TEA010 Matemática Aplicada I
Curso de Engenharia Ambiental
Departamento de Engenharia Ambiental, UFPR
P03, 19 nov 2021
Entrega em 20 nov 2021, 09:30.
Prof. Nelson Luís Dias

Prova com consulta exclusivamente ao livro-texto da disciplina
Declaro que segui o código de ética do Curso de Engenharia Ambiental ao realizar esta prova

NOME: Assinatura: ____________________

---

1 [25] Calcule

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{1/x}^{2/x}\frac{\operatorname{sen}(xt)}{t}\mathrm{d}t.$$

SOLUÇÃO DA QUESTÃO:
A regra de Leibnitz é

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{a(x)}^{b(x)} f(x,t)\,\mathrm{d}t = f(x,b)\frac{\mathrm{d}b}{\mathrm{d}x} - f(x,a)\frac{\mathrm{d}a}{\mathrm{d}x} + \int_{a(x)}^{b(x)}\frac{\partial f(x,t)}{\partial x}\,\mathrm{d}t.$$

Agora,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\int_{1/x}^{2/x}\frac{\operatorname{sen}(xt)}{t}\,\mathrm{d}t &= \frac{\operatorname{sen}(x(2/x))}{2/x}\frac{\mathrm{d}(2/x)}{\mathrm{d}x} - \frac{\operatorname{sen}(x(1/x))}{1/x}\frac{\mathrm{d}(1/x)}{\mathrm{d}x} + \int_{1/x}^{2/x}\frac{\partial}{\partial x}\frac{\operatorname{sen}(xt)}{t}\,\mathrm{d}t\\
&= \frac{\operatorname{sen}(2)}{2/x}\times\frac{-2}{x^2} - \frac{\operatorname{sen}(1)}{1/x}\times\frac{-1}{x^2} + \int_{1/x}^{2/x}\cos(xt)\,\mathrm{d}t\\
&= -\frac{\operatorname{sen}(2)}{x} + \frac{\operatorname{sen}(1)}{x} + \frac{1}{x}\int_1^2\cos(xt)\,\mathrm{d}(xt)\\
&= -\frac{\operatorname{sen}(2)}{x} + \frac{\operatorname{sen}(1)}{x} + \frac{1}{x}\left[\operatorname{sen}(2)-\operatorname{sen}(1)\right] = 0\ \blacksquare
\end{aligned}$$

Continue a solução no verso ⟹

2 [25] Se

$$\boldsymbol{F} = 1\boldsymbol{i}$$

dentro e sobre o círculo $\mathscr{L} : x^2 + y^2 = 1$, calcule

$$\oint_{\mathscr{L}} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}.$$

SOLUÇÃO DA QUESTÃO:
Use o Teorema de Green:

$$\begin{aligned}
\boldsymbol{F} &= (P, Q) = (1, 0),\\
\boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r} &= (1, 0) \cdot (\mathrm{d}x, \mathrm{d}y) = \mathrm{d}x,\\
\oint_{\mathscr{L}} P\mathrm{d}x + Q\mathrm{d}y &= \iint_{\mathscr{S}} \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] \mathrm{d}y\mathrm{d}x\\
&= \iint_{x^2+y^2 \le 1} 0 \, \mathrm{d}y\mathrm{d}x = 0 \;\blacksquare
\end{aligned}$$

Continue a solução no verso $\Longrightarrow$

3 [25] Encontre a solução geral de

$$
\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} = x.
$$

SOLUÇÃO DA QUESTÃO:

$$
\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} &= x,\\
y &= uv,\\
u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x} + \frac{uv}{x} &= x,\\
u\left[\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{v}{x}\right] + v\frac{\mathrm{d}u}{\mathrm{d}x} &= x,\\
\frac{\mathrm{d}v}{\mathrm{d}x} &= -\frac{v}{x},\\
\frac{\mathrm{d}v}{v} &= -\frac{\mathrm{d}x}{x}\\
\frac{\mathrm{d}v}{v} + \frac{\mathrm{d}x}{x} &= 0\\
\ln|v| + \ln|x| &= k_v,\\
\ln|xv| &= k_v,\\
|xv| &= \mathrm{e}^{k_v},\\
xv &= \pm\mathrm{e}^{k_v} = C_v,\\
v &= \frac{C_v}{x};\\
v\frac{\mathrm{d}u}{\mathrm{d}x} &= x,\\
\frac{C_v}{x}\frac{\mathrm{d}u}{\mathrm{d}x} &= x,\\
\frac{\mathrm{d}u}{\mathrm{d}x} &= \frac{1}{C_v}x^2,\\
u &= \frac{x^3}{3C_v} + C_u,\\
y = uv &= \left[\frac{x^3}{3C_v} + C_u\right]\frac{C_v}{x}\\
&= \frac{C}{x} + \frac{x^2}{3} \blacksquare
\end{aligned}
$$

Continue a solução no verso ⟹

4 [25] Obtenha a solução geral de

$$y'' + y = \operatorname{sen}(x).$$

Atenção: simplifique ao máximo sua resposta.

