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Traversable Wormholes, Stargates, and Negative Energy

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Traversable Wormholes, Stargates, and Negative Energy

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Traversable Wormholes, Stargates, and Negative Energy

I. Summary

Implementation of faster-than-light (FTL) interstellar travel via traversable wormholes generally requires the engineering of spacetime into very specialized local geometries. The analysis of these via Einstein's General Theory of Relativity, plus the resultant equations of state, demonstrates that such geometries require the use of "exotic" matter. It has been claimed that since such matter violates the energy conditions, FTL spacetimes are not plausible. However, it has been shown that this is a spurious issue. The identification, magnitude, and production of exotic matter are seen to be a key technical challenge, however. These issues are reviewed and summarized, and an assessment on the present state of their resolution is provided.

In 1985 CalTech physicists M. Morris and K. Thorne discovered the principle of traversable wormholes based on Einstein's General Theory of Relativity (published in 1915). Morris and Thorne (Reference 1) and Morris et al. (Reference 2) did this as an academic exercise at the request of Carl Sagan, who had completed the draft of his novel *Contact*. This little exercise led to the development of two new cottage industries in spacetime physics research: the study of traversable wormholes and the study of time machines. Wormholes are hyperspace tunnels through spacetime connecting either remote regions within our universe or two different universes; they even connect different dimensions and different times. Space travelers would enter one side of the tunnel and exit the other, passing through the throat along the way. The travelers would move at $\leq c$ (c is the speed of light, 3×10^8 m/s) through the wormhole and therefore not violate Special Relativity, but external observers would view the travelers as traversing multi-light-year distances through space at FTL speed; Figure 1 illustrates this effect. A "stargate" is a special class of traversable wormhole solutions to Einstein's general relativistic field equation that possesses very simple physics and flat entry and exit openings.

Traversable wormholes are unlike the well-known, non-traversable Einstein-Rosen Bridges or Schwarzschild wormholes that are formed from collapsed stellar matter (that is, black holes) or spherically symmetric vacuum regions. Black holes are collapsed stars that have all their mass concentrated at an infinitesimal point where the induced gravitational field crushes all matter and spacetime. However, even Einstein-Rosen bridges can be made traversable by an infinitesimal tweaking of their spacetime metric. In the case of black holes, the singularity of collapsed matter, along with its crushing gravity field, totally blocks the way through the tunnel. A traversable wormhole does not have a singularity blocking the tunnel or any crushing gravity field. Explorers would enter one side of the tunnel, travel through the throat, and exit the other side. Traversable wormholes also do not possess an event horizon, a region of high gravitational field strength separating the inside space surrounding the black hole's singularity from the outside universe. Once you go through a black hole's event horizon, you can never come back out because you will have to attain FTL speed to escape it. Not even light can escape from an event horizon.

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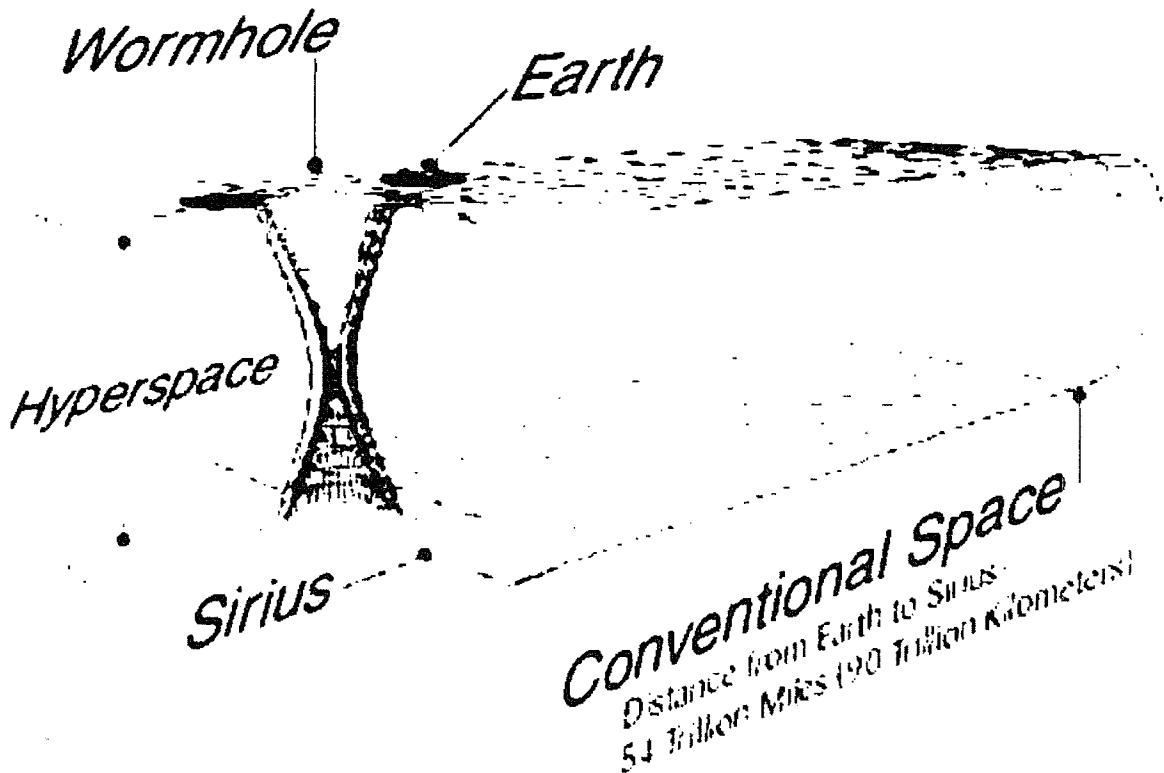


Figure 1. Intra-Universe Wormhole as a Hyperspace Shortcut Through Conventional Space

Traversable wormholes are creatures of classical general relativity theory allowing for very comfortable travel through the Cosmic Neighborhood. But from the viewpoint of modern physics, the Cosmic Neighborhood can encompass other universes, other space dimensions, and other times beyond the four-dimensional spacetime realm. Mankind has certainly not discovered all of the universe's facets and will need to continue to construct new experiments and technology in order to verify (or not) these undiscovered facets. Wormholes can possess normal or backward (in special cases) motion through time and normal or nonexistent gravitational stresses on space travelers, and their entry/exit openings (or throats) are spherically shaped, flat, cubic shaped, polyhedral shaped, generic shaped, and so forth.

Why consider wormholes for travel through space, time, and other dimensions? All standard space propulsion engineering is based on Newton's three laws of motion, which is dependent on the expenditure of propellant to induce thrust-generating momentum transfer on a spacecraft. Many investigators have proposed interstellar propulsion schemes based on a variety of nuclear (fission, fusion, and pulsed) rockets, electric (ion or plasma) rockets, matter-antimatter annihilation rockets, solar or laser sails, fusion or laser ramjets, interstellar ion scoops, beamed energy propulsion (sails, rockets, and ramjets), and so forth. Many of these modes either have been experimentally tested at one time or another in our recent history or remain as theoretical proposals,

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but all are based on Newtonian mechanics. The limiting speed of space flight, based on any of these modes, is the speed of light. It is important to point out that for the interstellar travel application, Newtonian rocket propulsion modes suffer from enormous mass ratios $> 10^5 - 10^{100}$ (depending on the specific impulse) for spacecraft cruise velocities $> 0.05c$, if the travel time is constrained to within 100 years for a one-way interstellar voyage. If the cruise velocity is increased to sub-relativistic, near-relativistic, or even ultra-relativistic speeds and thus reduces the one-way travel time, then the mass ratio increases (exponentially!). The mass ratio is the initial spacecraft mass (payload + structure + propellant) at launch divided by the final spacecraft mass (payload + structure) at "burnout." The large ratios given above show that Newtonian rockets consist mostly of propellant in order to propel the propellant, along with a given tiny payload, through interstellar space. The specific impulse is a measure of rocket propulsion system efficiency: how much impulse (thrust multiplied by time) is produced per unit of mass of propellant expenditure. It is desired that rocket propulsion systems possess a very high specific impulse in order to reduce the mass ratio, and hence propellant mass requirement, to reasonable levels.

The non-traditional propulsion modes (sails, ramjets, beamed power, etc.) have different efficiencies and constraints, but they are all still dependent on Newtonian mechanics, even though their mass ratio and specific impulse characteristics are slightly improved over that of the traditional modes. But all traditional and non-traditional propulsion modes come with a great cost in interstellar voyage travel time. At non-relativistic and sub-relativistic cruise speeds, it will take explorers several human lifetimes to reach stellar destinations. At low relativistic to ultra-relativistic cruise speeds, the travel time will be reduced to hours, days, weeks, months, or years. However, at these cruise speeds, relativistic time dilation will kick in, and the returning interstellar voyagers will find that decades to thousands of years have elapsed on Earth since their launch date and that their families and culture no longer exist or are unrecognizable. This is an undesirable outcome for any interstellar voyage. Furthermore, traditional Newtonian propulsion cannot transcend time or spacetime dimensions or universes.

The solution to this problem is to dispense entirely with long interstellar voyage times or the undesirable outcome of relativistic time dilation. Explorers could deploy a wormhole-stargate near the Earth's surface, in Earth's orbit, or anywhere in the solar system they like and just pass through the "stargate" and come out the other side in remote spacetime within seconds, moving through the throat at low cruise speeds (30 mph!) and with no time dilation effects. Explorers could travel through the wormhole-stargates in small scout ships or send probes unencumbered by either enormous propellant mass ratios or extensive life support provisions. Effective travel time through the Cosmic Neighborhood via stargates would become irrelevant but could be estimated to be many times or thousands of times the speed of light. Explorers could spend all day investigating the remote spacetime location and then return home through the stargate in time to have dinner with their families. If explorers were to really push the envelope, they would design their stargate so they could return from their voyage in time to wave goodbye to themselves

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as they see themselves depart on their journey. This is no longer recognized in classical general relativity physics as a time paradox issue. It is very easy to build a time machine, given a traversable wormhole. But time travel via wormhole is beyond the scope of this paper. Suffice it to say that classical general relativity theory is seriously infested with time machines; the theory both allows for and demands time travel in order to preserve self-consistency of dynamic spacetime solutions for just about every problem ever studied.

Implementation of FTL interstellar travel via traversable wormholes generally requires the engineering of spacetime into very specialized local geometries. Analysis of these via the general relativistic field equation, plus the resultant source matter equations of state, demonstrates that such geometries require the use of "exotic" matter in order to produce the requisite FTL spacetime modification. Exotic matter is generally defined by general relativity physics to be matter that possesses (renormalized) negative energy density (sometimes negative stress-tension = outward pressure, aka gravitational repulsion or antigravity). This term is very misunderstood and misapplied by the non-general-relativity community. This misconception can be cleared up by defining what negative energy is and where it can be found in nature and by reviewing the proposed experimental concepts for generating negative energy in the laboratory. In addition, it has been claimed that FTL spacetimes are not plausible because exotic matter violates the general relativistic energy conditions. However, this has been shown to be a spurious issue. The identification, magnitude, and production of exotic matter are seen as key technical challenges, however. FTL spacetimes also possess features that challenge the notions of causality, and quantum effects allegedly place constraints on them. These issues are reviewed and summarized, and an assessment on the present state of their resolution is provided.

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II. A Brief Review of Transversable Wormholes and the Stargate Solution

How does one study the physics of FTL spacetimes within the framework of general relativity theory? When studying spacetime physics, the normal philosophy is to take the general relativistic field equation, add some form of matter, make simplifying assumptions, and then solve to deduce what the geometry of spacetime will be.¹ This is very difficult to do because there are ten nonlinear second-order partial differential equations with four redundancies (arbitrary choice of spacetime coordinates) and four constraints (stress-energy conservation). There is a tremendous body of research that takes exactly this approach, either analytically or numerically. However, this is not the best strategy for understanding wormhole spacetimes. The appropriate strategy is to decide beforehand on a definition of the traversable wormhole that you desire and decide what the spacetime geometry should look like. Given the desired geometry, use the general relativistic field equation to calculate the distribution of matter required to set up this geometry. Then one needs to assess whether the required distribution of matter is physically reasonable and whether it violates any basic rules of physics, etc. The following sections briefly outline the key results for traversable wormholes.

