APPLICATION OF REAL-TIME BEAT TRACKING

Signal Processing Midterm-2

Summary

Creating of a real-time beat tracker using IIR and FIR filters and its application on a sample song.

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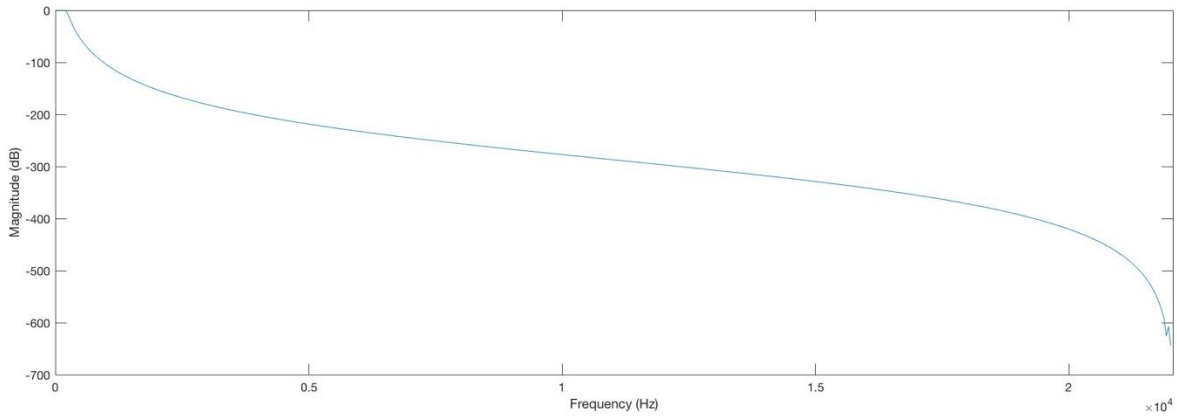
DESIGN OF AN IIR BUTTERWORTH FILTER

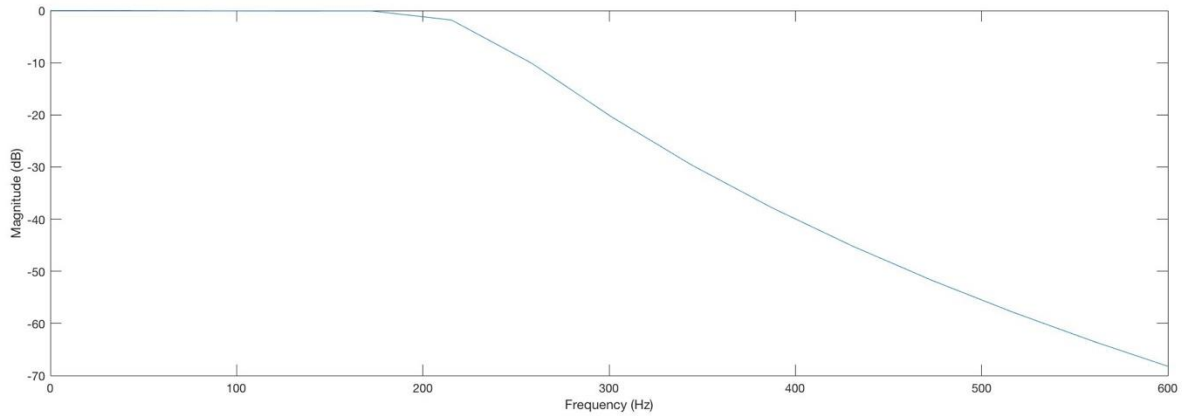
Our Butterworth filter has a 44100 sampling frequency. It has following properties;

- 200 Hz Pass-band edge frequency
- 400 Hz Stop-band edge frequency
- -1 dB ripple at pass-band
- -40 dB minimum attenuation at stop-band

The code of the filter will be as follows;

```
%% Initialize parameters - DON'T EDIT
Fs = 44100; Ts = 1/Fs;

%% Place your code here
wp = 2*200*Ts; % Pass-band edge frequency
ws = 2*400*Ts; % Stop-band edge frequency

rp = 1;  % Maximum pass band ripple 1dB
rs = 40;   % Minimum stop band attenuation 40 dB

%  Set bL and aL as coefficients of your filter

[N, wc] = buttord(wp, ws, rp, rs); % Getting the cutoff frequency and order of the filter values
[bL,aL] = butter(N, wc); % Getting the filter coefficients

% Plot the frequency response in 2 subplots
% Subplot 1 - Plot the frequency between [0 - Fs/2] Hz.
% Subplot 2 - Plot the frequency between [0 - 600] Hz (Zoomed version).
% x axes are in terms of Hz.
% y axes are in terms of dBs.

figure(1)
subplot(2,1,1)
[h,w]=freqz(bL,aL); % Getting the frequency and magnitude values from frequency analysis
plot(w/(2*pi*Ts),mag2db(abs(h))); % Plotting the magnitude response
xlim([0,22050]); % Plotting for frequencies between 0-22050

subplot(2,1,2)
[h,w]=freqz(bL,aL); % Getting the frequency and magnitude values from frequency analysis
plot(w/(2*pi*Ts),mag2db(abs(h))); % Plotting the magnitude response
xlim([0,600]); % Plotting for frequencies between 0-600

% Find the transfer function of the filter.
% Plot the pole-zero map of the filter.

Ts = 1; % Sampling period
Hb = tf(bL, aL, Ts); % Creating the transfer function with coefficient values and the sampling constant

figure (2)
pzmap(Hb); % Getting the pole-zero map of the filter

% End of your code

%% Print the coefficients and save the results - DON'T EDIT
clc
for k = 1:N+1
    fprintf('a(%d) = %8.6g, \t b(%d) = %8.6g \n', k-1, aL(k),k-1, bL(k));
end

% Clear needless variables and save the filter
clearvars -except bL aL
save low_pass_coefficients;
```





The magnitude response of the filter is like above.

