

# Deep Learning Methods for NLP

Machine Learning for Natural Language Processing, ENSAE 2022

Lecture 3

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# Lectures Outline

1. The Basics of Natural Language Processing (February 1st)
2. Representing Text with Vectors (February 1st)
- 3. Deep Learning Methods for NLP (February 8th)**
4. Language Modeling (February 8th)
5. Sequence Labelling (Sequence Classification) (February 15th)
6. Sequence Generation Tasks (February 15th)

# Today Lecture Outline

- Deep Learning Framework
- The Multi-Layer Perceptron
- Recurrent Neural Network
- Attention Mechanism
- Self-Attention Mechanism and the Transformer Architecture

# Motivations

So far, we have seen, **techniques to represent tokens with vectors**

Given a certain representations of tokens:

➔ **How can we model a sequence of tokens to perform a specific task?**

In the past 10 years, a “new” class of machine learning techniques has become very popular and successful in NLP: **Deep Learning**

*In this session, we introduce Deep Learning with a focus on the methods used in NLP*

# Framework

We want to model  $(X_1, \dots, X_T)$  i.e. find the correct label  $Y$

$$dnn_{\theta} : \mathbb{R}^{d,T} \rightarrow \mathbb{R}^p \text{ or } [0, K]^p$$

$$(X_1, \dots, X_T) \mapsto \hat{Y}$$

- Output space is  $\mathbb{R}^p$  for **Regression** tasks
- Output space is  $[0, K]^p$  for **Classification** tasks

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Questions: **when we do Deep Learning...**

- How do we **define**  $dnn_{\theta}$  ?
- How do we **train**  $dnn_{\theta}$  with data ?

# Framework

Given a sequence of vectors  $(X_1, \dots, X_T)$  we want to predict  $Y$

$$dnn_{\theta} : \mathbb{R}^{d,T} \rightarrow \mathbb{R}^p \text{ or } [0, K]^p$$

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Most Deep Learning Models (all the ones we will use in this course):

- are **parametric** (i.e.  $\theta \in \mathbb{R}^D$  )
- defined as a **composition of “simple” functions (linear & non-linear)**
- are trained in an **end-to-end** fashion with **backpropagation**

NB: In Deep Learning, **the parametrization of  $dnn$**  is called **the Architecture**

# Different Types of Architecture

**How can we define** our predictive function  $dnn_{\theta}$  ?

- Multi-Layer Perceptron
- Recurrent Layers
- Attention Layers
- Self-Attention Layers (in a Transformer Architecture)



# Different Types of Architecture

How can we define our predictive function  $dnn_{\theta}$  ?

- Multi-Layer Perceptron
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How do we **train our model**? (i.e. estimate the parameters of the model)

- **Stochastic Gradient Descent** also called **backpropagation** in this context

# The MultiLayer Perceptron (MLP)

*aka "the Most simple Deep Learning Architecture"*

The **MLP** works **on unidimensional data** (e.g. dimension  $d$ )

We present the **MLP in the regression case** (e.g. output space is  $\mathbb{R}^2$  ))

$$\begin{aligned} dnn_{\theta} : \quad \mathbb{R}^d &\rightarrow \mathbb{R}^2 \\ X &\mapsto \hat{Y} \end{aligned}$$

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$$dnn_{(W_1, b_1, W_2, b_2)}(X) = W_2 \varphi_1(W_1 X + b_1) + b_2$$

$W_1, b_1, W_2$  and  $b_2$  are trainable parameters.  $W_1 \in \mathbb{R}^{\delta \times d}$ ,  $b_1 \in \mathbb{R}^\delta$ ,  $W_2 \in \mathbb{R}^{2 \times \delta}$  and  $b_2 \in \mathbb{R}$

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- This model is a **2-layer MLP** model
- With **1 hidden layer** of dimension  $\delta$
- Taking as input a vector of **dimension  $d$**  to output a vector of **dimension 2**
- Such a model is also referred to as a **Feed-Forward Neural Network (FNN)**

