Experiment 8
Magnetic Moment

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In this lab we calculated of the magnetic moment of a permanent magnet inside of a cue ball using 5 different methods. Our results can be seen in TableI. And our agreement between our different methods can be seen in Table.II. Most of our methods agreed with each other. The only exceptions were that Method 1 did not agree with Method 2 or Method 5. All uncertainties are reported with a 95% confidence interval.

I. INTRODUCTION

The goal of this lab was to experimentally determine the magnetic moment of a permanent magnet using 5 different methods. We did this using an apparatus that could generate a magnetic field and a magnetic field gradient. The first method we used to determine the magnetic moment was by balancing the torque of the magnetic field on the magnetic moment with a gravitational torque on the moment. The second was by measuring the period of the moment oscillating in the magnetic field and modeling. The third was by measuring the precision of the moment spinning in the magnetic field. The fourth was by baling the force of the changing magnetic field. And the fifth method was by measuring the magnetic field of the magnet using a hall effect probe.

II. THEORETICAL BACKGROUND

The Magnetic Torque apparatus has been designed to facilitate measuring a magnetic moment using experimentation. We used the apparatus and 5 different methods to measure a magnetic moment. It is important to cover some theory behind the apparatus itself before going into the theory behind each experimental method.

A. The Apparatus

The magnet is the component of the instrument that houses the two co-axial coils, the air bearing and the strobe light.

The coils are copper wire wound on bobbins. Each coil has 195 turns. The coils are in series, so the same current runs through each turn. The current is displayed on the analog ammeter, and can be adjusted with the nob below it.

The coils have some resistance that is temperature dependent, so running the apparatus for a long time, or at high current will cause the temperature of the coils to rise. This causes the resistance of the coils to rise and the current will begin to decrease because he power supply isn't current-regulated. If this happens then the apparatus wont be able to be set to its highest current settings, so it is important to turn of the apparatus when not in use so that it doesn't get too hot.

To calculate the magnetic field at the center of the apparatus requires the use of an integral since each turn has a different radius and a different distance from the center. This is a difficult problem, so instead a single equivalent radius and separation between the coils has been calculated. Each coil can be treated as two different current loops, each with N = 195. For this apparatus the equivalent coil radius is R = 0.109 m, and the equivelent separation between coils is d =0.138 m. Then the Biot-Savart Law can be used to find the magnetic field at the center of the two coils. The Biot-Savart Law is

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \vec{r}}{4\pi r^2}. \tag{1}$$

For out setup, a current loop, $d\vec{L}$ is perpendicular to $\vec{r}$, so their cross products is $d\vec{L}$. And distance from the edge of the ring to some position z is given by

$$r^2 = z^2 + R^2. \tag{2}$$

For a current loop the x and y components of magnetic field so our magnetic field will just point in the z direction, so $dB_z = dB\sin\theta$, and from the geometry of the loop we can see that

$$\sin\theta = \frac{R}{\sqrt{z^2 + R^2}}. \tag{3}$$

We also know that I in the Biot-Savart law is really NI because we have N current loops.

Plugging these things back into the Biot-Savart law we see that

$$B_z = \oint \frac{\mu_0 N I dl}{4\pi} \frac{R}{\sqrt{z^2 + R^2}(z^2 + R^2)}. \tag{4}$$

Now we can evaluate this integral because most of it has no dependence on dl and see

$$B_z = \frac{\mu_0 N I R}{4\pi (z^2 + R^2)^{3/2}} \oint dl, \tag{5}$$

and the integral around dl is just the circumference of the ring $2\pi R$, so we get

$$B_z = \frac{\mu_0 NIR^2}{2(z^2+R^2)^{3/2}}. \tag{6}$$

This is the magnetic field for one ring, the ring that is positioned at z = 0. To find the magnetic field for the other loop is the exact same process, the only difference is that

$$r^2 = (d-z)^2 + R^2, \tag{7}$$

because the other ring is positioned at $z = d$ relative to the other ring, and we get a result of

$$B_z = \frac{\mu_0 NIR^2}{2((d-z)^2+R^2)^{3/2}}. \tag{8}$$

To get the total magnetic field of the two rings we just add those two results together giving us

$$B_z(z) = \frac{N\mu_0 IR^2}{2}\left(\frac{1}{(R^2+z^2)^{3/2}} + \frac{1}{[R^2+(d-z)^2]^{3/2}}\right). \tag{9}$$

