```
> restart;
  read("c:/appl/appl7.txt");
```

PROCEDURES:

AllPermutations(n), AllCombinations(n, k), Benford(X), BootstrapRV(Data),
CDF:CHF:HF:IDF:PDF:SF(X, [x])), CoefOfVar(X), Convolution(X, Y),
ConvolutionIID(X, n), CriticalPoint(X, prob), Determinant(MATRIX), Difference(X, Y),
Display(X), ExpectedValue(X, [g]), KSTest(X, Data, Parameters), Kurtosis(X),
Maximum(X, Y), MaximumIID(X, n), Mean(X), MGF(X), Minimum(X, Y),
MinimumIID(X, n), Mixture(MixParameters, MixRVs),
MLE(X, Data, Parameters, [Rightcensor]), MLENHPP(X, Data, Parameters, obstime),
MLEWeibull(Data, [Rightcensor]), MOM(X, Data, Parameters),
NextCombination(Previous, size), NextPermutation(Previous), OrderStat(X, n, r, ["wo"]),
PlotDist(X, [low], [high]), PlotEmpCDF(Data, [low], [high]),
PlotEmpCIF(Data, [low], [high]), PlotEmpSF(Data, Censor),
PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]),
PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]),
PlotEmpVsFittedSF(X, Data, Parameters, Censor, low, high),
PPPlot(X, Data, Parameters), Product(X, Y), ProductIID(X, n),
QQPlot(X, Data, Parameters), RangeStat(X, n, ["wo"]), Skewness(X), Transform(X, g),
Truncate(X, low, high), Variance(X), VerifyPDF(X)

Procedure Notation:
X and Y are random variables
Greek letters are numeric or symbolic parameters
x is numeric or symbolic
n and r are positive integers, n >= r
low and high are numeric
g is a function
Brackets [] denote optional parameters
"double quotes" denote character strings
MATRIX is a 2 x 2 array of random variables
A capitalized parameter indicates that it must be
entered as a list --> ex. Data := [1, 12.4, 34, 52.45, 63]

Variate Generation:

ArcTanVariate(alpha, phi), BinomialVariate(n, p, m), ExponentialVariate(lambda),
NormalVariate(mu, sigma), UniformVariate(), WeibullVariate(lambda, kappa, m)

DATA SETS:

BallBearing, HorseKickFatalities, Hurricane, MP6, RatControl, RatTreatment, USSHalfBeak

ArcSinRV(), ArcTanRV(alpha, phi), BetaRV(alpha, beta), CauchyRV(a, alpha), ChiRV(n),

ChiSquareRV(n), ErlangRV(lambda, n), ErrorRV(mu, alpha, d), ExponentialRV(lambda), ExponentialPowerRV(lambda, kappa), ExtremeValueRV(alpha, beta), FRV(n1, n2), GammaRV(lambda, kappa), GeneralizedParetoRV(gamma, delta, kappa), GompertzRV(delta, kappa), HyperbolicSecantRV(), HyperExponentialRV(p, l), HypoExponentialRV(l), IDBRV(gamma, delta, kappa), InverseGaussianRV(lambda, mu), InvertedGammaRV(alpha, beta), KSRV(n), LaPlaceRV(omega, theta), LogGammaRV(alpha, beta), LogisticRV(kappa, lambda), LogLogisticRV(lambda, kappa), LogNormalRV(mu, sigma), LomaxRV(kappa, lambda), MakehamRV(gamma, delta, kappa), MuthRV(kappa), NormalRV(mu, sigma), ParetoRV(lambda, kappa), RayleighRV(lambda), StandardCauchyRV(), StandardNormalRV(), StandardTriangularRV(m), StandardUniformRV(), TRV(n), TriangularRV(a, m, b), UniformRV(a, b), WeibullRV(lambda, kappa)

