

```
> restart;
read("c:/appl/appl7.txt");
```

#### PROCEDURES:

*AllPermutations(n), AllCombinations(n, k), Benford(X), BootstrapRV(Data),  
CDF:CHF:HF:IDF:PDF:SF(X, [x]), CoefOfVar(X), Convolution(X, Y),  
ConvolutionIID(X, n), CriticalPoint(X, prob), Determinant(MATRIX), Difference(X, Y),  
Display(X), ExpectedValue(X, [g]), KSTest(X, Data, Parameters), Kurtosis(X),  
Maximum(X, Y), MaximumIID(X, n), Mean(X), MGF(X), Minimum(X, Y),  
MinimumIID(X, n), Mixture(MixParameters, MixRVs),  
MLE(X, Data, Parameters, [Rightcensor]), MLENHPP(X, Data, Parameters, obstime),  
MLEWeibull(Data, [Rightcensor]), MOM(X, Data, Parameters),  
NextCombination(Previous, size), NextPermutation(Previous), OrderStat(X, n, r, ["wo"]),  
PlotDist(X, [low], [high]), PlotEmpCDF(Data, [low], [high]),  
PlotEmpCIF(Data, [low], [high]), PlotEmpSF(Data, Censor),  
PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]),  
PlotEmpVsFittedCDF(X, Data, Parameters, [low], [high]),  
PlotEmpVsFittedSF(X, Data, Parameters, Censor, low, high),  
PPPlot(X, Data, Parameters), Product(X, Y), ProductIID(X, n),  
QQPlot(X, Data, Parameters), RangeStat(X, n, ["wo"]), Skewness(X), Transform(X, g),  
Truncate(X, low, high), Variance(X), VerifyPDF(X)*

#### Procedure Notation:

*X and Y are random variables  
Greek letters are numeric or symbolic parameters  
x is numeric or symbolic  
n and r are positive integers,  $n \geq r$   
low and high are numeric  
g is a function  
Brackets [] denote optional parameters  
"double quotes" denote character strings  
MATRIX is a 2 x 2 array of random variables  
A capitalized parameter indicates that it must be  
entered as a list --> ex. Data := [1, 12.4, 34, 52.45, 63]*

#### Variate Generation:

*ArcTanVariate(alpha, phi), BinomialVariate(n, p, m), ExponentialVariate(lambda),  
NormalVariate(mu, sigma), UniformVariate(), WeibullVariate(lambda, kappa, m)*

#### DATA SETS:

*BallBearing, HorseKickFatalities, Hurricane, MP6, RatControl, RatTreatment, USSHalfBeak  
ArcSinRV(), ArcTanRV(alpha, phi), BetaRV(alpha, beta), CauchyRV(a, alpha), ChiRV(n),*

*ChiSquareRV(n), ErlangRV(lambda, n), ErrorRV(mu, alpha, d), ExponentialRV(lambda),  
 ExponentialPowerRV(lambda, kappa), ExtremeValueRV(alpha, beta), FRV(n1, n2),  
 GammaRV(lambda, kappa), GeneralizedParetoRV(gamma, delta, kappa),  
 GompertzRV(delta, kappa), HyperbolicSecantRV(), HyperExponentialRV(p, l),  
 HypoExponentialRV(l), IDBRV(gamma, delta, kappa), InverseGaussianRV(lambda, mu),  
 InvertedGammaRV(alpha, beta), KSRV(n), LaPlaceRV(omega, theta),  
 LogGammaRV(alpha, beta), LogisticRV(kappa, lambda), LogLogisticRV(lambda, kappa),  
 LogNormalRV(mu, sigma), LomaxRV(kappa, lambda), MakehamRV(gamma, delta, kappa),  
 MuthRV(kappa), NormalRV(mu, sigma), ParetoRV(lambda, kappa), RayleighRV(lambda),  
 StandardCauchyRV(), StandardNormalRV(), StandardTriangularRV(m),  
 StandardUniformRV(), TRV(n), TriangularRV(a, m, b), UniformRV(a, b),  
 WeibullRV(lambda, kappa)*

Error, attempting to assign to `DataSets` which is protected.  
 Try declaring `local DataSets`; see ?protect for details.

```

> bf := LogLogisticRV(1, 2);
bfname := "LogLogisticRV(1, 2)";
      bf :=  $\left[ \left[ x \rightarrow \frac{2x}{(x^2 + 1)^2} \right], [0, \infty], ["Continuous", "PDF"] \right]$ 
      bfname := "LogLogisticRV(1, 2)"

```

(1)

```

> #plot(1/csch(t)+1, t = 0..0.0010);
#plot(diff(1/csch(t),t), t=0..0.0010);
#limit(1/csch(t), t=0);
> solve(exp(-t) = y, t);
      -ln(y)

```

(2)

```

> # discarded -ln(t + 1), t-> csch(t), t->arccsch(t), t -> tan(t),
> glist := [t -> t^2, t -> sqrt(t), t -> 1/t, t -> arctan(t), t
-> exp(t), t -> ln(t), t -> exp(-t), t -> -ln(t), t -> ln(t+1),
t -> 1/(ln(t+2)), t -> tanh(t), t -> sinh(t), t -> arcsinh(t),
t-> csch(t+1), t->arccsch(t+1), t-> 1/tanh(t+1), t-> 1/sinh(t+1),
t-> 1/arcsinh(t+1), t-> 1/csch(t)+1, t-> tanh(1/t), t->csch
(1/t), t-> arccsch(1/t), t-> arctanh(1/t) ]:
base := t -> PDF(bf, t):
print(base(x)):

for i from 15 to 20(glist) do
    print( "i is", i, " -----"
-----" );
    g := glist[i]:
    l := bf[2][1];
    u := bf[2][2];
    Temp := Transform(bf, [[unapply(g(x), x)], [l,u]]);

    #print( "l and u", l, u );

```

```

#print("g(x)", g(x), "base", base(x), bfname);
print("f(x)", PDF(Temp, x));
#print("F(x)", CDF(Temp, x));
#print("IDF(x)", IDF(Temp));
#print("S(x)", SF(Temp, x));
print("h(x)", HF(Temp, x));
#print("mean and variance", Mean(Temp), Variance(Temp));
#assume(r > 0); mf := int(x^r*PDF(Temp, x), x = Temp[2][1] ..
Temp[2][2]);
#print("MF", mf);
#print("MGF", MGF(Temp));
PlotDist(PDF(Temp), 0, 40);
PlotDist(HF(Temp), 0, 40);
latex(PDF(Temp, x));
#print("transforming with", [[x->g(x)], [0, infinity]]);
#X2 := Transform(bf, [[x->g(x)], [0, infinity]]);
#print("pdf of X2 = ", PDF(X2, x));
#print("pdf of Temp = ", PDF(Temp, x));
od;

```

$$\frac{2x}{(x^2 + 1)^2}$$

"i is", 15,

"-----"

$$g := t \rightarrow \operatorname{arcsch}(t + 1)$$

$$l := 0$$

$$u := \infty$$

$$Temp := \left[ \left[ y \rightarrow \frac{2 (\sinh(y) - 1) \cosh(y) \sinh(y)}{-4 \cosh(y)^4 + 8 \sinh(y) \cosh(y)^2 - 4 \sinh(y) + 3} \right], [0, \ln(1 + \sqrt{2})], \right. \\ \left. ["Continuous", "PDF"] \right]$$

$$\text{"f(x)", } \frac{2 (\sinh(x) - 1) \cosh(x) \sinh(x)}{-4 \cosh(x)^4 + 8 \sinh(x) \cosh(x)^2 - 4 \sinh(x) + 3}$$

