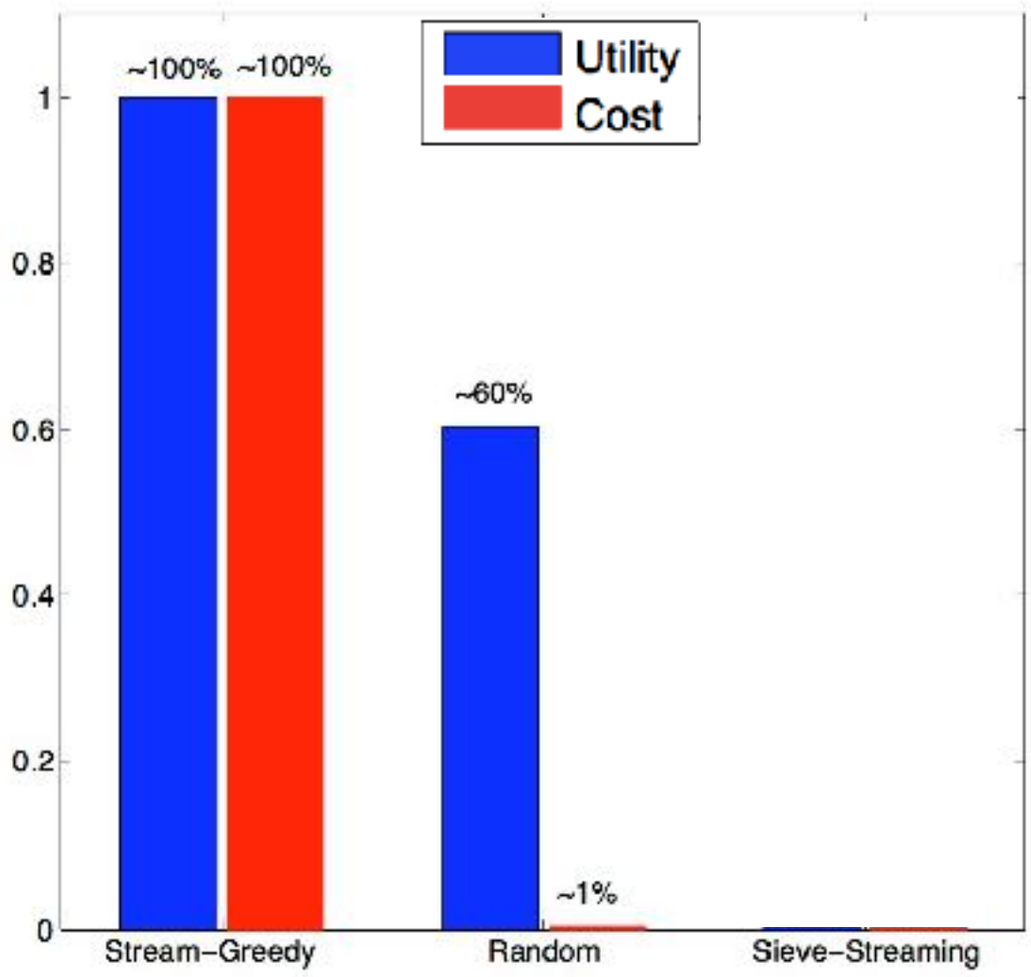

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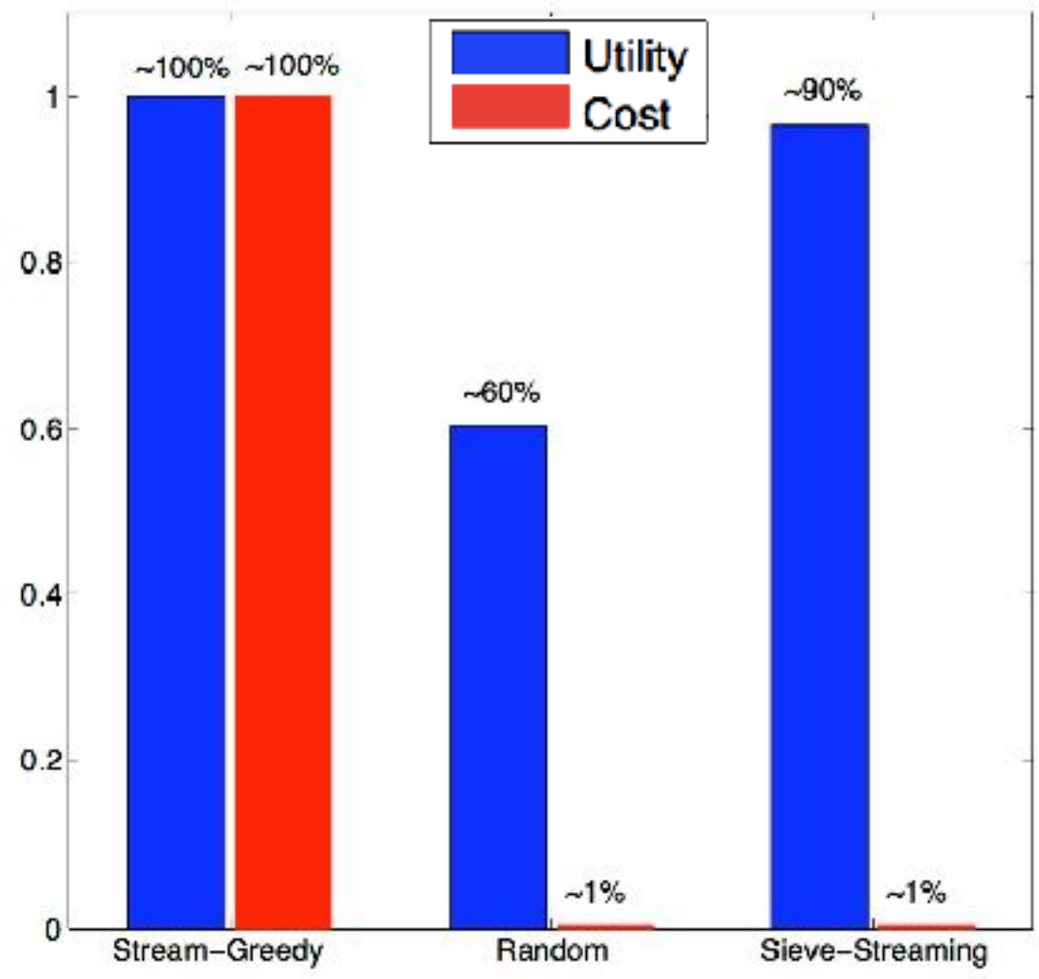
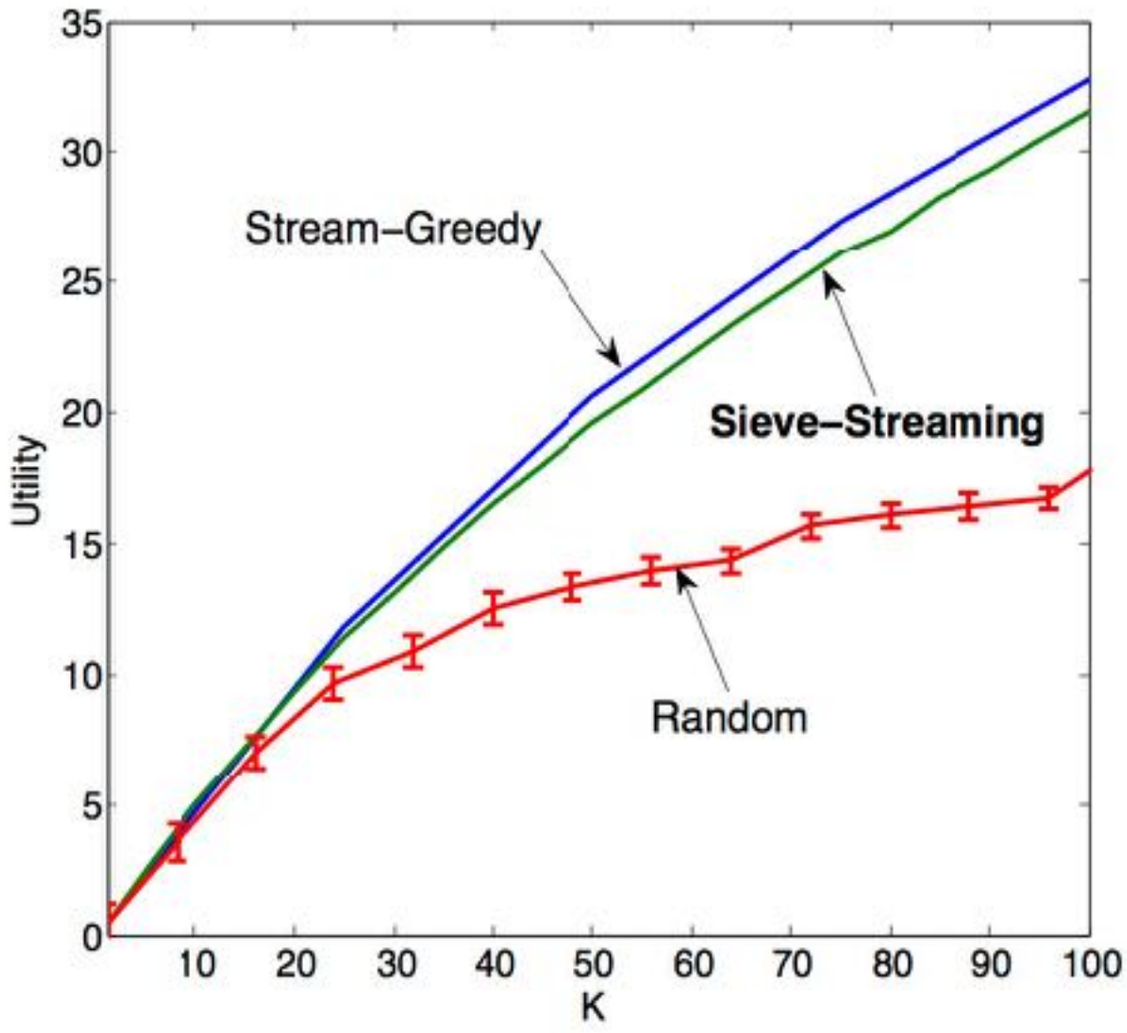




Open Question: Matroid

[Chakrabati, Kale]

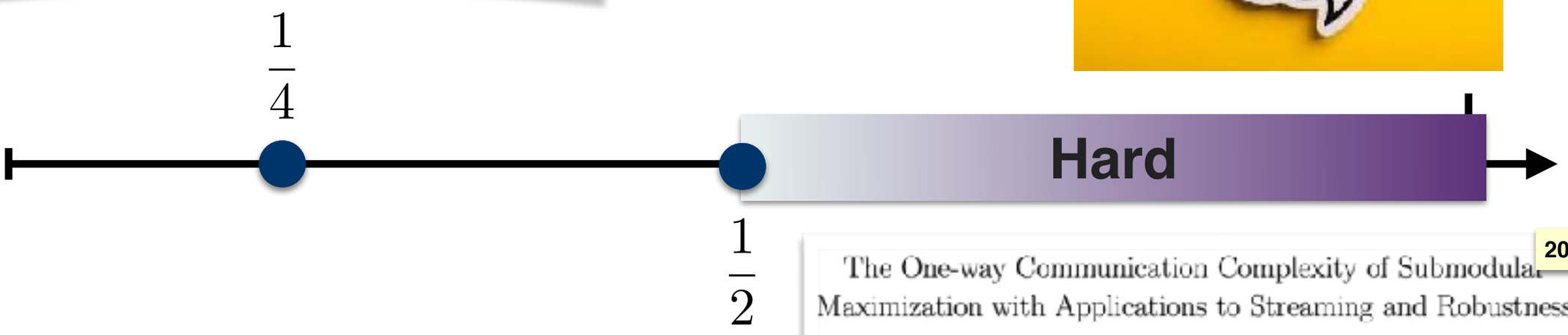
For monotone submodular functions, and subject to the intersection of p matroids, there is a one-pass streaming algorithm that achieves

$$f(S_{\text{MSIS}}) \geq \left(\frac{1}{4p}\right) \text{OPT}$$

Submodular Maximization Meets Streaming: Matchings, Matroids, and More

Amit Chakrabarti* Sagar Kale*

2014



The One-way Communication Complexity of Submodular Maximization with Applications to Streaming and Robustness

Moran Feldman* University of Haifa
Ashkan Norouzi-Fard† Google Research
Ola Svensson‡ EPFL
Rico Zenklusen§ ETH Zurich

2020

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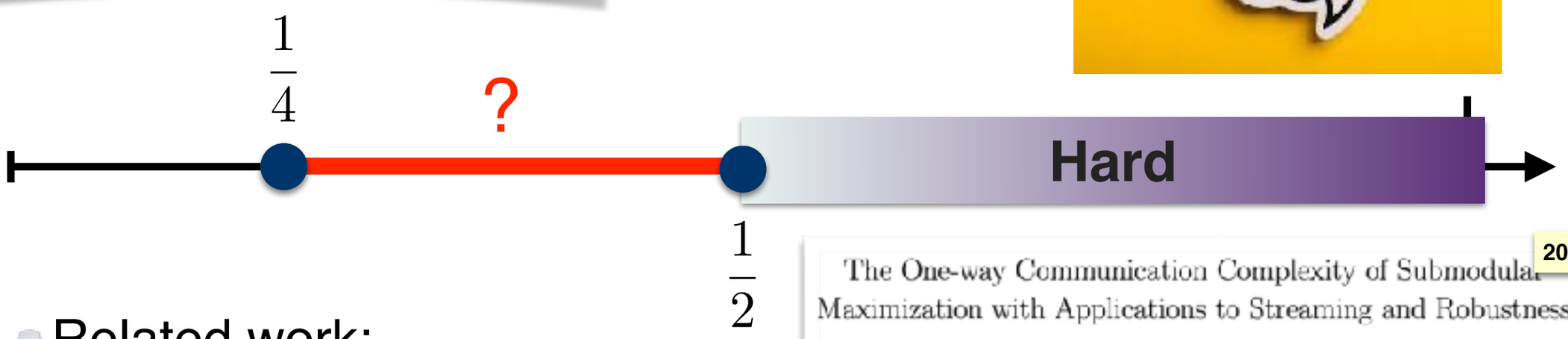

- Related work:

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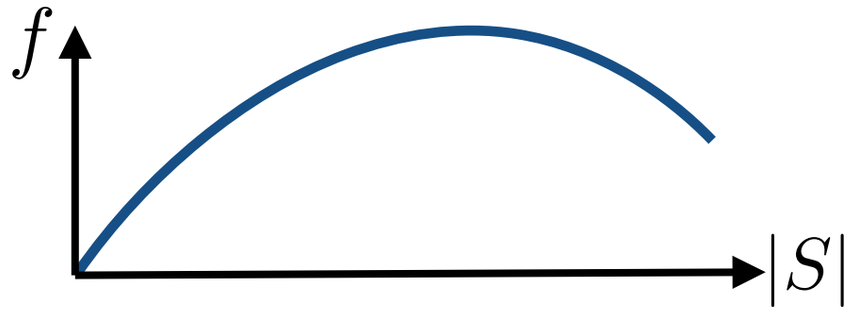
"Approximability of monotone submodular function maximization under cardinality and matroid constraints in the streaming model ", C. Huang, N. Kakimura, S. Mauras, Y. Yoshida

Cardinality Constrained Streaming Submodular Maximization

- When the utility functions f is non-monotone submodular

$$S^* = \arg\max_{|S| \leq k} f(S)$$



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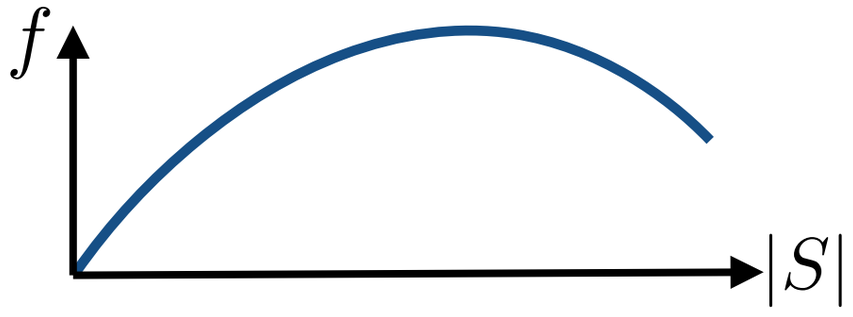

- Three ingredients
  - Thresholding
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"Optimal Streaming Algorithms for Submodular Maximization with Cardinality Constraints", Alaluf, Ene, Feldma, Nguyen Suh, 2020

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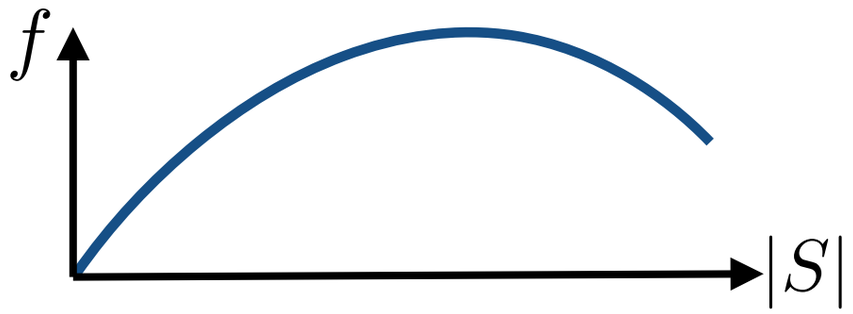
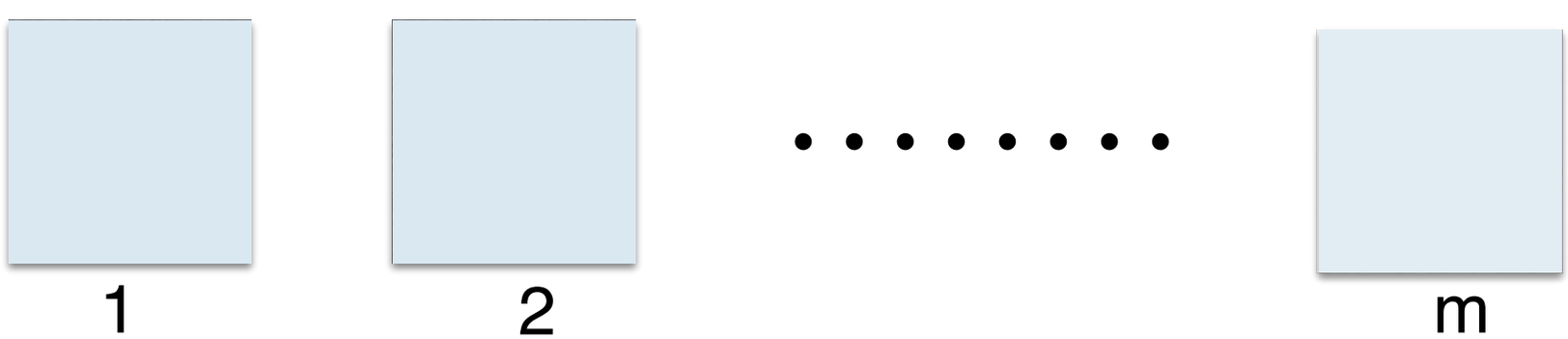


building block

Data Stream

Marginal Gain



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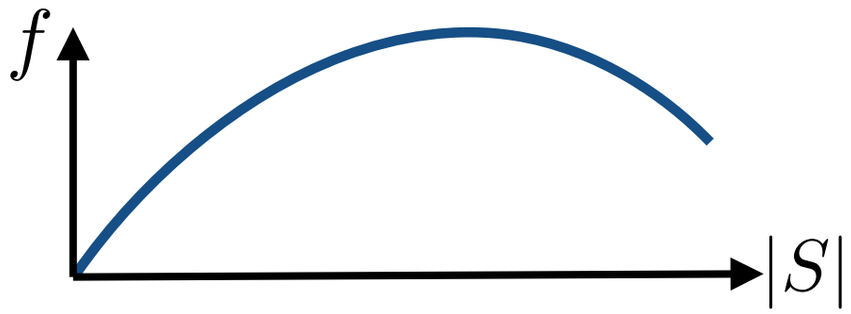

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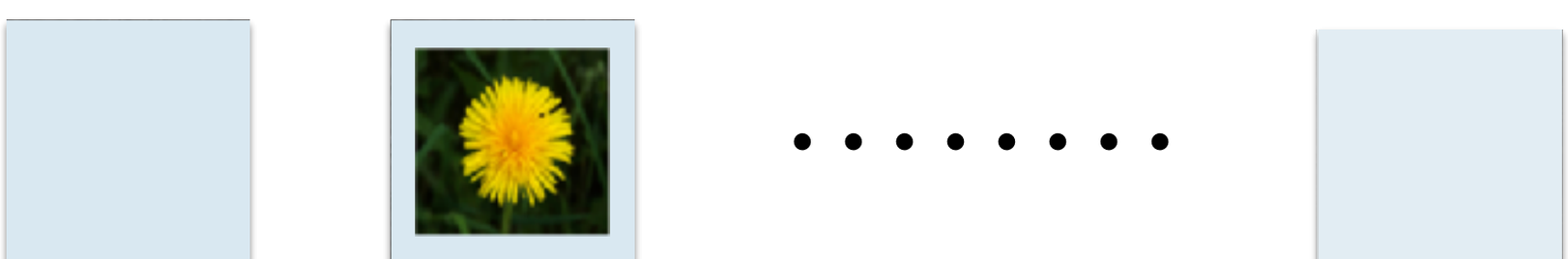

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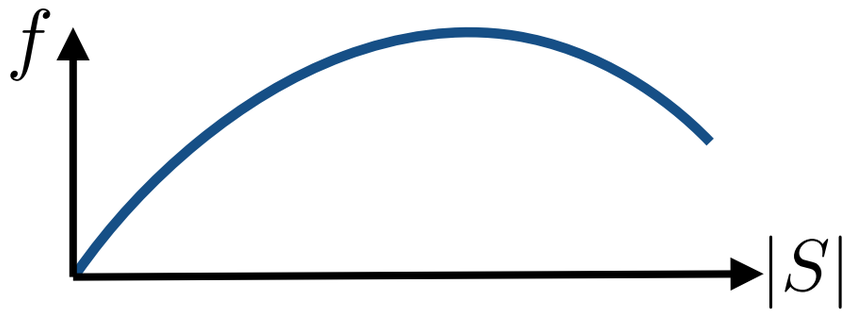

