

# 4.2 HOMEWORK

10/30/2023

$$2. \begin{bmatrix} 5 & 2 & 1 & 14 \\ 13 & 23 & 2 \\ 8 & 14 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} \approx \begin{matrix} 25 - 63 + 38 \\ 65 - 69 + 4 \\ 40 - 42 + 2 \end{matrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} \in \text{Nul } A$$

$$4. \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\sim \vec{x} = \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_4$$

$$\text{Nul } A = \text{span} \left\{ \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$3. \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \vec{x} = \begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -6 \\ 2 \\ 0 \\ 1 \end{bmatrix} x_4$$

$$\text{Nul } A = \text{span} \left\{ \begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$



$$5. \left[ \begin{array}{ccccc|c} 1 & -2 & 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & -9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$= \vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix} x_4 \Rightarrow \text{Nul } A = \text{Span} \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix} \right\}$$

7. Not subspace: no zero vector

$$8. 5r \boxed{-1} = s + 2t$$

$$r = s = t = 0 = \text{false}$$

Not subspace: no zero vector

$$18. a. k=3 \quad b. k=4$$

$$21. \left[ \begin{array}{cc|c} 2 & -6 & 0 \\ -1 & 3 & 0 \\ -4 & 12 & 0 \\ 3 & -9 & 0 \end{array} \right] = \left[ \begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} x_1 = 3x_2 \\ x_2 = x_2 \end{array}$$

non zero  $\vec{v}$  in  $\text{col } A =$

$$\text{non zero } \vec{v} \text{ in } \text{nul } A = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -1 \\ -4 \\ 3 \end{bmatrix}$$



22.

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{non } 0 \text{ col } A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{non } 0 \text{ row } A = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

23.

$$\begin{bmatrix} -6 & 12 & 0 & 1 \\ 3 & 6 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -6 & 12 & | & 0 \\ -3 & 6 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} x_1 = 2x_2 \\ x_2 = x_2 \end{matrix}$$

$$x = \begin{bmatrix} 2 \\ 1 \end{bmatrix} x_2$$

$$w \in \text{Nul } A$$

$$w \notin \text{Col } A$$

26.

- a. True
- b. True
- c. False
- d. True
- e. True
- f. True

THM 2

$$m \text{ rows} \Rightarrow \mathbb{R}^m$$

THM 3

Def. of Null space

Def. of Subspace

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## 4.2 S.T.

$\text{Nul } A = \text{solutions for } Ax=0$

$$\text{Col } A = \text{span} \{a_1, a_2, \dots, a_n\}$$

33. a.  $T(A+B) = A+B+A^T+B^T = T(A) + T(B)$

~~b.~~  $T$  is lin. trans.

~~c.~~ b.  $T(A) = A+A^T = B$

$$B = \frac{1}{2}B + \frac{1}{2}B = \frac{1}{2}B + \frac{1}{2}B^T$$

$$= T\left(\frac{1}{2}B\right) \Rightarrow A = \frac{1}{2}B$$

c.  $B = A+A^T \Rightarrow B^T = (A+A^T)^T =$

$$A^T + (A^T)^T = A^T + A = B$$

$$\Rightarrow B = B^T$$

d.  $A+A^T = \begin{bmatrix} 2a & c+b \\ c+b & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$T: \left\{ \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}, b \in \mathbb{R} \right\}$$