

Mathematical Programming in Robotics

Nicolas Mansard



Complex movement for complex robots

Complexity of the problem

VS

Needed computational efficiency

1 hour of
computation



1000 Hz
decisions / seconds

Number of degrees of freedom

Uncertainty

Instability of dynamics

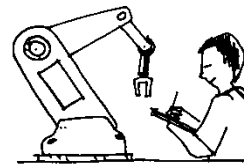
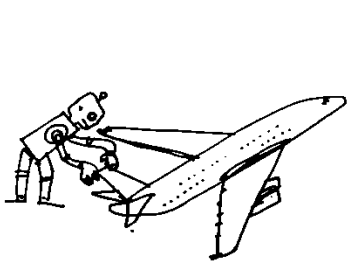
Linearity/convexity of problems

Environment dynamics

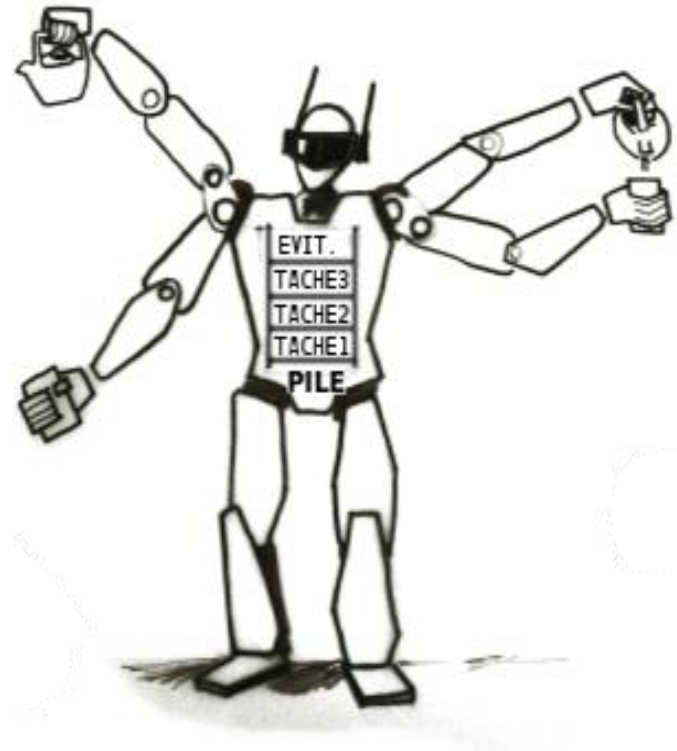
Robot speed

Bounded viability

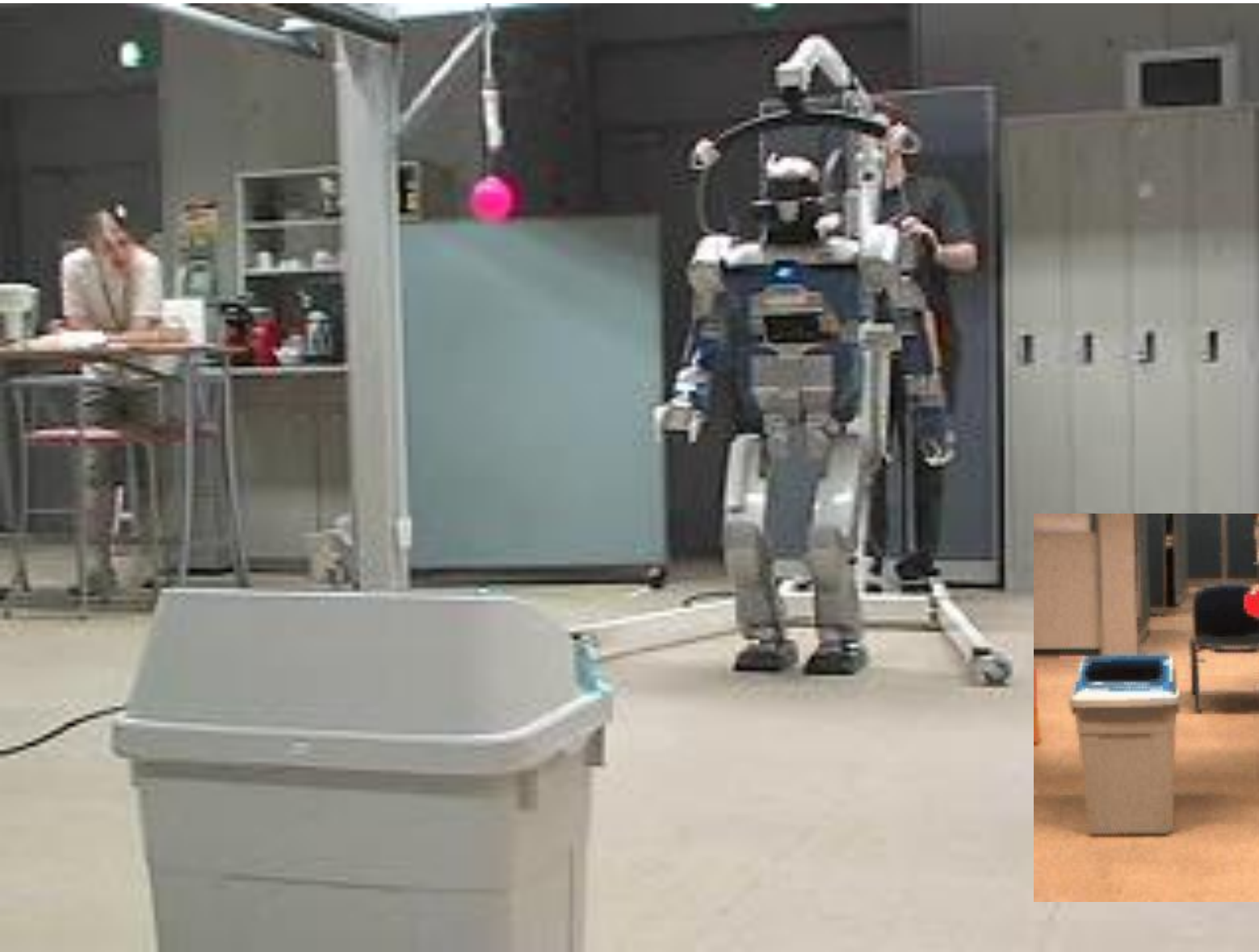
Control of a contact interface



Stacking tasks



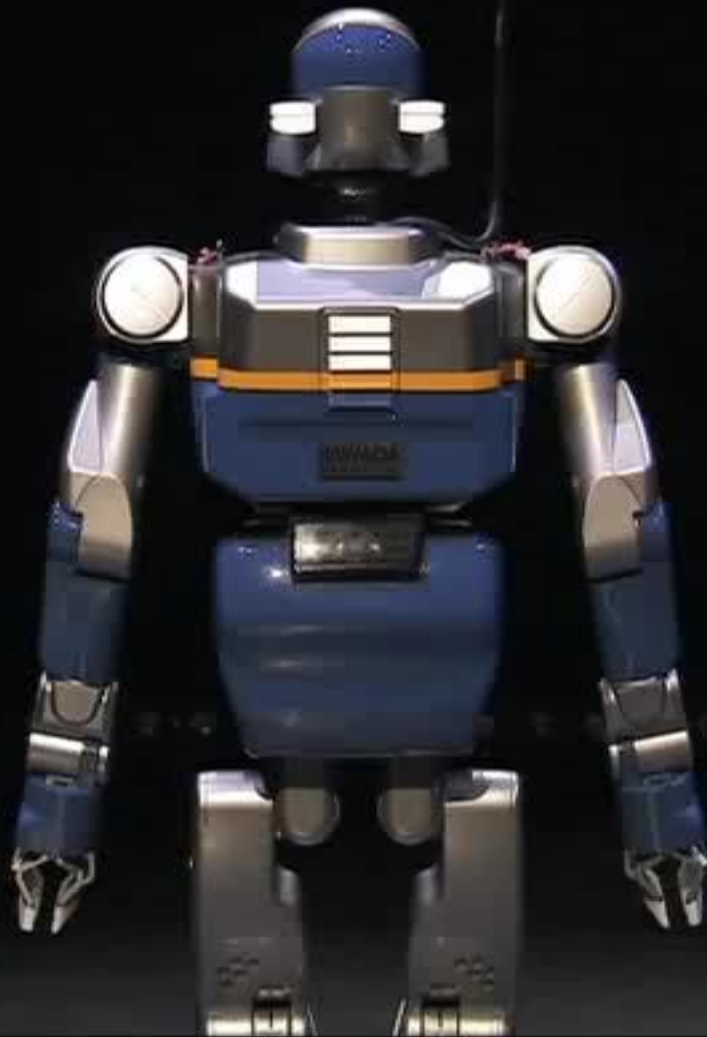
Vision control



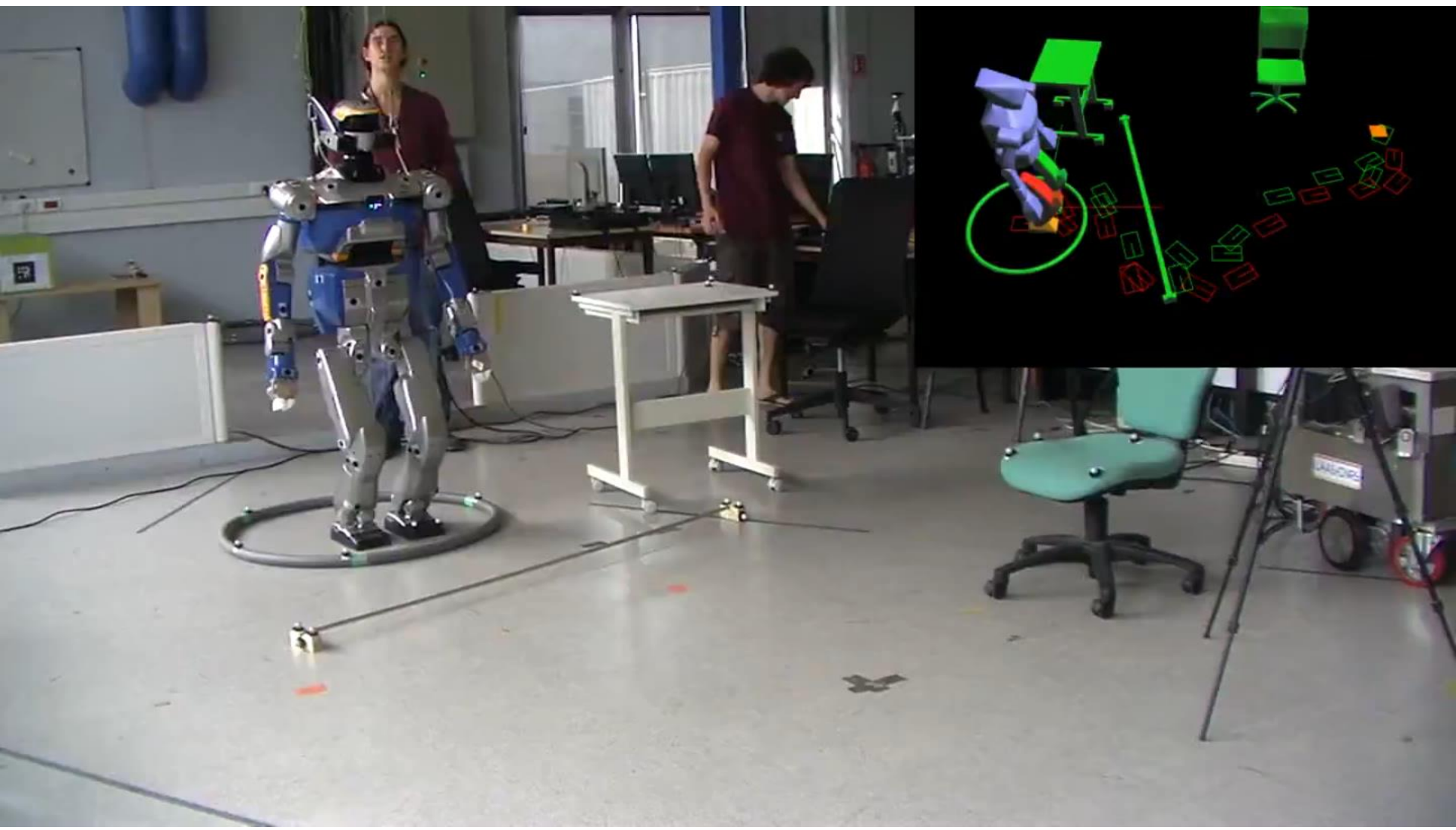
Force control (@JRL+TUM, with LAAS)



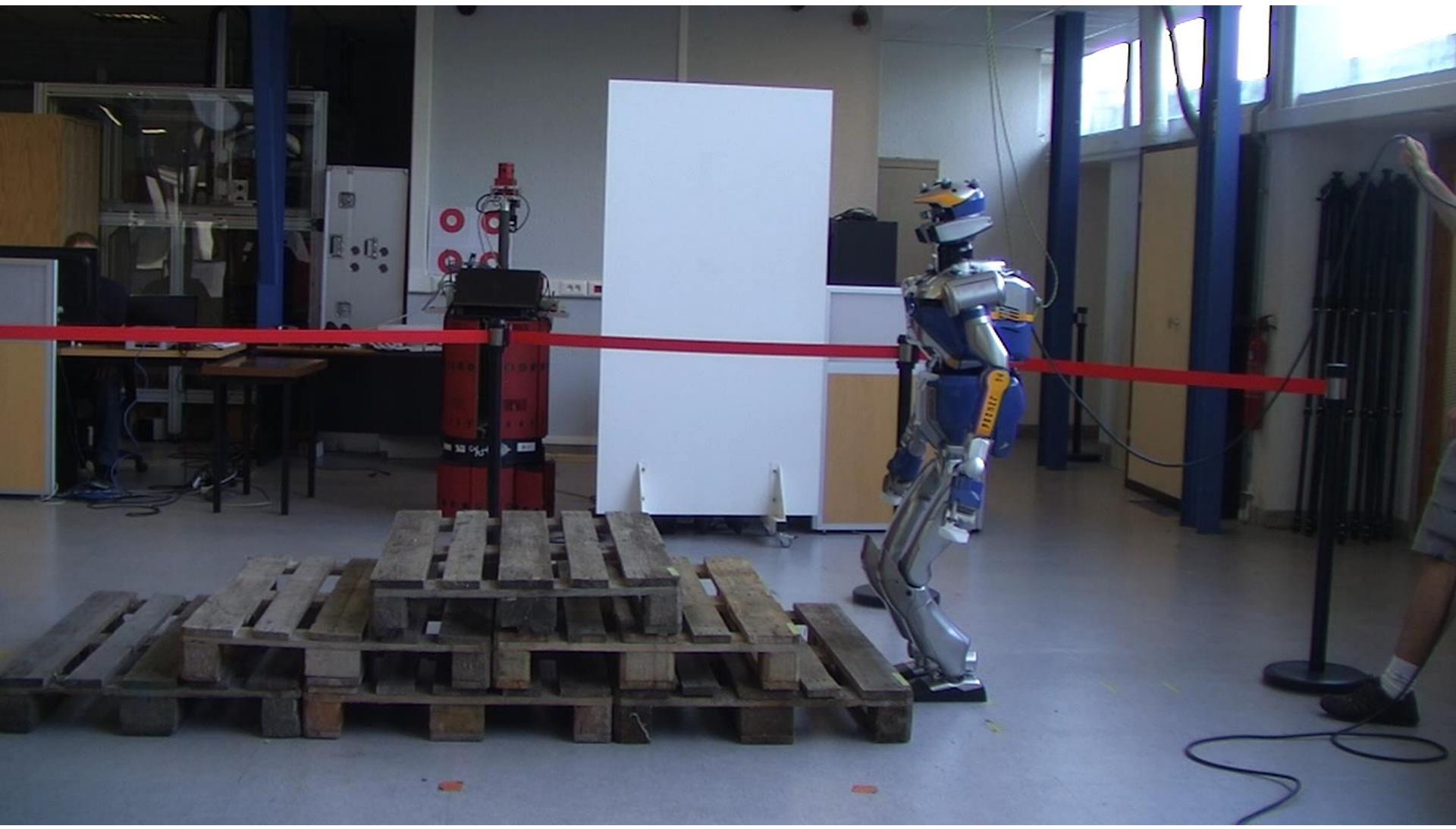
Dynamics



Reactive walk (@LAAS, with JRL)



