Homework 2

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Problem 2.71 a
P(A|B) = P(A $\cap$ B) / P(B) = 0.1 / 0.3 = 1/3

Problem 2.71 b
P(B|A) = P(A $\cap$ B) / P(A) = 0.1 / 0.5 = 1/5

Problem 2.71 c
P(A|A $\cup$ B) = P(A) / P(A $\cup$ B) = 0.5 / (0.5+0.3-0.1) = 0.5 / 0.7 = 5/7

Problem 2.71 d
P(A|A $\cap$ B) = 1

Problem 2.71 e
P(A $\cap$ B—A $\cup$ B) = P(A $\cap$ B) / P(A $\cup$ B) = 0.1 / (0.5 + 0.3 - 0.1) = 0.1/0.7 = 1/7

Problem 2.75 a
A = draw 2 spades from 52 cards, B = draw 3 spades from 50 cards
number of ways drawing 2 spades from 13 spades, n(A) = $\binom{13}{2}$, P(A) = $\binom{13}{2}/\binom{52}{2}$
number of ways drawing 3 spades from 11 spades, n(B) = $\binom{11}{3}$, P(B) = $\binom{11}{3}/\binom{50}{3}$
P(B $\cap$ A) = $\binom{13}{5}/\binom{52}{5}$
P(B|A) = P(B $\cap$ A) / P(A) = $\frac{\binom{13}{5}/\binom{52}{5}}{\binom{13}{2}/\binom{52}{2}}$ = 0.00842

Problem 2.75 b
A = draw 3 spades from 52 cards, B = draw 2 spades from 49 cards
number of ways drawing 3 spades from 13 spades, n(A) = $\binom{13}{3}$, P(A) = $\binom{13}{3}/\binom{52}{3}$
number of ways drawing 2 spades from 10 spades, n(B) = $\binom{10}{2}$, P(B) = $\binom{10}{2}/\binom{49}{2}$
P(B $\cap$ A) = $\binom{13}{5}/\binom{52}{5}$
P(B|A) = P(B $\cap$ A) / P(A) = $\frac{\binom{13}{5}/\binom{52}{5}}{\binom{13}{3}/\binom{52}{3}}$ = 0.03827

Problem 2.75 d
A = draw 4 spades from 52 cards, B = draw 1 spades from 48 cards
number of ways drawing 4 spades from 13 spades, n(A) = $\binom{13}{4}$, P(A) = $\binom{13}{4}/\binom{52}{4}$
number of ways drawing 1 spades from 9 spades, n(B) = $\binom{9}{1}$, P(B) = $\binom{9}{1}/\binom{48}{1}$
P(B $\cap$ A) = $\binom{13}{5}/\binom{52}{5}$
P(B|A) = P(B $\cap$ A) / P(A) = $\frac{\binom{13}{5}/\binom{52}{5}}{\binom{13}{4}/\binom{52}{4}}$ = 0.1875

Problem 2.83
$P(A \mid A \cup B) = P(A \cap (A \cup B))/P(A \cup B)$
A and B are mutually exclusive, $A \cap B = \emptyset$
$P(A \mid A \cup B) = P(A \cap A)/P(A \cup B) = P(A) / P(A \cup B)$
$P(A \cup B) = P((A \cup B) \cap A) + P((A \cup B) \cap B) = P(A) + P(B)$
$P(A \mid A \cup B) = P(A) / (P(A) + P(B))$. We've shown that when A and B are mutually exclusive events and $P(B) > 0$, then $P(A \mid A \cup B) = P(A)/(P(A) + P(B))$

Problem 2.86
Part a: Yes, it's possible for P(A *cap* B) to be 0.1. The probability of the intersection of the two events can be any value between 0 and the minimum of P(A) and P(B). In this case, the min of P(A) and P(B) is 0.7, so $P(A \cap B)$ can indeed be 0.1 which is less than 0.7.

Part b: The smallest possible value for $P(A \cap B)$ is 0. It can be 0 because the intersection of 2 events can be empty, meaning there are no outcomes that belong to both events A and B.

Part c: No, it's not possible for $P(A \cap B)$ to be 0.77 or any value greater than the minimum of P(A) and P(B). $P(A \cap B)$ can't exceed the smallest of P(A) and P(B) which in this case is 0.7.

Part d: The largest possible value for $P(A \cap B)$ is 0.7. $P(A \cap B)$ can't exceed the minimum of P(A) and P(B).

Problem 2.91
Case 1: $P(A)= 0.4$, $P(B) = 0.7$
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.7 - 0 = 1.1$
These events aren't mutually exclusive because the probability value is greater than 1, and doesn't satisfy the property of sum total probability.