The filter coefficients a_i, b_i are 9 element arrays, their values are;

$a_0 = 1$ $\quad b_0 = 3.9968\ x\ 10^{-15}$

$a_1 = -7.8357$ $\quad b_1 = 3.19744\ x\ 10^{-14}$

$a_2 = 26.8633$ $\quad b_2 = 1.1191\ x\ 10^{-13}$

$a_3 = -52.6297$ $\quad b_3 = 2.23821\ x\ 10^{-13}$

$a_4 = 64.4479$ $\quad b_4 = 2.79776\ x\ 10^{-13}$

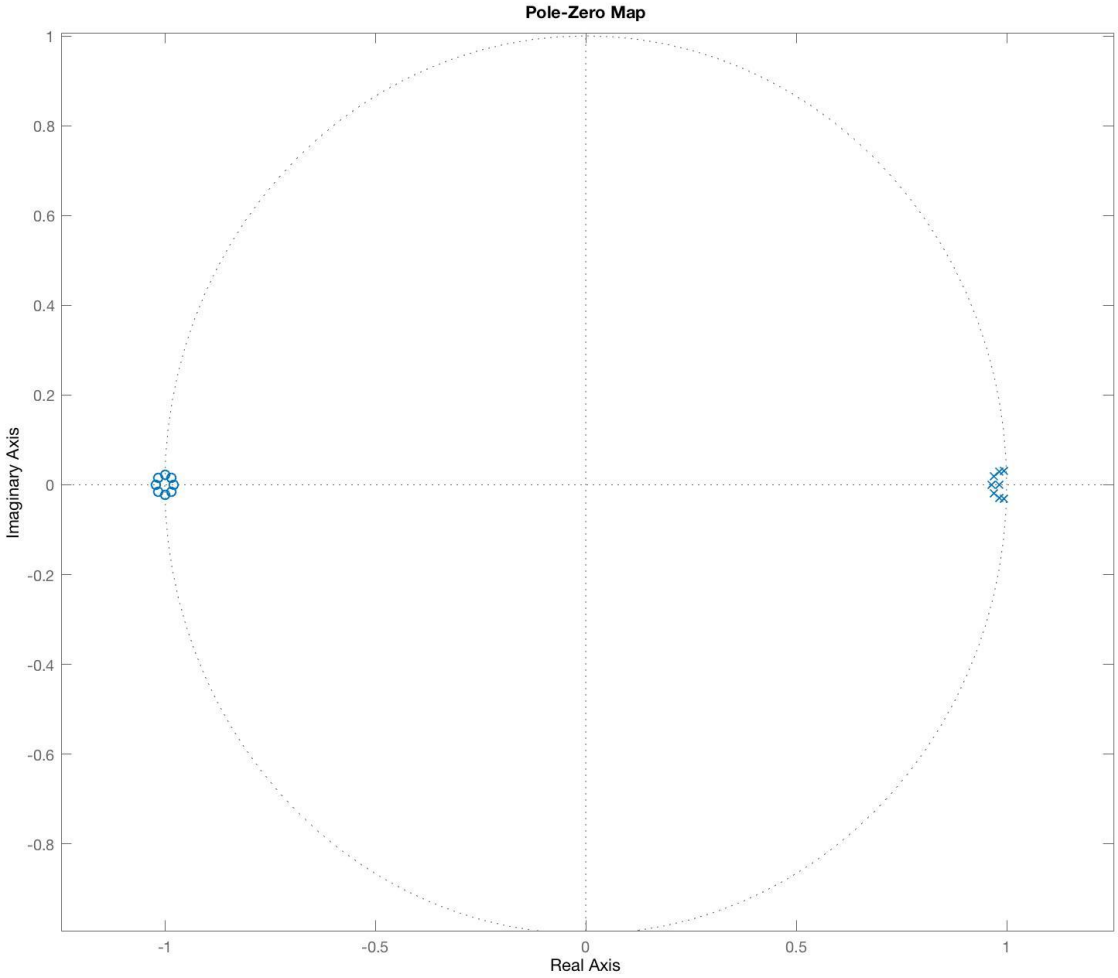
$a_5 = -50.5117$ $\quad b_5 = 2.23821\ x\ 10^{-13}$

$a_6 = 24.7447$ $\quad b_6 = 1.1191\ x\ 10^{-13}$

$a_7 = -6.92722$ $\quad b_7 = 3.19744\ x\ 10^{-14}$

$a_8 = 0.848476$ $\quad b_8 = 3.9968\ x\ 10^{-15}$

The filter's pole-zero map is like below;