Quasi-flat walk



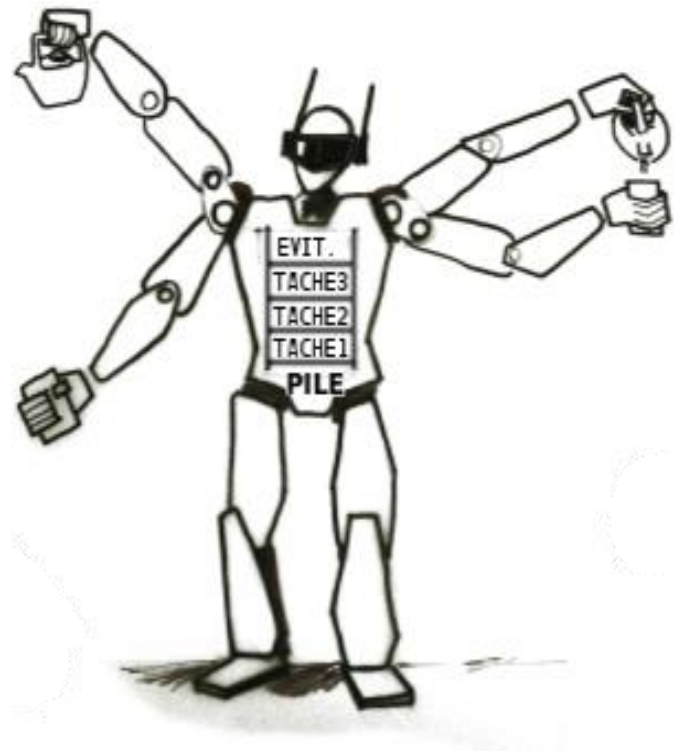
Hierarchy and sequencing



Stacking tasks



Visual control



Hierarchy



Force control



Reactive walk



Dynamics



Quasi-flat

Planning
versus
control



Control:
Instantaneous versus receding



Feedback
linearization



*Limit
behavior*



planning

*Work on
the same
object*



Receding
control

Stake in optimal control

Non-linear / non-convex

Cost and constraints

$$\min_{\substack{X=(Q,\dot{Q}), \\ U=\tau}} \int_0^T l_1(x_t, u_t) dt$$

so that $\forall t, \dot{x}(t) = f(x(t), u(t))$

Dynamics

Thousands of (sparse) variables

Current status of our DDP

Unconstraint sparse (ad-hoc) solver

Dedicated to locomotion

Fixed contact sequence and timings

Alternance with centroidal optimization

Analytic derivatives

Foot trajectory around obstacles

Target: 1 second horizon / 10 ms computation

3 seconds horizon / 7 ms computation



Current status of constrained optimization

Augmented lagrangian

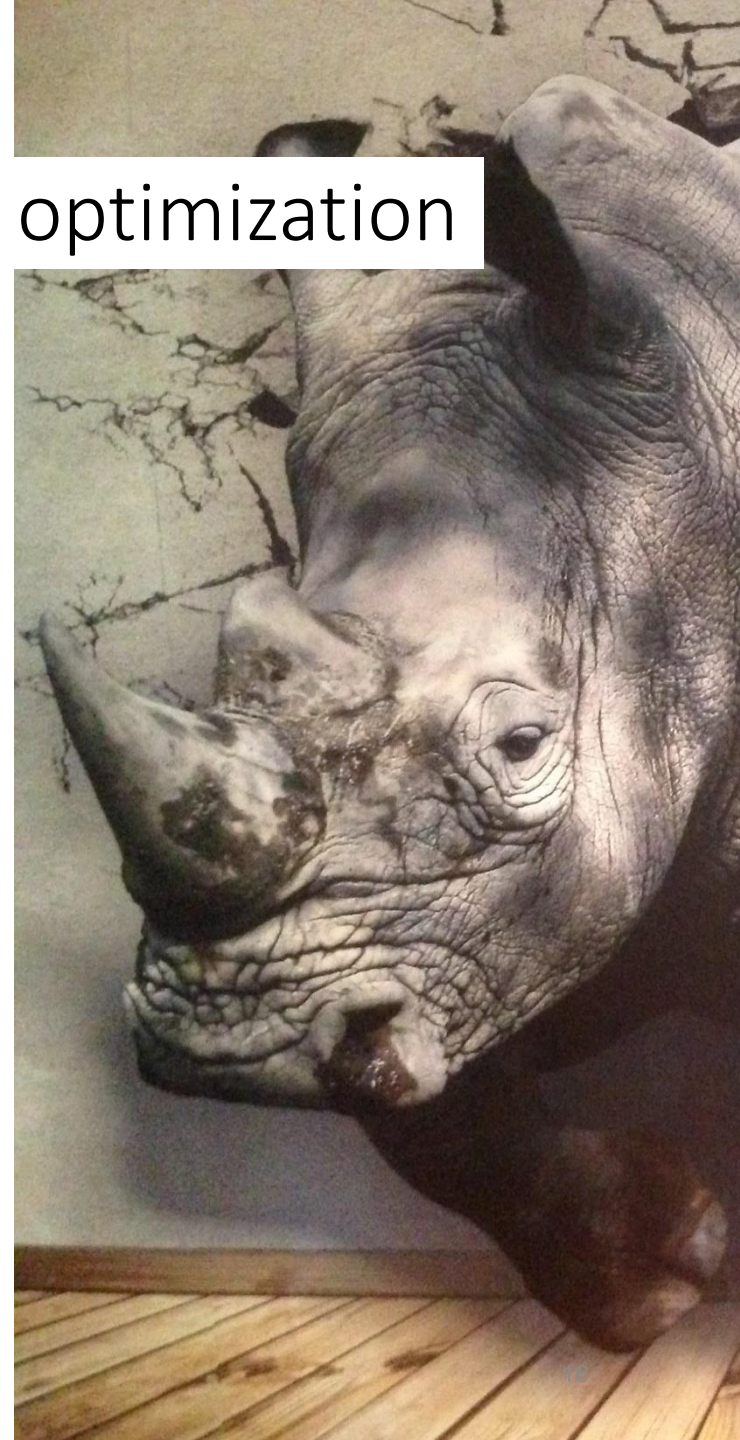
Stronger than SQP

Easier to initialize than Interior-points

Benchmark versus IPOPT and SciPy::SQP

Target: home-made sparse NLP

with optimization on manifolds



Accomplishment

~~Stake~~ in optimal control

Dimension: Thousand of variables

Dedicated DDP, release in Feb 2019

Dynamics: Fast forward simulation

Pinocchio 2.0 with derivative, released in beta

Complexity: Non-linearity and non-convexity

Constraints: physical limits and obstacles

On-going works, preliminary results

What is missing?

$$\begin{aligned} \min_{X,U} \int_0^T l(x_t, u_t) dt \\ \text{s.t. } \dot{x}_t = f(x_t, u_t) \end{aligned}$$

