STAT 477/STAT 577
Exam 1 Review Sheet

Binomial Distribution:

$$P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y} \qquad E(Y) = np \qquad V(Y) = np(1-p)$$

Multinomial Distribution:

$$P(Y_1 = y_1, Y_2 = y_2, \ldots, Y_j = y_j) = \frac{n!}{y_1! y_2! \cdots y_J!} p_1^{y_1} p_2^{y_2} \cdots p_j^{y_j} \qquad E(Y_j) = np_j$$

$$V(Y_j) = np_j(1-p_j) \qquad \mathrm{Cov}(Y_j, Y_k) = -np_j p_k \qquad \rho(Y_j, Y_k) = -\sqrt{\frac{p_j p_k}{(1-p_j)(1-p_k)}}$$

Sampling Distribution for $\hat{p}$:

$$\hat{p} = \frac{Y}{n} \qquad E(\hat{p}) = p \qquad V(\hat{p}) = \frac{p(1-p)}{n}$$

Shape is well approximated by Normal distribution when np and $n(1-p) \geq 10$

Hypothesis Tests for p:

$$H_0 : p = p_0 \qquad H_a : p < p_0 \text{ or } H_a : p > p_0 \text{ or } H_a : p \neq p_0$$

- Binomial Exact Test
  - Test Statistic: y
  - p-value for $H_a : p < p_0$ - $P(Y \leq y | p = p_0)$
  - p-value for $H_a : p > p_0$ - $P(Y \geq y | p = p_0)$
- Score Test
  - Test Statistic: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
  - p-value for $H_a : p < p_0$ - $P(Z < z)$
  - p-value for $H_a : p > p_0$ - $P(Z > z)$
  - p-value for $H_a : p \neq p_0$ - $2 * P(Z > |z|)$

Confidence Intervals for p

- Normal Approximation Method

$$\hat{p} \pm z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Wilson's Score Method

$$\frac{\hat{p} + \frac{1}{2n}z^2_{1-\alpha/2} \pm z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2_{1-\alpha/2}}{4n}}}{1 + \frac{1}{n}z^2_{1-\alpha/2}}$$

- Sample size calculations

$$n \geq \left(\frac{0.5z_{1-\alpha/2}}{M}\right)^2 \qquad n \geq \left(\frac{z_{1-\alpha/2}}{M}\right)^2 \hat{\hat{p}}(1-\hat{\hat{p}})$$

Goodness of Fit Test:

$$H_0 : p_1 = p_{1_0}, p_2 = p_{2_0}, \ldots, p_J = p_{J_0} \qquad H_a : \text{ at least one } p_j \neq p_{j_0} \text{ for } j = 1, 2, \ldots, J$$

$$E(Y_j) = np_{j_0} \qquad X^2 = \sum_{j=1}^{J} \frac{(Y_j - E(Y_j))^2}{E(Y_j)} \qquad \text{p-value } = P(\chi^2_{J-1} > X^2)$$