Merge sort

$C(n) = 2C(n/2) + \sigma(n)$, $n > 1$ e $C(1) = \sigma(1)$

$n = 2^M$ logo $M = \log_2 n$

$$C(2^M) = 2C(2^{M-1}) + \sigma(2^M)$$

$$\Leftrightarrow \frac{C(2^M)}{2^M} = \frac{C(2^{M-1})}{2^{M-1}} + \sigma(1) \qquad (1)$$

c.a.

$$\frac{2}{2^M} = \frac{1}{2^{-1} \cdot 2^M} = \frac{1}{2^{M-1}}$$

Cálculo de $C(2^{M-1})$: Substitui-se $n \leftarrow 2^{M-1}$ na eq. geral:

$$C(2^{M-1}) = 2C(2^{M-2}) + \sigma(2^{M-1})$$

$$\Leftrightarrow \boxed{\frac{C(2^{M-1})}{2^{M-1}}} = \frac{C(2^{M-2})}{2^{M-2}} + \sigma(1) \qquad (2)$$

Substituindo (2) em (1) temos que:

$$\frac{C(2^M)}{2^M} = \frac{C(2^{M-2})}{2^{M-2}} + \sigma(1) + \sigma(1)$$

$$= \dots$$

$$= \frac{C(2^0)}{2^0} + \sigma(M)$$

$$= \sigma(1) + \sigma(M) = \sigma(M)$$

Então, como $n = 2^M \Leftrightarrow M = \log_2 n$:

$$\frac{C(n)}{n} = \sigma(\log_2 n) \Leftrightarrow \boxed{C(n) = \sigma(n \times \log_2 n)}$$

Teorema Mestre

Ex: Merge sort

$$C(n) = \underbrace{2}_{a} \times C\left(\frac{n}{\underbrace{2}_{b}}\right) + n$$ ← "merge"

$a = 2$

$b = 2$

$f(n) = n$

$\varepsilon > 0$

Caso 1: $\log_2 2 = 1$

$$f(n) \overset{?}{=} O\left(n^{\log_b a - \varepsilon}\right) = O\left(n^{1-\varepsilon}\right)$$

$\overset{n}{h} = O(\overset{n^{1-\varepsilon}}{k})$ se e só se $h(n) \le c \times k(n)$

$n \le c \times n^{1-\varepsilon}$

$\exists n_0 \in \mathbb{N}_0, \exists c \in \mathbb{R}^+, \forall n > n_0$

Ex: $\varepsilon = 0.2 \Rightarrow n \le c \times n^{1-0.2}$?

Não é verdade

Não se aplica o caso 1

Caso 3:

$$f(n) \overset{?}{=} \Omega\left(n^{\log_b a + \varepsilon}\right)$$

$$= \Omega\left(n^{1+\varepsilon}\right)$$

$h = \Omega(k)$ se e só se

$\exists_{n_0} \in \mathbb{N}_0, \exists_c \in \mathbb{R}^+, \forall_{n > n_0}$ $\quad c \times \overbrace{k(n)}^{n^{1+\varepsilon}} \leq \overbrace{h(n)}^{n}$

Ex: $\varepsilon = 1 \Rightarrow c \times n^{1+\varepsilon} \leq n$

$$\Leftrightarrow \underbrace{c \times n^2 \leq n}_{\text{não é verdade}}$$

Caso 3
não se
aplica

Caso 2: $f(n) \overset{?}{=} \Theta(n^{\log_b a})$ **Sim**

$$= \Theta(n^1) = \Theta(n)$$

$\overset{n}{\widehat{h}} = \Theta(\overset{n}{\widehat{k}})$ se e só se $\overset{n}{\widehat{h}} = O(\overset{n}{\widehat{k}})$ e

$\overset{n}{\widehat{h}} = \Omega(\overset{n}{\widehat{k}})$

É verdade

Então: $C(n) = \Theta(n^{\log_b a} \times \log_2 n) =$

$$= \Theta(n \times \log_2 n)$$

T.P.C.

$$C(n) = 2 \times C(n/2) + \overset{\frown}{1} \to f$$