Assignment Module 2 - The LP Model Problem and Solution

Problem 1:

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000-square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provide 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. Clearly define the decision variables
b. What is the objective function?
c. What are the constraints?
d. Write down the full mathematical formulation for this LP problem.

Solution:

a. Decision Variables:
   Let x be the total number of collegiate to produce per week
   Let y be the total number of mini to produce per week

   Therefore, the two decision variables will be x (how many collegiate to produce per week) and y (how many mini to produce per week)

b. Objective function:
   Here the objective is to produce the maximum number of each type of backpack to obtain the maximum profit possible. $32 profit is generated for each collegiate and $24 profit is generated for each mini. Therefore, the objective function(Z) will be:

$$Z = 32x + 24y$$

c. Constraints:
   Below are all the constraints:

   1. Material (Nylon) constraint:
      The company receives a shipment of 5000 square feet of nylon each week, 3 square feet is required by each collegiate, and 2 square feet is required by each mini. Therefore, the material constraint will be:

$$3x + 2y \leq 5000$$

   2. Labor constraint:
      45 minutes of labor is required to produce each collegiate i.e., 45/60 = 3/4 hour
      40 minutes of labor is required to produce each mini i.e., 40/60 = 2/3 hour
      The company has 35 laborers that can work 40 hours per week i.e., 35 x 40 = 1400
      Therefore, the labor constraint will be:

$$(3/4)\, x + (2/3)\, y \leq 1400$$

   3. Sales constraint:
      As per the sales forecast, 1000 collegiate and 1200 mini only be sold per week. Therefore, the sales forecast constraint will be:

$$x \leq 1000$$
$$y \leq 1200$$

4. Non-negativity constraint:
   Also, the number of collegiate and mini can never be a negative value, it can be either 0 or greater than 0. Therefore, the non-negativity constraint will be:

$$x \geq 0$$
$$y \geq 0$$

d. After considering all the above constraints, the linear programming model for this problem will be
   Let x be the total number of collegiate to produce per week
   Let y be the total number of minis to produce per week

$$\text{Maximize } Z = 32x + 24y$$

subject to

$$3x + 2y \leq 5000$$
$$(3/4)x + (2/3)y \leq 1400$$
$$x \leq 1000$$
$$y \leq 1200$$
$$x \geq 0$$
$$y \geq 0$$

Problem 2:

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced
by each of the plants to maximize profit.
a. Define the decision variables
b. Formulate a linear programming model for this problem.

Solution:

a. Decision variables:
   Let x_{11} be the number of large-sized products produced in Plant 1
   x_{12} be the number of medium-sized products produced in Plant 1
   x_{13} be the number of small-sized products produced in Plant 1
   y_{11} be the number of large-sized products produced in Plant 2
   y_{12} be the number of medium-sized products produced in Plant 2
   y_{13} be the number of small-sized products produced in Plant 2
   z_{11} be the number of large-sized products produced in Plant 3
   z_{12} be the number of medium-sized products produced in Plant 3
   z_{13} be the number of small-sized products produced in Plant 3

Objective function:

Here the objective is to maximize the profit. For each plant, a large-sized product yields a profit of $420, a medium-sized product yields a profit of $360, and a small product yields a profit of $300.

Therefore, the objective function(Z) to maximize the profit will be:

$Z = 420x_{11} + 360x_{12} + 300x_{13} + 420y_{11} + 360y_{12} + 300y_{13} + 420z_{11} + 360z_{12} + 300z_{13}$

Constraints:

1. Plant production capacity constraint:
   Plant 1 can produce 750 units per day. Plant 2 can produce 900 units per day. Plant 3 can produce 450 units per day. Therefore, the production capacity constraint will be:

   $x_{11} + x_{12} + x_{13} \leq 750$
   $y_{11} + y_{12} + y_{13} \leq 900$
   $z_{11} + z_{12} + z_{13} \leq 450$

2. Storage space constraint:
   Plant 1 has 13000 square feet, Plant 2 has 12000 square feet, and Plant 3 has 5000 square feet of in-process storage space available per day for production. Also, each unit produced per day for large, medium, and small-size products requires 20, 15, and 12 square feet respectively. Therefore, the in-process storage space constraint will be:

   $20x_{11} + 15x_{12} + 12x_{13} \leq 13000$
   $20y_{11} + 15y_{12} + 12y_{13} \leq 12000$
   $20z_{11} + 15z_{12} + 12z_{13} \leq 5000$

3. Sales constraint:
   Also, 900 large, 1200 medium and 750 small size units per day can be sold. Therefore, the sales forecast constraint will be:

   $x_{11} + y_{11} + z_{11} \leq 900$
   $x_{12} + y_{12} + z_{12} \leq 1200$
   $x_{13} + y_{13} + z_{13} \leq 750$

4. Non-negativity constraint:
   $x_{11} \geq 0$
   $x_{12} \geq 0$
   $x_{13} \geq 0$
   $y_{11} \geq 0$
   $y_{12} \geq 0$
   $y_{13} \geq 0$
   $z_{11} \geq 0$
   $z_{12} \geq 0$
   $z_{13} \geq 0$

b. After considering all the above constraints, the linear programming model for this problem will be
   Let x_{11} be the number of large-sized products produced in Plant 1
   x_{12} be the number of medium-sized products produced in Plant 1
   x_{13} be the number of small-sized products produced in Plant 1
   y_{11} be the number of large-sized products produced in Plant 2
   y_{12} be the number of medium-sized products produced in Plant 2

y_{13} be the number of small-sized products produced in Plant 2
z_{11} be the number of large-sized products produced in Plant 3
z_{12} be the number of medium-sized products produced in Plant 3
z_{13} be the number of small-sized products produced in Plant 3

Maximize $Z = 420x_{11} + 360x_{12} + 300x_{13} + 420y_{11} + 360y_{12} + 300y_{13} + 420z_{11} + 360z_{12} + 300z_{13}$

subject to

$x_{11} + x_{12} + x_{13} \leq 750$
$y_{11} + y_{12} + y_{13} \leq 900$
$z_{11} + z_{12} + z_{13} \leq 450$
$20x_{11} + 15x_{12} + 12x_{13} \leq 13000$
$20y_{11} + 15y_{12} + 12y_{13} \leq 12000$
$20z_{11} + 15z_{12} + 12z_{13} \leq 5000$
$x_{11} + y_{11} + z_{11} \leq 900$
$x_{12} + y_{12} + z_{12} \leq 1200$
$x_{13} + y_{13} + z_{13} \leq 750$
$x_{11} \geq 0$
$x_{12} \geq 0$
$x_{13} \geq 0$
$y_{11} \geq 0$
$y_{12} \geq 0$
$y_{13} \geq 0$
$z_{11} \geq 0$
$z_{12} \geq 0$
$z_{13} \geq 0$