

## CS 230 : Discrete Computational Structures

**Spring Semester, 2021**

**HOMEWORK ASSIGNMENT #2**

**Due Date:** Monday, February 15

**Suggested Reading:** Rosen Sections 1.4 - 1.6; LLM Chapter 3

For the problems below, explain your answers and show your reasoning.

1. [4 Pts] Prove, by counterexample, that the propositions  $\forall x(P(x) \rightarrow Q(x))$  and  $\forall xP(x) \rightarrow \forall xQ(x)$  are not logically equivalent. *Hint:* Let your domain be the set of integers. Define  $P(x)$  to be ‘ $x$  is odd’ and  $Q(x)$  to be ‘ $x$  is even’.
2. [8 Pts] For the following problems, let  $S(x)$ ,  $F(x)$  and  $A(x, y)$  be the statements “ $x$  is a student”, “ $x$  is a faculty member” and “ $x$  has asked  $y$  a question”. Let the domain be all people at ISU. Translate into English or logic, as appropriate.
  - (a)  $\exists x \exists y \forall z (S(x) \wedge S(y) \wedge (x \neq y) \wedge (F(z) \rightarrow (A(x, z) \leftrightarrow A(y, z))))$
  - (b) There are at least two faculty members who have not asked questions to any students.
3. [4 Pts] State whether the following arguments are correct. Explain your answer briefly.
  - (a) All freshmen take online classes. Jim is not a freshman. Therefore, Jim does not take online classes.
  - (b) Mary likes all her computer science classes. Mary likes ‘Discrete Math’. Therefore, ‘Discrete Math’ is a computer science class.
4. [8 Pts] Define predicates and prove the following using two-column proofs and the appropriate rules of inference:
  - (a) [4 Pts] Jim, a student in class, owns a personal computer. Everyone who owns a personal computer types up their homework. Therefore, someone in class types up their homework.
  - (b) [4 Pts] There are college towns in the midwest. All college towns are fun places to live. There is a town in the midwest that is a fun to live in.
5. [14 Pts] Consider the following argument:  
*All bears are good swimmers. If you can catch fish, you will not go hungry. If you can't catch fish, you are not a good swimmer. Therefore, no bears go hungry.*
  - (a) [4 Pts] Define the predicates  $B(x)$ ,  $S(x)$ ,  $H(x)$  and  $F(x)$  to describe the sentences above using predicate logic. Then, prove the argument using two-column proofs and the rules of inference you have learned.

- (b) [3 Pts] Prove the **universal transitivity** rule, which states:  
if  $\forall x(P(x) \rightarrow Q(x))$  and  $\forall x(Q(x) \rightarrow R(x))$ , then  $\forall x(P(x) \rightarrow R(x))$ .
- (c) [3 Pts] Prove the **universal contrapositive** rule, which states:  
if  $\forall x(P(x) \rightarrow Q(x))$  then  $\forall x(\neg Q(x) \rightarrow \neg P(x))$ .
- (d) [4 Pts] Now, prove the previous argument using the **universal transitivity** rule and the **universal contrapositive** rule. Is your proof shorter? Notice that you no longer have to use instantiation and generalization rules.
6. [12 Pts] Given the universe of all non-negative integers, we define the two predicates  $A(k, m, n) : (k = m + n)$  and  $M(k, m, n) : (k = mn)$ . For example  $A(4, 3, 1)$  is true while  $A(5, 2, 2)$  is false. We can use  $A$  and  $M$  to define other predicates. For example,  $Zero(n) : A(n, n, n)$ . Note that  $n = 0$  is the only integer such that  $n = n + n$ . We can now use  $Zero$  to define  $Greater(m, n) : \exists k(\neg Zero(k) \wedge A(m, n, k))$ . Define the following predicates. You can use the predicates defined earlier in subsequent definitions.
- (a) [4 Pts]  $Equal(m, n)$
- (b) [4 Pts]  $One(n)$
- (c) [4 Pts]  $Two(n)$
- (d) **Extra Credit** [4 Pts]  $Prime(p)$

For more practice, work on the problems from Rosen Sections 1.4 - 1.6 and LLM Chapter 3.