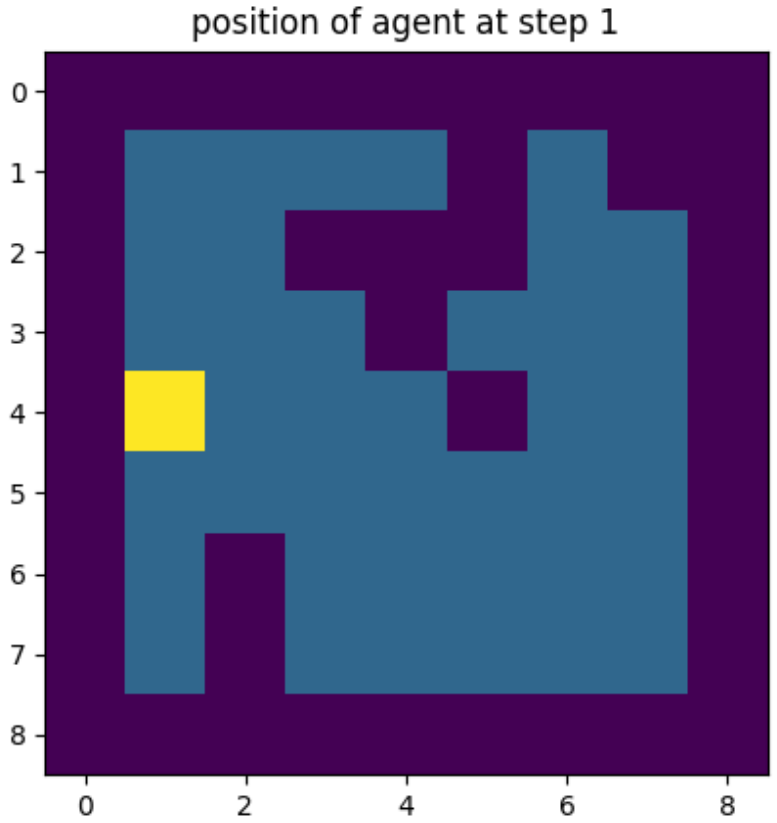
# Machine learning II, unsupervised learning and agents: reinforcement learning



- RL has many applications and is quite a hot topic.
- **Deep Reinforcement Learning** has received a lot of attention recently.

- Atari games

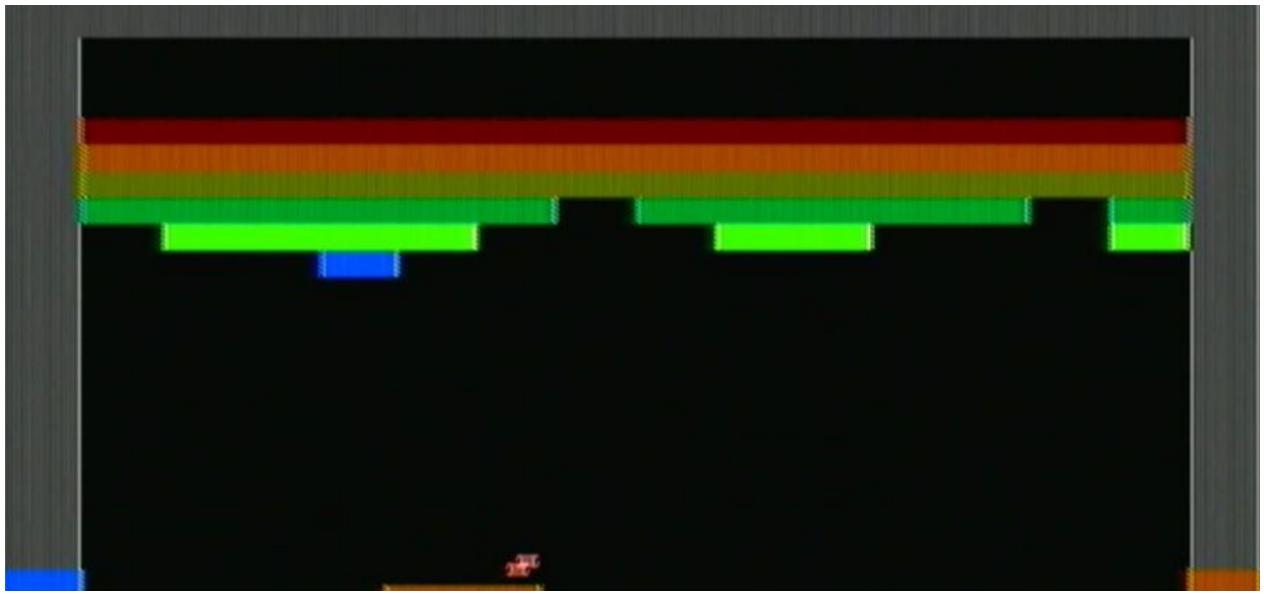


Figure – [?]

