~~1~~, ~~3~~, ~~5~~, ~~7~~, 10, 12, 13, 15, 22, 25, 26, 32, 40

S.T. 1.4

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1.4 Homework

1. $\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$ Ax = undefined because there are less columns than x's

3. $\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$

5. $\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$

$5x_1 \begin{bmatrix} 5 \\ -2 \end{bmatrix} - x_2 \begin{bmatrix} 1 \\ -7 \end{bmatrix} + 3x_3 \begin{bmatrix} -8 \\ 3 \end{bmatrix} - 2x_4 \begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$

7. $\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ 7 \end{bmatrix}$

10. $8x_1 - x_2 = 4$
$5x_1 + 4x_2 = 1$
$x_1 - 3x_2 = 2$
$= \begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

~~11~~, 13, 15, 22

12. $A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}$ $b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{array}\right]$ $3R_1 + R_3 \to R_2$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 5 & 3 & -1 \end{array}\right]$ $-R_2 + R_3 \to R_3$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & -2 & -2 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1/5 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $-2R_2 + R_1 \to R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2/5 \\ 0 & 1 & 1 & 1/5 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $R_3 + R_1 \to R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/5 \\ 0 & 1 & 1 & 1/5 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $-R_3 + R_2 \to R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/5 \\ 0 & 1 & 0 & -4/5 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$(x_1, x_2, x_3) = (3/5, -4/5, 1)$

13. $\left[\begin{array}{cc|c} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{array}\right]$ $R_3 \leftrightarrow R_1$ $\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 3 & -5 & 0 \\ -2 & 6 & 4 \end{array}\right]$ $-3R_1 + R_2 \to R_2$, $2R_1 + R_3 \to R_3$ $\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -8 & -12 \\ 0 & 8 & 12 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -8 & -12 \\ 0 & 0 & 0 \end{array}\right]$ 0 2 3

$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{array}\right]$ = has a solution: u is in $\mathbb{R}^3$

~~15~~, ~~22~~, ~~25~~, ~~26~~, ~~32~~, 40

15. $[A \mid b] = \left[\begin{array}{cc|c} 2 & -1 & b_1 \\ -6 & 3 & b_2 \end{array}\right]$ $3R_1 + R_2 \to R_2$

$\left[\begin{array}{cc|c} 2 & -1 & b_1 \\ 0 & 0 & b_2 + 3b_1 \end{array}\right]$

$\frac{1}{2}R_1 \to R_1$ $\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{b_1}{2} \\ 0 & 0 & b_2 + 3b_1 \end{array}\right]$

$\frac{1}{b_2+3b_1}R_2 \to R_2$

$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{b_1}{2} \\ 0 & 0 & 1 \end{array}\right]$ = no solutions

22. Yes because $\left[\begin{array}{ccc|c} 0 & 0 & 4 & b_1 \\ 0 & -3 & -1 & b_2 \\ -2 & 8 & -5 & b_3 \end{array}\right]$ has a solution

$Ax = b$

25. $(c_1, c_2, c_3) = (-3, -1, 2)$

26. $(x_1, x_2) = (3, -5)$

32. The matrix can only have n pivot columns $= \mathbb{R}^n$ and not $\mathbb{R}^m$ because $m > n$

40. Spans $\mathbb{R}^4$ because has pivot in every row

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$$Ax = [a_1, a_2 \dots a_n] \begin{bmatrix} x^1 \\ \vdots \\ x_n \end{bmatrix}$$

$$= x_1 a_1 + x_2 a_2 + \dots x_n a_n$$