A. TRAVERSABLE WORMHOLES

Traversable wormholes represent a class of exact metric solutions of the general relativistic field equation. The solutions are "exact" in the sense that no approximations requiring a plethora of physical assumptions have to be made to derive the appropriate spacetime geometry. To define a stable traversable wormhole one needs to define the desirable physical requirements it is to have in order to achieve the desired FTL travel benefit. The desired requirements are the following (Reference 1, 3):

- Travel time through the wormhole tunnel or throat should be ≤ 1 year as seen by both the travelers and outside static observers.
- Proper time as measured by travelers should not be dilated by relativistic effects.
- The gravitational acceleration and tidal-gravity accelerations between different parts of the travelers' body should be $\leq 1 g_0$ (g_0 is the acceleration of gravity near the Earth's surface, 9.81 m/s^2) when going through the wormhole.
- Travel speed through the tunnel/throat should be $< c$.
- Travelers (made of ordinary matter) must not couple strongly to the material that generates the wormhole curvature; the wormhole must be threaded by a vacuum tube through which the travelers can move.
- There is no event horizon at the wormhole throat.

¹The Einstein field equation is: $G_{\mu\nu} = R_{\mu\nu} - [(1/2) g_{\mu\nu} R] = -(8\pi G/c^4)T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein curvature tensor, $R_{\mu\nu}$ is the Ricci curvature tensor, $R = R^\nu_\nu$ (the trace of $R_{\mu\nu}$) is the Ricci scalar curvature, $T_{\mu\nu}$ is the stress-energy-momentum tensor (a matrix quantity that encodes the density and flux of a matter source's energy and momentum), G is Newton's universal gravitation constant ($6.673 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$), and c is the speed of light. In simplest terms, this relation states that gravity is a manifestation of the spacetime curvature ($G_{\mu\nu}$) induced by a source of matter ($T_{\mu\nu}$). The Greek Indices ($\mu, \nu = 0\dots3$) denote spacetime coordinates, $x_0\dots x_3$, such that $x_1\dots x_3$ = space coordinates and x_0 = time coordinate.

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- There is no singularity of infinitely collapsed matter residing at the wormhole throat.

These requirements then lead us to define a spherically symmetric Lorentzian spacetime metric, ds^2 ,² that prescribes the required traversable wormhole geometry (Reference 1, 3):

$$ds^2 = -e^{2\phi(r)}c^2dt^2 + [1 - b(r)/r]^{-1}dr^2 + r^2d\Theta^2 \quad (1)$$

where standard spherical-polar coordinates are used (r : $2\pi r$ = circumference; $0 \leq \theta \leq \pi$; $0 \leq \varphi \leq 2\pi$), t is time ($-\infty < t < \infty$), $d\Theta^2 = d\theta^2 + \sin^2\theta d\varphi^2$, $\phi(r)$ is the freely specifiable redshift function that defines the proper time lapse through the wormhole throat, and $b(r)$ is the freely specifiable shape function that defines the wormhole throat's spatial (hypersurface) geometry. The throat is spherically shaped. There are a large number of variations of Equation (1), which define traversable wormholes having different properties. The reader should consult (Reference 3) for further details. By inserting Equation (1) into the Einstein field equation and cranking through the math, one can derive the density and flux of energy and momentum (a.k.a. pressure) encoded by $T_{\mu\nu}$ for the source of matter that is required to produce the traversable wormhole. The results show that the source of matter must have zero or negative energy density and/or an outward radial tension (negative pressure) that is larger than the magnitude of the energy density (Reference 1-3). Travelers moving through the throat at very high speed will tend to measure a negative energy density. These exotic properties are required to create and thread open the wormhole, and stabilize it against collapse (see Section III for more details).

The technical description of a trip through a spherically symmetric traversable wormhole is simply given by the proper time and/or the proper distance of travel through its throat as measured by space travelers, while the (radial) travel velocity through the throat is $v = v(r) < c$. The proper time of travel as measured by space travelers going through the wormhole is given by $\Delta\tau = \int (\gamma v)^{-1} d\lambda$, where $\gamma = [1 - (v/c)^2]^{-1/2}$ and the integration (over the element of proper distance, $d\lambda$) is taken from the wormhole entrance to its exit. The proper distance of travel as measured by the space travelers is $\Delta\lambda = v\Delta\tau$. Remote static observers watching the space travelers go through the wormhole will measure their travel time to be $\Delta t = \int (ve^{\phi(r)})^{-1} d\lambda$ and their travel distance will be $\Delta\lambda = v\Delta t$, where the integration is taken over the same limits as before.

² A spacetime metric, ds^2 , is a Lorentz-invariant distance function between any two points in spacetime that is defined by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where $g_{\mu\nu}$ is the metric tensor which is a 4×4 matrix that encodes the geometry of spacetime and dx^μ is the infinitesimal coordinate separation between two points.

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Figure 2 shows two diagrams representing the embedded space (Flamm diagram) representation of Equation (1), which depicts the geometry of an equatorial ($\theta = \pi/2$) slice through space at a specific moment of time ($t = \text{const}$). The top of Figure 2 shows the embedding diagram for a traversable wormhole that connects two different universes (i.e., an inter-universe wormhole). The bottom diagram in the figure is an intra-universe wormhole with a throat that connects two distant regions of our own universe. These diagrams serve to aide in visualizing traversable wormhole geometry and are merely a geometrical exaggeration.

There was originally one other criterion for defining a traversable wormhole, which was that it must be embedded within the surrounding (asymptotically) flat spacetime. However, Hochberg and Visser (Reference 4) proved that it is only the behavior near the wormhole throat that is critical to understanding the physics, and that a generic throat can be defined without having to make all the symmetry assumptions and without assuming the existence of an asymptotically flat spacetime in which to embed the wormhole. Therefore, one only needs to know the generic features of the geometry near the throat in order to guarantee violations of the Null Energy Condition (NEC; see Section III for further detail) for certain open regions near the throat. So one is free to place our wormhole anywhere in spacetime because it is only the geometry and physics near the throat that matters for any analysis. This fact led to the development of a number of different traversable wormhole throat designs that are cubic shaped, polyhedral shaped, flat-face shaped, generic shaped, etc. The reader should consult (Reference 3) for a complete technical review of the various types (and shapes) of traversable wormhole solutions found in general relativity theory.

One knows that one needs exotic or negative energy to create and thread open a traversable wormhole. So in this regard, one asks what kind of wormhole one can make with less effort. To answer this question one can relate the local wormhole geometry to the global topological invariant of the spacetime via the Gauss-Bonnet Theorem (Reference 5). In the Gauss-Bonnet Theorem the local wormhole geometry is quantified by the energy density, U (in geometrodynamical units, $\eta = G = c = 1$), threading the wormhole throat plus a spatial curvature constant (for the throat). The global topological invariant of spacetime is quantified by the Euler Number, χ_e , which is itself defined in terms of the genus, g , representing the number of handles (or throats or tunnels) a wormhole can be assigned. These two topological quantities are related via $\chi_e = 2(1 - g)$. Therefore, the (static) wormhole Gauss-Bonnet relation is given by $U \leq \chi_e/4$ or $U \leq (1 - g)/2$ (Reference 5). (The case for dynamic traversable wormholes has

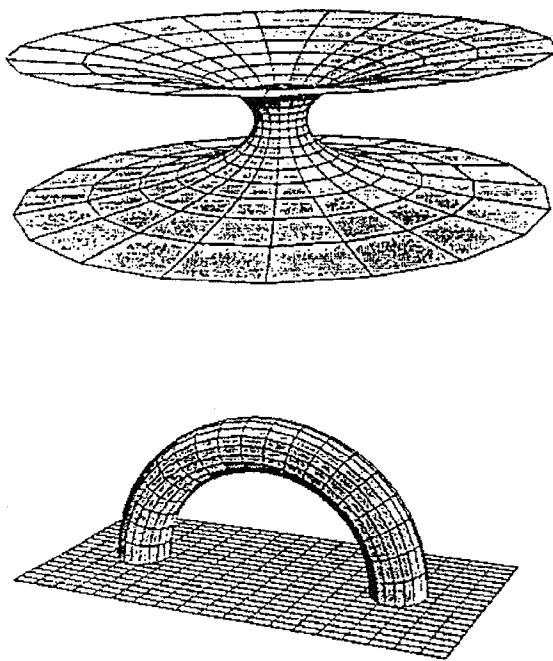


Figure 2. Inter-Universe Wormhole (top) and Intra-Universe Wormhole (bottom).

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results that are similar to the static case.) This relation will help to decide if a traversable wormhole having one throat, or two or more throats should be built and at what energy cost this will incur.

The following is the result of our analysis for traversable wormholes having:

- 1-handle/throat (i.e., flat torus or spherical wormhole topology) giving $g = 1$, thus $\chi_e = 0$, and so $U \leq 0$
- 2-handles/throats giving $g = 2$, thus $\chi_e = -2$, and so $U \leq -1/2$
- 3-handles/throats giving $g = 3$, thus $\chi_e = -4$, and so $U \leq -1$; and so on.

It is clear from this that as the number of wormhole handles/throats increases the amount of negative energy required to create the wormhole will grow larger in magnitude. This is an undesirable demand on any putative negative energy generator. It is clear then that item (a) defines the most desirable engineering solution one can hope for: a 1-handle/throat traversable wormhole that will require zero or (arbitrarily) little negative energy to create. The magnitude of energy condition violations and the amount of negative energy required to build a traversable wormhole will be addressed.

B. THE "STARGATE" SOLUTION

It is a straightforward exercise to design a real "stargate" from wormhole physics. A stargate is essentially a traversable wormhole with a flat-face shape for the throat as opposed to the spherical-shaped throat of the Morris and Thorne wormhole as discussed in the previous section. A traveler going through a stargate will simply be shunted into another remote spacetime region within our universe or into another universe.

The flat-face traversable wormhole solution is derived from the thin shell (a.k.a. junction condition or surface layer) formalism of the Einstein field equation (Reference 6, 7). The procedure is to take two copies of flat Minkowski space and remove from each identical regions of the form $\Omega \times \mathbb{R}$, where Ω is a three-dimensional compact spacelike hypersurface and \mathbb{R} is a timelike line (time axis). Then identify these two incomplete spacetimes along the timelike boundaries $\partial\Omega \times \mathbb{R}$. The resulting spacetime is geodesically complete and possesses two asymptotically flat regions connected by a traversable wormhole. The throat of the wormhole is just the junction $\partial\Omega$, which is a two-dimensional space-like hypersurface, at which the two original Minkowski spaces are identified (see Figures 3 and 4).

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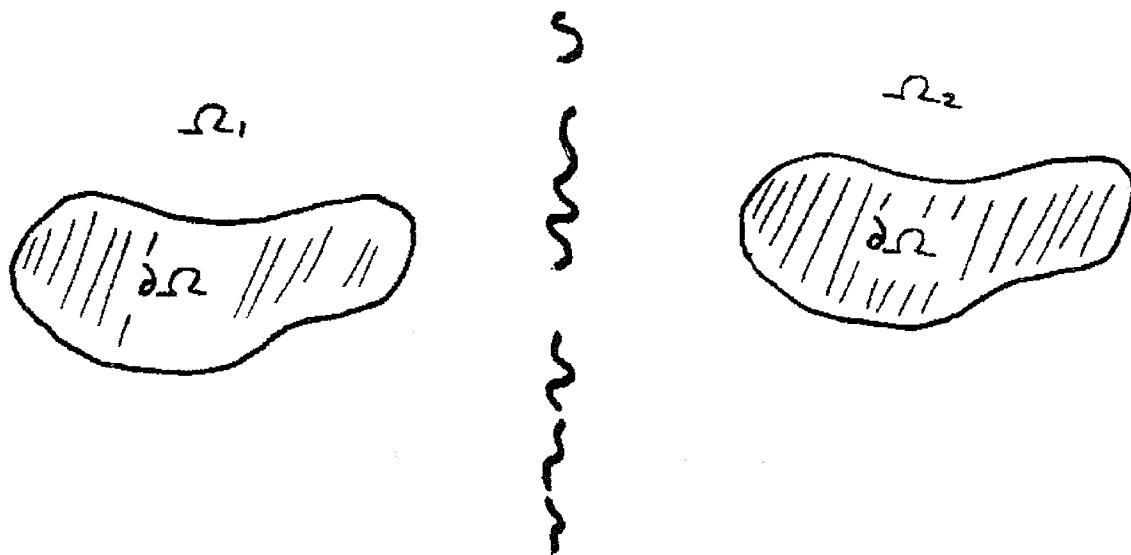


Figure 3. Diagram of a Simultaneous View of Two Remote Compact Regions, Ω_1 and Ω_2 , of Minkowski Space Used to Create the Wormhole Throat $\partial\Omega$ (time is suppressed in this diagram)

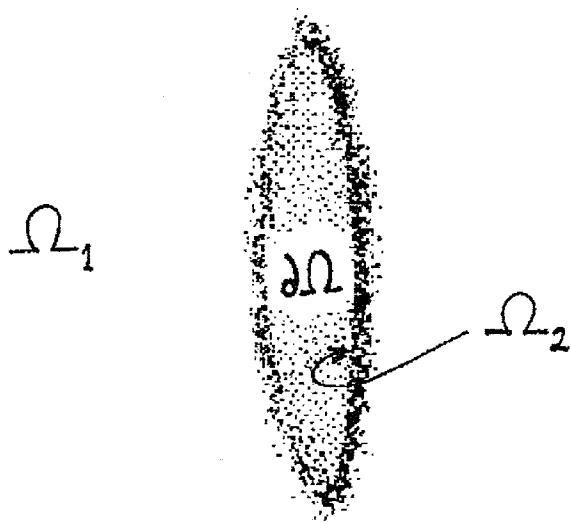


Figure 4. The Same Diagram as in Figure 3 Except as Viewed by an Observer Sitting in Region Ω_1 Who Looks Through the Wormhole Throat $\partial\Omega$ and Sees Remote Region Ω_2 (dotted area inside the circle) on the Other Side

It is a standard result of the thin shell formalism that the Einstein field equation may be cast in terms of the surface stress-energy tensor S^{μ}_{ν} of a thin shell of matter (or mass-energy) localized inside the wormhole throat $\partial\Omega$ (Reference 8):

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$$S^i_j = -\frac{c^4}{4\pi G} (K^i_j - \delta^i_j K^k_k) \quad (2)$$

where the second fundamental form K^i_j is a matrix that represents the extrinsic curvature of $\partial\Omega$ (telling how the wormhole throat is curved with respect to the enveloping four-dimensional spacetime), δ^i_j is the three-dimensional unit matrix, and K^k_k is the trace (sum of diagonal matrix elements) of K^i_j .³ K^i_j is a diagonal matrix having the two principal radii of curvature, ρ_1 and ρ_2 , of the thin shell as its components (see Figure 5). S^i_j may be interpreted in terms of the thin shell's surface energy density σ and principal surface tensions, β_1 and β_2 , which are also diagonal matrix components.

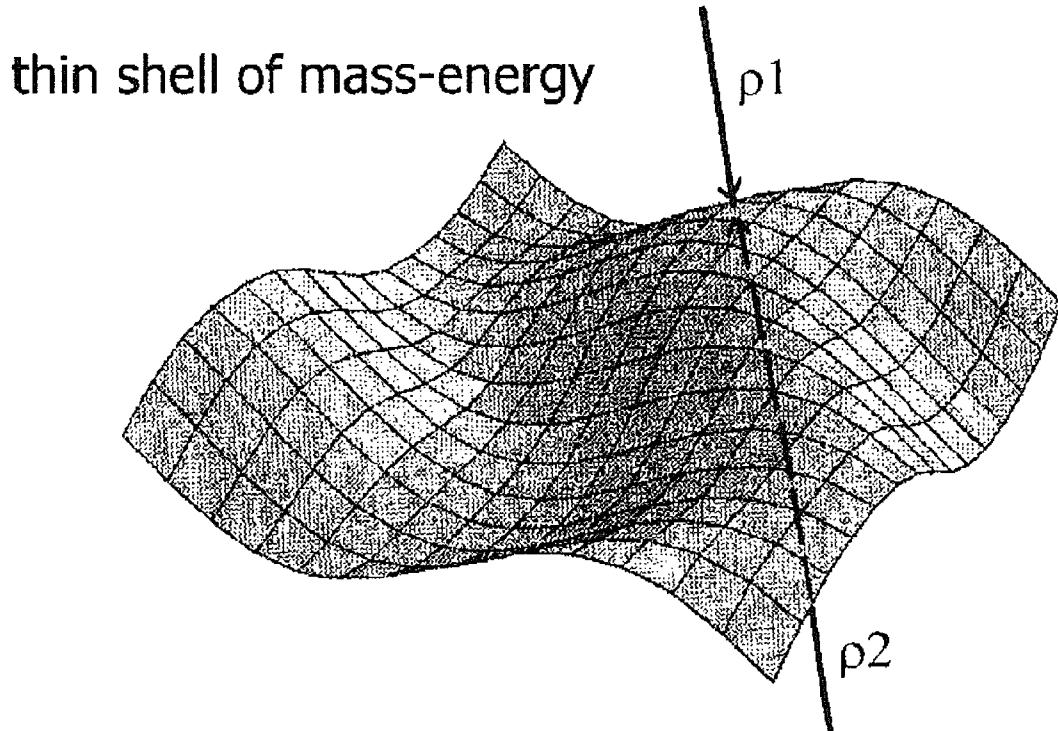


Figure 5. A Thin Shell of (Localized) Mass-Energy Possessing Two Principal Radii of Curvature, ρ_1 and ρ_2 .

Equation (2) is solved and the components of S^i_j are found to be (Reference 8):

$$\sigma = -\frac{c^4}{4\pi G} \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \quad (3a)$$

³ The Latin indices ($i, j, k = 0..2$) denote three-dimensional hypersurface coordinates, $x^0..x^2$, such that x^1, x^2 = space coordinates and x^0 = time coordinate.

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$$\theta_1 = -\frac{c^4}{4\pi G} \frac{1}{\rho_2} \quad (3b)$$

$$\theta_2 = -\frac{c^4}{4\pi G} \frac{1}{\rho_1} \quad (3c)$$

These are the Einstein field equations for a traversable wormhole that is produced by a thin shell of localized matter. Equations (3a-c) imply that (for $\partial\Omega$ a convex hypersurface) one is dealing with negative surface energy density and negative surface tensions. This is exotic matter! The negative surface tension (= positive outward pressure, a.k.a. gravitational repulsion) is required to keep the throat open and stable against collapse. To make this thin shell wormhole entirely flat requires that one chooses the throat $\partial\Omega$ to have at least one flat face (picture the thin shell in Figure 5 becoming flat). On that face the two principal radii of curvature become $\rho_1 = \rho_2 = \infty$ as required by standard three-dimensional geometry; therefore, substituting this requirement into Equations (3a-c) gives:

$$\sigma = \theta_1 = \theta_2 = 0 \quad (4)$$

which is a remarkable result. This means that a traveler encountering and going through such a wormhole-stargate will feel no tidal gravitational forces and see no exotic matter threading the throat. A traveler stepping through the throat will simply be shunted into another remote spacetime region or into another universe (note: the Einstein field equation does not fix the spacetime topology, so it is possible that wormholes are inter-universe as well as intra-universe tunnels). Therefore, one can construct a stargate by generating a thin shell or surface layer of exotic matter much like a thin film of soap stretched across a loop of wire.

C. WHAT A WORMHOLE LOOKS LIKE IN THE REAL WORLD

The exotic matter threading a traversable wormhole throat produces repulsive gravity, which will then deflect light rays going through and around it. The entrance to the spherically symmetric Morris & Thorne wormhole looks like a sphere that contains the mirror image of a whole other universe or remote region within our own universe, incredibly shrunken and distorted (see Figure 6). This is an

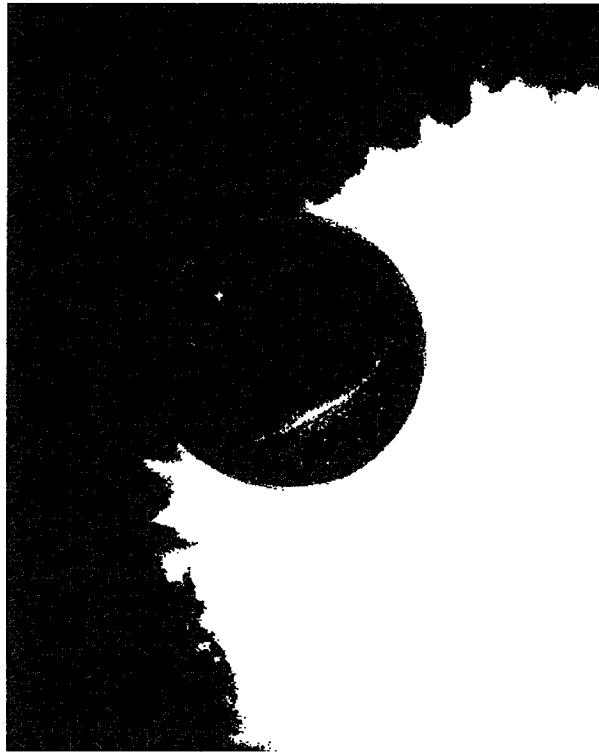


Figure 6. A Spherically Symmetric Traversable Wormhole Observed in Space

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example of the topological inversion manifested in wormhole geometry. The spherical wormhole entrance/exit (a.k.a. the throat) is called a hypersphere because it is the hyperspace surface of our four-dimensional spacetime. If one were to travel through the wormhole and look back at it from the other side, then one would see a sphere (the entry way back home) that seemed to contain the whole original universe or home region of space near Earth (within your universe). This would look just like a glass Christmas tree ornament, which is just a spherical mirror that reflects, in principle, the entire universe around it.

A flat-faced wormhole, or stargate, which is also a hypersurface, would not distort the mirror image of the remote space region or other universe seen through it because the negative surface energy density and negative surface tensions of the exotic matter threading its throat is zero as seen and felt by light and matter passing through it (recall Equation (4)). See Figures 7 and 8.

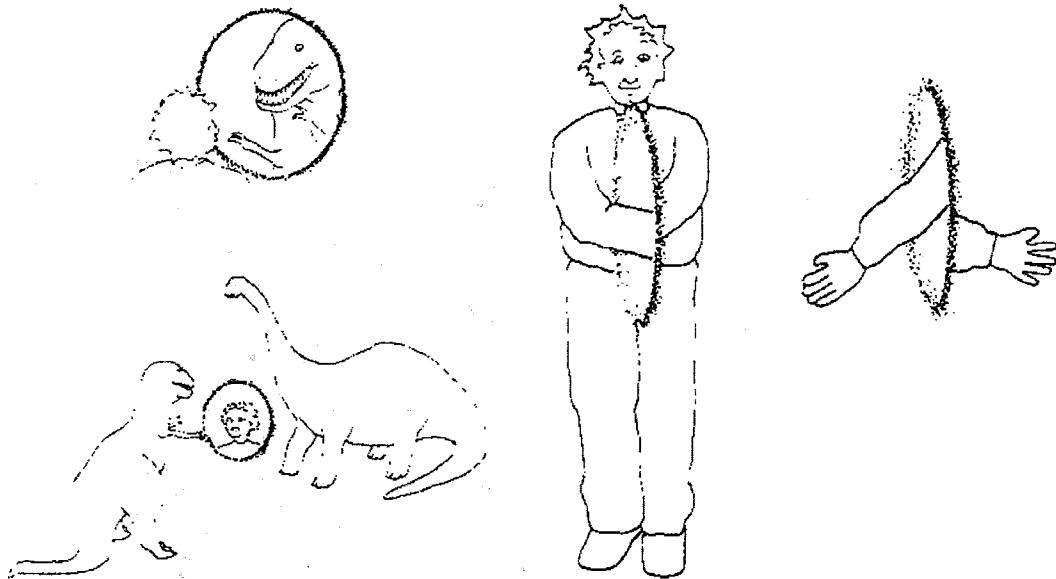


Figure 7. A Stargate (adapted from Reference 9)

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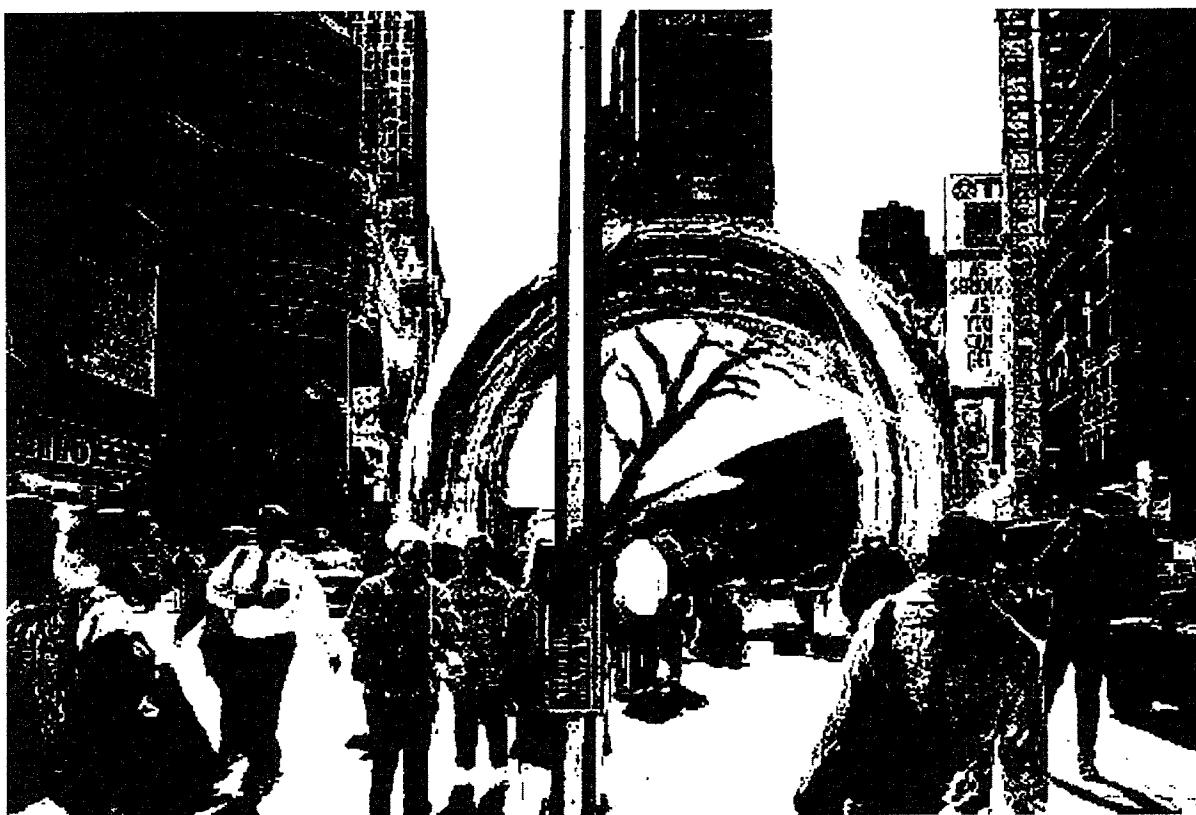


Figure 8. A Stargate in Times Square

If a small wormhole (three or more dimensional) were to begin to appear or even bump into our local space, one would perceive this process as the occurrence of an unusually bright spot in the sky. Blue and red Doppler shifting of this bright spot would manifest when the intersection of the wormhole with our local space grows or recedes, respectively.

III. The General Relativistic Definition of Exotic Matter and the Energy Conditions

This section will consider the physics of the exotic matter that is required to build traversable wormholes. What exactly is "exotic" matter? In classical physics the energy density of all observed forms of matter (fields) is non-negative. What is exotic about the type of matter that must be used to generate traversable wormhole spacetime is that it must have negative energy density and/or negative flux (Reference 10). The energy density is "negative" in the sense that the configuration of matter fields one must deploy to generate and thread a traversable wormhole throat must have an energy density, ρ_E ($= \rho c^2$, where ρ is the rest-mass density), that is less than or equal to its pressures/tensions, p_i (Reference 1, 3).⁴ In many cases, these equations of state are also known to possess an energy density that is algebraically negative, i.e., the energy density and flux are less than zero. It is on the basis of these conditions that

⁴ From this point forward in the text, all Latin indices (e.g., $i, j, k = 1...3$) that are affixed to physical quantities denote the usual 3-dimensional space coordinates, $x^1...x^3$, indicating the spatial components of vector or tensor quantities.

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one can call this material property "exotic." The condition for ordinary, classical (non-exotic) forms of matter that all are familiar with in nature is that $\rho_E > \rho_i$ and/or $\rho_E \geq 0$. These conditions represent two examples of what are variously called the "standard" energy conditions: Weak Energy Condition (WEC: $\rho_E \geq 0$, $\rho_E + p_i \geq 0$), Null Energy Condition (NEC: $\rho_E + p_i \geq 0$), Dominant Energy Condition (DEC), and Strong Energy Condition (SEC). These energy conditions forbid negative energy density between material objects to occur in nature, but they are mere hypotheses. Hawking and Ellis (Reference 11) formulated the energy conditions in order to establish a series of mathematical hypotheses governing the behavior of collapsed-matter singularities in their study of cosmology and black hole physics. More specifically, classical general relativity allows one to prove lots of general theorems about the behavior of matter in gravitational fields. The impact or implications of the DEC or SEC will not be considered because they add no new information beyond the WEC and NEC.

The bad news is that real physical matter is not "reasonable" because the energy conditions are in general violated by semiclassical quantum effects (occurring at order η) (Reference 3).⁵ More specifically, quantum effects generically violate the average NEC (ANEC). Furthermore, it was discovered in 1965 that quantum field theory has the remarkable property of allowing states of matter containing local regions of negative energy density or negative fluxes (Reference 12). This violates the WEC, which postulates that the local energy density is non-negative for all observers. And there are also general theorems of differential geometry that guarantee that there must be a violation of one, some, or all of the energy conditions (meaning exotic matter is present) for all traversable wormhole spacetimes. With respect to creating traversable wormhole spacetimes, "negative energy" has the unfortunate reputation of alarming physicists. This is unfounded since all the energy condition hypotheses have been experimentally tested in the laboratory and experimentally shown to be false – 25 years before their formulation (Reference 13).

Further investigation into this technical issue showed that violations of the energy conditions are widespread for all forms of both "reasonable" classical and quantum matter (Reference 14-18). Furthermore, Visser (Reference 3) showed that all (generic) spacetime geometries violate all the energy conditions. So the condition that $\rho_E > \rho_i$ and/or $\rho_E \geq 0$ must be obeyed by all forms of matter in nature is spurious. Violating the energy conditions commits no offense against nature. Negative energy has been produced in the laboratory and this will be discussed in the following sections.

A. EXAMPLES OF EXOTIC OR "NEGATIVE" ENERGY FOUND IN NATURE

The exotic (energy condition-violating) fields that are known to occur in nature are:

- Static, radially-dependent electric or magnetic fields. These are borderline exotic, if their tension were infinitesimally larger, for a given energy density (Reference 11, 19).
- Squeezed quantum vacuum states: electromagnetic and other (non-Maxwellian) quantum fields (Reference 1, 20).

⁵ Planck's reduced constant, $\eta = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$.

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- Gravitationally squeezed vacuum electromagnetic zero-point fluctuations (Reference 21).
- Casimir effect, i.e., the Casimir vacuum in flat, curved, and topological spaces (Reference 22-28).
- Other quantum fields/states/effects. In general, the local energy density in quantum field theory can be negative due to quantum coherence effects (Reference 12). Other examples that have been studied are Dirac field states: the superposition of two single particle electron states and the superposition of two multi-electron-positron states (Reference 29, 30). In the former (latter), the energy densities can be negative when two single (multi-) particle states have the same number of electrons (electrons and positrons) or when one state has one more electron (electron-positron pair) than the other.

Cosmological inflation (Reference 3), cosmological particle production (Reference 3), classical scalar fields (Reference 3), the conformal anomaly (Reference 3), and gravitational vacuum polarization (Reference 14-17) are among many other examples that also violate the energy conditions. Since the laws of quantum field theory place no strong restrictions on negative energies and fluxes, then it might be possible to produce exotic phenomena such as faster-than-light travel (Reference 31-33), traversable wormholes (Reference 1-3), violations of the second law of thermodynamics (Reference 34, 35), and time machines (Reference 2, 3, 36). There are several other exotic phenomena made possible by the effects of negative energy, but they lie outside the scope of the present study. This section will review the previously listed items 1 thru 4 and examine their applicability and technical maturity. Dirac field states are currently under study by investigators. Also, the issue of capturing and storing negative energy is not considered in what follows because free-space negative energy sources appear to be a more desirable option for inducing traversable wormholes than stored negative energy, and because there is very little technical literature that addresses how to capture and store negative energy (see, e.g., Reference 10). The issue of capturing and storing negative energy will be left for future investigations.

B. GENERATING NEGATIVE ENERGY IN THE LAB

1. Static Radial Electric & Magnetic Fields

It is beyond the scope of this study to include all the technical configurations by which one can generate static, radially-dependent electric or magnetic fields. Suffice it to say that ultrahigh-intensity tabletop lasers have been used to generate extreme electric and magnetic field strengths in the lab. Ultrahigh-intensity lasers use the chirped-pulse amplification (CPA) technique to boost the total output beam power. All laser systems simply repackage energy as a coherent package of optical power, but CPA lasers repackage the laser pulse itself during the amplification process. In typical high-power short-pulse laser systems, it is the peak intensity, not the energy or the fluence, which causes pulse distortion or laser damage. However, the CPA laser dissects a laser pulse according to its frequency components, and reorders it into a time-stretched lower-peak-intensity pulse of the same energy (Reference 37-39). This benign pulse can then be amplified safely to high energy, and then only afterwards reconstituted as a very short pulse of enormous peak power – a pulse which could never itself have passed safely through the laser system. Made more tractable in this way, the pulse can be

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amplified to substantial energies (with orders of magnitude greater peak power) without encountering intensity-related problems.

The extreme output beam power, fields and physical conditions that have been achieved by ultrahigh-intensity tabletop lasers are (Reference 39):

- Power Intensity $\approx 10^{19}$ to 10^{30} W/m² (10^{34} W/m² using SLAC as a booster).
- Peak Power Pulse $\leq 10^3$ fs.
- Electric field, $E \approx 10^{14}$ to 10^{18} V/m [note: compare this with the critical quantum electrodynamic (QED) vacuum breakdown E-field intensity, $E_c = 2m_e^2c^3/\eta e \approx 10^{18}$ V/m, defined by the total rest-energy of an electron-positron pair created from the vacuum divided by the electron's Compton wavelength]⁶.
- Magnetic field, $B \approx \text{several} \times 10^6$ Tesla (note: the critical QED vacuum breakdown B-field intensity is $B_c = E_c/c \approx 10^{10}$ Tesla).
- Ponderomotive Acceleration of Electrons $\approx 10^{17}$ to 10^{30} g₀ (g_0 is the acceleration of gravity near the Earth's surface, 9.81 m/s²).
- Light Pressure $\approx 10^9$ to 10^{15} bars.
- Plasma Temperatures $> 10^{10}$ K.

The vigilant reader might assert that the electric and magnetic fields generated by ultrahigh-intensity lasers are not static. But in fact, these fields are static over the duration of the pulse-width while at peak intensity. The data above illustrates that ultrahigh-intensity lasers can generate an electric field energy density $\sim 10^{16}$ to 10^{28} J/m³ and a magnetic field energy density $\sim 10^{19}$ J/m³. However, there remains the problem of engineering this type of experiment because classical electromagnetic theory states that every observer associated with the experiment will see a non-negative energy density that is $\propto E^2 + B^2$, where E and B are measured in an observer's reference frame. It is not known how to increase the tension in these fields using current physics, but some new physics may provide an answer. This technical problem must be left for future investigation.

