Are Stocks Really Less Volatile in the Long Run?

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According to conventional wisdom, stock returns are less volatile over longer investment horizons. The idea is that bull and bear markets partially offset each other, reducing long-horizon variance. This reasoning, supported by historical estimates of volatility, is often invoked to justify generous stock allocations for long-horizon investors.

Our JF article reaches the opposite conclusion: stocks are actually *more* volatile over longer investment horizons. The key to our conclusion is that we take an investor's perspective. Instead of calculating backward-looking historical estimates of volatility, we calculate forward-looking measures of volatility that are relevant to investors.

Recognizing uncertainty

Investors are uncertain about the extent to which future stock returns will behave similarly to historical estimates. Our measure of volatility incorporates this "parameter uncertainty," whereas historical volatility does not. The forward-looking volatility we calculate, commonly referred to as *predictive volatility* in Bayesian statistics, is the relevant volatility from an investor's perspective. We find empirically that the U.S. stock market's predictive volatility exceeds its historical volatility, especially at long investment horizons. Moreover, predictive volatility increases with the investment horizon, unlike historical volatility, which exhibits the opposite pattern.

A key parameter governing future stock returns is the equity premium, μ_t, the stock market return one should expect in year $t+1$ relative to a riskless investment. Even after observing two centuries of stock market returns, investors are uncertain about the current μ_t, as well as how it might change in the future. To compute predictive volatility, we must specify how μ_t can change over time. It is commonly assumed that μ_t depends on the investment environment via a set of observable predictors, x_t. Specifically, $\mu_t = a + b'x_t$. This is a useful assumption in many applications, but we relax it here because it understates the uncertainty faced by an investor assessing the volatility of future returns. No investor can be certain that μ_t is perfectly captured by x_t. It seems much more likely that a set of observed predictors is imperfect, in that $\mu_t = a + b'x_t + \pi_t$, where π_t can be viewed as an unobservable predictor at time t. To admit such predictor imperfection, we employ a *predictive system*, an econometric model that we developed in an earlier 2009 JF article. The predictive system assumes

$$r_{t+1} = a + b'x_t + \pi_t + u_{t+1} , \quad (1)$$

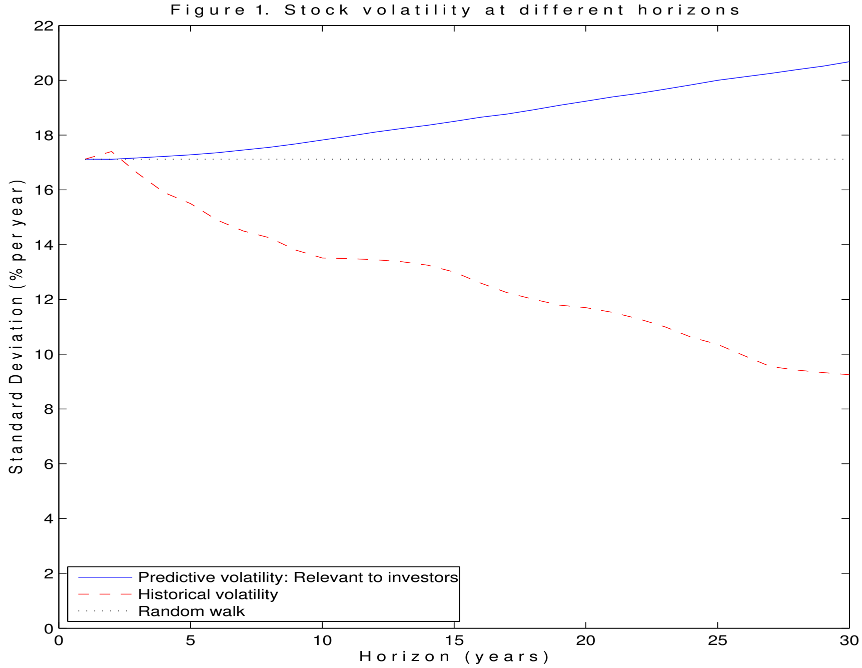
where r_{t+1} is the stock market return at time $t + 1$, and u_{t+1} is a random error. Recognizing uncertainty due to predictor imperfection is important in reaching our conclusions.

We estimate the predictive system on 206 years of annual real U.S. stock market returns, covering the period 1802 through 2007. We consider three observable predictors (x_t): the aggregate dividend yield on U.S equity, the term spread (i.e., the difference between the long-term high-grade bond yield and the short-term interest rate), and the change in the long-term bond yield. These predictors seem reasonable choices given the various predictors used in previous studies and the information available in the historical data set, for which we are grateful to Jeremy Siegel. All three predictors exhibit significant ability to predict next year's market return.

The main result.

