Mutual Fund Flows and Performance in Rational Markets[1]

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Abstract

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We derive a parsimonious rational model of active portfolio management that reproduces many regularities widely regarded as anomalous. Fund flows rationally respond to past performance in the model even though performance is not persistent and investments with active managers do not outperform passive benchmarks on average. The lack of persistence in returns does not imply that differential ability across managers is nonexistent or unrewarded, or that gathering information about performance is socially wasteful. The model can quantitatively reproduce many salient features in the data. The flow-performance relationship is consistent with high average levels of skills and considerable heterogeneity across managers.

One of the central mysteries facing financial economics is why financial intermediaries appear to be so highly rewarded, despite the apparent fierce competition between them and the uncertainty about whether they add value through their activities. Research into mutual fund performance has provided evidence that deepens this puzzle. Since Jensen (1968), studies have shown little evidence that mutual fund managers outperform passive benchmarks. Recent work has produced several additional findings. The relative performance of mutual fund managers appears to be largely unpredictable using past relative performance.[1] Nevertheless, mutual fund investors chase performance. Flows into and out of mutual funds are strongly related to lagged measures of excess returns (see Chevalier and Ellison (1997) or Sirri and Tufano (1998)).

The evidence that performance does not persist is widely regarded as implying that superior performance is attributable to luck rather than differential ability across managers.[2] This implication, were it true, would be troubling from an economic point of view. If all performance is due to luck, there should be no reason to reward these talents. Yet, in reality, managers appear to reap rich rewards from superior past performance. Together these findings have led researchers to raise questions about the rationality of investors, who place money with active managers despite their apparent inability to outperform passive strategies and who appear to devote considerable resources to evaluating past performance of managers.[3] In the face of this evidence many researchers have concluded that a consistent explanation of these regularities is impossible without appealing to behavioral arguments that depend on irrationality, or to elaborate theories based on asymmetric information or moral hazard. One thing that has been missing from this debate is a clear delineation of what a rational model, with no moral hazard or asymmetric information, implies about flows and performance. Before appealing to these additional effects we believe it makes sense to first establish which behaviors in the data are qualitatively and quantitatively consistent with more direct explanations.

Our simple model of active portfolio management and fund flows provides a natural benchmark against which to evaluate observed returns, flows, and performance outcomes in this important sector of the financial services industry. Using this model we show the effects discussed above are generally consistent with a rational and competitive market for capital investment, and with rational, self-interested choices by fund managers who have differential ability to generate abnormal returns.

[1] While some controversial evidence of persistence does exist (see Gruber (1996), Carhart (1997), Zheng (1999) and Bollen and Busse (2001)) it is concentrated in low liquidity sectors or at shorter horizons.

[2] Indeed, various researchers have interpreted this fact as evidence for "market efficiency." For example, see Malkiel (1995, p. 571) or Ross, Westerfield and Jaffe (2002, p. 353).

[3] For example, see Daniel *et al* (1997), Bollen and Busse (2001) and Gruber (1996)

The model combines three elements. There is competitive provision of capital by investors to mutual funds. There is differential ability to generate high average returns across managers, but decreasing returns to scale in deploying these abilities. Finally, there is learning about managerial ability from past returns. Some of these features have been examined in other models. Our contribution is to show that together they can reproduce the salient features of the empirical evidence as equilibrium outcomes in a rational model.

In our model investments with active managers do not outperform passive benchmarks because investors competitively supply funds to managers and there are decreasing returns for managers in deploying their superior ability. Managers increase the size of their funds, and their own compensation, to the point where expected returns to investors are competitive going forward. The failure of managers as a group to outperform passive benchmarks does not imply that they lack skill. Furthermore, the lack of persistence does not imply that differential ability across managers is unrewarded, that gathering information about performance is socially wasteful, or that chasing performance is pointless. It merely implies the provision of capital by investors to the mutual fund industry is competitive.

Performance is not persistent in the model precisely because investors chase performance and make full, rational, use of information about funds' histories in doing so. High performance is rationally interpreted by investors as evidence of the manager's superior ability. New money flows to the fund to the point that expected excess returns going forward are competitive. This process necessarily implies investors cannot expect to make positive excess returns, so superior performance cannot be predictable. The response of fund flows to performance is simply evidence that capital flows to investments where it is most productive.

The logic underlying these results is similar to standard models of corporation finance. Capital is supplied with perfect elasticity to managers whose differential ability allows them to identify positive net present value opportunities. Since these opportunities, or the ability to identify them, are the resource that is ultimately in scarce supply, the economic rents thus created flow through to the managers who create them, not to the investors who invest in them. In corporate finance new money is raised to fund investments on competitive terms. Investors supplying these funds do not earn high expected returns, but this is not evidence that managers are all the same or that firms lack valuable investment opportunities. Similarly, in our model, the competitive returns investors earn do not tell us managers lack ability. What is unique about mutual funds is the equilibrating mechanism. In corporate finance models the price of the firm's securities adjusts to ensure returns going forward are competitive. With open-ended mutual funds this cannot happen. The adjustment must come through quantities, or fund flows, rather than price.

Our model is related to several recent papers, though none of these are directly aimed at reconciling the lack of return persistence with the responsiveness of flows to past performance. Bernhardt, Davies, and Westbrook (2002) study short-term return persistence in a model where privately informed managers trade to maximize funds under management. The response of funds to performance in their model, however, is exogenous. In our model, the flow-performance relationship is endogenous, and consistent with no persistence in performance. Ippolito (1992) and Lynch and Musto (2002) endogenize the flow of funds. As in our paper, investors and managers learn about manager's abilities and the profitability of their strategies from past returns. However, in both of these models differences in ability lead to persistent differences in performance. We show that rational learning and strong response of flows to performance can be consistent with no persistence in performance. Nanda, Narayan and Warther (2000) develop a three-date model with heterogeneous managerial ability. They use the model to derive endogenous heterogeneous fee structures involving loads as managers compete for clients with different expected liquidity needs. In their model managerial ability is known, so there is no role for learning about managerial ability, and the response of the flow of funds to it. These are central concerns in our paper. Finally, in a corporate setting, Holmström (1999) considers a dynamic multi-period moral-hazard problem in which learning about managerial ability is very similar to that in our model. He characterizes the resulting optimal contract. Our goals are very different. There is no moral hazard or asymmetric information in our model, so the simple compensation scheme we consider is optimal, funds are allocated efficiently across managers, and all outcomes are first-best.

The paper is organized as follows. In the next section we lay out the elements of the model, and in Section 2 we develop its implications for the flow of funds and fund life cycles. Section 3 derives the flow of funds and performance relationship for a cross section of funds with different histories. In Section 4 we parameterize the model and evaluate its quantitative implications. Section 5 concludes.

1 The Model

We begin by describing the simplest version of our model that produces the two central behaviors the literature has struggled to reconcile: flows that are responsive to performance, and performance that is not persistent. We will then elaborate on the model to show these outcomes are not only consistent with investor rationality, but also with efficient provision of portfolio management services. For convenience, we include a summary of the important notation in the appendix.

