Methods in Experimental P. P.

convolution Integral Energy spectrum f(E) with a Gaussian resolution g(E)

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Evaluate numerically and plot graphically the convolution integral of the energy spectrum $f(E)$ with gaussian resolution $g(E)$ defined below

$$f(E) = a_1 f_1(E) + a_2 f_2(E) + a_3 f_3(E)$$

with $a_1 = 1$, $a_2 = 0.5$, $a_3 = 0.1$ and,

$$f_1(E) = \begin{cases} 1/E & \text{if } 0.1 < E < 0.95 \\ 0 & \text{if } E < 0.1 \text{ or } E > 0.95 \end{cases} \tag{1}$$

$$f_2(E) = G(\mu = 1.2 \text{ MeV}, \sigma = 0.01 \text{ MeV}) \tag{2}$$

$$f_3(E) = G(\mu = 0.5 \text{ MeV}, \sigma = 0.01 \text{ MeV}) \tag{3}$$

In the following 4 cases:

1. $g(E) = G(E, \sigma)$ with $\sigma/E = 1\%/\sqrt{E}$ MeV
2. $g(E) = G(E, \sigma)$ with $\sigma/E = 5\%/\sqrt{E}$ MeV
3. $g(E) = G(E, \sigma)$ with $\sigma/E = 10\%/\sqrt{E}$ MeV
4. $g(E) = G(E, \sigma)$ with $\sigma/E = 30\%/\sqrt{E}$ MeV

Solution:

For equations (2) and (3), it is possible to define the Gaussian function as follows.

$$G(E, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp - \frac{(E-\mu)^2}{2\sigma^2}$$

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For the four different cases, the Gaussian function is defined as follows.

$$G(E,\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp -\frac{E^2}{2\sigma^2}$$

Considering that sigma depends on the value of the energy in this way.

$$\sigma = \frac{\%}{100}\sqrt{E}$$

With the above established, it is possible to plot the function $f(E)$, yielding the following result.

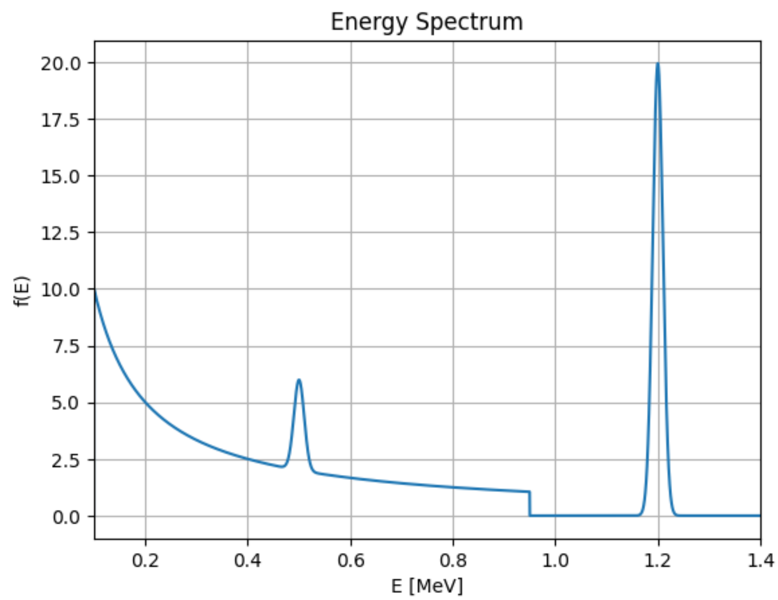

Figure 1: Energy spectrum f(E)

While the different Gaussian resolutions look as follows:

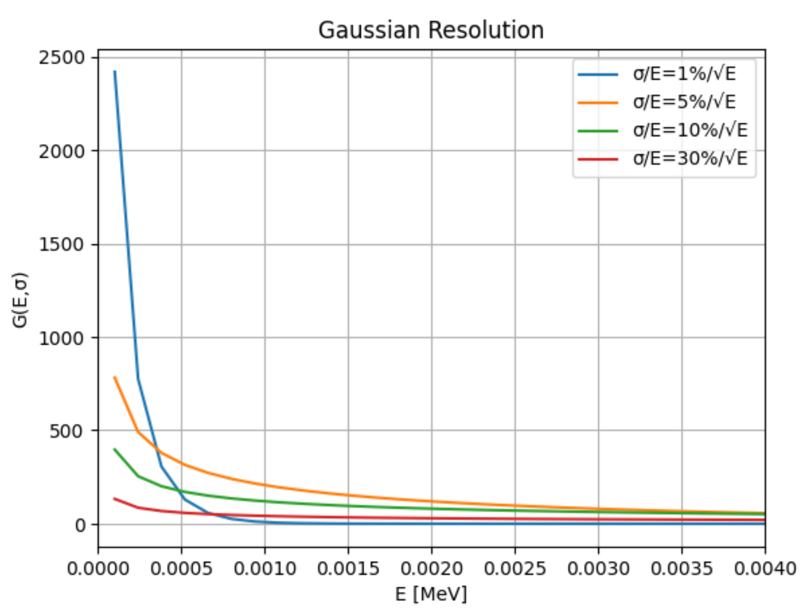

Figure 2: Energy-dependent Gaussian resolutions

The convolution between energy spectrum and Gaussian resolutions is commonly used to compare the data obtained from an experiment and the theory that attempts to explain it, in short, it folds the theory with the experimental resolution.

In this line of thought, for each of the different cases, the result of the convolution is as follows:

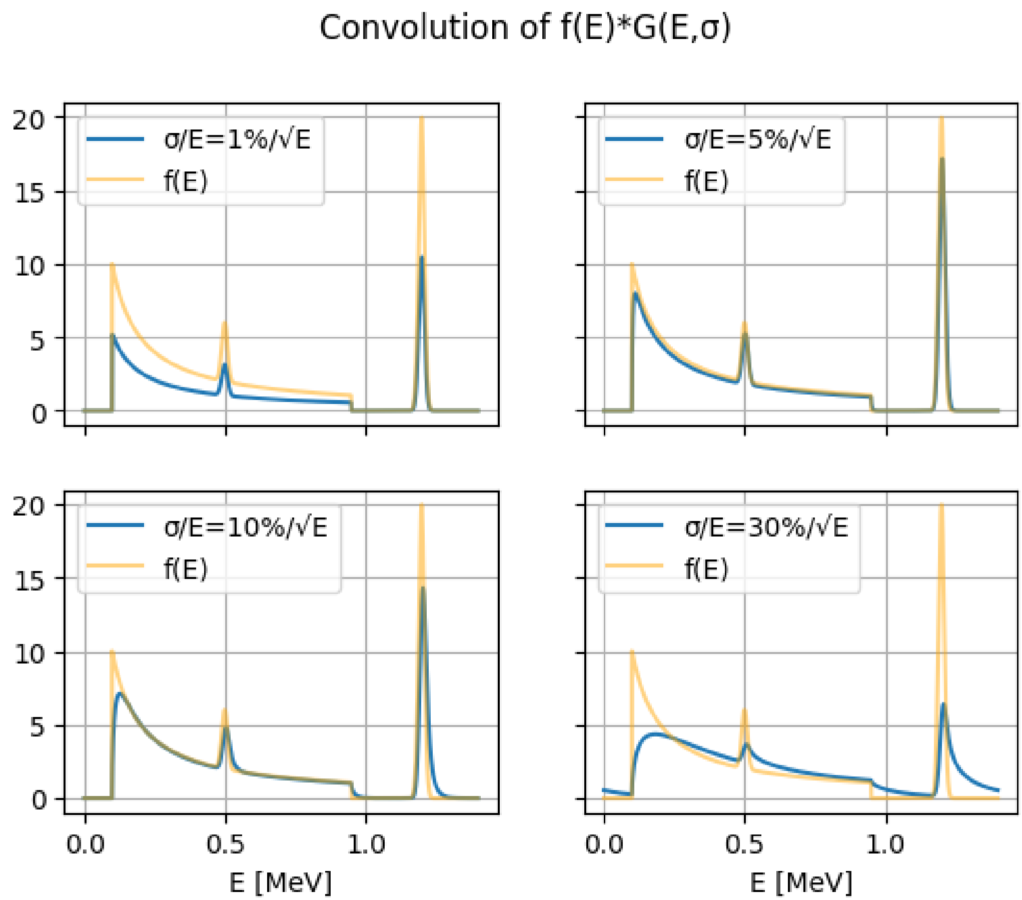

Figure 3: Convolution integral of the energy spectrum $f(E)$ with the different Gaussian resolutions.

