COMS 331: Theory of Computing, Spring 2023
Homework Assignment 5

Due at 10:00PM, Wednesday, March 1, on Gradescope.

Problem 30. Prove or disprove: If A and $A \cap B$ are regular, then B is regular.

Problem 31. Prove or disprove: If A is regular and $A \cap B$ is not regular, then B is not regular.

Problem 32. Prove or disprove: If A is regular and B is not regular, then $A \cap B$ is not regular.

Problem 33. Prove that the language

$$A = \{x \in \{0,1\}^* \mid \#(0,x) = \#(1,x)\}$$

is not regular.

Problem 34. Prove that the language

$$A = \{x \in \{0,1\}^* \mid \#(01,x) = \#(10,x)\}$$

is regular. (Here, for $a, b \in \{0,1\}$, $\#(ab,x)$ is the number of a's in x that are immediately followed by b.)

Recall that the *canonical equivalence relation* of a language $A \subseteq \Sigma^*$ is the binary relation $\equiv_A$ on Σ^* defined by

$$x \equiv_A y \text{ if and only if, for all } z \in \Sigma^*, [xz \in A \Longleftrightarrow yz \in A].$$

Problem 35. (a) What are the equivalence classes of $\equiv_A$ if $A = \{0^n 1^n \mid n \in \mathbb{N}\}$?

(b) Prove that your answer to (a) is correct.

(c) Explain why your answer to (a) implies that A is not regular.

Problem 36. Define Σ, *top*, and *bottom* as in Problem 19. Prove: If $A, B \subseteq \{0,1\}^*$ are regular, then the language

$$\begin{bmatrix} A \\ B \end{bmatrix} = \{z \in \Sigma^* \mid \mathit{top}(z) \in A \text{ and } \mathit{bottom}(z) \in B\}$$

is regular.