**Kinematics and Dynamics of Robotic Mechanisms**

Midterm 01 10 Kasım 2015 Salı

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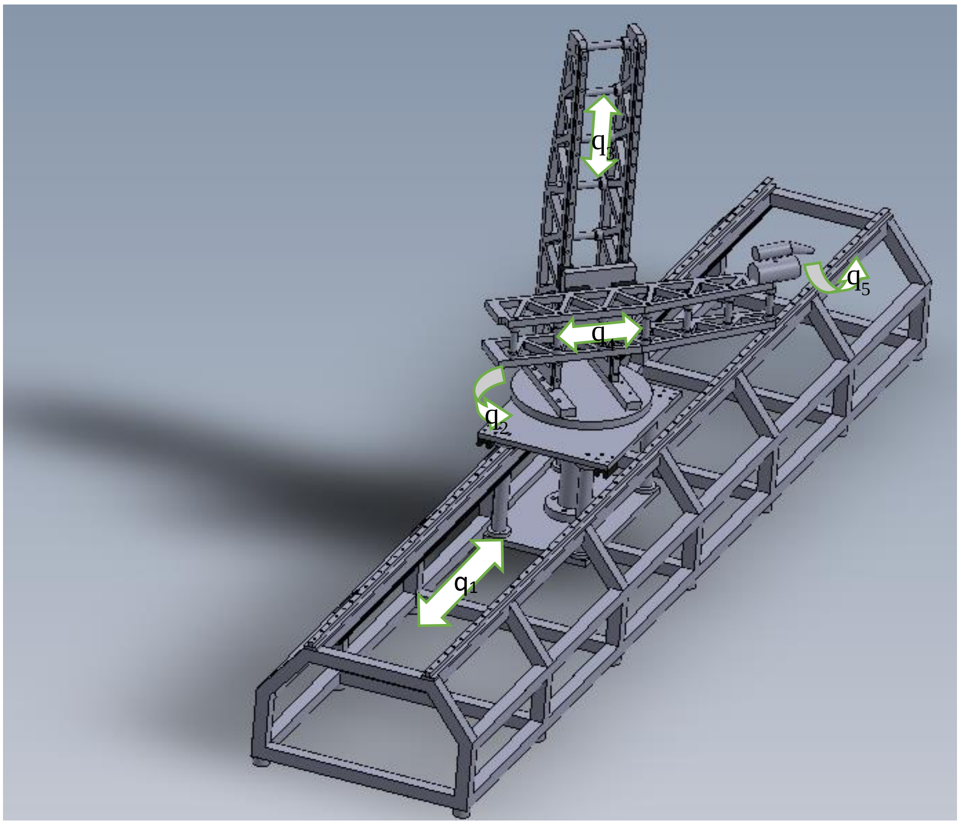
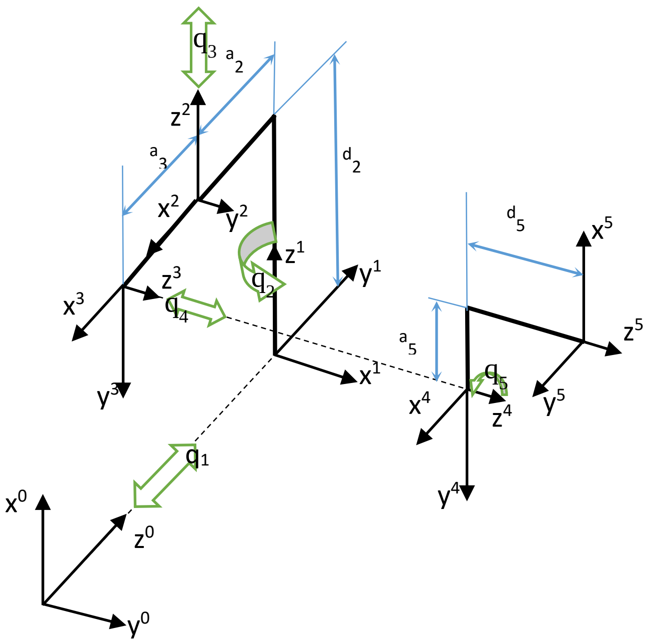
1. A welding robot is given as a solid model. **Joint variables** and **assigned coordinate frames** are also provided.