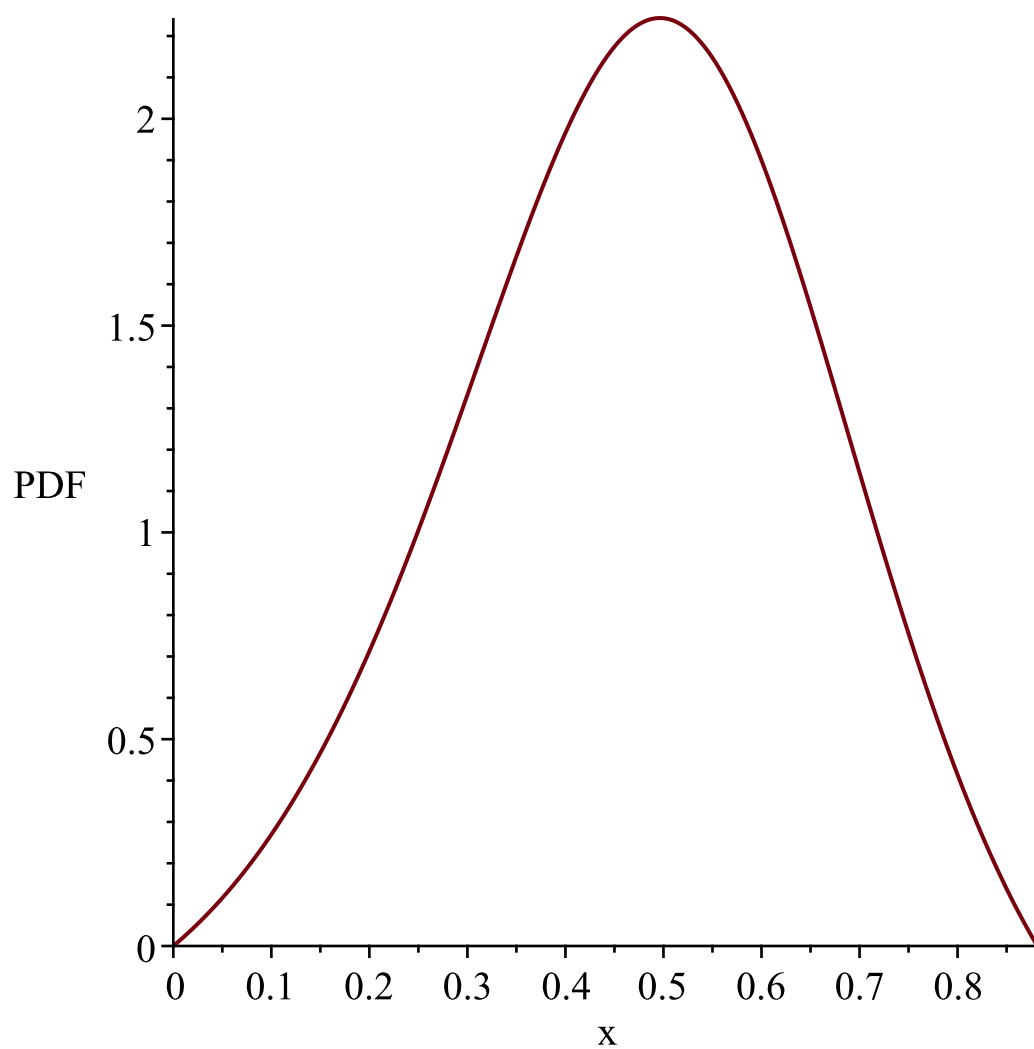
"h(x)",

$$\frac{(4 (\sinh(x) - 1) \cosh(x) \sinh(x) (e^{-4x} + 2 e^{-3x} - 2 e^{-x} + 1)) / ((-4 \cosh(x)^4 + 8 \sinh(x) \cosh(x)^2 - 4 \sinh(x) + 3) (e^{-4x} + 4 e^{-3x} + 2 e^{-2x} - 4 e^{-x} + 1))}{}$$

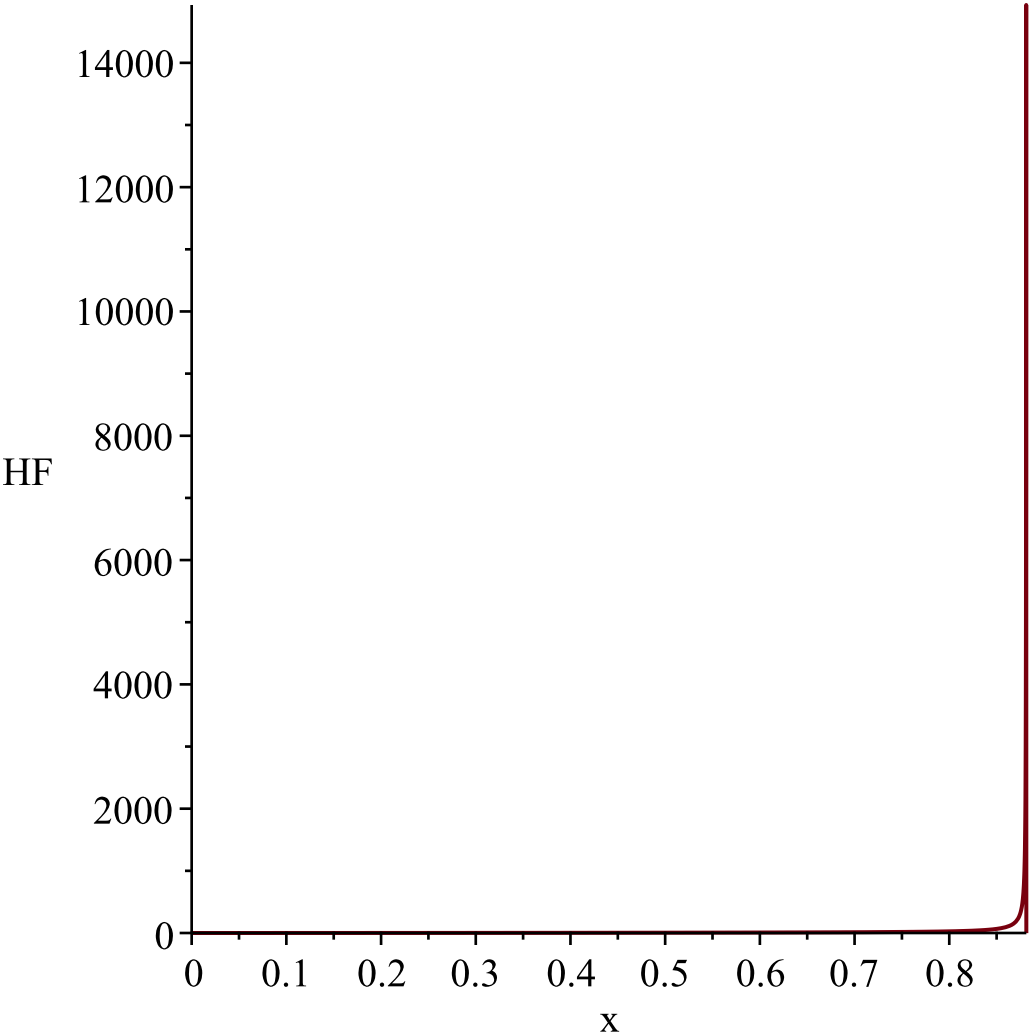
*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random*

*variable,  $\ln(1 + \sqrt{2})$*

*Resetting high to RV's maximum support value*



*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random  
variable,  $\ln(1 + \sqrt{2})$   
Resetting high to RV's maximum support value*



```

2\,{\frac { \left( \sinh \left( x \right) -1 \right) \cosh \left
( x
\right) \sinh \left( x \right) }{-4\, \left( \cosh \left( x
\right) ^{4}+8\,\sinh \left( x \right) \left( \cosh \left( x
\right) \right) ^{2}-4\,\sinh \left( x \right) +3}}