This equation can be simplified as

$$B_z(z) = C_1 I, \tag{10}$$

where C_1 is just a constant based on the geometry of our coils. We calculated $C_1 = 0.00135\frac{\text{kg}}{\text{s}^2\text{A}^2}$

If the current goes through the coils in opposite directions, a field gradient will be created in the z direction. Solving the Biot Savart setup from before, is exactly the same but the magnetic fields of the two rings will be opposite of each other so instead of adding the two fields together we take the difference of the two giving us

$$B_z(z) = \frac{N\mu_0 IR^2}{2}\left(\frac{1}{(R^2+z^2)^{3/2}} - \frac{1}{[R^2+(d-z)^2]^{3/2}}\right). \tag{11}$$

The field at the center will be zero, but there will be a non-zero field gradient. This can be found by taking the dirivitave of Eq.(11) with respect to z, then looking at the midpoint $z = d/2$. The resulting equation is

$$\frac{\partial B_z}{\partial z} = \frac{N\mu_0 IR^2}{2}\left(\frac{-3z}{(R^2+z^2)^{5/2}} - \frac{3(d-z)}{((d-z)^2+R^2)^{5/2}}\right), \tag{12}$$

then pluging in $z = d/2$ gives us

$$\frac{\partial B_z}{\partial z} = \frac{3N\mu_0 R^2 d}{2}\frac{1}{\left(R^2+\frac{d^2}{4}\right)^{5/2}} I. \tag{13}$$

This eqation can be simplified as

$$\frac{\partial B_z}{\partial z} = C_2 I, \tag{14}$$

where C_2 is a constant based on our geometry. We calculated $C_2 = 0.0169\frac{\text{kg}}{\text{ms}^2\text{A}^2}$

The air bearing is the spherical hollow in the cylindrical brass rod that is supported on the bottom coil. There is a narrow opening that allows air to be pumped into the spherical hollow. The ball sits in the hollow and floats on a cushion of air. This supports the ball with very little friction. The air pump is inside o the power supply, and is turned on and off with a switch on the power supply.

The strobe light is located on top of the upper coil. The frequency of the strobe can be controlled with a dial located on the power supply, and the frequency is displayed on the front panel of the power supply.

If the apparatus is not level a torque can be caused from unequal air flow from the air bearing. The apparatus can be leveled with a bulls eye level, and placing shims under the rubber feet below the magnet.

The controller houses the air pump and the electronic controls for the strobe light and a voltage controlled power supply.

An analog ammeter displays the current passing through the coils, and is controlled with a dial below the meter.

The field direction down or up can be controlled with a switch on the controller. This switch changes the direction of the current in both coils simultaneously.

There is a field gradient switch located on the controller that changes between a uniform field and a field gradient. When in the OFF position the current will flow in the same direction in both coils, in the ON position the current will flow in opposite directions. When the field gradient is on the magnitude of the field at the center of the system is zero.

There is an on/off switch for all of the components inside the controller located on the back panel of the controller.

The cue balls has a small cylindrical permanent magnet at its center. The magnet acts as a dipole. The handle is embedded in the ball with ts axis along the magnetic moment vector of the dipole. The ball has been manufactured so that the center of mass of the sphere/magnet/handle is still at the center of the ball.

There is a small hole drilled into the handle that can hold a thin aluminum tube. The rod has a steel-tipped end that attracted to the magnet inside of the ball, so that it doesn't easily fall out. A small plastic cylinder slides along the length of the tube is used to vary the gravitational torque on the ball/handle system.

B. Method 1 - Static Equilibrium

The magnetic field of the magnet inside the ball acts as a magnetic dipole fo magnitude μ aligned with the handle on the ball. When placed in a uniform magnetic field a magnetic dipole experiences a torque given by the expression

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \tag{15}$$

where $\vec{\tau}$ is the magnetic torque and $\vec{B}$ is the magnetic field. The magnetic field of the coils is either up or down. The torque on the magnetic moment will align the moment with the direction of the magnetic field. So if the field direction is set to up the handle of the ball will go straight up.

