227-0689-00L System Identification **15 October 2018**

Exercise 4: Smoothing ETFE Estimates and PRBS

Background reading

The background material for this exercise is Sections 6.4, 6.5 and 13.3 of Ljung (*System Identification; Theory for the User*, 2nd Ed., Prentice-Hall, 1999).

Problem 1:

Averaging the estimates $\hat{G}_r(e^{j\omega_n})$ from several experiments $r = 1, \ldots, R$ can improve the estimate $\hat{G}(e^{j\omega_n}) = \sum_{r=1}^{R} \alpha_r(e^{j\omega_n})\hat{G}_r(e^{j\omega_n})$ at frequency ω_n (cf. Eq. (6.41) in [Ljung, 1999] and Lecture Slide 4.2). The optimal weighting $\alpha_r(e^{j\omega_n})$ at each frequency ω_n is proportional to the inverse variance of $\hat{G}_r(e^{j\omega_n})$.

To demonstrate the optimality in a minimum variance sense, let p_r with $r = 1, \ldots, R$, be independent random variables with identical mean $E\{p_r\} = m$ and individual variance $E\{(p_r - m)^2\} = \lambda_r$. For $p = \sum_{r=1}^{R} \alpha_r p_r$ determine α_r, $r = 1, \ldots, R$, such that

(1.a) $E\{p\} = m$,

(1.b) $E\{(p - m)^2\}$ is minimized.

Hint: Use Lagrange multipliers to solve the minimization problem in (1.b).

Problem 2:

It is not straightforward to use PRBS for multi-input systems. Consider the following: Let $s^M = s(t)$, $t = 1, \ldots, M$, be a maximum length M PRBS signal. Consider an identification experiment over a multiple of $2M$ samples by letting $u_1 = u_2 = s^M$ for the first M samples, and $u_1 = -u_2 = s^M$ for the next M samples. Show that this gives the same input covariance matrix as exciting one input at a time with $\sqrt{2}s^M$, i.e., $\tilde{u}_1 = \sqrt{2}s^M, \tilde{u}_2 = 0$ for the first M samples and then $\tilde{u}_1 = 0, \tilde{u}_2 = \sqrt{2}s^M$ for the next M samples.

Matlab Exercise 1:

Plot periodograms for maximum length PRBS of order 5 to 8 and show how these spectral estimates change as the number of periods increases. What are advantages and disadvantages of PRBS signals?

MATLAB **Exercise 2:**

Complete the MATLAB Exercise from Exercise Sheet 3, this time using the time domain Hann window.