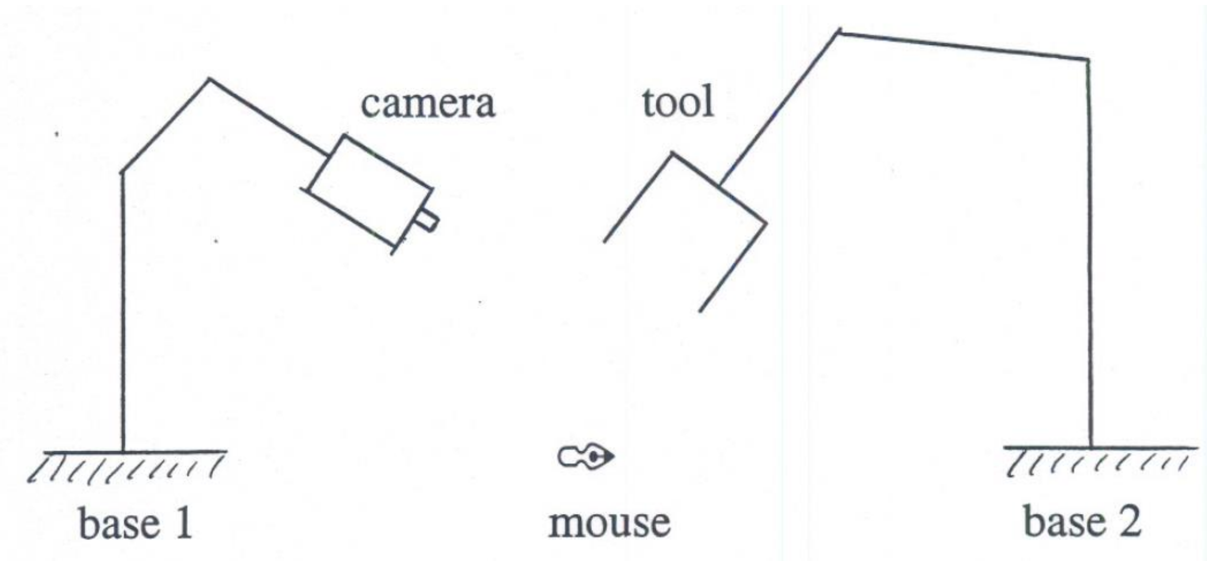
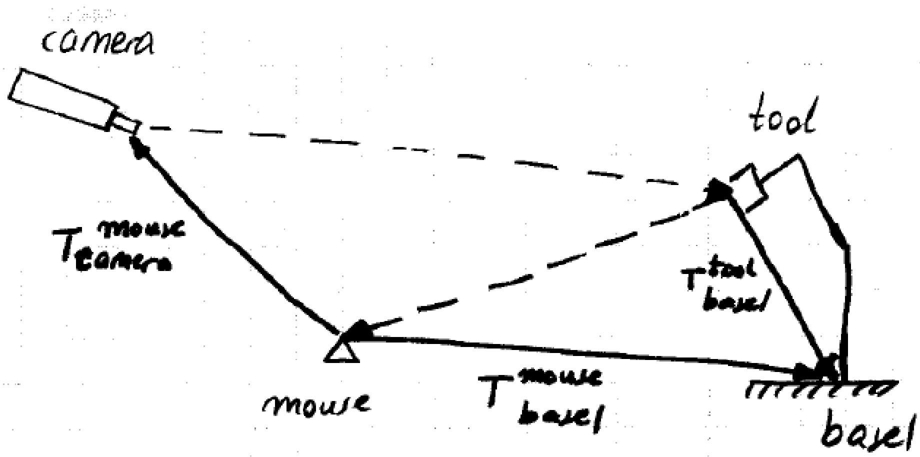
Kinematics and Dynamics of Robotic Mechanisms

Midterm 01 13 Nov 2017

Sur/name: #:

