Portfolio Managment: Homework 3

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April 11, 2017

A.1. Questions on Treasury Inflation-Protected Securities (TIPS).

a)

Regular bonds pay a fixed coupon throughout their life and return their pre-established face value at maturity. TIPS adjust to inflation – the principal value is pinned to the CPI, and coupon amounts are calculated based on their pre-established interest rate and the adjusted principal value.

b)

If we expect inflation to rise we can short treasuries and use the proceeds to purchase TIPS, profiting on the difference between the treasury rate and the TIPS rate on the inflation-adjusted principal amount. That is, TIPS are adjusted for the level of inflation and thus the spread between normal Treasuries and TIPS should diverge based upon the expected level of inflation, as measured by CPI.

A.2. Questions on HMC's portfolio.

a)

HMC considered three return characteristics when developing their capital market assumptions: expected future real returns, volatility of real returns, and correlation between real returns from each asset class with real returns with all other asset classes. When placed into a mean-variance framework, the optimal allocation was determined by modern portfolio theory, mean-variance analysis, (return vs variance, given covariance).

Naturally, HMC used twenty-year historical data to determine optimal weight allocation. To the extent to which they didn't have data (in the case of TIPS), they used as much historical data as was currently available.

Traditionally, HMC viewed the U.S. equity premium favorably, allocating 32% of their portfolio into U.S. equities. However, based upon expected future returns, variances, and covariances, HMC anticipated that the U.S. equity premium to be less profitable (in terms of its risk/return/covariance structure) in the future. As such, the firm reduced its U.S. equity exposure by 10% in its new TIPS incorporated portfolio.

b)

Although HMC's rationale for allocating funds to TIPS seemed sound we decided to run our own mean variance optimization analysis. We copied the data tables to excel and computed a portfolio that keeps all asset classes inside industry lows and highs range (as provided by exhibit 7) and minimizes the standard deviation for a return similar to what HMC's management is offering: 6.44%. We were able to construct a portfolio which has a lower standard deviation of 8.5% and which yields the same return of %6.44. Those generate a Sharpe ratio of 0.76. For returns and correlations, we used the assumed values presented in the case. See the Figure 1 on next page.

	weight	Return
Domestic equity	14.60%	7%
Foreign Equity	3.63%	7%
Emerging Markets	8.50%	9%
Private Equity	22.31%	10%
Absolute Return	0.00%	6%
High Yield	5.25%	6%
Commodities	5.70%	5%
Real Estate	15.20%	6%
Domestic Bonds	4.10%	4%
Foreign Bonds	10.10%	4%
Cash	10.00%	4%
TIPS	0.61%	4%
	100.00%	
Portfolio		
Return	**6.44%**	
SD	**8.50%**	
Sharpe	**0.76**	

Figure 1: Custom Portfolio

B.1. Relative performance of stocks and T-bills.

a)

Considering that an unskilled investor could choose a stock only strategy and be 'correct' (that is, select the higher return) 68.1% of the time, Claire's "skill" is unconvincing. Claire is 8% less skilled than an investor who simply chooses a stock only strategy.

```
length(which(data$stocks > data$tbills)) / NROW(data) * 100
```

```
[1] 68.1
```

b)

```
cumreturn <- (data + 1)
kable(t(apply(cumreturn, 2, prod)), digits = 6) # in dollars
```

tbills	stocks
20.5	4598

B.2. Perfect vs. random market timing.

a)

Naturally, the omniscient strategy performs better as it selects the highest return for each given year.

cumreturn_perfect	mean_perfect	sr_perfect	mean_market	sr_market
648122	0.166	0.964	0.117	0.405

b)

