PROBLEM SET 1: APPLIED MATHEMATICS 216

Due: Friday February 7 in class.

Goals for the week.

(1) Get some experience with coding in python. For those of you who do not know python, this is likely to be the main challenge in carrying out the homework problems below. The class notebook contains lots of background on how the functions work. Play with them! Start by looking for commands (plotting, etc.) on Stack Overflow. Don't be nervous to ask your classmates and us for help.
(2) Understand qualitatively how different types of norms can influence the results of regression problems, especially the robustness to outliers. Estimation of the errors in computing the coefficients.
(3) Learn about convolving and deconvolving images. An essential part of experimental science is image collection and analysis. Once obtained images often require different forms of manipulation to be useful.

Problems.

(1) *What polynomial did we pick?*

We have posted on the course web site one datafiles, called "Problem1.txt" containing 1000 points (x, y) that we have drawn from a tenth order polynomial of the form:

$$y = \sum_{i=1}^{10} a_i x^i$$

We have also added noise to the data that was chosen from one of the distributions that is a method in numpy.random (see here) for a list.

Your challenge is to solve the inverse problem and figure out which polynomial we have chosen. Your answer should give the coefficients a_i, and also your estimated errors in the coefficients $(a_i \pm \epsilon_i)$.

(2) *Beyond polynomials*

The problem above was easy in that we actually told you which functions we expanded in. Life is not usually so easy. For this problem, we are providing a datafile "Problem2.txt" that 1000 samples of the form (x, y) from the function $y = f(x)$.

Your goal here is to determine, to the best of your ability, what the function $f(x)$ is.

(3) *Guess who?*

Systems where a single input maps onto a single output are nice, but in science it is often convenient to use more complex representations for examining data. One such example are images. Commonly, when given an image, there is a need to refine and manipulate it. In this problem we provided a tutorial that explains one extremely powerful manipulation method: convolution. A common definition of the two-dimensional discrete convolution C of an image array I with a filter array F is:

$$C_{i,j} = \sum_{k,l} F_{k,l}\, I_{i-k,j-l}$$

Here, you will go through a notebook, playing with convolutions of a cat image, learning how to manipulate images (and, thus arrays) in python. Once done, you will be given an image that has been convolved and noised in an unknown manner.

You'll need to restore it at the best of your abilities and guess who the image belongs to! We have set up a competition on Kaggle. For this problem, you must submit your final restored image to Kaggle where it will be scored based on the original.

There will be more Kaggle competitions during the semester and the best performers will win a variety of prizes.

Submission Instructions.

(1) Submit 2 notebooks to Canvas:
    LASTNAME_FIRSTNAME_P1_2
    LASTNAME_FIRSTNAME_P3
(2) Submit your restored image to Kaggle