optimal control

=



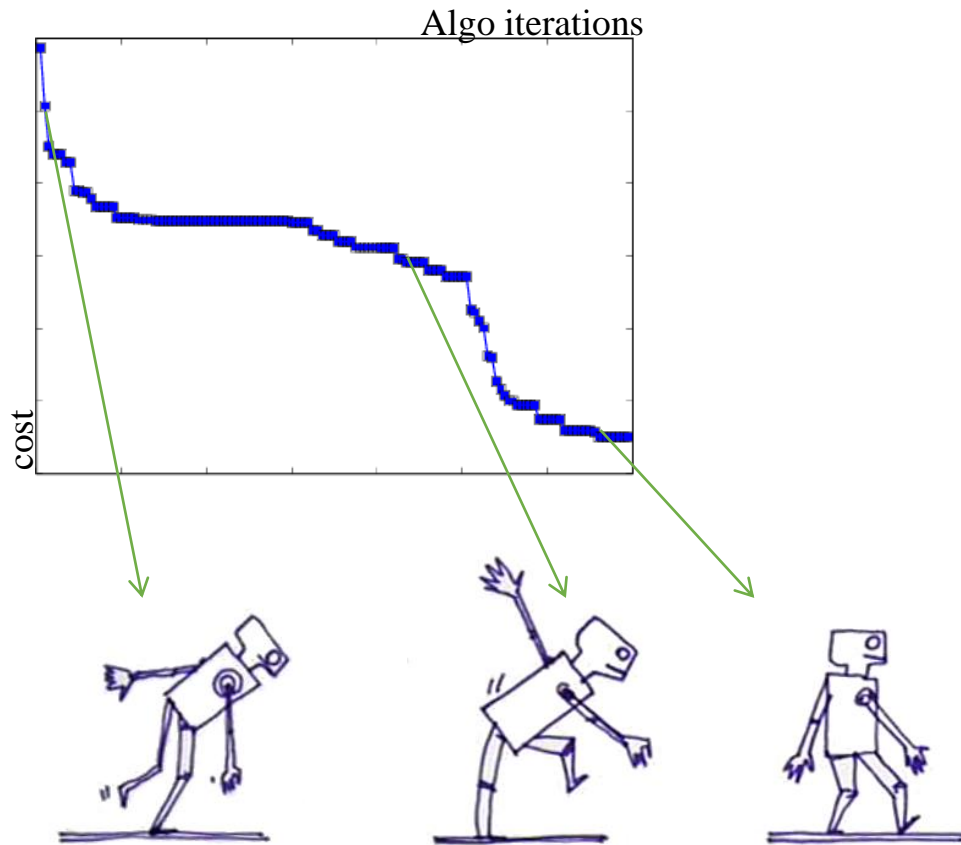
planning

control

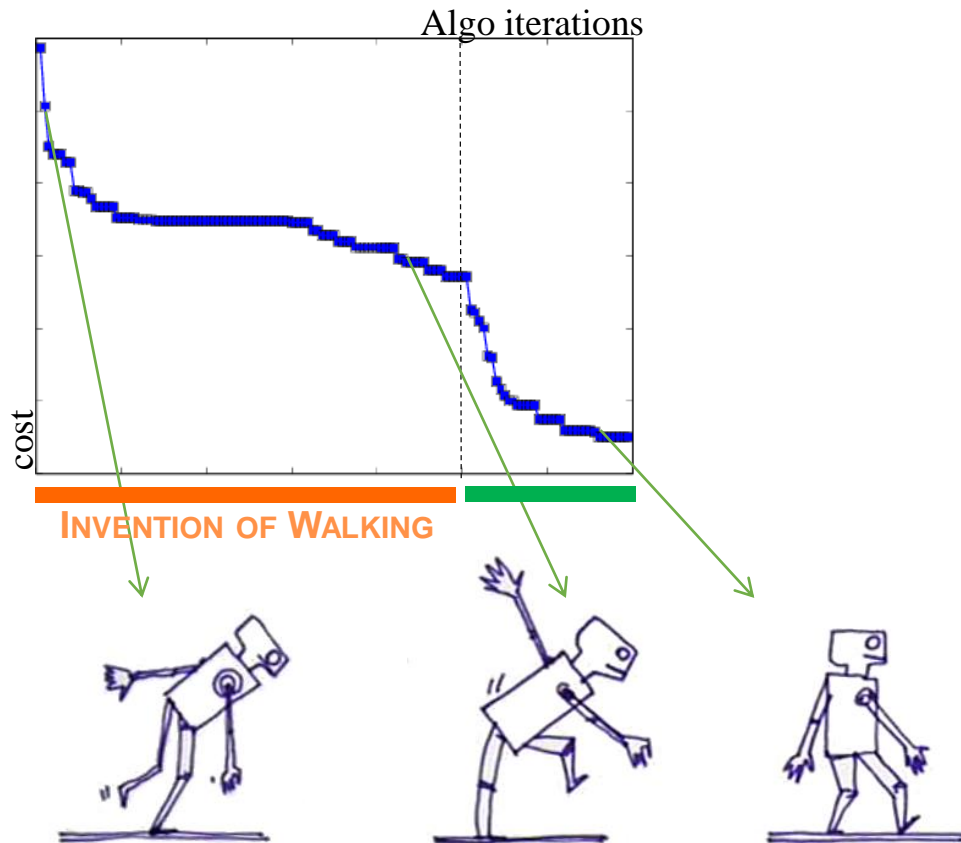
*... but it needs
guidance*



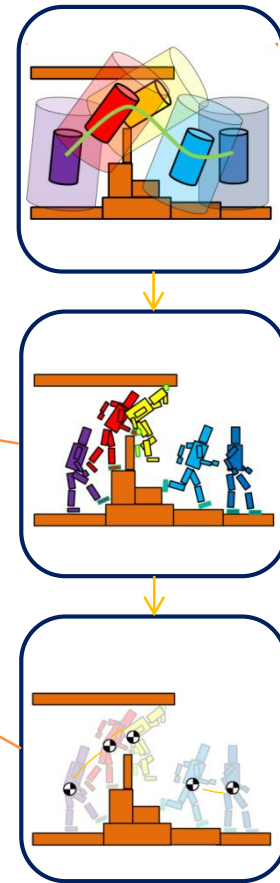
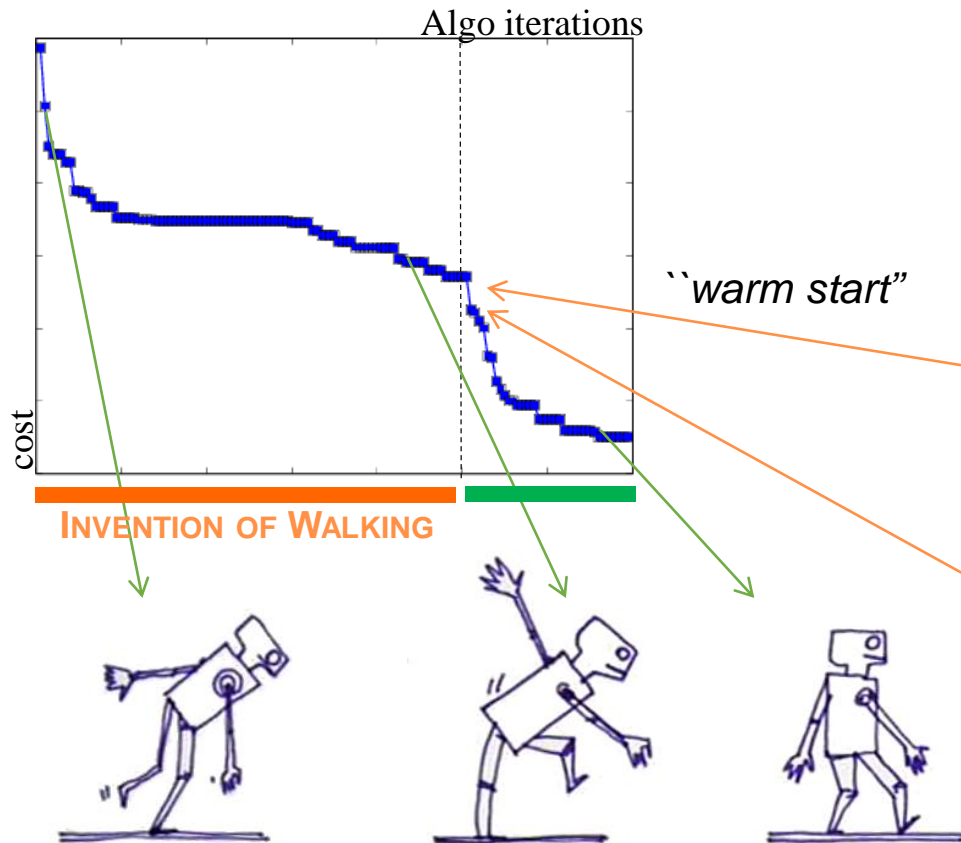
Example of locomotion



Example of locomotion

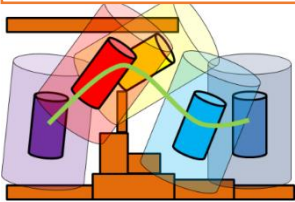


Example of locomotion

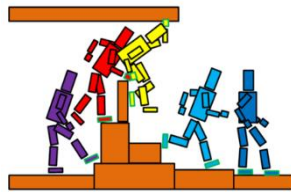


Model-based warm start

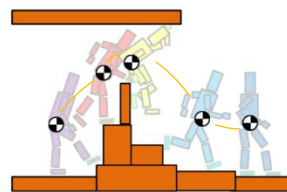
Reachability
planner



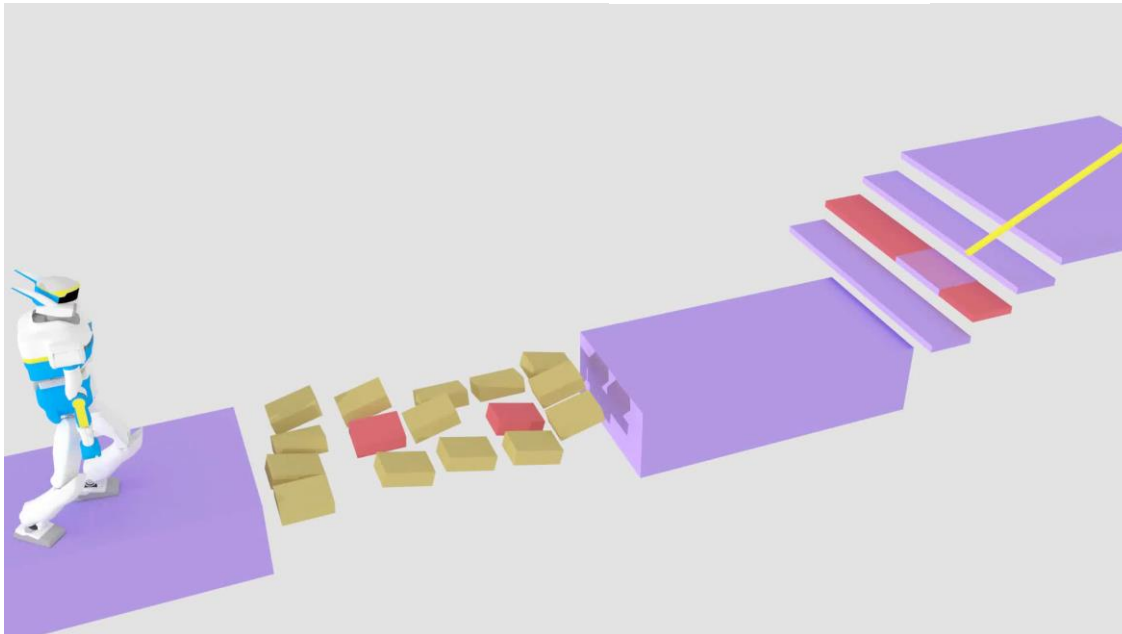
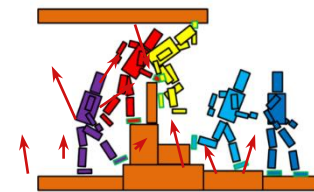
Sequence of
contacts



3D Pattern
generator



Whole-body
force control

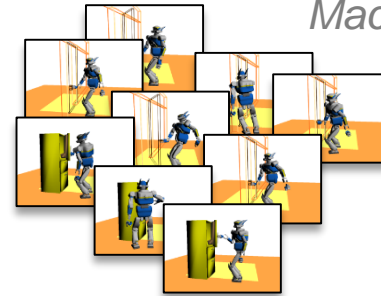


Memory of Motion

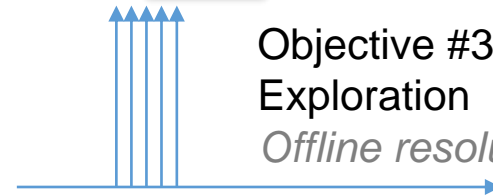
Objective #1. Data-driven
model predictive control
Targeting real platforms

$$\min_{x,u} \int_0^T l(x(t), u(t)) dt$$
$$s.t. \quad \forall t \quad \dot{x}(t) = f(x(t), u(t))$$

