MLR Inference Review

DS 301

Iowa State University

Today's Agenda

- HW 1 due by 11:59 pm today (on Canvas).
- HW 2 will be posted at midnight. Due next Wednesday.
- Hypothesis Testing Review / confidence intervals
- Multiple Testing Problem

Recap: How good are our least square estimates in the linear regression model?

$j = 0, 1, \dots, p.$

- Unbiasedness: $E(\hat{\beta}_j) = \beta_j$. Least square estimates are unbiased estimate of the true population parameters $\beta_0, \beta_1, \dots, \beta_p$.
  - Idea: Suppose we fit a linear regression line on a data set and we obtain $\hat{\beta}_1$. If we repeat this process for a huge number of datasets and average all of the $\hat{\beta}_1$'s we obtain, the average would exactly equal β_1.
  - An unbiased estimator does not systematically over or under-estimate the true parameter.
- Standard error: Quantifies the how far off a single estimate of $\hat{\beta}_j$ will be from β_j. We denote this as $se(\hat{\beta}_j)$. These depend on estimate of σ^2, which represents the variance of ϵ (and Y).

Correction from previous slides

$se(\hat{B_1})$, $se(\hat{B_2})$.

$\hat{\sigma}^2$

To keep things simple, the formula for $\hat{\sigma}^2$ we will be using is:

$$\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n - p + 1}$$

Review of Hypothesis Testing

Hypothesis tests provide a rigorous statistical framework for answering ‘yes-or-no’ questions about the data.

Our setting: Is the coefficient β_j in a linear regression of Y onto $X_1, \ldots, X_p$ equal to 0?

Framework:

1. Null/alternative hypothesis.
2. Test statistic.
3. Null distribution.
4. Compute the p-value.
5. Conclusion: decide whether or not to reject the null.

Define the null and alternative hypothesis

null $\qquad\qquad\qquad$ alternative

$Y = B_0 + B_1 X_1 + \mathcal{E}.$

(two-sided) $\qquad H_0 : \beta_j = 0$ versus $H_1 : \beta_j \neq 0 \qquad Y = B_0 + \mathcal{E}.$

(one-sided) $\qquad H_0 : B_j = 0$ vs. $H_1 : B_j > 0$

$H_0 : B_j = 0$ vs. $H_1 : B_j < 0.$

Treatment of H_0 and H_1 is asymmetric:

- H_0 is treated as the default state. We focus on using data to reject H_0. We can think of rejecting H_0 as making a **discovery** about our data.
- Rejecting the H_0 does not imply that the alternative hypothesis is true.

Test statistic

$Y = B_0 + B_1 X_1 + \varepsilon$

We assume that H_0 is true. The test statistic is a summary of our data. It provides evidence as to whether or not the H_0 holds.

$H_0: B_1 = 0$ vs. $H_1: B_1 \neq 0$.

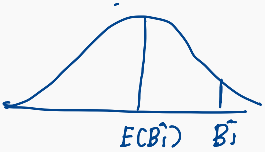
$$t_s = \frac{\hat{B}_1 - E(\hat{B}_1)}{se(\hat{B}_1)}$$

$$= \frac{\hat{B}_1 - B_1}{se(\hat{B}_1)}$$

holds b/c we assume H_0 is true

$$= \frac{\hat{B}_1 - 0}{se(\hat{B}_1)}$$

ex: $\hat{B}_1 = 5$



Null distribution

In order to decide whether or not our test statistic provides evidence in favor of H_0, we need to know the distribution of the test statistic.

Since we assume H_0 is true, we refer to this distribution as the null distribution.

$Y_i = B_0 + B_1 X_1 + \varepsilon_i$

(1) $\varepsilon_i \sim N(0, \sigma^2)$. $\Rightarrow$

(2) sample size is large enough for CLT to kick in.

OR

bootstrap (later)

$E(Y_i) = B_0 + B_1 X_1$

$Y_i \sim N(B_0 + B_1 X_1, \sigma^2)$

$\hat{B}$ are functions of Y

$\Rightarrow$ $\hat{B}$'s are also normal.

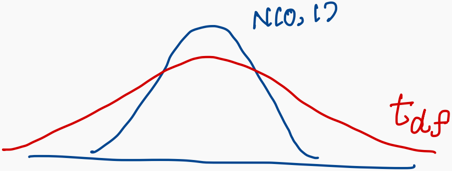
Null distribution

$\varepsilon_i \sim N(0, \sigma^2) \rightarrow Y_i \sim N(\beta_0 + \beta_1 X_i, \sigma^2)$

$\rightarrow t_s \sim N$

?

t-distribution.



σ^2 is unknown

↳ estimate σ^2 using $\hat{\sigma}^2$

↳ introduces additional uncertainty in your distribution

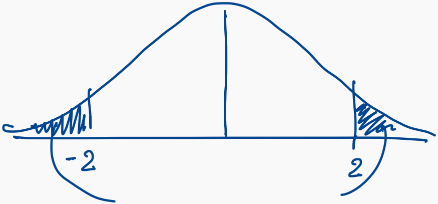
↳ to take into account this uncertainty, we use the t-distribution w/ n-p+1 df.

p-value

Given a value for our test statistic, does this provide strong evidence against H_0? $ts = 2 \longrightarrow$ prob?

p-value allows us to transform our test statistic into a probability that can answer this question.

p.value: probability of observing our test statistic or something more extreme, assuming H_0 is true.



$ts = 2$

$ts \overset{H_0}{\sim} t_{n-p+1}$

shaded area = p.value. $= \underline{0.01}$.