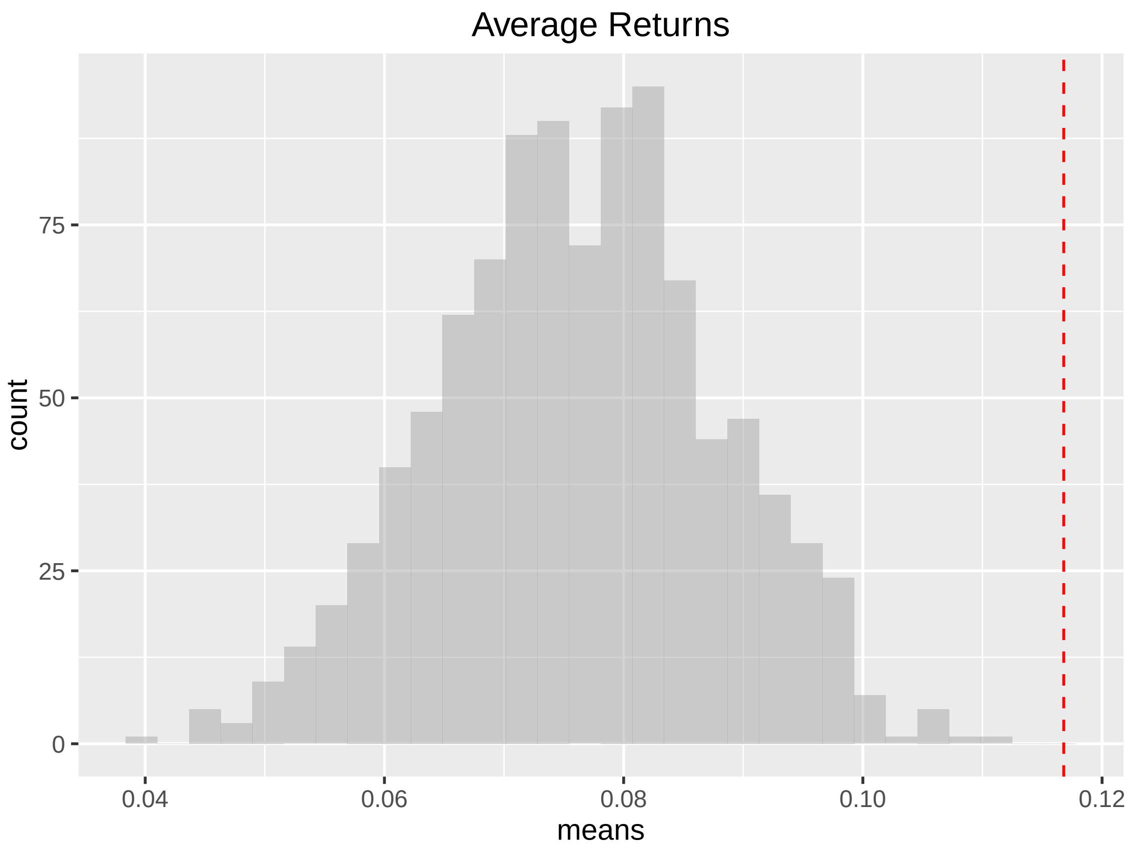
We write the function '`randomTiming`' that randomly selects between the treasury bill and market.

```
randomTiming <- function() as.matrix(apply(data, 1, function(x) sample(x, 1)))
# run 1000 simulations of random timing
simulations <- replicate(1000, randomTiming(), simplify = FALSE)

sim_means <- unlist(lapply(simulations, mean))
sim_sharperatios <- unlist(lapply(simulations,
                           function(x) mean(x - data$tbills) / sd(x - data$tbills)))
```

