Lunceford
11/20/2023

5.2 Homework

1. $\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$ $\det(A-\lambda I)=0$

$\Rightarrow \det\begin{bmatrix} 2-\lambda & 7 \\ 7 & 2-\lambda \end{bmatrix}=0$

$\Rightarrow \lambda^2-4\lambda-45=0$

evals $= 9, -5$

$\Rightarrow$ -4
9 -5
-45

3. $\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} \Rightarrow \lambda^2-2\lambda-1$

$\Rightarrow$ evals $= 1\pm\sqrt{2}$

7. $\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix} \Rightarrow \lambda^2-9\lambda+32=0$

evals $= \dfrac{9\pm\sqrt{81-128}}{2}$

$\Rightarrow$ no real evals

11. $\begin{bmatrix} 4 & 0 & 0 \\ 5 & 3 & 2 \\ -2 & 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 4-\lambda & 0 & 0 \\ 5 & 3-\lambda & 2 \\ -2 & 0 & 2-\lambda \end{bmatrix} \Rightarrow$

$(4-\lambda)\det\begin{bmatrix} 3-\lambda & 2 \\ 0 & 2-\lambda \end{bmatrix}=0$

$\Rightarrow (4-\lambda)(\lambda^2-5\lambda+6) = -\lambda^3+9\lambda^2-26\lambda+24$

15. $\begin{bmatrix} 4 & -7 & 0 & 2 \\ 0 & 3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ THM 5

evals = 1, 4, 3, 3

16. THM 5. evals = 5, 4, 1, 1

18. $(A-5I)x = 0$

$n-6$ $\sim \begin{bmatrix} 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \end{bmatrix}$

$n = 6$

19. THM 5 $\Rightarrow$ upper tri matrix
then pivots = evals
& also product
of pivots = det A

20. $\det(A - \lambda I) \Rightarrow \det(A^T) = \det(A)$

$\det((A-\lambda I)^T) \Rightarrow (A+B)^T = A^T + B^T$

$= \det(A^T - (\lambda I)^T) = (kA)^T = kA^T$

$= \det(A^T - \lambda I^T) = I^T = I$

24. if A and $B = m \times m$, they are similar if there is an invertible matrix P such that $A = P^{-1}BP$

$$\Rightarrow \det(A) = \det(P^{-1}BP)$$

$$\Rightarrow \det(P^{-1})\det(B)\det(P) = \det(P)\det(P^{-1})\det(B)$$

$$= \det(PP^{-1}B) = \det(IB) = \det(B)$$

$$\Longrightarrow \det(A) = \det(B)$$

S.T 5.2

- $\det(A - \lambda I) = 0$

- If A and $B = m \times m$ ~~then A is similar to B~~ then there is an invertible matrix P such that $P^{-1}AP = B$