https://github.com/nmansard/wan24

Optimal control for walking robots

Theory and practice with Crocoddyl

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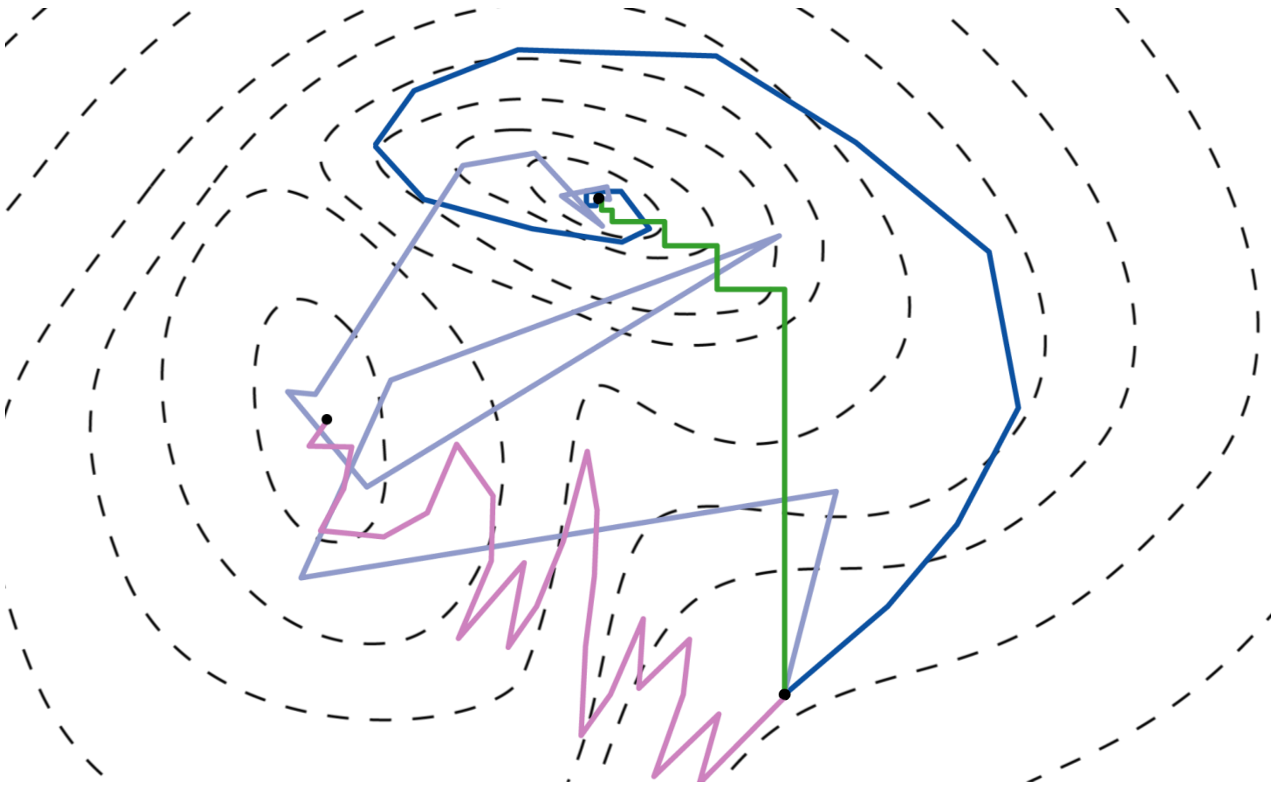
#01: Basis of $\left\{\begin{array}{c}\text{nonlinear}\\ \text{constrained}\end{array}\right\}$ optimization