- AlphaGo

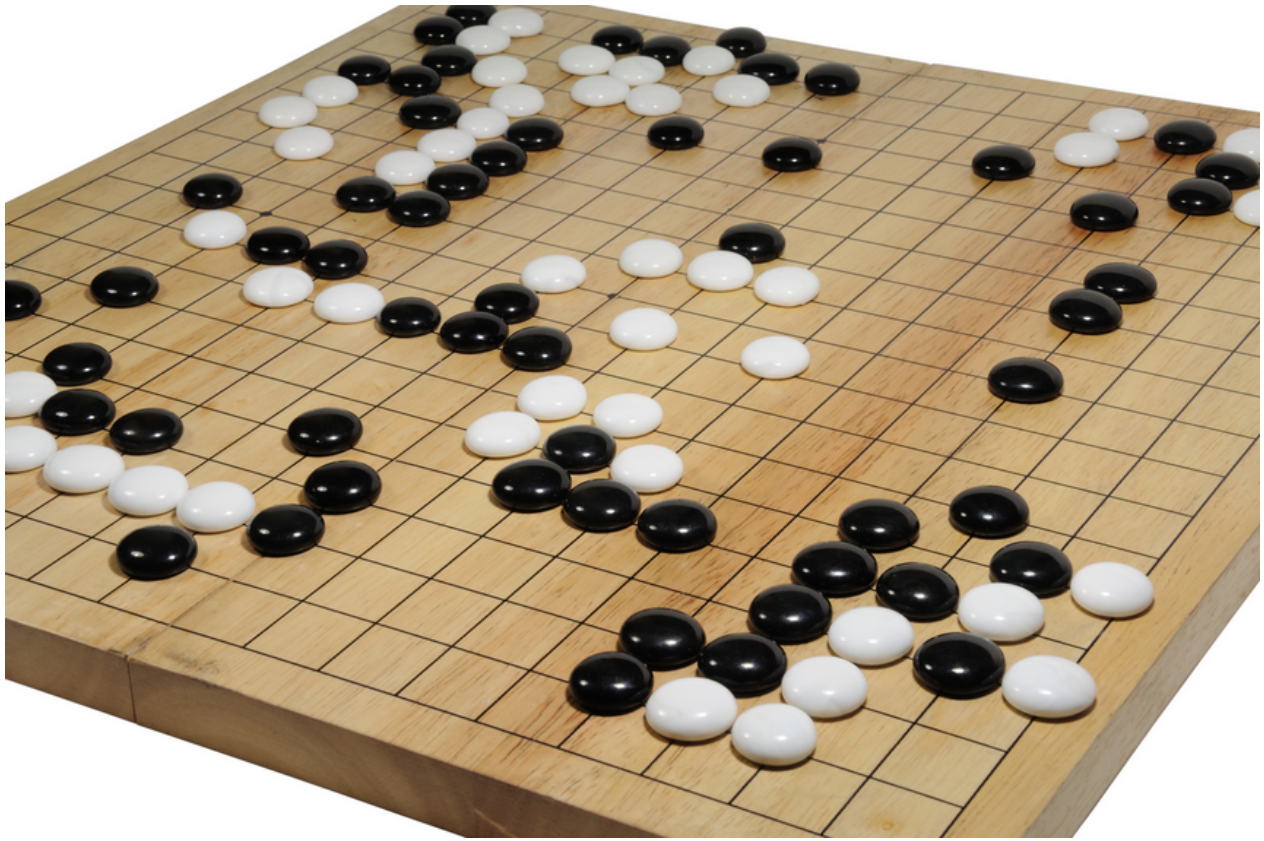


Figure – Go game, beaten by AlphaGo in 2017 [?]

- Reinforcement Learning is also being used in the community of **Computationnal neuroscience.**

# Overview

# Supervised learning and Correction

- In **supervised learning**, the supervisor indicates the **expected answer** the agent should give.
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- The feedback does not depend on the action performed by the agent (for instance the prediction from the agent)
- We say that the agent receives an **instructive feedback**.

# Supervised learning Correction

- In **supervised learning**, the supervisor indicates the **expected answer** the agent should give.
- The agent must then **correct its model** based on this answer.

# Cost sensitive learning

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- The agent receives an **evaluative feedback**. The feedack depends on the action performed by the agent.
- **Examples :**
  - AI playing a game and receiving "victory" or "defeat" as a feedback.
  - Child playing with an animal.

# Reinforcement learning

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- In reinforcement learning, the feedback is a **real number**.
- **Example :** amount of coins won after a poker turn.

