

Homework n.6

A) A fit with a function $f(x; \gamma)$ on a measured x -distribution yields the result for the parameter γ :

$$\gamma = (-0.34 \pm 0.48) \times 10^{-5}$$

with $\gamma > 0$. Put an upper limit at **90%** C.L. on γ using:

- The classical frequentist approach
- The unified approach (Feldman Cousins) with the mean of the Gaussian constrained to be non-negative
(see Feldman and Cousins Phys.Rev.D 57 3873 (1998) TABLE X)
- The Bayesian approach (briefly comment the choice of the prior)

B) a counting experiment yields $N=S+B=2$ event in the signal region with $B=5$ expected background. Assuming negligible uncertainty on B , evaluate an upper limit for S at **95%** C.L. using:

- The classical frequentist approach
- The unified approach (Feldman Cousins) for a Poisson signal
(see Feldman and Cousins Phys.Rev.D 57 3873 (1998) TABLES VI-VII)
- The Bayesian approach (briefly comment the choice of the prior)

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C) The parameter $\mu = \sigma / \sigma_{\text{THEORY}}$ for the signal strength of a new particle (with predicted cross section σ_{THEORY}) is measured with three different and independent methods yielding:

$$\mu_1 = 0.55 \pm 0.30$$

$$\mu_2 = 0.43 \pm 0.37$$

$$\mu_3 = 0.99 \pm 0.29$$

Assuming gaussian pdfs and errors:

- evaluate the best estimate of the parameter μ and its uncertainty;
- evaluate the corresponding p-value for the fit hypothesis (i.e. a χ^2 test, consistency of three measurements of the same true value);
- build an expression for the test statistics

$$q_\mu = -2 \ln \frac{L(\underline{x}|\mu)}{L(\underline{x}|\hat{\mu})}$$

- Evaluate with a toy MC simulation the pdfs $f(q_1|1)$, $f(q_1|0)$, and $f(q_0|0)$. (assuming the same gaussian uncertainties of the measurements).
- Evaluate the p-value of the null hypothesis ($\mu=0$) corresponding to the observed q_0^{obs} .
- Evaluate the p-values CL_{s+b} , CL_b and CL_s corresponding to the observed q_1^{obs} .
- Briefly comment the results.

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D) Repeat all the evaluations of point C) for the following set of measurements

$$\mu_1 = 1.55 \pm 0.30$$

$$\mu_2 = 1.43 \pm 0.37$$

$$\mu_3 = 0.99 \pm 0.29$$