Intensity Modulated Radiation Therapy (IMRT) as a Multi-Objective Optimization Problem

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1. **Problem Formulation**

Let us start by defining the notations. Let the specified region be divided into voxels i.e. the space is divided into small three-dimensional cuboids. Let each voxel be denoted by the coordinates of its center () for voxel. Now, the goal is to find the amount of energy absorbed by each voxel when the collimator of the linear accelerator is emitting the radiation at the body.

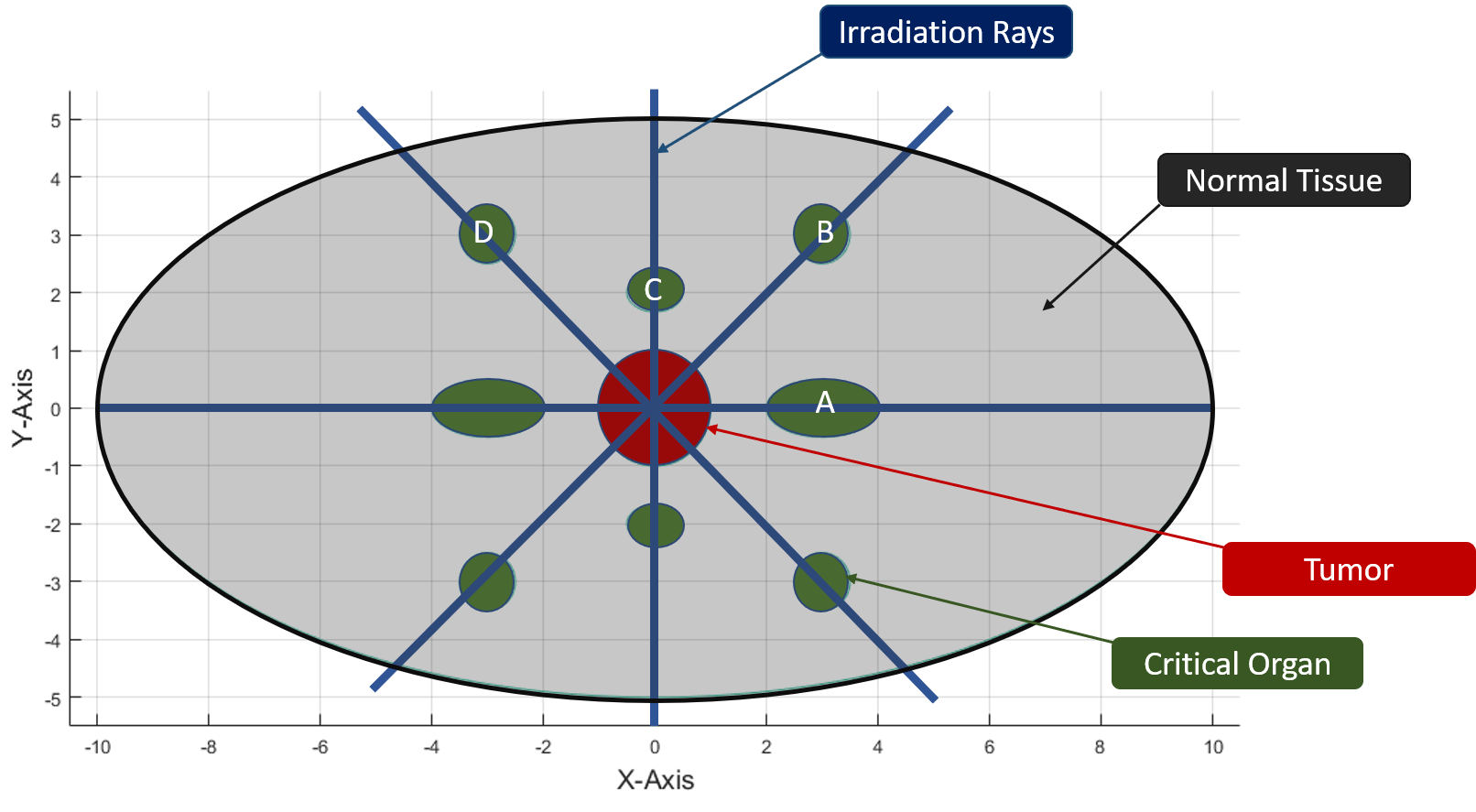


Figure 2. Depicting the tumor (red region), critical organ (dark green region) and normal tissue (rest of the area in the large ellipse marked in gray color).