# Reinforcement learning

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- A reward of $-10$ good be good or bad depending on the other rewards that are possible to obtain.
- Most of the time, the objective of the agent will be to optimize the **agregation of rewards**.

# Reinforcement learning

- The agent lives in a world $E$, and can be in several states $s$. The agent performs **actions** $a$ and receives rewards $r$.

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- **Examples :**
  - world $= \mathbb{R}^2$
  - state $=$ position
  - actions $=$ moving somewhere
  - reward $=$ amount of food found

# Formalization

- There are many aspects of the problem that we need to formalize. Several formalizations are possible depending on the situation.

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- We will consider **discrete spaces** :
  - the time will be discrete
  - the number of possible states will be **finite**
  - the number of possible actions will be **finite**
- Continuous spaces are also available for RL. In those cases the objects are slightly different, and the optimization procedures also differ. For an introductory course, discrete spaces are more suitable.

# Question

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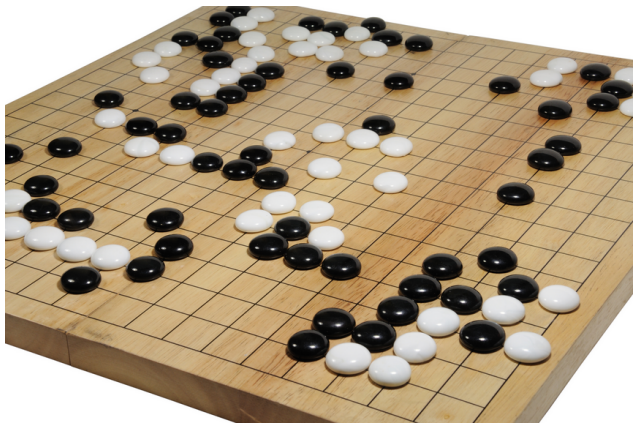
- Are these hypotheses valid in the case of AlphaGo ?



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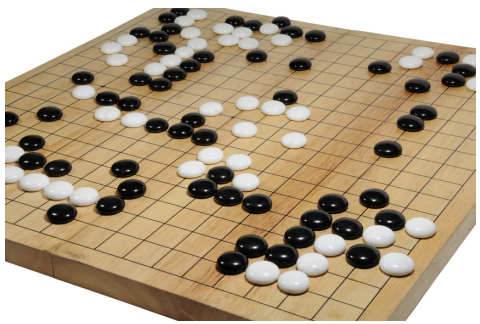

- Yes ! This shows that discrete spaces can still describe very complex problems.

# Formalization

- we will write :
  - $S_t$ : state at time $t$
  - $R_t$ : reward received at time $t$
  - $A_t$ : action performed at time $t$
- the actions are chosen according to a **policy** $\pi$

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- Or **stochastic :** the action performed in a given state is drawn from a **distribution**.

# Two levels of randomness

- The policy can be deterministic or stochastic.
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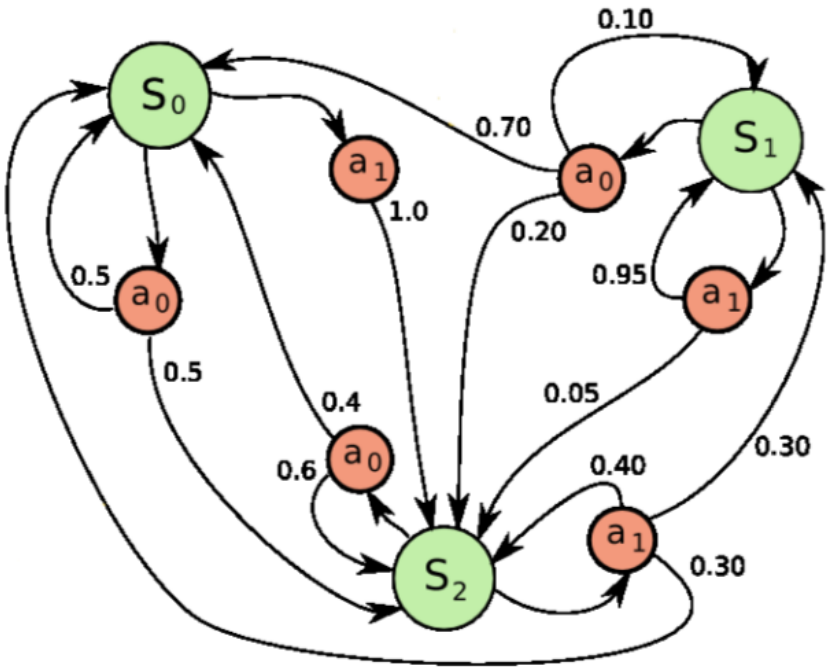
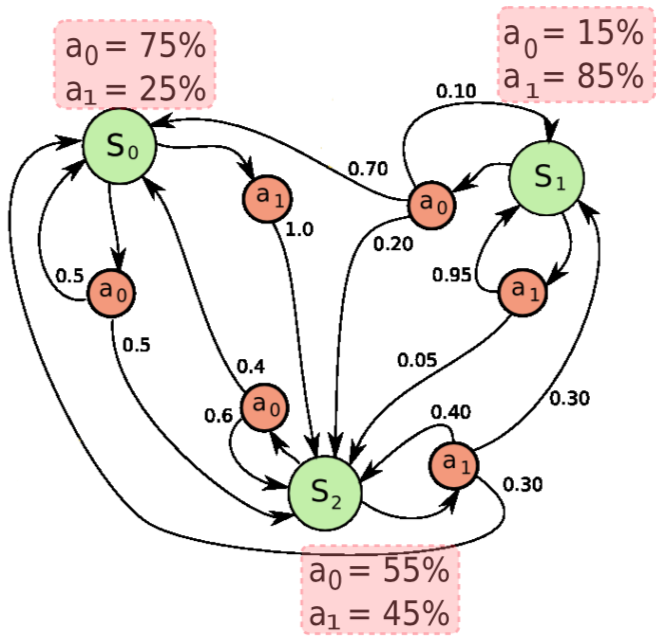



