Homework 1

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September 1, 2023

Problem 2.5 a
substitute S in 1st identity with 2nd identity using distributive law:
A = A $\cap$ (B $\cup$ $\bar{B}$)
A = (A $\cap$ B) $\cup$ (A $\cap$ $\bar{B}$)

Problem 2.5 b
B $\subset$ A
from part a, A = (A $\cap$ B) $\cup$ (A $\cap$ $\bar{B}$)
since B $\subset$ A, every element of B is in A. since B $\subseteq$ A, A $\cap$ B is just B
A $\cap$ B = B
A = (A $\cap$ B) $\cup$ (A $\cap$ $\bar{B}$)
A = B $\cup$ (A $\cap$ $\bar{B}$)

Problem 2.5 c
to show this, demonstrate that their intersection is empty.
(A $\cap$ B) $\cap$ (A $\cap$ $\bar{B}$) = $\emptyset$
(A $\cap$ B) $\cap$ (A $\cap$ $\bar{B}$) = A $\cap$ (B $\cap\bar{B}$)
since B and $\bar{B}$ are complements, B $\cap$ $\bar{B}$ = $\emptyset$, so
A $\cap$ (B $\cap$ $\bar{B}$) = A $\cap$ $\emptyset$ = $\emptyset$
this proves that (A $\cap$ B) and (A $\cap$ $\bar{B}$) are mutually exclusive. Since they are mutually exclusive, the union of these sets would be the same as the union of A and $\bar{B}$:
(A $\cap$ B) $\cup$ (A $\cap$ $\bar{B}$) = A $\cup$ $\bar{B}$ = A

Problem 2.5 d
Since B $\subset$ A, we have already established that (A $\cap$ B) and (A $\cap$ $\bar{B}$) are mutually exclusive. To show that B and (A $\cap$ $\bar{B}$) are mutually exclusive, we need to demonstrate that their intersection is empty.
B $\cap$ (A $\cap$ $\bar{B}$) = $\emptyset$
B $\cap$ (A $\cap$ $\bar{B}$) = (B $\cap$ A) $\cap$ $\bar{B}$
since B is a subset of A, B $\cap$ A = B:
(B $\cap$ A) $\cap$ $\bar{B}$ = B $\cap$ $\bar{B}$ = $\emptyset$
this proves that B and (A $\cap$ $\bar{B}$) are mutually exclusive. since they are mutually exclusive, the union of these sets would be the same as the union of B and A:
B $\cup$ (A $\cap$ $\bar{B}$) = B $\cup$ A = A

Problem 2.8 a
A = {(1,2), (1,4), (1,6), (2,2), (2,4), (2,6), (3,2), (3,4), (3,6), (4,2), (4,4), (4,6), (5,2), (5,4), (5,6),

(6,2), (6,4), (6,6)}
B = {(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6) }
C = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,3), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,3), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,3), (6,5) }

Problem 2.8 b
A = (1,2), (1,4), (1,6), (2,2), (2,4), (2,6), (3,2), (3,4), (3,6), (4,2), (4,4), (4,6), (5,2), (5,4), (5,6), (6,2), (6,4), (6,6)
$\bar{C}$ = (2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)
A $\cap$ B = (2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)
A $\cap$ $\bar{B}$ = (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)
$\bar{A}$ $\cup$ B = (1,1), (1,3), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,3), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,3), (5,5), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)
$\bar{A}$ $\cap$ C = (1,1), (1,3), (1,5), (2,1), (2,3), (2,5), (3,1), (3,3), (3,5), (4,1), (4,3), (4,5), (5,1), (5,3), (5,5), (6,1), (6,3), (6,5)

Problem 2.15 a
P(E_2) = 1 - (0.01 + 0.09 + 0.81)
= 1 - 0.91 = 0.09
P(E_2) = 0.09. This is the probability that the company will hit oil or gas on the first drill and miss on the second drill.

Problem 2.15 b
P(E) = P(E_1) + P(E_2) + P(E_3)
= 0.01 + 0.09 + 0.09 = 0.19
P(E) = 0.19. This is the probability that the company will hit oil on at least one of the two drillings.