**1.1 Geometry**

The equation of the large ellipse is given by the following. The diameter along the major axis is 20 units while the one along the minor axis is 10 units. Further, the gray shaded regions inside the large ellipse correspond to the normal tissue, red region in the center of the ellipse is the tumor, and dark green regions are critical organs at risk (OAR) as shown in Figure 2.

(large ellipse)

(red region – tumor)

(A – critical organ at risk)

(B – critical organ at risk)

(C – critical organ at risk)

**1.2 Multi-leaf collimator**

It is assumed that the irradiation can occur only along eight directions (separated by ) as shown in the Figure 2. The z-axis is assumed to be coming out of the paper. Further, the absorption coefficient of the normal tissue, OAR and tumor are given by and respectively. Let intensity of the radiation at be , then, the intensity at is given by the following equation, where is the absorption coefficient of the medium.

Next, we will assume that the multileaf collimator has the square cross section with the side of the square being 4 inches long. The thickness of the retractable rods in the collimator is 0.5 inches. Further, the energy flux of the linear accelerator is assumed to be Joules per inch2 per second. The density of the normal tissue, OAR and tumor are given by and kg/m3 respectively. Next, the safe dosage level for normal tissue and the OAR are given by and centi-gray (J/kg) respectively. Lastly, it is assumed that the safe dosage for the tumor is . Our first task is to discretize the collimator cross section as shown in the Figure 3. The default values of and are set to 0.5. Given the side of the collimator is 4 inches, the cross section is sub-divided into an 8x8 grid. Since the diameter of the collimator is 2 inches, it can be assumed that the side of the collimator is 2 inches. Hence, the collimator is represented as a 4 x 4 grid. The index for each unit is numbered in a raster scan order (1, 2, …, 16). Each unit is referred to as a beamlet.

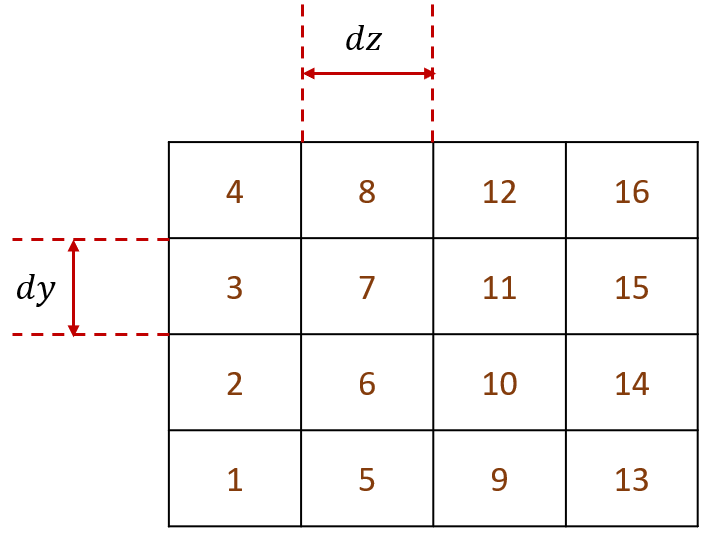


Figure 3. Cross section of the collimator is discretized along y-z axis.

**1.3 Dose influence matrix**

Now, our task is to identify the dose influence matrix , where indicates the index of the collimator cross section and indicates the index of the voxel when the irradiation occurs along the direction A. Similarly, and denote the dose influence matrices along the direction B and C respectively. Note that the large ellipse is discretized in a similar manner as shown in the Figure 4.

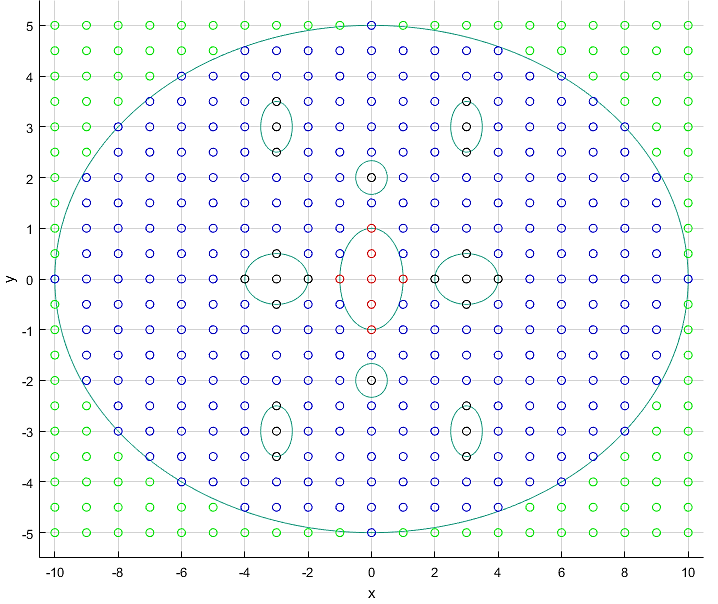


Figure 4. The green points indicate that the voxels are outside the ellipse and the blue points indicate the voxels that are inside the ellipse, red voxels are inside the tumor, black voxels are inside the organs at risk. Though the body is in 3D, we depict a cross section in the x-y plane.

Let be the values between 0 and 1 indicating the percentage of beamlet that is irradiated at the body along the horizontal direction A. If the collimator’s retractable rods cover the entire first column, then the corresponding values are set to be zero. Similarly, and , and be the variables along the directions B, C and D respectively. Only four directions were considered due to the symmetry of the ellipse, organs at risk and the tumor locations. In more words, the right-hand side of the constraints are halved as we considered only four directions.

**1.4 Optimization problem**

Our end goal is to determine the values of the variables that optimize for multiple objective functions while complying with a given set of linear constraints. Assuming that the energy is irradiated in three directions (A, B, C and D), the energy absorbed by voxel is given by . The values of are computed according to the equation: . Note that the , and are the projected area normal to the directions A, B and C respectively. Their values depend on how well the mesh is refined. Note that , and are grid dimensions of the ellipse along and axes.

, , ,

Now, this formulation is used to compute the energy absorbed by tumor (), normal tissue ( ) and the organs at risk (). Let and be the index variables that correspond to the voxels related to tumor, normal tissue and organs at risk.

**1.5 Objective functions**

MOOP objective functions that need to be minimized are given below:

There are three objectives in total. It can be represented as , where is a matrix containing the coefficients of . We want to maximize the dose on the tumor while minimizing the dose on the normal tissue and organs at risk. Since we want to construct a minimization problem we are going to minimize which is equivalent to maximizing the .

**1.6 MOOP Constraints**

MOOP constraints are given below. Note that , and are the volume of the tumor, normal tissue and organs at risk respectively.

This can be represented as . consists of the coefficients present in , and respectively. Similarly, consists of . Note that the values of , and are computed from the voxels (we know if the voxel belongs to one of the three categories: normal tissue, organ at risk and the tumor). We want the dosage on the tumor to be more than a certain threshold. On the other hand, we want the dosage on the normal tissue and the organs at risk to be less than a certain threshold. Hence, we need to multiply the with -1.

Where, the decision variables are: , , and .

In other words, , ,

Final MOOP looks is converted to the standard format:

**1.6 Assumptions**

* Following assumptions were made in solving the IMRT problem. It was assumed that the collimator has only four rods, each measuring 0.5 inches in width. Hence the collimator is divided into 4 x 4 grid, where each cell is square with side 0.5 inches.
* The x, y and z axes of the body were divided into voxels so that there were voxels in each direction. It was found that, when , the volume estimated by the voxels and the real volume of the ellipse (body) are almost equivalent.

1. **Visualization**

**2.1 Dosage on the body**

The dosage on the body when the collimator rods are fully open are computed in for all four directions (A, B, C and D). They are individually depicted in the figures below. Note that the lighter colors indicate that the intensity is maximum, and the darker colors indicate that the intensity is minimum.

