UNIVERSITY OF CHICAGO
Booth School of Business

Bus 35120 – Portfolio Management Prof. Lubos Pastor

Assignment #9

Solutions

B. DATA ANALYSIS.

1. No. Ford's stock has a lower Sharpe ratio than the market (the G subscript refers to Ford; I don't want to use F so as not to confuse Ford with the risk-free asset):

$$\mathrm{SR}_G^{20} = \frac{\mathrm{E}_G^{20} - R_f^{20}}{\sigma_G^{20}} = \frac{20\mathrm{E}_G - 20R_f}{\sqrt{20}\sigma_G} = 0.5306$$

$$\mathrm{SR}_M^{20} = \frac{\mathrm{E}_M^{20} - R_f^{20}}{\sigma_M^{20}} = \frac{20\mathrm{E}_M - 20R_f}{\sqrt{20}\sigma_M} = 1.5012$$

Thanks to diversification, the market portfolio's volatility is much smaller than Ford's, which results in a higher Sharpe ratio for the market.

2. You are willing to participate in FLOP if the Sharpe ratio of Ford after the discount is at least as large as the Sharpe ratio of the market. What is the minimum expected Ford return (denote it by $\bar{E}_G^{20}$) for which this happens?

$$\frac{\bar{E}_G^{20} - R_f^{20}}{\sigma_G^{20}} = \frac{\mathrm{E}_M^{20} - R_f^{20}}{\sigma_M^{20}},$$

from which we have

$$\bar{E}_G^{20} = R_f^{20} + \frac{\sigma_G^{20}}{\sigma_M^{20}}\left[\mathrm{E}_M^{20} - R_f^{20}\right] = 3.1038.$$

(Expected cumulative return is 310.38% over 20 years.)

Any discount received on the Ford stock increases expected return on the Ford stock. The higher the discount, the higher the expected return. If the discount is large enough, the expected return on the Ford stock exceeds $\bar{E}_G^{20}$ and you will choose to participate in FLOP. We need to solve for the discount d that sets Ford's expected 20-year log return equal to $\bar{E}_G^{20}$.

Let P_t denote Ford's stock price at time t. Without any discount, Ford's expected 20-year log return is equal to

$$E_{\text{no discount}}^{20} = \mathrm{E}\left[\ln\left(\frac{P_{20}}{P_0}\right)\right] = 1.6714,$$

as estimated from the data (this is 20 times Ford's sample average return).

With discount d, you pay $P_0(1-d)$ for the Ford stock, so your expected 20-year log return on Ford is

$$\begin{aligned} E^{20}_{\text{discount}} &= \mathrm{E}\left[\ln\left(\frac{P_{20}}{P_0(1-d)}\right)\right] = \mathrm{E}\left[\ln\left(\frac{P_{20}}{P_0}\right)\right] - \ln(1-d) \\ &= E^{20}_{\text{no discount}} - \ln(1-d). \end{aligned}$$

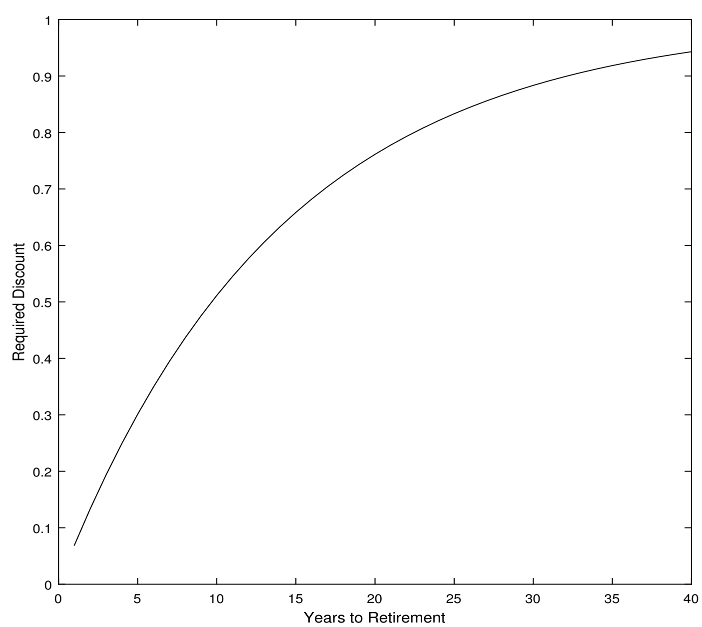
We want this quantity to be equal to $\bar{E}^{20}_G$, so we obtain d as

$$\begin{aligned} d &= 1 - \exp\left(E^{20}_{\text{no discount}} - E^{20}_{\text{discount}}\right) \\ &= 1 - \exp\left(E^{20}_{\text{no discount}} - \bar{E}^{20}_G\right) \\ &= 1 - \exp\left(1.6714 - 3.1038\right) = 0.7613 = 76.13\%. \end{aligned}$$

You would require a 76% discount on Ford stock in order to participate in FLOP. That is a huge discount! Ford is unlikely to offer such a good deal.

