STAT 477/STAT 577
Exam 2 Review Sheet

Inference for $p_1 - p_2$

- Descriptive Statistics

$$\hat{p}_1 = \frac{Y_1}{n_1} \qquad \hat{p}_2 = \frac{Y_2}{n_2} \qquad \hat{p}_{\text{pooled}} = \frac{Y_1 + Y_2}{n_1 + n_2}$$

- Hypothesis Test

  $H_0 : p_1 - p_2 = 0$
  $H_a : p_1 - p_2 > 0$ or $H_a : p_1 - p_2 < 0$ or $H_a : p_1 - p_2 \neq 0$

$$\text{Test Statistic: } z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_{\text{pooled}}(1-\hat{p}_{\text{pooled}})}{n_1} + \frac{\hat{p}_{\text{pooled}}(1-\hat{p}_{\text{pooled}})}{n_2}}}$$

  p-value for $H_a : p_1 - p_2 > 0$ - $P(Z > z)$
  p-value for $H_a : p_1 - p_2 < 0$ - $P(Z < z)$
  p-value for $H_a : p_1 - p_2 \neq 0$ - $2 * P(Z > |z|)$

- Confidence Interval

$$\hat{p}_1 - \hat{p}_2 \pm z_{1-\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Test of Equality of Multiple Proportions or Multiple Multinomial Distributions

H_0 : all proportions or multinomial distributions are the same
H_a : at least one of the proportions or multinomial distributions is different

$$E(Y_{ij}) = \frac{n_i Y_{.j}}{n} \qquad \text{Test Statistic: } X^2 = \sum_{i=1}^{I}\sum_{j=1}^{J}\frac{(Y_{ij} - E(Y_{ij}))^2}{E(Y_{ij})} \qquad \text{p-value: } P(\chi^2_{(I-1)(J-1)} > X^2)$$

Relative Risk

$$RR = \frac{p_1}{p_2} \qquad \widehat{RR} = \frac{\hat{p}_1}{\hat{p}_2} \qquad \exp\left(\ln \widehat{RR} \pm z_{1-\alpha/2}\sqrt{\frac{1-\hat{p}_1}{n_1\hat{p}_1} + \frac{1-\hat{p}_2}{n_2\hat{p}_2}}\right)$$

Odds Ratio

$$\phi = \frac{\frac{p_1}{1-p_1}}{\frac{p_2}{1-p_2}} \qquad \hat{\phi} = \frac{\frac{\hat{p}_1}{1-\hat{p}_1}}{\frac{\hat{p}_2}{1-\hat{p}_2}} = \frac{\frac{Y_{11}}{Y_{12}}}{\frac{Y_{21}}{Y_{22}}} = \frac{Y_{11}Y_{22}}{Y_{12}Y_{21}}$$

$$\exp\left(\ln \hat{\phi} \pm z_{1-\alpha/2}\sqrt{\frac{1}{Y_{11}} + \frac{1}{Y_{12}} + \frac{1}{Y_{21}} + \frac{1}{Y_{22}}}\right)$$

Test of Independence

H_0 : variables are independent $\qquad H_a$: variables are not independent

$$\widehat{E(Y_{ij})} = \frac{Y_{i.}Y_{.j}}{n} \qquad \text{Test Statistic: } X^2 = \sum_{i=1}^{I}\sum_{j=1}^{J}\frac{(Y_{ij}-E(Y_{ij}))^2}{E(Y_{ij})} \qquad \text{p-value: } P(\chi^2_{(I-1)(J-1)} > X^2)$$

Measures of Association

$$r_\varphi = \pm\sqrt{\frac{X^2}{n}} \qquad \hat{\varphi}_C = \sqrt{\frac{\frac{X^2}{n}}{min(I-1, J-1)}} \qquad \hat{\gamma} = \frac{P-Q}{P+Q}$$

McNemar's Test

$H_0 : p_{1.} = p_{.1}$ vs. $H_a : p_{1.} \neq p_{.1}$

Test Statistic: $z^2 = \frac{(Y_{12}-Y_{21})^2}{Y_{12}+Y_{21}}$ $\qquad$ p-value: $P(\chi^2_1 > z^2)$

Extension of McNemar's Test Statistic

$H_0 : p_{1.} = p_{.1}, p_{2.} = p_{.2}, \ldots, p_{J.} = p_{.J}$
H_a : at least one $p_{j.} \neq p_{.j}$ for $j = 1, 2, \ldots, J$

Test Statistic: $W = \hat{d}' V \hat{d}$ $\qquad$ p-value: $P(\chi^2_{J-1} > W)$

Measures of Agreement

- Cohen's Kappa

$$\hat{\kappa} = \frac{n\sum_{j=1}^{J} Y_{jj} - \sum_{j=1}^{J} Y_{j.}Y_{.j}}{n^2 - \sum_{j=1}^{J} Y_{j.}Y_{.j}}$$

- Weighted Cohen's Kappa

$$\hat{\kappa}_w = \frac{n\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}Y_{ij} - \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}Y_{i.}Y_{.j}}{n^2 - \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}Y_{i.}Y_{.j}} \qquad \text{where } w_{ij} = 1 - \frac{(i-j)^2}{(J-1)^2}$$