2. Squeezed Quantum Vacuum

Substantial theoretical and experimental work has shown that in many quantum systems the limits to measurement precision imposed by the quantum vacuum zero-point fluctuations (ZPF) can be breached by decreasing the noise in one observable (or measurable quantity) at the expense of increasing the noise in the conjugate observable; at the same time the variations in the first observable, say the energy, are reduced below the ZPF such that the energy becomes "negative." "Squeezing" is thus the control of quantum fluctuations and corresponding uncertainties, whereby one can squeeze/reduce the variance of one (physically important) observable quantity provided the variance in the (physically unimportant) conjugate variable is stretched/increased. The squeezed quantity possesses an unusually low variance, meaning less variance than would be expected on the basis of the equipartition theorem. One can in principle

⁶ Electron mass, $m_e = 9.11 \times 10^{-31}$ kg; electron charge, $e = 1.602 \times 10^{-19}$ C.

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exploit quantum squeezing to extract energy from one place in the ordinary vacuum at the expense of accumulating excess energy elsewhere (Reference 1).

The squeezed state of the electromagnetic field is a primary example of a quantum field that has negative energy density and negative energy flux. Such a state became a physical reality in the laboratory as a result of the nonlinear-optics technique of "squeezing," i.e., of moving some of the quantum-fluctuations of laser light out of the $\cos[\omega(t - z/c)]$ part of the beam and into the $\sin[\omega(t - z/c)]$ part (Reference 20, 40-44).⁷ The observable that gets squeezed will have its fluctuations reduced below the vacuum ZPF. The act of squeezing transforms the phase space circular noise profile characteristic of the vacuum into an ellipse, whose semimajor and semiminor axes are given by unequal quadrature uncertainties (of the quantized electromagnetic field harmonic oscillator operators). This applies to coherent states in general, and the usual vacuum is also a coherent state with eigenvalue zero. As this ellipse rotates about the origin with angular frequency ω , these unequal quadrature uncertainties manifest themselves in the electromagnetic field oscillator energy by periodic occurrences, which are separated by one quarter cycle, of both smaller and larger fluctuations compared to the unsqueezed vacuum.

Morris and Thorne (Reference 1) and Caves (Reference 45) point out that if one squeezes the vacuum, i.e., if one puts vacuum rather than laser light into the input port of a squeezing device, then one gets at the output an electromagnetic field with weaker fluctuations and thus less energy density than the vacuum at locations where $\cos^2[\omega(t - z/c)] \approx 1$ and $\sin^2[\omega(t - z/c)] \ll 1$; but with greater fluctuations and thus greater energy density than the vacuum at locations where $\cos^2[\omega(t - z/c)] \ll 1$ and $\sin^2[\omega(t - z/c)] \approx 1$. Since the vacuum is defined to have vanishing energy density, any region with less energy density than the vacuum actually has a negative (renormalized) expectation value for the energy density. Therefore, a squeezed vacuum state consists of a traveling electromagnetic wave that oscillates back and forth between negative energy density and positive energy density, but has positive time-averaged energy density.

For the squeezed electromagnetic vacuum state, the energy density $\rho_{E\text{-sqvac}}$ is given by (Reference 46):

$$\rho_{E\text{-sqvac}} = \left(\frac{2\hbar\omega}{L^3} \right) \sinh \xi \left[\sinh \xi + \cosh \xi \cos(2\omega(t - z/c) + \delta) \right] \quad (\text{J/m}^3) \quad (5)$$

where L^3 is the volume of a large box with sides of length L (i.e., the quantum field is placed in a box with periodic boundary conditions), ξ is the squeezed state amplitude (giving a measure of the mean photon number in a squeezed state), and δ is the phase of squeezing. Equation (5) shows that $\rho_{E\text{-sqvac}}$ falls below zero once every cycle when the condition $\cosh \xi > \sinh \xi$ is met. It turns out that this is always true for every nonzero value of ξ , so $\rho_{E\text{-sqvac}}$ becomes negative at some point in the cycle for a general squeezed vacuum state. On another note, when a quantum state is close to a squeezed vacuum state, there will almost always be some negative energy densities present.

Negative energy can be generated by an array of ultrahigh-intensity lasers using an ultra-fast rotating mirror system (Reference 47). In this scheme a laser beam is passed

⁷ ω is the angular frequency of light, t is time, and z denotes the z -axis direction of beam propagation.

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through an optical cavity resonator made of a lithium niobate (LiNbO_3) crystal that is shaped like a cylinder with rounded silvered ends to reflect light. The resonator will act to produce a secondary lower frequency light beam in which the pattern of photons is rearranged into pairs. The squeezed light beam emerging from the resonator will contain pulses of negative energy interspersed with pulses of positive energy.

In this concept both the negative and positive energy pulses are $\sim 10^{-15}$ second duration. In principle a set of rapidly rotating mirrors could be arranged to separate the positive and negative energy pulses from each other. The light beam would be set to strike each mirror surface at a very shallow angle while the rotation would ensure that the negative energy pulses would be reflected at a slightly different angle from the positive energy pulses. A small spatial separation of the two different energy pulses would occur at some distance from the rotating mirror. Another system of mirrors would be needed to redirect the negative energy pulses to an isolated location and concentrate them there. See Figure 9 for an illustration of this concept.

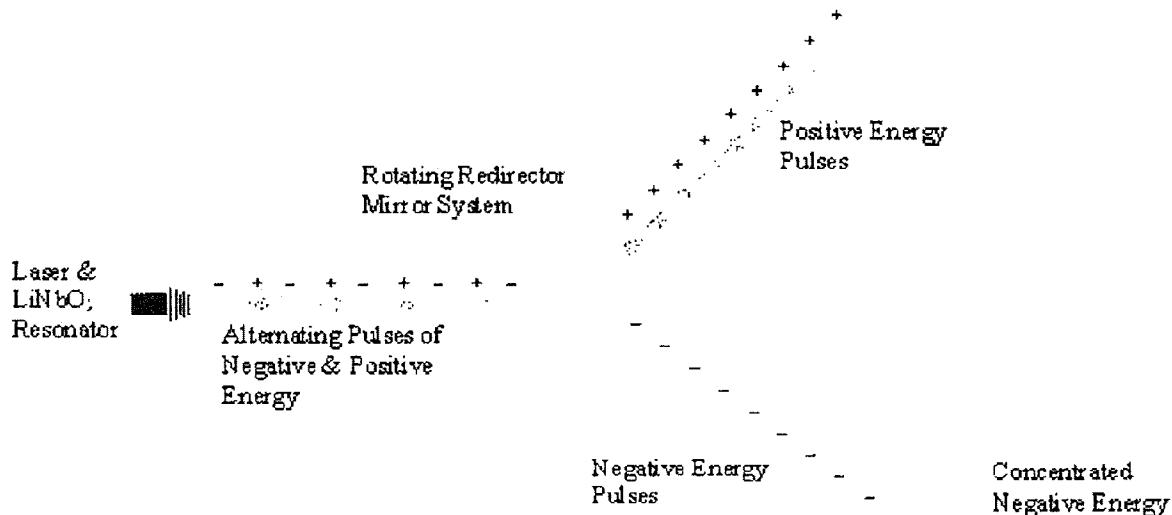


Figure 9. Conceptual Squeezed Light Negative Energy Generator

The rotating mirror system can actually be implemented via non-mechanical means. A chamber of sodium gas is placed within the squeezing cavity and a laser beam is directed through the gas. The beam is reflected back on itself by a mirror to form a standing wave within the sodium chamber. This wave causes rapid variations in the optical properties of the sodium thus causing rapid variations in the squeezed light so that one can induce rapid reflections of pulses by careful design (Reference 41). An illustration of this is shown in Figure 10.

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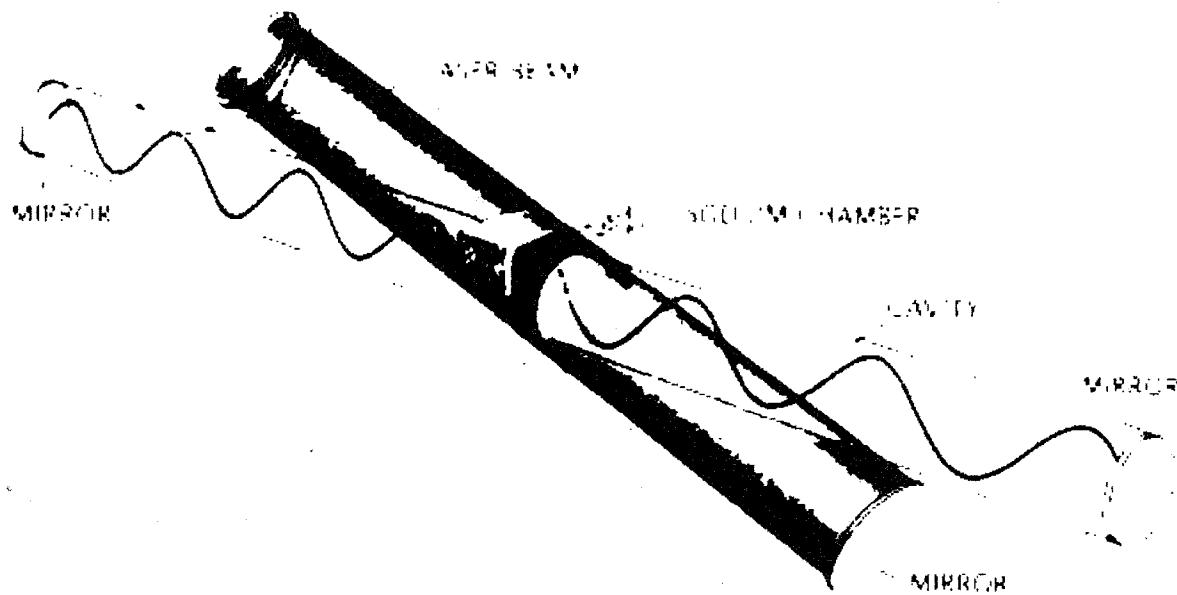
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Figure 10. Sodium Chamber Negative Energy Separator (Reference 41)

Another way to generate negative energy via squeezed light would be to manufacture extremely reliable light pulses containing precisely one, two, three, etc., photons apiece and combine them together to create squeezed states to order (Reference 47). Superimposing many such states could theoretically produce bursts of intense negative energy. See Figure 11 for a conceptual diagram of this concept. Photonic crystal research has already demonstrated the feasibility of using photonic crystal waveguides (mixing together the classical and quantum properties of optical materials) to engineer light sources that produce beams containing precisely one, two, three, etc., photons. For example, researchers at Melbourne University used a microwave oven to fuse a tiny diamond, just $1/1000^{\text{th}}$ of a millimeter long, onto an optical fiber, which could be used to create a single photon beam of light (Reference 48, 49). The combining of different beams containing different (finite integer) numbers of photons is already state-of-the-art practice via numerous optical beam combining methods that can readily be extended to our application.



Figure 11 Alternative Conceptual Squeezed Light Negative Energy Generator

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Finally, Ries et al. (Reference 50) experimentally demonstrated the very first simple, scalable squeezed vacuum source in the laboratory that consisted of a continuous-wave diode laser and an atomic rubidium vapor cell. The experimental tools one needs to begin exploring the generation of negative energy for the purpose of creating traversable wormholes are just now becoming available.

3. Gravitationally Squeezed Electromagnetic ZPF

A natural source of negative energy comes from the effect that gravitational fields (of astronomical bodies) in space have upon the surrounding quantum vacuum. For example, the gravitational field of the Earth produces a zone of negative energy around it by dragging some of the virtual quanta (a.k.a. vacuum ZPF) downward. This concept was initially developed in the 1970s as a byproduct of studies on quantum field theory in curved space (Reference 25). However, Hochberg and Kephart (Reference 21) derived an important application of this concept to the problem of creating and stabilizing traversable wormholes. They showed that one can utilize the negative energy densities, which arise from distortion of the vacuum ZPF due to the interaction with a prescribed gravitational background, for providing a violation of the energy conditions. The squeezed quantum states of quantum optics provide a natural form of matter having negative energy density.