It is noticeable that the convolutions that best fit the given energy spectrum are those of 1% and 5%. It is also observable that the sharp corners in the data have been smoothed out thanks to the convolution.

For the $1\%/\sqrt{E}$ resolution case, it's possible to notice that the convolution is almost the energy spectrum, this is the best resolution.

If the resolutions were independent of energy, it would be possible to observe that the resolution that best fits the energy spectrum is 1%, While the 5%, 10%, and 30% resolutions notably worsen. in figure 4.

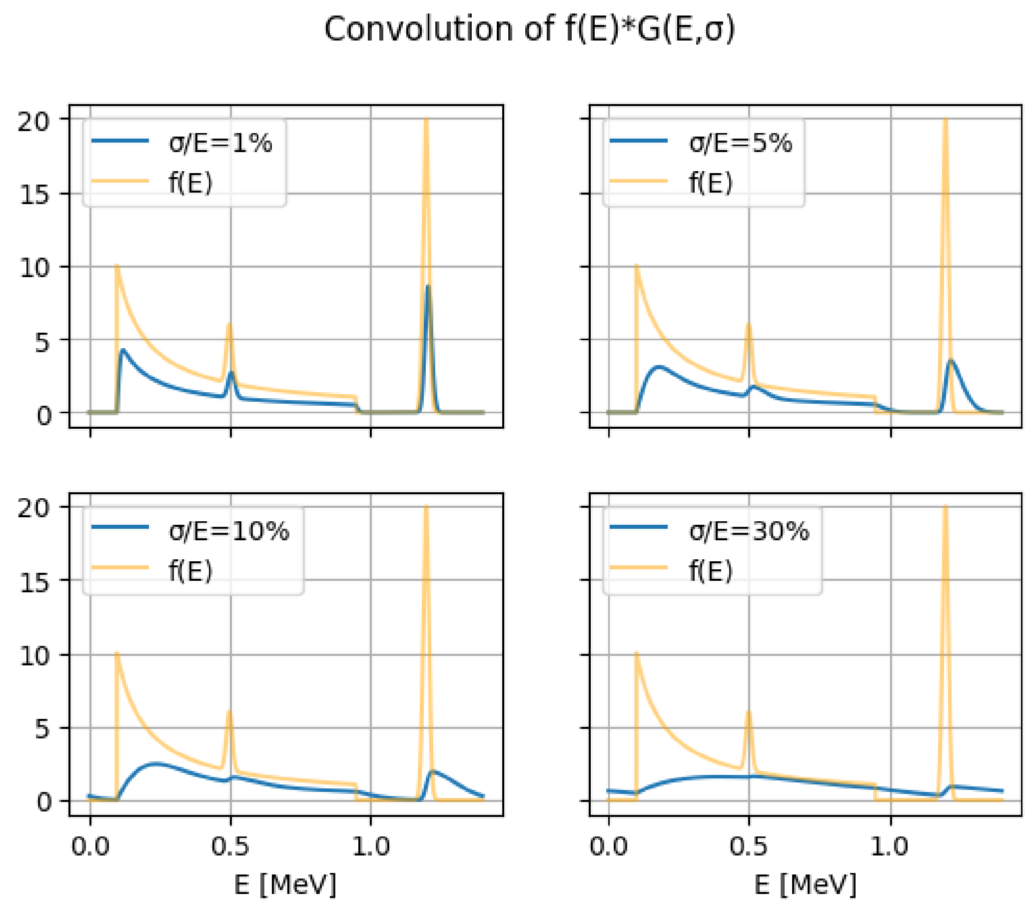

Figure 4: Resolutions not dependent on the square root of energy.

