Fall 2020 **Statistics 330 - Section 2** **Practice Exam I**

(100 points)

Please write your first and last name here:

Name ____

Instructions:

This practice exam contains similar types/styles of questions you will find on Exam 1. Expect between 8-10 questions of varying length and difficulty. There are many questions that could be asked, so this is not exhaustive. Refer back to Module 1 and 2 written homework for more review.

- Please read the Exam 1 Information
- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in lecture, so they are not necessarily ordered easiest to hardest.
- You may use any material from our Canvas Page, but nothing more

1. A business office orders paper supplies from one of three vendors, V_1, V_2, or V_3. Orders are to be placed on two successive days, one order per day, and the vendors are selected at random each day (same vendor can get both orders). For example, (V_2, V_3) denotes that vendor V_2 gets the order on the first day and vendor V_3 gets the order on the second day. Assume all outcomes are equally likely.

   (a) Write down the sample space of this experiment (of placing paper orders on two successive days). (4 points)

   (b) Let A denote the event that the same vendor gets both orders. List the outcomes in A, and find $\mathbb{P}(A)$. (4 points)

   (c) Let B denote the event that V_2 gets at least one order. List the outcomes in B, and find $\mathbb{P}(B)$. (4 points)

2. A box contains four red, three yellow, and seven green balls. Three balls are randomly selected from the box without replacement.

   (a) What is the probability that all three balls are the same color? (4 points)

   (b) What is the probability that the three balls are all different colors? (4 points)

   (c) What is the probability that the three balls have two colors (containing two balls of one color and one of another)? (4 points)

   (d) What is the probability of getting at least one red ball? (4 points)

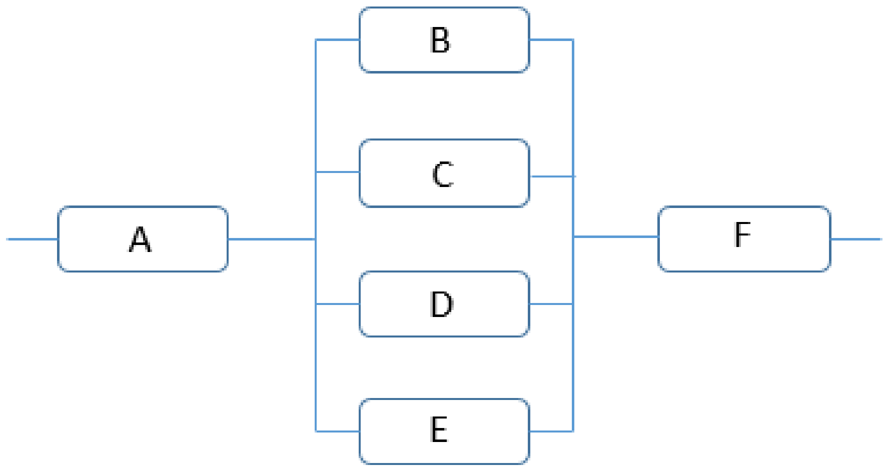
3. Suppose at a DUI checkpoint, random drivers are stopped and given a breathalyzer test. Let A be the event that a driver is over the legal limit and B be the event that the breathalyzer indicates they are over the legal limit. Suppose the breathalyzer test is calibrated such that the probability it correctly identifies someone who is over the legal limit is 0.96. Also, the probability that it correctly identifies someone who is *not* over the legal limit is 0.91. Assume that 15% of all drivers on the road are over the legal limit for the entire question, i.e. $\mathbb{P}(A) = 0.15$

   (a) Describe in words the meaning of $\mathbb{P}(B|\overline{A})$ in the context of the question. (2 points)

   (b) A car is randomly stopped and the driver is given a breathalyzer. What is the probability that the breathalyzer test indicates they are over the legal limit? (4 points)

   (c) A car is randomly stopped and the driver is given a breathalyzer. The breathalyzer indicates they are over the legal limit. What is the probability that they are actually over the legal limit? (4 points)

