Exam 2

Due Mar 31 at 11:59pm **Points** 80 **Questions** 7
Available Mar 31 at 12:01am - Mar 31 at 11:59pm about 24 hours
Time Limit 120 Minutes

Instructions

exam2_key.pdf (https://canvas.iastate.edu/courses/79674/files/15077352/download?download_frd=1)

This quiz was locked Mar 31 at 11:59pm.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	112 minutes	47 out of 80

Score for this quiz: **47** out of 80
Submitted Mar 31 at 12:20pm
This attempt took 112 minutes.

Question 1 — 0 / 12 pts

For the following functions, which ones are valid pdfs? (Check all that apply)

ou Answered

- [x] $f_X(x) = \frac{2x^3-1}{6} \quad 0 < x < 2$

- [ ] $f_X(x) = 3x^2 \quad 0 \le x \le 2$

- [ ] $f_X(x) = 4(1-x) \quad 0 < x < 1$

Correct!

- [x] $f_X(x) = \frac{x}{4} \quad 1 \le x \le 3$

Question 2

18 / 22 pts

Suppose the lifetime of a machine sold by a company can be modeled using the exponential distribution where the *average* lifetime of the machine is two years.
Define a random variable *X*= lifetime (in years) of the machine. Thus,

$$X \sim \exp(\lambda).$$

1. What is the value of λ? 1/2
2. What is the probability that a random machine's life time will be greater than three years? 0.22313
3. There is a 98% probability that the a random machine's lifetime will be less than 7.82 (in years).

Suppose the machines sell for $500 and the company has the following refund system: Full refund if the machine fails within half a year, half refund if it fails between half a year and one year, no refund after a year.
Define a new random variable $Y =$ refund received on a random machine.

4. What are the three values that *Y* can take on? 0, 250, 500

5. What is the probability of a full refund? 0.2212

6. What is the probability of a half refund? 0.2325

7. What is the probability of no refund? 0.60653

8. What is the expected refund amount for a machine?
250

ou Answered

Answer 1:

1/2

orrect Answer

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ou Answered

Answer 2:

0.22313

orrect Answer

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ou Answered

Answer 3:

7.82

orrect Answer

...

ou Answered

Answer 4:

0, 250, 500

orrect Answer

...

ou Answered

Answer 5:

0.2212

orrect Answer

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ou Answered

Answer 6:

0.2325

orrect Answer

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ou Answered

Answer 7:

0.60653

orrect Answer

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Answer 8:

ou Answered

250

orrect Answer

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Question 3

8 / 8 pts

The cross-sectional area of plastic tubing for use in pulmonary resuscitators is normally distributed with $\mu = 12.5\ \text{mm}^2$ and $\sigma = 0.2\text{mm}^2$.

1. What is the probability that a random tube has an area greater than $12.7\ \text{mm}^2$? 0.15866
2. When the area is less than $12.0\ \text{mm}^2$ or greater than $13.0\ \text{mm}^2$, the tube does not fit properly. What is the probability that a random tube *does not* fit properly. 0.0124
3. Continue the last part of the question. If the tubes are shipped in boxes of 1000, how many wrong-sized tubes per box would we expect to find? 13

Answer 1:

ou Answered

0.15866

orrect Answer

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Answer 2:

ou Answered

0.0124

orrect Answer

...

Answer 3:

ou Answered

13

orrect Answer

...

Question 4

5 / 7 pts

Suppose in an office building, the weights of the employees are distributed with a mean of 160 lbs, variance of 24 lbs^2, and are independent of each other. If we randomly select an employee and define $X_i =$ the weight of the *i*th employee, we have $X_i \sim f_X(x)$ where $\mathbb{E}(X_i) = 160$ and $\mathrm{Var}(X_i) = 24$. Suppose 40 random employees are going to get on the same elevator (it's a big elevator). Define $T =$ total weight of the 40 employees.

1. What is the (approximate) distribution of T? Give its name, its mean, and its variance N(6400,960) .

The elevator can hold a maximum of 6450 pounds. We would like to find the probability that the 40 employees can safely use the elevator. Use this information for the next two parts.

2. What is the *z* score? 1.61374

3. What is the probability that the 40 employees can safely use the elevator? 0.521

Answer 1:

ou Answered

N(6400,960)

orrect Answer

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Answer 2:

ou Answered

1.61374

orrect Answer

...

