

## Homework assignment

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Due on 29 June 2016

Collected in the classroom at the end of the class. For Problems 1 to 7, typing with a computer is desirable. If you handwrite, use pens. No pencils allowed. For problem (8), you can either email me or hand in a printed copy.

### Solow model with a Cobb–Douglas production function

Let  $F(K, AL) = K^\alpha (AL)^{1-\alpha}$ .

- (1) Compute the intensive form  $f(k) = F(k, 1)$ .
- (2) Prove that  $f$  has the following properties.
  - a.  $f(0) = 0$ ,
  - b.  $f'(k) > 0$ ,
  - c.  $f''(k) < 0$ ,
  - d.  $\lim_{k \rightarrow 0} f'(k) = +\infty$ ,
  - e.  $\lim_{k \rightarrow +\infty} f'(k) = 0$ .
- (3) Solve the steady state equation,  $sf(k^*) - (\delta + g + n)k^* = 0$ , for  $k^* > 0$ .
- (4) Similarly, find expressions for  $y^*$  and  $c^*$  in terms of  $s, \delta, g, n$  and  $\alpha$ , where  $y^*$  and  $c^*$  are the steady state levels of output and consumption per unit of effective labor, respectively.
- (5) Compute the golden rule values of  $k^*, y^*$  and  $c^*$ . [Hint: find  $s$  that maximizes  $c^* = c^*(s; \alpha, \delta, g, n)$  and plug that value into the solution of  $k^*, y^*$  and  $c^*$  obtained in (3) and (4).]
- (6) Let  $s_G$  be the golden-rule saving rate and  $s_1 < s_G$  the economy's saving rate given  $\alpha, \delta, g$  and  $n$ . Suppose that the economy is on the balanced growth path  $k^*(s_1)$  from  $t = 0$  to 1. At  $t = 1$ , the economy experiences a permanent increase of the saving rate to  $s'_1 < s_G$ . Draw rough sketches of the dynamics of  $K, C, Y$  and  $K/L, C/L, Y/L$ .
- (7) Do (6) again but, here, let the initial saving rate be  $s_2 > s_G$  and suppose that there is a **decrease**  $s_2 \rightarrow s'_2 > s_G$  instead of an increase.
- (8) You'll get a bonus of several points for simulating the dynamics for (6) and (7) by computer programs. Attach your figures and codes (in any programming language).