Inverted Gamma Distribution

$$f(x) = (x^{-a-1}e^{-1/(xb)})/(\Gamma(a)b^a)$$
 $x, a, b > 0$

| | General | | | | | | E | Examp | le: Inver | tedGamma(2 | ,3) | |
|----------------------------------|----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------------------|--------------|-------------|--------------------------------|-------------|
| Transformation | PDF | PDF | CDF | $_{ m HF}$ | IDF | μ | σ^2 | $\overline{\mathrm{MF}}$ | MGF | HF Shape | Support | Comment |
| x^2 | √ | √ | √ | √ | ✓ | ∞ | √ | √ | ✓ | DFR | $0, \infty$ | |
| \sqrt{x} | ✓ | ✓ | \checkmark | \checkmark | | \checkmark | \checkmark | \checkmark | \checkmark | $_{ m UBT}$ | $0, \infty$ | |
| x^{-1} | ✓ | ✓ | \checkmark | \checkmark | \checkmark | \checkmark | \checkmark | \checkmark | ∂ | $_{ m IFR}$ | $0, \infty$ | |
| $\arctan(x)$ | ✓ | ✓ | \checkmark | \checkmark | | ∂ | ∂ | ∂ | ∂ | $_{ m IFR}$ | $0,\pi/2$ | HF has peak |
| e^x | ✓ | \checkmark | \checkmark | \checkmark | \checkmark | ∞ | \checkmark | ∞ | ∂ | $_{ m UBT}$ | $1, \infty$ | |
| ln(x) | ✓ | ✓ | \checkmark | \checkmark | ∂ | ∂ | ∂ | ∂ | ∂ | | $-\infty,\infty$ | |
| e^{-x} | ✓ | ✓ | \checkmark | \checkmark | \checkmark | \checkmark | \checkmark | \checkmark | ∂ | $_{ m IFR}$ | 0, 1 | |
| $-\ln(x)$ | ✓ | \checkmark | \checkmark | \checkmark | ∂ | ∂ | ∂ | ∂ | ∂ | | $-\infty, \infty$ | |
| $\ln(x+1)$ | ✓ | \checkmark | \checkmark | \checkmark | \checkmark | ∂ | ∂ | ∂ | ∂ | $_{ m UBT}$ | $0, \infty$ | |
| $1/\ln(x+2)$ | ✓ | \checkmark | \checkmark | | \checkmark | | | | | | $0, 1/\ln(2)$ | |
| tanh(x) | ✓ | \checkmark | ∂ | ∂ | | ∂ | ∂ | ∂ | ∂ | $_{ m IFR}$ | 0, 1 | HF has peak |
| $\sinh(x)$ | ✓ | ✓ | ∂ | \checkmark | | ∞ | \checkmark | ∞ | ∂ | $_{ m UBT}$ | $0, \infty$ | |
| $\operatorname{arcsinh}(x)$ | ✓ | \checkmark | \checkmark | \checkmark | | ∂ | ∂ | ∂ | ∂ | | $0, \infty$ | |
| $\operatorname{csch}(x+1)$ | ✓ | ✓ | ∂ | ∂ | | ∂ | ∂ | ∂ | ∂ | | $0,2/(-e+e^{-1})$ | |
| $\operatorname{arccsch}(x+1)$ | ✓ | \checkmark | \checkmark | \checkmark | ∂ | ∂ | ∂ | ∂ | ∂ | | $0, \ln(1+\sqrt{2})$ | |
| $1/\tanh(x+1)$ | ✓ | \checkmark | ∂ | ∂ | | | | | | | $1, (e + e^{-1})/(e - e^{-1})$ | |
| $1/\sinh(x+1)$ | ✓ | \checkmark | ∂ | | | | | | | | $2,2/(e-e^{-1})$ | |
| $1/\operatorname{arcsinh}(x+1)$ | ✓ | ✓ | \checkmark | \checkmark | ∂ | \checkmark | ∂ | ∂ | ∂ | IFR | $0, 1/\ln(1+\sqrt{2})$ | |
| $1/\operatorname{csch}(x) + 1$ | ✓ | ✓ | ∂ | ∂ | | ∞ | \checkmark | ∞ | ∂ | UBT | $1,\infty$ | |
| $\tanh(x^{-1})$ | ✓ | ✓ | \checkmark | \checkmark | ∂ | ∂ | ∂ | ∂ | ∂ | $_{ m IFR}$ | 0, 1 | |
| $\operatorname{csch}(x^{-1})$ | ✓ | ✓ | ∂ | ∂ | ∂ | ∂ | ∂ | ∂ | ∂ | DFR | $1, \infty$ | |
| $\operatorname{arccsch}(x^{-1})$ | ✓ | ✓ | ✓ | ✓ | | ∂ | ∂ | ∂ | ∂ | | $0, \infty$ | |

Legend

| Symbol | Meaning |
|------------|-------------------------|
| √ | Exists, Closed Form |
| ∂ | Exists, Not Closed Form |
| Ø | Not Possible |
| | Not Calculated |