CW 04A Solutions

1. (a) $P(1)$: $2(1) = 1(1+1)$

$2(1) = 2$ and $1(1+1) = 2$. So, $P(1)$.

(b) $P(k)$: $2 + 4 + \cdots + 2k = k(k+1)$

(c) $P(k+1)$: $2 + 4 + \cdots + 2k + 2(k+1) = (k+1)(k+2)$

(d) $2 + 4 + \cdots + 2k + 2(k+1)$

$= k(k+1) + 2(k+1)$ by IH

$= (k+2)(k+1)$

2. $a_0 = 8$, $a_{n+1} = a_n + 4$ for $n \geq 0$

3. Base: $3 \in S$

Ind: if $x \in S$ then $3x \in S$

or

if $x \in S$ and $y \in S$ then $xy \in S$

CW 04B Solutions

1. (a) $P(1)$: $4(1) = 2(1)(1+1)$

$4(1) = 4$ and $2(1)(1+1) = 4$. So, $P(1)$.

(b) $P(k)$: $4 + 8 + \cdots + 4k = 2k(k+1)$

(c) $P(k+1)$: $4 + 8 + \cdots + 4k + 4(k+1) = 2(k+1)(k+2)$

(d) $4 + 8 + \cdots + 4k + 4(k+1)$

$= 2k(k+1) + 4(k+1)$ by IH

$= (2k+4)(k+1)$

$= 2(k+1)(k+2)$

2. $a_0 = 3$, $a_{n+1} = a_n + 5$ for $n \geq 0$

3. Base: $5 \in S$

Ind: if $x \in S$ then $5x \in S$

or

if $x \in S$ and $y \in S$ then $xy \in S$