Homework 02

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1.7.4

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1\cdot 0)+(2\cdot 1) & (1\cdot 1)+(2\cdot 0) \\ (0\cdot 0)+(1\cdot 1) & (0\cdot 1)+(1\cdot 0) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} (0\cdot 1)+(1\cdot 0) & (0\cdot 2)+(1\cdot 1) \\ (1\cdot 1)+(0\cdot 0) & (1\cdot 2)+(0\cdot 1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}}$$

1.12.1

$V = \mathbb{R}^3$

$$H = \left\{ t\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \right\}$$

~~1. let $t = 0$~~

$$\cancel{0\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

H contains $\vec{0}$

H basis is $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

1-D

1.12.9

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 1 & 0 \end{array}\right] = \xrightarrow{-2R1+R2} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 1 & 0 \end{array}\right] \xrightarrow{-2R_2+R_1} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$$

$x_1 = 0$ independent

$x_2 = 0$

yes S is a basis for $\mathbb{R}^2$

1.12.13

No, you need at least 4 vectors to span $\mathbb{R}^4$. 3 vectors can at most make a 3d object in $\mathbb{R}^4$.

1.12.14

No, 7th vector would be a lin. combo of other six.

1.1.3

$H = \left\{ t \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} : t \in \mathbb{R} \right\}$

1. let $t = 0$

$$0 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

contains $\vec{0}$ ✓

2. Let $c \in \mathbb{R}$

$$c\left(t \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}\right) = (ct) \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

let $c \cdot t = r = r \in \mathbb{R}$

so closed scalar mult. ✓

3. $$t_1 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} + t_2 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = (t_1 + t_2) \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$t_1 + t_2 = t_3$ and $t_3 \in \mathbb{R}$

so closed under vector addition

Yes subspace

10/11/17

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$(2-\lambda)^2 - (-1)^2 = 0$

$(2-\lambda)^2 - 1 = 0$

$(2-\lambda)^2 = 1$

$2-\lambda = 1$ $\quad$ $2-\lambda = -1$

$-\lambda = -1$ $\quad$ $-\lambda = -3$

$\lambda = 1$ $\quad$ $\lambda = 3$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$x_1 - x_2 = 0$

$-x_1 + x_2 = 0$

$x_1 = x_2$

$= t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} = x_1 = -x_2$$

$= \Big[\, 1$

1.11.29

$$\boxed{\begin{aligned} y' &= 3y-3 \\ y &= \frac{3}{2}y^2 - 3y \end{aligned}}$$

$y = Ce^{3t}+1$

$y' = 3Ce^{3t}$

$\boxed{3Ce^{3t}} = 3(Ce^{3t}+1)-3$

$= 3Ce^{3t}+3-3$

$= \boxed{3Ce^{3t}}$

1. contains $\vec{0}$

$Ce^{3t}+1=0$

No C that makes ~~&~~ this true

Not subspace

1.10.1

$A = \begin{bmatrix} 5 & 1 \\ 0 & 3 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$\begin{vmatrix} 5-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix}$

$(5-\lambda)(3-\lambda) - 0 = 0$

$(5-\lambda)(3-\lambda) = 0$

$\lambda = 5, 3$

$\left[\begin{array}{cc|c} 5-5 & 1 & 0 \\ 0 & 3-5 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & -2 & 0 \end{array}\right]$ $x_2 = 0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$x_1 = x_1$

$\left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $2x_1 + x_2 = 0$

$x_2 = -2x_1 \Rightarrow \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

1.10.5

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{vmatrix} \Rightarrow (2-\lambda)^3 = 0$$

$$\lambda = 2$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x_2 = 0 \\ x_3 = 0 \\ x_1 = x_1 \end{array} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

1.10.9

$\lambda_1 = 5$, $\lambda_2 = -1$ $\Rightarrow Ax$ $\quad x = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$

$\vec{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$x = -5\vec{v} + 4\vec{u}$$

$$Ax = A(-5\vec{v} + 4\vec{u})$$

$$Ax = -5A\vec{v} + 4A\vec{u}$$

$$A\vec{v} = \lambda_2 v, \quad A\vec{u} = \lambda_1 \vec{u}$$

$$= (-1)\vec{v} \qquad = 5\vec{u}$$

$$Ax = (-5)(-\vec{v}) + 4(5\vec{u})$$

$$= 5\vec{v} + 20\vec{u}$$

$$= \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 5 \\ 20 \end{bmatrix}$$

1.10.11

$$A=\begin{bmatrix}-2 & 1 & 1\\ 1 & -2 & 1\\ 1 & 1 & -2\end{bmatrix}=$$

a) $$\begin{vmatrix}-2-\lambda & 1 & 1\\ 1 & -2-\lambda & 1\\ 1 & 1 & -2-\lambda\end{vmatrix} \Rightarrow (\lambda^2+6\lambda+9)\lambda=0$$

$$\Rightarrow (\lambda+3)^2\lambda=0$$

$$\cancel{(-2-\lambda)^3-(-6-3\lambda)-2=0}$$

$$\cancel{(-2-\lambda)^3+6+3\lambda+2=0}$$

$$\cancel{(-2-\lambda)^3+3\lambda+8=0}$$

$$\cancel{(-2-\lambda)^3+3\lambda=-8}$$

$\lambda_1=0$

$\lambda_2=-3$

$\lambda_2=-3$

1.9.1

$A = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \quad 4 - 2 = 2$

1.9.5

$$A = \begin{bmatrix} -3 & 1 & 0 & 5 \\ 0 & 2 & -4 & 0 \\ 0 & 0 & -7 & 1 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$(-3)(2)(-7)(6)$

$(-6)(-7)(6)$

$(-36)(-7) = 252$

1.9.17

$\det(A^2) = 0$

$\det(A^2) = (\det(A))^2$

$\det(A)$ must $= 0$

1.8.2

$$\begin{bmatrix} 5 & 0 \\ 0 & -3 \end{bmatrix} \quad \cancel{\det(A) = -15}$$

$$I = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix}$$

$$AA^{-1} = I$$

1.8.5

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 5 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 & 5 & 1 \end{array}\right] \Rightarrow \text{Not invertible}$$