If the aluminum rod is placed in the handle, there is a now another torque due to the earth's gravitational field. The expression for this torque is

$$\vec{\tau} = \vec{r} \times m\vec{g}, \tag{16}$$

where $\vec{r}$ is the vector extending from the center of the ball to the center of the plastic mass on the aluminum rod and m is the mass of the plastic. The gravitational torque causes the ball to rotate so that the handle will point downwards. If there is a net torque the ball will rotate, but when the torques cancel out the ball will not rotate. The balanced configuration is described by $\tau_{\text{net}} = 0$ or

$$\mu B \sin(\theta) = rmg \sin(180 - \theta) = rmg(\sin(\theta). \tag{17}$$

The $\sin(\theta)$ factors cancel out. In our setup there are actually two masses, the plactic amss and the aluminum rod, and the two values for r being measured form the center of the ball to the center of mass of either the plastic mass or the aluminum rod. So, there are reall two rmg terms on the left hand side of the equation. Let r and m be the location and mass of the plastic mass, and R and M describe dthe center of mass and mass of the aluminum rod. Solvint for r gives us

$$r = \frac{\mu}{mg} B - \frac{MR}{m}. \tag{18}$$

This is the relationship that we will used to determine a value for μ.

C. Method 2 - Harmonic Oscillation

The net torque on an object causes a change in that object's angular momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt} \tag{19}$$

For our system if the cue ball is placed in the air bearing with a uniform magnetic field, and displaced an angle θ away from its equilibrium position,the ball will experince a net torque and will change its =angularmomentum. This is the same torque described in Eq(15) and will cause the cue ball to accelerate back toward the quilibrum position. As it rotates it will have an angular velocity of

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}. \tag{20}$$

The cue ball has an angular momentum

$$L = \mathscr{I}\vec{\omega} = \mathscr{I}\frac{d\vec{\theta}}{dt}, \tag{21}$$

where $\mathscr{I}$ is the moment of inertia of the cue ball. $\mathscr{I}$ is given by $\mathscr{I}_{\text{sphere}} = \frac{2}{5}MR^2$. We calculated $\mathscr{I}_{\text{sphere}} = 4.0503 \times 10^{-5} \pm 0.15 \times 10^{-5}[\text{kgm}^2]$. We can plug this into Eq.(19) to get

$$-\mu B \sin(\theta) = \mathscr{I}\frac{d^2\theta}{dt^2}. \tag{22}$$

The minus sign is because the torque is a restoring torque and θ can be positive or negative depending on which side the handle is displaced to. Now we use the small angle approximation we find that

$$\frac{d^2\theta}{dt^2} = -\frac{\mu B}{\mathscr{I}}\theta. \tag{23}$$

This is the equation of Simple Harmonic Motion $\theta(t) = A\cos(\omega_{\text{osc}} t)$, where $\omega_{\text{osc}} = 2\pi/T$ is the osculation frequency and is seen to be

$$\omega_{\text{osc}} = \frac{2\pi}{T} = \sqrt{\frac{\mu B}{\mathscr{I}}}, \tag{24}$$

where T is the period of the oscillatin motio. We can solve for T^2 and we get

$$T^2 = \frac{4\pi^2 \mathscr{I}}{\mu B}. \tag{25}$$

If period data is collected for different magnetic fields, we can plot a relationship between T^2 and $1/B$ which will have a linear relationship and the magnetic moment can be extracted from its slope.

D. Method 3 - Precession of a Spinning Sphere

This method is similar to method 2, but the sphere is placed in a uniforme magnetic field at some angleθ and is also spinning about its handle with and angular speed of ω_{handle}. The sphere has angular momentum $L = \mathscr{I}\omega_{\text{handle}}$ about the handle. Eq.19 is still true so this anglular momentum vector will change due to the net torque on the ball. Since $\vec{L}_{\text{handle}}$ and $\vec{\mu}$ are both pointing in the same direction the net torque will is perpendicular to the direction of the angular momentum about the handle. Only the horizontal component of the angular momentum will change, its magnitude wont change, but just its direction. This causes the cue ball to rotate round in a circle. It will rotate through a small angle $d\phi$ resulting in a perpendicular change in the horizontal component of the amount dL_{horiz}. This is the only change in angular momentum so i is equal to the change in angular momentum of the handle. Approximating this change as the arch length of a circle we get,

$$dL_{\text{handle}} = d\phi L_{\text{handle}} \sin(\theta). \tag{26}$$

By dividing both sides by dt, defining the precession angular frequency $\Omega_p = \frac{d\phi}{dt}$, and using Eq.(15) we get

$$\frac{dL_{\text{handle}}}{dt} = \Omega_p L_{\text{handle}} \sin(\theta) = \mu B \sin(\theta). \tag{27}$$