# The MultiLayer Perceptron: Diagram View

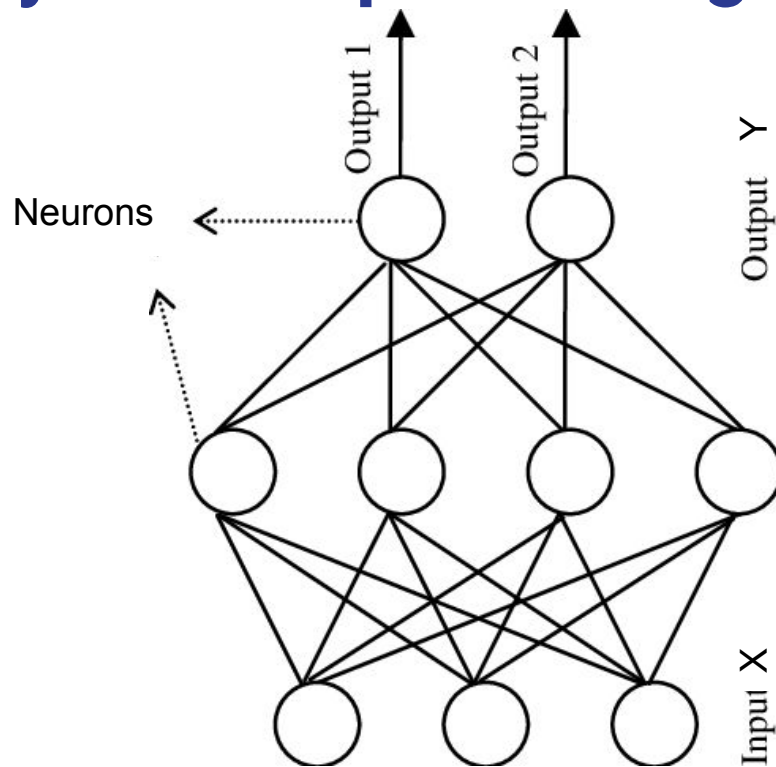
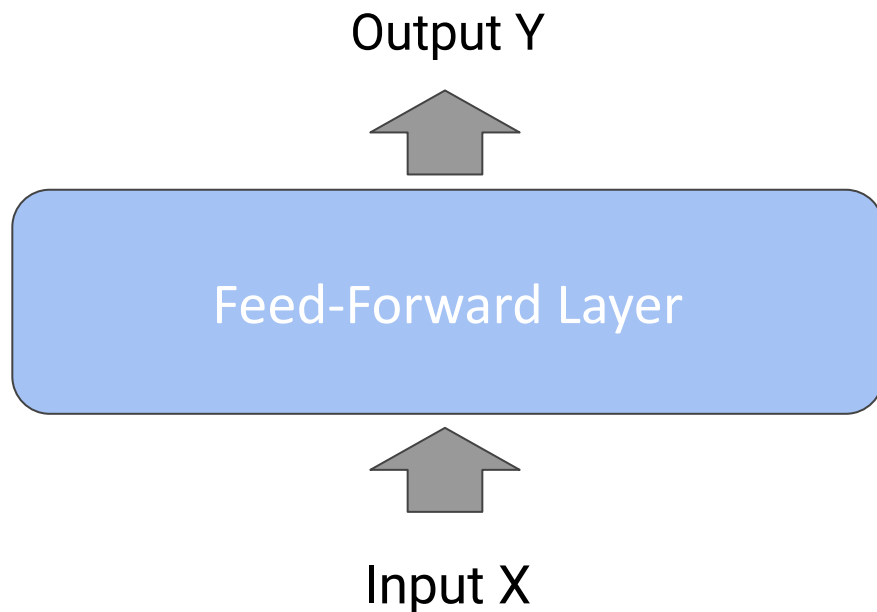


Figure from (R. Rezvani et. al. 2012)

In Deep Learning, it is usual to represent equations **with diagrams**



# The MultiLayer Perceptron: Diagram View



In Deep Learning, it is usual to represent equations **with diagrams**

# The MultiLayer Perceptron:

We have defined a 2-layers MLP model

We can define in the same way a **3-layers**, **4-layers**, **L-layers** MLP

$$dnn_{(W_i, b_i, i \in [1, L])}(X) = W_L \varphi_{L-1}(\dots \varphi_2 \circ W_2 \varphi_1(W_1 X + b_1) + b_2) \dots) + b_L$$

$W_l$  and  $b_l$  are trainable parameters.  $W_l \in \mathbb{R}^{\delta_{l-1} \times \delta_l}$ ,  $b_l \in \mathbb{R}^{\delta_l}$ , with  $\delta_l \in \mathbb{N}^*$ ,  $\forall l \in [1, L]$

$\varphi_l$  fixed non-linear functions,  $\varphi_l : \mathbb{R}^{\delta_{l-1}} \rightarrow \mathbb{R}^{\delta_l}$ ,  $\forall l \in [1, L-1]$

# The MultiLayer Perceptron

The same equation with a loop...

$$h_{i+1} = \varphi_i(W_i h_i + b_i), \forall i \in [1, L-1]$$

with  $h_1 = X$  and  $\hat{Y} = dnn(X) = h_L$

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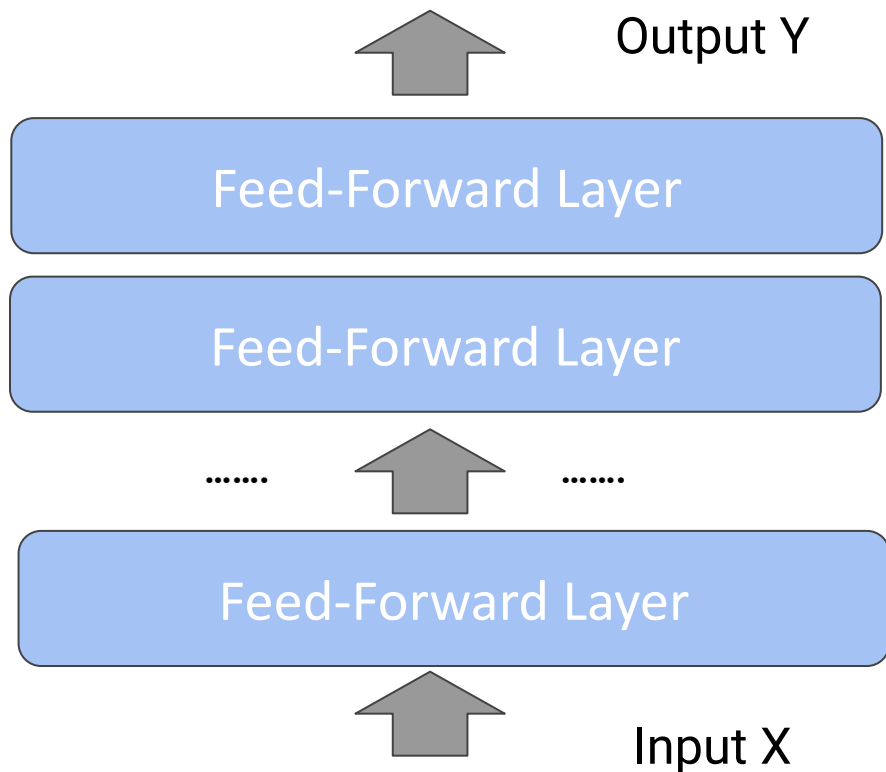
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$h_i$  are called hidden states ( $h_i \in \mathbb{R}^{\delta_i}$ ).

