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Statistical Inference & Confidence Intervals

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Agenda

- Statistical Inference
- Confidence Intervals
- Small Sample Estimates

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Statistical Inference

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What is Statistical Inference?

Statistical inference is the process of using data from a sample to make conclusions about a population. It answers questions like:

- What is the average height of all adults in a city?
- Is a new drug effective?
- Will this marketing strategy improve sales?

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Key Components of Statistical Inference

Recall the following definitions/quantities:

Population: The entire group of interest.

Sample: A subset of the population.

Parameter: A value that describes the population (e.g., population mean μ).

Statistic: A value calculated from the sample (e.g., sample mean $\bar{X}$).

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Types of Statistical Inference

Parameter Estimation

- Point Estimation: Single best guess (e.g., sample mean $\bar{X}$ estimates population mean μ).
- Interval Estimation: Range of values (confidence interval) likely to contain the parameter.

Hypothesis Testing

- Procedure to test claims about a population.

  e.g. Is the average weight of bags < 50 kg?

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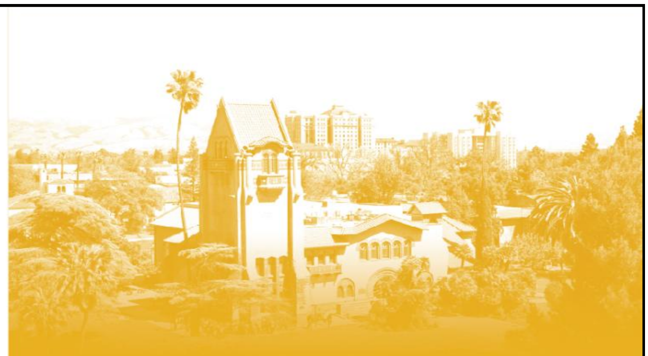
Confidence Intervals



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What is Confidence Intervals?

Confidence Intervals (CI) - a plausible range of values for the population parameter

- Use sampling to estimate the mean of a population μ.
- However, this estimate is unlikely to be exactly μ.
- How confident are we that our estimate is "close" to μ?

- Instead of a point estimate, we can specify an interval and claim that we are 95% confident that the interval contains the value of μ?
- We can construct a confidence interval at the 95% confidence level for each sample.

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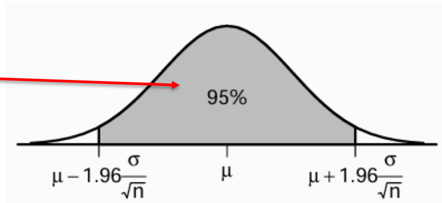
Confidence Intervals

- Recall that CLT says, for large n, $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$

95% of its area is within
1.96 SDs from the center



So, 95% of the time, $\bar{X}$ will be within $1.96\frac{\sigma}{\sqrt{n}}$ from μ

Alternatively, 95% of the time, μ will be within $1.96\frac{\sigma}{\sqrt{n}}$ from $\bar{X}$

$$\bar{X} - 1.96 \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}$$

95% confidence interval for μ

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How To Construct Confidence Intervals?

- Take a simple random sample (or i.i.d. sample) of size n and find he sample mean $\bar{X}$
- If n is large, the 95% confidence interval for is given by:

$$\bar{X} \pm (z \text{ critical value}) \cdot \frac{\sigma}{\sqrt{n}}$$

- If the population SD σ is unknown, we replace it with our best guess — the sample SD s

$$\bar{X} \pm (z \text{ critical value}) \cdot \frac{s}{\sqrt{n}}$$

- However, this replacement is dangerous because
  - s is a poor estimate of if the sample size n is small
  - s is very sensitive to outliers

Do this only if:
1) $n \geq 30$, and
2) sample doesn't have any outlier nor be too skewed

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Other Conditions Required to Use a Confidence Interval

Observations in the sample must be independent

- If the observations are from a simple random sample and consist of < 10% of the population, then they are nearly independent.
- Subjects in an experiment are considered independent if they undergo random assignment to the treatment groups.
- If a sample is from a seemingly random process, e.g. the lifetimes of wrenches used in a particular manufacturing process, checking independence is more difficult. In this case, use your best judgement.