The sine terms cancel and we find

$$\Omega_p = \frac{\mu}{L_{\text{handle}}} B. \tag{28}$$

If the cue ball is spinning about the handle and is in a uniform magnetic field B, it will precess about the vertical axis at an angular frequency of Ω_p By measuring the percession frequency for different values of B, a graph of Ω_p as a function of B can be made, and from the slope of that graph we will be able to determine a value for μ.

E. Method 4 - Net Force in a A Magnetic Field Gradient

The magnetized disk inside of the cue ball can be modeled as a current loop with current I. When placed in a uniform magnetic field that is directed along the xis of the loop the force on an infinitesimal section $d\vec{l}$ of the loop is given by

$$d\vec{F} = I d\vec{l} \times \vec{B}. \tag{29}$$

The same amount of current passes through each infinitesimal section of the loop. Using the right hand rule we can see that the force $d\vec{F}$ on opposite sides for the loop will be equal and opposite so the net force on the loop will be zero. There is no net force on a current loop in a uniform magnetic field. But if there is a changing magnetic field then there will be a net force.

The resulting force on the current loop in the z direction is given by

$$F_z = \mu \frac{dB_z}{dz}. \tag{30}$$

F. Method 5 - $1/z^3$ Dependence

The Biot-Savart Law can be usedto calculate the axial magntic field of acurrent loop as a function of position along the axis. Assume the current loop is centered in the xy-plane with the magnetic moment pointing in the +z direction. Assume a single current loop of radius R carrying current I. Using the Biot-Savart Law to determine the magnetic field we find that the components in the xy-plane cancel our while the z components all point in the same direction resulting in a magnetic field in the z direction. The magnitude of which is

$$B_z(z) = \frac{\mu_0 I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}} \tag{31}$$

We are interested in value of z much larger than R. This allows us to simplify the expression. Factoring out z^2 we get

$$B_z(z) = \frac{\mu_0 I R^2}{2z^3} \left(1 + \frac{R^2}{z^2}\right)^{-3/2} \tag{32}$$

R/z is very small, so we can make the simplifying approximation to get

$$B_z(z) = \frac{\mu_0 I R^2}{2z^3}. \tag{33}$$

For a single current loop the magnetic moment is defined to be $\mu = IA$ where $A = \pi R^2$ is the area inclosed in the loop. Plugging this in we see that

$$B_z(z) = \frac{\mu_0 \mu}{2\pi} \frac{1}{z^3}. \tag{34}$$

By measuring the magnetic field due to a dipole at different value z along the z-axis, a graph of B as a function of $1/z^3$ would be linear and from the slope we μ can be determined.

III. EXPERIMENTAL DESIGN AND PROCEDURE

A. Method 1

For method one the settings on the apparatus we used were: a constant magnetic field, with direction up, and air bearing on.

We then measured all the physical properties of the ball, aluminum rod and sliding plastic mass.

We inserted the aluminum rod into the ball with the plastic mass on the rod. Then we set the ball onto the air bearing and turned on the apparatus.

With no current there is no magnetic field and the aluminum rod will rest at the bottom. We then adjusted the current until the aluminum tube is horizontal. The rod tends to oscillate when you adjust the current too quickly so we found it useful to steady the rod with you hand and then see if it stayed horizontal. We then recorded the current and position of the sliding mass.

We repeated this process 7 more times for a total of 8 different positions of the sliding mass.

B. Method 2

For method two the settings on the apparatus we used were: constant magnetic field, with direction up, and air bearing on.

We placed the cue ball onto the air bearing and set the current at 1 amp. The handle of the ball was pointing straight up. We then displaced the handle a small amount and released it. This caused the ball to oscillate. We then measured the time for the ball to oscillate through 20 periods.

We repeated this process for currents from 1-2 A in increments of 0.2 A, 2-3 A in increments of 0.25 A and 3-4 A in increments of 0.5 A, for a total of 12 trials at different currents.

