Homework 4 Solution

3.38

Each judge's decision independently follows a Bernoulli distribution with $p = P(B) = 1/3$ and $P(A) = 1 - p = 2/3$. The number of judges preferring B over A follows a Binomial distribution with $n = 4$ trials and $p = 1/3$:

(a) $P(Y = y) = \binom{4}{y}(1/3)^y(2/3)^{4-y}, y = 0, 1, 2, 3, 4$

(b) $P(Y \geq 3) = P(Y = 3) + P(Y = 4) = 8/81 + 1/81 = 1/9$

(c) $E(Y) = np = 4 * 1/3 = 4/3$

(d) $Var(Y) = np(1 - p) = 4 * 1/3 * 2/3 = 8/9$

3.40

The number of recoveries follows a binomial distribution with $n = 20$ and $p = 0.8$.

(a) $P(Y = 14) = \binom{20}{14}0.8^{14}0.2^{20-14} = 0.109$

(b) $P(Y \geq 10) = \sum_{y=10}^{20} \binom{20}{y}0.8^y0.2^{20-y} = 0.999$

(c) $P(14 \leq Y \leq 18) = \sum_{y=14}^{18} \binom{20}{y}0.8^y0.2^{20-y} = 0.844$

(d) $P(Y \leq 16) = \sum_{y=0}^{16} \binom{20}{y}0.8^y0.2^{20-y} = 0.589$

3.41

The number of correct answers Y follows a binomial distribution with $n = 15$ and $p = 1/5$. $P(Y \geq 10) = \sum_{y=10}^{15} \binom{15}{y}0.2^y0.8^{15-y} = 0.0001$

3.44

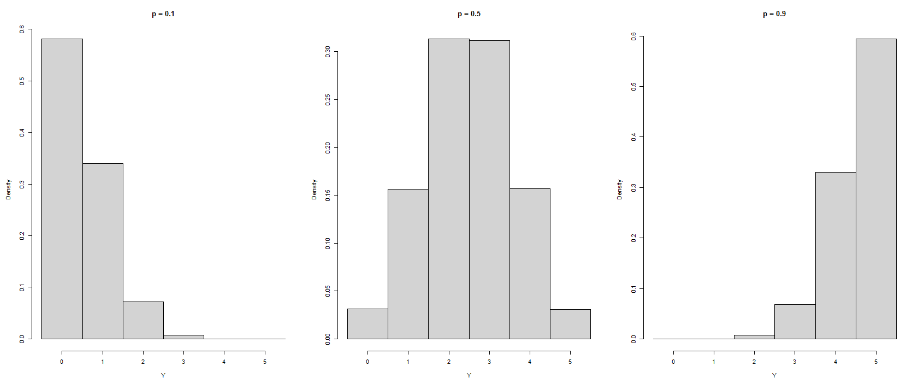
The number of successful operations follows a binomial distribution with $n = 5$ and p

(a) $p = 0.8$: $P(Y = 5) = \binom{5}{5}0.8^5 0.2^0 = 0.328$

(b) $p = 0.6$: $P(Y = 4) = \binom{5}{4}0.6^4 0.4^1 = 0.259$

(c) $p = 0.3$: $P(Y < 2) = \binom{5}{1}0.3^1 0.7^4 + \binom{5}{0}0.3^0 0.7^5 = 0.528$

3.46



3.60

The number of fishes survived Y follows a binomial distribution with $n = 20$ and $p = 0.8$.

(a) $P(Y = 14) = \binom{20}{14}0.8^{14}0.2^6 = 0.109$

(b) $P(Y \geq 10) = \sum_{y=10}^{20} \binom{20}{y}0.8^y 0.2^{20-y} = 0.999$

(c) $(Y \leq 16) = \sum_{y=0}^{16} \binom{20}{y}0.8^y 0.2^{20-y} = 0.589$

(d) $E(Y) = np = 16, Var(Y) = np(1-p) = 3.2$

3.66

(a) $\sum_{y=1}^{\infty} q^{y-1}p = p\sum_{y=1}^{\infty}(1-p)^{y-1} = p\frac{1}{1-(1-p)} = p\frac{1}{p} = 1$

(Used the fact that infinite sum of a geometric series $\{1, x, x^2, x^3, \cdots\}, |x| < 1$ is equal to $\frac{1}{x}$)

(b) Obviously $\frac{q^{y-1}p}{q^{y-2}p} = q < 1, y \geq 2$

The highest possible value for Y is $Y = 1$ with $P(Y = 1) = p$

3.70

The first successful drill Y follows a geometric distribution with $p = 0.2$

(a) $P(Y = 3) = (1 - p)^2 p = 0.128$

(b) $P(Y > 10) = P(\text{failed in the first 10 drills}) = (1 - p)^{10} = 0.107$

3.73

The number of accounts audited until first error is found Y follows a geometric distribution with $p = 0.9$

(a) $P(Y = 3) = (1 - p)^2 p = 0.009$

(b) $P(Y \geq 3) = P(Y > 2) = (1 - p)^2 = 0.01$

3.81

The number of tosses to get a first head Y follows a geometric distribution with $p = 0.5$: $E(Y) = \frac{1}{p} = 2$

3.90

The number of employees need to test for finding 3 positives Y follows a negative binomial distribution with $r = 3$ and $p = 0.4$:

$P(Y = 10) = \binom{10-1}{3-1} p^3 (1 - p)^{10-3} = 0.064$

3.97

(a) Geometric with $p = 0.2$ (or Negative Binomial with $r = 1$, $p = 0.2$):

$P = (1 - p)^2 p = 0.128$

(b) Negative Binomial with $r = 3$, $p = 0.2$:

$P = \binom{7-1}{3-1}(1 - p)^{7-3} p^3 = 0.049$

(c) All drills are independent with each other, with probability 0.2

(d) Negative Binomial with $r = 3$, $p = 0.2$:

$E(Y) = r\frac{1}{p} = 15$

$Var(Y) = r\frac{1-p}{p^2} = 60$