Homework 3

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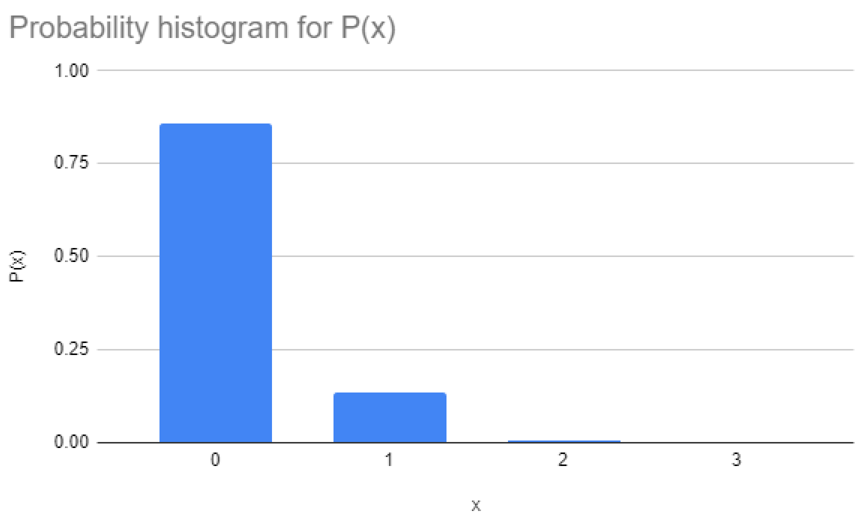
Problem 3.1
A = well has impurity A. B = well has impurity B. We know that P(A) = 0.4, P(B) = 0.5 and $P(\bar{A} \cap \bar{B}) = 0.2$.
Then, $P(A \cup B) = 1\text{-}0.2 = 0.8$ and $P(A \cap B) = 0.4+0.5-0.8 = 0.1$
$P(X=2) = P(A \cap B) = 0.1$
$P(X=1) = P(A \cup B)\text{-}P(A \cap B) = 0.8\text{-}0.1 = 0.7$
$P(X=0) = P(\bar{A} \cap \bar{B}) = 0.2$

Problem 3.4
A = valve 1 fails. B = valve 2 fails. C = valve 3 fails
$p(0) = P(Y=0) = P(A \cap (B \cup C))$
$p(0) = 0.2(0.2+0.2\text{-}(0.2)^2) = (0.20)(0.36) = 0.072$
$p(1) = P(Y=1)=P(A \cap (\bar{B} \cap \bar{C})) + P(\bar{A} \cap (B \cup C))$
$p(1) = (0.2)(0.8)^2+(0.8)(0.2+0.2\text{-}(0.2)^2) = (0.2)(0.64)+(0.8)(0.36)=0.416$
$p(2) = P(Y=2) = P(\bar{A} \cap \bar{B} \cap PC) = (0.8)^3 = 0.512$

Problem 3.9
Part a:
Using binomial distribution with n=3 and p=0.05, the probability for Y=P(Y=y) = $\binom{3}{y}(0.05)^y(0.95)^{3-y}$

Part b:
below is the pmf:

x	0	1	2	3
P(x)	0.8574	0.1354	0.0071	0.0001



Part c:
Probability that the auditor will detect more than one error = P(X>1) = P(X=2)+P(X=3) = 0.0071 + 0.0001 = 0.0072

Problem 3.12
E(Y) = 1(0.4)+2(0.3)+3(0.2)+4(0.1) = 2
E(1/Y) = 1/1(0.4) + 1/2(0.3) + 1/3(0.2) + 1/4(0.1) = 0.6146
E(Y^2-1) = (1^2*0.4 + 2^2*0.3 + 3^2*0.2 + 4^2*0.1)-1 = (0.4+4*0.3+9*0.2+16*0.1)-1 = 5 - 1 = 4
V(Y) = E(Y^2)-$(E(Y))^2$ = 5 - 2^2 = 5 - 4 = 1

Problem 3.14
Part a:
$\mu = E(y) = \sum_{n=3}^{13} x_i * P(x_i)$ = 3*0.03+4*0.05+5*0.07 ... +13*0.01 = 7.9

Part b:
$\sigma = \sqrt{var(y)}$
var(y) = ($\sum_{n=3}^{13} x^2{}_i * P(x_i)$) - (E(y))^2 = (($3^2$*0.03)+($4^2$*0.05)...+($13^2$*0.01)) - 7.9^2 = 67.14-62.41 = 4.73
$\sigma = \sqrt{4.73}$ = 2.17

Part c:
interval = [7.9-2(2.17), 7.9+2(2.17)] = [3.56, 12.24] $\approx$ [4,12]
P(4 < X < 12) = P(x_4) + ... + P($x_1$2) = 0.05 + ... + 0.03 = 0.96

Problem 3.21
E(r^2) = 21^2*0.05 + 22^2*0.20 + 23^2*0.30 + 24^2*0.25 + 25^2*0.15 + 26^2*0.05 = 549.1
N = 8*3.1416*r^2
E(N) = E(8*3.1416*r^2) = 8*3.1416*E(r^2) = 8*3.1416*549.1 = 13800.42

Problem 3.23
There is a 8/52 chance of drawing a jack or a queen. A 8/52 chance of drawing a king or ace. There is 36/52 chance of drawing any other card. X will be the monetary gain. $15 gain for 8/52 chance of jack or queen. $5 gain for 8/52 chance of king or ace. $4 loss for 36/52 chance of any other card.

E(X) = (15)(8/52)+(5)(8/52)+(-4)(36/52) = \$0.31

Problem 3.30
Part a:
The mean of X is more than the mean of Y since E(X)=E(Y+1)=E(Y)+E(1)=E(Y)+1

Part b:
Yes this is agreed to by part a.
Theorem 3.3 states that if you add a constant c to a random variable Y, the mean of the resulting random variable is given by: E(Y+c)=E(Y)+c.
In this case, c=1, so apply the theorem to E(X) = E(Y+1) = E(Y)+1. So E(X) is equal to the mean of Y plus 1.

Part c:
Var(X) is equal to σ^2 which is the variance of Y
Var(X) = Var(Y+1) = Var(Y) as Var(aX+b) = a^2(Var(X))

Part d:
Definition 3.5 gives the formula for the variance of a random variable X as V(X) = E$\{$(X-E(X))$^2\}$
V(X) = E$\{$(Y+1-(μ+1))$^2\}$
V(X) = E($(Y-\mu)^2$)
E($(Y-\mu)^2$) = σ^2
V(X) = E$\{$(X-E(X))$^2\}$ = E($(Y-\mu)^2$) = σ^2
This shows that X and Y have equal variances.