```

"i is",16,  
" -----  
-----"

$$\begin{aligned}
&g:=t\rightarrow \frac{1}{\tanh(t+1)}\\
&l:=0\\
&u:=\infty\\
Temp:=&\left[\left[y\rightsquigarrow \frac{-2+2\operatorname{arctanh}\left(\frac{1}{y\sim}\right)}{\left(\operatorname{arctanh}\left(\frac{1}{y\sim}\right)^2-2\operatorname{arctanh}\left(\frac{1}{y\sim}\right)+2\right)^2(y\sim^2-1)}\right],\left[1,\frac{e+e^{-1}}{e-e^{-1}}\right],
\end{aligned}$$

["Continuous", "PDF"]

$$\text{"f(x)", } \frac{-2 + 2 \operatorname{arctanh}\left(\frac{1}{x}\right)}{\left(\operatorname{arctanh}\left(\frac{1}{x}\right)^2 - 2 \operatorname{arctanh}\left(\frac{1}{x}\right) + 2\right)^2 (x^2 - 1)}$$

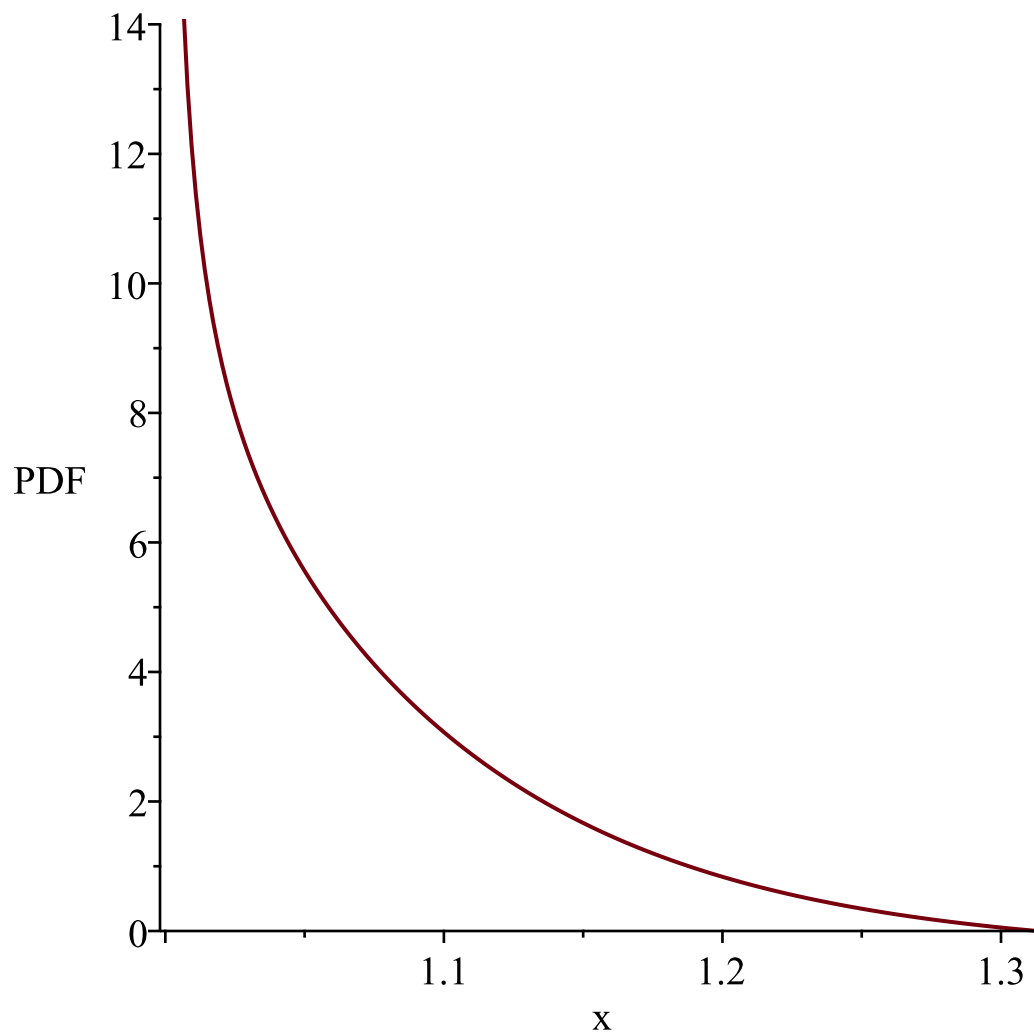
$$\text{"h(x)", } \frac{2}{\left(-1 + \operatorname{arctanh}\left(\frac{1}{x}\right)\right) (x^2 - 1) \left(\operatorname{arctanh}\left(\frac{1}{x}\right)^2 - 2 \operatorname{arctanh}\left(\frac{1}{x}\right) + 2\right)}$$

*WARNING(PlotDist): Low value provided by user, 0  
is less than minimum support value of random variable*  
1

*Resetting low to RV's minimum support value*  
*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random*

$$\text{variable, } \frac{e + e^{-1}}{e - e^{-1}}$$

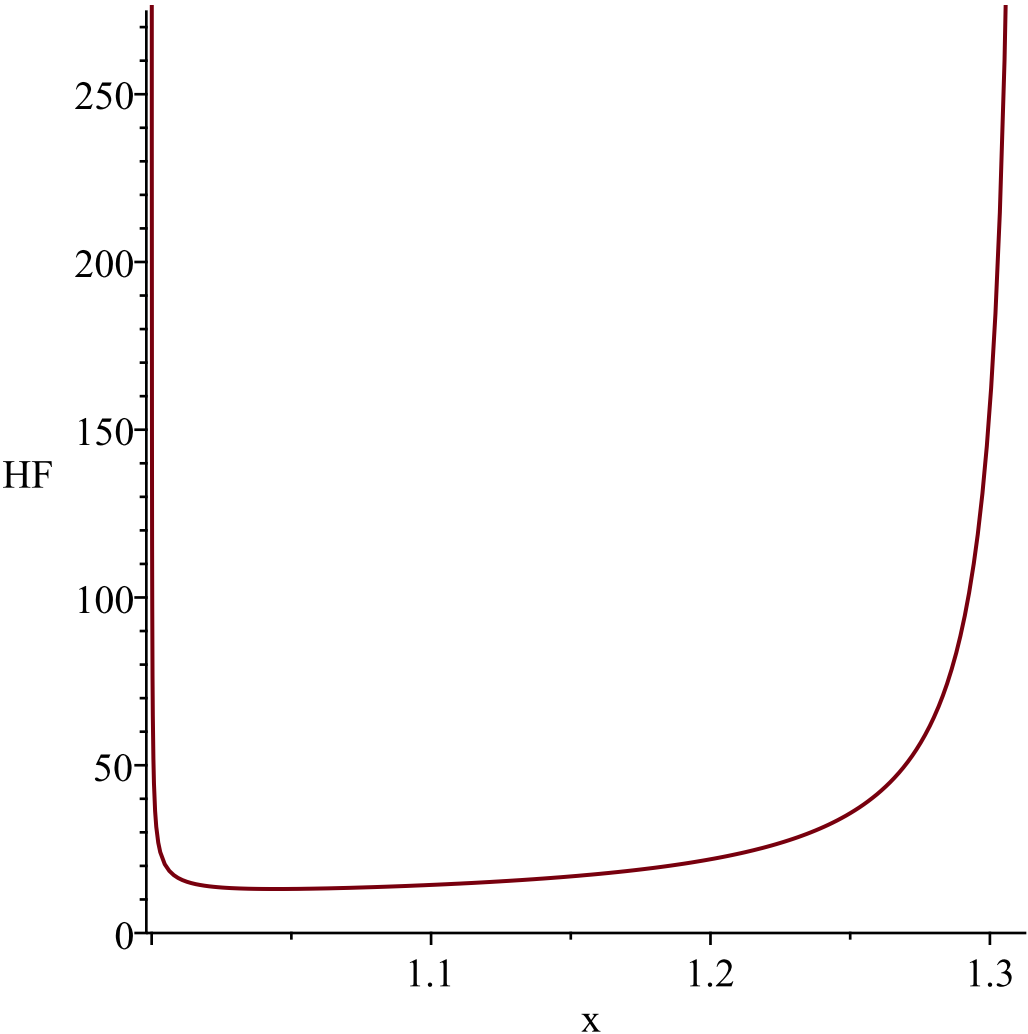
*Resetting high to RV's maximum support value*



