COMS 331: Theory of Computing, Spring 2023
Homework Assignment 4

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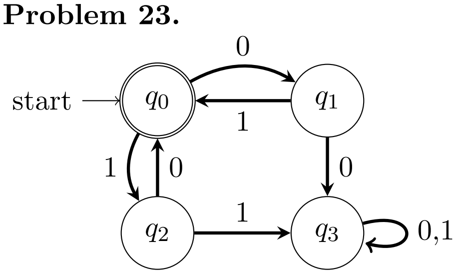
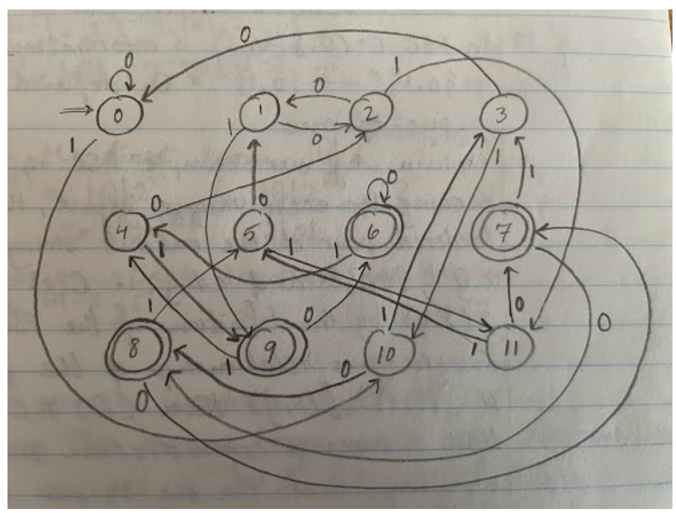
Due at 10:00PM, Wednesday, February 22, on Gradescope.

Problem 23.



Problem 24.

$\delta(q, a)$	0	1
0	0	10
1	2	9
2	1	11
3	0	10
4	2	9
5	1	11
6	6	4
7	8	3
8	7	5
9	6	4
10	8	3
11	7	5



Q = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
$\Sigma = \{0, 1\}$
δ = table above where rows 0-11 are q and columns 0-1 are a
s = 0
F = $\{6, 7, 8, 9\}$

Problem 25.

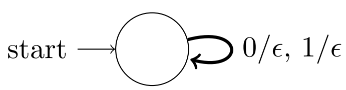
$|C(0^n)| = n/4$ (integer division)
$|C(1^n)| = 2n$
All strings that C compresses start with 0000. There are a lot less 1's than 0's in the compressed strings, and 4 0's in a row allow there to be more 1's in the string.

Problem 26a.
$\{0, 1, 2, 3\}$

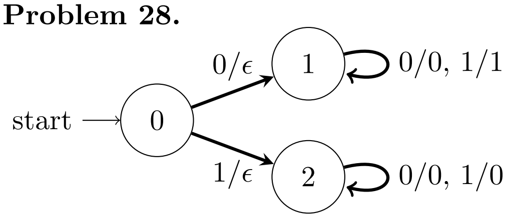
Problem 26b.
0: w = 0000000001
1: w = 00000000010
2: w = 000000000100
3: w = 0000000001000

Problem 27a. Because for each unique input, there is a different g(w) (ordered pair of the output string and end state). For $x, y \in \{0,1\}^*$ and $x \neq y$, if C(x) = C(y), then $\hat{\delta}(s, x) \neq \hat{\delta}(s, y)$ by the definition of the FSC which states the one-to-one relationship of the function defining IL. If $\hat{\delta}(s, x) = \hat{\delta}(s, y)$ then C(x) $\neq$ C(y) but if C(x) = C(y) then there's a contradiction and x = y, either way showing that the one-to-one quality holds in the function that defines IL.

Problem 27b.



Problem 28.



Problem 29.
Assume that $|C(w)| < n - log_2|Q|$. We know that there are 2^n possible input strings of length n, and the number of possible output strings (C(w)) is $\sum_{i=0}^{n-log_2|Q|-1} 2^i$. The number of possible g(w) is $|Q| \cdot \#C(w)$. We have $|Q| \cdot \#C(w) = |Q|(\sum_{i=0}^{n-log_2|Q|-1} 2^i) = |Q|(2^{n-log_2|Q|-1} + \sum_{i=0}^{n-log_2|Q|-2} 2^i) = |Q|2^{n-log_2|Q|-1} + |Q|\sum_{i=0}^{n-log_2|Q|-2} 2^i$

Now we'll simplify the first term. $|Q|2^{n-log_2|Q|-1} = |Q|2^{n-(log_2|Q|+1)} = \frac{|Q|2^n}{2 \cdot 2^{log_2|Q|}} = \frac{|Q|2^n}{2|Q|} = \frac{2^n}{2} = 2^{n-1}$

Putting the expression back together, we have $2^{n-1} + |Q|\sum_{i=0}^{n-log_2|Q|-2} 2^i$

If we repeat the process we took to simplify the 1st term by pulling the highest term out of the summation and simplifying, we get $2^{n-1} + 2^{n-2} + ... + 2^0$. Then we compare the number of possible input strings (2^n) and the number of possible combinations of output strings and end states ($2^{n-1} + 2^{n-2} + ... + 2^0$), we have $2^n > 2^{n-1} + 2^{n-2} + ... + 2^0$ or $\#w > \#g(w)$. This inequality holds for all $n \in N$.

However, we have a contradiction because if there are more input strings than possible combinations of unique output strings and end states, there must be some input string that has the same g(w) as a different input string, not satisfying the definition of being IL. Therefore, for every IL FSC C=(Q, δ,v,s) and every $n \in N$, there is an input string w$\in \{0,1\}^n$ such that $|C(w)| \geq n - log_2|Q|$. ■