## Beta Distribution

$$f(x) = \frac{\Gamma(1+b)x^{a-1}(1-x)^{b-1}}{\Gamma(a)\Gamma(b)} \qquad 0 < x < 1 \quad a,b > 0$$

| Transformation | General PDF | PDF | CDF | HF | IDF | $\mu$ | $\sigma^2$ | MF | MGF | HF Shape | Support | Comment |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | Example: Beta(2,3) | | | |
| $x^2$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | ✓ | ✓ | IFR | $0,1$ | |
| $\sqrt{x}$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | ✓ | ✓ | IFR | $0,1$ | |
| $x^{-1}$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | $\infty$ | $\partial$ | $\partial$ | UBT | $0,1$ | |
| $\arctan(x)$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | $\partial$ | $\partial$ | IFR | $1,\pi/4$ | |
| $\mathrm{e}^{x}$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | ✓ | $\partial$ | IFR | $1,\mathrm{e}$ | |
| $\ln(x)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | $\partial$ | $\partial$ | IFR | $-\infty,0$ | |
| $\mathrm{e}^{-x}$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | ✓ | $\partial$ | IFR | $0,1$ | |
| $-\ln(x)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | ✓ | $\partial$ | IFR | $-\infty,0$ | |
| $\ln(x+1)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | $\partial$ | ✓ | IFR | $0,\ln(2)$ | |
| $1/\ln(x+2)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | $\partial$ | $\partial$ | IFR | $1/\ln(2),1/\ln(3)$ | |
| $\tanh(x)$ | ✓ | ✓ | ✓ | ✓ | ✓ | $\partial$ | $\partial$ | $\partial$ | $\partial$ | IFR | $1,\tanh(1)$ | |
| $\sinh(x)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | $\partial$ | $\partial$ | IFR | $1,\sinh(1)$ | |
| $\operatorname{arcsinh}(x)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | $\partial$ | $\partial$ | IFR | $0,-\ln(\sqrt{2}-1)$ | |
| $\operatorname{csch}(x+1)$ | ✓ | ✓ | $\partial$ | $\partial$ | | $\partial$ | $\partial$ | $\partial$ | $\partial$ | IFR | $-2/(\mathrm{e}^{-2}-\mathrm{e}^{2}),2/(\mathrm{e}-\mathrm{e}^{-1})$ | |
| $\operatorname{arccsch}(x+1)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | | | | | IFR | $\ln(2)-\ln(\sqrt{5}-1),\ln(1+\sqrt{2})$ | |
| $1/\tanh(x+1)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | $\partial$ | $\partial$ | $\partial$ | $\partial$ | IFR | $(-\mathrm{e}^{-2}-\mathrm{e}^{2})/(\mathrm{e}^{-2}-\mathrm{e}^{2}),(\mathrm{e}+\mathrm{e}^{-1})/(\mathrm{e}-\mathrm{e}^{-1})$ | |
| $1/\sinh(x+1)$ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | $\partial$ | $\partial$ | | $-2/(\mathrm{e}^{-2}-\mathrm{e}^{2}),2/(\mathrm{e}-\mathrm{e}^{-1})$ | |
| $1/\operatorname{arcsinh}(x+1)$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | ✓ | ✓ | $\partial$ | $\partial$ | UBT | $-1/\ln(\sqrt{5}-2),1/\ln(1+\sqrt{2})$ | |
| $1/\operatorname{csch}(x)+1$ | ✓ | ✓ | $\partial$ | $\partial$ | | $\partial$ | $\partial$ | $\partial$ | $\partial$ | | $1,-1/2\mathrm{e}^{-1}+1/2\mathrm{e}+1$ | |
| $\tanh(x^{-1})$ | ✓ | ✓ | ✓ | ✓ | $\partial$ | $\partial$ | $\partial$ | $\partial$ | $\partial$ | IFR | $(\mathrm{e}-\mathrm{e}^{-1})/(\mathrm{e}+\mathrm{e}^{-1}),1$ | |
| $\operatorname{csch}(x^{-1})$ | ✓ | ✓ | $\partial$ | $\partial$ | | $\partial$ | $\partial$ | $\partial$ | $\partial$ | | $0,2/(\mathrm{e}-\mathrm{e}^{-1})$ | |
| $\operatorname{arccsch}(x^{-1})$ | ✓ | ✓ | $\partial$ | $\partial$ | | $\partial$ | $\partial$ | $\partial$ | $\partial$ | IFR | $0,\ln(1+\sqrt{2})$ | |

### Legend

| Symbol | Meaning |
|---|---|
| ✓ | Exists, Closed Form |
| $\partial$ | Exists, Not Closed Form |
| $\varnothing$ | Not Possible |
| | Not Calculated |