Homework 2 Solution

2.71

Note that:

$$\begin{aligned} A \cap (A \cup B) &= A \\ A \cup (A \cap B) &= A \\ P(A \cap B) &= P(A)P(B|A) = P(B)P(A|B) \\ P(A \cap B) + P(A \cup B) &= P(A) + P(B) \end{aligned}$$

(a) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$

(b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.5} = \frac{1}{5}$

(c) $P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{P(A)}{P(A)+P(B)-P(A \cap B)} = \frac{0.5}{0.5+0.3-0.1} = \frac{5}{7}$

(d) $P(A|A \cap B) = \frac{P(A \cap (A \cap B))}{P(A \cap B)} = \frac{P(A \cap B)}{P(A \cap B)} = 1$

(e) $P(A \cap B|A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.1}{0.5+0.3-0.1} = \frac{1}{7}$

2.75

(a) First 2 cards are spades: $13 - 2 = 11$ spades remaining in the deck.
Total possible ways of drawing the next 3 cards: $\binom{52-2}{3}$
Possible ways of drawing the next 3 cards: $\binom{11}{3}$
$P = \frac{\binom{11}{3}}{\binom{50}{3}} = 0.00842$

(b) First 3 cards are spades: $13 - 3 = 10$ spades remaining in the deck.
Total possible ways of drawing the next 2 cards: $\binom{52-3}{2}$
Possible ways of drawing the next 2 cards: $\binom{10}{2}$
$P = \frac{\binom{10}{2}}{\binom{49}{2}} = 0.0383$

(c) First 4 cards are spades: $13 - 4 = 9$ spades remaining in the deck.

Total possible ways of drawing the next card: $\binom{52-4}{1}$

Possible ways of drawing the next card: $\binom{9}{1}$

$P = \frac{\binom{9}{1}}{\binom{48}{1}} = 0.1875$

2.83

A and B mutually exclusive: No event in both A & B, $P(A \cup B) = P(A) + P(B), P(A \cap B) = 0$.

$$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)}$$

2.86

(a) Not possible: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.8 - 0.1 = 1.4 > 1$, probability cannot be greater than 1.

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.8 - P(A \cap B) \leq 1$: $P(A \cap B) \geq 0.5$

(c) Not possible: $P(A \cap B) \leq P(B) = 0.7$ must hold

(d) $P(A \cap B) \leq \min(P(A), P(B))$: $P(A \cap B)_{max} = P(B) = 0.7$

2.91

1. A and B cannot be exclusive if $P(A) = 0.4$ and $P(B) = 0.7$, since $P(A \cup B) = P(A) + P(B) = 1.1 > 1$ which is impossible.

2. Yes, it is possible.

2.98

1. Series circuit: Both relays must be activated, $P = 0.9 * 0.9 = 0.81$

2. Parallel circuit: Current will flow if either relay activates, $P = 1 - (1 - 0.9) * (1 - 0.9) = 0.99$

2.124

	Republicans	Democrats	Total
Favor	0.4*0.3 = 0.12	0.6*0.7 = 0.42	0.54
Non-favor	0.4*(1-0.3) = 0.28	0.6*(1-0.7) = 0.18	0.46
Total	0.4	0.6	1

$$\begin{aligned} P(Democrat|Favor) &= \frac{P(Democrat \ \cap \ Favor)}{P(Favor)} \\ &= \frac{0.42}{0.54} \\ &= \frac{7}{9} \end{aligned}$$

2.130

Define events C: having lung cancer; S: worked at the shipyard. Define $\bar{A}$ to be the complement of A.

$$\begin{aligned} P(S|C) &= 0.22 \\ P(S|\bar{C}) &= 0.14 \\ P(C) &= 0.0004 \end{aligned}$$

By Bayesian rule:

$$\begin{aligned} P(C|S) &= \frac{P(S|C)P(C)}{P(S)} \\ &= \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|\bar{C})P(\bar{C})} \\ &= \frac{0.22 * 0.0004}{0.22 * 0.0004 + 0.14 * (1 - 0.0004)} \\ &= 0.0006 \end{aligned}$$

2.134

Let F denote the event of failure to learn the skill:

$$
\begin{aligned}
P(F|A) &= 0.2 \\
P(F|B) &= P(F|\bar{A}) = 0.1 \\
P(A) &= 0.7 \\
P(B) &= P(\bar{A}) = 0.3
\end{aligned}
$$

(A and B are complementary, so we can use $\bar{A}$ to denote B)

So the probability of the worker taught by A after knowing she has failed is (by Bayesian rule):

$$
\begin{aligned}
P(A|F) &= \frac{P(F|A)P(A)}{P(F)} \\
&= \frac{P(F|A)P(A)}{P(F|A)P(A) + P(F|\bar{A})P(\bar{A})} \\
&= \frac{0.2 * 0.7}{0.2 * 0.7 + 0.1 * 0.3} \\
&= \frac{14}{17}
\end{aligned}
$$

2.135

	major	private	commercial	Total
business	0.6*0.5	0.3*0.6	0.1*0.9	0.57
non-business	0.6*0.5	0.3*0.4	0.1*0.1	0.43
Total	0.6	0.3	0.1	1

(a) $P(business) = 0.6 * 0.5 + 0.3 * 0.6 + 0.1 * 0.9 = 0.57$

(b) $P(business \ \cap \ private) = 0.3 * 0.6 = 0.18$

(c)

$$
\begin{aligned}
P(private|business) &= \frac{P(business \ \cap \ private)}{P(business)} \\
&= \frac{0.18}{0.57} = \frac{6}{19}
\end{aligned}
$$

(d)

$$
\begin{aligned}
P(business|commercial) &= \frac{P(business \ \cap \ commercial)}{P(commercial)} \\
&= \frac{0.1 * 0.9}{0.1} \\
&= 0.9
\end{aligned}
$$