```
> restart;
  read("c:/appl/appl7.txt");
```

PROCEDURES:

AllPermutations(n), AllCombinations(n, k), Benford(X), BootstrapRV(Data), CDF:CHF:HF:IDF:PDF:SF(X, [x])), CoefOfVar(X), Convolution(X, Y), ConvolutionIID(X, n), CriticalPoint(X, prob), Determinant(MATRIX), Difference(X, Y), Display(X), ExpectedValue(X, [g]), KSTest(X, Data, Parameters), Kurtosis(X), Maximum(X, Y), MaximumIID(X, n), Mean(X), MGF(X), Minimum(X, Y), MinimumIID(X, n), Mixture(MixParameters, MixRVs), MLE(X, Data, Parameters, [Rightcensor]), MLENHPP(X, Data, Parameters, obstime), MLEWeibull(Data, [Rightcensor]), MOM(X, Data, Parameters), NextCombination(Previous, size), NextPermutation(Previous), OrderStat(X, n, r, ["wo"]), PlotDist(X, [low], [high]), PlotEmpCDF(Data, [low], [high]), PlotEmpCIF(Data, [low], [high]), PlotEmpSF(Data, Censor), PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]), PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]), PlotEmpVsFittedSF(X, Data, Parameters, Censor, low, high), PPPlot(X, Data, Parameters), Product(X, Y), ProductIID(X, n), QQPlot(X, Data, Parameters), RangeStat(X, n, ["wo"]), Skewness(X), Transform(X, g), Truncate(X, low, high), Variance(X), VerifyPDF(X)

Procedure Notation:
X and Y are random variables
Greek letters are numeric or symbolic parameters
x is numeric or symbolic
n and r are positive integers, n >= r
low and high are numeric
g is a function
Brackets [] denote optional parameters
"double quotes" denote character strings
MATRIX is a 2 x 2 array of random variables
A capitalized parameter indicates that it must be
entered as a list --> ex. Data := [1, 12.4, 34, 52.45, 63]

Variate Generation:

ArcTanVariate(alpha, phi), BinomialVariate(n, p, m), ExponentialVariate(lambda), NormalVariate(mu, sigma), UniformVariate(), WeibullVariate(lambda, kappa, m)

DATA SETS:

BallBearing, HorseKickFatalities, Hurricane, MP6, RatControl, RatTreatment, USSHalfBeak

ArcSinRV(), ArcTanRV(alpha, phi), BetaRV(alpha, beta), CauchyRV(a, alpha), ChiRV(n), *ChiSquareRV(n), ErlangRV(lambda, n), ErrorRV(mu, alpha, d), ExponentialRV(lambda), ExponentialPowerRV(lambda, kappa), ExtremeValueRV(alpha, beta), FRV(n1, n2), GammaRV(lambda, kappa), GeneralizedParetoRV(gamma, delta, kappa), GompertzRV(delta, kappa), HyperbolicSecantRV(), HyperExponentialRV(p, l), HypoExponentialRV(l), IDBRV(gamma, delta, kappa), InverseGaussianRV(lambda, mu), InvertedGammaRV(alpha, beta), KSRV(n), LaPlaceRV(omega, theta), LogGammaRV(alpha, beta), LogisticRV(kappa, lambda), LogLogisticRV(lambda, kappa), LogNormalRV(mu, sigma), LomaxRV(kappa, lambda), MakehamRV(gamma, delta, kappa), MuthRV(kappa), NormalRV(mu, sigma), ParetoRV(lambda, kappa), RayleighRV(lambda), StandardCauchyRV(), StandardNormalRV(), StandardTriangularRV(m), StandardUniformRV(), TRV(n), TriangularRV(a, m, b), UniformRV(a, b), WeibullRV(lambda, kappa)*

```
Error, attempting to assign to `DataSets` which is protected.
Try declaring `local DataSets`; see ?protect for details.
```

```
> bf := LogNormalRV(1, 2);
  bfname := "LogNormalRV(1, 2)";
```

$$bf := \left[\left[x \to \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$bfname := \text{"LogNormalRV(1, 2)"} \qquad (1)$$

```
> #plot(1/csch(t)+1, t = 0..0.0010);
  #plot(diff(1/csch(t),t), t=0..0.0010);
  #limit(1/csch(t), t=0);
> solve(exp(-t) = y, t);
```

$$-\ln(y) \qquad (2)$$

```
> # discarded  -ln(t + 1),  t-> csch(t),t->arccsch(t),t -> tan(t),
> #name of the file for latex output
  filename := "C:/Latex_Output_2/LogNormal.tex";


  glist := [t -> t^2 , t -> sqrt(t),  t -> 1/t,  t -> arctan(t),  t
  -> exp(t), t -> ln(t), t -> exp(-t), t -> -ln(t),  t -> ln(t+1),
  t -> 1/(ln(t+2)), t -> tanh(t), t -> sinh(t), t -> arcsinh(t),
  t-> csch(t+1),t->arccsch(t+1), t-> 1/tanh(t+1), t->  1/sinh(t+1),
   t->  1/arcsinh(t+1), t->  1/csch(t)+1, t-> tanh(1/t), t->csch
  (1/t), t-> arccsch(1/t), t-> arctanh(1/t)  ]:

  base := t -> PDF(bf, t):

  print(base(x)):

  #begin latex file formatting
  appendto(filename);
    printf("\\documentclass[12pt]{article} \n");
    printf("\\usepackage{amsfonts} \n");
```

```
  printf("\\begin{document} \n");
  print(bfname);
  printf("$$");
  latex(bf[1]);
  printf("$$");
writeto(terminal);

#begin loopint through transformations
for i from 1 to 13 do
#for i from 1 to 3 do
   print( "i is", i, "  ----------------------------------------
-----------------------------------------------------------------
-------------");

   g := glist[i]:
   l := bf[2][1];
   u := bf[2][2];
   Temp := Transform(bf, [[unapply(g(x), x)],[l,u]]);

  #terminal output
  print( "l and u", l, u );
  print("g(x)", g(x), "base", base(x),bfname);
  print("f(x)", PDF(Temp, x));
  print("F(x)", CDF(Temp, x));
  if i=9 then print("IDF did not work") elif i=10 then print("IDF
did not work") elif i=11 then print("IDF did not work") else
print("IDF(x)", IDF(Temp)) end if;
  print("S(x)", SF(Temp, x));
  print("h(x)", HF(Temp, x));
  print("mean and variance", Mean(Temp), Variance(Temp));
  assume(r > 0); mf := int(x^r*PDF(Temp, x), x = Temp[2][1] ..
Temp[2][2]):
  print("MF", mf);
  if i=13 then print("MGF did not work") else print("MGF", MGF
(Temp)) end if;
  #PlotDist(PDF(Temp), 0, 40);
  #PlotDist(HF(Temp), 0, 40);
  latex(PDF(Temp,x));
  #print("transforming with", [[x->g(x)],[0,infinity]]);
  #X2 := Transform(bf, [[x->g(x)],[0,infinity]]);
  #print("pdf of X2 = ", PDF(X2,x));
  #print("pdf of Temp = ", PDF(Temp,x));


  #latex output
  appendto(filename);
  printf("-----------------------------------------------------------
------------------------------------  \\\\");
  printf("$$");
  latex(glist[i]);
  printf("$$");
  printf("Probability Distribution Function \n$$  f(x)=");
  latex(PDF(Temp,x));
  printf("$$");
  printf("Cumulative Distribution Function  \n $$F(x)=");
  latex(CDF(Temp,x));
  printf("$$");
```

```
  printf(" Inverse Cumulative Distribution Function \n ");
  printf(" $$F^{-1} = ");
  if i=9 then print("Unable to find IDF") elif i=10 then print
("Unable to find IDF") elif i=11 then print("Unable to find IDF")
else latex(IDF(Temp)[1]) end if;
  printf("$$");
  printf("Survivor Function \n $$ S(x)=");
  latex(SF(Temp, x));
  printf("$$ Hazard Function \n $$ h(x)=");
  latex(HF(Temp,x));
  printf("$$");
  printf("Mean \n $$ \mu=");
  latex(Mean(Temp));
  printf("$$ Variance \n $$ \sigma^2 = ");
  latex(Variance(Temp));
  printf("$$");
  printf("Moment Function \n $$ m(x) = ");
  latex(mf);
  printf("$$ Moment Generating Function \n $$");
  if i=13 then print("Unable to find MGF") else latex(MGF(Temp)
[1]) end if;
  printf("$$");
  #latex(MGF(Temp)[1]);

  writeto(terminal);


od;

#begin loopint through transformations
for i from 16 to 22 do
#for i from 1 to 3 do
   print( "i is", i, "  --------------------------------------
-----------------------------------------------------------------
-------------");

   g := glist[i]:
   l := bf[2][1];
   u := bf[2][2];
   Temp := Transform(bf, [[unapply(g(x), x)],[l,u]]);

  #terminal output
  print( "l and u", l, u );
  print("g(x)", g(x), "base", base(x),bfname);
  print("f(x)", PDF(Temp, x));
  print("F(x)", CDF(Temp, x));
  #if i=9 then print("IDF did not work") elif i=10 then print
("IDF did not work") elif i=11 then print("IDF did not work")
else print("IDF(x)", IDF(Temp)) end if;
  print("S(x)", SF(Temp, x));
  print("h(x)", HF(Temp, x));
  print("mean and variance", Mean(Temp), Variance(Temp));
  assume(r > 0); mf := int(x^r*PDF(Temp, x), x = Temp[2][1] ..
Temp[2][2]):
  print("MF", mf);
  if i=13 then print("MGF did not work") else print("MGF", MGF
(Temp)) end if;
```

```
  #PlotDist(PDF(Temp), 0, 40);
  #PlotDist(HF(Temp), 0, 40);
  latex(PDF(Temp,x));
  #print("transforming with", [[x->g(x)],[0,infinity]]);
  #X2 := Transform(bf, [[x->g(x)],[0,infinity]]);
  #print("pdf of X2 = ", PDF(X2,x));
  #print("pdf of Temp = ", PDF(Temp,x));


  #latex output
  appendto(filename);
  printf("-------------------------------------------------------
------------------------------------  \\\\");
  printf("$$");
  latex(glist[i]);
  printf("$$");
  printf("Probability Distribution Function \n$$  f(x)=");
  latex(PDF(Temp,x));
  printf("$$");
  printf("Cumulative Distribution Function  \n $$F(x)=");
  latex(CDF(Temp,x));
  printf("$$");
  printf(" Inverse Cumulative Distribution Function \n ");
  printf(" $$F^{-1} = ");
  #if i=9 then print("Unable to find IDF") elif i=10 then print
("Unable to find IDF") elif i=11 then print("Unable to find IDF")
else latex(IDF(Temp)[1]) end if;
  printf("$$");
  printf("Survivor Function \n $$ S(x)=");
  latex(SF(Temp, x));
  printf("$$ Hazard Function \n $$ h(x)=");
  latex(HF(Temp,x));
  printf("$$");
  printf("Mean \n $$ \mu=");
  latex(Mean(Temp));
  printf("$$ Variance \n $$ \sigma^2 = ");
  latex(Variance(Temp));
  printf("$$");
  printf("Moment Function \n $$ m(x) = ");
  latex(mf);
  printf("$$ Moment Generating Function \n $$");
  if i=13 then print("Unable to find MGF") else latex(MGF(Temp)
[1]) end if;
  printf("$$");
  #latex(MGF(Temp)[1]);

  writeto(terminal);


od;

#final latex output
appendto(filename);
printf("\\end{document}\n");
writeto(terminal);
```

