$$
\begin{aligned}
\boldsymbol{v} \times \boldsymbol{u} &= \epsilon_{ijk} v_i u_j \boldsymbol{e}_k \\
&= \epsilon_{ijk} u_j v_i \boldsymbol{e}_k \\
&= -\epsilon_{jik} u_j v_i \boldsymbol{e}_k \\
&= -\boldsymbol{u} \times \boldsymbol{v} \blacksquare
\end{aligned}
$$

$$
\begin{aligned}
|\boldsymbol{u} \times \boldsymbol{v}|^2 &= [\boldsymbol{u} \times \boldsymbol{v}] \cdot [\boldsymbol{u} \times \boldsymbol{v}] \\
&= [\epsilon_{ijk} u_i v_j \boldsymbol{e}_k] \cdot [\epsilon_{lmn} u_l v_m \boldsymbol{e}_n] \\
&= (\epsilon_{ijk} u_i v_j \epsilon_{lmn} u_l v_m)(\boldsymbol{e}_k \cdot \boldsymbol{e}_n) \\
&= (\epsilon_{ijk} u_i v_j \epsilon_{lmn} u_l v_m)\delta_{kn} \\
&= (\epsilon_{ijk} u_i v_j (\epsilon_{lmn}\delta_{kn}) u_l v_m) \\
&= (\epsilon_{ijk} u_i v_j \epsilon_{lmk} u_l v_m) \\
&= \underbrace{[\epsilon_{ijk}\epsilon_{lmk}]}_{\text{identidade polar}} u_i v_j u_l v_m \\
&= \underbrace{[\epsilon_{ijk}\epsilon_{lmk}]}_{\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}} u_i v_j u_l v_m \\
&= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) u_i v_j u_l v_m \\
&= \delta_{il}\delta_{jm} u_i v_j u_l v_m - \delta_{im}\delta_{jl} u_i v_j u_l v_m \\
&= \delta_{jm} u_i v_j u_i v_m - \delta_{im} u_i v_j u_j v_m \\
&= u_i v_j u_i v_j - u_i v_j u_j v_i \\
&= (u_i u_i)(v_j v_j) - (u_i v_i)(u_j v_j) \\
&= (\boldsymbol{u} \cdot \boldsymbol{u})(\boldsymbol{v} \cdot \boldsymbol{v}) - (\boldsymbol{u} \cdot \boldsymbol{v})(\boldsymbol{u} \cdot \boldsymbol{v}) \\
&= |\boldsymbol{u}|^2|\boldsymbol{v}|^2 - |\boldsymbol{u}||\boldsymbol{v}|\cos(\theta)|\boldsymbol{u}||\boldsymbol{v}|\cos(\theta) \\
&= |\boldsymbol{u}|^2|\boldsymbol{v}|^2(1 - \cos^2(\theta)) \\
&= |\boldsymbol{u}|^2|\boldsymbol{v}|^2 \operatorname{sen}^2(\theta); \qquad \Rightarrow \\
|\boldsymbol{u} \times \boldsymbol{v}| &= |\boldsymbol{u}||\boldsymbol{v}| \operatorname{sen}\theta \blacksquare
\end{aligned}
$$

$$f \text{ é linear} \Leftrightarrow f = cx.$$

$$\Leftarrow \text{ é fácil!}$$

Suponha que f seja linear. Então,

$$f(\alpha x) = \alpha f(x)$$

Derive o lado *direito*:

$$\frac{\mathrm{d}[\alpha f(x)]}{\mathrm{d}x} = \alpha \frac{\mathrm{d}f(x)}{\mathrm{d}x} = \alpha f'(x).$$

Derive o lado *esquerdo*:

$$
\begin{aligned}
u &= \alpha x; \\
\frac{\mathrm{d}f(u)}{\mathrm{d}x} &= \frac{\mathrm{d}f}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} \\
&= f'(\alpha x)\alpha
\end{aligned}
$$

Portanto,

$$
\begin{aligned}
\alpha f'(\alpha x) &= \alpha f'(x) \\
f'(\alpha x) &= f'(x) \\
f'(x) &= c \\
f(x) &= cx \blacksquare
\end{aligned}
$$