# The MultiLayer Perceptron: Diagram View



# Output Activation Function for Classification

When we do a classification task the goal is to learn a distribution of probability on the output label space

To do so, **we usually use the softmax function** as the last activation function

$$\text{softmax}(s) = \left( \frac{e^{s_i}}{\sum_k e^{s_k}} \right)_{i \in [1, K]}, \text{ for } s \in \mathbb{R}^K$$

# Loss Functions

Based on the task we aim at modeling, we can use:

**For Regression: Mean-Square Error**

$$l(y, \hat{y}) = \|y - \hat{y}\|_2^2 = \sum_i (y_i - \hat{y}_i)^2 \text{ assuming } y_i, \hat{y}_i \in \mathbb{R}$$

**For Classification: Cross-Entropy Loss**

$$l(y, \hat{y}) = CE(y, \hat{y}) = \sum_i y_i \log(\hat{y}_i) \text{ assuming } y_i, \hat{y}_i \in [0, 1]$$

Most NLP tasks will be based on the **Cross-Entropy loss**

# The MultiLayer Perceptron: Hyperparameters

- Number of **hidden layers**
- Hidden layers **dimensions**
- Initialization of the trainable parameters/weights



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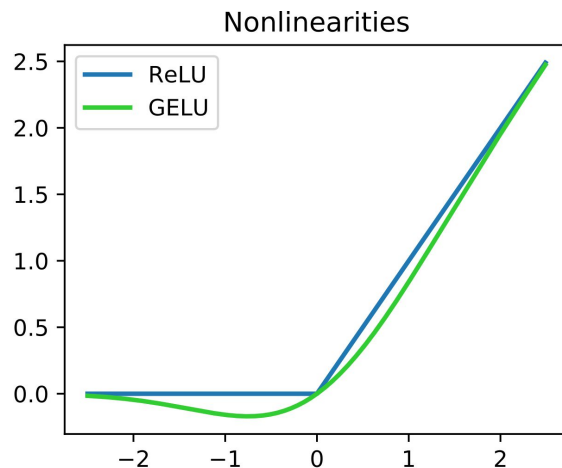
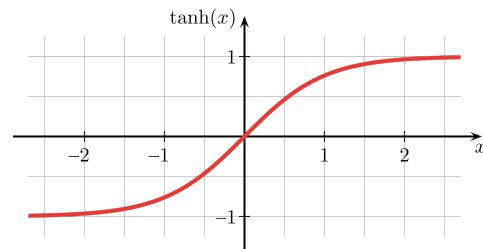
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  - They should be **non-linear**
  - **Differentiable**
  - **Standard ones are:**  
*Relu, tanh, sigmoid*

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# The MultiLayer Perceptron: Hyperparameters

- Number of hidden layers
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- Initialization
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## How to define them?

- Look for **best practices** to choose which are the best
- In most DL libraries, the **“good” hyperparameters are usually the default**
- If no best practices/default: **you have to find the best ones empirically**

# Intuition

*playground*

# Training Deep Learning Models

- Nearly all Deep Learning models are trained with (some version of) **Stochastic Gradient Descent (SGD)**

## Stochastic Gradient Descent

- The goal is find the set of **parameters/weights** that **minimizes the loss function**
- To do so, SGD estimates the true gradient of a function with **one sample at time**
- **Repeat** this process multiple times

**NB:** in deep learning, we usually train all the parameters together **“end-to-end”**

# Stochastic Gradient Descent

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## Algorithm 2 Stochastic Gradient Descent

---

Given observations  $((x_i), (y_i))$  of two variables  $(X, Y)$

Given a loss function  $l$ . An architecture  $dnn_{\theta}$

**The goal is to find the best  $\theta$  s.t.  $E(l(Y, dnn_{\theta}(X)))$  is small.** Given a learning rate  $\alpha$

**for**  $step < max$  **do**

    Sample  $(x, y)$

    # Forward pass:

$\hat{y} = dnn_{\theta}(x)$  and  $l(y, \hat{y})$

    # Backward pass:

$\nabla_{\theta} l(y, \hat{y})$  # compute loss

$\theta := \theta - \alpha \nabla_{\theta} l(y, \hat{y})$  # parameter update

**end**

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    # Backward pass:

$\nabla_{\theta} l(y, \hat{y})$  # compute gradients

$\theta := \theta - \alpha \nabla_{\theta} l(y, \hat{y})$  # parameter update

**end**

---

# Stochastic Gradient Descent

## Optimization Hyperparameters

### Learning Rate

- Can be refined with **variable learning rate**  
*E.g. increasing during the first steps (**warmup**) then decreasing*

### Number of steps

- Usually defined with based on the validation loss  
*When it stops decreasing we can stop training (= **early stopping**)*

# Stochastic Gradient Descent

Optimizing large Deep Learning Models **is challenging**

- **Unstable training**
- **Overfitting**
- **Take a lot of steps/epochs**

**To make training better, many refinement of the SGD have been proposed**

- In practice, we (nowadays) **use the ADAM optimizer**  
(cf. Kingma et. al 2015)

# Stochastic Gradient Descent for MLP

Let  $(X, Y) \in \mathbb{R}^d \times \mathbb{R}$ , the MSE loss  $l(y, \hat{y}) = (y - \hat{y})^2$ .

