

1, 3, 4, 7, 16, 18, 20, 21, 26, 33, 36
S.T. 1.7

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1.7 Homework

$$1. \begin{bmatrix} 5 & 7 & 9 \\ 0 & 2 & 4 \\ 0 & -6 & -8 \end{bmatrix} \begin{bmatrix} 1 & \frac{7}{5} & \frac{9}{5} \\ 0 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & \frac{7}{5} & \frac{9}{5} \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$Ax = 0$ only trivial

linearly independent $\frac{-2x_3}{-2} = \frac{0}{-2}$

3. linearly dependent $x_3 = 0$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix} \cdot -3 = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$$4. \begin{bmatrix} 1 & -2 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

linearly independent

* only trivial solution to $Ax = 0$

7. $\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$ = linear dependent because more columns than rows = free variable = dependent

16, 18, 20, 21, 26, 33, 36

16. linearly dependent

$$\vec{v}_1 \cdot x_1 = \vec{v}_2 = \vec{v}_1 \cdot -x_1 + v_2 = 0$$

18. linearly dependent

$A = m \times n$; $n > m$ = free variable
= dependent

20. linearly dependent ~~$A \in \mathbb{R}^n$~~ $\exists \in A =$
free variable = dependent

21. a. False $A \leftrightarrow$ trivial solution = independent

b. False all vectors unique to be independent

c. True 4×5 = columns > rows = dependent

d. True $x \cdot x_1 + y \cdot x_2 = z$

26. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ = pivots in each column
since a_1, a_2, a_3 = independent

33. True $v_3 = 2v_1 + v_2 = \cancel{v_1 + v_2 + (2v_1 + v_2)} +$
 ~~$v_4 = 0$~~

$$v_3 = 2v_1 + v_2 + 0v_4$$

$$= 0 = 2v_1 + v_2 + 0v_4 - v_3$$

= dependent

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36. False v_3 ~~does~~ does not guarantee linear independence for (v_1, v_2, v_4)

1.7 S.T.

$$v_1 x_1 + v_2 x_2 + \dots + v_n x_n = 0$$

then dependent if not trivial solution

if only solution to above is trivial solution then independent