CM1015 - Computational Mathematics

BSc Computer Science

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Number Bases

Decimal

Number bases relate to the amount of digits used to represent a number. The decimal system is base 10, meaning it uses 10 digits (0-9) to represent numbers. The value of a number is calculated by multiplying each digit by the base raised to the power of its position. For example, the number 123 in decimal is calculated as follows:

10^2	10^1	10^0
1	2	3
100	10	1

$$= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$$

$$= 123$$

Binary

The binary system is base 2, meaning it uses 2 digits (0-1) to represent numbers. Binary numbers are calculated in the same way as decimal numbers, but using powers of 2 instead of 10. For example, the binary number 101 in decimal is calculated as follows:

2^2	2^1	2^0
1	0	1
4	2	1

$$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 5$$

Hexadecimal

The hexadecimal system is base 16, meaning it uses 16 digits (0-9, A-F) to represent numbers. Hexadecimal numbers are calculated in the same way as decimal numbers, but using powers of 16 instead of 10. For example, the hexadecimal number 1A3 in decimal is calculated as follows:

16^2	16^1	16^0
1	10	3
256	16	1

$$= 1 \times 16^2 + 10 \times 16^1 + 3 \times 16^0$$

$$= 419$$

Generic Base Conversion to Decimal

To convert any base to decimal, the following method can be used:

1. Write the number in the base you are converting from.
2. Multiply each digit by the base raised to the power of its position.
3. Add the results together to get the decimal value.

Formula: $d_n \times b^n + d_{n-1} \times b^{n-1} + \ldots + d_1 \times b^1 + d_0 \times b^0$

Where: d_n is the digit at position n, b is the base, and n is the position of the digit.

Example, shown with above symbols:

b^2	b^1	b^0
d_2	d_1	d_0

$$= d_2 \times b^2 + d_1 \times b^1 + d_0 \times b^0$$

Example with base 5 number 234, here $b = 5$, $d_2 = 2$, $d_1 = 3$, $d_0 = 4$:

5^2	5^1	5^0
2	3	4
25	5	1

$$= 2 \times 5^2 + 3 \times 5^1 + 4 \times 5^0$$

$$= 69$$

Self-assessment Questions from Foundation Maths Book

Worked Example 14.2

Convert 83_{10} to binary.

Solution:

The general method for converting a decimal number to binary is to repeatedly divide the number by 2 and record the remainder. The binary number is then read from the remainders in reverse order.

The steps for converting 83 to binary are as follows:

1. $83 \div 2 = 41$ remainder 1
2. $41 \div 2 = 20$ remainder 1
3. $20 \div 2 = 10$ remainder 0
4. $10 \div 2 = 5$ remainder 0
5. $5 \div 2 = 2$ remainder 1
6. $2 \div 2 = 1$ remainder 0
7. $1 \div 2 = 0$ remainder 1
8. The binary number is read from the remainders in reverse order: 1010011

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	0	1	1
64	32	16	8	4	2	1

$$1 \times 64 + 1 \times 16 + 1 \times 2 + 1 \times 1$$

$$= 83$$

Exercise 14.2:

Convert the following decimal numbers to binary:

(a): 19 (b): 36 (c): 100 (d): 796 (e): 5000

Solutions:

(a): 19_{10}

1. $19 \div 2 = 9$ remainder 1
2. $9 \div 2 = 4$ remainder 1
3. $4 \div 2 = 2$ remainder 0
4. $2 \div 2 = 1$ remainder 0
5. $1 \div 2 = 0$ remainder 1
6. The binary number is read from the remainders in reverse order: 10011
7. $19_{10} = 10011_2$
8. **Answer:** $19_{10} = 10011_2$

(b): 36_{10}

1. $36 \div 2 = 18$ remainder 0
2. $18 \div 2 = 9$ remainder 0
3. $9 \div 2 = 4$ remainder 1
4. $4 \div 2 = 2$ remainder 0
5. $2 \div 2 = 1$ remainder 0
6. $1 \div 2 = 0$ remainder 1
7. The binary number is read from the remainders in reverse order: 100100
8. $36_{10} = 100100_2$
9. **Answer:** $36_{10} = 100100_2$

(c): 100_{10}

1. $100 \div 2 = 50$ remainder 0
2. $50 \div 2 = 25$ remainder 0
3. $25 \div 2 = 12$ remainder 1
4. $12 \div 2 = 6$ remainder 0
5. $6 \div 2 = 3$ remainder 0
6. $3 \div 2 = 1$ remainder 1
7. $1 \div 2 = 0$ remainder 1

8. The binary number is read from the remainders in reverse order: 1100100
9. $100_{10} = 1100100_2$
10. **Answer:** $100_{10} = 1100100_2$

(d): 796_{10}

1. $796 \div 2 = 398$ remainder 0
2. $398 \div 2 = 199$ remainder 0
3. $199 \div 2 = 99$ remainder 1
4. $99 \div 2 = 49$ remainder 1
5. $49 \div 2 = 24$ remainder 1
6. $24 \div 2 = 12$ remainder 0
7. $12 \div 2 = 6$ remainder 0
8. $6 \div 2 = 3$ remainder 0
9. $3 \div 2 = 1$ remainder 1
10. $1 \div 2 = 0$ remainder 1
11. The binary number is read from the remainders in reverse order: 1100011100
12. $796_{10} = 1100011100_2$
13. **Answer:** $796_{10} = 1100011100_2$

(e): 5000_{10}

1. $5000 \div 2 = 2500$ remainder 0
2. $2500 \div 2 = 1250$ remainder 0
3. $1250 \div 2 = 625$ remainder 0
4. $625 \div 2 = 312$ remainder 1
5. $312 \div 2 = 156$ remainder 0
6. $156 \div 2 = 78$ remainder 0
7. $78 \div 2 = 39$ remainder 0
8. $39 \div 2 = 19$ remainder 1
9. $19 \div 2 = 9$ remainder 1
10. $9 \div 2 = 4$ remainder 1
11. $4 \div 2 = 2$ remainder 0
12. $2 \div 2 = 1$ remainder 0
13. $1 \div 2 = 0$ remainder 1
14. The binary number is read from the remainders in reverse order: 1001110001000
15. $5000_{10} = 1001110001000_2$
16. **Answer:** $5000_{10} = 1001110001000_2$