2. Inverse Geometry

5 minutes trailer

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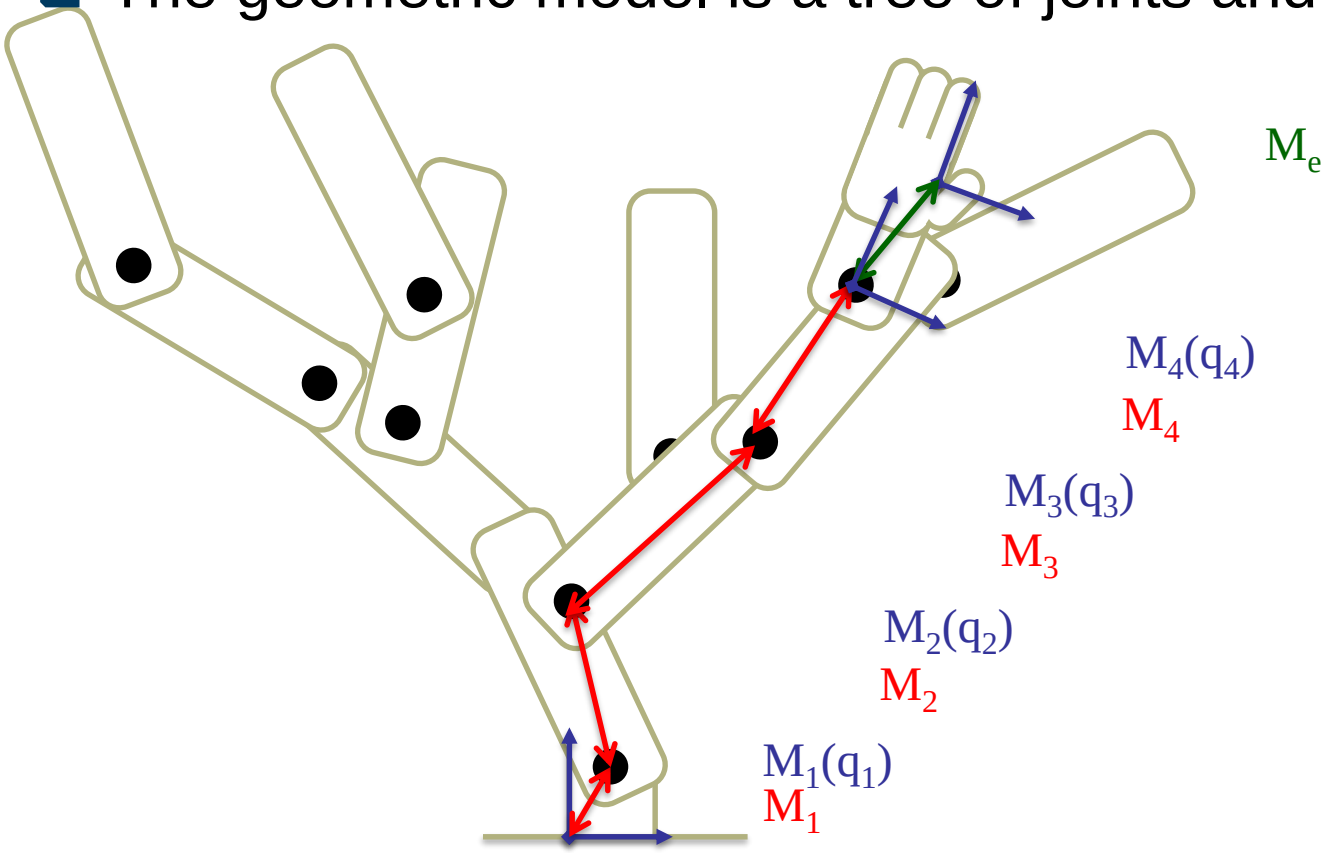
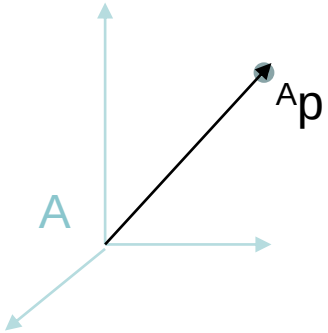
Direct geometry

- The geometric model is a tree of joints and bodies



$$M(q) = M_1 \oplus M_1(q_1) \oplus M_2 \oplus \ldots \oplus M_4 \oplus M_4(q_4) \oplus M_e$$

Representation!



