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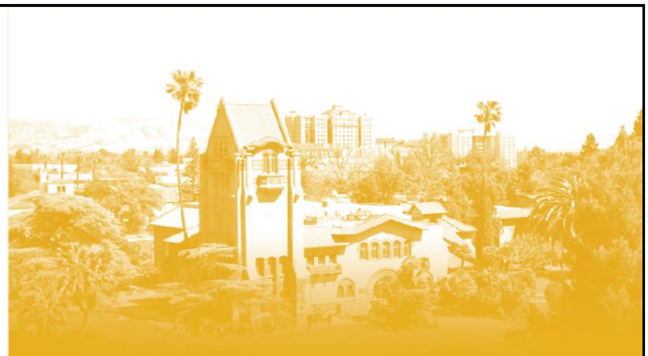
Hypothesis Testing



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Agenda

- Introduction to Hypothesis Testing
- Steps of Hypothesis Testing
- Types of Errors, Significance Levels & Power
- Examples, Tests, and Practice Problems

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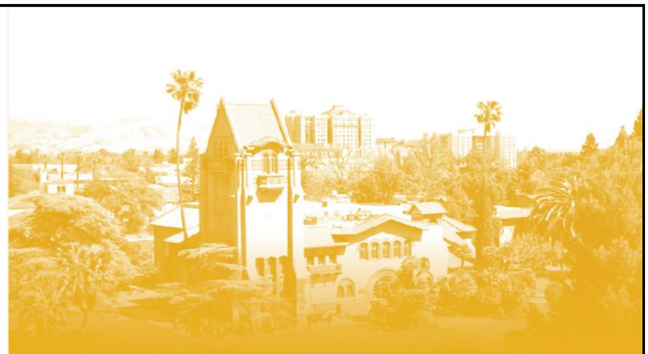
Introduction to Hypothesis Testing



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What is Hypothesis Testing?

- A statistical method to make or test claims decisions about population parameters.
- Uses sample data to test assumptions.
- Central to scientific reasoning, quality control, and evidence-based decision-making.
- Compares hypotheses using probabilistic reasoning.

- Hypothesis test is also known as test of significance.

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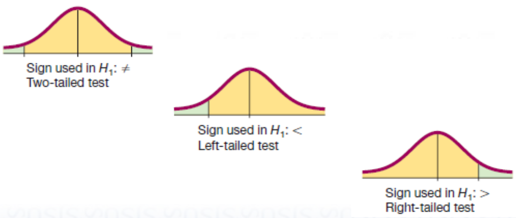
Null and Alternative Hypothesis

The first step is to create the two possibilities in hypothesis testing:

- Null hypothesis (H_0): Default claim or "Status Quo"
  - A statement that the value of a population parameter is equal to some claimed value.
- Alternative hypothesis (H_A or H_1): The claim that we seek evidence
  - A statement that the parameter has a value that somehow differs from the null hypothesis.
- Types:
  - Two-tailed: H_A: $\mu \neq \mu_0$
  - Left-tailed: H_A: $\mu < \mu_0$
  - Right-tailed: H_A: $\mu > \mu_0$



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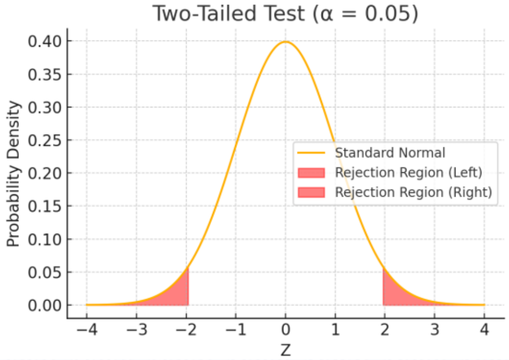
One-Tailed vs. Two-Tailed Tests

- Two-tailed: Difference in either direction
- One-tailed: Directional difference



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Examples of Hypotheses

- Lifespan of LED bulbs:
  - H_0: $\mu \geq 1000$ hrs
  - H_A: $\mu < 1000$ hrs
- Drug effectiveness:
  - H_0: μ = placebo mean
  - H_A: $\mu \neq$ placebo mean
- Defect rate:
  - H_0: $p \leq 0.05$
  - H_A: $p > 0.05$

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Example: Defining the Null and Alternative Hypothesis

Consider the claim that a medical procedure designed to increase the likelihood of a baby girl is effective, so that the probability of a baby girl is $p > 0.5$.

