Midterm 2 Solution

1

(a) (a) 0.9243, (b) 2.3263, (c) 0

(b) $P(-2 < X < 5.74) = P(\frac{-2-1}{2} < Z < \frac{5.74-1}{2}) = 0.9243$

2

(a) $f_2(y_2) = 2y_2, 0 \leq y_2 \leq 1$

(b)

$$\begin{aligned}
P(Y_2 \geq 3/4) &= \int_{3/4}^{1} f_2(y_2) dy_2 \\
&= \frac{7}{16} \\
P(Y_1 \leq 1/2, Y_2 \geq 3/4) &= \int_{3/4}^{1} \int_{0}^{1/2} f(y_1, y_2) dy_1 dy_2 \\
&= \frac{7}{64} \\
P(Y_1 \leq 1/2 | Y_2 \geq 3/4) &= \frac{P(Y_1 \leq 1/2, Y_2 \geq 3/4)}{P(Y_2 \geq 3/4)} \\
&= \frac{1}{4}
\end{aligned}$$

(c) $f_{1|2}(y_1|y_2) = \frac{f(y_1,y_2)}{f_2(y_2)} = 2y_1, 0 \leq y_1 \leq 1, \forall y_2$

(d)

$$\begin{aligned}
E(Y_1 Y_2) &= \int_0^1 \int_0^1 y_1 y_2 f(y_1, y_2) dy_1 dy_2 \\
&= 4 \int_0^1 \int_0^1 y_1^2 y_2^2 dy_1 dy_2 = \frac{4}{9} \\
E(Y_1) &= \int_0^1 y_1 f_1(y_1) dy_1 = \frac{2}{3} \\
E(Y_2) &= \frac{2}{3} \\
Cov(y_1, y_2) &= E(Y_1 Y_2) - E(Y_1)E(Y_2) = 0
\end{aligned}$$

Y_1, Y_2 are independent (since marginal densities are the same as conditional densities), thus the covariance is zero.

3

(a)

$$\begin{aligned} F_X(x) &= P(X < x) \\ &= P(-(1/3)\log Y < x) = P(Y > e^{-3x}) \\ &= 1 - F_Y(e^{-3x}) = 1 - e^{-3x} \\ f_X(x) &= 3e^{-3x}, x \geq 0 \end{aligned}$$

(b)

$$E(e^X) = \int_0^\infty e^x * 3e^{-3x} dx = \frac{3}{2}$$

4

$$\begin{aligned} F(y) &= 1 - e^{-y/\beta} \\ F_{(n)}(y) &= F(y)^n = (1 - e^{-y/\beta})^n \end{aligned}$$

$$\begin{aligned} P(Y_{(n)} \geq 4) &= 1 - F_{(n)}(4) \\ &= 1 - (1 - e^{-4/2})^5 = 0.5167 \end{aligned}$$

5

$$\begin{aligned} P(|\bar{X} - \mu| < 1) &= P(\frac{|\bar{X} - \mu|}{\sigma/\sqrt{n}} < \frac{1}{\sigma/\sqrt{n}}) \\ &= P(|Z| < \frac{1}{2/\sqrt{25}}) \\ &= P(|Z| < \frac{5}{2}) \\ &= 0.9876 \end{aligned}$$