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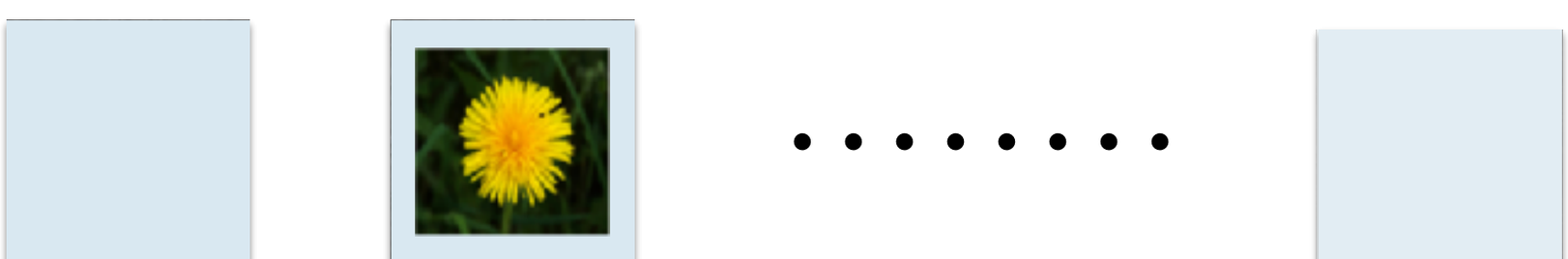

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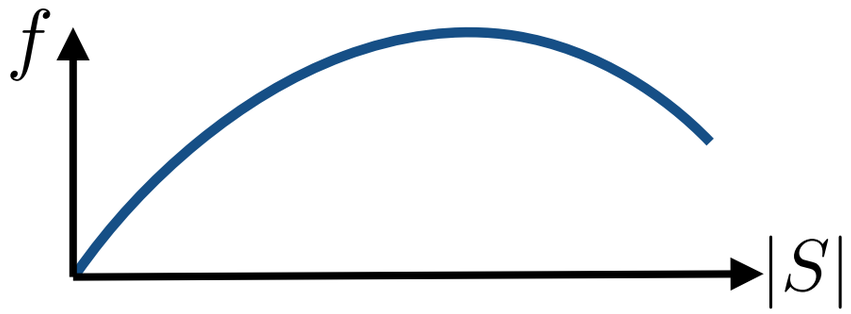

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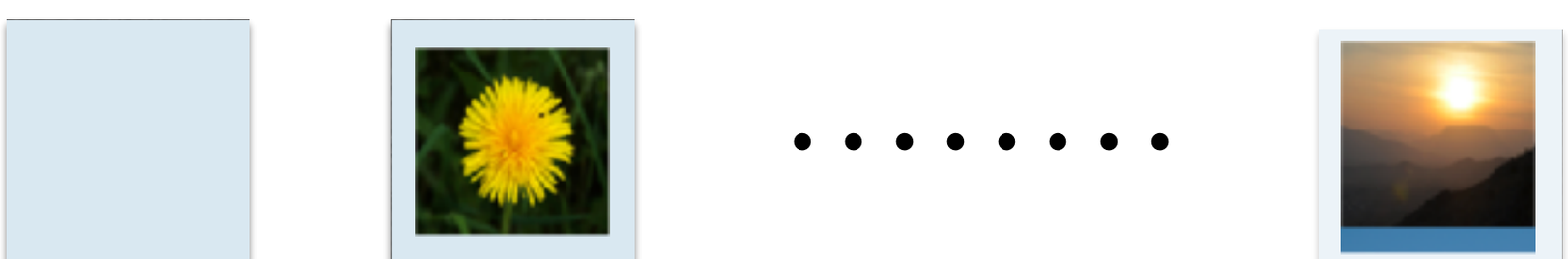
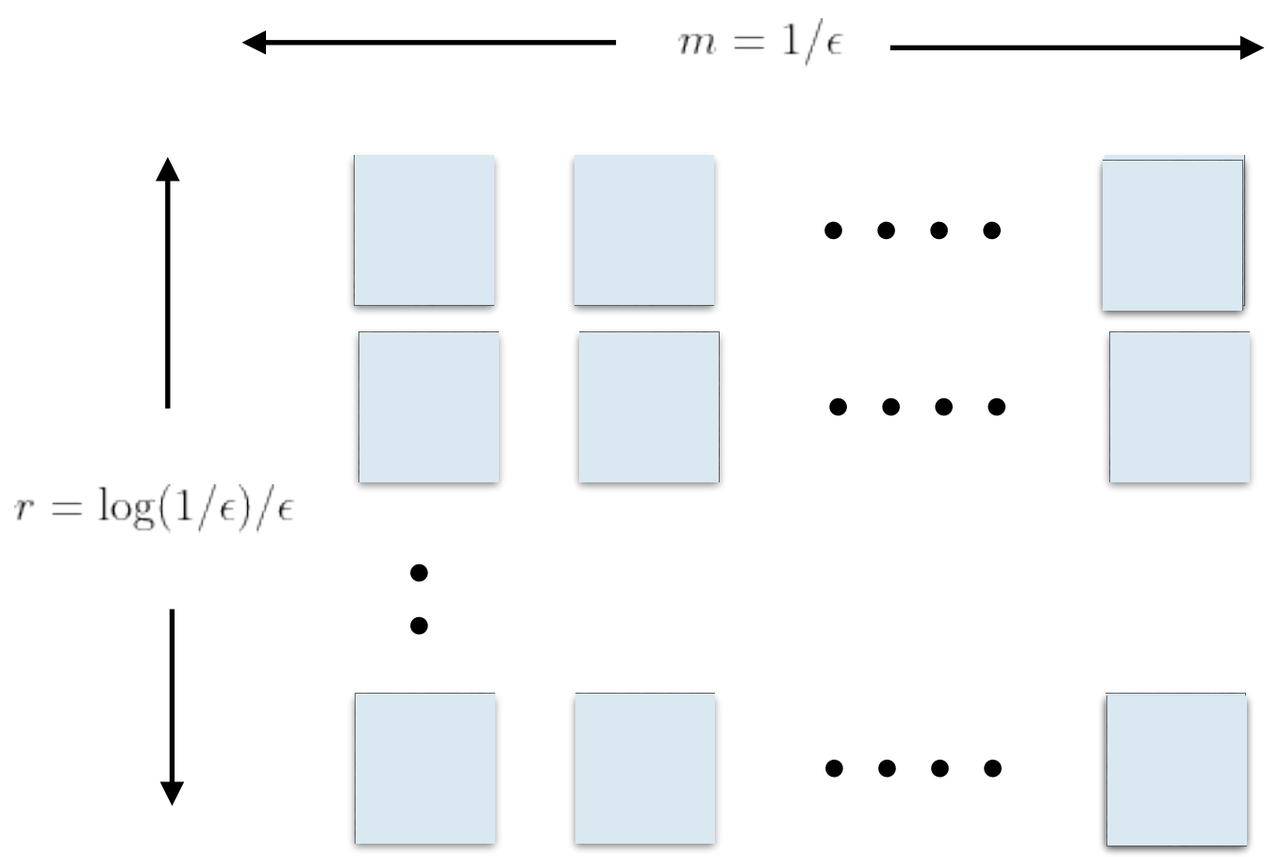


Cardinality Constrained Streaming Submodular Maximization

- Partition the data stream into $m = \frac{1}{\epsilon}$ parts.
- Run $r = \frac{\log(1/\epsilon)}{\epsilon}$ instances of the building block in parallel



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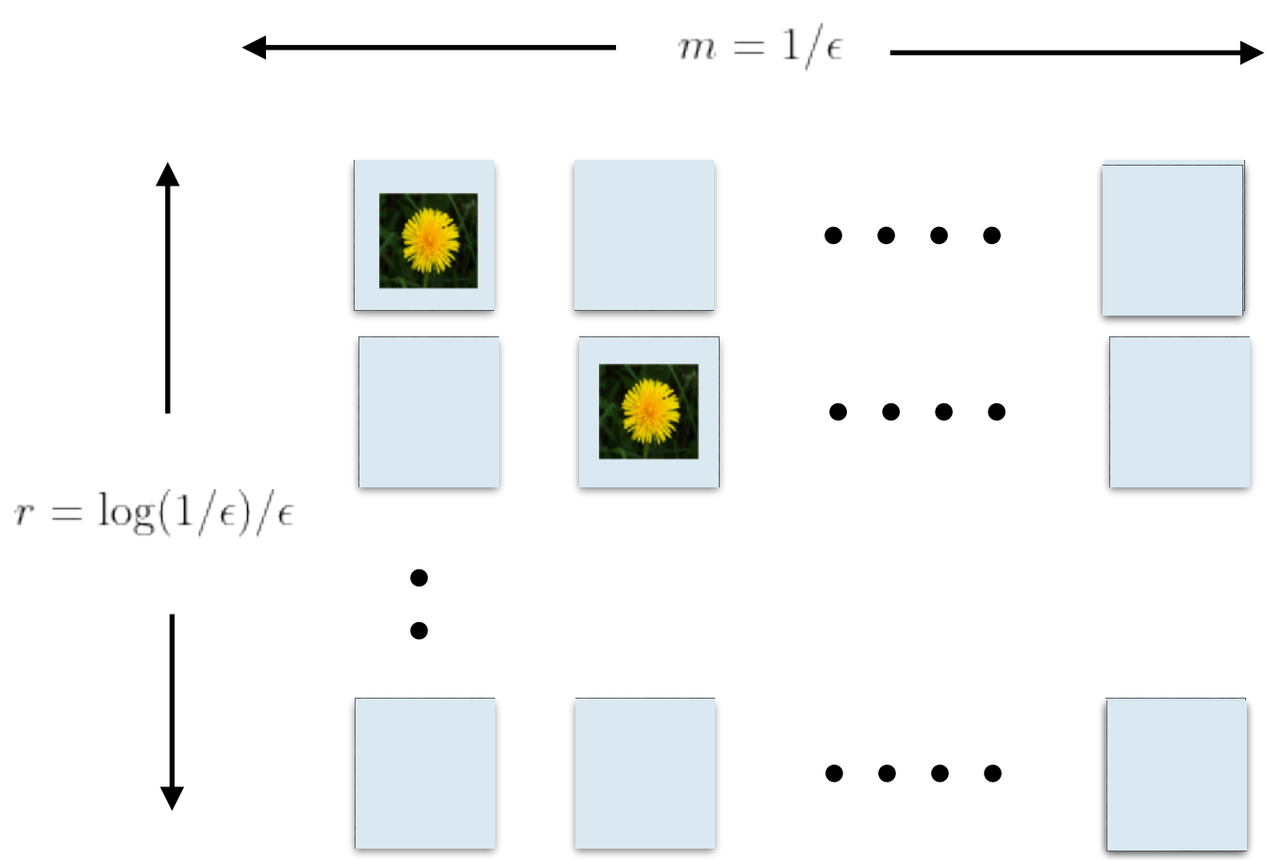

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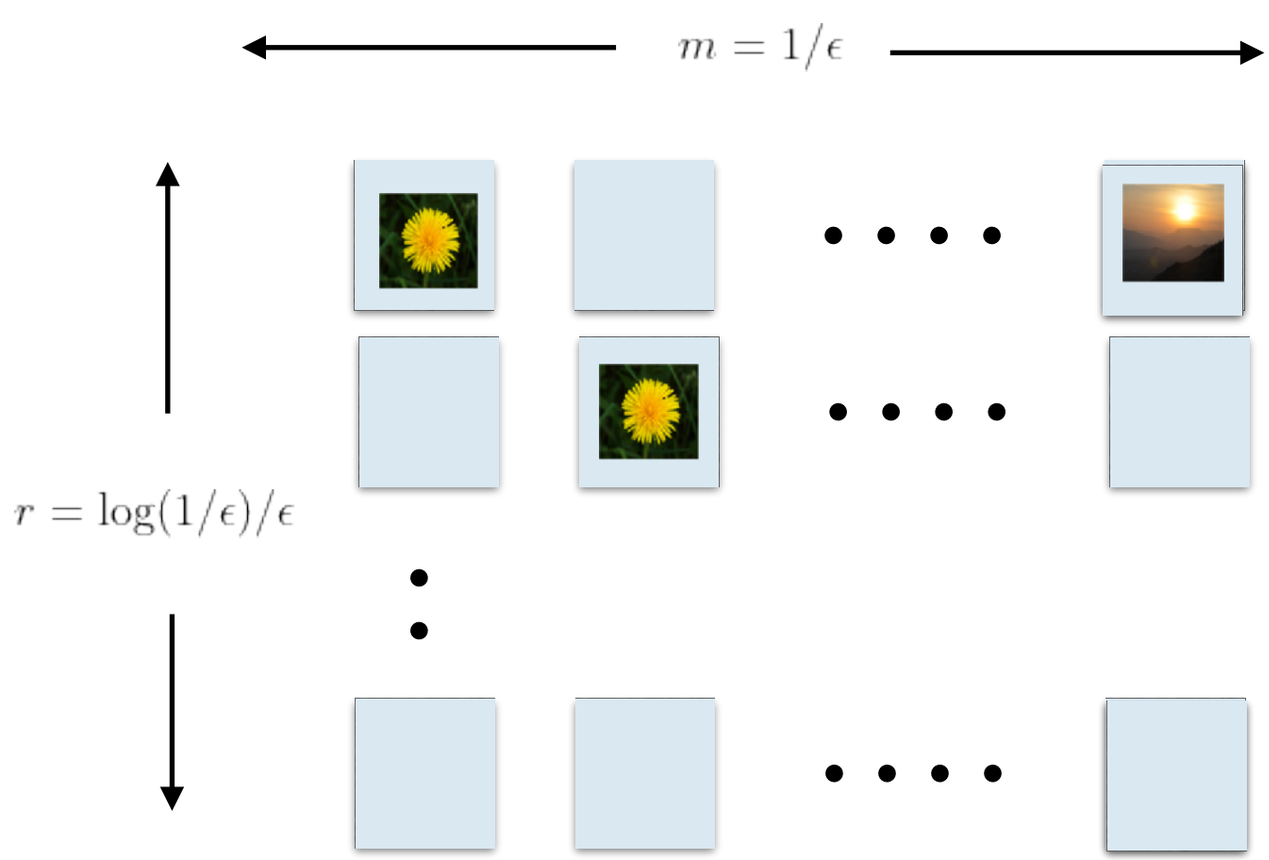

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- Upon termination:
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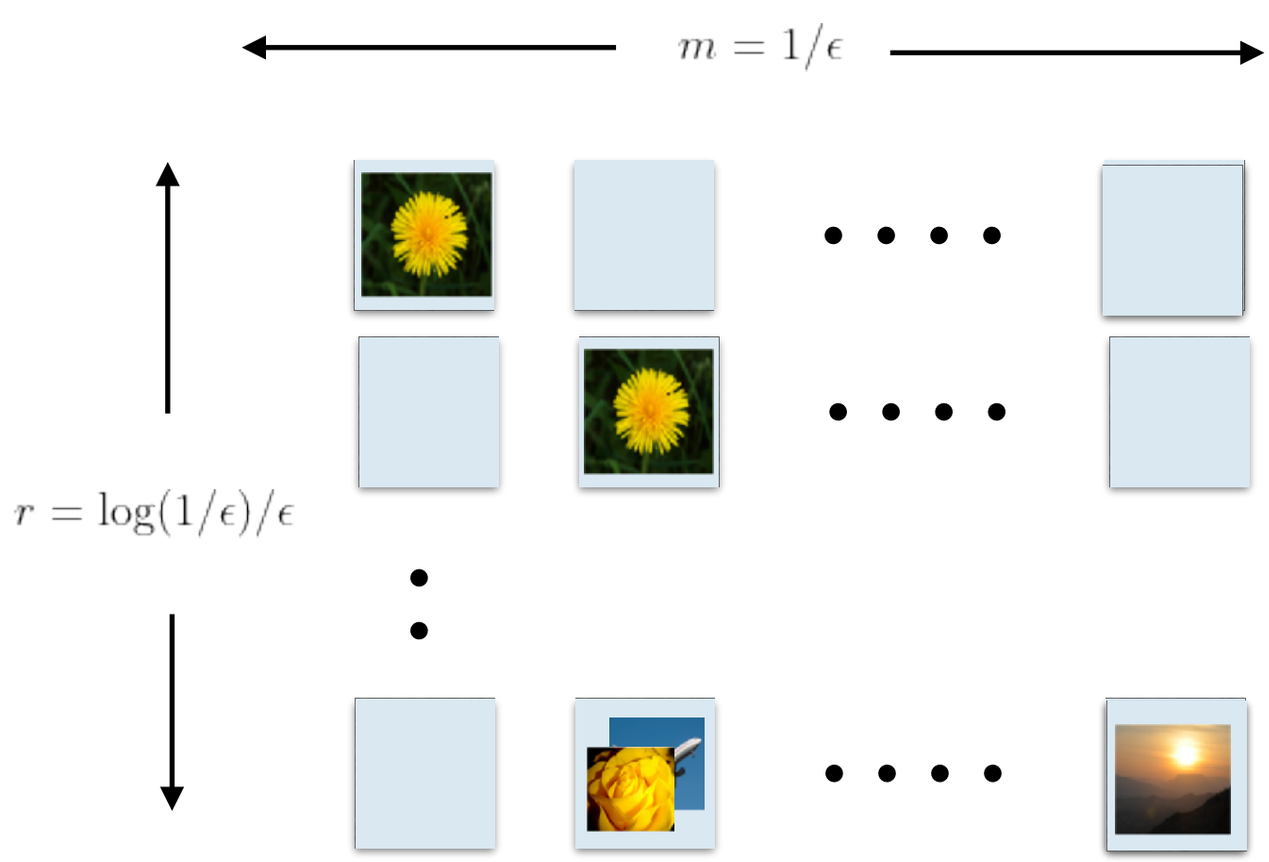

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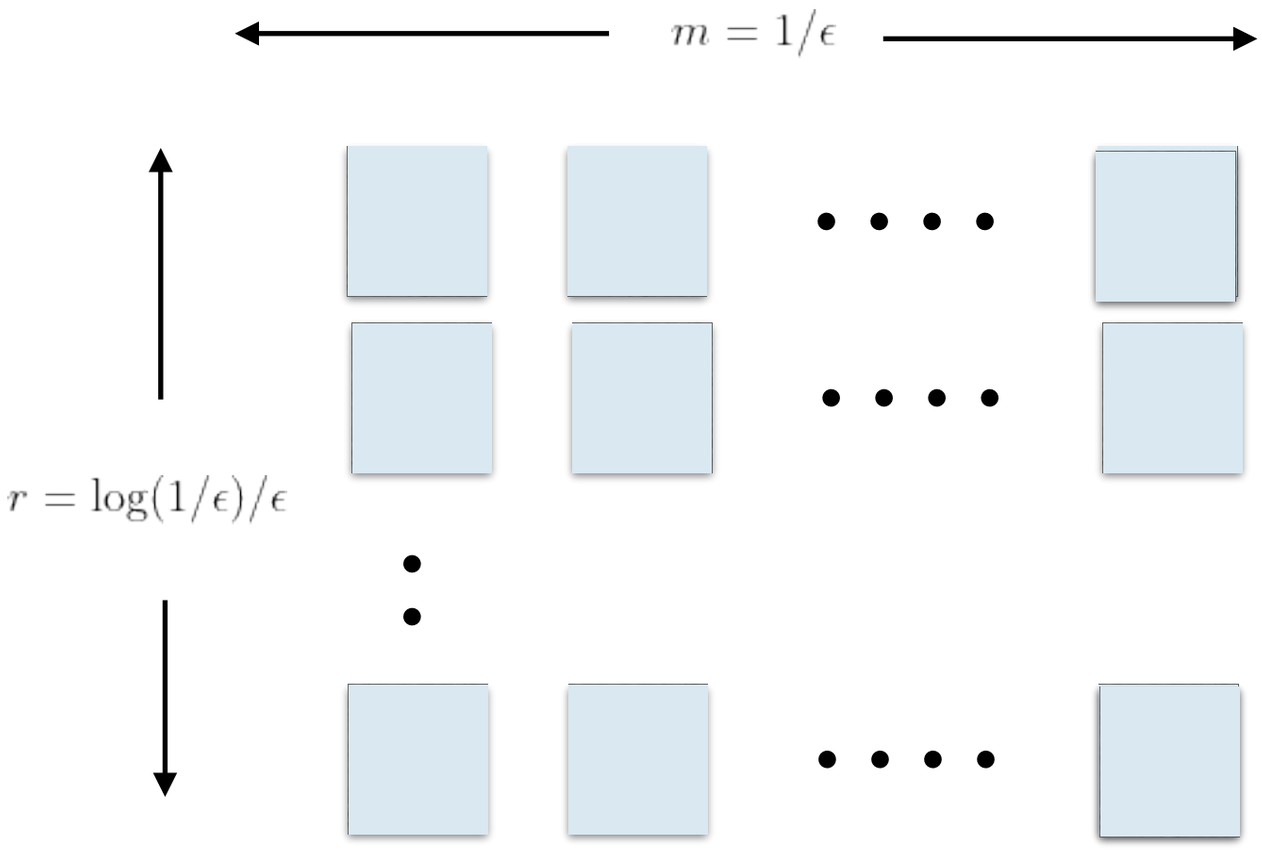

Offline Algorithm

Cardinality Constrained Streaming Submodular Maximization

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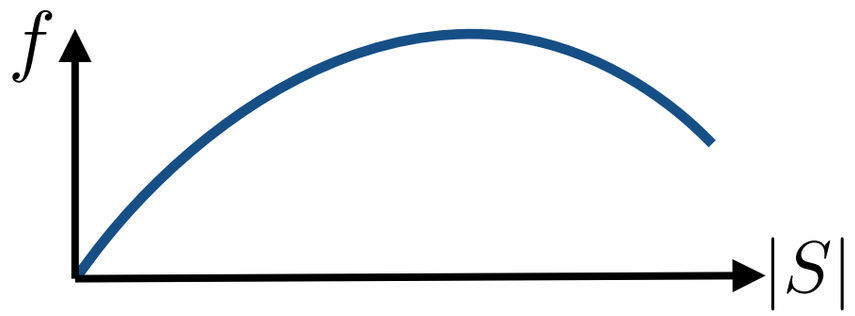

[Alaluf, Ene, Feldma, Nguyen Suh, 2020]

For non-monotone submodular functions, and subject to a cardinality constraint, there is a one-pass streaming algorithm that achieves

$$\mathbb{E}[f(S)] \geq \left(\frac{\alpha}{1+\alpha} - \epsilon\right) \mathrm{OPT}$$

where α is the approximation guarantee of the best offline algorithm.