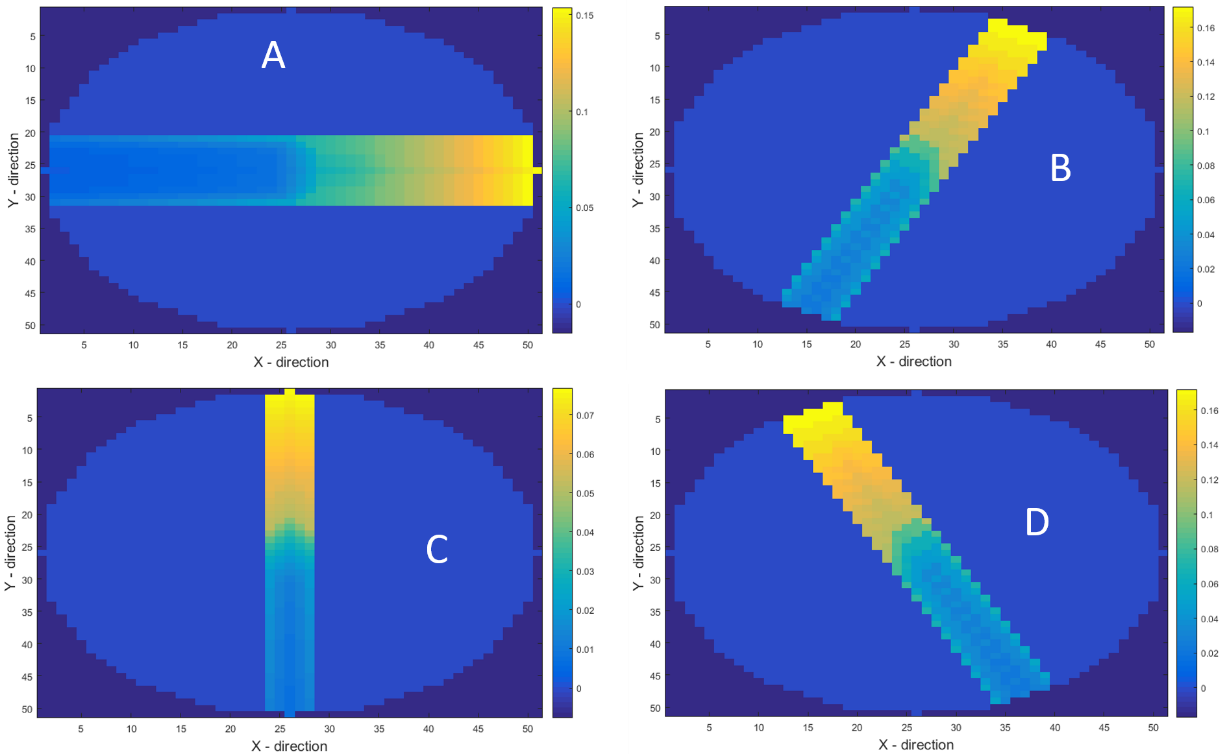


Figure. Dosage on the body when the collimator is placed in A, B, C and D positions, respectively.

When the all the collimator rods in all positions are open, the dosage on the body is shown in the figure below. Note that the dosage on the tumor is highest (voxels are lighter in color) as tumor is the common target point for all four directions.

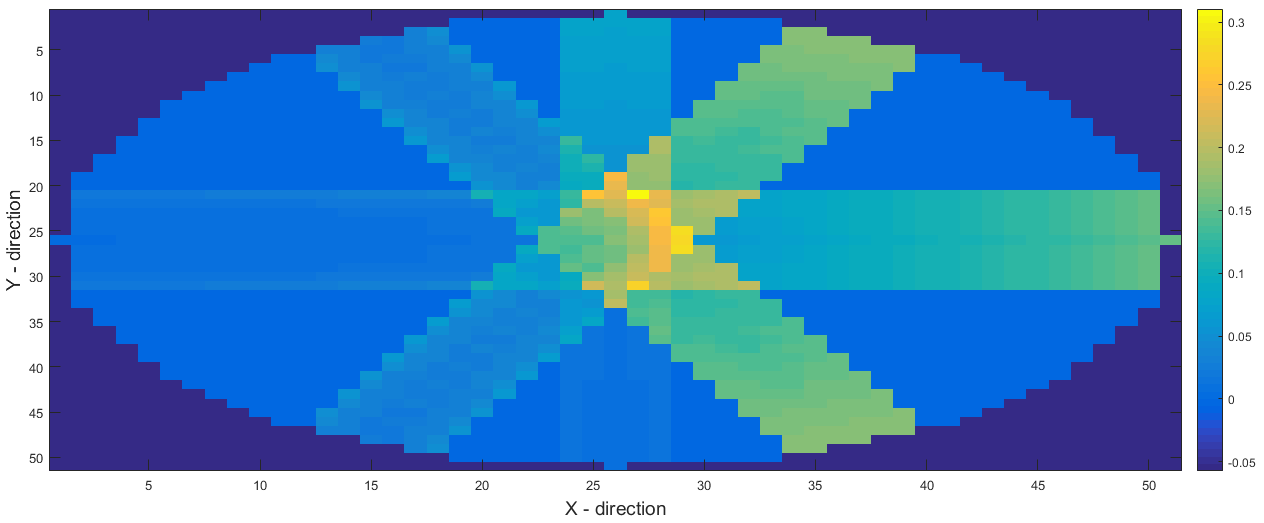


Figure. The dosage on the body when all rods in the collimator are open in all four positions (A, B, C and D).