The solid line in Figure 1 plots predictive volatility as a function of the investment horizon k. Predictive volatility is the annualized standard deviation of market returns over the following k years, calculated based on all available data at the end of our sample.[1] The figure shows that predictive volatility rises with the investment horizon, from about 17% per year at the one-year horizon to almost 21% per year at the 30-year horizon. Long-horizon stock investors clearly face more volatility than short-horizon investors on a per-year basis.

Figure 1. Stock volatility at different horizons



For comparison, the dashed line in Figure 1 plots the historical volatility as a function of the in-

[1]Formally, annualized predictive volatility is given by Std$(r_{T,T+k}|D_T)/\sqrt{k}$, where $r_{T,T+k}$ is the cumulative return between times T and $T + k$, and D_T contains all return and predictor data available at time T.

vestment horizon. Historical volatility at a given horizon k is computed as the annualized standard deviation of cumulative market returns over all k-year periods in our sample. For example, for $k = 10$ years, we compute the cumulative return for each of the 197 overlapping 10-year periods 1802–1811, 1803–1812, ..., 1998–2007, then we calculate the sample standard deviation of the 197 returns, and finally we annualize the standard deviation by dividing it by the square root of 10. The figure shows that historical volatility decreases with the investment horizon, from 17% per year at the one-year horizon to 9.3% per year at the 30-year horizon. The dashed line is the source of the conventional wisdom—historically, stocks have been less volatile in the long run.[2]

Where does our main result come from?

To understand the difference between the solid and dashed lines, it is useful to begin by considering a hypothetical world in which stock prices follow a random walk—μ_t is constant over time—and there is no parameter uncertainty. In such a world, annualized predictive volatility does not depend on the investment horizon, as shown by the flat dotted line in Figure 1.

The real world differs from the hypothetical world in two important ways. First, stock prices do not follow a random walk. Instead, they exhibit a certain degree of "mean reversion," in that unexpectedly high returns tend to be followed by a lower μ_t, and low returns tend to be followed by a higher μ_t. Mean reversion pulls long-horizon volatilities down. Second, investors face substantial parameter uncertainty. Investors are uncertain about the current and future values of μ_t as well as other parameters of stock returns such as volatility or persistence. Parameter uncertainty pulls long-horizon volatilities up.

Historical volatility reflects mean reversion but not parameter uncertainty; hence the dashed line is downward-sloping. Predictive volatility reflects both mean reversion and parameter uncertainty. These two forces pull in opposite directions, but our results show that parameter uncertainty prevails; hence the solid line is upward-sloping. That is our main result.

For more intuition, consider the trend from which stock prices randomly depart. Historical volatility is computed around the historical trend, which is known. The future trend is unknown, however, so forward-looking volatility must reflect not only random departures from the trend but also uncertainty about the trend itself. Due to the latter uncertainty, predictive volatility exceeds historical volatility. Moreover, the wedge between the two volatilities increases with the investment horizon. Trend uncertainty does not matter much at short horizons, but it compounds quickly as the horizon lengthens.

[2] The same conclusion obtains based on "conditional" volatility, which conditions on information useful in predicting returns but also ignores parameter uncertainty (e.g., see work by John Campbell and Luis Viceira).

We also find that long-run predictive volatility is substantially higher compared to the framework in which predictors are perfect (i.e., when π_t is omitted from equation (1)). Predictor imperfection and parameter uncertainty interact—once predictor imperfection is admitted, parameter uncertainty is more important in general. In particular, when the conditional mean is not observed, learning about its properties is harder compared to the perfect-predictor case.

We show that our main result is robust to various modifications of the basic framework. We consider three econometric models: two predictive systems as well as a predictive regression with an uncertain set of observable predictors. We split the sample in half and run the analysis on both sub-samples. We replace our annual sample with a quarterly sample of post-war returns. We consider different observable predictors. All of these exercises lead to the same basic conclusion: stocks are more volatile over longer horizons from an investor's perspective.

Implications for investors.

Our conclusion makes stocks less appealing to long-horizon investors than conventional wisdom would suggest. As a result, buy-and-hold investors who have invested based on historical estimates of volatility might reconsider their stock allocations. We take a closer look at the implications of our findings for investors in target-date retirement funds, which have become very popular recently. These funds follow a pre-determined asset allocation policy that gradually reduces the stock allocation as the target date approaches, with the aim of providing a more conservative asset mix to investors approaching retirement.

