Homework 1 Solution

2.5

(a) $(A \cap B) \cup (A \cap \bar{B}) = A \cap (B \cup \bar{B}) = A \cap S = A$

(b) $B \cup (A \cap \bar{B}) = (B \cup A) \cap (B \cup \bar{B}) = A \cap S = A$

(c) $(A \cap B) \cap (A \cap \bar{B}) = A \cap (B \cap \bar{B}) = A \cap \emptyset = \emptyset$
From part (a), $A = (A \cap B) \cup (A \cap \bar{B})$

(d) $B \cap (A \cap \bar{B}) = (B \cap A) \cap (B \cap \bar{B}) = (B \cap A) \cap \emptyset = \emptyset$
From part (b), $A = B \cup (A \cap \bar{B})$

2.8

	undergrad	grad	Total
off campus	3	9 - 3 = 6	9
on campus	36 - 3 = 33	24 - 6 = 18	60 - 9 = 51
Total	36	60 - 36 = 24	60

(a) undergrad **or** off campus: $33 + 3 + 6 = 42$

(b) undergrad **and** on campus: 33

(c) grad **and** on campus: 18

2.15

(a) Hit on first and missed on second:
$1 - 0.01 - 0.09 - 0.81 = 0.09$

(b) hit on at least one drilling: complement of "missed in both drillings"
$1 - 0.81 = 0.19$

2.23

$B \subset A$: All possible events in B is inside A, but not vice versa. $P(A) = P(B) + P(A \backslash B)$, where $A \backslash B$ represents "events outside B but inside A". Probability is always non-negative, thus $P(B) \leq P(A)$

2.33

(a) Let H denote income higher than \$43318; L less than or equal to \$43318

Each family has 2 possible outcomes of H or L

Sample space (All possible outcomes for the combination of the 4 households):

$$\begin{aligned} &HHHH, HHHL, HHLH, HHLL, \\ &HLHH, HLHL, HLLH, HLLL, \\ &LHHH, LHHL, LHLH, LHLL, \\ &LLHH, LLHL, LLLH, LLLL \end{aligned}$$

(b) A: $HHHH, HHHL, HHLH, HHLL, HLHH, HLHL, HLLH, LHHH, LHHL, LHLH, LLHH$

B: $HHLL, HLHL, HLLH, LHHL, LHLH, LLHH$

C: $HHHL, HHLH, HLHH, LHHH$

(c) By definition of the median: $P(H) = P(L) = \frac{1}{2}$. Thus each of the points in the sample space has probability $p = \frac{1}{2^4} = \frac{1}{16}$

$$\begin{aligned} P(A) &= \frac{11}{16} \\ P(B) &= \frac{6}{16} = \frac{3}{8} \\ P(C) &= \frac{4}{16} = \frac{1}{4} \end{aligned}$$

2.39

(a) Each dice has 6 possible outcomes, results from the two dices are independent: $6 * 6 = 36$ points in the sample space,

(b) Event $A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$. 6 points in event A, thus $P(A) = 6/36 = \frac{1}{6}$

2.41

- 1^{st} digit non-zero: $1 \sim 9$, 9 possibilities
- 2^{nd} to 7^{th} digit: each can be $0 \sim 9$, each has 10 possibilities

Number of possible telephone numbers: $9 * 10 * 10 * 10 * 10 * 10 * 10 = 9 * 10^6$

2.51

There are $\binom{50}{4}$ total number of ways to pick 4 tickets among the 50.

(a) If you want to win all 3 prizes with your 4 tickets, you must have 3 tickets chosen among those 3 prizes, and $4 - 3 = 1$ ticket chosen among the rest 47 non-prize tickets. That is, $\binom{3}{3} * \binom{47}{1}$ cases can win all 3 prizes.

The probability is then $P = \binom{3}{3} * \binom{47}{1} / \binom{50}{4} = 1/4900$

(b) $\binom{3}{2} * \binom{47}{2}$ cases can win exactly 2 prizes. $P = 69/4900$

(c) $\binom{3}{1} * \binom{47}{3}$ cases can win exactly 1 prize. $P = 207/980$

(d) $\binom{3}{0} * \binom{47}{4}$ cases can win no prize. $P = 759/980$

2.64

6 independent rolls, each roll with possible outcomes $\{1, 2, 3, 4, 5, 6\}$ of equal probability $\frac{1}{6}$.

Total possible outcomes: 6^6, each outcome with equal probability $1/6^6$

Number of outcomes with exactly $\{1, 2, 3, 4, 5, 6\}$ in any order (number of ways to arrange $1 \sim 6$ in a row): 6!

$$P = \frac{6!}{6^6} = \frac{5}{324}$$

2.69

Recall that $(n+1)! = 1 * 2 \cdots * n * (n+1) = n! * (n+1)$:

$$
\begin{aligned}
\binom{n+1}{k} &= \frac{(n+1)!}{k!(n+1-k)!} \\
&= \frac{n!}{k!(n-k)!} * \frac{n+1}{n+1-k} \\
&= \frac{n!}{k!(n-k)!} * (1 + \frac{k}{n+1-k}) \\
\binom{n}{k} + \binom{n}{k-1} &= \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n+1-k)!} \\
&= \frac{n!}{k!(n-k)!} + \frac{n!}{k!(n-k)!} * \frac{1}{(n+1-k)/k} \\
&= \frac{n!}{k!(n-k)!} * (1 + \frac{k}{n+1-k})
\end{aligned}
$$

Thus $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$