UNIVERSITY OF CHICAGO
Booth School of Business

Bus 35120 – Portfolio Management | Prof. Lubos Pastor

Assignment #1
Solutions

The solutions given below were obtained in MATLAB using the program *hwk1_solutions.m*. You can download this program from Canvas into your current directory and run it by typing "*hwk1_solutions*" at the command line. You can also examine the program to learn more of MATLAB – there are many helpful comments in there. Note that the program is much longer than necessary because I wanted it to produce very user-friendly output.

2. and 3.

The table below summarizes the estimates based on daily data.

	Stock returns	Bond returns
Mean	0.0004	0.0000
Variance	0.0001	0.0000
St.dev	0.0102	0.0052
Skewness	-0.5717	0.0968
Kurtosis	22.5726	11.3377

The covariance based on daily data is < 0.0001. The correlation is also small: 0.0208.

Next, we move on to monthly figures:

	Stock returns	Bond returns
Mean	0.0101	0.0048
Variance	0.0018	0.0007
St.dev	0.0421	0.0268
Skewness	-0.4229	0.5558
Kurtosis	4.5275	7.1305

The monthly covariance is 0.0001 and the correlation is 0.1175.

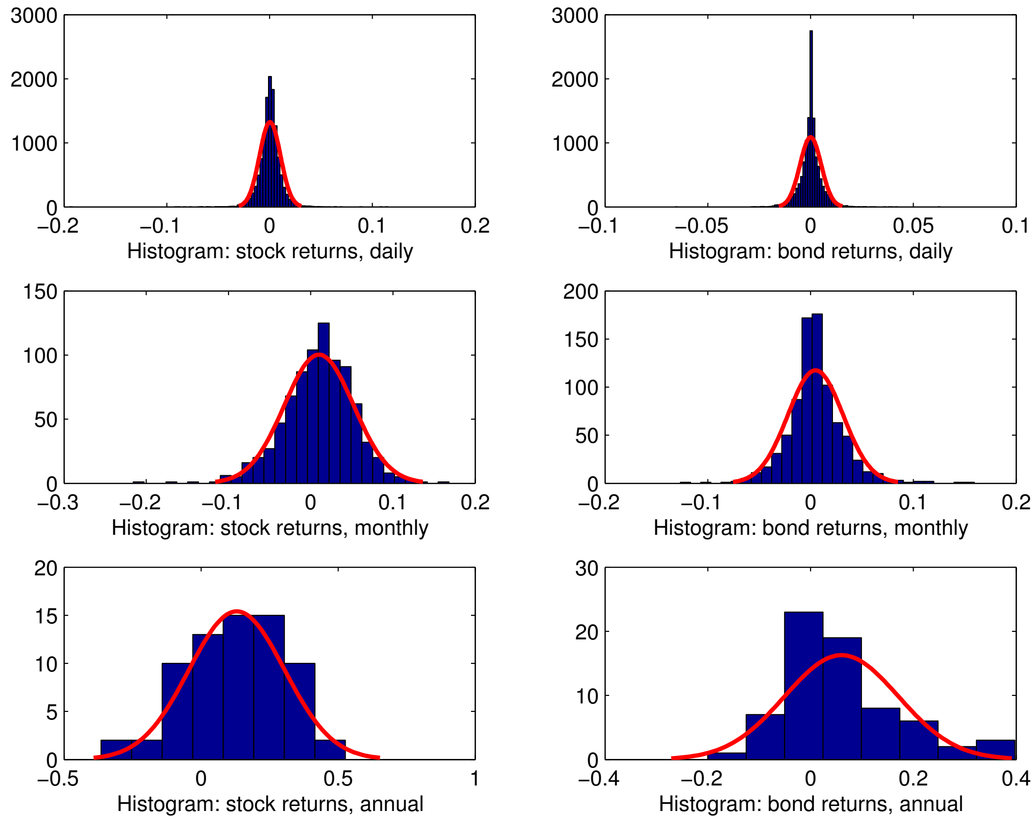
The final matrix here is based on annual returns.

	Stock returns	Bond returns
Mean	0.1303	0.0602
Variance	0.0303	0.0122
St.dev	0.1741	0.1105
Skewness	-0.4233	0.8491
Kurtosis	2.9944	4.0099

The annual covariance is -0.0002 and the correlation is -0.0121.

4.

The graph below summarizes the histograms of the returns.



What conclusions about normality can we draw from the tables in parts 2) and 3) and from the above histograms? Let's start with daily data. Both stock and bond returns are highly non-normal. Recall from your stats courses that skewness and kurtosis of a normal distribution are 0 and 3, respectively. The skewness of daily returns is close to 0, but kurtosis is much different from 3. Moreover, the histograms imply that the empirical distribution of daily returns is "fat-tailed" (or "leptokurtic," for those of you whose undergraduate major was in ancient languages). The fractions of observations that fall close to or far away from the mean are much too high given the fraction that falls "moderately close" to the mean.

The fat tails are easily observable: note how quickly the imposed normal curve goes to zero and how many return observations are still "out there" in the tails. For the record, a formal Jarque-Bera test (*jbtest.m*; I did this test on the side but you don't have to worry about it) resolutely rejects the assumption of normality for daily stock and bond returns.

Monthly returns come a little closer to normality. Their histograms are closer to the imposed normal pdf's, although fat tails are still apparent; in fact, kurtosis is even larger than in daily data. The Jarque-Bera test still rejects normality for both stocks and bonds.