Figure – A stochastic policy with a stochastic transition function.

## Exercice 1 :

- What is the probability of staying in state $s$ when performing an action from $s$ ? and from $S_1$ and $S_2$ ?

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# Agregation of rewards

- Remember that our agent want to optimize the **agregation of the rewards.**
- There are several ways to agregate the rewards.

# Returns

We introduce the **return** $G_t$.

- Episodic case (finite number $N$ of steps) :

$$G_t = R_{t+1} + R_{t+2} + \cdots + R_{t+N} \tag{1}$$

- Continuing tasks :

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \tag{2}$$

$\gamma \in [0, 1[$ is the **discount factor**

# Value function

Given a fixed policy $\pi$, the value function $v_\pi(s)$ quantifies how good a state $s$ is.

$$v_\pi(s) = E[G_t | S_t = s] \tag{3}$$

(the expectation is taken over the next actions, following the policy $\pi$).

# The Bellman equation

Given a fixed policy $\pi$, and a state $s$, we can write a recursive relationship between $v_\pi(s)$ and the values $v_\pi$ of the next possible successor states (see the Sutton book for the general form of the equation 3.12 page 85).

# More considerations

- The Markov hypothesis
- Exploitation exploration compromise

# $\epsilon$-greedy policy

A way to tackle the exploitation-exploration compromise.

- with probability $1 - \epsilon$ : go to the best known reward (exploitation).
- with probability $\epsilon$ : perform a random action (exploration).

# Art

"RL is a science, but dealing with the exploration-exploitation compromise is an art" (Sutton)

# Dynamic programming

- Today we will study a simple case of Reinforcement learning
- Deterministic transition function.

# World

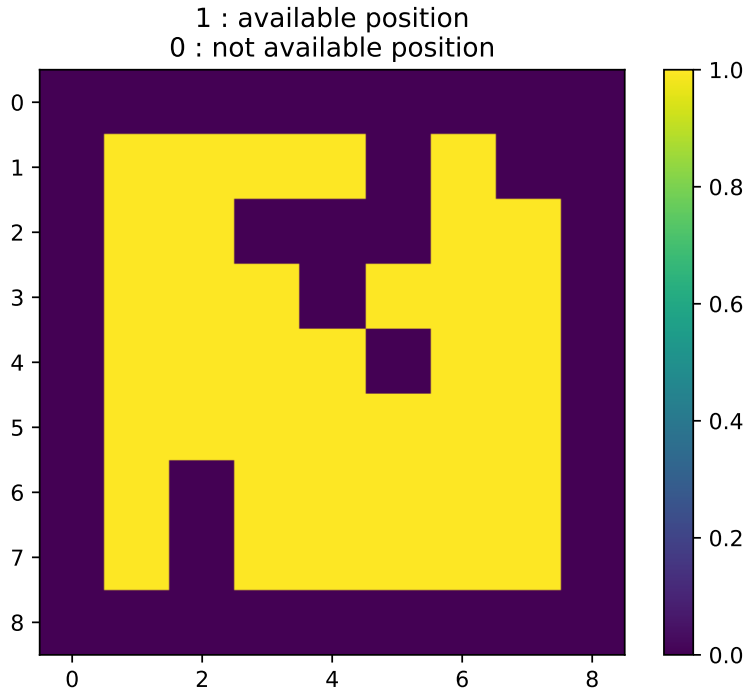



Figure – 2 dimensional world.