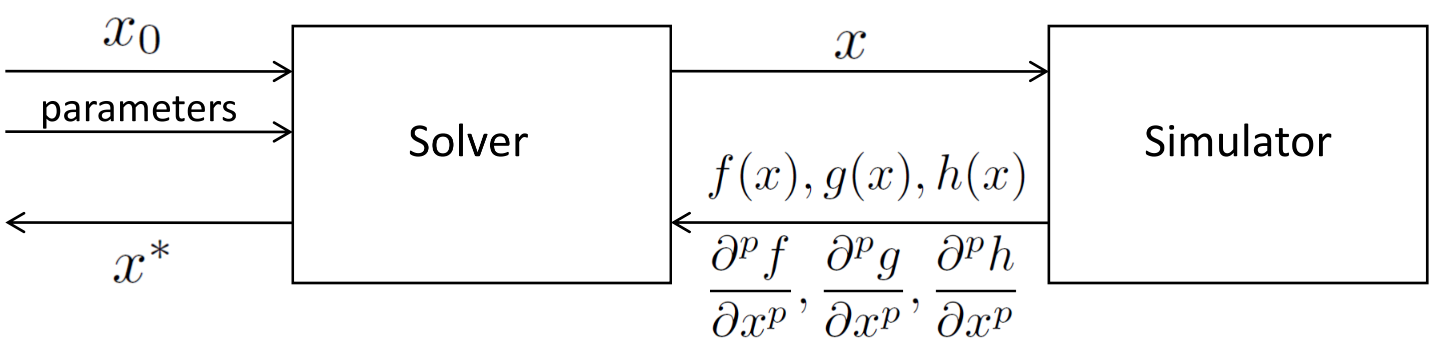
Follow the slope

- Decreasing sequence: $f(x_{k+1}) < f(x_k)$



Problem specifications

- Problem specification
  - Computing *f(x)* is easy
  - We can differentiate $f: x \to f(x)$
  - We know the distance to the reference value