*WARNING(PlotDist): Low value provided by user, 0  
is less than minimum support value of random variable  
1*

*Resetting low to RV's minimum support value  
WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random  
variable,  $\frac{e+e^{-1}}{e-e^{-1}}$*

*Resetting high to RV's maximum support value*



```
{\frac {-2+2\,{\rm arctanh} \left({x}^{-1}\right)}{\left(
\left(
{\rm arctanh} \left({x}^{-1}\right) \right) ^{2}-2\,{\rm
arctanh}
\left({x}^{-1}\right)+2 \right) ^{2} \left( {x}^{2}-1 \right) }}
"i is",17,
```

"-----"

$$g:=t\rightarrow \frac{1}{\sinh(t+1)}$$

$$l:=0$$

$$u:=\infty$$

$$Temp:=\left[\left[y\rightsquigarrow \frac{2\left(-1+\operatorname{arcsinh}\left(\frac{1}{y\sim}\right)\right)}{\sqrt{y\sim^2+1}\left(\operatorname{arcsinh}\left(\frac{1}{y\sim}\right)^2-2\operatorname{arcsinh}\left(\frac{1}{y\sim}\right)+2\right)^2|y\sim|}\right],\left[0,\frac{2}{e-e^{-1}}\right],$$



["Continuous", "PDF"]

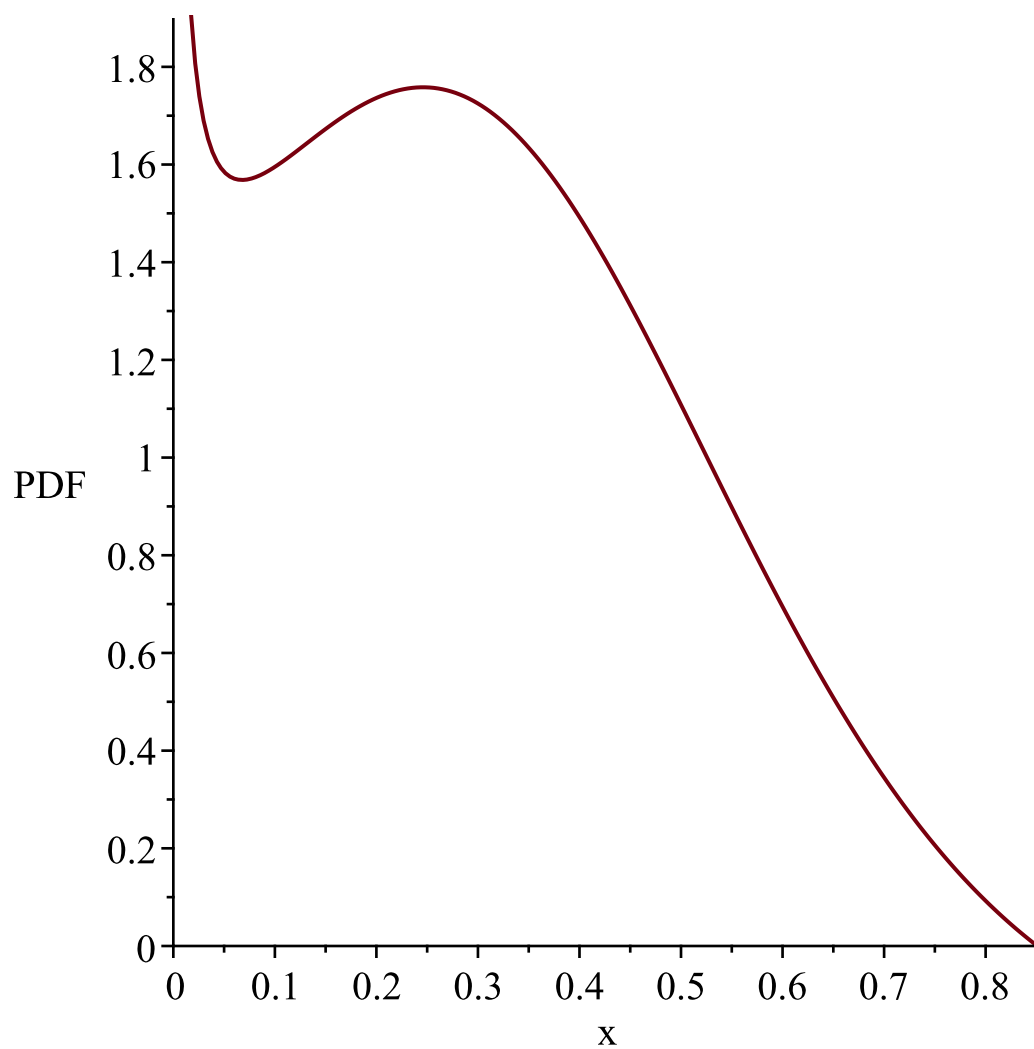
$$\text{"f(x)", } \frac{2 \left( -1 + \operatorname{arcsinh}\left(\frac{1}{x}\right) \right)}{\sqrt{x^2 + 1} \left( \operatorname{arcsinh}\left(\frac{1}{x}\right)^2 - 2 \operatorname{arcsinh}\left(\frac{1}{x}\right) + 2 \right)^2 |x|}$$

$$\begin{aligned} \text{"h(x)", } & \left( 2 \left( -1 + \operatorname{arcsinh}\left(\frac{1}{x}\right) \right) \left( \ln(\sqrt{x^2 + 1} + 1)^2 - 2 \ln(\sqrt{x^2 + 1} + 1) \ln(x) + \ln(x)^2 \right. \right. \\ & \left. \left. - 2 \ln(\sqrt{x^2 + 1} + 1) + 2 \ln(x) + 2 \right) \right) / \left( \sqrt{x^2 + 1} \left( \operatorname{arcsinh}\left(\frac{1}{x}\right)^2 \right. \right. \\ & \left. \left. - 2 \operatorname{arcsinh}\left(\frac{1}{x}\right) + 2 \right)^2 |x| \left( \ln(\sqrt{x^2 + 1} + 1)^2 - 2 \ln(\sqrt{x^2 + 1} + 1) \ln(x) + \ln(x)^2 \right. \right. \\ & \left. \left. - 2 \ln(\sqrt{x^2 + 1} + 1) + 2 \ln(x) + 1 \right) \right) \end{aligned}$$