All participants in the model are symmetrically informed. Funds differ in their managers' ability to generate expected returns in excess of those provided by a passive benchmark—an alternative investment opportunity available to all investors with the same risk as the manager's portfolio. The model is partial equilibrium. Manager's actions do not affect the benchmark returns, and we do not model the source of successful managers' abilities. A manager's ability to beat this benchmark is unknown to both the manager and investors, who learn about this ability by observing the history of the managed portfolio's returns. Let

$$R_t = \alpha + \epsilon_t$$

denote the return, in excess of the passive benchmark, on the actively managed funds, absent costs and fees. This is not the return actually earned and paid out by the fund, which is net of costs and fees (see below). The parameter α is the source of differential ability across managers. The error term, ϵ_t, is normally distributed with mean zero and variance σ^2, and is independently distributed through time. We further assume that this uncertainty is idiosyncratic to the manager—by investing with a large number of these managers, investors can diversify away this risk. Denote the precision of this uncertainty as $\omega = \frac{1}{\sigma^2}$. Participants learn about α by observing the realized excess returns the manager produces. This learning is the source of the relationship between performance and the flow of funds.

To achieve high returns management must identify undervalued securities and trade to exploit this knowledge without moving the price adversely. To do this managers must expend resources and pay bid-ask spreads that diminish the return available to pay out to investors. We assume these costs are independent of ability, and are increasing and convex in the amount of funds under active management. Denote the costs incurred when actively managing a fund of size q_t as $C(q_t)$, and assume for all $q \geq 0$ $C(q) \geq 0$, and $C''(q) > 0$, while $C(0) = 0$, and $\lim_{q \to \infty} C'(q) = \infty$. These assumptions capture the notion that with a sufficiently large fund a manager will spread his information gathering activities too thin, or that large trades will be associated with a larger price impact and higher execution costs.

Assume the fund managers are simply paid a fixed management fee, f, expressed as a fraction of the assets under management, q_t. (We will relax this assumption shortly.) Managers accordingly accept and invest all the funds investors are willing to allocate to them. The amount investors will invest in the fund depends on their assessment of the managers' ability, and on the costs they know managers face in expanding the fund's scale.

The excess total payout to investors over what would be earned on the passive benchmark is:

$$TP_{t+1} = q_t R_{t+1} - C(q_t) - q_t f.$$

Let r_t denote the excess return over the benchmark that investors in the fund receive in period t. Then

$$\begin{aligned} r_{t+1} &= \frac{TP_{t+1}}{q_t} = R_{t+1} - \frac{C(q_t)}{q_t} - f \\ &= R_{t+1} - c(q_t), \end{aligned} \tag{1}$$

where

$$c(q_t) \equiv \frac{C(q_t)}{q_t} + f \tag{2}$$

denotes the unit cost associated with investing in the actively managed fund. The return r_t corresponds to the return empirically observed.

At the birth of the fund the participants' priors about the ability of the fund's management, α, are normally distributed with mean ϕ_0 and variance η^2. Since all participants are assumed to have rational expectations, this is also the distribution of skills across new funds. The precision of the prior will be denoted $\gamma = \frac{1}{\eta^2}$. Investors (and the manager) update their posteriors based on the history of observed returns as Bayesians. The costs faced by the funds are common knowledge, and the resources each fund has under management are observable. Thus, by observing the history $\{r_s, q_s\}_{s=0}^t$ investors can infer the history $\{R_s\}_{s=0}^t$. Let the posterior mean of management ability at time t be denoted

$$\phi_t \equiv E(R_{t+1} \mid R_1, \ldots, R_t).$$

The timing convention is as follows. The fund enters period t with q_{t-1} funds under management and an estimate of managerial ability, ϕ_{t-1}. Managers and investors observe r_t (from which they can infer R_t), and update their estimate of the manager's ability by calculating ϕ_t. Then capital flows into or out of the fund to determine q_t. In the next section we will calculate this flow of funds explicitly.

2 The Flow of Funds and Performance

We assume investors supply capital with infinite elasticity to funds that have positive excess expected returns. This can be justified as long as ϵ_t is idiosyncratic risk—by diversifying across funds with positive excess expected returns, investors can achieve the average excess expected return with certainty. Similarly, they remove all funds from any fund that has a negative excess expected return. At each point in time, then, funds flow to and from each fund so that the expected excess return to investing in any surviving fund is zero:

$$E_t(r_{t+1}) = 0. \tag{3}$$

As in any equilibrium with perfect competition, the marginal return on the last dollar invested must be zero. In this case, however, since all investors in open-ended mutual funds earn the same return, *all* investors earn zero expected excess return in equilibrium. Condition (3) clearly implies that there is no predictability or persistence in funds' excess returns, and that the average excess return of all managers will be zero, regardless of their overall level of skill. Finally, we will now show it implies flows in and out of the fund respond to performance.

Taking expectations of both sides of (1), and requiring expected excess returns of zero as in (3), gives

$$\phi_t = c(q_t) = \frac{C(q_t)}{q_t} + f. \tag{4}$$

As ϕ_t changes, q_t changes to ensure that this equation is satisfied at all points in time. The following proposition derives the flow of funds relation.

Proposition 1 *For any fund in operation at dates $t-1$ and t, the evolution of ϕ_t and the change in the amount of money under management at any date, as a function of the prior performance, is the solution to*

$$\phi_t = \phi_{t-1} + r_t \frac{\omega}{\gamma + t\omega} \tag{5}$$

$$c(q_t) = c(q_{t-1}) + r_t \frac{\omega}{\gamma + t\omega} \tag{6}$$

Proof: It is straightforward to show from DeGroot (1970) (Theorem 1, p. 167), that the mean of investor's posteriors will satisfy the following recursion:

$$\phi_t = \frac{\gamma + (t-1)\omega}{\gamma + t\omega}\phi_{t-1} + \frac{\omega}{\gamma + t\omega}R_t \tag{7}$$

where ϕ_0 is the mean of the initial prior. Next, use (4), evaluated at both t and $t-1$ to substitute for ϕ_t and ϕ_{t-1} in this expression. Finally, use the fact that the realized return is given by $r_t = R_t - c(q_{t-1})$. These substitutions yield equation (6). Appealing, again, to (4) gives (5). QED

Equation (6) reproduces the observed relationship between performance and fund flows. Note that $c'(q) = \frac{1}{q}\left(C'(q) - \frac{C(q)}{q}\right) > 0$, so good performance (a positive realization of r_t) results in an inflow of funds ($q_t > q_{t-1}$).[4] Similarly, bad performance results in an outflow of funds. A smaller t increases the weight on r_t, so flows to younger funds respond much more dramatically to performance than flows to more mature funds.

2.1 Efficient Provision of Portfolio Management Services

The assumption that fees are fixed may appear restrictive. A manager could, presumably, raise his compensation by choosing the fee optimally in each period, and in this way always invest efficiently. We now show that if managers can expand the fund by investing a portion of it in the passive benchmark (i.e., "closet indexing"[5]), efficient outcomes can be achieved with a proportional fee that does not change over time and is not contingent on performance.