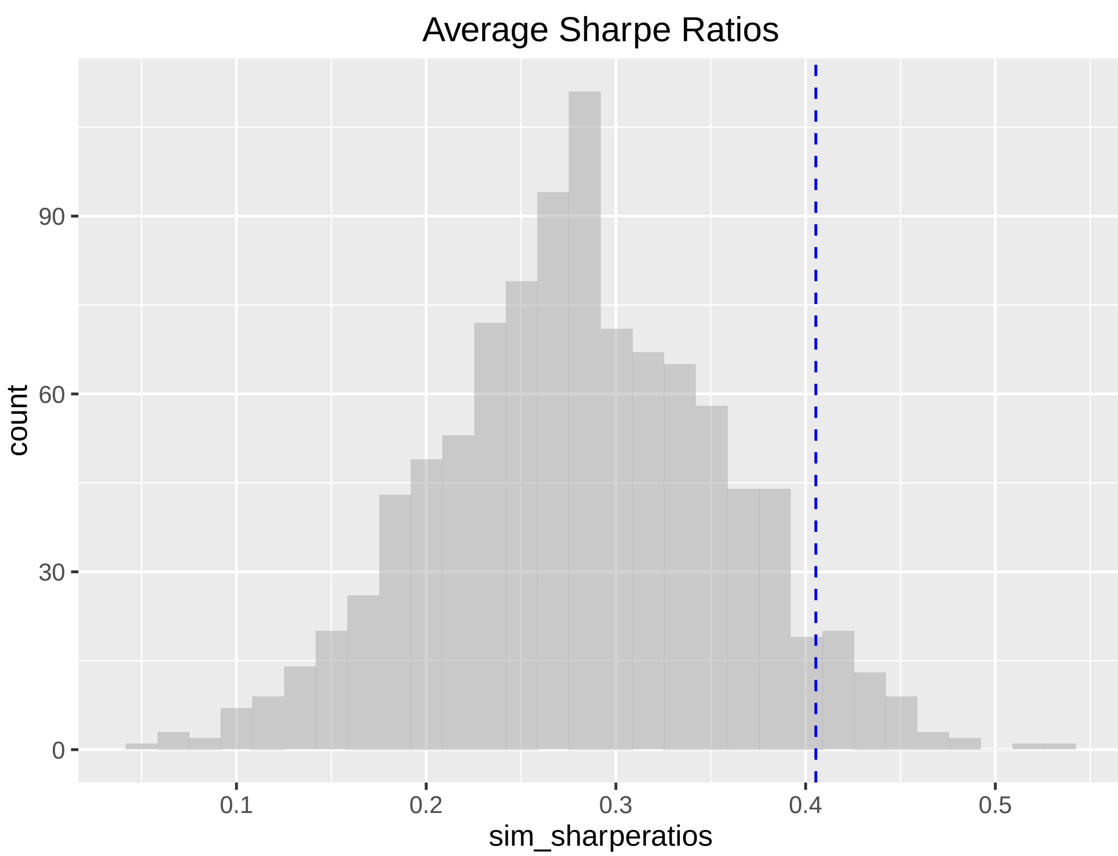
The expected return of Claire's random strategy is 7.62%. The average return of the stock only portfolio is shown in red.

```
[1] 0.0762
```



The expected Sharpe Ratio of Claire's random strategy is 0.28. The Sharpe Ratio of the stock only portfolio is shown in blue.

```
[1] 0.281
```



B.3. Benefits of imperfect market timing.

a)

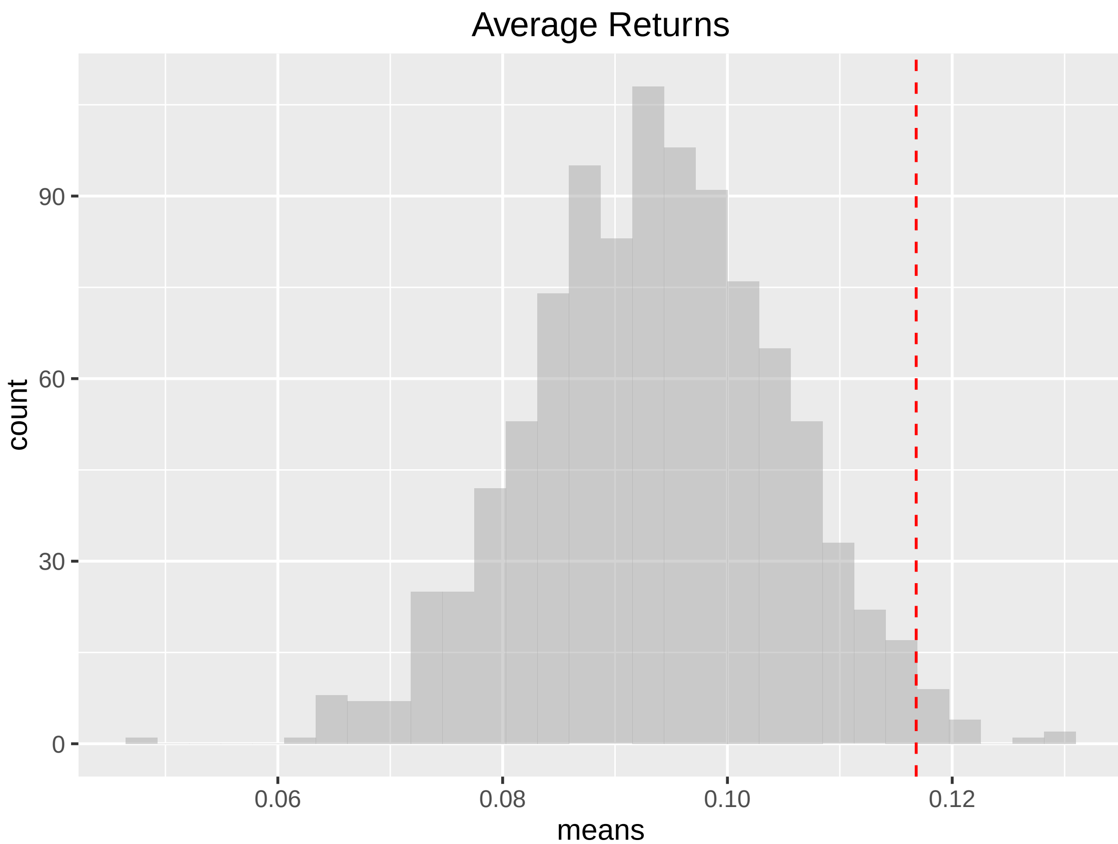
We write the function '`skillTiming`' that selects the 'correct' secruity 60% of the time.

```
skillTiming <- function() as.matrix(apply(data, 1,
               function(x) sample(c(max(x), min(x)),
                                  prob = c(0.60, 0.40), 1)))
              # get the 'correct', aka max 60% of time
simulations <- replicate(1000, skillTiming(), simplify = FALSE)
# run 1000 simulations of random timing

sim_means <- unlist(lapply(simulations, mean))
sim_sharperatios <- unlist(lapply(simulations,
                           function(x) mean(x - data$tbills) / sd(x - data$tbills)))
```

