Award yourself partial credit as you see fit

#8,9: 3pts: Correctly identify as consistent/inconsistent
2pts: Correct expression of solution.

Homework #1.2 : 8, 9, 11, 17

#8

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -3 & & 5 \\ 0 & 0 & 1 & -2 & & 4 \\ 0 & 0 & 0 & 0 & & 0 \end{array}\right] \rightarrow \begin{cases} x_1 = 5+3x_4 \\ x_2 = x_2 \\ x_3 = 4+2x_4 \\ x_4 = x_4 \end{cases} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 4 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Parametric form: Let $x_4 = t$, $(5+3t, 0, 4+2t, 1+t)$

consistent (infinite solutions)

#9

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 4 & 0 & -1 \\ 0 & 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{cases} x_1 = -1+2x_2-4x_4 \\ x_2 = x_2 \\ x_3 = 2-3x_4 \\ x_4 = x_4 \\ x_5 = -5 \end{cases} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 0 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$$

Parametric form: Let $x_2 = t$, $x_4 = s$, $(-1+2t-4s, t, 2-3s, s, -5)$

consistent (infinite solutions)

#11

#11: 5pts Correctly identify that no solution exists

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 3 \\ 0 & 1 & 1 & -2 \end{array}\right]$$

$\rightarrow$ No solution exits

#17

$$\left[\begin{array}{cc|c} 1 & h & 3 \\ 2 & h & 6 \end{array}\right] \xrightarrow[-2R1+R2]{RREF} \begin{array}{cc} x_1 & x_2 \\ \end{array} \left[\begin{array}{cc|c} 1 & h & 3 \\ 0 & -h & 0 \end{array}\right] \rightarrow \begin{array}{l} x_1 + x_2 h = 3 \\ -h x_2 = 0 \end{array} \rightarrow \begin{cases} h = 0 \\ \text{or} \\ x_2 = 0 \end{cases}$$

if $h=0 \Rightarrow x_1 = 3$, x_2 can be anything

if $x_2 = 0 \Rightarrow x_1 = 3$, h can be anything

$\Rightarrow$ if $h \neq 0$, $(x_1, x_2) = (3, 0)$ is unique

if $h = 0$, $(x_1, x_2) = (3, x_2)$, infinite solutions

#17: 1pt : Correct row reductions

4pts : Convincing argument regarding solutions

Homework #1.3: 1, 9, 11, 17

#1.

$A = \begin{bmatrix} 1 & -3 & 2 \\ -4 & 1 & 0 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

3 pts: for undefined
2 pts: Explanation

undefined.
Because A has 3 columns but $\vec{x}$ has 2 rows

#9 2 pts. for consistent, 3 pts: stating that multiple (infinite) number of solutions.

$$\begin{array}{cccc} & & & \vec{b} \\ \end{array}$$
$$\left[\begin{array}{ccc|c} -1 & 3 & 1 & 0 \\ 2 & 1 & 5 & 7 \\ 1 & 1 & 3 & 4 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

consistent
so $\vec{b}$ is a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$

there are infinite many solutions would work

#11

$A = \begin{bmatrix} 4 & 5 & -1 \\ 3 & 1 & 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 13 \\ -4 \end{bmatrix}$ $A\vec{x} = \vec{b} \Rightarrow \left[\begin{array}{ccc|c} 4 & 5 & -1 & 13 \\ 3 & 1 & 2 & -4 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & -1 & 5 \end{array}\right]$

infinite solutions

$x_1 = -3 - x_3$
$x_2 = 5 + x_3$
$x_3 = x_3$

$\rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} x_3$

Parametric form: Let $x_3 = t$. $(-3-t, 5+t, t)$

2 pts: Parametric expression of solution

$\vec{b}$ is a linear combination of columns of A. 3 pts: for determining $\vec{b}$ is a linear combination.

#17

$A = \begin{bmatrix} 4 & 5 & -1 \\ 3 & 1 & 2 \end{bmatrix}$ $A\vec{x} = 0 \rightarrow \left[\begin{array}{ccc|c} 4 & 5 & -1 & 0 \\ 3 & 1 & 2 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$

$x_1 = -x_3$
$x_2 = x_3$
$x_3 = x_3$

$\rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} x_3$

Parametric form: Let $x_3 = t$.

$(-t, t, t) \leftarrow$ 3 pts: Parametric expression of solution.

3 pts: Concluding there is more than one solution

Homework #1.4: 1, 2, 9, 14

#1

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -4 & 1 & 0 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{2}{11} & 0 \\ 0 & 1 & -\frac{8}{11} & 0 \end{array}\right]$$

infinite solutions

$$\underbrace{\begin{cases} x_1 = \frac{2}{11}x_3 \\ x_2 = \frac{8}{11}x_3 \\ x_3 = x_3 \end{cases} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} \frac{2}{11} \\ \frac{8}{11} \\ 1 \end{bmatrix}}$$

solution set $= \text{Span}\left\{\left[\frac{2}{11}, \frac{8}{11}, 1\right]^T\right\}$

3pts: correct solution

2pts: correct span

#2

$$\left[\begin{array}{cc|c} -4 & 2 & 0 \\ 1 & -3 & 0 \\ 6 & 5 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} x_1 = 0 \\ x_2 = 0 \end{array}$$

5pts: Only solution is the trivial solution.

