# Momentum

Fun With Fiziks

### What is momentum?

- Basically how hard something is to stop
- Represent momentum using ρ
- Is a vector quantity (direction/magnitude)
- Units: kg\*m/s
- $\bullet$   $\rho = mv$

### Conservation of momentum

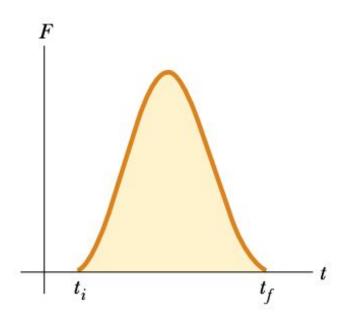
- Just like energy, momentum is conserved!
- Initial momentum = final momentum
  - Momentum is only conserved when there are no external forces! (just like energy)
- Can write conservation of momentum as:
  - $\circ$   $\rho_i = \rho_f = constant$
  - $\rho_{1i} + \rho_{2i} = \rho_{1f} + \rho_{2f}$

### What is impulse?

- Change in momentum
- Represent impulse using I
- Is a vector
- Units: kg\*m/s

### Impulse cont.

- Impulse momentum theorem:  $I = \Delta \rho$
- Area under force time curve
- Another representation:
  - $\circ$  I = F $\triangle$ t
- This looks similar to work-energy theorem!!
  - Momentum related to time
  - Work related to distance



### Practice problem

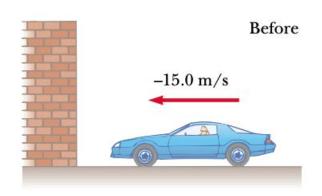
Bob throws a 10 kg ball at Joe at 100 m/s and Joe hits it back at Bob at 1 m/s. What is the magnitude of the impulse on the ball from Joe?

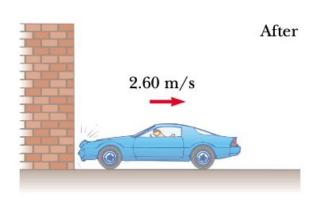
Use I = 
$$\Delta \rho$$
  
I =  $\Delta \rho = \rho_f - \rho_i = m_{ball} v_f - m_{ball} v_i$   
Plug in numbers:  
I =  $\Delta \rho = 10*1 - 10*(-100) = 1010 \text{ kg*m/s}$ 



### Another practice problem

In a crash test, a car of mass 1500 kg collides with a wall. The initial and final velocities are 15 m/s and 2.6 m/s, respectively. If the collision lasts for 0.15 s, find the impulse caused by the collision and the force exerted on the car.





Use both definitions of impulse

$$\circ$$
  $I = \Delta \rho$ 

$$\circ$$
  $I = F_{net} \Delta t$ 

- $I = \Delta \rho = mv_f mv_i = 1500*2.6 1500*(-15)$ 
  - $\circ$  I = 2.64\*10^4 kg\*m/s
- $I = F_{net} \Delta t = 2.64*10 \Delta t = F_{net}*0.15$ 
  - $\circ$  F<sub>net</sub> = 1.76\*10 $^{1}$ 5 N

When do we use momentum (the most)?

# COLLISIONS!

### Types of collisions

- Elastic: KE and momentum conserved (impossible)
- Inelastic: KE not constant because energy lost
- Perfectly inelastic: objects stick together after colliding (car crash)
- Momentum is constant in all collisions, but KE is not

### **Elastic collisions**

- Momentum conserved:
- $\rho_{1i} + \rho_{2i} = \rho_{1f} + \rho_{2f}$
- Kinetic energy is also conserved:
- $1/2m_1v_{1i}^2 + 1/2m_2v_{2i}^2 = 1/2m_1v_{1f}^2 + 1/2m_2v_{2f}^2$

# Practice problem

A 2 kg ball is moving right at 2 m/s and a 3 kg ball moving left at 3 m/s collide. The collision is elastic. After, the 3 kg ball moves right at 1 m/s. Find the velocity of the 2 kg ball.

