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Stat330 HW3 Neha Maddali

1a) $\int_0^1 kx(1-x)dx = 1$

$k\int_0^1 x - x^2 dx = 1$

$k(\frac{1}{2} - \frac{1}{3}) = 1$

$\frac{1}{6}k = 1$

$k = 6$

1b) $F_x(x) = 3x^2 - 2x^3$

1c) $\mathbb{P}(0.5 \le x \le 1) = \int_{0.5}^1 6x(1-x)dx \approx 0.499$

1d) $\mathbb{P}(0 \le X \le 0.75)$

$= \int_0^{0.75} 6x(1-x)dx = 0.844$

1e) $\mathbb{E}(x) = \int_0^1 x\, 6x(1-x)dx = 0.5$

1f) $Var(x) = \int_0^1 x^2 6x(1-x) = 0.3$ $\quad 0.3 - 0.5^2 = 0.05$

2a) $F_x(t) = \int \frac{3}{2}x^2 dx = \underline{t^3 - 1}$

2b) $\mathbb{P}(|x - \frac{1}{2}| < \frac{1}{4})$

$\mathbb{P}(\frac{1}{4} < x < \frac{3}{4})$

$\int_{1/4}^{3/4} \frac{3}{2}x^2 dx = \frac{13}{64} \approx 0.203$

2c) $\mathbb{P}(x \le t) = 0.80$

$0.80 = \int_{-1}^{t} \frac{3}{2}x^2 dx \approx 0.843$

3a) $F_x(x) = \begin{cases} 0 & x < 0 \\ x^2/2 & 0 \le x \le 1 \\ x - 1/2 & 1 < x \le 1.5 \\ 1 & x > 1.5 \end{cases}$

3b) $\mathbb{P}(0.5 \le x \le 1.2) = \int_{0.5}^1 x\,dx + \int_1^{1.2} dx \approx 0.575$

3c) $\mathbb{E}(x) = \int_0^1 x^2 dx = 1/3$

$+ \int_1^{1.5} x\,dx = 5/8$ $\quad 1/3 + 5/8 = 23/24 \approx 0.958$

4a) $E(x) = \frac{0+30}{2} = 15$ minutes

4b) $\mathbb{P}(X > 10) = \int_{10}^{30} 1/30\, dx = 2/3$

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4c) $\mathbb{P}(X > 25 \mid X > 15)$

$$= \frac{\mathbb{P}(X>25)}{\mathbb{P}(X>15)} = \frac{\int_{25}^{30} 1/30\,dx}{\int_{15}^{30} 1/30\,dx} = \frac{5}{15} = \frac{1}{3}$$

4d) $\frac{1}{10} = \int_{a}^{30} 1/30\,dx$

$a = 27 \Rightarrow 30 - 27 = 3$ minutes $\Rightarrow$ 10:03

5a) $\lambda = 20$

5b) $E(X) = 1/20 = 3$ minutes

5c) $\mathbb{P}(X \leq 20/60) = 1 - e^{-20/60 \cdot 20} = 0.9987$

5d) $Y \sim \text{Gamma}(2, 20)$

5e) $W = \sum_{i=1}^{5} X_i$ so $W \sim \text{Gamma}(5, 20)$

5f) $E[W] = \alpha/\lambda = 5/20 = 0.25$ hours

5g) $N \sim \text{Pois}(5)$, $\mathbb{P}(N \leq 3) = 0.44$

6) prove $\mathbb{P}(Y \leq s+t \mid Y \geq s) = \mathbb{P}(Y \leq t)$

$$\mathbb{P}(Y \leq s+t \mid Y \geq s) = \frac{\mathbb{P}[(Y \leq s+t) \cap (Y \geq s)]}{\mathbb{P}(Y \geq s)}$$

$$= \frac{e^{-\lambda s} - e^{-\lambda(s+t)}}{e^{-\lambda s}}$$

$$= 1 - e^{-\lambda(t)}$$

$\mathbb{P}(Y \leq s+t \mid Y \geq s) = \mathbb{P}(Y \leq t)$ ✓ so the statement is proved true

7a) $\lambda = 1/\mu$ $\lambda = 1/5 = 0.20$

$X \sim \text{Exp}(0.20)$

7b) $f_X(x) = \begin{cases} 0.20e^{-0.20x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$F_X(t) = \begin{cases} 1 - e^{-0.20t} & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

7c) $\mathbb{P}(X < 5) = 1 - e^{-0.20(5)} = 0.632$

7d) $\mathbb{P}(X < 5 \mid X > 2) = \mathbb{P}(X < 5) = 0.632$

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8a) $T \sim \text{Gamma}(25, 6)$

8b) $\mathbb{E}(x) = 25/6 = 4.166$ hours

8c) $\mathbb{P}(x < 5) = 0.84276$

9a) $x \sim N(-3, 2)$, $\mathbb{P}(x \leq 2.39) = 0.996$

9b) $\mathbb{P}(\frac{x+3}{2} \leq -0.99) = \mathbb{P}(X \leq -4.98) = 0.161$

9c) $\mathbb{P}(|x| \geq 2.39) = 0.77$

9d) $\mathbb{P}(|x+3| \geq 2.39) = 0.1654$

9e) $\mathbb{P}(x < 5) = 0.999$

9f) $\mathbb{P}(|x| < 5) = 0.8413$

9g) $\mathbb{P}(X > x) = 0.33$

$\Rightarrow x = -2.12$

10a) $\mathbb{P}(x < 490) = \mathbb{P}(x < 490 - 500/15) = 0.254$

10b) $\mathbb{P}(x > 530) = \mathbb{P}(x > 530 - 500/15) = \mathbb{P}(x > 2) = 0.02275$

10c) $\mathbb{P}(490 \leq x \leq 530) = 0.7248$

10d) $\mathbb{P}(Z \leq z) = 0.90 \Rightarrow z = 1.29$

$$\frac{x - 500}{15} = 1.29 \qquad x = \$519.35$$

10e) $\mu = \$500$ $\sigma/\sqrt{n} = 15/\sqrt{25} = \$3$

$$\mathbb{P}(x > 510) = \mathbb{P}\left(x > \frac{510 - 500}{3}\right) = 0.00043$$

11a) $\int_{0.3}^{0.5} \frac{4}{3}(1 - x^3) = 0.2485$

11b) Central Limit theorem

$\bar{n}_{30} \sim N(\mu, \sigma^2/n)$

$\mu = \int_0^1 x \, 4/3 (1 - x^3) = 0.4$

$Var(x) = 0.0622 = \int_0^1 x^2 \, 4/3 (1 - x^3)$

11c) $N(0.4, 0.249)$

$\mathbb{P}(0.3 \leq x \leq 0.5) = 0.312$

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12) $n = 82$ $Var(X) = 16\ sec^2$

$\mu = 15$ $x = 20\ min = 1200\ sec$

$$P(S_n \le 1200) \approx P\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \le \frac{1200-1230}{\sqrt{1312}}\right) =$$

$$\mathbb{P}(Z \le -0.83) = 0.203776$$

13) $F_Y(t) = \mathbb{P}(Y \le t) = \mathbb{P}(F_X^{-1}(U) \le t) = \mathbb{P}(X \le F(y)) = F(F_X^{-1}(U)) = F_X(t)$