COMS 331: Theory of Computing, Spring 2023
Homework Assignment 7

Due at 10:00PM, Wednesday, April 5, on Gradescope.

Problem 44. Prove that $A \leq_m A \sqcup B$ where $A \sqcup B = \{0x \mid x \in A\} \cup \{1y \mid y \in B\}$.

Problem 45. Let $A, B, C \subseteq \{0,1\}^*$. Prove: If $A \leq_m C$ and $B \leq_m C$, then $A \sqcup B \leq_m C$.

Problem 46. Let $A \subseteq \{0,1\}^*$, and let $A^c = \{0,1\}^* - A$ be the complement of A. Prove: If A is $\leq_m$-complete for CE, then $A \sqcup A^c$ is neither c.e. nor co-c.e.

Recall that we have used the string-pairing function

$$\langle \cdot, \cdot \rangle : \{0,1\}^* \times \{0,1\}^* \longrightarrow \{0,1\}^*$$

defined by

$$\langle x, y \rangle = 0^{|x|}1xy$$

for all $x, y \in \{0,1\}^*$.

Given a language $B \subseteq \{0,1\}^*$, define the language $\exists B \subseteq \{0,1\}^*$ by

$$\exists B = \{x \in \{0,1\}^* \mid (\exists w \in \{0,1\}^*)\langle x, w \rangle \in B\}.$$

Problems 47 and 48 characterize computable enumerability in terms of *unbounded existential search.*

Problem 47. Let $B \subseteq \{0,1\}^*$. Prove: If B is c.e., then $\exists B$ is c.e.

Problem 48. Prove: For every c.e. language $A \subseteq \{0,1\}^*$, there is a decidable language $B \subseteq \{0,1\}^*$ such that $A = \exists B$.

Problem 49. Show that $A = \{\langle M \rangle \mid M \text{ is a TM, and } M \text{ accepts } w^{\mathcal{R}} \text{ whenever it accepts } w\}$ is undecidable.

Problem 50. Prove that $\{k \in \mathbb{N} \mid M_k(3) \uparrow\}$ is $\leq_m$-complete for coCE.