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a. Determine the **kinematic parameters** (2 joint par. + 2 link par.) and show them in **kinematic parameter table**.
b. Find out the **link coordinate transformation matrices** for each link
c. Find out the **arm matrix** that relates the tip of the end effector to the base
**d. Cross check every transformation stage and final base→tool tip arm matrix** so that they give correct results.

| | θ mafsal açısı | d mafsal uzunluğu | a kol uzunluğu | α burgu açısı |
|---|---|---|---|---|
| 1 | 90° | $q_1$ | 0 | 90° |
| 2 | $q_2$ | $d_2$ | $a_2$ | 0 |
| 3 | 0 | $q_3$ | $a_3$ | -90° |
| 4 | 0 | $q_4$ | 0 | 0 |
| 5 | $q_5$ | $d_5$ | $a_5$ | 0 |

| ***Koordinat sistemleri arası dönüşümler*** | **Açıklama** | ***Tabana göre dönüşümler*** |
|---|---|---|
| $T_0^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | **z ekseni x'e, x ekseni y'ye, y ekseni z'ye dönüşüyor. $L_1$, $L_0$'a göre, z ekseninde $q_1$ mesafesinde** | $T_0^2 = \begin{bmatrix} 0 & 0 & 1 & d_2 \\ C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & q_1 + a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $T_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | **$L_2$, $L_1$ ile aynı yönde z eksenine sahip** | |
| $T_2^3 = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | **x ekseni x'e, z ekseni y'ye, -y ekseni z'ye dönüşüyor. $L_3$, $L_2$'a göre, z ekseninde $q_3$ ve x yönünde $a_3$ mesafesinde** | $T_0^3 = \begin{bmatrix} 0 & -1 & 0 & d_2 + q_3 \\ C_2 & 0 & -S_2 & C_2(a_2 + a_3) \\ S_2 & 0 & C_2 & q_1 + S_2(a_2 + a_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $T_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | **x ekseni x'e, y ekseni y'ye, z ekseni z'ye dönüşüyor. $L_4$, $L_3$'a göre, z ekseninde $q_4$ mesafesinde** | $T_0^4 = \begin{bmatrix} 0 & -1 & 0 & d_2 + q_3 \\ C_2 & 0 & -S_2 & C_2(a_2 + a_3) - q_4S_2 \\ S_2 & 0 & C_2 & q_1 + S_2(a_2 + a_3) + q_4C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $T_4^5 = \begin{bmatrix} C_5 & -S_5 & 0 & a_5C_5 \\ S_5 & C_5 & 0 & a_5S_5 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | **$L_5$, $L_4$ ile aynı yönde z eksenine sahip** | $T_0^5 = \begin{bmatrix} -S_5 & -C_5 & 0 & d_2 + q_3 - a_5S_5 \\ C_2C_5 & -C_2S_5 & -S_2 & C_2(a_2 + a_3) - S_2(d_5 + q_4) + a_5C_2C_5 \\ S_2C_5 & -S_2S_5 & C_2 & q_1 + S_2(a_2 + a_3) + C_2(d_5 + q_4) + a_5S_2C_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |

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2. Arm matrix of a robot is given below.

$$T_0^5 = \begin{bmatrix} -S_5 & -C_5 & 0 & d_2 + q_3 - a_5 S_5 \\ C_2 C_5 & -C_2 S_5 & -S_2 & C_2(a_2 + a_3) - S_2(d_5 + q_4) + a_5 C_2 C_5 \\ S_2 C_5 & -S_2 S_5 & C_2 & q_1 + S_2(a_2 + a_3) + C_2(d_5 + q_4) + a_5 S_2 C_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Writing down **tool configuration vector**, obtain equations for joint variables in any order. (**inverse kin**.)

$q_1$, $q_2$, $q_3$, $q_4$, $q_5$ = ?

$$w(q) = \begin{bmatrix} d_2 + q_3 - a_5 S_5 \\ C_2(a_2 + a_3) - S_2(d_5 + q_4) + a_5 C_2 C_5 \\ q_1 + S_2(a_2 + a_3) + C_2(d_5 + q_4) + a_5 S_2 C_5 \\ 0 \\ -exp(q_5/\pi) S_2 \\ exp(q_5/\pi) C_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}$$

