UNIVERSITY OF CHICAGO
Booth School of Business

Bus 35120 – Portfolio Management Prof. Lubos Pastor

# Assignment #5
Solutions

## B. DATA ANALYSIS.

The solutions given below were obtained in MATLAB using the program *hwk5_solutions.m*, which you can download from Canvas.

**1.**

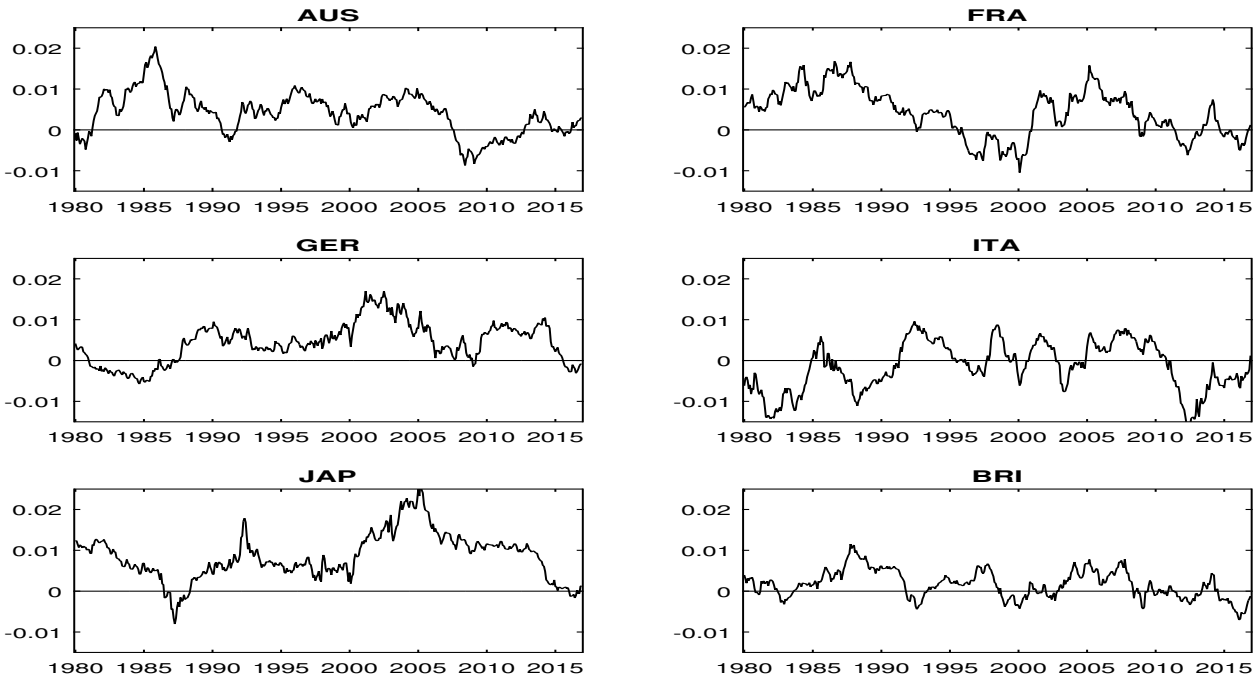
The table below summarizes the monthly means, standard deviations, and $t$-statistics of the six international value portfolios. (Averages and standard deviations are in percent.)

| | AUS | FRA | GER | ITA | JAP | UK |
|---|---|---|---|---|---|---|
| Average | 0.4365 | 0.3763 | 0.4091 | -0.1228 | 0.8436 | 0.1610 |
| Std dev | 4.1089 | 4.5787 | 3.9894 | 5.2747 | 4.5108 | 3.7359 |
| t-stat | 2.3847 | 1.8449 | 2.3021 | -0.5225 | 4.1987 | 0.9673 |

The value effect is significantly positive for Australia, Germany, and Japan; it is positive but insignificant for France and UK, and insignificantly negative for Italy. While the value effect clearly does not hold in every country, I would agree with the casewriter in that the value effect seems fairly robust internationally.