"Optimal Streaming Algorithms for Submodular Maximization with Cardinality Constraints", 2020

Non-Monotone Streaming: Cardinality Constraint

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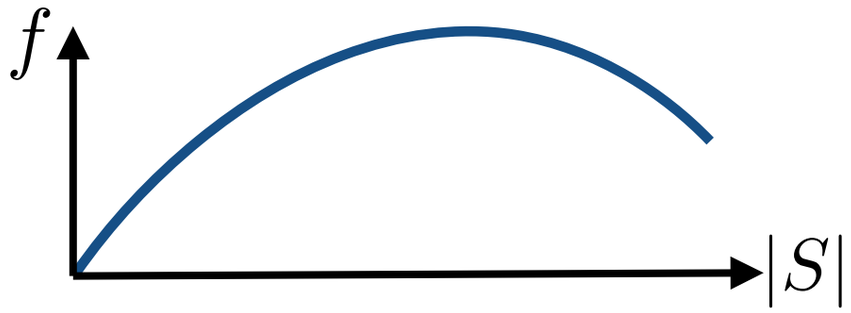


Optimal Streaming Algorithms for Submodular Maximization with Cardinality Constraints

2020

Naor Alaluf* Alina Ene† Moran Feldman‡ Huy L. Nguyen§ Andrew Suh¶

$\alpha = 0.385$

?

Hard

0.277

0.491

Submodular Maximization by Simulated Annealing

Shayan Oveis Gharan* Jan Vondrák†

2010

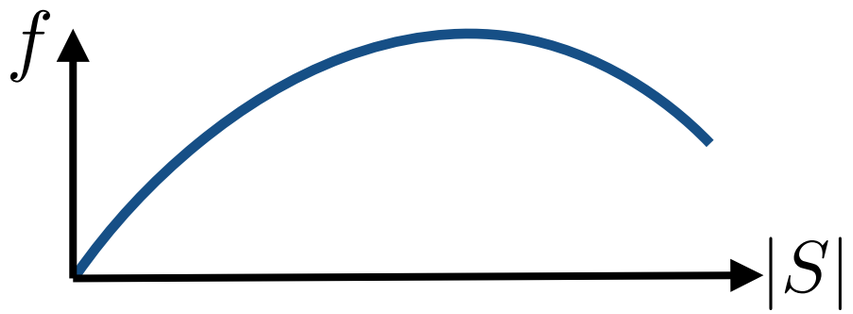
Streaming Non-Monotone Submodular Maximization

- Consider the following problem where f is submodular:

$$S^* = \arg\max_{S \in \mathcal{I}} f(S)$$

constraints

- When the utility functions f is non-monotone submodular



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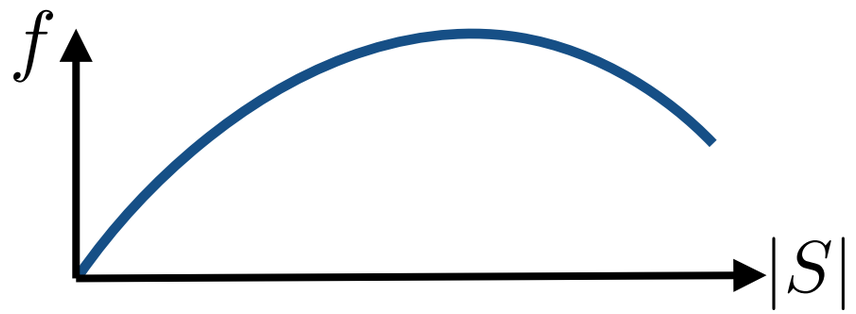
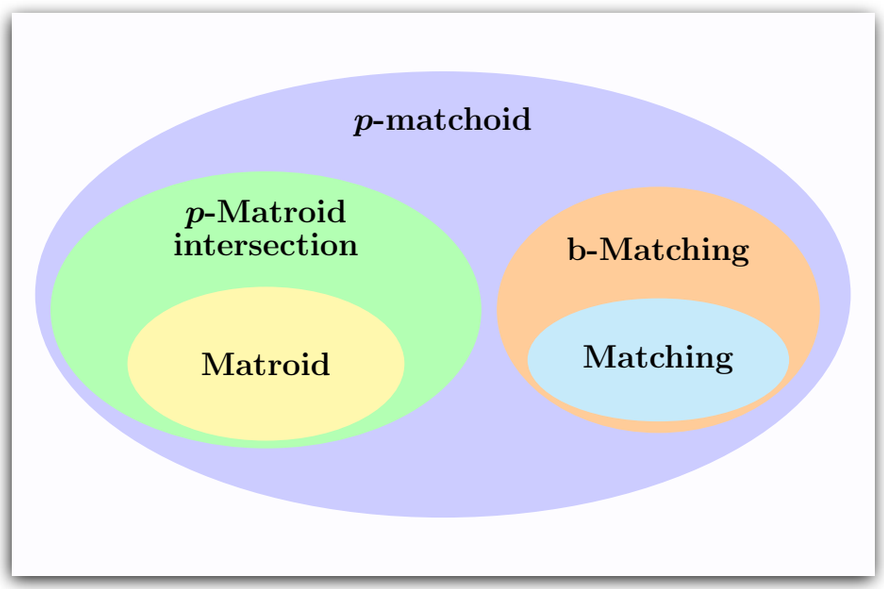


- and constraints imposed by the application

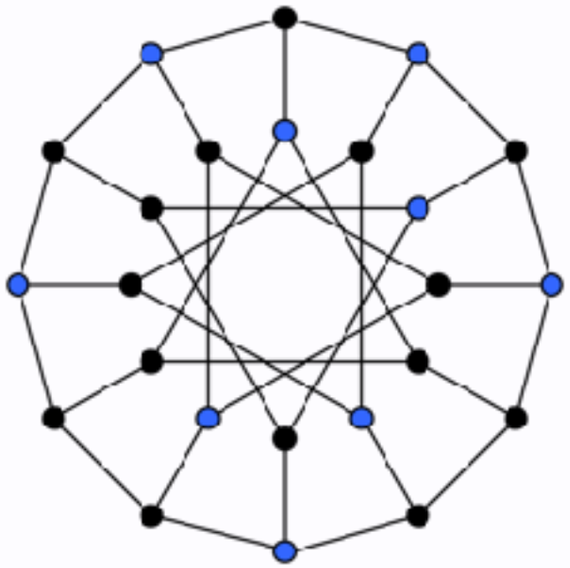
p-matchoid: a set system $(V, \mathcal{I})$ where there exist m matroids $(V_i, \mathcal{I}_i)$ such that every element of V appears in the ground set of at most p matroids and

$$\mathcal{I} = \left\{ S \subseteq 2^V | \forall_{1 \leq i \leq m} S \cap V_i \in \mathcal{I}_i \right\}$$



P-extendible

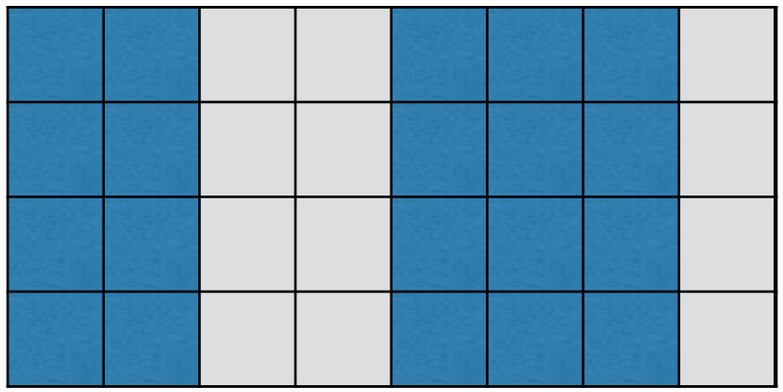
Independent Set $p = d_{\max}$



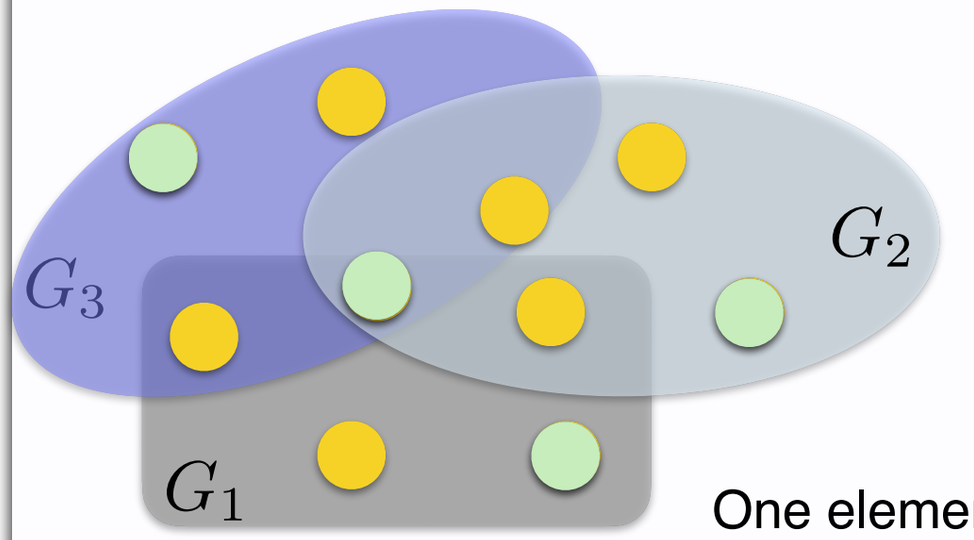
Matroid $p = 1$

$\forall A \subset B$ if $B \in \mathcal{I} \Rightarrow A \in \mathcal{I}$

$A, B \in \mathcal{I}$ & $|A| < |B| \Rightarrow \exists v \in B$ s.t. $A \cup \{v\} \in \mathcal{I}$

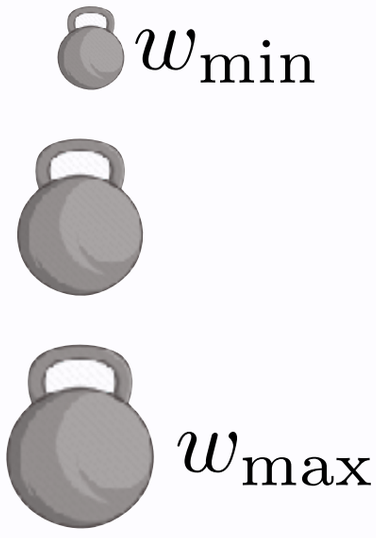


Intersection of k matroids $p = k$



One element from each group

Knapsack $p = \left\lceil \frac{w_{\max}}{w_{\min}} \right\rceil$



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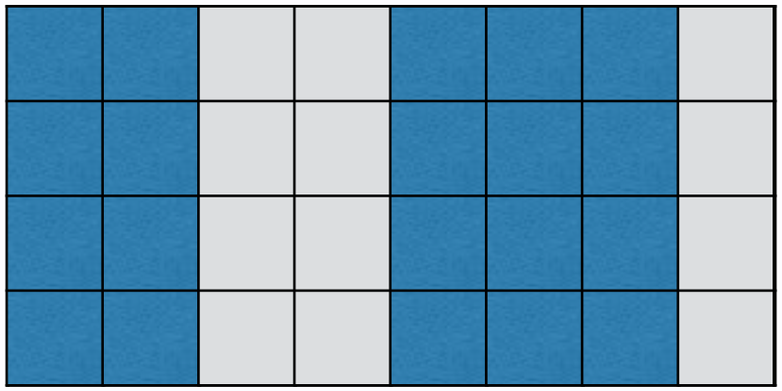

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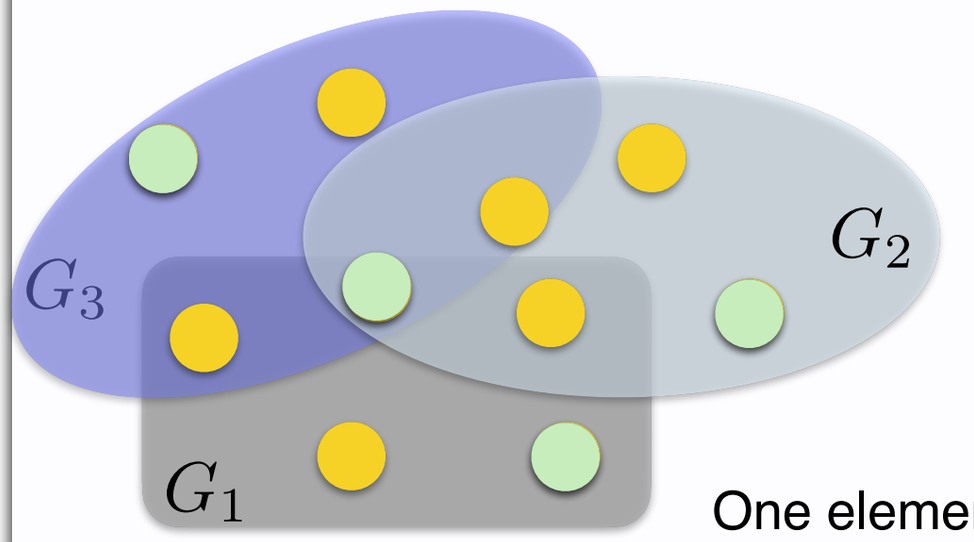

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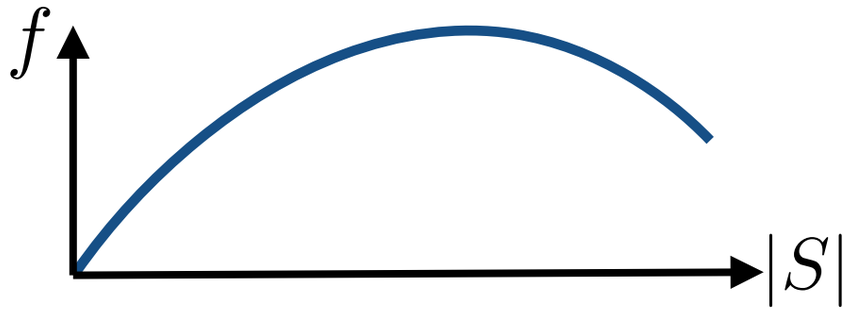

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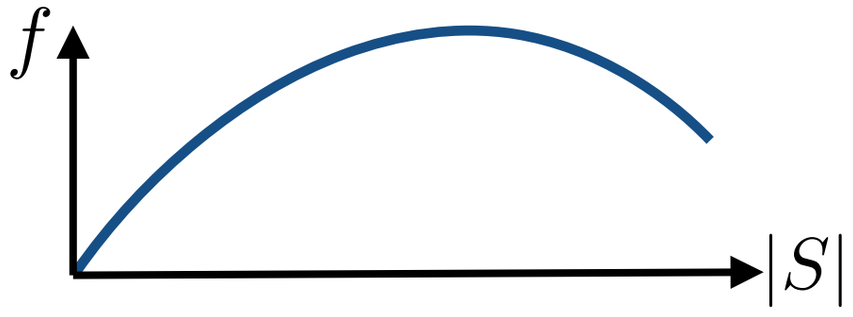

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"Streaming Algorithms for Submodular Function Maximization", Chekuri, Gupta, Quanrud, 2015.

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Yale

Penn

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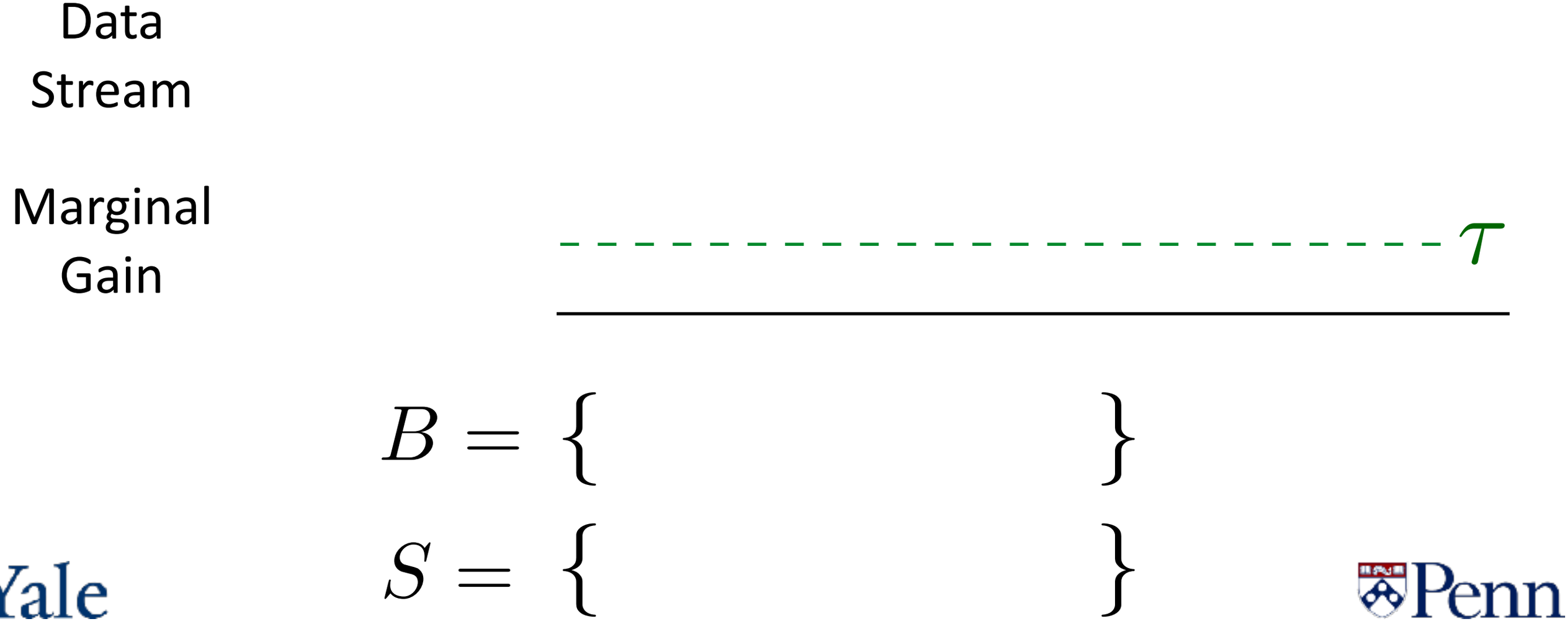

Yale

Penn

Cardinality Constrained Streaming Submodular Maximization

- For each item e in the stream:
  - If the marginal gain of e is greater than the threshold τ, add e to the buffer B.
  - If the buffer B becomes full:
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Data Stream

Marginal Gain

Yale

Penn

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$$B = \{ \quad \quad \quad \}$$

$$S = \{ \quad \}$$

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New Marginal Gains

τ

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Yale

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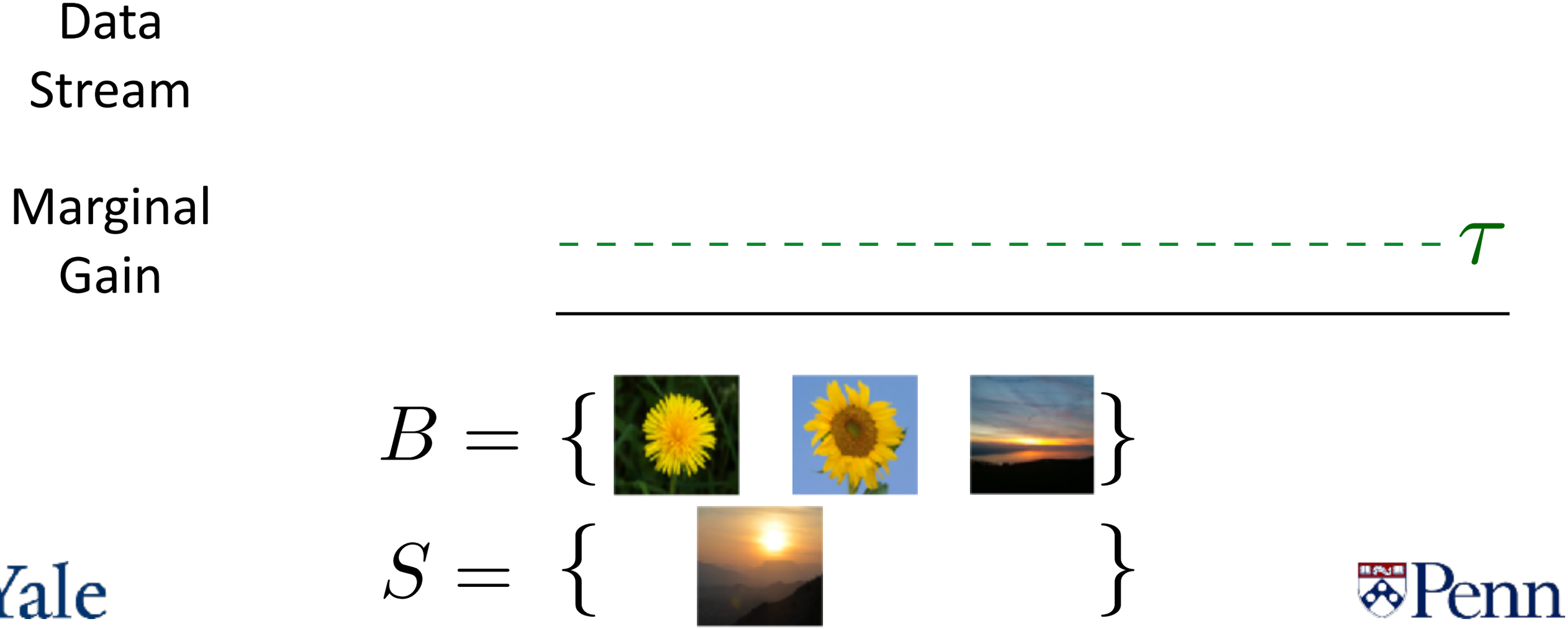

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Yale

Penn

Randomized Streaming Greedy

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[Chekuri, Gupta, Quanrud]

For non-negative submodular functions under a p-matchoid constraint, Streaming-Greedy with an $O(k \log k/\epsilon^2)$ memory gives constant factor approximation:

- if f is **monotone**:

$$\mathbb{E}[f(S_{\text{Streaming-greedy}})] \geq \frac{1}{4p}\text{OPT}$$

- if f is **non-monotone**:

$$\mathbb{E}[f(S_{\text{Streaming-greedy}})] \geq \frac{1-\epsilon}{5p+2+1/p}\text{OPT}$$

"Streaming Algorithms for Submodular Function Maximization", 2015

Yale

Penn

The Sample-Streaming Algorithm

- For each item u in the stream:
  - Keep u with probability $q = \frac{1}{p+\sqrt{p(p+1)}+1}$
  - If we kept u: swap u into S so that we maintain feasibility and check that the marginal gain is large enough.