SOLUÇÃO DA QUESTÃO:

$$y'' + y = \operatorname{sen}(x);$$

Tente

$$\begin{aligned}
y &= A(x)\cos(x) + B(x)\operatorname{sen}(x);\\
y' &= -A\operatorname{sen}(x) + B\cos(x) + \underbrace{A'\cos(x) + B'\operatorname{sen}(x)}_{=0}\\
y'' &= -A\cos(x) - B\operatorname{sen}(x) - A'\operatorname{sen}(x) + B'\cos(x).
\end{aligned}$$

Substitua na EDO:

$$\begin{aligned}
y'' + y &= \operatorname{sen}(x);\\
[-A\cos(x) - B\operatorname{sen}(x) - A'\operatorname{sen}(x) + B'\cos(x)] + A\cos(x) + B\operatorname{sen}(x) &= \operatorname{sen}(x);\\
-A'\operatorname{sen}(x) + B'\cos(x) &= \operatorname{sen}(x).
\end{aligned}$$

Agora resolva o sistema de EDOs

$$\begin{aligned}
A'\cos(x) + B'\operatorname{sen}(x) &= 0,\\
-A'\operatorname{sen}(x) + B'\cos(x) &= \operatorname{sen}(x);
\end{aligned}$$

Para eliminar B:

$$\begin{aligned}
A'\cos^2(x) + B'\operatorname{sen}(x)\cos(x) &= 0,\\
A'\operatorname{sen}^2(x) - B'\operatorname{sen}(x)\cos(x) &= -\operatorname{sen}^2(x);\\
\frac{\mathrm{d}A}{\mathrm{d}x} &= -\operatorname{sen}^2(x);\\
A(x) &= \frac{\operatorname{sen}(2x) - 2x}{4} + k_1.
\end{aligned}$$

Para eliminar A:

$$\begin{aligned}
A'\operatorname{sen}(x)\cos(x) + B'\operatorname{sen}^2(x) &= 0,\\
-A'\operatorname{sen}(x)\cos(x) + B'\cos^2(x) &= \operatorname{sen}(x)\cos(x);\\
\frac{\mathrm{d}B}{\mathrm{d}x} &= \operatorname{sen}(x)\cos(x);\\
\frac{\mathrm{d}B}{\mathrm{d}x} &= \frac{2\operatorname{sen}(x)\cos(x)}{2};\\
\frac{\mathrm{d}B}{\mathrm{d}x} &= \frac{2\operatorname{sen}(2x)}{2};\\
\mathrm{d}B &= \frac{2\operatorname{sen}(2x)(2\mathrm{d}x)}{4};\\
B(x) &= \frac{-\cos(2x)}{4} + k_2.
\end{aligned}$$

Continue a solução no verso ⟹

Reunindo tudo,

$$
\begin{aligned}
y &= A(x)\cos(x) + B(x)\operatorname{sen}(x)\\
&= \left[\frac{\operatorname{sen}(2x) - 2x}{4} + k_1\right]\cos(x) + \left[\frac{-\cos(2x)}{4} + k_2\right]\operatorname{sen}(x)\\
&= -\frac{x\cos(x)}{2} + k_1\cos(x) + \frac{2\operatorname{sen}(x)\cos^2(x)}{4} + k_2\operatorname{sen}(x) - \frac{(\cos^2(x) - \operatorname{sen}^2(x))\operatorname{sen}(x)}{4}\\
&= -\frac{x\cos(x)}{2} + k_1\cos(x) + \frac{2\operatorname{sen}(x)\cos^2(x)}{4} + k_2\operatorname{sen}(x) - \frac{(\cos^2(x) - \operatorname{sen}^2(x) - \cos^2(x) + \cos^2(x))\operatorname{sen}(x)}{4}\\
&= -\frac{x\cos(x)}{2} + k_1\cos(x) + \frac{2\operatorname{sen}(x)\cos^2(x)}{4} + \left(k_2 + \frac{1}{4}\right)\operatorname{sen}(x) - \frac{2\cos^2(x)\operatorname{sen}(x)}{4}\\
&= C_1\cos(x) + C_2\operatorname{sen}(x) - \frac{x\cos(x)}{2} \blacksquare
\end{aligned}
$$

Continue a solução no verso $\Longrightarrow$