Objective #2. Memory encoding
Machine learning



Objective #3.
Exploration
Offline resolution



CS #1. Humanoid in
factory of the future



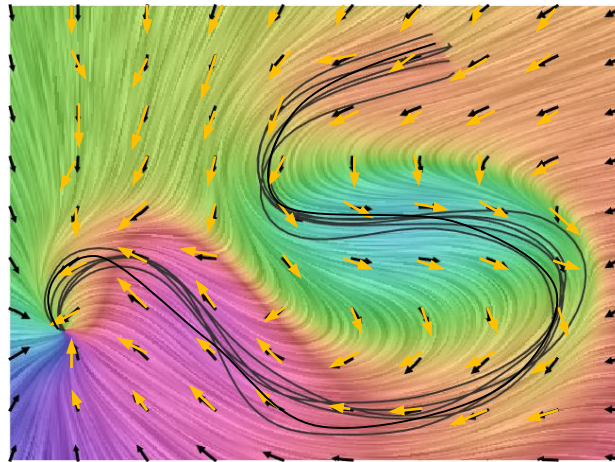
CS #2. Exoskeleton
for disabled people



CS #3. Quadruped for
inspection

$$\min_{\substack{X=(Q,\dot{Q}), \\ U=\tau}} \int_0^T l_1(x_t, u_t) dt$$

so that $\forall t, \dot{x}(t) = f(x(t), u(t))$



Optimizing a trajectory

$U: t \rightarrow u(t)$

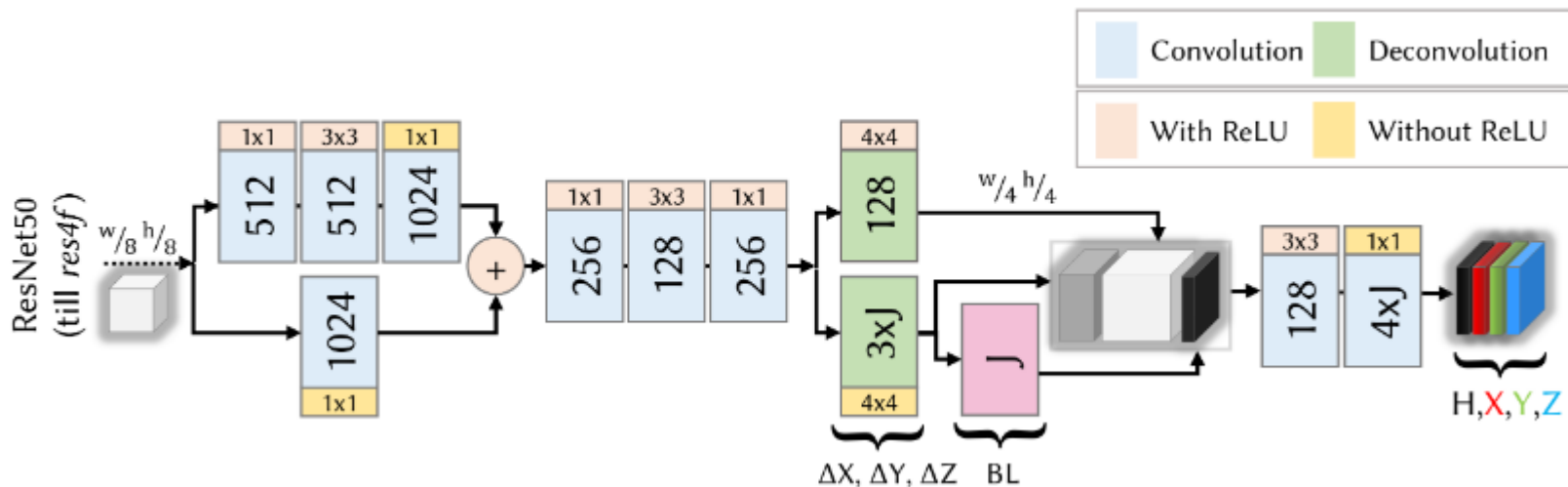
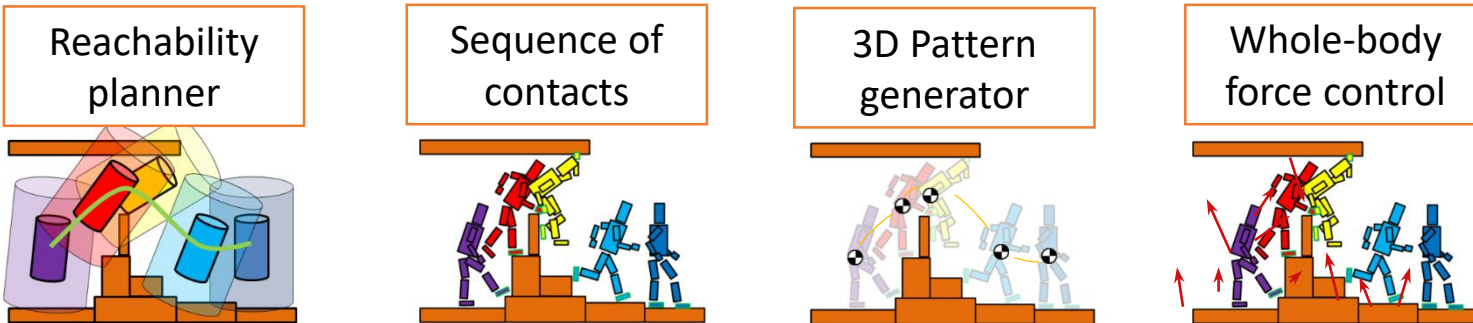
Motion planning

Optimizing a policy

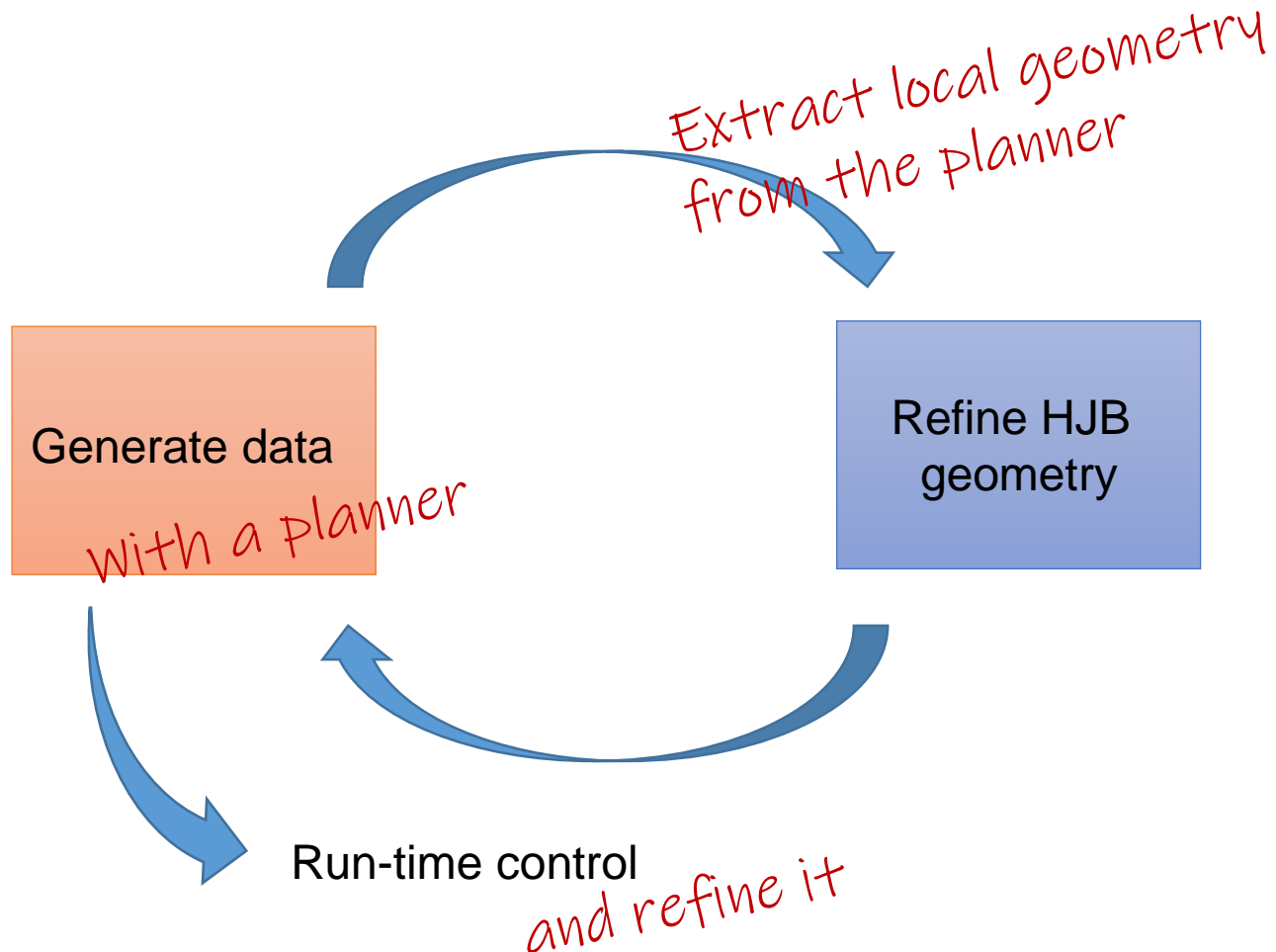
$\Pi: x \rightarrow u = \Pi(x)$

Reinforcement learning

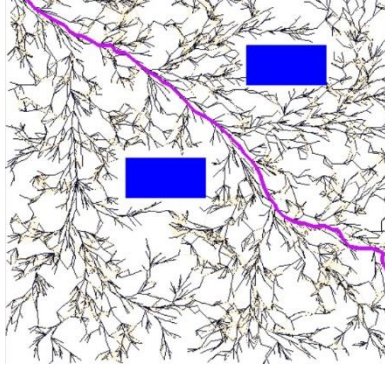
First track: learn HPP-Loco



Second track: reinforcement learning



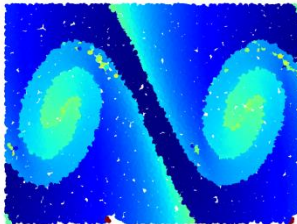
Roadmap/approximation co-training



Kino-dynamic
Probabilistic Roadmap

30-50 states, dense connect

↑ Roadmap
extension



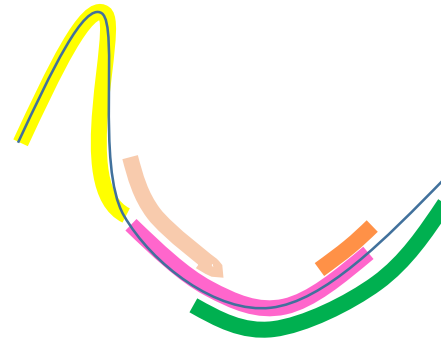
HJB approximation

Value function as metric

Policy function as warm-start



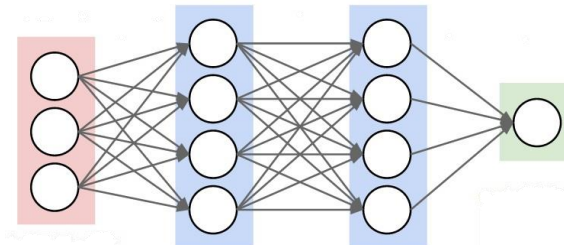
Sampling



Dataset of subtrajectories

10-100k items

↓ Regression
(stoch.grad.)

