Com S 230 Discrete Computational Structures

Fall Semester 2019

Exam 1

Thursday, October 3, 2019

Time: 90 minutes

Name: ____________________________________

The exam is **closed book. No notes or calculators are allowed.** Please go over all the questions in the exam before you start working on it. Attempt the questions that seem easier first. The exam has a total of 120 points but you will only be scored over 100 points. This means that you can get a total of 20 **extra credit** points. If you see yourself getting stuck on one question, continue on and come back to it later if you have time. Try to stay as brief and to the point as possible. Good luck!

1	2	3	4	5	6	7	8	Total
18	12	28	12	9	9	14	18	100

1. Short Answers [**18 Points**]

   (a) [**6 Pts**] Define $C = \{3, 8, 13, \ldots\}$ using set-builder notation. Now, enumerate $D = \{2n + 2 \mid n \in C\}$.

   (b) [**6 Pts**] What do we mean when we say that $\{\wedge, \vee, \neg\}$ is functionally complete? Now, explain why this implies that $\{\vee, \neg\}$ is also functionally complete.

   (c) [**6 Pts**] What is the domain and co-domain of the function $f(A, B) = A \cup B$ where A and B are sets of integers?

2. Propositions, Predicates and Quantifiers [**12 Points**]

Consider the predicates defined below, where the domain is the ISU community.

$F(x, y) = x$ and y are friends
$S(x) = x$ is a software engineering (SE) major
$C(x) = x$ is a computer science (CS) major

(a) [**8 Pts**] Express the English statement below using predicate logic. All quantifiers should be on the left of your formula with no negations to the left of the quantifiers.

There is an SE major who is not friends with any CS majors.

(b) [**4 Pts**] Now, negate the statement so that a negation appears only to the left of a predicate.

3. Making Logical Arguments [**28 Points**]

Prove the following using the appropriate rules of inference. Remember to instantiate! You may assume that your universe is all students in Com S 230.

Any student who fails the test will be sad and angry. Every student that goes to the party tonight is happy (not sad). All students go to the party tonight. Therefore, no student fails the test.

(a) [**4 Pts**] Define the predicates $F(x)$, $S(x)$, $A(x)$, and $P(x)$.

(b) [**8 Pts**] Convert the three premises and the conclusion into logical notation. Use quantifiers.

(c) [**16 Pts**] Now, prove the conclusion from the premises using rules of inference.

4. Properties of Sets [**12 Points**]

(a) [**6 Pts**] Disprove using a counter-example: *If* $A \cap B = A$ *then* $A = B$.

(b) [**6 Pts**] What can you say about the relationship between A and B if $A \cap B = A$? You do not need to justify your answer.

5. Proof Methods I [**9 Points**]

   Let a and b be real numbers. Prove: If $a + b = s$, then $a \geq s/2$ or $b \geq s/2$. What proof method did you use?

6. Proof Methods II [**9 Points**]

   Prove that if x is a non-zero rational and y is irrational, then x/y is irrational. What proof method did you use?

7. Proofs about Sets [**14 Points**]

Prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$ using the subset argument.

(a) First, prove that $(A \cup B) - (A \cap B) \subseteq (A - B) \cup (B - A)$.
Let $x \in (A \cup B) - (A \cap B)$. We prove that $x \in (A - B) \cup (B - A)$ by cases.

i. We consider the two cases: (I) $x \in A$, (II) $x \in B$. *Explain why.*

ii. Note that the two cases are symmetric. Complete case (I) below.

(b) Next, prove that $(A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$.
Let $x \in (A - B) \cup (B - A)$. We prove that $x \in (A \cup B) - (A \cap B)$ by cases.

i. We consider the two cases: (I) $x \in A - B$, (II) $x \in B - A$. *Explain why.*

ii. Note that the two cases are symmetric. Complete case (I) below.

8. Functions [**18 Points**]

(a) [**9 Pts**] Let $f(n) = 3n + 9$, where the domain and co-domain of f is $\mathcal{N}$. Is f one-to-one? Is f onto? Prove or give a counterexample in each case.

(b) [**9 Pts**] Let $f(m, n) = mn$ where the domain of f is $\mathcal{R} \times \mathcal{R}$ and the co-domain of f is $\mathcal{R}$. Is f one-to-one? Is f onto? Prove or give a counterexample in each case.