COMS 331: Theory of Computing, Spring 2023
Midterm Exam

8:50 AM - 10:00 PM Wednesday, March 22, 2023, on Gradescope.

You have TWO hours to complete and submit this exam. There are six problems, worth 50 points each. Any problem (or part of problem) that you leave blank or indicate clearly that you do not want graded will receive 30% credit. Every other problem will be graded and recieve 0%-100% credit. All answers should be explained, at least briefly.

While taking the exam you MAY

- consult the textbook, your own notes on the class lectures, and your graded homework.
- contact the instructor or TA (by email, NOT by posting to canvas) with questions about the exam. (We'll try to answer promptly, but cannot guarantee that.)

While the exam is open (8:50 AM - 10:00 PM) you may NOT

- work with anyone else or access any other resources or websites.
- post questions or any information about the exam.
- discuss the exam with anyone except the instructor and TAs.

Problem 1. Let A be the set of all strings $x \in \{0,1\}^*$ such that, every time the bits 001 occur consecutively in x, they are immediately followed by the bits 1101. (Note: If 001 does not occur in x, then $x \in A$.) Design a DFA M such that $L(M) = A$.

Problem 2. Let $A = \{x \in \{0,1\}^* \mid |x| \text{ is even if and only if } \#(0,x) \text{ is a multiple of } 3\}$. Design a DFA M such that $L(M) = A$.

Problem 3. Prove: For every $k \in \mathbb{N}$, the language

$$A_k = \{0^n 1^n | n \leq k\}$$

is regular.

Problem 4. Prove: For every $n \in \mathbb{N}$, there is a regular language $A \subseteq \{0,1\}^*$ that is not decided by any n-state DFA.

Problem 5. Prove that the language

$$A = \{0^k 10^n 1y | x, y \in \{0,1\}^* \text{ and } k + |y| = n\}$$

is not regular.
Note: a proof using the pumping lemma will receive full credit if correct, but is not eligible for partial credit. The ordinal extension nonregularity method is recommended here.

Problem 6. In each of the following, either give an expample of languages A and B with the indicated properties or state that no such languages A and B exist.

(a) A is regular and B is not regular.

(b) A is regular, $B \subseteq A$, and B is not regular.

(c) A is regular, B is regular, and $A \cap B$ is not regular.

(d) A is regular, B is not regular, and $A \cap B$ is regular.