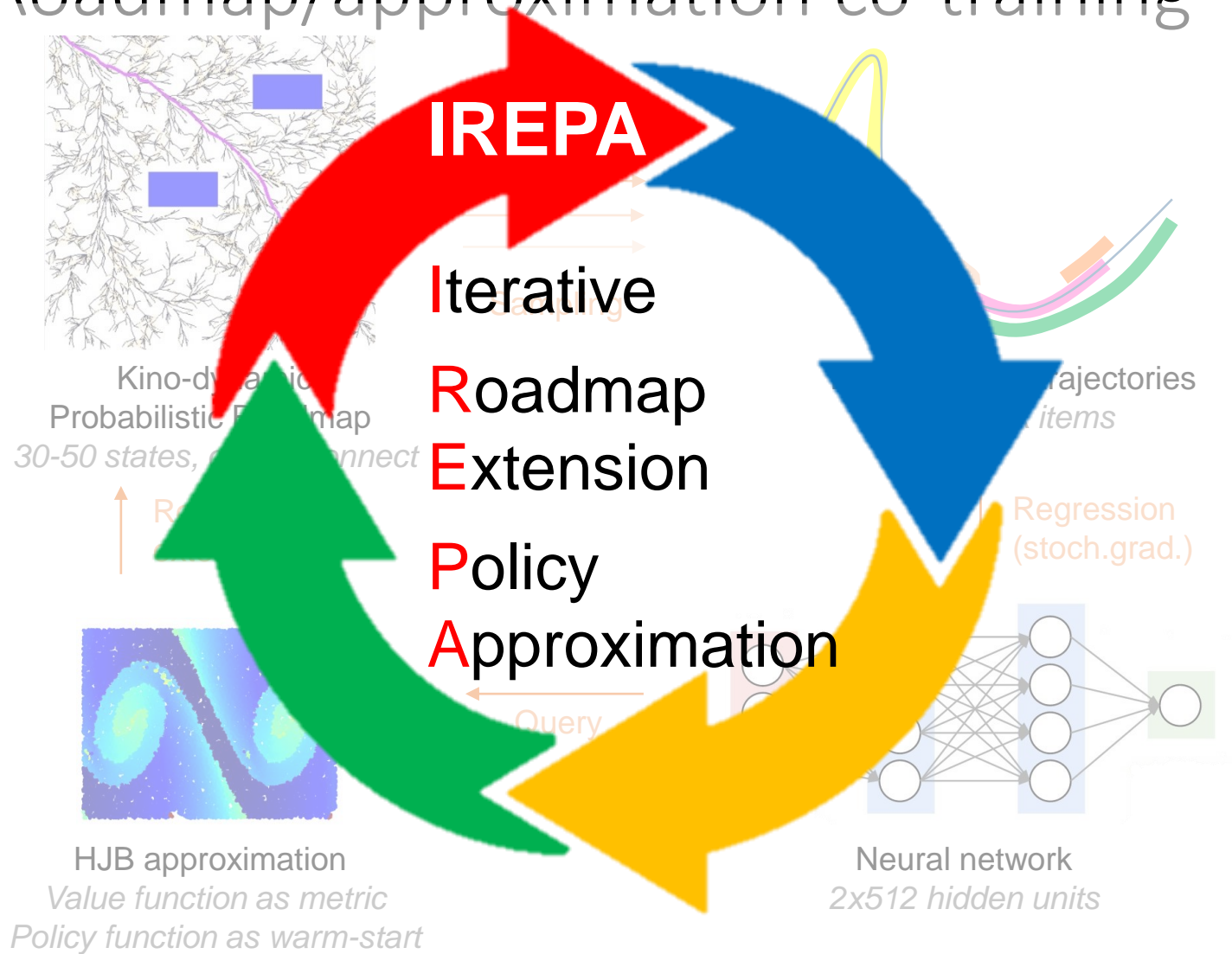


Neural network

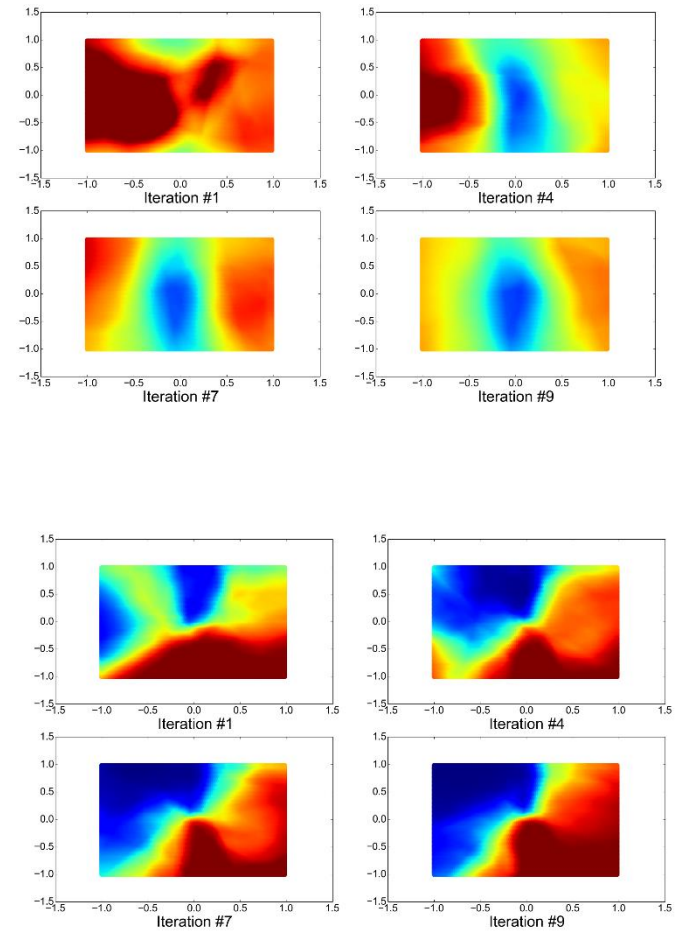
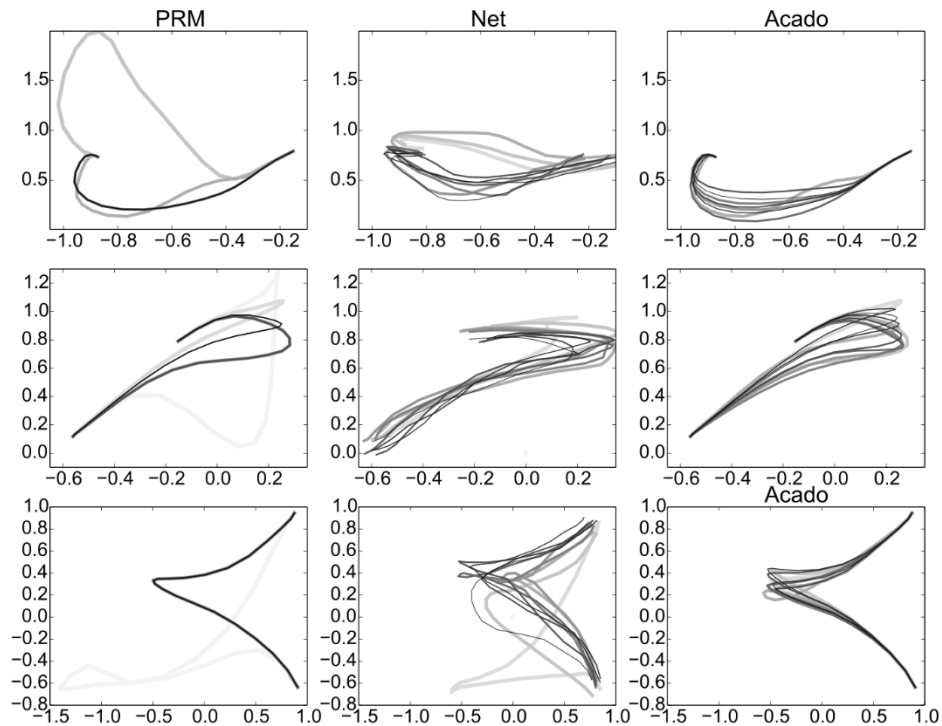
2x512 hidden units

← Query

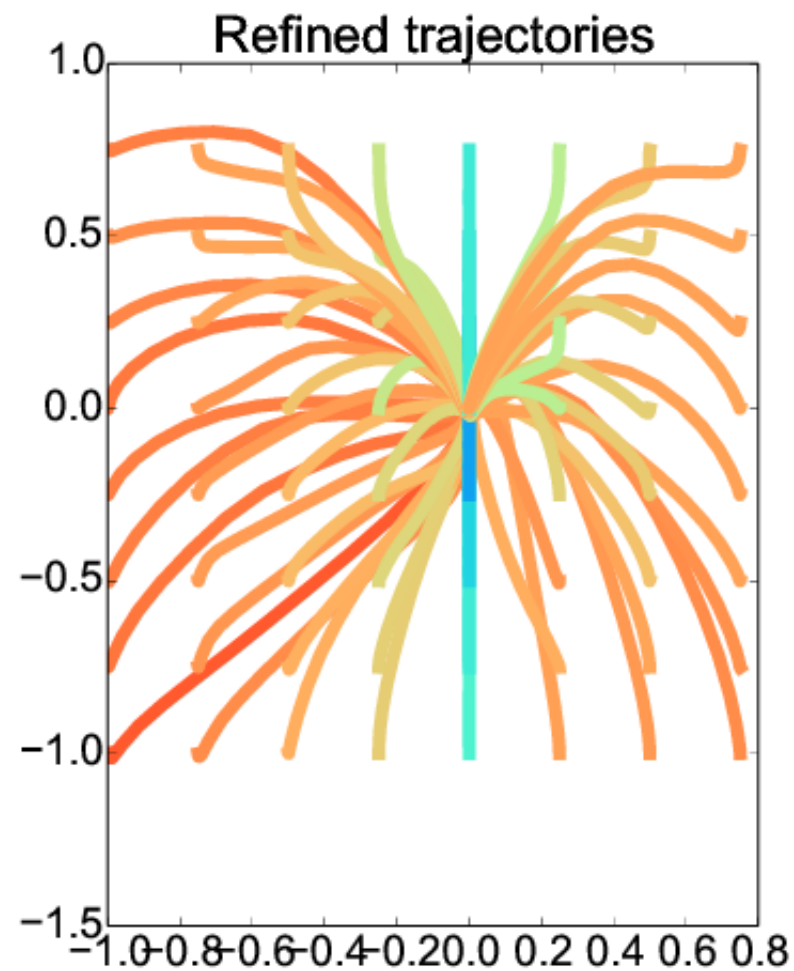
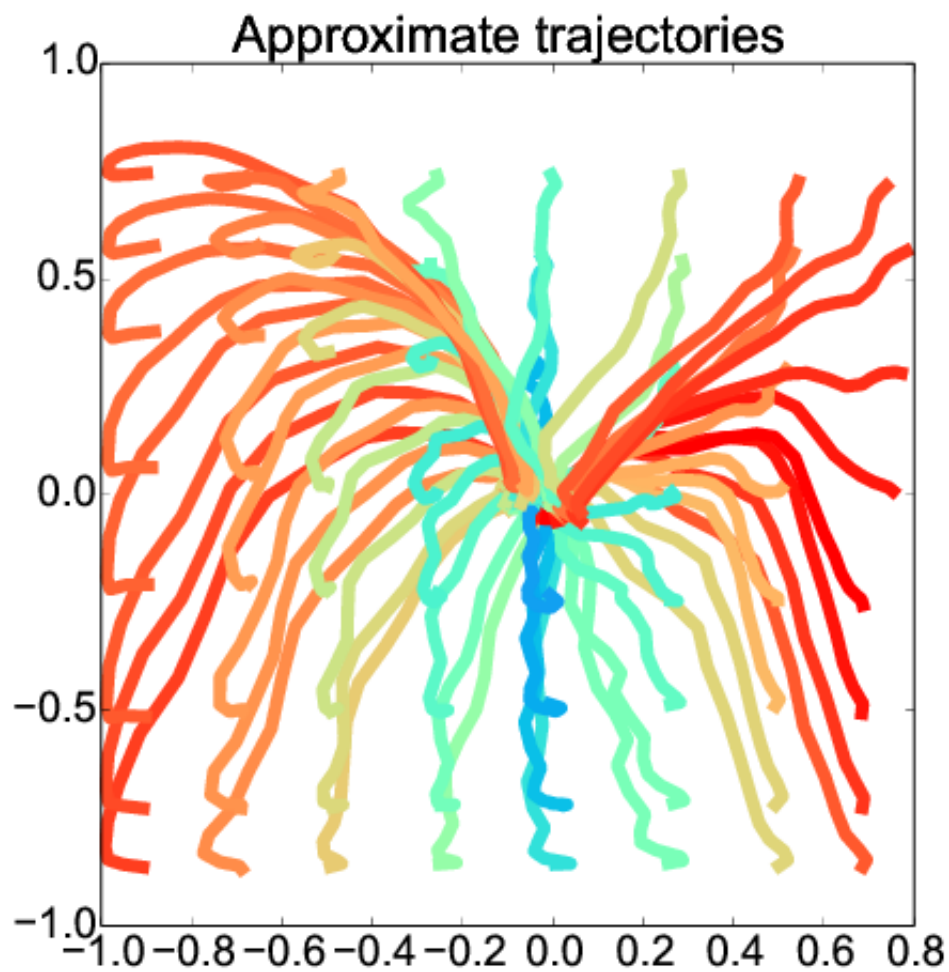
Roadmap/approximation co-training



