

Homework 8

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Problem 5.47

Y1 and Y2 are dependent. $P(Y1 = 1 \mid Y2 = 2) \neq P(Y1 = 1 \mid Y2 = 1)$

Problem 5.50

Part a:

Y1 and Y2 are independent.

$$f_1(y_1) = 1, 0 \leq y_1 \leq 1, f_2(y_2) = 1, 0 \leq y_2 \leq 1. f(y_1, y_2) = f_1(y_1)f_2(y_2)$$

Problem 5.59

We know that the domain is $0 \leq y_2 \leq y_1 < \infty$ so Y1 and Y2 are dependent.

Problem 5.63

numerator:

$$\frac{1}{49} \int_0^\infty \int_{y_1/2}^{y_1} e^{-\frac{y_1+y_2}{7}} dy_2 dy_1 = -\frac{1}{7} \left(\frac{7}{2} - \frac{14}{3} \right) = \frac{1}{6}$$

denominator:

$$\int_0^\infty \int_{y_1/2}^\infty f_{Y1,Y2}(y_1, y_2) dy_2 dy_1 = \frac{2}{3}$$

$$\text{so, } P(Y_1 > Y_2 \mid Y_1 < 2Y_2) = \frac{1/6}{2/3} = \frac{1}{4}$$

Problem 5.72

Part a:

$$E(Y1) = 4/9 * 0 + 4/9 * 1 + 1/9 * 2 = 2/3$$

Part b:

$$V(Y1) = E(Y1^2) - E(Y1)^2 = 8/9 - (2/3)^2 = 4/9$$

Part c:

$$E(Y1 - Y2) = E(Y1) - E(Y2) = 2/3 - 2/3 = 0$$

Problem 5.77

We know that $f_1(y_1) = 3(1 - y_1)^2, 0 \leq y_1 \leq 1, f_2(y_2) = 6y_2(1 - y_2), 0 \leq y_2 \leq 1$

Part a:

$$E(Y1) = \int_0^1 y_1 f_1(y_1) dy_1 = 1/4$$

$$E(Y2) = \int_0^1 y_2 f_2(y_2) dy_2 = 1/2$$

Part b:

$$\text{Var}(Y1) = E(Y1^2) - E(Y1)^2 = 3/80$$

$$\text{Var}(Y2) = E(Y2^2) - E(Y2)^2 = 1/20$$

Part c:

$$E(Y1 - 3Y2) = E(Y1) - 3E(Y2) = 1/4 - 3(1/2) = -5/4$$

Problem 5.89

From problem 5.72, we know that $E(Y1)$ and $E(Y2) = 2/3$

$$\text{Cov}(Y1, Y2) = E(Y1Y2) - E(Y1)E(Y2) = 2/9 - 2/3 * 2/3 = -2/9$$

I think it's not surprising that Cov is negative because from the table, the value of Y2 tends to be smaller as Y1 increases.

Problem 5.92

$$E(Y1) = \int_0^1 \int_0^{y2} y1 f(y1, y2) dy1 dy2 = 1/4$$

$$E(Y2) = \int_0^1 \int_0^{y2} y2 f(y1, y2) dy1 dy2 = 1/2$$

$$E(Y1, Y2) = \int_0^1 \int_0^{y2} y1 y2 f(y1, y2) dy1 dy2 = 3/20$$

$$\text{Cov}(Y1, Y2) = E(Y1, Y2) - E(Y1)E(Y2) = 3/20 - 1/4 * 1/2 = 3/20 - 1/8 = 1/40$$

$\text{Cov}(Y1, Y2) \neq 0$ so Y1, Y2 are not independent