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RADIO MEASUREMENTS

A METHOD OF MEASURING CURRENT–VOLTAGE CHARACTERISTICS BASED ON THE HAMMERSTEIN–CHEBYSHEV MODEL

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A method of measuring the current–voltage characteristics of nonlinear components is proposed, which involves applying a sinusoidal voltage to the object being measured, finding the Fourier spectrum to read out the output current, and a calculation using the spectrum of the coefficients of the expansion of the measured characteristic in Chebyshev polynomials. The properties of the Hammerstein–Chebyshev model are considered, a block diagram of a measuring instrument is presented, and recommendations are made for minimizing the systematic errors due to approximating their characteristics by Chebyshev polynomials.

Key words: *mathematical model of the object being measured, current–voltage characteristic.*

When measuring the input–output characteristics of nonlinear components, the effectiveness of the result depends very much on the choice of the mathematical model of the object being measured [1–3]. As analysis has shown, when measuring the current–voltage characteristics the Hammerstein–Chebyshev model (Fig. 1) is very effective. In general, it enables one to describe nonlinear systems in the form of dynamic (operator) mathematical models [4–6]. In the model, the nonlinearity is taken into account by functional-conversion sections with nominal static characteristics in the form of Chebyshev polynomials: $T_i(X)$ (i = 0, ..., n), where $X \in [-1, 1]$ is the normalized input quantity. The dynamic properties of the model are described by sections of continuous linear systems with amplitude–phase characteristics $W_i(j\omega)$. The Hammerstein–Chebyshev model is an interpreting-type model [1] and is linear in the parameters with respect to a system of known Chebyshev basis functions $T_n(X) = \cos(n \arccos X)$, which can be represented in the form of polynomials:

$$\left.\begin{aligned} &T_0(X) = 1; \\ &T_1(X) = X; \\ &T_2(X) = 2X^2 - 1; \\ &T_3(X) = 4X^3 - 3X; \\ &\dots\dots\dots\dots\dots\dots\dots\dots \\ &T_{n+1}(X) = 2XT_n(X) - T_{n-1}(X);\ (n > 1). \end{aligned}\right\} \tag{1}$$

Hence, according to Fig. 1, the expression for the output quantity can be represented in the form

$$Y = \sum_{n=0}^{M} W_n(j\omega) T_n(X),$$

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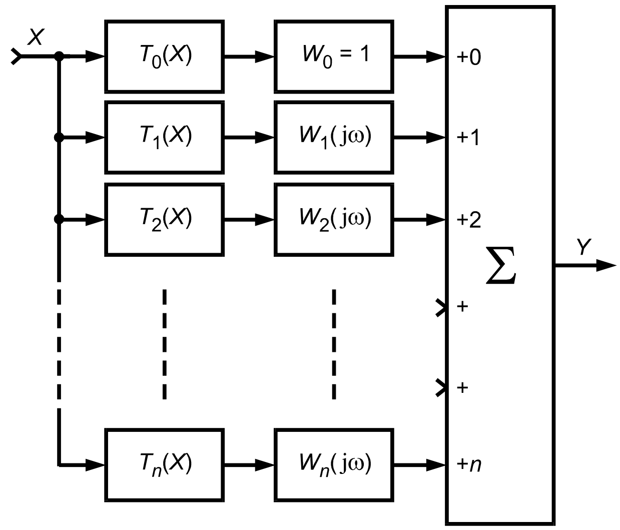

Fig. 1. Block diagram of the Hammerstein–Chebyshev model.

where M is the number of terms used in the expansion. When $W_i(j\omega)$ = const, the Hammerstein–Chebyshev model is essentially a static (functional) mathematical model [3, 6], i.e., it corresponds to the requirements needed to measure nonlinearity characteristics.

Of all the useful properties of the Hammerstein–Chebyshev model, we will consider one important one, which is the basis of the proposed method of measuring the current–voltage characteristics of semiconductor components. This property manifests itself in the case of a sinusoidal input signal, i.e., when $X = \sin\omega t$, when there is a very simple relation between the Chebyshev expansion and the Fourier-series expansion of the output signal Y, which does not require calculations. One can transfer from one expansion to the other merely by changing the signs in front of the coefficients. We will investigate this property in more detail using the delayless Hammerstein–Chebyshev model, i.e., we will assume that $W_n(j\omega) = W_n$ = const. In this case, the conversion function will be described by the expression

$$Y(X) = F(X) = \sum_{n=0}^{M} W_n T_n(X), \tag{2}$$

where $F(X)$ is the current–voltage characteristic of the nonlinear component.