1. For given transformation matrices of T_{base1}^{tool}, T_{base1}^{mouse} and T_{camera}^{mouse}, find T_{tool}^{camera} and T_{mouse}^{tool}. (15 points)





$$T_{base1}^{mouse} = T_{base1}^{tool} \cdot T_{tool}^{camera} \, T_{camera}^{mouse}$$

$$\boxed{(T_{base1}^{tool})^{-1} \, T_{base1}^{mouse} \cdot (T_{camera}^{mouse})^{-1} = T_{tool}^{camera}}$$

$$T_{base1}^{tool} = T_{base1}^{mouse} \, T_{mouse}^{tool}$$

$$\boxed{T_{mouse}^{tool} = (T_{base1}^{mouse})^{-1} \, T_{base1}^{tool}}$$

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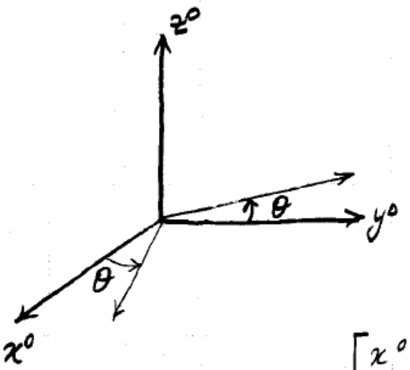
2. Given the homogenous transformation matrix
   a. Sketch the location of frame 1 relative to base frame. Label joint variable. Assume the home position is zero . (8 points)
   b. Consider a point in L_1 frame: $\{x^1, y^1, z^1\} = \{1,2,4\}$. Find the point in base coordinates when θ_1=45° (7 points)

$$T_0^1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$T_0^1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rotation around z + no translation

$$\begin{bmatrix} x^0 \\ y^0 \\ z^0 \\ 1 \end{bmatrix} = T_0^1 \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 - 2\sqrt{2}/2 \\ \sqrt{2}/2 + 2\sqrt{2}/2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 \\ 3/2\sqrt{2} \\ 4 \\ 1 \end{bmatrix}$$

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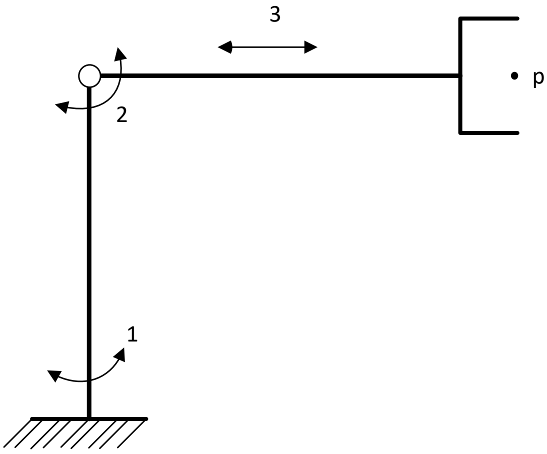
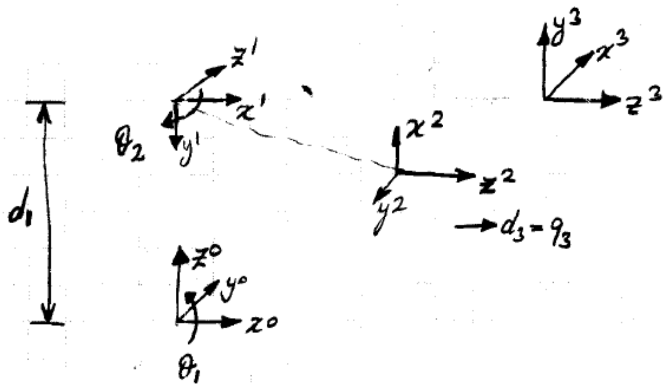
3. A three axis spherical robot is sketched. Joints 1 and 2 are revolute and Joint 3 is prismatic. Assign link coordinate frames using Denavit-Hartenberg representation. Label the link parameters θ_i, d_i, α_i, and a_i. (15 points)





