10/16/2023

2.4 Homework

1. $\begin{bmatrix} IA+O\cdot C & IB+O\cdot D \\ EA+IC & EB+ID \end{bmatrix}$

$\begin{matrix} A & B \\ EA+C & EB+D \end{matrix}$

7. $\begin{bmatrix} X & O & O \\ Y & O & I \end{bmatrix} \begin{bmatrix} A & Z \\ C & O \\ B & I \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix}$

$\begin{bmatrix} XA & XZ \\ YA+B & YZ+I \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix}$

$XA = I \rightarrow X = A^{-1}$

$XZ = O \rightarrow A^{-1}Z = O \rightarrow Z = O(A) = O$

$YA+B = O \rightarrow YA = -B \rightarrow Y = -BA^{-1}$

$YZ+I = I$

$X = A^{-1}$

$Y = -BA^{-1}$

$Z = O$

10. $\begin{bmatrix} I & 0 & 0 \\ C & I & 0 \\ A & B & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ Z & I & 0 \\ X & Y & I \end{bmatrix}$

$= \begin{bmatrix} I^2 & 0 & 0 \\ IC+IZ & I^2 & 0 \\ AI+BZ+IX & IB+IY & I^2 \end{bmatrix}$

$= \begin{bmatrix} I & 0 & 0 \\ C+Z & I & 0 \\ A+BZ+X & B+Y & I \end{bmatrix} = \begin{matrix} I\,0\,0 \\ 0\,I\,0 \\ 0\,0\,I \end{matrix}$

$C+Z=0 \qquad B+Y=0$

$Z=-C \qquad Y=-B$

$A+BZ+X=0$

~~$A+B \cdot X \equiv A-BZ$~~

$A+B(-C)+X=0$

$X=-A+BC$

$Y=-B$

$Z=-C$

X

~~13, 14, 21~~

13. $A = \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}$ $A^{-1} = \begin{bmatrix} D & E \\ F & G \end{bmatrix}$

$$AA^{-1} = \begin{bmatrix} BD & BE \\ CF & CG \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$BD = I$ $CG = I$

$D = B^{-1}$ $G = C^{-1}$

$\equiv$ A is invertible

17. $G_k = X_k \cdot X_k^T =$

$col_1(X_k) \cdot row_1(X_k^T) + \ldots + col_k(X_k) \cdot row_k(X_k^T)$

$G_{k+1} \equiv G_k + col_{k+1}(X_{k+1}) row_{k+1}(X_{k+1}^T$

21. a. $A^2 = \begin{pmatrix} 1+0 \cdot 3 & 1 \cdot 0 + 0 \cdot (-1) \\ 3 \cdot 1 + -1 \cdot 3 & 3 \cdot 0 + (-1)(-1) \end{pmatrix}$

$\equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b. $M^2 = \begin{bmatrix} A^2 + 0 & 0+0 \\ A-A & 0+(-A)^2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$

2-4 S.T.

onto = at least 1 output for every input

one-to-one = exactly one input for every output