IREPA: iterations



IREPA with MPC



Double Pendulum

Number of states: 4

Number of controls: 2

Torque limites:

- joint 1: 5 Nm**
- joint 2: 10 Nm**

Mass: 6 kg

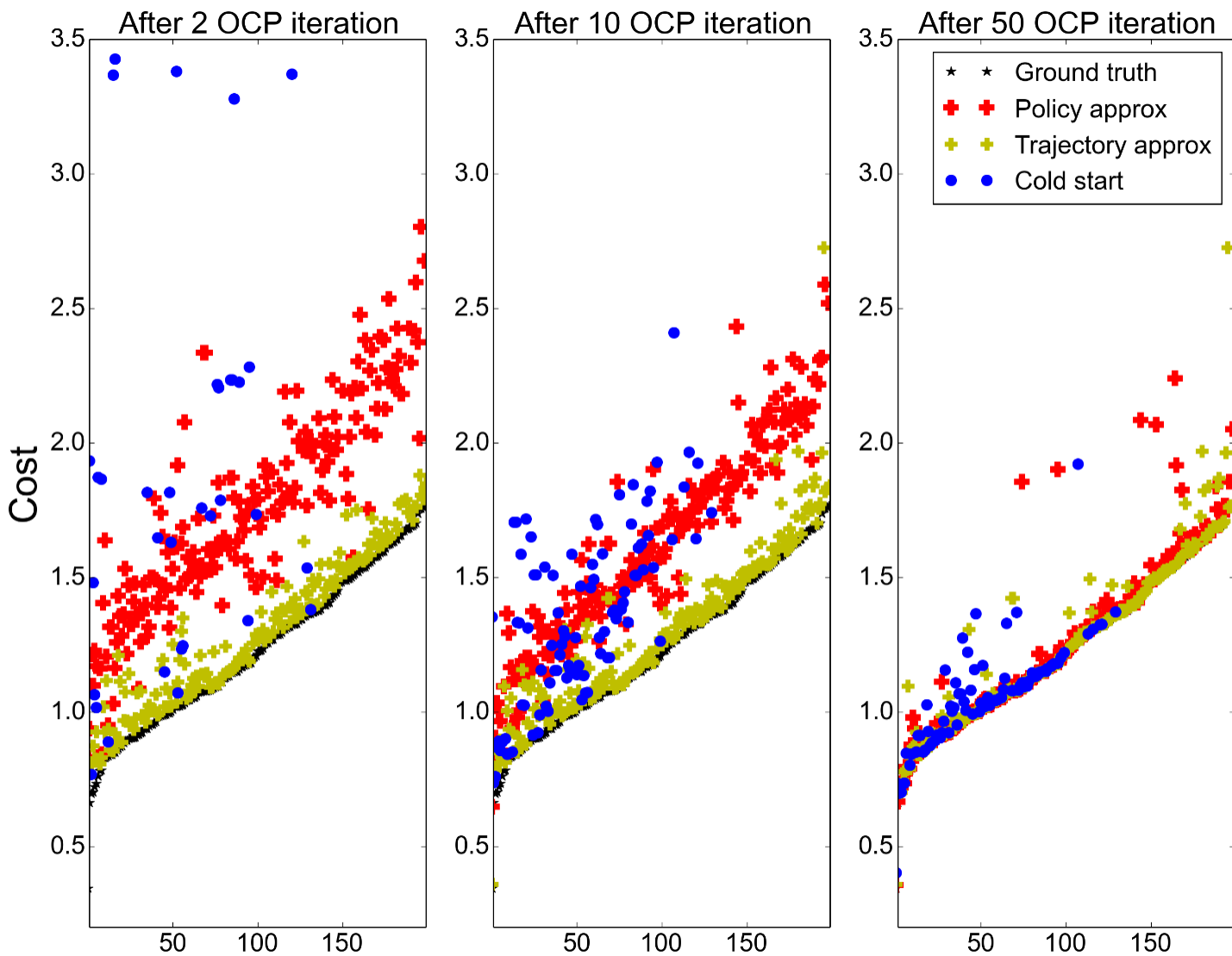


PRM: 30 nodes

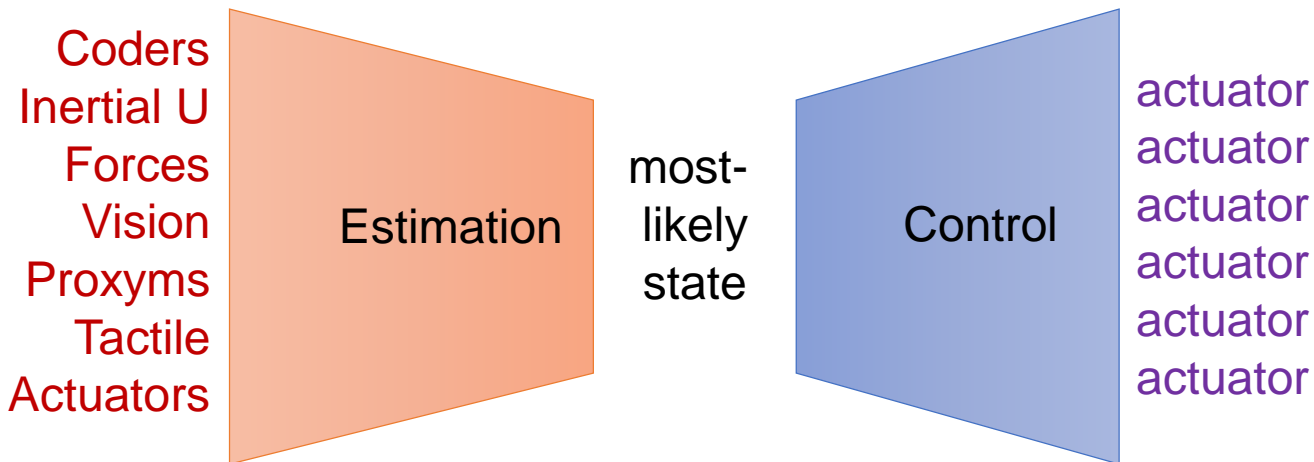
IREPA: 6 iterations

Training time: 55 mins

IREPA with MPC



End-to-end control?



1. State-dependent force model
 2. Learning local sensori-motor models ...
- ... and use them for control-oriented predictions



Force-based control of balance



Take-home messages

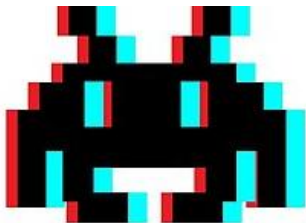
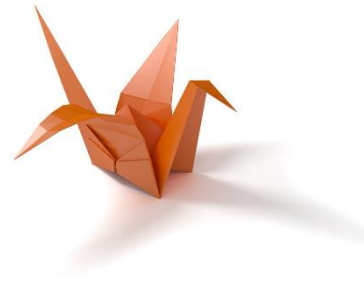


Numerical problems (few/none discrete constraints)

- nonconvex ... warm start needed
- very constrained ... mostly feasibility problems

