$$
\begin{aligned}
T &= 1.418\mathrm{s}\\
L &= 0.5\mathrm{m}\\
g &= 9.81\mathrm{m\,s^{-2}}
\end{aligned}
$$

Se eu mudar do MKS (SI) para o GCS:

$$
\begin{aligned}
T' &= 1.418\mathrm{s}\\
L' &= 50\mathrm{cm}\\
g' &= 981\mathrm{cm\,s^{-2}}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{T} &= 1,\\
\mathsf{L} &= 100,\\
\mathsf{M} &= 1000.
\end{aligned}
$$

O que acontece quando mudamos de sistema de unidades com o grupo

$$
\Pi = \frac{T}{\sqrt{\frac{L}{g}}}?
$$

$$
\begin{aligned}
\Pi' &= \frac{T'}{\sqrt{\frac{L'}{g'}}}\\
&= \frac{\mathsf{T}T}{\sqrt{\frac{\mathsf{L}L}{\mathsf{L}\mathsf{T}^{-2}g}}}\\
&= \frac{\mathsf{T}T}{\mathsf{T}\sqrt{\frac{L}{g}}}\\
&= \frac{T}{\sqrt{\frac{L}{g}}} = \Pi = 2\pi.
\end{aligned}
$$

Para concretizar o teorema dos Pis, faça

$$
\begin{aligned}
\frac{F}{\rho U^2 D^2} &= \phi\left(\frac{UD}{\nu}, \frac{U}{\sqrt{gD}}\right),\\
F &= \rho U^2 D^2 \phi\left(\frac{UD}{\nu}, \frac{U}{\sqrt{gD}}\right).
\end{aligned}
$$

Para garantir similaridade entre modelo e protótipo em que usamos o mesmo planeta (g) e o mesmo fluido (ρ, g), devemos ter

$$
\begin{aligned}
\frac{U_m D_m}{\nu} &= \frac{U_p D_p}{\nu},\\
\frac{U_m^2}{g D_m} &= \frac{U_p^2}{g D_p}.
\end{aligned}
$$

Isolando U_m na 1a equação, obtenho:

$$U_m = U_p \frac{D_p}{D_m}$$

Levando na 2a equação, obtenho:

$$\begin{aligned}
\frac{U_m^2}{gD_m} &= \frac{U_p^2}{gD_p} \\
\left[U_p \frac{D_p}{D_m}\right]^2 \frac{1}{gD_m} &= \frac{U_p^2}{gD_p} \\
\left[U_p \frac{D_p}{D_m}\right]^2 \frac{1}{D_m} &= \frac{U_p^2}{D_p} \\
\frac{U_p^2 D_p^2}{D_m^3} &= \frac{U_p^2}{D_p} \\
\frac{D_p^2}{D_m^3} &= \frac{1}{D_p} \\
D_p^2 &= D_m^3 \Rightarrow D_p = D_m.
\end{aligned}$$