# COMS 331: Theory of Computing, Spring 2023
# Homework Assignment 11

Neha Maddali

Due at 10:00PM, Wednesday, May 3, on Gradescope.

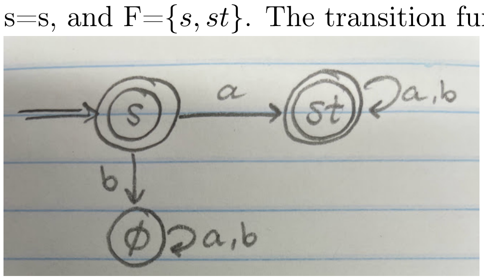
**Problem 71.**
Let M be a DFA for A so L(M)=A. Then we can design an NFA N using M that shows rev A is regular. The start state of M becomes the final state of N. The final state(s) of M becomes the start state(s) of N. Q and $\Sigma$ stay the same. To get the transitions in N, the transitions in M are reversed. A DFA can be constructed in place of an NFA by having 1 start state and using epsilon transition to the start states of the NFA. So rev A is regular.

**Problem 72.**
Let M be a DFA for A, so L(M)=A. Then we can construct a DFA M' such that L(M')=$N_2(A)$. M'=(Q',$\Sigma$,$\delta'$,s,F') where $\Sigma$ and s are the same as they are in M. Q'=Q $\times\{0,1,2\}$ and F'=F $\times\{0,1,2\}$. The idea for $\delta'$ is that we have M, but every time a bit is an error, we transition to the next "level" of the DFA, where each level represents another error. So going up two levels means that the string has two errors. If there is another error while a the highest level, which in this case is 2, then transition to a trash state because there are too many errors to be in the language. This idea can be more formally defined as $\delta'(q\times n,a)=p\times n$ if $\delta(q,a)=p$, where $q,p\in Q$; $q\times n, p\times n\in Q'$; and $a\in\Sigma$ is the "correct" bit. $\delta'(q\times n,a)=p\times(n+1)$ if $\delta(q,b)=p$, where $b\in\Sigma$ is an "error" bit, indicating to go to the next "level" of the DFA. $\delta'(q\times n,a)$ =some trash state if n=2 and $\delta(q,b)=p$ meaning there is another error but there have already been 2 errors.

**Problem 73.**
First we can make a chart of transitions in the NFA to determine which states of the equivalent DFA are accessible. The chart is as follows:

| $(Q\backslash\Sigma)$ | a | b |
|---|---|---|
| s | st | $\emptyset$ |
| st | st | st |

So, let M=(Q, $\Sigma$, $\delta$, s, F) be an equivalent DFA of the given NFA. Then Q=$\{s,st,\emptyset\},\Sigma=\{a,b\}$, s=s, and F=$\{s,st\}$. The transition function is given in the following diagram:

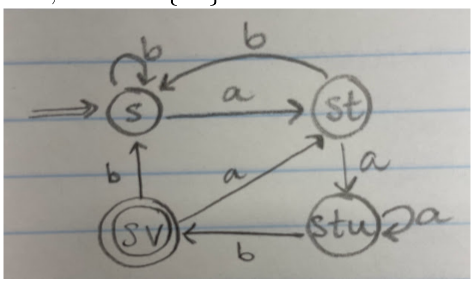


**Problem 74.a**

First we can make a chart of transitions in the NFA to determine which states of the equivalent DFA are accessible. The chart is as follows:

| $(Q\backslash\Sigma)$ | a | b |
|---|---|---|
| s | st | s |
| st | stu | s |
| stu | stu | sv |
| sv | st | s |

So let M=(Q, Σ, $\delta$, s, F) be an equivalent DFA of the given NFA. Then Q=$\{s, st, stu, sv\}, \Sigma = \{a, b\}$, s=s, and F=$\{sv\}$. The transition function is given in the following diagram:




**Problem 74.b**

First, we can make a chart of transitions in the NFA to determine which states of the equivalent DFA are accessible. The chart is as follows:

| $(Q\backslash\Sigma)$ | a | b |
|---|---|---|
| s | st | s |
| st | stu | s |
| stu | stu | sv |
| sv | stv | sv |
| stv | stuv | sv |
| stuv | stuv | sv |

So let M=(Q, Σ, $\delta$, s, F) be an equivalent DFA of the given NFA. Then Q=$\{s, st, stu, sv, stv, stuv\}$, $\Sigma = \{a, b\}$, s=s, and F=$\{sv, stv, stuv\}$. The transition function is given in the following diagram: