

COMS 331: Theory of Computing, Spring 2023

Homework Assignment 2

Due at 10:00PM, Thursday, February 9, on Gradescope.

Problems 8 – 12 can be found in your textbook *Automata and Computability* by Dexter Kozen.

Problem 8. Page 301 #1 part (a).

Problem 9. Page 301 #1 part (c).

Problem 10. Page 301 #1 part (d).

Problem 11. Page 301 #1 part (e).

Problem 12. Page 301 #3. (Note: Use induction on y , not $|y|$.)

Recall that a language $A \subseteq \{0, 1\}^*$ is *prefix-free* if no element of A is a prefix of any *other* element of A . Recall also that we proved the Kraft inequality, which says that, for every prefix-free language A ,

$$\sum_{x \in A} 2^{-|x|} \leq 1.$$

Define a prefix-free language A to be *maximal* if it is not a proper subset of any prefix-free language. Define a prefix-free language to be *full* if

$$\sum_{x \in A} 2^{-|x|} = 1.$$

Problem 13. Prove that every full prefix-free language is maximal.

Problem 14. How many DFAs $M = (Q, \Sigma, \delta, s, F)$ are there with $\Sigma = \{0, 1\}$ and $Q = \{1, 2, \dots, n\}$?

Problem 15. Prove that there is a language $A \subseteq \{0, 1\}^*$ with both of the following properties:

- (i) For all $x \in A$, $|x| \leq 5$.
- (ii) Every DFA that decides A has more than 8 states.