Nathan Laroebo[illegible]
4/10/2023

Homework 4.6

1. rank A + dim Nul $A = n = 4$

basis col $A = \left\{ \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 6 \end{bmatrix} \right\}$

basis row $A = \left\{ \begin{bmatrix} 1 & 0 & -1 & 5 \end{bmatrix}, \begin{bmatrix} 0 & -2 & 5 & -6 \end{bmatrix} \right\}$

" Nul $A = \left\{ \begin{bmatrix} 1 \\ 2.5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

rank $A = 2$ dim Nul $A = A$

2. col $A = \left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ -10 \\ -3 \\ 0 \end{bmatrix} \right\}$

Row $A = \begin{bmatrix} 1, -3, 0, 5 & -7 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 2 & -3 & 8 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 0 & 0 & 5 \end{bmatrix}$

Nul $A = \begin{bmatrix} A & 0 \end{bmatrix} = \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 3/2 \\ 1 \\ 0 \end{bmatrix} \right.$

Rank $A = 3$
dim Nul $A = 2$

5. $\dim \text{Nul} A = 5$ $\dim \text{Row} A = 3$

$\text{rank} A^T = 3$

6. $\dim \text{Nul} A = \cancel{1}\ 0$

$\dim \text{Row} A = 3$

$\text{rank} A^T = 3$

7. yes $\Rightarrow$ 4 pivot colums $\Rightarrow$ Rank $= 4$

so col$A$ spans $\mathbb{R}^4$

No $A = 4 \times 7$ matrix $\Rightarrow n = 7$

and

$$\text{rank}(A) + \dim \text{Nul} A = n$$

$$\Downarrow \qquad \Downarrow$$

$$4 + 3 = n = 7$$

$\dim \text{Nul} A = 3$ but lives in $\mathbb{R}^7$

so not quite $\mathbb{R}^3$

8. $\dim \text{Nul} A = 6 - \text{rank} A = 6 - 4 = 2$

No, $\text{Col} A \neq \mathbb{R}^4$ $\text{Col} A =$ subspace of $\mathbb{R}^5$

15. $\text{rank} \leq 6 \Rightarrow \text{rank} + \dim \text{Nul} A = 8$

smallest Nul is $8 - 6 = 2$

19. $A = 5\times6$ matrix

Let x_1 = solution to $Ax=0$
and $x_1 \neq 0$

Let $c \in \mathbb{R}$ and $A(cx_1)=0$

$\Rightarrow$ solution for $Ax=0$ = span of
one element : $\{x_1\}$ so
$\dim \mathrm{Nul}\, A = 1$

$\Rightarrow \operatorname{rank} A + \dim \mathrm{Nul}\, A = n$

$\Rightarrow$ ~~x + = n~~ $6-1=n=5$
$\operatorname{rank} A = 5$

$= \dim \mathrm{Col}\, A = 5$
$\mathrm{col}\, A$ = subspace of $\mathbb{R}^5$
so 5 dimensional subspace
of $\mathbb{R}^5 = \mathbb{R}^5$

yes

so $Ax=b$ = has solution for every b

26. Let A = $m\times n$ Matrix where $m > n$
so Full rank $A = n$
$\Rightarrow \operatorname{rank} A = \dim \mathrm{Col}\, A = n$
$\Rightarrow \dim \mathrm{Nul}\, A = n - \operatorname{rank} A = n-n = 0$
$\Rightarrow \mathrm{Nul}\, A = \{0\} \Rightarrow$ trivial solution
to $Ax=0$ $\Rightarrow$ linearly independent

31. $\begin{matrix} 2a & 2b & -2c \\ -3a & -3b & -3c \\ 5a & 5b & 5c \end{matrix}$

$$= \begin{bmatrix} a & b & c \\ a & b & c \\ a & b & c \end{bmatrix} \sim \begin{bmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \operatorname{rank} A \le 1$

S. I. 4.6

- $\operatorname{rank} A = \dim \operatorname{Col} A$
- $\operatorname{Rank} A + \dim \operatorname{Nul} A = n$