# Reward

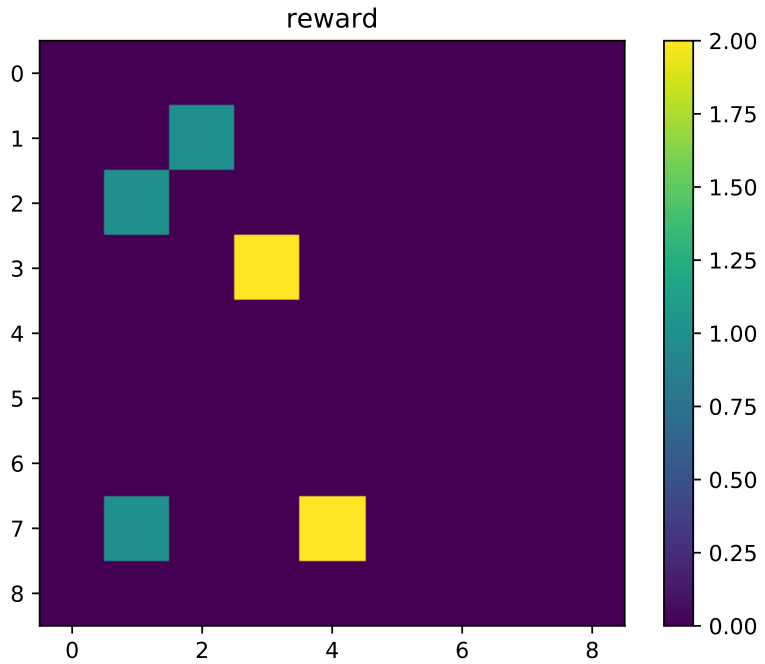



Figure – Reward function.

# 2D world

- Our agent can move in the 4 directions, one step at a time.

# Optimal value functions

We look for the value of the **optimal policy** $\pi^*$, defined by the fact that it has the best value function among all policies.

$$
\begin{aligned}
v_*(s) &= \max_{a \in \text{available actions}} E[G_t | S_t = s, A_t = a] \\
&= \max_{a \in \text{available actions}} E[\sum_{k \geq 0} \gamma^k R_{t+k+1} | S_t = s, A_t = a] \\
&= \max_{a \in \text{available actions}} E[R_{t+1} + \gamma \sum_{k \geq 1} \gamma^{k-1} R_{t+k+1} | S_t = s, A_t = a] \\
&= \max_{a \in \text{available actions}} E[R_{t+1} + \gamma \sum_{k \geq 0} \gamma^k R_{t+k+2} | S_t = s, A_t = a] \\
&= \max_{a \in \text{available actions}} E[R_{t+1} + \gamma v_*(S_{t+1}) | S_t = s, A_t = a]
\end{aligned} \tag{4}
$$

# Bellman optimality equation

In the case of our simple 2D deterministic world, the Bellman optimality equation ?? takes a simpler form !

$$v_*(s) = \max_{a \in \text{available actions}} R_{t+1} + \gamma v_*(S_{t+1}) \tag{5}$$

The expected values are replaced by deterministic values.

# Value Iteration

- Value iteration belongs to dynamic programming methods. They are a specific case of RL where a perfect model of the environment is assumed.
- In value iteration, equation ?? is used as an update rule at each time step.

# Value Iteration

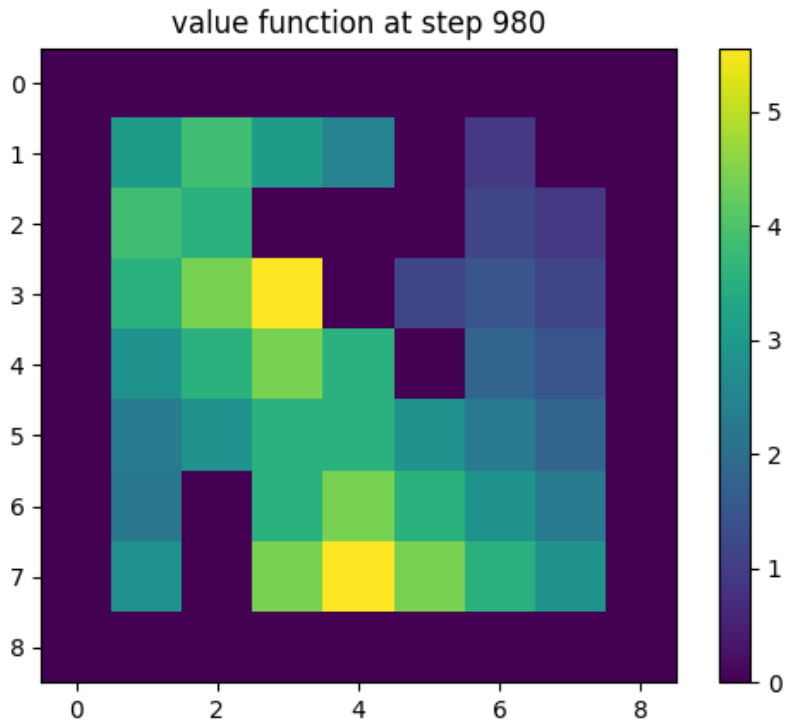
- First, the initial Value function for all the states is 0.
- Then we propagate the information about the rewards between the states, in order to **update the value function** (sweep)

