Homework 6

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Problem 4.42
P(X≤x) = $\int_{\theta_1}^{x} \frac{1}{\theta_2-\theta_1}\,dx = [\frac{x}{\theta_2-\theta_1}]_{\theta_1}^{x} = \frac{x-\theta_1}{\theta_1-\theta_2}$
Median is P(X≤x) = 0.5
Median = $\frac{\theta_1+\theta_2}{2}$

Problem 4.43
E(A) = πE(R^2) = $\pi\int_0^1 r^2\,dr = \frac{\pi}{3}$
Var(A) = π^2Var(R^2) = $\pi^2[E(R^4) - (\frac{1}{3})^2] = \pi^2[\int_0^1 r^4\,dr$-$(\frac{1}{3})^2] = \pi^2[\frac{1}{5} - (\frac{1}{3})^2] = \frac{4\pi^2}{45}$

Problem 4.58
Part a:
P($0 \leq Z \leq 1.2$) = F(1.2)-F(0) = 0.8849-0.5 = 0.3849

Part b:
P($-0.9 \leq Z \leq 0$) = F(0)-F(-0.9) = 0.5-0.1841 = 0.3159

Part c:
P($0.3 \leq Z \leq 1.56$) = F(1.56)-F(0.3) = 0.9406-0.6179 = 0.3227

Part d:
P($-0.2 \leq Z \leq 0.2$) = F(0.2)-F(-0.2) = 0.5793 - 0.4207 = 0.1586

Part e:
P($-1.56 \leq Z \leq -0.2$) = F(-0.2)-F(-1.56) = 0.4207 - 0.0594 = 0.3613

Part f:
P($0 \leq Z \leq 1.2$) = 0.38493. This is the desired probability for a standard normal.

Problem 4.59
Part a:
P($Z > z_0$) = 0.5
1-P($Z > z_0$) = 0.5
P($Z < z_0$) = 0.5
$z_0 = 0$

Part b:
P($Z < z_0$) = 0.8643
$z_0 = 1.1$

Part c:

$P(-z_0 < Z < z_0) = 0.9$
$P(Z < z_0)$ - $P(Z < -z_0) = 0.9$
$P(Z < z_0)$ - $P(Z > z_0) = 0.9$
$P(Z < z_0)$ - [1-$P(Z < z_0)] = 0.9$
$2P(Z < z_0)$-1 $= 0.9$
$P(Z < z_0) = \frac{1+0.9}{2}$
$P(Z < z_0) = 0.95$
$z_0 = 1.645$

Part d:
$P(-z_0 < Z < z_0) = 0.99$
$P(Z < z_0)$ - $P(Z < -z_0) = 0.99$
$P(Z < z_0)$ - $P(Z > z_0) = 0.99$
$P(Z < z_0)$ - [1-$P(Z < z_0)] = 0.99$
$2P(Z < z_0)$-1 $= 0.99$
$P(Z < z_0) = \frac{1+0.99}{2}$ $P(Z < z_0) = 0.995$
$z_0 = 2.576$

Problem 4.71
Part a:
$P(0.12 \leq Y \leq 0.14) = P(\frac{0.12-0.13}{0.005} \leq Z \leq \frac{0.14-0.13}{0.005}) = P(-2 \leq Z \leq 2) = 0.9544$

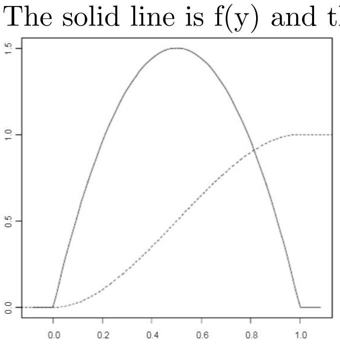
Part b:
X = number of wires that don't meet specifications. X Bin(4, 0.9544)
P(X=4)=$0.9544^4 = 0.8297$

Problem 4.89
Part a:
$\int_2^\infty \frac{1}{\beta} e^{-y/\beta}\, dy = e^{-2/\beta} = 0.0821$
Therefore, $\beta = 0.8$

Part b:
$P(Y \leq 1.7) = 1$ - $e^{-1.7/0.8} = 0.5075$

Problem 4.126
Part a:
F(y) = $P(Y \leq y) = \int_0^y 6t(1-t)\, dt = [\frac{t^2}{2} - \frac{t^3}{3}]_0^y = 3y^2 - 2y^3$

Part b:
The solid line is f(y) and the dashed line is F(y):



Part c:
P($0.5 \leq Y \leq 0.8$) = F(0.8) - F(0.5) = $(3 * 0.8^2 - 2 * 0.8^3)$ - $(3 * 0.5^2 - 2 * 0.5^3)$ = $(1.9^2 - 1.024)$ - $(0.75 - 0.25) = 0.396$

Problem 4.144
Part a:
$\int_{-\infty}^{\infty} ke^{-y^2/2}\, dy = 1k\int_{-\infty}^{\infty} e^{-y^2/2}\, dy = 1$
so $k\sqrt{2\pi} = 1$
k = $1/\sqrt{2\pi}$

Part b:
$m_y(t)$ = E(e^{ty}) = $1/\sqrt{2}\int_{-\infty}^{\infty} e^{-y^2/2+ty}\, dy$
mgf is m(t) = $e^{t^2/2}$

Part c:
E($e^{t^2/2}$)
E(Y) = 0
Var(Y) is the coefficient of $t^2/2!$
Var(Y) = 1