# Homework 6 Solution

## 4.42

For $Uniform(\theta_1, \theta_2)$ distribution, the cdf is $F(y) = \frac{y-\theta_1}{\theta_2-\theta_1}, \theta_1 \leq y \leq \theta_2$. Thus medium is $F(m) = 0.5$, $m = \frac{\theta_1+\theta_2}{2}$

## 4.43

Let R denote the random variable of radius: R follows a $Uniform(0, 1)$ distribution, $f(r) = 1, 0 \leq r \leq 1$. Thus:

$$
\begin{aligned}
E(A) &= E(\pi r^2) = \int_0^1 \pi r^2 * 1 * dr \\
&= \frac{\pi}{3} \\
Var(A) &= Var(\pi r^2) = \pi^2 Var(r^2) \\
&= \pi^2 [E(r^4) - E(r^2)^2] \\
&= \pi^2 [\int_0^1 r^4 dr - (\int_0^1 r^2 dr)^2] \\
&= \pi^2 (\frac{1}{5} - \frac{1}{9}) \\
&= \frac{4}{45} \pi^2
\end{aligned}
$$

## 4.58

Standard Normal distribution is symmetric with $z = 0$.

(a) $P(0 \leq Z \leq 1.2) = 0.5 - 0.1151 = 0.3849$

(b) $P(-0.9 \leq Z \leq 0) = 0.5 - 0.1841 = 0.3159$

(c) $P(0.3 \leq Z \leq 1.56) = 0.3821 - 0.0594 = 0.3227$

(d) $P(-0.2 \leq Z \leq 0.2) = 1 - 2 * 0.4207 = 0.1586$

(e) $P(-1.56 \leq Z \leq -0.2) = 0.4207 - 0.0594 = 0.3613$

(f) $P(0 \leq Z \leq 1.2) = 0.3849$

## 4.59

(a) $z_0 = 0$

(b) $P(Z > z_0) = 0.1357, z_0 = 1.10$

(c) $P(Z > z_0) = 0.05, z_0 = 1.645$

(d) $P(Z > z_0) = 0.005, z_0 = 2.576$

## 4.71

(a) Let $X$ denote the random variable of actual resistance: $X \sim N(0.13, 0.005^2)$

$$\begin{aligned} P(X \in [0.12, 0.14]) &= P(|X - 0.13| \leq 0.01) \\ &= P(\frac{|X - 0.13|}{0.005} \leq 2) \\ &= P(-2 \leq Z \leq 2), Z \sim N(0,1) \\ &= 0.9544 \end{aligned}$$

(b) Let $Y$ denote number of meeting specifications: $Y \sim Binomial(4, 0.9544)$

$$P(Y = 4) = 0.9544^4 = 0.8297$$

## 4.89

For exponential distribution with parameter $\beta$ and pdf $\frac{1}{\beta}e^{-y/\beta}$: cdf $F(y) = 1 - e^{-y/\beta}$

(a) $1 - F(2) = e^{-2/\beta} = 0.0821$: $\beta = 0.8$

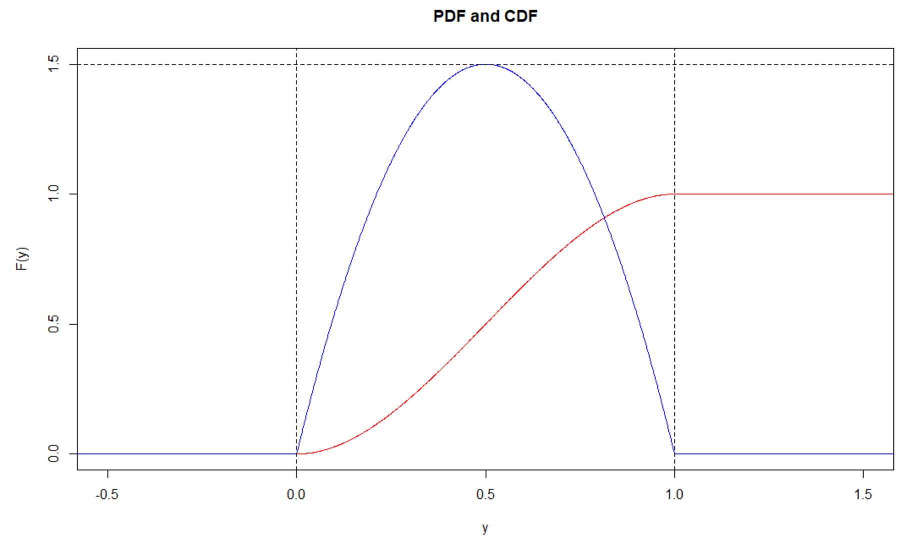
(b) $F(1.7) = 1 - e^{-1.7/0.8} = 0.8806$

## 4.126

(a)

$$F(y) = \begin{cases} 0, y < 0 \\ 3y^2 - 2y^3, 0 \leq y < 1 \\ 1, y \geq 1 \end{cases}$$

(b) The PDF (blue) and CDF (red):




(c) $F(0.8) - F(0.5) = 0.396$

## 4.144

(a) Standard Normal density: $\frac{1}{\sqrt{2\pi}}e^{-y^2/2}$. Thus $k = \frac{1}{\sqrt{2\pi}}$

(b)

$$\begin{aligned}
m_Y(t) &= E(e^{tY}) \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{ty} e^{-y^2/2} dy \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp(-y^2/2 + ty) dy \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp(\frac{-(y-t)^2 + t^2}{2}) dy \\
&= e^{t^2/2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp(\frac{-(y-t)^2}{2}) dy \\
&= e^{t^2/2}
\end{aligned}$$

The last step uses the fact that the function within the integration sign is the pdf for a $N(t, 1)$ random variable, thus the integral is equal to 1.

(c) $Y \sim N(0, 1)$: $E(Y) = 0, Var(Y) = 1$