Lab Report 1: Supersonic Wind Tunnel Block Calibration

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Nomenclature

P_{atm}	=	atmospheric pressure
P_{01}	=	stagnation pressure before shock
P_1	=	static pressure before shock
P_{02}	=	stagnation pressure after shock
γ	=	specific heat of fluid
M	=	Mach number

I. Introduction

THIS document is a report on the first MAE 352 lab in the Spring 2019 semester. The goal of the lab is to create a calibration curve for the supersonic wind tunnel using two methods of Mach number calculation and compare the results. Pressure data at different points in the wind tunnel was collected during each run. After each test, the block number was increased by 200, leaving the total range of tested block numbers to be 400 to 2600. The block number controls the nozzle width in the wind tunnel, with a higher block number representing a higher nozzle area, and therefore a theoretically slower flow. MATLAB was used to analyze and compare the two data sets collected during experimentation.

II. Calculations

A. Addition of atmospheric pressure

The pressure readings taken from the transducers in the lab do not account for the atmospheric pressure, which is recorded separately. For accurate calculation of Mach number, the atmospheric pressure, P_{atm} , was added to all other pressure readings, which includes $P_{01}, P_1,$ and P_{02}. This basic addition is shown for the stagnation pressure before the shock below to serve as an example. These total pressure values were used to make the calculations for Mach number, but the subscripted $P_{1,total}$ is only present here to distinguish it from the pressure reading P_1. Note that in later calculations, the additional subscript is dropped and P_1 is used to refer to the new $P_{1,total}$ value.

$$P_{1,total} = P_1 + P_{atm} \tag{1}$$

B. Mach Number by Isentropic Relation

The first method of Mach number calculation was with the use of the isentropic relation equation below.

$$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{2}$$

C. Mach Number by Rayleigh Pitot Tube Equation

Alternatively, the Mach number was calculated using the Rayleigh pitot tube equation.

$$\frac{P_{02}}{P_1} = \left(\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2(\gamma-1)}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1-\gamma+2\gamma M^2}{\gamma+1}\right) \tag{3}$$

D. Percent Difference

Percent difference between the isentropic relation Mach numbers and the Rayleigh equation Mach numbers was calculated with the formula below.

$$Percent\ Difference = \frac{|M_{isentropic} - M_{Rayleigh}|}{\frac{1}{2}(|M_{isentropic} + M_{Rayleigh})} * 100 \tag{4}$$

III. Data and Results

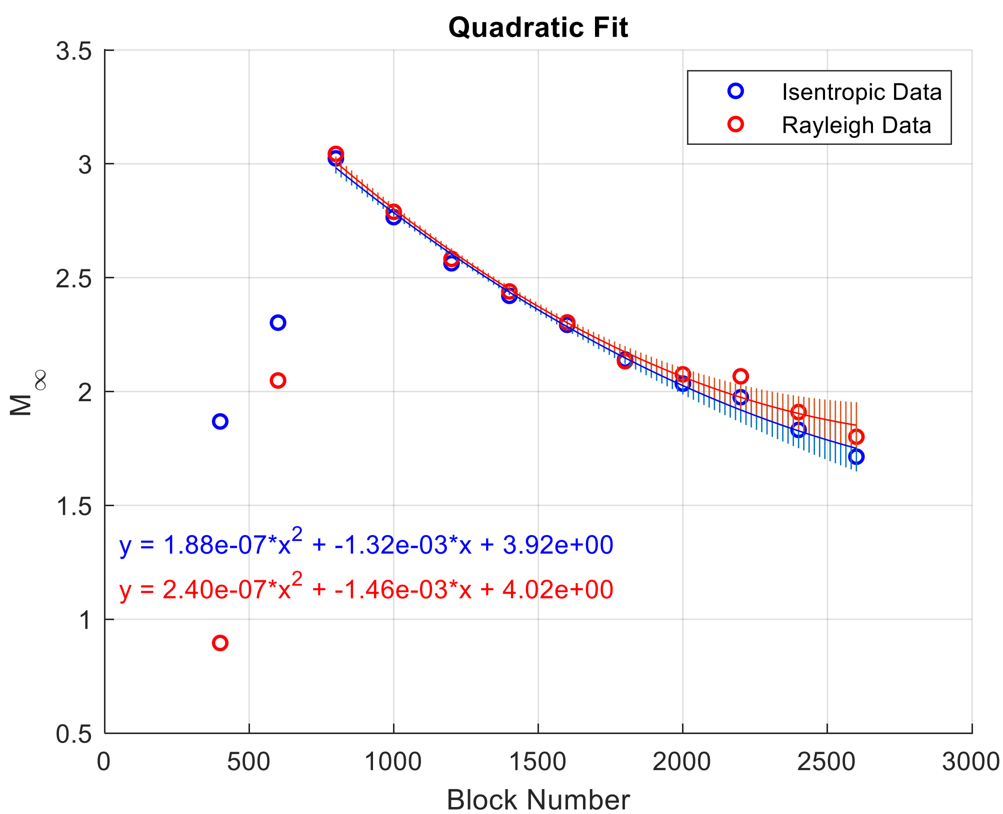

Figure 1. Quadratic Regression of Mach Number Data. *The quadratic regression ignores the first two points of each data set. In addition, error bars are displayed along each of the 100 evenly-spaced points used in the regression. Magnitude of the bar at each point is equal to the difference of the method regression values.*