**2.2 Results obtained from the MOOP**

The results obtained by the MOOP solver at four efficient solutions are given as follows.

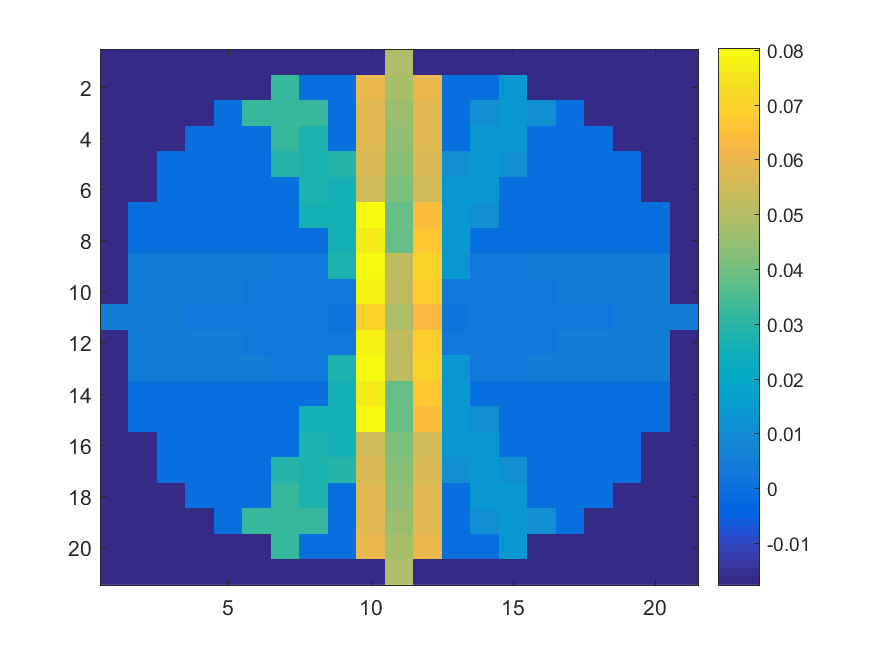


Figure. Solution 8. The intensity absorbed by the tumor, normal tissue and OAR are 1.8, 57.62 and 0.82 J/s/inch2 respectively. The number of voxels corresponding to the normal tissue is significantly higher than that of the tumor and the organs at risk.

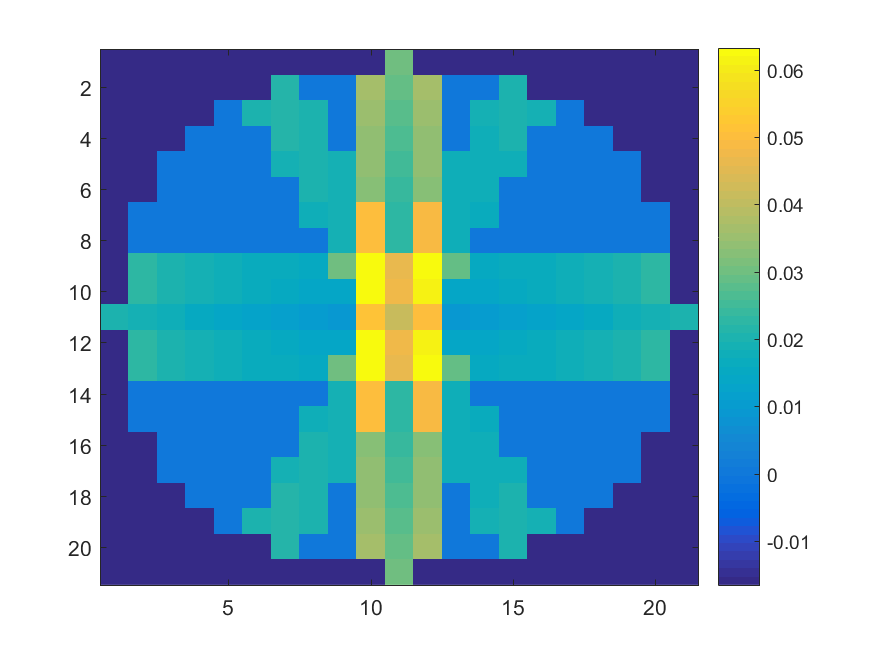


Figure. Solution 10. The intensity absorbed by the tumor, normal tissue and OAR are 1.55, 56.26 and 0.94 J/s/inch2 respectively. The number of voxels corresponding to the normal tissue is significantly higher than that of the tumor and the organs at risk.

