

Homework 14 Solution

10.17

- (a) $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_1 > \mu_2$
- (b) Reject if $Z > 2.326$, where Z is the test statistic
- (c) $Z = \frac{9017-5853}{\sqrt{7162^2/130+1961^2/80}} = 4.756$
- (d) Reject H_0 , there is sufficient evidence to indicate that the average number of meters per week spent practicing breaststroke is greater for exclusive breaststrokers than it is for those swimming individual medley
- (e) Two groups have very different sample means.

10.18

$H_0 : \mu = 13.2$ vs. $H_a : \mu < 13.2$. $Z = \frac{12.2-13.2}{2.5/\sqrt{40}} = -2.53 < Z_{0.01} = -2.326$.

Reject the null hypothesis, there is evidence that the company is paying less than average.

10.21

$H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_1 \neq \mu_2$
 $Z = \frac{1.65-1.43}{\sqrt{0.26^2/30+0.22^2/35}} = 3.648 > Z_{0.995} = 2.576$

Reject the null hypothesis, the soils don't have equal mean shear strengths.

10.33

Define p_1 : proportion of republicans; p_2 : proportion of democrats.

$H_0 : p_1 = p_2$ vs. $H_a : p_1 > p_2$
 $\hat{p}_1 = 0.23, \hat{p}_2 = 0.17, \hat{\sigma}_1^2 = 0.23(1 - 0.23), \hat{\sigma}_2^2 = 0.17(1 - 0.17)$
 $Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{\sigma}_1^2/n_1 + \hat{\sigma}_2^2/n_2}} = 1.504 < Z_{0.95} = 1.645$

We fail to reject the null hypothesis, under 95% confidence level there is no evidence that proportion for republicans are higher.

10.40

$H_0 : p_1 = p_2$ vs. $H_a : p_1 > p_2$

$$\beta = 0.2, \text{ minimum detection} = 0.1: \frac{\hat{p}_1 - \hat{p}_2 - 0.1}{\sqrt{p_1(1-p_1)/n + p_2(1-p_2)/n}} = Z_{0.2} = -0.842$$

$$\alpha = 0.05: \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p_1(1-p_1)/n + p_2(1-p_2)/n}} = Z_{0.95} = 1.645$$

$$\text{Thus } \frac{0.1}{\sqrt{p_1(1-p_1)/n + p_2(1-p_2)/n}} = 1.645 + 0.842$$

$$\text{Plugging in } p_1 = p_2 = 0.5: n = 308.76 = 309$$

10.43

(a) $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_1 > \mu_2$

$$Z = \frac{32.19 - 31.68}{\sqrt{4.34^2/37 + 4.56^2/37}} = 0.49 < Z_{0.95} = 1.645$$

Do not reject the null hypothesis, where is no difference.

(b) $\bar{Y}_1 - \bar{Y}_2 > Z_{0.95} \sqrt{4.34^2/37 + 4.56^2/37} = 1.702$

Rejection region: $(1.702, \infty)$

$$\beta = P(\bar{Y}_1 - \bar{Y}_2 \leq 1.702 | \mu_1 - \mu_2 = 3) = P(Z \leq \frac{1.702 - 3}{\sqrt{4.34^2/37 + 4.56^2/37}}) = 0.105$$