$$filename := \text{"C:/Latex\_Output\_2/LogNormal.tex"}$$

$$\frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(x)-1)^2}}{\sqrt{\pi}\,x}$$

"i is", 1,
" ---
----------"

$$g := t \to t^2$$
$$l := 0$$
$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{8}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{32}\,(\ln(y\sim)-2)^2}}{\sqrt{\pi}\,y\sim}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, x^2, \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{8}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{32}\,(\ln(x)-2)^2}}{\sqrt{\pi}\,x}$$

$$\text{"F(x)"}, \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{8}\,\sqrt{2}\,(\ln(x)-2)\right)$$

$$\text{"IDF(x)"}, \left[\left[s \to \mathrm{e}^{2+4\sqrt{2}\,RootOf(-\mathrm{erf}(\_Z)-1+2s)}\right], [0, 1], [\text{"Continuous"}, \text{"IDF"}]\right]$$

$$\text{"S(x)"}, \frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{8}\,\sqrt{2}\,(\ln(x)-2)\right)$$

$$\text{"h(x)"}, -\frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{32}\,(\ln(x)-2)^2}}{\sqrt{\pi}\,x\left(-1+\mathrm{erf}\left(\frac{1}{8}\,\sqrt{2}\,(\ln(x)-2)\right)\right)}$$

$$\text{"mean and variance"}, \mathrm{e}^{10}, \mathrm{e}^{36} - \mathrm{e}^{20}$$

$$mf := \mathrm{e}^{8r\sim^2+2r\sim}$$

$$\text{"MF"}, \mathrm{e}^{8r\sim^2+2r\sim}$$

$$\text{"MGF"}, \int_0^\infty \frac{1}{8}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{32}\,\ln(x)^2+t\,x}}{\sqrt{\pi}\,x^{7/8}}\,dx$$

```
1/8\,{\frac {\sqrt {2}{{\rm e}^{-1/32\, \left( \ln  \left( x
\right) -
2 \right) ^{2}}}}{\sqrt {\pi}x}}
```

"i is", 2,
" ---

----------"

$$g := t \to \sqrt{t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{2}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(y\sim^2\right)-1\right)^2}}{\sqrt{\pi}\,y\sim}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \sqrt{x}, \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln(x)-1\right)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{2}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(x^2\right)-1\right)^2}}{\sqrt{\pi}\,x}$$

$$\text{"F(x)"}, \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(2\ln(x)-1)\right)$$

$$\text{"IDF(x)"}, \left[\left[s \to \mathrm{e}^{\frac{1}{2} + \sqrt{2}\,RootOf(-\mathrm{erf}(\_Z) - 1 + 2s)}\right], [0, 1], [\text{"Continuous"}, \text{"IDF"}]\right]$$

$$\text{"S(x)"}, \frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(2\ln(x)-1)\right)$$

$$\text{"h(x)"}, -\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(x^2\right)-1\right)^2}}{\sqrt{\pi}\,x\left(-1 + \mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(2\ln(x)-1)\right)\right)}$$

$$\text{"mean and variance"}, \mathrm{e}, \mathrm{e}^3 - \mathrm{e}^2$$

$$mf := \int_0^\infty \frac{1}{2}\,\frac{x^{r\sim}\,\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(x^2\right)-1\right)^2}}{\sqrt{\pi}\,x}\,\mathrm{d}x$$

$$\text{"MF"}, \int_0^\infty \frac{1}{2}\,\frac{x^{r\sim}\,\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(x^2\right)-1\right)^2}}{\sqrt{\pi}\,x}\,\mathrm{d}x$$

$$\text{"MGF"}, \int_0^\infty \frac{1}{2}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8} - \frac{1}{2}\ln(x)^2 + t\,x}}{\sqrt{\pi}\,\sqrt{x}}\,\mathrm{d}x$$

```
1/2\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( {x}^
{2}
 \right) -1 \right) ^{2}}}}{\sqrt {\pi}x}}
```

"i is", 3,

" --

----------"

$$g := t \rightarrow \frac{1}{t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \rightarrow \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(\frac{1}{y\sim}\right)-1\right)^2}}{\sqrt{\pi}\,y\sim}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \frac{1}{x}, \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(\frac{1}{x}\right)-1\right)^2}}{\sqrt{\pi}\,x}$$

$$\text{"F(x)"}, \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(\ln(x)+1)\right)$$

$$\text{"IDF(x)"}, \left[\left[s \rightarrow \mathrm{e}^{-1+2\sqrt{2}\,RootOf(-\mathrm{erf}(\_Z)-1+2s)}\right], [0, 1], [\text{"Continuous"}, \text{"IDF"}]\right]$$

$$\text{"S(x)"}, \frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(\ln(x)+1)\right)$$

$$\text{"h(x)"}, -\frac{1}{2}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(\frac{1}{x}\right)-1\right)^2}}{\sqrt{\pi}\,x\left(-1+\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(\ln(x)+1)\right)\right)}$$

$$\text{"mean and variance"}, \mathrm{e}, \mathrm{e}^6 - \mathrm{e}^2$$

$$mf := \int_0^\infty \frac{1}{4}\,\frac{x^{r\sim}\,\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(\frac{1}{x}\right)-1\right)^2}}{\sqrt{\pi}\,x}\,\mathrm{d}x$$

$$\text{"MF"}, \int_0^\infty \frac{1}{4}\,\frac{x^{r\sim}\,\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(\frac{1}{x}\right)-1\right)^2}}{\sqrt{\pi}\,x}\,\mathrm{d}x$$