We define a 1-hidden-layer MLP with a RELU activation function of dimension  $\delta$ .

$$\hat{y} = \text{nn}_{W_1, W_2}(x) = W_2 \max(W_1 x, 0) \text{ and } W_1 \in \mathbb{R}^{d \times \delta} \text{ and } W_2 \in \mathbb{R}^{1 \times \delta}$$

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→ **Goal: Apply SGD to  $dnn$**



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**1. Forward pass: Compute  $\hat{y}$**

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1. Forward pass
2. Compute Gradients

$$\nabla_{W_1} l(y, \hat{y}) \quad \nabla_{W_2} l(y, \hat{y})$$

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1. **Forward pass**
2. **Compute Gradients**
3. **Backward pass (parameter update)**

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Idea: we use **the chain rule** to decompose **the gradient** starting from the **top layers**

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$$\nabla_{W_2} l(y, \hat{y}) = \frac{\partial l}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial W_2} = 2(y - \hat{y}) h_1$$

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$$\nabla_{W_1} l(y, \hat{y}) = \frac{\partial l}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h_1} \frac{\partial h_1}{\partial W_1} = 2(y - \hat{y}) W_2 1_{W_1 X > 0}$$



# Backpropagation and Deep Learning in practice

In practice, we use Deep Learning Libraries

- Define **the Architecture with *tensor* operators**
- Backpropagation is done **seamlessly using automatic differentiation**

# Deep Learning & Backpropagation in practice

In practice, we use Deep Learning Libraries (e.g. pytorch, tensorflow, jax)

- Define **the Architecture with *tensor* operators**
- Backpropagation is done **seamlessly using automatic differentiation**
- Standard layers **are pre-implemented** (Feed-Forward Layers, LSTM, Attention, Self-Attention...)

[See code example with pytorch](#)

# Recurrent Neural Network

# Vanilla Recurrent Neural Network

We would like to model sequences (e.g. words)  $(X_1, \dots, X_T)$  in  $\mathbb{R}^{d,T}$

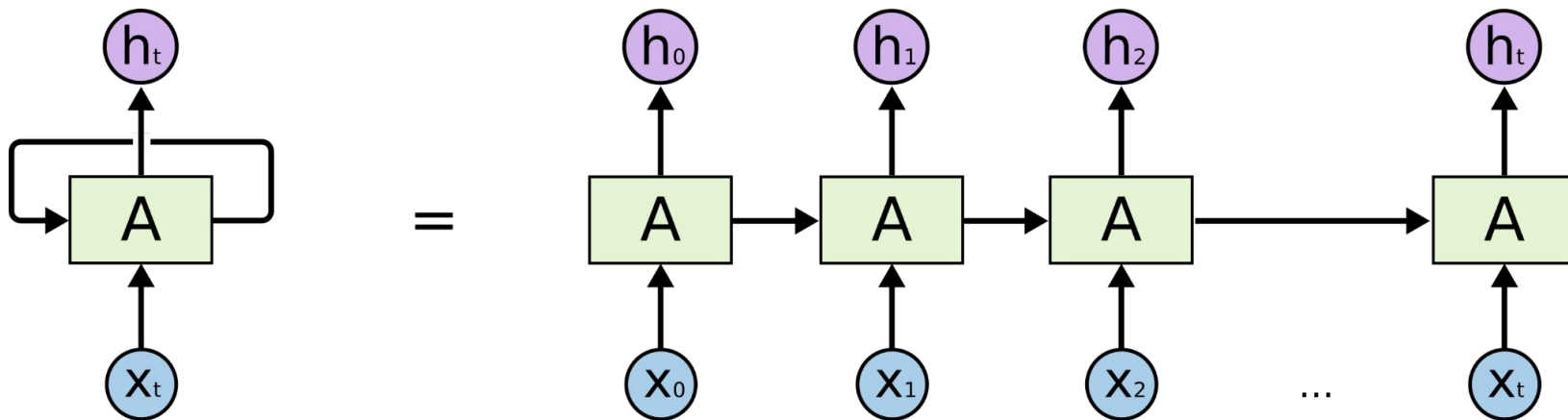
We can introduce **a recurrence relation** into our MLP to model it:

$$h_{i+1,t+1} = \varphi_i(W_i h_{i,t} + U_i h_{i+1,t} + b_i), \forall i \in [|1, L - 1|]$$

with  $h_{1,t} = X_t$  and  $\hat{Y}_t = \text{dnn}(X_t) = h_{L,t} \forall t \in [|1, T - 1|]$

