$$
\begin{aligned}
g &= c^2 L T^{-2},\\
T^2 &= c^2 \frac{L}{g}\\
T &= c\sqrt{\frac{L}{g}},\\
\Pi_1 &= \frac{T}{\sqrt{\frac{L}{g}}}.
\end{aligned}
$$

Note que

$$\Pi_1' = \frac{T^2}{\frac{L}{g}}$$

também é um grupo adimensional!

Verifique:

$$\frac{\mathsf{T}^2}{\frac{\mathsf{L}}{\mathsf{L}\mathsf{T}^{-2}}} = \frac{\mathsf{T}^2}{\mathsf{T}^2} = 1$$

Vamos fazer o Exemplo 1:

$$
\begin{aligned}
\Pi_1 &= F D^a U^b \rho^c\\
[\![\Pi_1]\!] &= \left[\mathsf{MLT}^{-2}\right] [\mathsf{L}]^a \left[\mathsf{LT}^{-1}\right]^b \left[\mathsf{ML}^{-3}\right]^c\\
1 &= \mathsf{M}^{1+c} \mathsf{L}^{1+a+b-3c} \mathsf{T}^{-2-b}
\end{aligned}
$$

Mas agora todos os expoentes têm que ser iguais a zero:

$$
\begin{aligned}
0a + 0b + 1c &= -1,\\
1a + 1b - 3c &= -1,\\
0a - 1b + 0c &= 2,\\
c &= -1,\\
b &= -2,\\
a - 2 - 3(-1) &= 0,\\
a &= 2 - 4 = -2.\\
\Pi_1 &= \frac{F}{\rho U^2 D^2};
\end{aligned}
$$

Isso foi um exemplo intermediário:

$$
\begin{aligned}
a &= \frac{\ell}{t^2}\\
[\![a]\!] &= \frac{[\![\ell]\!]}{[\![t^2]\!]} = \frac{\mathsf{L}}{[\mathsf{T}]^2} = \mathsf{LT}^{-2}.
\end{aligned}
$$

E agora para o Π_2:

$$\begin{aligned}
\Pi_2 &= \nu^1 D^a U^b \rho^c, \\
[\![\Pi_2]\!] &= \left[\mathsf{L}^2\mathsf{T}^{-1}\right]^1 [\mathsf{L}]^a \left[\mathsf{L}\mathsf{T}^{-1}\right]^b \left[\mathsf{M}\mathsf{L}^{-3}\right]^c \\
1 &= \mathsf{M}^c\mathsf{L}^{2+a+b-3c}\mathsf{T}^{-1-b}
\end{aligned}$$

Monto então o sistema de equações

$$\begin{aligned}
0a + 0b + 1c &= 0, \\
1a + 1b - 3c &= -2, \\
0a - 1b + 0c &= 1, \\
c &= 0, \\
b &= -1, \\
1a - 1 &= -2, \\
a &= -2 + 1 = -1.
\end{aligned}$$

$$\Pi_2 = \frac{\nu}{UD} = \frac{1}{\mathrm{Re}}$$

$$\mathrm{Re} = \frac{UD}{\nu}$$

O Teorema dos Pis agora me "diz" que

$$\frac{F}{\rho U^2 D^2} = f\left(\frac{UD}{\nu}\right) \blacksquare$$