STAT 330 (Spring 2021) - Exam 1

March 5, 2021

1. Counting (3 random questions)

   (a) How many different passwords are there that are 8 digits long and only contain lowercase letters?

   (b) How many debit card pins are there if you can only use unique numbers (0-9) and the pin is 6 numbers long?

   (c) One coin is flipped and one 6-sided die is rolled. How many different combinations of coin flips and die rolls are there?

   (d) A combination lock has 3 dials each containing the numbers 0 to 9. How many different combinations are there?

   (e) A game of pool contains 15 balls numbered from 1 to 15 and the goal is to knock these balls into the pockets of the pool table. How many different orders are there for knocking all 15 balls into the pockets?

2. Consider rolling two 4-sided dice. (3 random questions)

   (a) What is the sample space for this experiment?

   (b) What is the probability of rolling a sum of 3?

   (c) What is the probability that exactly one die has a 4?

   (d) What is the probability at least one of the dice is an even number?

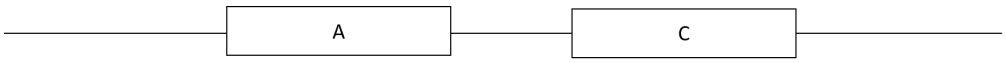
   (e) What is the probability the sum of the two dice is less than or equal to 5?

3. Reliability. For the following questions assume the reliability of component A is 0.9, component B is 0.8, and component C is 0.7.

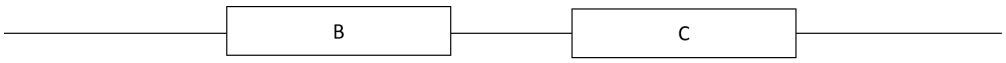
   (a) Reliability serial (1 random question)

      i. What is the reliability of a system that has components A and B in serial?

      ii. What is the reliability of a system that has components A and C in serial?

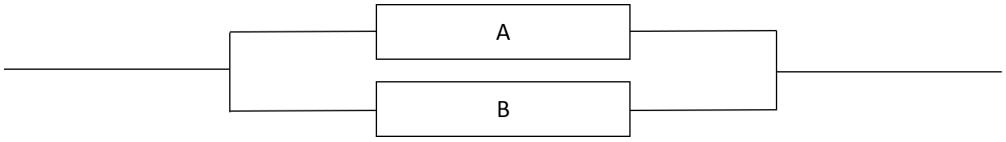


      iii. What is the reliability of a system that has components B and C in serial?

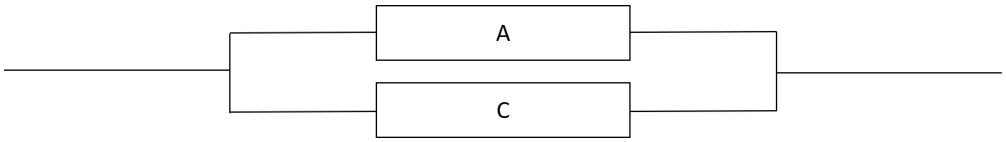


(b) Reliability parallel (1 random question)

i. What is the reliability of a system that has components A and B in parallel?



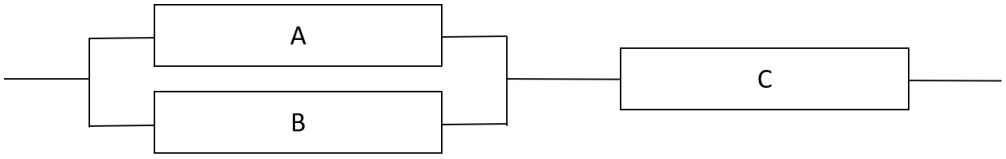
ii. What is the reliability of a system that has components A and C in parallel?