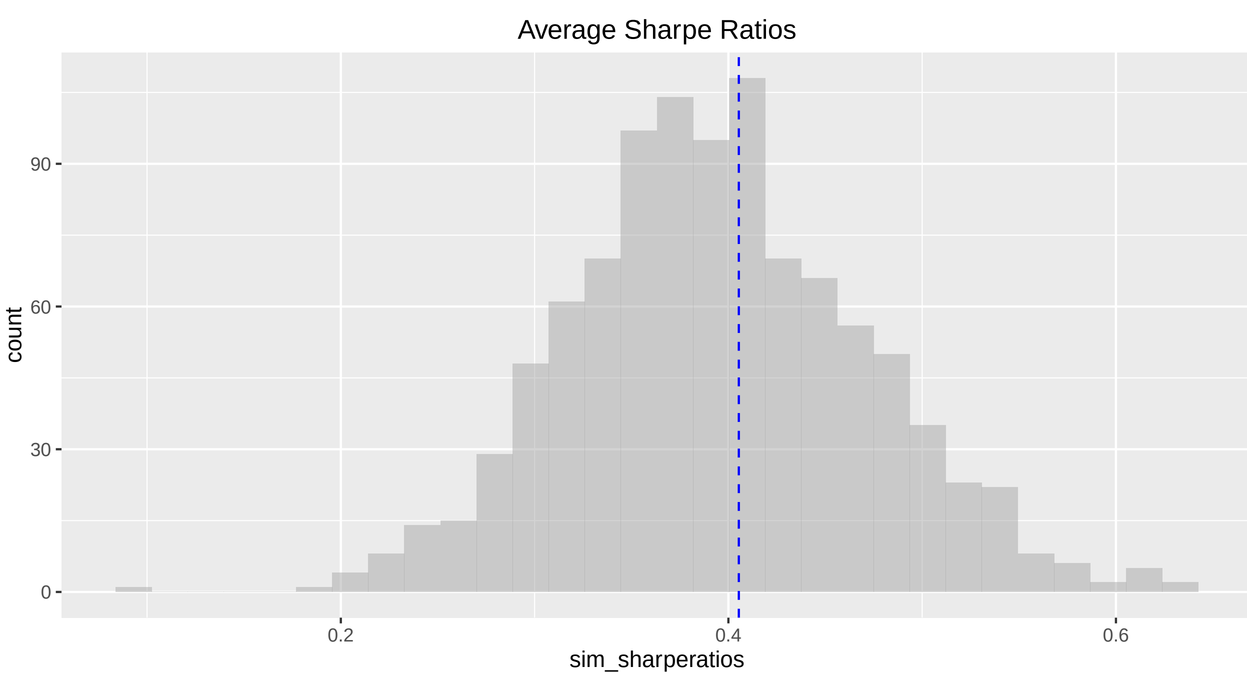
Claire's skill strategy yields an average return of 9.35%. Naturally, the skill strategy improves upon the random strategy but is still inferior to the market only portfolio.

```
[1] 0.0935
```



Equally, the 60% skill strategy yields an improved average Sharpe Ratio of 0.394.

```
[1] 0.394
```



b)

With 2% fees, Claire's skill strategy yields a 7.35% average return and 0.261 Sharpe Ratio. As this is no better than randomly selecting between stocks and treasury bills, and worse yet than selecting a market only portfolio, we would not hire Claire based upon this fee arrangement.

```
[1] 0.0735
```

```
[1] 0.261
```

B.4. Imperfect market timing with different forecasting accuracies.

