1) 15 students 8 questions $\binom{15}{8} \to \dfrac{\binom{15}{8}}{\binom{22}{14}}$

1 question probability $\to \binom{15\ 15-1}{15-7}$

Probability no student will have to answer more than one question

$$\frac{\binom{15}{8}}{\binom{22}{14}} = \frac{15}{15} \times \frac{14}{15} \times \frac{13}{15} \times \frac{12}{15} \times \frac{11}{15} \times \frac{10}{15} \times \frac{9}{15} \times \frac{8}{15} = \boxed{0.10}$$

unique digit

2) $\dfrac{1}{5} \times \dfrac{1}{4} \times \underbrace{\dfrac{7}{10} \times \dfrac{6}{10}}_{\text{unique digit}} \times \dfrac{1}{5} = \dfrac{32}{10000}$

chance of one odd digit

chance of second odd digit

even digit

5 digits $\to 10^5$

$5 \times 5 \times 4 \times 7 \times 6 = \dfrac{4200}{10^5}$

probability even

prob of odd

prob of 2nd odd

Rest of criteria

amount of digits to meet criteria

$= 0.042$ chance

3) $P(A) = 3/6 \times 3/6 = 25\%$

$P(B) = \dfrac{6}{6} \times 1/6 \times 1/6 = 1/36$ all dice have same value

$P(A) \cdot P(B) = 0.25\left(\frac{36}{36}\right) \times 1/36 = \dfrac{9}{36} \times \dfrac{1}{36} = \dfrac{9}{1296} = \dfrac{3}{432}$

$P(A \cap B) = \dfrac{3}{216}$ no, they are not independent

4) $13/52$ $\binom{13}{5}$ The expected of getting a flush:

Getting a flush $= \dfrac{\binom{13}{5} \cdot 4}{\binom{52}{5}}$ ← from the 4 different suits of a card

Expected hands $\displaystyle\sum_{x=1}^{\infty} x \left(1 - \frac{\binom{13}{5} \cdot 4}{\binom{52}{5}}\right)^{x-1} \left(\frac{\binom{13}{5} \cdot 4}{\binom{52}{5}}\right) = \boxed{\dfrac{1}{\frac{\binom{13}{5} \cdot 4}{\binom{52}{5}}}}$

Chance not getting flush for x-1 hands

5) $P(S) \to$ superstar plays game Team $\to T$ wins 4/5 games

$$P(S \mid T) = \frac{P(S \cap T)}{P(S)} = \frac{P(S \mid T) \cdot P(S)}{P(S)}$$

$$= \frac{\left(\binom{5}{4} \cdot 0.7^4 \cdot 0.3\right) \cdot 0.75}{\left(\binom{5}{4} \cdot 0.7^4 \cdot 0.3\right) + \left(\binom{5}{4} \cdot 0.5^4 \cdot 0.5\right)}$$