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Question 1)

a) $(\forall s) \in D, \sim C(s) \wedge E(s)$
b) $(\exists s) \in D, C(s) \wedge M(s)$

Question 2)

Given nephew (X, Y): - son (X, Z), sibling (Z, Y) is a conditional statement, where nephew (X, Y) is only true if and only son (X, Z), sibling (Z, Y) is true

a) |?- nephew(U,jo) will return NO because there is no existed entry such as son(U, x) where x is a arbitrary variable. Also, there is a transitive relation between son and siblings, so if son is not true then siblings will be not true. Therefore, the query |?- nephew(U,jo) is false and will return NO as the result.

b) |?- nephew(tom, ann) will return YES because there is existed entry such as son(tom,paul) and siblings(paul,ann) where x = tom and y = ann and z = paul. Since both son and siblings hold a true condition, the query |?- nephew(tom, ann) is true and will return YES as the result.

Question 3)

a) Given $\sim(\exists x \in D(\exists y \in E(P(x,y)))) \equiv \forall x \in D(\forall y \in E(\sim P(x,y)))$
We have:
$\sim(\exists x \in D(\exists y \in E(P(x,y)))) \equiv \forall x \in D, \sim(\exists y \in E(P(x,y)))$ [Negation of $\exists$ is $\forall$]
$\forall x \in D, \sim(\exists y \in E(P(x,y))) \equiv \forall x \in D, (\forall y \in E (\sim(P(x,y))))$ [Negation of $\exists$ is $\forall$]
Therefore,
$\sim(\exists x \in D(\exists y \in E(P(x,y)))) \equiv \forall x \in D(\forall y \in E(\sim P(x,y)))$

b)
i. Invalid – converse error. Consider the arguments into the formal statements:
- $(\forall s) \in D$, if P(s) then Q(s); where P is the back seats and Q is the cheater.
- Monty sits in the back can be written as Q(Monty).
- Q(Monty) => P(s), which is converse to the first statement

- The third statement is the converse of the first which is not necessarily true.

ii. Invalid – inverse error. Consider the arguments into the formal statements:
  - ($\forall$s) $\in$ D, if P(s) then Q(s); where P is the honest and Q is the tax payers.
  - Darth is not honest can be written as ~P(Darth).
  - ~P(Darth) then ~Q(s), which is inverse to the first statement.
  - The third statement is the inverse of the first which is not necessarily true.

iii. Valid – universal modus ponens

iv. Valid – universal modus ponens

v. Valid – universal modus Tollens

Question 4)

a) Some primes are even: $(\exists x)\ [P(x) \wedge Q(x)]$

b) All even numbers are greater than 1. Let F(x,1) denotes "x is greater than 1"

   $(\forall x)[(x \wedge Q(x)) \rightarrow F(x,1)]$

c) There is no prime less than 3. Let T(x,3) denotes "x is not less than 3"

   $\sim(\exists x)[((x>1) \wedge P(x)) \rightarrow T(x,3)]$

Question 5)

a) In $p(x) \wedge \sim r(y,a)$ the free variables are x, a and y

b) In $\exists x.r(x,y)$ the free variable is y

Question 6)

According to the definition of odd integer = 2n + 1, where n is any integer.
According to the definition of even integer = 2n, where n is any integer.

Sum of odd and even integer = 2n + 1 + 2m = 2(n + m) + 1
Let m + n = x
Therefore, sum of odd and even integer = 2x +1, which is an odd integer.