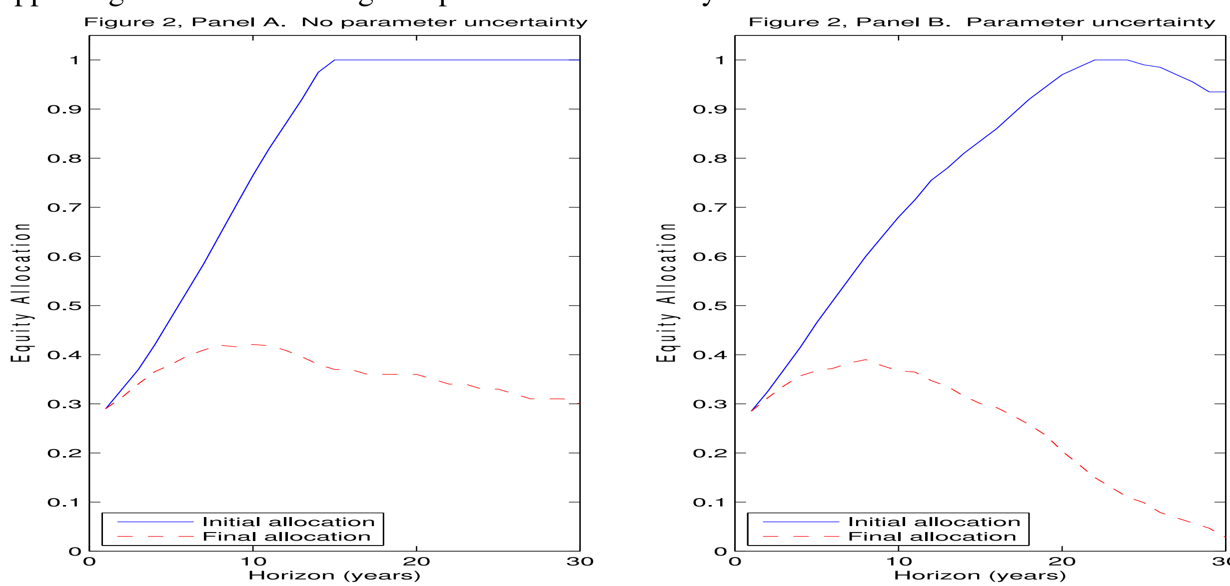
We analyze target-date funds using a simple model in which a risk-averse investor can invest in only two assets, the stock market and a real riskless asset. The investor focuses on the final level of wealth achieved at the end of a K-year horizon.[3] The investor commits at the outset to a pre-determined investment strategy in which the stock allocation evolves linearly from the first-period allocation w_1 to the final-period allocation w_K. The investor chooses the values of w_1 and w_K within the $(0, 1)$ interval. We allow the investor to save from labor income, which we calibrate to match the observed hump-shaped pattern of labor income over a typical American's life cycle. We obtain the same conclusions in the absence of labor income.

Figure 2 plots the investor's optimal initial and final stock allocations, w_1 (solid line) and w_K (dashed line), respectively, for investment horizons ranging from one to 30 years. We incorporate parameter uncertainty in Panel B but not in Panel A.

The optimal allocations in Panel A are strikingly similar to those selected by real-world target-

[3]Specifically, the investor maximizes the expected value of utility $W_K^{1-A}/(1-A)$ for end-of-horizon wealth W_K.

date funds. The initial allocation w_1 decreases as the investment horizon shortens, declining from 100% at horizons longer than 15 years to about 30% at the one-year horizon, whereas the final allocation w_K is roughly constant at about 30% to 40% across all horizons. Investors in real-world target-date funds similarly commit to a stock allocation schedule, or "glide path," that decreases steadily to a given level at the target date. The final stock allocation in a target-date fund does not depend on when investors enter the fund, but the initial allocation does—it is higher for investors entering longer before the target date. Not only the patterns but also the magnitudes of the optimal allocations in Panel A resemble those of target-date funds. In short, target-date funds seem appealing to investors who ignore parameter uncertainty.



In contrast, Panel B shows that target-date funds do not appear desirable if the same investors acknowledge a realistic amount of parameter uncertainty. For short investment horizons, the results look similar to those in Panel A, but for horizons longer than 23 years, both w_1 and w_K decrease with K. For example, an investor with a 23-year horizon chooses to glide from $w_1 = 100\%$ to $w_{23} = 14\%$, whereas an investor with a 30-year horizon glides from $w_1 = 93\%$ to $w_{30} = 3\%$. Parameter uncertainty clearly matters more at longer investment horizons.

Figure 2 shows that parameter uncertainty makes target-date funds undesirable when they would otherwise be virtually optimal for investors who desire a pre-determined asset allocation policy. It would be premature, however, to conclude that parameter uncertainty makes target-date funds undesirable in all settings. Our analysis abstracts from many important considerations faced by investors, such as intermediate consumption, additional risky assets, housing, etc. Our objective is simply to illustrate how parameter uncertainty can reduce the stock allocations of long-horizon investors, consistent with our results about long-horizon volatility.