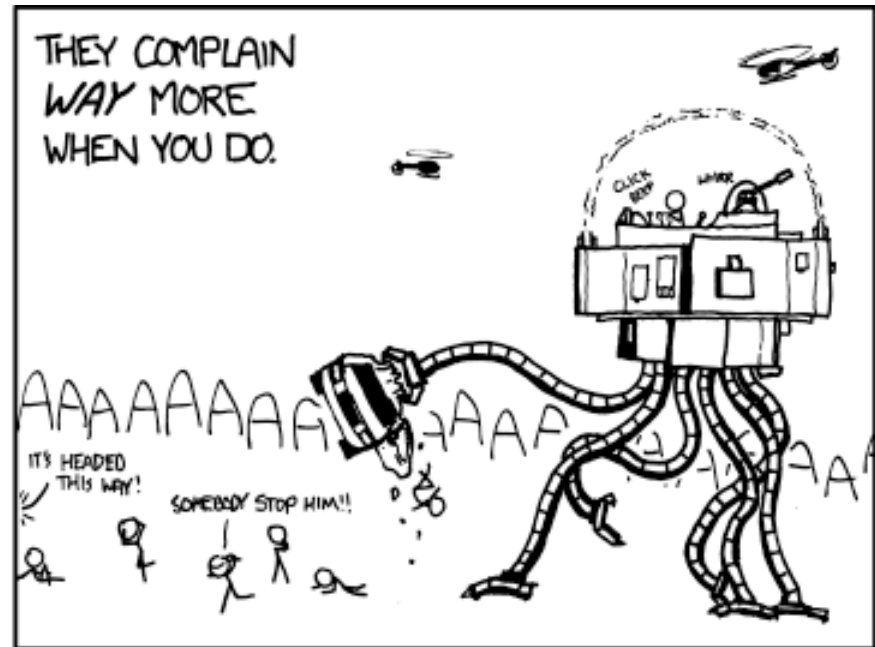
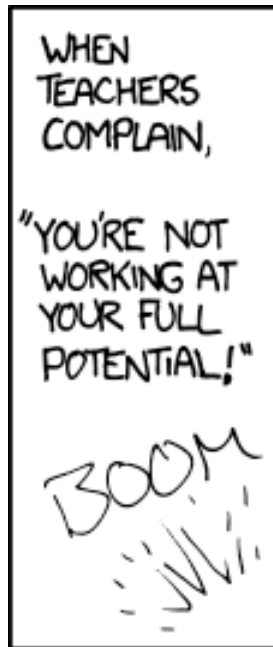
The formulation/transcription is our central problem

- expert+math knowledge
- keep generalization

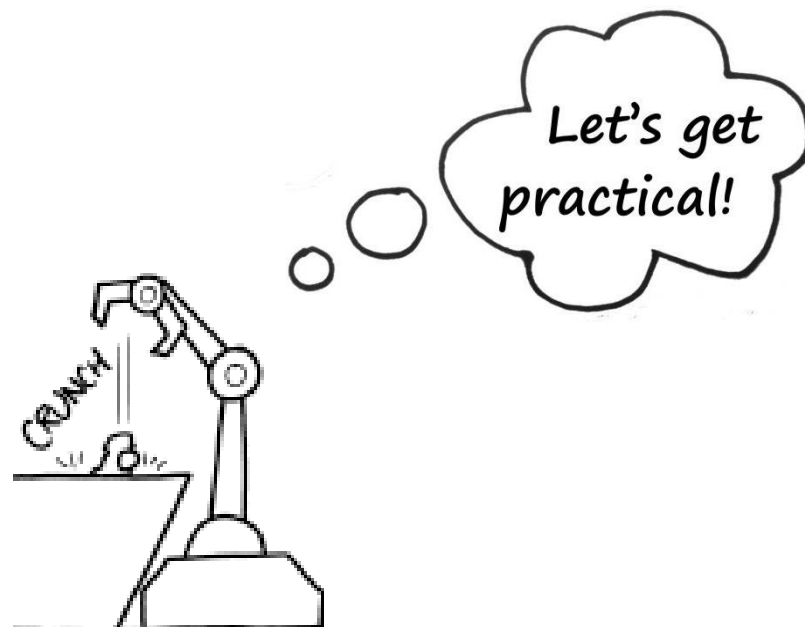


Optimal control = reinforcement learning

- train offline
- generalize online



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Agenda 2019-2020

1. Geometry and inverse geometry
2. Kinematics and inverse kinematics
3. Dynamics and control
4. Optimal control and reinforcement learning

- 7/11 Introduction
- 8/11 Inverse geometry (1) -- pract.work #0
- ~~14/11 Inverse geometry (2) -- pract.work #1~~
- ~~21/11 Inverse kinematics -- pract.work #2~~
- 28/11 Dynamics: simulation and control (1)
- 5/12 Experimental work: with Tiago -- by Olivier Stasse @ AIP
- 13/12 Industrial conference – Wandercraft, Airbus, Kineo Siemens
- 20/12 Experimental work: with Tiago – by Olivier Stasse @AIP
- 10/1 Dynamics: simulation and control (2) – pract.work #3
- 17/1 Dynamics: actuation and control – by Thomas Flayols
- 24/1 Experimental work: with Open Dynamic Robot Initiative – by Thomas Flayols @ LAAS
- 31/1 Optimal control – pract.work #4
- 6/2 Reinforcement learning – pract.work #5

Practical and experimental work

- In simulation

 - Geometry: parallel robotics

 - Kinematics: mobile manipulation

 - Dynamics: dexterous manipulation

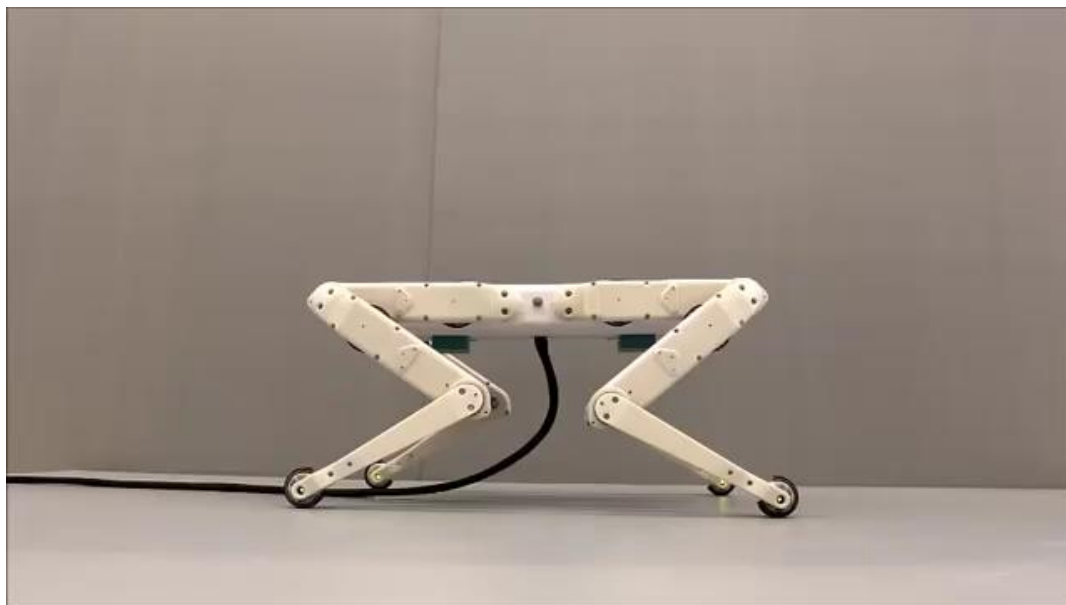
 - Optimal control: flying machines

 - Reinforcement learning: autonomous cars

- On hardware

 - With Tiago: ROS, navigation and inverse kinematics

 - With ODRI: actuation, low level control and force feedback



Scoring

- Presentation of a paper
 - 10 minutes presentation
 - During each class, as a recap of previous lesson
- Practical work
 - 1h homework at each lesson (6 in total)
 - Each homework is scored between 0 and 5
 - Send the code – by mail? ... maybe not
- Group evaluation VS individual evaluation
 - Likely: presentation in group, practical work individual
 - Collaboration in the class is encouraged

Paper presentation

- **Geometry (14/11) ---Hanoune Hervier Bernard**
Learning the problem-optimum map: Analysis and application to global optimization in robotics, by Kris Hauser (TRO 2016)
- **Kinematics (28/11) – Corderes Bhada Vachon**
Visual servo control, Part I: Basic approaches, by François Chaumette, S. Hutchinson (RAM 2016)
- **Dynamics (10/1) – Debeunne Noel**
Feature-Based Locomotion Controllers, by Martin de Lasa
Igor Mordatch, Aaron Hertzmann (TOG 2010)
- **Simulation (17/1) – Niu Sun**
Staggered Projections for Frictional Contact in Multibody Systems, by D. Kaufman et al (TOG 2008)
Interactive Simulation of Rigid Body Dynamics in Computer Graphics, by Jan Bender, Kenny Erleben , Jeff Trinkle and Erwin Coumans (STAR 2011)
- **Actuation (24/1) – Creuse Arlaud Valette**
MIT Cheetah Proprioceptive Actuator Design in the MIT Cheetah: Impact Mitigation and High-Bandwidth Physical Interaction for Dynamic Legged Robots, by Patrick Wensing et al (TRO 2016)
- **Trajectory optimization (31/1) – Zerah Herlmer**
A tutorial on Newton methods for constrained trajectory optimization and relations to SLAM, Gaussian Process smoothing, optimal control, and probabilistic inference, by Marac Toussaint (Book 2017)
Multi-contact Locomotion of Legged Robots by Justin Carpentier, Nicolas Mansard (TRO 2018)
Control-Limited Differential Dynamic Programming, by Yuval Tassa, Nicolas Mansard and Emo Todorov (ICRA 2014)
- **Reinforcement learning (6/2) – Vidal Consiglieri Templier**
End-to-End Training of Deep Visuomotor Policies, by Levine, Finn, Abeel (JMLR 2017)
Using a Memory of Motion to Efficiently Warm-Start a Nonlinear Predictive Controller by Nicolas Mansard, Andrea del Prete, Mathieu Geisert, Steve Tonneau, Olivier Stasse (ICRA 2016)
Interactive Control of Diverse Complex Characters with Neural Networks by Igor Mordatch, Kendall Lowrey, Galen Andrew, Zoran Popovic, Emanuel Todorov (NeurIPS 2016)

Web page of the class

- <https://gepettoweb.laas.fr/index.php/Teach/Supaero2020>
- Alias: <https://frama.link/supaero2020>
- Chat room for the class
<https://frama.link/supaero2020chat>