We write the function '`skillTimingn`' that selects the 'correct' secruity n% of the time. Next, we simulate 1000 trails at an accuracy of [0.50 - 1.00], and parallize the operation over multiple cores.

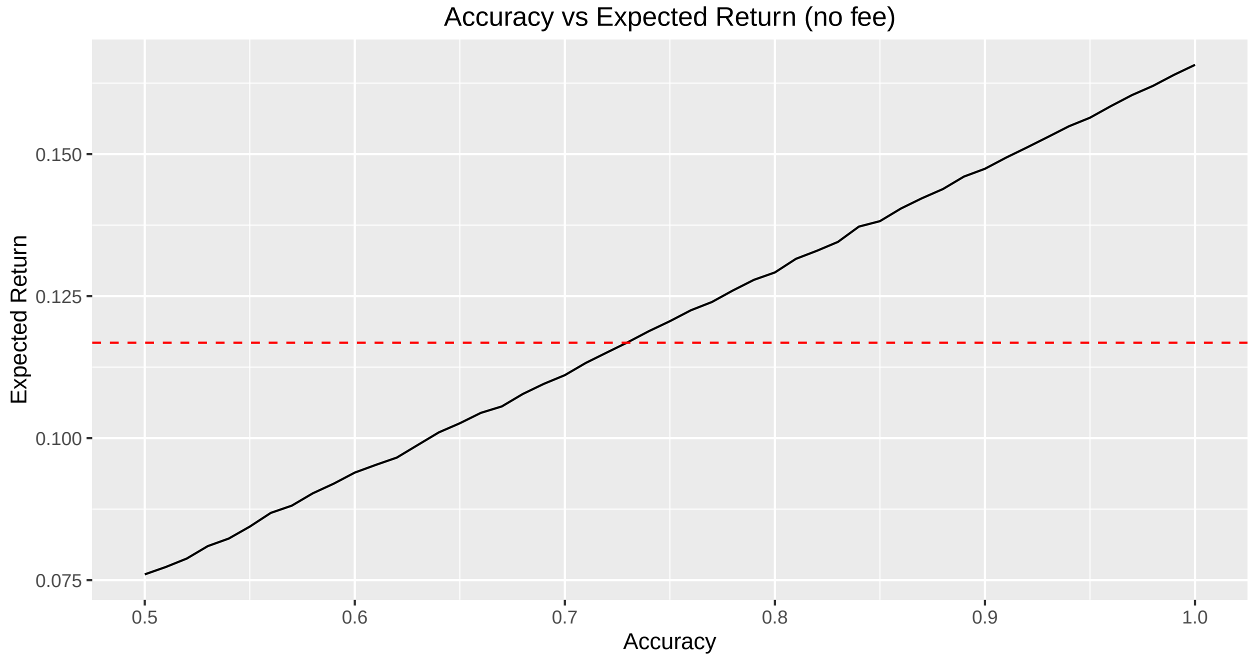
```
ns <- rep(50:100) / 100
skillTiming_n <- function(n) as.matrix(apply(data, 1,
                 function(x) sample(c(max(x), min(x)), prob = c(n, 1 - n), 1)))
# all(skillTiming_n(1.00) == perfect) # sanity check

registerDoMC(detectCores() - 1)
simulations <- foreach(i = ns) %dopar% {
  replicate(1000, skillTiming_n(i), simplify = FALSE)
  } # for each n, run a simulation

sim_means_ns <- unlist(lapply(simulations,
                      function(x) mean(unlist(lapply(x, mean)))))
sim_sharperatios <- unlist(lapply(simulations,
                      function(y) mean(unlist(lapply(y,
                      function(x) mean(x - data$tbills) / sd(x - data$tbills))))))
frame <- cbind.data.frame(ns, sim_means_ns, sim_sharperatios)
```

