

12/08/2023

6.4 Homework

$$\begin{aligned}
 3. \quad v_1 = x_1 &= \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \Rightarrow v_2 = x_2 - \frac{x_2 \cdot v_1}{v_1 \cdot v_1} v_1 = \\
 &= \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} - \frac{8 + 5 + 2}{4 + 25 + 1} \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ -2.5 \\ .5 \end{bmatrix} \\
 &= \begin{bmatrix} 3 \\ 1.5 \\ 1.5 \end{bmatrix} \Rightarrow \text{ortho } B = \left\{ \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1.5 \\ 1.5 \end{bmatrix} \right\}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad x_1 &= \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix} = v_1 \quad v_2 = x_2 - \frac{x_2 \cdot v_1}{v_1 \cdot v_1} v_1 \\
 &= \begin{bmatrix} 7 \\ -7 \\ -4 \\ 1 \end{bmatrix} - \frac{7 + 28 - 0 + 1}{1 + 16 + 1} \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -4 \\ -1 \end{bmatrix} \\
 \text{ortho. } B &= \left\{ \begin{bmatrix} 1 \\ -4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -4 \\ -1 \end{bmatrix} \right\}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad v_1 &= \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{4 + 25 + 1}} \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{30} \\ -5/\sqrt{30} \\ 1/\sqrt{30} \end{bmatrix} \\
 v_2 &= \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{9 + 2.25 + 2.25}} \begin{bmatrix} 3 \\ 1.5 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 3/\sqrt{13.5} \\ 1.5/\sqrt{13.5} \\ 1.5/\sqrt{13.5} \end{bmatrix}
 \end{aligned}$$

$$9. \quad v_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} \Rightarrow v_2 = \begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix} - \frac{-40}{20} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix} - \frac{30}{20} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ -1 \\ 3 \end{bmatrix}$$

$$\text{ortho. } \beta \text{ for } \mathcal{A} = \left\{ \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix} \right\}$$

17. a. True, since $A = \beta$ then β also β for W
 b. False, Gram-Schmidt ortho process def
 c. True, $QQ^T = I \Rightarrow Q^T = Q^{-1}$

$$18. \quad Rx = 0 \Rightarrow A = QR \Rightarrow Ax = QRx = Q0 = 0$$

$$x = \vec{0} \Rightarrow \text{cols of } R = \text{lin independent}$$