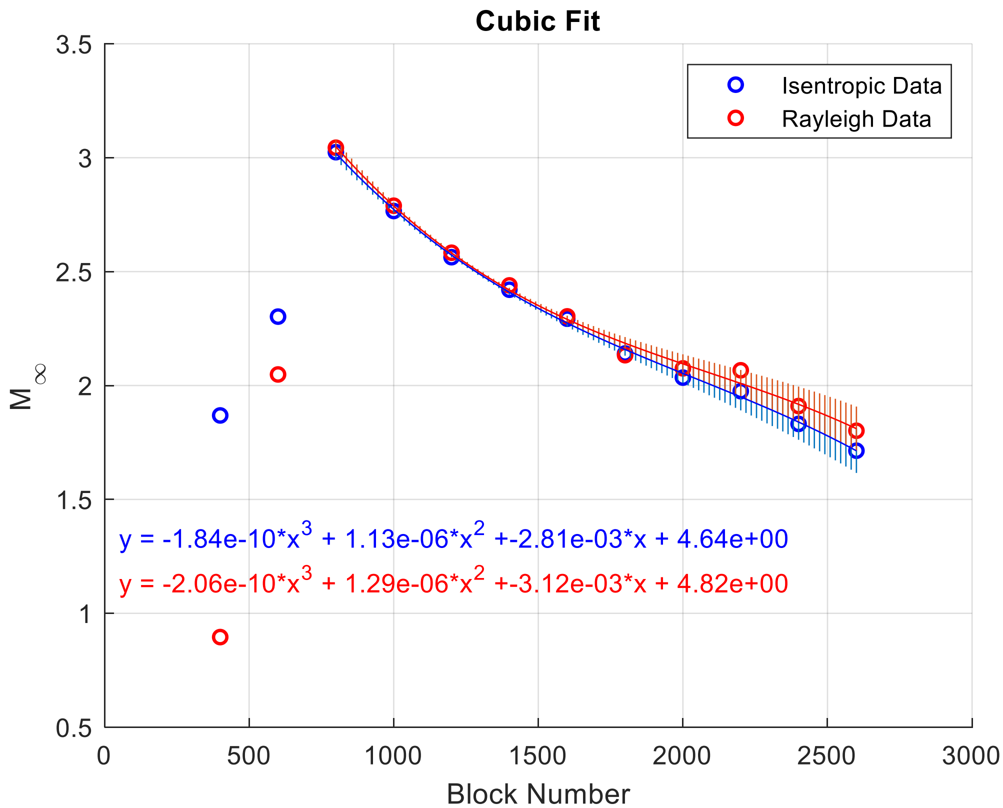

Figure 2. Cubic Regression of Mach Number Data. *The cubic regression also ignores the first two points of each data set. Again, the error bars are displayed in the same fashion. Note that the coefficient of the leading term is smaller than before.*