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Data Stream

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Yale

Penn

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Yale

Penn

Sample-Streaming: Guarantee

[Feldman, Karbasi, Kazemi]

For non-negative submodular functions under a p-matchoid constraint, Sample-Streaming with an $O(k)$ memory gives constant factor approximation using only $O(km/p)$ function evaluation per element:

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- if f is **non-monotone**:

$$\mathbb{E}[f(S_{\text{SAMPLE-STREAMING}})] \geq \frac{1}{4p+2-o(1)}\text{OPT}$$

"Do Less, Get More: Streaming Submodular Maximization with Subsampling", 2018

Summary

"Streaming non-monotone submodular maximization: Personalized video summarization on the fly", Mirzasoleiman, Jegelka, Krause, 2018

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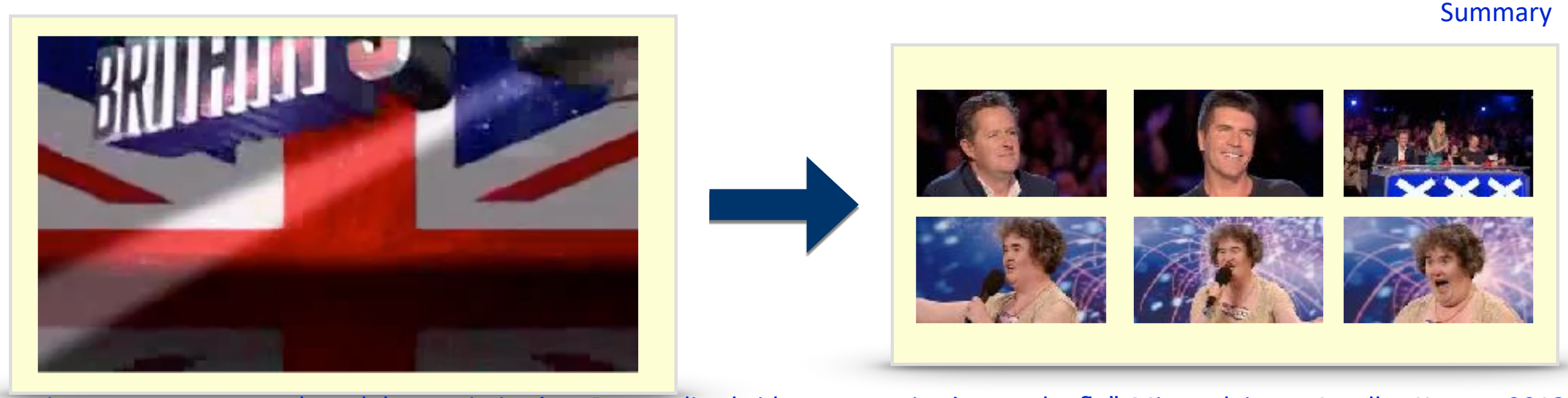

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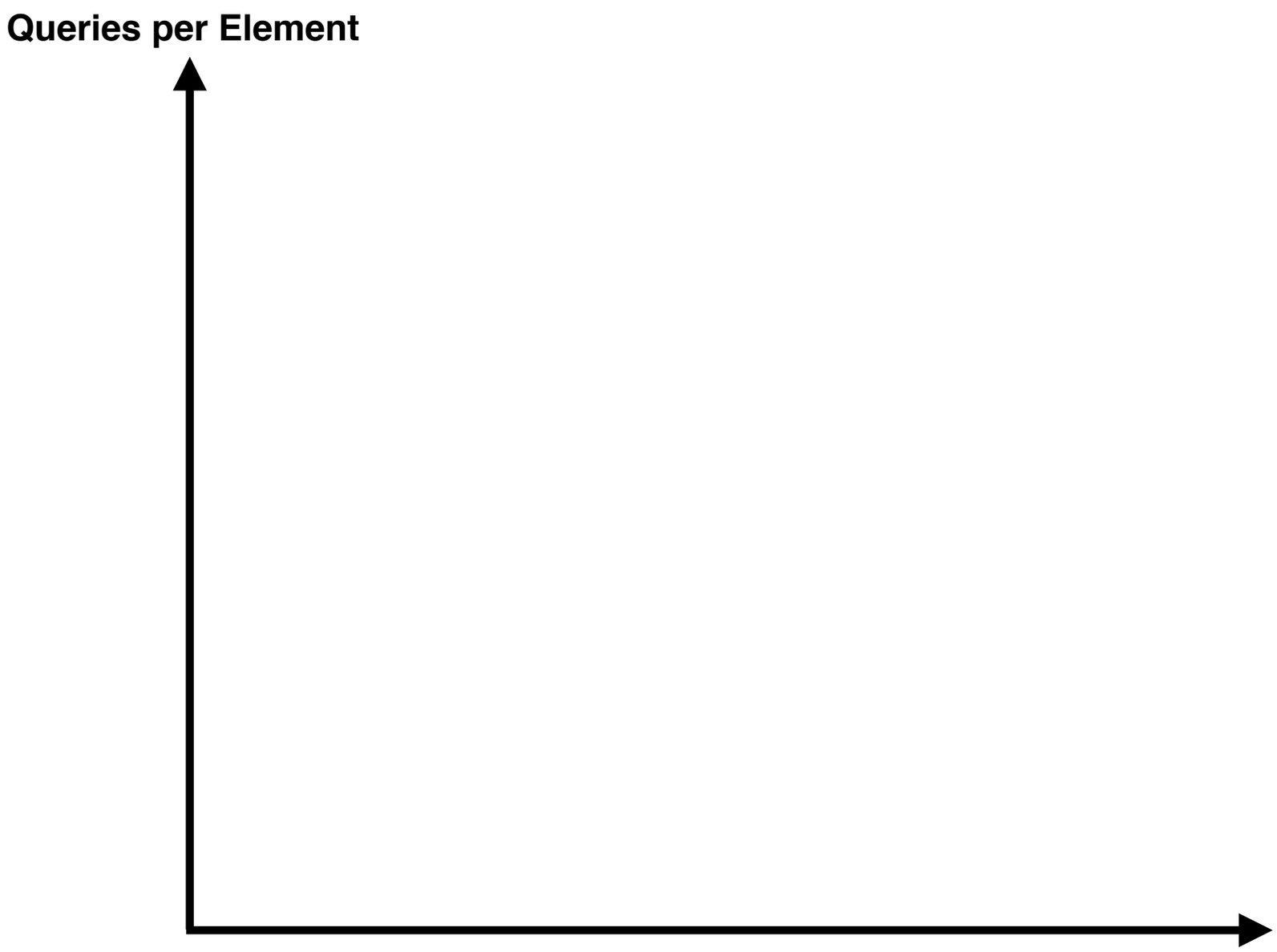
Streaming Algorithms Subject to p-matchoid

Monotone



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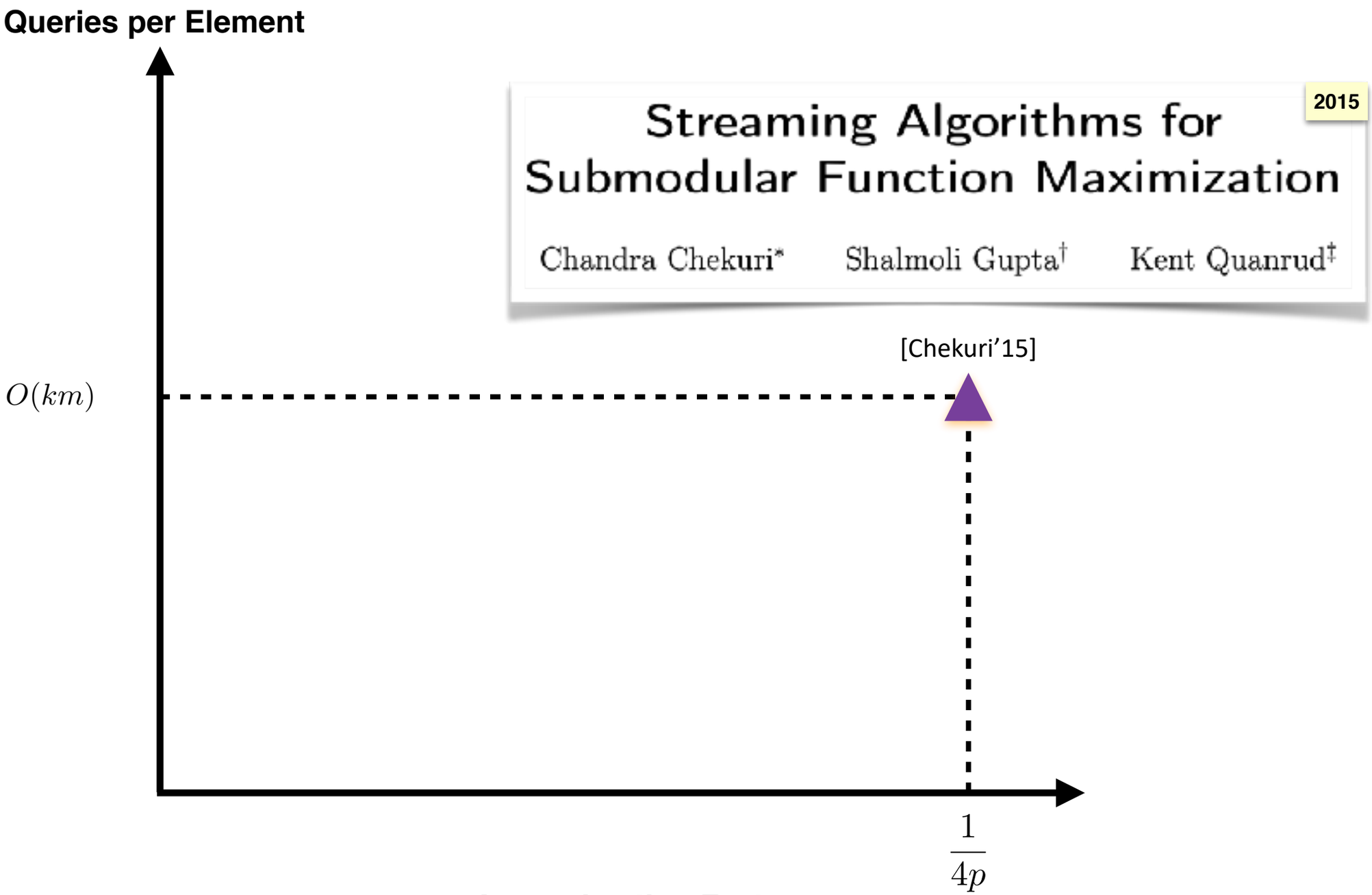

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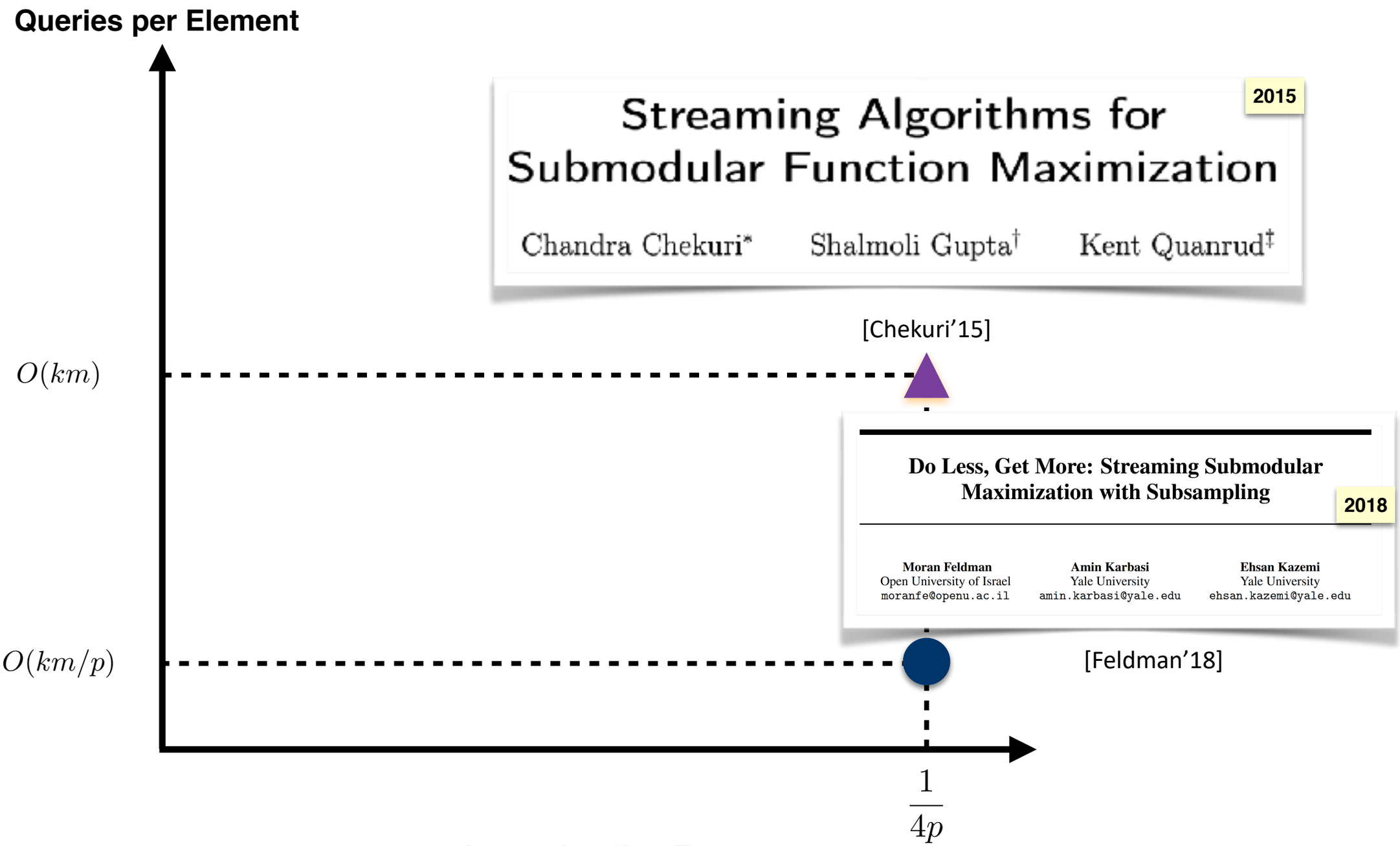
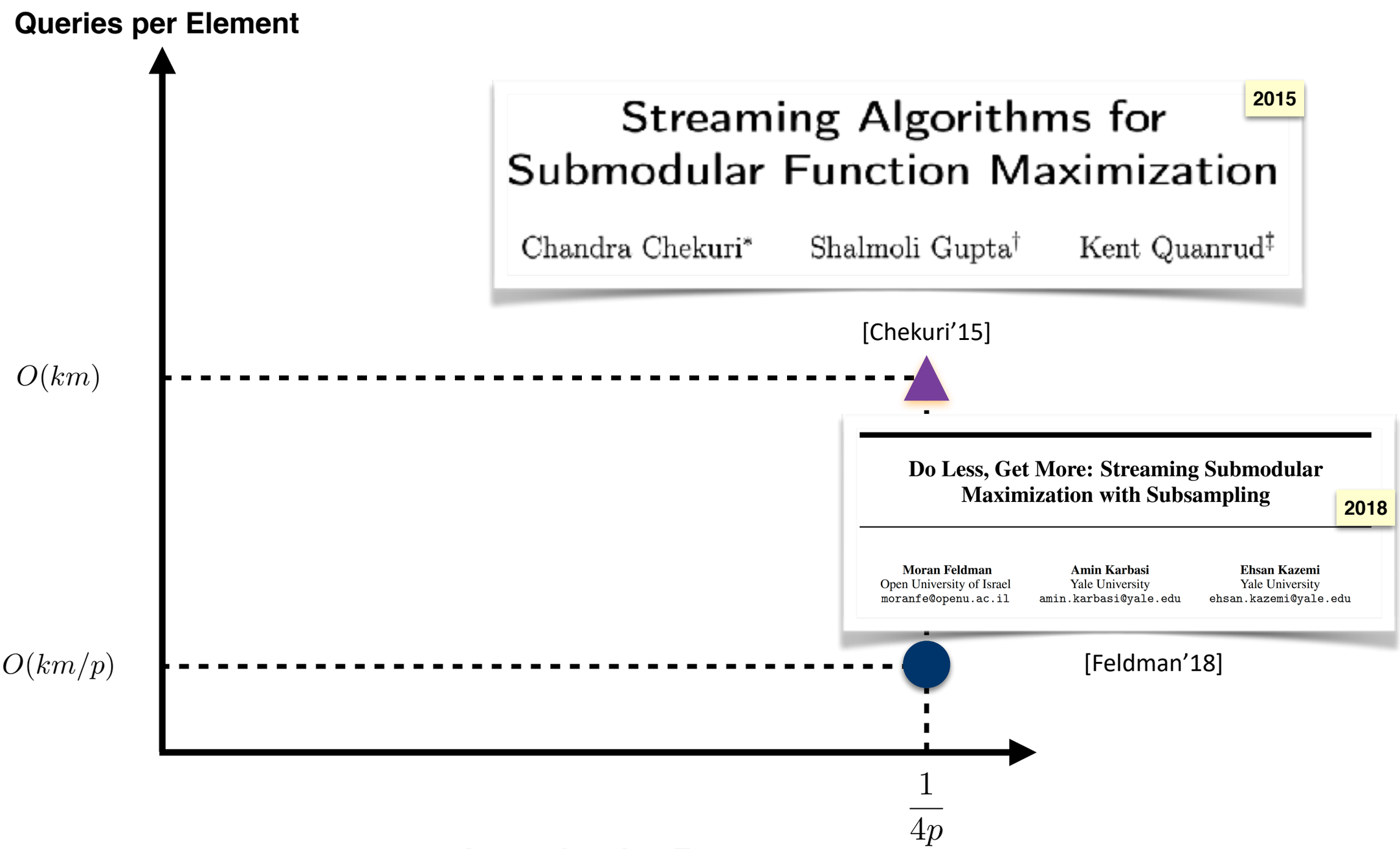


Streaming Algorithms Subject to p-matchoid

Monotone → Non-montone



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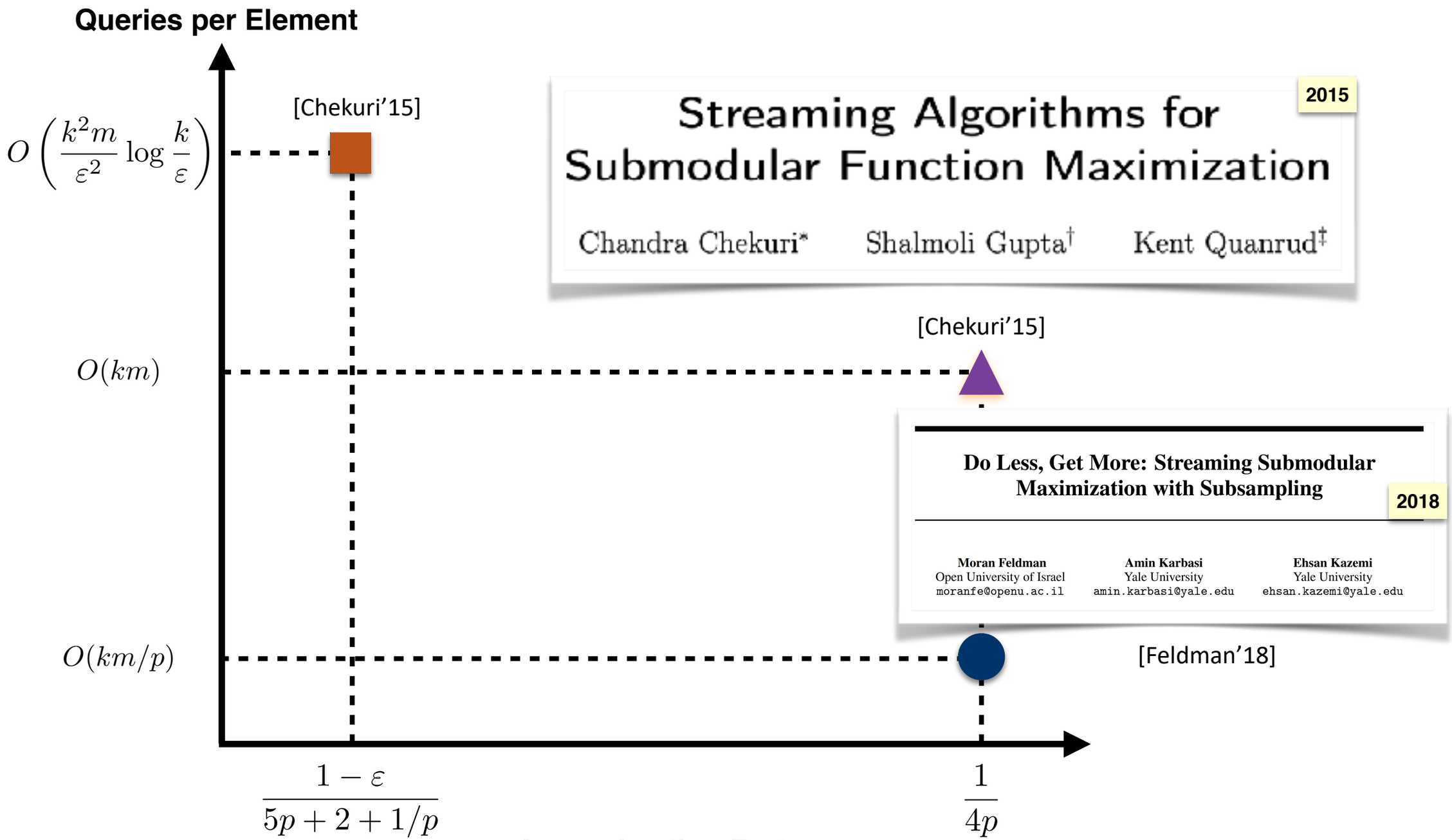

