Homework 4

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Problem 3.38
Part a:
P(A) = 2/3 and P(B) = 1/3
P(Y = y) = $\binom{4}{y}(\frac{1}{3})^y(\frac{2}{3})^{4-y}$

Part b:
P(Y≥3) = P(Y=3)+P(Y=4) = $\binom{4}{3}(\frac{1}{3})^3(\frac{2}{3})^1 + \binom{4}{4}(\frac{1}{3})^4(\frac{2}{3})^0$ = 1/9

Part c:
p = 1/3 and n = 4
E[Y] = np = 4/3

Part d:
Var(Y) = np(1-p) = 8/9

Problem 3.40
Part a:
P(X=14) = $\binom{20}{14} * 0.8^{14} * 0.2^6$ = 0.1091

Part b:
P(X $\geq$ 10) = 1 - (P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9)) = 1 - 0.0006 = 0.9994

Part c:
P(14≤X≤) = P(X=14)+P(X=15)+P(X=16)+P(X=17)+P(X=18) = $(\binom{20}{14} * 0.8^{14} * 0.2^6) + (\binom{20}{15} * 0.8^{15} * 0.2^5) + (\binom{20}{16} * 0.8^{16} * 0.2^4) + (\binom{20}{17} * 0.8^{17} * 0.2^3) + (\binom{20}{18} * 0.8^{18} * 0.2^2)$ = 0.1091 + 0.1746 + 0.2182 + 0.2054 + 0.1369
= 0.8441

Part d:
P(X≤16) = 1- P(X>16) = 1 - (P(X=17)+P(X=18)+P(X=19)+P(X=20)) = 1 - (0.2054 + 0.1369 + 0.0576 + 0.0115) = 1 - 0.4114 = 0.5886

Problem 3.41
probability of getting a correct answer = p = 1/5 = 0.2
P(x $\geq$ 10) = P(10) + P(11) + P(12) + P(13) + P(14) + P(15)
$= \frac{15!}{10!(15-10)!}(0.2)^{10} * (0.8)^5 + \frac{15!}{11!(15-11)!}(0.2)^{11} * (0.8)^4 + \frac{15!}{12!(15-12)!}(0.2)^{12} * (0.8)^3 + \frac{15!}{13!(15-13)!}(0.2)^{13} * (0.8)^2 + 15 * 0.2^{14} * 0.8^1 + 1 * 0.2^{15}$
= 0.00011323

Problem 3.44

Part a:

P(X=5)=$\binom{5}{5}*0.8^5*0.2^0$=0.32768

Part b:

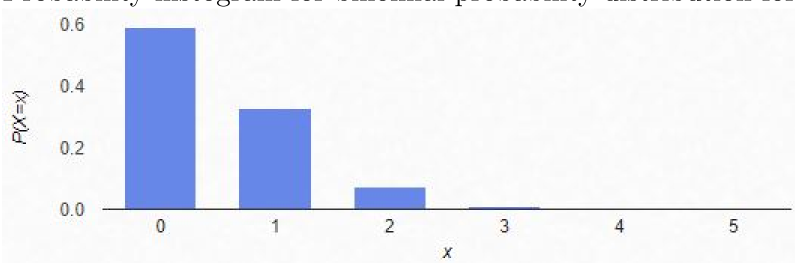
P(X=4)=$\binom{5}{4}*0.6^4*0.4^1$=0.2592

Part c:

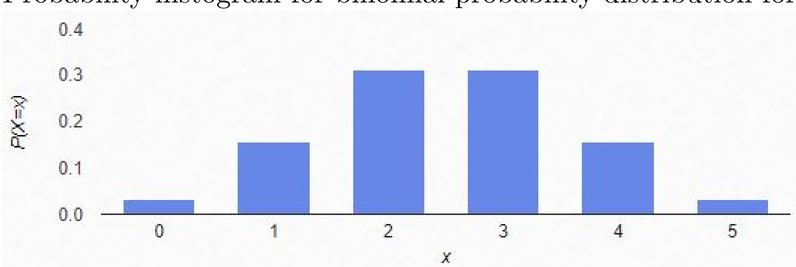
P(X<2) = P(X≤1) = P(X=0) + P(X=1) = 0.16807 + 0.36015 = 0.52822

