Homework 8 Solution

5.47

They are dependent: $P(Y_1 = 1|Y_2 = 2) \neq P(Y_1 = 1|Y_2 = 1)$

5.50

(a) $f_1(y_1) = 1, 0 \leq y_1 \leq 1$, $f_2(y_2) = 1, 0 \leq y_2 \leq 1$. $f(y_1, y_2) = f_1(y_1)f_2(y_2)$, They are independent.

(b) Yes. Since they are independent, the conditional pdf is the same as marginal pdf.

5.59

Since the domain is $0 \leq y_2 \leq y_1 < \infty$, they are dependent.

5.63

$$P(Y_1 > Y_2|Y_1 < 2Y_2) = \frac{P(Y_1 > Y_2, Y_1 < 2Y_2)}{P(Y_1 < 2Y_2)}$$

$$P(Y_1 > Y_2, Y_1 < 2Y_2) = \int_0^\infty e^{-y_2} \int_{y_2}^{2y_2} e^{-y_1} dy_1 dy_2 = \frac{1}{6}$$

$$P(Y_1 < 2Y_2) = \int_0^\infty e^{-y_2} \int_0^{2y_2} e^{-y_1} dy_1 dy_2 = \frac{2}{3}$$

Thus $P(Y_1 > Y_2|Y_1 < 2Y_2) = \frac{1}{4}$

5.72

(a) $E(Y_1) = \frac{4}{9} * 0 + \frac{4}{9} * 1 + \frac{1}{9} * 2 = \frac{2}{3}$

(b) $V(Y_1) = E(Y_1^2) - E(Y_1)^2 = \frac{8}{9} - (\frac{2}{3})^2 = \frac{4}{9}$

(c) $E(Y_1 - Y_2) = E(Y_1) - E(Y_2) = 0$

5.77

$f_1(y_1) = 3(1 - y_1)^2, 0 \leq y_1 \leq 1,\ f_2(y_2) = 6y_2(1 - y_2), 0 \leq y_2 \leq 1$

(a) $E(Y_1) = \int_0^1 y_1 f_1(y_1) dy_1 = \frac{1}{4}$
$E(Y_2) = \int_0^1 y_2 f_2(y_2) dy_2 = \frac{1}{2}$

(b) $V(Y_1) = E(Y_1^2) - E(Y_1)^2 = \frac{3}{80}$
$V(Y_2) = E(Y_2^2) - E(Y_2)^2 = \frac{1}{20}$

(c) $E(Y_1 - 3Y_2) = E(Y_1) - 3E(Y_2) = -\frac{5}{4}$

5.89

$$\begin{aligned} Cov(Y_1, Y_2) &= E(Y_1Y_2) - E(Y_1)E(Y_2) \\ &= \frac{2}{9} - \frac{2}{3} * \frac{2}{3} \\ &= -\frac{2}{9} \end{aligned}$$

Not surprising, since from the table the value of Y_2 tend to be smaller as Y_1 increases.

5.92

$E(Y_1) = 1/4, E(Y_2) = 1/2, E(Y_1Y_2) = \int_0^1 \int_0^{y_2} 6y_1y_2(1 - y_2) dy_1 dy_2 = \frac{3}{20}$

Thus $Cov(Y_1, Y_2) = \frac{3}{20} - \frac{1}{2} * \frac{1}{4} = \frac{1}{40}$, consistent with the fact that Y_1, Y_2 are not independent.