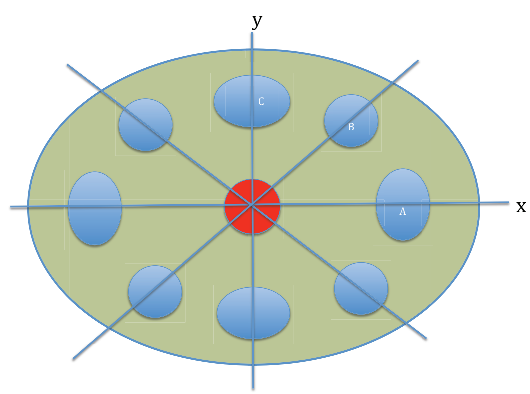
Project

IE 630: Multiobjective Optimization

1 Data Model

1. We will use the following synthetic data for the project.
2. Note that the diagram below is NOT drawn to scale. Rely on the equations and inequalities listed below to infer the exact sizes and locations of the tumor and the organs at risk.
3. The cross section is shown below.



4. Assume that the irradiation can occur only along the eight directions shown in the figure, which are separated by angular increments of 45^o.
5. The green area inside the cross section represents normal tissue.
6. The equation of the large ellipse–the proxy for a cross section of the human body–is

$$\frac{x^2}{100} + \frac{y^2}{25} = 1$$

In all of the equations presented in this document, the unit of length is taken to be an inch. Assume that the z-axis is coming out of the plane of the paper.

7. The red sphere at the center represents the tumor. Assume that it is described by the inequality

$$x^2 + y^2 + z^2 \leqslant 1$$

8. The blue ellipsoids and spheres represent critical organs or Organs At Risk (OARs). Assume that the OARs are located symmetrically about each of the four axes shown. Thus it is adequate to provide the equations of the organs labeled A, B and C.

$$\begin{aligned} A: &\quad (x-3)^2 + 4y^2 + \frac{z^2}{4} &\leqslant 1 \\ B: &\quad 4(x-3)^2 + 4(y-3)^2 + 4z^2 &\leqslant 1 \\ C: &\quad 4x^2 + 9(y-2)^2 + 16z^2 &\leqslant 1 \end{aligned}$$

9. We will make the simplifying assumption that the absorption coefficient for the normal tissue, OAR and tumor are given by α_n, α_o and α_t respectively. Thus, if radiation travels through a normal tissue for a unit length (1 inch) then a fraction α_n of the radiation is absorbed by the tissue.

10. Finally we will assume that the multileaf collimator has a square cross section with the side of the square being 4 inches long. Assume that the thickness of the retractable rods in the collimator is 0.5 inches.

11. Assume that the energy flux of the linear accelerator that generates the high energy radiation is F Joules per inch^2 per second.

12. Assume that the normal tissue, OARs and tumor have densities μ_n kg/m^3, μ_o kg/m^3 and μ_t kg/m^3 respectively.

13. The safe dosage level–measured in centiGray–for normal tissue is s_n and for OARs is s_o respectively.

14. The minimum dosage for the tumor is d_T.

2 Formulation

1. List your decision variables and explain the meaning of each variable clearly.

2. Formulate and describe the objective functions as well as each of the constraints of the problem.