$$\text{"MGF"}, \int_0^\infty \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(x)^2+t\,x}}{\sqrt{\pi}\,x^{5/4}}\,\mathrm{d}x$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( {x}^
{-1}
 \right) -1 \right) ^{2}}}}{\sqrt {\pi}x}}
```

"i is", 4,

" --
----------"

$$g := t \to \arctan(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(\tan(y\sim))-1)^2}\left(1+\tan(y\sim)^2\right)}{\sqrt{\pi}\,\tan(y\sim)}\right], \left[0, \frac{1}{2}\,\pi\right], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \arctan(x), \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(\tan(x))-1)^2}\left(1+\tan(x)^2\right)}{\sqrt{\pi}\,\tan(x)}$$

$$\text{"F(x)"}, \frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(\ln(\tan(x))-1)\right)$$

$$\text{"IDF(x)"}, \left[\left[\arctan@\left(s \to \mathrm{e}^{1+2\sqrt{2}\,RootOf(-\mathrm{erf}(\_Z)-1+2s)}\right)\right], [0, 1], [\text{"Continuous"}, \text{"IDF"}]\right]$$

$$\text{"S(x)"}, \frac{1}{2}-\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(\ln(\tan(x))-1)\right)$$

$$\text{"h(x)"}, -\frac{1}{2}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(\tan(x))-1)^2}\left(1+\tan(x)^2\right)}{\sqrt{\pi}\,\tan(x)\left(-1+\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}\,(\ln(\tan(x))-1)\right)\right)}$$

$$\text{"mean and variance"}, \frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^{\frac{1}{2}\pi}\frac{\mathrm{e}^{-\frac{1}{8}\,(\ln(\sin(x))-\ln(\cos(x))-1)^2}\,x}{\cos(x)\,\sin(x)}\,\mathrm{d}x\right)}{\sqrt{\pi}}, \frac{1}{8}\,\frac{1}{\pi^{3/2}}\left(2\sqrt{2}\left(\int_0^{\frac{1}{2}\pi}\frac{\mathrm{e}^{-\frac{1}{8}\,(\ln(\sin(x))-\ln(\cos(x))-1)^2}\,x^2}{\cos(x)\,\sin(x)}\,\mathrm{d}x\right)\pi-\left(\int_0^{\frac{1}{2}\pi}\frac{\mathrm{e}^{-\frac{1}{8}\,(\ln(\sin(x))-\ln(\cos(x))-1)^2}\,x}{\cos(x)\,\sin(x)}\right.\right.$$

$$\left.\left. dx\right)^2 \sqrt{\pi}\right)$$

$$mf := \int_0^{\frac{1}{2}\pi} \frac{1}{4} \frac{x^{r\sim} \sqrt{2}\, \mathrm{e}^{-\frac{1}{8} (\ln(\tan(x)) - 1)^2} \left(1 + \tan(x)^2\right)}{\sqrt{\pi}\, \tan(x)}\, dx$$

$$\text{"MF"}, \int_0^{\frac{1}{2}\pi} \frac{1}{4} \frac{x^{r\sim} \sqrt{2}\, \mathrm{e}^{-\frac{1}{8} (\ln(\tan(x)) - 1)^2} \left(1 + \tan(x)^2\right)}{\sqrt{\pi}\, \tan(x)}\, dx$$

$$\text{"MGF"}, \frac{1}{4} \frac{\sqrt{2} \left(\int_0^{\frac{1}{2}\pi} \frac{\cos(x)\, \mathrm{e}^{\frac{1}{4}\ln(\sin(x)) - \frac{5}{4}} \mathrm{e}^{-\frac{1}{8} + t\, x - \frac{1}{8}\ln(\sin(x))^2 - \frac{1}{8}\ln(\cos(x))^2}}{\sin(x)^{3/4}}\, dx \right)}{\sqrt{\pi}}$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( \tan
\left(
x \right)  \right) -1 \right) ^{2}}} \left( 1+ \left( \tan \left
( x
 \right)  \right) ^{2} \right) }{\sqrt {\pi}\tan \left( x
\right) }}
```

"i is", 5,
" ---
----------"

$$g := t \rightarrow \mathrm{e}^t$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[\left[y\sim \rightarrow \frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8} (\ln(\ln(y\sim)) - 1)^2}}{\sqrt{\pi}\, \ln(y\sim)\, y\sim}\right], [1, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]\right.$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \mathrm{e}^x, \text{"base"}, \frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8} (\ln(x) - 1)^2}}{\sqrt{\pi}\, x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8} (\ln(\ln(x)) - 1)^2}}{\sqrt{\pi}\, \ln(x)\, x}$$

$$"F(x)", \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\,(\ln(\ln(x)) - 1)\right)$$

$$"IDF(x)", \left[\left[\exp@\left(s \to e^{1+2\sqrt{2}\,RootOf(-\operatorname{erf}(\_Z)-1+2s)}\right)\right], [0, 1], ["Continuous", "IDF"]\right]$$

$$"S(x)", \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\,(\ln(\ln(x)) - 1)\right)$$

$$"h(x)", -\frac{1}{2}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(\ln(x))-1)^2}}{\sqrt{\pi}\,\ln(x)\,x\left(-1+\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\,(\ln(\ln(x))-1)\right)\right)}$$

$$"mean and variance", \infty, \mathit{undefined}$$

$$mf := \infty$$

$$"MF", \infty$$

$$"MGF", \int_1^{\infty} \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}-\frac{1}{8}\ln(\ln(x))^2+t\,x}}{\sqrt{\pi}\,\ln(x)^{3/4}\,x}\,dx$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( \ln  \left(
x \right)  \right) -1 \right) ^{2}}}}{\sqrt {\pi}\ln  \left( x
 \right) x}}
```

"i is", 6,

" --"

$$g := t \to \ln(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(y\sim-1)^2}}{\sqrt{\pi}}\right], [-\infty, \infty], ["Continuous", "PDF"]\right]$$

$$"l and u", 0, \infty$$

$$"g(x)", \ln(x), "base", \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, "LogNormalRV(1, 2)"$$

$$"f(x)", \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(x-1)^2}}{\sqrt{\pi}}$$

$$"F(x)", \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\,x\sqrt{2} - \frac{1}{4}\sqrt{2}\right)$$

$$"IDF(x)", \left[\left[s \to \frac{1}{2}\left(\sqrt{2} + 4\,RootOf(-\operatorname{erf}(\_Z) - 1 + 2s)\right)\sqrt{2}\right], [0, 1], ["Continuous", "IDF"]\right]$$

$$\text{"S(x)"}, \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{1}{4}x\sqrt{2} - \frac{1}{4}\sqrt{2}\right)$$

$$\text{"h(x)"}, -\frac{1}{2}\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(x-1)^2}}{\sqrt{\pi}\left(-1 + \operatorname{erf}\left(\frac{1}{4}x\sqrt{2} - \frac{1}{4}\sqrt{2}\right)\right)}$$

$$\text{"mean and variance"}, 1, 4$$

$$mf := \int_{-\infty}^{\infty} \frac{1}{4}\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(x-1)^2}}{\sqrt{\pi}}\,\mathrm{d}x$$

$$\text{"MF"}, \int_{-\infty}^{\infty} \frac{1}{4}\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(x-1)^2}}{\sqrt{\pi}}\,\mathrm{d}x$$

$$\text{"MGF"}, \mathrm{e}^{t(2t+1)}$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( x-1 \right) ^{2}}}
}{
\sqrt {\pi}}}
```

"i is", 7,

" ---"

$$g := t \to \mathrm{e}^{-t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to -\frac{1}{4}\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(-\ln(y\sim))-1)^2}}{\sqrt{\pi}\,\ln(y\sim)\,y\sim}\right], [0, 1], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \mathrm{e}^{-x}, \text{"base"}, \frac{1}{4}\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, -\frac{1}{4}\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(-\ln(x))-1)^2}}{\sqrt{\pi}\,\ln(x)\,x}$$

$$\text{"F(x)"}, \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\,(\ln(-\ln(x)) - 1)\right)$$

$$\text{"IDF(x)"}, \left[\left[s \to \mathrm{e}^{-\mathrm{e}^{1 + 2\sqrt{2}\,RootOf(\operatorname{erf}(\_Z) - 1 + 2s)}}\right], [0, 1], [\text{"Continuous"}, \text{"IDF"}]\right]$$

$$\text{"S(x)"}, \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\,(\ln(-\ln(x)) - 1)\right)$$

$$\text{"h(x)"},\ -\frac{1}{2}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln(-\ln(x))-1\right)^2}}{\sqrt{\pi}\,\ln(x)\,x\left(1+\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\,\left(\ln(-\ln(x))-1\right)\right)\right)}$$

$$\text{"mean and variance"},\ -\frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^1 \frac{\mathrm{e}^{-\frac{1}{8}\left(\ln(-\ln(x))-1\right)^2}}{\ln(x)}\,\mathrm{d}x\right)}{\sqrt{\pi}},$$

$$-\frac{1}{8}\,\frac{\left(\int_0^1 \frac{\mathrm{e}^{-\frac{1}{8}\left(\ln(-\ln(x))-1\right)^2}}{\ln(x)}\,\mathrm{d}x\right)^2\sqrt{\pi}+2\sqrt{2}\left(\int_0^1 \frac{x\,\mathrm{e}^{-\frac{1}{8}\left(\ln(-\ln(x))-1\right)^2}}{\ln(x)}\,\mathrm{d}x\right)\pi}{\pi^{3/2}}$$

$$mf := \int_0^1 \left(-\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln(-\ln(x))-1\right)^2}}{\sqrt{\pi}\,\ln(x)\,x}\right)\mathrm{d}x$$

$$\text{"MF"},\ \int_0^1 \left(-\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln(-\ln(x))-1\right)^2}}{\sqrt{\pi}\,\ln(x)\,x}\right)\mathrm{d}x$$

$$\text{"MGF"},\ \frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^1 \frac{\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(-\ln(x))^2+t\,x}}{x\,(-\ln(x))^{3/4}}\,\mathrm{d}x\right)}{\sqrt{\pi}}$$

```
-1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( -\ln
 \left( x \right)  \right) -1 \right) ^{2}}}}{\sqrt {\pi}\ln
\left( x
 \right) x}}
```