3. See the figure below. The required discount increases from 6.91% for $T = 1$ year to 94.30% for $T = 40$ years until retirement! For any realistic discount that Ford might offer, only the workers closest to retirement are likely to be interested.

   When T increases, the required discount increases because when a given discount is "amortized" over a longer period, the "per-period" discount is lower.



4. You are deciding between two portfolios. Portfolio 1 combines L (labor income) with M (market portfolio), portfolio 2 combines L with G (Ford stock). It is easy to verify that portfolio 1 has a higher Sharpe ratio, so in the absence of a discount on the Ford stock, you should not participate in FLOP.

What is the minimum expected return on portfolio 2 (denote it by $\bar{E}_2^{20}$) for which the Sharpe ratio of portfolio 2 (after the Ford discount) is at least as large as the Sharpe ratio of portfolio 1?

$$\frac{\bar{E}_2^{20} - R_f^{20}}{\sigma_2^{20}} = \frac{\mathrm{E}_1^{20} - R_f^{20}}{\sigma_1^{20}},$$

where E_1^{20}, σ_1^{20}, and σ_2^{20} are the return moments of portfolios 1 and 2. Hence,

$$\bar{E}_2^{20} = R_f^{20} + \frac{\sigma_2^{20}}{\sigma_1^{20}} \left[\mathrm{E}_1^{20} - R_f^{20}\right].$$

Denote the minimum required Ford return by $\bar{\bar{E}}_G^{20}$. Since $\bar{\mathrm{E}}_2^{20} = (0.5)\bar{\bar{E}}_G^{20} + (0.5)\mathrm{E}_L^{20}$,

$$\bar{\bar{E}}_G^{20} = 2\left(\bar{\mathrm{E}}_2^{20} - \frac{1}{2}E_L^{20}\right) = 3.9060.$$

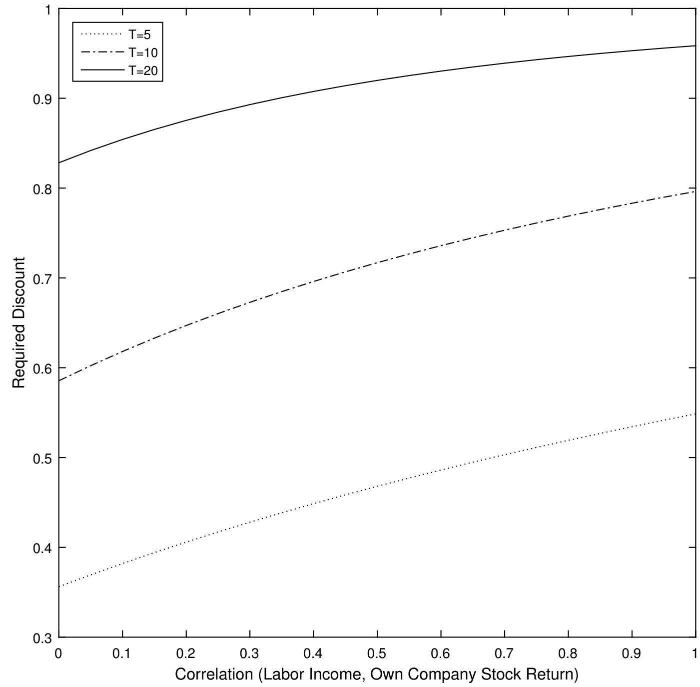
Following the same logic as before, we obtain d as

$$\begin{aligned} d &= 1 - \exp\left(E^{20}_{\text{no discount}} - E^{20}_{\text{discount}}\right) \\ &= 1 - \exp\left(E^{20}_{\text{no discount}} - \bar{\bar{E}}_G^{20}\right) \\ &= 1 - \exp\left(1.6714 - 3.9060\right) = 0.8930 = 89.30\%. \end{aligned}$$

The discount is even larger than the discount computed in part 2. In part 2, the Ford stock was unattractive because of its high volatility relative to the market (i.e., because of the lack of diversification). Here, there is an additional reason for Ford being unattractive, namely its positive correlation with your labor income. When we put both reasons together, they make the Ford stock so unattractive that you would require an 89.30% discount to be willing to participate in the FLOP plan for 20 years!

As shown in the figure below, the required discount increases with $\rho_{L,F}$, for all three T's.

5. High correlation: Investment banker. Low correlation: Janitor in a utility firm.
6. It is costly to hold the stock of your company, for at least two reasons. One is underdiversification – a single stock is generally much riskier than a portfolio of stocks. Two, the return on the company stock is positively correlated with your labor income, which deepens the underdiversification problem. (If Ford happens to get in trouble, its stock plunges, and at the same time your bonus will be cut and you might even lose your job.) We have computed the discount you should require to be willing to hold the stock of your company. This discount is large, unless you are very close to retirement.



