COM S 311 SPRING 2022
HOMEWORK 4

Due: April 7, 11:59 p.m.

Late Submission Due: April 8, 11:59 p.m. (25% penalty)

(1) (Exercise 15.1-3 of CLRS) Consider a modification of the rod-cutting problem in which, in addition to a price p_i for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts. Give a dynamic-programming algorithm to solve this modified problem. Argue that your algorithm is correct and analyze its running time.

(2) (Problem 15-2 of CLRS) A ***palindrome*** is a nonempty string over some alphabet that reads the same forward and backward. Examples of palindromes are all strings of length 1, `civic`, `racecar`, and `aibohphobia` (fear of palindromes). Give an $O(n^2)$ algorithm to find the longest palindrome that is a subsequence of a given input string, where n is the length of the input string. For example, given the input `character`, your algorithm should return `carac`. Argue that your algorithm is correct and analyze its running time.

(3) Dan, the owner of Dan's Burgers (DB), is considering opening a series of restaurants along Scenic Valley Highway (SVH). The n possible locations are along a straight line, and the distances, in miles, of these locations from the start of SVH are $m_1 < m_2 < \cdots < m_n$. The constraints are as follows:
- At each location, DB may open at most one restaurant. The expected profit from opening a restaurant at location i is p_i, where $p_i > 0$, $i = 1, 2, \ldots, n$.
- Any two restaurants should be at least k miles apart, where k is a positive integer.

Give an efficient algorithm to compute the maximum expected total profit subject to the given constraints. Argue that your algorithm is correct and analyze its running time.

(4) Consider the following game. A dealer produces a sequence $s_1, s_2, \ldots, s_n$ of cards, face up, where each card s_i has a value v_i. Then two players take turns picking a card from the sequence, but can only pick the first or the last card of the (remaining) sequence. The winner of the game is the player who collect cards of largest total value. Assume n is even.
   (a) A natural greedy strategy for the first player is to start by picking up the available card of larger value. Show that the natural greedy strategy is not guaranteed to be a winning strategy for the first player.
   (b) Design an $O(n^2)$ algorithm to compute a winning strategy for the first player. Given the initial sequence, your algorithm should precompute in $O(n^2)$ time some information, and then the first player should be able to make each move optimally in $O(1)$ time by looking up the precomputed information. Argue that your algorithm is correct and analyze its running time.

(5) (Exercise 16.2-5 of CLRS) Describe an efficient algorithm that, given a set $\{x_1, x_2, \ldots, x_n\}$ of points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Argue that your algorithm is correct and analyze its running time.

Guidelines

- For each problem, if you write the statement "I do not know how to solve this problem" (and nothing else), you will receive 20% credit for that problem. If you do write a solution, then your grade could be anywhere between 0% to 100%. To receive this 20% credit, you must explicitly state that you do not know how to solve the problem.
- You may discuss homework with classmates, but you must write the final solutions alone, without consulting anyone. Your writing should demonstrate that you completely understand the solution you present.
- Remember that, often, the clearest way to describe an algorithm is to give a brief overview of the algorithm, followed by pseudocode.
- When presenting an algorithm, always argue its correctness and analyze its running time.
- When writing a proof, make sure it clear and rigorous.
- *Homework solutions must be typed.* We can make exceptions for certain diagrams, which can be hand-drawn and scanned. We reserve the right not to grade homework that does not follow the formatting requirements.
- Submit a pdf version of your assignment via Canvas. Please make sure that the file you submit is not corrupted and that its size is reasonable (no more than, say, 10-11 MB).

If we cannot open your file, your homework will not be graded.

- Any concerns about grading should be expressed within one week of returning the homework.

Note. We reserve the right to grade only a subset of the problems assigned. Which problems will be graded will be decided after the submission deadline.