**2.**

The plots are on the following page. By and large, value underperformed growth during the 2007-2008 financial crisis; between July 2007 and December 2008, the average return on the value-minus-growth portfolio was negative in five of the six countries (as well as in the U.S.). This evidence suggests that value stocks perform poorly in bad states of the world, which is broadly consistent with the risk-based explanation for the value premium.




**3.**

| | USA | AUS | FRA | GER | ITA | JAP | UK |
|---|---|---|---|---|---|---|---|
| USA | 1 | 0.2250 | 0.2748 | 0.3257 | 0.1360 | 0.1704 | 0.2902 |
| AUS | | 1 | 0.1200 | 0.0921 | 0.0555 | 0.0198 | 0.2143 |
| FRA | | | 1 | 0.2908 | 0.3105 | 0.1600 | 0.3765 |
| GER | | | | 1 | 0.1346 | -0.0174 | 0.2103 |
| ITA | | | | | 1 | 0.1657 | 0.1740 |
| JAP | | | | | | 1 | 0.1130 |
| UK | | | | | | | 1 |

I would disagree with the casewriter, who writes about a "high level of correlation among value-growth portfolios." While most correlations are indeed positive, they seem fairly low.

How do we judge whether the correlations are low? Recall from statistics that correlation squared between $X$ and $Y$ can be interpreted as the $R^2$ from the regression of $X$ on $Y$. The highest correlation in the table is 0.3765; this implies $R^2 = 0.1418$, so that only 14% of the variation in the French HML can be explained by the variation in its UK counterpart. The $R^2$s are even lower for all other pairs of value-growth spreads.

**4.**

Definitely. Given the low correlations, substantial diversification can be achieved by investing in international value-growth spreads.

**5.**

The weights in the minimum variance portfolio of the seven spreads are

| USA | AUS | FRA | GER | ITA | JAP | UK |
|---|---|---|---|---|---|---|
| 0.2928 | 0.1692 | 0.0125 | 0.1564 | 0.0739 | 0.1594 | 0.1357 |

The standard deviation of this portfolio is 2.12% per month, vs. 2.91% for the U.S. HML. No surprise. This must be true, by construction.

**6.**

Recall that the optimal weight in the market in combination with a T-bill is

$$w^* = \frac{E_M - r_f}{\gamma \sigma_M^2}$$

Here, $w^* = 1$, from which $\gamma = (E_M - r_f)/\sigma_M^2 = 3.4297$.

**7.**

The weights in the optimal portfolio are

| US Market | HML | T-bill |
|---|---|---|
| 1.2622 | 1.6623 | -1.9245 |

This portfolio involves shorting 192% ($1.92m) in T-bills. This is a highly levered position; the investor borrows heavily (we assume borrowing at the T-bill rate is possible) to be able to take advantage of the remarkable risk-return tradeoff on HML. In practice, an investor is unlikely to be willing to take such an extreme position, especially since it has been computed based on historical sample estimates.

**8.**

The Sharpe ratio increases from 0.1527 (market only) to 0.2217 (market plus HML). The squared Sharpe ratio increases by 0.0259.

**9.**

The information ratio, $\alpha/\sigma_e$, is computed from a regression of HML on MKT: 0.1608. Its square is 0.0259, which is equal to the improvement in the squared Sharpe ratio computed in the previous part. This is not a coincidence—it must always be the case!

**10.**

The weights in the optimal portfolio of the eight assets and the T-bill are

| USA | HML | AUS | FRA | GER | ITA | JAP | UK | T-bill |
|---|---|---|---|---|---|---|---|---|
| 1.4245 | 1.1679 | 0.6986 | -0.0457 | 0.6108 | -0.5018 | 1.3589 | -0.4197 | -3.2934 |

Another highly levered position, with the same caveat as before.

