exercises

(1) Use the formula of (co)limits in Ani to show that $\mathfrak{L}: \mathcal{C} \to P(\mathcal{C})$ is limit-preserving

(2) For $(\mathcal{C}, W) \in RelCat_\infty$, show $\mathcal{C} \to \mathcal{C}[W^{-1}]$ is cofinal & coinitial.

(3) Suppose $\operatorname{colim} \mathcal{C}_\lambda \xrightarrow[i_\lambda]{\sim} \mathcal{C}$ and $\mathcal{C} \xrightarrow{f} \mathcal{D}$ (or assume $\mathcal{D} = Ani$)
$\qquad \lambda \in \Omega$

Prove · $\varprojlim_{\mathcal{C}} f \xrightarrow{\sim} \varprojlim_\lambda \varprojlim_{\mathcal{C}_\lambda} f \circ i_\lambda$

$\qquad$ · $\operatorname*{colim}_\lambda \operatorname*{colim}_{\mathcal{C}_\lambda} f \circ i_\lambda \longrightarrow \operatorname*{colim}_{\mathcal{C}} f$, (for this sub-ex : localization is cofinal)

(4) Show $Fun^R(\mathcal{C}, \mathcal{D})^{op} \simeq Fun^L(\mathcal{D}, \mathcal{C})$

$\qquad$ (possibly hard.) $\xrightarrow{\sim} Cat^{Cart \& cocart}_{/\Delta^1} \times_{Cat_{/\partial \Delta^1}} * \mathcal{L}(\mathcal{D}, \mathcal{C})$

(5) Show a right-adjoint is cofinal (& left adjoint is coinitial)

(6) Show $\Delta^{inj} \hookrightarrow \Delta$ is coinitial.
$\qquad$ $\llcorner$ mor: $[n] \underset{inj}{\to} [m]$

(7) Show $\mathcal{C}: K$-filtered $\iff$ diag: $\mathcal{C} \to Fun(I, \mathcal{C})$ is cofinal for any K-small I.

(8) Show $\Delta \xrightarrow{\delta} \Delta \times \Delta$ is cofinal. $\left(\begin{array}{l} \text{if you like.} \\ \text{you may use the fact that } \Delta \hookrightarrow Cat \text{ is dense} \end{array} \right)$

(9) Consider $\mathcal{C} \xrightarrow{f} \mathcal{D}$ fully faithful, where $\mathcal{D}$ has K-filt colim, $\mathcal{C}$ small
$\qquad \rightsquigarrow \underset{Ind_K(\mathcal{C})}{\searrow} \nearrow_{\bar{f}}$
$\qquad$ So that $Ind_K(\mathcal{C}) = \left\{ \begin{array}{l} \tilde{\mathcal{C}} \\ \downarrow \text{ rfib. } \tilde{\mathcal{C}}: \text{filtered} \\ \mathcal{C} \end{array} \right\}$
$\qquad\qquad \subset P(\mathcal{C}) \simeq Cat^{rfib}_{/\mathcal{C}}$

$\qquad$ (1) if $\mathcal{C} \subset \mathcal{D}^K$, $\bar{f}$ is fully faithful
$\qquad$ (2) if moreover $\mathcal{C}$ generates $\mathcal{D}$ under K-filt colim (i.e. the closure of $\mathcal{C}$ is $\mathcal{D}$) then $\bar{f}$: equiv.
$\qquad\qquad\qquad\qquad\qquad X \cdot \operatorname{colim} dx$;

$\qquad$ (similar claims are true for $P(\mathcal{C})$, $P_\Sigma(\mathcal{C})$)

(10) Use (9) to recognize your favorite category as an Ind-completion