# ON THE RISK OF STOCKS IN THE LONG RUN

*Zvi Bodie*

## *ABSTRACT*

This paper examines the proposition that investing in common stocks is less risky the longer an investor plans to hold them. If the proposition were true, then the cost of insuring against earning less than the risk-free rate of interest should decline as the length of the investment horizon increases. The paper shows that the opposite is true even if stock returns are "mean-reverting" in the long run. The case for young people investing more heavily in stocks than old people cannot therefore rest solely on the long-run properties of stock returns. For guarantors of money-fixed annuities, the proposition that stocks in their portfolio are a better hedge the longer the maturity of their obligations is unambiguously wrong.

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*Zvi Bodie is Professor of Finance at Boston University*

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## *1. Introduction*

The conventional "wisdom" in the professional investment community seems to be that investors with a long time horizon should invest more heavily in stocks than investors with a short time horizon. The idea is that the riskiness of stocks diminishes with the length of one's time horizon, so that the risk-reward tradeoff faced, for example, by a young person saving for retirement is more favorable to stocks than that faced by an older person who is already retired.[1]

The following excerpt from the Spring 1990 issue of *In The Vanguard*, a publication of The Vanguard Group (of mutual funds), is typical of the proposition and the reasoning behind it:

> "Over the past six decades, stocks have achieved an average annual rate of return of 9.7% — far exceeding the 5.2% average return on corporate bonds and the 3.6% average return on U.S. Treasury bills. Yet it's no secret that the stock market is subject to wide and unpredictable price swings in any given year. Consider, however, that *the volatility of stock market returns diminishes markedly over time.....*
> During any one-year period between 1960 and 1989, the maximum spread in annual returns of stocks (as measured by the unmanaged Standard and Poor's 500 Composite Stock Price Index) was 64% (from a high of 37.2% to a low of 26.5%). Over ten-year holding periods, the difference in annual rates of return decreased to 16% (17.5% to 1.2%) and, over 25 years, less than 2% (10.2% to 8.4%). Note that for ten-year periods and beyond, the returns were all positive.
> *Clearly, over time, stock market risk hardly seems excessive—even for the most cautious long-term investor.*
> So, take stock of time when investing in stocks."[2]

Robert C. Merton and Paul A. Samuelson have written numerous articles over the years showing the fallacy in such statements.[3] The proofs and counterarguments

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[1]For example, a popular proposed rule is to set the portfolio proportion invested in stocks equal to 100% less one's age. Thus, a 40 year old should invest 60% in stocks, and a 60 year old should invest 40% in stocks.

[2]B. G. Malkiel makes virtually the same statement in *A Random Walk Down Wall Street* (New York: Norton, paperback edition 1991): 343. For a similar view presented in the recent academic literature, see J. F. Marshall, "The Role of the Investment Horizon in Optimal Portfolio Sequencing," *The Financial Review* (November 1994): 557-576.

[3] See R.C. Merton and P. A. Samuelson, "Fallacy of the Log-Normal Approximation to Portfolio Decision-Making Over Many Periods," *Journal of Financial Economics* (March 1974): 67-94; and P. A. Samuelson, "Risk and Uncertainty: A Fallacy of Large Numbers," *Scientia* (April-May 1963): 1-6; "The Fallacy of Maximizing the Geometric Mean in Long Sequences of Investing or Gambling," *Proceedings of the National Academy of Science* (1971): 207-211; "The Judgement of Economic Science on Rational Portfolio Management: Timing and Long-Horizon Effects," *Journal of Portfolio Management* (Fall 1989):

presented by Merton and Samuelson rely on the theory of expected utility maximization. In this paper I use option pricing theory to demonstrate the fallacy. Taking as the measure of the riskiness of an investment the cost of insuring it against earning less than the risk-free rate of return over the investor's time horizon, the paper shows that the riskiness of stocks *increases* rather than *decreases* with the length of that horizon. This is so both under the assumption of a "random-walk" process for stock returns and for the kinds of "mean-reverting" processes that have been reported in the economics and finance literature.[4]

The paper then briefly discusses the investment implications of this finding for individuals and for guarantors of money-fixed annuities. It shows that the case for young people investing more heavily in stocks than old people cannot rest solely on the long-run properties of stock returns. For guarantors of money-fixed annuities, the proposition that stocks in their portfolio are a better hedge the longer the maturity of their obligations is unambiguously wrong.