We can get the transfer function of the filter with z-domain coefficients like below;

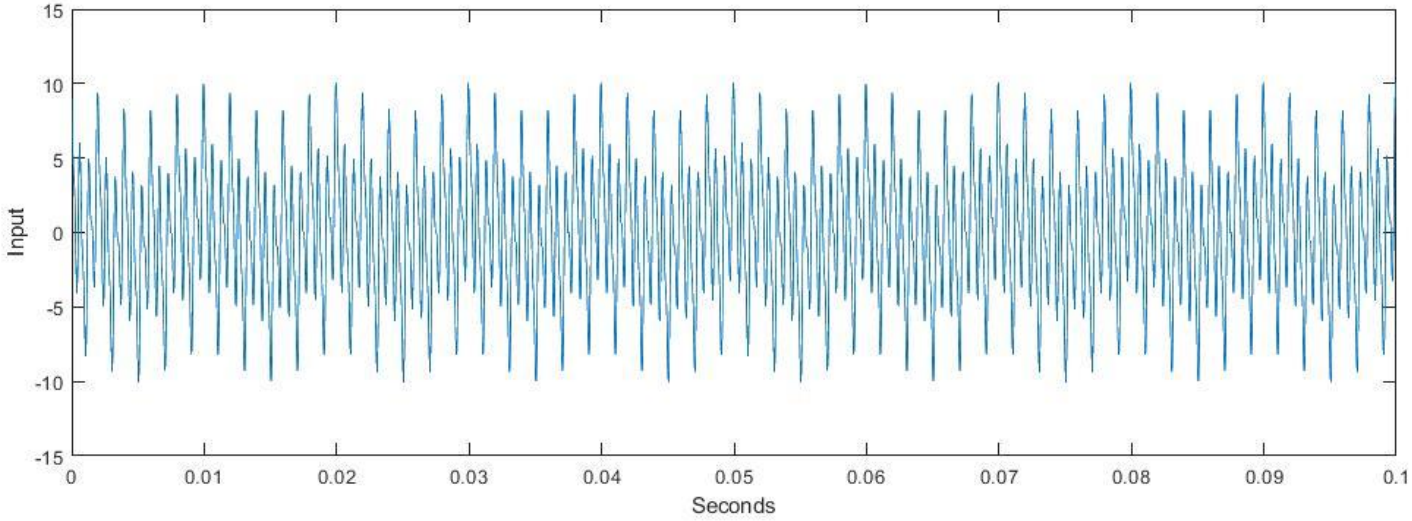
$$H(z) = \frac{b_0 z^8 + b_1 z^7 + b_2 z^6 + b_3 z^5 + b_4 z^4 + b_5 z^3 + b_6 z^2 + b_7 z + b_8}{a_0 z^8 + a_1 z^7 + a_2 z^6 + a_3 z^5 + a_4 z^4 + a_5 z^3 + a_6 z^2 + a_7 z + a_8}$$

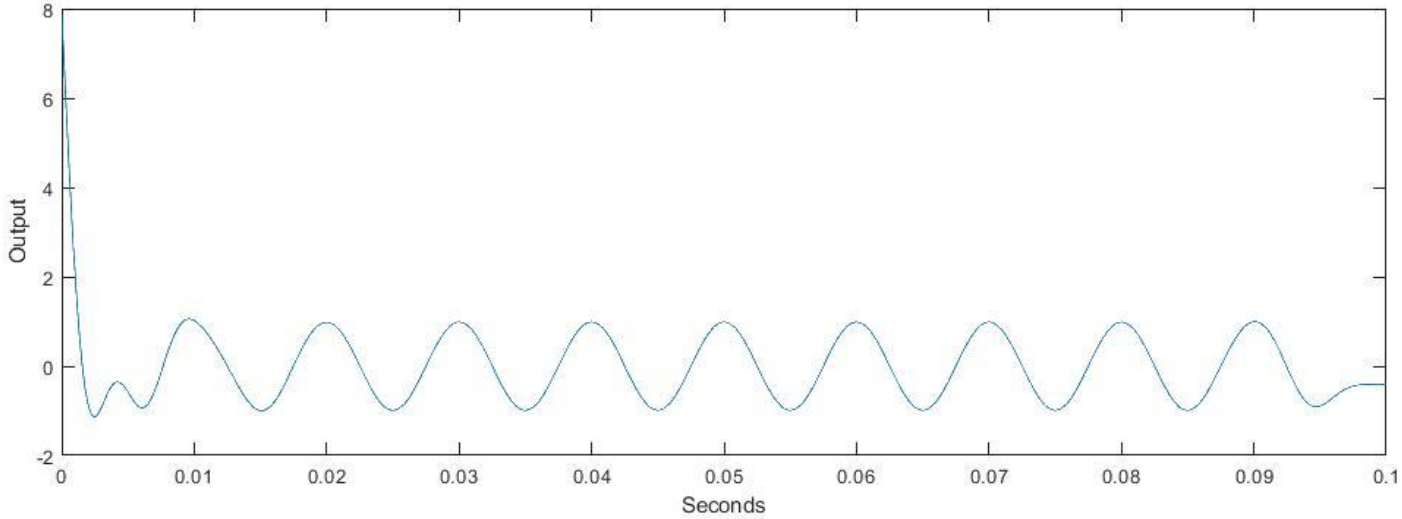
APPLICATION OF THE IIR FILTER

In this task, we are going to apply our IIR Butterworth filter to a noisy signal. The code of the application is below;

