Homework 7

Neha Maddali
October 14, 2023

Problem 5.3
joint probability function: P$(y_1, y_2) = \frac{\binom{4}{y_1}\binom{3}{y_2}\binom{2}{3-y_1-y_2}}{\binom{9}{3}} = 0 \leq y_1, 0 \leq y_2, 1 \leq y_1 + y_2 \leq 3$

Problem 5.7
Part a:
P$(Y1 < 1, Y2 > 5) = \int_5^\infty \int_0^1 e^{-(y_1+y_2)}\, dy_1 dy_2 = e^{-5}(1 - e^{-1}) = 0.00426$

Part b:
P$(Y1 + Y2 < 3) = \int_0^3 \int_0^{3-y_1} e^{-(y_1+y_2)}\, dy_1 dy_2 = \int_0^3 e^{-y_1}(1 - e^{y1-3})\, dy_1 =$ 1-4e^{-3} $= 0.8009$

Problem 5.9
Part a:
$\int_0^1 \int_0^{y^2} (1 - y_2)\, dy_1 dy_2 = \int_0^1 (1 - y_2) \int_0^{y^2} dy_1 dy_2 = \int_0^1 (1 - y_2) y_2\, dy_2 = \frac{1}{2} - \frac{1}{3} = 1/6$
k=6

Part b:
P$(Y_1 \leq 3/4, Y_2 \geq 1/2) =$ P$(Y_1 < 1/2, Y_2 \geq 1/2)$ + P$(1/2 \leq Y_1 \leq 3/4, Y_2 > 1/2)$
$= \int_{1/2}^1 \int_0^{1/2} 6(1 - y_2)\, dy_1 dy_2 + \int_{1/2}^{3/4} \int_{y_1}^1 6(1 - y_2)\, dy_1 dy_2 = \frac{3}{8} + \frac{7}{64} = 0.484375$

Problem 5.15
Part a:
P$(Y_1 < 2, Y_2 > 1) = \int_1^2 \int_1^{y_1} e^{-y_1}\, dy_2 dy_1$
$= \int_1^2 (y_1 - 1) e^{-y1}\, dy_1 = e^{-1} - 2e^{-2} = 0.0972$

Part b:
P$(Y_1 \geq 2Y_2) = \int_0^\infty \int_{2y_2}^\infty e^{-y_1}\, dy_1 dy_2 = \frac{1}{2} \int_0^\infty y_1 e^{-y_1}\, dy_1 = 1/2$

Part c:
P$(Y_1 - Y_2 \leq 1) = \int_0^\infty \int_{y2+1}^\infty e^{-y_1}\, dy_1 dy_2 = \int_0^\infty e^{-(y_2+1)}\, dy_2 = e^{-1}$

Problem 5.21
Part a:
Y_1 has hypergeometric distribution with N=9, r=4, n=3.
pdf: P(Y_1=y) $= \frac{\binom{4}{y}\binom{5}{3-y}}{\binom{9}{3}}$ where $0 \leq y \leq 3$

Part b:

$\mathrm{P}(Y_1 = 1|Y_2 = 2) = \frac{P(Y_1=1,Y_2=2)}{P(Y_2=2)} = \frac{\frac{\binom{4}{1}\binom{3}{2}\binom{9-4-3}{0}}{\binom{9}{3}}}{\frac{\binom{3}{2}\binom{6}{1}}{\binom{9}{3}}} = 2/3$

Part c:

$\mathrm{P}(Y_3 = 1|Y_2 = 1) = \mathrm{P}(Y_1 = 1, Y_2 = 1) = \frac{P(Y_1=1,Y_2=1)}{P(Y_2=1)} = \frac{\frac{\binom{4}{1}\binom{3}{1}\binom{9-4-3}{1}}{\binom{9}{3}}}{\frac{\binom{3}{1}\binom{6}{2}}{\binom{9}{3}}} = 2/3$

Part d:
The two are the same.

Problem 5.23
Part a:
$Y_2 < Y_1$. The marginal density function is... $f_2(y_2) = \int_{y_2}^{1} 3y_1\, dy_1 = \frac{3}{2}(1 - y_2^2)$, $0 \leq y_2 \leq 1$

Part b:
$y_2 \in [0, y_1]$

Part c:
$\mathrm{P}(Y_2 > \frac{1}{2}|Y_1 = \frac{3}{4}) =$ 1-$\mathrm{P}(Y_2 \leq \frac{1}{2}|Y_1 = \frac{3}{4}) =$ 1-$\int_0^{1/2} f_{Y_2|Y_1}(y_2|y_1 = 3/4)dy_1\, dy_1$
$=$ 1-$\int_0^{1/2}(\frac{1}{3/4})dy_1\, dy_1$
$=$ 1-$\frac{4}{3}\int_0^{1/2} 1dy_1\, dy_1$
$=$ 1-$\frac{4}{3}(y_1)_0^{1/2} = 1 - (4/3)(1/2 - 0) = 1 - 2/3 = 1/3$ $\mathrm{P}(Y_2 > \frac{1}{2}|Y_1 = \frac{3}{4}) = 1/3$

Problem 5.25
Part a:
$f_1(y_1) = e^{-y_1}, y_1 > 0$
$f_2(y_2) = e^{-y_2}, y_2 > 0$

Part b:
$\mathrm{P}(1 < Y_1 < 2.5) = \int_1^{2.5} e^{-y_1}\, dy_1 = e^{-1} - e^{-2.5} = 0.2858$
$\mathrm{P}(1 < Y_2 < 2.5) = \int_1^{2.5} e^{-y_2}\, dy_2 = e^{-1} - e^{-2.5} = 0.2858$

Part c:
values $y_2 \in (0, \infty)$

Part d:
$f_{y1|y2}(y_1|y_2) = f_1(y_1) = e^{-y1}, y_1 > 0$

Part e:
$f_{y2|y1}(y_2|y_1) = \frac{f(y_1,y_2)}{f(y_1)} = \frac{e^{-(y1+y2)}}{e^{-y1}}$
$= e^{-y2}$, $y_2 > 0$.

Part f:
The two are the same.

Part g:
From part f, the marginal and conditional probabilities that Y1 falls in any interval will be equal.

Problem 5.27

Part a:

marginal density functions...

$f_1(y_1) = \int_{y1}^{1} 6(1-y_1)\,dy_1 = 3(1-y_1)^2, 0 \leq y_1 \leq 1$

$f_2(y_2) = \int_{0}^{y2} 6(1-y_2)\,dy_2 = 6y_2(1-y_2), 0 \leq y_2 \leq 1$

Part b:

$\mathrm{P}(Y_2 \leq 1/2 | Y_1 \leq 3/4) = \frac{\int_0^{1/2}\int_0^{y_2} 6(1-y_2) dy1 dy2}{\int_0^{3/4} 3(1-y_1)^2} = 0.5079$

Part c:

$f_{y1|y2}(y1|y2) = \frac{f(y1,y2)}{f_2(y2)} = \frac{1}{y2}, 0 \leq y1 \leq y2 \leq 1$

Part d:

$f_{y2|y1}(y2|y1) = \frac{f(y1,y2)}{f_1(y1)} = \frac{2(1-y_2)}{(1-y_1)^2}, 0 \leq y1 \leq y2 \leq 1$

Part e:

$\mathrm{P}(Y_2 \geq 3/4 | Y_1 = 1/2) = \int_{3/4}^{1} f_{2|1}(y2|y1 = 1/2)\,dy_2 = \int_{3/4}^{1} 8(1-y2)\,dy_2 = 1/4$