## 2. Measuring the Risk of Stocks

In discussions of the performance of stocks over various time horizons, a widely-used concept of risk is that of a "shortfall." A shortfall occurs when the value of a stock portfolio at the horizon date is less than some value determined by a specified "target" rate of return.[5] A natural choice for this target rate of return is the rate of interest on default-free zero-coupon bonds maturing on the horizon date. Since such bonds are free of risk over the relevant time horizon, they represent a logical benchmark against which to measure the risk of stocks.

The basis for the proposition that stocks are less risky in the long run appears to be the observation that the longer the time horizon, the smaller the *probability* of a shortfall. If the *ex ante* mean rate of return on stocks exceeds the risk-free rate of interest, it is indeed true that the probability of a shortfall declines with the length of the investment time horizon. For example, suppose the rate of return on stocks is lognormally distributed with a risk premium of 8% per year and an annualized standard deviation of 20%. With a time horizon of only 1 year, the probability of a shortfall is 34%, whereas at 20 years that probability is only 4%.

But as has been shown in the literature, the probability of a shortfall is a flawed measure of risk because it completely ignores how large the potential shortfall might be.[6] It is easiest to see this point if we assume that in any 1-year period, the rate of return on stocks can take only one of two values. For example, assume that the rate of return will either be +20% or  20%. Consider the worst possible outcome for time horizons of

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4-12; and "The Long-Term Case for Equities and How It Can Be Oversold," in *Journal of Portfolio Management* (Fall 1994): 15-24.

[4]For example, see A. Lo and C. MacKinlay, "Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test," *Review of Financial Studies* (1988): 41-66.

[5]See, for example, M. L. Leibowtiz and W.S. Krasker, "The Persistence of Risk: Shortfall Probabilities Over the Long Term," *Financial Analysts Journal* (November/December 1988).

[6]See W. V. Harlow, "Asset Allocation in a Downside Risk Framework," *Financial Analysts Journal* (September/October 1991): 28-40.

increasing length. For a 1-year horizon one can lose 20% of the initial investment, for a 2-year period 36%, and for a 20-year period as much as 99%. Using the probability of a shortfall as the measure of risk, no distinction is made between a loss of 20% or a loss of 99%.

## 3. The Cost of Insuring Against a Shortfall

If it were true that stocks are less risky in the long run, then the cost of insuring against earning less than the risk-free rate of interest should decline as the length of the investment horizon increases. But the opposite is true.

To see this, define the cost of shortfall insurance, *P*, as the additional amount of money one has to add at the investment starting date to assure that at the horizon date the portfolio will have a value at least as great as it would have earning the risk-free interest rate. Thus, for each dollar insured against a shortfall, the total amount actually invested at the starting date is $1 + *P*.

To find *P*, we use modern option pricing methodology.[7] Insurance against shortfall risk is effectively a *put* option. The put is of the European type (i.e., it can only be exercised at the expiration date), and it matures in *T* years. The put's exercise price is the insured value of the portfolio. If at the expiration date *T* years from now the portfolio's value exceeds its insured value, then the put expires worthless. If, however, there is a shortfall, then the put's payoff is equal to the shortfall.

Because we are insuring against earning less than the risk-free interest rate, the exercise price of the put equals the future value of the underlying stock portfolio compounded at the risk-free *T*-year interest rate. Therefore the *put-call parity theorem* tells us that the price of the put equals the price of the corresponding call.[8]

To show that the value of the put increases with *T*, we could use any option pricing model based on the condition that the financial markets do not allow anyone to earn risk-free arbitrage profits.[9] Because it is so compact and so widely used in practice, we will

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[7] The reference here is to the option-pricing theory originally developed by F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* (May-June 1973), and R. C. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* (Spring 1973). There is an extensive literature on using option-pricing models to estimate the value of financial guarantees. For a comprehensive list of references, see R. C. Merton and Z. Bodie, "On the Management of Financial Guarantees," *Financial Management* (Winter 1992).