Answer 3:

ou Answered

0.521

orrect Answer

...

Question 5

5 / 9 pts

Classify the four stochastic processes below. There could be multiple or no answers in each category. Each process may belong to multiple categories.

A.Let $X_t = 1$ if two countries are at war in year *t*, and $X_t = 2$ if the two countries are in peace. Whether the two countries are at war or not at year *t* depends on their interaction in the most recent decade.

B. Let X_t be the bet a gambler places at a roulette in the *t*th bet, where the bet is either red (1) or black (2). The color of the *t*th bet only depends on the previous bet.

C. Let X_t be the total number of residents infected with COVID-19 in a city at time *t* reported by a real-time updated system. COVID-19 is highly contagious and thus the outbreak events tend to cluster.

D. Let X_t be the total number of crashes in a laptop up until time *t* after its purchase. The time between successive crashes follows independent and identical Exponential distribution with rate λ.

1. Discrete-time, discrete-state stochastic process: C
2. Markov chain: A,B

3. Homogeneous Poisson process: D

Answer 1:

ou Answered C

orrect Answer AB

Answer 2:

ou Answered A,B

orrect Answer B

Answer 3:

Correct! D

Question 6

7 / 12 pts

Sports teams can have long streaks of winning (or losing) seasons, but occasionally a team's fortunes change quickly. Suppose that each team in a college football conference can be classified as either (1) medium or (2) strong, and that the following one-step transition probabilities apply to the Markov chain X_n which equals a team's strength n seasons from now.

$$P = \begin{bmatrix} .8 & .2 \\ .3 & .7 \end{bmatrix}$$

1. Is X_n a regular Markov chain? (Yes or no) Yes
2. What are the entries in the two-step transition probability matrix? Enter the values of $p_{11}^{(2)}$, $p_{12}^{(2)}$, $p_{21}^{(2)}$, $p_{22}^{(2)}$ in this order, using commas to separate them. 0.7, 0.3, 0.45, 0.55

3. If a team is strong this season, what is the probability it will also be strong two seasons from now? 0.475
4. A team has undetermined strength before the start of the season this year, and it is thought that this team has a .5 probability to be classified as strong this year. Find the distribution of the strength of this team next season. Exp(0.5)
5. Continue the last part of the question: What is the probability the team will be strong next season? 0.35
6. Write down two equations solving which will give you the steady-state distribution associated with P. Equation 1 is 0.8(pi1)+0.3(pi2) and equation 2 is 0.2(pi1)+0.7(pi2) .
7. In the long run, what is the probability for any given team to be strong? 0.4

Answer 1:

ou Answered

Yes

orrect Answer

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Answer 2:

ou Answered

0.7, 0.3, 0.45, 0.55

orrect Answer

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Answer 3:

ou Answered

0.475

orrect Answer

...

Answer 4:

ou Answered

Exp(0.5)

orrect Answer ...

Answer 5:

ou Answered 0.35

orrect Answer ...

Answer 6:

ou Answered 0.8(pi1)+0.3(pi2)

orrect Answer ...

Answer 7:

ou Answered 0.2(pi1)+0.7(pi2)

orrect Answer ...

Answer 8:

ou Answered 0.4

orrect Answer ...

Question 7

4 / 10 pts

Packets arrive at a network gateway at a rate of 5 packets per minute, on average. Assume that the number of packets arrived follows a Poisson process.

1. Find the distribution of packets that arrive in the next three minutes.

Po(15)

2. Find the probability that the number of packets arrived in the next two minutes is at least 2. 0.99950
3. Find the distribution of the waiting time from receiving the 4th packet until receiving the 5th. Po(5)
4. Calculate the probability that the next packet will arrive in 15 seconds. 0.2212
5. Find the distribution of the waiting time until the 4th packet arrives. Po(20)
6. Calculate the probability that the 4th packet arrives in less than 45 seconds. 0.51623

Answer 1:

ou Answered Po(15)

orrect Answer .

Answer 2:

ou Answered 0.99950

orrect Answer .

Answer 3:

ou Answered Po(5)

orrect Answer .

Answer 4:

ou Answered 0.2212

orrect Answer .

Answer 5:

ou Answered

Po(20)

orrect Answer

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Answer 6:

ou Answered

0.51623

orrect Answer

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Quiz Score: **47** out of 80