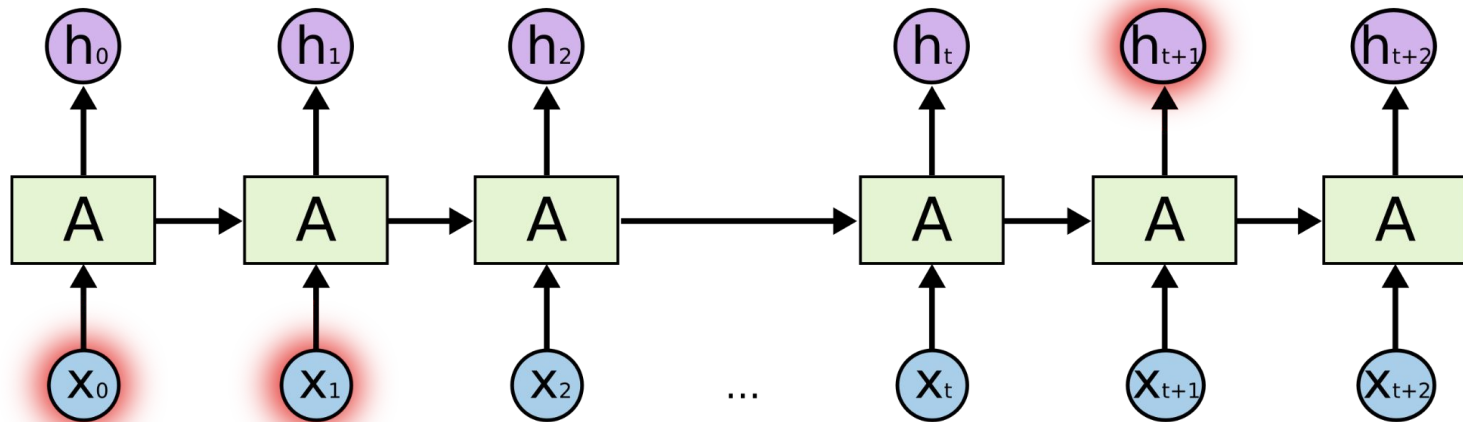
# Recurrent Neural Network

## Illustration of a 1-layer Recurrent Neural Network



# Recurrent Neural Network

## Illustration of a 1-layer Recurrent Neural Network



# Training Recurrent Neural Network

Recurrent Neural Network are trained with an extension of the Backpropagation algorithm

→ Backpropagation Through Time (BPTT)

BPTT follows exactly the same ideas as backpropagation

- SGD
- Chain Rule starting from the last layer and the last hidden state
- **With extra derivative dependencies between state  $t$  and  $t+1$**

# Limits of Recurrent Neural Networks

Vanilla Recurrent Neural Network have trouble to capture long-term dependencies

Idea:

- Encode **explicitly in a vector a “memory” in the recurrent architecture**
- Control what is memorized and forgotten
- Train all those parameters **end-to-end**



# LSTM: Long-Short Term Memory Network

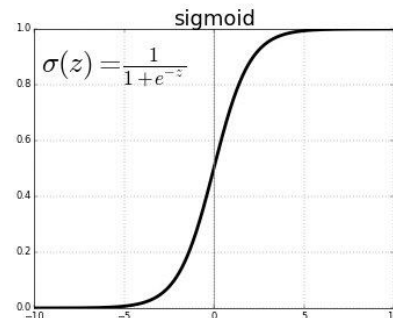
Introduce a memory vector  $C_t$

$C_t$  is designed to **capture long term dependencies**

The output state  $h_t$  of each LSTM cell is based on  $C_t$  and an **output gate**  $o_t$

$$o_t = \sigma(W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$



# LSTM: Long-Short Term Memory Network

Introduce a memory vector  $C_t$

$C_t$  is designed to **capture long term dependencies**

$C_t$  is define recurrently based on the previous step and the input and the forget gate. Those gates control what is memorized and forgotten.

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$i_t = \sigma (W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