[8] The put-call parity theorem for European options says that:

$$P + S = C + E\,e^{-rT}$$

where *P* is the price of the put, *S* is the price of the underlying stock, *C* is the price of the corresponding call, *E* is the exercise price, and *r* is the risk-free interest rate. In our case:

$$E = Se^{rT}$$

By substituting into the put-call parity relation we get:

$$P = C$$

[9] The option price is derived by considering a dynamic investment strategy involving only the underlying stock and the risk-free asset, which has as its objective to produce at the horizon date a payoff equal to that of the put. The strategy is self-financing, that is, no additional infusions of money beyond the original $*P* are required. As is well known in the literature, an option's price can also be expressed using a "risk-neutral" valuation method. This method makes explicit that the cost of shortfall insurance reflects a weighting of the possible shortfall magnitudes.

use the Black-Scholes formula. In our special case a *simplified* form of the formula can be used to compute *P*. Moreover, with no loss of generality, we can express the put as a fraction of the price of the stock:

$$P/S = N(d_1) - N(d_2)$$

$$d_1 = \frac{s\sqrt{T}}{2}$$

$$d_2 = -\frac{s\sqrt{T}}{2}$$

where:
*C* = *price of the call*
*S* = *price of the stock*
*E* = *exercise price*
*r* = *risk-free interest rate (the annualized continuously compounded rate on a safe asset with the same maturity as the option)*
*T* = *time to maturity of the option in years*
*s* = *standard deviation of the annualized continuously compounded rate of return on the stock*
ln = *natural logarithm*
*e* = *the base of the natural log function (approximately 2.71828)*
*N*(*d*) = *the probability that a random draw from a standard normal distribution will be less than d.*

Note that *P/S* is independent of the risk-free interest rate; it depends only on σ and *T*. *Table 1* and *Figure 1* show the result of applying the formula to compute *P/S* assuming the annualized standard deviation of stock returns (σ ) is .2.[10]

The cost of the insurance rises with *T*. For a one-year horizon, the cost is 8% of the investment. For a 10-year horizon, it is 25%, and for a 50-year horizon it is 52%. As the length of the horizon grows without limit, the cost of the insurance approaches 100% of the investment. In other words, it can never cost more than \$1 to insure that a dollar invested in stocks will earn the risk free rate. This is because one can always invest the \$1 insurance premium in risk-free bonds maturing in *T* years, so that even if the value of stocks falls to zero, the investor still will have the guaranteed minimum.

## 4. Mean Reversion in Stock Returns Does Not Matter

Some financial economists and other observers of the stock market have claimed that stock returns do not follow a random walk in the long run. Rather, they argue, the

[10]An alternative procedure would be to deduct the cost of insurance from the initial investment. In that case, the insured value of the portfolio at the horizon date becomes (\$1-*P*)$e^{rT}$ instead of $e^{rT}$.

behavior of stock returns is best characterized as a mean-reverting process.[11] It is mean reversion in stock returns, some say, that is the reason stocks are less risky for investors with a long time horizon.[12]

But our result is valid for mean reverting processes too. The reason is that arbitrage-based option pricing models, such as the Black-Scholes-Merton or binomial models, are valid regardless of the process for the mean. They are based on the law of one price and the condition of no-arbitrage profits. Investors who disagree about the mean rate of return on stocks, but agree about the variance, will therefore agree about the option price. This is a feature of these models that may at first seem "counter-intutitive," but is nonetheless true.[13]

For the relation depicted in *Figure 1* to be invalid, mean reversion is not enough. Stock prices would have to behave just like the price of a $T$-period zero-coupon bond that converges towards the bond's face value as the horizon date approaches. In other words, stocks would have to be indistinguishable from the risk-free asset for a $T$-period horizon.

