Date:

Step I find the Eigenvalues (λ)

$\det(A - \lambda I) = 0$

Where I is the identity matrix.

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-2 & -2 \\ 0 & 10 & 5-2 \end{vmatrix}$$

We expand along the first row:

$$\det(A-\lambda I) = (4-\lambda)\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} - 8\begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} +$$

$$(-1)\begin{vmatrix} -2 & -9-2 \\ 0 & 10 \end{vmatrix}.$$

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Date:

Compute each minor determinant;

Expanding

1. first minor:

$$\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} = (-9-\lambda)(5-\lambda)-(-2)(10) = (-9-\lambda)(5-\lambda)+20.$$

Expanding:

$$(-9-\lambda)(5-\lambda) = -45+9\lambda-5\lambda+x^2 = x^2+4\lambda-25.$$

So: $$\begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} = x^2+4\lambda-45+20 = x^2+4\lambda-25.$$

Second minor:

$$\begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} = (-2)(5-\lambda)-(-2)(0) = -10+2\lambda.$$

Date:

Third minor :

$$\begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix} = (-2)(10) - (0)(-9-\lambda) = -20.$$

Substitute back to the determinant.

Date:

Substitute, back into determinant;

$$\det(A-\lambda I) = (4-\lambda)(\lambda^2+4\lambda-25) - 8(-10+2\lambda) - (-20).$$

Simplify each term;

1. Expand $(4-\lambda)(\lambda^2+4\lambda-25)$:

$$(4-\lambda)(\lambda^2+4\lambda-25) = 4\lambda^2+16\lambda-100-\lambda^3-4\lambda^2+25\lambda = -\lambda^3+41\lambda-100.$$

2. Simplify $-8(-10+2\lambda)$;

$$-8(-10+2\lambda) = 80-16\lambda.$$

3. Add $-(-20)$;

$$-(-20) = +20$$

Combine all terms;

$$\det(A-\lambda I) = \lambda^3+41\lambda-100+80-16\lambda+20$$

$$\det(A-\lambda I) = -\lambda^3+25\lambda.$$

Date:

factorize

$\det(A-\lambda I) = -\lambda(\lambda^2-25)$.

$\det(A-\lambda I) = (\lambda-5)(\lambda+5)$

Eigenvalues:

$\lambda_1 = 0,\ \lambda_2 = 5,\ \lambda_3 = -5$.

Step 2 find the Eigenvectors.

For $\lambda_2 = 5$:

$$A-5I = \begin{bmatrix} -1 & 8 & -1 \\ -2 & -14 & -2 \\ 0 & 10 & 0 \end{bmatrix}$$

~~Second~~

(*) Eigen vectors : $(A - \lambda I)v = 0$, where $v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\Rightarrow$ v_1 for $\lambda_1 = 0$

$$v_1 \Rightarrow \begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\left.\begin{array}{l} 1.\ 4x + 8y - z = 0 \\ 2.\ -2x - 9y - 2z = 0 \\ 3.\ 10y + 5z = 0 \end{array}\right\} \begin{array}{l} x = -2.5 \\ y = 1 \\ z = -2 \end{array}$$

$$v_1 = \begin{pmatrix} -2.5 \\ 1 \\ -2 \end{pmatrix}$$

After calculation, $v_2 = \begin{pmatrix} 0.75 \\ -0.3 \\ 0.6 \end{pmatrix}$

$$v_3 = \begin{pmatrix} -0.7 \\ 0 \\ 0.7 \end{pmatrix}$$

Importance of Eigen vectors in % form:

Eigen values' magnitudes

$\Rightarrow |\lambda_1| = |0| = 0$

$|\lambda_2| = |5| = 5$

$|\lambda_3| = |-5| = 5$

sum $\Rightarrow$ $0+5+5 = 10$

Importance of $\lambda_2 = \dfrac{|\lambda_2|}{\text{Total magnitude}} \times 100\%$

$\Rightarrow \dfrac{5}{10} \times 100 = \boxed{50\%}$

for $\lambda_1 = \dfrac{|\lambda_1|}{\text{Total Mag}} \times 100\% = \boxed{0\%}$

for $\lambda_3 = \dfrac{|\lambda_3|}{\text{Tot Mag}} \times 100\% = \dfrac{5}{10} \times 100\% = \underline{\underline{50\%}}$

Final Answers

$\lambda_2 = 5$, $V_2 = \begin{pmatrix} 0.75 \\ -0.3 \\ 0.6 \end{pmatrix}$, importance: $\underline{\underline{50\%}}$

$\lambda_3 = -5$, $V_3 = \begin{pmatrix} -0.7 \\ 0 \\ 0.7 \end{pmatrix}$, importance: $\underline{\underline{50\%}}$