# LSTM: Long-Short Term Memory Network

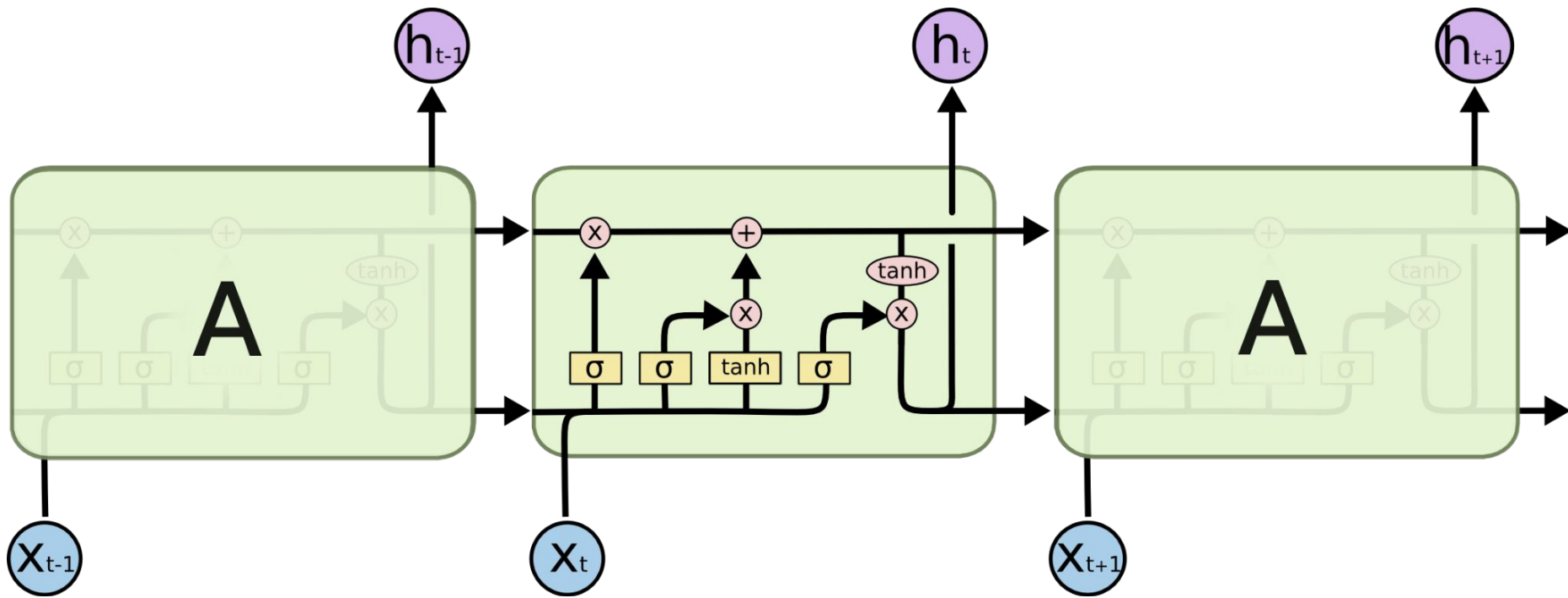


Figure from [colah](#)

# LSTM: Long-Short Term Memory Network

- We train LSTM with Backpropagation (through time)
- LSTM cells are usually combined with Feed-Forward Layers

**NB:** Until recently (2018), LSTM-based models were delivering **State-of-the-art performance for most sequence modelling tasks**

# Attention Mechanism

## Motivation for Attention Mechanisms

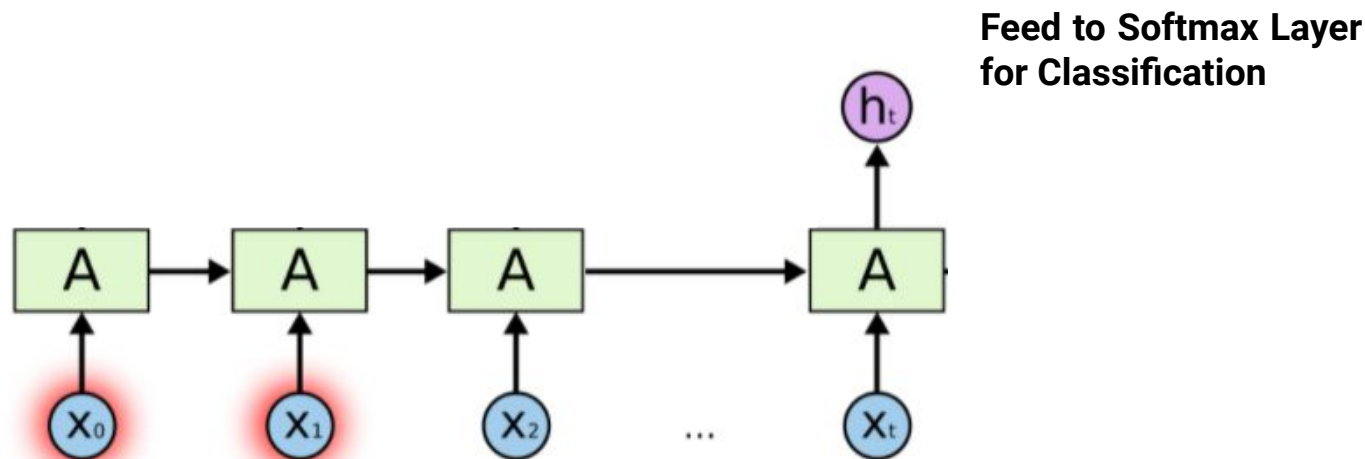
- The Deep Learning Architecture that we have seen so far are **hard to interpret (black-box)**
- Recurrent Network provide a fixed vector encoding of a sequence at each step

→ **Attention Mechanisms**

# Attention Mechanism for Sequence Classification

We want to classify  $(X_0, X_t)$  sequences (e.g. sentiment analysis)

**Solution 1:** Use a LSTM model  $\rightarrow$  Problem (not interpretable)

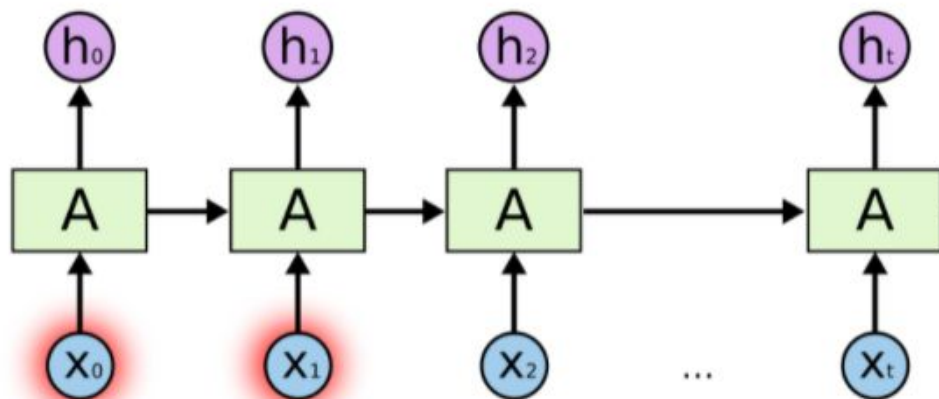


# Attention Mechanism for Sequence Classification

We want to classify  $(X_0, X_t)$  sequences (e.g. sentiment)

