COMS W3261
Computer Science Theory
Chapter 11 Notes

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Additional Classes of Problems

Complements of Languages in $\mathcal{NP}$

The class of languages $\mathcal{P}$ is closed under complementation. It is not known whether $\mathcal{NP}$ is closed under complementation.

The Class of Lagnuages Co-$\mathcal{NP}$

Co-$\mathcal{NP}$ is the set of languages whose complements are in $\mathcal{NP}$Every language $\mathcal{P}$ has its complement also in $\mathcal{P}$, and therefore in $\mathcal{NP}$. We believe that none of the NP- Complete problem have their complements in $\mathcal{NP}$, and therefore no NP- Complete problem is in Co-$\mathcal{NP}$. Likewise, we believe the complements of NP-Complete problems, which are by definition in Co-$\mathcal{NP}$, are not in $\mathcal{NP}$.

NP-Complete Problems and Co-$\mathcal{NP}$

Let us assume that $\mathcal{P} \neq \mathcal{NP}$. We could have $\mathcal{NP}$ and co-$\mathcal{NP}$ are equal, but larger than $\mathcal{P}$.

Theorem 1. $\mathcal{NP}$ = Co-$\mathcal{NP}$ if an only if there is some NP-complete problem whose complement is in $\mathcal{NP}$.