Error, attempting to assign to `DataSets` which is protected. Try declaring `local DataSets`; see ?protect for details.

```
> bf := MakehamRV(a,b,c);
  bfname := "MakehamRV(a,b,c)";
Originally b, renamed b~:
  is assumed to be: RealRange(Open(0),infinity)

Originally c, renamed c~:
  is assumed to be: RealRange(Open(1),infinity)

Originally a, renamed a~:
  is assumed to be: RealRange(Open(0),infinity)
```

$$bf := \left[\left[x \to \left(a\sim + b\sim\, c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}\right], [0, \infty], ["\text{Continuous}", "\text{PDF}"]\right]$$

$$bfname := \text{"MakehamRV(a,b,c)"} \quad \textbf{(1)}$$

```
> #plot(1/csch(t)+1, t = 0..0.0010);
  #plot(diff(1/csch(t),t), t=0..0.0010);
  #limit(1/csch(t), t=0);
> solve(exp(-t) = y, t);
```

$$-\ln(y) \quad \textbf{(2)}$$

```
> # discarded  -ln(t + 1),  t-> csch(t),t->arccsch(t),t -> tan(t),
> #name of the file for latex output
  filename := "C:/Latex_Output_2/Makeham_Gen.tex";


  glist := [t -> t^2 , t -> sqrt(t),  t -> 1/t,  t -> arctan(t),  t
  -> exp(t), t -> ln(t), t -> exp(-t), t -> -ln(t),  t -> ln(t+1),
  t -> 1/(ln(t+2)), t -> tanh(t), t -> sinh(t), t -> arcsinh(t),
  t-> csch(t+1),t->arccsch(t+1), t-> 1/tanh(t+1), t->  1/sinh(t+1),
   t->  1/arcsinh(t+1), t->  1/csch(t)+1, t-> tanh(1/t), t->csch
  (1/t), t-> arccsch(1/t), t-> arctanh(1/t)  ]:
```

```
base := t -> PDF(bf, t):

print(base(x)):

#begin latex file formatting
appendto(filename);
  printf("\\documentclass[12pt]{article} \n");
  printf("\\usepackage{amsfonts} \n");
  printf("\\begin{document} \n");
  print(bfname);
  printf("$$");
  latex(bf[1]);
  printf("$$");
writeto(terminal);

#begin loopint through transformations
for i from 1 to 22 do
#for i from 1 to 3 do
   print( "i is", i, "  ----------------------------------------
----------------------------------------------------------------
-------------");

   g := glist[i]:
   l := bf[2][1];
   u := bf[2][2];
   Temp := Transform(bf, [[unapply(g(x), x)],[l,u]]);

  #terminal output
  print( "l and u", l, u );
  print("g(x)", g(x), "base", base(x),bfname);
  print("f(x)", PDF(Temp, x));



  #latex output
  appendto(filename);
  printf("----------------------------------------------------------
-----------------------------------  \\\\");
  printf("$$");
  latex(glist[i]);
  printf("$$");
  printf("Probability Distribution Function \n$$  f(x)=");
  latex(PDF(Temp,x));
  printf(" \\qquad");
  latex(Temp[2][1]);
  printf(" < x < ");
  latex(Temp[2][2]);
  printf("$$");



  writeto(terminal);


od;

#final latex output
```

```
appendto(filename);
printf("\\end{document}\n");
writeto(terminal);
```

$$filename := \text{"C:/Latex\_Output\_2/Makeham\_Gen.tex"}$$

$$\left(a\sim + b\sim\, c\sim^{x}\right)\, \mathrm{e}^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$$

"i is", 1,

" ----------"

$$g := t \to t^{2}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{2}\, \frac{\left(a\sim + b\sim\, c\sim^{\sqrt{y\sim}}\right)\, \mathrm{e}^{-\frac{a\sim\sqrt{y\sim}\,\ln(c\sim) + b\sim\, c\sim^{\sqrt{y\sim}} - b\sim}{\ln(c\sim)}}}{\sqrt{y\sim}}\right], [0,\ \infty], [\text{"Continuous"},\ \text{"PDF"}]\right]$$

"l and u", 0, ∞

"g(x)", x^{2}, "base", $\left(a\sim + b\sim\, c\sim^{x}\right)\, \mathrm{e}^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{1}{2}\, \frac{\left(a\sim + b\sim\, c\sim^{\sqrt{x}}\right)\, \mathrm{e}^{-\frac{a\sim\sqrt{x}\,\ln(c\sim) + b\sim\, c\sim^{\sqrt{x}} - b\sim}{\ln(c\sim)}}}{\sqrt{x}}$