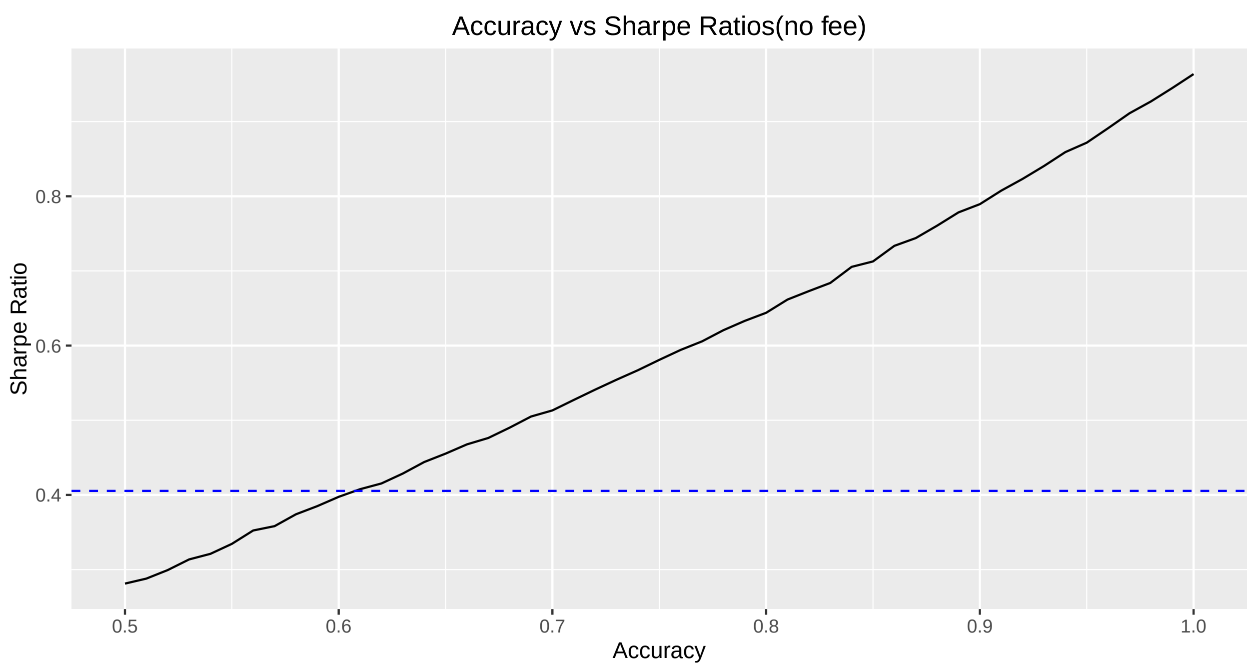
We plot the two cases: one without fees, and one with 2% management fees. To beat the stock-only strategy's expected return (11.68%), Claire requires a minimum accuracy of 73%. To exceed the market's Sharpe ratio (0.4054) requires an accuracy level of 61%. To beat the stock-only strategy's expected return (11.68%), Claire requires a minimum accuracy of 84%. To exceed the market's Sharpe ratio (0.4054) requires an accuracy level of 72%.

```
[1] 0.73
```