**11.**

The Sharpe ratio of the above international portfolio is 0.3235, which is much higher

than the ratios achievable domestically. This improvement reflects the large benefits of going global.

Caveat: these are in-sample Sharpe ratios, obtained with the benefit of hindsight. The out-of-sample Sharpe ratios achievable by investors are likely to be lower. Nonetheless, going global seems to be the way to go.

**C. EXAM-LIKE QUESTIONS.**

**1.**

If the CAPM holds, the true alphas ($\alpha$) in the regression should all be equal to zero. Given that there is noise in the data, we only expect that the estimated alphas ($\hat{\alpha}$) be statistically indistinguishable from zero.

The CAPM does not make any predictions for stock betas.

The only restriction on the betas is that their value-weighted average across all stocks must be equal to one (because the market portfolio is value-weighted, and its beta is one, $\beta = cov(R_M, R_M)/var(R_M) = 1$). However, this is true whether the CAPM holds or not.

**2.**

First, note that the CAPM is a special case of the Fama-French model in which the risk premia on SMB and HML are both zero, $\lambda_{SMB} = \lambda_{HML} = 0$. (We know that the sample estimates $\hat{\lambda}_{SMB}$ and $\hat{\lambda}_{HML}$ are not zero (they are positive), but that does not prove with 100% certainty that $\lambda_{SMB}$ and $\lambda_{HML}$ are also nonzero.)

So, if the CAPM holds, then a special case of the Fama-French model holds. And if (some case of) the Fama-French model holds, then $\alpha$ from the regression of any stock's excess returns on the three Fama-French factors must be zero. See a remark in Lecture 5.

The CAPM does not make any predictions for the three factor betas. Many students wrote that the CAPM implies zero betas on SMB and HML; that is incorrect. Under the CAPM, SMB and HML can be factors (i.e., stocks can have non-zero betas with respect to SMB and HML), they just cannot be priced factors (i.e., their risk premia, $\lambda_{SMB}$ and $\lambda_{HML}$, cannot be different from zero). The distinction between factors and priced factors is discussed in Lecture 5.

**3.**

They have high betas with respect to the Fama-French factors. Specifically, small stocks tend to have high betas with respect to SMB (because small stocks tend to covary more with small stocks than with big stocks, so their covariance with small-minus-big is positive), and value stocks tend to have high betas with respect to HML (because value stocks tend to covary more with value stocks than with growth stocks, so their covariance with value-minus-growth (HML) is positive).

Fama and French argue that SMB and HML are proxies for aggregate risks that investors cannot diversify and thus require to be compensated for. They conjecture that size and value are related to the risk of distress. Companies more likely to become distressed (with small size or high B/M ratio) are less valuable to investors, and thus command lower prices (i.e., higher expected returns).

**4.**

The estimated Sharpe ratio is the ratio of the sample average excess return and the sample standard deviation. The $t$-statistic for the test whether the average excess return is zero equals the Sharpe ratio times the square root of the number of observations:

$$\begin{aligned} SR &= \frac{\text{Avg}(R - R_f)}{\sigma(R)} \\ t &= \frac{\text{Avg}(R - R_f)}{\frac{\sigma(R)}{\sqrt{T}}} = \frac{\text{Avg}(R - R_f)}{\sigma(R)}\sqrt{T} = SR \times \sqrt{T} \end{aligned}$$

Lecture 5 provides the average annual returns over the 90 years from 1927 to 2016 – MKT: 8.4% ($t = 3.9$), SMB: 3.3% ($t = 2.3$), HML: 5.1% ($t = 3.4$). Only the $t$-stats are important. Given those, the Sharpe ratio of MKT is $3.9/\sqrt{90} = 0.41$, the Sharpe ratio of SMB is $2.3/\sqrt{90} = 0.24$, and the Sharpe ratio of HML is $3.4/\sqrt{90} = 0.36$.

