COMS 331: Theory of Computing, Spring 2023
Final Exam

8:50 AM - 10:00 PM Tuesday, May 9, 2023, on Gradescope.

You have TWO hours to complete and submit this exam. There are seven problems, worth the indicated number of points. Any problem (or part of problem) that you leave blank or indicate clearly that you do not want graded will receive 30% credit. Every other problem will be graded and recieve 0%-100% credit. All answers should be explained, at least briefly.

While taking the exam you MAY

- consult the textbook, your own notes on the class lectures, and your graded homework.
- contact the instructor or TA (by email, NOT by posting to canvas) with questions about the exam. (We'll try to answer promptly, but cannot guarantee that.)

While the exam is open (8:50 AM - 10:00 PM) you may NOT

- work with anyone else or access any other resources or websites.
- post questions or any information about the exam.
- discuss the exam with anyone except the instructor and TAs.

Two notes:

- $\#(1, x)$ is the number of 1's in the string x.
- Thanks to the Church-Turing thesis, you do *not* need to explicitly define any Turing machines to complete this exam.

Problem 1. (50 points) Let

$$A = \{x \in \{0,1\}^* \mid |x| \text{ and } \#(1,x) \text{ are odd}\}.$$

Design a DFA M that decides A.

Problem 2. (50 points) Prove that the language

$$B = \{0^{2n}1^{3n} \mid n \in \mathbb{N}\}$$

is not regular.
Note: a proof using the pumping lemma will receive full credit if correct, but is not eligible for partial credit. The ordinal extension nonregularity method is recommended here, but not required.

Problem 3. (60 points) Prove or disprove: The language

$$C = \{0^n 1^n \mid n \in \mathbb{N}\}$$

is decidable.

Problem 4. (60 points) Prove or disprove: The language

$$D = \{k \in \mathbb{N} \mid M_k(k) \text{ and } M_k(2k) \text{ both halt}\}$$

is decidable.

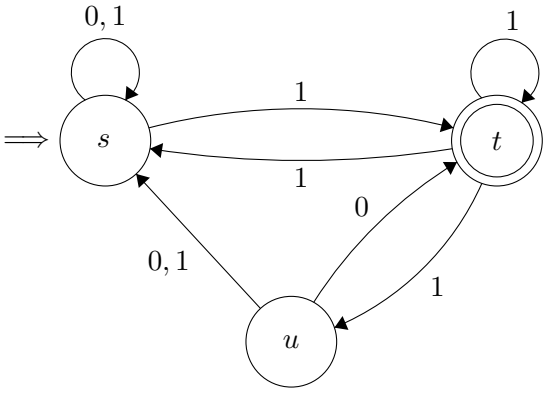
Problem 5. (60 points) Let

$$E = \{x^{|x|^2} \mid x \in \{0,1\}^*\}.$$

Prove that there is a constant $c \in \mathbb{N}$ such that, for all $y \in E$,

$$C(y) \leq \sqrt[3]{|y|} + c.$$

Problem 6. (60 points) Design a DFA that is equivalent to the following NFA.



Problem 7. (60 points) In each of the following either give an example with the indicated property or state that no such example exists.

(a) A language A that is regular but not decidable.

(b) A c.e. language B that is $\leq_m$-reducible to $\{0,1\}^* \setminus B$.

(c) A c.e. language C that is not decidable.

(d) An infinite language D such that $C(x) \geq 2|x|$ for all $x \in D$.

(e) A regular language E such that $\{0,1\}^* \setminus E$ is not regular.

(f) A c.e. language F that is not reducible to the diagonal halting problem K.