The analysis, via quantum optics, showed that gravitation itself provides the mechanism for generating the squeezed vacuum states needed to support stable traversable wormholes. The production of negative energy densities via a squeezed vacuum is a necessary and unavoidable consequence of the interaction or coupling between ordinary matter and gravity, and this defines what is meant by gravitationally squeezed vacuum states. The magnitude of the gravitational squeezing of the vacuum can be estimated from the quantum optics squeezing condition for given transverse momentum and (equivalent) energy eigenvalues, j , of two electromagnetic ZPF field modes, such that this condition is subject to $j \rightarrow 0$, and it is defined as (Reference 21):

$$j \equiv \frac{4\pi c^2}{\lambda g_0} \left(\frac{r}{R_0} \right)^2 \frac{M_0}{M} = \frac{8\pi r_s}{\lambda} \quad (6)$$

where λ is the ZPF mode wavelength, r is the radial distance from the center of the astronomical body in question, R_0 is the radius of the Earth (6.378×10^6 m), M_0 is the mass of the Earth (5.972×10^{24} kg), M is the mass of the astronomical body, and r_s is the Schwarzschild radius of the astronomical body.⁸ Note that r_s is only a convenient radial distance parameter for any object under examination and so there is no black hole collapse involved in this analysis. Any radial distance from the body in question can be chosen to perform this analysis, but using r_s makes the equation simpler in form. Also note that Equation (6) contains an extra factor of two (compared to the j derived in Reference 21) in order to account for the photon spin. The squeezing condition plus Equation (6) simply states that substantial gravitational squeezing of the vacuum occurs for those ZPF field modes with $\lambda \geq 8\pi r_s$ of the mass in question (whose

⁸ $r_s = 2GM/c^2$. According to general relativity theory, this is the critical radius at which a spherically symmetric massive body becomes a black hole, i.e., at which light is unable to escape from the body's surface.

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gravitational field is squeezing the vacuum). The corresponding local vacuum state energy density is: $\rho_{E-gsvac} = -2\pi^2\eta c/\lambda^4$.

The general result of the gravitational squeezing effect is that as the gravitational field strength increases, the negative energy zone (surrounding the body) also increases in strength. Table 1 shows when gravitational squeezing becomes important for sample bodies and their associated $\rho_{E-gsvac}$. The table shows that in the case of the Earth, Jupiter and the Sun, the squeezing effect is extremely feeble because only ZPF mode wavelengths above 0.2 m to 78 km are affected, each having very minute $\rho_{E-gsvac}$. For a solar mass black hole (radius of 2.95 km), the effect is still feeble because only ZPF mode wavelengths above 78 km are affected. But note that Planck mass bodies will have an enormously strong negative energy zone surrounding them because all ZPF mode wavelengths above 8.50×10^{-34} m will be squeezed, in other words, all wavelengths of interest for vacuum fluctuations. Protons will have the strongest negative energy zone in comparison because the squeezing effect includes all ZPF mode wavelengths above 6.50×10^{-53} m. Furthermore, a body smaller than a nuclear diameter ($\approx 10^{-16}$ m) and containing the mass of a mountain ($\approx 10^{11}$ kg) has a fairly strong negative energy zone because all ZPF mode wavelengths above 10^{-15} m will be squeezed. In each of these cases, the magnitude of the corresponding $\rho_{E-gsvac}$ is very large.

However, the estimates for the wavelengths in Table 1 might be too small. Ford (private communication, 2007) argues that Reference 21 is in error because spacetime is flat on scales smaller than the local radius of curvature, which is defined by the inverse square root of the typical Riemann curvature tensor component in a local orthonormal frame, or $\lambda_C \approx (r^3 c^2/GM)^{1/2}$. According to Ford, only ZPF modes with $\lambda \geq \lambda_C$ will be squeezed by the gravitational field. This leads to a different local vacuum state energy density (for $r \gg r_s$) (Reference 15):

$$\begin{aligned}\rho_{E-gsvac} &= -\frac{2\pi^2\hbar c}{\lambda^4} \\ &\approx -\frac{2\pi^2\hbar c}{\lambda_C^4} \\ &\approx -\frac{2\pi^2\hbar G^2 M^2}{c^3 r^6} \text{ (J/m}^3)\end{aligned}\tag{7}$$

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Table 1. Substantial Gravitational Squeezing Occurs for Vacuum ZPF When $\lambda \geq 8\pi r_s$

Mass of body (kg)	r_s (m)	λ (m)	$\rho_{E-gsvac}$ (J/m ³)
Sun = 2.00×10^{30}	2.95×10^3	$\geq 78.0 \times 10^3$	-1.69×10^{-44}
Jupiter = 1.90×10^{27}	2.82	≥ 74	-2.08×10^{-32}
Earth = 5.98×10^{24}	8.87×10^{-3}	≥ 0.23	-2.23×10^{-22}
Typical mountain $\approx 10^{11}$	$\approx 10^{-16}$	$\geq 10^{-15}$	-6.25×10^{35}
Planck mass = 2.18×10^{-8}	3.23×10^{-35}	$\geq 8.50 \times 10^{-34}$	-1.20×10^{108}
Proton = 1.67×10^{-27}	2.48×10^{-54}	$\geq 6.50 \times 10^{-53}$	-3.50×10^{184}

For example, near the surface of the Earth ($r \approx R_0, M = M_0$), $\lambda_c \approx 2.42 \times 10^{11}$ m and hence, Equation (7) gives $\rho_{E-gsvac} \approx -1.82 \times 10^{-70}$ J/m³. Compare these values with $\lambda \geq 0.23$ m and $\rho_{E-gsvac} \approx -2.23 \times 10^{-22}$ J/m³ in Table 1. The resolution of this disagreement remains an open question.

One is presently unaware of any way to artificially generate gravitational squeezing of the vacuum in the laboratory. This will be left for future investigation. However, it is predicted to occur in the vicinity of astronomical matter. Naturally occurring traversable wormholes in the vicinity of astronomical matter would therefore become possible.

4. Vacuum Field Stress: Negative Energy from the Casimir Effect

The Casimir effect is by far the easiest and most well known way to generate negative energy in the lab. The Casimir effect that is familiar to most people is the force that is associated with the electromagnetic quantum vacuum (Reference 51). This is an attractive force that must exist between any two neutral (uncharged), parallel, flat, conducting surfaces (e.g., metallic plates) in a vacuum. This force has been well measured and it can be attributed to a minute imbalance in the vacuum electromagnetic zero-point energy density inside the cavity between the conducting surfaces versus the vacuum electromagnetic zero-point energy density in the free-space region outside of the cavity (Reference 52-54). See Figure 12 for an illustration of this effect.

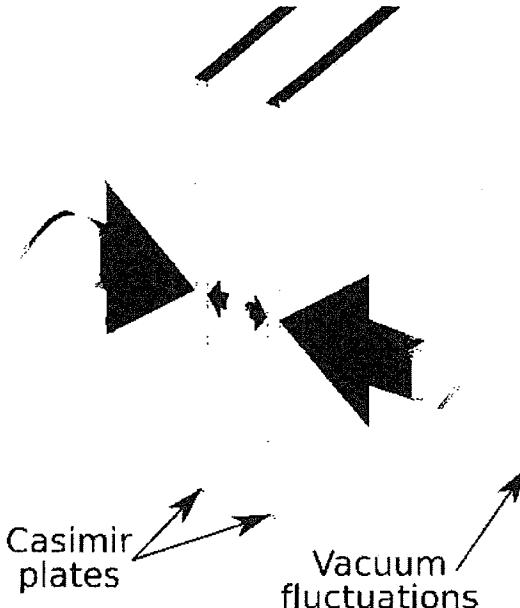


Figure 12. Schematic of the Casimir Effect

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It turns out that there are many different types of Casimir effects found in quantum field theory (Reference 22-24, 28, 55). For example, if one introduces a single infinite plane conductor into the Minkowski (flat spacetime) vacuum by bringing it adiabatically from infinity so that whatever quantum fields are present suffer no excitation but remain in their ground states, then the vacuum (electromagnetic) stresses induced by the presence of the infinite plane conductor produces a Casimir effect. This result holds equally well when two parallel plane conductors (with separation distance d) are present, which gives rise to the familiar Casimir effect inside a cavity. Note that in both cases, the spacetime manifold is made incomplete by the introduction of the plane conductor boundary condition(s). The vacuum region put under stress by the presence of the plane conductor(s) is called the Casimir vacuum. The generic expression for the energy density of the Casimir effect is $\rho_{CE} = -A(\eta c)d^{-4}$, where $A = \zeta(D)/8\pi^2$ in spacetimes of arbitrary dimension D (Reference 22-24). The appearance of the zeta-function $\zeta(D)$ is characteristic of expressions for vacuum stress-energy tensors, $T_{vac}^{\mu\nu}$. In our familiar four-dimensional spacetime ($D = 4$), $A = \pi^2/720$. To calculate $T_{vac}^{\mu\nu}$ for a given quantum field is to calculate its associated Casimir effect.

Analogs of the Casimir effect also exist for fields other than the electromagnetic field. When considering the vacuum state of other fields, one must consider boundary conditions that are analogous to the perfect-conductor boundary conditions for the electromagnetic field at the surfaces of the plates (Reference 22-24, 28). Other fields are not electromagnetic in nature, that is to say they are non-Maxwellian, and so the perfect-conductor boundary conditions do not apply to them. It turns out that complete manifolds exhibit what is called the topological Casimir effect for any non-Maxwellian fields. In order to define boundary conditions for other fields the conductor boundary conditions are replaced and Minkowski spacetime by a manifold of the form $\mathbb{R} \times \Sigma$ (i.e., a product space), where \mathbb{R} is the real line defining the time dimension for this particular product space and Σ is a flat three-dimensional manifold having any one of the following topologies: $\mathbb{R}^2 \times S^1$, $\mathbb{R} \times T^2$, T^3 , $\mathbb{R} \times K^2$, etc., \mathbb{R} being the real line that defines any linear space dimension (e.g., \mathbb{R} = line, \mathbb{R}^2 = two-dimensional plane, etc.), T^n being the n-torus, K^2 the two-dimensional Klein bottle, S^1 the circle, etc.

The case $\Sigma = \mathbb{R}^2 \times S^1$ has the closest resemblance to the electromagnetic Casimir effect, the difference being that instead of imposing conductor boundary conditions, one imposes periodic boundary conditions on some of the space coordinates in the three-dimensional manifold. When imposing this topological constraint on the field theoretic calculation of the topological Casimir effect (for linear massless fields), one finds that the generic expression for the energy density is also $\rho_{CE} = -A(\eta c)d^{-4}$, where $A = \pm d_f (\pi^2/90)$, d_f is the number of degrees of freedom (e.g., helicity states) per spatial point, the plus sign holds for boson fields (giving a negative energy density) and the negative sign for fermion fields (giving a positive energy density).

If one were to admit spin structure in the manifolds described above and the field is spinorial, then there is another important subtlety that must be taken into account when evaluating $T_{vac}^{\mu\nu}$. However, this introduces an additional complexity involving the relationship between the spin structure and the global structure (i.e., the configuration space or fibre bundle) of the field in question whereby the topology not only of the base

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manifold, but of the fibre bundle itself has an effect on $T_{vac}^{\mu\nu}$. In addition to this, there are (compactified) extra-space dimensional quantum field (i.e., D-Brane or "brane world") analogs of the Casimir effect yet to be explored. But a detailed consideration of these for producing traversable wormholes is beyond the scope of this report and will be left for future investigation.

As a final note, the methods used to obtain the electromagnetic $T_{vac}^{\mu\nu}$ between parallel plane conductors can also be used when the conductors are not parallel but are joined together along a line of intersection. If the conductors have curved surfaces instead, then one obtains results that are similar to the case of intersecting conductors. These geometries have also been evaluated for the case of dielectric media. These particular cases will not be considered further since there are technical subtleties involved that complicate the calculations and application of the different approaches. This topic will also be left for future investigation.