C. Method 3

For this method the settings we used were: strobe light on, air bearing on, field direction up, and we flipped between a constant field and a field gradient. Field gradient for setting up the ball to get it spinning at a constant frequency, and a constant field to actually take data.

We set the strobe light frequency to 5 Hz. And turned off all of the lights in the room so only the strobe light illuminated the white dot. We also turn on the field gradient, because when the field gradient is on the magnitude of the magnetic field at the center is zero so we could get the ball spinning without it starting to process. We set the current of the apparatus to 1 A.

We spun the ball on the air bearing. we oriented it so that the handle faces the strobe. With our strobe set to 5 Hz I was able to get the ball spinning fast enough with just my fingers. When we used the wind up a string around the handle and insert a paper clip method that we heard other groups used, we found that we always spun the ball much too fast with that method and it would take a really long time for the ball to slow down to the frequency of our strobe light.

After spinning the ball it would be a bit off axis but we found it easy to fix by touching the aluminum rod from method 1 to the handle of the ball, and then the ball would line itself up with the rod and spin at a constant angle.

Once the ball was spinning at a constant angle and had slowed down to the frequency of our strobe light we turned off the field gradient and simultaneously started a stop watch and recorded the time for the ball to spin 2π radians.

We repeated this process with currents from 1 to 4 A in 0.5 A increments.

D. Method 4

For this method we used the apparatus settings: air bearing off, strobe light off, magnetic field gradient on, and we set up the Magnetic Force Balance on the apparatus.

To set up the Magnetic Force Balance we attached the ball to the balance beam by inserting the handle in to the black Delrin cup. We installed the V-groove bearing block on top of a brass magnet support. We checked that the ball was directly over the center of the air bearing, the balance beam is not rubbing against the support and the beam is approximately horizontal. This took some careful adjustments.

We balanced the unit without a field gradient. To do this we had to add a few brass ball weights to the counterweight. We used the level on my phone to make sure the balance beam was level. This was a little tricky because you couldn't rest the phone on the beam so we just kind of had to hold it behind the beam and try to line it up.

Before making any measurements with the apparatus we measured the mass of the brass balls we were going to use on the counter balance. There are 1/4" diameter balls and 3/16" diameter balls. We assumed that the bearing were manufacture well enough that they can be treated as interchangeable without affecting results. We weighted all the 1/4" balls together and divided the total weight by the number of balls to get the weight of one ball, and did the same for the 3/16" balls.

With the beam balanced and the apparatus off we used the adjustable height marker to located the position of the bottom of the counter weight. Then we added some balls to the counter weight and adjusted the current until the balance was leveled again. We recorded the current and the number of balls that we added.

E. Method 5

For this method we only used the apparatus to calibrate the hall effect probe. To do that we set the apparatus to a constant magnetic field, and all the other settings on it were off.

A Hall Effect probe gives an output voltage signal proportional to the magnitude of the magnetic field the sensor is located in. To calibrate the probe we put the Hall Effect probe in a known magnetic field, the field generated by our apparatus. We stuck the probe into the field from the top so that the sensor was at the center, where the ball was located for our other methods. Then we adjusted the current of the apparatus and recorded the voltage output of the probe. We measured currents ranging from 0-4 A in 0.5 A increments. When you graph the magnetic field as a function of output voltage you will get a linear relationship and the slope of that line is the conversion factor to get the magnetic field from the hall effect probe. Now when we measure an unknown magnetic field with the hall effect probe we just multiply the voltage reading by the slope of our calibration graph to get the magnitude of the magnetic field. When calibrating we had the the probe set to 10x and radial on the two switches on the probe.

To take data we placed the cue ball on a holder, a small piece of PVC piping, and tapped a piece of graph paper under it. We placed the hall effect probe as close to the ball as possible. The height of the hall effect probe is adjustable we positioned the height so that it was at the center of the ball. We marked the position of the hall effect probe and position of the ball on the graph paper. Then we rotated the ball on its holder until we were getting a maximum voltage output on the hall effect probe. Then we moved the ball away from the Hall Effect probe in increments of 1 square of graph paper which is about 0.5 cm. We marked the position of the ball at each movement and recorded the output voltage. When doing this process we had the probe set to 10x and trans on the two switches on the probe.