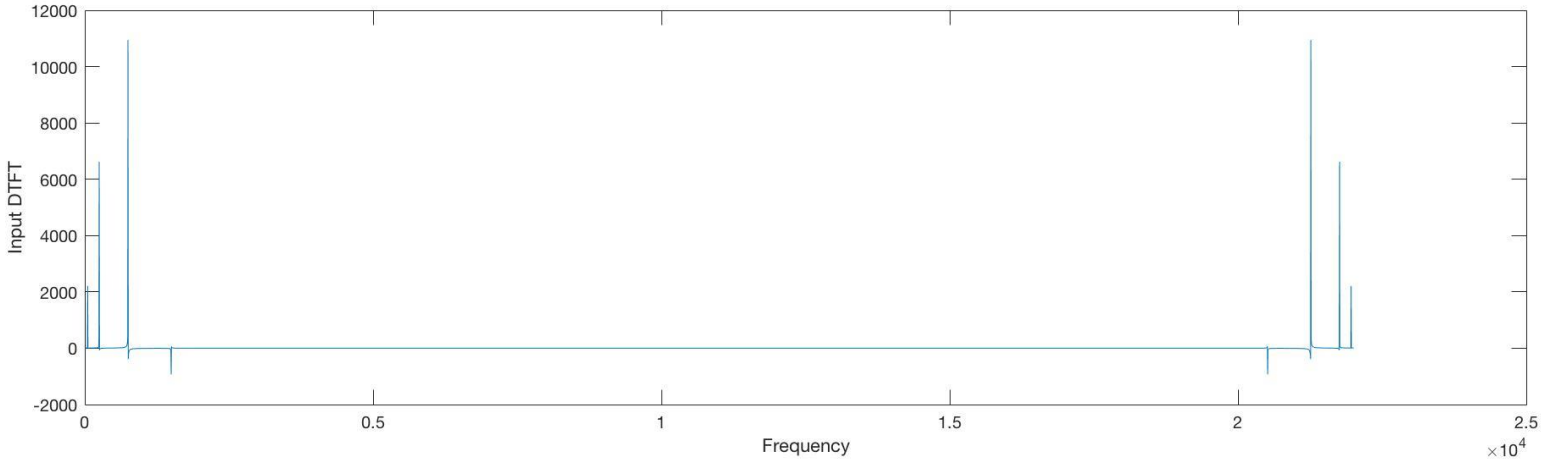
```
%% Initialize parameters - DON'T EDIT
load question2_signal;
load low_pass_coefficients;

Fs = 44100; Ts = 1/Fs;
%
%% Place your code here... - EDIT HERE
wp = 2*200*Ts; % Pass-band edge frequency
ws = 2*400*Ts; % Stop-band edge frequency
[h,w]=freqz(bL,aL); % The magnitude and phase frequency values from the frequency analysis.

% Plot the input and output in two subplots
figure(1)

subplot(2,1,1)
plot(t, x); % The graph of the value of the input signal.
xlabel('Seconds'); ylabel('Input');

subplot(2,1,2)
y=filtfilt(Bl,aL,x); % Filtering the input signal with Butterworth digital filter.
plot(t,y); % The graph of the value of the filtered signal (output).
xlabel('Seconds'); ylabel('Output');

% Plot the input and output Fourier plots (FFT plots) in two subplots

expanded_w=w(1):(w(512)-w(1))/4410:w(512); % Expanding of frequency array to the same length input signal's matrix has.
maxfrequency=expanded_w/(2*pi*Ts); % Setting the new frequency value of recently expanded frequency matrix's length.

figure(2)

input_FFT = fft(x); % The DTFT of the input signal.

subplot(2,1,1)
plot(maxfrequency, input_FFT); % The graph of DTFT of the input signal respect to the frequency
xlabel('Frequency'); ylabel('Input DTFT');

output_FFT = fft(y); % The DTFT of the output signal.

subplot(2,1,2)
plot(maxfrequency, output_FFT); % The graph of DTFT of the output signal respect to the frequency
xlabel('Frequency'); ylabel('Output DTFT');

% End of your code - END OF EDIT
```