"i is", 8,

" --
----------"

$$g := t \to -\ln(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(y\sim+1)^2}}{\sqrt{\pi}}\right], [-\infty, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0,\ \infty$$

$$\text{"g(x)"},\ -\ln(x),\ \text{"base"},\ \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(x)-1)^2}}{\sqrt{\pi}\,x},\ \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"},\ \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(x+1)^2}}{\sqrt{\pi}}$$

$$\text{"F(x)"},\ \frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,x\sqrt{2}+\frac{1}{4}\,\sqrt{2}\right)$$

$$\text{"IDF(x)"},\ \left[\left[s\to\frac{1}{2}\,\left(-\sqrt{2}+4\,\mathit{RootOf}(-\mathrm{erf}(\_Z)-1+2\,s)\right)\sqrt{2}\right],\ [0,1],\ [\text{"Continuous"},\ \text{"IDF"}]\right]$$

$$\text{"S(x)"},\ \frac{1}{2}-\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{4}\,x\sqrt{2}+\frac{1}{4}\,\sqrt{2}\right)$$

$$\text{"h(x)"},\ -\frac{1}{2}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(x+1)^2}}{\sqrt{\pi}\,\left(-1+\mathrm{erf}\left(\frac{1}{4}\,x\sqrt{2}+\frac{1}{4}\,\sqrt{2}\right)\right)}$$

$$\text{"mean and variance"},\ -1,\ 4$$

$$mf:=\int_{-\infty}^{\infty}\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(x+1)^2}}{\sqrt{\pi}}\,\mathrm{d}x$$

$$\text{"MF"},\ \int_{-\infty}^{\infty}\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(x+1)^2}}{\sqrt{\pi}}\,\mathrm{d}x$$

$$\text{"MGF"},\ \mathrm{e}^{t\,(2\,t-1)}$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( x+1 \right) ^{2}}}
}{
\sqrt {\pi}}}
```

"i is", 9,

" ---"

$$g:=t\to\ln(t+1)$$

$$l:=0$$

$$u:=\infty$$

$$Temp:=\left[\left[y\sim\to\frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\,\ln\left(\mathrm{e}^{y\sim}-1\right)^2+y\sim}}{\sqrt{\pi}\,\left(\mathrm{e}^{y\sim}-1\right)^{3/4}}\right],\ [0,\infty],\ [\text{"Continuous"},\ \text{"PDF"}]\right]$$

$$\text{"l and u"},\ 0,\ \infty$$

$$\text{"g(x)"}, \ln(x+1), \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^x-1)^2+x}}{\sqrt{\pi}\,(\mathrm{e}^x-1)^{3/4}}$$

$$\text{"F(x)"}, \frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^x \frac{\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^t-1)^2+t}}{(\mathrm{e}^t-1)^{3/4}}\,\mathrm{d}t\right)}{\sqrt{\pi}}$$

$$\text{"IDF did not work"}$$

$$\text{"S(x)"}, \frac{1}{4}\,\frac{-\sqrt{2}\left(\int_0^x \frac{\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^t-1)^2+t}}{(\mathrm{e}^t-1)^{3/4}}\,\mathrm{d}t\right)+4\sqrt{\pi}}{\sqrt{\pi}}$$

$$\text{"h(x)"}, \frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^x-1)^2+x}}{(\mathrm{e}^x-1)^{3/4}\left(-\sqrt{2}\left(\int_0^x \frac{\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^t-1)^2+t}}{(\mathrm{e}^t-1)^{3/4}}\,\mathrm{d}t\right)+4\sqrt{\pi}\right)}$$

$$\text{"mean and variance"}, \int_0^\infty \frac{1}{4}\,\frac{x\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^x-1)^2+x}}{\sqrt{\pi}\,(\mathrm{e}^x-1)^{3/4}}\,\mathrm{d}x,$$

$$\int_0^\infty \frac{1}{4}\,\frac{x^2\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^x-1)^2+x}}{\sqrt{\pi}\,(\mathrm{e}^x-1)^{3/4}}\,\mathrm{d}x-\left(\int_0^\infty \frac{1}{4}\,\frac{x\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^x-1)^2+x}}{\sqrt{\pi}\,(\mathrm{e}^x-1)^{3/4}}\,\mathrm{d}x\right)^2$$

$$mf := \int_0^\infty \frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^x-1)^2+x}}{\sqrt{\pi}\,(\mathrm{e}^x-1)^{3/4}}\,\mathrm{d}x$$

$$\text{"MF"}, \int_0^\infty \frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln(\mathrm{e}^x-1)^2+x}}{\sqrt{\pi}\,(\mathrm{e}^x-1)^{3/4}}\,\mathrm{d}x$$

$$"MGF", \int_0^{\infty} \frac{1}{4} \frac{\sqrt{2}\, e^{t x - \frac{1}{8} - \frac{1}{8} \ln(e^x - 1)^2 + x}}{\sqrt{\pi}\, (e^x - 1)^{3/4}} \, dx$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8-1/8\, \left( \ln  \left( {
{\rm e}}
^{x}}-1 \right)  \right) ^{2}+x}}}{\sqrt {\pi} \left( {{\rm e}^
{x}}-1
 \right) ^{3/4}}}
```

"i is", 10,

" ---
----------"

$$g := t \to \frac{1}{\ln(t+2)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[\left[y\sim \to \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{y\sim}} - 2\right)^2 y\sim - 2 \ln\left(e^{\frac{1}{y\sim}} - 2\right) y\sim + y\sim - 8}{y\sim}}}{\sqrt{\pi} \left(e^{\frac{1}{y\sim}} - 2\right) y\sim^2}\right], \left[0, \frac{1}{\ln(2)}\right], \left["Continuous", "PDF"\right]\right]\right]$$

"l and u", 0, ∞

$$"g(x)", \frac{1}{\ln(x+2)}, "base", \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8} (\ln(x) - 1)^2}}{\sqrt{\pi}\, x}, "LogNormalRV(1, 2)"$$

$$"f(x)", \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{x}} - 2\right)^2 x - 2 \ln\left(e^{\frac{1}{x}} - 2\right) x + x - 8}{x}}}{\sqrt{\pi} \left(e^{\frac{1}{x}} - 2\right) x^2}$$

$$"F(x)", \frac{1}{4} \frac{\sqrt{2} \left(\int_0^x \frac{e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{t}} - 2\right)^2 t - 2 \ln\left(e^{\frac{1}{t}} - 2\right) t + t - 8}{t}}}{\left(e^{\frac{1}{t}} - 2\right) t^2} \, dt\right)}{\sqrt{\pi}}$$

"IDF did not work"

$$"S(x)", \frac{1}{4} \frac{-\sqrt{2}\left(\int_{0}^{x} \frac{e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{t}}-2\right)^{2} t-2 \ln\left(e^{\frac{1}{t}}-2\right) t+t-8}{t}}}{\left(e^{\frac{1}{t}}-2\right) t^{2}} \mathrm{d}t\right)+4\sqrt{\pi}}{\sqrt{\pi}}$$

$$"h(x)", \frac{\sqrt{2}\, e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{x}}-2\right)^{2} x-2 \ln\left(e^{\frac{1}{x}}-2\right) x+x-8}{x}}}{\left(e^{\frac{1}{x}}-2\right) x^{2}\left(-\sqrt{2}\left(\int_{0}^{x} \frac{e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{t}}-2\right)^{2} t-2 \ln\left(e^{\frac{1}{t}}-2\right) t+t-8}{t}}}{\left(e^{\frac{1}{t}}-2\right) t^{2}} \mathrm{d}t\right)+4\sqrt{\pi}\right)}$$

$$"mean and variance", \frac{1}{4} \frac{\sqrt{2}\left(\int_{0}^{\frac{1}{\ln(2)}} \frac{e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{x}}-2\right)^{2} x-2 \ln\left(e^{\frac{1}{x}}-2\right) x+x-8}{x}}}{x\left(e^{\frac{1}{x}}-2\right)} \mathrm{d}x\right)}{\sqrt{\pi}},$$

$$-\frac{1}{8} \frac{1}{\pi^{3/2}}\left(\left(\int_{0}^{\frac{1}{\ln(2)}} \frac{e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{x}}-2\right)^{2} x-2 \ln\left(e^{\frac{1}{x}}-2\right) x+x-8}{x}}}{x\left(e^{\frac{1}{x}}-2\right)} \mathrm{d}x\right)^{2} \sqrt{\pi}-2\sqrt{2}\left(\int_{0}^{\frac{1}{\ln(2)}} \frac{e^{-\frac{1}{8} \frac{\ln\left(e^{\frac{1}{x}}-2\right)^{2} x-2 \ln\left(e^{\frac{1}{x}}-2\right) x+x-8}{x}}}{e^{\frac{1}{x}}-2} \mathrm{d}x\right)\pi\right)$$

$$mf := \int_0^{\frac{1}{\ln(2)}} \frac{1}{4} \frac{x^{r\sim} \sqrt{2}\, \mathrm{e}^{-\frac{1}{8} \frac{\ln\left(\mathrm{e}^{\frac{1}{x}}-2\right)^2 x - 2\ln\left(\mathrm{e}^{\frac{1}{x}}-2\right) x + x - 8}{x}}}{\sqrt{\pi} \left(\mathrm{e}^{\frac{1}{x}}-2\right) x^2} \, dx$$

