

# BUS 41204 Review Session 5

## Variable Selection

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# Plan

- ▶ Why variable selection
- ▶ Two main approaches
  - ▶ wrapper methods
  - ▶ filter methods
- ▶ Example using **recursive feature elimination** method

# Why Variable Selection

- ▶ Determining which predictors to include in a model is crucial
  - ▶ model is more interpretable with fewer predictors
  - ▶ some models negatively affected by non-informative/redundant predictors
  - ▶ some models such as regression trees and LASSO are naturally resistant to non-informative predictors

# Main Approaches For Reducing the Number of Predictors

- ▶ Wrapper methods
  - ▶ Evaluate multiple models using procedures that add and/or remove predictors to find the optimal combination that maximizes model performance
  - ▶ Pros: Selection criterion directed related to model effectiveness
  - ▶ Cons: computationally intensive; risk of over-fitting
- ▶ Filter methods
  - ▶ Evaluate the relevance of predictors outside of the predictive models and subsequently model only the predictors that pass some threshold criterion
  - ▶ Correlation matrix is helpful for detecting relevant predictors
  - ▶ Pros: computationally efficient
  - ▶ Cons: redundant predictors may be selected; interactions hard to quantify

# Wrapper Methods

- ▶ Classical forward selection for linear regression
  - ▶ Problems
    1. Greedy search algorithm, hence does not reevaluate past solutions
    2. Repeated hypothesis tests invalidates many statistical properties since the same data are being evaluated multiple times
    3. Maximizing statistical significant  $\neq$  predictive accuracy
- ▶ Stepwise selection
- ▶ Backward selection via recursive feature elimination (RFE)

# Naive Recursive Feature Elimination

## ► Algorithm

- 1 Tune/train the model on the training set using all  $P$  predictors
- 2 Calculate model performance
- 3 Calculate variable importance or rankings
- 4 **for** *each subset size*  $S_i$ ,  $i = 1 \dots S$  **do**
- 5     Keep the  $S_i$  most important variables
- 6     [Optional] Pre-process the data
- 7     Tune/train the model on the training set using  $S_i$  predictors
- 8     Calculate model performance
- 9     [Optional] Recalculate the rankings for each predictor
- 10 **end**
- 11 Calculate the performance profile over the  $S_i$
- 12 Determine the appropriate number of predictors (i.e. the  $S_i$  associated with the best performance)
- 13 Fit the final model based on the optimal  $S_i$

# Selection Bias

- ▶ The same idea as over-fitting. Feature selection is part of the model selection process!!
- ▶ The risk of over-fitting increases if
  1. The dataset is small
  2. The number of predictors is large
  3. The predictive model is powerful, which is more likely to overfit the data
  4. No independent test set is available
- ▶ Solution
  - ▶ For large data set, separate data sets for selecting features (training set), tuning models (validation set), and validating the final model (test set)
  - ▶ For small data set, use **cross-validation**

# Recursive Feature Elimination with Cross Validation

## ► Algorithm

```
1 for each resampling iteration do
2   Partition data into training and test/hold-back set via
   resampling
3   Tune/train the model on the training set using all  $P$  predictors
4   Calculate model performance
5   Calculate variable importance or rankings
6   for Each subset size  $S_i$ ,  $i = 1 \dots S$  do
7     Keep the  $S_i$  most important variables
8     [Optional] Pre-process the data
9     Tune/train the model on the training set using  $S_i$  predictors
10    Calculate model performance using the held-back samples
11    [Optional] Recalculate the rankings for each predictor
12  end
13 end
14 Calculate the performance profile over the  $S_i$  using the held-back
   samples
15 Determine the appropriate number of predictors
16 Determine the final ranks of each predictor
17 Fit the final model based on the optimal  $S_i$  using the original
   training set
```



# Hands-on Implementation

See the companion R file

- ▶ TA5\_code\_my.R: coding from scratch (to demonstrate what's inside the blackbox)
- ▶ TA5\_code\_caret.R: using caret package

# References

- ▶ RFE implementation in R: caret package

[https://topepo.github.io/caret/  
recursive-feature-elimination.html](https://topepo.github.io/caret/recursive-feature-elimination.html)

- ▶ RFE implementation in Python: sklearn.feature\_selection.RFECV module

[http:  
//scikit-learn.org/stable/modules/feature\\_selection.html](http://scikit-learn.org/stable/modules/feature_selection.html)

- ▶ Book on feature selection

Kuhn, Max, and Kjell Johnson. Applied predictive modeling. Vol. 26. New York: Springer, 2013