"i is", 2,

" ----------"

$$g := t \to \sqrt{t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to 2\, y\sim\, \mathrm{e}^{-\frac{a\sim\, y\sim^{2}\ln(c\sim) + b\sim\, c\sim^{y\sim^{2}} - b\sim}{\ln(c\sim)}}\, \left(a\sim + b\sim\, c\sim^{y\sim^{2}}\right)\right], [0,\ \infty], [\text{"Continuous"},\ \text{"PDF"}]\right]$$

"l and u", 0, ∞

"g(x)", $\sqrt{x}$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right)\, \mathrm{e}^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

$$\text{"f(x)"},\ 2\,x\,e^{-\frac{a\sim x^2\ln(c\sim)\ +b\sim c\sim^{x^2}-b\sim}{\ln(c\sim)}}\left(a\sim+b\sim c\sim^{x^2}\right)$$

"i is", 3,

" ----------"

$$g := t\to\frac{1}{t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim\to\frac{e^{-\frac{b\sim y\sim c\sim^{\frac{1}{y\sim}}+a\sim\ln(c\sim)-b\sim y\sim}{y\sim\ln(c\sim)}}\left(a\sim+b\sim c\sim^{\frac{1}{y\sim}}\right)}{y\sim^2}\right],[0,\ \infty],[\text{"Continuous"},\ \text{"PDF"}]\right]$$

$$\text{"l and u"},\ 0,\ \infty$$

$$\text{"g(x)"},\ \frac{1}{x},\ \text{"base"},\ \left(a\sim+b\sim c\sim^{x}\right)\ e^{-a\sim x-\frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}},\ \text{"MakehamRV(a,b,c)"}$$

$$\text{"f(x)"},\ \frac{e^{-\frac{b\sim x c\sim^{\frac{1}{x}}+a\sim\ln(c\sim)-b\sim x}{x\ln(c\sim)}}\left(a\sim+b\sim c\sim^{\frac{1}{x}}\right)}{x^2}$$

"i is", 4,

" ----------"

$$g := t\to\arctan(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim\to\left(a\sim+b\sim c\sim^{\tan(y\sim)}\right)\ e^{-\frac{a\sim\tan(y\sim)\ln(c\sim)\ +b\sim c\sim^{\tan(y\sim)}-b\sim}{\ln(c\sim)}}\left(1+\tan(y\sim)^2\right)\right],\left[0,\ \frac{1}{2}\,\pi\right],[\text{"Continuous"},\ \text{"PDF"}]\right]$$

$$\text{"l and u"},\ 0,\ \infty$$

$$\text{"g(x)"},\ \arctan(x),\ \text{"base"},\ \left(a\sim+b\sim c\sim^{x}\right)\ e^{-a\sim x-\frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}},\ \text{"MakehamRV(a,b,c)"}$$

$$\text{"f(x)"},\ \left(a\sim+b\sim c\sim^{\tan(x)}\right)\ e^{-\frac{a\sim\tan(x)\ln(c\sim)\ +b\sim c\sim^{\tan(x)}-b\sim}{\ln(c\sim)}}\left(1+\tan(x)^2\right)$$

"i is", 5,

" ----------"

$$g := t \to e^{t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{e^{-\frac{b\sim\left(y\sim^{\ln(c\sim)}-1\right)}{\ln(c\sim)}}\left(y\sim^{-a\sim}\,a\sim + y\sim^{-a\sim+\ln(c\sim)}\,b\sim\right)}{y\sim}\right], [1, \infty], ["Continuous", "PDF"]\right]$$

"l and u", 0, ∞

"g(x)", e^{x}, "base", $\left(a\sim + b\sim\, c\sim^{x}\right)\, e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{e^{-\frac{b\sim\left(x^{\ln(c\sim)}-1\right)}{\ln(c\sim)}}\left(x^{-a\sim}\,a\sim + x^{-a\sim+\ln(c\sim)}\,b\sim\right)}{x}$

"i is", 6,

" ----------"

$$g := t \to \ln(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to e^{-\frac{a\sim\, e^{y\sim}\ln(c\sim) + b\sim\, c\sim^{e^{y\sim}} - y\sim\ln(c\sim) - b\sim}{\ln(c\sim)}}\left(a\sim + b\sim\, c\sim^{e^{y\sim}}\right)\right], [-\infty, \infty], ["Continuous", "PDF"]\right]$$