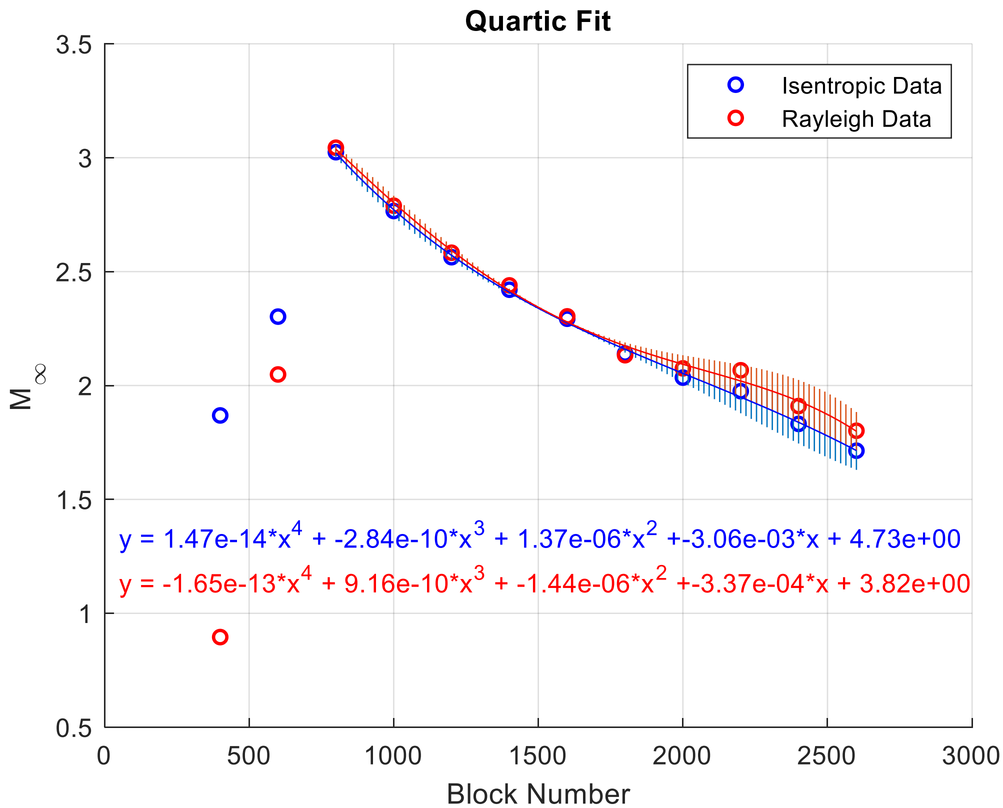

Figure 3. Quartic Regression of Mach Number Data. *The quartic regression follows the same format as the other two plots. Note that the leading term coefficient is much smaller than that of the quadratic regression.*

Regression	Mean percent difference
Quadratic	0.7937
Cubic	0.7915
Quartic	0.8470

Table 1. Mean Percent Differences. *Here it can be seen that percent difference values were low for all three regressions tested.*

IV. Discussion

Knowing that the supersonic wind tunnel at NCSU has the capability to operate between Mach numbers of 1.5 and 3.5, it can be noted that the Mach numbers from both methods of calculation are reasonable values as they fall within the range. As the block number increases, the nozzle in the wind tunnel widens, and therefore should reflect a decrease in Mach number due to the reduced speed of the flow. This idea also reinforces the validity of the experimental values as the plots display that higher block number results in lower Mach number.

The first two and a half seconds of each wind tunnel run were excluded from the data set to account for the time the flow requires to gain stability. The remainder of the values at each block number were then averaged and converted from pounds per square inch to Pascals. One-hundred equidistant values were used when calculating the regressions for both the isentropic and Rayleigh methods, but the first two data points from both sets were excluded from the range as the flow did not appear to have gained stability at that block number.

The isentropic method overall yielded slightly lower Mach numbers at every point, but this difference is only very slight, as can be seen in Table 1. The regression values did not differ by even one percent, though when high Mach numbers are concerned, one percent may cause drastic impacts. The isentropic method makes an idealistic assumption that the flow does not generate entropy, when in reality it does, however, the amount of entropy may be insignificant. The Rayleigh pitot tube method however does not make this assumption and accounts for the pressure after the shockwave. While the irreversibility in the flow is likely negligible, the results of the Rayleigh equation are more reliable.

The lack of stability could be explained by the limitations of the wind tunnel. At block number 600, it is reasonable that the flow would exceed a Mach number of 3.5, which is the maximum that the wind tunnel can support. Shown in each figure is the equation for its regression. Each regression has a small leading term, but this is especially true for the cubic and quartic regressions. As the block number to Mach number relationship is not linear, it is likely quadratic in nature, and therefore the data set is best represented by the quadratic regression shown in Fig. 1. Even if the Mach number limitation is only approached by block number 600, it is possible that the data may be somewhat inaccurate unless given more time to stabilize.

V. Appendix

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%% Experimental Aerodynamics II: Supersonic Wind Tunnel Block Calibration

%% Load in data
calibs = {'400','600','800','1000','1200','1400', ...
    '1600','1800','2000','2200','2400','2600'};

% Constants and variables
gamma = 1.4;
Patm = 1:length(calibs);
P01 = Patm;
P02 = Patm;
P1 = Patm;
isentropM = Patm;
rayleighM= Patm;

dataSet = {};
% Column definitions:
% time (s) | P_01 (psi) | P_1 (psi) | P_02 (psi) | Patm (psi) | T_01 (degree
F)
for i = 1:length(calibs)
    % Load data
    filename = sprintf('calibration_%s.txt',calibs{i}); % string
concatenation
    dataSet{i} = importdata(filename);
    thisData = dataSet{1,i}.data;
    % Getting only values of time >= 2.5s
    timeFixDex = thisData(:,1) >= 2.5; % logical array of what rows to
extract
    thisData = thisData(timeFixDex,:);

