CMPT 260 Assignment 3   Due date: 31st July 2017

1. You have 2 parents, 4 grandparents, 8 great-grandparents, and so forth. If all of your ancestors were distinct, what would be the total number of your ancestors for the past 40 generations, counting your parent's generation as number 1? Hint: What kind of sequence is this? Use the sum formula for that sequence to solve the problem. Show your work. (3 marks).

2. Give a proof by contradiction to show that there does not exist a constant c such that for all integers n≥1, $(n+1)^2-n^2<c$ (3 marks)

3. Given the fact that $\lceil x \rceil < x + 1$, give a proof by contradiction that if n items are placed in m boxes then at least one box must contain at least ceiling(n/m) items. (3 marks)

4. Use mathematical induction to prove the following statement is true for all integers n≥2. Clearly identify the base case, the induction hypothesis and the induction step you are using in your proof. (3 marks)

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

5. Use the Euclidian Algorithm (outlined in Epp pages 220 – 224) to hand-calculate the greatest common denominator (gcd) of 832 and 10933 (2 marks)

6. Prove, by contraposition, that if (n(n-1) + 3(n-1) - 2) is even then n is odd. Assume only the definition of odd/even. (3 marks)

7. Use mathematical induction to prove that $\sum_{i=1}^{n}$ (5i-4)=n(5n−3)/2. Clearly identify the base case, the induction Hypothesis and the induction step you are using in your proof. (3 marks)