Nuclear Physics Term Paper
Big Bang Nucleosynthesis - The Deuterium Bottleneck

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The Deuterium Bottleneck is a time constraint on when Big Bang Nucleosynthesis can being. Forming Deuterium is the first step of Big Bang Nucleosynthesis, but the early universe had so many high energy photons that any Deuterium that was formed was broken by a photon as soon as it was made. The universe need to cool to a point so that deuterium could form and then the rest of Big Bang Nucleosynthesis could occur. In this paper I am going to go through how to calculate the temperature and time that the Deuterium Bottleneck ended, and explain some implications of the bottleneck.

I. INTRODUCTION

The early universe was a hot dense mass with temperatures high enough for fusion reactions to occur. Initially it was hot enough that there was a equilibrium between nucleon pair production and annihilation. The universe quickly cooled as it expanded and annihilation began to dominate. Soon the universe was mostly photons and a some left over matter that didn't annihilate because of an asymmetry between matter and antimatter. After this point the universe was hot enough that fusion reactions could occur, and larger nuclei could start to form.

The first step of Big Bang nucleosynthesis after proton-neutron freezout is complete is the fusion of protons and neutrons to form deuterium.

$$p + n \rightleftharpoons D + \gamma \tag{1}$$

The Q value of this fusion reaction is 2.22MeV. A photon with more energy than this can dissociate a deuteron back into a proton and neutron. The universe was still hot enough to disassociate deuterium, so the universe was at an equilibrium again like it was with nucleon pair production an annihilation in the earlier universe. The universe needed to expand and cool off before deuterium could form, and no heavier elements could be formed until deuterium formed. This is known as the Deuterium Bottleneck.

II. COMPLICATIONS

One would think that deuterium could start to form once the average energy of photons in the universe was less than the binding energy of deuterium. However, there are many more photons in the universe than particles, and they have a distribution of energies. So, even when the average energy of photons is lower than the binding energy of deuterium, there are still plenty of photons with enough energy to disassociate the deuterium as soon as it forms. The baryon-to-photon ratio η, basically the ratio of number protons and neutrons to the number photons is $\eta = 6.1 \times 10^{-11}$. This means that for every proton or neutron there are about 600 billion photons. So we need another way to determine the temperature at which the bottleneck ended other than the temperature at which the average photon energy was less than the binding energy of deuterium.

III. WHEN DID THE BOTTLENECK END

The way to solve when the deuterium bottleneck ended would be to compare the rate of production vs the rate of disassociation. We need to compare the number of deuterons in the universe to the number of neutrons in the universe.

The early universe had a lot of thermal energy, so the particles in the early universe had a very high velocity, but were traveling at non-relativistic speeds. The number density of particles can be described with a Maxwell-Boltzmann distribution of particle speeds. The total number density of a particle is:

$$n_x = g_x\left(\frac{m_x kT}{2\pi\hbar^2}\right)^{3/2}\exp\left(\frac{-m_x c^2+\mu_x}{kT}\right), \quad (2)$$

Where n_x is the number density of a particle, g_x is its statistical weight, m_x is its mass, k is Boltzmann's constant, T is the temperature of the universe, and μ_x is the particles chemical potential.

Using this equation and assuming $\mu_D = \mu_p + \mu_n$ we can get a equation relating the number densities of deuterium, protons and neutrons in the early universe. This equation is given by:

$$\frac{n_D}{n_p n_n} = \frac{g_D}{g_p g_n}\left(\frac{m_D}{m_p m_n}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2}\exp\left(\frac{Q}{kT}\right), \quad (3)$$

$g_p = g_n = 2$, and $g_D = 3$, and Q is the binding energy of deuterium. This equation can be simplified more by plugging in the respective values for g_p, g_n and g_D, and also by making the estimation that $m_p \approx m_n \approx m_d/2$. Doing this gives:

$$\frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{2\pi\hbar^2}\right)^{-3/2}\exp\left(\frac{Q}{kT}\right). \quad (4)$$

From here we are looking to find a temperature where $n_D/n_n = 1$. This would be when half of the free neutrons in the universe have been fused into deuterium. Eq.(4) can be rewritten as:

$$\frac{n_D}{n_n} = 6n_p\left(\frac{m_n kT}{2\pi\hbar^2}\right)^{-3/2}\exp\left(\frac{Q}{kT}\right). \quad (5)$$

Now we can rewrite n_p in terms of the baryon-to-photon ratio. We know that about 80% of baryons were in the form of unbound protons, and $\eta = n_{\text{bary}}/n_\gamma$, so we can write n_p as:

$$n_p \approx 0.8n_{\text{bary}} = 0.8\eta n_\gamma = 0.8\eta\left[0.2436\left(\frac{kT}{\hbar c}\right)^3\right], \quad (6)$$

where n_γ is:

$$n_\gamma = 0.2436\left(\frac{kT}{\hbar c}\right)^3, \quad (7)$$

which is the number density of photons for a blackbody, this applies because the early universe was a blackbody.