Define the null and alternative hypotheses.

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Example: Can Dogs Smell Bladder Cancer?

- A study by M. Willis et al. considered whether dogs could be trained to detect if a person has bladder cancer by smelling his/her urine.
- 6 dogs of varying breeds were trained to discriminate between urine from patients with bladder cancer and urine from control patients without it.
- The dogs were taught to indicate which among several specimens was from the bladder cancer patient by lying beside it.
- Once trained, the dogs' ability to distinguish cancer patients from controls was tested using urine samples from subjects not previously encountered by the dogs.

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Example: Can Dogs Smell Bladder Cancer?

- The researchers blinded both dog handlers and experimental observers to the identity of urine samples.
- Each of the 6 dogs was tested with 9 trials. In each trial, one urine sample from a bladder cancer patient was randomly placed among 6 control urine samples.
- Outcome: In the total of 54 trials with the 6 dogs, the dogs made the correct selection 22 times.
- The dogs were correct for 22/54 ≈ 41% of the time ➔ not fabulous
- If the dogs just guessed at random, they were only expected to be correct for 1/7 ≈ 14% of the time
- Is this difference (41% vs 14%) surprising?

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Example: Can Dogs Smell Bladder Cancer?

Let p be the probability that a dog makes the correct selection on a given trial.

- Null hypothesis (H_0): $p = 1/7$

  "There is nothing going on." ➔ "null" means "nothing surprising is going on".

  The dogs just guessed at random ➔ lucky to make more correct selections than expected.

- Alternative hypothesis (H_A or H_1): $p > 1/7$

  "There is something going on."

  Dogs can do better than random guessing

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Weighing Evidence Using a Test Statistic

After defining H_0 and H_A, the next step of hypothesis testing is to weigh the evidence ➔

How likely the observed data could have occurred if H_0 was true?

- If the observed result was very unlikely to have occurred under the H_0, then the evidence raises more than a reasonable doubt in our minds about the H_0.

The **test statistic** is a summary of the data that best reflects the evidence for or against the hypotheses.

For the example, the test statistics that is chosen:

X = the total number of correct selections in the 54 trials

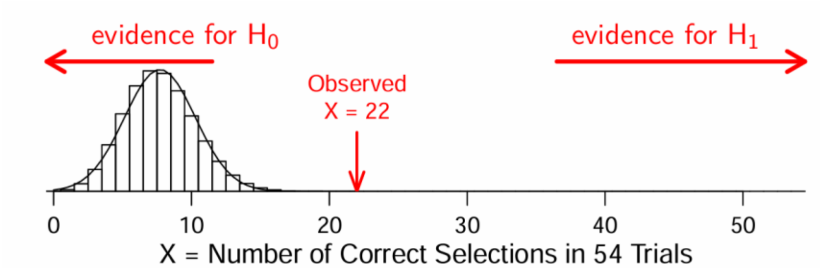
A larger X value is a stronger evidence for H_A and against H_0

Distribution of the Test Statistic Under H_0

For the "Dogs Smell Cancer" example, if H_0 is true, then

$$X \sim \text{Binomial}(n = 54, p = 1/7)$$

which implies



Example: Can Dogs Smell Bladder Cancer?

- What's the conclusion of this example?
- Any evidence that dogs have some ability to smell bladder cancer?
- If so, how practical is it?