IV. CALCULATIONS AND ANALYSIS

A. Method 1

For method we made a graph of r, the distance from the center of mass of the plastic slider to the center of the ball, as a function of the magnitude of the magnetic field. This graph can be seen in Fig.1. Doing a linear regression of this graph and comparing it to Eq.(18) we can calulate a value of for μ and we calculated a value of 0.432 ± 0.004[Am^2]. This can also be seen in Table.I and compared to results from other methods in Table.II.

Looking at Eq.(18) we can calculate an expected y-intercept. We calculated an expected value of -0.0446[m] and our measured value was -0.0461 ± 0.0013[m]. Our measured value does not agree with the expected value. However, the lower bound of our measured value was only 0.0002[m] of agreeing with the expected. This error is probably due to poor execution of the experiment. It is hard to tell if the aluminum rod is perfectly horizontal you just eyeball if it is horizontal. So a better procedure could help fix this error.

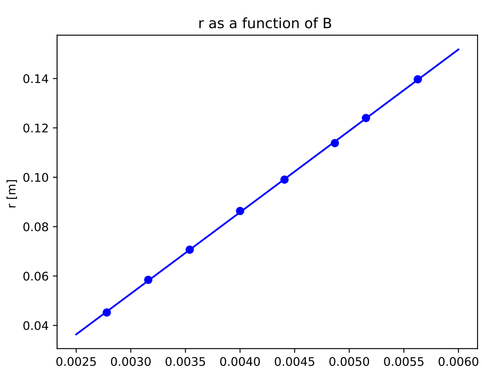

FIG. 1. r as a function of B.
Slope= 33.0 ± 0.3[m/T], Y-int= -0.4607 ± 0.0013[m]

B. Method 2

For method 2 we graphed a the period of oscillation squared as a function of the inverse of the magnitude of the magnetic field. This graph can be seen in Fig.2. Doing a linear regression on this graph and looking at Eq.(25) we can calculate a value of μ. We calculated $\mu = 0.418 \pm 0.008$[Am^2]. This is also shown in Table.I and compared to results from other methods in Table.II.

Looking at Eq.(25) we see that the expected y-intercept should be 0[1/T]. Our measured value was 0.02 ± 0.03[1/T]. Our measured value does agree with the theoretical value.

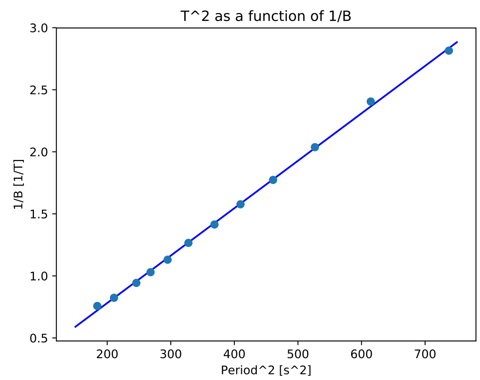

FIG. 2. T^2 as a function of 1/B.
Slope= 0.00382 ± 0.00007[1/Ts^2], Y-int= 0.02 ± 0.3[1/T]

C. Method 3

For method 3 we graphed the angular frequency of the ball as a function of the magnitude of the magnetic field divided by the angular momentum of the ball. This graph can be seen in Fig.3. Doing a linear regression of this graph and looking at Eq.(28), we see that the slope of this graph is μ. So for this method we got found $\mu = 0.39 \pm 0.06$[Am^2] This is also shown in Table.I and compared to results from other methods in Table.II.

Looking at Eq.(28) we see that our expected y-intercept is 0[Hz]. Our measured value is 0.05 ± 0.19[Hz]. Our measured value does agree with the expected value.

D. Method 4

For method 4 we made a graph of the force of gravity due to the added weights as a function of the magnitude of the magnetic field gradient. Looking at Eq.(30) we see that if we do a regression of this graph the slope will be μ. We calculated $\mu = 0.43 \pm 0.03$[Am^2]. This is also shown in Table.I and compared to results from other methods in Table.II.

