

12/06/23

6.3 Homework

$$1. \quad -\frac{16}{18} \begin{bmatrix} 0 \\ 1 \\ 4 \\ -1 \end{bmatrix} + \frac{8}{36} \begin{bmatrix} 3 \\ 5 \\ 1 \\ 1 \end{bmatrix} + \frac{12}{18} \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} 0 \\ -2 \\ 4 \\ 2 \end{bmatrix} \Rightarrow x - \hat{x} = \begin{bmatrix} 10 \\ -6 \\ -2 \\ 2 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 \\ -2 \\ 4 \\ -2 \end{bmatrix} + \begin{bmatrix} 10 \\ -6 \\ -2 \\ 2 \end{bmatrix}$$

$$x = -\frac{8}{9}u_1 + \frac{2}{9}u_2 + \frac{6}{9}u_3 + 2u_4$$

$$\frac{10}{36} \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \hat{x} = \begin{bmatrix} 10 \\ -6 \\ -2 \\ 2 \end{bmatrix}$$

$$x - \hat{x} = \begin{bmatrix} 10 \\ -8 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 10 \\ -6 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 4 \\ -2 \end{bmatrix}$$

$$x = \begin{bmatrix} 10 \\ -6 \\ -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 4 \\ -2 \end{bmatrix}$$

$$x = -\frac{8}{9}u_1 + \frac{2}{9}u_2 + \frac{6}{9}u_3 + 2u_4$$

$$x = \begin{bmatrix} 10 \\ -6 \\ -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 4 \\ -2 \end{bmatrix}$$

$$5. u_1 u_2 = 3 + 1 - 4 = 0$$

$$\frac{7}{14} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \frac{15}{6} \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$$

$$8. y \cdot u_1 = y^T u_1 = (-1)(1) + 4 \cdot 1 + 3 \cdot 1 = 6$$

$$\left. \begin{array}{l} y u_2 = y^T u_2 = 7 \\ u_1 \cdot u_1 = 3 \\ u_2 \cdot u_2 = 14 \end{array} \right\} = \hat{y} = \frac{6}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{7}{14} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix}$$

$$z = y - \hat{y} = \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$$

$$y = \hat{y} + z = \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix} + \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$$

$$12. \quad y \cdot u_1 = 30, \quad y \cdot u_2 = 26$$

$$u_1 \cdot u_1 = 10$$

$$u_2 \cdot u_2 = 26$$

$$\frac{30}{10} \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix} + \frac{26}{26} \begin{bmatrix} 4 \\ 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ -3 \\ 9 \end{bmatrix}$$

$$16. \quad \hat{y} = \begin{bmatrix} -1 \\ 5 \\ -3 \\ 9 \end{bmatrix} \quad y - \hat{y} = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$$

$$\|y - \hat{y}\|^2 = 64$$

$$y - \hat{y} = \sqrt{64} = 8$$

$$19. \quad u = \frac{u_3 u_1}{u_1 u_1} u_1 + \frac{u_3 u_2}{u_2 u_2} u_2$$

$$= \frac{(0, 0, 1) \cdot (1, 1, -2)}{(1, 1, -2) \cdot (1, 1, -2)} (1, 1, -2) + \frac{(0, 0, 1) \cdot (5, -1, 2)}{(5, -1, 2) \cdot (5, -1, 2)} (5, -1, 2)$$

$$= \left(0, \frac{2}{5}, \frac{1}{5}\right)$$

21. a. True, Example 1 \Rightarrow Section 6.1
b. True, Orthogonal Decomposition THM
c. False, proof of THM 8
d. True, props of orthogonal proj Pg. 352