Optimality principles

$$\min_x c(x)$$

- First order

$$\nabla c = 0$$

- Second order

$$\nabla c = 0$$
$$\nabla^2 c \geq 0$$

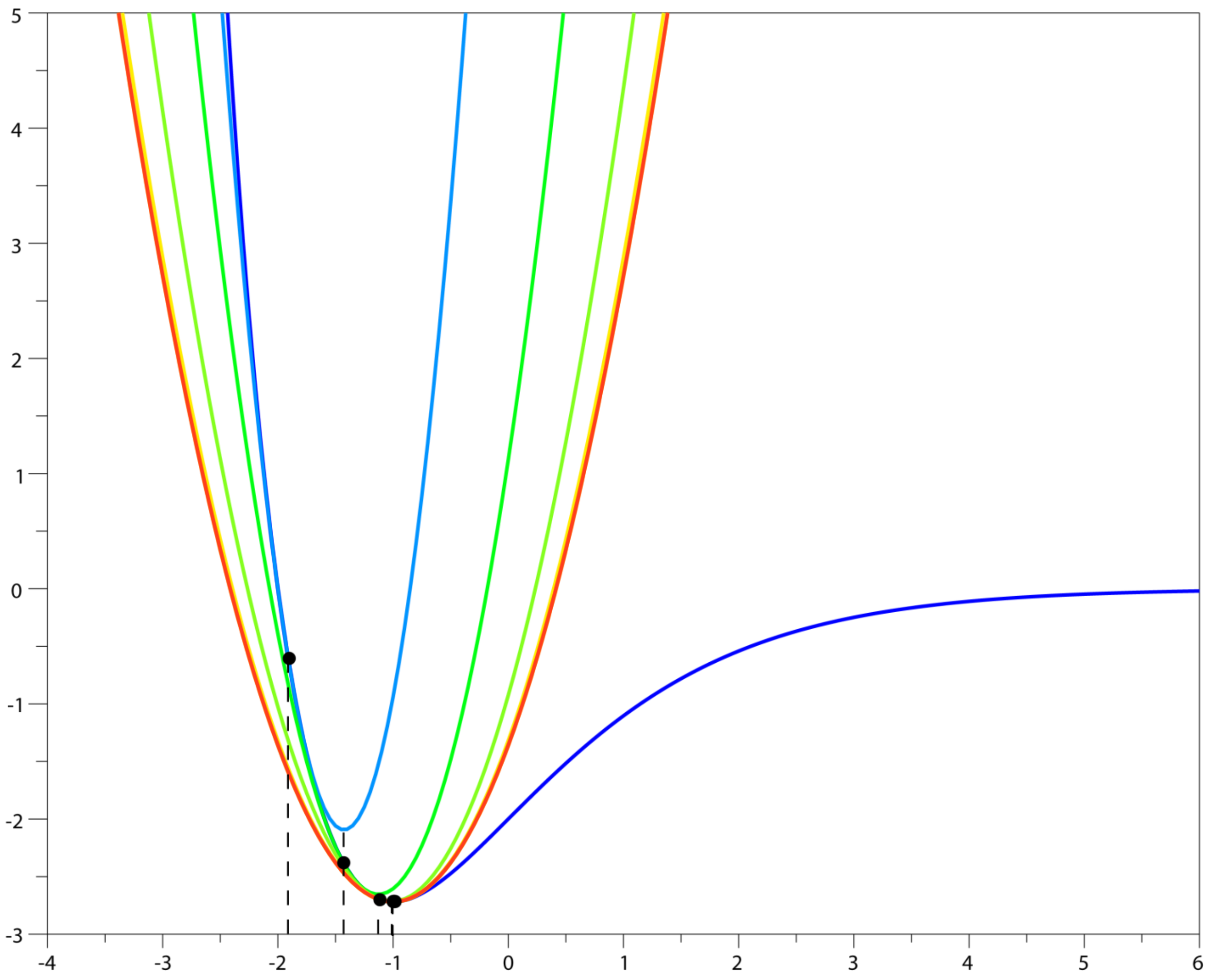
necessary

$$\nabla c = 0$$
$$\nabla^2 c > 0$$

sufficient

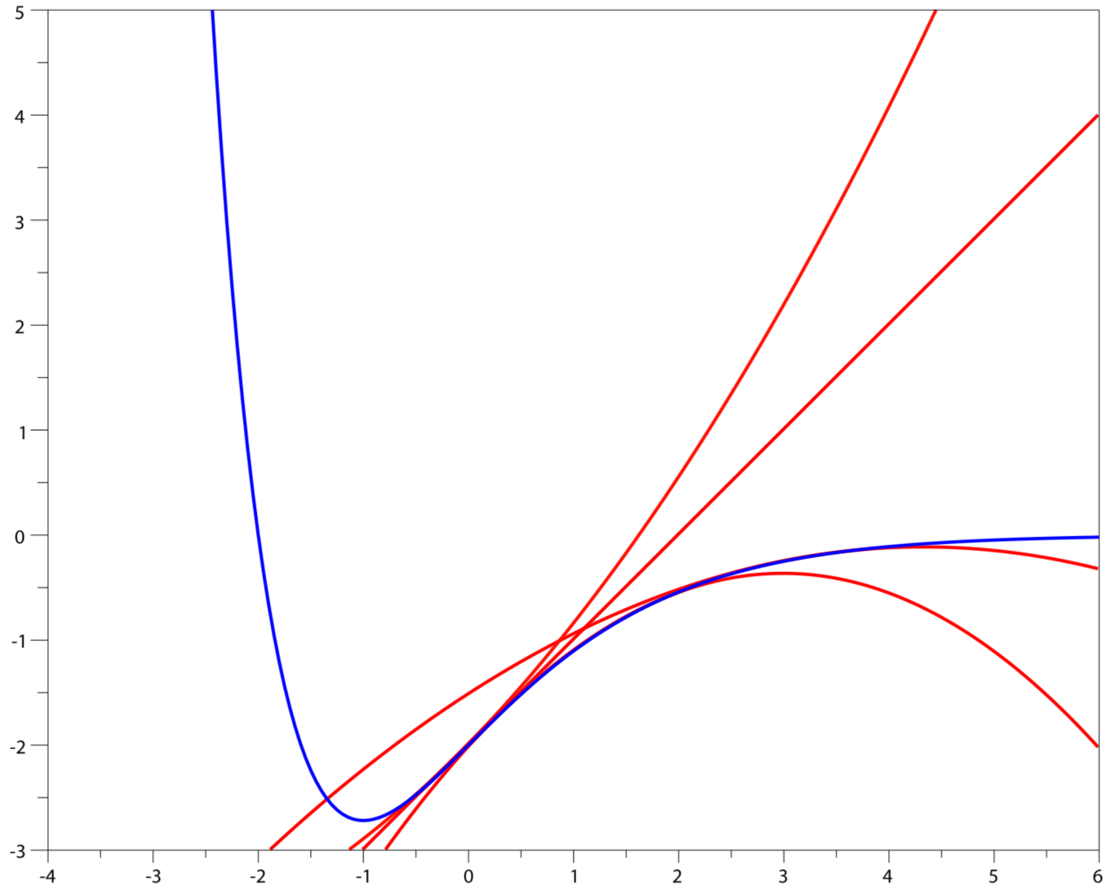
Newton method (unconstrained)