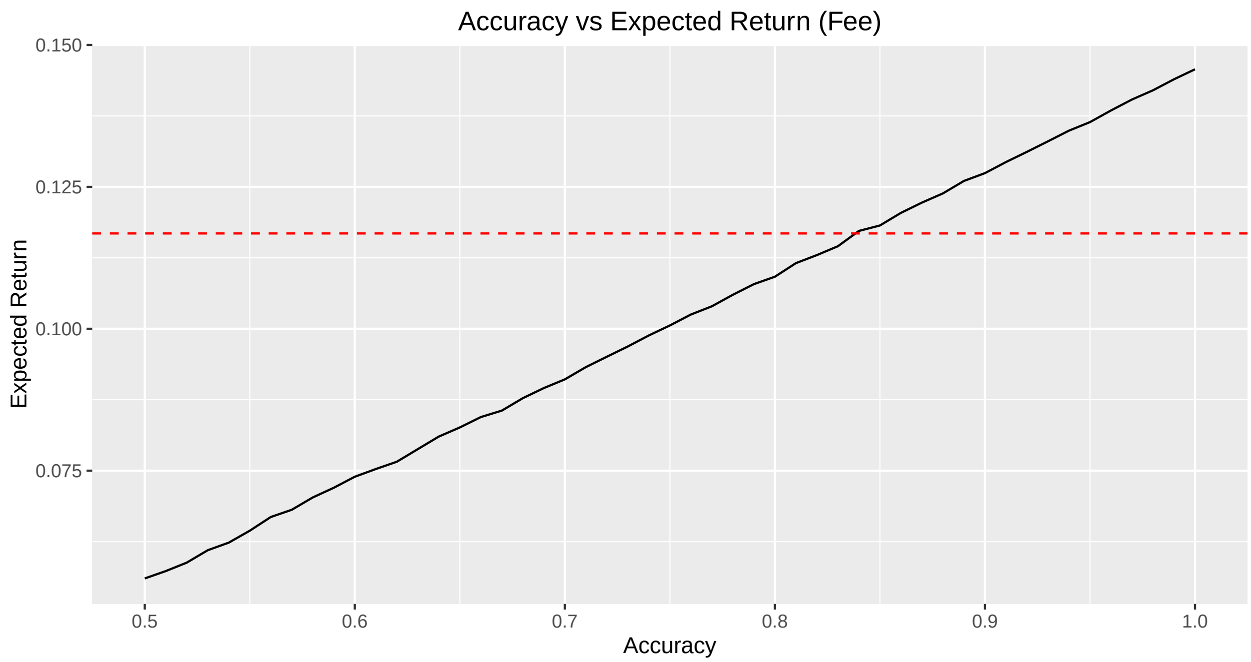
Accuracy vs Expected Return (no fee)
Expected Return
0.075
0.100
0.125
0.150
0.5
0.6
0.7
0.8
0.9
1.0
Accuracy


```
[1] 0.61
```



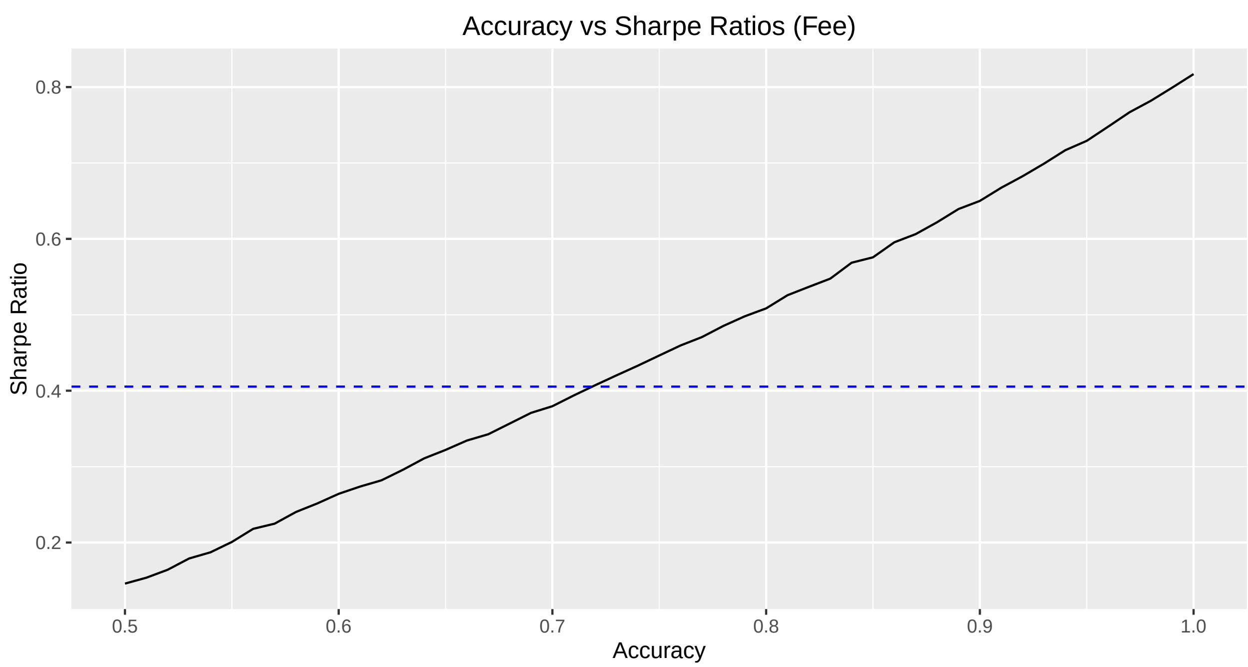
Accuracy vs Sharpe Ratios(no fee)
Sharpe Ratio
0.4
0.6
0.8
0.5
0.6
0.7
0.8
0.9
1.0
Accuracy


```
[1] 0.84
```



