Methods in Experimental Particles Physics

Analysis of the "Evidence for the 2π decay of the K_2^0 meson" paper

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Abstract

The results obtained in the experiment demonstrating CP violation by studying the decay of K_2^0 into 2π will be analyzed. Concepts such as event selection, background, significance, efficiency, among others, will be studied.

1 After Final Selection for Signal Events

1.1 Number of candidate events after final selection

One of the criteria for data selection involved the mass of the neutral K meson $(497.611 \pm 0.013)\,\mathrm{MeV/c}$. In Figure 1b, this ranged from $490 < m^* < 510$ MeV, while in Figure 2, it ranged from $494 < m^* < 504$ MeV. Additionally, the data were sorted based on their angular distribution: in Figure 1b for $0.998 < \cos\theta < 1$ and in Figure 2 for $0.9996 < \cos\theta < 1.0000$. Figure 2 shows the results of the more accurate measuring machine, where the number of candidates obtained simply by counting the number of events in the 3 and 4 bins closest to 1.0000 was around:

$$N_{\text{cand(3 bin)}} = 57 \pm 9 \text{ events.} \tag{1}$$
$$N_{\text{cand(4 bin)}} = 62 \pm 10 \text{ events.} \tag{2}$$

1.2 Number of signal events after final selection

The report [1] states that after subtracting the background,

$$N_{\text{signal}} = 45 \pm 9 \text{ events} \tag{3}$$

are observed in the forward peak in the mass of the K^0.

1.3 Number of background events after final selection

In the signal region, which includes only the 3 and 4 bins closest to 1.0000 in the angular distribution that contains K^0 mass (fig. 2).

$$N_{\text{b}} = N_{\text{cand}} - N_{\text{signal}} \tag{4}$$

then,

$$N_{\text{b(3 bin)}} = 12 \pm 12 \text{ events.} \tag{5}$$
$$N_{\text{b(4 bin)}} = 17 \pm 13 \text{ events.} \tag{6}$$

From sidebands in figure 2, the background level is extrapolated between $B = 4/\text{bin}$ and $B = 4.25/\text{bin}$.

1.4 The purity of the sample after final selection

Purity is often defined as the ratio of the number of signal events to the total number of events in the signal region (after the cuts).

$$\text{Purity}_{\text{(3 bin)}} = \frac{N_{\text{signal}}}{N_{\text{cand}}} \tag{7}$$
$$= \frac{45 \pm 9}{57 \pm 9} \tag{8}$$
$$= (78.9 \pm 20.1)\% \tag{9}$$
$$\text{Purity}_{\text{(4 bin)}} = \frac{N_{\text{signal}}}{N_{\text{cand}}} \tag{10}$$
$$= \frac{45 \pm 9}{62 \pm 10} \tag{11}$$
$$= (72.6 \pm 18.6)\% \tag{12}$$

A purity of $70\% - 80\%$ means that while a significant portion of the sample consists of signal events, there is also a contamination from background events, about $20 - 30\%$.

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1.5 The significance of the observed signal

The significance is

$$\frac{N_{\text{signal(3 bin)}}}{\sigma(N_{\text{signal}})} = \frac{N_{\text{signal}}}{\sqrt{N_{\text{signal}} + N_{\text{b(3 bin)}}}} \quad (13)$$
$$= \frac{45 \pm 9}{\sqrt{(45 \pm 9) + (12 \pm 12)}} \quad (14)$$
$$= 5.96 \pm 1.21 \quad (15)$$
$$\frac{N_{\text{signal(4 bin)}}}{\sigma(N_{\text{signal}})} = \frac{N_{\text{signal}}}{\sqrt{N_{\text{signal}} + N_{\text{b(4 bin)}}}} \quad (16)$$
$$= \frac{45 \pm 9}{\sqrt{(45 \pm 9) + (17 \pm 13)}} \quad (17)$$
$$= 5.72 \pm 1.16 \quad (18)$$

A significance greater than 5 means that an observation of a signal is accepted.