Plug in numbers in 
$$\rho_{1i}$$
+  $\rho_{2i}$  =  $\rho_{1f}$ +  $\rho_{2f}$   
 $\rho_{1i}$  = 2\*2,  $\rho_{2i}$  = 3\*(-3),  $\rho_{2f}$  = 3\*1,  $\rho_{1f}$  = ?  
4 - 9 = 3 +  $\rho_{1f}$   
 $\rho_{1f}$  = -8  $\Rightarrow$  -8 = 2\* $\nu_{1f}$   
 $\nu_{1f}$  = -4 m/s, which means the ball goes 4 m/s to the left

# What if you don't know either of the final speeds?

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

### Inelastic collisions

- What happens in reality energy is not conserved because there is friction
- Momentum is still conserved
- We don't really deal with inelastic collisions much

### Perfectly inelastic collisions

- When two things stick together after a collision
- The conservation of momentum equation:

• 
$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) * v_f$$



### Ballistic pendulum

- Collision is perfectly inelastic
- $m_1$  = mass of bullet,  $m_2$  = mass of block,  $m_{1i}$  = initial velocity of bullet,  $v_{2i}$  = initial velocity of block,  $v_f$  = final velocity
- $V_{2i} = 0$

- $m_1 v_{1i} = (m_1 + m_2) * v_f$
- $KE_f = 1/2(m_1 + m_2)*v_f \wedge 2$
- Solve momentum equation for  $v_f = m_1 v_{1i} / (m_1 + m_2)$
- Plug in v<sub>f</sub> into KE equation

$$K_f = \frac{m_1^2 v_{1i}^2}{2(m_1 + m_2)}$$

### All KE becomes PE (conservation of energy)

$$\frac{m_1^2 v_{1i}^2}{2(m_1 + m_2)} = (m_1 + m_2)gh$$

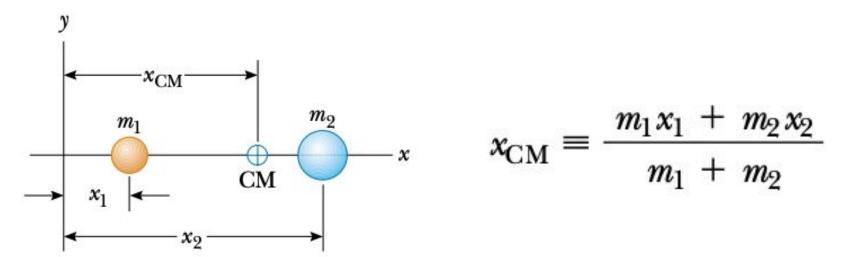
Solve for v<sub>1i</sub>

$$v_{1\,i}=igg(rac{m_1\,+\,m_2}{m_1}igg)\sqrt{2gh}$$

### Center of Mass

### What is CM?

- Where something balances on 1 finger
- Average location of all the mass in an object



### General formula

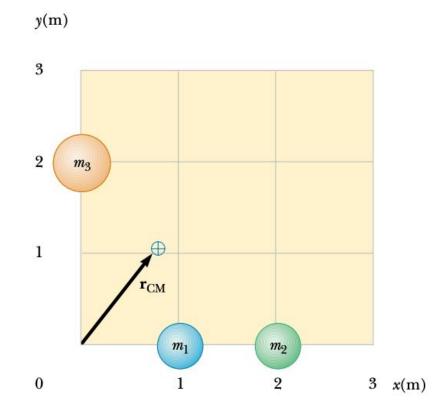
$$x_{\text{CM}} \equiv \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n} = \frac{\sum_{i} m_i x_i}{\sum_{i} m_i}$$

- This is x coordinate of CM
- Do the same but use y or z coordinates to find y and z coordinate of CM
- Basically multiply mass by the position for all points, add them up, and divide by total mass



#### **Practice Problems**

Find center of mass of this system of 3 particles:  $m_1 = m_2 = 1 \text{ kg}$  $m_3 = 2 \text{ kg}$ 



### **Answer**

- Total mass = 1 kg + 1 kg + 2 kg = 4 kg
- $x_{cm} = 1*1 + 1*2 + 2*0/4 = 0.75 \text{ m}$
- $y_{cm} = 1*0 + 1*0 + 2*2/4 = 1 \text{ m}$
- Coordinates of center of mass:  $(x_{cm}, y_{cm}) = (0.75 \text{ m}, 1 \text{ m})$
- Vector of center of mass: <0.75 m, 1 m>

