exercises (1) · Show $\mathrm{Gpd}_1^{str} \hookrightarrow \mathrm{Cat}_1^{str} \xhookrightarrow[N]{} \mathrm{sSet}$ fully faithful, (or at least you can reconstruct a category from its nerve)

· Show the horn-filling characterization of the essential image

(2) If $\mathcal{C} \in \mathrm{Kan}\text{-}\mathrm{Cat}$, show that

$$\begin{tikzcd} \Lambda^3_1 \arrow[r] \arrow[d, hook] & N_{hc}(\mathcal{C}) \\ \Delta^3 \arrow[ur, dashed, "\exists"'] \end{tikzcd}$$

(this will indicate the proof for the general case)

(3) From the definitions, check left adjoints preserve colim
right adjoints preserve lim

(4) prove the following formula if I didn't.

Prop $\mathcal{C} \in \mathrm{Cat}_\infty$.

(i) $F: \mathcal{C} \to \mathrm{Ani} \rightsquigarrow \operatorname{colim} F = |\int F|$, $\lim F \simeq \mathrm{Map}_{/\mathcal{C}}(\mathcal{C}, \int F)$ $(\simeq \mathrm{Fun}_{/\mathcal{C}}(\mathcal{C}, \int F))$

(ii) $F: \mathcal{C} \to \mathrm{Cat} \rightsquigarrow \operatorname{colim} F = (\int F)[\mathrm{cocart}^{-1}]$, $\underbrace{\lim F \simeq \mathrm{Fun}^{\mathrm{cocart}}_{/\mathcal{C}}(\mathcal{C}, \int F)}_{\text{cocart sections}}$

$\rightsquigarrow \begin{array}{c} \int F \\ \downarrow \text{cocart} \\ \mathcal{C} \end{array}$

Rmk (ii) specializes to (i) because $\forall$ mor in left fib is cocart.

(5) Construct the functor $\mathrm{Map}_{\mathcal{C}}(-,-): \mathcal{C}^{op} \times \mathcal{C} \to \mathrm{Ani}$ if I didn't.
(or $よ: \mathcal{C} \to \mathcal{P}(\mathcal{C}) := \mathrm{Fun}(\mathcal{C}^{op}, \mathrm{Ani})$)

prove $よ$ is limit-preserving.

(6) Assume the Yoneda lemma that $よ$: fully faithful.

· Show that if $f: \mathcal{C} \to \mathcal{D}$ has a local left / right adjoint for $\forall d \in \mathcal{D}$, they assemble into a functor $f^L, f^R: \mathcal{D} \to \mathcal{C}$.
and moreover show that these are uniquely determined from f.

· Show the equivalence to another definition of adj

(7) Suppose $\operatorname{colim}_{\lambda \in \Lambda} \mathcal{C}_\lambda \xrightarrow[i_\lambda]{\sim} \mathcal{C}$ and $\mathcal{C} \xrightarrow{f} \mathcal{D}$ (or assume $\mathcal{D} = \mathrm{Ani}$)

Prove · $\lim_{\mathcal{C}} f \xrightarrow{\sim} \lim_{\lambda} \lim_{\mathcal{C}_\lambda} f \circ i_\lambda$

· $\operatorname{colim}_{\lambda} \operatorname{colim}_{\mathcal{C}_\lambda} f \circ i_\lambda \longrightarrow \operatorname{colim}_{\mathcal{C}} f$. (for this sub-ex : localization is cofinal)