2 Background

2.1 Total number of events collected

As can be seen in Figure 1a, the total number of collected events was 5211 in a range of $330 < m^* < 600$ MeV.

2.2 The total number of background events in the experiment before the event selection

From Figure 1a, it can be observed that within the range of m^* covering the mass of K^0, i.e., $490 < m^* < 510$ MeV, there are approximately 500 ± 50 events. Therefore, it can be estimated that the total number of background events before the event selection was

$$N_{\text{Tb}} = 5211 - (500 \pm 50) = 4711 \pm 50 \text{ events.} \quad (19)$$

2.3 The total background rejection factor

The background rejection factor can be calculated as the ratio of the number of background events rejected by the selection criteria to the total number of background events that were expected in the signal region.

$$R_{\text{(3 bin)}} = \frac{4711 \pm 50}{12 \pm 12} = 392.58 \quad (20)$$
$$R_{\text{(4 bin)}} = \frac{4711 \pm 50}{17 \pm 13} = 277.12 \quad (21)$$

The background rejection factor quantifies how effectively the selection criteria reduce background contamination in the signal region. A higher background rejection factor indicates better rejection of background events.

2.4 The total number of decays to be considered as normalization for the Branching Ratio R evaluation

The branching ratio (BR) is defined as the fraction of decays that result in the specific final state of interest (in this case, 2π). It is calculated as the ratio of the number of signal events to the total number of neutral kaons that is 22700:

$$BR(K_2^0 \to 2\pi) = \frac{N_{\text{signal}}}{N_{K_2^0}} = \frac{45 \pm 9}{22700} \quad (22)$$
$$= (2.0 \pm 0.4) \times 10^{-3} \quad (23)$$

3 Referring to fig 1a

1. Which is the significance of the signal considering a region on the distribution of the invariant mass alone, wide 1 (or 2) bin(s)?

   Due to the neutral Kaon mass being around $497.7\,\text{MeV}$, it is possible to take a bin ranging from $490 < m^* < 500$ MeV which, according to Figure 1a, contains approximately 300 candidates, of which 45 ± 9 would correspond to the signal, resulting in a significance of

   $$\frac{N_{\text{signal}}}{\sigma(N_{\text{signal}})} = 2.60 \quad (24)$$

   Meanwhile, with two bins, meaning a range of $490 < m* < 510$ MeV, there would be around 500 candidates, resulting in a significance of

   $$\frac{N_{\text{signal}}}{\sigma(N_{\text{signal}})} = 2.01 \quad (25)$$

   Due to the low significance, indicating that the candidates could be a mere fluctuation of the background, it is understandable why the authors decided to conduct the experiment with a more precise measuring machine, which increased the significance to over 5σ.

2. What resolution on the invariant mass would have been necessary to get a significance on the observed signal based on the observation of the invariant mass alone (i.e. without the measurement of the angle θ) at the same level of the final result in the paper?

   The effect of mass resolution on signal significance makes N_{b} given by $6b\sigma_{\text{M}}$, where σ_{M} is the mass resolution and b represents the background events per unit mass, calculated as the number of events falling in the so-called sideband regions, which are the regions outside the signal region, divided by the sidebands amplitude.

   The sidebands amplitude represents the width or size of the sidebands regions. This can be obtained by counting the number of bins or the width of the invariant mass distribution covered by the sidebands.

   Then, for the two bins covering $490 < m^* < 510$ MeV (amplitude $20\,\text{MeV}$).

$$b = \frac{\text{Number of events in sidebands}}{\text{Sidebands amplitude}} \tag{26}$$
$$= \frac{5211 - 500}{(550\,\mathrm{MeV} - 330\,\mathrm{MeV}) - 20\,\mathrm{MeV}} \tag{27}$$
$$= \frac{4711}{200\,\mathrm{MeV}} \tag{28}$$
$$= 23.55\,\frac{1}{\mathrm{MeV}} \tag{29}$$