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Steps in Hypothesis Testing

- Formulate H_0 and H_A
- Choose significance level α
- Select appropriate test statistic (e.g. z, t, etc)
- Compute the test statistic and p-value
- Decision: **Reject** or **Fail** to reject H_0

Parameter	Sampling Distribution	Requirements	Test Statistic
Proportion p	Normal (z)	$np \geq 5$ and $nq \geq 5$	$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$
Mean μ	t	σ not known and normally distributed population *or* σ not known and $n > 30$	$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
Mean μ	Normal (z)	σ known and normally distributed population *or* σ known and $n > 30$	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
St. dev. σ or variance σ^2	χ^2	Strict requirement: normally distributed population	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

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Test Procedure & Rejection Region

A test procedure is specified by the following:

- a test statistic
- a rejection region

The null hypothesis H_0 will be rejected if & only if the test statistic falls in the rejection region.

A sensible rejection region is of the form

$X \geq k$ for some cutoff k

and the test procedure is reject H_0 if $X \geq k$

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Type I and Type II Errors

In a hypothesis test, we make a decision about which of H_0 or H_A might be true, but our decision might be incorrect.

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type I Error*
	H_1 true	*Type II Error*	✓

- Type I Error (α): rejecting H_0 when it is true.
- Type II Error (β): failing to reject H_0 when it is false.

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Example: Type 1 and 2 Errors

Consider the claim that a medical procedure designed to increase the likelihood of a baby girl is effective, so that the probability of a baby girl is $p > 0.5$.

What are the Type I and Type II Errors?

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Interpretations of Type 1 and 2 Errors

Type 1 and Type 2 errors are different sorts of mistakes and have different consequences:

- Usually H_0 is the status quo, thing we generally believe to be true
- If H_0 is not rejected → the status quo is fine. No action needs to be taken
- Rejecting H_0 means something we used to believe is overturned. It might be a scientific breakthrough (e.g. discovery of a new drug).
- A Type 1 error introduces a false conclusion that can lead to a tremendous waste of resources before further research invalidates the original finding.
- A Type 2 error represents a missed opportunity for scientific progress
- Type 2 errors can be costly as well, but generally go unnoticed

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Significance Level of Hypothesis Testing

- The Significance Level α for a hypothesis test is the probability value used as the cutoff for determining when the sample evidence constitutes significant evidence against the null hypothesis H_0.

$$\text{Significance Level } \alpha = P(\text{rejecting } H_0 \text{ when } H_0 \text{ is true}) = P(\text{Type 1 error})$$

- The probability of rejecting the null hypothesis H_0 when it is actually true (Type I error).

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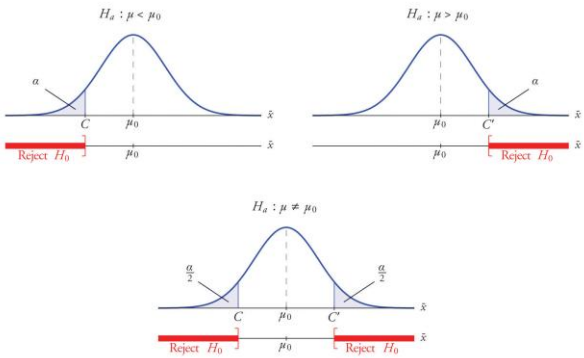
Significance Level of Hypothesis Testing



- Common choices of α are 0.01, **0.05**, 0.10.
- Controls how "strict" the test is in ruling out random chance (Smaller α means stricter criteria for significance)

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Different Significance Levels

- Using a different value for α can change whether we reject H_0 or not.

