Homework 12

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Problem 8.40
Part a:
P$(-1.96 \leq Y - \mu \leq 1.96) =$ P$(Y - 1.96 \leq \mu \leq Y + 1.96) = 0.95$
So the 95 percent CI is (Y-1.96, Y+1.96)

Part b:
P$(Y - \mu \leq 1.645) =$ P$(\mu \leq Y + 1.645) = 0.95$
So the value Y+1.645 is the 95 percent upper limit for μ

Part c:
Like part b, Y-1.645 is the 95 percent lower limit for μ

Problem 8.56
Part a:
$\hat{p} \pm z_{0.01}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.45 \pm 2.335\sqrt{\frac{0.45*0.55}{800}} = 0.45 \pm 0.041.$
The 98 percent CI is 0.45 ± 0.041

Part b:
0.50 is not in the interval, so there is not any compelling evidence that a majority of adults feel the movies are getting better.

Problem 8.60
Part a:
$98.25 \pm 2.57(\frac{0.73}{\sqrt{130}}) = 98.25 \pm 2.57(0.0640)$
The 99 percent CI is 98.25 ± 0.1645

Part b:
98.6 is not in the interval. So there is evidence to claim that the average temperature for healthy humans is not 98.6 degrees.

Problem 9.19
E(Y) $= \frac{\theta}{\theta+1}$
Var(Y) $= \frac{\theta}{(\theta+2)(\theta+1)^2}$
Y follows beta distribution with $\alpha = \theta$ and $\beta = 1$. So ...
$E(\bar{Y}) = \frac{\theta}{\theta+1}$
$Var(\bar{Y} = \frac{\theta}{n(\theta+2)(\theta+1)^2}$
So $\bar{Y}$ is a consistent estimator.

Problem 9.37
$L(x_1, x_2..., x_n|p) = P(x_1|p)P(x_2|p)...P(x_n|p)$
$= p\sum_{i=1}^n x_i(1-p)^{n-\sum_{i=1}^n x_i}$
By Theorem 9.4...
$\sum_{i=1}^n x_i$ is sufficient for p with $g(\sum_{i=1}^n x_i, p) = p^{\sum_{i=1}^n x_i}(1-p)^{n-\sum_{i=1}^n x_i}$ and h(y) = 1.

Problem 9.39
$P(Y_1 = y_1, ..., Y_n = y_n|U = u) = \frac{P(Y_1=y_1,...,Y_n=y_n)}{P(U=u)}$
$= \frac{\prod_{i=1}^n \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}}{\frac{(n\lambda)^u e^{-n\lambda}}{u!}}$
$= \frac{\frac{\lambda^{\sum y_i} e^{-n\lambda}}{\prod y_i!}}{\frac{(n\lambda)^u e^{-n\lambda}}{u!}}$
$= P(Y_1 = y_1, ..., Y_n = y_n|U = u) = \frac{u!}{n^u \prod y_i!}$ if $\sum y_i = u$ and 0 otherwise.
Here, the conditional distribution is free of λ. So the statistic is sufficient.

Problem 9.62
$E(Y_{(1)}) = \int_\theta^\infty nye^{-n(y-\theta)}\, dy = \int_0^\infty n(u+\theta)e^{-nu}\, du = \theta + 1/n.$
The MVUE for θ is $Y_{(1)} - 1/n$

Problem 9.63
Part a:
F(y) = $\frac{y^3}{\theta^3}, 0 \le y \le \theta$. Then, we can get:
$f_{(n)}(y) = n[F(y)]^{n-1}f(y) = 3ny^{3n-1}/\theta^{3n}, 0 \le y \le \theta.$

Part b:
Using part a, $E(Y_{(n)}) = \frac{3n}{3n+1}\theta$. So $\frac{3n+1}{3n}Y_{(n)}$ is the MVUE.

Problem 9.71
$E(Y) = \mu_1' = 0$
$E(Y^2) = \mu_2' = Var(Y) = \sigma^2$
$\hat{\sigma}^2 = 1/n\sum_{i=1}^n Y_i^2$ is the method of moments estimator.

Problem 9.77
E(Y) = $\mu_1' = 1.5\theta$
$\hat{\theta} = 2/3\bar{Y}$ is the method of moments estimator

Problem 9.81
$\hat{\theta} = \bar{Y}$
We can use the invariance property of MLE to find MLE of θ^2: $\bar{Y}^2$

Problem 9.97
Part a:
$\mu_1' = \frac{1}{p}$. So the method of moment estimator for p is $\hat{p} = \frac{1}{\bar{Y}}$

Part b:
$L(p0 = p^n(1-p)^{\sum y_i - n}$ is the likelihood function. So the log-likelihood function is:
$lnL(p_= nlnp + (\sum_{i=1}^n y_i - n)ln(1-p)$
Take the derivative next:
$\frac{d}{dp}lnL(p) = n/p - \frac{1}{1-p}(\sum_{i=1}^n y_i - n).$

$n/p - \frac{1}{1-p}(\sum_{i=1}^{n} y_i - n) = 0$ to solve for p.
$\hat{p} = \frac{1}{\bar{Y}}$ is the MLE