CS 230 : Discrete Computational Structures
Spring Semester, 2021
HOMEWORK ASSIGNMENT #6
Due Date: Monday, March 22

Suggested Reading: Rosen Section 2.5

For the problems below, explain your answers and show your reasoning.

1. **[14 Pts]** Show that the following sets are countably infinite, by defining a bijection between $\mathcal{N}$ (or $\mathcal{Z}^+$) and that set. You do not need to prove that your function is bijective.

   (a) **[4 Pts]** the set of non-negative integers divisible by 5

   (b) **[5 Pts]** the set of integers divisible by 5

   (c) **[5 Pts]** $\{0, 1, 2, 3\} \times \mathcal{N}$

2. **[14 Pts]** Determine whether the following sets are countable or uncountable. Prove your answer. To prove countable, describe your enumeration precisely, There is no need to define a bijection.

   (a) **[7 Pts]** the set of real numbers with decimal representation consisting of all 5's (5.55 and $55.555\ldots$ are such numbers).

   (b) **[7 Pts]** the set of real numbers with decimal representation consisting of 1's, 3's and 5's

3. **[6 Pts]** Prove that the set of functions from $\mathcal{N}$ to $\mathcal{N}$ is uncountable, by using a diagonalization argument.

4. **[6 Pts]** Argue that a countably infinite union of countable infinite sets is countably infinite.

For more practice, you are encouraged to work on other problems, like the ones below.

1. Give an example of two uncountable sets A and B such that $A - B$ is (a) finite, (b) countably infinite, (c) uncountably infinite.

2. Show that any infinite set has a countably infinite subset.

3. Show that there is no infinite set A such that $|A| < |\mathcal{Z}|$.