CS230-HW2Sol

1. **Ying 4 pts**

   Define $P(x)$: x is odd and $Q(x)$: x is even over the integers. Then $\forall x(P(x) \rightarrow Q(x))$ is false because $P(1) \rightarrow Q(1) \equiv True \rightarrow False \equiv False$.

   Also, $\forall x P(x)$ is false because 2 is not an odd integer, so $P(2)$ is false. Similarly, $\forall x Q(x)$ is false since $Q(1)$ is false. Then, $\forall x P(x) \rightarrow \forall x Q(x) \equiv False \rightarrow False \equiv True$. Since $\forall x(P(x) \rightarrow Q(x))$ and $\forall x P(X) \rightarrow \forall x Q(X)$ evaluate to different truth values, the two statements are not logically equivalent.

2. **Modeste 8 pts**

   a) There are at least two different students who asked a question to the same set of faculty members.

   b) $\exists x \exists y \forall z(F(x) \wedge F(y) \wedge (x \neq y) \wedge (S(z) \rightarrow (\neg A(x,z) \wedge \neg A(y,z)))$

3. **Ying 4 pts**

   a) False. The first statement is only specified for freshmen. The first statement says nothing about whether people who are not freshmen can take online classes. Therefore, it is not known if Jim is taking an online class or not.

   b) False. The first statement states that Mary likes all of her computer science classes. Mary may like other classes that are not computer science classes. Therefore, Mary liking Discrete Mathematics does not mean Discrete Mathematics is a computer science course (in this example).

4. **Modeste 8 pts**

   a) We have the following predicates:

   $C(x)$: x owns a personal computer.

   $S(x)$: x is a student in class.

   $T(x)$: x types up his homework.

   Translation of statements:

   1) Jim, a student in class, owns a personal computer: $S(Jim) \wedge C(Jim)$

   2) Everyone who owns a personal computer types up their homework: $\forall x(C(x) \rightarrow T(x))$

   3) Someone in class types up their homework: $\exists x(S(x) \wedge T(x))$

   Now assuming 1) and 2) we need to prove 3) using the appropriate rules of inference.

	Steps	Reason
1)	$S(Jim) \wedge C(Jim)$	Premise 1
2)	$C(Jim)$	Simplification from 1)
3)	$S(Jim)$	Simplification from 1)
4)	$\forall x(C(x) \to T(x))$	Premise 2
5)	$C(Jim) \to T(Jim)$	Universal instantiation from 4)
6)	$T(Jim)$	Modus ponens from 2) and 5)
7)	$S(Jim) \wedge T(Jim)$	Conjunction from 3) and 6)
8)	$\exists x(S(x) \wedge T(x))$	Existential generalization from 7)

b) We define the following predicates:

$C(x)$: x is a college town.

$M(x)$: x is in the Midwest.

$F(x)$: x is fun to live in.

Translation

1)There are college towns in the Midwest: $\exists x(C(x) \wedge M(x))$

2) All college towns are fun places to live: $\forall x(C(x) \to F(x))$

3)There is a town in the Midwest that is fun to live in: $\exists x(M(x) \wedge F(x))$

Assuming (1) and (2) let prove (3) using the appropriate rules of inference.

	Steps	Reason
1)	$\exists x(C(x) \wedge M(x))$	Premise 1
2)	$C(a) \wedge M(a)$	Existential instantiation from 1)
3)	$C(a)$	Simplification from 2)
4)	$M(a)$	Simplification from 2)
5)	$\forall x(C(x) \to F(x))$	Premise 2
6)	$C(a) \to F(a)$	Universal instantiation from 5)
7)	$F(a)$	Modus ponens from 3) and 6)
8)	$F(a) \wedge M(a)$	Conjunction from 4) and 7)
9)	$\exists x(F(x) \wedge M(x))$	Existential generalization from 8)

5. **Jonathan 14 pts**

a) $B(x)$: x is a bear.

$S(X)$: x is a good swimmer.

$H(x)$: x goes hungry.

$F(x)$: x can catch fish.

1) $\forall x(B(x) \rightarrow S(x))$ Premise 1
2) $\forall x(F(x) \rightarrow \neg H(x))$ Premise 2
3) $\forall x(\neg F(x) \rightarrow \neg S(x))$ Premise 3
4) $B(a) \rightarrow S(a)$ for any a Universal Instantiation of 1)
5) $F(a) \rightarrow \neg H(a)$ for any a Universal Instantiation of 2)
6) $\neg F(a) \rightarrow \neg S(a)$ for any a Universal Instantiation of 3)
7) $S(a) \rightarrow F(a)$ for any a Contrapositive of 6)
8) $B(a) \rightarrow F(a)$ for any a Hypothetical Syllogism of 4) and 7)
9) $B(a) \rightarrow \neg H(a)$ for any a Hypothetical Syllogism of 5) and 9)
10) $H(a) \rightarrow \neg B(a)$ for any a Contrapositive of 9)
11) $\forall x(H(x) \rightarrow \neg B(x))$ Universal Generalization of 10)

b) Universal Transitivity

1) $\forall x(P(x) \rightarrow Q(x))$ Premise 1
2) $\forall x(Q(x) \rightarrow R(x))$ Premise 2
3) $P(a) \rightarrow Q(a)$ for any a Universal Instantiation of 1)
4) $Q(a) \rightarrow R(a)$ for any a UI of 2)
5) $P(a) \rightarrow R(a)$ for any a HS of 3) and 4)
6) $\forall x(P(x) \rightarrow R(x))$ UG of 5)

c) Universal Contrapositive

1) $\forall x(P(x) \rightarrow Q(x))$ Premise 1
2) $P(a) \rightarrow Q(a)$ for any a UI of 1)
3) $\neg Q(a) \rightarrow \neg P(a)$ for any a Contrapositive of 2)
4) $\forall x(\neg Q(x) \rightarrow \neg P(x))$ UG of 4)

d) Same predicates are used as in part a)

1) $\forall x(B(x) \rightarrow S(x))$ Premise 1
2) $\forall x(F(x) \rightarrow \neg H(X))$ Premise 2
3) $\forall x(\neg F(x) \rightarrow \neg S(x))$ Premise 3
4) $\forall x(S(x) \rightarrow F(x))$ Universal Contrapositive of 3)
5) $\forall x(B(x) \rightarrow F(x))$ Universal Transitivity of 1) and 4)
6) $\forall x(B(x) \rightarrow \neg H(x))$ UT of 2) and 5)
7) $\forall x(H(x) \rightarrow \neg B(x))$ UC of 6)

6. **Ling 12 pts**

a) $Equal(m,n) \equiv \exists k(Zero(k) \wedge A(m,n,k))$

b) $One(n) \equiv M(n,n,n) \wedge \neg Zero(n) \equiv \forall k((\neg Zero(k) \wedge \neg Equal(n,k)) \rightarrow Greater(k,n))$

c) $Two(n) \equiv \exists k(One(k) \wedge A(n,k,k)) \equiv \forall k((\neg Zero(k) \wedge \neg One(k) \wedge \neg Equal(n,k)) \rightarrow Greater(k,n))$

d) $Prime(p) \equiv \forall m \forall n(M(p,m,n) \rightarrow (Equal(p,m) \vee One(m)))$