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Example: Average Number of Exclusive Relationships

A random sample of 50 college students were asked how many electronic devices they use on a daily basis. This sample yielded a mean of 3.2 and a standard deviation of 1.74. Estimate the true average number of electronic devices using this sample.

Example: Average Number of Exclusive Relationships

True/False: We are 95% confident that the average number electronic devices the college students in this sample have been in is between 2.7 and 3.7.

Explain your choice.

True/False: 95% of college students have 2.7 to 3.7 electronic devices.

Explain your choice.

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What does "95% confidence" mean?

- It is the procedure to construct the 95% confidence interval.
- About 95% of the C.I. constructed following the procedure ($\bar{X} \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}}$) will cover the true population mean μ.
- After taking the sample and a C.I. is constructed, the C.I. either covers the population mean μ or it doesn't. No one knows…
- Just like lottery, before you pick the numbers and buy a lottery ticket, you have some chance to win the price. After you get the ticket, you either win or lose.

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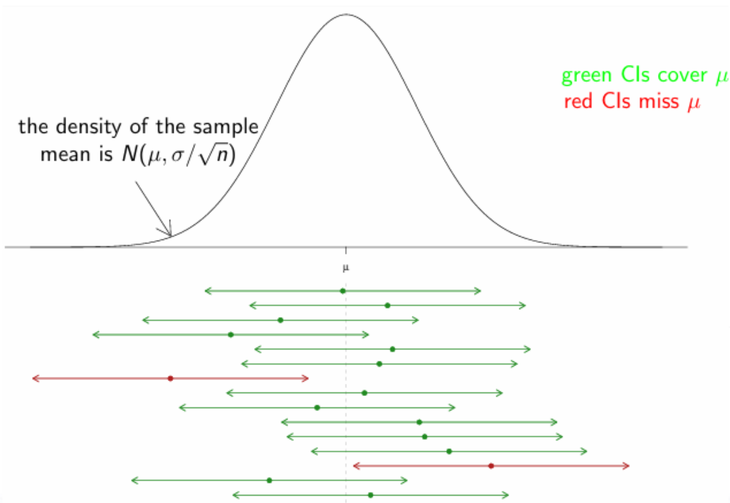
Confidence Intervals



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More Confidence Interval Questions

True/False: If a new random sample of size 50 is taken, we are 95% confident that the new sample mean will be between 2.7 and 3.7.

Explain your choice.

True/False: This confidence interval $\bar{X} \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}}$ is not valid since the number of exclusive relationships is integer-valued. Neither the population nor sample is normally distributed.

Explain your choice.

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Other Confidence Intervals at Different Confidence Levels

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	[illegible]
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	[illegible]
$\boxed{-2.5}$	~~.00621~~	~~.00604~~	~~.00587~~	~~.00570~~	~~.00554~~	~~.00539~~	~~.00523~~	~~.00508~~	$\boxed{.00494}$	[illegible]
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	[illegible]
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	[illegible]
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	[illegible]	[illegible]	[illegible]
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	[illegible]	[illegible]	[illegible]
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	[illegible]	[illegible]	[illegible]
$\boxed{-1.9}$	~~.02872~~	~~.02807~~	~~.02743~~	~~.02680~~	~~.02619~~	~~.02559~~	$\boxed{.02500}$	[illegible]	[illegible]	[illegible]
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	[illegible]	[illegible]	[illegible]
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	[illegible]	[illegible]	[illegible]	.03673
$\boxed{-1.6}$	~~.05480~~	~~.05370~~	~~.05262~~	~~.05155~~	~~.05050~~	$\boxed{.04947}$	[illegible]	[illegible]	[illegible]	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	[illegible]	[illegible]	[illegible]	.05592

90%: $z = 1.645$
95%: $z = 1.96$
99%: $z = 2.58$

At the 99% confidence level, the area in the tails should each be 0.01 / 2 = 0.005. The critical z value is **2.58**.

