

11/3/2023

4.3 Homework

1. is a base for \mathbb{R}^3
pivot in every row + square

2. not a base, zero vector makes them dependent

8. row reduces to:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{3}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{array} \right] \Rightarrow \text{independent}$$

So = base for \mathbb{R}^3

$$B. \vec{x} = \begin{bmatrix} -6 \\ -5\frac{1}{2} \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -5 \\ -3\frac{1}{2} \\ 0 \\ 1 \end{bmatrix} x_4$$

$$\text{Nul } A = \left\{ \begin{bmatrix} -6 \\ -5\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ -3\frac{1}{2} \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\text{Col } A = \left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 9 \end{bmatrix} \right\}$$

$$14 \quad \sim \quad \vec{x} = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 0 \end{bmatrix} x_4$$

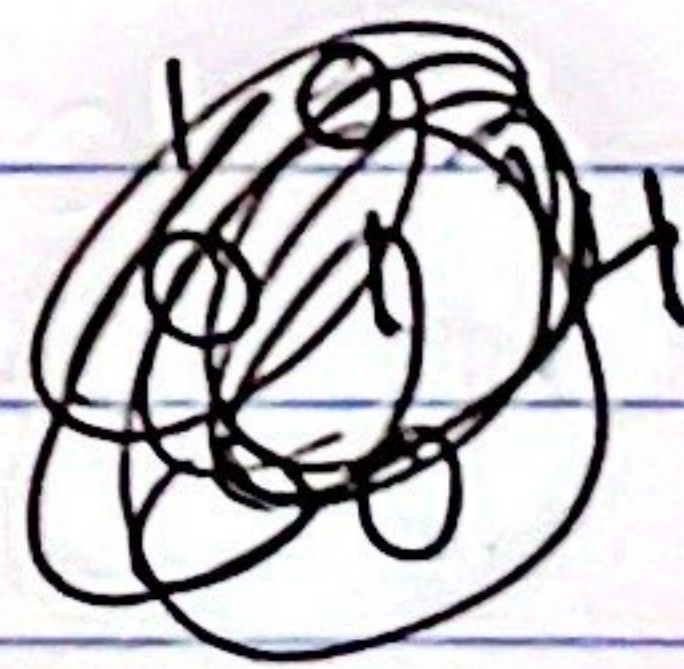
$$\text{Nul } A = \left\{ \begin{array}{c} \downarrow \\ \downarrow \end{array} \right\}$$

$$\text{Col } A = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 5 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 5 \\ 2 \end{bmatrix} \right\}$$

$$14. \quad 4v_1 + 5v_2 - 3v_3 = 0 \Rightarrow v_3 = \frac{4}{3}v_1 + \frac{5}{3}v_2$$

$$\alpha v_1 + \beta v_2 = 0$$

$$\left[\begin{array}{cc|c} 4 & 1 & 0 \\ -3 & 5 & 0 \\ 7 & -2 & 0 \end{array} \right] \sim$$



$$\beta = 0$$

$$\alpha = 0$$

$$\{v_1, v_2\}$$

is a basis for H

$$\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}$$

27.

a. False

Ex. 10

b. True

THM 5

c. True

Two views of a basis section

29. $M = n \times m : m < n$

M can have at most m pivot columns.

THM 4 \Rightarrow if m pivot columns
and n rows
and $n < m$ then

$M \in$ pivot in every column
then $M \neq \text{Span } \mathbb{R}^n$

30.

= dependent because
not square

THM 8

33.

independent because

neither equation is
a scalar combination of
the other

S.T. 4.3

- $H = \text{subset of vector space } V$

Basis = linearly independent set \wedge that spans H
in H

- linearly independent means only trivial solution to $v_1 + v_2 + \dots + v_n = 0$

~~set~~ all combinations of vectors in set reach all points of said points