Substituting $X = \sin\omega t$ into (2) and using formulas (1), we will consider the signals at the inputs of the adder Σ (see Fig. 1). For the input +1, we have

$$W_1 T_1(\sin\omega t) = W_1 X \Big|_{X=\sin\omega t} = W_1 \sin\omega t. \tag{3}$$

Using this procedure further, taking into account well-known trigonometric conversions [7], we obtain

$$W_2 T_2(\sin\omega t) = W_2(2X^2 - 1)\Big|_{X=\sin\omega t} = -W_2 \cos 2\omega t; \tag{4}$$

$$W_3 T_3(\sin\omega t) = W_3(4X^3 - 3X)\Big|_{X=\sin\omega t} = -W_3 \sin 3\omega t; \tag{5}$$

$$W_4 T_4(\sin\omega t) = W_4(8X^4 - 8X^2 + 1)\Big|_{X=\sin\omega t} = W_4 \cos 4\omega t; \tag{6}$$

$$W_5 T_5(\sin\omega t) = W_5(16X^5 - 2X^3 + 5X)\Big|_{X=\sin\omega t} = W_5 \sin 5\omega t; \tag{7}$$

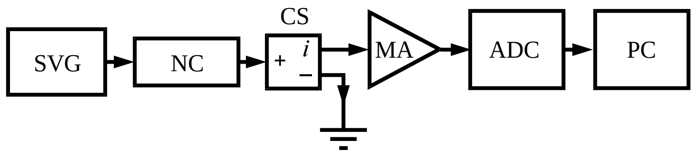

Fig. 2. Block diagram of a system for measuring the current–voltage characteristic: SVG is a sinusoidal-voltage generator; NC is the nonlinear component, whose characteristics are to be measured; CS is a current sensor; MA is a measuring amplifier; ADC is an analog-to-digital converter; and PC is a personal computer.

$$W_6 T_6(\sin\omega t) = W_6(32X^6 - 12X^4 + 18X^2 - 1)\Big|_{X=\sin\omega t} = -W_6 \cos 6\omega t\,; \tag{8}$$

$$W_7 T_7(\sin\omega t) = W_7(64X^7 - 40X^5 + 38X^3 - 7X)\Big|_{X=\sin\omega t} = -W_7 \sin 7\omega t\,. \tag{9}$$

If there is a constant component present in the expansion of $F(X)$ in Chebyshev polynomials, then, being equal to W_0, it will appear at the input +0. Hence, at the first input of the adder there will be a signal at the frequency of the fundamental (first) harmonic, at the second there will be a signal at the frequency of the second harmonic, etc. For comparison, we will consider the expansion of the output signal in a Fourier series:

$$Y(t) = \frac{A_0}{2} + \sum_{n=1}^{M}(A_n \cos n\omega t + B_n \sin n\omega t)\,, \tag{10}$$

where $\omega = 2\pi/T$, $n = 0, 1, 2, \ldots, M$; and T is the expansion interval.

Comparing expressions (3)–(9) with expression (10), we can write the following series of obvious equalities:

$$W_0 = A_0;\quad W_1 = B_1;\quad W_2 = -A_2;\quad W_3 = -B_3;\quad W_4 = A_4;\quad W_5 = B_5;\quad W_6 = -A_6;\quad W_7 = -B_7, \tag{11}$$

which show that by applying a sinusoidal voltage to the input of the Hammerstein–Chebyshev model and knowing the spectrum of the output signal, one can easily (by merely a change in the signs of the coefficients) determine the values of the coefficients of the expansion of the static characteristic of the system being modeled in terms of Chebyshev polynomials.

Formulas (11) enable us to explain the behavior of the alternation in the signs and to write two equations relating the coefficients of the Fourier series and the coefficients of the Chebyshev polynomial:

$$W_{2k} = (-1)^{k+2} A_{2k}; \tag{12}$$

$$W_{2k+1} = (-1)^{k} A_{2k+1}, \tag{13}$$

where $k = 0, 1, 2, \ldots, (0.25M - 1)$.

Hence, the procedure for measuring the current–voltage characteristic consists of the following:

1) a sinusoidal voltage is applied to the two-terminal network being measured;

2) the spectrum (A_n, B_n) of the current is determined in terms of the two-terminal network;

3) using formulas (12) and (13), we find the coefficients W_n of the expansion of the static characteristic in terms of Chebyshev polynomials;

4) using formula (2), we construct the current–voltage characteristic of the two-terminal network being measured.

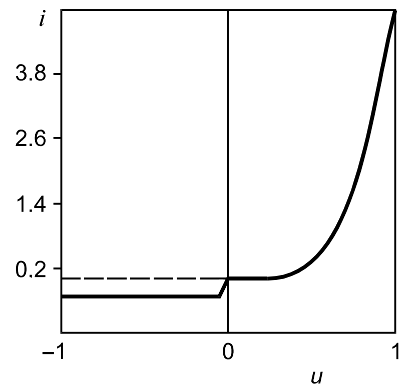

Fig. 3. Current–voltage characteristic with an anomaly in the region of $u = 0$.

