CS 230 : Discrete Computational Structures

Spring Semester, 2021

HOMEWORK ASSIGNMENT #4

Due Date: Wednesday, March 3

Suggested Reading: Rosen Sections 2.1 - 2.3; Lehman et al. Chapter 4.1, 4.3, 4.4.

For the problems below, explain your answers and show your reasoning.

1. **[6 Pts]** Let A and B be non-empty sets. Prove that if $A \neq B$, then $A \times B \neq B \times A$.
2. **[4 Pts]** Prove that $(A \cup B) - C = (A - C) \cup (B - C)$ using iff arguments and logical equivalences.
3. **[8 Pts]** Disprove the statements below.
   (a) If $A \cup C \subseteq B \cup C$ then $A \subseteq B$.
   (b) If $A \cap C \subseteq B \cap C$ then $A \subseteq B$.
4. **[8 Pts]** Prove by contradiction that if $A \cup C \subseteq B \cup C$ and $A \cap C \subseteq B \cap C$ then $A \subseteq B$.
5. **[8 Pts]** Prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$ using subset argument. You *may not* use logical equivalences in your proof. Use general proof techniques like 'proof by contradiction' and 'proof by cases'.
6. **[4 Pts]** Prove that $f(n) = 5n + 9$ is one-to-one, where the domain and co-domain of f is $\mathcal{Z}^+$. Show that f is not onto.
7. **[4 Pts]** Prove that $f(m, n) = m + n + mn$ is onto, where the domain of f is $\mathcal{Z} \times \mathcal{Z}$ and the co-domain of f is $\mathcal{Z}$. Show that f is not one-to-one.
8. **[8 Pts]** Let g be a total function from A to B and f be a total function from B to C.
   (a) If $f \circ g$ is one-to-one, then is g one-to-one? Prove or give a counter-example.
   (b) If $f \circ g$ is onto, then is g onto? Prove or give a counter-example.

For more practice, work on the problems from Sections 2.1 - 2.3; Lehman et al. Chapter 4.1, 4.3, 4.4.