"l and u", 0, ∞

"g(x)", $\ln(x)$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right)\, e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $e^{-\frac{a\sim\, e^{x}\ln(c\sim) + b\sim\, c\sim^{e^{x}} - x\ln(c\sim) - b\sim}{\ln(c\sim)}}\left(a\sim + b\sim\, c\sim^{e^{x}}\right)$

"i is", 7,

" ----------"

$$g := t \to e^{-t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{e^{-\frac{b\sim\left(y\sim^{-\ln(c\sim)}-1\right)}{\ln(c\sim)}}\left(y\sim^{a\sim}\,a\sim + y\sim^{a\sim-\ln(c\sim)}\,b\sim\right)}{y\sim}\right], [0, 1], ["Continuous",$$

"PDF"]]

"l and u", 0, ∞

"g(x)", e^{-x}, "base", $(a\sim + b\sim c\sim^{x})\ e^{-a\sim x - \frac{b\sim (c\sim^{x}-1)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{e^{-\frac{b\sim (x^{-\ln(c\sim)}-1)}{\ln(c\sim)}}\ (x^{a\sim}\ a\sim + x^{a\sim-\ln(c\sim)}\ b\sim)}{x}$

"i is", 8,

" ----------"

$g := t \to -\ln(t)$

$l := 0$

$u := \infty$

$Temp := [[y\sim \to e^{-\frac{a\sim e^{-y\sim}\ln(c\sim) + b\sim c\sim^{e^{-y\sim}} + y\sim\ln(c\sim) - b\sim}{\ln(c\sim)}}\ (a\sim + b\sim c\sim^{e^{-y\sim}})], [-\infty, \infty],$

["Continuous", "PDF"]]

"l and u", 0, ∞

"g(x)", $-\ln(x)$, "base", $(a\sim + b\sim c\sim^{x})\ e^{-a\sim x - \frac{b\sim (c\sim^{x}-1)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $e^{-\frac{a\sim e^{-x}\ln(c\sim) + b\sim c\sim^{e^{-x}} + x\ln(c\sim) - b\sim}{\ln(c\sim)}}\ (a\sim + b\sim c\sim^{e^{-x}})$

"i is", 9,

" ----------"

$g := t \to \ln(t+1)$

$l := 0$

$u := \infty$

$Temp := [[y\sim \to e^{-\frac{a\sim e^{y\sim}\ln(c\sim) + b\sim c\sim^{e^{y\sim}-1} - a\sim\ln(c\sim) - y\sim\ln(c\sim) - b\sim}{\ln(c\sim)}}\ (a\sim + b\sim c\sim^{e^{y\sim}-1})], [0, \infty],$

["Continuous", "PDF"]]

"l and u", 0, ∞

"g(x)", $\ln(x+1)$, "base", $(a\sim + b\sim c\sim^{x})\ e^{-a\sim x - \frac{b\sim (c\sim^{x}-1)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $e^{-\frac{a\sim e^{x}\ln(c\sim) + b\sim c\sim^{e^{x}-1} - a\sim\ln(c\sim) - x\ln(c\sim) - b\sim}{\ln(c\sim)}}\ (a\sim + b\sim c\sim^{e^{x}-1})$

"i is", 10,

" ----------
----------"

$$g := t \to \frac{1}{\ln(t+2)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{e^{-\frac{a\sim e^{\frac{1}{y\sim}} \ln(c\sim)\, y\sim + b\sim c\sim^{e^{\frac{1}{y\sim}}} - 2 y\sim - 2\, a\sim \ln(c\sim)\, y\sim - b\sim y\sim - \ln(c\sim)}{\ln(c\sim)\, y\sim}} \left(a\sim + b\sim c\sim^{e^{\frac{1}{y\sim}}} - 2\right)}{y\sim^2}\right], \left[0, \frac{1}{\ln(2)}\right], \left["Continuous", "PDF"\right]\right]$$

