1 Properties of Context-Free Languages (CFLs)

1. Consider the following CFG:

$$S \to A1B$$
$$A \to 0A \mid \epsilon$$
$$B \to 0B \mid 1B \mid \epsilon$$

   a) Convert it to the Chomsky Normal Form (CNF), showing the intermediate simplification steps.

   b) Show how the CYK algorithm accepts the string 0101.

2. Consider the following CFG:

$$S \to 0S00 \mid 0B0 \mid B$$
$$B \to 11B22 \mid 12 \mid C$$
$$C \to 0 \mid \epsilon$$

   a) Convert it to the Chomsky Normal Form (CNF), showing the intermediate simplification steps.

   b) Show how the CYK algorithm accepts the string 000.

3. Show, using the pumping lemma for context-free languages, that the language $\{a^n b^n c^i \mid n \leq i \leq 2n\}$ is not a context-free language.

4. Show, using the pumping lemma for context-free languages, that the language $\{0^p \mid p \text{ prime}\}$, is not a context-free language. [Note: this language does not satisfy the pumping lemma for regular languages; use a similar strategy to prove that this language does not meet the pumping lemma for context-free languages.]

5. Show, using the pumping lemma for context-free languages, that the language $\{0^i 1^j \mid j = i^2\}$, is not a context-free language.

6. Given two string w and x, let us call $inter(w, x)$ to the set of strings obtained interchanging symbols of w and x by the order they occur in w and in x. We can extend the operation to two languages $L1$ and $L2$, naming $inter(L1, L2)$ to the union, for all the pairs of strings w from $L1$ and x from $L2$, of $inter(w, x)$.

   a) Determine the value of $inter(00, 111)$.

   b) Determine the value of $inter(L1, L2)$ with $L1 = L(0^*)$ and $L2 = \{0^n 1^n, n \geq 0\}$.

   c) Show that if $L1$ and $L2$ are both regular languages then $inter(L1, L2)$ is a regular language. [Suggestion: consider the DFAs of $L1$ and $L2$.]

   d) Show that if L is a context-free language and R a regular language then $inter(L, R)$ is a context-free language. [Suggestion: consider a PDA for L and a DFA for R.]