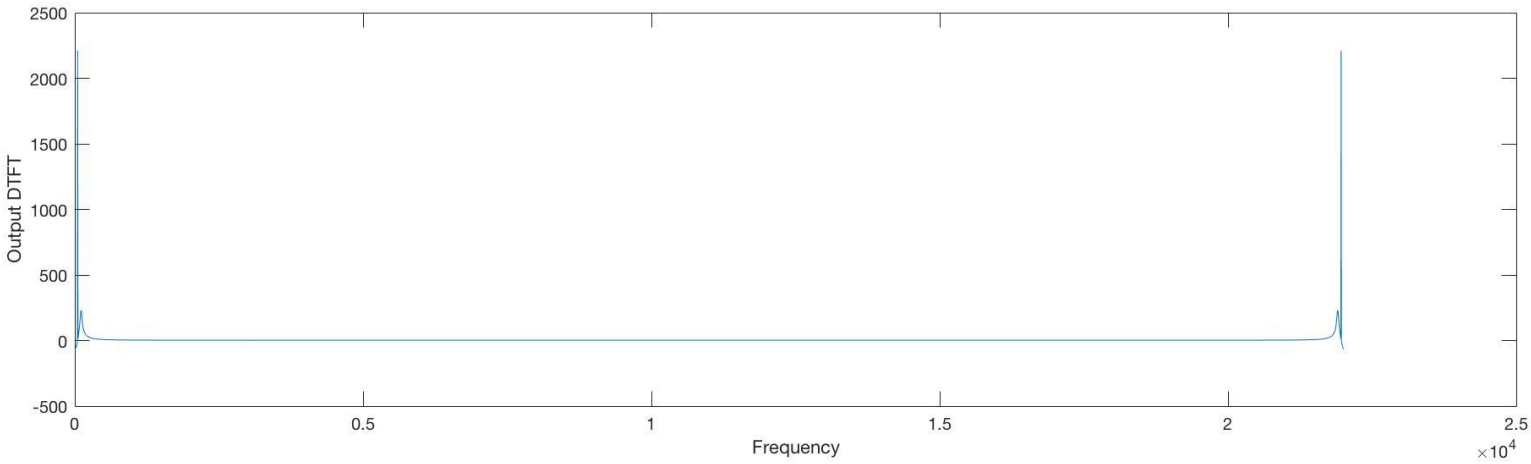
The raw (input) and filtered (output) signals are as below;





The DTFT of raw (input) and filtered (output) signals are as below;





DESIGN OF A FIR FILTER

In this task, we are going to create a FIR filter using Parks-McClellan algorithm. The filter will be seventh order. The ideal differentiator is;

$$H(j\Omega) = j\Omega$$

$$|H(j\Omega)| = \Omega$$

We know that $\omega = \Omega T_s$. The sampling frequency is $F_s = \frac{44100}{40} = 1102.5\ Hz$. Then, the digital differentiator filter specifications are;

$$\left|H\left(e^{j\omega}\right)\right| = \omega F_s, \qquad |H(e^{j0})| = 0\ , \qquad |H(e^{j\pi})| = \pi F_s$$

The code of this filter is like below;

```
%% Initialize parameters
W = 40;
Fs = 44100/W;
Ts = 1/Fs;
N = 7; % The order of the filter

f = [0 1];      % Frequency band edges (as pi)
a = [0 pi*Fs];   % Desired amplitudes


%% Place your code here...
%  Design a FIR differentiator with Parks-Mccellan
%  Set bD as your filter coefficients

bD = firpm(N,f,a,'d'); % The coefficients of the filter, d parameter has been added for optimizing the filter

% Plot the frequency response of your filter
% y axis is magnitude (not in dB)
% x axis is frequency in Hz. Plot for the range [0 - Fs/2]
% Place your plot code here,

[H,W]=freqz(bD); % Getting the frequency and magnitude values from the frequency analysis
plot(W/(2*pi*Ts),abs(H)); % Drawing of the designed filter's response until the Fs/2
hold on;
plot([0 Fs/2], [0 pi*Fs], '--r'); % Drawing of the ideal filter's response
hold off;
legend('Filter', 'Ideal Differentiator', 'Location', 'NorthWest')

% End of your code - END OF EDIT
%% Plot the results - DON'T EDIT


% Print the coefficients
clc
for k = 1:N+1
    fprintf('b(%d) = %8.6g, \n', k-1, bD(k));
end

% Clear needless variables and save the filter
clearvars -except bD
save differentiator_coefficients;
```

The filter coefficient b_i is a 8 element array and its values are;