This is a point

This is not a point
This is the representation of a point

Rotation

- Rotation matrices

$$R = \begin{pmatrix} r00 & r01 & r02 \\ r10 & r11 & r12 \\ r20 & r21 & r22 \end{pmatrix}$$

- Derivation of a matrix

$$\dot{R} = \cdots$$

Angular velocity / Angle vector

- Formal definition

$$\dot{R} = \omega \times R$$

- From rotation to velocity
  - R -> w
- From velocity to rotation?
  - w -> R … integrate

- Meaning of w ?
- Angle axis representation

Quaternions

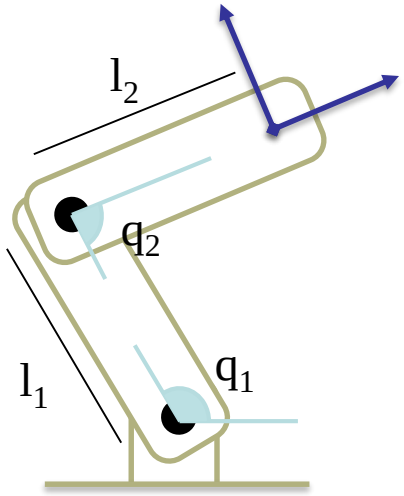
- Start from complex
  - 1, i, -1, -i, 1 …
  - X, Y, -X, -Y, X …
  - Complexs can map the 2D plan , and the 2D rotation
- Hamilton (again!) says: let's do it more complex
  - j so that $j^2 = -1$ and ij = -ji
  - ij = k , jk = i …
  - $x \times y = z$, $y \times z = x$ …
- Unit quaternions map 3D rotations
  - q = [w, x, y, z] = cos(α/2), sinα/2)[a,b,c]

Inverse geometry

- Being given a x^* …
- what is q such that $h(q) = x^*$

$$M^{-1}: x^* \rightarrow q = M^{-1}(x^*)$$

$$M(q) = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1+q_2) \\ l_1 \sin(q_1) + l_2 \sin(q_1+q_2) \end{bmatrix}$$



Numerical inversion of the geometry

- Computing analytically h^{-1} is difficult and tedious
- We can compute it numerically!

