227-0689-00L System Identification **08 October 2018**

Exercise 03: Smoothing ETFE Estimates

Background reading

The background material for this exercise is Section 6.4 of Ljung (*System Identification; Theory for the User*, 2nd Ed., Prentice-Hall, 1999).

Problem 1:

Prove that if $r(k)$ is a fixed signal (for example a transient) and $u(k)$ is a periodic signal (with period $= M$), then

$$\lim_{N \to \infty} \frac{|R_N(e^{jw_n})|}{|U_N(e^{jw_n})|} = 0$$

and converges with a rate of $1/N$.

Problem 2:

We are given data $\{y(k), u(k)\}$, $k = 1, \ldots, N$, from a noise corrupted identification experiment:

$$y(k) \; = \; G(z)\, u(k) \; + \; v(k),$$

with $v(k)$ a zero-mean stochastic signal.

Suppose that we filter our experimental data with stable LTI filters (or transfer functions) for the input and output signals giving,

$$\begin{aligned} y^F(k) &= L_y(z)\, y(k), \\ u^F(k) &= L_u(z)\, u(k). \end{aligned}$$

The DFTs of the filtered signals are $Y_N^F(\omega)$ and $U_N^F(\omega)$. Now calculate an ETFE estimate using the filtered signals,

$$\hat{G}^F(\mathrm{e}^{j\omega}) \; := \; \frac{Y_N^F(\omega)}{U_N^F(\omega)}.$$

Hint: Start with:

$$Y_N(\omega) = G(e^{j\omega})U_N(\omega) + R_N^y(\omega) + V_N(\omega)$$

and use the Fourier Transforms resulting from Theorem 2.1 in the textbook of Ljung:

$$U_N^F(\omega) = L_u(e^{j\omega})U_N(\omega) + R_N^{uf}(\omega)$$
$$Y_N^F(\omega) = L_y(e^{j\omega})Y_N(\omega) + R_N^{yf}(\omega)$$

Questions:

a) Calculate the expected value of the filtered estimate,

$$E\{\hat{G}^F(e^{j\omega})\},$$

and its mean-square error at frequency ω,

$$E\left\{\left|\hat{G}^F(e^{j\omega}) - G(e^{j\omega})\}\right|^2\right\}.$$

b) What are the asymptotic properties of these quantities as $N \to \infty$? You may use $\lim_{N\to\infty} E\{\frac{1}{N}|X(\omega)|^2\} = \Phi_x(\omega)$.

c) Can you think of reasonable choices for $L_y(z)$ and $L_u(z)$?

Matlab exercises:

Consider the discrete time system,

$$G(z) = \frac{0.1z}{(z^2 - 1.7z + 0.72)(z^2 - 0.98z + 0.9)},$$

and noise model,

$$H(z) = \frac{0.5(z - 0.9)}{(z - 0.25)}.$$

The sample time can be taken as 1 second. The noise system, $H(z)$, is driven by a $N(0,1)$ white noise, and the measured output is the sum of the outputs of $G(z)$ and $H(z)$.

a) Generate a 1024 point Matlab simulation for an identification “experiment.”

b) Estimate $G(z)$ via the unsmoothed ETFE and compare it to the true system; particularly in the frequency range around the resonant peak. Plot both the transfer functions and the magnitude of the errors.

c) Split the data record in 4 and calculate a “smoothed” ETFE by averaging the 4 ETFEs. Compare this to the true system as well as the original 1024 point ETFE. To compare these, calculate the Mean Square Error (MSE) in each case (making sure to normalize for the different data lengths). Again plot the transfer functions and the magnitude of the errors.

We will also look at the effect of windowing the data by smoothing in both the time and frequency domain. To achieve this write two MATLAB functions to calculate a Hann window response in both the time and frequency domains:

`WHfdom(gamma,omega)`	frequency domain Hann window
`WHtdom(gamma,tau)`	time domain Hann window

where `gamma` is the width parameter.

d) Plot the frequency response of the Hann window for $\gamma = 5$, 10, 50 and 100, on a frequency grid of 1024 points.

e) Plot the time domain weight for the same values of γ.

f) Calculate the smoothed ETFE for each of these values of γ. Do this in both the time and frequency domains to check that your functions are working correctly.

g) Experiment and find a value of γ that gives the best resolution of the resonant peak.