We begin by first allowing managers to optimally choose the fee they charge at each point in time, f_t. Every manager is constrained by the investor participation constraint (4)—investors in the fund must earn (at least) zero excess returns. Rewriting (4) now gives:

$$q_t\phi_t - C(q_t) = q_t f_t. \tag{8}$$

The manager's objective is to maximize compensation—the right-hand side of (8). He does this by first determining what level of q_t maximizes the left-hand side of (8). This choice will set the expected excess return on the marginal dollar equal to the marginal cost of expansion:

$$\phi_t = C'(q_t). \tag{9}$$

Let $q_t^*(\phi_t)$ be the solution of this equation.[6] The manager then selects a fee to ensure that investors choose to invest $q_t^*(\phi_t)$ (by using (8)):

$$f_t^* = \phi_t - \frac{C(q_t^*(\phi_t))}{q_t^*(\phi_t)}. \tag{10}$$

This compensation scheme allows managerial compensation to depend on history, but it is

[4] We know by the mean value theorem that for any q there exists a point $\bar{q}, 0 \leq \bar{q} \leq q$, such that $\frac{C(q)-C(0)}{q} = C'(\bar{q})$. Strict convexity of $C(\cdot)$ and the assumption $C(0) = 0$ then imply $C'(q) > C'(\bar{q}) = \frac{C(q)}{q}$.

[5] While used in the industry and financial press, the expression "closet indexing" should be interpreted with caution in this setting. There is no moral hazard in the model, and thus no cost or inefficiency associated with the indexing. In the model nothing is hidden in the "closet." Investing in the passive benchmark serves simply as a means of compensating managers for value they are creating.

[6] Existence of a solution is guaranteed under our assumptions on $C(q)$.

not contingent on current performance. The fee, f_t, depends on a conditional expectation, so it is implicitly a function of $r_s, s < t$, but it is not a function of r_t. It is nevertheless efficient. The left-hand side of (8) is the value the manager adds. Since this contract maximizes this value and then delivers it all to the manager, no other contract exists that could make the manager better off. Investors are constrained by competition to always make zero excess expected returns in any case. The contract is also Pareto efficient: no other contract could make both investors and the manager better off.

If we allow managers to index part of their funds, we can use *any* fixed fee, $f > 0$, to implement exactly the same compensation to the manager and returns to investors. Assume that of the total amount of money under management, q_t, the manager always chooses to invest $q_t^*(\phi_t)$ in active management, and let q_{It} denote the remaining amount of money that managers choose to index. Investors still pay the managment fee, f, on this money but because it is not actively managed, it does not earn excess returns and does not contribute to the costs, $C(\cdot)$. Under these conditions the return investors earn on their money is

$$r_{t+1} = h(q_t)R_{t+1} - c(q_t) \tag{11}$$

where $h(q_t)$ is the proportion of funds under active management, $q_t^*(\phi_t)/q_t$ and the unit cost function is now given as:

$$c(q_t) = \frac{C(q_t^*(\phi_t))}{q_t} - f. \tag{12}$$

Taking expectations of (11) and imposing the participation condition, (3), gives

$$h(q_t)\phi_t = c(q_t) \tag{13}$$

Writing out $c(q_t)$ and $h(q_t)$ explicitly gives:

$$q_t^*(\phi_t)\phi_t - C(q_t^*(\phi_t)) = (q_t^*(\phi_t) + q_{It})f. \tag{14}$$

The left-hand side of (14) is identical to the left-hand side of (8), so the right hand sides must also be the same:

$$q_t^*(\phi_t)f_t^* = (q_t^*(\phi_t) + q_{It})f. \tag{15}$$

Because investors compete away the economic rents, managers earn the same compensation under either contract. Since the contract under which the manager is allowed to set his fee is efficient, the fixed-fee contract must be efficient as well. Solving (15) for q_{It} gives the amount of money the manager chooses to manage passively:

$$q_{It} = q_t^*(\phi_t) \frac{f_t^* - f}{f}. \tag{16}$$

If $f > f_t^*$, then $q_{It} < 0$, implying the manager borrows or short sells the benchmark portfolio. As we will discuss further in the next section, by simply choosing a fee $f \leq f_t^*$, a manager can ensure that he will never need to short sell, and this involves no cost in terms of the manager's total compensation.

Adding the active and passive part of the manager's portfolio together (using (14)) gives the total funds under management using the fixed fee contract:

$$q_t = q_t^*(\phi_t) + q_{It} = \frac{q_t^*(\phi_t)\phi_t - C(q_t^*(\phi_t))}{f}. \tag{17}$$

In light of the revenue equivalence between the two contracts, we could choose either in our subsequent analysis. We focus on the fixed-fee contract for several reasons. First, it comes closer to the institutional setting for retail mutual funds, and these are the primary source of data for empirical studies. Mutual funds show relatively little variation in fees, through time and across funds. As discussed in Christoffersen (2001), both historical practice and regulatory constraints limit the ability of retail mutual funds to use performance contingent fees. Our theory suggests, instead, that rents can be collected through the flow of funds. Second, allowing for indexing in this way augments the responsiveness of fund flows to performance. Third, it leads to interesting and empirically realistic life-cycle effects for funds. As funds survive, age, and grow, they will have an ever larger portion of their portfolio passively invested. They will exhibit less idiosyncratic volatility and lower attrition rates. These behaviors have often been attributed to an irrationally sluggish response by investors to mediocre performance, and the opportunistic exploitation of it by fund managers. In our model these behaviors arise as a natural consequence of learning and of rents accruing to the talents that are their source.

Using (11) through (13), application of the same steps that prove Proposition 1 gives the following expressions for updating expectations about managerial ability when the fee is fixed and managers index part of their funds:

$$\phi_t = \phi_{t-1} + \frac{r_t}{h(q_{t-1})} \frac{\omega}{\gamma + t\omega} \tag{18}$$

$$\frac{c(q_t)}{h(q_t)} = \frac{c(q_{t-1})}{h(q_{t-1})} + \frac{r_t}{h(q_{t-1})} \frac{\omega}{\gamma + t\omega} \tag{19}$$

These expressions are the basis for our analysis going forward. Note all the variables governing the flow of funds are observable to investors. There is no asymmetric information or moral hazard in the model.

2.2 Entry and Exit of Funds

We now introduce assumptions that govern how funds enter and exit. This allows us to examine other behaviors that have been studied empirically for mutual funds, such as survival rates.

Suppose managers incur known fixed costs of operation, denoted F, each period. These costs can be viewed as overhead, back-office expenses, and the opportunity cost of the managers' time. Managers will choose to shut down their funds when they cannot cover their fixed costs. From (14), the manager's compensation is, $q_t^*(\phi_t)\phi_t - C(q_t^*(\phi_t))$, so the fund will be shut down whenever this expression is less than F. Let $\bar{\phi}$ be defined as the lowest expectation of management ability for which the manager will still remain in business. It solves,

$$q_t^*(\bar{\phi})\bar{\phi} - C(q_t^*(\bar{\phi})) = F.$$

Funds shut down the first time $\phi_t < \bar{\phi}$. In each period a cohort of new managers enter, and their abilities are distributed according to the market's initial prior. We will also assume that at the point a manager goes into business for the first time, he incurs additional costs he must recover from his fees, so that when a cohort enters $\phi_0 > \bar{\phi}$.