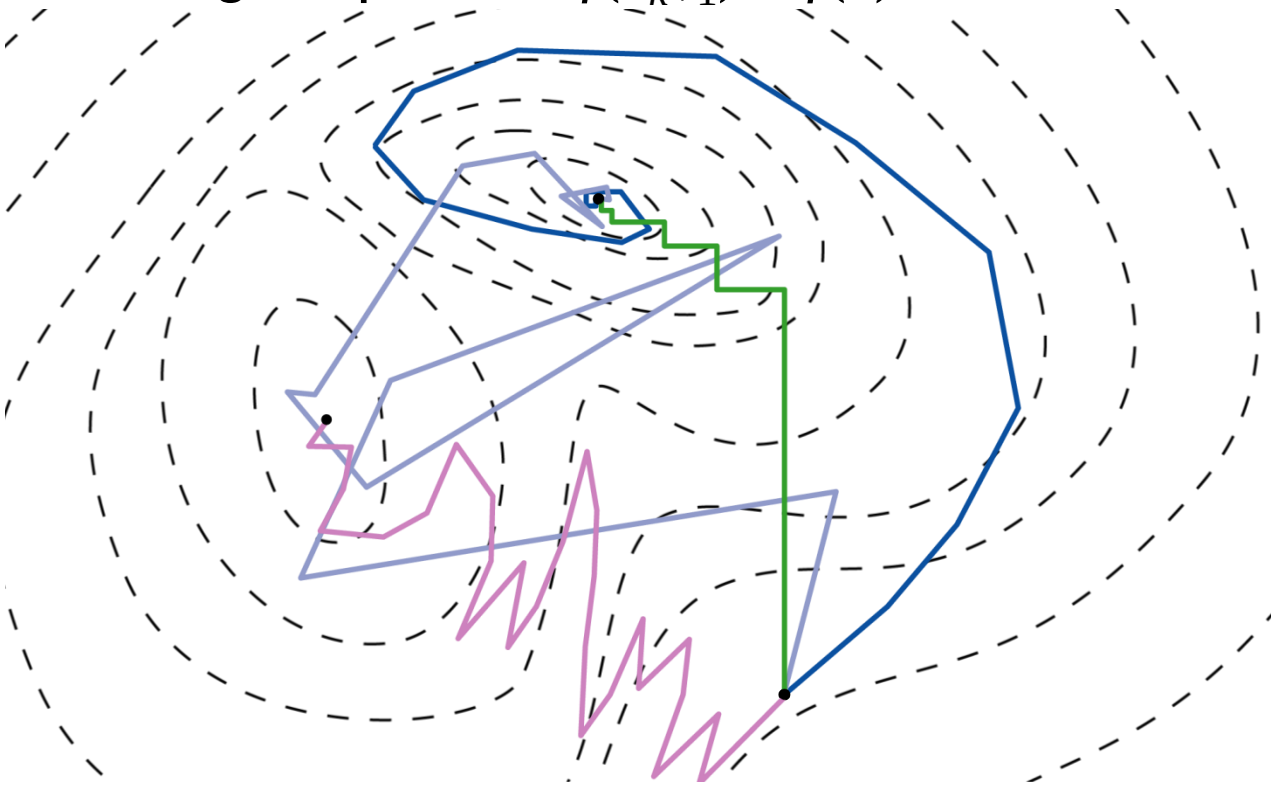
- Problem definition

$$search\ f(x) - f^* = 0$$

$$min\ \| f(x) - f^* \|$$

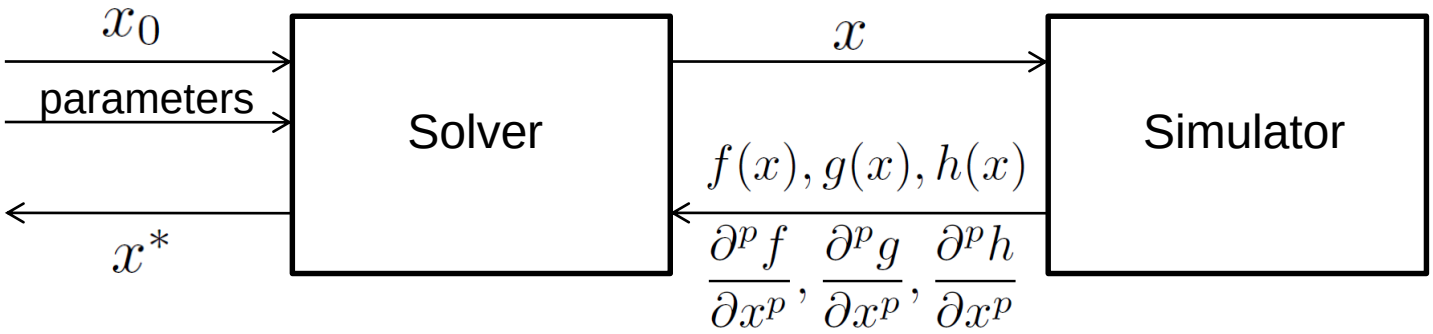
Follow the slope

- Decreasing sequence: $f(x_{k+1}) < f(x)$



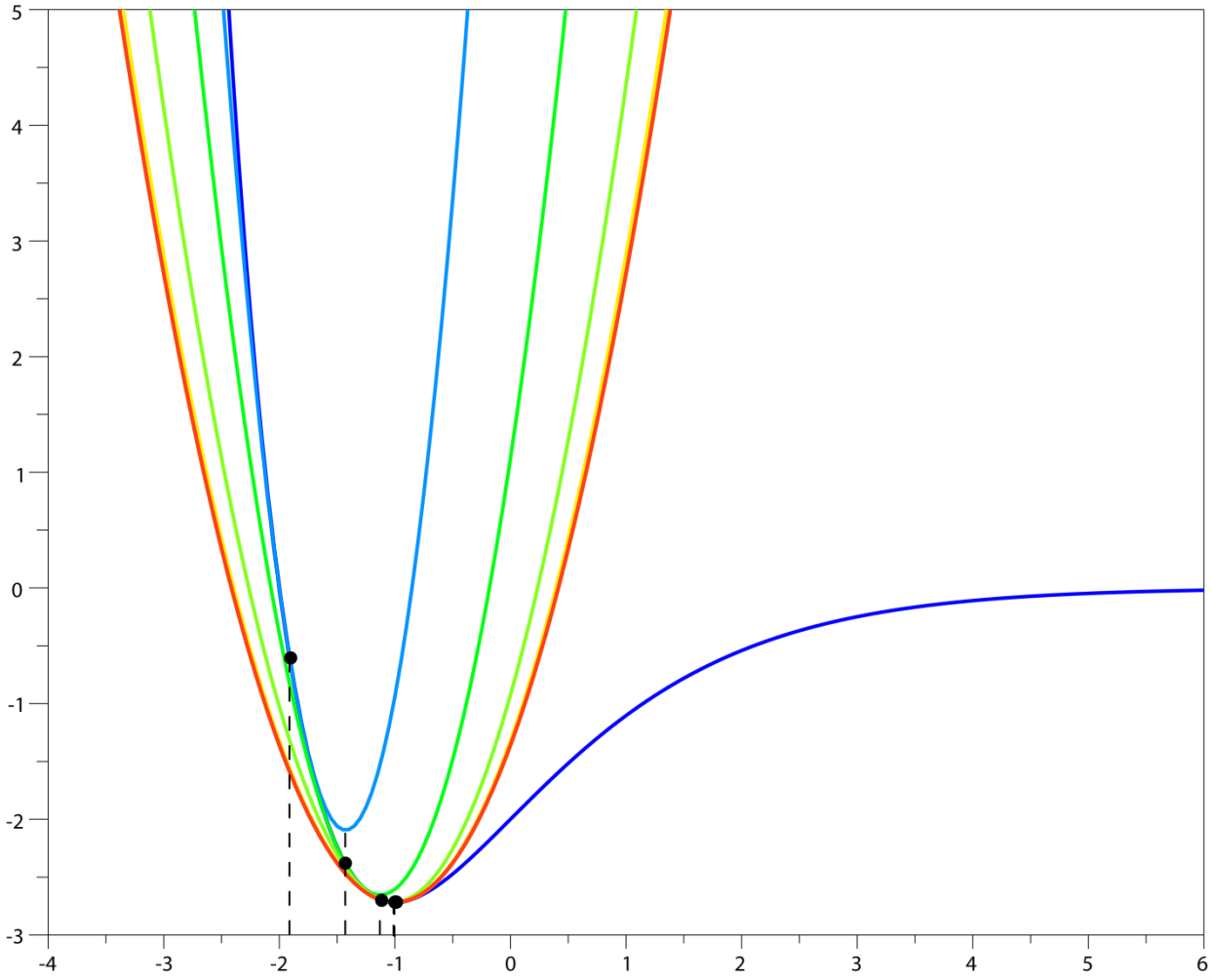
Problem specifications

- Problem specification
  - Computing $f(x)$ is easy
  - We can derivate $f: x \rightarrow f(x)$
  - We know the distance to the reference value



Newton method (unconstrained)

x0=-1.9
x1=-1.4263158
x2=-1.1274228
x3=-1.0144015
x4=-1.0002045
x5=-1.00000004
x6=-1.



