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Step 1 find the Eigenvalues (λ)

$\det(A - \lambda I) = 0$

where I is the identity matrix.

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{vmatrix}$$

We expand along the first row:

$$\det(A-\lambda I) = (4-\lambda)\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} - 8\begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} +$$

$$(-1)\begin{vmatrix} -2 & -9-2 \\ 0 & 10 \end{vmatrix}.$$

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Compute each minor determinant;

Expanding

1. first minor:

$$\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} = (-9-\lambda)(5-\lambda)-(-2)(10) =$$

$$(-9-\lambda)(5-\lambda)+20.$$

Expanding:

$$(-9-\lambda)(5-\lambda) = -45+9\lambda-5\lambda+\lambda^2 = \lambda^2+4\lambda-25.$$

So: $$\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} = \lambda^2+4\lambda-45+20 = \lambda^2+4\lambda-25.$$

Second minor:

$$\begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} = (-2)(5-\lambda)-(-2)(0) = -10+2\lambda.$$

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Third minor:

$$\begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix} = (-2)(10) - (0)(-9-\lambda) = -20.$$

Substitute back to the determinant.

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Substitute back into determinant:

$$\det(A-\lambda I) = (4-\lambda)(\lambda^2+4\lambda-25) - 8(-10+2\lambda) - (-20).$$

Simplify each term:

1. Expand $(4-\lambda)(\lambda^2+4\lambda-25)$:

$$(4-\lambda)(\lambda^2+4\lambda-25) = 4\lambda^2+16\lambda-100-\lambda^3-4\lambda^2+25\lambda = -\lambda^3+41\lambda-100.$$

2. Simplify $-8(-10+2\lambda)$:

$$-8(-10+2\lambda) = 80-16\lambda.$$

3. Add $-(-20)$:

$$-(-20) = +20$$

Combine all terms:

$$\det(A-\lambda I) = \lambda^3+41\lambda-100+80-16\lambda+20$$

$$\det(A-\lambda I) = -\lambda^3+25\lambda.$$

Date:

factorize

$\det(A-\lambda I) = -\lambda(\lambda^2 - 25).$

$\det(A-\lambda I) = (\lambda-5)(\lambda+5)$

Eigenvalues;

$\lambda_1 = 0, \ \lambda_2 = 5, \ \lambda_3 = -5.$

Step 2 find the Eigenvectors.

For $\lambda_2 = 5$:

$$A - 5I = \begin{bmatrix} -1 & 8 & -1 \\ -2 & -14 & -2 \\ 0 & 10 & 0 \end{bmatrix}$$

Eigen Vectors

Formula : $(A - \lambda I)v = 0$, where $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

V_2 for $\lambda_2 = 5$

$$\Rightarrow \left[\begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -1 & 8 & -1 \\ -2 & -14 & -2 \\ 0 & 10 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\Rightarrow \begin{cases} -x + 8y - z = 0 \\ -2x - 14y - 2z = 0 \\ 10y = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x = -z \\ 2z - 2z = 0 \end{cases}$$

⚠ since $x = -z$, the equation holds for any non-zero x and z,

so let take $x = 1$, $z = -1$, $V_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

V_3 , for $\lambda = -5$

$$\Rightarrow \left[\begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{cases} 9x + 8y - z = 0 \\ -2x - 4y - 2z = 0 \\ 10y + 10z = 0 \end{cases} \Rightarrow \begin{cases} y + z = 0, y = -z \\ -2x + 2z = 0 \\ x = z \end{cases}$$

since eq is always true let's assign $z = 1$

$$\Rightarrow V_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Importance of Eigen vectors in % form:

Eigen values : $\lambda_2 = 5$, $\lambda_3 = -5$

their Magnitudes <=> $|\lambda_2| = 5$

$|\lambda_3| = 5$

Sum = $5+5 = 10$

Importance of $\lambda_2 = \frac{|\lambda_2|}{\text{Tot Magnitude}} \times 100\% =$

$\Rightarrow \frac{5}{10} \times 100\% = \underline{\underline{50\%}}$

Importance of $\lambda_3 = \frac{5}{10} \times 100\% = \underline{\underline{50\%}}$

Final Answers

$\lambda_2 = 5$, $v_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, importance = $\underline{\underline{50\%}}$

$\lambda_3 = -5$, $v_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, importance = $\underline{\underline{50\%}}$