*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random*

$$\text{variable, } \frac{2}{e - e^{-1}}$$

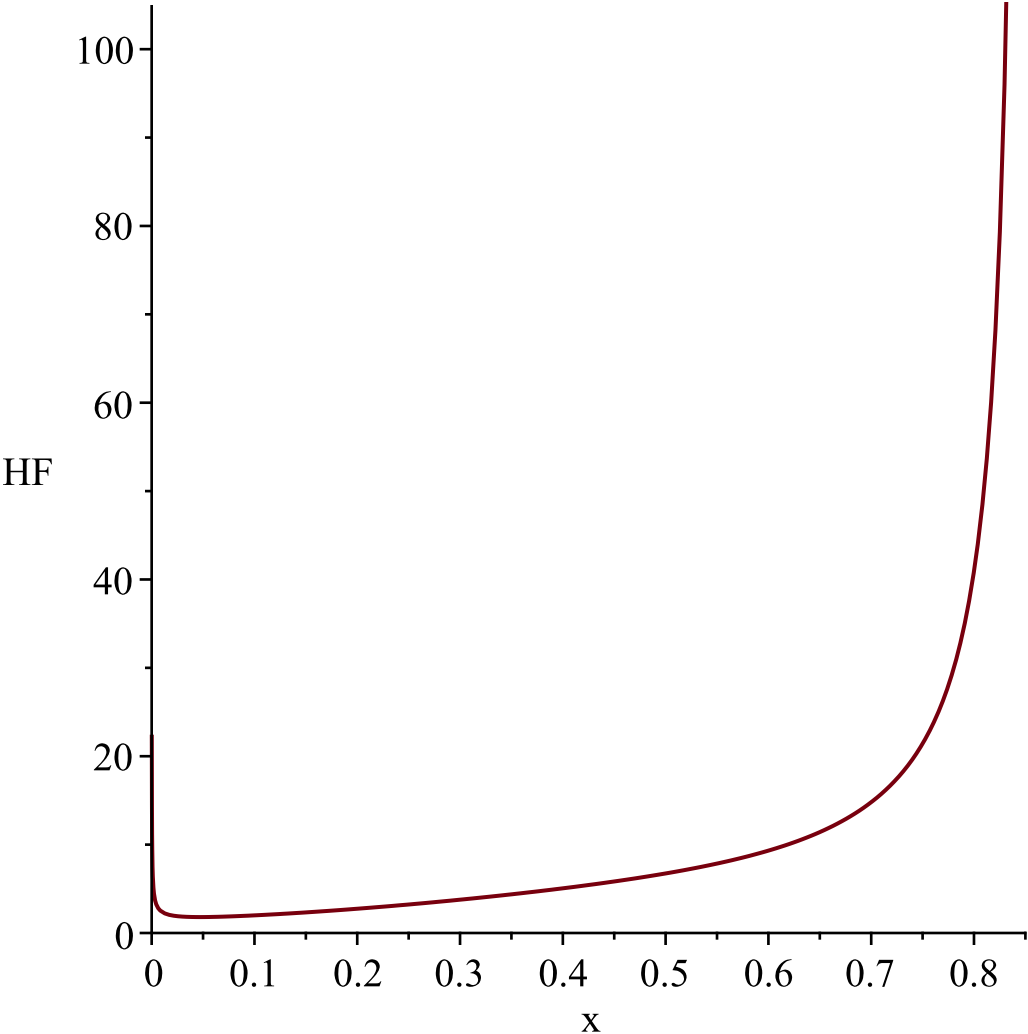
*Resetting high to RV's maximum support value*



*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random*

*variable,  $\frac{2}{e - e^{-1}}$*

*Resetting high to RV's maximum support value*



```

2\,{\frac {-1+{\rm arcsinh} \left({x}^{-1}\right)}{\sqrt {{x}^
{2}+1}}
\left( \left( {\rm arcsinh} \left({x}^{-1}\right) \right) ^{2}
-2\,
{\rm arcsinh} \left({x}^{-1}\right)+2 \right) ^{2} \left| x
\right| \}}
"i is",18,
" -----
-----"

```

$$\begin{aligned}
g &:= t \rightarrow \frac{1}{\operatorname{arcsinh}(t+1)} \\
l &:= 0 \\
u &:= \infty
\end{aligned}$$

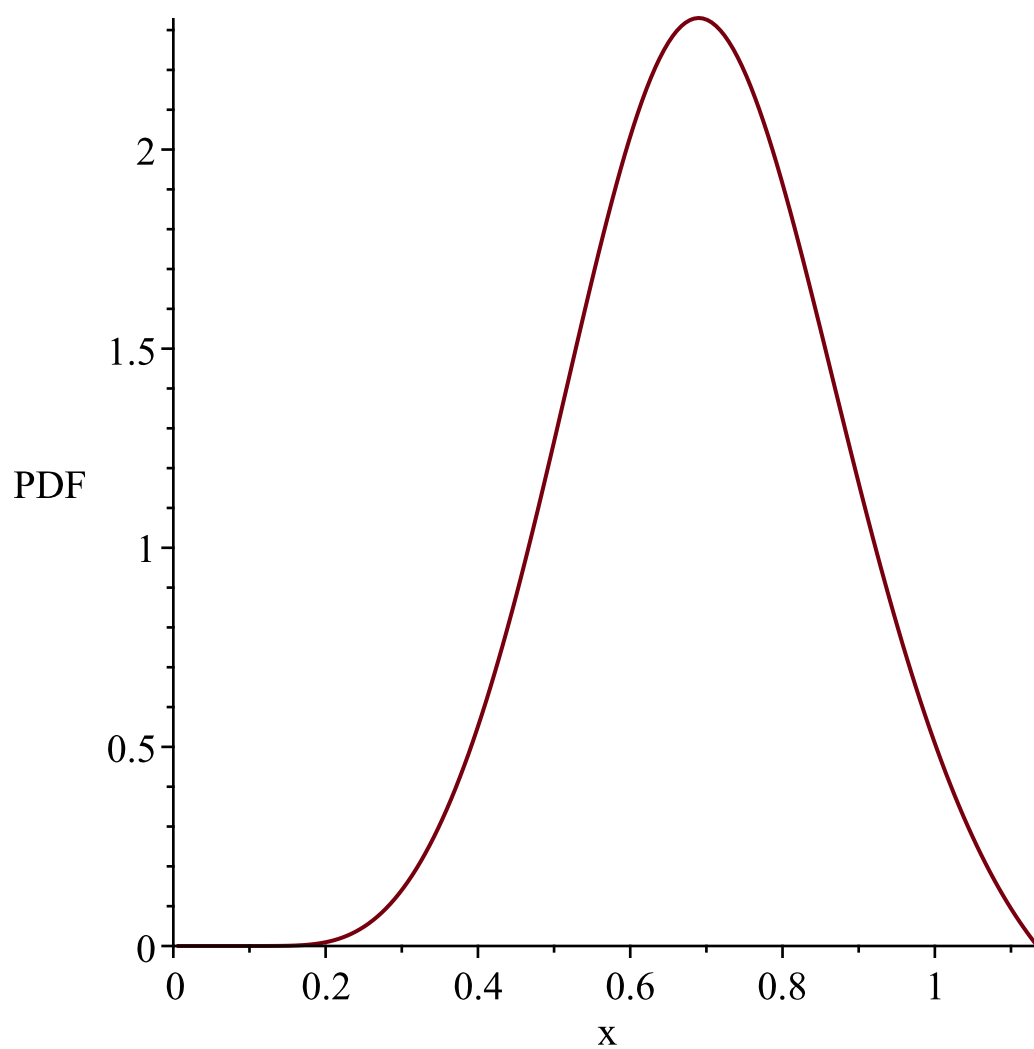
$$\begin{aligned}
Temp &:= \left[ \left[ y_{\sim} \right. \right. \\
&\rightarrow \left( 2 \left( -1 + \sinh \left( \frac{1}{y_{\sim}} \right) \right) \cosh \left( \frac{1}{y_{\sim}} \right) \right) \Big/ \left( y_{\sim}^2 \left( \cosh \left( \frac{1}{y_{\sim}} \right) \right)^4 \right.
\end{aligned}$$