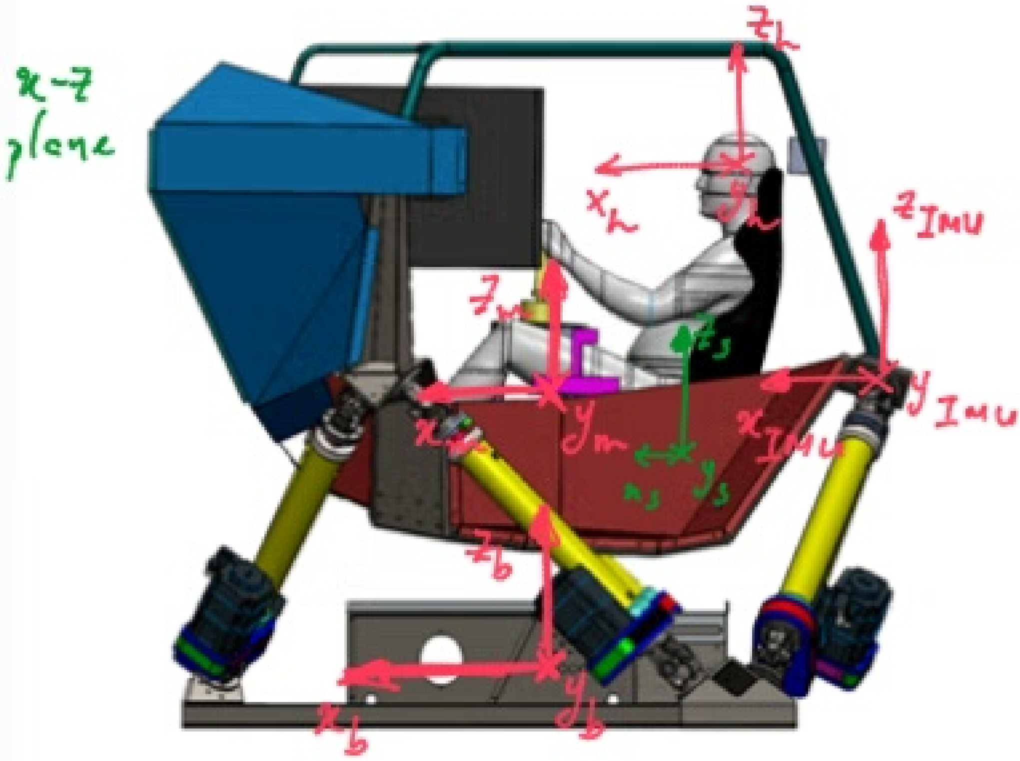
$$q_5 = \frac{pi}{2} \ln({w_5}^2 + {w_6}^2)$$

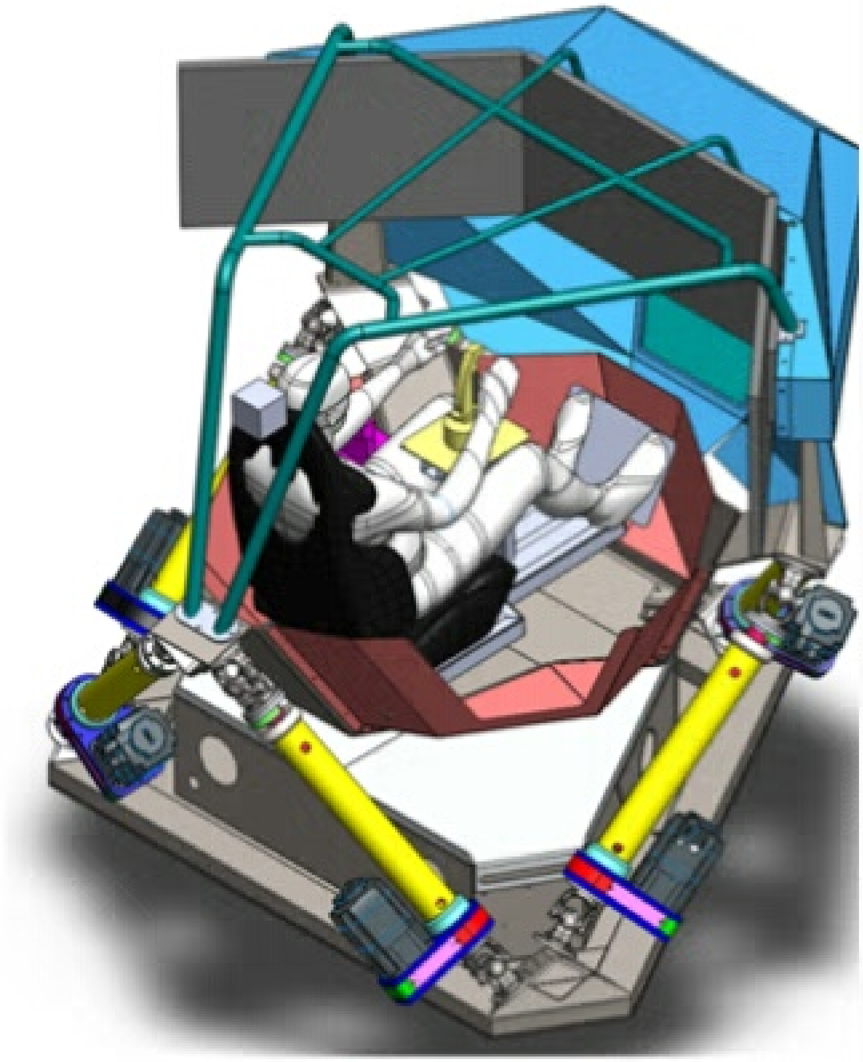
$$q_3 = w_1 - d_2 + a_5 Sin\left[\frac{pi}{2} \ln({w_5}^2 + {w_6}^2)\right]$$

