Part a) $\log \log n$

```
void f1 (int n)
{
  int i = 2
  while (i < n) {
      do O(1) time
      i = i * i;
  }
}
```

k iterations	n	i
0	2	2
1	4	4
2	16	16
3	256	256
4	65356	65...

$i = 1$

$i = 2^{2^k} \Rightarrow 2^{(2^k)} = n$

$2^k = \log_2 n$

Runtime $= \boxed{\Theta(\log(\log n))}$

Part b)

```
void f2(int n)
{
  for (int i=1; i<=n; i++){
    if ( i % (int) sqrt(n)) ==0) {
      for (int k=0; k< pow(i,3); k++) {
      }
    }
  }
}
```

k iterations	i
1	3
2	6
3	9
4	12

$n = 9$, $\sqrt{9} = 3$

$i = k\sqrt{n}$

$$\sum_{i=0}^{n} \left(\Theta(1) + \Theta\left(\sum_{k=0}^{i^2} \Theta(1) \right) \right) = \sum_{i=0}^{n} \left(\Theta(1) + \sum_{i}^{n-1} \sum_{i=0} \Theta(1) \right)$$

$$\Theta(n) + \sum_{k=1}^{\sqrt{n}} \sum_{i=0}^{k^3-1} \Theta(1) = \Theta(n) + \sum_{k=1}^{\sqrt{n}} \Theta(k^3)$$

$$= \Theta(n) + \Theta\left(n^3 \cdot \sqrt{n}\right) = \Theta(n) + \Theta\left(n^{\frac{7}{2}}\right)$$

Runtime $= \Theta(n^{7/2})$

Part ( )

k iterations	i

```
for (int i=1; i<=n; i++){                    ← n^2
    for (int k=1; k<=n; k=k*2){
        if (A[k] == i){                      n
            for (int m=1; m<=n; m=m*m){      log n
                O(1)
            }
        }
    }
}
```

$$\left(\sum_{i=1}^{n} \Theta(1) \cdot \sum_{k=1}^{n} \Theta(1)\right) + \left(O\left(\sum_{m=1}^{\log_2 n} \Theta(1)\right)\right)$$

$$2^m = n$$

$$\log_2 n = m$$

$$(\log n)$$

$$\Theta(n \cdot n) + \sum_{n} \left(\sum_{m=1}^{\log n} \Theta 1\right)$$

$$\Theta(n^2) + \Theta(n \log n) = \Theta(n^2)$$

Runtime $= \Theta(n^2)$

Part d)

```
int f(int n)
{ int *a = new int[10];
  int size = 10;
  for (int i=0; i<n; i++)
  { if (i==size)
    { int newsize = 3*size/2
      int *b = new int [newsize];
      for (int j=0; j<size; j++)
      { b[j] = a[j];
      }
      delete [] a;
      a = b;
      size = newsize;
    }
    a[i] = i*i;
  }
```

$\Theta(1)$

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$\Theta(1)$

$\sum_{i=0}^{n-1} \Theta(1) = \Theta(n)$

$n = 12 \quad k = 1 \quad n < 10 \quad size = 10$

size	10	15	22	33	49	73	109	
n	10	15	22	40	50	60	70	80
k iter	1	2	3	4	5	5	5	6

$\sum_{j=0}^{\log n} \frac{3}{2}^j$

$10\left(\frac{3}{2}^{k-1}\right) = n \qquad \frac{n}{10} = \frac{3}{2}^{k-1}$

$k-1 = \log_{\frac{3}{2}} \frac{n}{10} \qquad k = \log_{\frac{3}{2}} \frac{n}{10} + 1$

$k = \Theta \log n$

$\sum_{i=0}^{n-1} \Theta(2) + \sum_{j=0}^{\log n} \frac{3}{2}^j$

$\Theta(n) + \Theta\left(\frac{3}{2}^{\log n}\right)$

$\Theta(n) + \sum_{k=0}^{\log n}$

$\Theta\left(\frac{3}{2}^{\log n}\right)$