Convergence rate

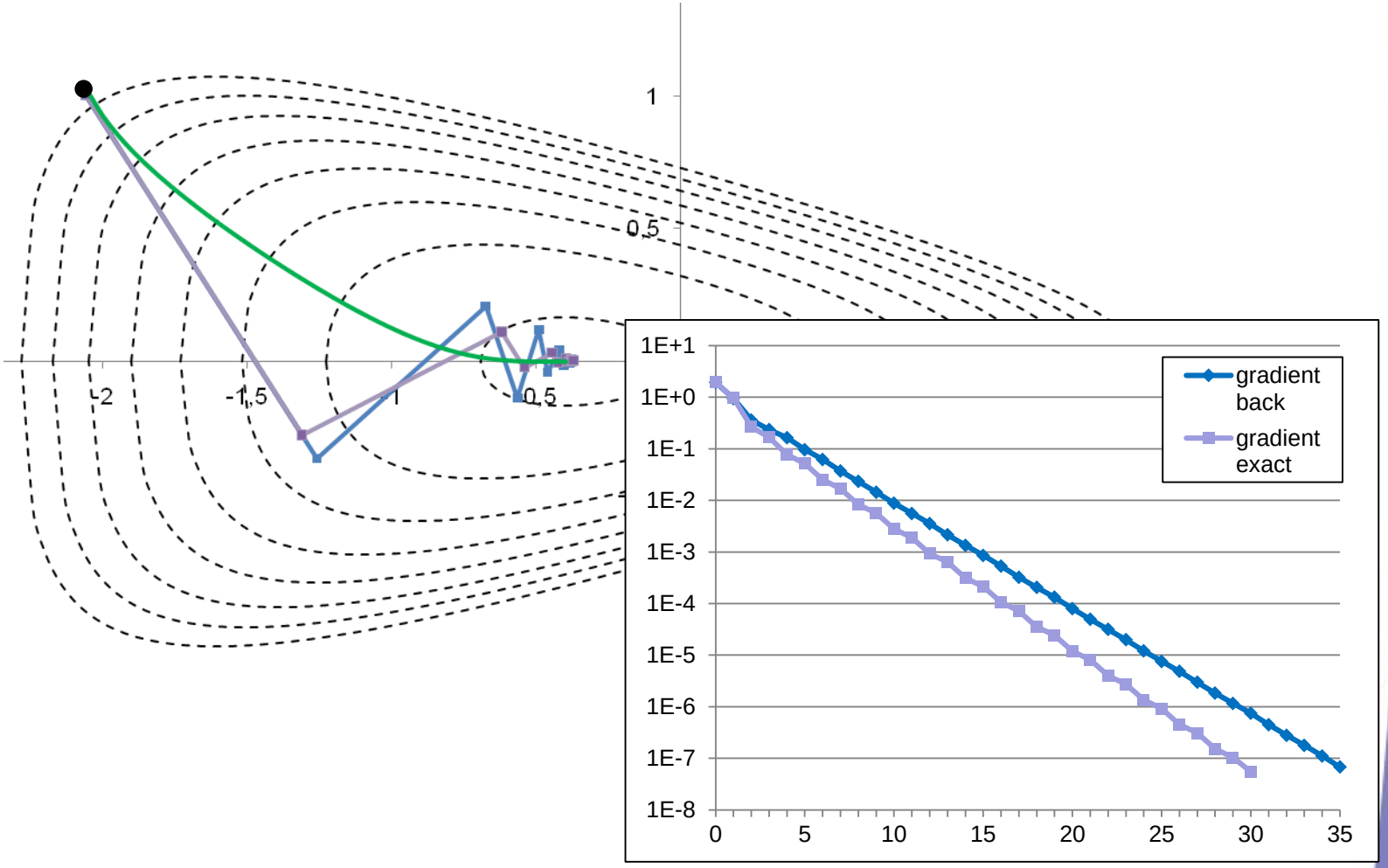
	linear	superlinear	quadratic
0	1.	1.	1.
1	1.2071067811865475244008444	1.2071067811865475244008444	1.5
2	1.3106601717798212866012674	1.3451779686442458740014084	1.4178932188134524755991560
3	1.3624368670764581677014784	1.3969546639408827551016194	1.4142032033230885458074630
4	1.3883252147247766082515834	1.4107617826866525900616754	1.4142135623960110638920294
5	1.4012693885489358285266364	1.4136382657586879723450204	1.4142135623730950488019524
6	1.4077414754610154386641634	1.4141313771424654664507364	1.4142135623730950488016894
7	1.4109775189170552437329264	1.4142032892192663510078204	
8	1.4125955406450751462673074	1.4142124209115585268245924	
9	1.4134045515090850975344984	1.4142134482269413966039804	
10	1.4138090569410900731680944	1.4142135519961719895109884	
11	1.4140113097278032391035464	1.4142135615083514605274644	
12	1.4141124360504491439526184	1.4142135623065763112421334	
13	1.4141629992117720963771534	1.4142135623683437104045784	
14	1.4141882807217228944707664	1.4142135623727782929085484	
15	1.4142009215474039716362284	1.4142135623730752515583684	
16	1.4142072419602495102189584	1.4142135623730938842579644	
17	1.4142104021666722795103244	1.4142135623730949841048164	
18	1.4142119822698836641560064	1.4142135623730950453965904	
19	1.4142127722507781783601934	1.4142135623730950486314344	
20	1.4142131673119366135809414	1.4142135623730950487935824	
21	1.4142133649132265093099704	1.4142135623730950488013214	
22	1.4142134636431607790558294	1.4142135623730950488016734	
23	1.4142135130081279139287594	1.4142135623730950488016894	
81	1.4142135623730950488016887		