$$b_0 = -31.8985$$

$$b_1 = 73.6609$$

$$b_2 = -162.2599$$

$$b_3 = 1408.4$$

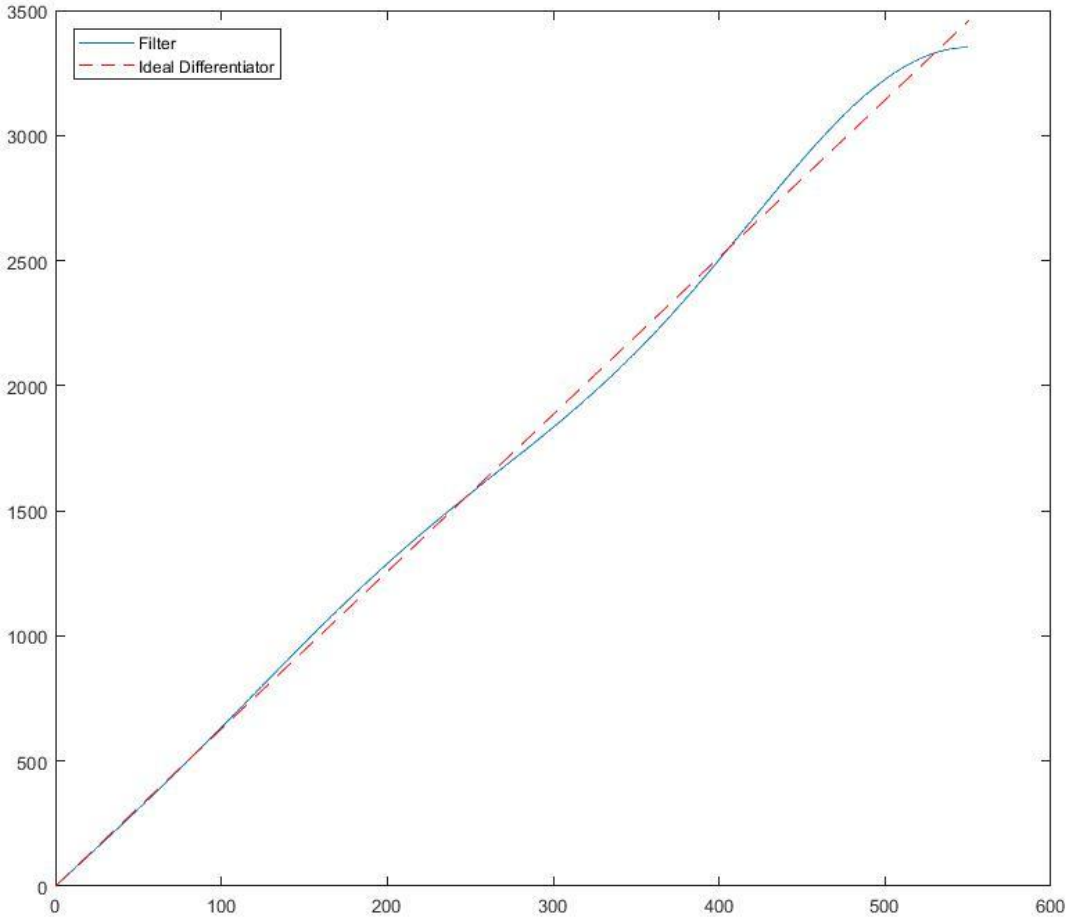
$$b_4 = 1408.4$$

$$b_5 = -162.2599$$

$$b_6 = 73.6609$$

$$b_7 = -31.8985$$

The magnitude response of designed and ideal filter is below;

