Midterm 1 Solution

1. There are $\binom{50}{4}$ total number of ways to pick 4 tickets among the 50. Let X denote the random variable for number of tickets won.

(a) If you want to win all 3 prizes with your 4 tickets, you must have 3 tickets chosen among those 3 prizes, and $4-3=1$ ticket chosen among the rest 47 non-prize tickets. That is, $\binom{3}{3} * \binom{47}{1}$ cases can win all 3 prizes.

The probability is then $P(X=3) = \binom{3}{3} * \binom{47}{1} / \binom{50}{4} = 1/4900 = 0.0002$

(b) $\binom{3}{2} * \binom{47}{2}$ cases can win exactly 2 prizes. $P(X=2) = 0.0141$. The largest possible number of prizes we can win is 3, thus:

$P(X \geq 2) = P(X=3) + P(X=2) = 1/70 = 0.0143$

2. The number of recoveries X follows a binomial distribution with $n = 20$ and $p = 0.8$.

(a) $E(X) = 20 * 0.8 = 16$

(b) $Var(X) = 20 * 0.8 * 0.2 = 3.2$
$sd(X) = \sqrt{3.2} = 1.7889$

3. Let X denote the random variable for the number of interviews until first applicant with advanced training: X follows a geometric distribution with $p = 0.3$.

$P(X=5) = 0.7^{(5-1)} * 0.3 = 0.0720$

4. Suppose the number of customers arrive follows a Poisson distribution with parameter λ. Then $\frac{\lambda^0 e^{-\lambda}}{0!} = \frac{\lambda^1 e^{-\lambda}}{1!}$: $\lambda = 1$.

(a) $P(X=2) = \frac{\lambda^2 e^{-\lambda}}{2!} = 0.1839$

(b) $P(X > 1) = 1 - P(X=1) - P(X=0) = 0.2642$

5. Here,

(a) $P(Y < 0.5) = \int_0^{0.5} (1.5y^2 + y)dy = 3/16 = 0.1875$

(b) $E(Y) = \int_0^1 y(1.5y^2 + y)dy = \frac{17}{24}$

(c) $E(Y^2) = \int_0^1 y^2(1.5y^2 + y)dy = \frac{11}{20}$
$Var(Y) = E(Y^2) - E(Y)^2 = 0.0487$