"l and u", 0, ∞

"g(x)", $\frac{1}{\ln(x+2)}$, "base", $\left(a\sim + b\sim c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{e^{-\frac{a\sim e^{\frac{1}{x}} \ln(c\sim)\, x + b\sim c\sim^{e^{\frac{1}{x}}} - 2x - 2\, a\sim \ln(c\sim)\, x - b\sim x - \ln(c\sim)}{\ln(c\sim)\, x}} \left(a\sim + b\sim c\sim^{e^{\frac{1}{x}}} - 2\right)}{x^2}$

"i is", 11,

" ----------
----------"

$$g := t \to \tanh(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to -\frac{e^{-\frac{a\sim \operatorname{arctanh}(y\sim) \ln(c\sim) + b\sim c\sim^{\operatorname{arctanh}(y\sim)} - b\sim}{\ln(c\sim)}} \left(a\sim + b\sim c\sim^{\operatorname{arctanh}(y\sim)}\right)}{y\sim^2 - 1}\right], [0, 1], \left["Continuous", "PDF"\right]\right]$$

"l and u", 0, ∞

"g(x)", $\tanh(x)$, "base", $\left(a\sim + b\sim c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

$$\text{"f(x)"},\ -\frac{e^{-\frac{a\sim \operatorname{arctanh}(x)\ \ln(c\sim)\ +\ b\sim\ c\sim^{\operatorname{arctanh}(x)}\ -\ b\sim}{\ln(c\sim)}}\left(a\sim + b\sim\ c\sim^{\operatorname{arctanh}(x)}\right)}{x^2-1}$$

"i is", 12,

" ----------"

$$g := t \to \sinh(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{\left(a\sim + b\sim\ c\sim^{\operatorname{arcsinh}(y\sim)}\right) e^{-\frac{a\sim \operatorname{arcsinh}(y\sim)\ \ln(c\sim)\ +\ b\sim\ c\sim^{\operatorname{arcsinh}(y\sim)}\ -\ b\sim}{\ln(c\sim)}}}{\sqrt{y\sim^2+1}}\right], [0,\ \infty],\right.$$

$$\left.[\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"},\ 0,\ \infty$$

$$\text{"g(x)"}, \sinh(x), \text{"base"}, \left(a\sim + b\sim\ c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}, \text{"MakehamRV(a,b,c)"}$$

$$\text{"f(x)"},\ \frac{\left(a\sim + b\sim\ c\sim^{\operatorname{arcsinh}(x)}\right) e^{-\frac{a\sim \operatorname{arcsinh}(x)\ \ln(c\sim)\ +\ b\sim\ c\sim^{\operatorname{arcsinh}(x)}\ -\ b\sim}{\ln(c\sim)}}}{\sqrt{x^2+1}}$$

"i is", 13,

" ----------"

$$g := t \to \operatorname{arcsinh}(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \left(a\sim + b\sim\ c\sim^{\sinh(y\sim)}\right) e^{-\frac{a\sim \sinh(y\sim)\ \ln(c\sim)\ +\ b\sim\ c\sim^{\sinh(y\sim)}\ -\ b\sim}{\ln(c\sim)}} \cosh(y\sim)\right], [0,\ \infty],\right.$$

$$\left.[\text{"Continuous"}, \text{"PDF"}]\right]$$

$$\text{"l and u"},\ 0,\ \infty$$

$$\text{"g(x)"}, \operatorname{arcsinh}(x), \text{"base"}, \left(a\sim + b\sim\ c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}, \text{"MakehamRV(a,b,c)"}$$

$$\text{"f(x)"},\ \left(a\sim + b\sim\ c\sim^{\sinh(x)}\right) e^{-\frac{a\sim \sinh(x)\ \ln(c\sim)\ +\ b\sim\ c\sim^{\sinh(x)}\ -\ b\sim}{\ln(c\sim)}} \cosh(x)$$

"i is", 14,

" ----------"

$$g := t \to \operatorname{csch}(t+1)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{\left(a\sim + b\sim\, c\sim^{-1 + \operatorname{arccsch}(y\sim)}\right) e^{-\frac{\ln(c\sim)\, a\sim \operatorname{arccsch}(y\sim) + b\sim\, c\sim^{-1 + \operatorname{arccsch}(y\sim)} - a\sim \ln(c\sim) - b\sim}{\ln(c\sim)}}}{\sqrt{y\sim^2 + 1}\ |y\sim|} \right], \left[0, \frac{2}{e - e^{-1}}\right], \left["Continuous", "PDF"\right]\right]$$