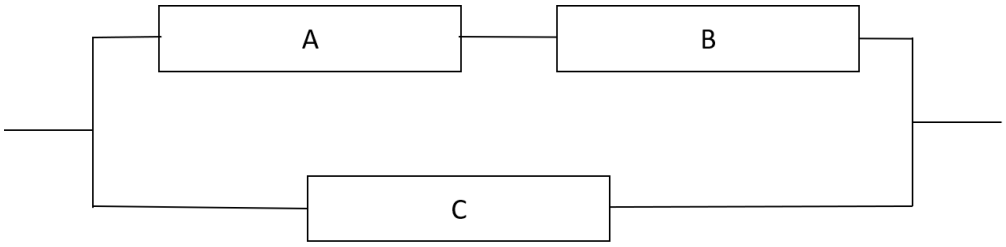
iii. What is the reliability of a system that has components B and C in parallel?

(c) Reliability both (1 random question)

i. What is the reliability of a system that has components A and B parallel and then serial with component C?



ii. What is the reliability of a system that has components A and B serial and then parallel with component C?



4. In Candy Crush, players can use boosters to help defeat levels. For a particular level, the following table provides the joint probability of players beating the level and number of boosters used.

	Number of boosters		
Beat level	0	1	2
Yes	0.00	0.20	0.50
No	??	0.10	0.05

Answer the following questions based on this table. (3 random questions)

(a) What is the probability that a user uses 2 boosters and beats the level?

(b) What is the probability that a user uses no boosters and does not beat the level?

(c) What is the marginal probability a user beats the level?

(d) What is the conditional probability that a user beats the level given that they use 2 boosters?

(e) Are the number of boosters and whether or not the user beats the level independent?

5. Diagnostic testing (1 random question)

   (a) Suppose a diagnostic test has a sensitivity of 0.99 and a specificity of 0.95. The disease being tested for has an overall prevalence of 0.1. If a test result comes back positive, what is the probability of having the disease?

   (b) Suppose a diagnostic test has a sensitivity of 0.95 and a specificity of 0.99. The disease being tested for has an overall prevalence of 0.1. If a test results comes back positive, what is the probability the individual has the disease?

6. Let $X \sim Bern(1/3)$. (all questions)

   (a) $E[X]$

   (b) $Var[X]$

   (c) $P(X = 0)$

7. Let $Y \sim Bin(16, 0.4)$. (3 random questions)

(a) $E[Y]$

(b) $Var[Y]$

(c) $P(Y = 4)$

(d) $P(Y \leq 10)$

8. Let $X \sim Geo(0.8)$. (3 random questions)

   (a) $E[X]$

   (b) $Var[X]$

   (c) $P(X = 1)$

   (d) $P(X > 3)$

9. Let $Y \sim Po(5)$. (3 random questions)

(a) $E[Y]$

(b) $Var[Y]$

(c) $P(Y = 3)$

(d) $P(Y < 5)$

10. Suppose you roll a fair 20-sided die 2 times and you win if you get a 20 in either roll. (3 random questions)

(a) What is the expected number of 20s?

(b) What is the variance in the expected number of 20s?

(c) What is the probability you win?

11. A particular website has 15 visitors per hour. Assume each hour is independent of all other hours. (3 random questions)

(a) What is the expected number of visitors in the next hour?

(b) What is the variance of the number of visitors in the next hour?

(c) What is the probability there will be exactly 13 visitors in the next hour?

(d) What is the probability there will be more than 10 visitors in the next hour?

12. In Minecraft, you can trade with Piglins to obtain Ender Pearls. You continue trading with a Piglin until you obtain one Ender Pearl. For the following questions, assume the probability of obtaining an Ender Pearl is 5%. (3 random questions)

(a) What is the expected number of trades when you receive your first Ender Pearl?

(b) What is the variance in the number of trades when you receive your first Ender Pearl?

(c) What is the probability you get the Ender Pearl on your first trade?

(d) What is the probability you will have to trade more than 30 times before you get your first Ender Pearl?