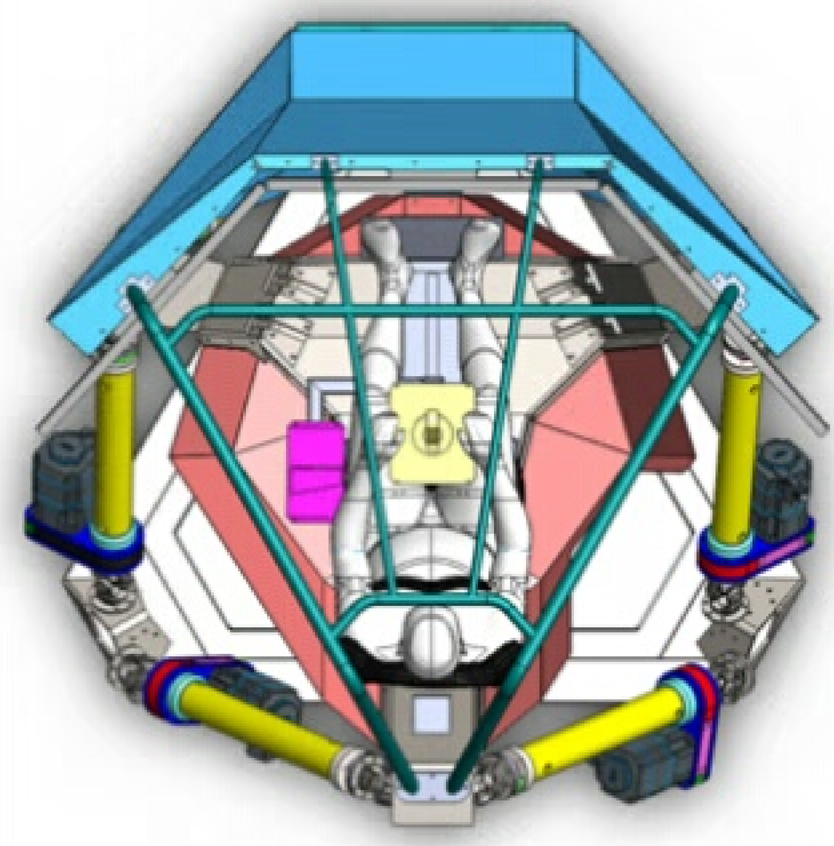
$$q_2 = atan2\left(-\frac{w_5}{w_6}\right)$$

$$q_4 = \frac{a_5 C_2 C_5 + C_2(a_2 + a_3) - d_5 S_2 - w_2}{S_2}$$

$$q_1 = w_3 - S_2(a_2 + a_3) - C_2(d_5 + q_4) - a_5 S_2 C_5$$





since coordinate frame centers on x-z plane:

$\vec{r}_{m/b} = \{0, 0, z_m\}$

$\vec{r}_{s/b} = \{-a_s, 0, c_s\}$

$\vec{r}_{h/b} = \{-a_h, 0, c_h\}$

$\vec{r}_{IMU/b} = \{-a_{IMU}, 0, c_{IMU}\}$

$L_b = \{x_b, y_b, z_b\}$

$L_m = \{x_m, y_m, z_m\}$

$L_s = \{x_s, y_s, z_s\}$

$L_{IMU} = \{x_{IMU}, y_{IMU}, z_{IMU}\}$

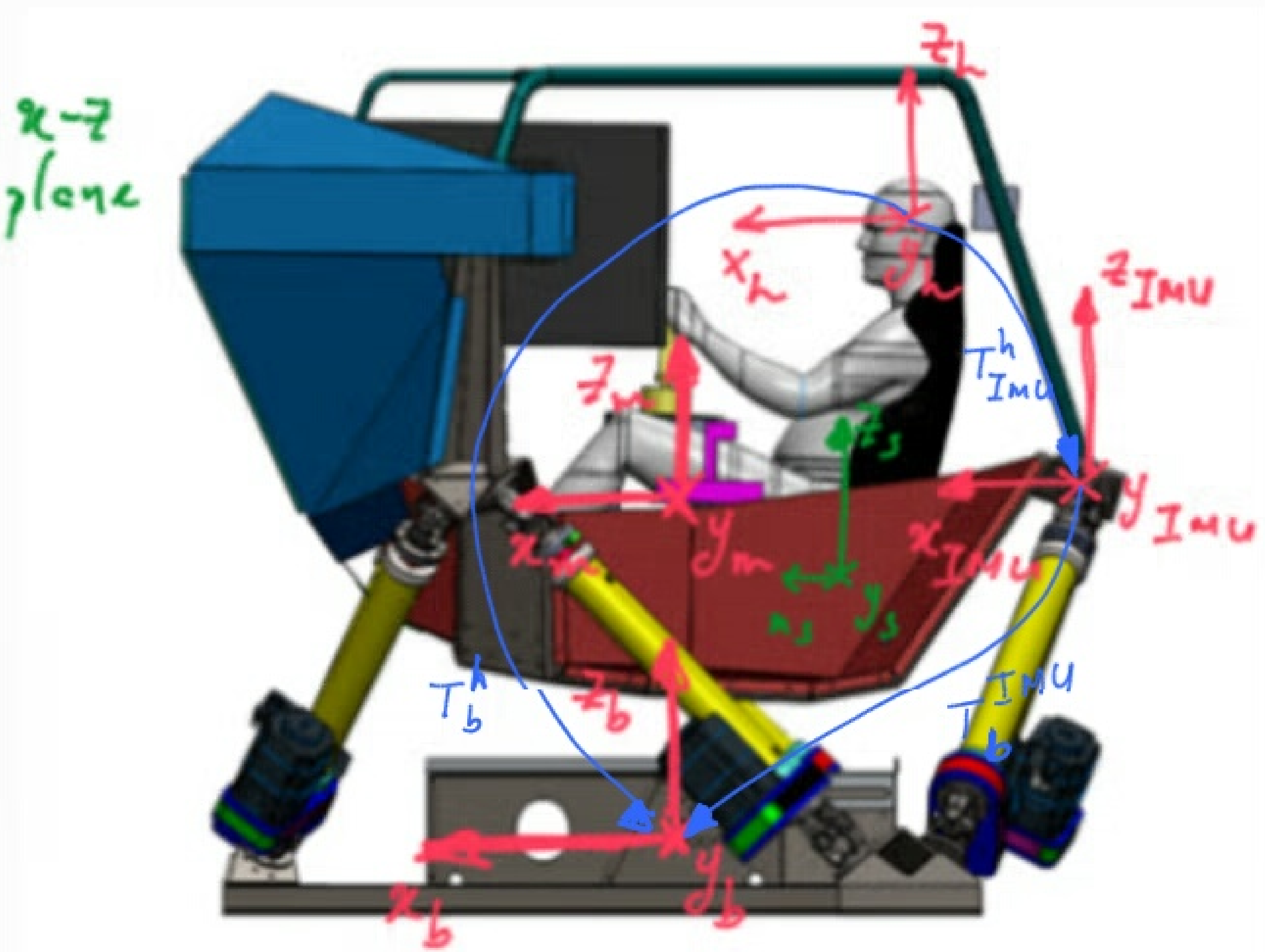
$L_h = \{x_h, y_h, z_h\}$

$$T_b^m = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_m \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$T_b^s = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -a_s \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_s \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$T_b^{IMU} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -a_{IMU} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_{IMU} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$T_b^h = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -a_h \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_h \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$




$$T_b^{IMU} \cdot T_{IMU}^h = T_b^h \Longrightarrow T_{IMU}^h = \left(T_b^{IMU}\right)^{-1} T_b^h$$

$$T_{IMU}^h = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -a_{IMU} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_{IMU} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]^{-1} \cdot \left[\begin{array}{ccc|c} 1 & 0 & 0 & -a_h \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_h \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a_{IMU} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -c_{IMU} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$T^{-1} = \begin{bmatrix} & R^T & & -R^T p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{h}_{IMU} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & a_{IMU} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -c_{IMU} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & -a_h \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_h \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\boxed{T^{h}_{IMU} = \begin{bmatrix} 1 & 0 & 0 & a_{IMU} - a_h \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_h - c_{IMU} \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$