Now, the same procedure will be carried out with a different and slightly more complex function.

$$f(E) = a_1 f_1(E) + a_2 f_2(E) + a_3 f_3(E)$$

with $a_1 = 1$, $a_2 = 0.3$, $a_3 = 0.2$ and,

$$f_1(E) = \begin{cases} 1/\sin E & \text{if } 0.1 < E < 1.0 \\ 0 & \text{if } E < 0.1 \text{ or } E > 0.95 \end{cases} \tag{4}$$

$$f_2(E) = G(\mu = 0.7 \text{ MeV}, \sigma = 0.02 \text{ MeV}) \tag{5}$$

$$f_3(E) = G(\mu = 0.5 \text{ MeV}, \sigma = 0.01 \text{ MeV}) \tag{6}$$

The result of the convolution was as follows:

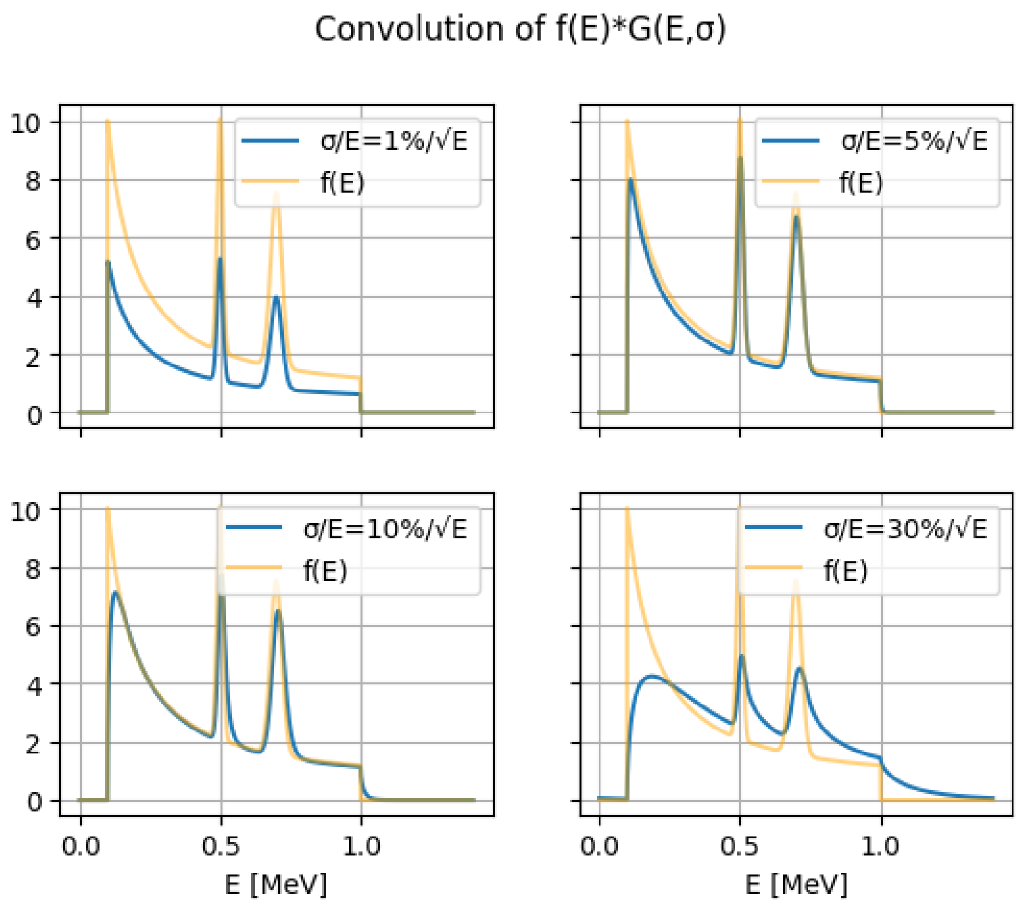

Figure 5: Convolution integral of the energy spectrum $f(E)$ with the different Gaussian resolutions.