**5.**

Most overpriced: small growth stocks (highest negative alpha). Most underpriced: big growth stocks and small value stocks (highest positive alphas). The overpricing of small growth stocks is the biggest problem of the FF model.

**6.** The typical stock market investor (a criminal) would now want to buy insurance against crime going down, not up. Gun stock prices would drop relative to the BMW stock price. (Gun stock expected returns would go up relative to the BMW expected return.)

Lesson: Different investors have different exposures to systematic risks (such as crime). A stock may look fairly priced to the average investor, but investors have different risk exposures, so some investors will find the stock attractive at the current price and others will find it unattractive. This argument helps explain why some people want to buy a given stock and others want to sell even in a perfectly efficient market where the "price is right".

**7.** True.

$$S_{B+P}^2 = S_B^2 + \left(\frac{\alpha}{\sigma_\epsilon}\right)^2,$$

where $S$ denotes the Sharpe ratio, $B$ is the benchmark, and $\frac{\alpha}{\sigma_\epsilon}$ is the information ratio.

**8.** False. Slicing actually reduces flexibility because it allows one to update only $1/N$ of the portfolio at once, where $N$ is the number of slices. The main benefit of slicing is instead to reduce transaction costs.

**9.** When Standard & Poor's computes the total earnings of the S&P 500 index, they add up the earnings of the 500 individual firms. Jeremy Siegel claims that they should value-weight earnings instead, weighting each company's earnings by its market capitalization. I don't understand why it would make sense to value-weight earnings.

Suppose you want to compute the total earnings of a company that has 500 divisions. The right thing to do is to add up the earnings of all of the company's divisions. It would make little sense to value-weight the earnings of the 500 divisions. Suppose that 499 largest divisions all make a profit of \$1, but the smallest division (a mortgage-backed trading desk?) produces a loss of \$499. The firm's total profit is obviously zero, not the 99 cents or so that you would get by value-weighting across divisions. It does not matter whether the big loss was incurred by a small division or a large division—the company's total profits are reduced equally in both cases.

The above analogy seems to suggest that Siegel's argument is plain wrong. There is, however, a difference between divisions comprising the same company and firms comprising the index. Whereas losses of one division do cancel out profits of another division of the same company, losses of one firm do not necessarily cancel out profits of another firm in the same index. Most of the time they do, but now and then they don't.

The reason is that shareholders have limited liability, and if a firm goes bankrupt, some of the losses are borne by the bondholders (or the taxpayers). Shareholders holding the S&P 500 index did not care whether Lehman Brothers lost \$100 billion or \$500 billion; their Lehman stock was worth zero either way, and the losses beyond those causing bankruptcy did not offset the earnings of any other firms in the index. (In contrast, a single division of a firm cannot go bankrupt; the divisions are cross-subsidized within the firm.) If we do not recognize limited liability, we understate the value of the index. Recall that a stock can be viewed as a call option on the firm's assets, with the strike price equal to the face value of the firm's debt. A portfolio of options on many individual firms is worth more than a single option on the portfolio of all firms.

This bankruptcy-related argument is the only argument I can think of for why adding up earnings (as the S&P does) is not necessarily the right thing to do for the purpose of valuing an index. I therefore find it surprising that Siegel's article does not mention bankruptcy at all. Perhaps it was supposed to be an obvious point.

But even after we recognize the bankruptcy issue, it is not clear why value-weighting should fix the problem. After all, even large firms can go bankrupt. It would make sense to value-weight earnings per dollar of market value (or, equivalently, to share-weight earnings per share), but that brings you back to adding up earnings, which is what S&P does.

Most of the time, it does not make much difference whether you add earnings up or value-weight them because larger firms tend to have larger earnings. The bankruptcy issue should make you cautious when using the S&P's aggregate earnings for valuation purposes, but it is not clear that value-weighting is an improvement, nor what the optimal way of aggregating earnings is. On balance, I think the S&P's approach is just fine.