5. Dynamical Casimir Effect: Moving Mirrors

Negative energy can be created by a single moving reflecting (conducting) surface (a.k.a. a moving mirror). A mirror moving with increasing acceleration generates a flux of negative energy that emanates from its surface and flows out into the space ahead of the mirror (Reference 25, 56). This is essentially the simple case of an infinite plane conductor undergoing acceleration perpendicular to its surface. If the acceleration varies with time, the conductor will generally emit or absorb photons (i.e., exchange energy with the vacuum), even though it is neutral. This is an example of the well-known quantum phenomenon of parametric excitation. The parameters of the electromagnetic field oscillators (e.g., their frequency distribution function) change with time owing to the acceleration of the mirror (Reference 57). However, this effect is known to be exceedingly small, and it is not the most effective way to produce negative energy. This scheme will not be considered any further.

6. Casimir Effect: Negative Energy for Traversable Wormholes

The electromagnetic Casimir effect can be used in principle to create a traversable wormhole. The energy density $\rho_{CE} = -(\pi^2 \hbar c / 720) d^{-4}$ within a Casimir cavity is negative and manifests itself by producing a force of attraction between the cavity walls. But cavity dimensions must be made exceedingly small in order to generate a significant amount of negative energy. In order to use the Casimir effect to generate a spherically symmetric traversable wormhole throat of radius r_{throat} , there is need to design a cavity made of perfectly conducting spherically concentric thin plates with a plate separation d of (Reference 2):

$$d = \left(\frac{\pi^3}{30} \right)^{\frac{1}{4}} \left(r_{throat} \sqrt{\frac{\hbar G}{c^3}} \right)^{\frac{1}{2}} \\ = (4.05 \times 10^{-18}) \sqrt{r_{throat}} \quad (\text{m}) \quad (8)$$

To counteract the collapse of the cavity due to the Casimir Force acting between the plates, the plates will have equal electric charges placed upon them to establish

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adequate Coulomb repulsion.⁹ Equation (8) shows that a 1 km radius throat will require a cavity plate separation of 1.28×10^{-16} m (smaller than a nuclear diameter), which gives $\rho_{EC} = -1.62 \times 10^{36}$ J/m³ for this configuration. In contrast, a wormhole with a throat radius of 1 AU will require a plate separation of 1.57×10^{-12} m (or 35% smaller than the electron's Compton wavelength), which results in an energy density of -7.14×10^{19} J/m³.¹⁰ There is no technology known today that can engineer a cavity with such minuscule plate separations. In addition, such minuscule plate separations are unrealistic because the Casimir effect switches over to the non-retarded field behavior ($\sim d^{-3}$) of van der Waals forces when plate separations go below the wavelength (≈ 10 nm) where they are no longer perfectly conducting (Reference 58). This scheme will not be considered any further. However, future work will be necessary to elucidate whether the various quantum field analogs of the Casimir effect can provide a more reasonable technical solution to this problem.

IV. Constructing a Traversable Wormhole is not Easy

A. NEGATIVE ENERGY REQUIREMENTS AND ENERGY CONDITION VIOLATIONS

One knows how to make small quantities of negative energy in the lab. But one does not know if it is possible to make large quantities of negative energy. It was pointed out in Section III that one, some, or all of the classical energy conditions must be violated in order to build a traversable wormhole. And it was also cautioned that this was not a showstopper because the energy conditions have all been violated by nature or by lab experiment prior to their formulation. However, the reader should be forewarned that there are a number of published claims that the energy condition violations can be avoided. These claims are just semantic games whereby investigators universally invoke the following scenario: divide the total stress-energy into weird matter plus normal matter, push all the energy condition violations into the weird matter so that the normal matter does not violate the energy conditions. Given that the energy conditions are not absolute, such rearranging approaches are not necessary.

Traversable wormhole throats violate the NEC (or ANEC). So how big a violation is required? The answer is that there is only need to calculate the amount of negative energy that will be needed to generate and hold open a wormhole throat. A simple formula for short-throat wormholes using the thin shell formalism gives this quantity in terms of the equivalent mass (note: the energy density derived from the general relativistic field equation is too complex to use for this mass comparison) (Reference 3):

⁹ In a detailed analysis the electrostatic energy required to support the Coulomb repulsion between the plates would be considered separately.

¹⁰ Mean Earth-Sun distance, 1 AU = 1.50×10^{11} m.

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$$\begin{aligned}
 M_{\text{wh}} &= -\frac{\frac{1}{\lambda_{\text{throat}}} c^2}{G} \\
 &= -(1.35 \times 10^{27} \text{ kg}) \frac{\frac{1}{\lambda_{\text{throat}}}}{1 \text{ meter}} \\
 &= -(0.71 M_J) \frac{\frac{1}{\lambda_{\text{throat}}}}{1 \text{ meter}}
 \end{aligned} \tag{9}$$

where M_{wh} is the (equivalent) mass required to build the wormhole, λ_{throat} is a suitable measure of the linear dimension (width or diameter) of the throat, and M_J is the mass of the planet Jupiter. One can also obtain the required energy, E_{wh} , by multiplying both sides of Equation (9) by c^2 . Equation (9) shows that a mass of $-0.71 M_J$ will be required to build a wormhole 1-m in size. As the wormhole size increases, the mass requirement grows negative-large. Table 2 presents a tabulation of the required negative (equivalent) mass as a function of sample wormhole throat sizes. After being alarmed by the magnitude of the results, one should note that M_{wh} is not the total mass of the wormhole as seen by remote observers. The non-linearity of the general relativistic field equation dictates that the total mass is zero (actually, the total net mass being positive, negative or zero in the Newtonian approximation depending on the details of the negative energy configuration constituting the wormhole system). Finally, Visser et al. (Reference 59) demonstrated the existence of spacetime geometries containing traversable wormholes that are supported by arbitrarily small quantities of negative energy, and this was proved to be a general result. The next section will expand on this further.

Table 2. Negative Equivalent Mass Required for Traversable Wormhole

$\lambda_{\text{throat}} (\text{m})$	M_{wh}
1000	$-709.9 M_J$
100	$-71 M_J$
10	$-7.1 M_J$
1	$-0.71 M_J$
0.1	$-22.6 M_{\oplus}$
0.01	$-2.3 M_{\oplus}$

$$M_J = 1.90 \times 10^{27} \text{ kg}, M_{\oplus} = 5.98 \times 10^{24} \text{ kg}$$

B. PHYSICAL CONSTRAINTS ON NEGATIVE ENERGY

The Quantum Inequalities (QI) conjecture is an extension of the Heisenberg Uncertainty Principle to curved spacetimes. Much research has been conducted around this one topic alone. The literature is too numerous to cite here but the reader should consult (Reference 10) and (Reference 46) for detailed information. The QI conjecture relates (via model dependent time integrals of the energy density along geodesics) the energy density of a free quantum field and the time during which this energy density is observed. This conjecture was devised as an attempt to quantify the amount of

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negative energy or energy condition violations required to build a traversable wormhole spacetime. Investigators have invoked the QI to rule out many of the macroscopic wormhole spacetimes. When generating negative energy the QI postulate that: a) the longer the pulse of negative energy lasts, the weaker it must be; b) a pulse of positive energy must follow and the magnitude of the positive pulse must exceed that of the initial negative pulse; and c) the longer the time interval between the two pulses, the larger the positive pulse must be. This actually sounds quite reasonable on energy conservation grounds until one discovers that the Casimir effect and its non-Maxwellian quantum field analogs violate all three conditions. There are also a number of squeezed vacuum sources and Dirac field states that manifestly violate all three conditions. Cosmological inflation, cosmological particle production, classical scalar fields, the conformal anomaly, and gravitational vacuum polarization are among the many other examples that also violate the QI. Visser (Reference 60) also points out that observational data indicate that large amounts of "exotic matter" are required to exist in the universe in order to account for the observed cosmological evolution parameters. The QI have also not been verified by laboratory experiments. The assumptions used to derive the QI and the efficacy of their derivation for various cases has been called into question by numerous investigators. Krasnikov (Reference 61) constructed an explicit counterexample for generalized FTL spacetimes showing that the relevant QI breaks down even in the simplest FTL cases. And he also addressed Fewster's (Reference 62) technical arguments on this issue. It is important to point out that the QIs have been mainly proven for free massless scalar fields in flat two-dimensional Minkowski spacetime, so there remains the unanswered questions of extending the QI into a four-dimensional curved spacetime model (with or without boundaries) and how much negative energy density can arise for interacting fields.

It turns out that Visser and coworkers (Reference 59, 63, 64) developed a superior way to properly quantify the amount of negative energy or energy condition violations required to build a traversable wormhole spacetime. They propose a quantifier in terms of a spatial volume integral, which amounts to calculating the following definite integrals (Reference 59, 63, 64):

$$\int \rho_E dV \leq 0; \quad \int (\rho_E + p_i) dV \leq 0 \quad (10)$$

with an appropriate choice of the integration measure dV ($= 4\pi r^2 dr$ or $g^{1/2} dr d\theta d\phi$, where $g = \det(g_{\mu\nu})$ is the matrix determinant of $g_{\mu\nu}$). The amount of energy condition violation is defined as the extent to which Equation (10) can become negative. The value of Equation (10) provides information about the total amount of energy condition violating matter that must exist for any given FTL spacetime under study (e.g., warp drives and traversable wormholes). It was further shown that Equation (10) can be adjusted to become vanishingly small by appropriate choice of parameters; therefore, examples can be constructed whereby the energy condition violation can be made arbitrarily small. But the violation cannot be made to vanish entirely.

Equation (10) also gives the result that traversable wormholes require arbitrarily small amounts of negative energy to build (whereby Equation (9) serves only as a gross upper limit) such that within a wormhole spacetime (Reference 59):

$$\rho_E = 0; \quad \int_C p_r dV \rightarrow 0 \quad (11)$$

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where p_r is the outward radial pressure required to hold a wormhole throat open. The Gauss-Bonnet Theorem (discussed in Section II-A) predicted this result beforehand. Equation (11) is a result that is also due to the intrinsic nonlinearity of the general relativistic field equation. This nonlinearity also impacts the coupling of a finite spaceship mass with each side of a wormhole's throat (or the mouth on each side of the throat) leading to a specialized mass conservation law for the combined system of spacecraft and wormhole: when finite mass spaceships traverse a wormhole they alter the (equivalent) mass of the wormhole mouths they pass through (Reference 3). The entrance mouth absorbing the spacecraft gains (equivalent) mass while the exit mouth emitting it loses (equivalent) mass.¹¹ (This mass coupling and conservation law takes into account the possibility that spaceships traversing the wormhole may lose or gain some momentum and kinetic energy in the process, and it is assumed that the two mouths are sufficiently far apart that their mutual gravitational interaction is negligible.) This unusual result suggests, but does not prove, the possibility of a fundamental limit on the total mass that can traverse a wormhole. The coupled mass conservation law shows that for a sufficiently large net transfer of mass the final (equivalent) mass of the exit mouth becomes negative. This is actually a beneficial result because ANEC violations are required just to hold the wormhole throat open in the first place. If it appears that a runaway reaction might occur, then it would be prudent for wormhole engineers to simply "turn off" the wormhole for a brief moment and then "turn it back on" (i.e., "reset" the wormhole) to restart space transportation operations.

It is on the basis of the foregoing discussion that traversable wormholes appear to be the most viable form of FTL transport. However, one still does not know how to construct a traversable wormhole because general relativity theory only provides a recipe for the essential geometric and material ingredients required to open and maintain one, but not the required assembly instructions. Will one need to pull a traversable wormhole out of the quantum spacetime foam and enlarge it to macroscopic scale or will there be need to use extremely large spacetime curvatures to "punch a hole" through space? Or are there construction techniques yet to be identified? The author is convinced that the answer can only be found through empirical studies designed to decide whether the present general relativistic recipe is enough to work with or an additional construction mechanism will be required.

On physical grounds Equation (10) appears to be the correct negative energy/energy condition violation quantifier. However, further work is needed to establish whether Equation (10) is the correct quantifier to use overall and whether all (averaged) energy condition theorems can be extended to include it.