axis	θ_i	d_i	a_i	α_i
1	θ_1	d_1	0	-90
2	θ_2	0	0	-90
3	0	d_3	0	0

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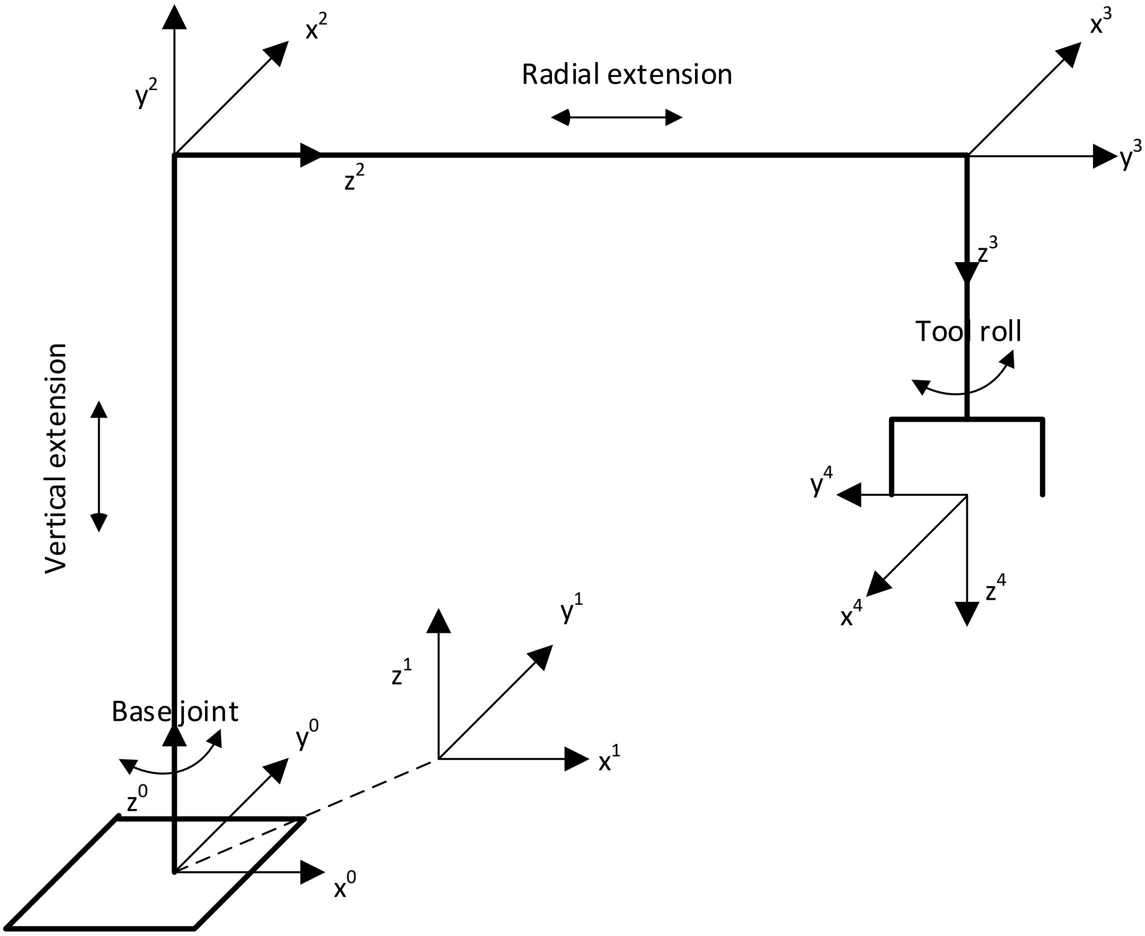
4. A four axis cylindirical robot is sketched below. Link coordinate frames have been assisgned. Fill in the link parameter table. (15 points)



Joint	θ	d	a	α
1	θ_1	0	0	0
2	$\pi/2$	$q_2 = d_2$	0	$\pi/2$
3	0	$q_3 = d_3$	0	$\pi/2$
4	θ_4	d_4	0	0