To be consistent with the institutional setting for retail mutual funds, we will assume managers cannot borrow or short sell the benchmark asset: $q_{It} \geq 0$. This rules out dynamic strategies where managers with low expected ability borrow in hopes of building or restoring a reputation that would lead to high fees in the future. Managers can ensure the short-sale constraint will never bind by choosing a fee $f \leq f_t^*$. It is optimal from their perspective to make this choice. Since managers extract all the surplus in the model, they bear the cost of any inefficient outcomes, and a binding short sale constraint would impose such costs. At any fee lower than f_t^*, the manager's compensation is given by $q_t^*(\phi_t)\phi_t - C(q_t^*(\phi_t))$, the left hand side of (14), which is independent of the fee. The particular fee chosen in the range $0 < f < f_t^*$ is therefore a matter of indifference to the manager. In our numerical implementation we set the fee equal to the highest level that will ensure the short sale constraint will never bind over the range of outcomes where the fund remains in business:

$$f = \frac{F}{q_t^*(\bar{\phi})} \tag{20}$$

At this fee, the fund goes out of business whenever $q_t < q_t^*(\bar{\phi})$. Define

$$\bar{q} \equiv q_t^*(\bar{\phi})$$

as the lowest value of q for which the fund remains viable.

3 Cross-Sectional Distribution of Managerial Talent

So far we have analyzed the relation between performance and flows in time series for a single manager. The empirical evidence concerning fund flows and survival rates, however, analyzes funds by age cohort based on the cross-sectional variation in returns across funds and the reaction of flows to these returns. In this section we derive the model's implications for the cross-sectional distribution of funds by cohort. The sections following this one uses these results to predict cross-sectional survival rates and the unconditional flow of funds.

All funds start with the same expectation of managerial ability, ϕ_0. From this point posteriors diverge depending on the manager's track record. So long as perceived ability in period t, ϕ_t, is greater than or equal to $\bar{\phi}$, we will say that *the fund survived through period* t. For such a fund, if perceived ability in period $t+1$, ϕ_{t+1}, is less than $\bar{\phi}$, then the *fund goes out of business in period* $t+1$.

For a fund that starts at time 0, let $G_t(\phi)$ be the probability, conditional on the fund surviving though period t, that the perceived talent of the manager at t, ϕ_t, is less than ϕ. The conditioning event means there is zero probability assigned to values of $\phi < \bar{\phi}$. To derive this distribution, we will first consider the joint probability that a fund born at date 0 both survives through period t *and* has $\phi_t \leq \phi$. We denote this joint probability as $\hat{G}_t(\phi)$. Let $g_t(\phi) \equiv \frac{dG_t(\phi)}{d\phi}$ and $\hat{g}_t(\phi) \equiv \frac{d\hat{G}_t(\phi)}{d\phi}$ be the associated densities.

We begin by conditioning on α, the actual talent of the manager. Let $\hat{G}_t^\alpha(\phi)$ be the probability conditional on having a manager with talent α, of the fund surviving through period t *and* infering, at time t, that the perceived talent of the manager is ϕ or less. Let $\hat{g}_t^\alpha(\phi) \equiv \frac{d\hat{G}_t^\alpha(\phi)}{d\phi}$ be the associated density.

Proposition 2 *Suppose a manager with true ability α begins operating at time 0 when the market's prior on her ability is ϕ_0. Then $\hat{g}_t^\alpha(\phi)$ is defined recursively as follows:*

$$\hat{g}_t^\alpha(\phi) = \frac{\gamma + t\omega}{\omega} \int_{\bar{\phi}}^{\infty} \hat{g}_{t-1}^\alpha(v)\, n^\alpha\left(\frac{\gamma + t\omega}{\omega}\phi - \frac{\gamma + (t-1)\omega}{\omega}v\right) dv \tag{21}$$

with the boundary condition $\hat{g}_1^\alpha(\phi) = \frac{\omega+\gamma}{\omega} n^\alpha(\frac{\gamma+\omega}{\omega}(\phi - \phi_0) + \phi_0)$ and $n^\alpha(\cdot)$ is a Normal density *function with mean α and precision ω.*

Proof:

$$\begin{aligned}
\Pr\left[\phi_t \geq \phi\right] &= \Pr\left[\phi_t \geq \phi \mid \phi_{t-1} \geq \bar{\phi}\right] \Pr[\phi_{t-1} \geq \bar{\phi}] \\
&= \Pr\left[\frac{\gamma + (t-1)\omega}{\gamma + t\omega}\phi_{t-1} + \frac{\omega}{\gamma + t\omega} R_t \geq \phi \mid \phi_{t-1} \geq \bar{\phi}\right] \Pr[\phi_{t-1} \geq \bar{\phi}] \\
&= \int_{\bar{\phi}}^{+\infty} \int_{\frac{\gamma + t\omega}{\omega}(\phi - v) + v}^{\infty} n^{\alpha}(u)du \; \hat{g}_{t-1}^{\alpha}(v)dv
\end{aligned}$$

Thus

$$\Pr\left[\phi_t < \phi\right] = 1 - \Pr\left[\phi_t \geq \phi\right] = 1 - \int_{\bar{\phi}}^{+\infty} \int_{\frac{\gamma + t\omega}{\omega}(\phi - v) + v}^{\infty} n^{\alpha}(u)du \; \hat{g}_{t-1}^{\alpha}(v)dv$$

Differentiating this expression with respect to ϕ, and thus eliminating the inner integral, gives the density in the proposition. Finally, we need to derive $\hat{g}_1$. By (7) we have that

$$\phi_1 = \frac{\gamma}{\gamma + \omega}\phi_0 + \frac{\omega}{\gamma + \omega} R_1$$

where R_1 is distributed Normal$[\alpha, \sqrt{\frac{1}{\omega}}]$. This implies that $\hat{g}_1(\phi)$ (the density of ϕ_1) is Normal $[\frac{\gamma}{\gamma+\omega}\phi_0 + \frac{\omega}{\gamma+\omega}\alpha, \frac{\sqrt{\omega}}{\gamma+\omega}]$ over the range $[\bar{\phi}, +\infty)$ and zero for $\phi < \bar{\phi}$. QED

The expression for $\hat{G}_t$, the joint probability pooling all types, now follows immediately:

Proposition 3 *Suppose a fund begins operating at time* 0 *when the market's prior on the funds manager's ability is ϕ_0. Then*

$$\hat{g}_t(\phi) = \int_{-\infty}^{\infty} \hat{g}_t^u(\phi) n^{\phi_0}(u)du \tag{22}$$

and

$$\hat{G}_t(\phi) = \int_{\bar{\phi}}^{\phi} \hat{g}_t(u)du \tag{23}$$

where $n^{\phi_0}(u)$ is a Normal density function with mean ϕ_0 and precision γ.

Proof: Follows immediately from Proposition 2 and the fact that the prior distribution for α is Normal with mean ϕ_0 and precision γ. QED

The distribution of ϕ_t, conditional on survival through date t, can now be computed directly using the unconditional probabilities:

$$G_t(\phi) = \frac{\int_{\bar{\phi}}^{\phi} \hat{g}_t(u)du}{\int_{\bar{\phi}}^{\infty} \hat{g}_t(u)du} \tag{24}$$

with associated density function

$$g_t(\phi) = \frac{\hat{g}_t(\phi)}{\int_{\bar{\phi}}^{\infty} \hat{g}_t(u)du}. \tag{25}$$

Recall that the conditioning event, survival *through* period t, implies that when $\phi_t < \bar{\phi}$, $G(\phi_t) = g(\phi_t) = 0$.