### Center of gravity

- Very closely related to CM
- CG is the point at which gravity acts on an object
- CG = CM in a uniform gravitational field, which is assumed in mechanics problems

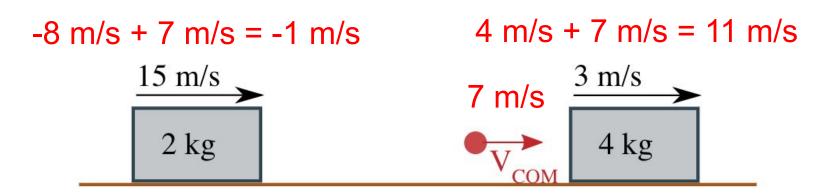
# How is center of mass related to momentum?

- When multiple objects move at constant velocities, their center of mass is moving at v<sub>cm</sub>
- v<sub>cm</sub> = sum of momentums/total mass
- Makes solving momentum problems harder



### Make $v_{cm} = 0$ to solve problems

- To make  $v_{cm} = 0$ , we have to "scale" the other velocities
- Subtract  $v_{cm}$  (7 m/s in this case) from all the velocities
- When  $v_{cm} = 0$ , velocities are the opposites after collision
- ullet To get the actual final velocities, add back  $v_{cm}$



# Check the previous problem using the formula

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

This is really hard to memorize

$$v_{1f} = \left(\frac{2-4}{2+4}\right) \cdot 15 + \left(\frac{2\cdot 4}{2+4}\right) \cdot 3 = -1$$

$$v_{2f} = \left(\frac{2 \cdot 2}{6}\right) \cdot 15 + \left(\frac{2}{6}\right) \cdot 3 = 11$$

But we get the same answer as the  $v_{cm} = 0$  method!

## **2D Collisions**



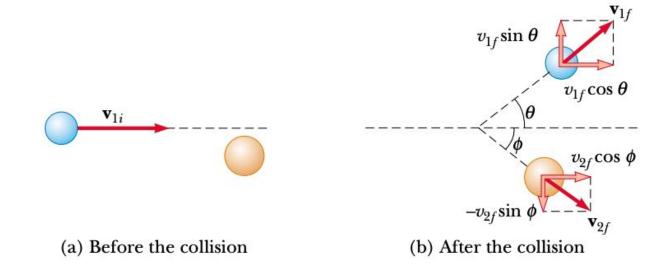
### 2D collisions

- Same as 1D collisions but solve for x and y components separately
- Since velocity is vector, split vector into x and y components

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$
  
 $m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$ 

### Glancing collisions

- One particle is moving and another is at rest
- After, the two particles move at an angle



### Glancing collisions cont.

- Since  $v_{2i} = 0$ ,  $m_2 v_{2i}$  term = 0
- x components of final velocity are  $mvcos\theta$
- Since initially the particles aren't moving up or down, momentum in y direction initial and final is 0

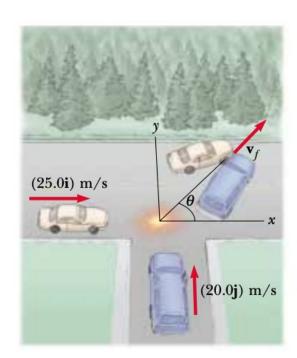
$$m_1 v_{1i} = m_1 v_{1f} \cos \theta + m_2 v_{2f} \cos \phi$$
$$0 = m_1 v_{1f} \sin \theta - m_2 v_{2f} \sin \phi$$

### Practice problem

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.15. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).

Direction:  $\theta = 53.1^{\circ}$ 

Magnitude:  $v_f = 15.6 \text{ m/s}$ 



### **Explanation**

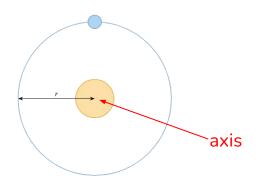
- Do x and y components of velocity separately
- Momentum in x direction:
  - $\circ$  1500\*25 = (1500 + 2500)\* $v_f$ \* $\cos\theta$
- Momentum in y direction:
  - $\circ$  2500\*20 = (1500 + 2500)\* $v_f$ \*sin $\theta$
- Have 2 unknowns and 2 equations so solve for  $\theta$  and  $v_{_{\rm f}}$