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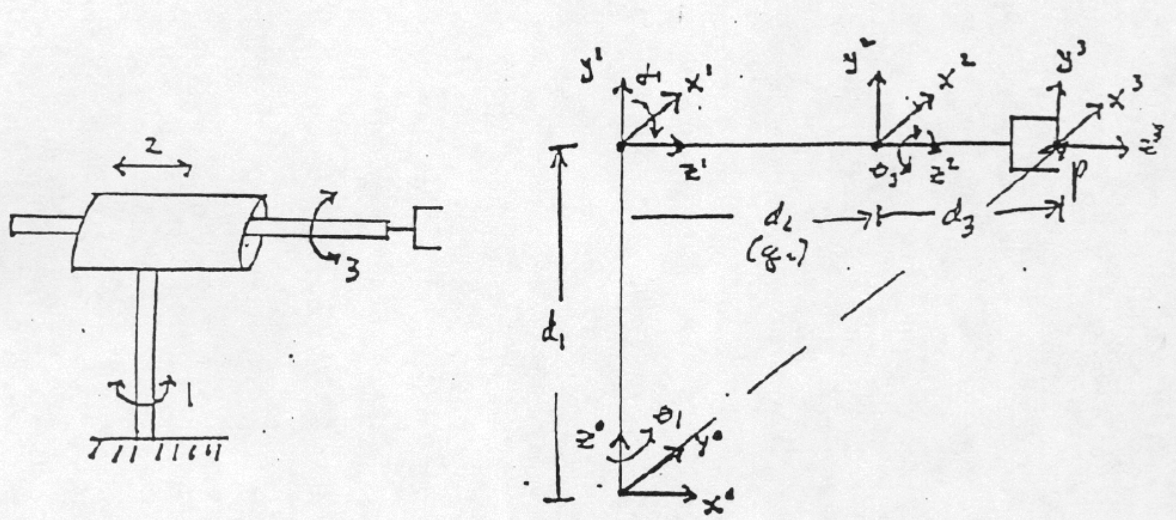
5. The figure below shows a three degree of freedom robot with a revolute joint (#1), a prismatic joint (#2) and another revolute joint (#3) that provides tool roll. The following arm matrix transforms tool coordinates to the base.
    a. find the tool configuration vector (5 points)
    b. find the inverse kinematics, q_1, q_2 and q_3 (15 points)

$$T_{base}^{tool} = T_0^3 = \begin{bmatrix} C_1C_3 & -C_1S_3 & S_1 & S_1(d_2+d_3) \\ S_1C_3 & -S_1S_3 & -C_1 & -C_1(d_2+d_3) \\ S_3 & C_3 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

note: $d_2 = q_2$



$$w(q) = \begin{bmatrix} S_1(d_2+d_3) \\ -C_1(d_2+d_3) \\ d_1 \\ S_1 e^{\frac{q_3}{\pi}} \\ -C_1 e^{\frac{q_3}{\pi}} \\ 0 \end{bmatrix}$$

$q_3 = \pi \ln \sqrt{w_4^2 + w_5^2}$

Check: $w_4^2 + w_5^2 = e^{2q_3/\pi}$

$\sqrt{w_4^2+w_5^2} = e^{q_3/\pi}$

$\ln \sqrt{w_4^2+w_5^2} = \frac{q_3}{\pi}$

Other possibilities: $q_2 = S_1 w_1 - C_1 w_2 - d_3$

$q_1 = \text{atan2}(w_4, -w_5)$, $q_3 = \pi \ln [S_1 w_4 - C_1 w_5]$

$q_1 = \text{atan2}(w_1, -w_2)$

the projection of the p-vector onto the x^0-y^0 plane only depends on d_2 $(= q_2)$.