Problem 3.46

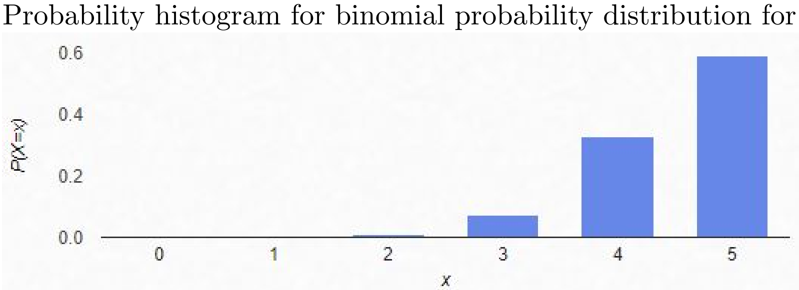
Probability histogram for binomial probability distribution for n=5, p=0.1. Right skewed



Probability histogram for binomial probability distribution for n=5, p=0.5



Probability histogram for binomial probability distribution for n=5, p=0.9. Left skewed



Problem 3.60

Part a:

P(X=14) = $\binom{20}{14}(0.8)^{14}(0.2)^6$ = 0.109

Part b:

P(X≥10) = 1-P(X≤9) = 1 - 0.001 = 0.999

Part c:

P(X $\leq$ 16) = $\sum_{n=0}^{16}(0.8)^n(0.2)^{20-n}$ = 0.589

Part d:

mean of X is μ=np=20(0.8)=16

Var(X) = np(1-p) = 20(0.8)(0.2)=3.2

Problem 3.66
Part a:
The random variable Y has a geometric distribution with probability of the first success on the kth trial given by p(k)=$(1-p)^{k-1} * p = q^{k-1} * p$ for k=1,2,3...
$\sum_{k=1}^{\infty} p(k) = \sum_{k=1}^{\infty} q^{k-1} * p = p * \sum_{k=1}^{\infty} q^{k-1} = p * \sum_{n=0}^{\infty} q^n = p * \frac{1}{1-q} = p * 1/p = 1$

Part b:
For any k=2,3,4...we have $0 < \frac{p(k)}{p(k-1)} = \frac{q^k * p}{q^{k-1} * p} = q < 1$ and the sequence $p(k)_{k \geq 1}$ is monotonic decreasing. The largest value occurs for k=1, so the value Y=1 has the largest probability of occuring: P(Y=1)=p

Problem 3.70
Part a:
P(X=3) = $0.8^2 * 0.2 = 0.128$

Part b:
P(X>10) = $0.8^{10} = 0.1074$

Problem 3.73
Part a:
$= 0.9 * (1-0.9)^{3-1} = 0.009$

Part b:
$= (1-0.9)^{3-1} = 0.01$

Problem 3.81
X is getting the first head. Given that X $\sim$ Geom(p). p = 1/2
E[X] = 1/p = 1/(1/2) = 2

Problem 3.90
probability that ten employees must be tested to find three positives = P(till 9 employees, 2 are positive and 3rd positive is on 10th employee) = $\binom{9}{2} * (0.4)^3 * (0.6)^7 = 0.0645$

Problem 3.97
Part a:
p=0.2, 1-p=0.8, r=1, x=3
P(X=3)=$\binom{3-1}{1-1}(0.8)^{3-1}(0.2)^1 = (0.8)^2 * (0.2)^1 = 0.128$

Part b:
P(X=7)=$\binom{7-1}{3-1}(0.8)^4(0.2)^3 = \binom{6}{2}0.8^4 * 0.2^3 = 0.049$

Part c:
The assumption made is that the random variable X is equal to the number of trials on which the rth success occurs. The number of trials is assumed to be independent.

Part d:
μ = E(X) = r/p = 3/0.2 = 15
Var(X) = $\frac{r(1-p)}{p^2} = \frac{3*0.8}{0.2^2} = 60$