2015

Streaming Algorithms for Submodular Function Maximization

Chandra Chekuri* Shalmoli Gupta† Kent Quanrud‡

2018

Do Less, Get More: Streaming Submodular Maximization with Subsampling

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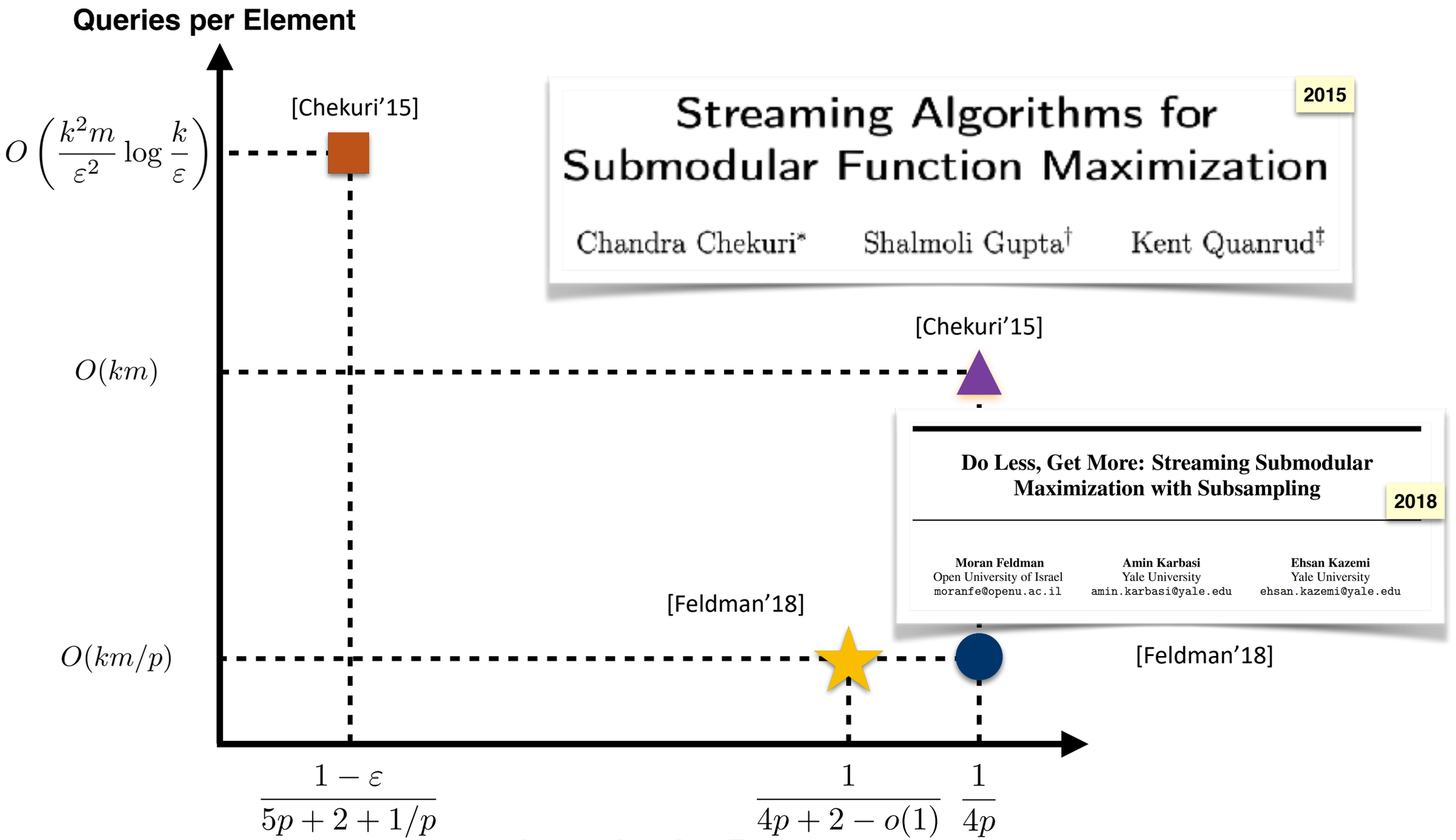

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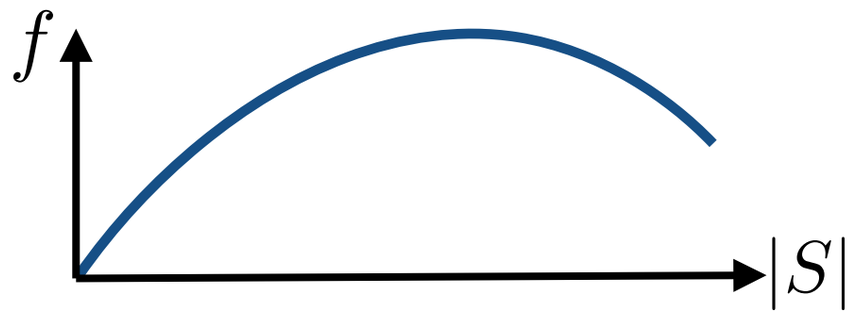
Constrained Non-Monotone Submodular Maximization

- Consider the following problem where f is submodular:

$$S^* = \arg\max_{S \in \mathcal{I}} f(S)$$

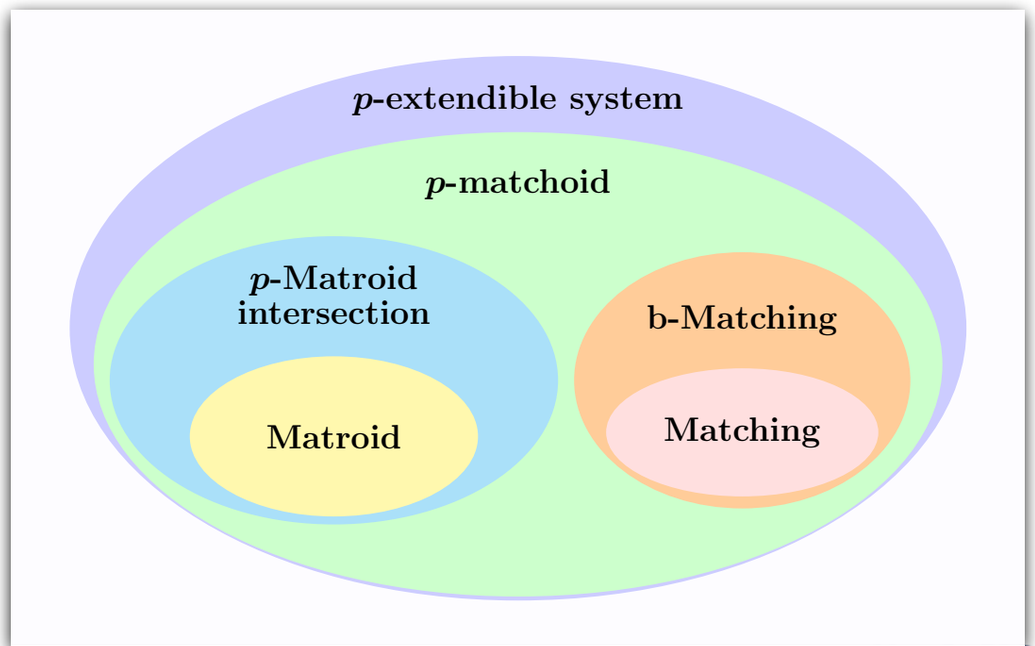
constraints

- When the utility functions f is non-monotone submodular



- and constraints imposed by the application

p-extendible system: adding an element e to a feasible set B requires the removal of at most p elements in order to keep the resulting set feasible.



Open Question: K-Extendible and K-System

Streaming Submodular Maximization under a k-Set System Constraint

Ran Haba* Ehsan Kazemi† Moran Feldman† Amin Karbasi§

2020

$$O\left(\frac{1}{p \log p}\right)$$

Hard

$$\frac{1}{p + 1/2 - \varepsilon}$$

Greed Is Good: Near-Optimal Submodular Maximization via Greedy Optimization

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2017

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2017

This Talk

$$S^* = \arg\max_{S \subseteq V, S \in \mathcal{I}} f(S)$$

Perfect Oracle

Discrete Methods

- **Centralized**
- **Streaming**
- **Distributed**

Imperfect Oracle

Continuous Methods

This Talk

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Perfect Oracle

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- **Centralized**
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Continuous Methods

Scaling Up

- On massive data (45M users' logs) the greedy policies take a few days/ weeks to complete

$$S^* = \arg\max_{S \subseteq V, S \in \mathcal{I}} f(S)$$

Can we parallelize the greedy approach?

Setting

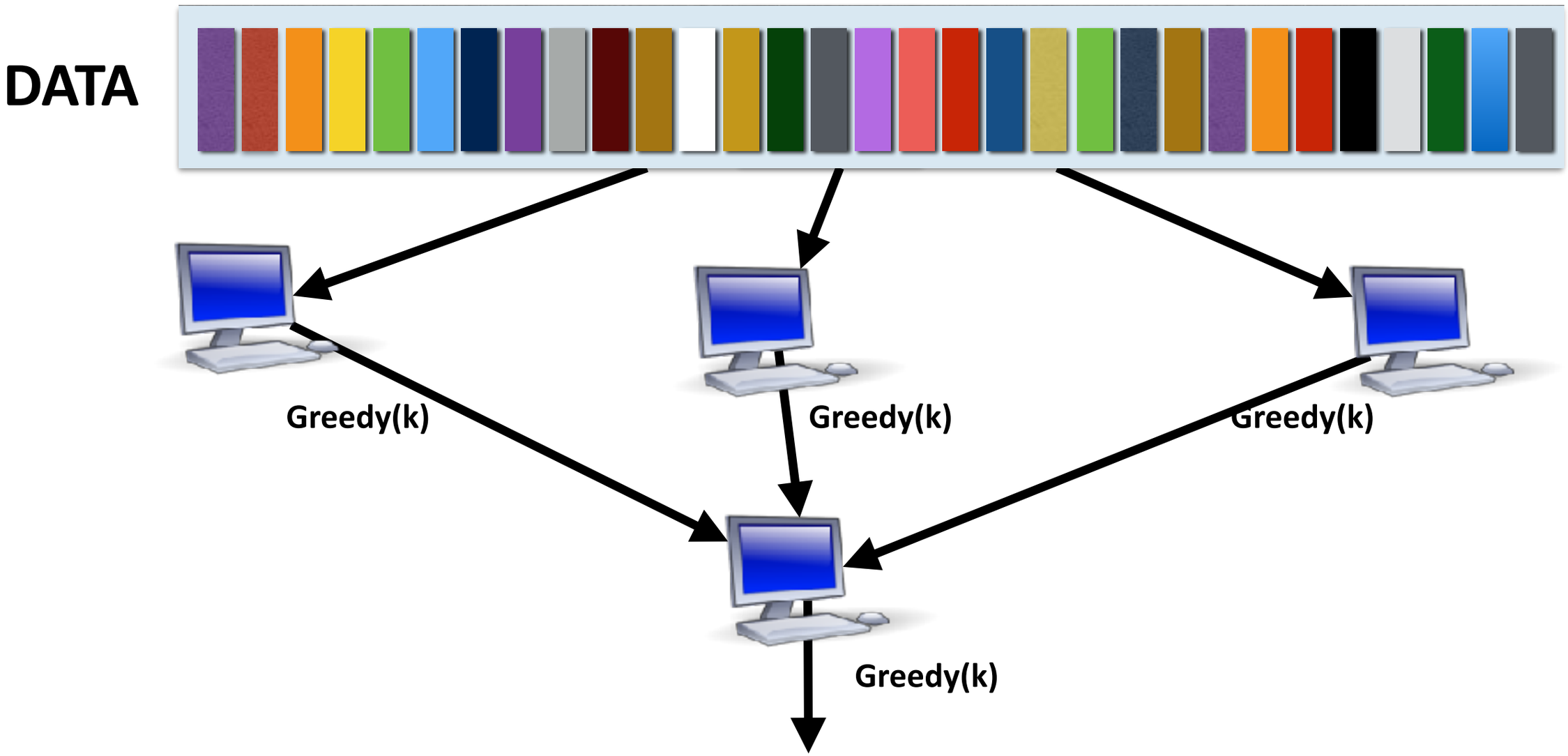
- Distributed

"Fast Greedy Algorithms in MapReduce and Streaming", 2013, Kumar, Moseley, Vassilvitskii, Vattani

Two-Stage Distributed Algorithm (GreeDi)

- Consider the following problem where f is monotone submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$



"Distributed Submodular Maximization: Identifying Representative Elements in Massive Data", Mirzasoleiman et al, 2013

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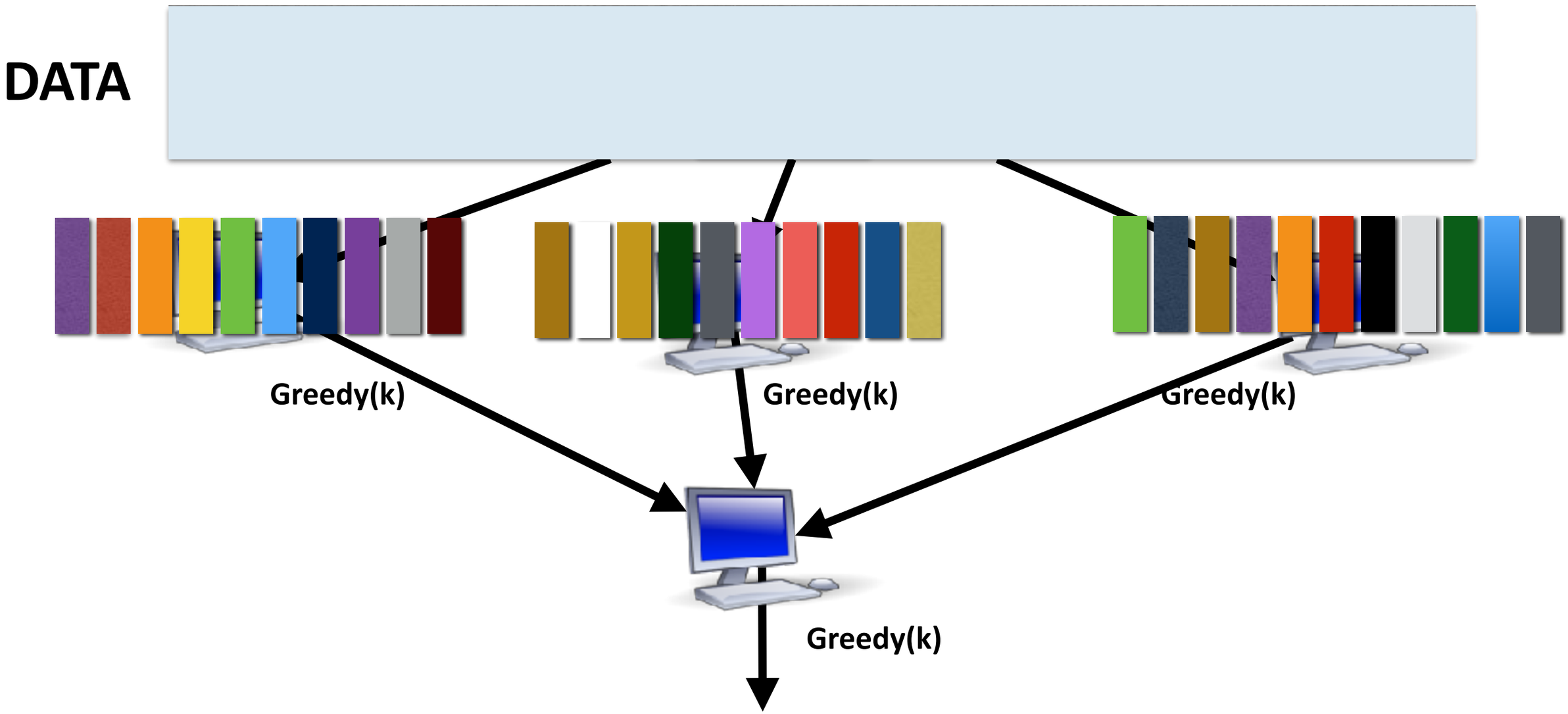

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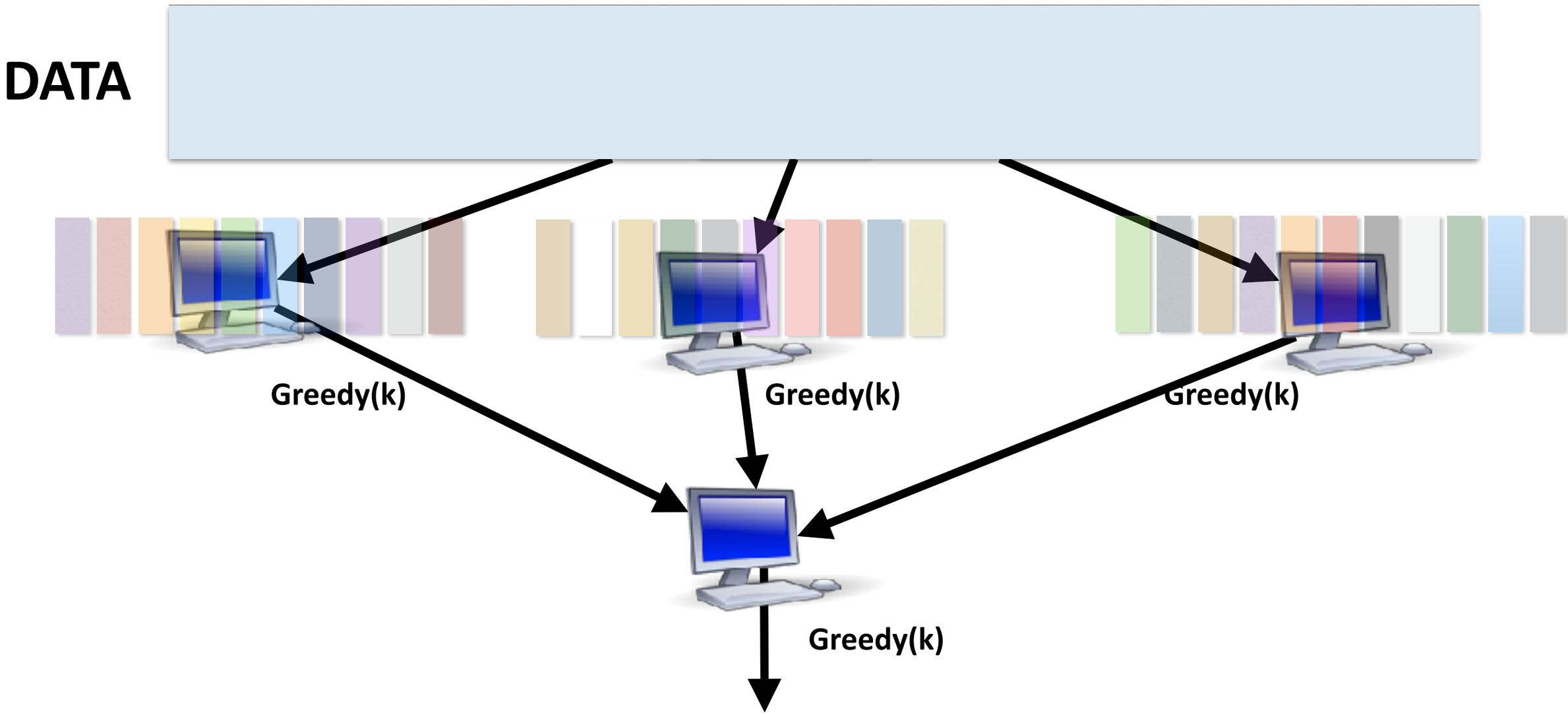