## 5. Investment Implications for Individuals

We have seen that despite the fact that the probability that stocks will earn less than the risk-free rate of interest *decreases* with the length of the time horizon, the cost of insuring against this eventuality *increases*. What are the investment implications of this finding? In particular, what about the popular notion that because of their longer time horizon, the young should invest more in stocks than the old?

Finance theory indicates that there is no such simple rule that applies in all cases. If investors act so as to rationally maximize the expected utility of consumption over their lifetimes, then an investor's age *per se* has no predictable effect on the optimal proportion to invest in stocks.[14] Asset allocation for individuals should be viewed in the broader context of deciding on an allocation of *total* wealth between risk-free and risky assets.

A critical determinant of optimal asset allocation for individuals is the time and risk profile of their human capital. A person faces an expected stream of labor income over the working years, and human capital is the present value of that stream. One's human capital, is a large proportion of total wealth (human capital + other assets) when one is young, and eventually decreases as one ages. From this perspective, it *may* be optimal to start out in the early years with a higher proportion of one's investment portfolio in stocks

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[11]See, for example, J. M. Poterba and L. Summers, "Mean Reversion in Stock Returns: Evidence and Implications," *Journal of Financial Economics* (1988): 27-60.

[12]See, for example, the footnote on page 344 of B. Malkiel, *A Random Walk Down Wall Street* (New York: Norton, 1990).

[13] There is an extensive literature, including many text books, explaining why the formula holds regardless of the mean rate of return on stocks. We will not attempt to repeat those explanations here. While mean reversion does not affect the Black-Scholes formula, it does affect the *measured* variance over the observation period. See A. Lo and J. Wang, "Implementing Option Pricing Models When Asset Returns Are Predictable," (National Bureau of Economic Research Working Paper No. 4720, 1994), for a discussion of this effect.

[14] For example, for constant relative risk aversion utility functions, the proportion of total wealth to invest in stocks is *independent* of the investor's age. See R. C. Merton, R.C., "Lifetime Portfolio Selection by Dynamic Stochastic Programming: The Continuous Time Case," *Review of Economics and Statistics* (August 1969) and P. A. Samuelson, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," *Review of Economics and Statistics* (August 1969).

and decrease it over time as suggested by the conventional wisdom. However, the conventional wisdom may not apply to broad classes of individuals who face substantial human-capital risk early in their careers. For such individuals, the *opposite* policy may be optimal, i.e., to start out with a relatively low fraction of the investment portfolio in stocks and increase it over time.[15]

Another critical determinant of the optimal investment in stocks is how close people are to some minimum "subsistence" level of consumption. People should be expected to insure against falling below such a level through their asset allocation policy.[16]

## 6. Implications for Guarantors of Annuities

What does our finding imply about investment policy for a guarantor of money-fixed annuities, such as the Pension Benefit Guaranty Corporation (PBGC)? The pension annuities that are guaranteed by the PBGC are annuities fixed in dollar amount, and their present value is extremely sensitive to changes in long-term interest rates. At the same time, the pension fund assets securing the promised benefits are heavily invested in stocks. As guarantor of the promised pension benefits, the PBGC bears the risk of a shortfall between the value of insured benefits and the assets securing those benefits. Thus the PBGC is exposed to two types of risk — interest-rate risk and stock-market risk.[17]

The magnitude of the PBGC's exposure to shortfall risk because of the mismatch between pension fund assets and liabilities appears not to be well understood.[18] It is apparently a widespread belief among policymakers that a well-diversified pension portfolio of stocks provides an effective long-run hedge against liabilities of defined-benefit pension plans, so that there is no mismatch problem.

As we have seen, this belief is mistaken. Stocks are not a hedge against fixed-income liabilities even in the long run. Exactly the opposite is the case: When a pension plan sponsor invests the pension assets in stocks, the actuarial present value cost to the PBGC of providing a guarantee against a shortfall *increases* rather than decreases with the length of the time horizon, even for plans that might start out fully funded.