A version of a system for measuring the current–voltage characteristic is shown in Fig. 2. The operating principle of the system does not require detailed explanations: it follows from the function of each of the components in the system. We merely note that in the scheme shown in Fig. 2 the nonlinear element operates at a specified voltage; a measuring voltage generator is used, which ensures the required stability of the amplitude and frequency of the oscillations, and also the required nonsinusoidality coefficient; for a specific form of the measuring system, it is possible to combine the functions of individual units.

The measurement information is fed into a personal computer, where it is processed. All the remaining problems connected with the construction of the conversion characteristics of the nonlinear component are solved by appropriate software and the operation is carried out by means of a program.

As investigations have shown, if the current–voltage characteristic of the diode being measured is smooth and monotonic (this is an inherent feature, for example, of silicon diodes), the errors in approximating it with Chebyshev polynomials are small, and to obtain acceptable accuracy it is sufficient to use information solely on the first four-six harmonics of the current spectrum. The situation is much more complicated in the case when the functional relationships $F(X)$ have anomalies: breaks in the functions and derivatives. We will consider this case using a specific example of measurements of the current–voltage characteristic of a germanium diode (Fig. 3). As follows from Fig. 3, the characteristic has a kink at zero voltage, which is logically interpreted as a discontinuity of the first derivative of the function $F(X)$.

Because of this feature of the current–voltage characteristic in the neighborhood of the point $u = 0$, the approximation error reaches its maximum value and its behavior has an oscillatory form, similar to the Gibbs effect for expansion in a Fourier series [8]. In practice, the error in question cannot be eliminated either by increasing the number of readouts N in the time T, or by increasing the degree M of the Chebyshev polynomial employed. Examples of typical curves for the approximation error are shown in Fig. 4.

To minimize the errors due to approximation by Chebyshev polynomials, we can employ an approximation over a number of local parts with subsequent "joining" of the partial results, rather than an approximation over the whole interval in which the function is defined. This method can be realized by instrumental means, by making measurements individually for the forward and reverse branches, but it is more effective to use an algorithmic solution by appropriate data processing.

Briefly, the basic principle of the method is to divide the initial data matrix I in half and to process the data matrices Id (direct) and Ir (reverse) separately for the direct and reverse branches of the current–voltage characteristic respectively. Each half of the initial data matrix is unrolled as a "butterfly" with inversion of the signs of the values of the readouts. The use of this method enables one to reduce the error in approximating the characteristic a hundredfold.

In conclusion, we make a number of observations, regarding the advantages of this method of measuring current–voltage characteristics.

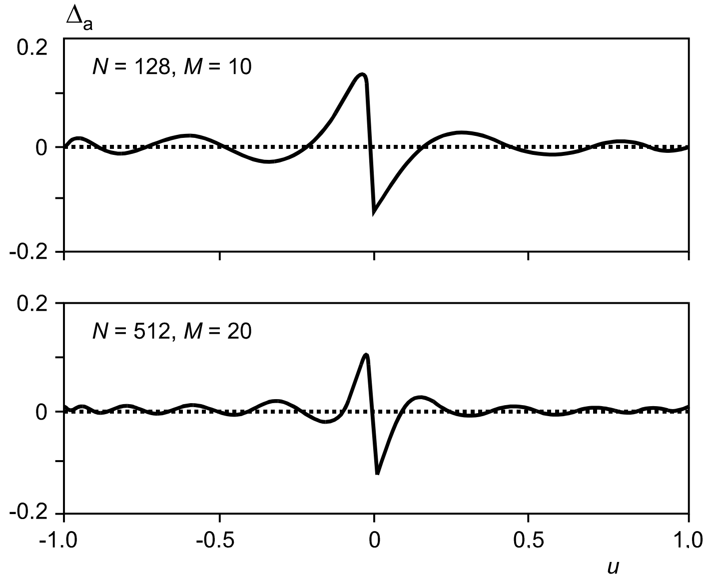

Fig. 4. Graph of the approximation error.

First, the area of application of the method is fairly wide, and it is possible to use it to measure both the current–voltage characteristics of power semiconductor components, when the voltages are measured in kilovolts, and the currents in kiloamperes, and for intracircuit diagnostics of hybrid and integrated microcircuits, when the intracircuit diagnostics is combined with methods of measuring the nonlinear parameters of semiconductor test structures (the voltages are measured in microvolts and millivolts, while the currents are measured in microamperes and milliamperes), made in a single technological cycle with the integrated circuit as the components (resistors, capacitors, *p*–*n*-junctions, diodes, transistors and connections) [9].

Second, the method requires much simpler apparatus compared with existing methods, since one only needs to use a single measuring channel to measure the current.

Third, among the advantages of the method is that the data on the current–voltage characteristic are presented in parametric form, which is much more compact for data storage compared with nonparametric data in the volume required to obtain acceptable accuracy in describing the properties of a nonlinear component.

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