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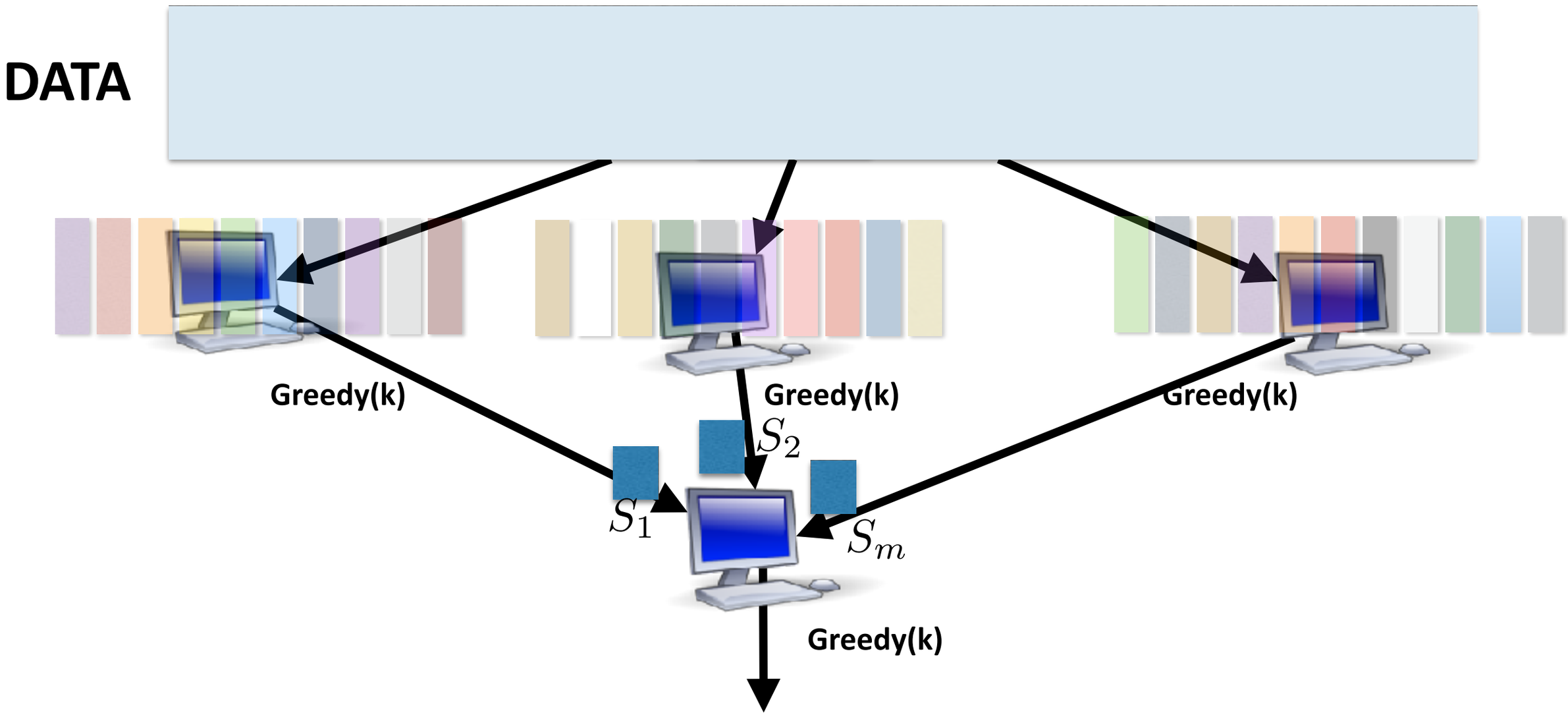

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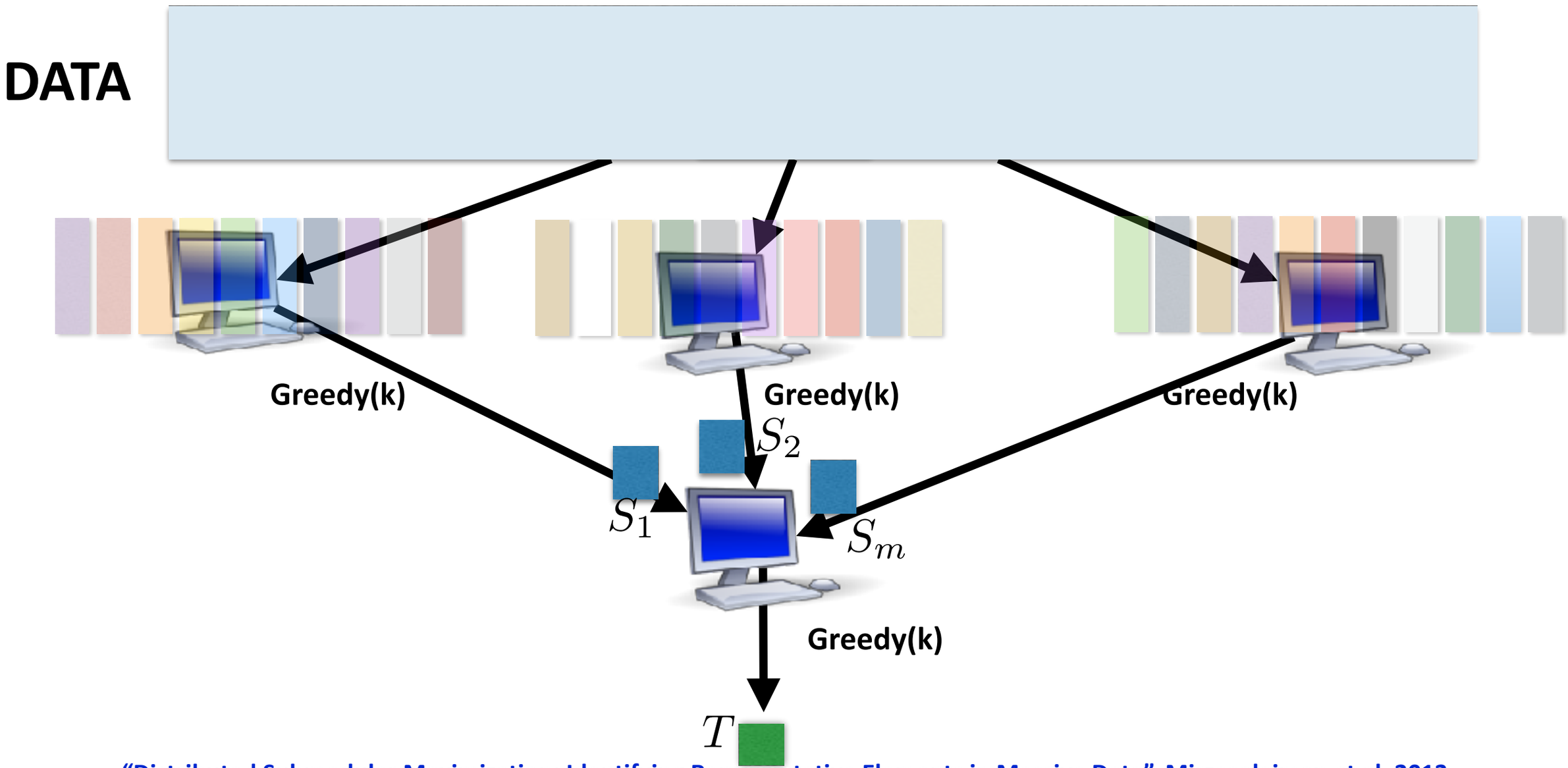

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Theoretical Guarantees

- Split data (arbitrarily) among *m* machine

[Mirzasoleiman, Karbasi, Sarkar, Krause]

For monotone submodular functions, GreeDi gives

$$f(S_{\mathrm{GreeDi}}) \geq \frac{1-1/e}{\min(m,k)} \cdot \mathrm{OPT}$$

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[Mirzasoleiman, Karbasi, Badanidiyuru, Krause] [Barbosa, Ene, Nguyen, Ward]

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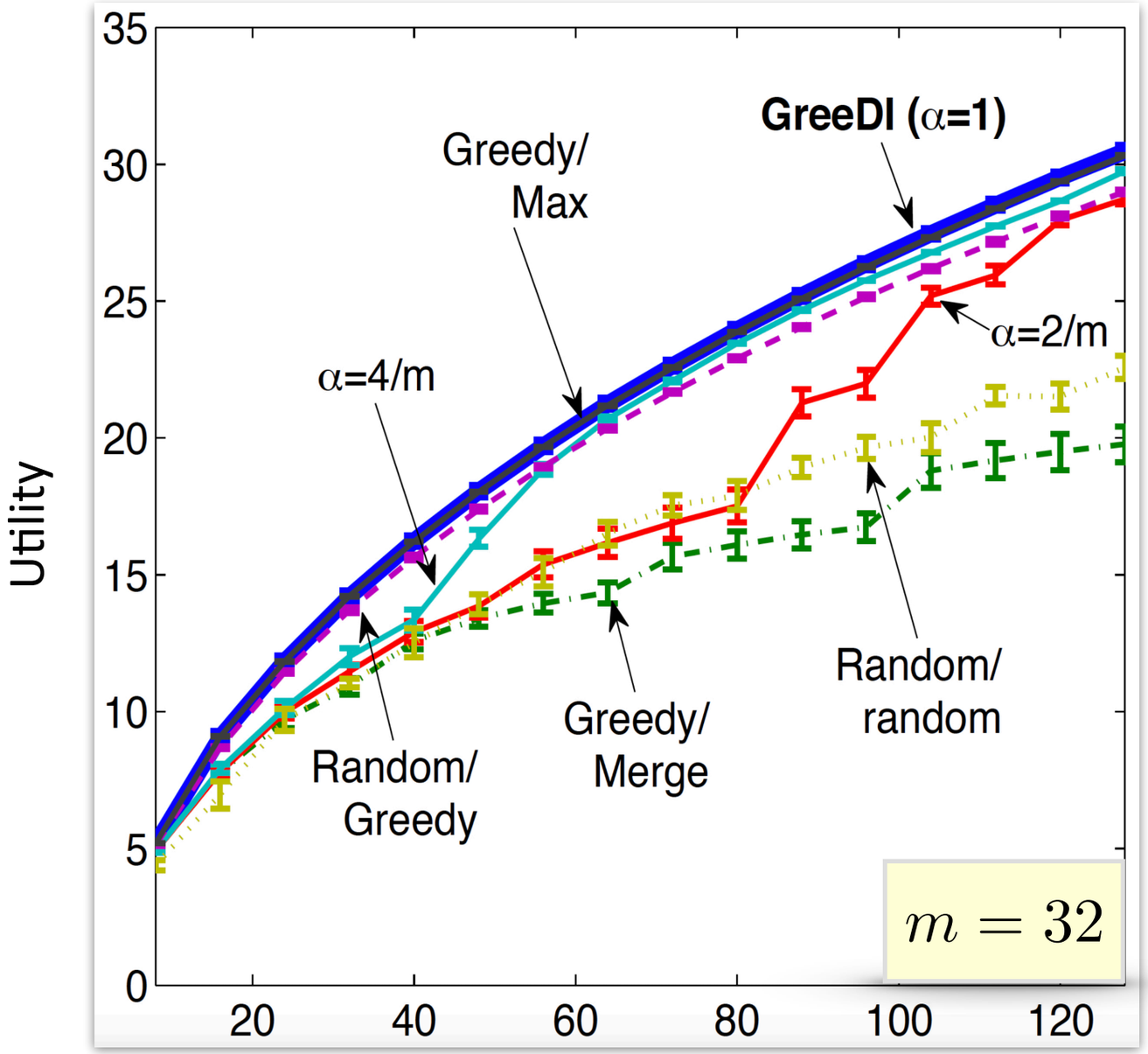
and there exists no algorithm that can do better in general.

"Distributed Submodular Cover: Succinctly Summarizing Massive Data", 2015
"The Power of Randomization: Distributed Submodular Maximization on Massive Datasets", 2015.

Nonparametric Regression on Hadoop

- **Yahoo! Webscope** dataset: 45,811,883 user click logs for news articles displayed in Yahoo! Front Page



The Power of Randomization

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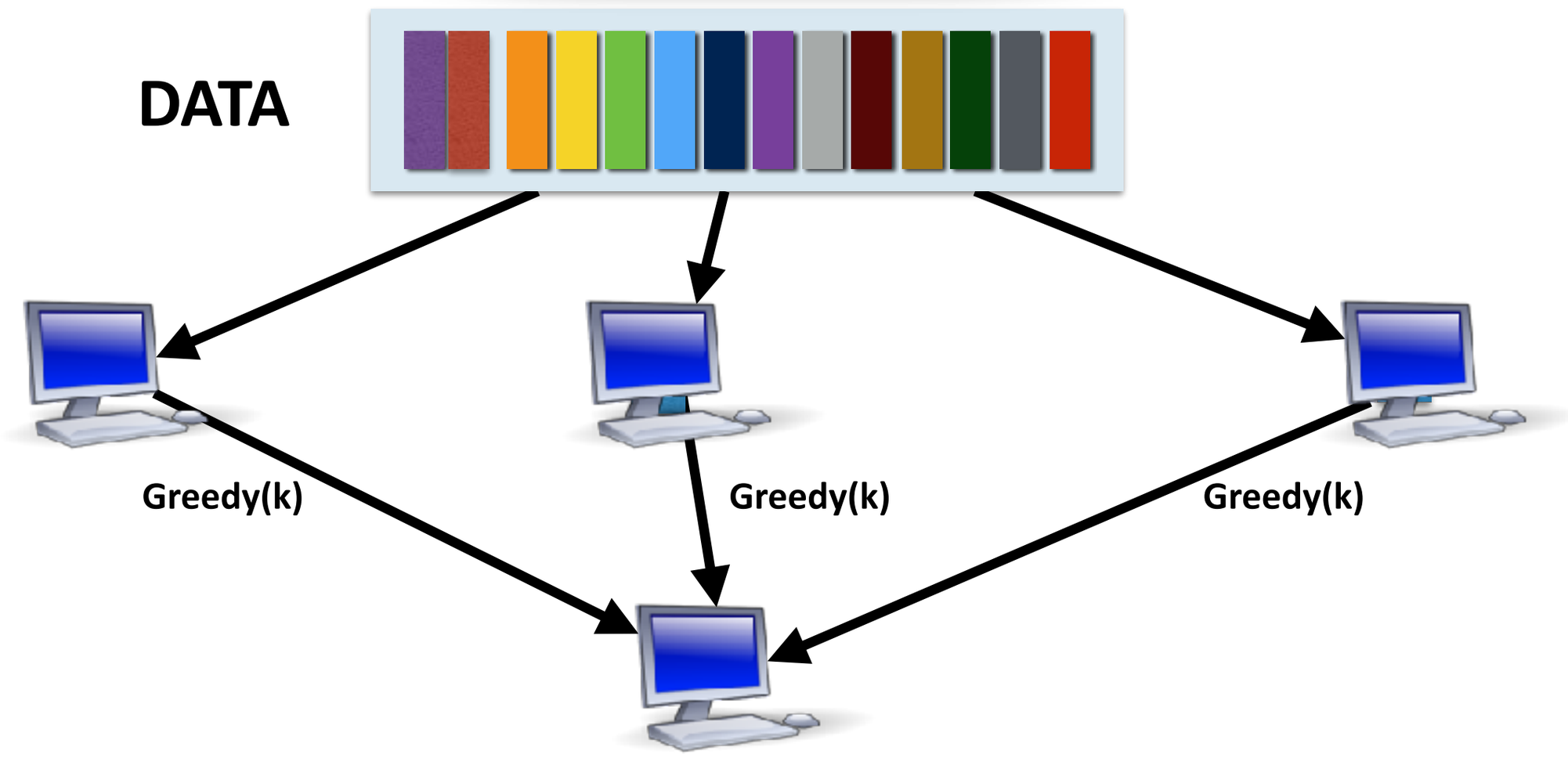

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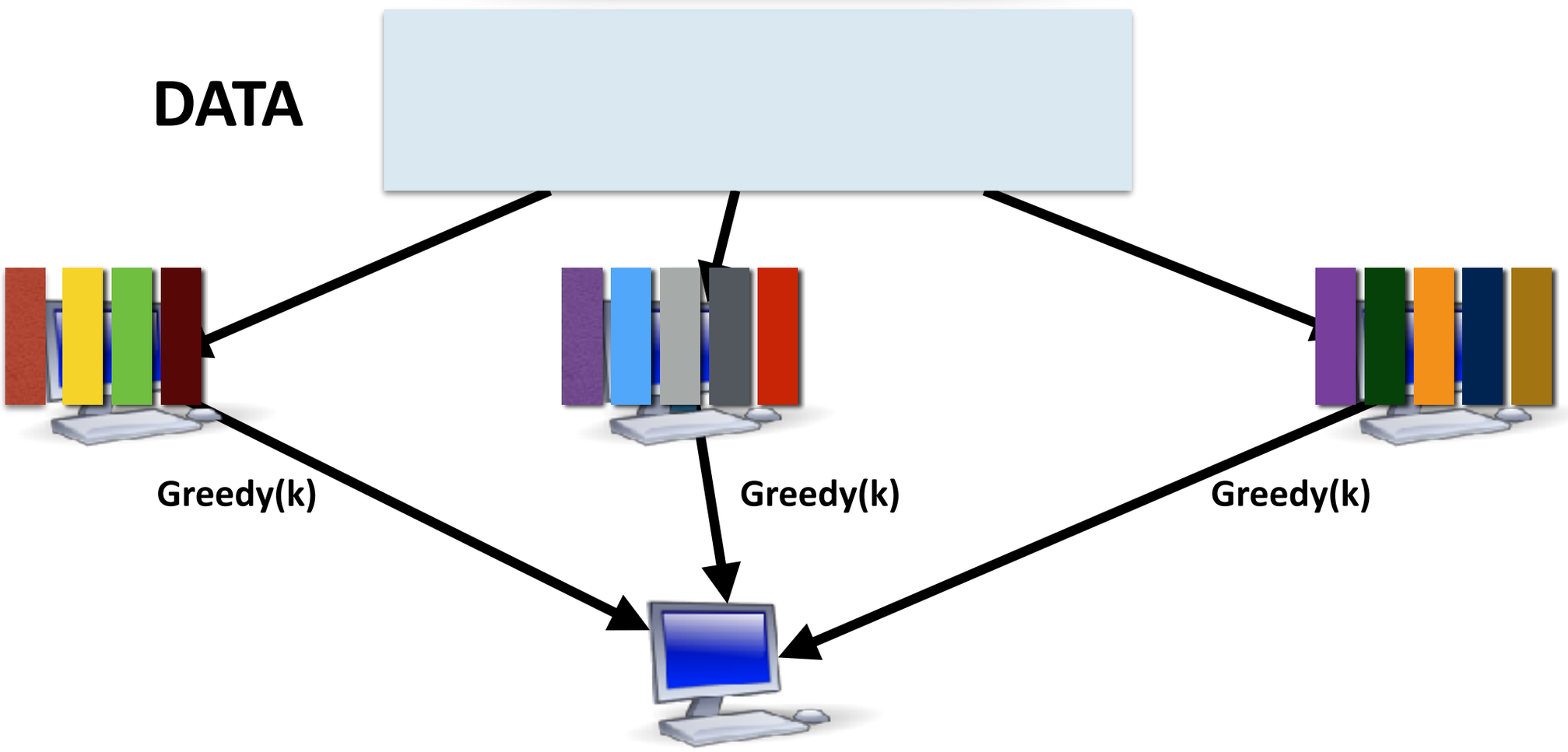

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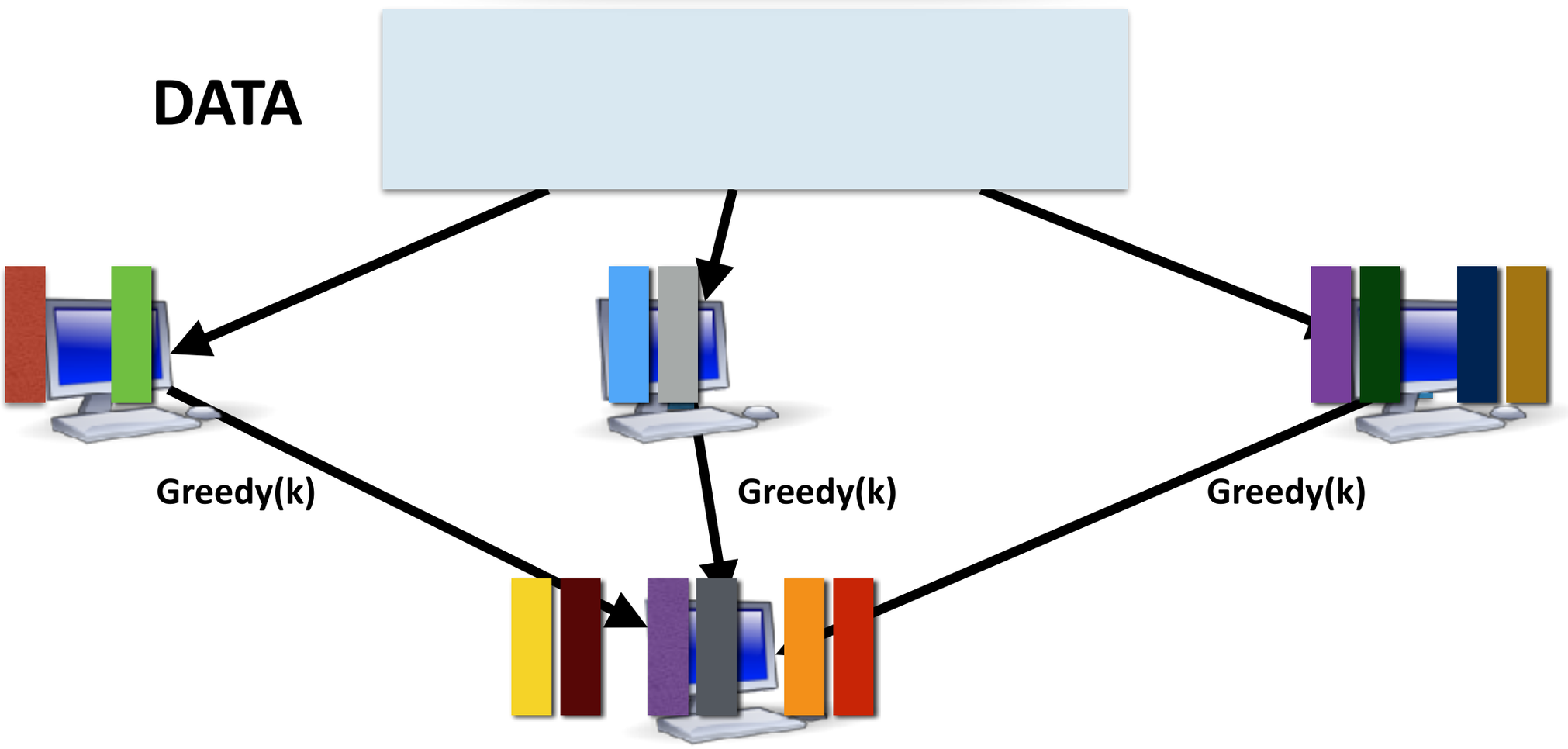