$$\forall s \in V(s) \leftarrow \max_a \left(R_{t+1} + \gamma V(s_{t+1}) | S_t = s, A_t = a\right) \quad (6)$$

- In parallel, we explore the world to learn about the distribution of rewards.

# Value iteration

- After learning, we will obtain a value function



- **cd reinforcement_learning/**
- Use the file **create_world.py** in order to generate your own environment.
- You can also use the one that is already there if you prefer.
- We store the data about the world in **.npy** files.

Exercice 2 :

- In **value_iteration.py**, modify the function **move_agent()** so that the agent is randomly moved at each time step.

Exercice 3 :

- In **value_iteration.py**, modify the function **update_value_function()** in order to update the value function according to the Bellman equation, and run the algorithm until convergence of the value function.

# Optimal policy

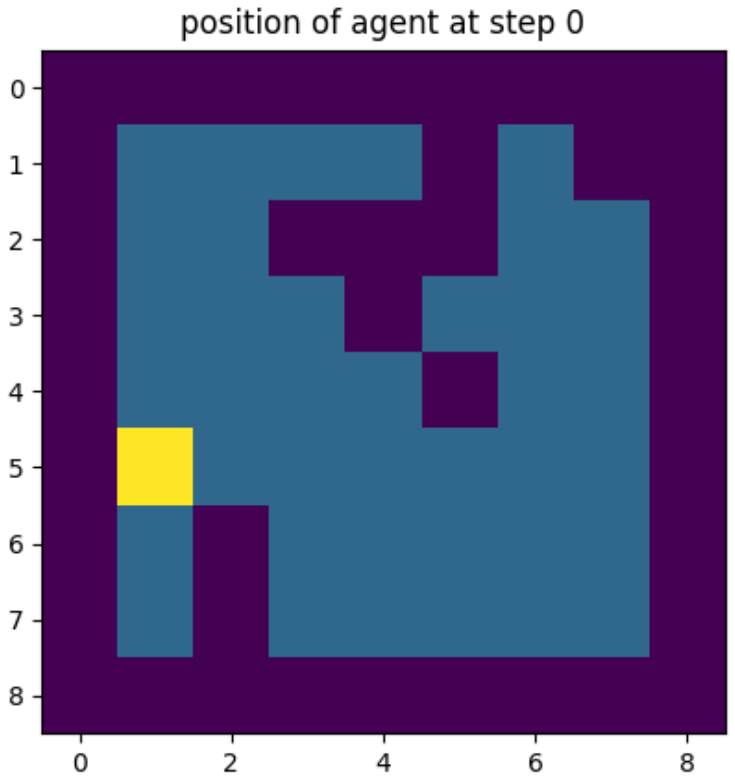



Figure – After learning hte optimal policy, the agent can go to the reward.

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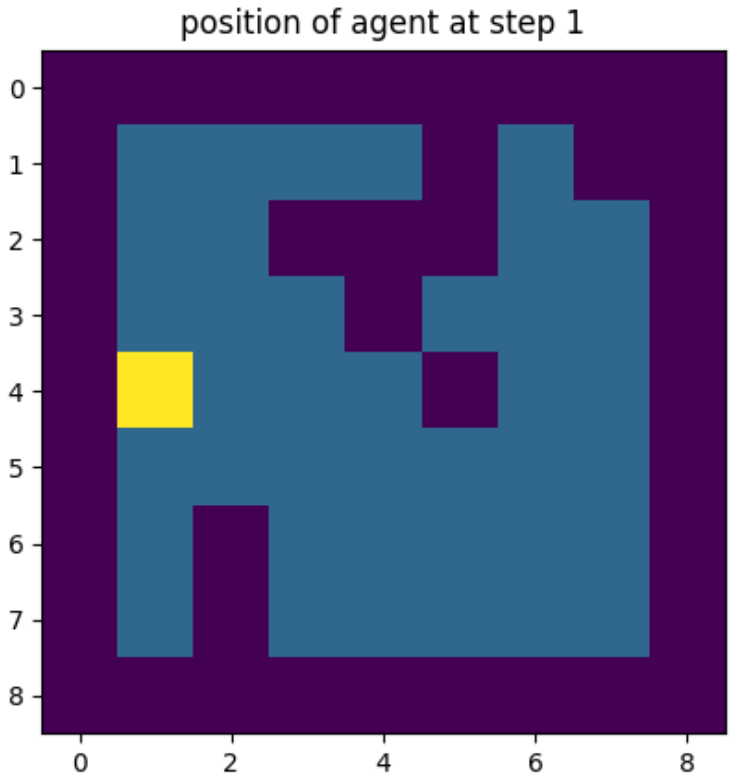