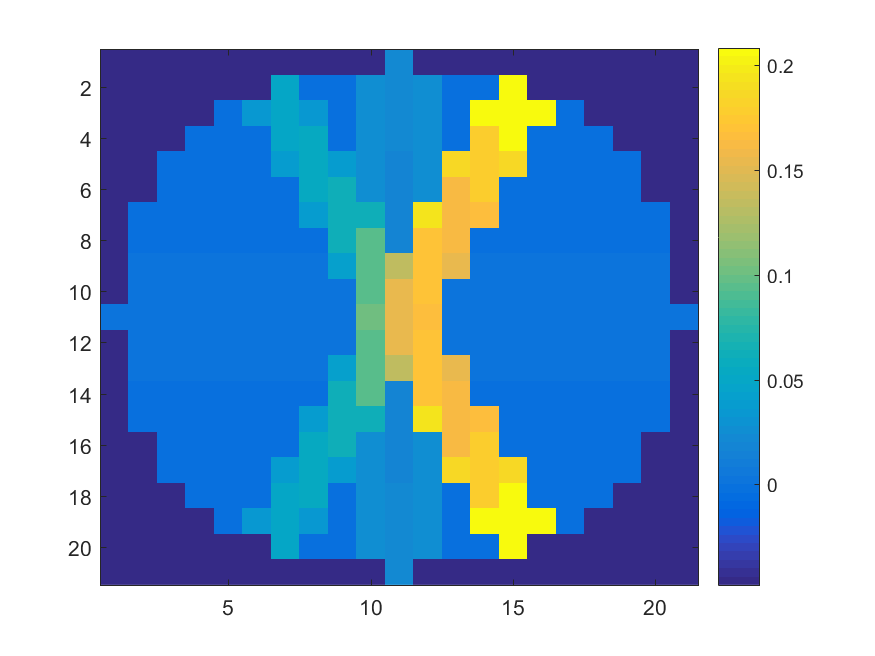


Figure. Solution 17. The intensity absorbed by the tumor, normal tissue and OAR are 4.77, 132.25 and 3.16 J/s/inch2 respectively. The number of voxels corresponding to the normal tissue is significantly higher than that of the tumor and the organs at risk.

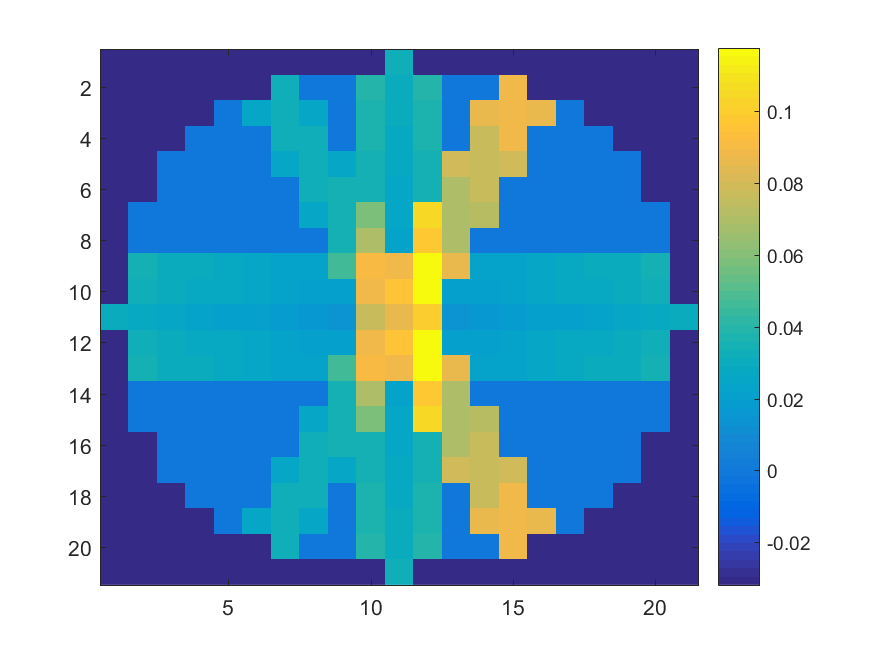


Figure. Solution 22. The intensity absorbed by the tumor, normal tissue and OAR are 3.0, 100.58 and 2.01 J/s/inch2 respectively. The number of voxels corresponding to the normal tissue is significantly higher than that of the tumor and the organs at risk.

**References**

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