$p=2$

$$\frac{r_{k+1}}{r_k^{\,p}} \underset{+\infty}{\longrightarrow} \alpha \qquad 0<\alpha<1,\ p>1$$

$$r_{k+1} \sim \alpha\, r_k\,, \qquad 0<\alpha<1$$

Gradient descent



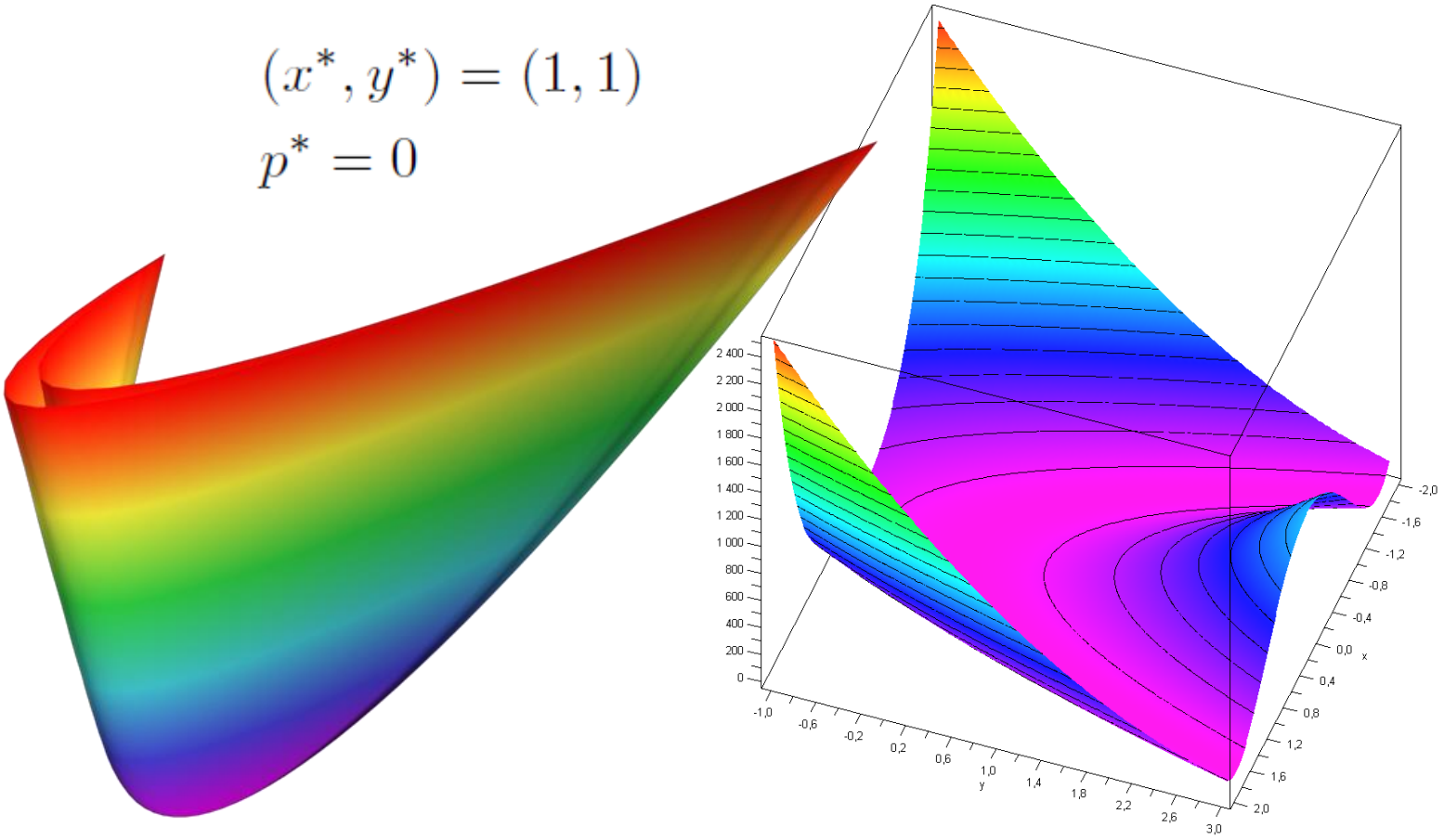
Gradient descent

- The nemesis: the Rosenbrock's banana function

$$f(x,y) = (1-x)^2 + 100 * (y - x^2)^2$$

$$(x^*, y^*) = (1, 1)$$

$$p^* = 0$$



Examples

- Ill-conditionned hessian
- Non positive hessian