$$\begin{aligned}
& -4 \cosh\left(\frac{1}{y_{\sim}}\right)^2 \sinh\left(\frac{1}{y_{\sim}}\right) + 6 \cosh\left(\frac{1}{y_{\sim}}\right)^2 - 4 \sinh\left(\frac{1}{y_{\sim}}\right) - 3 \Big) \Big], \Big[ 0, \\
& \frac{1}{\ln(1 + \sqrt{2})} \Big], ["Continuous", "PDF"] \Big] \\
& \text{"f(x)", } \frac{2 \left( -1 + \sinh\left(\frac{1}{x}\right) \right) \cosh\left(\frac{1}{x}\right)}{x^2 \left( \cosh\left(\frac{1}{x}\right)^4 - 4 \cosh\left(\frac{1}{x}\right)^2 \sinh\left(\frac{1}{x}\right) + 6 \cosh\left(\frac{1}{x}\right)^2 - 4 \sinh\left(\frac{1}{x}\right) - 3 \right)} \\
& \text{"h(x)", } - \left( 2 \left( e^{\frac{4}{x}} - 4 e^{\frac{3}{x}} + 6 e^{\frac{2}{x}} + 4 e^{\frac{1}{x}} + 1 \right) \left( -1 + \sinh\left(\frac{1}{x}\right) \right) \cosh\left(\frac{1}{x}\right) \right) \Big/ \left( x^2 \left( e^{\frac{4}{x}} \right. \right. \\
& \left. \left. - 4 e^{\frac{3}{x}} + 2 e^{\frac{2}{x}} + 4 e^{\frac{1}{x}} + 1 \right) \left( -\cosh\left(\frac{1}{x}\right)^4 + 4 \cosh\left(\frac{1}{x}\right)^2 \sinh\left(\frac{1}{x}\right) - 6 \cosh\left(\frac{1}{x}\right)^2 \right. \right. \\
& \left. \left. + 4 \sinh\left(\frac{1}{x}\right) + 3 \right) \right) \Big)
\end{aligned}$$

*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random*

*variable,  $\frac{1}{\ln(1 + \sqrt{2})}$*

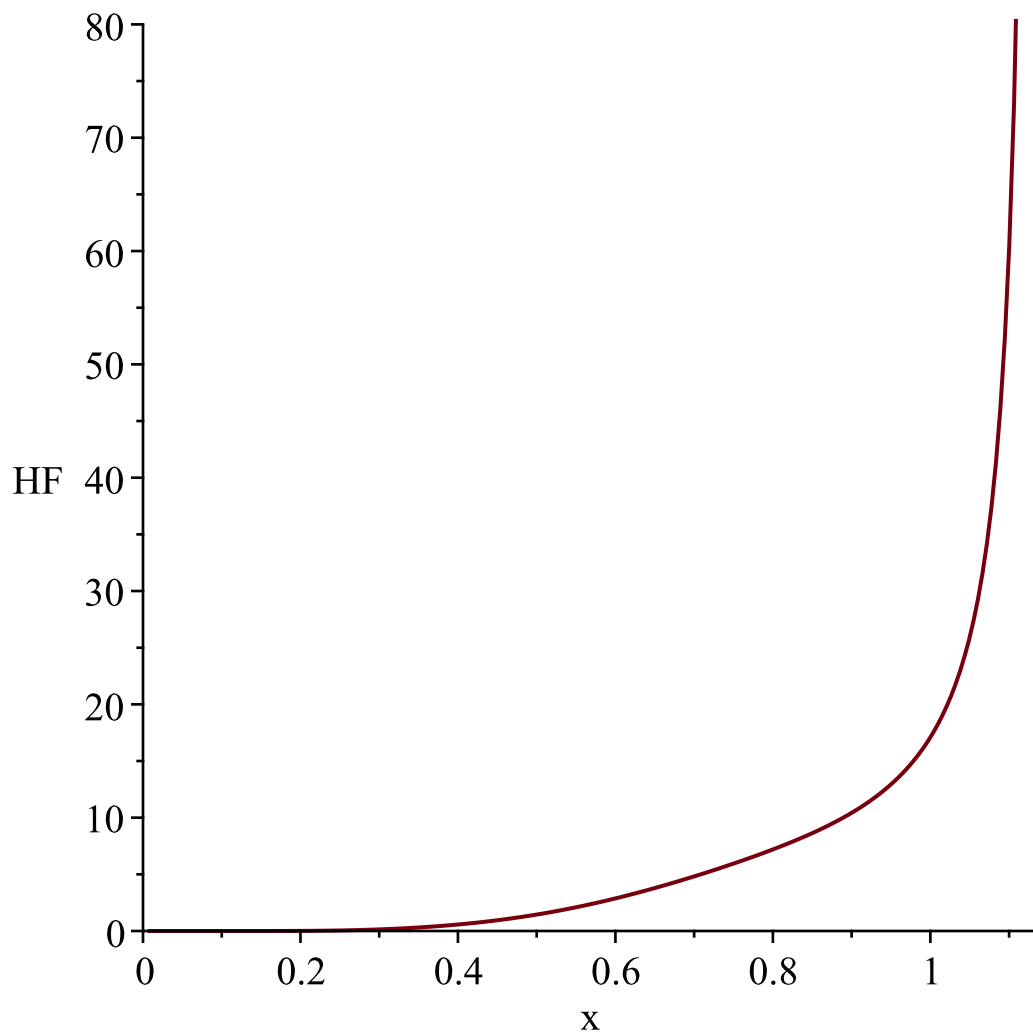
*Resetting high to RV's maximum support value*



*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random*

*variable,  $\frac{1}{\ln(1 + \sqrt{2})}$*

*Resetting high to RV's maximum support value*



```

2\,{\frac { \left( -1+\sinh \left( {x}^{-1} \right) \right) \cosh
\left( {x}^{-1} \right) }{{x}^2 \left( \left( \cosh \left(
{x}^{-1} \right) \right) ^4-4\, \left( \cosh \left( {x}^{-1} \right)
\right) ^2\sinh \left( {x}^{-1} \right) +6\, \left( \cosh
\left( {x}^{-1} \right) \right) ^2-4\, \sinh \left( {x}^{-1} \right) -3
\right) }}
"i is", 19,

```

```

" -----
-----"