Accuracy vs Expected Return (Fee)
Expected Return
0.150
0.125
0.100
0.075
0.5
0.6
0.7
0.8
0.9
1.0
Accuracy

[1] 0.72



Accuracy vs Sharpe Ratios (Fee)
Sharpe Ratio
0.8
0.6
0.4
0.2
0.5
0.6
0.7
0.8
0.9
1.0
Accuracy

C.1.

a)

Given four possible states of the world and no market timing ability on part of the money manager, we calculate the expected return and standard deviation of the portfolio as follows:

Prob	Rs	Rc	Ws	Ra
0.25	0.05	0.01	0.7	0.038
0.25	0.05	0.01	0.3	0.022
0.25	-0.02	0.01	0.7	-0.011
0.25	-0.02	0.01	0.3	0.001

Thus,

E_A = 0.25(0.038 + 0.022 - 0.011 + 0.001) = 1.25%

$\sigma_A = \sqrt{0.25[(0.038-0.0125)^2+(0.022-0.0125)^2+(-0.011-0.0125)^2+(0.001-0.0125)^2]^2} = 1.89\%$

$S_a = \frac{(1.25-1)}{1.8875} = 0.1325$

b)

In english, stocks return -2% or 5% with equal probability, so on expectation our portfolio yields 1.5% with a standard deviation of 3.5%.

$E_s = \mathbb{E}[R_s]$ = 0.50(-0.02) + 0.50(0.05) = 1.5%

$\sigma_s = \text{Std}(R_s) = \sqrt{[(0.05-0.015)^2+(-0.02-0.015)^2]/2} = 3.5\%$

Our goal is to construct a portfolio with a standard deviation equal to 1.89%. Thus, we find the optimal weights such that standard deviation is satisfied.

$\sigma_P = \text{Std}(R_P)$ = w(0.035) + 1-w(0) = 0.0189

w = 0.54

$E_P = \mathbb{E}(R_P)$ = 0.54(0.015) + (1 - 0.54)(0.01) = 1.26%

$S_a = \frac{(1.2697-1)}{1.8875} = 0.1429$

Indeed, we realize a higher expected return and Sharpe Ratio while maintaining a comparable standard deviation.

C.2.

a)

1 - Prob(z $< \sqrt{10}(\frac{0.10-0.04}{0.20})$)

= 0.171

b)

Using the put-call parity:

2 * pnorm($\frac{0.20\sqrt{10}}{2}$) - 1

= 0.248

c)

Naturally, as T grows the probability goes to zero.

1 - Prob(z $< \sqrt{20}(\frac{0.10-0.04}{0.20})$)

= 0.09

And the cost of insurance increases with T.

2 * pnorm($\frac{0.20\sqrt{20}}{2}$) - 1

= 0.346

d)

In regards to the probability of underperformance when T = 10

1 - Prob(z $< \frac{0.10-0.04}{\sqrt{0.03^2+\frac{0.20^2}{10}}}$)

= 0.196

Likewise, the probability of underperformance when T = 20

1 - Prob(z $< \frac{0.10-0.04}{\sqrt{0.03^2+\frac{0.20^2}{20}}}$)

= 0.133

Naturally, the greater uncertainty increased the probability of underperformance. However, the effect of μ on the cost of shortfall insurance is zero. That is, the value of the option does not depend on μ, so there is no change when T is either 10 or 20.

C.3.

The authors speak to time diversification. The idea being that risk associated with high-expected-return portfolios (and thus high variance) can be reduced over longer periods of time. For this to properly work, however, the critical assumption is that returns must not be perfectly correlated over time. The returns—as the article alludes to, must be i.i.d.

C.4.

Though the authors give a convincing argument for a behaviorist's irrational exuberance. We don't know which way the causation goes. While the Harvard MBA indicator may be significant, we can't say whether the influx of Harvard MBAs into the finance industry causes a financial crisis, or, perhaps more likely, an unseen omitted variable(s) actually causes the financial crisis. Either way, it certainly appears to be correlated, and at the least serves as an indicator. Given that Booth and Harvard MBAs are largely from similiar socioeconomic circles, I would expect a similar result to hold–if not more so, given Booth's finance reputation.