**Solution 2:** Integrate an Attention Mechanism to interpret what input impacts the prediction

→ Learn a ponderation/weighting of the hidden states  $h_t$



# Attention Mechanism for Sequence Classification

We want to classify  $(X_0, X_t)$  sequences (e.g. sentiment)

**How to learn this weighting?**

1. Define a specific type of layer to learn the ponderation
2. Train this layer end-to-end with all the other parameters of the model



# Attention Mechanism for Sequence Classification

We want to classify  $(x_0, x_t)$  sequences (e.g. sentiment)

How to learn this weighting?

Given  $(h_1, \dots, h_T)$  hidden representations of  $(x_1, \dots, x_T)$  (e.g. output of a LSTM Layer).

$$q_i = \tanh(W_a h_i + b_a), \text{ with } W_a \in \mathbb{R}^{\delta \times \delta_a}$$

$$s_t = \frac{e^{q_t q_T}}{\sum_j e^{q_j q_T}}, \text{ i.e. } \sum_{t \in [1, T]} s_t = 1$$

$$\tilde{h}_T = \sum_{t \in [1, T]} s_t \cdot h_t$$

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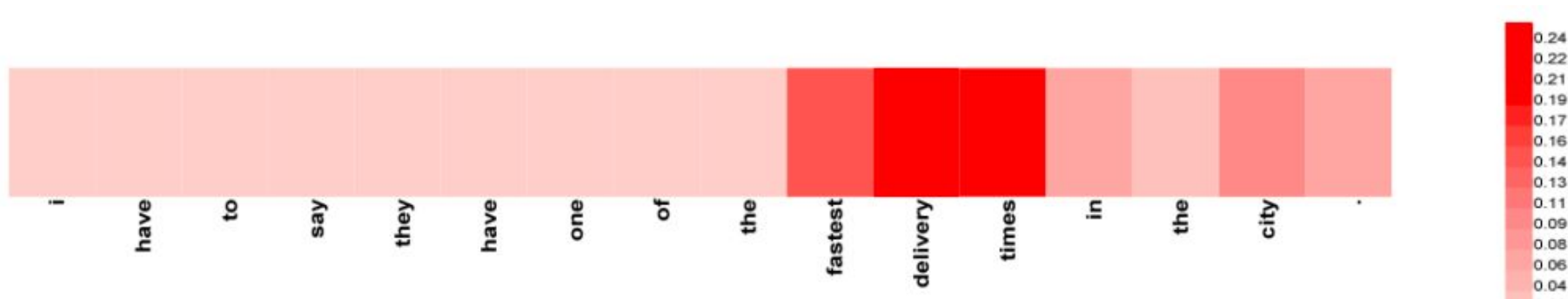
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# Attention Mechanism for Sequence Classification

We want to classify  $(X_0, X_t)$  sequences (e.g. sentiment classification)

After we trained the model, **Attention scores** can be used to **interpret the model** behavior and **what input vector impacted the decision**



(Wang et. al 2016)

# Attention Mechanism for Sequence Classification

Many variant of Attention Mechanisms (in combination with LSTM layers) have been designed

## Design Choices

- How to define the *query vectors*?
- How to define the *scoring function*?

Many variants exists but the principles are the same.

# The Transformer Architecture

# *Attention might be all we need*

## Do we really need recurrent layers?

RNN models (such as vanilla RNN, LSTM...) were designed to model sequential data

Still, for most tasks, we **need both left and right context** (e.g. **sequence classification, sequence labelling..**)

Why not modelling sequences in a bi-directional way directly

→ **Using Self-Attention Mechanism**

# Self-Attention Layers

Given a sequence of input vectors  $(x_1, \dots, x_T) \in \mathbb{R}^\delta$  (noted  $(h_{0,1}, \dots, h_{0,T})$ ).

## Objective:

- Build a representation of the input vectors based on the **surrounding vectors** (both right-and left-context)

## Idea:

- No need of recurrent cells

→ **Self-Attention**



# Self-Attention Layers: Intuition

Given a sequence of input vectors  $X = (x_1, \dots, x_T) \in \mathbb{R}^\delta$  (noted  $H = (h_{0,1}, \dots, h_{0,T})$ )

We build 3 new vectorial representation of our sequence  $H = (h_1, \dots, h_T)$ .

The *query*  $Q = (q_1, \dots, q_T)$ , the *key*  $K = (k_1, \dots, k_T)$  and the *value*  $V = (v_1, \dots, v_T)$  vectors.

- For a given vector  $h_t$  and its query vector  $q_t$  we want to build the new representation vector  $\tilde{h}_t$
- Using the best ponderation of the information encoded in  $(v_1, \dots, v_T)$
- This ponderation being computed by finding the key vectors in  $(k_1, \dots, k_T)$  that are more similar to the query vector  $q_t$  (that encodes relevant information from  $h_t$ ).