x0=-1.9
x1=-1.4263158
x2=-1.1274228
x3=-1.0144015
x4=-1.0002045
x5=-1.00000004
x6=-1.



Newton method (unconstrained)

- Ill-conditionned hessian
- Non positive hessian



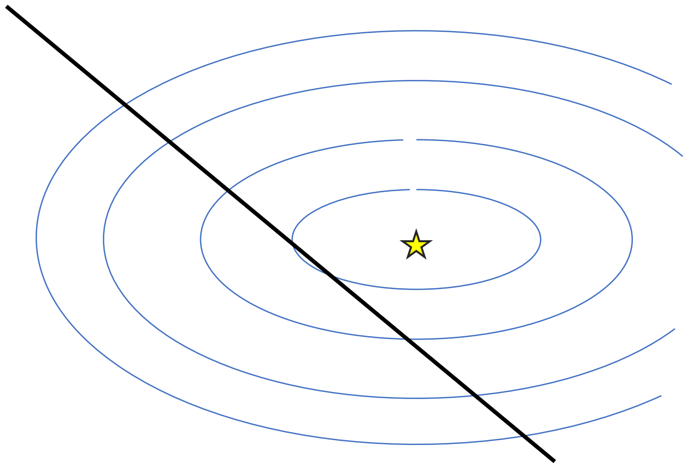
Main algorithms to understand

- Gradient descent
- Newton descent
- Quasi-newton principle (BFGS example)
- Guauss-Newton and Levenberg-Marquardt descent

Optimization with constraint

- Linearly-Constrained Quadratic Program (LCQP)

$$\min_x ||Ax - b||^2$$
$$s.t \;\; Cx = d$$



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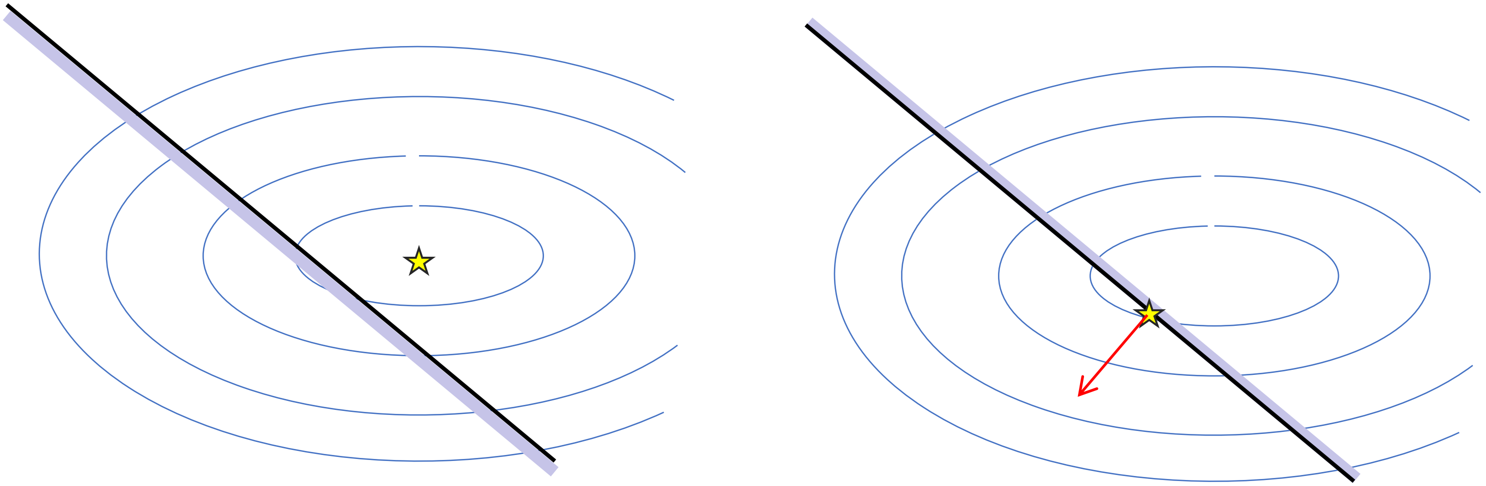