Survivorship bias for mutual funds has been an important area of empirical investigation. Clearly, empirical studies that ignore funds that failed will have biased estimates of the expected returns for the surviving funds (see, for example, Carhart *et al* (2001)). More importantly, survival rates themselves communicate information about the abilities of active managers. For example, one could examine the hypothesis that no skilled managers exist by comparing the fit of our model with $\alpha = 0$ to its performance with parameters of the prior distribution freely estimated. Survival rates are natural moments to focus on in such a test. Our model provides explicit expressions for them.

Proposition 4 *The unconditional probability a fund that starts at time* 0 *survives through period* t *is*

$$P_t = \int_{\bar{\phi}}^{\infty} \hat{g}_t(\phi)d\phi = \hat{G}(\infty)$$

and the unconditional probability that a fund shuts down in period t *is* $P_{t-1} - P_t$, *where* $P_0 = 1$.

Proof: This result follows immediately from the definitions of $\hat{g}_t(\phi)$ and $\hat{G}_t(\phi)$. QED

Now that we have derived the cross-sectional distribution of ϕ_t, we can use the results to analytically describe, within an age cohort of funds, the cross-sectional relation between past performance and fund flows and compare these to the empirical findings in the literature. That is the goal of the next section.

4 Parameterization

We begin by first specifying a cost function and then using it to derive an explicit expression for the relation between performance and the flow of funds. We then parameterize this model and quantitively compare it to the salient features of the data.

4.1 Parametric Cost Function

Assume $C(q) = aq^2$, so the variable cost function is quadratic. Under our assumption that the manager sets his fees so the constraint $q_{It} \geq 0$ does not bind,[7] (9) implies

$$q_t^*(\phi_t) = \frac{\phi_t}{2a}. \tag{26}$$

The total amount of money under management will, by (18), be

$$q_t = \frac{\phi_t^2}{4af}. \tag{27}$$

Inverting this relationship and substituting the result into (26), gives the fraction of money under active management:

$$h(q_t) = \sqrt{\frac{f}{aq_t}}. \tag{28}$$

Equation (27) together with (13) gives

$$\frac{c(q_t)}{h(q_t)} = 2\sqrt{aq_t f}. \tag{29}$$

The percentage change in funds at time t as a function of the manager's performance (so long as the fund survived to time t) can now be obtained by substituting (28) and (29) into (19) and simplifying:

$$\frac{q_t - q_{t-1}}{q_{t-1}} = \frac{r_t}{f}\left(\frac{\omega}{\gamma + t\omega}\right) + \frac{r_t^2}{4f^2}\left(\frac{\omega}{\gamma + t\omega}\right)^2 \tag{30}$$

Each of the parameters affecting the responsiveness of flows to performance has an intuitive role. As the noise in observed returns increases (ω falls) relative to the precision of the priors, investors learn less from returns about ability, and a given return triggers less response in flows. As the age of the fund, t, increases investors have more information about the

[7] The extension to the general case where the constraint may bind is straightforward, but tedious. It can be found in Berk and Green (2002)

fund's performance, and flows respond less to the next return. As fees increase, the fund becomes less attractive relative to passive alternatives and the manager earns his equilibrium compensation with a smaller amount of funds under management, making flows less sensitive to returns.

The flow of funds is zero at an excess return of zero. This suggests the empirically observed shape of the flows-performance relationship. Because of the quadratic term, flows respond more dramatically to extreme performance than to mediocre performance. While this quadratic behavior slows down the response to extreme negative performance, that will be offset in the cross section by funds closing when performance is extremely bad. That is, for r_t negative enough, the flow of funds is simply $-q_t$.

Either the flow of funds responds continuously to r_t (given by (30)), or r_t is so bad that the fund shuts down and investors withdraw all funds. The latter will occur for any r_t below a critical realization. Denote this critical realization as $r^*(\phi_{t-1})$. It is determined by finding the r_t such that the market's posterior on the manager's ability falls to $\bar{\phi}$. Using equation (18), this value is given by:

$$r^*(\phi_{t-1}) = (\bar{\phi} - \phi_{t-1})h(q_{t-1})\frac{\gamma + t\omega}{\omega}$$

Using (27) and (28) to simplify this expression gives

$$r^*(\phi_{t-1}) = 2(\frac{\bar{\phi} - \phi_{t-1}}{\phi_{t-1}})\left(\frac{\gamma + t\omega}{\omega}\right)f. \tag{31}$$

In summary, the overall change in the assets under management is given by

$$\frac{q_t - q_{t-1}}{q_{t-1}} = \begin{cases} -1 & \text{if } r_t < 2(\frac{\bar{\phi}-\phi_{t-1}}{\phi_{t-1}})\left(\frac{\gamma+t\omega}{\omega}\right)f \\ \frac{r_t}{f}\left(\frac{\omega}{\gamma+t\omega}\right) + \frac{r_t^2}{4f^2}\left(\frac{\omega}{\gamma+t\omega}\right)^2 & o.w. \end{cases} \tag{32}$$

Empirical studies, however, commonly consider the flow of *new* funds, that is, the percentage change in new assets, which is typically defined to be

$$n_t(r_t, q_t) \equiv \frac{q_t - q_{t-1}(1 + r_t)}{q_{t-1}} \tag{33}$$

Note that this measure uses q_{t-1} in the denominator rather than $q_{t-1}(1+r_t)$. Unfortunately, this definition distorts the implications of very large negative returns that cause liquidation of the fund. For any $r_t < r^*(\phi_t)$, $q_t = 0$ and $\frac{q_t - q_{t-1}(1+r_t)}{q_{t-1}} = -(1 + r_t)$. That is, under this definition, for these returns, the measure becomes *less* responsive the worse the performance.

In the limit when $r_t = -100\%$, the measure gives no response in the flow of funds. In an effort to address this issue while still maintaining consistency with the empirical estimates for other returns, we set our measure of the percentage flow of funds equal to -1 whenever liquidation occurs. With this caveat, (32) implies that:

$$n_t(r_t, \phi_{t-1}) = \begin{cases} -1 & \text{if } r_t < 2(\frac{\bar{\phi}-\phi_{t-1}}{\phi_{t-1}})\left(\frac{\gamma+t\omega}{\omega}\right) f \\ \left[\frac{1}{f}\left(\frac{\omega}{\gamma+t\omega}\right) - 1\right] r_t + \frac{1}{4f^2}\left(\frac{\omega}{\gamma+t\omega}\right)^2 r_t^2 & o.w. \end{cases} \tag{34}$$