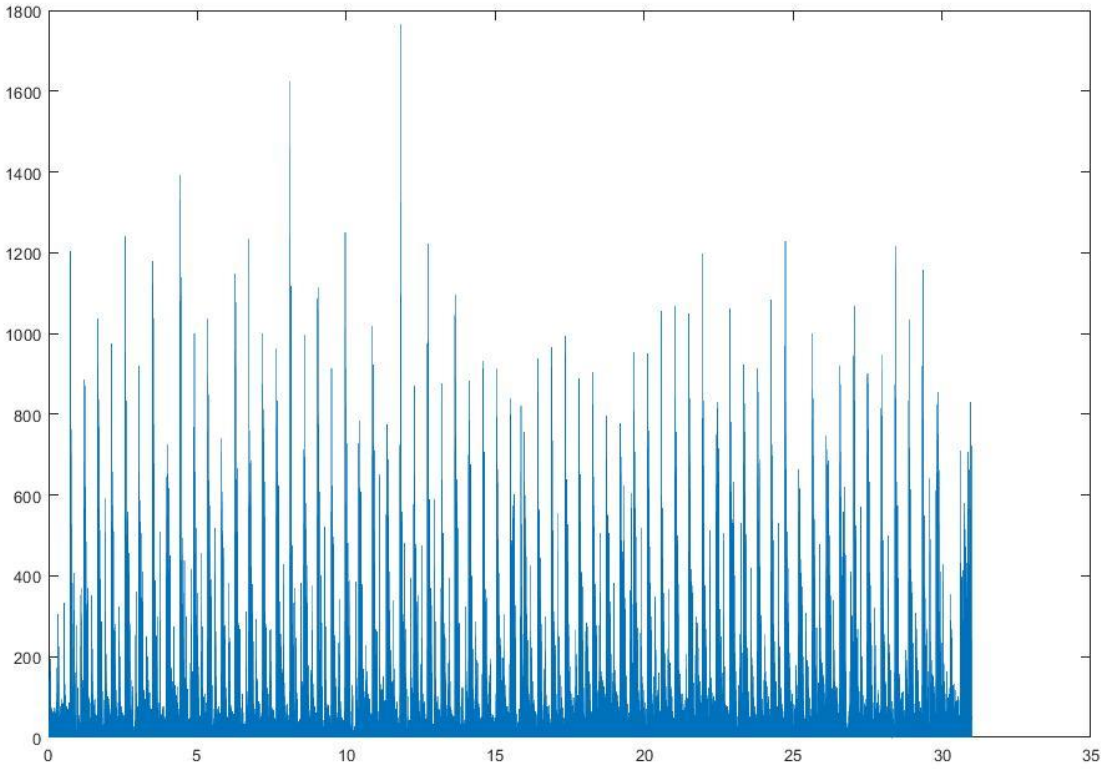


APPLICATION OF BEAT-TRACKER

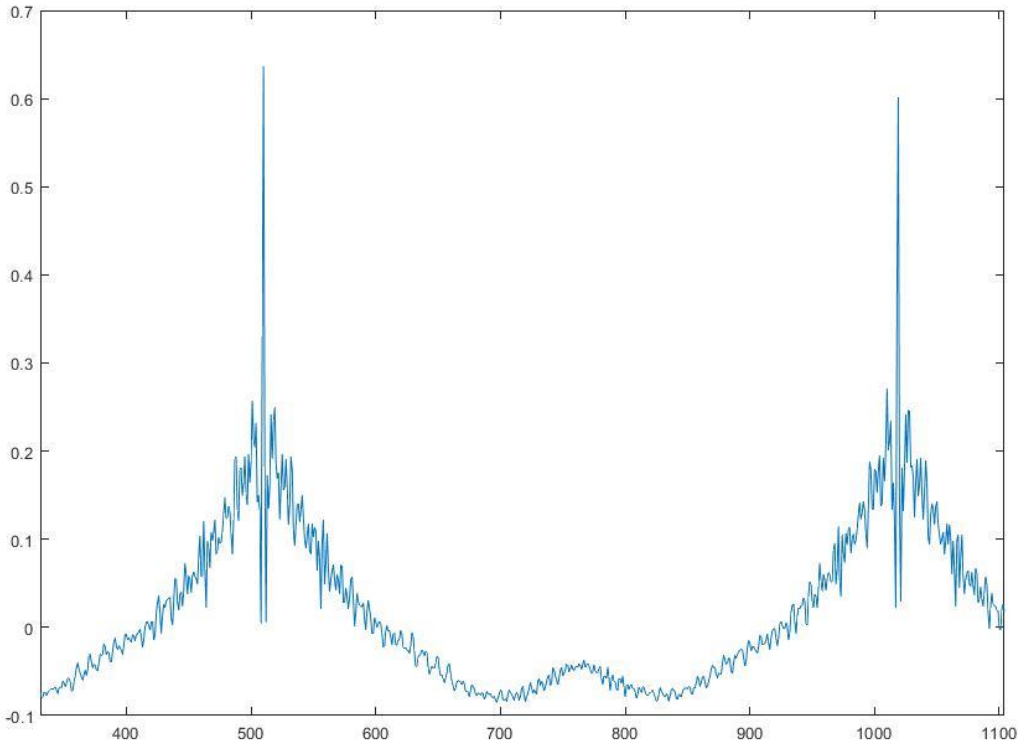
In this task, we are going to implement a tempo estimation algorithm. Following codes are going to do the job;

```
load barbie_girl;
load low_pass_coefficients;
load differentiator_coefficients;

Fs=44100;
Ts = 1/Fs;
xL=filtfilt(bL,aL,x); % Application of the IIR filter

L = length(xL);
W = 40;

% Calculating the energy of the signal

for k = 1:floor(L/W)
    xp = xL( (k-1)*W+1 : k*W );
    E(k) = sum(xp.^2);
end

Fs = 44100/W;
Ts = 1/Fs;
Ef=filter(bD,1,E); % Application of the FIR filter

Edh=Ef;

% Rectifying the signal

for k=1:length(Edh)
    if Edh(k)<=0
        Edh(k)=0;
    end
end

expanded_t=t(1):(t(length(t))-t(1))/(length(Edh)-1):t(length(t)); % Arranging the time matrix for
plotting

figure(1)
plot(expanded_t,Edh); % Plotting the rectified signal

AC=autocorr(Edh,1103,331,3); % Auto correlating the rectified signal
ACwoO=AC;
ACwoO(1)=0; % Neglecting the first value of the matrix, because it is the max value(1)

[value,index] = max(ACwoO); % Getting the maximum value's indice

bpm=round(60/((index-1)*Ts)); % Calculating the BPM

figure(2)
autocorr(Edh,1103,331); % Drawing of the auto correlation
xlim([331 length(AC)]); % Limiting the x-axis in the drawing

trainmat=zeros(size(Edh));

% Identifying the impulse train matrix
for i=1:length(Edh)
    if mod(i,index-1)==0
        trainmat(i)=1;
    end
end

CC=crosscorr(Edh,trainmat,1500); % Cros correlation of the rectified signal

figure(3)
crosscorr(Edh,trainmat,1500); % Drawing of the cross correlation
xlim([-1500 0]); % Limiting the x-axis in the drawing

max_val=299; % The first maximum is at -304 as can be seen from the cross correlation plot

time_start=max_val*Ts; % Calculating the starting time of the song

fprintf('BPM = %d, Start = %0.2ds', bpm, time_start); % Printing the bpm and the starting time
values
```