Example: A cigarette company claims that the average nicotine content is 1.5 mg/cigarette.

$$H_0: \mu = 1.5$$
$$H_A: \mu > 1.5$$

Significance Level α	Rejection Region	Conclusion for z = 2.10	Significance level preferred by
.05	$z_1 > 1.645$	Reject H_0	Anti-tobacco activists
.025	$z_2 > 1.96$	Reject H_0	Anti-tobacco activists
.01	$z_3 > 2.33$	Fail to reject H_0	Tobacco company
.005	$z_4 > 2.58$	Fail to reject H_0	Tobacco company

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Different Significance Levels

Significance Level α	Rejection Region	Conclusion for z = 2.10	Significance level preferred by
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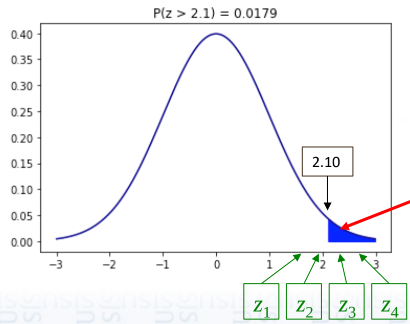
.025	$z_2 > 1.96$	Reject H_0	Anti-tobacco activists
.01	$z_3 > 2.33$	Fail to reject H_0	Tobacco company
.005	$z_4 > 2.58$	Fail to reject H_0	Tobacco company

H_0: $\mu = 1.5$
H_a: $\mu > 1.5$

The tobacco company can claim that they failed to reject H_0. But the company probably won't tell you what significance level it used for testing.



This area 0.0179 is the smallest α at which we can reject H_0. Any smaller area would not include 2.10

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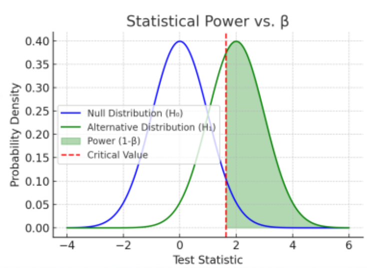
Power of Hypothesis Testing

- The **Power** for a hypothesis test is the probability correctly rejecting the null hypothesis H_0 when the alternative hypothesis is true H_A.

Power = 1 - P(Type II error) = 1 - β

- The probability of avoiding a Type II error (Correction rejection)
- Higher power means a greater ability to detect true effects.
- Power increases with larger sample sizes and lower variability.
- Higher α ➔ higher power



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Controlling Type 1 Errors

How to Control Type I Error:

- Set a Smaller Significance Level α
  - Common values: 0.05, 0.01, 0.001
  - Example: Use α = 0.01 to reduce false positives in critical medical trials
- Use Two-Tailed Tests When Appropriate
  - Less aggressive than one-tailed, splits α between both tails
- Ensure Assumptions of the Test Are Met
  - Incorrect assumptions (e.g., non-normality) can inflate error rates
- Apply Multiple Comparison Corrections (when testing many hypotheses)

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Controlling Type 2 Errors

How to Control Type II Error:

- Increase Sample Size n
  - Reduces standard error → increases test sensitivity → lower β.
- Increase Effect Size (if feasible)
  - Sometimes through better instrumentation, experimental design, or stronger treatments.
- Choose a Higher Significance Level α
  - Increases power, but also increases Type I error risk ➔ trade-off must be justified
- Use More Powerful Tests
  - Parametric tests (t-test, z-test) are more powerful than non-parametric ones when assumptions are satisfied

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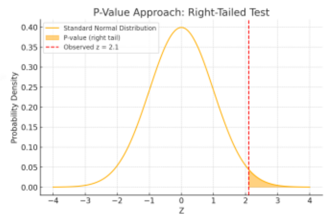
The P-Values and Hypothesis Testing

- The p-value of test is the probability of getting a value of the test statistic that is at least as extreme as the test statistic obtained from the sample data, assuming that the null hypothesis H_0 is true.
- It quantifies the "strength" of the evidence against H_0 (helps us decide whether to reject H_0)
  - A small p-value (usually less than the significance level, such as 0.05) indicates strong evidence against H_0.
  - A large p-value suggests weak evidence against H_0 (fail to reject it).
- Compare the p-value to the significance level α:
  - If p-value $\leq \alpha$: Reject H_0 (evidence supports H_A) at level α.
  - If p-value $> \alpha$: Fail to reject H_0 (evidence is insufficient to support H_A).
- The p-value is the smallest level of significance at which H_0 can be rejected.



