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11/10/2023

4.9 Homework

2. a. $M = \begin{bmatrix} .5 & .25 & .25 \\ .25 & .5 & .25 \\ .25 & .25 & .5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$p_0 = \begin{bmatrix} .5 \\ .25 \\ .25 \end{bmatrix}$ ⇓ $= \begin{bmatrix} .375 \\ .3125 \\ .3125 \end{bmatrix}$

.3125

3. a. # $M = \begin{bmatrix} .95 & .45 \\ .05 & .55 \end{bmatrix}$

b. $\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} .8 \\ .2 \end{bmatrix} = \begin{bmatrix} .85 \\ .15 \end{bmatrix}$

$\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} .85 \\ .15 \end{bmatrix} = \begin{bmatrix} .8725 \\ .1275 \end{bmatrix}$

Tue = 15%
Wed = 12.5%

c. 92.5%

5. $Px = x \; ; (P - I)x = 0$

$$\Downarrow = \begin{bmatrix} -.9 & .6 \\ .9 & -.6 \end{bmatrix} x = 0$$

$$\sim \begin{bmatrix} 1 & -2/3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\frac{2}{3}x_2 + x_2 = 1 \Rightarrow \frac{5}{3}x_2 = 1$

$\Rightarrow x_2 = \frac{3}{5}$

$$x = \begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix}$$

9. yes entries add to 1 for each vector and P^2 has only positive entries

12. $P - I = \begin{bmatrix} -.4 & .2 & .2 \\ .2 & -.4 & .2 \\ .2 & .2 & -.4 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_3$

$x_2 = x_3$

$x_3 = x_3$

after many trials each food preferred equally

B. a. $P - I = \begin{bmatrix} .05 & .45 \\ .05 & -.45 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -9 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = 9x_2 = \begin{bmatrix} 9/10 \\ 1/10 \end{bmatrix}$

$x_2 = x_2$

b. prob. of $1/10$ and it doesn't matter if person is ill today

S.T 4.9

- Markov Chain = sequence of prob. vectors.
- ~~Steady~~ Steady State solution = vector q for P ~~that~~ such $Pq = q$