COMS 331: Theory of Computing, Fall 2021
Midterm Exam

8:50AM-10:00PM, Friday, October 15, on Gradesccope.

You have TWO hours to complete and submit this exam. There are five problems, worth 100 points for the last question, and 50 points for each of the others. Any problem that you leave blank or indicate clearly that you do not want graded will receive 30% credit. Every other problem will be graded and receive 0-50 points (first four questions) or 0-25 (each part of the last question). All answers should be explained, at least briefly.

While taking the exam you MAY

- consult your own notes and the lecture notes;
- consult the textbook and the class lectures;
- contact the instructor or TA with questions about the exam. (We'll try to answer promptly, but cannot guarantee that.)

While taking the exam you may NOT

- post questions or any information about the exam;
- discuss the exam with anyone except the instructor and TAs;
- work with anyone else or access any other resources or websites.

If you want the tex file of this exam, please go to
`https://iastate.box.com/s/0lhv1umtpx8cr70pdwwrk27vdrug9e5h`

Problem 1. (50 points) Let A be the set of all strings $x \in \{0,1\}^*$ such that the string 001 (with these three bits consecutive) occurs in exactly two places in x. (Examples: $\underline{001}\,\underline{001} \in A$ and $1\underline{001}01\underline{001}0 \in A$, but $001 \notin A$ and $10010010010 \notin A$.)
Design a DFA M such that $L(M) = A$.

Problem 2. (50 points) Prove: For every $k \in \mathbb{N}$, the language

$$A_k = \{x \in \{0,1\}^* \mid \#(1,x) \leq k\}$$

is regular, where $\#(1,x)$ is the number of 1's in x.

Problem 3. (50 points) Let

$$A = \{x \in \{0,1\}^* \mid \#(1,x) \text{ is even if and only if } |x| \text{ is a multiple of } 3\}.$$

(Recall that P if and only if Q means that P and Q are both true or both false.)
Design a DFA M such that $L(M) = A$.

Problem 4. (50 points) Prove: For every $n \in \mathbb{N}$, there is a regular language $A \subseteq \{0,1\}^*$ such that A is not decided by any n-state DFA.

Problem 5. (100 points) In each of the following, either give an example of languages A and B with the indicated property or state that no such languages A and B exist.

(a) A is regular and B is not regular.

(b) A is regular, $B \subseteq A$, and B is not regular.

(c) A is regular, B is regular, and $A \cap B$ is not regular.

(d) A is regular, B is not regular, and $A \cap B$ is regular.