On another note, Börde et al. (Reference 65) have recast the QI conjecture into a new program which seeks to study the allowed spatial distributions of negative energy density in quantum field theory. Their study models free massless scalar fields in flat two-dimensional Minkowski spacetime. Several explicit examples of spacetime averaged QI were studied to allow or rule out some particular model (spatial) distributions of negative energy. Their analysis showed that some geometric configurations of negative energy can either be ruled out or else constrained by the QI restrictions placed upon

¹¹ Similar coupling and conservation results hold for the case of electrically charged matter that traverse a (charged or uncharged) wormhole.

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the allowable spatial distributions of negative energy. And there were found to be allowable negative energy distributions in which observers would never encounter the accompanying positive energy distribution so long as the QI restrictions and corresponding energy conditions are violated. The extent to which the results of Börde et al.'s analysis can be generalized to a four-dimensional curved spacetime (with or without boundaries) and interacting fields remain unsolved.

C. OBSERVING NEGATIVE ENERGY IN THE LAB

Negative energy should be observable in lab experiments. The presence of naturally occurring negative energy regions in space is predicted to produce a unique signature corresponding to lensing, chromaticity and intensity effects in micro- and macro-lensing events on galactic and extragalactic/cosmological scales (Reference 66-71). It has been shown that these effects provide a specific signature that allows for discrimination between ordinary (positive energy) and negative energy lenses via the spectral analysis of astronomical lensing events. Theoretical modeling of negative energy lensing effects has led to intense astronomical searches for naturally occurring traversable wormholes in the universe. Computer model simulations and comparison of their results with recent satellite observations of gamma ray bursts (GRBs) has shown that putative negative energy (i.e., traversable wormhole) lensing events very closely resemble the main features of some GRBs. Other research has found that current observational data suggests that large amounts of naturally occurring "exotic matter" must have existed sometime between the epoch of galaxy formation and the present in order to (properly) quantitatively account for the "age-of-the-oldest-stars-in-the-galactic halo" problem and the cosmological evolution parameters (Reference 60).

When background light rays strike a negative energy lensing region, they are swept out of the central region thus creating an umbra region of zero intensity. At the edges of the umbra the rays accumulate and create a rainbow-like caustic with enhanced light intensity. The lensing of a negative energy region is not analogous to a diverging lens because in certain circumstances it can produce more light enhancement than does the lensing of an equivalent positive energy region. Real background sources in lensing events can have non-uniform brightness distributions on their surfaces and a dependency of their emission with the observing frequency. These complications can result in chromaticity effects, i.e., in spectral changes induced by differential lensing during the event. The quantification of such effects is quite lengthy, somewhat model dependent, and with recent application only to astronomical lensing events. Suffice it to say that future work is necessary to scale down the predicted lensing parameters and characterize their effects for lab experiments in which the negative energy will not be of astronomical magnitude. Present ultrahigh-speed optics and optical cavities, lasers, photonic crystal (and related switching) technology, sensitive nano-sensor technology, and other techniques are very likely capable of detecting the very small magnitude lensing effects expected in lab experiments.

A non-optical scheme for detecting negative energy in experiments was recently reported by Davies and Ottewill (Reference 72) who studied the response of switched particle detectors to static negative energy densities and negative energy fluxes. Their model is based on a free (massless) scalar field in flat four-dimensional Minkowski spacetime and utilized a simple generalization of the standard monopole detector, which is switched on and off to concentrate the measurements on periods of isolated negative energy density (or negative energy flux). The detector model includes an

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explicit switching factor whereby five different switching functions (based on data windowing theory) are defined and evaluated. In order to isolate the effects of negative energy a comparison is made for the response of a detector switched on and off during a period of negative energy density (or negative energy flux) and that switched on and off in the vacuum. The results shed light on the response of matter (detectors) to pulses of negative energy of finite duration, and they showed that negative energy should have the effect of enhancing deexcitation (i.e., induce cooling) of the detector. This is the opposite of our experience with detectors that undergo excitation when encountering “normal” matter or energy, and isolated detectors placed in a vacuum naturally cool due to the usual thermodynamic reasons. But Davies and Ottewill point out that the enhanced cooling effect they discovered cannot be used to draw a thermodynamic conclusion because their modeling was restricted to first order in perturbation theory. It is not possible at first order to determine whether the enhanced cooling effects are due to the small violation of energy conservation expected in any process in which a general quantum state collapses to an energy eigenstate, or whether they predict a systematic reduction in the energy of the detector which has serious thermodynamic implications. However, Davies and Ottewill point out that their results are model dependent and they found for their standard monopole detector model that there is not always a simple relationship between the strength of the negative energy density/flux and the behavior of the detector. Further research will be necessary to resolve these issues.

V. Conclusion: The Way Forward

More than 40 years elapsed between the late 1890s when the Curies first identified radioactive substances in their laboratory and when a neutron-catalyzed fission chain reaction – the world’s first nuclear reactor – was demonstrated at the University of Chicago in 1939 by Enrico Fermi and Leo Szilard. Six more years would pass before the world’s first nuclear bomb was successfully tested in New Mexico. The progress of science and technology is rapid, but highly dependent on adequate and sustained focus, effort, and support. On this basis, it is possible that a traversable wormhole can be demonstrated in the laboratory as long as there is a focused, sustained level of long-term research support.

A game changer may appear that could dramatically accelerate or alter the direction of an experimental traversable wormhole program. Such a game changer could entail new physics that is predicted by a complete, comprehensive quantum gravity theory, or a quantum gravity theory that is a subset of a larger unified field theory (i.e., a finalized quantum superstring theory, or some other theory that replaces it), or a completely new theory for the quantum vacuum and its related spacetime physics (e.g., “emergent” spacetime/gravity theories (Reference 73, 74)). The new field of “emergent” spacetime/gravity suggests that gravitation is not a fundamental force of nature because, among many other considerations, of its extreme weakness relative to the other forces of nature. Instead, spacetime and gravitation are seen as emergent low-energy phenomenon, which arises from the collective action of much higher-energy phenomenon occurring in the quantum vacuum where Lorentz invariance and energy conservation may be violated in the trans-Planckian regime. One now knows empirically that the “emergent” low-energy vacuum within which one exists is in fact a rich quantum ether comprised of zero-point fluctuation fields that make it behave like a nonlinear optical medium endowed with paramagnetic, dichroic, birefringent, condensed

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matter, and many other fascinating properties (Reference 22-24, 26, 73-76). Therefore, if the emergent spacetime/gravity approach turns out to be correct, then there will likely be a direct consequence to the physics of traversable wormholes that could dramatically alter the mechanism by which they are created and/or mitigate the requirement for negative energy.

Until such new approaches are established and testable predictions published by their proponents, one cannot speculate on how the physics of traversable wormholes will be affected. Therefore, it is beneficial to stick to the outcome of the present study in terms of quantum field theory and general relativity theory, and outline what needs to be accomplished going forward in order to demonstrate a traversable wormhole in the lab.

Going forward toward the demonstration of a traversable wormhole will require the following:

- Generating Negative Energy in the Lab: Our assessment concludes that we already make small amounts of negative energy in the lab, but we do not yet know if we can access larger amounts for extended periods of time over extended spatial distributions for the purpose of engineering a traversable wormhole. In this regard we propose the following options for further exploration.
- Squeezed quantum vacuum generators: A dedicated research program to develop the two negative energy generator concepts described in Section III-B-2 will need to be established in order to evolve state-of-the-art quantum optics technology towards producing higher magnitudes of negative energy as well as special techniques required to separate out any positive energy fluxes that accompany the negative energy fluxes. Specifically, the Rabeau et al. (Reference 48, 49) and Ries et al. (Reference 50) experimental programs should be followed as a template toward this goal. Quantum optics technology via high power fiber lasers, resonators, amplifier stages, beam conditioning stages, etc., are rapidly advancing. So research should be conducted in parallel to invent additional ways to produce negative energy via innovative quantum optics.
- Casimir effect: Even though the standard electromagnetic Casimir effect is feeble, and thus not likely to contribute to a traversable wormhole engineering program, there are still a number of other electromagnetic and non-electromagnetic Casimir effects described in Section III-B-4 that require further study. These other Casimir effects have not been explored with an eye toward testing them in the lab, and so there could be important new information yet to be uncovered.
- Moving Mirrors (a.k.a. the dynamical Casimir effect): Even though this concept was identified (Section III-B-5) as being too feeble to produce any useful flux of negative energy, the observable effects due to the change in the boundary conditions (e.g., moving mirrors/cavity walls) of quantum fields provide crucial information on the quantum vacuum at the macroscopic level. Theoretical and laboratory efforts are underway to understand the dissipative effects of vacuum fluctuations (Reference 77-78). This dissipation mechanism should induce irradiation of photons, a phenomenon also known as the dynamical Casimir effect. This can be understood both as the creation of particles under non-adiabatic changes in the boundary conditions of quantum fields, or as classical parametric amplification with the zero-point energy of a vacuum field mode as an input state. More recent developments

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include models for the super-radiant amplification of photons with particular emphasis on its dynamics and the optimization of the involved parameters. Experimental concepts being pursued will try to reveal directly the presence of a non-empty vacuum by using a specifically designed device to amplify the virtual vacuum photons and produce real electromagnetic radiation via the parametric amplification of the vacuum fluctuations in an electromagnetic cavity. The 'amplifier' is a boundary undergoing an oscillation, and hence radiates energy due to the dissipative action against the vacuum photons. This line of investigation could serve as a very useful probe to explore the possibility of generating large fluxes of negative energy. It may be expected that a laboratory demonstration of the dynamical Casimir effect will occur before 2012.

- Dirac field states: As described in Section III-A, this involves either the superposition of two single particle electron states or the superposition of two multi-electron-positron states (Reference 29, 30). This is still a nascent topic of study in quantum field theory. However, mankind already possesses a great deal of technology that is dedicated to the manipulation and storage of electrons and positrons via solid state/condensed matter devices and particle accelerators. This research topic should be supported in order to establish how it could contribute to an experimental traversable wormhole program.
- Quantum coherence effects: Other types of quantum coherence effects not already identified or invented should be theoretically developed and explored for the possibility of finding new free-field or interacting field configurations that produce a significant magnitude of negative energy which could be produced by technological means.
- Detecting Negative Energy in the Lab: In Section IV-C this paper identified proposals for observing negative energy in outer space and in the laboratory, but further work is needed to downscale astronomical techniques for use at the lab scale, and we need to firm up our understanding of how lab detectors will respond to negative energy in situ. A first step in the latter direction was recently proposed by Marecki (Reference 79) who generalized the analysis of the output of balanced homodyne detectors (BHDs). The most important feature of these devices is their ability to quantify the quantum vacuum fluctuations of the electric field because the output of BHDs provides information on the one- and two-point functions of arbitrary states of quantum fields. Marecki computed the two-point function and the associated spectral density for the ground state of the quantum electric field in Casimir geometries, and predicts a position- and frequency-dependent pattern of BHD responses if a device of this type is placed inside a Casimir cavity. The proposed device allows for the direct detection of quantum vacuum fluctuations and provides a spatial mapping of the negative energy contained inside the cavity. This offers a potential new characterization of ground states in Casimir geometries, which would provide an understanding of the negative energy densities present in some regions in these geometries.
- Trapping and Storing Negative Energy: Ford and Roman (Reference 10) have only superficially addressed this topic, and there is very little technical literature that addresses it fully. A theoretical program to develop the physics and technology of trapping and storing negative energy will need to be supported, and such a program should be guided by the use of laboratory detectors such as the one proposed in the

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previous section. However, it is the opinion of the author that free-space negative energy sources appear to be a more desirable option for building traversable wormholes than stored negative energy.

- Constructing Traversable Wormholes in the Lab: Einstein's General Theory of Relativity does not provide instructions on how to construct a traversable wormhole in space or inside a laboratory vacuum vessel. The Einstein general relativistic field equation only provides a prescription for designing a special, localized spacetime geometry and calculating the physical characteristics of a source of matter that is required to induce it. If one "zaps" a region of empty space with a beam of negative energy, will a traversable wormhole appear? One doesn't know. Maybe one has to poke a hole in space with an intense beam of negative energy, or maybe we have to use the negative energy to inflate a quantum spacetime fluctuation (allegedly in the form of a "geometric foam"). Theoretical studies need to be implemented to address this question and the author believes that empirical studies will be necessary to find the answer once we develop an intense source of negative energy.

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