"l and u", 0, ∞

"g(x)", $\operatorname{csch}(x + 1)$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x} - 1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{\left(a\sim + b\sim\, c\sim^{-1 + \operatorname{arccsch}(x)}\right) e^{-\frac{\ln(c\sim)\, a\sim \operatorname{arccsch}(x) + b\sim\, c\sim^{-1 + \operatorname{arccsch}(x)} - a\sim \ln(c\sim) - b\sim}{\ln(c\sim)}}}{\sqrt{x^2 + 1}\ |x|}$

"i is", 15,

" ----------"

$$g := t \to \operatorname{arccsch}(t + 1)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{1}{\sinh(y\sim)^2} \left(\left(a\sim + b\sim\, c\sim^{-\frac{\sinh(y\sim) - 1}{\sinh(y\sim)}} \right) e^{\frac{-a\sim \ln(c\sim) + a\sim \sinh(y\sim) \ln(c\sim) - b\sim\, c\sim^{-\frac{\sinh(y\sim) - 1}{\sinh(y\sim)}} \sinh(y\sim) + b\sim \sinh(y\sim)}{\sinh(y\sim) \ln(c\sim)}} \cosh(y\sim) \right) \right], \left[0, \ln\left(1 + \sqrt{2}\right)\right], \left["Continuous", "PDF"\right]\right]$$

"l and u", 0, ∞

"g(x)", $\operatorname{arccsch}(x + 1)$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x} - 1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)",

$$\frac{1}{\sinh(x)^2}\left(\left(a\sim + b\sim\, c\sim^{-\frac{\sinh(x)-1}{\sinh(x)}}\right) e^{\frac{-a\sim\ln(c\sim)+a\sim\sinh(x)\ln(c\sim)-b\sim\, c\sim^{-\frac{\sinh(x)-1}{\sinh(x)}}\sinh(x)+b\sim\sinh(x)}{\sinh(x)\ln(c\sim)}}\cosh(x)\right)$$

"i is", 16,

" ----------"

$$g := t \to \frac{1}{\tanh(t+1)}$$
$$l := 0$$
$$u := \infty$$

$$Temp := \left[\left[\left[y\sim \to \frac{e^{-\frac{\ln(c\sim)\,a\sim\operatorname{arctanh}\left(\frac{1}{y\sim}\right)-a\sim\ln(c\sim)+b\sim\, c\sim^{-1+\operatorname{arctanh}\left(\frac{1}{y\sim}\right)}-b\sim}{\ln(c\sim)}}\left(a\sim + b\sim\, c\sim^{-1+\operatorname{arctanh}\left(\frac{1}{y\sim}\right)}\right)}{y\sim^2-1}\right], \left[1, \frac{e+e^{-1}}{e-e^{-1}}\right], \left["\text{Continuous}", "\text{PDF}"\right]\right]$$

"l and u", 0, ∞

"g(x)", $\frac{1}{\tanh(x+1)}$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right) e^{-a\sim x-\frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{e^{-\frac{\ln(c\sim)\,a\sim\operatorname{arctanh}\left(\frac{1}{x}\right)-a\sim\ln(c\sim)+b\sim\, c\sim^{-1+\operatorname{arctanh}\left(\frac{1}{x}\right)}-b\sim}{\ln(c\sim)}}\left(a\sim + b\sim\, c\sim^{-1+\operatorname{arctanh}\left(\frac{1}{x}\right)}\right)}{x^2-1}$

"i is", 17,

" ----------"

$$g := t \to \frac{1}{\sinh(t+1)}$$
$$l := 0$$
$$u := \infty$$

$$Temp := \left[\left[y\sim \rightarrow \frac{\left(a\sim + b\sim\, c\sim^{-1+\operatorname{arcsinh}\left(\frac{1}{y\sim}\right)}\right) e^{-\frac{\ln(c\sim)\, a\sim \operatorname{arcsinh}\left(\frac{1}{y\sim}\right) - a\sim \ln(c\sim) + b\sim\, c\sim^{-1+\operatorname{arcsinh}\left(\frac{1}{y\sim}\right)} - b\sim}{\ln(c\sim)}}}{\sqrt{y\sim^2+1}\ |y\sim|} \right], \left[0, \frac{2}{e - e^{-1}}\right], [\text{"Continuous"}, \text{"PDF"}]\right]$$

