

COMS 331: Theory of Computing, Spring 2023

Homework Assignment 3

Due at 10:00PM, Wednesday, February 15, on Gradescope.

Problem 16. Prove that no 2-state DFA decides the language

$$A = \{x \in \{0, 1\}^* \mid \text{bnum}(x) \text{ is divisible by } 3\}.$$

Problem 17. Prove that for all $x \in \Sigma^*$, the singleton language $\{x\}$ is regular.

Problem 18. Prove that every finite language $A \subseteq \Sigma^*$ is regular.

Problem 19. Prove or disprove: Every subset of a regular language is regular.

Problem 20. Consider the four-symbol alphabet

$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

Define the functions

$$\text{top}, \text{bottom} : \Sigma^* \rightarrow \{0, 1\}^*$$

By the recursion

- $\text{top}(\lambda) = \text{bottom}(\lambda) = \lambda$
- for all $x \in \Sigma^*$ and $a, b \in \{0, 1\}$,

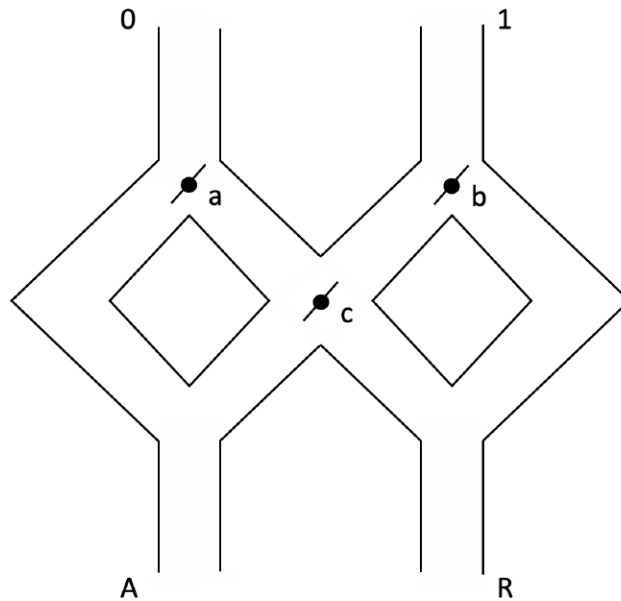
$$\begin{aligned} \text{top}\left(x \begin{bmatrix} a \\ b \end{bmatrix}\right) &= \text{top}(x)a, \\ \text{bottom}\left(x \begin{bmatrix} a \\ b \end{bmatrix}\right) &= \text{bottom}(x)b. \end{aligned}$$

Design a DFA M such that

$$L(M) = \{x \in \Sigma^* \mid \text{bnum}(\text{top}(x)) < \text{bnum}(\text{bottom}(x))\}.$$

Problem 21. Prove or disprove: If $A_n \subseteq \Sigma^*$ is regular for each $n \in \mathbb{N}$ then $\bigcup_{n=0}^{\infty} A_n$ is regular.

Problem 22. Consider the following toy. Successive marbles are dropped into the toy at 0 or 1, corresponding to the bits of an input string $x \in \{0, 1\}^*$.



The levers at a, b and c cause a marble to fall left in the indicated position, but when the marble hits a lever, the lever moves so that the next marble goes in the opposite direction. The levers start in the indicated position. The string x is accepted if its last marble falls out of the toy at A and is rejected otherwise. Design a DFA that simulates this toy.