```

$$g := t \rightarrow \frac{1}{\operatorname{csch}(t)} + 1$$

$$l := 0$$

$$u := \infty$$

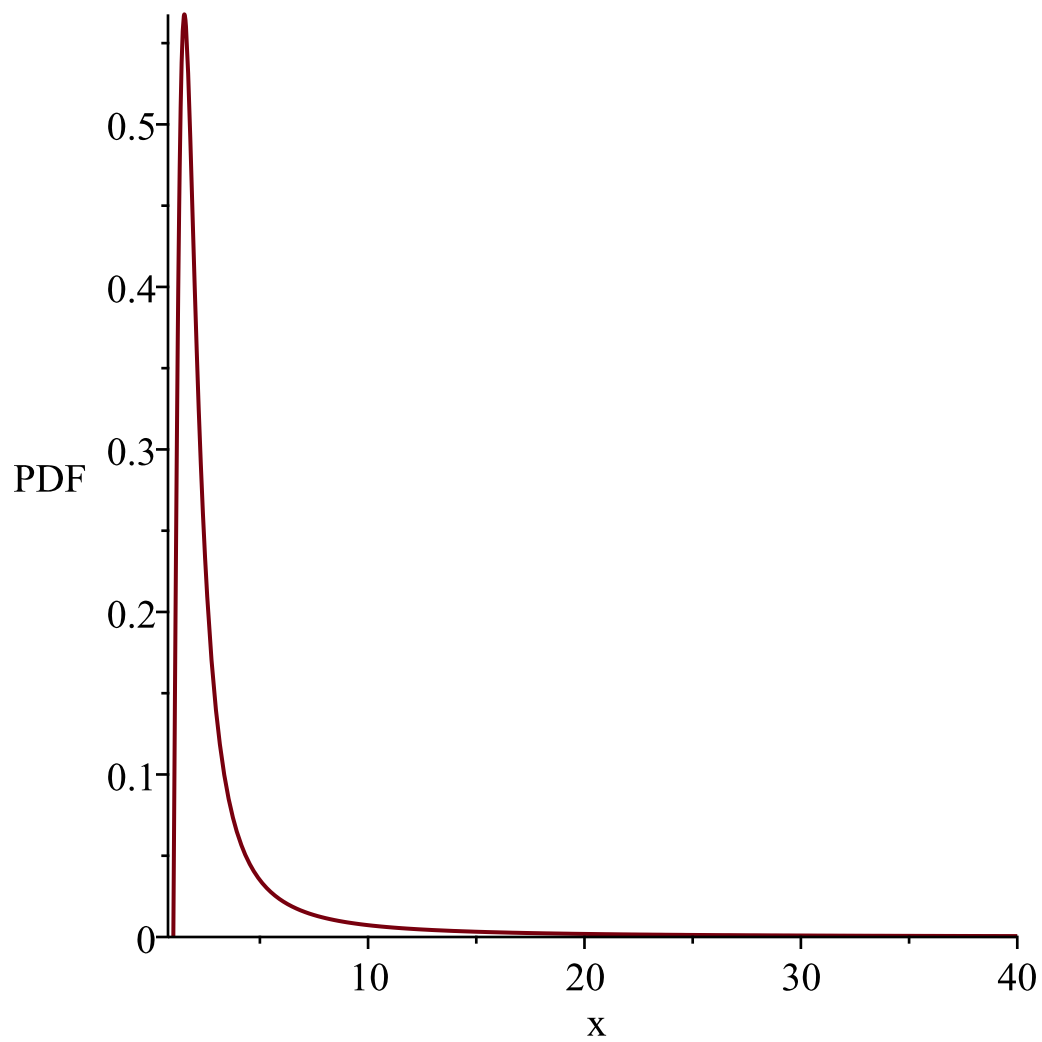
$$Temp := \left[ \left[ y \rightarrow \frac{2 \operatorname{arccsch}\left(\frac{1}{y-1}\right)}{\sqrt{y^2-2y+2} \left(\operatorname{arccsch}\left(\frac{1}{y-1}\right)^2+1\right)^2}, [1, \infty], ["Continuous", "PDF"] \right] \right]$$

$$\begin{aligned} & \text{"f(x)", } \frac{2 \operatorname{arccsch}\left(\frac{1}{x-1}\right)}{\sqrt{x^2-2x+2} \left(\operatorname{arccsch}\left(\frac{1}{x-1}\right)^2+1\right)^2} \\ & \text{"h(x)", } -\left(2 \operatorname{arccsch}\left(\frac{1}{x-1}\right)\right) \bigg/ \left(\sqrt{x^2-2x+2} \left(\operatorname{arccsch}\left(\frac{1}{x-1}\right)^2+1\right)^2\right) \left(-1+2\left(\int_1^x \frac{\operatorname{arccsch}\left(\frac{1}{t-1}\right)}{\sqrt{t^2-2t+2} \left(\operatorname{arccsch}\left(\frac{1}{t-1}\right)^2+1\right)^2} dt\right)\right) \end{aligned}$$

*WARNING(PlotDist): Low value provided by user, 0 is less than minimum support value of random variable*

1

*Resetting low to RV's minimum support value*

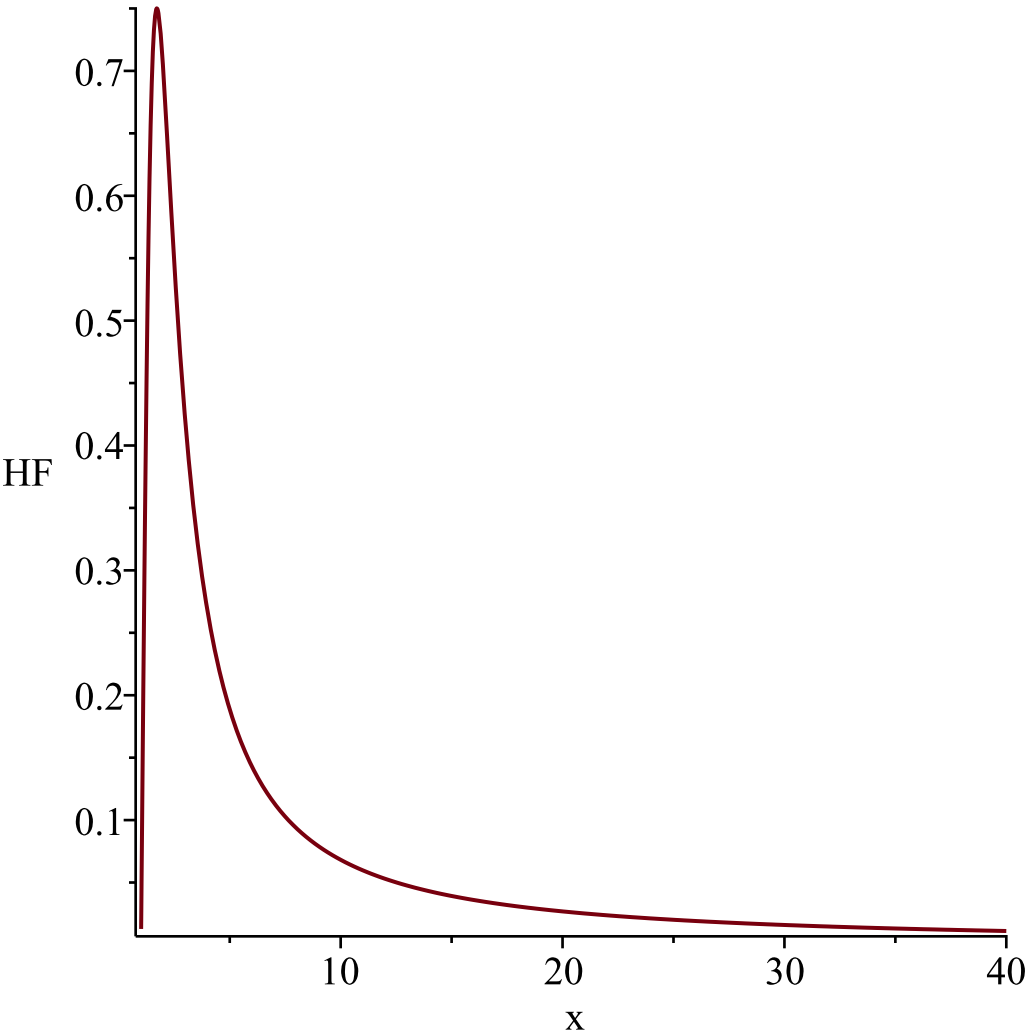


*WARNING(PlotDist): Low value provided by user, 0  
is less than minimum support value of random variable*

1

*Resetting low to RV's minimum support value*





```

2\,{\frac {{\rm arccsch} \left( \left( x-1 \right) ^{-1}\right) }{
\sqrt {{x}^{2}-2\,{x+2}} \left( \left( {\rm arccsch} \left( \left(
( x-1
\right) ^{-1}\right) \right) ^{2}+1 \right) ^{2}}}}