Optimization with constraint

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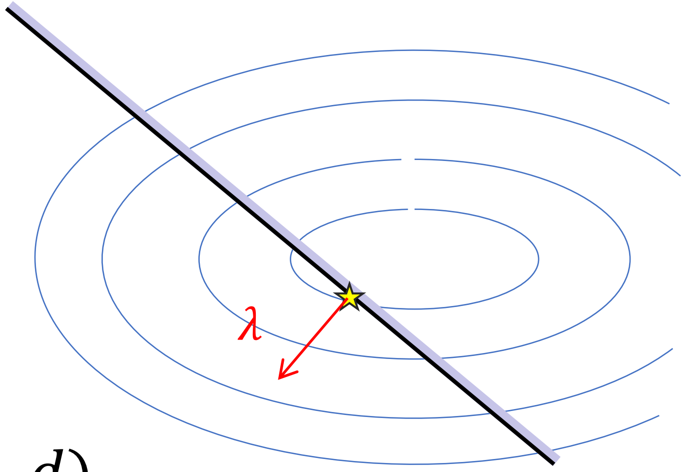


λ is called a Lagrange multiplier

$$\mathcal{L}(x, \lambda) = ||Ax - b||^2 + \lambda^T (Cx - d)$$

Duality in optimization

- The dual variable is an auxiliary quantity
  - Needed to assert the optimality condition

$$\min_x \max_{\lambda} \mathcal{L}(x, \lambda)$$

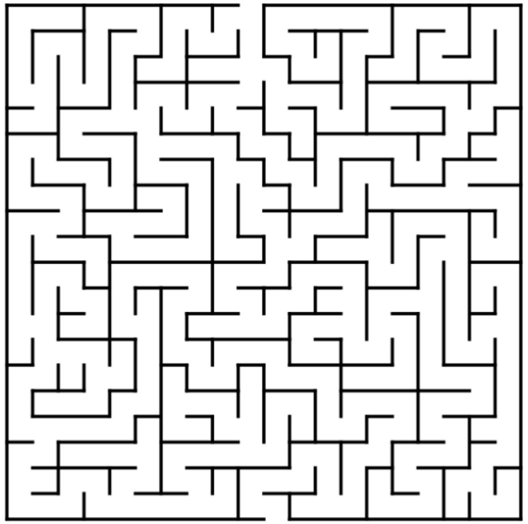
$$\begin{aligned} &\min_x c^T x \\ s.t. \quad &Ax \geq b \end{aligned} \qquad \text{(LP)}$$

$$\begin{aligned} &\max_{\lambda} b^T \lambda \\ s.t. \quad &A^T \lambda = c, \lambda \geq 0 \end{aligned} \qquad \text{(LP*)}$$

