Com S 230 Discrete Computational Structures

Spring Semester 2021

Exam 1

Monday, March 8, 2021

Time: 90 minutes

Name: ______________________________

The exam is **open book, open notes, open Canvas course notes. No other materials are allowed, including browsing on internet. Your signature on Canvas pledges that you have not violated these conditions.**

Please go over all the questions in the exam before you start working on it. Attempt the questions that seem easier first. The exam has a total of 120 points but you will only be scored over 100 points. This means that you can get a total of 20 **extra credit** points. If you see yourself getting stuck on one question, continue on and come back to it later if you have time. Try to stay as brief and to the point as possible. Good luck!

The exam is set for 90 minutes but you are given two hours to complete it, including the time you need to upload your exam to Canvas. Please give yourself sufficient time!

1	2	3	4	5	6	7	Total
18	16	26	8	14	18	20	100

1. Short Answers [**18 Points**]

   (a) [**6 Pts**] Define $A = \{-6, -2, 2, 6, \ldots\}$ using set-builder notation. Now, enumerate $B = \{5n + 3 \mid n \in A\}$.

   (b) [**6 Pts**] Define $C = B \cap \mathbf{Z}^{-}$. Now, enumerate $C \times C$ and $\mathbf{P}(C)$.

   (c) [**6 Pts**] Given $f : A \to B$ and $g : B \to C$, prove that if f and g are one-to-one, then $g \circ f$ is one-to-one.

2. Propositions, Predicates and Quantifiers [**16 Points**]

   Suppose a group of friends are sitting on adjacent seats in a single row in a movie theater. Let $left(x, y)$ be x is sitting somewhere to the left of y. You can assume that your universe is the group of friends. You can use earlier predicates to define subsequent ones.

   (a) [**5 Pts**] Define $right(x, y)$ is 'x is sitting somewhere to the right of y'.

   (b) [**5 Pts**] Define $leftmost(x)$ is 'x is at the left end of the row'.

   (c) [**6 Pts**] Define $middle(x)$ is 'x is neither at the left end nor at the right end'.

3. Making Logical Arguments [**26 Points**]

   Prove the following using the appropriate rules of inference. Remember to instantiate! You may assume that your universe is all ISU students.

   Every student plays basketball or chess. Every student who plays chess is good at logic and loves arguing. Sam does not love arguing. Therefore, there is a student who plays basketball.

   (a) [**4 Pts**] Define the predicates $B(x)$, $C(x)$, $L(x)$, and $A(x)$.

   (b) [**8 Pts**] Convert the three premises and the conclusion into logical notation. Use quantifiers.

   (c) [**14 Pts**] Now, prove the conclusion from the premises using rules of inference.

4. Proofs of Sets I [**8 Points**]

   Prove that $A - (B - C) = (A - B) \cup (A \cap C)$ using two-column iff arguments and logical equivalences, giving reasons for each step.
   Note: You can use $x \notin A$ as short form for $\neg(x \in A)$.

5. Proofs about Sets II [**14 Points**]

We prove De Morgans's Law, that $\overline{A \cup B} = \overline{A} \cap \overline{B}$ using the subset argument, giving reasons for each step. Remember that you **may not** use logical equivalences in your proof, including De Morgan's Law of logic.

(a) First, we prove that $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$, as follows.

Let $x \in \overline{A \cup B}$. We prove that $x \in \overline{A} \cap \overline{B}$. By definition of intersection, we need to prove $x \in \overline{A}$ and $x \in \overline{B}$.
Prove, by contradiction, $x \in \overline{A}$. (Proving $x \in \overline{B}$ is symmetric, so you do not need to repeat the argument).

(b) Next, we prove that $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$.
Let $x \in \overline{A} \cap \overline{B}$. **Prove, by contradiction,** $x \in \overline{A \cup B}$.

6. Proof Methods [**18 Points**]

   Prove the following statements. State the proof method you used (eg. direct, contradiction, contrapositive, constructive existential, non-constructive existential).

   (a) [**9 Points**] The product of a non-zero rational and an irrational number is always irrational.

   (b) [**9 Points**] The product of two irrational numbers may be rational or irrational.

7. Functions [**20 Points**]

(a) [**10 Pts**] Let $f(n) = 5n + 10$, where the domain and co-domain of f is $\mathbf{R}$. Is f one-to-one? Is f onto? Prove or give a counterexample in each case.

(b) **10 Pts**] Let $f(A) = A \cup \{1\}$ where the domain and co-domain of f is $\mathbf{P}(\mathbf{N})$. Is f one-to-one? Is f onto? Prove or give a counterexample in each case.