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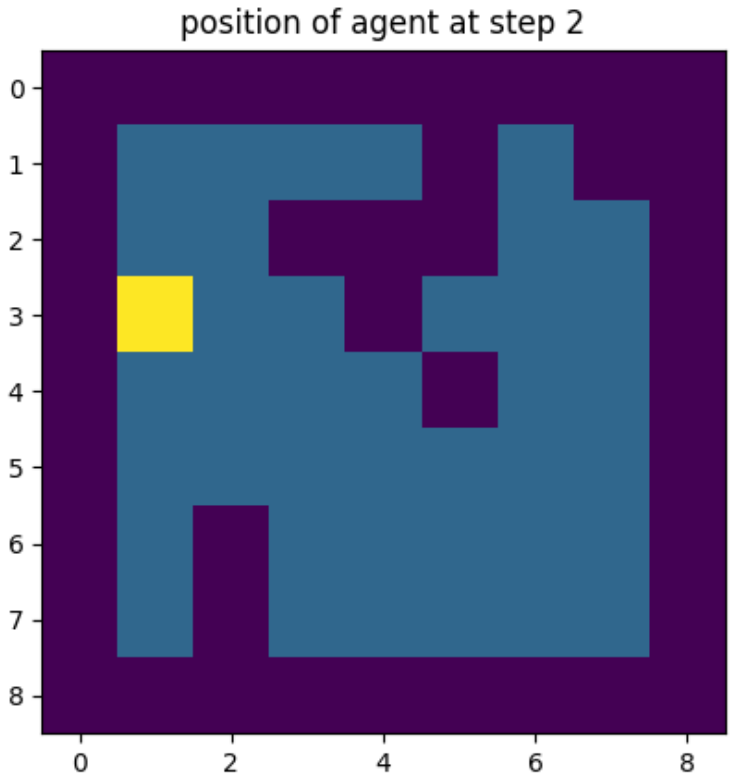


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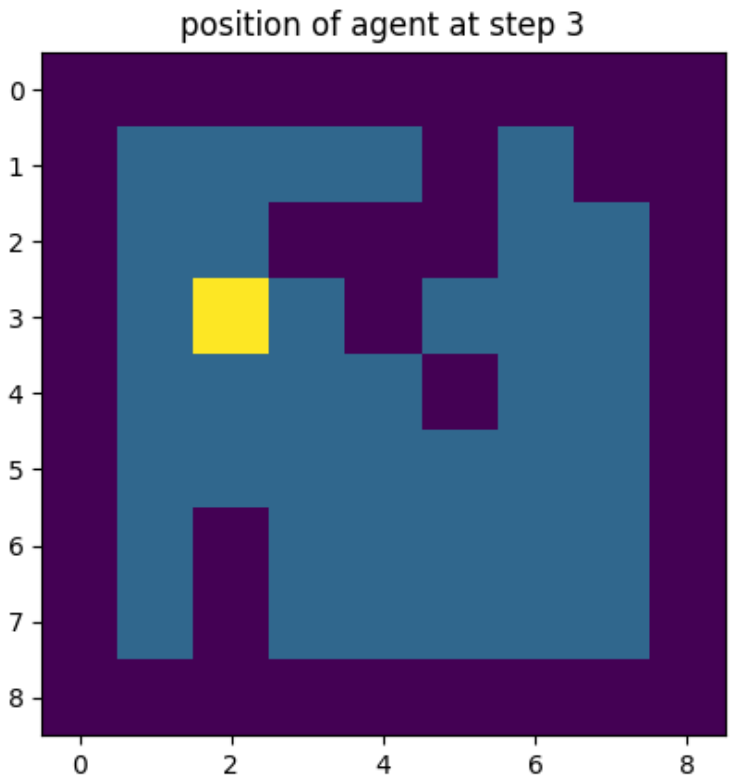


