Stat 330 HW1   Neha Maddali

1a) $\Omega = 2^3 = \boxed{8}$
      $\{HHH, TTT, HTT, HHT, TTH, THH, THT, HTH\}$

1b)   i) A exactly 2 tails = $\{HTT, TTH, THT\}$ —▹ 3 possibilities
      ii) B at least 2 tails = $\{TTT, HTT, TTH, THT\}$ —▹ 4 possibilities
      iii) C last two tosses heads = $\{HHH, THH\}$ —▹ 2 possibilities

1c)   i) $\overline{A}$ = $\{HHH, TTT, HHT, THH, HTH\}$ —▹ 5 possibilities
      ii) $A \cup B$ = $\{HTT, TTH, THT, TTT\}$ —▹ 4 possibilities
      iii) $A \cap B$ = $\{HTT, TTH, THT\}$ —▹ 3 possibilities
      iv) $A \cap C$ = $\{\emptyset\}$ —▹ 0 possibilities

2a) $P(MB \cup HD) = P(MB) + P(HD) - P(MB \cap HD)$ =
            $0.35 + 0.3 - 0.2 = 0.45$
            $P(M \cap H) = 1 - 0.45 = \boxed{0.55}$
2b) $1 - P(M \cup H) = \boxed{0.45}$

3a) $12 \cdot 11 \cdot 10 = \boxed{1320}$ possibilities
3b) $12C_3 = \boxed{220}$ possibilities
3c) $3 \cdot 2 \cdot 1 = \boxed{6}$ possibilities

4)   total possible = 65^8 $26 + 26 + 10 + 3 = 65^8$ possible passwords
      possible w/o uppercase = 39^8
      possible w/o lowercase = 39^8 w/ only uppercase = 26^8
      possible w/o digits = 55^8 w/ only lowercase = 26^8
      possible w/o special chars = 62^8 w/ only digits = 10^8
                                               w/ only special = 3^8

      w/o uppercase & lowercase = 13^8 = $65^8 - 39^8 - 39^8 - 55^8 - 62^8 +$
      w/o upper & digits = 29^8 $13^8 + 29^8 + 36^8 + 29^8 + 36^8 + 52^8 -$
      w/o upper & special chars = 36^8 $26^8 - 26^8 - 10^8 - 3^8$
      w/o lower & digits = 29^8 = $65,552,253,661,440$
      w/o lower & special chars = 36^8 = $\boxed{6.5 \times 10^{13}}$ possible passwords
      w/o digits & special chars = 52^8

5a) $\dfrac{C(7,4)}{C(12,4)} = \boxed{0.07}$

5b) 1/7 comet 260s $C(12,4) = \Omega = 495$ $\dfrac{C(7,1) * C(4,2) * C(1,1)}{C(12,4)}$

   2/4 Nimbus 2000s $\dfrac{C(7,1) \quad C(1,1)}{C(4,2)}$

   1/1 Firebolt $= \boxed{0.084}$

5c) $1 - \dfrac{C(7,4)}{C(12,4)} = \boxed{0.93}$

6)
R R
W W
W W

R R
R W

$P(W_2) = P(A) + P(B)$

$P(W_2) = P(W_1)\, P(W_{2_1}) + P(R_1)\, P(W_{2_2})$

$P(R_1) = 2/6 = 1/3$

$P(W_1) = 4/6 = 2/3$

red chip urn 1 to 2   white chip urn 1 to 2

$r = 4$ $P(R_{2_1}) = 4/5$ $r = 3$ $P(R_{2_2}) = 3/5$

$w = 1$ $P(W_{2_1}) = 1/5$ $w = 2$ $P(W_{2_2}) = 2/5$

total = 5   total = 5

$P(W_2) = \dfrac{2}{3} \cdot \dfrac{1}{5} + \dfrac{1}{3} \cdot \dfrac{2}{5} + \dfrac{2}{6} \cdot \dfrac{1}{5}$

$= \boxed{5/15}$

7) 2 dice (1 red, 1 green)

   A: 1 or 2 on red die

   B: 3,4 or 5 on green die $\Omega = $ sample space $= 6^2 = 36$

   C: sum of dice is 4, 11 or 12

$P(A) = 12/36$ $P(B) = 18/36$ $P(C) = 6/36$ $P(A \cap B \cap C) = 1/36$

                                                      ∟ A, B and C have possibility of
                                                        roll (1,3)

① $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$
   $1/36 \quad = 12/36 * 18/36 * 6/36$
   $1/36 = 1/36$ ✓

② $P(A \cap B) = P(A)\,P(B)$ $P(B \cap C) = P(B)\,P(C)$
   $1/6 = 1/6$ ✓ $1/18 = 1/12$ ✗

   $P(A \cap C) = P(A)\,P(C)$
   $1/36 = 1/18$ ✗

* the events are not Mutually
  Independent b/c they
  satisfy eq 1 but not 2 *

8a) positive test

$$= 0.15 * 0.95 \text{ (people with disease)} + (1-0.15) * 0.10 \text{ (people w/o disease)} = 0.2275$$

P(positive) = 22.75%

8b) $\dfrac{(0.15)(0.95)}{(0.15)(0.95)+(0.85)(0.10)} = \dfrac{0.1425}{0.2275} = 0.6264 = $ 62.64%

8c) P(no disease | positive test) = $\dfrac{(1-0.15)(0.10)}{0.2275} = 0.3736$

= 37.36%

9a) A & B are in a series, so reliability = $P(A)P(B)$

$$= 0.95 * 0.9 = 0.855$$

9b) A & B are in a series parallel to series C & D

$P(A \cap B) = 0.855$

$P(C \cap D) = 0.56$

$1 - P(A \cap B)' P(C \cap D)'$

$= 1-(1-0.855)(1-0.56)$

$= 1-(0.145)(0.44) = 0.9362$

9c) $1-(1-0.95)(1-0.9)(1-0.8)$

$1-0.001 = 0.999$