Case 2: $P(A)=0.4$, $P(B)=0.3$
By definition, if the two events are mutually exclusive, then $P(A \cap B) = 0$
$P(A \cup B) = P(A)+P(B)-P(A \cap B) = 0.4 + 0.3 - 0 = 0.7$
The events A and B are mutually exclusive because the probability value is less than 1, satisfying the property of sum total probability.

Problem 2.98
Let's suppose that the relays act independently of one another and close properly when activated with a probability of 0.90. So, E1 = relay 1 is activated, E2 = relay 2 is activated. Then $P(E1) = 0.90$, $P(E2) = 0.90$.
The probability that current will flow in the series system when the relays are activated is $P(E1 \cap E2) = P(E1)P(E2) = (0.90)(0.90) = 0.81$.
The probability that current will flow in the parallel system when the relays are activated is $P(E1 \cup E2) = 1\text{-}(\overline{E1 \cup E2}) = (1\text{-}P(\overline{E1})P(\overline{E2})) = 1\text{-}(0.1)(0.1) = 0.99$.
Given the current flowed when the relays were activated, the probability that relay 1 functioned in the series system is $P(E1 \mid E1 \cap E2) = P((E1) \cap (E1 \cap E2))/P(E1 \cap E2) = 1$.
Given the current flowed when the relays were activated, the probability that relay 1 functioned in the parallel system is $P(E1 \mid E1 \cup E2) = P((E1) \cap (E1 \cup E2))/P(E1 \cup E2) = P(E1)/P(E1 \cup E2) = 0.9/0.99 = 0.9091$

Problem 2.124
R = population of Republican voters, D = population of Democrat voters, E = population of voters refers favor an election issue.
P(R) = 0.4, P(D) = 0.6, P(E|R) = 0.3, P(E|D) = 0.7, P(E) = P(E|R)P(R) + P(E|D)P(D) = (0.3)(0.4)+(0.7)(0.6) = 0.54
P(D|E) = (P(E|D)P(D))/P(E) = ((0.6)(0.7)/(0.54) = 0.42/0.54 = 0.778

Problem 2.130
S = worked in shipyard, C = person had lung cancer, $\overline{C}$ = person had no lung cancer, S | C = person who had lung cancer some prior time in shipyard, S | $\overline{C}$ = person who had no lunch cancer some prior time in shipyard.
P(C) = 0.0004, P(S|C) = 0.22, P(S|$\overline{C}$ = 0.14, P($\overline{C}$) = 1-0.0004 = 0.9996
P(C|S) = (P(S|C)P(C))/(P(S|C)P(C) + P(S|$\overline{C}$)P($\overline{C}$)) = ((0.22)(0.0004)/((0.22)(0.0004)+(0.14)(0.9996)) = 0.000088/0.140032 = 0.0006
The percentage of Georgians living during the same period who will contract (or have contracted) lung cancer, given that they have at some prior time worked in a shipyard is 0.06%

Problem 2.134
F = failure to learn, A = method A was used, B = method B was used. P(F|A) = 0.2, P(F|B) = 0.1, P(A) = 0.7, P(B) = 0.3
P(A|F) = ((P(F|A))P(A))/((P(F|A)P(A)+(P(F|B)P(B)) = ((0.2)(0.7))/((0.2)(0.7)+(0.1)(0.3)) = 0.14/0.17 = 0.8235

Problem 2.135
M = traveling on major airlines, P = traveling on privately owned airlines, C = traveling on commercially owned planes not belonging to a major airline, B = traveling for business reasons, NB = not traveling for business reasons.
P(M) = 0.6, P(P) = 0.3, P(C) = 1-P(M)-P(P) = 0.1, P(B|M) = 0.5, P(B|P) = 0.6, P(B|C) = 0.9

Part a:
P(B) = P(B|M)P(M) + P(B|P)P(P) + P(B|C)P(C) = (0.5*0.6)+(0.6*0.3)+(0.9*0.1) = 0.3 + 0.18 + 0.09 = 0.57
The probability that the person is traveling on business is 0.57.

Part b:
P(B $\cap$ P) = P(B|P)P(P) = 0.6*0.3 = 0.18
The probability that the person is traveling for business on a privately owned plane is 0.18

Part c:
P(P|B) = (P(B|P)*P(P)) / P(B) = (0.6*0.3)/0.57 = 0.18/0.57 = 0.3158
The probability that the person arrived on a privately owned plane, given that the person is traveling for business reasons is 0.3158.

Part d:
P(B|C) = (P(C)P(B|C)) / (P(M)*P(B|M) +P(P)*P(B|P) + P(C)*P(B|C)) = (0.1*0.9)/(0.6*0.5 + 0.3*0.6 + 0.1*0.9) = 0.7714 The probability that the person is traveling on business, given that the person is flying on a commercially owned plane is 0.7714.