$$\text{"MF"}, \int_0^{\frac{1}{\ln(2)}} \frac{1}{4} \frac{x^{r\sim} \sqrt{2}\, \mathrm{e}^{-\frac{1}{8} \frac{\ln\left(\mathrm{e}^{\frac{1}{x}}-2\right)^2 x - 2\ln\left(\mathrm{e}^{\frac{1}{x}}-2\right) x + x - 8}{x}}}{\sqrt{\pi} \left(\mathrm{e}^{\frac{1}{x}}-2\right) x^2} \, dx$$

$$\text{"MGF"}, \frac{1}{4} \frac{\sqrt{2} \left(\int_0^{\frac{1}{\ln(2)}} \frac{\mathrm{e}^{-\frac{1}{8} \frac{\ln\left(\mathrm{e}^{\frac{1}{x}}-2\right)^2 x - 8 t x^2 - 2\ln\left(\mathrm{e}^{\frac{1}{x}}-2\right) x + x - 8}{x}}}{\left(\mathrm{e}^{\frac{1}{x}}-2\right) x^2} \, dx \right)}{\sqrt{\pi}}$$

```
1/4\,{\frac {\sqrt {2}}{\sqrt {\pi} \left( {{\rm e}^{{x}^{-1}}}
-2
 \right) {x}^{2}}{{\rm e}^{-1/8\,{\frac { \left( \ln  \left( {
{\rm e}^
{{x}^{-1}}}-2 \right)  \right) ^{2}x-2\,\ln  \left( {{\rm e}^{
{x}^{-1}
}}-2 \right) x+x-8}{x}}}}}
```

"i is", 11,

" --"

$$g := t \rightarrow \tanh(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \rightarrow -\frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8} (\ln(\operatorname{arctanh}(y\sim)) - 1)^2}}{\sqrt{\pi} \operatorname{arctanh}(y\sim) \left(y\sim^2 - 1\right)} \right], [0, 1], [\text{"Continuous"}, \text{"PDF"}] \right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \tanh(x), \text{"base"}, \frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8} (\ln(x) - 1)^2}}{\sqrt{\pi}\, x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, -\frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))-1)^2}}{\sqrt{\pi}\,\operatorname{arctanh}(x)\,(x^2-1)}$$

$$\text{"F(x)"}, -\frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^x \frac{\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(t))-1)^2}}{\operatorname{arctanh}(t)\,(t^2-1)}\,\mathrm{d}t\right)}{\sqrt{\pi}}$$

"IDF did not work"

$$\text{"S(x)"}, \frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^x \frac{\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(t))-1)^2}}{\operatorname{arctanh}(t)\,(t^2-1)}\,\mathrm{d}t\right)+4\sqrt{\pi}}{\sqrt{\pi}}$$

$$\text{"h(x)"}, -\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))-1)^2}}{\operatorname{arctanh}(x)\,(x^2-1)\left(\sqrt{2}\left(\int_0^x \frac{\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(t))-1)^2}}{\operatorname{arctanh}(t)\,(t^2-1)}\,\mathrm{d}t\right)+4\sqrt{\pi}\right)}$$

$$\text{"mean and variance"}, -\frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^1 \frac{x\,\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))-1)^2}}{\operatorname{arctanh}(x)\,(x^2-1)}\,\mathrm{d}x\right)}{\sqrt{\pi}},$$

$$-\frac{1}{8}\,\frac{\left(\int_0^1 \frac{x\,\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))-1)^2}}{\operatorname{arctanh}(x)\,(x^2-1)}\,\mathrm{d}x\right)^2\sqrt{\pi}+2\sqrt{2}\left(\int_0^1 \frac{x^2\,\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))-1)^2}}{\operatorname{arctanh}(x)\,(x^2-1)}\,\mathrm{d}x\right)\pi}{\pi^{3/2}}$$

$$mf := \int_0^1 \left(-\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))-1)^2}}{\sqrt{\pi}\,\operatorname{arctanh}(x)\,(x^2-1)}\right)\mathrm{d}x$$

$$\text{"MF"}, \int_0^1 \left(-\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))-1)^2}}{\sqrt{\pi}\,\operatorname{arctanh}(x)\,(x^2-1)}\right)\mathrm{d}x$$

$$\text{"MGF"}, -\frac{1}{4}\,\frac{\sqrt{2}\left(\displaystyle\int_0^1 \frac{e^{-\frac{1}{8}-\frac{1}{8}\ln(\operatorname{arctanh}(x))^2+t\,x}}{\operatorname{arctanh}(x)^{3/4}\,(x^2-1)}\,dx\right)}{\sqrt{\pi}}$$

```
-1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left(
{\rm arctanh} \left(x\right) \right) -1 \right) ^{2}}}}{\sqrt
{\pi}
{\rm arctanh} \left(x\right) \left( {x}^{2}-1 \right) }}
```

"i is", 12,

" --"

$$g := t \rightarrow \sinh(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \rightarrow \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(\operatorname{arcsinh}(y\sim))-1)^2}}{\sqrt{\pi}\,\operatorname{arcsinh}(y\sim)\,\sqrt{y\sim^2+1}}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \sinh(x), \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(\operatorname{arcsinh}(x))-1)^2}}{\sqrt{\pi}\,\operatorname{arcsinh}(x)\,\sqrt{x^2+1}}$$

$$\text{"F(x)"}, \frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\left(\ln\left(-\ln\left(-x+\sqrt{x^2+1}\right)\right)-1\right)\right)$$

$$\text{"IDF(x)"}, \left[\left[s \rightarrow \frac{1}{2}\,e^{e^{1+2\sqrt{2}\,RootOf(-\operatorname{erf}(\_Z)-1+2s)}}-\frac{1}{2}\,e^{-e^{1+2\sqrt{2}\,RootOf(-\operatorname{erf}(\_Z)-1+2s)}}\right], [0, 1], [\text{"Continuous"}, \text{"IDF"}]\right]$$

$$\text{"S(x)"}, \frac{1}{2}-\frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\left(\ln\left(-\ln\left(-x+\sqrt{x^2+1}\right)\right)-1\right)\right)$$

"h(x)",

$$-\frac{1}{2}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(\operatorname{arcsinh}(x))-1)^2}}{\sqrt{\pi}\,\operatorname{arcsinh}(x)\,\sqrt{x^2+1}\,\left(-1+\operatorname{erf}\left(\frac{1}{4}\sqrt{2}\left(\ln\left(-\ln\left(-x+\sqrt{x^2+1}\right)\right)-1\right)\right)\right)}$$

$$\text{"mean and variance"}, \infty, \mathit{undefined}$$

$$mf := \infty$$

$$\text{"MF"}, \infty$$

$$\text{"MGF"}, \int_0^\infty \frac{1}{4} \frac{e^{-\frac{1}{8} - \frac{1}{8}\ln(\operatorname{arcsinh}(x))^2 + t x}\sqrt{2}}{\operatorname{arcsinh}(x)^{3/4}\sqrt{x^2+1}\sqrt{\pi}}\, dx$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left(
{\rm arcsinh} \left(x\right) \right) -1 \right) ^{2}}}}{\sqrt
{\pi}
{\rm arcsinh} \left(x\right)\sqrt {{x}^{2}+1}}}
```

"i is", 13,

" ---"

$$g := t \rightarrow \operatorname{arcsinh}(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \rightarrow \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\sinh(y\sim)) - 1)^2} \cosh(y\sim)}{\sqrt{\pi}\, \sinh(y\sim)}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \operatorname{arcsinh}(x), \text{"base"}, \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(x) - 1)^2}}{\sqrt{\pi}\, x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\sinh(x)) - 1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)}$$

$$\text{"F(x)"}, \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{1}{4}\sqrt{2}\left(-\ln(e^x - 1) - \ln(e^x + 1) + \ln(2) + x + 1\right)\right)$$

$$\text{"IDF(x)"}, \left[\left[s \rightarrow \ln\left(RootOf\left(\_Z^2 + \left(-2\, e^{\frac{1}{2}\sqrt{2}\left(-4\, RootOf(\operatorname{erf}(\_Z) - 1 + 2 s) + \sqrt{2}\right)} - 2\right)\_Z + 2\, e^{\frac{1}{2}\sqrt{2}\left(-4\, RootOf(\operatorname{erf}(\_Z) - 1 + 2 s) + \sqrt{2}\right)} - 1\right)\right)\right], [0, 1], [\text{"Continuous"}, \text{"IDF"}]\right]$$

$$\text{"S(x)"}, \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{1}{4}\sqrt{2}\left(-\ln(e^x - 1) - \ln(e^x + 1) + \ln(2) + x + 1\right)\right)$$

$$\text{"h(x)"}, \frac{1}{2} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\sinh(x)) - 1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)\left(1 + \operatorname{erf}\left(\frac{1}{4}\sqrt{2}\left(-\ln(e^x - 1) - \ln(e^x + 1) + \ln(2) + x + 1\right)\right)\right)}$$

$$\text{"mean and variance"}, \int_0^\infty \frac{1}{4} \frac{x\sqrt{2}\, e^{-\frac{1}{8}(\ln(\sinh(x)) - 1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)}\, dx,$$

$$\int_0^\infty \frac{1}{4}\,\frac{x^2\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(\sinh(x))-1)^2}\cosh(x)}{\sqrt{\pi}\,\sinh(x)}\,dx$$

$$-\left(\int_0^\infty \frac{1}{4}\,\frac{x\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(\sinh(x))-1)^2}\cosh(x)}{\sqrt{\pi}\,\sinh(x)}\,dx\right)^2$$

$$mf := \int_0^\infty \frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(\sinh(x))-1)^2}\cosh(x)}{\sqrt{\pi}\,\sinh(x)}\,dx$$

$$\text{"MF"}, \int_0^\infty \frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(\sinh(x))-1)^2}\cosh(x)}{\sqrt{\pi}\,\sinh(x)}\,dx$$

"MGF did not work"

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( \sinh
 \left( x \right)  \right) -1 \right) ^{2}}}\cosh \left( x
\right) }{
\sqrt {\pi}\sinh \left( x \right) }}
```