$w_1^2 + w_2^2 = (d_2+d_3)^2 \Rightarrow d_2 = q_2 = \sqrt{w_1^2+w_2^2} - d_3$

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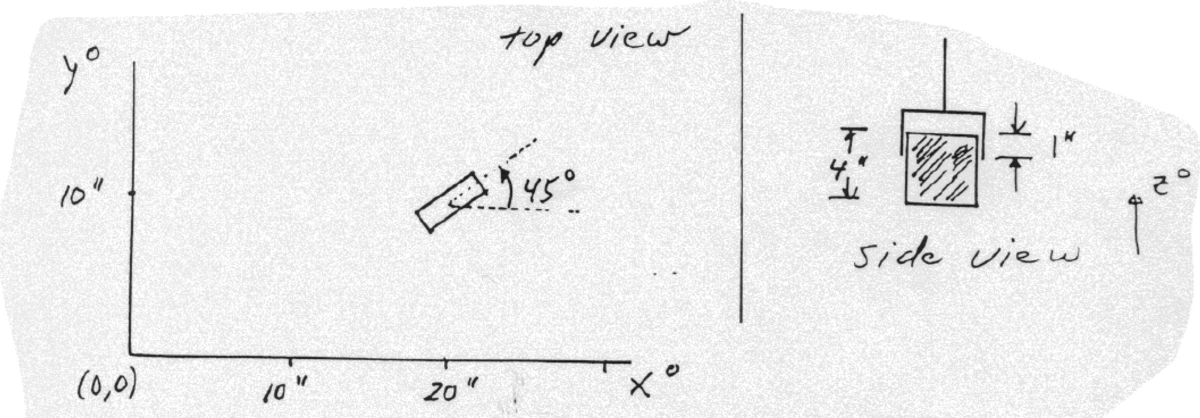
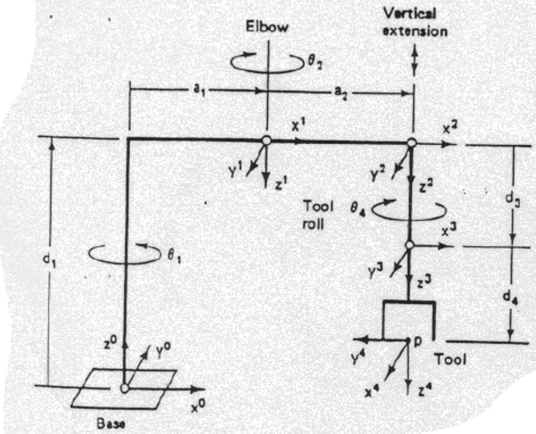
6. A scara robot is used to insert a printed circuit board in a slot as shown below. The printed circuit board is 4”x4”. Assume the gripper grasps the board on the edges and 1” below the top. Find the tool configuration vector, w, to carry out this task. Specify w just as the printed circuit board is about to enter the slot. (20 points)





$$e^{q_n/\pi} \Rightarrow q_n = q_4 = -\pi/4 \qquad w = \begin{bmatrix} 20 \\ 10 \\ 3 \\ 0 \\ 0 \\ -e^{-1/4} \end{bmatrix}$$

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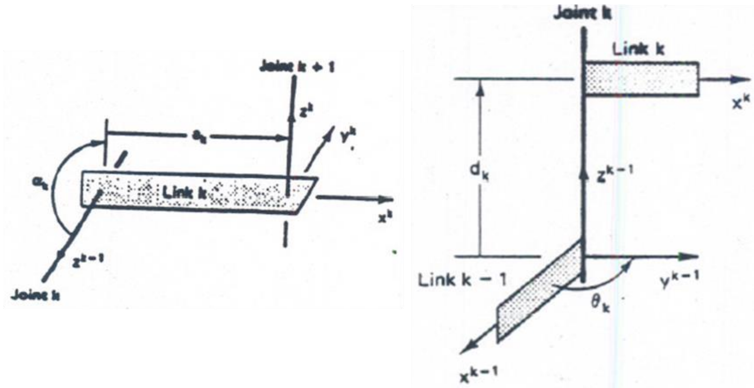
$$Rot(x,\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & C\theta & -S\theta\\ 0 & S\theta & C\theta\end{bmatrix}\quad Rot(y,\theta)=\begin{bmatrix}C\theta & 0 & S\theta\\ 0 & 1 & 0\\ -S\theta & 0 & C\theta\end{bmatrix}\quad Rot(z,\theta)=\begin{bmatrix}C\theta & -S\theta & 0\\ S\theta & C\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

Kinematic Parameters: 2 joint (*Joint angle* (θ_k) ve *Joint distance* (d_k)) + 2 link (*Link length* (a_k) ve *Twist angle* (α_k))



$$[q]^{k-1} = T_{k-1}^{k}\cdot[q]^{k}$$

$$T_{k-1}^{k}=\begin{bmatrix}C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k\\ S\theta_k & C\alpha_k C\theta_k & -C\theta_k S\alpha_k & a_k S\theta_k\\ 0 & S\alpha_k & C\alpha_k & d_k\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix} & R & & p\\ & & & \\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$T_{base}^{tool}(q)$$

task conf. space — tool conf. space

$$w=\begin{bmatrix}w_1\\ w_2\\ w_3\\ w_4\\ w_5\\ w_6\end{bmatrix}=\begin{bmatrix}p_1\\ p_2\\ p_3\\ r_1^3 e^{q_n/\pi}\\ r_2^3 e^{q_n/\pi}\\ r_3^3 e^{q_n/\pi}\end{bmatrix}$$