

4.5 HOMEWORK

11/06/2023

1. a. $s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \right\}$$

b. $\dim A = 2$

10. $\begin{bmatrix} 2 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 10 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} 2 & -4 & -3 \\ -5 & 10 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dim H = 2$$

11. $\begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

~~$\dim H =$~~ $\dim \text{Col } A = 2$

13. $\dim \text{Col } A = 3$

$$\dim \text{Nul } A = 2$$

$$14. \dim \text{Col } A = 3$$

$$\dim \text{Null } A = 3$$

$$17. \dim \text{Col } A = 3$$

$$\dim \text{Null } A = 0$$

$$21. \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 8 \\ 0 \\ -12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -12 & 0 & 2 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \sim I_4$$

so $\text{Col } A = 4 \Rightarrow$ spans $\mathbb{P}^3 \Rightarrow$
 4 vectors \Rightarrow basis for \mathbb{P}_3

28. $H_n = \text{span} \{1, x, x^2, \dots, x^n\} =$ linearly independent
 $H_n = \text{subspace of } C(\mathbb{R})$ \star THM. 11

$$\dim H_n = n+1 \leq \dim C(\mathbb{R})$$

$$\Rightarrow \dim C(\mathbb{R}) = \infty$$

29.

- a. True, less than \Rightarrow remove dependent vectors
- b. True, if set spans V then $V = P$
- c. True, but will make set dependent

30.

- a. False, can't equal $P \Rightarrow$ dependent
- b. True, needs more vectors to span
- c. False, ex. $\dim V = 3 \Rightarrow \{v_1, v_2, v_3\} = B$
set of $3-1=2$ vectors $\{v_1, 2v_1\} = \text{dependent}$

4.5 S.T.

$\dim V =$ number of vectors of
basis for V