Neha Maddali

Stat 330 HW 4 — Neha Maddali

1a) continuous time, continuous space
1b) discrete time, continuous space
1c) continuous time, discrete space
1d) discrete time, discrete space
1e) discrete time, discrete space

2a) {full, part, broken} - discrete space

2b) $\begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.6 & 0 & 0.4 \end{pmatrix}$ 1 = full 2 = part 3 = broken — transition matrix

2c) broken day after tomorrow = $P_{3a} = 0.12$

3a) $\begin{pmatrix} 0.96 & 0.04 \\ 0.05 & 0.95 \end{pmatrix}$

3b) $V_0 = [0.8, 0.2]$

$V_1 = V_0 P = [0.8, 0.2] \times \begin{pmatrix} 0.96 & 0.04 \\ 0.05 & 0.95 \end{pmatrix} = [0.778 \quad 0.222]$

3c) $P^2 = P \cdot P = \begin{pmatrix} 0.96 & 0.04 \\ 0.05 & 0.95 \end{pmatrix} \times \begin{pmatrix} 0.96 & 0.04 \\ 0.05 & 0.95 \end{pmatrix} = \begin{pmatrix} 0.9236 & 0.0764 \\ 0.0955 & 0.9045 \end{pmatrix}$

$P^3 = P^2 \cdot P = \begin{pmatrix} 0.9236 & 0.0764 \\ 0.0955 & 0.9045 \end{pmatrix} \times \begin{pmatrix} 0.96 & 0.04 \\ 0.05 & 0.95 \end{pmatrix} = \begin{pmatrix} 0.890476 & 0.109524 \\ 0.136905 & 0.863095 \end{pmatrix}$

$V_3 = V_0 P^3 = [0.8, 0.2] \times \begin{pmatrix} 0.890476 & 0.109524 \\ 0.136905 & 0.863095 \end{pmatrix} = [0.7398 \quad 0.2602]$

4a) $\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}$

4b) Yes, this is a regular Markov Chain

4c) $\pi = [1/3, 1/3, 1/3]$

4d) 1/2

5a) $\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix}$

Neha Maddali

5b) $P_0 = (1/3 \quad 1/3 \quad 1/3)$

$$P \times P = \left(\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \right) \times (1/3 \quad 1/3 \quad 1/3) =$$

$(0.4167 \quad 0.25 \quad 0.\overline{33})$

5c) Yes because all values are positive

5d) $(0.44, 0.22, 0.33)$

6a) $\text{Pois}(X_3 = 6 \mid X_6 = 3)$

$\text{Pois}(2/12 \,(12-6))$

$= \text{Pois}(1) = 0.14108$

6b) 4 expected radio blackouts

6c) $0.5 = \text{Exp}(2)$

$\mathbb{P}(X \leq 0.5) = 1 - 0.34657 = 0.65343$ years

6d) Gamma(4,2)

$\mathbb{P}(x \leq 2) = 0.566653$

7a) $X \sim \text{Pois}(12)$

$P(X = 0) = 0.00001$

7b) $P(X > 25) = 1 - P(X \leq 25) = 0.00308$

7c) Exp(4)

$\mathbb{P}(X \leq 5) = 0.86466$

8a) $\lambda = 1$ customer per min

$\mathbb{E}(x) = 60$ customers

8b) Gamma(10,1)

$\mathbb{P}(x > 10) = 0.45$

8c) Exp(1)

$\mathbb{P}(x < 1/2) = 0.3934$

8d) Gamma(100,1)

$\mathbb{E}(X) = 100/1 = 100$ min = 1:40 pm

$SD = \sqrt{100} = 10$ minutes

9) $Y \sim \text{Exp}(\text{Min}(B, D))$