

```
> restart;  
read("c:/appl/appl7.txt");
```

#### PROCEDURES:

*AllPermutations(n), AllCombinations(n, k), Benford(X), BootstrapRV(Data),  
CDF:CHF:HF:IDF:PDF:SF(X, [x]), CoefOfVar(X), Convolution(X, Y),  
ConvolutionIID(X, n), CriticalPoint(X, prob), Determinant(MATRIX), Difference(X, Y),  
Display(X), ExpectedValue(X, [g]), KSTest(X, Data, Parameters), Kurtosis(X),  
Maximum(X, Y), MaximumIID(X, n), Mean(X), MGF(X), Minimum(X, Y),  
MinimumIID(X, n), Mixture(MixParameters, MixRVs),  
MLE(X, Data, Parameters, [Rightcensor]), MLENHPP(X, Data, Parameters, obstime),  
MLEWeibull(Data, [Rightcensor]), MOM(X, Data, Parameters),  
NextCombination(Previous, size), NextPermutation(Previous), OrderStat(X, n, r, ["wo"]),  
PlotDist(X, [low], [high]), PlotEmpCDF(Data, [low], [high]),  
PlotEmpCIF(Data, [low], [high]), PlotEmpSF(Data, Censor),  
PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]),  
PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]),  
PlotEmpVsFittedSF(X, Data, Parameters, Censor, low, high),  
PPPlot(X, Data, Parameters), Product(X, Y), ProductIID(X, n),  
QQPlot(X, Data, Parameters), RangeStat(X, n, ["wo"]), Skewness(X), Transform(X, g),  
Truncate(X, low, high), Variance(X), VerifyPDF(X)*

#### Procedure Notation:

*X and Y are random variables  
Greek letters are numeric or symbolic parameters  
x is numeric or symbolic  
n and r are positive integers,  $n \geq r$   
low and high are numeric  
g is a function  
Brackets [] denote optional parameters  
"double quotes" denote character strings  
MATRIX is a 2 x 2 array of random variables  
A capitalized parameter indicates that it must be  
entered as a list --> ex. Data := [1, 12.4, 34, 52.45, 63]*

#### Variate Generation:

*ArcTanVariate(alpha, phi), BinomialVariate(n, p, m), ExponentialVariate(lambda),  
NormalVariate(mu, sigma), UniformVariate(), WeibullVariate(lambda, kappa, m)*

#### DATA SETS:

*BallBearing, HorseKickFatalities, Hurricane, MP6, RatControl, RatTreatment, USSHalfBeak  
ArcSinRV(), ArcTanRV(alpha, phi), BetaRV(alpha, beta), CauchyRV(a, alpha), ChiRV(n),*

*ChiSquareRV(n), ErlangRV(lambda, n), ErrorRV(mu, alpha, d), ExponentialRV(lambda),  
 ExponentialPowerRV(lambda, kappa), ExtremeValueRV(alpha, beta), FRV(n1, n2),  
 GammaRV(lambda, kappa), GeneralizedParetoRV(gamma, delta, kappa),  
 GompertzRV(delta, kappa), HyperbolicSecantRV(), HyperExponentialRV(p, l),  
 HypoExponentialRV(l), IDBRV(gamma, delta, kappa), InverseGaussianRV(lambda, mu),  
 InvertedGammaRV(alpha, beta), KSRV(n), LaPlaceRV(omega, theta),  
 LogGammaRV(alpha, beta), LogisticRV(kappa, lambda), LogLogisticRV(lambda, kappa),  
 LogNormalRV(mu, sigma), LomaxRV(kappa, lambda), MakehamRV(gamma, delta, kappa),  
 MuthRV(kappa), NormalRV(mu, sigma), ParetoRV(lambda, kappa), RayleighRV(lambda),  
 StandardCauchyRV(), StandardNormalRV(), StandardTriangularRV(m),  
 StandardUniformRV(), TRV(n), TriangularRV(a, m, b), UniformRV(a, b),  
 WeibullRV(lambda, kappa)*

Error, attempting to assign to `DataSets` which is protected.  
 Try declaring `local DataSets`; see ?protect for details.

```

> bf := InvertedGammaRV(a,b) ;
  bfname := "InvertedGammaRV(a,b)";
Originally a, renamed a~:
  is assumed to be: RealRange(Open(0),infinity)

Originally b, renamed b~:
  is assumed to be: RealRange(Open(0),infinity)

```

$$bf := \left[ \left[ x \rightarrow \frac{x^{-a\sim} - 1}{\Gamma(a\sim)} e^{-\frac{1}{x b\sim}}, [0, \infty], ["Continuous", "PDF"] \right] \right]$$

bfname := "InvertedGammaRV(a,b)"

(1)

```

> #plot(1/csch(t)+1, t = 0..0.0010);
  #plot(diff(1/csch(t),t), t=0..0.0010);
  #limit(1/csch(t), t=0);
> solve(exp(-t) = y, t);

```

-ln(y)

(2)

```

> # discarded -ln(t + 1), t-> csch(t), t->arccsch(t), t -> tan(t),
> #name of the file for latex output
  filename := "C:/LatexOutput/InvertedGamma_Gen.tex";

```

```

glist := [t -> t^2, t -> sqrt(t), t -> 1/t, t -> arctan(t), t
-> exp(t), t -> ln(t), t -> exp(-t), t -> -ln(t), t -> ln(t+1),
t -> 1/(ln(t+2)), t -> tanh(t), t -> sinh(t), t -> arcsinh(t),
t-> csch(t+1), t->arccsch(t+1), t-> 1/tanh(t+1), t-> 1/sinh(t+1),
t-> 1/arcsinh(t+1), t-> 1/csch(t)+1, t-> tanh(1/t), t->csch
(1/t), t-> arccsch(1/t), t-> arctanh(1/t) ]:

```

```

base := t -> PDF(bf, t):

```

```

print(base(x)):

#begin latex file formatting
appendto(filename);
printf("\\documentclass[12pt]{article} \n");
printf("\\usepackage{amsfonts} \n");
printf("\\begin{document} \n");
print(bfname);
printf("$");
latex(bf[1]);
printf("$");
writeto(terminal);

#begin loopint through transformations
for i from 1 to 22 do
#for i from 1 to 3 do
    print( "i is", i, " -----"
-----" );

    g := glist[i];
    l := bf[2][1];
    u := bf[2][2];
    Temp := Transform(bf, [[unapply(g(x), x)], [l,u]]);

#terminal output
print( "l and u", l, u );
print("g(x)", g(x), "base", base(x), bfname);
print("f(x)", PDF(Temp, x));

#latex output
appendto(filename);
printf("----- \\\");
printf("$");
latex(glist[i]);
printf("$");
printf("Probability Distribution Function \n$ f(x)=");
latex(PDF(Temp,x));
printf("$");

    writeto(terminal);

od;

#final latex output
appendto(filename);
printf("\\end{document}\n");
writeto(terminal);

```

*filename* := "C:/LatexOutput/InvertedGamma\_Gen.tex"

$$\frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}$$

"i is", 1,

"-----  
-----"

$$g := t \rightarrow t^2$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{1}{2} \frac{y\sim^{-\frac{1}{2} a\sim-1} e^{-\frac{1}{\sqrt{y\sim} b\sim}}}{\Gamma(a\sim)} b\sim^{-a\sim} \right], [0, \infty], ["Continuous", "PDF"] \right]$$

"l and u", 0,  $\infty$

$$\text{"g(x)", } x^2, \text{"base", } \frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{1}{2} \frac{x^{-\frac{1}{2} a\sim-1} e^{-\frac{1}{\sqrt{x} b\sim}}}{\Gamma(a\sim)} b\sim^{-a\sim}$$

"i is", 2,

"-----  
-----"

$$g := t \rightarrow \sqrt{t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{2 (y\sim^2)^{-a\sim} e^{-\frac{1}{y\sim^2 b\sim}}}{y\sim \Gamma(a\sim)} b\sim^{-a\sim} \right], [0, \infty], ["Continuous", "PDF"] \right]$$

"l and u", 0,  $\infty$

$$\text{"g(x)", } \sqrt{x}, \text{"base", } \frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{2 (x^2)^{-a\sim} e^{-\frac{1}{x^2 b\sim}}}{x \Gamma(a\sim)} b\sim^{-a\sim}$$

"i is", 3,

"-----  
-----"

$$g := t \rightarrow \frac{1}{t}$$

$$\begin{aligned}
& l := 0 \\
& u := \infty \\
& Temp := \left[ \left[ y \rightarrow \frac{\left( \frac{1}{y} \right)^{-a} e^{-\frac{y}{b}} b^{-a}}{y \Gamma(a)}, [0, \infty], ["Continuous", "PDF"] \right] \right. \\
& \quad "l \text{ and } u", 0, \infty \\
& \quad "g(x)", \frac{1}{x}, "base", \frac{x^{-a-1} e^{-\frac{1}{xb}}}{\Gamma(a) b^a}, "InvertedGammaRV(a,b)" \\
& \quad "f(x)", \frac{\left( \frac{1}{x} \right)^{-a} e^{-\frac{x}{b}} b^{-a}}{x \Gamma(a)}
\end{aligned}$$

"i is", 4,

"-----"

$$\begin{aligned}
& g := t \rightarrow \arctan(t) \\
& l := 0 \\
& u := \infty \\
& Temp := \left[ \left[ y \rightarrow \frac{\tan(y)^{-a-1} e^{-\frac{1}{\tan(y) b}} b^{-a} (1 + \tan(y)^2)}{\Gamma(a)}, \left[ 0, \frac{1}{2} \pi \right], ["Continuous", \right. \right. \\
& \quad "PDF"] \left. \right]
\end{aligned}$$

$$\begin{aligned}
& "l \text{ and } u", 0, \infty \\
& "g(x)", \arctan(x), "base", \frac{x^{-a-1} e^{-\frac{1}{xb}}}{\Gamma(a) b^a}, "InvertedGammaRV(a,b)" \\
& "f(x)", \frac{\tan(x)^{-a-1} e^{-\frac{1}{\tan(x) b}} b^{-a} (1 + \tan(x)^2)}{\Gamma(a)}
\end{aligned}$$

"i is", 5,

"-----"

$$\begin{aligned}
& g := t \rightarrow e^t \\
& l := 0 \\
& u := \infty \\
& Temp := \left[ \left[ y \rightarrow \frac{\ln(y)^{-a-1} e^{-\frac{1}{\ln(y) b}} b^{-a}}{\Gamma(a) y}, [1, \infty], ["Continuous", "PDF"] \right] \right. \\
& \quad "l \text{ and } u", 0, \infty
\end{aligned}$$

$$\text{"g(x)", } e^x, \text{"base", } \frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{\ln(x)^{-a\sim-1} e^{-\frac{1}{\ln(x) b\sim}}}{\Gamma(a\sim) x} b\sim^{-a\sim}$$

"i is", 6,

"-----"  
 -----"

$$g := t \rightarrow \ln(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{e^{-\frac{y\sim b\sim a\sim + e^{-y\sim}}{b\sim}}}{\Gamma(a\sim)} b\sim^{-a\sim} \right], [-\infty, \infty], ["Continuous", "PDF"] \right]$$

$$\text{"l and u", } 0, \infty$$

$$\text{"g(x)", } \ln(x), \text{"base", } \frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{e^{-\frac{xb\sim a\sim + e^{-x}}{b\sim}}}{\Gamma(a\sim)} b\sim^{-a\sim}$$

"i is", 7,

"-----"  
 -----"

$$g := t \rightarrow e^{-t}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{(-\ln(y\sim))^{-a\sim-1} e^{\frac{1}{\ln(y\sim) b\sim}}}{\Gamma(a\sim) y\sim} b\sim^{-a\sim} \right], [0, 1], ["Continuous", "PDF"] \right]$$

$$\text{"l and u", } 0, \infty$$

$$\text{"g(x)", } e^{-x}, \text{"base", } \frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{(-\ln(x))^{-a\sim-1} e^{\frac{1}{\ln(x) b\sim}}}{\Gamma(a\sim) x} b\sim^{-a\sim}$$

"i is", 8,

"-----"  
 -----"

$$\begin{aligned}
g &:= t \rightarrow -\ln(t) \\
l &:= 0 \\
u &:= \infty \\
Temp &:= \left[ \left[ y \sim \rightarrow \frac{e^{-\frac{-y \sim b \sim a \sim + e^{y \sim}}{b \sim}}}{\Gamma(a \sim)} b \sim^{-a \sim} \right], [-\infty, \infty], ["Continuous", "PDF"] \right] \\
&\quad "l \text{ and } u", 0, \infty \\
&\quad "g(x)", -\ln(x), "base", \frac{x^{-a \sim - 1} e^{-\frac{1}{x b \sim}}}{\Gamma(a \sim) b \sim^{a \sim}}, "InvertedGammaRV(a,b)" \\
&\quad "f(x)", \frac{e^{-\frac{-x b \sim a \sim + e^x}{b \sim}}}{\Gamma(a \sim)} b \sim^{-a \sim}
\end{aligned}$$

"i is", 9,

"-----"

$$\begin{aligned}
g &:= t \rightarrow \ln(t + 1) \\
l &:= 0 \\
u &:= \infty \\
Temp &:= \left[ \left[ y \sim \rightarrow \frac{(e^{y \sim} - 1)^{-a \sim - 1} e^{\frac{y \sim b \sim e^{y \sim} - y \sim b \sim - 1}{(e^{y \sim} - 1) b \sim}}}{\Gamma(a \sim)} b \sim^{-a \sim} \right], [0, \infty], ["Continuous", "PDF"] \right] \\
&\quad "l \text{ and } u", 0, \infty \\
&\quad "g(x)", \ln(x + 1), "base", \frac{x^{-a \sim - 1} e^{-\frac{1}{x b \sim}}}{\Gamma(a \sim) b \sim^{a \sim}}, "InvertedGammaRV(a,b)" \\
&\quad "f(x)", \frac{(e^x - 1)^{-a \sim - 1} e^{\frac{x b \sim e^x - x b \sim - 1}{(e^x - 1) b \sim}}}{\Gamma(a \sim)} b \sim^{-a \sim}
\end{aligned}$$

"i is", 10,

"-----"

$$\begin{aligned}
g &:= t \rightarrow \frac{1}{\ln(t + 2)} \\
l &:= 0 \\
u &:= \infty \\
Temp &:= \left[ \left[ y \sim \rightarrow \frac{\left( \frac{1}{e^{y \sim}} - 2 \right)^{-a \sim - 1} e^{\frac{b \sim e^{y \sim} - 2 b \sim - y \sim}{\left( \frac{1}{e^{y \sim}} - 2 \right) b \sim y \sim}}}{\Gamma(a \sim) y \sim^2} b \sim^{-a \sim} \right], \left[ 0, \frac{1}{\ln(2)} \right], ["Continuous",
\end{aligned}$$

"PDF"]

"l and u", 0, ∞

"g(x)",  $\frac{1}{\ln(x+2)}$ , "base",  $\frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}$ , "InvertedGammaRV(a,b)"

"f(x)",  $\frac{\left(e^{\frac{1}{x}}-2\right)^{-a\sim-1} e^{\frac{\frac{1}{b\sim}e^x-2b\sim-x}{\left(\frac{1}{e^x}-2\right)b\sim x}}}{\Gamma(a\sim) x^2} b\sim^{-a\sim}$

"i is", 11,

"-----"

$g := t \rightarrow \tanh(t)$

$l := 0$

$u := \infty$

$Temp := \left[ \left[ y\sim \rightarrow -\frac{\operatorname{arctanh}(y\sim)^{-a\sim-1} e^{-\frac{1}{\operatorname{arctanh}(y\sim) b\sim}}}{(y\sim^2-1) \Gamma(a\sim)} b\sim^{-a\sim} \right], [0, 1], ["Continuous", "PDF"] \right]$

"l and u", 0, ∞

"g(x)",  $\tanh(x)$ , "base",  $\frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}$ , "InvertedGammaRV(a,b)"

"f(x)",  $-\frac{\operatorname{arctanh}(x)^{-a\sim-1} e^{-\frac{1}{\operatorname{arctanh}(x) b\sim}}}{(x^2-1) \Gamma(a\sim)} b\sim^{-a\sim}$

"i is", 12,

"-----"

$g := t \rightarrow \sinh(t)$

$l := 0$

$u := \infty$

$Temp := \left[ \left[ y\sim \rightarrow \frac{\operatorname{arcsinh}(y\sim)^{-a\sim-1} e^{-\frac{1}{\operatorname{arcsinh}(y\sim) b\sim}}}{\Gamma(a\sim) \sqrt{y\sim^2+1}} b\sim^{-a\sim} \right], [0, \infty], ["Continuous", "PDF"] \right]$

"l and u", 0, ∞



$$\text{"g(x)", sinh(x), "base", } \frac{x^{-a\sim - 1} e^{-\frac{1}{x b\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{\operatorname{arcsinh}(x)^{-a\sim - 1} e^{-\frac{1}{\operatorname{arcsinh}(x) b\sim}}}{\Gamma(a\sim) \sqrt{x^2 + 1}} b\sim^{-a\sim}$$

"i is", 13,

"-----"  
 -----"

$$g := t \rightarrow \operatorname{arcsinh}(t)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{\sinh(y\sim)^{-a\sim - 1} e^{-\frac{1}{\sinh(y\sim) b\sim}} b\sim^{-a\sim} \cosh(y\sim)}{\Gamma(a\sim)}, [0, \infty], ["Continuous", \right. \right. \\ \left. \left. \text{"PDF"} \right] \right]$$

$$\text{"l and u", } 0, \infty$$

$$\text{"g(x)", arcsinh(x), "base", } \frac{x^{-a\sim - 1} e^{-\frac{1}{x b\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{\sinh(x)^{-a\sim - 1} e^{-\frac{1}{\sinh(x) b\sim}}}{\Gamma(a\sim)} b\sim^{-a\sim} \cosh(x)$$

"i is", 14,

"-----"  
 -----"

$$g := t \rightarrow \operatorname{csch}(t + 1)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{(-1 + \operatorname{arccsch}(y\sim))^{-a\sim - 1} e^{-\frac{1}{(-1 + \operatorname{arccsch}(y\sim)) b\sim}}}{\sqrt{y\sim^2 + 1} \Gamma(a\sim) |y\sim|} b\sim^{-a\sim} \right], \left[ 0, \frac{2}{e - e^{-1}} \right], \right. \\ \left. ["Continuous", "PDF"] \right]$$

$$\text{"l and u", } 0, \infty$$

$$\text{"g(x)", csch(x + 1), "base", } \frac{x^{-a\sim - 1} e^{-\frac{1}{x b\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{(-1 + \operatorname{arccsch}(x))^{-a\sim - 1} e^{-\frac{1}{(-1 + \operatorname{arccsch}(x)) b\sim}} b\sim^{-a\sim}}{\sqrt{x^2 + 1} \Gamma(a\sim) |x|}$$

"i is", 15,  
 "-----"  
 -----"

$$g := t \rightarrow \operatorname{arccsch}(t + 1)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow - \frac{b\sim^{-a\sim} \left( - \frac{\sinh(y\sim) - 1}{\sinh(y\sim)} \right)^{-a\sim} e^{\frac{\sinh(y\sim)}{b\sim (\sinh(y\sim) - 1)}} \cosh(y\sim)}{\Gamma(a\sim) \sinh(y\sim) (\sinh(y\sim) - 1)} \right], \left[ 0, \ln(1 \right.$$

$$\left. + \sqrt{2} \right) \right], \left[ \text{"Continuous"}, \text{"PDF"} \right]$$

$$\text{"l and u", } 0, \infty$$

$$\text{"g(x)", } \operatorname{arccsch}(x + 1), \text{"base", } \frac{x^{-a\sim - 1} e^{-\frac{1}{x b\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } - \frac{b\sim^{-a\sim} \left( - \frac{\sinh(x) - 1}{\sinh(x)} \right)^{-a\sim} e^{\frac{\sinh(x)}{b\sim (\sinh(x) - 1)}} \cosh(x)}{\Gamma(a\sim) \sinh(x) (\sinh(x) - 1)}$$

"i is", 16,  
 "-----"  
 -----"

$$g := t \rightarrow \frac{1}{\tanh(t + 1)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{\left( -1 + \operatorname{arctanh}\left(\frac{1}{y\sim}\right) \right)^{-a\sim - 1} e^{-\frac{1}{\left( -1 + \operatorname{arctanh}\left(\frac{1}{y\sim}\right) \right) b\sim}} b\sim^{-a\sim}}{\Gamma(a\sim) (y\sim^2 - 1)} \right], \left[ 1, \frac{e + e^{-1}}{e - e^{-1}} \right], \right.$$

$$\left. \left[ \text{"Continuous"}, \text{"PDF"} \right] \right]$$

$$\text{"l and u", } 0, \infty$$

$$\text{"g(x)", } \frac{1}{\tanh(x+1)}, \text{"base", } \frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{\left(-1 + \operatorname{arctanh}\left(\frac{1}{x}\right)\right)^{-a\sim-1} e^{-\frac{1}{\left(-1 + \operatorname{arctanh}\left(\frac{1}{x}\right)\right) b\sim}} b\sim^{-a\sim}}{\Gamma(a\sim) (x^2-1)}$$

"i is", 17,

"-----"  
 -----"

$$g := t \rightarrow \frac{1}{\sinh(t+1)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y\sim \rightarrow \frac{\left(-1 + \operatorname{arcsinh}\left(\frac{1}{y\sim}\right)\right)^{-a\sim-1} e^{-\frac{1}{\left(-1 + \operatorname{arcsinh}\left(\frac{1}{y\sim}\right)\right) b\sim}} b\sim^{-a\sim}}{\sqrt{y\sim^2+1} \Gamma(a\sim) |y\sim|} \right], \left[ 0, \frac{2}{e-e^{-1}} \right],$$

["Continuous", "PDF"]

"l and u", 0,  $\infty$

$$\text{"g(x)", } \frac{1}{\sinh(x+1)}, \text{"base", } \frac{x^{-a\sim-1} e^{-\frac{1}{xb\sim}}}{\Gamma(a\sim) b\sim^{a\sim}}, \text{"InvertedGammaRV(a,b)"}$$

$$\text{"f(x)", } \frac{\left(-1 + \operatorname{arcsinh}\left(\frac{1}{x}\right)\right)^{-a\sim-1} e^{-\frac{1}{\left(-1 + \operatorname{arcsinh}\left(\frac{1}{x}\right)\right) b\sim}} b\sim^{-a\sim}}{\sqrt{x^2+1} \Gamma(a\sim) |x|}$$

"i is", 18,

"-----"  
 -----"

$$g := t \rightarrow \frac{1}{\operatorname{arcsinh}(t+1)}$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y \rightarrow \frac{\left( -1 + \sinh\left(\frac{1}{y}\right) \right)^{-a-1} e^{-\frac{1}{\left( -1 + \sinh\left(\frac{1}{y}\right) \right) b}} b^{-a} \cosh\left(\frac{1}{y}\right)}{\Gamma(a) y^2}, \left[ 0, \right. \right. \right. \\ \left. \left. \left. \frac{1}{\ln(1 + \sqrt{2})} \right] \right], ["Continuous", "PDF"] \right]$$

"l and u", 0,  $\infty$

"g(x)",  $\frac{1}{\operatorname{arcsinh}(x+1)}$ , "base",  $\frac{x^{-a-1} e^{-\frac{1}{xb}}}{\Gamma(a) b^a}$ , "InvertedGammaRV(a,b)"

"f(x)",  $\frac{\left( -1 + \sinh\left(\frac{1}{x}\right) \right)^{-a-1} e^{-\frac{1}{\left( -1 + \sinh\left(\frac{1}{x}\right) \right) b}} b^{-a} \cosh\left(\frac{1}{x}\right)}{\Gamma(a) x^2}$

"i is", 19,

"-----"  
 -----"

$$g := t \rightarrow \frac{1}{\operatorname{csch}(t)} + 1$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y \rightarrow \frac{\operatorname{arccsch}\left(\frac{1}{y-1}\right)^{-a-1} e^{-\frac{1}{\operatorname{arccsch}\left(\frac{1}{y-1}\right) b}} b^{-a}}{\sqrt{y^2 - 2y + 2} \Gamma(a)}, \left[ 1, \infty \right], \right. \right. \\ \left. \left. ["Continuous", "PDF"] \right] \right]$$

