Stat 330: Exam 2 Solutions

Question 1–version 1.

$$f_X(x) = 3x^2,\ 0 \leq x < 1$$

Question 1–version 2.

$$f_X(x) = \frac{x}{4},\ 1 \leq x \leq 3$$

Question 1–version 3.

$$f_X(x) = 3x^2,\ -1 \leq x \leq 0$$

Question 2–version 1.
Answer:

1. $\lambda = 1/2$.
2. $P(X > 3) = 1 - P(X \leq 3) = 0.223$.
3. $P(X < q) = 98\%$; therefore $q = 7.82 \approx 8$ years.
4. $Im\{Y\} = \{500, 250, 0\}$.
5. $P(Y = 500) = P(X < 0.5) = 0.221$.
6. $P(Y = 250) = P(0.5 < X < 1) = P(X < 1) - P(X < 0.5) = 0.1723$.
7. $P(Y = 0) = P(X > 1) = 1 - P(X \leq 1) = 0.6065$.
8. $E(Y) = \sum_{y \in \{Im(Y)\}} yP(Y = y) = 500 \times 0.221 + 250 \times 0.1723 + 0 \times 0.6065 = 153.575$.

Question 2–version 2
Answer:

1. $\lambda = 1/2$.
2. $P(X > 4) = 1 - P(X \leq 4) = 0.1353$.
3. $P(X < q) = 92\%$; therefore $q = 5.051 \approx 5$ years.
4. $Im\{Y\} = \{600, 300, 0\}$.
5. $P(Y = 600) = P(X < 0.5) = 0.221$.
6. $P(Y = 300) = P(0.5 < X < 1) = P(X < 1) - P(X < 0.5) = 0.1723$.
7. $P(Y = 0) = P(X > 1) = 1 - P(X \leq 1) = 0.6065$.
8. $E(Y) = \sum_{y \in \{Im(Y)\}} yP(Y = y) = 600 \times 0.221 + 300 \times 0.1723 + 0 \times 0.6065 = 184.29$.

Question 2–version 3
Answer:

1. $\lambda = 1/2$.
2. $P(X > 2) = 1 - P(X \leq 2) = 0.3679$.
3. $P(X < q) = 95\%$; therefore $q = 5.99 \approx 6$ years.
4. $Im\{Y\} = \{400, 200, 0\}$.

5. $P(Y = 400) = P(X < 0.5) = 0.221$.
6. $P(Y = 200) = P(0.5 < X < 1) = P(X < 1) - P(X < 0.5) = 0.1723$.
7. $P(Y = 0) = P(X > 1) = 1 - P(X \leq 1) = 0.6065$.
8. $E(Y) = \sum_{y \in \{Im(Y)\}} yP(Y = y) = 400 \times 0.221 + 200 \times 0.1723 + 0 \times 0.6065 = 122.86$.

Question 3–version 1
Answer:

1. $P(X > 12.7) = 0.1587$.
2. $P(X < 12) + P(X > 13) = 0.0124$.
3. $1000 \times 0.0124 = 12.4$.

Question 3–version 2
Answer:

1. $P(X < 12.3) = 0.1587$.
2. $P(X < 12) + P(X > 13) = 0.0124$.
3. $800 \times 0.0124 = 9.92$.

Question 3–version 3
Answer:

1. $P(X < 12.7) = 0.8413$.
2. $P(X < 12.1) + P(X > 12.9) = 0.046$.
3. $1200 \times 0.046 = 55.2$.

Question 4–version 1
Answer:

1. $T \sim \text{Normal}(6400, 960)$.
2. $Z = \frac{6450-6400}{\sqrt{960}} = 1.6137$.
3. $P(Z < 1.6137) = 0.947$.

Question 4–version 2
Answer:

1. $T \sim \text{Normal}(5760, 864)$.
2. $Z = \frac{6400-5760}{\sqrt{864}} = 21.77$.
3. $P(Z < 21.77) = 1$.

Question 4–version 3
Answer:

1. $T \sim \text{Normal}(6720, 1008)$.
2. $Z = \frac{6650-6720}{\sqrt{1008}} = -2.205$.

3. $P(Z < -2.205) = 0.0137$.

Question 5–version 1
Answer:

1. A, B
2. A
3. C

Question 5–version 2
Answer:

1. A, B
2. B
3. D

Question 5–version 3
Answer:

1. A, D
2. D
3. B

Question 6–version 1
Answer:

1. Yes
2. $P^2 = \begin{bmatrix} 0.70 & 0.30 \\ 0.45 & 0.55 \end{bmatrix}$
3. 0.55
4. $\begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \times \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} = [0.55, 0.45]$
5. 0.45
6. $\pi_1 + \pi_2 = 1$ and one of $\begin{cases} 0.8\pi_1 + 0.3\pi_2 = \pi_1 \\ 0.2\pi_1 + 0.7\pi_2 = \pi_2 \end{cases}$
7. 0.4

Question 6–version 2
Answer:

1. Yes
2. $P^2 = \begin{bmatrix} 0.7675 & 0.2325 \\ 0.4650 & 0.5350 \end{bmatrix}$
3. 0.5350

4. $\begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.85 & 0.15 \\ 0.3 & 0.7 \end{bmatrix} = [0.63, 0.37]$

5. 0.37

6. $\pi_1 + \pi_2 = 1$ and one of $\begin{cases} 0.85\pi_1 + 0.3\pi_2 = \pi_1 \\ 0.15\pi_1 + 0.7\pi_2 = \pi_2 \end{cases}$

7. 0.33

Question 6–version 3
Answer:

1. Yes

2. $P^2 = \begin{bmatrix} 0.55 & 0.45 \\ 0.30 & 0.70 \end{bmatrix}$

3. 0.70

4. $\begin{bmatrix} 0.4 & 0.6 \end{bmatrix} \times \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} = [0.4, 0.6]$

5. 0.6

6. $\pi_1 + \pi_2 = 1$ and one of $\begin{cases} 0.7\pi_1 + 0.2\pi_2 = \pi_1 \\ 0.3\pi_1 + 0.8\pi_2 = \pi_2 \end{cases}$

7. 0.6

Question 7–version 1
Answer:

1. $X \sim Poi(\lambda = 10)$

2. $P(X \leq 2) = 0.00277$

3. $Y \sim Exp(\lambda = 5)$

4. $P(Y < \frac{1}{6}) = 0.5654$

5. $W \sim Gamma(\alpha = 5, \lambda = 5)$

6. $P(W < \frac{3}{4}) = P(W \leq \frac{3}{4}) = P(T \geq 5) = 0.3225$ where $T \sim Poi(0.375)$

Question 7–version 2
Answer:

1. $X \sim Poi(\lambda = 15)$

2. $P(X' \geq 2) = 0.9995$ where $X' \sim Poi(10)$

3. $Y \sim Exp(\lambda = 5)$

4. $P(Y < \frac{1}{4}) = 0.7135$

5. $W \sim Gamma(\alpha = 4, \lambda = 5)$

6. $P(W < \frac{3}{4}) = P(W \leq \frac{3}{4}) = P(T \geq 4) = 0.51623$ where $T \sim Poi(0.375)$

Question 7–version 1

Answer:

1. $X \sim Poi(\lambda = 10)$
2. $P(X \leq 5) = 0.0671$
3. $Y \sim Exp(\lambda = 5)$
4. $P(Y < \frac{1}{3}) = 0.8111$
5. $W \sim Gamma(\alpha = 6, \lambda = 5)$
6. $P(W < 1) = P(W \leq 1) = P(T \geq 6) = 0.3840$ where $T \sim Poi(5)$