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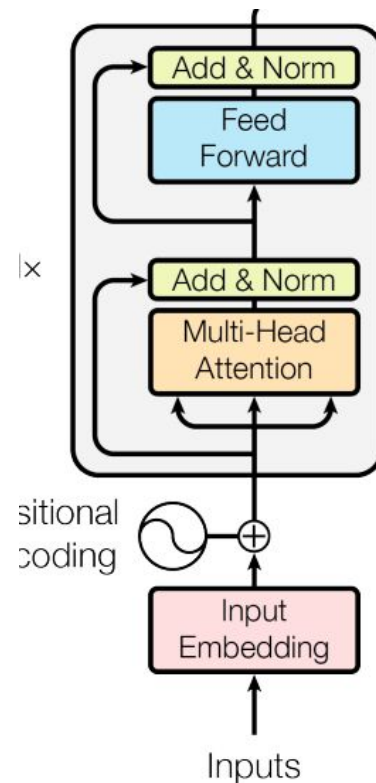
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# The Transformer Architecture

## The Transformer Architecture is

- Stack of [Self-Attention + FF Layer]
- With Skip-Layer and Normalization between Layers
- Encoding the position with positional vector



# Positional Embedding Vector

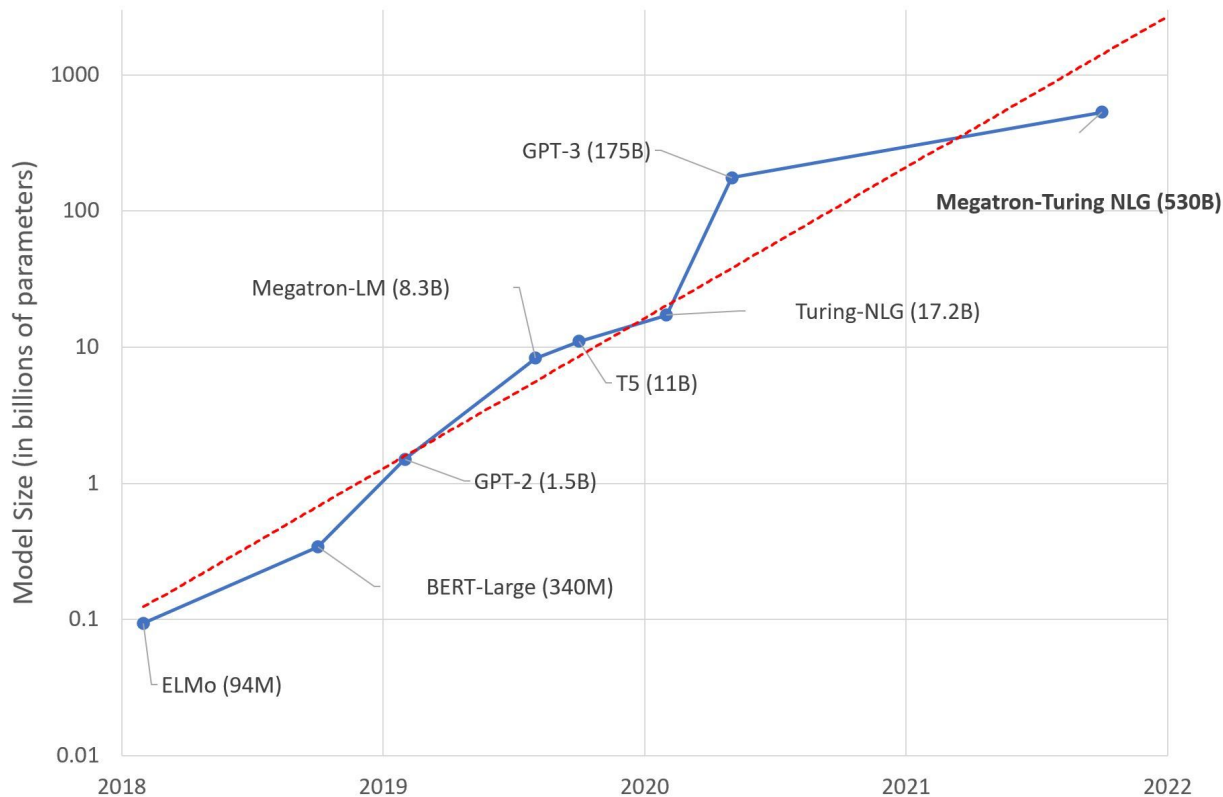
- **Limitation:** self attention does not take position into account!
- Indeed, shuffling the input gives the same results
- **Solution:** add position encodings.
- Replace the matrix  $\mathbf{W}$  by  $\mathbf{W} + \mathbf{E}$ , where  $\mathbf{E} \in \mathbb{R}^{d \times T}$
- $\mathbf{E}$  can be learned, or defined using sin and cos:

$$e_{2i,j} = \sin\left(\frac{j}{10000^{2i/d}}\right)$$
$$e_{2i+1,j} = \cos\left(\frac{j}{10000^{2i/d}}\right)$$

# Scaling Laws Intuition

- The larger the dimension of the weight matrices
- The larger the number of parameters in the model
- The more “expressive” is the model
- The better it will generalize

# Typical Architecture Sizes



# Lecture Summary

Deep Learning is a powerful and general modelling approach

- **Designing Architectures**, i.e. composition of linear transformation and non-linear transformation (possibly including recurrences)
- All those transformations **should be differentiable**
- All the parameters of the model **are trained with backpropagation**
- **Toward a specific task** s.t. regression or classification
- All the hyperparameters are chosen based on **best-practices** or empirical research