Neha Maddali

Stat330 HW5

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1a) mean: 56.9 IQR: 11.5 (Q1 = 45 + Q3 = 56.5)

median: 50.5 SD: 26.50975

1b) IQR × 1.5 = 17.25 + Q3 = 73.75

Q1 − 17.25 = 27.75

outliers: 130

1c) mean: 48.778 IQR: 7 (Q1 = 45 + Q3 = 52)

median: 50 SD: 6.960204

1d) outliers have great effect on mean and standard deviation of the data set. Outliers can skew data sets making calculations like the mean not representative of the data set

2a) The histogram is right-skewed (positively-skewed). The majority of diamond prices are a lower dollar value. The diamond count gets smaller as the price in dollars increased

2b) Exponential distribution as the decrease in diamond count as price increases follows that of an exponential data set

2c) There is a strong, positive, linear relation between carat and price of diamonds. The price increases as the number of carats increase. Higher variability as carat increases

3a) $\frac{2}{n}\sum_{i=1}^{n} x_i$

$2\sum_{i=1}^{n} x_i/n$

$E(x)$ of unif $= \frac{a+b}{2}$

$= \frac{0+\theta}{2}$

$E(\theta) = \frac{\theta}{2}$

$E(\hat{\theta}) = E\left(\frac{2}{n}\sum_{i=1}^{n} x_i\right)$

$= \frac{2}{n} E\left(\sum_{i=1}^{n} x_i\right)$

$= \frac{2}{n}\sum_{i=1}^{n} E(x_i)$

$= \frac{\cancel{2}}{\cancel{n}} \cancel{n} \frac{\theta}{\cancel{2}} = \theta$

$\hat{\theta}$ is an unbiased estimator for θ

3b) $var(x)$ for unif $\frac{(a+b)^2}{12}$

$Var(x_i) = \theta^2/12$

$\frac{2}{n}\sum_{i=1}^{n} x_i$

$4/n^2 \, Var\left(\sum_{i=1}^{n} x_i\right)$

$4/n^2 \cdot \theta/12 \cdot n = \theta^2/3n$

$Var(\hat{\theta}) = \theta^2/3n$

$se(\hat{\theta}) = \sqrt{\theta^2/3n} = \theta\sqrt{1/3n}$

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4a) $E(x_1+x_2+x_3+x_4/4)=p$

$\rightarrow \text{Bias}(x_1+x_2+x_3+x_4/4) = E(\overbrace{x_1+x_2+x_3+x_4/4}^{p}) - p = 0$

$E(x_1+2x_2+x_3/4)=p$

$\rightarrow \text{Bias}(x_1+2x_2+x_3/4) = E(\overbrace{x_1+2x_2+x_3/4}^{p}) - p = 0$

both estimators are <u>unbiased</u> estimators of p

4b) $MSE(\hat{\theta}) = Var(\hat{\theta})$

$MSE(\hat{p}_1) = (x_1+x_2+x_3+x_4/4)$

$= 1/16\ Var(x_1+x_2+x_3+x_4)$

$= 1/16 \cdot 4(p(1-p))$

$MSE(\hat{p}_1) = 1/4\ p(1-p)$

$MSE(\hat{p}_2) = x_1+2x_2+x_3/4$

$= 1/16\ Var(x_1+2x_2+x_3)$

$= 1/16\ Var(x_1) + Var(2x_2) + Var(x_3)$

$= 1/16\ Var(x_1) + 4Var(x_2) + Var(x_3)$

$MSE(\hat{p}_2) = 3/8\ p(1-p)$

$MSE(\hat{p}_1) < MSE(\hat{p}_2)$

<u>$\hat{p}_1$ is the better estimator</u>

5) $\sum p_X(x) = \dfrac{\sum_{i=1}^{n} \frac{1}{i}}{N} \qquad x \in \{1, \ldots, N\}$

$\underline{\hat{N}_{MOM}} = \dfrac{\sum_{i=1}^{N} \frac{1}{i}}{\bar{x}}$

6a) $\mu_1 = E(x) = \lambda$

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$\lambda = \bar{x} \longrightarrow \hat{\lambda}_{MOM} = \bar{X}$

6b) i. $L(\lambda) = \prod_{i=1}^{n} f_X(x_i) = \prod_{i=1}^{n} \dfrac{e^{-\lambda}\lambda^{x_i}}{x_i!} = \dfrac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!}$

ii. $\log L(\lambda) = \left(\sum_{i=1}^{n} x_i\right)\log\lambda - n\lambda - \prod_{i=1}^{n} \log(x_i!)$

iii. $d/d\lambda \log L(\lambda) = \dfrac{\sum_{i=1}^{n} x_i}{\lambda} - n$

iv. $\dfrac{\sum_{i=1}^{n} x_i}{\lambda} - n = 0$

$\lambda = \frac{1}{n}\sum_{i=1}^{n} x_i$

v. $\underline{\hat{\lambda}_{MLE}} = \frac{1}{n}\sum_{i=1}^{n} x_i$

6c) $\hat{\lambda}_{MOM} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \frac{1}{5}(7+6+7+2+4) = 5.2$

$\underline{\hat{\lambda}_{MLE}} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \frac{1}{5}(7+6+7+2+4) = 5.2$

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7a) $f(t) = \begin{cases} \theta t^{\theta-1} & , 0 < t < 1 \\ 0 & , \text{otherwise} \end{cases}$

$\theta \int_0^1 t \cdot t^{\theta-1}$

$\frac{\theta}{2} = \bar{\mu}$

$\underline{\hat{\theta}_{MOM}} = 2\bar{\mu}$

$= 1.333$

7b) $L(\theta) = \prod_{i=1}^{3} \theta t_i^{\theta-1} = \theta^3 \left(\prod_{i=1}^{3} t_i\right)^{\theta-1}$

$\log L(\theta) = 3\log\theta + (\theta-1)\sum_{i=1}^{3} \log(t_i)$

$\frac{d}{d\theta}\log L(\theta) = \frac{3}{\theta} + \sum_{i=1}^{3} \log t_i$

$\hat{\theta} = -3 / \sum_{i=1}^{3} \log t_i = -3/-1.378326 = 2.17655$

8a) Gamma $(3, \lambda)$

$\int_0^\infty \frac{\lambda^3}{2} x^2 e^{-\lambda x}$

$= \frac{3}{x^2}$

$\underline{\hat{\lambda}_{MOM}} = \frac{\bar{x}}{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2}$

$E(x) = \alpha/\lambda$

$= 3/\lambda = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$E(x^2) = \frac{\alpha(\alpha+1)}{\lambda^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$

8b) $L(\lambda) = \prod_{i=1}^{n} \frac{\lambda^3}{2} x^2 e^{-\lambda x}$

$\log L(\lambda) = 3n\log(\lambda) - n\log(2) - x\lambda + 2\log(x)$

$\frac{d}{d\lambda}\log L(\lambda) = \frac{3n}{\lambda} - x$

$\frac{3n}{\lambda} - x = 0$

$3n/\lambda = x$

$\lambda = x/3n$

$\underline{\hat{\lambda}_{MLE}} = x/3n$