## Gompertz Distribution

$$f(x) = ab^x e^{-a(b^x - 1)/\ln(b)}$$
  $x, a, b > 0$ 

|                                  | General  |          |              |              |              |            |              |            | Examp      | le: Gompert | z(2,3)                         |                       |
|----------------------------------|----------|----------|--------------|--------------|--------------|------------|--------------|------------|------------|-------------|--------------------------------|-----------------------|
| Transformation                   | PDF      | PDF      | CDF          | $_{ m HF}$   | IDF          | $\mu$      | $\sigma^2$   | MF         | MGF        | HF Shape    | Support                        | Comment               |
| $x^2$                            | ✓        | <b>√</b> | ✓            | <b>√</b>     | ✓            | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $0, \infty$                    |                       |
| $\sqrt{x}$                       | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\partial$   | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $0, \infty$                    |                       |
| $x^{-1}$                         | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\infty$   | $\checkmark$ | $\partial$ | $\partial$ | UBT         | $0, \infty$                    |                       |
| $\arctan(x)$                     | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\partial$   | $\partial$ | $\partial$   | $\partial$ | $\partial$ |             | $0,\pi/2$                      | piecewise CDF, HF, SF |
| $e^x$                            | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | $_{ m IFR}$ | $1, \infty$                    |                       |
| $\ln(x)$                         | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ |            |              |            |            | $_{ m IFR}$ | $-\infty, \infty$              |                       |
| $e^{-x}$                         | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | $_{ m IFR}$ | 0, 1                           |                       |
| $-\ln(x)$                        | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ |            |              |            |            |             | $-\infty, \infty$              |                       |
| $\ln(x+1)$                       | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ |            |              |            |            | $_{ m IFR}$ | $0,\infty$                     |                       |
| $1/\ln(x+2)$                     | <b>√</b> | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $0, 1/\ln(2)$                  |                       |
| tanh(x)                          | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\partial$   | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | 0, 1                           |                       |
| $\sinh(x)$                       | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $0, \infty$                    |                       |
| $\operatorname{arcsinh}(x)$      | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | $_{ m IFR}$ | $0, \infty$                    |                       |
| $\operatorname{csch}(x+1)$       | ✓        | ✓        | $\partial$   | $\partial$   |              | $\partial$ | $\partial$   | $\partial$ | $\partial$ | $_{ m IFR}$ | $0, 2/(-e + e^{-1})$           |                       |
| $\operatorname{arccsch}(x+1)$    | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $0, \ln(1+\sqrt{2})$           |                       |
| $1/\tanh(x+1)$                   | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $1, (e + e^{-1})/(e - e^{-1})$ |                       |
| $1/\sinh(x+1)$                   | ✓        | ✓        | $\partial$   | $\partial$   |              | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $2, 2/(e - e^{-1})$            |                       |
| $1/\operatorname{arcsinh}(x+1)$  | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | $_{ m IFR}$ | $0, 1/\ln(1+\sqrt{2})$         |                       |
| $1/\operatorname{csch}(x) + 1$   | ✓        | ✓        | $\partial$   | $\partial$   | $\partial$   | $\partial$ | $\partial$   | $\partial$ | $\partial$ |             | $1, \infty$                    |                       |
| $\tanh(x^{-1})$                  | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | 0, 1                           |                       |
| $\operatorname{csch}(x^{-1})$    | ✓        | ✓        | $\partial$   | $\partial$   |              |            |              |            |            |             | $1, \infty$                    |                       |
| $\operatorname{arccsch}(x^{-1})$ | ✓        | ✓        | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\partial$ | $\partial$   | $\partial$ | $\partial$ | IFR         | $0, \infty$                    |                       |

Legend

| Symbol     | Meaning                 |
|------------|-------------------------|
| <b>√</b>   | Exists, Closed Form     |
| $\partial$ | Exists, Not Closed Form |
| Ø          | Not Possible            |
|            | Not Calculated          |

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