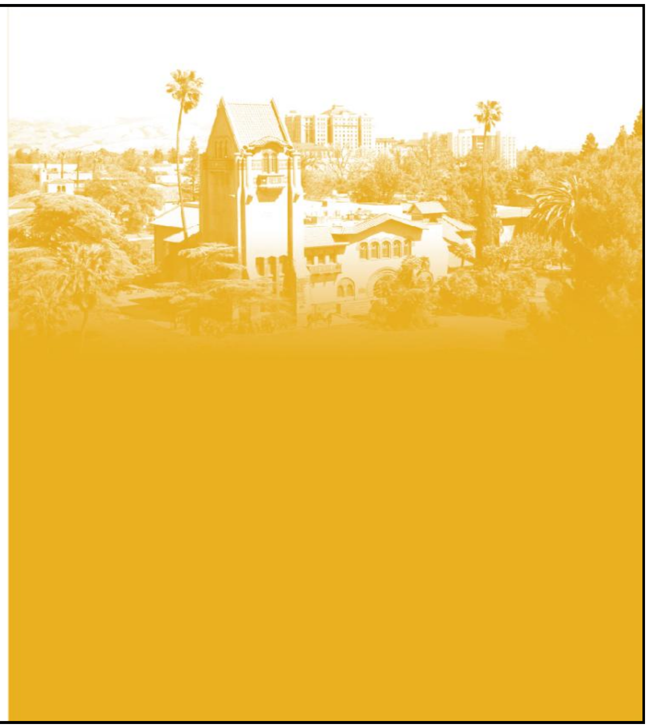
Parameter Estimation &
Central Limit Theorem



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Agenda

- Sampling Distributions & Parameter Estimations
- Central Limit Theorem

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Sampling Distributions

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Example of a Population Distribution

- Suppose a certain movie has a bipolar distribution of ratings, that in a 1 to 10 scale, of those having watched the movie, 13 gave 9 points, 13 gave 2 points, and the remaining 13 gave 1 points.
- So, the population distribution is:

X	1	2	9
$P(X)$	1/3	1/3	1/3

Population Distribution

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Histogram of Samples

- In practice, since the population are difficult (or impossible) to examine completely, we take a sample to learn about the population. Will the makeup of the sample mimic the makeup of the population?
- First, the sampling method must be appropriate. A biased sample won't give us the correct information about the population.
- Suppose we take a simple random sample of size n (say n =400) from the population. What will the histogram of the ratings of the movie given by subjects in the sample look like?

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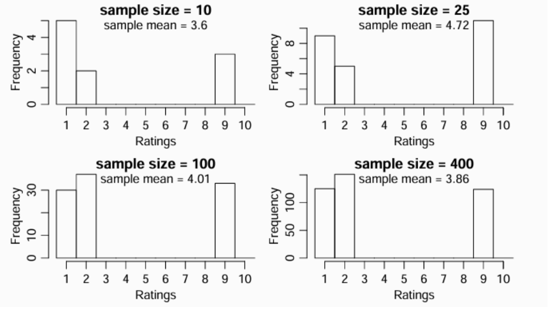
Histogram of Samples



- The histogram of the sample looks somewhat like the histogram of the population. The larger the sample size, the higher the resemblance.

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Estimation of the Population Mean

In practice, the population distribution is usually unknown. We are often interested in population parameters, like the population mean.

- As all we know about the population is the sample, we can only use the sample to estimate the population parameter of interest, called *statistic*.
- A commonly used estimate of the population mean is the sample mean. Thus, the sample mean is one of such statistic.
- Sample statistics vary from sample to sample.
- **How close is the sample mean to the population mean?**

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Variability of the Sample Means

To determine the variability of the sample mean of a sample of size n = 25, we pretend that we know the population:

X	1	2	9
$P(X)$	1/3	1/3	1/3

Then, we perform the following simulations:

- We take a random sample of size n = 25 from the population, compute and record the sample mean, and the put the sample back.
- We repeat the previous step 10,000 times, and then obtain 10,000 sample means.
- What will the histogram of the 10,000 (n = 25) samples means look like?