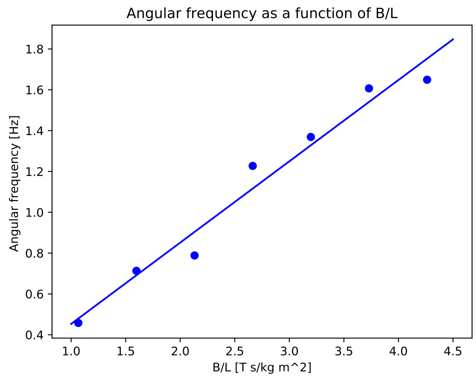

FIG. 3. Angular frequency as a function of B/L
Slope= 0.39 ± 0.06[Am^2], Y-int= 0.05 ± 0.19[Hz]

Eq.(30) tells us that the expected y-intercept is 0[N]. Our measured value is 0.0487 ± 0.0014[N]. This measured value does not agree with the expected y-intercept. This was probably due to poor execution of the experiment. It was really difficult to make sure the balance was perfectly level initially. A better procedure of setting up this experiment would could help fix this error.

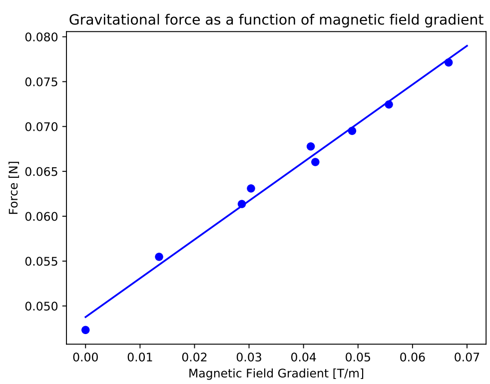

FIG. 4. Gravitational force as a function of magnetic field gradient. Slope= 0.43 ± 0.03[Am^2], Y-int= 0.0488 ± 0.0014[N]

E. Method 5

For method 5 we first calibrated the Hall Effect probe by measuring a known magnetic field. We made a graph of the known magnetic field as a function of the voltage output of the Hall Effect probe. This graph can be seen in Fig.5. The slope of this graph can be used as a conversion between the voltage output of the probe to what magnitude of magnetic field that corresponds too.

After we calibrated the probe we made a function of the magnitued of magnetic field as a function of the inverse cube of the distance between the probe and the magnetic moment. This graph can be seen in Fig.6. Looking at a linear regression of this graph and looking at Eq.(34) we can calculate μ. We calculated a value of $\mu = 0.408 \pm 0.019$[Am^2]. This can also be seen in Table.I and compared to results from other methods in Table.II.

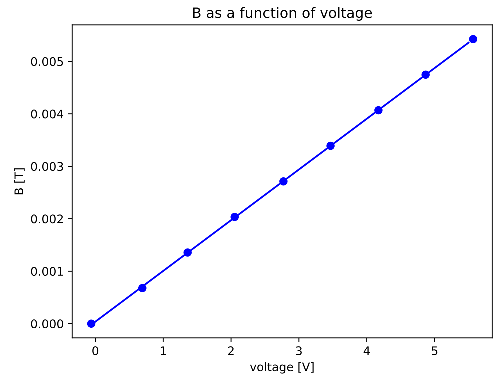

FIG. 5. B as a function of voltage.
Slope= 0.000967 ± 0.000005[T/V],
Y-int= 0.000037 ± 0.000017[T], This is graph used to calibrate the Hall Effect probe.

Looking at Eq.(34) we see that the expected y-intercept is 0[T]. Our measured y-intercept is $-4 \times 10^{-5} \pm 2 \times 10^{-5}$ [T]. This does not agree with the expected value. This could be due to difficulty in making measurements for this procedure. We didn't know exactly where the end of probe was. We were just told that it was about 1 centimeter from the physical edge of the probe. We marked a mark 1 centimeter from the edge of the probe to denote where the actual sensor was. It was also hard to project this onto the graph paper where we were actually making measurements. The actual probe was about 4 inches or so above the paper and we had to make a mark on graph paper under it. We used a level to try to make sure we made a mark perfectly vertically underneath the probe. But with a more refined process for taking the measurements this error could be fixed.