Substituting Eq.(6) into Eq.(5) we see that:

$$\frac{n_D}{n_n} \approx 6.5\eta\left(\frac{kT}{m_n c^2}\right)^{3/2}\exp\left(\frac{Q}{kT}\right). \quad (8)$$

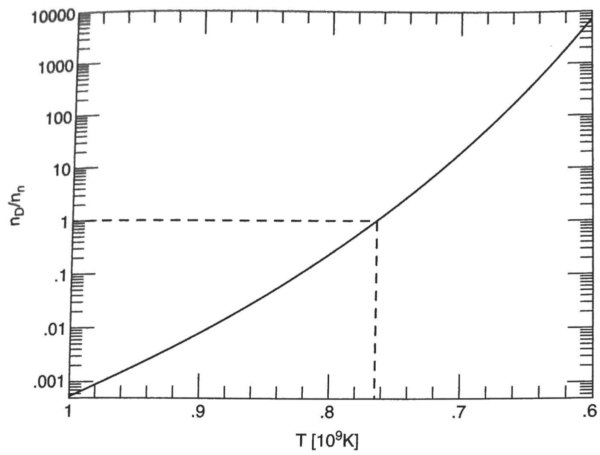

FIG. 1: Deuteron-to-neutron ratio plotted as a function of Temperature. Critical value of $n_D/n_n = 1$ marked with a dotted line. [2].

This is an equation that gives us the deuteron-to-neutron ratio as a function of the temperature of the universe. The point at which the deuterium bottleneck ended would be the point when $n_D/n_n = 1$. This ratio being 1 means that there are the same number of deuterons in the universe as their are neutrons, and once this ratio is greater than 1 then deuterons are being produced faster than they can be disassociated, and therefore Big Bang nucleosynthesis can continue to larger nuclei. Setting n_D/n_n to 1, and solving for T we find that $T = 7.6 \times 10^8$K, and this corresponds to a time of $t \approx 200$s or about 3 minutes after the big bang. This relation ship is graphed in FIG.1, and the critical value of $n_D/n_n = 1$ is shown with a dotted line. So, Big Bang nucleosynthesis couldn't really start until about 3 minutes after the big bang.

IV. IMPLICATIONS OF THE DEUTERIUM BOTTLENECK

The major implication of the Deuterium Bottleneck is that free neutrons decay, and while the universe is in the Deuterium Bottleneck all of the neutrons in the universe are free neutrons, so they are decaying. This effects the the ratio of protons and neutrons in the universe, the longer the bottleneck lasts the less neutrons their are in the universe, and if there are less neutrons then less helium will be made in Big Bang nucleosynthesis. The primordial abundance of Helium (Y) is the fraction of baryonic mass in the form of ^{4}He in the universe. The Deuterium Bottleneck causes the max value of Y to drop from $Y \approx 0.33$ to $Y \approx 0.27$. So the Deuterium Bottleneck puts a limit on the maximum amount of helium that can be created by Big Bang Nucleosynthesis.

V. CONCLUSION

The early universe was a very hot mess of fundamental particles. There was enough thermal energy for nuclear fusion to occur. But there was also enough energy in photons to disassociate composite nuclei. So the universe needed to expand and cool off enough so that larger nuclei could form and not be destroyed by high energy photons. The process of fusion in the early universe was called Big Bang Nucleosynthesis and the first step of this process is to form Deuterium Eq.(1). This reaction has a Q value of 2.22 MeV. So the universe needs to cool to a point that the majority of photons in the universe have less energy in this value. This time period is called the Deuterium Bottleneck. Calculating the time length of the Deuterium Bottleneck yields the result of the 200s, or about 3 minutes. So Big Bang Nucleosynthesis couldn't really start until 3 minutes after the big bang. This doesn't seem like a big deal considering how old the universe is. However, free neutrons decay, and before Big Bang Nucleosynthesis all neutrons are free neutrons. So, the amount of time the Deuterium Bottleneck lasts puts a limit of the number of neutrons in the universe, and this limits the amount of Helium created in Big Bang Nucleosynthesis.

[1] Lilley, John. *Nuclear Physics Principles and Applications.* Chichester, West Sussex, England, John Wiley & Sons Ltd, 2001

[2] Ryden, Barbara. *Introduction to Cosmology.* Pearson Education, Inc., 2003