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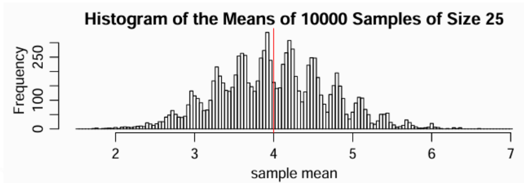
Variability of the Sample Means

- For a sample of size of n = 25, the distribution of the sample means is <u>not very normal</u>, with a number of hills and valleys.



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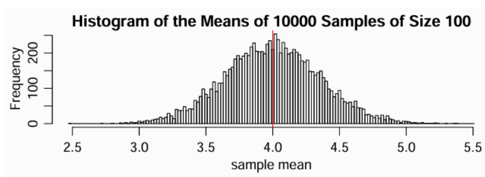
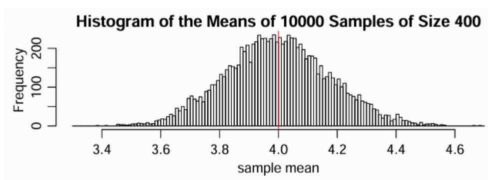
Variability of the Sample Means

- But when increase to n = 100, then…



The red vertical line marks the position of the population mean = 4

- And n = 400 :



The red vertical line marks the position of the population mean = 4

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Sampling Distributions

- The probability distribution of a statistic is called the sampling distribution of the statistic.
- What we just constructed is the sampling distribution of the sample mean.

A few observations:

- The sampling distribution of the sample mean may not be normal when the sample size is small, but it gets more normal when the sample size gets larger.
- The sample mean may not be equal to the population mean, but its distribution centers at the population mean.
- With a larger sample, the variability sample mean around the population gets smaller.

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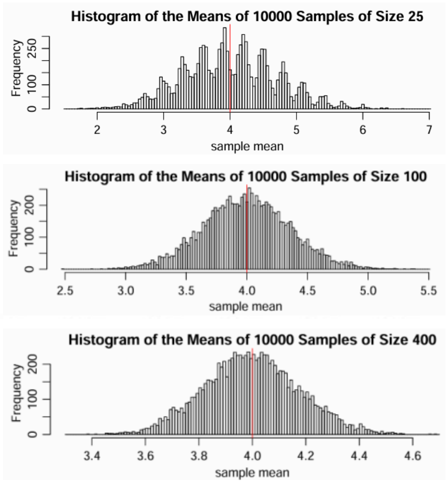
Sampling Distributions

What are the SDs of the sample means?

Sample Size	Mean	Std Dev
25	3.998	0.707
100	4.001	0.358
400	3.999	0.177



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Expected Value and Standard Deviation of the Sample Mean

- Given random variables $X_1, X_2, \cdots, X_n$ from a population with mean μ and SD σ^2 that are independent and identical probability distributions (aka i.i.d. – independent & identically distributed), the sample mean is simply:

$$\bar{X}_n = \frac{(X_1 + X_2 + \cdots + X_n)}{n}$$

- The expected value and standard deviation of the sample mean are:

$$E(\bar{X}_n) = \mu \qquad SD(\bar{X}_n) = \frac{\sigma}{\sqrt{n}}$$

- Observations in a simple random sample is nearly i.i.d. if the sample size is less than 10% of the population size.
- Standard deviation of the sample mean is also called the standard error.

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Example: Movie Rating Revisited

- For the movie rating example, recall the population distribution is:

X	1	2	9
P(X)	1/3	1/3	1/3

- The mean, variance and SD of the population distribution are:

$$\mu = 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 9 \cdot \frac{1}{3} = 4$$

$$\sigma = \sqrt{(1-4)^2 \cdot \frac{1}{3} + (2-4)^2 \cdot \frac{1}{3} + (9-4)^2 \cdot \frac{1}{3}} = \sqrt{\frac{38}{3}} = 3.56$$

- The sample means and SD's are:

$$E(\bar{X}_n) = \mu$$

$$SD(\bar{X}_n) = \frac{\sigma}{\sqrt{n}}$$

Sample Size	$E(\bar{X}_n)$	$SD(\bar{X}_n)$
25	4	$3.56/\sqrt{25} = 0.712$
100	4	$3.56/\sqrt{100} = 0.356$
400	4	$3.56/\sqrt{400} = 0.178$