"l and u", 0, ∞

"g(x)", $\frac{1}{\sinh(x+1)}$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{\left(a\sim + b\sim\, c\sim^{-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)}\right) e^{-\frac{\ln(c\sim)\, a\sim \operatorname{arcsinh}\left(\frac{1}{x}\right) - a\sim \ln(c\sim) + b\sim\, c\sim^{-1+\operatorname{arcsinh}\left(\frac{1}{x}\right)} - b\sim}{\ln(c\sim)}}}{\sqrt{x^2+1}\ |x|}$

"i is", 18,

" ----------"

$$g := t \rightarrow \frac{1}{\operatorname{arcsinh}(t+1)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \rightarrow \frac{1}{y\sim^2}\left(\left(a\sim + b\sim\, c\sim^{-1+\sinh\left(\frac{1}{y\sim}\right)}\right) e^{-\frac{\ln(c\sim)\, a\sim \sinh\left(\frac{1}{y\sim}\right) - a\sim \ln(c\sim) + b\sim\, c\sim^{-1+\sinh\left(\frac{1}{y\sim}\right)} - b\sim}{\ln(c\sim)}} \cosh\left(\frac{1}{y\sim}\right)\right)\right],\right.$$

$$\left[\left[0,\ \frac{1}{\ln\left(1+\sqrt{2}\right)}\right],\ \left["\text{Continuous}",\ "\text{PDF}"\right]\right]$$

"l and u", 0, ∞

"g(x)", $\frac{1}{\operatorname{arcsinh}(x+1)}$, "base", $\left(a\sim+b\sim\, c\sim^{x}\right)\ \mathrm{e}^{-a\sim x-\frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)",

$$\frac{\left(a\sim+b\sim\, c\sim^{-1+\sinh\left(\frac{1}{x}\right)}\right)\ \mathrm{e}^{-\frac{\ln(c\sim)\ a\sim\sinh\left(\frac{1}{x}\right)-a\sim\ln(c\sim)+b\sim\, c\sim^{-1+\sinh\left(\frac{1}{x}\right)}-b\sim}{\ln(c\sim)}}\cosh\left(\frac{1}{x}\right)}{x^{2}}$$

"i is", 19,

" ----------"

$$g := t\to\frac{1}{\operatorname{csch}(t)}+1$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[\left[y\sim\ \to\frac{\left(a\sim+b\sim\, c\sim^{\operatorname{arccsch}\left(\frac{1}{y\sim-1}\right)}\right)\ \mathrm{e}^{-\frac{a\sim\operatorname{arccsch}\left(\frac{1}{y\sim-1}\right)\ln(c\sim)+b\sim\, c\sim^{\operatorname{arccsch}\left(\frac{1}{y\sim-1}\right)}-b\sim}{\ln(c\sim)}}}{\sqrt{y\sim^{2}-2\,y\sim+2}}\right],\ [1,\ \infty\ ]\right],\ \left["\text{Continuous}",\ "\text{PDF}"\right]\right]$$

"l and u", 0, ∞

"g(x)", $\frac{1}{\operatorname{csch}(x)}+1$, "base", $\left(a\sim+b\sim\, c\sim^{x}\right)\ \mathrm{e}^{-a\sim x-\frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{\left(a\sim+b\sim\, c\sim^{\operatorname{arccsch}\left(\frac{1}{x-1}\right)}\right)\ \mathrm{e}^{-\frac{a\sim\operatorname{arccsch}\left(\frac{1}{x-1}\right)\ln(c\sim)+b\sim\, c\sim^{\operatorname{arccsch}\left(\frac{1}{x-1}\right)}-b\sim}{\ln(c\sim)}}}{\sqrt{x^{2}-2\,x+2}}$