"i is", 16,

" ---
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$$g := t \to \frac{1}{\tanh(t+1)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[\left[y\sim \to \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{y\sim}\right)\right)-1\right)^2}}{\sqrt{\pi}\left(-1+\operatorname{arctanh}\left(\frac{1}{y\sim}\right)\right)(y\sim^2-1)}\right], \left[1, \frac{\mathrm{e}+\mathrm{e}^{-1}}{\mathrm{e}-\mathrm{e}^{-1}}\right], [\text{"Continuous"}, \text{"PDF"}]\right]\right]$$

"l and u", 0, ∞

$$\text{"g(x)"}, \frac{1}{\tanh(x+1)}, \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\,(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{\pi}\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)\left(x^2-1\right)}$$

$$\text{"F(x)"}, \frac{1}{4} \frac{\sqrt{2}\left(\int_1^x \frac{e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{t}\right)\right)-1\right)^2}}{\left(-1+\operatorname{arctanh}\left(\frac{1}{t}\right)\right)\left(t^2-1\right)}\, dt\right)}{\sqrt{\pi}}$$

$$\text{"S(x)"}, \frac{1}{4} \frac{-\sqrt{2}\left(\int_1^x \frac{e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{t}\right)\right)-1\right)^2}}{\left(-1+\operatorname{arctanh}\left(\frac{1}{t}\right)\right)\left(t^2-1\right)}\, dt\right)+4\sqrt{\pi}}{\sqrt{\pi}}$$

$$\text{"h(x)"}, \left(\sqrt{2}\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)-1\right)^2}\right) \Bigg/ \left(\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)\left(x^2-1\right)\left(-\sqrt{2}\left(\int_1^x \frac{e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{t}\right)\right)-1\right)^2}}{\left(-1+\operatorname{arctanh}\left(\frac{1}{t}\right)\right)\left(t^2-1\right)}\, dt\right)+4\sqrt{\pi}\right)\right)$$

$$\text{"mean and variance"}, \frac{1}{4} \frac{\sqrt{2}\left(\int_1^{\frac{e^2+1}{e^2-1}} \frac{x\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)\left(x^2-1\right)}\, dx\right)}{\sqrt{\pi}},$$

$$-\frac{1}{8} \frac{1}{\pi^{3/2}}\left(\left(\int_1^{\frac{e^2+1}{e^2-1}} \frac{x\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)\left(x^2-1\right)}\, dx\right)^2 \sqrt{\pi} - 2\sqrt{2}\left(\right.\right.$$

$$\int_{1}^{\frac{e^2+1}{e^2-1}} \frac{x^2\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)\left(x^2-1\right)}\,dx\Bigg)\,\pi\Bigg)$$

$$mf := \int_{1}^{\frac{e+e^{-1}}{e-e^{-1}}} \frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{\pi}\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)\left(x^2-1\right)}\,dx$$

$$\text{"MF"}, \int_{1}^{\frac{e+e^{-1}}{e-e^{-1}}} \frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{\pi}\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)\left(x^2-1\right)}\,dx$$

$$\text{"MGF"}, \frac{1}{4}\,\frac{\sqrt{2}\left(\int_{1}^{\frac{e^2+1}{e^2-1}} \frac{e^{-\frac{1}{8}-\frac{1}{8}\ln\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)^2+t\,x}}{\left(-1+\operatorname{arctanh}\left(\frac{1}{x}\right)\right)^{3/4}\left(x^2-1\right)}\,dx\right)}{\sqrt{\pi}}$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( -1+
{\rm arctanh} \left({x}^{-1}\right) \right) -1 \right) ^{2}}}}
{\sqrt {
\pi} \left( -1+{\rm arctanh} \left({x}^{-1}\right) \right)
\left( {x}
^{2}-1 \right) }}
```

"i is", 17,

" --
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$$g := t \to \frac{1}{\sinh(t+1)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{4}\,\frac{\sqrt{2}\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{y\sim}\right)\right)-1\right)^2}}{\sqrt{\pi}\sqrt{y\sim^2+1}\left(-1+\operatorname{arcsinh}\left(\frac{1}{y\sim}\right)\right)|y\sim|}\right], \left[0, \frac{2}{e-e^{-1}}\right],\right.$$

$$["\text{Continuous}", "\text{PDF}"]$$

$$"\text{l and u}", 0, \infty$$

$$"\text{g(x)}", \frac{1}{\sinh(x+1)}, "\text{base}", \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8} (\ln(x)-1)^2}}{\sqrt{\pi}\, x}, "\text{LogNormalRV(1, 2)}"$$

$$"\text{f(x)}", \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{\pi}\sqrt{x^2+1}\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)|x|}$$

$$"\text{F(x)}", \frac{1}{4} \frac{\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{t}\right)\right)-1\right)^2}}{\sqrt{t^2+1}\left(-1+\operatorname{arcsinh}\left(\frac{1}{t}\right)\right)|t|}\, dt\right)}{\sqrt{\pi}}$$

$$"\text{S(x)}", \frac{1}{4} \frac{-\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{t}\right)\right)-1\right)^2}}{\sqrt{t^2+1}\left(-1+\operatorname{arcsinh}\left(\frac{1}{t}\right)\right)|t|}\, dt\right)+4\sqrt{\pi}}{\sqrt{\pi}}$$

$$"\text{h(x)}", \left(\sqrt{2}\, e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)-1\right)^2}\right) \Bigg/ \left(\sqrt{x^2+1}\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)|x| \left(-\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{t}\right)\right)-1\right)^2}}{\sqrt{t^2+1}\left(-1+\operatorname{arcsinh}\left(\frac{1}{t}\right)\right)|t|}\, dt\right)+4\sqrt{\pi}\right)\right)$$

$$"\text{mean and variance}", \frac{1}{4} \frac{\sqrt{2}\left(\int_0^{\frac{2e}{e^2-1}} \frac{e^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{x^2+1}\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)}\, dx\right)}{\sqrt{\pi}},$$

$$-\frac{1}{8}\frac{1}{\pi^{3/2}}\left(\left(\int_0^{\frac{2\,\mathrm{e}}{\mathrm{e}^2-1}}\frac{\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{x^2+1}\,\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)}\,\mathrm{d}x\right)^2\sqrt{\pi}-2\sqrt{2}\left(\int_0^{\frac{2\,\mathrm{e}}{\mathrm{e}^2-1}}\frac{x\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{x^2+1}\,\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)}\,\mathrm{d}x\right)\pi\right)$$

$$mf:=\int_0^{\frac{2}{\mathrm{e}-\mathrm{e}^{-1}}}\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{\pi}\,\sqrt{x^2+1}\,\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)\,|x|}\,\mathrm{d}x$$

$$\text{"MF"},\int_0^{\frac{2}{\mathrm{e}-\mathrm{e}^{-1}}}\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)-1\right)^2}}{\sqrt{\pi}\,\sqrt{x^2+1}\,\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)\,|x|}\,\mathrm{d}x$$

$$\text{"MGF"},\frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^{\frac{2\,\mathrm{e}}{\mathrm{e}^2-1}}\frac{\mathrm{e}^{-\frac{1}{8}-\frac{1}{8}\ln\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)^2+t\,x}}{\left(-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)\right)^{3/4}x\sqrt{x^2+1}}\,\mathrm{d}x\right)}{\sqrt{\pi}}$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( -1+
{\rm arcsinh} \left({x}^{-1}\right) \right) -1 \right) ^{2}}}}
{\sqrt {
\pi}\sqrt {{x}^{2}+1} \left( -1+{\rm arcsinh} \left({x}^{-1}
\right)
 \right)  \left| x \right| }}
```