4. In the following system, the probability of the individual components A, B, C, D, E, and F working are 0.95, 0.80, 0.80, 0.80, 0.80, and 0.95 respectively. Compute the system's reliability. (8 points)



5. A video game store has one copy of a game that it rents out in whole hour increments with a maximum check out time of five hours. When a person comes by to rent the game we can define a random variable, X, as: X = time (in whole hours) that the game will be rented for. From past experience, the distribution of X is given by the following table.

x	1	2	3	4	5
$p_X(x)$	0.04	0.14	0.28	0.32	0.22

(a) Compute $\mathbb{E}(X)$ and $Var(X)$. (8 points)

(b) The game store has the following pricing scheme for the game rental. \$5 flat fee plus \$2 **per** hour the game is rented for. Let Y = profit when a person come to rent the game. Write Y as a function of X and use the rules of expectation to find $\mathbb{E}(Y)$. (3 points)

(c) In an attempt to make more money, the store changes the game rental pricing to: \$2 flat fee upfront. \$3 extra **per** hour if rented for one, two, or three hours; \$4 extra **per** hour if rented for four or five hours. Because of this change, the demand for the game has changed. Suppose the new distribution for X, the time the game is rented for (in hours), is now:

x	1	2	3	4	5
$p_X(x)$	0.11	0.20	0.41	0.19	0.09

Let Z = profit when a random person rents the game with the new pricing scheme described in (c). Fill in the rest of the pmf for Z and calculate the store's new expected profit. Did the new pricing scheme increase expected profit? (4 points)

z					
$p_Z(z)$	0.11	0.20	0.41	0.19	0.09

6. An internet search engine puts 25% of its advertisements on the left of web contents, and the rest on the right. Six advertisements are randomly and independently selected. Define random variable

$$X_i = \begin{cases} 1 & \text{if the } i\text{th advertisement is on the left} \\ 0 & \text{otherwise,} \end{cases}$$

for $i = 1, \ldots, 6$.

(a) Name the distribution of X_i, and give the name(s) and value(s) of the parameter(s). (3 points)

(b) Let Y be the total number of advertisements placed on the left, i.e. $Y = X_1 + X_2 + \cdots + X_6$. Name the distribution of Y, and give the name(s) and value(s) of the parameter(s). (3 points)

(c) Find the expected value and variance of Y. (2 points)

(d) What is the probability that at least 3 advertisements are on the left? (4 points)

7. Suppose in a fiber optic communication system, there is an average of 1.5 transmission errors per ten seconds. Define a random variable X as the number of errors in a ten second period, then $X \sim \text{Pois}(1.5)$.

   (a) What is the probability of *at most* one error in a ten second period? (3 points)

   (b) What is the probability that less than four errors occur in a half-minute period? (3 points)

   (c) Suppose you start monitoring ten second periods, one after another. Call them periods 1, 2, 3, ... etc. You record the number of transmission errors per period. Assume time periods are independent of each other in terms of errors etc. Define a success for each period as "time period had more than one error". Let $Y =$ the time period in which the first success occurs. Find $\mathbb{P}(Y > 3)$. (3 points)

8. Let X be the grade point for a major course and Y be that for a general education (GE) course. Consider the following joint distribution for X and Y.

	Y		
X	2	3	4
2	0.05	0.1	0
3	0.1	0.15	0.1
4	0.1	0.1	0.3

(a) Find the marginal probability mass functions for X and Y. (4 points)

(b) Find the probability that the major course has a higher grade point than the GE course. (3 points)

(c) Calculate the covariance between X and Y. (4 points)

(d) Calculate the correlation between X and Y. Are they correlated? (3 points)

(e) Are X and Y independent? (2 points)

(f) A job application asks for a weighted grade point average, calculated as $0.8X + 0.2Y$. Find the expectation of the weighted grade point average of X and Y. (2 points)