#9

$$\left[\begin{array}{cc|c} 3 & -9 & 11 \\ -2 & 6 & -4 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 0 & 1 \end{array}\right] \rightarrow \text{inconsistent.}$$

so $\vec{b}$ isn't in span $\{\vec{a}_1, \vec{a}_2\}$

5 pts: Determine $\vec{b}$ is not in the span

#14

$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} x_1 = 0 \\ x_2 = 0 \end{array}$$

2pts: No. since span of 2 vectors cannot span R^3.

(Note: this is a minimal expression)

ex: Let $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\quad \left[\begin{array}{cc|c} 1 & -3 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$

inconsistent.

2pts: Vector $\notin W$

1pt: Show/explain how vector is not in W.

Homework #1.5 : 3, 7, 13, 21, 23, 26

#3

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

3pts: Correct justification

pivots in both rows

So columns of A span R^2

2pts: correct answer

consistent

#7

$$A = \begin{bmatrix} 1 & 8 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

3pts: Correct justification

No pivot in Row 4

so columns of A doesn't span R^4.

2pts: correct answer

Not consistent

#13

$$\vec{b} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\begin{pmatrix} 6 \\ -2 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

3pts: Explicitly write $\vec{b}$ as linear combination

$A\vec{x} = \vec{b}$:
$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 6 \\ 0 & 1 & -3 & -2 \end{array}\right] \to \begin{aligned} x_1 &= 6-2x_3 \\ x_2 &= -2+3x_3 \\ x_3 &= x_3 \end{aligned} \to \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix}}_{x_p} + \underbrace{x_3\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}}_{x_h}$$

Yes, $\vec{b}$ is in span of columns of A.

2pts: correct answer

Let $x_3 = 1$.

then $x_2 = 1$

$x_1 = 4$

#21

$$A = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 4 & -2 & 6 & 0 \\ -7 & 3 & -10 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

consistent.

Correct rref solution 3pts

$x_1 = -x_3$

$x_2 = x_3$

$x_3 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\boxed{x_3\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}}_{x_h}$$

general solution.

Correct general solution 2pts

#23

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 3 & 1 & 5 & -7 & 3 \\ 4 & -1 & 10 & -13 & 5 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -5 & -1 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

consistent

$x_1 = -1 + 5x_4$
$x_2 = 1 - 3x_4$
$x_3 = 1 - x_4$
$x_4 = x_4$

$$\rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}}_{x_p} + \underbrace{x_4 \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix}}_{x_h}$$

2 pts: Correct homogeneous solution $\vec{x}_h$
2 pts: Correct nonhomogeneous solution $\vec{x}_p$
1 pt: Correct form: $\vec{x} = \vec{x}_p + \vec{x}_h$

#26

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 4 & -2 & 6 & 16 \\ -7 & 3 & -10 & -27 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

2 pts: Correct homogeneous solution $\vec{x}_h$
2 pts: Correct nonhomogeneous solution $\vec{x}_p$
1 pt: Correct form: $\vec{x} = \vec{x}_p + \vec{x}_h$

$$\begin{cases} x_1 = 3 - x_3 \\ x_2 = -2 + x_3 \\ x_3 = x_3 \end{cases} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}}_{x_p} + \underbrace{x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}_{x_h}$$

\# Homework # ~~1~~6: 1, 5, 10, 12, 14, 15

(#1) 5pts: Completion

$S = \{\vec{v}_1, \vec{v}_2\}$, $\vec{v}_1 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $\vec{v}_2 \begin{bmatrix} -9 \\ 6 \end{bmatrix}$.

$\vec{v}_2 = -3\vec{v}_1$, So linear dependent

(#5) $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, $\vec{v}_1 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ $\vec{v}_3 = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$

$\begin{bmatrix} -1 & 3 & 1 \\ 2 & 1 & 5 \\ 1 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

2 pivots 3pts: A correct reason

so linear dependent. 2pts: A correct solution

(#10). suppose S is a set of 3 vectors in $\mathbb{R}^5$. is it possible for S to span $\mathbb{R}^5$?

No. 2pts: Correct solution

Because we'll at most have 3 pivots positions, 3pts: A correct reason

(#12) S is a set of four vectors in $\mathbb{R}^3$, is it possible for S to be linear independent? is it possible for S to span $\mathbb{R}^3$?

2pts: Correct solution

3pts: A correct reason

No. It's not possible for S to be linear independent, at most 3 are L.I.

Yes it's possible for S to span $\mathbb{R}^3$ so long as 3 of the vectors are L.I.

(#14) if A is $m \times n$. for what relationship between n and m are the columns of A guaranteed to not span $\mathbb{R}^m$?

For what relationship between n and m will the column have to be linear dependent?

if $n < m$. columns of A is guaranteed to not span $\mathbb{R}^m$

if $n > m$ columns of A is guaranteed to be linear dependent.

5pts: Completion

(#15) prove any set that contains zero vector must be linear dependent.

If there is a zero vector, the zero vector ($\begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}$) will always be a linear combination of the other vectors. ($0 = 0\vec{v}$).

5pts: Completion