When empiricists study the relation between past returns and the flow of funds, they often condition on age, but do not condition on perceived managerial ability. To derive this unconditional relation between the flow of funds and past returns, we integrate over the cross-sectional distribution of surviving funds in a cohort:

$$\begin{aligned} N_t(r) &= \int_{\bar{\phi}}^{\infty} n_t(r,\phi) g_{t-1}(\phi) d\phi \\ &= -\int_{\bar{\phi}}^{\rho(r)} g_{t-1}(\phi) d\phi + \int_{\rho(r)}^{\infty} \left(\left[\frac{1}{f}\left(\frac{\omega}{\gamma+t\omega}\right) - 1 \right] r + \frac{1}{4f^2}\left(\frac{\omega}{\gamma+t\omega}\right)^2 r^2 \right) g_{t-1}(\phi) d\phi \\ &= \left(\frac{1}{f}\left(\frac{\omega}{\gamma+t\omega}\right) - 1 \right)\left(1 - G_{t-1}\left[\rho(r)\right]\right) r \\ &\quad + \frac{1}{4f^2}\left(\frac{\omega}{\gamma+t\omega}\right)^2 \left(1 - G_{t-1}\left[\rho(r)\right]\right) r^2 - G_{t-1}\left(\rho(r)\right). \end{aligned} \tag{35}$$

where

$$\rho(r) \equiv \begin{cases} \frac{\bar{\phi}}{1+\frac{r}{2f}\left(\frac{\omega}{\gamma+t\omega}\right)} & r > -2\left(\frac{\gamma+t\omega}{\omega}\right) f \\ \infty & r \leq -2\left(\frac{\gamma+t\omega}{\omega}\right) f \end{cases}.$$

For a given realization of r_t, $G_{t-1}\left[\rho(r_t)\right]$ gives the unconditional probability that the fund will go out of business at time t. Clearly, when $r_t \geq 0$ no fund goes out of business ($\rho(r_t) < \bar{\phi}$) so $G_{t-1}\left[\rho(r)\right] = 0$ and the flow of funds is quadratic for any positive excess return. For a given negative excess return, the flow of funds is no longer quadratic because the unconditional probability that a fund will go out of business is positive. For very large negative returns (less than $-2\left(\frac{\gamma+t\omega}{\omega}\right) f$), shutting down is certain.

It is clear that larger fees imply less sensitivity. Fund age also attenuates fund sensitivity. Figure 1 plots the flow of funds relationship for funds of different ages. The shapes of the relation between performance and flows are reminiscent of what researchers have found empirically (see, for example, Chevalier and Ellison (1997, Fig. 1)).

With this cost function the conditional volatility of funds' excess returns as a function of

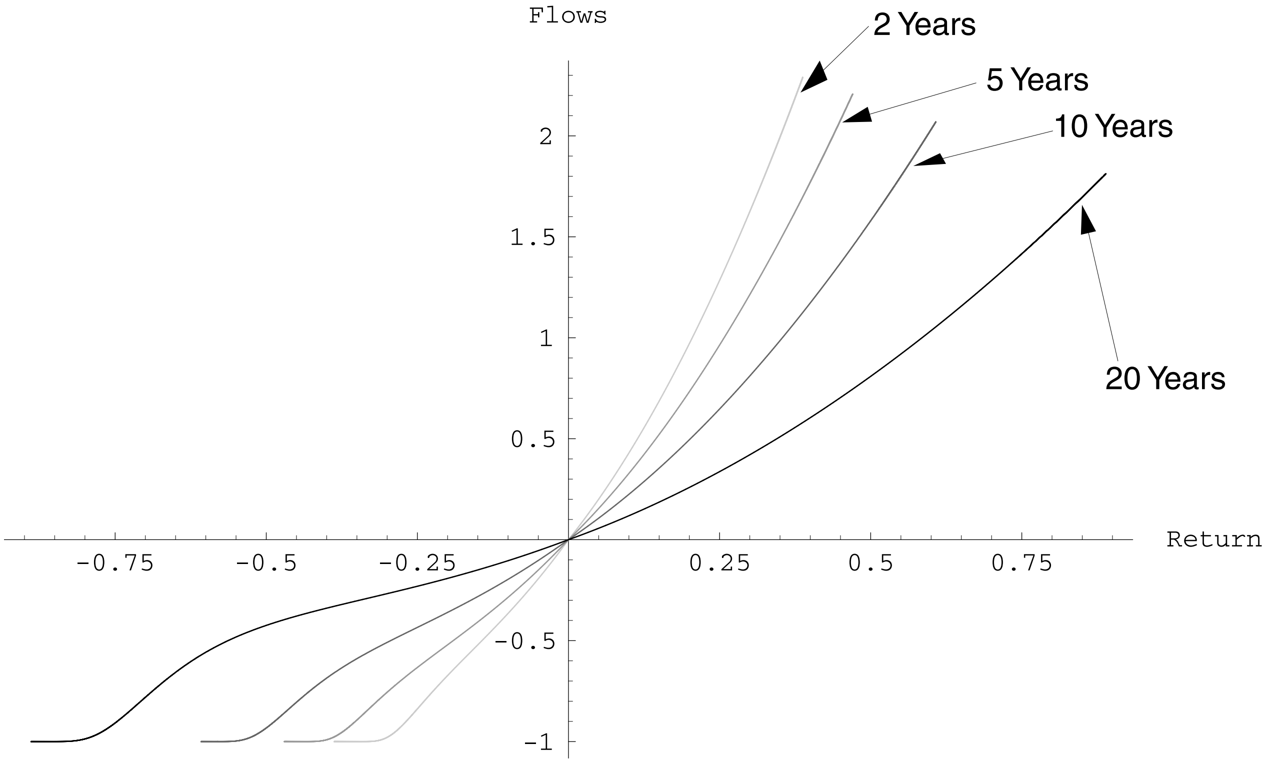

Figure 1: **Flow of New Funds as a Function of Return:** The curves plot the response in the flow of new funds to the previous period's return (i.e., equation (35)). The steepest curve shows the response for a 2 year old fund (i.e., the return is from year 2 to year 3). The remaining curves show the response for a 5, 10 and 20 year old fund respectively. The parameter values used are as follows: $\bar{\phi} = 0.03, \phi_0 = 0.065, f = 0.015, \gamma = 277$ and $\omega = 25$.

the assets under management (or the perceived quality of the manager) is:

$$\operatorname{var}_t(r_{t+1}) = \operatorname{var}_t\left(\frac{q_t^*(\phi_t)}{q_t}R_{t+1}\right) = \left(\frac{q_t^*(\phi_t)}{q_t}\right)^2 \frac{1}{\omega} = \frac{f}{aq_t\omega} = \frac{1}{\omega}\left(\frac{2f}{\phi_t}\right)^2. \tag{36}$$

The conditional volatility is a decreasing function of the size of the fund. Since past positive returns increase the size of the fund, the volatility of the fund will decrease when managers do well. The opposite occurs with negative returns. Thus, the model delivers the relation between risk and past performance that has been attributed in past research to attempts by managers to mislead investors. In our model, funds with superior past performance invest a larger portion of their new capital in passive strategies and thus lower their overall volatility, or "tracking error." Similarly, funds with poor performance increase their volatility because as funds flow out, they preferentially liquidate capital that was allocated to passive strategies.

Variable	Symbol	Value
Percentage fee	f	1.5%
Prior precision	γ	277
(Prior stan. dev.)	(η)	(6%)
Return precision	ω	25
(Return stand. dev.)	(σ)	(20%)
Mean of prior	ϕ_0	6.5%
Exit mean	$\bar{\phi}$	3%

Table 1: **Parameter Values**

4.2 Implementation

We begin by tying down the model parameters that can be inferred directly from existing evidence. The parameters that govern the distribution of skill level (ϕ_0 and γ) are then inferred by matching two moments we have derived closed from expression for—the survival probabilities and the flow of funds.