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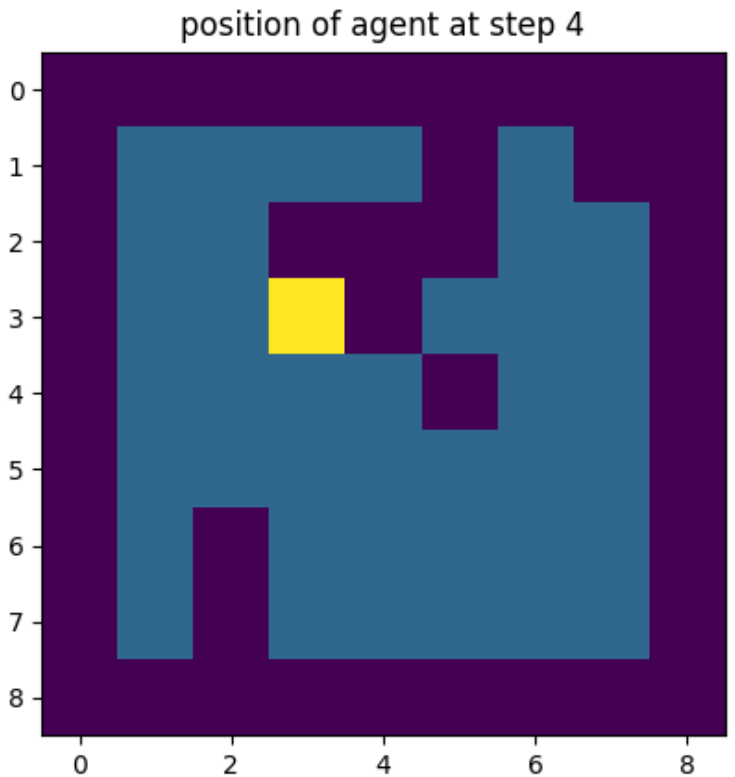


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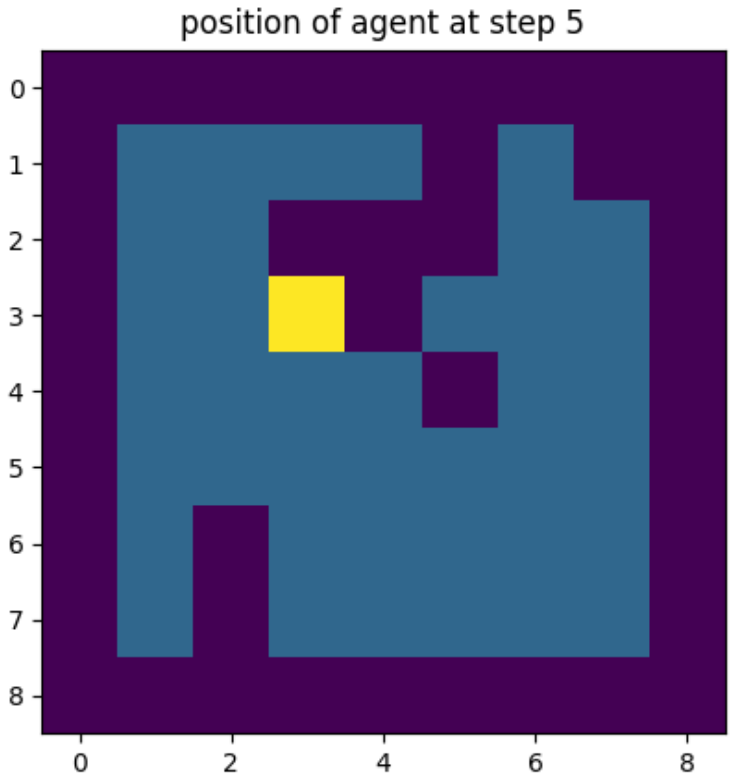


Figure – After learning, the agent can go to the reward.

# Multiple paradigms

- Reinforcement learning has many variants.
- In the ones we studied, a model of the consequence of our actions was known.
- This is not always the case.

# Temporal difference learning

- In temporal difference learning, the agent does not know a **model** of its world.
- But it can still learn the value function with the **TD (temporal difference) updates**

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$$V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)] \quad (7)$$

# Monte Carlo methods

Monte Carlo methods can be used in Reinforcement Learning to estimate some expected values (such as the expected reward in a given state).

# Actor critic methods

- Sometimes you can use **two** policies
  - the **behavior policy** provides actions and guarantees exploration
  - the **target policy** is the optimal policy learned in parallel by the agent, that would be used in exploitation mode.

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- However, RL can also be applied to continuous state / discrete action spaces (DQN)
- And even to continous state / continous action spaces (DDPG) [?] .

# References I