12/08/2023

6.5 Homework

1. a. $A^TA = \begin{bmatrix} 6 & -11 \\ -11 & 22 \end{bmatrix} \Rightarrow A^Tb = \begin{bmatrix} -4 \\ 11 \end{bmatrix}$

b. $\left[\begin{array}{cc|c} 6 & -11 & -4 \\ -11 & 22 & 11 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$

$x_1 = 3 \quad x_2 = 2 \quad \hat{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

3. a. $A^TA = \begin{bmatrix} 6 & 6 \\ 6 & 42 \end{bmatrix} \quad A^Tb = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$

$\left[\begin{array}{cc|c} 6 & 6 & 6 \\ 6 & 42 & -6 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 4/3 \\ 0 & 1 & -1/3 \end{array}\right]$

b. $\hat{x} = \begin{bmatrix} 4/3 \\ -1/3 \end{bmatrix}$

5. $A^TA = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \Rightarrow A^Tb = \begin{bmatrix} 14 \\ 4 \\ 10 \end{bmatrix}$

$A\vec{x} = b \quad \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 10 \end{bmatrix}$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\left.\begin{array}{l} x_1 = 5 - x_3 \\ x_2 = -3 + x_3 \\ x_3 = x_3 \end{array}\right\} \Rightarrow \vec{x} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} x_3$

$x = \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} x_3$

7. ~~$A\hat{x}=$~~ $b=\begin{bmatrix}3\\1\\-4\\2\end{bmatrix}$

$A\hat{x}=\begin{bmatrix}2\\-2\\-1\\1\end{bmatrix}$

$b-A\hat{x}=\begin{bmatrix}1\\3\\-3\\1\end{bmatrix}$

$\|b-A\hat{x}\| = \sqrt{1+9+9+1} = \sqrt{20} = 2\sqrt{5}$

B. $Au=\begin{bmatrix}11\\-11\\11\end{bmatrix}$, $Av=\begin{bmatrix}7\\-12\\7\end{bmatrix}$

$\|Au-b\|^2=\sqrt{40} \neq \|Av-b\|^2=\sqrt{29}$

$Au \neq$ least square solution

17. a. True, $\|Ax-b\|$
b. True, comments on equation (1)
c. False, def. least-squares solution
d. True, THM. 13
e. True THM. 14