Gamma Distribution

$$f(x) = \frac{a\ (a\,x)^{b-1}\,\mathrm{e}^{-a\,x}}{\Gamma\,(b)} \qquad x, a, b > 0$$

Transformation	General PDF	Example: Gamma(2,2) PDF	CDF	HF	IDF	μ	σ^2	MF	MGF	HF Shape	Support	Comment
x^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	DFR	$0, \infty$	general ✓
$\sqrt{x}$	✓	✓	✓	✓	∂	✓	✓	✓	✓	IFR	$0, \infty$	general ✓
x^{-1}	✓	✓	✓	✓	✓	✓	✓	✓	✓	UBT	$0, \infty$	inverted gamma
$\arctan(x)$	✓	✓	✓	✓	✓	∂	∂	∂	∂	IFR	$0, \pi/2$	
e^x	✓	✓	✓	✓	✓	✓	∞	∂	∂	UBT	$1, \infty$	
$\ln(x)$	✓	✓	✓	✓	∂	∂	∂	∂	∂	IFR	$-\infty, \infty$	log gamma
e^{-x}	✓	✓	✓	✓	✓	✓	✓	✓	✓	IFR	$0, 1$	
$-\ln(x)$	✓	✓	✓	✓	✓	∂	∂	∂	∂		$-\infty, \infty$	
$\ln(x+1)$	✓	✓	✓	✓	✓	∂	∂	∂	∂	IFR	$0, \infty$	
$1/\ln(x+2)$	✓	✓	✓	✓	✓	∂	∂	∂	∂	IFR	$0, 1/\ln(2)$	
$\tanh(x)$	✓	✓	✓	✓	∂	✓	✓	∂	∂	IFR	$0, 1$	
$\sinh(x)$	✓	✓	✓	✓	✓	✓	∞	∂	∂	UBT	$0, \infty$	memoryless HF
$\mathrm{arcsinh}(x)$	✓	✓	✓	✓	∂	∂	∂	∂	∂	IFR	$0, \infty$	
$\mathrm{csch}(x+1)$	✓	✓	∂	∂		∂	∂	∂	∂	IFR	$0, 2/(-\mathrm{e}+\mathrm{e}^{-1})$	
$\mathrm{arccsch}(x+1)$	✓	✓	✓	✓		∂	∂	∂	∂	IFR	$0, \ln(1+\sqrt{2})$	
$1/\tanh(x+1)$	✓	✓	∂	∂		∂	∂	∂	∂	BT	$1, (\mathrm{e}+\mathrm{e}^{-1})/(\mathrm{e}-\mathrm{e}^{-1})$	
1/sinh(x+1)	✓	✓	✓	✓						IFR	$2, 2/(\mathrm{e}-\mathrm{e}^{-1})$	
$1/\mathrm{arcsinh}(x+1)$	✓	✓	✓	✓	∂			∂		IFR	$0, 1/\ln(1+\sqrt{2})$	
$1/\mathrm{csch}(x)+1$	✓	✓	∂	∂		∂	∂	∂	∂	UBT	$1, \infty$	
$\tanh(x^{-1})$	✓	✓	∂	∂		∂	∂	∂	∂	IFR	$0, 1$	
$\mathrm{csch}(x^{-1})$	✓	✓	∂	∂		∂	∂	∂	∂		$1, \infty$	
$\mathrm{arccsch}(x^{-1})$	✓	✓	✓	✓	∂	∂	∂	∂	∂	IFR	$0, \infty$	

Legend

Symbol	Meaning
✓	Exists, Closed Form
∂	Exists, Not Closed Form
$\varnothing$	Not Possible
	Not Calculated