At the 95% confidence level, the area in the tails should each be 0.05 / 2 = 0.025. The critical z value is **1.96**.

At the 90% confidence level, the area in the tails should each be 0.10 / 2 = 0.05. The critical z value is **1.645**.

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The Confidence Interval Compromise

- Why settle for 90% confidence when 95% or even 99% is possible?
- Higher confidence levels have wider confidence intervals.
- The cost of higher reliability is less precision.
- The 95% confidence interval is a good compromise between reliability and precision.

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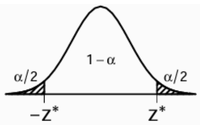
Significance Level

- The significance level α is defined as:

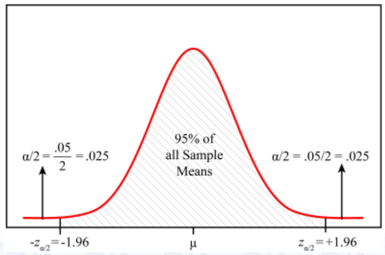
$$\alpha = 1 - confidence\ level\ (prob\ \%)$$

$P(-z^* < Z < z^*) = 1 - \alpha$ or



α is the probability of making a Type I error

- Common confidence levels:
  - 90% C.I.: $\alpha = 0.10\ (10\%)$, $z_{\alpha/2} = 1.645$
  - 95% C.I.: $\alpha = 0.05\ (5\%)$, $z_{\alpha/2} = 1.96$
  - 99% C.I.: $\alpha = 0.01\ (1\%)$, $z_{\alpha/2} = 2.58$



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Margin of Error

- A confidence interval is a range of values used to estimate a population parameter. It's associated with a specific confidence level (e.g. 95%).
- The margin of error (MOE) determines the width of the confidence interval and provides a measure of the precision of the estimate.
- It quantifies the <u>uncertainty</u> around a point estimate in a confidence interval.
- It's a measure of <u>sampling error</u> in the results of a survey or poll (sampling).

$$MOE = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Example: Margin of Error

Collect a random sample of 30 bowls of cereals to calculate a 95% confidence interval for the average amount of carbohydrates in a bowl. The sample mean and the SD are determined as 29 g and 8.74 g, respectively. Determine the margin of error.

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Small Sample Estimates

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Small Sample Estimates of Mean

- If the sample size n is small, say $n \leq 30$, then the Central Limit Theorem does not apply.
- We can use sampling to estimate the population mean μ if the population is normal.
  - Recall that if the population distribution is normal, even when n is small, the $\bar{X}$ distribution is normal.
- We estimate the population standard deviation σ with the sample standard deviation s to obtain the standardized variable:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

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Student's *t* Distribution

- The standardized variable t has the Student's t distribution.
- When n is small, the value of s may not be close to σ, so that introduces extra variability.
  - There will be greater variability in both $\bar{X}$ and s across different samples.
- There are many t distributions which are indexed by their degrees of freedom (df).
  - A df is a whole number 1, 2, 3, etc.
- William Sealy Gosset, an English statistician, chemist etc pioneered small experimental design and analysis and developed the t distribution under the author name "Student."

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Properties of the t Distribution

- The t curve corresponding to a given degree of freedom df is bell-shaped and centered at 0 (similar to the standard normal z curve)
- Any t curve is more spread out than the z curve ➔ It gives less accurate estimates.
- As the df increases, the spread of the corresponding t curve decreases.
  - The t curves approach the z curve.

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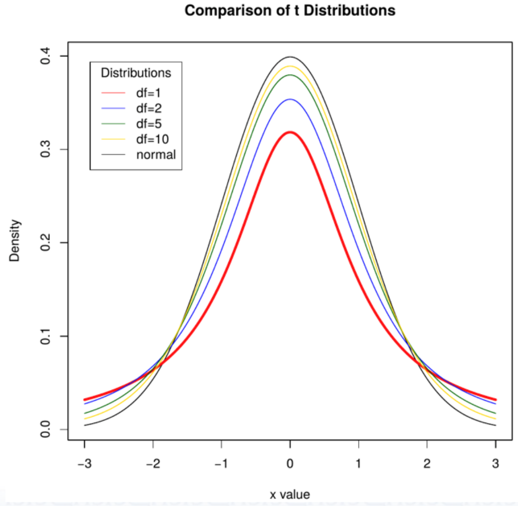
Properties of the t Distribution



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The t Confidence Interval

- Let $x_1, x_2, \ldots x_n$ be a random sample from a normal population distribution with mean μ. Then the sampling distribution of the standardized variable:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

is the t distribution with $\mathrm{df} = n - 1$ degrees of freedom.