Primal problem

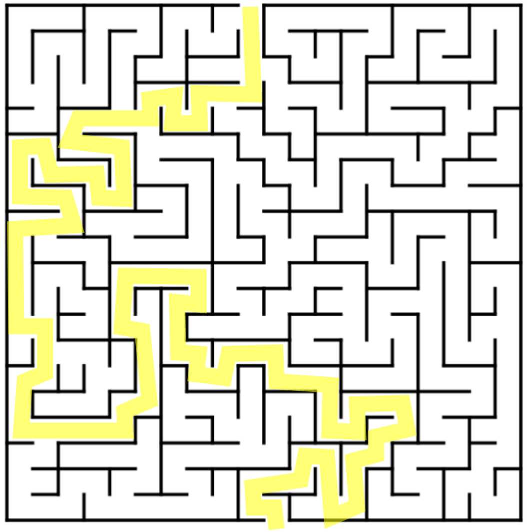
Dual problem

Duality in optimization



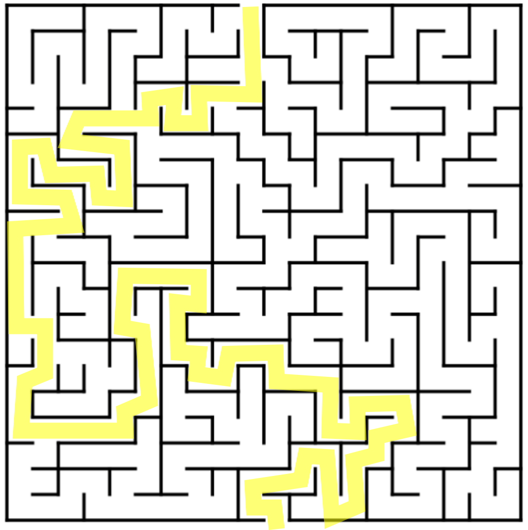
IS IT FEASIBLE ?

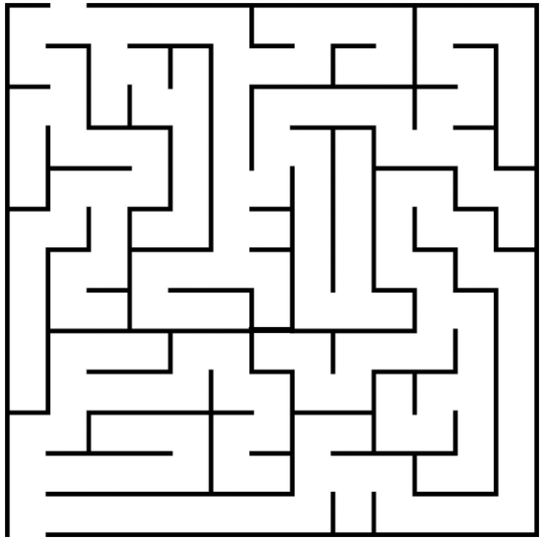
Duality in optimization



Feasible = I can demonstrate the existence of a path

Duality in optimization

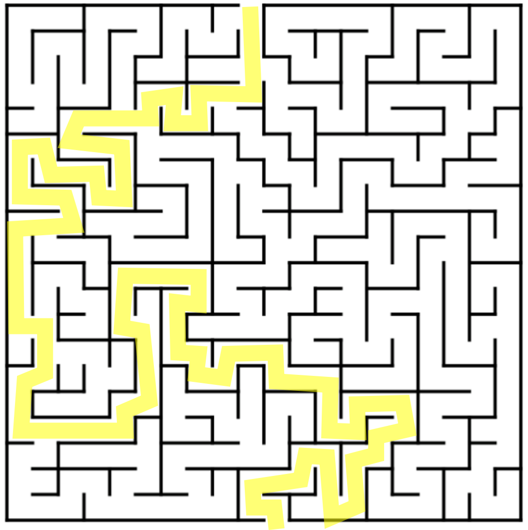


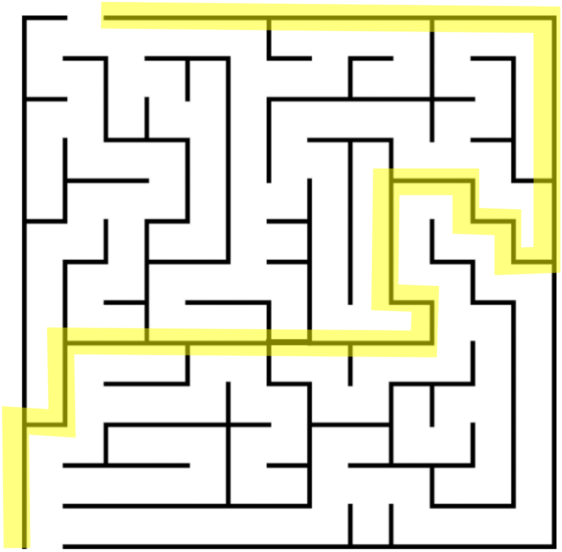


Feasible = I can demonstrate the existence of a path

IS THIS ONE FEASIBLE ?

