HW 01

1)

a.) Find x^2 in frame 1 coordinates.

Using T_1^2, $x^2 = C_2 x^1 + S_2 y^1$

b.) Find the vector (in base coordinates) from the origin of the base to the origin of frame 3.

Using T_0^3, $\vec{v} = q_3 C_1 S_2 x^0 + q_3 S_1 S_2 y^0 + (q_3 C_2 + d_1) z^0$

c.) Specify the tool tip, **p**, in frame 3 coordinates.

Need $T_3^5 = T_3^4 T_4^5 = \begin{bmatrix} C_4 C_5 & -C_4 S_5 & S_4 & d_5 S_4 \\ S_4 C_5 & -S_4 S_5 & -C_4 & -d_5 C_4 \\ S_4 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$p = d_5 S_4 x^3 - d_5 C_4 y^3$

2)

a.) Find the homogeneous transformation matrix $T_0^1(q)$.

$$T_0^1(q) = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $$T_1^2(q) = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint	θ	d	a	α
1	q_1	d_1	0	$-\pi/2$
2	q_2	d_2	0	$\pi/2$
3	$-\pi/2$	q_3	0	0
4	q_4	0	0	$-\pi/2$
5	q_5	0	0	$\pi/2$
6	q_6	d_6	0	0

c) $$T_1^3 = T_1^2 \, T_2^3$$

$$= \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -C_2 & S_2 & d_3 S_2 \\ 0 & -S_2 & -C_2 & -d_3 C_2 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -C\theta_k S\alpha_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3)

$$w(q) = \begin{bmatrix} S_1(d_2+d_3) \\ -C_1(d_2+d_3) \\ d_1 \\ S_1 e^{\frac{q_3}{\pi}} \\ -C_1 e^{\frac{q_3}{\pi}} \\ 0 \end{bmatrix}$$

note: $d_2 = q_2$

1. The projection of the p-vector onto the x^0-y^0 plane only depends on d_2 ($= q_2$).

$$w_1^2 + w_2^2 = (d_2+d_3)^2 \Rightarrow \underline{d_2 = q_2 = \sqrt{w_1^2+w_2^2} - d_3}$$

2. $q_1 = \operatorname{atan2}(w_1, -w_2)$

3. $q_3 = \pi \ln \sqrt{w_4^2 + w_5^2}$

Check: $w_4^2 + w_5^2 = e^{2q_3/\pi}$

$$\sqrt{w_4^2+w_5^2} = e^{q_3/\pi}$$

$$\ln \sqrt{w_4^2+w_5^2} = \frac{q_3}{\pi}$$

4)

a.) Find q_1 (which is θ_1).

$$w(q) = \begin{bmatrix} -q_2 S_1 \\ q_2 C_1 \\ d_1 \end{bmatrix}$$

$$q_1 = \operatorname{atan2}(-w_1, w_2)$$

b.) Find q_2 (which is d_2).

$$w_1^2 + w_2^2 = q_2^2 S_1^2 + q_2^2 C_1^2$$

$$q_2 = \pm\sqrt{w_1^2 + w_2^2}$$