C. EXAM-LIKE QUESTIONS.

1. (a) For the portfolio value to fall below \$4 billion, the return on S&P needs to be 4/4.6-1=-13.04% or lower. Suppose that the index returns are normally distributed. Then the probability of observing such a return under the new volatility assumption is:

$$\begin{aligned} Pr\left\{R \le -0.1304\right\} &= Pr\left\{\frac{R-0.12}{0.32} \le \frac{-0.1304-0.12}{0.32}\right\} \\ &= Pr\left\{z \le -0.7825\right\} \\ &= \Phi\left\{-0.7825\right\} = 0.2170 \end{aligned}$$

The new probability is larger than the 1/6 (or 16.67%) threshold, so the risk is too large – something needs to be done!

(b) Suppose that \$X billion is added. The portfolio's new value is \$(4.6+X) billion and the level of return required to reduce the value to \$4 is 4/(4.6+X)-1. Using the same argument as before, we look for X that solves

$$\Phi\left(\frac{\frac{4}{4.6+X}-1-0.12}{0.32}\right) = 1/6.$$

We now use Matlab's *norminv* function: *norminv*(1/6)=-0.9674. Thus,

$$\frac{\frac{4}{4.6+X}-1-0.12}{0.32} = -0.9674$$

or

$$X = 0.3357.$$

Quantie needs to add \$335.7 million to reach the 1/6 threshold exactly.

(c) Suppose that the new portfolio assigns the weight of α to S&P and $1-\alpha$ to the T-bill. That portfolio's returns are normally distributed with mean

$$E(R) = 0.12\alpha + 0.04(1-\alpha) = 0.08\alpha + 0.04$$

and standard deviation of 0.32α. To reach the 1/6 threshold, α needs to solve

$$\Phi\left(\frac{-0.1304 - (0.08\alpha + 0.04)}{0.32\alpha}\right) = 1/6.$$

Since *norminv*(1/6)=-0.9674, we have

$$\frac{-0.1304 - (0.08\alpha + 0.04)}{0.32\alpha} = -0.9674.$$

The solution is $\alpha = 0.7423$. Thus, at least $\$(1-0.7423)\times 4.6 = 1.1854$ billion needs to be transferred to the T-bill.

(d) A shift from stocks to long-term bonds may well create shareholder value, for at least two main reasons. One, it better aligns the pension fund's assets and liabilities. By guaranteeing the minimum level of pension payments, a corporation provides insurance to the pension plan members, and the value of this insurance is rarely fully offset by the value of the potential surplus in the pension plan to the corporation. Two, holding stocks in a pension plan is less advantageous than holding bonds, from the tax perspective. Recall our discussion of the Boots case.

2. Probably in continental Europe, which has significantly lower projected population growth and substantially less developed private saving schemes (so the government-provided pensions still account for the bulk of the retirement income).

3. Here is a partial list of the UK-U.S. differences:

   - The UK equivalent of Social Security is different from U.S. Social Security
     * It is a flat-rate system
     * The payments are substantially lower than in the U.S.
     * There are also secondary, earnings-based government pensions, from which employees can opt out if they have private alternatives
   - The importance of DB (versus DC) schemes is relatively higher in UK (although the trend from DB to DC is the same in both countries)
   - In UK, DB pensions must be indexed to inflation, by law
   - Under UK accounting (FRS 17), both DB assets and liabilities are to be included on the balance sheet (under U.S.'s SFAS 87, only liabilities)

4. True. Lower interest rates inflate the present value of future pension liabilities. They often also increase the value of the pension fund assets, but the increase in liabilities tends to outweight the potential increase in assets because pension liabilities generally have higher duration than pension assets.

5. John Ralfe's view is presented from the perspective of the Boots' shareholders. This view agrees with what we discussed in class.

   Chris Daykin argues that a pension fund "should be diversified" instead of being invested only in bonds. This view has its merits, but it seems that a company should pay attention to the diversification/riskiness of its assets-minus-liabilities position, rather than just the asset side (the pension fund). Since pension fund liabilities resemble long-term bonds more than they resemble stocks, buying long-term bonds on the asset side should produce a less risky assets-minus-liabilities position than buying stocks.

   Daykin also argues that since DB benefits are linked to future wages, the pension fund should hold "a real claim on the economy," read stocks. Again, this makes some sense, but academic research shows that nominal stock returns are only weakly correlated with inflation (in fact, over short horizons, they are weakly negatively correlated). TIPS might provide a more reliable way of protecting a pension portfolio against inflation.

   Note that Mr. Daykin is a government official, and thus less concerned about the welfare of the Boots' shareholders than Mr. Ralfe (head of Boot's corporate finance) is.

   The article concludes that "Equities should remain a mainstay of pension funds' holdings because both have long time horizons." But long-term bonds do exist in Britain; in fact, there even exist consol bonds with infinite maturity. According to the Boots case, the duration of the Boots' pension fund liabilities is about 17 years. There are plenty of British government bonds with comparable or longer maturities.

   The article also concludes that "If you are planning 40 years ahead, short-term volatility becomes less of a worry." Such a statement is misleading, because it implicitly suggests that the objective of the Boots' decision to hold bonds was to reduce short-term volatility. That is a deep misunderstanding. Long-term bonds exhibit nontrivial short-term volatility. Their usefulness is instead in helping Boots immunize its long-term pension liability.

   This superficial article also ignores many additional arguments regarding the stock vs. bond choice that we discussed in class.