"i is", 18,

" ---
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$$g:=t\rightarrow\frac{1}{\operatorname{arcsinh}(t+1)}$$

$$l:=0$$

$$u:=\infty$$

$$Temp := \left[\left[y\sim \rightarrow \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{y\sim}\right)\right)-1\right)^2}\cosh\left(\frac{1}{y\sim}\right)}{\sqrt{\pi}\,\left(-1+\sinh\left(\frac{1}{y\sim}\right)\right)y\sim^2}\right], \left[0, \frac{1}{\ln\left(1+\sqrt{2}\right)}\right], \right.$$

$$\left.[\text{"Continuous", "PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \frac{1}{\operatorname{arcsinh}(x+1)}, \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4}\,\frac{\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)-1\right)^2}\cosh\left(\frac{1}{x}\right)}{\sqrt{\pi}\,\left(-1+\sinh\left(\frac{1}{x}\right)\right)x^2}$$

$$\text{"F(x)"}, \frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\,\frac{\sqrt{2}\,\left(\ln(2)\,x-\ln\left(\mathrm{e}^{\frac{2}{x}}-2\,\mathrm{e}^{\frac{1}{x}}-1\right)x+x+1\right)}{x}\right)$$

$$\text{"S(x)"}, \frac{1}{2}-\frac{1}{2}\operatorname{erf}\left(\frac{1}{4}\,\frac{\sqrt{2}\,\left(\ln(2)\,x-\ln\left(\mathrm{e}^{\frac{2}{x}}-2\,\mathrm{e}^{\frac{1}{x}}-1\right)x+x+1\right)}{x}\right)$$

$$\text{"h(x)"}, -\frac{1}{2}\left(\sqrt{2}\,\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)-1\right)^2}\cosh\left(\frac{1}{x}\right)\right) \Big/ \left(\sqrt{\pi}\,\left(-1+\sinh\left(\frac{1}{x}\right)\right)x^2\left(-1\right.\right.$$

$$\left.\left.+\operatorname{erf}\left(\frac{1}{4}\,\frac{\sqrt{2}\,\left(\ln(2)\,x-\ln\left(\mathrm{e}^{\frac{2}{x}}-2\,\mathrm{e}^{\frac{1}{x}}-1\right)x+x+1\right)}{x}\right)\right)\right)$$

$$\text{"mean and variance"}, \frac{1}{4}\,\frac{\sqrt{2}\left(\displaystyle\int_0^{\frac{1}{\ln\left(1+\sqrt{2}\right)}} \frac{\mathrm{e}^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)-1\right)^2}\cosh\left(\frac{1}{x}\right)}{x\left(-1+\sinh\left(\frac{1}{x}\right)\right)}\,\mathrm{d}x\right)}{\sqrt{\pi}},$$

$$-\frac{1}{8}\frac{1}{\pi^{3/2}}\left(\left(\int_0^{\frac{1}{\ln(1+\sqrt{2})}}\frac{e^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)-1\right)^2}\cosh\left(\frac{1}{x}\right)}{x\left(-1+\sinh\left(\frac{1}{x}\right)\right)}\,dx\right)^2\sqrt{\pi}-2\sqrt{2}\left(\int_0^{\frac{1}{\ln(1+\sqrt{2})}}\frac{e^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)-1\right)^2}\cosh\left(\frac{1}{x}\right)}{-1+\sinh\left(\frac{1}{x}\right)}\,dx\right)\pi\right)$$

$$mf:=\int_0^{\frac{1}{\ln(1+\sqrt{2})}}\frac{1}{4}\frac{x^{r\sim}\sqrt{2}\,e^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)-1\right)^2}\cosh\left(\frac{1}{x}\right)}{\sqrt{\pi}\left(-1+\sinh\left(\frac{1}{x}\right)\right)x^2}\,dx$$

$$\text{"MF"},\int_0^{\frac{1}{\ln(1+\sqrt{2})}}\frac{1}{4}\frac{x^{r\sim}\sqrt{2}\,e^{-\frac{1}{8}\left(\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)-1\right)^2}\cosh\left(\frac{1}{x}\right)}{\sqrt{\pi}\left(-1+\sinh\left(\frac{1}{x}\right)\right)x^2}\,dx$$

$$\text{"MGF"},\frac{1}{4}\frac{\sqrt{2}\left(\int_0^{\frac{1}{\ln(1+\sqrt{2})}}\frac{e^{-\frac{1}{8}+tx-\frac{1}{8}\ln\left(-1+\sinh\left(\frac{1}{x}\right)\right)^2}\cosh\left(\frac{1}{x}\right)}{\left(-1+\sinh\left(\frac{1}{x}\right)\right)^{3/4}x^2}\,dx\right)}{\sqrt{\pi}}$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( -1+
\sinh
 \left( {x}^{-1} \right)  \right) -1 \right) ^{2}}}\cosh \left(
{x}^{-
1} \right) }{\sqrt {\pi} \left( -1+\sinh \left( {x}^{-1} \right)

 \right) {x}^{2}}}
```

"i is", 19,
" ---
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$$g := t \rightarrow \frac{1}{\operatorname{csch}(t)} + 1$$
$$l := 0$$
$$u := \infty$$
$$Temp := \left[\left[\left[y\sim \rightarrow \frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}\left(\ln\left(\operatorname{arccsch}\left(\frac{1}{y\sim - 1}\right)\right) - 1\right)^2}}{\sqrt{\pi}\, \sqrt{y\sim^2 - 2\, y\sim + 2}\, \operatorname{arccsch}\left(\frac{1}{y\sim - 1}\right)}\right], [1, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]\right]$$

$$\text{"l and u"}, 0, \infty$$
$$\text{"g(x)"}, \frac{1}{\operatorname{csch}(x)} + 1, \text{"base"}, \frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}(\ln(x) - 1)^2}}{\sqrt{\pi}\, x}, \text{"LogNormalRV(1, 2)"}$$
$$\text{"f(x)"}, \frac{1}{4} \frac{\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}\left(\ln\left(\operatorname{arccsch}\left(\frac{1}{x - 1}\right)\right) - 1\right)^2}}{\sqrt{\pi}\, \sqrt{x^2 - 2\, x + 2}\, \operatorname{arccsch}\left(\frac{1}{x - 1}\right)}$$
$$\text{"F(x)"}, \frac{1}{4} \frac{\sqrt{2} \left(\int_1^x \frac{\mathrm{e}^{-\frac{1}{8}\left(\ln\left(\operatorname{arccsch}\left(\frac{1}{t - 1}\right)\right) - 1\right)^2}}{\sqrt{t^2 - 2\, t + 2}\, \operatorname{arccsch}\left(\frac{1}{t - 1}\right)} \, \mathrm{d}t\right)}{\sqrt{\pi}}$$
$$\text{"S(x)"}, \frac{1}{4} \frac{-\sqrt{2} \left(\int_1^x \frac{\mathrm{e}^{-\frac{1}{8}\left(\ln\left(\operatorname{arccsch}\left(\frac{1}{t - 1}\right)\right) - 1\right)^2}}{\sqrt{t^2 - 2\, t + 2}\, \operatorname{arccsch}\left(\frac{1}{t - 1}\right)} \, \mathrm{d}t\right) + 4\sqrt{\pi}}{\sqrt{\pi}}$$
"h(x)",
$$\left(\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}\left(\ln\left(\operatorname{arccsch}\left(\frac{1}{x - 1}\right)\right) - 1\right)^2}\right) \Bigg/ \left(\sqrt{x^2 - 2\, x + 2}\, \operatorname{arccsch}\left(\frac{1}{x - 1}\right) \left(-\sqrt{2} \left(\right.\right.\right.$$

$$\int_1^x \frac{e^{-\frac{1}{8}\left(\ln\left(\operatorname{arccsch}\left(\frac{1}{t-1}\right)\right)-1\right)^2}}{\sqrt{t^2-2t+2}\ \operatorname{arccsch}\left(\frac{1}{t-1}\right)}\,dt\Bigg) + 4\sqrt{\pi}\Bigg)\Bigg)$$

"mean and variance", ∞, *undefined*

$$mf := \infty$$

"MF", ∞

$$\text{"MGF"}, \int_1^\infty \frac{1}{4}\,\frac{e^{-\frac{1}{8}-\frac{1}{8}\ln\left(\operatorname{arccsch}\left(\frac{1}{x-1}\right)\right)^2+tx}\sqrt{2}}{\operatorname{arccsch}\left(\frac{1}{x-1}\right)^{3/4}\sqrt{x^2-2x+2}\,\sqrt{\pi}}\,dx$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left(
{\rm arccsch} \left(  \left( x-1 \right) ^{-1}\right) \right) -1
 \right) ^{2}}}}{\sqrt {\pi}\sqrt {{x}^{2}-2\,x+2}{\rm arccsch}
\left(
 \left( x-1 \right) ^{-1}\right)}}
```

"i is", 20,

" --"

$$g := t \to \tanh\left(\frac{1}{t}\right)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to -\frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(y\sim)}\right)-1\right)^2}}{\sqrt{\pi}\,\operatorname{arctanh}(y\sim)\,(y\sim^2-1)}\right], [0, 1], [\text{"Continuous"}, \text{"PDF"}]\right]$$