- The t confidence interval for μ is

$$\bar{X} \pm (t \text{ critical value}) \cdot \frac{s}{\sqrt{n}}$$

where the critical value is based on $\mathrm{df} = n - 1$ degrees of freedom.

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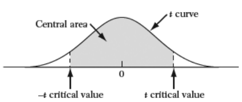
Table of t Critical Values



Central Area Captured: Confidence Level:		.80 80%	.90 90%	.95 95%	.98 98%	.99 99%	.998 99.8%	.999 99.9%
	1	3.08	6.31	12.71	31.82	63.66	318.31	636.62
	2	1.89	2.92	4.30	6.97	9.93	23.33	31.60
	3	1.64	2.35	3.18	4.54	5.84	10.21	12.92
	4	1.53	2.13	2.78	3.75	4.60	7.17	8.61
	5	1.48	2.02	2.57	3.37	4.03	5.89	6.86
	6	1.44	1.94	2.45	3.14	3.71	5.21	5.96
	7	1.42	1.90	2.37	3.00	3.50	4.79	5.41
	8	1.40	1.86	2.31	2.90	3.36	4.50	5.04
	9	1.38	1.83	2.26	2.82	3.25	4.30	4.78
	10	1.37	1.81	2.23	2.76	3.17	4.14	4.59
	11	1.36	1.80	2.20	2.72	3.11	4.03	4.44
	12	1.36	1.78	2.18	2.68	3.06	3.93	4.32
	13	1.35	1.77	2.16	2.65	3.01	3.85	4.22
	14	1.35	1.76	2.15	2.62	2.98	3.79	4.14
	15	1.34	1.75	2.13	2.60	2.95	3.73	4.07
	16	1.34	1.75	2.12	2.58	2.92	3.69	4.02
	17	1.33	1.74	2.11	2.57	2.90	3.65	3.97
Degrees of Freedom	18	1.33	1.73	2.10	2.55	2.88	3.61	3.92
	19	1.33	1.73	2.09	2.54	2.86	3.58	3.88
	20	1.33	1.73	2.09	2.53	2.85	3.55	3.85
	21	1.32	1.72	2.08	2.52	2.83	3.53	3.82
	22	1.32	1.72	2.07	2.51	2.82	3.51	3.79
	23	1.32	1.71	2.07	2.50	2.81	3.49	3.77
	24	1.32	1.71	2.06	2.49	2.80	3.47	3.75
	25	1.32	1.71	2.06	2.49	2.79	3.45	3.73
	26	1.32	1.71	2.06	2.48	2.78	3.44	3.71
	27	1.31	1.70	2.05	2.47	2.77	3.42	3.69
	28	1.31	1.70	2.05	2.47	2.76	3.41	3.67
	29	1.31	1.70	2.05	2.46	2.76	3.40	3.66
	30	1.31	1.70	2.04	2.46	2.75	3.39	3.65
	40	1.30	1.68	2.02	2.42	2.70	3.31	3.55
	60	1.30	1.67	2.00	2.39	2.66	3.23	3.46
	120	1.29	1.66	1.98	2.36	2.62	3.16	3.37
z Critical Values	∞	1.28	1.645	1.96	2.33	2.58	3.09	3.29

The t critical value for a 99% confidence interval based on 19 df.

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Example: The *t* Confidence Interval

Consider the following 20 random values. Compute the confidence interval of the mean.

.95	.85	.92	.95	.93	.86	1.00	.92	.85	.81
.78	.93	.93	1.00	.93	1.06	1.06	.96	.81	.96