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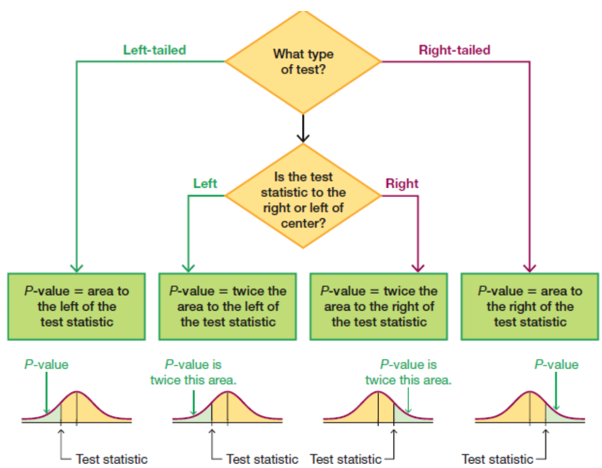
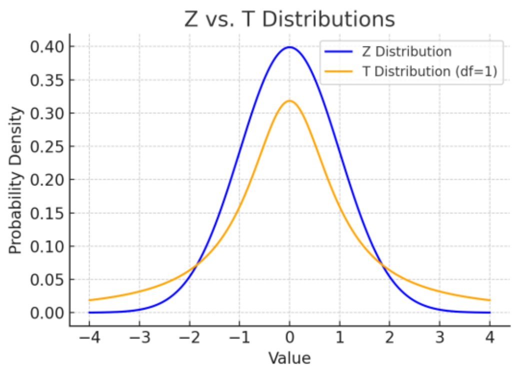
Finding P-Values



Recall z vs t Tests

- z-test: Known σ or large n
- t-test: Unknown σ, small n
- Use *t-distribution* for small samples



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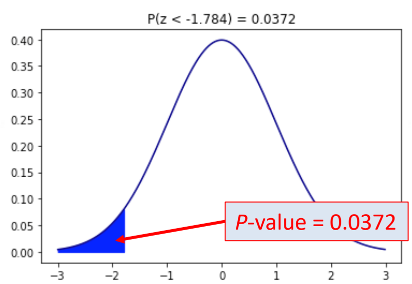
Example: P-Value

Given that:

H_0: $\mu = 125$
H_A: $\mu < 125$

It's reported that the p-value = 0.0372. What's the smallest α at which we can reject H_0?



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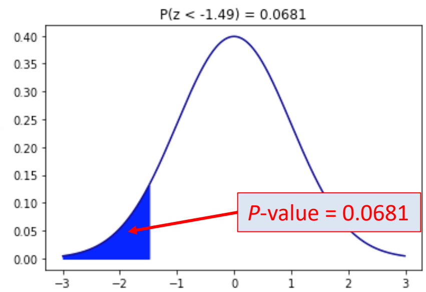
Example: P-Value

Given that:

H_0: $\mu = 10$
H_A: $\mu < 10$

Suppose that a z test for testing results in the test statistic z = -1.49. What's the smallest α at which we can reject H_0?



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Example

A company claims that their light bulbs last an average of 1,000 hours. You suspect the true mean lifespan is less than 1,000 hours. You take a sample of 40 bulbs and find that the sample mean and standard deviation are 980 hours and 50 hours, respectively. The significance level is 0.05. Prove or disprove your suspicion.