The parameter f is reasonably straightforward to determine from past empirical studies of mutual funds. We use $f = 1.5\%$. This is a bit higher than the averages reported in the literature for the expense ratio. For example, Chen and Pennacchi (2002) report average expense ratios for the funds in their study of 1.14%. Our use of a slightly higher number is intended to account for the amortized loads that are not included in the expense ratio. We set σ at 20%, which reflects historical levels of portfolio volatility. The natural interpretation of the volatility of returns in the model is as tracking error around a benchmark return. Empirical estimates of tracking error for mutual funds are lower than this. The value of σ in our model will be higher than the volatility of observed returns, however, since indexing reduces the variance of a fund's return. Simulating returns histories for 5,000 funds, this level of σ produces an average standard deviation of returns (r_t) of 9.12%. This number is consistent with empirical estimates of average levels of tracking error and idiosyncratic volatility.[8] From (20), the quadratic cost function and (26) gives $\bar{\phi} = 2f = 3\%$. These parameter values are summarized in Table 1 below. It straightforward to see (using (22) and (35)) that the only remaining parameters that affect the survival probabilities and flow of funds relationship are ϕ_0 and γ.

Our approach is to match the two parameters governing the distribution of skill level

[8]Table III in Koski and Pontiff (1999) reports mean levels of idiosyncratic risk for funds with different investment styles. When annualized, these vary from 9.97% for aggressive growth funds to 3.67% for Equity-Income funds. Chen and Pennacchi (2002) provide estimates of unconditional tracking error ($d_0 + d_1$ in the model estimated in Table III of that paper). The median value for all funds when annualized is 9.66%.

using an "eyeball metric" that matches the empirical survival rates and relation between the flow of funds and performance. Table 1 contains the values of ϕ_0 and γ that resulted from this process.

The lighter bars in Figure 2 represent quantity-weighted averages of survival rates for all funds in the CRSP mutual fund data base from 1969 to 1999. Each bar represents the fraction of funds that survived for one, two,..., or nineteen years over this interval, that is, the ratio of the number of surviving funds over the total number of funds that could have survived.[9] The dark bars are the matched survival rates computed using the probabilities from Proposition 3. In our model survival rates drop off geometrically with age, as they will in any model based on learning, while in the data they fall in a more linear fashion. We chose to match the shorter term survival probabilities. For obvious reasons there is much more data for younger funds (see Figure 2) which means the early data is estimated much more precisely. The behavior of the longer term survival rates is puzzling, and may be due to managerial turnover within the mutual fund. Good managers might be promoted or defect to other firms. Thus, the low survival rates over longer periods may reflect a renewal in the learning process our model does not capture.[10]

Matching the relatively high survival rates for the first few years requires that funds begin operating at a scale that is considerably higher than the size at which they liquidate, with correspondingly high expectations for managerial ability. The priors are that managers are expected to earn an annual excess return of 6.5% when they begin operating.

High precision in priors relative to the variability in returns is required to reproduce the fund-flow and performance relationship. The slope of this relationship is determined by the fees, f, and the ratio of the precision in priors and returns. Figure 3 provides the explicit comparison between the model's results and behaviors documented in empirical studies. In it the flow of funds relationship for two-year old funds is superimposed over the plot of the non-parametric estimates and 90% confidence intervals in Chevalier and Ellison (1997). The curve from the model seems to pick up the general curvature in the relationship. A notable discrepancy in this case is that the empirical curve does not go through the origin, as it does in our model. A natural explanation for this is the general growth in the mutual fund sector during their sample period, something that is missing in our model.

An important question in financial economics is whether active portfolio managers have skill. Alternatively, are other market participants enforcing such efficiency in financial mar-

[9]We are very grateful to Hsiu-lang Chen and George Pennacchi for providing us with the raw survival rates.

[10]In contrast, for hedge funds where there is little distinction between the manager and the fund company, Getmansky (2003) finds that survival rates do drop off geometrically.

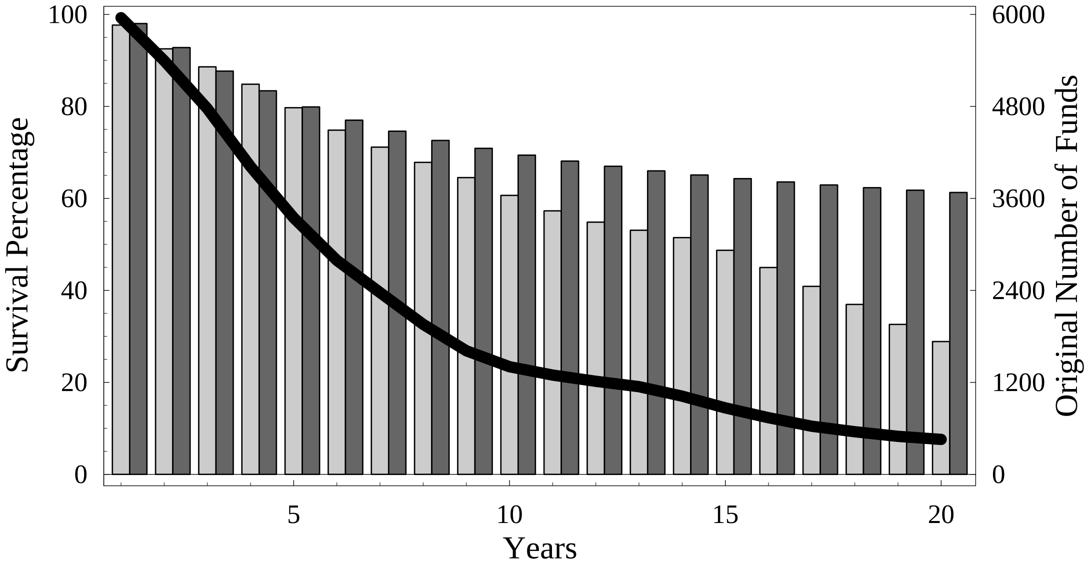

Figure 2: **Percentage of Surviving Funds:** The bars show the fraction of funds as a function of the number of years in business. Light bars are the actual survival rates computed from the CRSP mutual fund data base. The dark bars are what the model predicts the survival rates should be. The scale is marked on the left hand axis. The line marks the total number of funds that could have survived at each age. The scale for this line is marked on the right hand axis.

kets that there is no opportunity for active managers to add value or create rents for themselves? Figure 4 plots the prior distribution over management ability using our parameter values. It also shows the level of the management fee (1.5%). If level of skill in the economy is defined as the fraction of managers who can generate an α in excess of the fees they charge, then this fraction is the area of the curve to the right of the line. About 80% of managers satisfy this criteria—they generate value in that they can beat their fees on at least the first dollar they manage—and the average manager has an α of 6.5%.

