https://github.com/nmansard/wan24

Optimal control for walking robots

Theory and practice with Crocoddyl

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#05: Quick overview of DDP with constraints

Optimal control problem with constraints

$$\min_{\{x\},\{u\}} \sum_{t=0}^{T-1} l(x_t, u_t) + l_T(x_T)$$

Find control inputs to minimize cost

stage costs

terminal cost

$$x_0 = \hat{x}$$

initial dynamics

$$x_{t+1} = f(x_t, u_t)$$

deterministic dynamics

$$\mathrm{g}\,(x_t, u_t) \geq 0$$

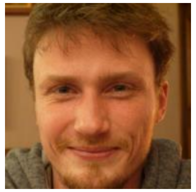
state and control constraints

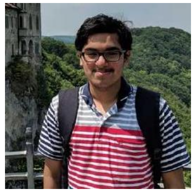
By projection

SQP, active set

By penalty

Interior point, augmented Lagrangian…





Stagewise implementation of SQP for MPC

- Sequential quadratic program approach
- QP based on the **O**perator-**S**plitting solver for **Q**uadratic **P**rogram

$$
\begin{aligned}
\min_{\Delta\boldsymbol{x},\Delta\boldsymbol{u}} &\sum_{k=0}^{T-1} \begin{bmatrix}\Delta x_k\\ \Delta u_k\end{bmatrix}^T \begin{bmatrix}Q_k & S_k\\ S_k^T & R_k\end{bmatrix} \begin{bmatrix}\Delta x_k\\ \Delta u_k\end{bmatrix} + \begin{bmatrix}q_k\\ r_k\end{bmatrix}^T \begin{bmatrix}\Delta x_k\\ \Delta u_k\end{bmatrix}\\
&+ \Delta x_T^T Q_T \Delta x_T + \Delta x_T^T q_T + \frac{\rho}{2}\left\| D_T \Delta x_T - z_T^j + \rho^{-1} y_T^j \right\|_2^2\\
&+ \sum_{k=0}^{T-1} \frac{\rho}{2}\left\| D_k \Delta x_k + E_k \Delta u_k - z_k^j + \rho^{-1} y_k^j \right\|_2^2\\
&+ \sum_{k=0}^{T} \frac{\sigma}{2}\left\| \Delta x_k - \Delta x_k^j \right\|_2^2 + \sum_{k=0}^{T-1} \frac{\sigma}{2}\left\| \Delta u_k - \Delta u_k^j \right\|_2^2 \qquad (15a)\\
\text{s.t.} \quad & \Delta x_{k+1} = A_k \Delta x_k + B_k \Delta u_k + \gamma_{k+1}. \qquad (15b)
\end{aligned}
$$

- The nonlinear search is a direct adaptation of Nocedal

$$\mathcal{L}_\mu(x; y_e) \stackrel{\text{def}}{=} \frac{1}{2}x^\top Qx + q^\top x + y_e^\top(Ax+b) + \frac{1}{2\mu}\|Ax+b\|_2^2$$
$$= -\underbrace{\min_y \left\{-\mathcal{L}(x,y) + \frac{\mu}{2}\|y-y_e\|_2^2\right\}}_{\text{proximal!}}$$

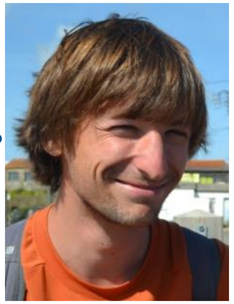
Proximal DDP

- Rewrite the constraints as **shifted penalties**
- Observe you obtained a unconstrained OCP
- Solve with care (ie with a **proximal** operator)
- Update the multipliers using a first-order step
- Update

Proximal optimization

- Hire Adrien Taylor for a training session

Nice guy …
… much more
able than me



- Main idea of the approach:

replace $\min_{x \in \mathbb{R}^n} f(x)$ by $$x_{k+1} = \operatorname*{argmin}_{x \in \mathbb{R}^n} \left\{ f(x) + \frac{1}{2\gamma} \|x - x_k\|^2 \right\}$$

Proximal method of multipliers … for QP

$$\min_{x} x^T H x + g^T x$$

$$\text{s.t } Ax \geq b$$

The Lagrangian is

$$\mathfrak{L}(x, \lambda) = x^T H x + g^T x - \lambda^T (Ax - b)$$

Replace

$$\min_{x} \max_{\lambda \geq 0} \mathfrak{L}(x, \lambda)$$

by

$$\min_{x} \max_{\lambda \geq 0} \{ \mathfrak{L}(x, \lambda) + \|x - x_k\|^2 + \|\lambda - \lambda_k\|^2 \}$$

Software status

- The stagewise SQP is implemented as a Crocoddyl ad-on
  https://github.com/machines-in-motion/mim_solvers

- ProxDDP is the main back-end of the Aligator package
  https://github.com/Simple-Robotics/aligator
  (a part of the front-end depends on Pinocchio 3, release in progress)