$x = (0,1,0,1)$, $y = (1,0,1,0)$

$$Cov(X,Y) = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{N}$$

$\bar{x} = 2/4 = 1/2$

$\bar{y} = 2/4 = 1/2$

$$\Rightarrow (0-1/2)(1-1/2) + (1-1/2)(0-1/2) + (0-1/2)(1-1/2) + (1-1/2)(0-1/2)$$

$-1/2 \times 1/2 = -1/4$ (4)

$$= \left[(0-1/2)(1-1/2)\times 2 + (1-1/2)(0-1/2)\times 2\right] 1/4$$

$= -1/4 = cov(x,y)$ $\quad \bar{x}, \bar{y} = 1/2$

$$\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{N}}$$

$$\sigma_y = \sigma_x = \sqrt{\frac{(0-1/2)^2 + (1-1/2)^2 + (0-1/2)^2 + (1-1/2)^2}{4}} = \frac{1}{2}$$

$$\rho(x,y) = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{1/4}{1/2 \cdot 1/2} = \frac{1/4}{1/4}$$

$$\rho(x,y) = 1$$

Q.E.D.

$\therefore cov(x,y) = 1/4$.

$$\rho(x,y) = 1$$