CS 230 : Discrete Computational Structures
Spring Semester, 2021
HOMEWORK ASSIGNMENT #8
Due Date: Wednesday, Apr 7

Suggested Reading: Rosen Sections 5.3; Lehman et al. Chapters 5, 6.1 - 6.2, 7

For the problems below, explain your answers and show your reasoning.

1. **[8 Pts]** Prove that $f_0 - f_1 + f_2 - \cdots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$ where f_i are the Fibonacci numbers.

2. **[12 Pts]** Let S defined recursively by (1) $4 \in S$ and (2) if $s \in S$ and $t \in S$, then $st \in S$. Let $A = \{4^i \mid i \in \mathcal{Z}^+\}$. Prove that

   (a) **[6 Pts]** $A \subseteq S$ by mathematical induction.

   (b) **[6 Pts]** $S \subseteq A$ by structural induction.

3. **[5 Pts]** Define the set $S = \{2^k 3^m 5^p \mid k, m, p \in \mathcal{Z}\}$ inductively. You do not need to prove that your construction is correct.

4. **[10 Pts]** Consider the following state machine with five states, labeled 0, 1, 2, 3 and 4. The start state is 0. The transitions are $0 \to 1$, $1 \to 2$, $2 \to 3$, $3 \to 4$, and $4 \to 0$.

   Prove that if we take n steps in the state machine we will end up in state 0 if and only if n is divisible by 5. Argue that to prove the statement above by induction, we first have to *strengthen the induction hypothesis*. State the strengthened hypothesis and prove it.

5. **[10 Pts]** A robot wanders around a 2-dimensional grid. He starts out at (0,0) and can take the following steps: (-1,+3), (+2,-2) and (+4,0). Define a state machine for this problem. Then, define a Preserved Invariant and prove that the robot will never get to (2,0).

6. **[15 Pts]** Let $L = \{(a, b) \mid a, b \in \mathcal{Z}, (a - b) \bmod 4 = 0\}$. We want to program a robot that can get to each point $(x, y) \in L$ starting at $(0, 0)$.

   (a) **[5 Pts]** Give an inductive definition of L. This will describe the steps you want the robot to take to get to points in L starting at $(0, 0)$. Let L' be the set obtained by your inductive definition.

   (b) **Extra Credit [5 Pts]** Prove inductively that $L' \subseteq L$, i.e., every point that the robot can get to is in L.

   (c) **Extra Credit [5 Pts]** Prove that $L \subseteq L'$, i.e., the robot can get to every point in L. To prove this, you need to give the path the robot would take to get to every point in L from $(0, 0)$, following the steps defined by your inductive rules.

For more practice, work on the problems from Rosen Sections 5.3; Lehman et al. Chapters 5, 6.1 - 6.2, 7