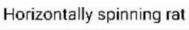
- Divide 2 equations to cancel out  $v_f$  solve for  $\theta$ 
  - 0 2500\*20/1500\*25 = (1500 + 2500)\* $v_f$ \*sinθ/(1500 + 2500)\* $v_f$ \*cosθ
  - $\circ$  1.33 = tan $\theta$
  - $\theta = 53.1^{\circ}$
- Plug in 53.1° for  $\theta$  in either equation to solve for  $v_f$ 
  - 0.1500\*25 = (1500 + 2500)\*v<sub>f</sub>\*cos(53.1)
  - $\circ$  v<sub>f</sub> = 15.6 m/s

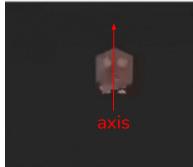
### **Rotational Motion**

#### What is rotation?

- Rotation is when an object moves around a stationary point.
- This stationary point is called the axis.
- All points on the object move around the axis in a circular path.

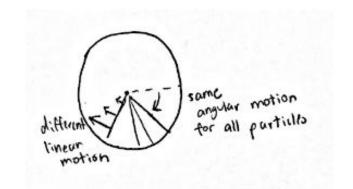






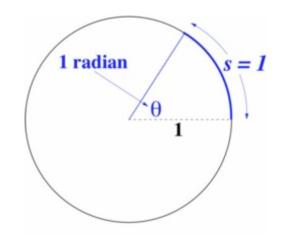
### "Angular"

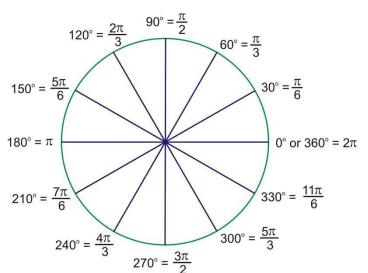
- When describing rotations, we use the word "angular" (angular momentum, angular velocity, angular acceleration, etc.).
- Unlike linear motion, during a rotation different parts of the object move at different velocities. However, every particle rotates through the same angle, hence the word "angular".



### Radians

- Another way to measure angles
- Usually preferred compared to degrees
- If you walk 1 unit along a unit circle, the angle you have traveled is 1 radian!
- Basically however far you walk along the circle is your angle in radians





### Why do we use radians?

- It makes it easy to convert between linear and angular quantities
- We know on a unit circle,  $s = \theta$ 
  - How far you walk = angle
- For a circle of radius r,  $s = r\theta$ 
  - You walk r times further but the angle you travel still stays the same

• Displacements  $s = \theta r$ 

• Speeds  $v = \omega r$ 

• Accelerations  $a = \alpha r$ 

### Angular vs linear quantities

 Basically the same as normal kinematics, just replace the linear quantity with the corresponding angular quantity

Linear quantity	symb.	Angular quantity	symb.
distance	d	angle	θ
velocity	V	angular vel.	ω
acceleration	a	angular accel. $\alpha$	

### **Kinematics for Rotations**

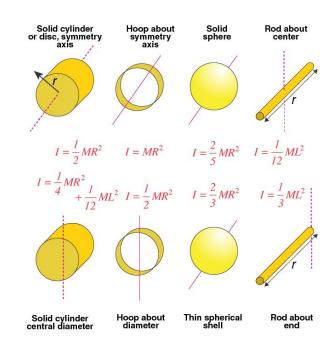
Translational Motion	Rotational Motion
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$\Delta x = \frac{1}{2}at^2 + v_i t$	$\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_i t$
$\Delta x = \frac{1}{2} (v_f + v_i) t$	$\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)t$
$v_f^2 = v_i^2 + 2a\Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

### Angular momentum

- The rotational version of normal momentum
- If momentum = how hard something is to stop moving, then
   rotational momentum = how hard something is to stop rotating

### What is the moment of inertia?

- Rotational version of mass
- How hard something is to spin
  - Normal mass: more mass → harder to move
  - Moment of inertia: more rotational inertia → harder to rotate
- Different from normal mass because it depends on the shape of the object and the axis of rotation



### Moment of inertia cont.

- The moment of inertia depends on the radius
  - If R increases, then
     rotational inertia increases
- This tells us that the further the mass is from the axis of rotation, the harder something is to spin

Two batons have equal mass and length. Which will be "easier" to spin?

- A) Mass on ends
- B) Same
- C) Mass in center