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[15]For example, see Z. Bodie, R. C. Merton, and W. Samuelson, "Labor Supply Flexibility and Portfolio Choice in a Life-Cycle Model," *Journal of Economic Dynamics and Control* (1992): 427-449.

[16]For a discussion of dynamic investment strategies designed to guarantee a minimum level of consumption while preserving the upside potential of the portfolio, see F. Black and A. Perold, "Theory of Constant Proportion Portfolio Insurance," *Journal of Economic Dynamics and Control* (1992): 403-426.

[17]Under certain assumptions, it is also exposed to the risk of a rise in wages.

[18]In this respect, there are some important lessons that the PBGC can learn from the experience of the Federal Savings and Loan Insurance Corporation (FSLIC). FSLIC was the government agency that insured deposits at savings and loan associations until it was replaced in 1989 leaving a massive deficit to be financed by taxpayers. For greater detail, see Z. Bodie, "Is the Pension Benefit Guaranty Corporation the FSLIC of the Nineties?" *Contingencies* (March/April 1992) and "What the Pension Benefit Guaranty Corporation Can Learn from the Federal Savings and Loan Insurance Corporation," Harvard Business School Working Paper No. 94-070, 1994.

## 7. Stocks as a Long-Run Inflation Hedge

It is often pointed out that investing in bonds exposes the investor to inflation risk — the risk of depreciation in the purchasing power of the currency in which the bond payments are denominated. One straightforward way to address this problem is to denominate the bonds in terms of a unit of *constant* purchasing power. Indeed, the governments of the United Kingdom and more recently Australia and Canada have issued long-term bonds linked to an index of consumer prices with precisely this purpose in mind, i.e., to offer investors a safe way to eliminate both interest rate risk and inflation risk over a long horizon.[19]

One sometimes gets the impression from reading popular articles on stocks as an inflation hedge, that their authors view stocks as if they were long-term real bonds. But there is a very big difference between stocks and long-term real bonds. With real bonds, the investor knows that regardless of what happens to the price of the bond *prior* to its maturity date, *at* maturity it will pay its holder a known number of units of purchasing power. With stocks there is no certainty of value — real or nominal — at *any* date in the future.

In the academic finance literature, researchers investigating whether stocks are an inflation hedge in the long run usually hypothesize that real stock returns are *unaffected* by inflation in the long run. By this they mean that the real return on stocks is uncorrelated with inflation.[20] They do *not* mean that stocks offer a risk-free real rate of return, even in the long run.[21]

[19]For a discussion of these bonds and their use in retirement portfolios, see Z. Bodie, "Inflation, Index-Linked Bonds, and Asset Allocation," *Journal of Portfolio Management* (Winter 1990).

[20] See J. Boudoukh and M.P. Richardson, "Stock Returns and Inflation: A Long-Horizon Perspective," *American Economic Review*, forthcoming 1994.

[21]The author thanks the following people for helpful comments and suggestions: Dwight Crane, Michael Edleson, Ken Froot, W. V. Harlow III, Robert C. Merton, Andre Perold, and Paul A. Samuelson.

**Table 1. Cost of Shortfall Insurance as a Function of Time Horizon**

| ***Length of Time Horizon in Years*** | ***Cost as Percentage of Investment*** |
|---|---|
| 0 | 0 |
| 1 | 7.98 |
| 5 | 17.72 |
| 10 | 24.84 |
| 20 | 34.54 |
| 30 | 41.63 |
| 50 | 52.08 |
| 75 | 61.35 |
| 100 | 68.27 |
| 200 | 84.27 |

*NOTES:*

*The table was derived using the simplified Black-Scholes formula with* $\sigma = .2$ *per year. The cost of the insurance is independent of the risk-free rate.*

**Figure 1.** **Cost of Shortfall Insurance as a Function of Time Horizon**