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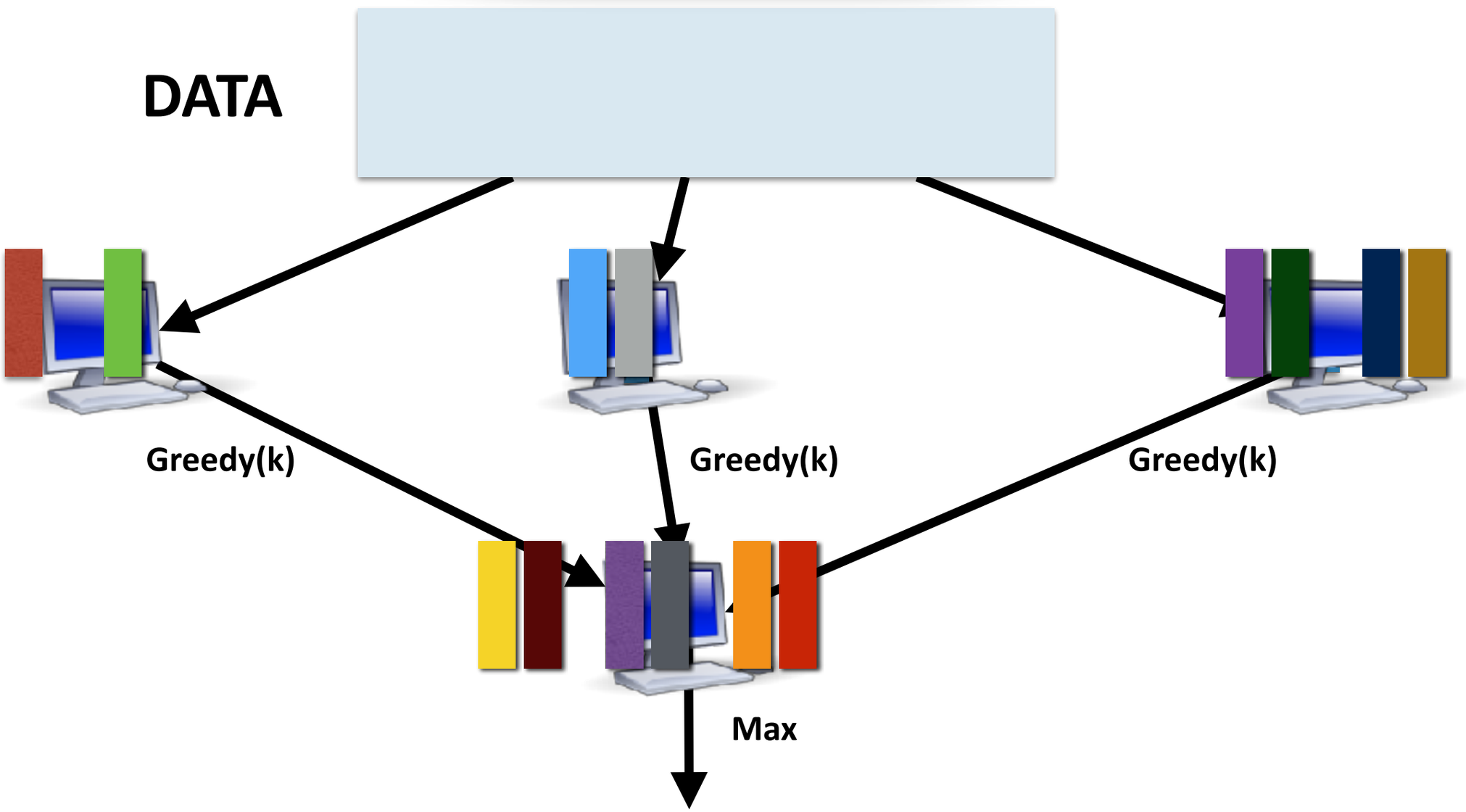

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The Power of Randomization

- Consider the following problem where f is non-negative submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$

[Barbosa, Ene, Nguyen, Ward] [Mirrokni, Zadimoghaddam]

When f is montone, radomized GreeDi returns a solution S of size k such that

$$\mathbb{E}[f(S_{\text{GreeDi}})] \geq \left(\frac{1-1/e}{2}\right) \text{OPT}$$

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"Randomized composable core-sets for distributed submodular maximization" Mirrokni, Zadimoghaddam, 2015.

The Power of Randomization

- Consider the following problem where f is non-negative submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$

[Barbosa, Ene, Nguyen, Ward] [Mirrokni, Zadimoghaddam]

When f is montone, radomized GreeDi returns a solution S of size k such that

$$\mathbb{E}[f(S_{\text{GreeDi}})] \geq \left(\frac{1-1/e}{2}\right) \text{OPT}$$

"The Power of Randomization: Distributed Submodular Maximization on Massive Datasets", 2015.

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"A New Framework for Distributed Submodular Maximization", 2016

Distributed Submodular Maximization

Arbitrary Partition

Distributed Submodular Maximization

Arbitrary Partition

Distributed Submodular Maximization: Identifying Representative Elements in Massive Data

Baharan Mirzasoleiman ETH Zurich
Amin Karbasi ETH Zurich
Rik Sarkar University of Edinburgh
Andreas Krause ETH Zurich

2013

Distributed Submodular Cover: Succinctly Summarizing Massive Data

Baharan Mirzasoleiman ETH Zurich
Amin Karbasi Yale University
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Random Partition (2 Rounds)

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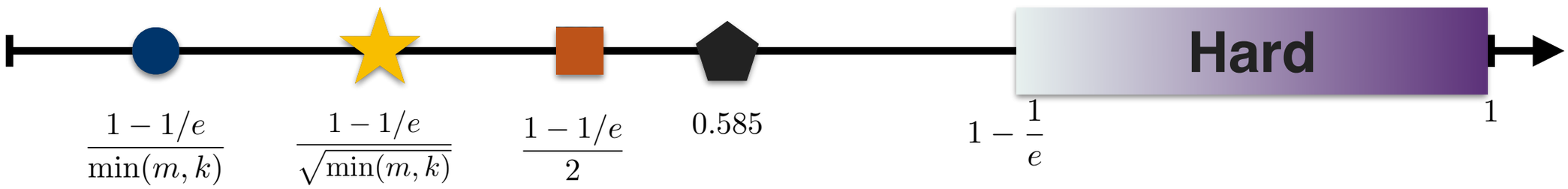

Distributed Submodular Cover: Succinctly Summarizing Massive Data

2015

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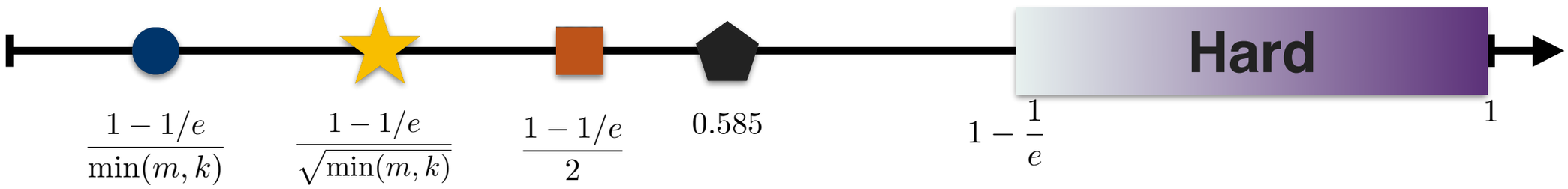

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Randomized Composable Core-sets for Distributed Submodular Maximization

Vahab Mirrokni Morteza Zadimoghaddam

- Is it possible to achieve 1-1/e?

Multi-Stage Distributed Greedy

- Randomly distribute the data amongst m outer machines. Each machine then runs a greedy algorithm on the subset of data assigned to it.
- The central machine aggregates all these sub-solutions into one large solution.
- Randomly re-distribute the data amongst the m outer machines. Each machine then runs another greedy algorithm on the subset of data assigned to it *plus* the combined solution from the previous step.

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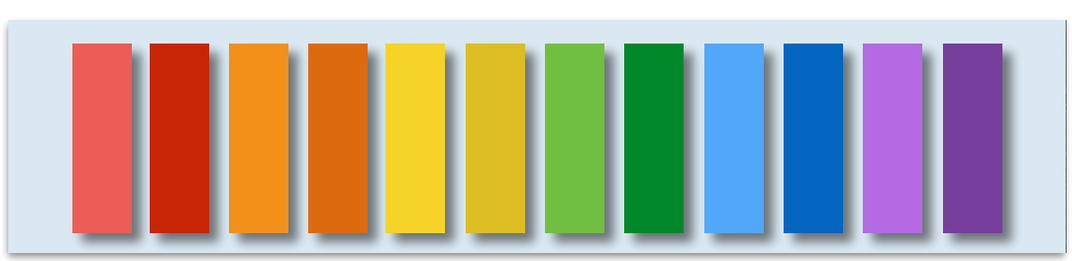

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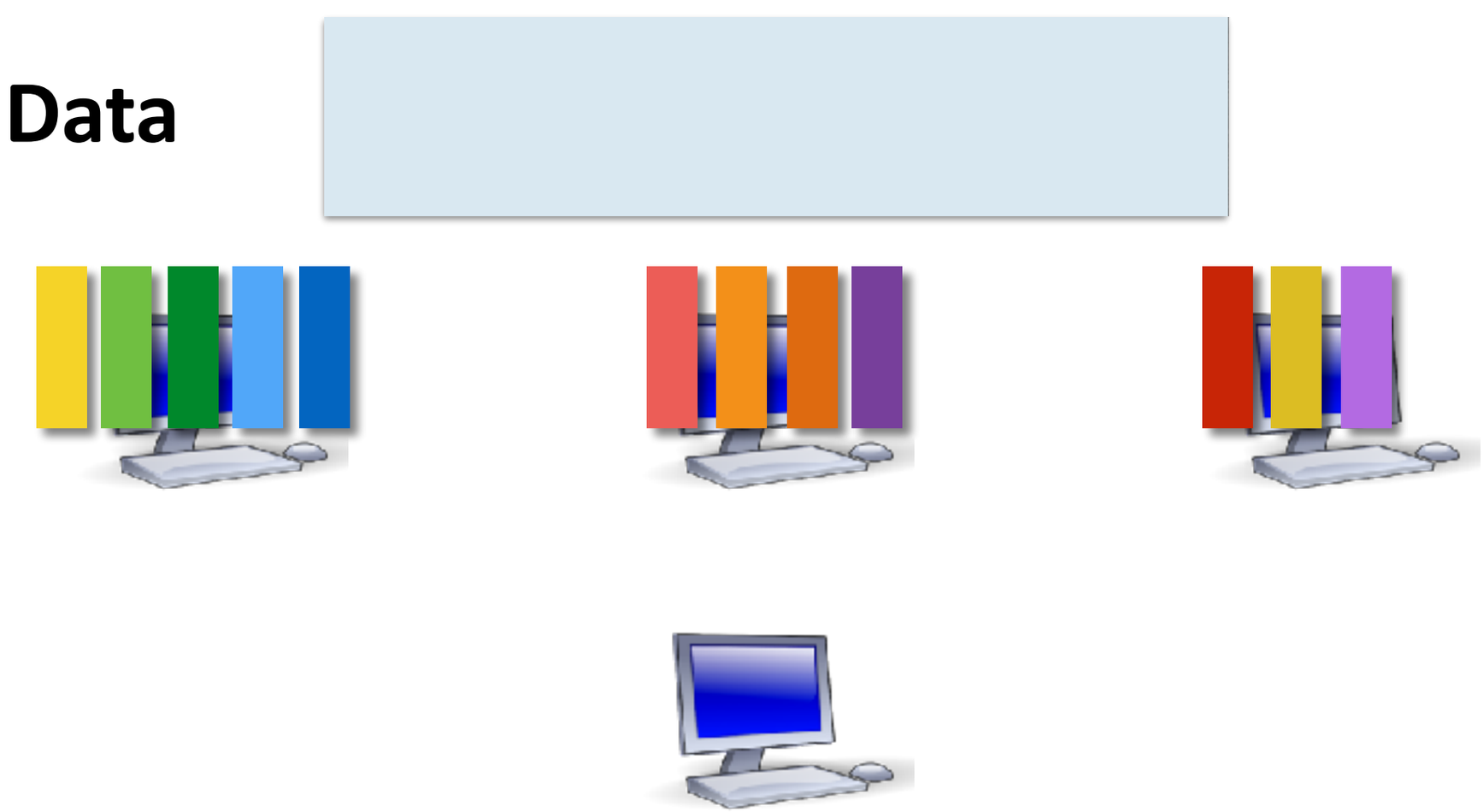

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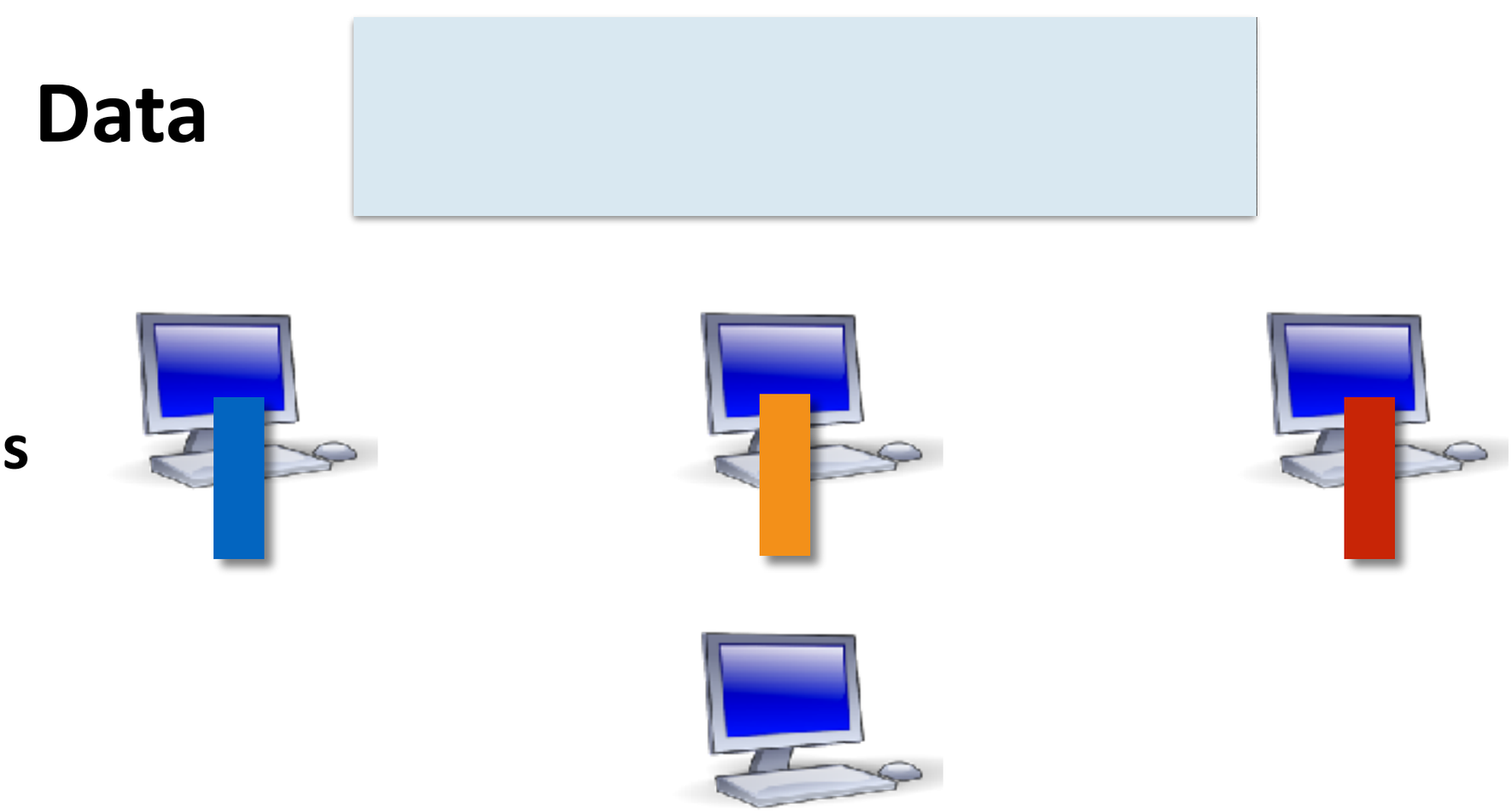

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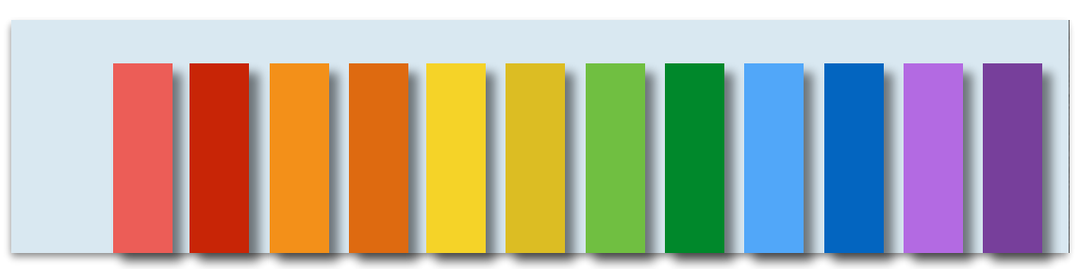

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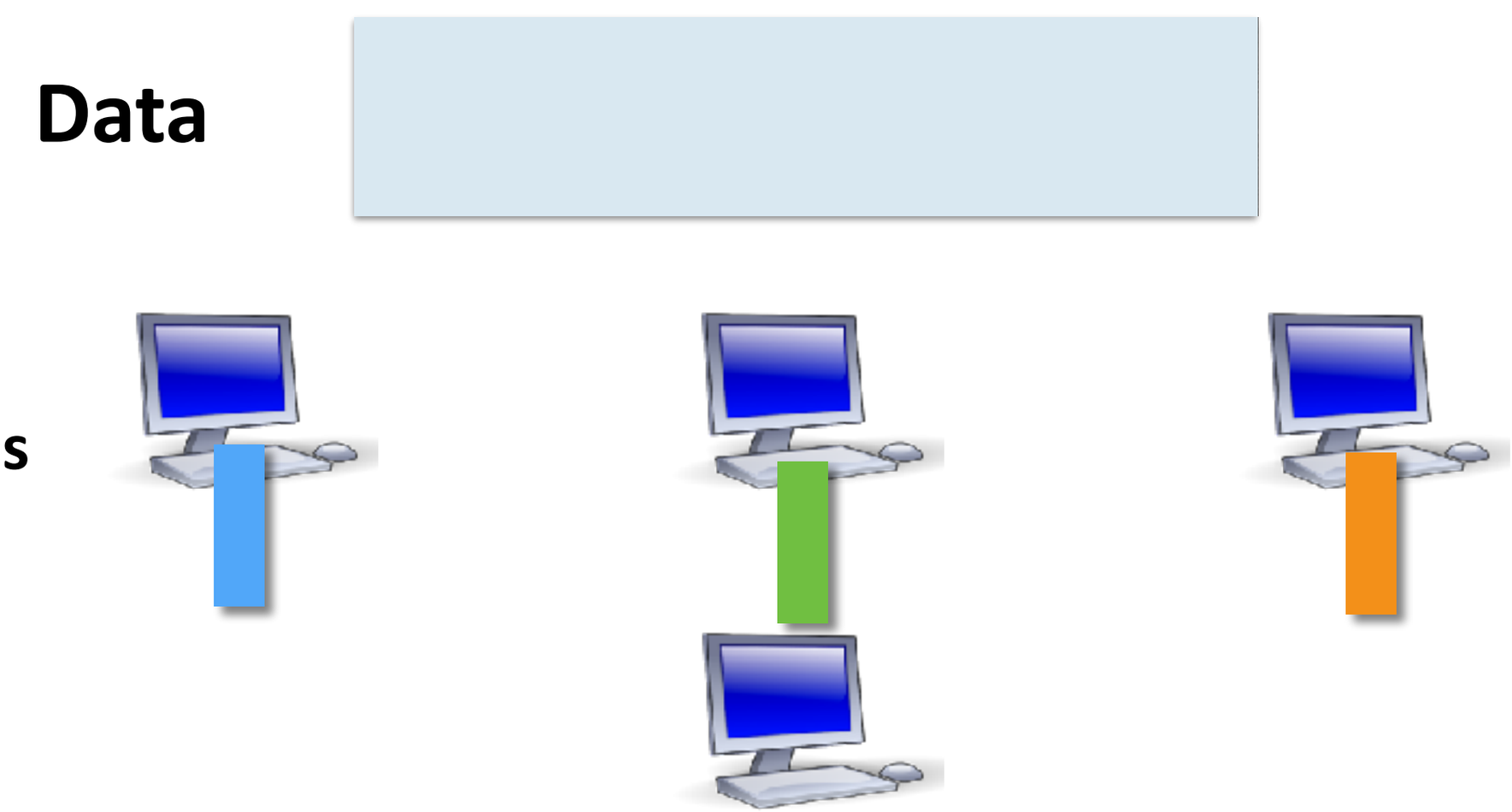

Distributed Submodular Maximization

Arbitrary Partition

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Baharan Mirzasoleiman
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Google

Andreas Krause
ETH Zurich

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Distributed Submodular Maximization

Arbitrary Partition → Random Partition (2 Rounds) → Multi-Stage Algs

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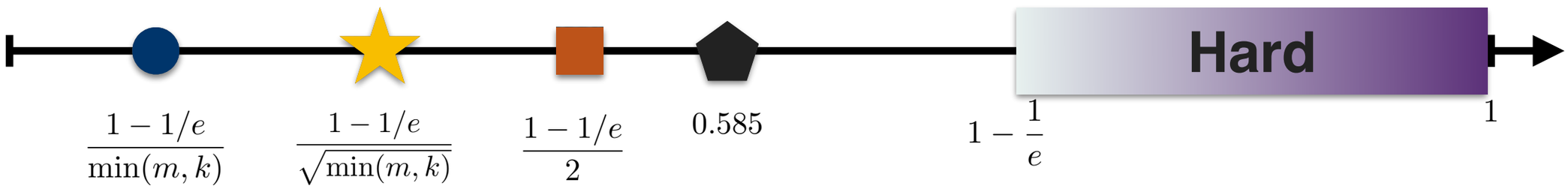