Annual returns are the closest to normality, at least for stocks. The estimated kurtosis is very close to 3; skewness is also reasonably low. According to the histogram, the distribution "hides" underneath the normal curve quite well. The Jarque-Bera test cannot reject normality. For bonds, returns are still a bit more fat-tailed and right-skewed than the normal distribution would predict.

5.

The 95% confidence intervals for a typical stock and bond return are:

	Stock returns	Bond returns
Daily	[-0.0195; 0.0204]	[-0.0101; 0.0101]
Monthly	[-0.0724; 0.0926]	[-0.0476; 0.0573]
Annual	[-0.2110; 0.4715]	[-0.1563; 0.2768]

Note how wide the intervals are! The stock market is unlikely to move by much more than 2% in any given day, but its annual return can be as large as 47.2% or as low as -21.1% (not to speak of the 5% tail probability). Similarly, bonds generally move by less than 1% per day, but the annual bond return can be anywhere between -15.6% and 27.7%.

The 95% confidence intervals for the arithmetic averages of 30-period stock and bond returns are:

	Stock returns	Bond returns
Daily	[-0.0032; 0.0041]	[-0.0018; 0.0019]
Monthly	[-0.0050; 0.0251]	[-0.0048; 0.0144]
Annual	[0.0680; 0.1926]	[0.0207; 0.0998]

For both stocks and bonds, average 30-year annual returns are reliably positive, though the same cannot be said about average 30-month returns or average 30-day returns.

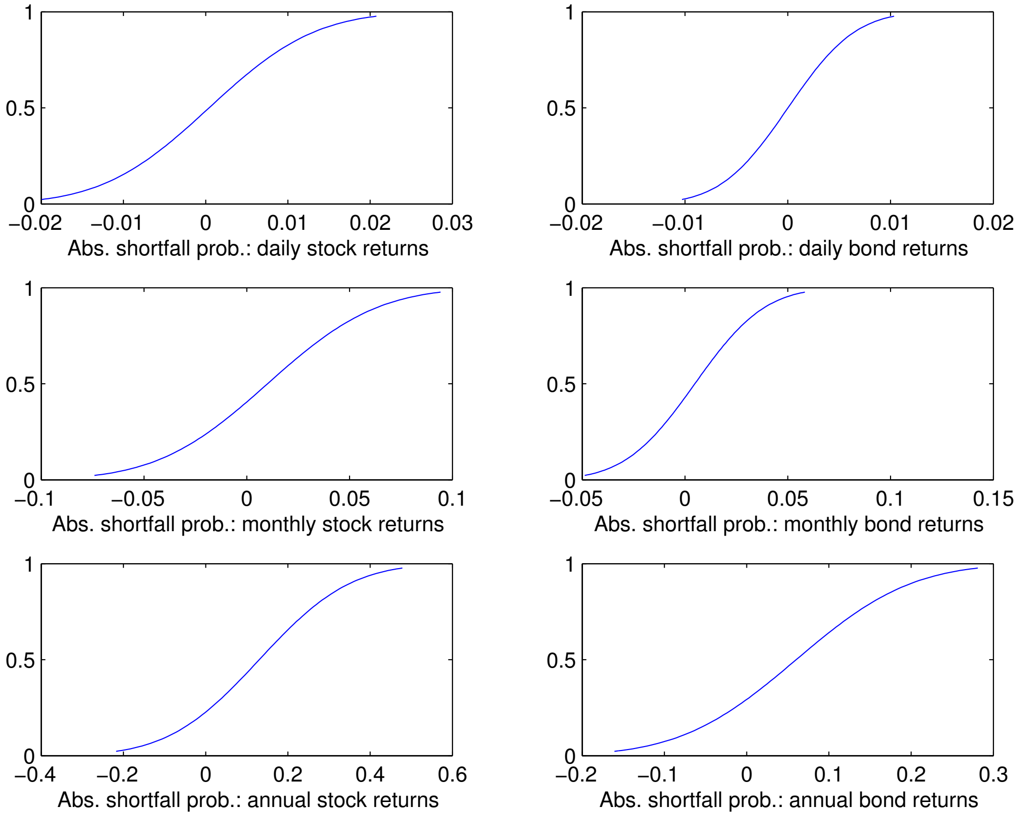
6.

Given that returns follow a normal distribution with mean μ and standard deviation σ, the probability that they are lower than a constant k is

$$Prob\,(R \leq k) = Prob\left(\frac{R-\mu}{\sigma} \leq \frac{k-\mu}{\sigma}\right) = N\left(\frac{k-\mu}{\sigma}\right),$$

where $N(x)$ is the cumulative function of the standard normal distribution. The values of $N(x)$ can be evaluated using statistical tables or using the *normcdf* command in MATLAB.

The shortfall probabilities are depicted in the following figure.



All curves have the same general shape, which follows from our normality assumption.

7.

Create a series of the differences between stock and bond returns and estimate the mean and the standard deviation of this difference. Under the assumption of normality, we can estimate the probability that stocks underperform bonds by using the standard normal cdf.

Estimated probability that daily stock returns will be lower than daily bond returns is 0.4863 – almost 0.5, which essentially corresponds to a coin flip. The monthly figure, 0.4555, and the annual one, 0.3677, indicate that stocks are less and less likely to underperform bonds as the time horizon increases, although such underperformance is very well possible.