Homework 10

Neha Maddali
November 3, 2023

Problem 6.74

Part a:

The probability density function is

$f_Y(y) = \begin{cases} \frac{1}{\theta} & 0 \le y \le \theta \\ 0 & \text{otherwise} \end{cases}$

The distribution of Y is

$F_Y(y) = \begin{cases} 0 & y < \theta \\ \frac{y}{\theta} & 0 \le y \le \theta \\ 1 & y \ge \theta \end{cases}$

The probability distribution function of $Y_{(n)}$ is $F(Y_{(n)}) = [F(y)]^n = [\frac{y}{\theta}]^n$

Part b:

The probability density function of $Y_{(n)}$ is $f(X_{(n)}) = n[F(x)]^{n-1}f(x) = n[\frac{y}{\theta}]^{n-1}\frac{1}{\theta} = n\frac{y^{n-1}}{\theta^{n-1+1}}$
$= n\frac{y^{n-1}}{\theta^n}$

Part c:

$E(Y_{(n)}) = \int_0^\theta yn\frac{y^{n-1}}{\theta^n}\,dy = \frac{n\theta}{n+1}$

$Var(X_{(n)}) = E(X_{(n)}^2) - [E(X_{(n)})]^2 = \frac{n\theta^2}{n+1} - [\frac{n\theta}{n+1}]^2 = \frac{n\theta^2}{(n+1)^2}$

Problem 6.81

Part a:

$F_{Y(1)}(y) = 1 - P(Y(1) > y) = 1 - P(Y1, Y2, ...Yn > y) = 1 - P(Y1 > y) * P(Y2 > y)...$
$= 1 - (1 - F_{Y1}) * (1 - F_{Y2})... = 1 - exp(-y/\beta) * exp(-y/\beta)... = 1 - exp(-ny/\beta)$

Differentiating we have,

$f_{Y1}(y) = (n/\beta)exp(-ny/\beta)$ which is an exponential distribution with mean β/n

Part b:

with n=5, $\beta = 2$

$P(Y(1) \le 3.6) = F_{Y(1)}(3.6) = 1 - exp(-n * 3.6/\beta) = 1 - exp(-9) = 0.999$

Problem 7.11

we have $\sigma = 4, n = 9$

consider $P(-2 \le (\bar{x} - \mu) \le 2)$

$= P(\frac{-2}{\frac{4}{\sqrt{9}}} \le Z \le \frac{2}{\frac{4}{\sqrt{9}}})$

$= P(\frac{-2}{1.3333333} \le Z \le \frac{2}{1.3333333})$

$= P(-1.5 \le Z \le 1.5) = P(Z \le 1.5) - P(Z \le -1.5) = 0.93319 - 0.06681 = 0.86639$

Problem 7.12
$P(-0.25\sqrt{n} \le Z \le 0.25\sqrt{n}) = 0.9$
$0.25\sqrt{n} = 1.645$ so n=43.296 which is a sample of approximately 44 trees.

Problem 7.43
Let's say that $\bar{Y}$ is the mean height where $\sigma = 2.5$ inches. We know from the central limit theorem that $P(|\bar{Y} - \mu| \le 0.5) = P(-0.5 \le \bar{Y} - \mu \le 0.5) = P(-2 \le Z \le 2) = 0.9544$

Problem 7.44
$P(|\bar{Y} - \mu| \le 0.4) = P(-0.4 \le \bar{Y} - \mu \le 0.4) = 0.95$
$\frac{5\sqrt{n}}{2.5} = 1.96$ so $n = 150.0625$. Thus 151 men should be sampled.

Problem 7.50
$P(|\bar{Y} - \mu| < 1) = P(|Z| < \frac{1}{\sigma/\sqrt{n}}) = P(\frac{-1}{10/\sqrt{n}} < Z < \frac{1}{10/\sqrt{n}}) = 0.99$
$\frac{1}{10/\sqrt{n}} = z_{0.005} = 2.576$. So our n=663.57. 664 measurements should be taken.

Problem 7.73
Y is the number of people that show up for a flight. Y Binomial(160,0.95). P($Y \le 155$) gives the probability that the airline will be able to accommodate all the passengers.
$P(Y \le 155) = P(Z \le \frac{155.5 - 160(0.95)}{\sqrt{160(0.95)(0.05)}}) = P(Z \le 1.27) = 0.898$ is the probability everyone will have a seat