Duality in optimization





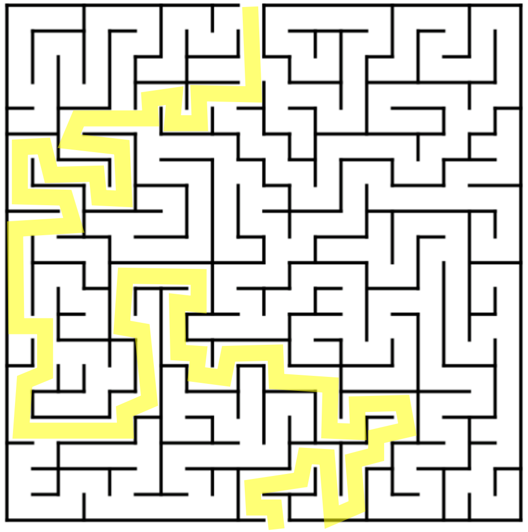
Feasible = I can demonstrate the existence of a path

white

Unfeasible = I can demonstrate the existence of a ~~wall~~

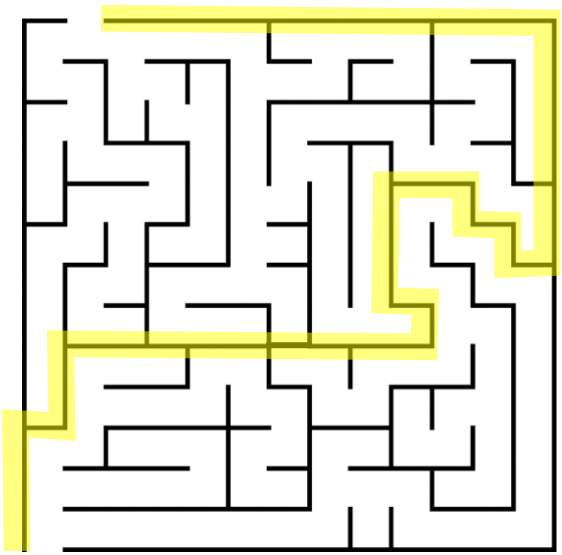
black path

Duality in optimization



Feasible = I can demonstrate the existence of a path

Primal problem



Unfeasible = I can demonstrate the existence of a wall

Dual problem

Optimality conditions (KKT conditions)

$$\min_x f(x) \quad s.t. \quad g(x) \geq 0$$

- x, λ are solutions if they respects:

$$\mathcal{L}(x,\lambda) = f(x) - \lambda^T g(x)$$ Lagrangian

$$\nabla_x \mathcal{L} = \nabla f - \lambda^T \nabla g = 0$$ Gradient normal to constraints

$$\nabla_\lambda \mathcal{L} = g(x) \geq 0$$ Constraint satisfied

$$\lambda^T g(x) = 0$$ Complementarity

Dynamics for simulation

- Complementarity problem

$$\ddot{q} = M^{-1}(\tau - b + J^T f)$$

$$J\ddot{q} + \dot{J}\dot{q} \geq 0 \quad \perp \quad f \geq 0$$

no penetration *no pulling*

one or the other

- Equivalent to a principled QP

$$\min_{\ddot{q}} \left\| \ddot{q} - \ddot{q}_{free} \right\|_M \text{ s.t. } \quad J\ddot{q} + \dot{J}\dot{q} \geq 0$$

Main algorithms to understand

- Projection algorithms
  - Sequential quadratic programming
- Penalty algorithms
  - Soft penalties
  - Interior points … not good for warm-start
  - Augmented Lagrangian