"l and u", 0,  $\infty$

"g(x)",  $\frac{1}{\operatorname{csch}(x)} + 1$ , "base",  $\frac{x^{-a-1} e^{-\frac{1}{xb}}}{\Gamma(a) b^a}$ , "InvertedGammaRV(a,b)"

"f(x)",  $\frac{\operatorname{arccsch}\left(\frac{1}{x-1}\right)^{-a-1} e^{-\frac{1}{\operatorname{arccsch}\left(\frac{1}{x-1}\right) b}} b^{-a}}{\sqrt{x^2 - 2x + 2} \Gamma(a)}$

"i is", 20,

"-----"  
-----"

$$\begin{aligned} g &:= t \rightarrow \tanh\left(\frac{1}{t}\right) \\ l &:= 0 \\ u &:= \infty \\ Temp &:= \left[ \left[ y \rightarrow -\frac{\left(\frac{1}{\operatorname{arctanh}(y)}\right)^{-a} e^{-\frac{\operatorname{arctanh}(y)}{b}} b^{-a}}{\operatorname{arctanh}(y) (y^2 - 1) \Gamma(a)}, [0, 1], ["Continuous", "PDF"] \right] \right. \\ &\quad "l \text{ and } u", 0, \infty \\ &\quad "g(x)", \tanh\left(\frac{1}{x}\right), "base", \frac{x^{-a-1} e^{-\frac{1}{xb}}}{\Gamma(a) b^a}, "InvertedGammaRV(a,b)" \\ &\quad "f(x)", -\frac{\left(\frac{1}{\operatorname{arctanh}(x)}\right)^{-a} e^{-\frac{\operatorname{arctanh}(x)}{b}} b^{-a}}{\operatorname{arctanh}(x) (x^2 - 1) \Gamma(a)} \end{aligned}$$

"i is", 21,

"-----"  
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$$\begin{aligned} g &:= t \rightarrow \operatorname{csch}\left(\frac{1}{t}\right) \\ l &:= 0 \\ u &:= \infty \\ Temp &:= \left[ \left[ y \rightarrow \frac{\operatorname{arccsch}(y)^{a-1} e^{-\frac{\operatorname{arccsch}(y)}{b}} b^{-a}}{\sqrt{y^2 + 1} \Gamma(a) |y|}, [0, \infty], ["Continuous", "PDF"] \right] \right. \\ &\quad "l \text{ and } u", 0, \infty \\ &\quad "g(x)", \operatorname{csch}\left(\frac{1}{x}\right), "base", \frac{x^{-a-1} e^{-\frac{1}{xb}}}{\Gamma(a) b^a}, "InvertedGammaRV(a,b)" \\ &\quad "f(x)", \frac{\operatorname{arccsch}(x)^{a-1} e^{-\frac{\operatorname{arccsch}(x)}{b}} b^{-a}}{\sqrt{x^2 + 1} \Gamma(a) |x|} \end{aligned}$$

"i is", 22,

"-----"  
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$$\begin{aligned} g &:= t \rightarrow \operatorname{arccsch}\left(\frac{1}{t}\right) \\ l &:= 0 \end{aligned}$$

$$\begin{aligned}
& u := \infty \\
Temp &:= \left[ \left[ y \rightsquigarrow \frac{\sinh(y \sim)^{-a \sim - 1} e^{-\frac{1}{\sinh(y \sim) b \sim}} b \sim^{-a \sim} \cosh(y \sim)}{\Gamma(a \sim)}, [0, \infty], ["Continuous", \right. \right. \\
& \quad \left. \left. "PDF"] \right] \right. \\
& \quad \left. \begin{aligned} & "l \text{ and } u", 0, \infty \\ & "g(x)", \operatorname{arccsch}\left(\frac{1}{x}\right), "base", \frac{x^{-a \sim - 1} e^{-\frac{1}{x b \sim}}}{\Gamma(a \sim) b \sim^{a \sim}}, "InvertedGammaRV(a,b)" \\ & "f(x)", \frac{\sinh(x)^{-a \sim - 1} e^{-\frac{1}{\sinh(x) b \sim}} b \sim^{-a \sim} \cosh(x)}{\Gamma(a \sim)} \end{aligned} \right] \quad (3)
\end{aligned}$$