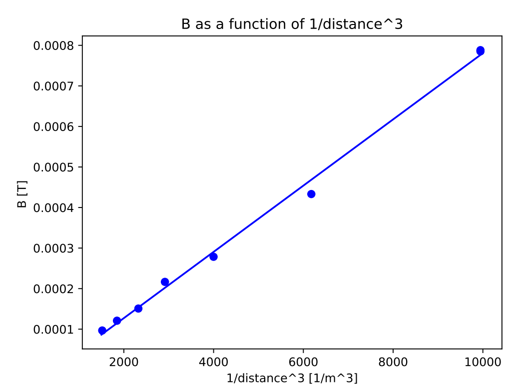

FIG. 6. B as a function of 1/distance^3.
Slope = $8.2 \times 10^{-8} \pm 0.4 \times 10^{-8}$ [T/m^3],
Y-int = $-4 \times 10^{-5} \pm 2 \times 10^{-5}$ [T]

F. Agreement between Methods

2 line summaries of each method can be seen in Table.I, and our agreement matrix between summaries can be seen in Table.II.

Taking an average of all methods gives us a value of $\mu = 0.42 \pm 0.03[\mathrm{Am}^2]$. All the individual methods agree with this value.

A weighted average of all the methods yields a value of $\mu = 0.428 \pm 0.003[\mathrm{Am}^2]$. This also agrees with all the other individual methods.

V. CONCLUSION

For this lab we used 5 different methods to calculate the magnetic moment of a permanent magnet. Method 1 was based on static equilibrium, method 2 used simple harmonic motion, method 3 was based on the precession of a spinning sphere, method 4 was based on the net force in a magnetic field gradient, and method 5 looked at the $1/z^3$ dependence of the magnetic field created by a magnetic dipole. Our results for each method can be seen in Table.I, and a comparison of the different methods can be seen in Table.II.

Most all of our methods agreed with each other. Method 1 did not agree with Method 2 or Method 5, but all other methods agreed. How ever our disagreement in the few cases was by a very small margin. Methods 1 and 2 were $0.002[\mathrm{Am}^2]$ from agreeing and Methods 1 and 5 were $0.001[\mathrm{Am}^2]$ from agreeing. This error was probably due to not performing the experiments perfectly. Some of the measurements were in some of the experiments were pretty tricky to measure. There was also significantly more uncertainty for some of the methods. This was also because of some tricky measurements.

Those tricky measurements were as follows. For method 1 it was difficult to determine if the rod was perfectly horizontal. There wasn't anything particularly tricky about method 2. As always when measuring periods it does take a little practice to get good at starting and stopping the stopwatch at the perfect moment. In method 3 it was hard to measure the period because you could only measure one period because the ball's angular velocity was slowly chaining, so that adds a lot of uncertainty. It was also difficult because the ball didn't only precess around its z-axis it also precessed a little bit around the axis of the handle making it difficult to tell when it got back to its initial position. Method 4 it was difficult to make sure the balance was perfectly level in the initial setup, because you couldn't set a level on top of the level without affecting the balance. For method 5 we it was hard to tell exactly where the sensor was inside of the probe. The lab handout said that it was about a centimeter from the edge of the probe but we couldn't measure that position ourselves because it was embedded in the probe. It was also hard to project the position of the probe and center of the cue ball onto the graph paper where we measured the change in positions. Improving on these parts of the procedure could lead to better results.

We also computed an average and weighted average for all 5 methods. We calculated $\mu_{\mathrm{avg}} = 0.42 \pm 0.03[\mathrm{Am}^2]$, and $\mu_{\mathrm{weighted\ avg}} = 0.428 \pm 0.003[\mathrm{Am}^2]$. I am most confident in our weighted average as the actual value for the magnetic moment of the magnet, because it averaged all our values together and weighted the methods with the smallest uncertainties the heaviest.

TABLE I. Two line summaries of calculated magnetic moments for each different method.

Method	Value $[\mathrm{Am}^2]$	Range $[\mathrm{Am}^2]$
Method 1	0.432 ± 0.004	$0.428 \to 0.436$
Method 2	0.418 ± 0.008	$0.410 \to 0.426$
Method 3	0.39 ± 0.06	$0.33 \to 0.45$
Method 4	0.43 ± 0.03	$0.40 \to 0.46$
Method 5	0.408 ± 0.019	$0.389 \to 0.427$

TABLE II. Agreement matrix between different methods.

	Method 1	Method 2	Method 3	Method 4	Method 5
Method 1	N/A	NO	YES	YES	NO
Method 2	NO	N/A	YES	YES	YES
Method 3	YES	YES	N/A	YES	YES
Method 4	YES	YES	YES	N/A	YES
Method 5	NO	YES	YES	YES	N/A