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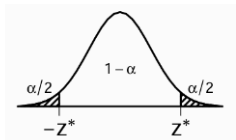
Relationship Between Confidence Intervals and 2-Sided Hypothesis Testing

For a two-sided test:

H_0: $\mu = \mu_0$
H_A: $\mu \neq \mu_0$

The following are equivalent:



- p-value > α (and hence H_0: $\mu = \mu_0$ is not rejected at level α)
- |z-statistic| = $|(\bar{X} - \mu_0)/SE| < z^*$, where z^* is a value such that
- μ_0 is in the $100(1-\alpha)\%$ confidence interval for μ:

$$\bar{X} - z^* \cdot SE < \mu_0 < \bar{X} + z^* \cdot SE$$

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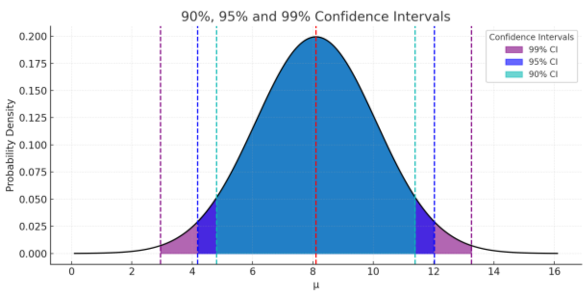
Relationship Between Confidence Intervals and 2-Sided Hypothesis Testing

Suppose in a study, we have the following

- 90% CI for μ: (4.81, 11.39)
- 95% CI for μ: (4.18, 12.02)
- 99% CI for μ: (2.95, 13.25)



Then

- H_0: $\mu = 4$ is rejected at 5% level but not at 1% level (2-sided p-value is between 1% and 5%)
- H_0: $\mu = 4.5$ is rejected at 10% level but not at 5% level

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Discussions on Hypothesis Testing

- Rejecting H_0 doesn't mean we are 100% sure that H_0 is false. We might make Type 1 errors. Setting a significance level just guarantee we won't make Type 1 error too often.
- P-value is not P(H_0 is true | data) but it is P(data | H_0 is true).
- Another mistake is to conclude from a high p-value that the H_0 is probably true.
- If our p-value is high, can we conclude that H_0 is true?
  - No, we could make a Type 2 error when failing to reject H_0
  - Moreover, unlike Type 1 error rate is controlled at a low level, Type 2 error rate is usually quite high. It is quite often that H_0 is not true but the data fail to reject it.
- When we fail to reject H_0, often it just means the data are not able to distinguish between H_0 and H_A (because the data are to noisy, etc)

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Summary: Hypothesis Testing Procedure

- Start with a null hypothesis (H_0) that represents the status quo or default claim.
- State an alternative hypothesis (H_A) that represents our research question, i.e. what we're testing for.
- Collect data and often summarize the data as a test statistic, which is usually a measure gauging whether H_0 or H_A are more plausible.
- Determine the sampling distribution of the test statistic assuming H_0 is true.
  - If the test statistic is too far away from what the H_0 predicts, then reject the H_0 in favor of the H_A.
- Choose a significance level α = maximal P (Type I error) that we can tolerate.
- Set the rejection region based on the significance level.
- Reject H_0 if the test statistic falls in the rejection region, and do not reject otherwise.

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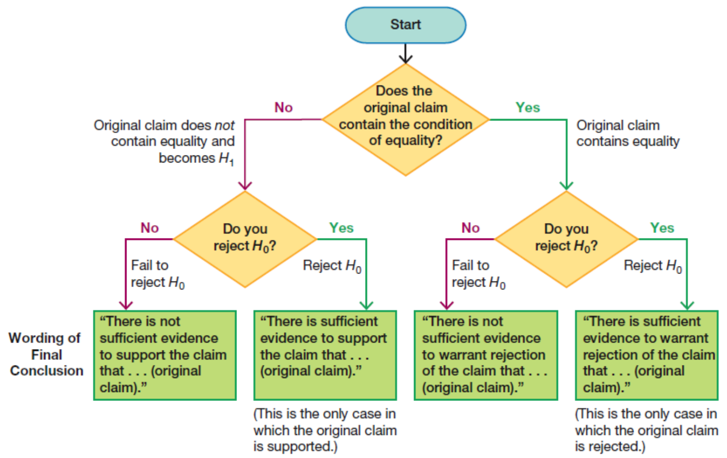
Hypothesis Tests: Wording of Final Conclusion



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