

1.2 Homework

1. a. Reduced Row Echelon form
b. Reduced Row Echelon form
c. None
d. Echelon form

3. $\begin{bmatrix} 1 & 2 & 3 & | & 4 \\ 4 & 5 & 6 & | & 7 \\ 6 & 7 & 8 & | & 9 \end{bmatrix}$

$-4R_1 + R_2 \rightarrow R_2$

$-\frac{5}{3}R_2 + R_3 \rightarrow R_3$

$\begin{bmatrix} 1 & 2 & 3 & | & 4 \\ 0 & -3 & 6 & | & -9 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

Pivot columns: C_1, C_2

$-\frac{1}{3}R_2 \rightarrow R_2$

$\begin{bmatrix} 1 & 2 & 3 & | & 4 \\ 0 & 1 & 2 & | & 3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

$-2R_2 + R_1 \rightarrow R_1$

$\begin{bmatrix} 1 & 0 & -1 & | & -2 \\ 0 & 1 & 2 & | & 3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 3 & 5 & | & 7 \\ 3 & 5 & 7 & | & 9 \\ 5 & 7 & 9 & | & 1 \end{bmatrix}$

$-3R_1 + R_2 \rightarrow R_2$

$-5R_1 + R_3 \rightarrow R_3$

$-\frac{1}{4}R_2 \rightarrow R_2$

$\begin{bmatrix} 1 & 3 & 5 & | & 7 \\ 0 & 1 & 2 & | & 3 \\ 0 & -8 & -16 & | & -34 \end{bmatrix}$

$8R_2 + R_3 \rightarrow R_3$

$\begin{bmatrix} 1 & 3 & 5 & | & 7 \\ 0 & 1 & 2 & | & 3 \\ 0 & 0 & 0 & | & -10 \end{bmatrix}$

$-3R_2 + R_1 \rightarrow R_1$

$-\frac{1}{10}R_3 \rightarrow R_3$

$\begin{bmatrix} 1 & 0 & -1 & | & -2 \\ 0 & 1 & 2 & | & 3 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$

5, 7, 8, 12

5. $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix} \begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 3 & 4 & | & 7 \\ 3 & 9 & 7 & | & 6 \end{bmatrix} \xrightarrow{-3R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 3 & 4 & | & 7 \\ 0 & 0 & -5 & | & -15 \end{bmatrix} \xrightarrow{-\frac{1}{5}R_2 \rightarrow R_2} \begin{bmatrix} 1 & 3 & 4 & | & 7 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$
 $\xrightarrow{-4R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 3 & 0 & | & -5 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$

$$x_1 + 3x_2 = -5$$

$$x_3 = 3$$

$$(x_1, x_2, x_3) = (-5 - 3x_2, x_2, 3)$$

8. $\begin{bmatrix} 1 & 4 & 0 & | & 7 \\ 2 & 7 & 0 & | & 10 \end{bmatrix} \xrightarrow{-2R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 4 & 0 & | & 7 \\ 0 & -1 & 0 & | & -4 \end{bmatrix} \xrightarrow{-4R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 0 & 0 & | & -9 \\ 0 & 1 & 0 & | & 4 \end{bmatrix}$

$$(x_1, x_2, x_3) = (-9, 4, x_3)$$

12. $\begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 1 & -2 & | & -3 \\ -1 & 7 & -4 & 2 & | & 7 \end{bmatrix} \xrightarrow{R_1 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 1 & -2 & | & -3 \\ 0 & 0 & -4 & 8 & | & 12 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_3 \rightarrow R_3} \begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & -2 & | & -3 \end{bmatrix}$
 $\xrightarrow{-R_3 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & -2 & | & -3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 1 & -2 & | & -3 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$

$$x_1 - 7x_2 + 6x_4 = 5$$

$$x_3 - 2x_4 = -3$$

$$(x_1, x_2, x_3, x_4) = (5 + 7x_2 - 6x_4, x_2, -3 + 2x_4, x_4)$$

14, 16, 17, 24, 29

14.
$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{-2R_2 + R_1 \rightarrow R_1} \left[\begin{array}{ccccc|c} 1 & 0 & 7 & 0 & 0 & -9 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$(x_1, x_2, x_3, x_4, x_5)$

$$= (-9 - 7x_3, 2 + 6x_3 + 3x_4, x_3, x_4, 0)$$

$x_1 + 7x_3 = -9$

$x_2 - 6x_3 - 3x_4 = 2$

$x_5 = 0$

16.a.
$$\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & 0 \end{bmatrix}$$

Consistent, unique.

b. Consistent, not unique. (free column)

17.
$$\left[\begin{array}{cc|c} 2 & 3 & h \\ 4 & 6 & 7 \end{array} \right] \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \left[\begin{array}{cc|c} 2 & 3 & h \\ 4 & 6 & 7 \end{array} \right] \xrightarrow{-2R_2 + R_1 \rightarrow R_1} \left[\begin{array}{cc|c} 2 & 3 & h \\ 0 & 0 & 7-2h \end{array} \right]$$

$$\left[\begin{array}{cc|c} 4 & 6 & 7 \\ 2 & 3 & h \end{array} \right]$$

$$\left[\begin{array}{cc|c} 4 & 6 & 7 \\ 0 & 0 & h - \frac{7}{2} \end{array} \right]$$

$-\frac{7}{2} + h$

$h - \frac{7}{2} = 0$

$h = \frac{7}{2}$

24, 29

24.

$$\left[\begin{array}{cccc|c} & & & & \\ & & & & \\ & & & & \\ 0 & 0 & 0 & 0 & * \end{array} \right]$$

inconsistent,

0 can't equal

anything but

0

29. When a system has more unknowns than equations you will only ever be able to reduce the system to have as many pivots as equations leaving some columns as free, containing free variables, meaning we can assign them any value thus making the system consistent but not unique.

~~29.~~

1.2 S.T.

1. a. Pivot Position: The leading non-zero value of a row in a matrix
- b. Pivot Column: In Row-Echelon form: the column in which pivots are
2. I can do this &
3. Theorem 2: The right most column (the constants) ~~is not~~ can not be a pivot column