Then, the significance should be equals to (15) and (18). For simplicity, they are called in terms of bins from section 1.3 to distinguish them.

$$\frac{N_{\text{signal(3 bins)}}}{\sigma(N_{\text{signal}})} = \frac{N_{\text{signal}}}{\sqrt{N_{\text{signal}} + N_{\text{b}}}} \tag{30}$$
$$= \frac{N_{\text{signal}}}{\sqrt{N_{\text{signal}} + 6b\sigma_{\text{M}}}} = 5.96 \tag{31}$$
$$\frac{N_{\text{signal(4 bins)}}}{\sigma(N_{\text{signal}})} = 5.72 \tag{32}$$

Solving the equation with $N_{\text{signal}} = 45$ and b from equation (29), the mass resolution is

$$\sigma_{\text{M (3 bins)}} = 0.085 \qquad \sigma_{\text{M (4 bins)}} = 0.12 \tag{33}$$

So for a region surrounding the mass of the neutral kaon at $497.611\,\mathrm{MeV}$, the window should be $[M_{K^0} - 3\sigma_M, M_{K^0} + 3\sigma_M]$, that is, $[497.356, 497.866]$ for 3 bins and $[497.25, 497.97]$ fot 4 bins, with a background of

$$B_{\text{(3 bins)}} = 6b\sigma_{\text{M (3 bins)}} \approx 12 \tag{34}$$
$$B_{\text{(4 bins)}} = 6b\sigma_{\text{M (3 bins)}} \approx 17 \tag{35}$$

(34) and (35) match what was found in section 1.3.

Additionally,

$$\frac{N_{\text{signal}}}{6b} = 0.31 >> \sigma_{\text{M}} \tag{36}$$

So the effect of mass resolution can be negligible on the uncertainty on N_{signal}.

4 General

1. Did the authors make an absolute or a relative measurement?

   The authors conducted a relative measurement because in the context of studying neutral Kaon decays, a relative measurement could involve comparing the branching ratio of the two pions decay mode, with respect to the total branching ratio of all possible decay modes of neutral kaons. This comparison provides information about the relative abundance of the decay mode of interest compared to other possible decay modes. This information is given in the paper [1].

2. Spot the typo in the formula for the evaluation of the CP violation parameter $|\epsilon|$ (the numerical value of $|\epsilon|$ is correct).

   Epsilon is the parameter that quantifies the degree of CP violation, therefore, it should be dimensionless. However, in the paper, it has the dimension of time, which indicates that the correct formula is:

$$|\epsilon| \approx R_T \frac{\tau_1}{\tau_2} \tag{37}$$

   with

$$R_T = \frac{3}{2} R. \tag{38}$$

   Then,

$$|\epsilon| \approx \frac{3}{2}(2.0 \times 10^{-3}) \frac{0.89 \times 10^{-10}\,\mathrm{s}}{5.2 \times 10^{-8}\,\mathrm{s}} \tag{39}$$
$$= 2.3 \times 10^{-3} \tag{40}$$

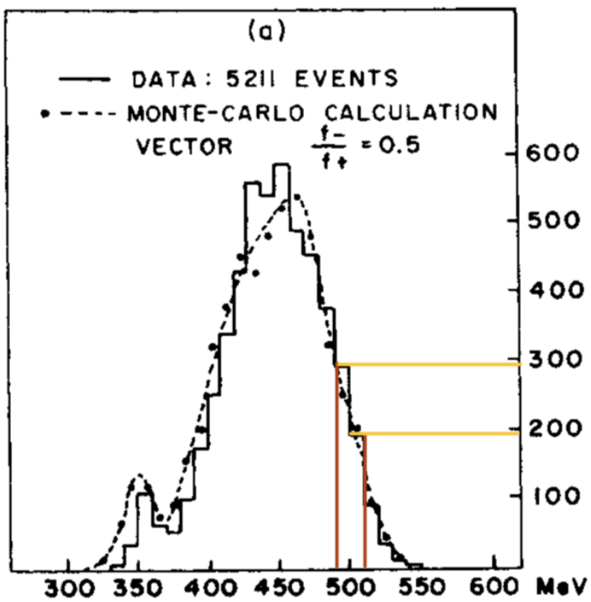
   That corresponds with what was found in the paper.