"i is", 20,

" ----------"

$$g := t \to \tanh\left(\frac{1}{t}\right)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to -\frac{e^{-\frac{b\sim \operatorname{arctanh}(y\sim)\, c\sim^{\frac{1}{\operatorname{arctanh}(y\sim)}} + a\sim \ln(c\sim) - b\sim \operatorname{arctanh}(y\sim)}{\operatorname{arctanh}(y\sim)\ln(c\sim)}} \left(a\sim + b\sim\, c\sim^{\frac{1}{\operatorname{arctanh}(y\sim)}}\right)}{\operatorname{arctanh}(y\sim)^2 \left(y\sim^2 - 1\right)}\right], [0, 1],\right.$$

$$\left.["\text{Continuous}", "\text{PDF}"]\right]$$

"l and u", 0, ∞

"g(x)", $\tanh\left(\frac{1}{x}\right)$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right) e^{-a\sim x - \frac{b\sim\left(c\sim^{x} - 1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $-\frac{e^{-\frac{b\sim \operatorname{arctanh}(x)\, c\sim^{\frac{1}{\operatorname{arctanh}(x)}} + a\sim \ln(c\sim) - b\sim \operatorname{arctanh}(x)}{\operatorname{arctanh}(x)\ln(c\sim)}} \left(a\sim + b\sim\, c\sim^{\frac{1}{\operatorname{arctanh}(x)}}\right)}{\operatorname{arctanh}(x)^2 \left(x^2 - 1\right)}$

"i is", 21,

" ----------"

$$g := t \to \operatorname{csch}\left(\frac{1}{t}\right)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[\left[y\sim \to \frac{\left(a\sim + b\sim\, c\sim^{\frac{1}{\operatorname{arccsch}(y\sim)}}\right) e^{-\frac{b\sim \operatorname{arccsch}(y\sim)\, c\sim^{\frac{1}{\operatorname{arccsch}(y\sim)}} - b\sim \operatorname{arccsch}(y\sim) + a\sim \ln(c\sim)}{\operatorname{arccsch}(y\sim)\ln(c\sim)}}}{\sqrt{y\sim^2 + 1}\ \operatorname{arccsch}(y\sim)^2\, |y\sim|}\right],\right.$$

$$\left.[0, \infty], ["\text{Continuous}", "\text{PDF}"]\right]$$

"l and u", 0, ∞

"g(x)", $\operatorname{csch}\left(\frac{1}{x}\right)$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right)\, e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\frac{\left(a\sim + b\sim\, c\sim^{\frac{1}{\operatorname{arccsch}(x)}}\right)\, e^{-\frac{b\sim \operatorname{arccsch}(x)\, c\sim^{\frac{1}{\operatorname{arccsch}(x)}} - b\sim \operatorname{arccsch}(x) + a\sim \ln(c\sim)}{\operatorname{arccsch}(x)\, \ln(c\sim)}}}{\sqrt{x^2+1}\, \operatorname{arccsch}(x)^2\, |x|}$

"i is", 22,

" ----------"

$g := t \to \operatorname{arccsch}\left(\frac{1}{t}\right)$

$l := 0$

$u := \infty$

$Temp := \left[\left[y\sim \to \left(a\sim + b\sim\, c\sim^{\sinh(y\sim)}\right)\, e^{-\frac{a\sim \ln(c\sim)\, \sinh(y\sim) + b\sim\, c\sim^{\sinh(y\sim)} - b\sim}{\ln(c\sim)}}\, \cosh(y\sim)\right], [0, \infty],\right.$

$\left.\left["\text{Continuous}", "\text{PDF}"\right]\right]$

"l and u", 0, ∞

"g(x)", $\operatorname{arccsch}\left(\frac{1}{x}\right)$, "base", $\left(a\sim + b\sim\, c\sim^{x}\right)\, e^{-a\sim x - \frac{b\sim\left(c\sim^{x}-1\right)}{\ln(c\sim)}}$, "MakehamRV(a,b,c)"

"f(x)", $\left(a\sim + b\sim\, c\sim^{\sinh(x)}\right)\, e^{-\frac{a\sim \sinh(x)\, \ln(c\sim) + b\sim\, c\sim^{\sinh(x)} - b\sim}{\ln(c\sim)}}\, \cosh(x)$ **(3)**