"l and u", 0, ∞

$$\text{"g(x)"}, \tanh\left(\frac{1}{x}\right), \text{"base"}, \frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\,x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, -\frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(x)}\right)-1\right)^2}}{\sqrt{\pi}\,\operatorname{arctanh}(x)\,(x^2-1)}$$

$$\text{"F(x)"}, -\frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(t)}\right)-1\right)^2}}{\operatorname{arctanh}(t)\,(t^2-1)}\,dt\right)}{\sqrt{\pi}}$$

$$\text{"S(x)"},\ \frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(t)}\right)-1\right)^2}}{\operatorname{arctanh}(t)\,(t^2-1)}\,dt\right)+4\sqrt{\pi}}{\sqrt{\pi}}$$

$$\text{"h(x)"},\ -\frac{\sqrt{2}\,e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(x)}\right)-1\right)^2}}{\operatorname{arctanh}(x)\,(x^2-1)\left(\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(t)}\right)-1\right)^2}}{\operatorname{arctanh}(t)\,(t^2-1)}\,dt\right)+4\sqrt{\pi}\right)}$$

$$\text{"mean and variance"},\ -\frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^1 \frac{x\,e^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))+1)^2}}{\operatorname{arctanh}(x)\,(x^2-1)}\,dx\right)}{\sqrt{\pi}},$$

$$-\frac{1}{8}\,\frac{\left(\int_0^1 \frac{x\,e^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))+1)^2}}{\operatorname{arctanh}(x)\,(x^2-1)}\,dx\right)^2\sqrt{\pi}+2\sqrt{2}\left(\int_0^1 \frac{x^2\,e^{-\frac{1}{8}(\ln(\operatorname{arctanh}(x))+1)^2}}{\operatorname{arctanh}(x)\,(x^2-1)}\,dx\right)\pi}{\pi^{3/2}}$$

$$mf := \int_0^1 \left(-\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(x)}\right)-1\right)^2}}{\sqrt{\pi}\,\operatorname{arctanh}(x)\,(x^2-1)}\right)dx$$

$$\text{"MF"},\ \int_0^1 \left(-\frac{1}{4}\,\frac{x^{r\sim}\sqrt{2}\,e^{-\frac{1}{8}\left(\ln\left(\frac{1}{\operatorname{arctanh}(x)}\right)-1\right)^2}}{\sqrt{\pi}\,\operatorname{arctanh}(x)\,(x^2-1)}\right)dx$$

$$\text{"MGF"},\ -\frac{1}{4}\,\frac{\sqrt{2}\left(\int_0^1 \frac{e^{-\frac{1}{8}-\frac{1}{8}\ln(\operatorname{arctanh}(x))^2+t\,x}}{\operatorname{arctanh}(x)^{5/4}\,(x^2-1)}\,dx\right)}{\sqrt{\pi}}$$

```
-1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left(
\left(
{\rm arctanh} \left(x\right) \right) ^{-1} \right) -1 \right) ^
{2}}}}{
\sqrt {\pi}{\rm arctanh} \left(x\right) \left( {x}^{2}-1 \right)
}}
```

"i is", 21,

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$$g := t \to \operatorname{csch}\left(\frac{1}{t}\right)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(y\sim))+1)^2}}{\sqrt{\pi}\,\sqrt{y\sim^2+1}\,\operatorname{arccsch}(y\sim)\,|y\sim|}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \operatorname{csch}\left(\frac{1}{x}\right), \text{"base"}, \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(x)-1)^2}}{\sqrt{\pi}\, x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(x))+1)^2}}{\sqrt{\pi}\,\sqrt{x^2+1}\,\operatorname{arccsch}(x)\,|x|}$$

$$\text{"F(x)"}, \frac{1}{4} \frac{\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(t))+1)^2}}{\sqrt{t^2+1}\,\operatorname{arccsch}(t)\,|t|}\, dt\right)}{\sqrt{\pi}}$$

$$\text{"S(x)"}, \frac{1}{4} \frac{-\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(t))+1)^2}}{\sqrt{t^2+1}\,\operatorname{arccsch}(t)\,|t|}\, dt\right) + 4\sqrt{\pi}}{\sqrt{\pi}}$$

$$\text{"h(x)"}, \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(x))+1)^2}}{\sqrt{x^2+1}\,\operatorname{arccsch}(x)\,|x|\left(-\sqrt{2}\left(\int_0^x \frac{e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(t))+1)^2}}{\sqrt{t^2+1}\,\operatorname{arccsch}(t)\,|t|}\, dt\right) + 4\sqrt{\pi}\right)}$$

$$\text{"mean and variance"}, \int_0^\infty \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(x))+1)^2}}{\sqrt{\pi}\,\sqrt{x^2+1}\,\operatorname{arccsch}(x)}\, dx, \int_0^\infty \frac{1}{4} \frac{x\sqrt{2}\, e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(x))+1)^2}}{\sqrt{\pi}\,\sqrt{x^2+1}\,\operatorname{arccsch}(x)}$$

$$dx - \left(\int_0^\infty \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8}(\ln(\operatorname{arccsch}(x))+1)^2}}{\sqrt{\pi}\,\sqrt{x^2+1}\,\operatorname{arccsch}(x)}\, dx\right)^2$$

$$mf := \int_0^{\infty} \frac{1}{4} \frac{x^{r\sim} \sqrt{2}\, e^{-\frac{1}{8} (\ln(\operatorname{arccsch}(x)) + 1)^2}}{\sqrt{\pi} \sqrt{x^2+1}\, \operatorname{arccsch}(x)\, |x|}\, dx$$

$$\text{"MF"}, \int_0^{\infty} \frac{1}{4} \frac{x^{r\sim} \sqrt{2}\, e^{-\frac{1}{8} (\ln(\operatorname{arccsch}(x)) + 1)^2}}{\sqrt{\pi} \sqrt{x^2+1}\, \operatorname{arccsch}(x)\, |x|}\, dx$$

$$\text{"MGF"}, \int_0^{\infty} \frac{1}{4} \frac{e^{-\frac{1}{8} - \frac{1}{8} \ln(\operatorname{arccsch}(x))^2 + t x} \sqrt{2}}{\operatorname{arccsch}(x)^{5/4} x \sqrt{x^2+1} \sqrt{\pi}}\, dx$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left(
{\rm arccsch} \left(x\right) \right) +1 \right) ^{2}}}}{\sqrt
{\pi}
\sqrt {{x}^{2}+1}{\rm arccsch} \left(x\right) \left| x \right| }
}
```

"i is", 22,

" ---
----------"

$$g := t \to \operatorname{arccsch}\left(\frac{1}{t}\right)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8} (\ln(\sinh(y\sim)) - 1)^2} \cosh(y\sim)}{\sqrt{\pi} \sinh(y\sim)}\right], [0, \infty], [\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"}, 0, \infty$$

$$\text{"g(x)"}, \operatorname{arccsch}\left(\frac{1}{x}\right), \text{"base"}, \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8} (\ln(x) - 1)^2}}{\sqrt{\pi}\, x}, \text{"LogNormalRV(1, 2)"}$$

$$\text{"f(x)"}, \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{8} (\ln(\sinh(x)) - 1)^2} \cosh(x)}{\sqrt{\pi} \sinh(x)}$$

$$\text{"F(x)"}, \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{1}{4} \sqrt{2} \left(-\ln(e^x - 1) - \ln(e^x + 1) + \ln(2) + x + 1\right)\right)$$

$$\text{"S(x)"}, \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{1}{4} \sqrt{2} \left(\ln(e^x - 1) + \ln(e^x + 1) - \ln(2) - x - 1\right)\right)$$

$$\text{"h(x)"}, \frac{1}{2} \frac{\sqrt{2}\, e^{-\frac{1}{8} (\ln(\sinh(x)) - 1)^2} \cosh(x)}{\sqrt{\pi} \sinh(x) \left(1 + \operatorname{erf}\left(\frac{1}{4} \sqrt{2} \left(-\ln(e^x - 1) - \ln(e^x + 1) + \ln(2) + x + 1\right)\right)\right)}$$

$$\text{"mean and variance"}, \int_0^{\infty} \frac{1}{4} \frac{x\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}(\ln(\sinh(x))-1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)}\, dx,$$

$$\int_0^{\infty} \frac{1}{4} \frac{x^2\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}(\ln(\sinh(x))-1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)}\, dx$$

$$-\left(\int_0^{\infty} \frac{1}{4} \frac{x\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}(\ln(\sinh(x))-1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)}\, dx\right)^2$$

$$mf := \int_0^{\infty} \frac{1}{4} \frac{x^{r\sim}\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}(\ln(\sinh(x))-1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)}\, dx$$

$$\text{"MF"}, \int_0^{\infty} \frac{1}{4} \frac{x^{r\sim}\sqrt{2}\, \mathrm{e}^{-\frac{1}{8}(\ln(\sinh(x))-1)^2} \cosh(x)}{\sqrt{\pi}\, \sinh(x)}\, dx$$

$$\text{"MGF"}, \int_0^{\infty} \frac{1}{4} \frac{\mathrm{e}^{-\frac{1}{8}+t x-\frac{1}{8}\ln(\sinh(x))^2} \cosh(x)\sqrt{2}}{\sinh(x)^{3/4}\sqrt{\pi}}\, dx$$

```
1/4\,{\frac {\sqrt {2}{{\rm e}^{-1/8\, \left( \ln  \left( \sinh
 \left( x \right)  \right) -1 \right) ^{2}}}\cosh \left( x
\right) }{
\sqrt {\pi}\sinh \left( x \right) }}
```