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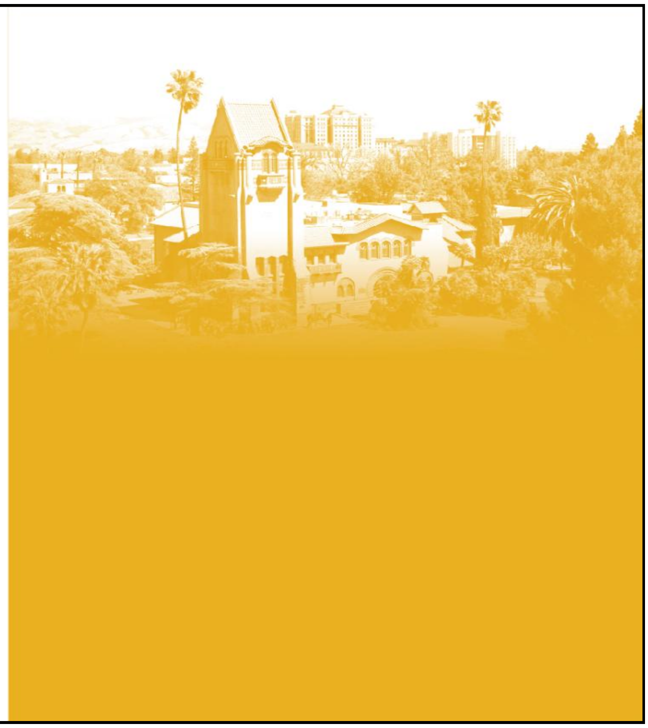
Central Limit Theorem



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Central Limit Theorem (CLT)

Let $X_1, X_2, \cdots, X_n$ be a sequence of i.i.d. random variables (discrete or continuous) with mean μ and SD σ^2. Then, when n is large,

- the distribution of the sample mean

$$\bar{X}_n = \frac{(X_1 + X_2 + \cdots + X_n)}{n} \quad \text{is approximately} \quad N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

- the distribution of the sum $S_n = X_1 + X_2 + \cdots + X_n$ is approximately

$$N(n\mu, \sqrt{n}\sigma)$$

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Example: Movie Rating Revisited

- For the movie rating example, recall that:

X	1	2	9
$P(X)$	1/3	1/3	1/3

$\mu = 4, \qquad \sigma = 3.56$

- The sampling distribution of $\bar{X}_{100}$ is approximately:

$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N(4, 0.356)$$

- The probability of $\bar{X}_{100} > 4.5$ is:

$$P(\bar{X}_{100} > 4.5) = P\left(Z > \frac{4.5 - 4}{0.356}\right) \approx P(Z > 1.40) = 0.08$$

- In the simulation 804 of the 10,000 simulated $\bar{X}_{100}$ exceeds 4.5, which agrees with the CLT approximation that $\bar{X}_{100}$ exceeds 4.5 for about 8% of the time.

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What's the Sample Size Required for CLT?

- Provided the sample size is large enough, the sampling distributions of the sample mean will be approximately normal, <u>even when the population distribution is not normal</u>.
- If the population distribution is normal, then so does the sampling distributions of the sample mean, regardless of the sample size.
- If population distribution is symmetric, then <u>n should be at least 30</u> or so.
- If the population distribution is skewed or has outliers, then sample size <u>n should be moderate (at least 100 or so)</u>, or even larger depending on how skewed or irregular the population distribution is.

Example: Central Limit Theorem

A housing survey was conducted to determine the price of a typical home in Topanga, CA. The mean price of a house was roughly $1.3 million with an SD of $0.3 million. There were no houses listed below $0.3 million but a few houses above $3 million.

1) Can we find an approximate probability that a randomly chosen house in Topanga costs more than $1.4 million using the normal distribution?
2) Can we find an approximate probability that the mean of 60 randomly chosen houses in Topanga is more than $1.4 million using the normal distribution? If yes, compute the approximate probability.

What Does the CLT Say?

<u>True or False</u>: The central limit theorem says that as you take larger and larger samples from a population, the histogram of the sample values looks more and more normal. **Explain.**

What is the quantity that becomes more and more normal as the sample size gets larger and larger?

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Python CLT Demo!!!