Conclusion

A small p-value indicates that such a large value of the test statistic is unlikely to occur under H_0, and thereby provides evidence against H_0. $\longrightarrow$ reject Ho

How small is small enough to reject H_0?

answer is in the eye of beholder.

p.value $<$ α $\Rightarrow$ reject Ho.

$\alpha = 0.05$

0.10

0.01

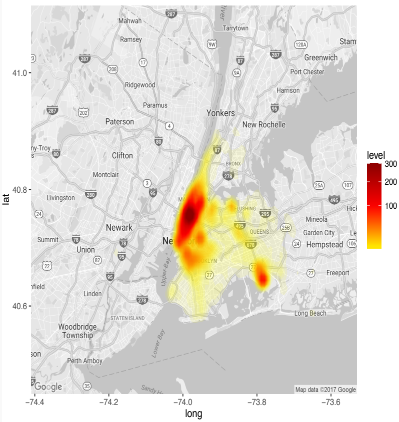
if we can reject Ho, we say our results are statistically significant.

Conclusion

If we reject H_0, that means we have evidence that β_j is significantly different from 0, at significance level α.

If we do not reject H_0, that means we do not have evidence that β_j is significantly different from 0, at significance level α.

Oct. 20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Yonkers
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New Rochelle
Clifton
BRONX
Montclair
Livingston
FLUSHING
Mineola
Newark
QUEENS
Garden City
Summit
Hempstead
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Freeport
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Long Beach
STATEN ISLAND
Woodbridge
Township
Perth Amboy
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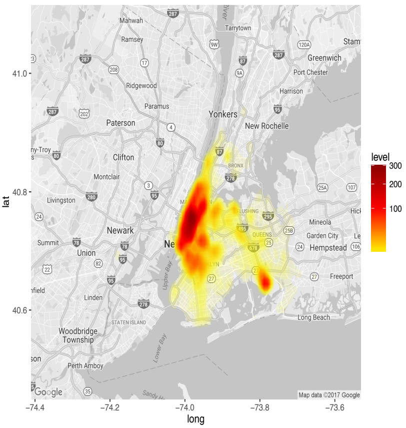
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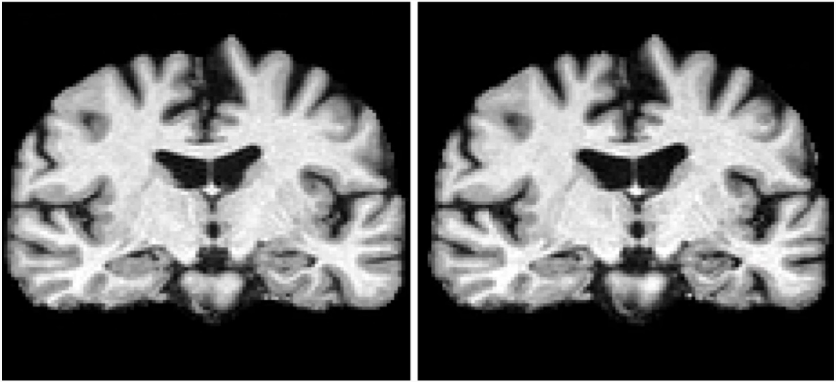


Confidence Intervals

$(10, 20) \rightarrow$ reject H_0

Confidence intervals are close cousins to hypothesis tests. Duality between 2-sided hypothesis tests and confidence intervals.

$1-\alpha$ CI for B_1:

$H_0: B_1 = 0$ vs. $H_1: B_1 \neq 0$.

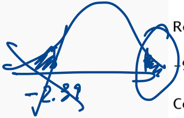
$\boxed{\text{estimate:}} \pm \boxed{\text{critical value; quantile}} \times \boxed{\text{standard error of your estimate}}$

$\Rightarrow \hat{B_1} \pm t_{n-p+1; 1-\alpha/2} \times se(\hat{B_1})$

95% CI for B_1: $(-3, 5)$.

95% confident the true B_1 falls btwn $(-3, 5)$.

$\Rightarrow$ if I repeated this many, many, many, many times, 95% of intervals would contain B_1.




```
> summary(lm(crim~.,data=Boston))

Call:
lm(formula = crim ~ ., data = Boston)

Residuals:
   Min     1Q Median     3Q    Max
-9.924 -2.120 -0.353  1.019 75.051

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  17.033228   7.234903   2.354 0.018949 *
zn            0.044855   0.018734   2.394 0.017025 *
indus        -0.063855   0.083407  -0.766 0.444294
chas         -0.749134   1.180147  -0.635 0.525867
nox         -10.313535   5.275536  -1.955 0.051152 .
rm            0.430131   0.612830   0.702 0.483089
age           0.001452   0.017925   0.081 0.935488
dis          -0.987176   0.281817  -3.503 0.000502 ***
rad           0.588209   0.088049   6.680 6.46e-11 ***
tax          -0.003780   0.005156  -0.733 0.463793
ptratio      -0.271081   0.186450  -1.454 0.146611
black        -0.007538   0.003673  -2.052 0.040702 *
lstat         0.126211   0.075725   1.667 0.096208 .
medv         -0.198887   0.060516  -3.287 0.001087 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.439 on 492 degrees of freedom
Multiple R-squared:  0.454,	Adjusted R-squared:  0.4396
F-statistic: 31.47 on 13 and 492 DF,  p-value: < 2.2e-16
```

X_1: zn

X_2: indus

$\alpha = 0.05$

$H_0: B_1 = 0$ vs. $H_1: B_1 \neq 0$.

$H_0: B_2 = 0$ vs. $H_1: B_2 \neq 0$

$ts = 2.394$

Null distr: t_{492}

p.value $= \frac{0.017025}{2}$

conclusion: we reject Ho at sig. level $\alpha = 0.05$

See R script `MLR_Inference.R`