

CS 230 : Discrete Computational Structures
Spring Semester, 2021
HOMEWORK ASSIGNMENT #5
Due Date: Monday, March 15

Suggested Reading: Rosen Sections 9.1 and 9.5; Lehman et al. Chapter 10.5, 10.6 and 10.10

For the problems below, explain your answers and show your reasoning.

1. [10 Pts] For each of these relations decide whether it is reflexive, anti-reflexive, symmetric, anti-symmetric and transitive. Justify your answers. R_1 and R_2 are over the set of real numbers.
 - (a) $(x, y) \in R_1$ if and only if $xy \geq 0$
 - (b) $(x, y) \in R_2$ if and only if $x = 2y$
2. [8 Pts] Let R_3 be the relation on $\mathcal{Z}^+ \times \mathcal{Z}^+$ where $((a, b), (c, d)) \in R_3$ if and only if $ad = bc$.
 - (a) Prove that R_3 is an equivalence relation.
 - (b) Define a function f such that $f(a, b) = f(c, d)$ if and only if $((a, b), (c, d)) \in R_3$.
 - (c) Define the equivalence class containing $(1, 1)$.
 - (d) Describe the equivalence classes. How many classes are there and how many elements in each class?
3. [8 Pts] Are these relations on the set of 5 digit numbers equivalence relations? If so, prove the properties satisfied, describe the equivalence classes and describe a new equivalence relation which is a refinement of the relation given. If not, describe which properties are violated.
 - (a) $(a, b) \in R_4$ if and only if a and b start with the same two digits
 - (b) $(a, b) \in R_5$ if and only if a and b have the same k th digit, where k is a number from 1 to 5
4. [12 Pts] Prove that these relations on the set of all functions from \mathcal{Z} to \mathcal{Z} are equivalence relations. Describe the equivalence classes.
 - (a) $R_6 = \{(f, g) \mid f(0) = g(0) \text{ and } f(1) = g(1)\}$
 - (b) $R_7 = \{(f, g) \mid \exists C \in \mathcal{Z}, \forall x \in \mathcal{Z}, f(x) - g(x) = C\}$

5. [12 Pts] Consider the following relations on the set of positive real numbers. One is an equivalence relation and the other is a partial order. Which is which? For the equivalence relation, describe the equivalence classes. What is the equivalence class of 2? of π ? Justify your answers.

(a) $(x, y) \in R_8$ if and only if $x/y \in \mathbb{Z}$

(b) $(x, y) \in R_9$ if and only if $x - y \in \mathbb{Z}$