Problem 2.23
Given that events A and B, where $B \subset A$, we want to prove that P(B) $\leq$ P(A). Since $B \subset A$, every outcome that satisfies event B also satisfies event A. The probability of an event is calculated as the ratio of the number of favorable outcomes to the total number of possible outcomes: P(E) = fav outcomes / total possible outcomes. Since $B \subset A$, the set of outcomes that satisfy event B is a subset of the set of outcomes that satisfy event A. So the number of favorable outcomes for event B is less than or equal to the number of favorable outcomes for event A: number of favorable outcomes for B $\leq$ number of favorable outcomes for A.
P(B) $\leq$ P(A) based on the subset relationship between events B and A, and the principles of probability calculation.

Problem 2.33 a
probability of both tested systems not being defective = (4/6)*(3/5) = 12/30 = 2/5
probability that at least one of the two systems tested will be defective = 1-(2/5) = 3/5
probability of both tested systems being defective = (2/6)*(1/5) = 2/30 = 1/15

Problem 2.33 b
probability of both tested systems not being defective = (2/6)*(1/5) = 2/30 = 1/15
probability that at least one of the two systems tested will be defective = 1-(1/15) = 14/15
probability of both tested systems being defective = (4/6) * (3/5) = 12/30 = 2/5

Problem 2.39 a
n = 6 which is the faces of the die
r = 2 which is the number of dice
sample = $n^r = 6^2 = 36$ sample points

Problem 2.39 b
sample space for sum of 7 = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)
$P_r = \frac{6}{36} = \frac{1}{6} = 0.1667$ is the probability that the sum of the number on the dice is 7.

Problem 2.41
For the first digit, we have 9 possibilities since we are excluding 0. And for the rest, we have 10 possibilities. So the number of different seven-digit telephone numbers possible is = 9*10*10*10*10*10*10 = 9000000

Problem 2.51 a
Since there are four organizers and each buy one ticket. There are only three prizes to be awarded.
If they win all of the prizes, number of ways = $\binom{4}{3}$
probability = $\binom{4}{3}$ / $\binom{50}{3} = \frac{4}{19600}$

Problem 2.51 b
If four organizers win exactly two prizes, 2 out of 4 organizers got prizes, number of ways = $\binom{4}{2}$
The remaining 1 prize goes to 50-4 = one of the 46 customers, number of ways = $\binom{46}{1}$
probability = [$\binom{4}{2}$ * $\binom{46}{1}$] / $\binom{50}{3} = \frac{276}{19600}$

Problem 2.51 c
If four organizers win exactly one prize, 1 out of 4 organizers got prizes, number of ways = $\binom{4}{1}$
The remaining 2 prizes goes to 50-4 = two of the 46 customers, number of ways = $\binom{46}{2}$
probability = [$\binom{4}{1}$ * $\binom{46}{2}$] / $\binom{50}{3} = \frac{4140}{19600}$

Problem 2.51 d
If four organizers win none of the prizes, 0 out of 4 organizers got prizes, number of ways = $\binom{4}{0}$
The remaining 3 prizes goes to 50-4 = 3 of the 46 customers, number of ways = $\binom{46}{3}$
probability = [$\binom{4}{0}$ * $\binom{46}{3}$] / $\binom{50}{3} = \frac{15180}{19600}$

Problem 2.64
P = $\frac{6}{6} * \frac{5}{6} * \frac{4}{6} * \frac{4}{6} * \frac{3}{6} * \frac{2}{6} * \frac{1}{6}$
= 720/46656 = 0.0154 is the probability of rolling a 1, 2, 3, 4, 5, and 6 sequences in any order.

Problem 2.69
we know that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!}$
$= \frac{n!(n-k+1)!}{k!(n+1-k)!} + \frac{kn!}{k!(n+1-k)!}$
$= \frac{(n+1-k+k)n!}{k!(n+1-k)!}$
$= \frac{(n+1)!}{k!(n+1-k)!}$
$= \binom{n+1}{k}$