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6.1 Homework

1. $u \cdot u = 5$, $v \cdot u = 8$, $\frac{v \cdot u}{u \cdot u} = 8/5$

5. $\left(\frac{u \cdot v}{v \cdot v}\right) v = \left(2/13\right) v = \frac{2}{13}\begin{bmatrix}4\\6\end{bmatrix} = \begin{bmatrix}8/13\\12/13\end{bmatrix}$

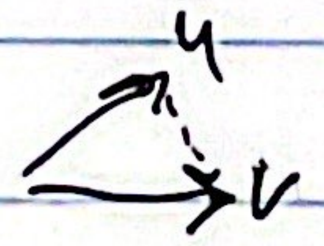
7. $\|w\| = \sqrt{35}$

9. $\hat{u} = \frac{u}{\|u\|} \Rightarrow \|u\| = \sqrt{(-30)^2+40^2} = \sqrt{2500} = 50$

$\Rightarrow \frac{1}{50}\begin{bmatrix}-30\\40\end{bmatrix} = \begin{bmatrix}-3/5\\4/5\end{bmatrix}$

11. $\|u\| = \sqrt{\left(\frac{7}{4}\right)^2 + \left(\frac{1}{2}\right)^2 + 1} = \frac{u}{\frac{\sqrt{69}}{4}} u$

$\Rightarrow \begin{bmatrix}7/\sqrt{69}\\2/\sqrt{69}\\4/\sqrt{69}\end{bmatrix}$

14. 

$\|u - z\| = \left\|\begin{bmatrix}4\\-4\\-6\end{bmatrix}\right\| = \sqrt{68}$

15. $u \cdot v = -1 \neq 0 \Rightarrow$ not ortho.

16. $u \cdot v = 0 \Rightarrow$ yes ortho.

19. a. True, $||v|| = \sqrt{v \cdot v} \Rightarrow ||v||^2 = v \cdot v$
b. True, THM. 1(c)
c. True, $u \cdot v = 0 \Rightarrow$ ortho
d. False, ~~A~~ $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow$ not true
e. True, any $\vec{v}$ can be written as combo of β

28. $\text{Span}\{u,v\} = \{au+bv : a,b \in \mathbb{R}\}$

Let $x = \text{span}\{u,v\} \Rightarrow$ exist some scalar a and b such that $x = au+bv$
$\Rightarrow y \cdot x = y(au+bv) = y(au) + y(bv)$
$= a(y \cdot u) + b(y \cdot u) = 0 + 0 = 0$

30. a. $z \in W^\perp$, $c \in \mathbb{R}$
for any $u \in W : z \cdot u = 0$
$(cz)u = c(z \cdot u) = c \cdot 0 = 0$
$\Rightarrow cz \in W^\perp$
$z_1, z_2 \in W^\perp$
for an $u \in W : z_1 \cdot u = 0, z_2 \cdot u = 0$
$z_1, z_j =$ ortho.

$(z_1 + z_2)u = z_1 u + z_2 u = 0 + 0 = 0$
$\Rightarrow z_1 + z_2 \in W^\perp$

$0 \in W^\perp \Rightarrow 0 \cdot x = 0$: any vector x
$\Rightarrow W^\perp =$ subspace $\mathbb{R}^n$

S.T. 6.1

- $\|v\| = \sqrt{v \cdot v}$, $\|v\|^2 = v \cdot v$
- $dist(u,v) = \|u-v\|$
- $u = \frac{1}{\|v\|} v$