

DATA 556

Question 5 HW 1 - A copy machine is used to make n pages of copies per day. The machine has two trays in which paper gets loaded, and each page is taken randomly and independently from one of the other trays. At the beginning of the day, the trays are refilled so that they each have m pages. Using simulations in R, find the smallest value of m for which there is at least a 95% chance that both trays have enough paper on a particular day, for $n = 10$, $n = 100$, $n = 1000$, and $n = 10000$.

I will start by creating a function to measure the probability of success:

```
#setting a seed so its reproducible
set.seed(10284)

#function for success
success <- function(n, m){
  #labeling the number of simulations
  nsims=100000
  #create a record of the simulations
  records = rep(0,nsims)
  for(i in 1:nsims){
    #using sample takes a sample of size n from either 0 or 1
    left <- sum(sample(c(0,1), size = n, replace = TRUE))
    #right side is the ones pulled when left is not pulled
    right <- n-left
    #take only when left and right are above m
    records[i] <- (left <= m & right <= m)
  }
  #take an average of the records
  mean(records)
}
```

Now I will look at $n = 10$ and play around with the m until I get a probability over .95. We find that for $n = 10$ the smallest m with probability of at least .95 is $m = 8$.

```
#trying with different m's until we find the one that is over .95 but the
#smallest possible
success(10, 1)
```

```
## [1] 0
```

```
success(10,5)
```

```
## [1] 0.24632
```

```
success(10,7)
```

```
## [1] 0.891
```

```
success(10,8)
```

```
## [1] 0.97811
```

We continue with $n = 100$ and find that $m = 60$ is the smallest m to satisfy our requirements.

```
success(100, 50)
```

```
## [1] 0.07864
```

```
success(100,75)
```

```
## [1] 1
```

```
success(100,60)
```

```
## [1] 0.96589
```

```
success(100, 59)
```

```
## [1] 0.94344
```

Looking at $n = 1000$ we find that $m = 531$ is the smallest m with a probability of 95% or over.

```
success(1000, 500)
```

```
## [1] 0.02461
```

```
success(1000,750)
```

```
## [1] 1
```

```
success(1000,600)
```

```
## [1] 1
```

```
success(1000, 550)
```

```
## [1] 0.99856
```

```
success(1000, 520)
```

```
## [1] 0.80426
```

```
success(1000, 530)
```

```
## [1] 0.94571
```

```
success(1000, 531)
```

```
## [1] 0.95296
```

Doing this with $n = 10,000$ and finding that $m = 5098$.

```
success(10000, 5000)
```

```
## [1] 0.00812
```

```
success(10000, 6000)
```

```
## [1] 1
```

```
success(10000, 5500)
```

```
## [1] 1
```

```
success(10000, 5200)
```

```
## [1] 0.99997
```

```
success(10000, 5100)
```

```
## [1] 0.95554
```

```
success(10000, 5099)
```

```
## [1] 0.95333
```

```
success(10000, 5098)
```

```
## [1] 0.95157
```

```
success(10000, 5097)
```

```
## [1] 0.94768
```