CS230-HW1Sol

1. **Ying 6 pts**

   f: passes the final, a: attends the class, c: passes the class

   a) $f \vee a \to c$

   b) $c \to a$

   c) $c \leftrightarrow (a \wedge f)$

2. **Ying 6 pts**

p	q	r	$p \to q$	$q \to r$	$(p \to q) \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

   The last two columns are not the same on line 6 and 8, therefore $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

3. **Ling 5 pts**

$$
\begin{aligned}
(p \to \neg q) \wedge (q \to \neg r) &\equiv (\neg p \vee \neg q) \wedge (\neg q \vee \neg r) && \text{Implication Rule} \\
&\equiv (\neg p \vee \neg q) \wedge (\neg r \vee \neg q) && \text{Commutative Law} \\
&\equiv (\neg p \wedge \neg r) \vee \neg q && \text{Distributive Law} \\
&\equiv \neg(p \vee r) \vee \neg q && \text{De Morgan's Law} \\
&\equiv (p \vee r) \to \neg q && \text{Implication Rule}
\end{aligned}
$$

4. **Ling 6 pts**

   Assume that Tom is telling the truth, then Sue must be also telling the truth. It contradicts to the fact that one of them is lying. Therefore, our assumption is incorrect, which means Tom is lying. We know that only one of them is lying and Tom is lying, then Sue must be telling the truth. Therefore, it is easy to see that Tom is a SE major and Sue is a CS major.

5. **Jonathan 6 pts**

Proof Idea: Given a compound proposition A, construct the corresponding truth table. From each line that results in A being false, take the negation (or canceling a double negation) of each proposition and "or" them together. "And" each of the negated proposition lines together. This results in a CNF formula that is equivalent to A because each line that results in A being false cannot be satisfied ("And"), so as long as at least one of the corresponding literals ("Or") is different ("negation") the line cannot be satisfied.

Example:

p	q	r	A
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

CNF: $\overset{\text{line 2}}{(\neg p \vee \neg q \vee r)} \wedge \overset{\text{line 5}}{(p \vee \neg q \vee \neg r)} \wedge \overset{\text{line 6}}{(p \vee \neg q \vee r)}$

6. **Modeste 6 pts**

   To prove that {XOR, AND, TRUE} is functionally complete, we must prove that any propositional formula is equivalent to one whose only connectives are XOR and AND, along with the constant TRUE. To prove this, we need to construct $\neg$ and $\vee$ to get {AND, NOT, OR}. It is given that XOR is logically equivalent to $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$. Let p be any proposition. We need to show that $\neg p$ is logically equivalent to ($p \oplus$ TRUE) Then also show that (($p\oplus$ TRUE) $\wedge$($q\oplus$ TRUE)) $\oplus$ TRUE is equivalent to $p \vee q$.

$$
\begin{aligned}
(1)\quad p \oplus T &\equiv (p \wedge \neg T) \vee (\neg p \wedge T) && \text{given} \\
&\equiv (p \wedge F) \vee (\neg p \wedge T) && \text{Negation} \\
&\equiv F \vee (\neg p \wedge T) && \text{Domination laws} \\
&\equiv F \vee (\neg p) && \text{Identity laws} \\
&\equiv \neg p && \text{Identity laws}
\end{aligned}
$$

$$
\begin{aligned}
((p \oplus T) \wedge (q \oplus T)) \oplus T &\equiv ((\neg p) \wedge (\neg q)) \oplus T && \text{From (1)} \\
&\equiv (\neg (p \vee q)) \oplus T && \text{DeMorgan's laws} \\
&\equiv \neg(\neg(p \vee q)) && \text{From (1)} \\
&\equiv p \vee q && \text{Double Negation}
\end{aligned}
$$