    % Do calculations
    % Pull pressures and convert from psi to Pa
    Patm(i) = mean(thisData(:,5))*6894.76;
    P01(i) = mean(thisData(:,2))*6894.76 + Patm(i);
    P1(i) = mean(thisData(:,3))*6894.76 + Patm(i);
    P02(i) = mean(thisData(:,4))*6894.76 + Patm(i);


    syms M
    isentrop = (1 + ((gamma-1)/2)*M^2)^(gamma/(gamma-1)) == P01(i)/P1(i);
    %isentrop = (1 + (1.2)*M^2)^(7/2) == 0.1037;
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```
    rayleigh = ((gamma+1)^2*M^2/(4*gamma*M^2-2*(gamma-1)))^(gamma/(gamma-
1))...
    *(1-gamma+2*gamma*M^2/(gamma+1)) == P02(i)/P1(i);

    isentropM(i) = double(vpasolve(isentrop,M,[0,5])); % guess between 0 and
5 to eliminate negative answers
    rayleighM(i) = double(vpasolve(rayleigh,M,[0,5]));
end

%% Plot
blockNumsRaw = 400:200:2600;
isentropMRaw = isentropM;
rayleighMRaw= rayleighM;


blockNums = blockNumsRaw(3:end);
isentropM = isentropM(3:end);
rayleighM = rayleighM(3:end);

% Ignoring first 2 data points

% Best fit lines
numtest = 4; % maximum degree polynomial to test
isenFit = [];
rayFit = isenFit;
xVals = linspace(min(blockNums),max(blockNums),100);
error = {};
percDiff = error;
meanError = error;
meanPercDiff = error;
h = [];
for k = 2:numtest
    % Polynomial fitting
    isenFit{k} = polyfit(blockNums, isentropM, k);
    rayFit{k} = polyfit(blockNums, rayleighM, k);
    isenVals = polyval(isenFit{k}, xVals);
    rayVals = polyval(rayFit{k}, xVals);
    error{k} = abs(isenVals - rayVals);

    h{k - 1} = figure(k - 1);
    hold on
    grid on
    plot(blockNumsRaw, isentropMRaw, 'bo','linewidth',1.25,'markersize',5); %
isentropic data
    plot(blockNumsRaw, rayleighMRaw, 'ro','linewidth',1.25,'markersize',5); %
rayleigh data
    errorbar(xVals, isenVals, error{k},'capsize',0);
    errorbar(xVals, rayVals, error{k},'capsize',0);
    plot(xVals,isenVals,'b') % isentropic fit
    plot(xVals,rayVals,'r') % rayleigh fit
    legend('Isentropic Data','Rayleigh Data');
    xlabel('Block Number');
    ylabel('M_\infty');

    meanError{k} = mean(error{k});
    percDiff{k} = error{k}./(0.5*isenVals(k)+rayVals(k)).*100;
```

```
    meanPercDiff{k} = mean(percDiff{k});
end
grid on
xlabel('Block Number');
ylabel('M_\infty');

% Polynomials on plots
figure(1)
 % power of 2
icap1 = sprintf('y = %.2s*x^2 + %.2s*x + %.2s',isenFit{2}(1),isenFit{2}(2),
isenFit{2}(3));
rcap1 = sprintf('y = %.2s*x^2 + %.2s*x +
%.2s',rayFit{2}(1),rayFit{2}(2),rayFit{2}(3));
text(50,1.35,icap1,'color','b')
text(50,1.15,rcap1,'color','r')
title('Quadratic Fit')

figure(2)
 % power of 3
icap2 = sprintf('y = %.2s*x^3 + %.2s*x^2 +%.2s*x +
%.2s',isenFit{3}(1),isenFit{3}(2),isenFit{3}(3),isenFit{3}(4));
rcap2 = sprintf('y = %.2s*x^3 + %.2s*x^2 +%.2s*x +
%.2s',rayFit{3}(1),rayFit{3}(2),rayFit{3}(3),rayFit{3}(4));
text(50,1.35,icap2,'color','b')
text(50,1.15,rcap2,'color','r')
title('Cubic Fit')

figure(3)
 % power of 4
icap3 = sprintf('y = %.2s*x^4 + %.2s*x^3 + %.2s*x^2 +%.2s*x +
%.2s',isenFit{4}(1),isenFit{4}(2),isenFit{4}(3),isenFit{4}(4),isenFit{4}(5));
rcap3 = sprintf('y = %.2s*x^4 + %.2s*x^3 + %.2s*x^2 +%.2s*x +
%.2s',rayFit{4}(1),rayFit{4}(2),rayFit{4}(3),rayFit{4}(4),rayFit{4}(5));
text(50,1.35,icap3,'color','b')
text(50,1.15,rcap3,'color','r')
title('Quartic Fit')
```

VI. References

[1]Narsipur, Shreyas. "MAE 352 – Experimental Aerodynamics II General Information and Lab 1 (Supersonic Wind Tunnel Block Calibration)". *NCSU,* January 2, 2019.