

Homework 3 Solution

3.1

$$P(Y = 0) = P(\text{neither impurity}) = 0.2$$

$$P(Y = 2) = P(\text{both impurities}) = 0.1$$

$$P(Y = 1) = P(\text{impurity } A \text{ or } B) = 1 - P(Y = 0) - P(Y = 2) = 0.7$$

3.4

Let A_1, A_2, A_3 denote the events that valve 1, 2, 3 open, $P(A_i) = 0.8, i = 1, 2, 3$, A_1, A_2, A_3 independent:

$$P(Y = 2) = P(A_1 \cap A_2 \cap A_3) = 0.8^3 = 0.512$$

$$P(Y = 0) = P(\bar{A}_1 \cap (\bar{A}_2 \cup \bar{A}_3)) = P(\bar{A}_1)P(\bar{A}_2 \cup \bar{A}_3) = 0.2(1 - 0.8^2) = 0.072$$

$$P(Y = 1) = 1 - P(Y = 0) - P(Y = 2) = 0.416$$

3.9

(a) Let E denote error for one entry, N denote no error:

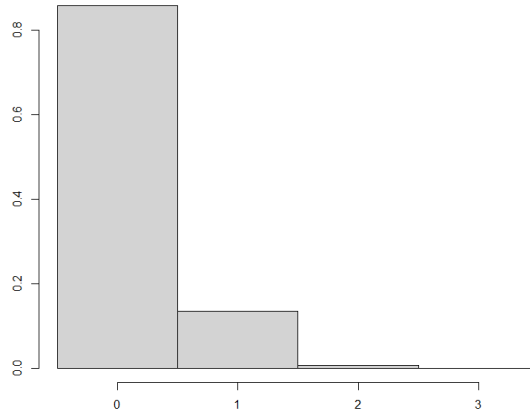
$$P(Y = 3) = P(EEE) = 0.05^3 = 0.000125$$

$$P(Y = 2) = P(EEN) + P(ENE) + P(NEE) = 3 * 0.95 * 0.05^2 = 0.007125$$

$$P(Y = 1) = P(ENN) + P(NEN) + P(NEE) = 3 * 0.95^2 * 0.05 = 0.135375$$

$$P(Y = 0) = P(NNN) = 0.95^3 = 0.857375$$

(b) The histogram:



(c) $P(Y > 1) = P(Y = 2) + P(Y = 3) = 0.00725$

3.12

- $E(Y) = 0.4 * 1 + 0.3 * 2 + 0.2 * 3 + 0.1 * 4 = 2$
- $E(1/Y) = 0.4 * 1/1 + 0.3 * 1/2 + 0.2 * 1/3 + 0.1 * 1/4 = 0.6417$
- $E(Y^2 - 1) = E(Y^2) - 1 = 0.4 * 1^2 + 0.3 * 2^2 + 0.2 * 3^2 + 0.1 * 4^2 - 1 = 4$
- $V(Y) = E(Y^2 - E(Y)^2) = 5 - 2^2 = 1$

3.14

(a) $E(y) = \sum_{y=3}^{13} y * p(y) = 3 * 0.03 + 4 * 0.05 + \dots + 13 * 0.01 = 7.9$

(b)

$$E(Y^2) = 3^2 * 0.03 + 4^2 * 0.05 + \dots + 13^2 * 0.01 = 67.14$$

$$V(Y) = E(Y^2) - E(Y)^2 = 67.14 - 7.9^2 = 4.73$$

$$\sigma = \sqrt{V(Y)} = 2.17$$

(c)

$$(\mu - 2\sigma, \mu + 2\sigma) = (7.9 - 2 * 2.17, 7.9 + 2 * 2.17) = (3.56, 12.24)$$

$$P(Y \in (\mu - 2\sigma, \mu + 2\sigma)) = P(Y \in \{4, 5, \dots, 12\}) = 0.96$$

3.21

$$E(N) = E(8\pi R^2) = 8\pi E(R^2) = 8\pi(0.02 * 21^2 + 0.20 * 22^2 + \dots + 0.05 * 26^2) = 8\pi * 549.1 = 13800.388$$

3.23

Let A denote event of drawing a jack or queen, B denote drawing a king or an ace: $A \cap B = \emptyset$.

Let Y denote the gain:

$$\begin{aligned}P(A) &= 2 * 4/52 = \frac{2}{13} \\P(B) &= 2 * 4/52 = \frac{2}{13} \\E(Y) &= 15 * P(A) + 5 * P(B) - 4 * (1 - P(A \cup B)) \\&= \frac{15 * 2 + 5 * 2 - 4 * 9}{13} = \frac{4}{13}\end{aligned}$$

3.30

- (a) $E(X) = E(Y + 1) = E(Y) + 1 > E(Y) > \mu$ The expectation of X is greater than Y .
- (b) $E(X) = E(Y + 1) = E(Y) + 1 = \mu + 1$
- (c) $V(X) = V(Y + 1) = V(Y)$. The variance will be the same.
- (d) $V(X) = E[(X - E(X))^2] = E[(Y + 1 - \mu - 1)^2] = E[(Y - \mu)^2] = V(Y) = \sigma^2$