```

"i is", 20,  
 " -----  
 -----"

$$g:=t\rightarrow \tanh\left(\frac{1}{t}\right)$$

$$l:=0$$

$$u:=\infty$$

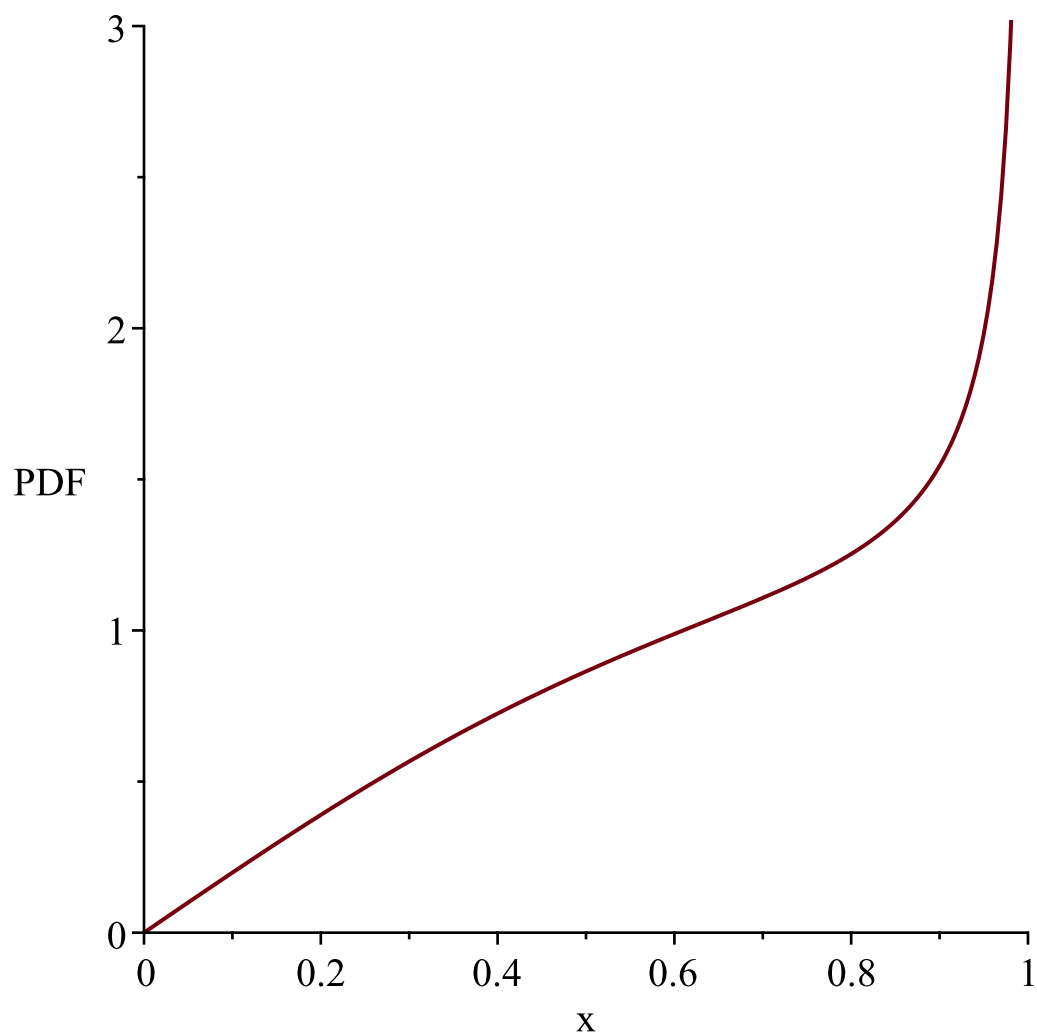
$$Temp:=\left[\left[y\rightsquigarrow-\frac{2\operatorname{arctanh}(y\sim)}{\left(\operatorname{arctanh}(y\sim)^2+1\right)^2\left(y\sim^2-1\right)}\right],\left[0,1\right],\left["Continuous","PDF"\right]\right]$$

$$\text{"f(x)", -}\frac{2\operatorname{arctanh}(x)}{\left(\operatorname{arctanh}(x)^2+1\right)^2\left(x^2-1\right)}$$

$$\text{"h(x)", -}\frac{2\operatorname{arctanh}(x)}{\left(\operatorname{arctanh}(x)^2+1\right)\left(x^2-1\right)}$$

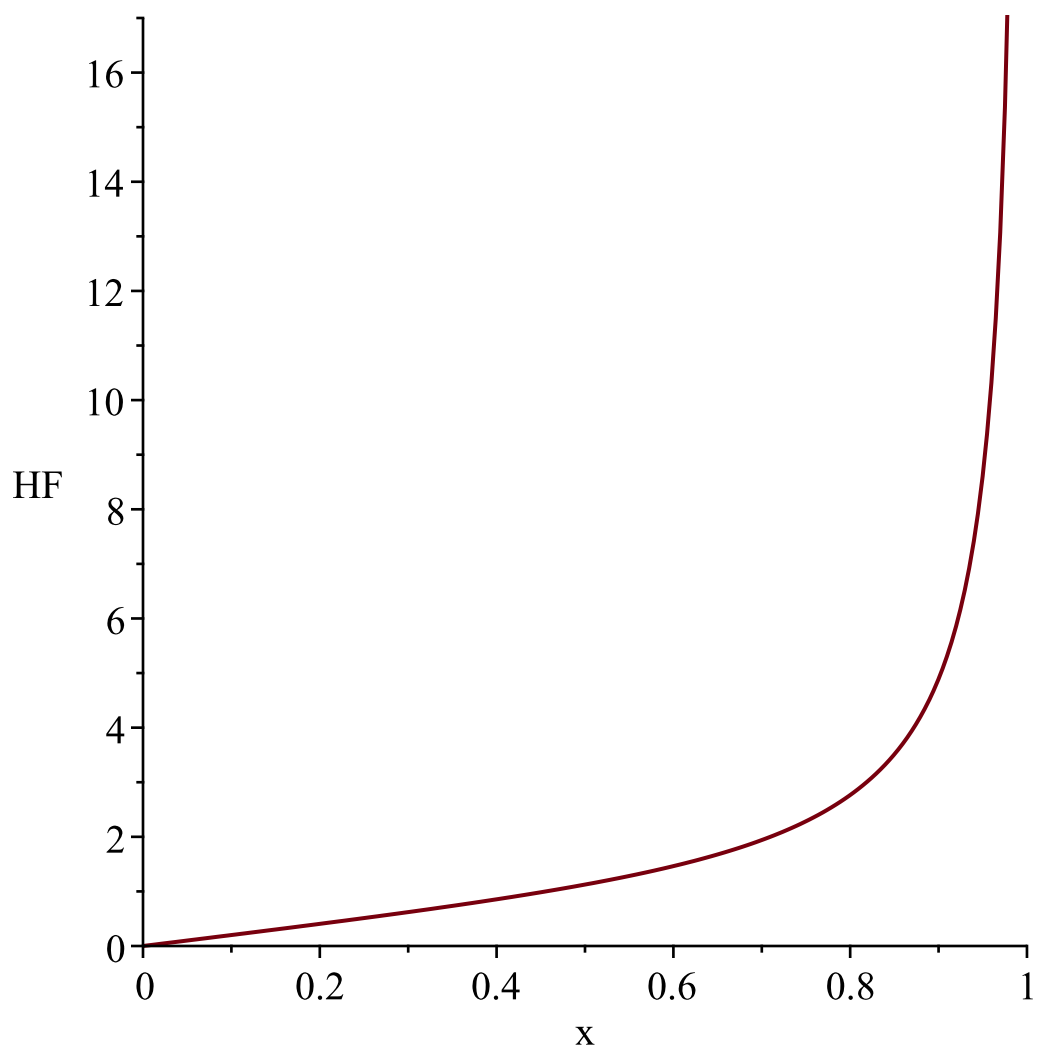
*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random  
variable, 1*

*Resetting high to RV's maximum support value*



*WARNING(PlotDist): High value provided by user, 40  
is greater than maximum support value of the random  
variable, 1*

*Resetting high to RV's maximum support value*



$$-2\sqrt{\frac{\operatorname{arctanh}(x)}{\left(\operatorname{arctanh}(x)\right)^2+1}}\left(x^2\operatorname{arctanh}(x)-1\right)$$