3. What are the approximations for R in this formula?

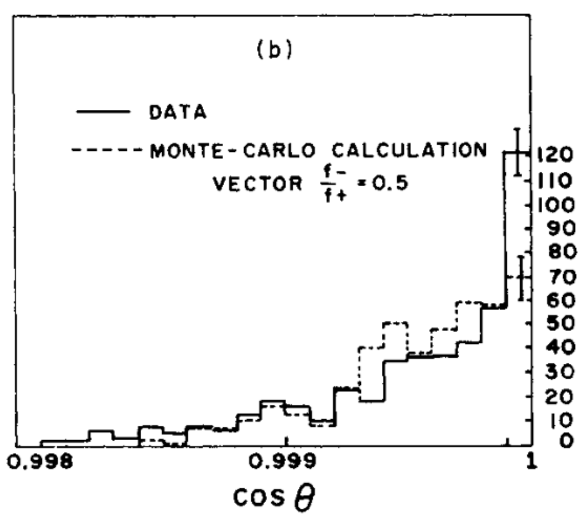
   The factor $\frac{3}{2}$ in (38) indicates that for every 3 decays of K_0^2 into all charged modes, 2 of them are decays into two pions. This is the approximation.

References

[1] J. H. Christenson et al. "Evidence for the 2π Decay of the K_2^0 Meson". In: *Phys. Rev. Lett.* 13 (1964), pp. 138–140. DOI: 10.1103/PhysRevLett.13.138.



(a) Experimental distribution in m^* compared with Monte Carlo calculation. the calculated distribution is normalized to the total number of observed events. [1]



(b) Angular distribution of those events in the range of $490 < m^* < 510$ MeV. The calculated cure is normalized o the number of events in the complete sample. [1]

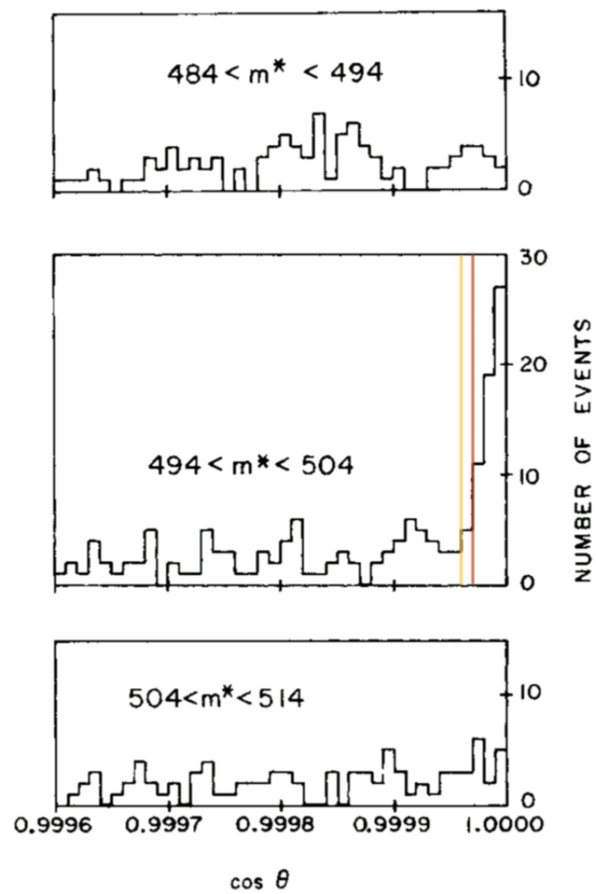


Figure 2: Angular distribution in three mass ranges for events with $\cos\theta > 0.9995$. [1]. The red and yellow lines represent a selection of 3 and 4 bins, respectively.