Homework 14

Neha Maddali
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Problem 10.17
Part a:
Null hypothesis: $H_0 : \mu_1 = \mu_2$
Alternative hypothesis: $H_a : \mu_1 > \mu_2$

Part b:
Since both sample size are greater than n=30, use normal distribution, and a one-tailed test. Given a = 0.01, the critical value is Z(0.01) = 2.33 from the standard normal table. So the rejection region is $Z > 2.33$

Part c:
z = 0.075

Part d:
Fail to reject the null hypothesis because there is not enough evidence to conclude the mean distance for breaststroke is larger than individual medley.

Part e:
The sample variances used in the test statistic were too large to be able to detect a difference

Problem 10.18
$H_0 : \mu = 13.20, H_a : \mu < 13.20$
With the large sample test for a mean, z=-2.53, $\alpha = 0.01, -z_{0.01} = -2.326$
H_0 is rejected since there is evidence that the company is paying substandard wages.

Problem 10.21
$H_0 : \mu_1 = \mu_2,\ H_a : \mu_1 \neq \mu_2$
$s_p^2 = \frac{29*(0.26)^2+34*(0.22)^2}{30+35-2} = \frac{1.9604+1.6456}{63} = 0.0572$
$z = \frac{1.65-1.43}{\sqrt{0.0572*(1/30+1/35)}} = 3.6871$
With $\alpha = 0.01$, the test rejects if $|z| > 2.576$. So reject the hypothesis that the soils have equal mean shear strengths

Problem 10.33
p_1 = proportion of Republicans strongly in favor of the death penalty
p_2 = proportion of Democrats strongly in favor of the death penalty
$H_0 : p_1 = p_2, H_a : p_1 > p_2$
$\alpha = 0.05$
$\hat{p} = \frac{46+34}{400} = 0.20$

reject H_0 if $z_{0.05} > 1.645$
$z = \frac{46/200-34/200}{\sqrt{0.2*0.8*(1/200+1/200)}} = \frac{0.23-0.17}{0.04} = 1.5$
We fail to reject H_0 since there is not enough evidence to support the researcher's belief

Problem 10.40
$H_0 : p_1 = p_2, H_a : p_1 > p_2$
Find a common sample size n such that $\alpha = P(rejectH_0|H_0true) = 0.05$ and where $\beta = P(failtorejectH_0|H_atrue)$
0.20 For $\alpha = 0.05$, use test statistic Z such that we can reject H_0 if $Z \geq z_{0.05} = 1.645$
For $\beta = 0.20$, fail to reject H_0 if $\frac{\hat{p_1}-\hat{p_2}-0.1}{\sqrt{\frac{p_1q_1}{n}+\frac{p_2q_2}{n}}} = -0.84$ where $-0.84 = z_{0.2}$
$-0.84 = \frac{1.645\sqrt{\frac{p_1q_1}{n}+\frac{p_2q_2}{n}}-0.1}{\sqrt{\frac{p_1q_1}{n}+\frac{p_2q_2}{n}}} = 2.485 = \frac{0.1}{\sqrt{\frac{p_1q_1}{n}+\frac{p_2q_2}{n}}}$
set $p_1 = p_2 = 0.5$ as the worst case scenario to find that $2.485 = \frac{0.1}{\sqrt{0.5*0.5*(1/n+1/n)}}$
the common sample size for the researcher's test should be n=309

Problem 10.43
Part a:
$H_0 : \mu_1 = \mu_2, H_a : \mu_1 > \mu_2$ with $\alpha = 0.05$, with a rejection region of $z > 1.645$.
the test statistic is $z = \frac{32.19-31.68}{\sqrt{\frac{4.34^2}{37}+\frac{4.56^2}{37}}} = 0.49$
H_0 is not rejected since there is not enough evidence to show that the mean dexterity score for students participating in sports is larger

Part b:
$\bar{Y_1} - \bar{Y_2} > 1.645\sqrt{\frac{4.34^2}{37} + \frac{4.56^2}{37}} = 1.702$
so $\beta = P(\bar{Y_1} - \bar{Y_2} \leq 1.702|\mu_1 - \mu_2 = 3) = P(Z \leq -1.25) = 0.1056$