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Step I find the Eigenvalues (λ)

$\det(A - \lambda I) = 0$

Where I is the identity matrix.

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{vmatrix}$$

We expand along the first row:

$$\det(A - \lambda I) = (4-\lambda)\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} - 8\begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} +$$

$$(-1)\begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix}.$$

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Compute each minor determinant;

Expanding

1. first minor:

$$\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} = (-9-\lambda)(5-\lambda) - (-2)(10) =$$

$$(-9-\lambda)(5-\lambda) + 20.$$

Expanding:

$$(-9-\lambda)(5-\lambda) = -45 + 9\lambda - 5\lambda + x^2 = x^2 + 4\lambda - 25.$$

So:

$$\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} = x^2 + 4\lambda - 45 + 20 = x^2 + 4\lambda - 25.$$

Second minor:

$$\begin{vmatrix} -2 & -2 \\ 0 & 5\lambda \end{vmatrix} = (-2)(5-\lambda) - (-2)(0) = -10 + 2\lambda.$$

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Third minor:

$$\begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix} = (-2)(10) - (0)(-9-\lambda) = -20.$$

Substitute back to the determinant.

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Substitute back into determinant:

$$\det(A-\lambda I) = (4-\lambda)(\lambda^2+4\lambda-25) - 8(-10+2\lambda) - (-20)$$

Simplify each term:

1. Expand $(4-\lambda)(\lambda^2+4\lambda-25)$:

$$(4-\lambda)(\lambda^2+4\lambda-25) = 4\lambda^2+16\lambda-100-\lambda^3-4\lambda^2+25\lambda = -\lambda^3+41\lambda-100.$$

2. Simplify $-8(-10+2\lambda)$:

$$-8(-10+2\lambda) = 80-16\lambda.$$

3. Add $-(-20)$:

$$-(-20) = +20$$

Combine all terms:

$$\det(A-\lambda I) = -\lambda^3+41\lambda-100+80-16\lambda+20$$

$$\det(A-\lambda I) = -\lambda^3+25\lambda.$$

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factorize

$\det(A-\lambda I) = -\lambda(\lambda^2-25)$.

$\det(A-\lambda I) = (\lambda-5)(\lambda+5)$

Eigenvalues;

$\lambda_1=0,\ \lambda_2=5,\ \lambda_3=-5$.

Step 2 find the Eigenvectors.

For $\lambda_2=5$:

$$A-5I = \begin{bmatrix} -1 & 8 & -1 \\ -2 & -14 & -2 \\ 0 & 10 & 0 \end{bmatrix}$$

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Row 1: $-1x + 8y - z = 0 \Rightarrow x = 8y - z$

Row 2: $-2x - 14y - 2z = 0 \Rightarrow x = -7y - z$

$8y - z = -7y - z \Rightarrow 15y = 0 \quad y = 0$

$x = -z$

let $z = t$, then:

$$V_2 = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Eigenvector for $\lambda_2 = 5$; $V_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Eigenvector for $\lambda_3 = -5$:

$$V_3 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

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Step 3. Importance of Eigenvalues in % form

Eigenvalues: $= \lambda_1 = 0, \lambda_2 = 5, \lambda_3 = -5$

Magnitudes $= |\lambda_1| = 0 \quad |\lambda_2| = 5, \quad |\lambda_3| = 5$

Sum:

$= 0 + 5 + 5 = 10.$

Importance of $\lambda_2 = \dfrac{|\lambda_2|}{\text{Total Magnitude}} \times 100\%$

$= \frac{5}{10} \times 100\% = 50\%$

Importance of $\lambda_3 = \frac{5}{10} \times 100\% = 50\%$

Final Answers:

$\lambda_2 = 5, \quad v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, Importance $= 50\%$

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Final Answers:

$\lambda_2 = 5$, $V_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, Importance $\lambda_2 = 50\%$

$\lambda_3 = -5$, $V_3 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$, Importance of λ_3 $= 50\%$