These estimates might appear high, given the skepticism in the academic literature about whether active managers add value at all. It is a direct consequence, however, of the very high survival rates and empirically observed flow of funds relationship. Furthermore, the other parameters implied by these estimates seem reasonable. For example, the implied value of the ratio $\frac{q_0}{\bar{q}}$ (the size of a new fund over the minimum fund size) is 4.7. So, for example, if the minimum fund size is \$5 million, the model implies that new funds would start around \$25 million. At $\bar{q}$, the fees equal the fixed costs, $F = \bar{q}f$, so $\bar{q} =$ \$5 million implies periodic fixed costs F of \$75,000. Far from implying that most managers are unskilled, therefore, the data seem consistent with a high level of skill for active managers.

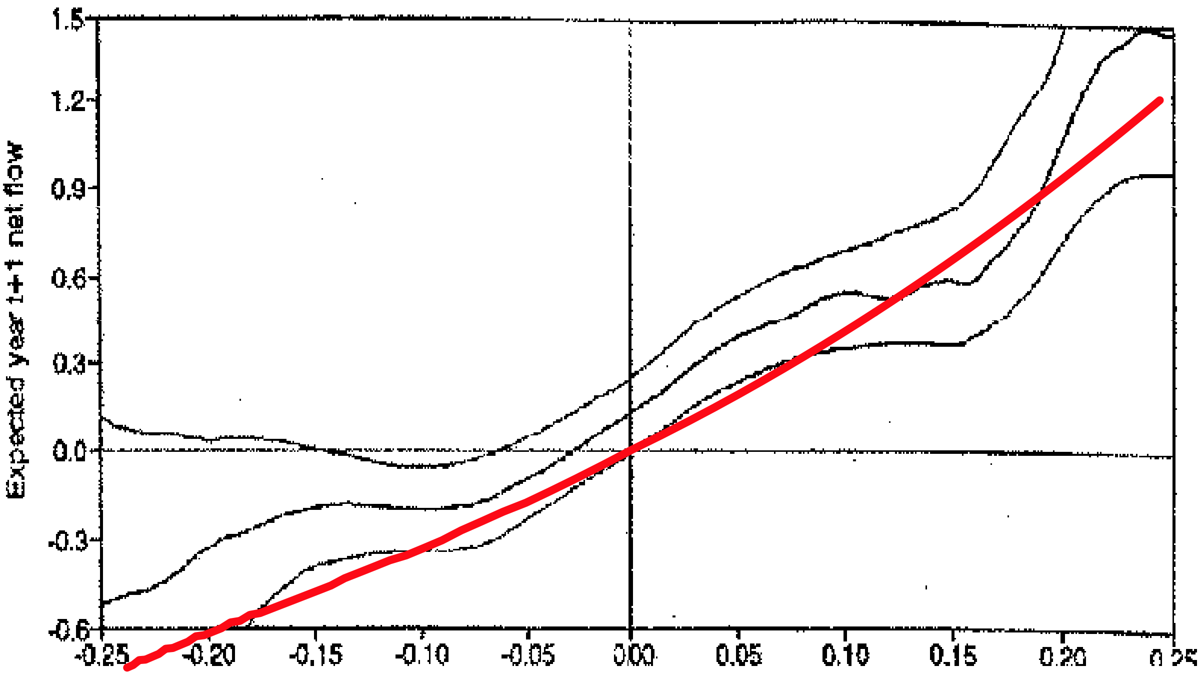

Figure 3: **Flow of Funds:** The dashed line shows the flow of funds for 2 year old funds produced by the model (using the parameters reported in Table 1) superimposed over the actual flow of funds plot (solid lines) for these funds as reported in Chevalier and Ellison(1997, Fig.1). Chevalier and Ellison report the estimated curve (middle line) as well as the 90% confidence intervals, the outer lines.

5 Conclusion

In this paper we derive a number of empirical predictions of a rational model for active portfolio management when managerial talent is a scarce resource and is dissipated as the scale of operations increases. Many of these predictions reproduce empirical regularities that often have been taken as evidence of investor irrationality or agency costs between managers and investors. Not only is the rational model consistent with much of the empirical evidence, but it is also consistent with a high level of skill amongst active managers.

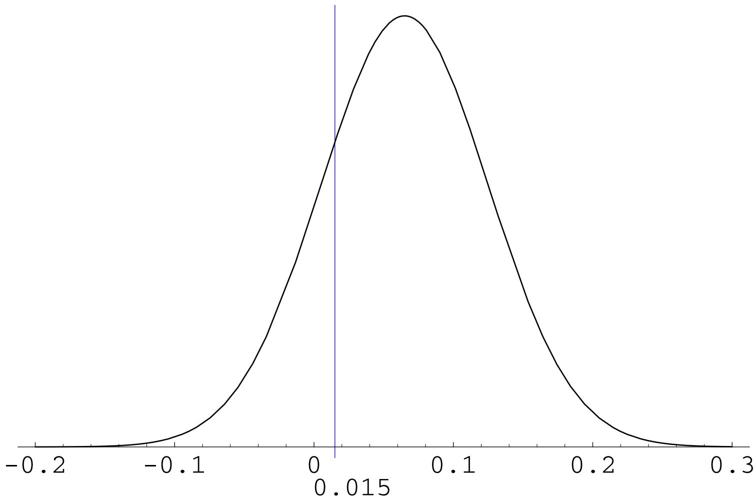

Figure 4: **Distribution of Management Skill:** The vertical line marks the level of the management fee — 1.5%. Approximately 80% of the area below the curve lies to the right of this line. The parameter values (mean and precision) are $\bar{\phi} = 0.065$ and $\gamma = 277$.

Appendix

A Important Notation

R_t Excess return earned on the first dollar actively managed by the fund in period t.

α Expected excess return earned on the first dollar actively managed by the fund in period t: our measure of the manager's ability.

σ^2 Variance of R_t.

ω Precision of R_t, $\omega = \frac{1}{\sigma^2}$

η^2 Variance of market's prior over managerial ability, α.

γ Precision of market's prior over managerial ability, α, $\gamma = \frac{1}{\eta^2}$.

ϕ_t Market's expectation of managerial ability, conditional on the return history up to date t.

ϕ_0 Mean of market's initial prior over managerial ability.

$\bar{\phi}$ The minimal expectation of managerial ability at which the fund can recover its fixed costs and continue to operate.

q_t Funds under management at date t.

$q^*(\phi_t)$ Optimal amount of funds under active management, given the market's expectations of managerial ability.

$\bar{q} \equiv q^*(\bar{\phi})$ The lowest value of q for which the fund remains viable.

q_{It} Funds the manager raises but passively invests in the benchmark portfolio: $q_t = q^*(\phi_t) + q_{It}$.

$h(q_t)$ Proportion of funds under active management: $h(q_t) = \frac{q^*(\phi_t)}{q_t}$.

f_t Management fees, where the optimal level of fees is denoted f_t^*, and when fees are fixed through time $f_t = f$.

F Fixed costs of operating the fund each period.

$C(q)$ The total variable costs of actively managing a fund of size q.

$c(q)$ The unit costs and fees born by investors in a fund of size q: $c(q) = \frac{C(q)}{q} + f$.

r_t Return to investors per dollar invested in the fund: $r_t = R_t - c(q_t)$.

P_t Survival rate: the probability a fund born at date zero survives until date t.

$r^*(\phi_{t-1})$ The minimal return realization such that a fund with expected managerial abiity ϕ_{t-1} survives through the next period.

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