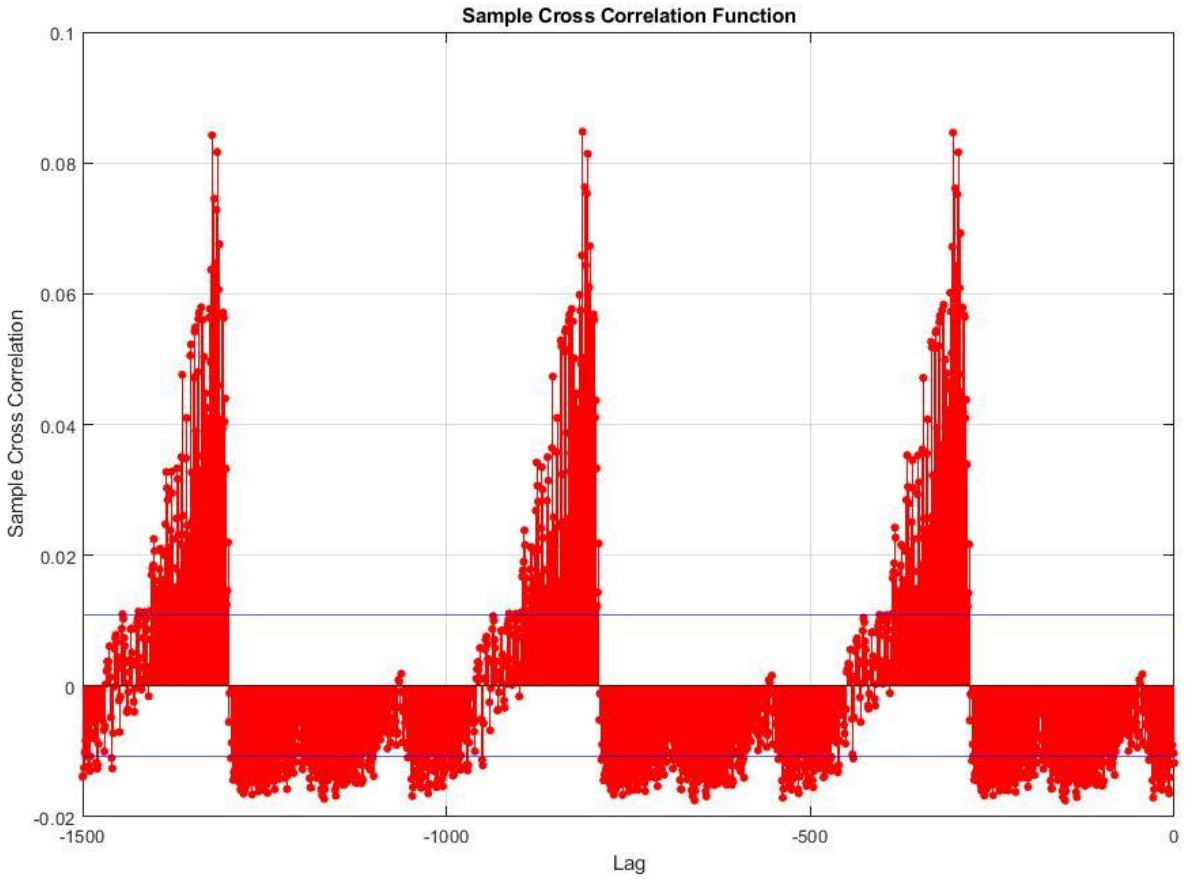
The rectified half-wave signal is below;



The auto-correlated function plot is below;



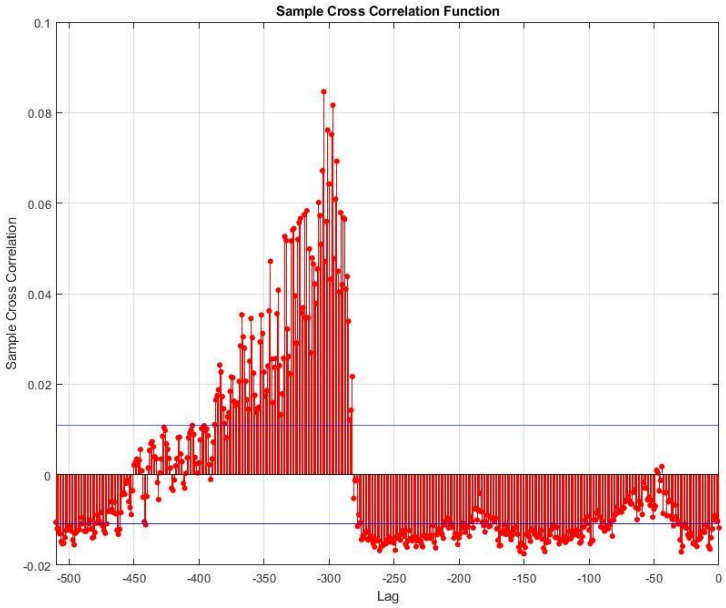
The cross-correlated function plot is below;



The interval between two beats is defined as;

$$\Delta N = round\left(\frac{60}{BPMxT_s}\right)$$

Since our ΔN value is 509, we will get the BPM value as **130**.



The first beat time is defined as $t_1 = N_1 T_s$, we can get the N_1 value from the $-\Delta N, 0$ intervaled cross-corelation plot. The x-axis value at the maximum is **-304**. So our N_1 value is going to be **304**. Our starting time will be **0.276s**.