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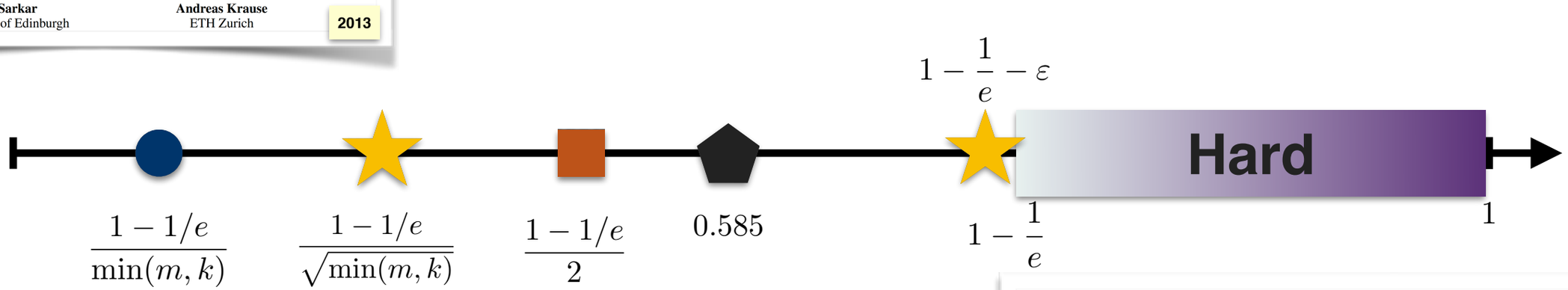

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Rafael da Ponte Barbosa* Alina Ene† Huy L. Nguyễn‡ Justin Ward§

2017

$O(\frac{1}{\varepsilon})$ Rounds (with duplications)

Distributed Submodular Maximization

Arbitrary Partition → Random Partition (2 Rounds) → Multi-Stage Algs

Distributed Submodular Maximization: Identifying Representative Elements in Massive Data

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2015

Regularized Submodular Maximization at Scale

Ehsan Kazemi* Shervin Minaee[†] Moran Feldman[‡] Amin Karbasi[§]

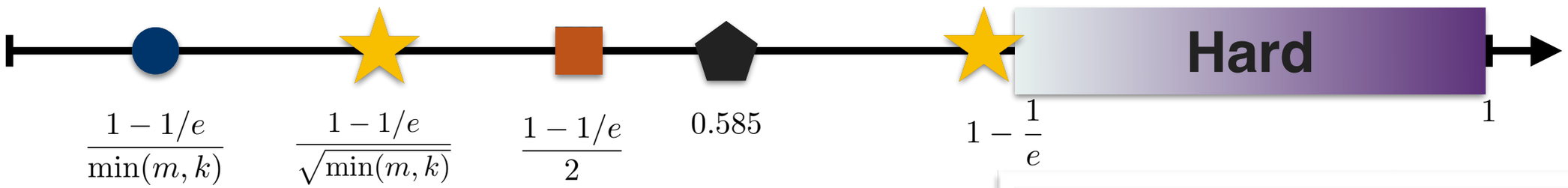
2020

Submodular Optimization in the MapReduce Model

Paul Liu and Jan Vondrak

2019

$1 - \frac{1}{e} - \varepsilon$ $O(\frac{1}{\varepsilon})$ Rounds (no duplications)



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$O(\frac{1}{\varepsilon})$ Rounds (with duplications)

Multi-Stage Distributed Greedy: Open Question

- Consider the following problem where f is monotone submodular:

$$S^* = \arg\max_{|S| \leq k} f(S)$$

[Liu, Vondrak] [Kazemi, Minaee, Feldman, Karbasi]

Multi-Stage GreeDi, after $O(1/\epsilon)$ iterations, and without duplicating the data returns a solution of size at most k such that

$$\mathbb{E}[f(S_{\text{Multi-GreeDi}})] \geq (1-\epsilon)\left(1-\frac{1}{e}\right)\text{OPT}$$

"Submodular Optimization in MapReduce Model", 2019

"Regularized Submodular Maximiza at Scale", 2020

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- Can we obtain (1-1/e) with fewer iteration?

Distributed Submodular Maximization: Extensions

- More Complex constraints

  "Distributed Submodular Maximization", Mirzasoleiman, Karbasi, Sarkar, Krause, 2016.

  "The Power of Randomization: Distributed Submodular Maximization on Massive Datasets" Barbosa, Ene, Nguyen, Ward, 2015.

- Non-monotone:

  "Parallel Double Greedy Submodular Maximization", Pan, Jegelka, Gonzalez, Bradley, Jordan, 2014.

  "The Power of Randomization: Distributed Submodular Maximization on Massive Datasets" Barbosa, Ene, Nguyen, Ward, 2015.

- Filteration:

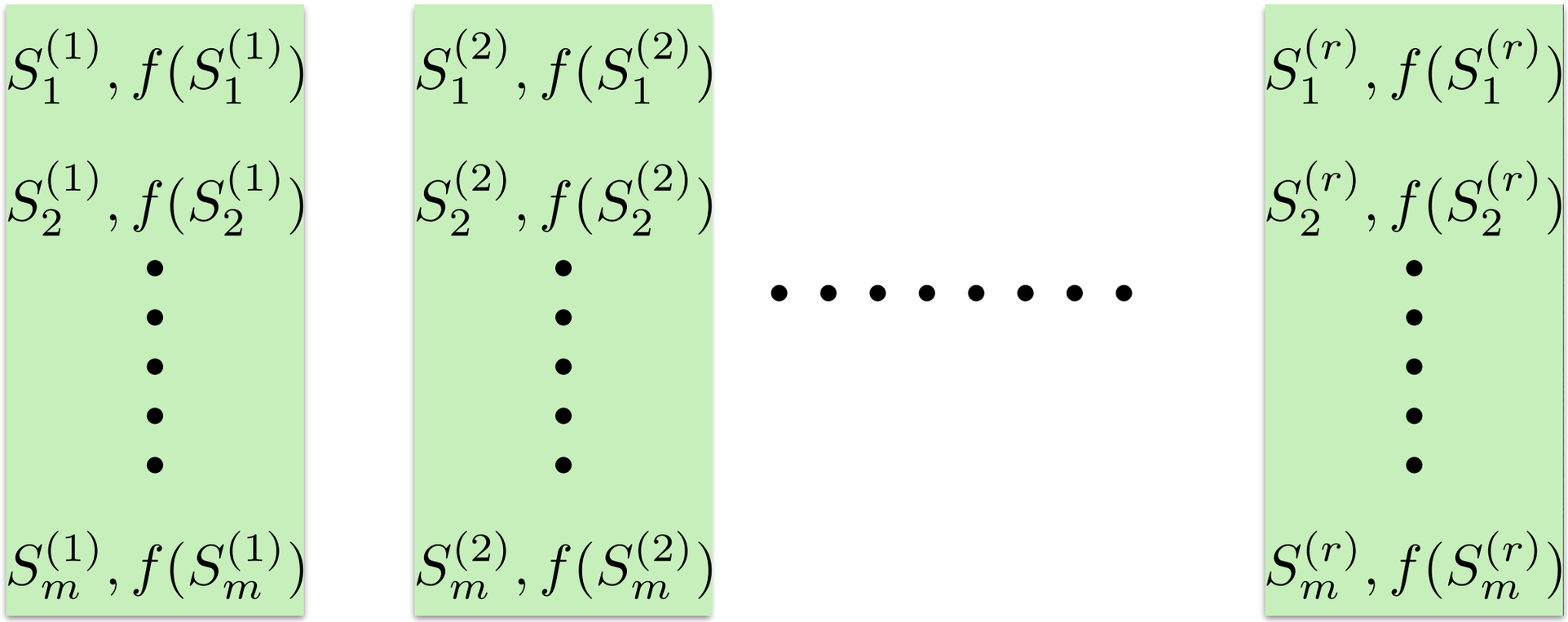
  "Fast Greedy Algorithms in MapReduce and Streaming", Kumar, Moseley, Vassilvitskii, Vattani, 2013.

- Sketching:

  "Optimal Distributed Submodular Optimization via Sketching", Bateni, Esfandiari, Mirrokni, 2018.

Adaptivity Complexity

- **r-adaptive**: An algorithm is r-adaptive if it makes r rounds of parallel computations. In each round, the algorithm is allowed to make polynomially many queries.



"The adaptive complexity of maximizing a submodular function", Balkanski, Singer, 2018

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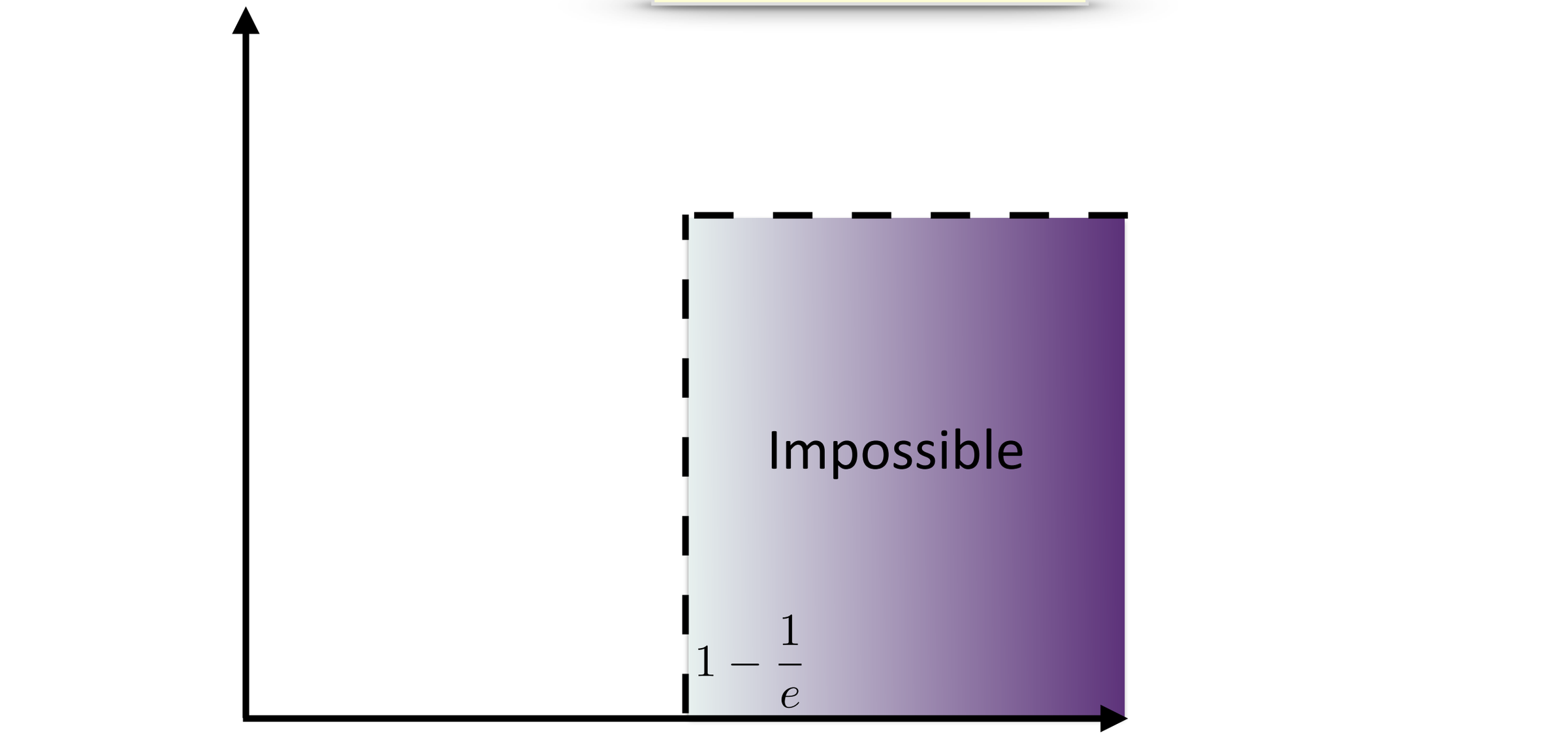

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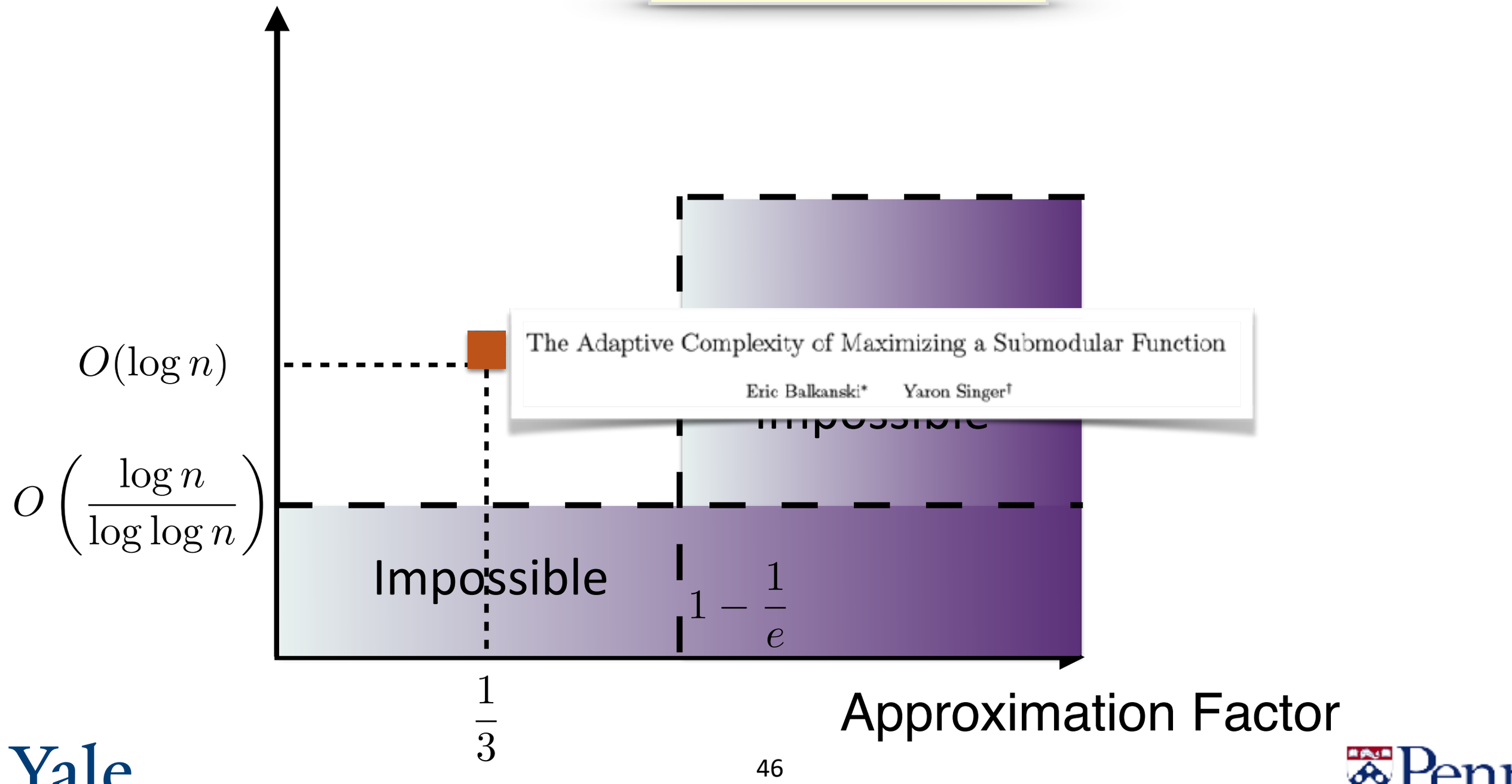

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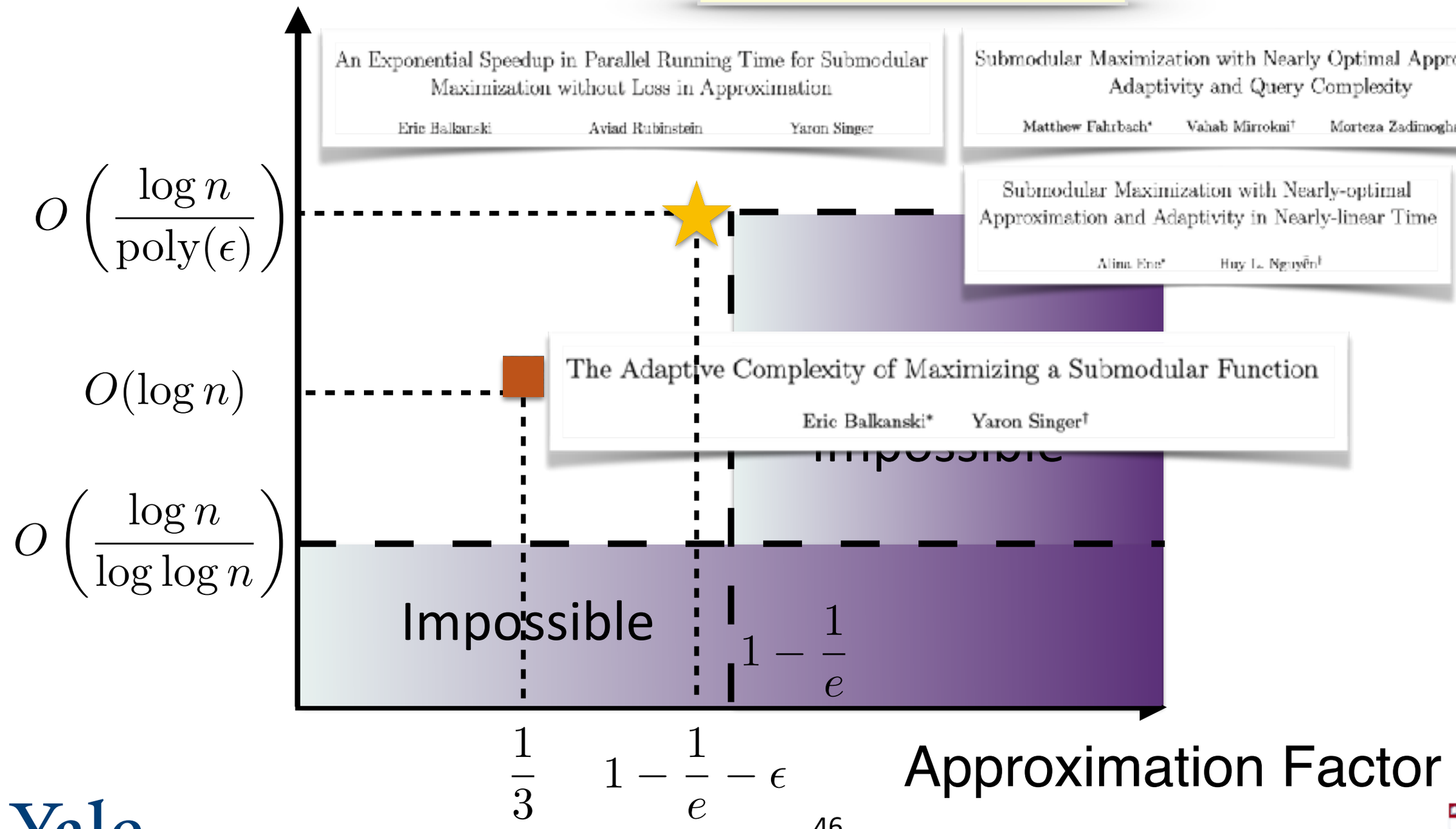
Adaptivity Complexity



Adaptivity Complexity: Extensions

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  "An optimal approximation for submodular maximization under a matroid constraint in the adaptive complexity model", Balkanski, Rubinstein, Singer , 2019

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  "Submodular maximization with matroid and packing constraints in parallel", Ene, Nguyen, Vladu, 2019.

- Unconstrained Submodular Maximization

  "Unconstrained submodular maximization with constant adaptive complexity", Chen, Feldman, Karbasi, 2019.

- Non-montone Submodular Maximization

  "Non-monotone submodular maximization in exponentially fewer iterations", Balkanski, Breuer, Singer, 2018

  "Non-monotone submodular maximization with nearly optimal adaptivity and query complexity", Fahrbach, Mirrokni, Zadimoghaddam, 2019.

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- Convex Minimization

  "Parallelization does not accelerate convex optimization: Adaptivity lower bounds for non-smooth convex minimization" Balkanski, Singer, 2019

  "Lower Bounds for Parallel and Randomized Convex Optimization", Diakonikolas, Guzmán, 2019

  "Complexity of Highly Parallel Non-Smooth Convex Optimization", Bubeck , Jiang , Lee, Li, Sidford, 2019

- Interactive Stochastic Optimization

  "Adaptivity in Adaptive Submodularity", Hossein Esfandiari, Karbasi, Mirrokni, 2019

Where Is the Oracle?

There is no **oracle**

Stochastic Oracle

- Very often, we cannot evaluate the function *exactly*

Stochastic Oracle

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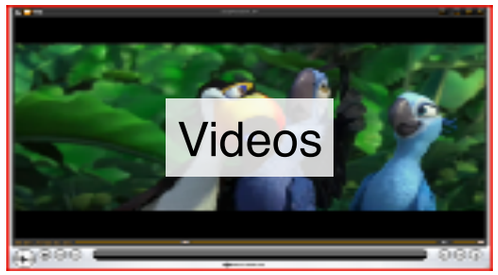
  - The value has to be estimated from the data (empirical mean)

$$f(S) = \frac{1}{n}\sum_{i=1}^{n} f_i(S)$$



Data Summarization:

Extract small, representative subset out of a massive data set

Yale

Penn

Stochastic Oracle

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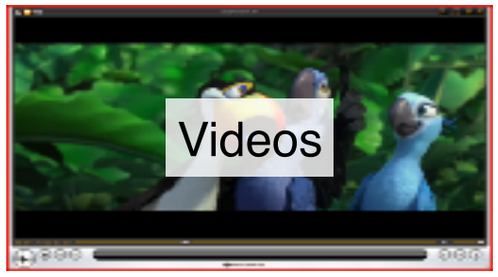
$$f(S) = \frac{1}{n}\sum_{i=1}^{n} f_i(S)$$

  - It is defined through a stochastic process

$$f(S) = \mathbb{E}_{\theta \sim D}[f_\theta(S)]$$



Data Summarization:

Extract small, representative subset out of a massive data set

Influence Maximization:

Find a small subset nodes with large influence

Part Two

$$S^* = \arg\max_{S \subseteq V, S \in \mathcal{I}} f(S)$$

Perfect Oracle

Discrete Methods

- **Centralized**
- **Streaming**
- **Distributed**

Imperfect Oracle

Continuous Methods