# UNIVERSITY OF CHICAGO Booth School of Business

Bus 35120 – Portfolio Management

Prof. Lubos Pastor

# Assignment #2

Due: April 7 by 8:15am

Be as clear and brief as possible. The data for this assignment can be downloaded from the course website.

- **A.1. CASE STUDY:** "The Risk of Stocks in the Long Run: The Barnstable College Endowment." Read the case study and answer the following questions.
  - 1. "If stock market movements are serially uncorrelated, then the risk of holding stocks diminishes as the holding period lengthens." Comment.
  - 2. Reconstruct as much as you can of Exhibits 1 and 2, and describe how you did it.
  - 3. Which of the two proposals (if any) should Ms. Adams accept?
- **A.2. CASE STUDY:** "The Vanguard Group in 2006 and Target Retirement Funds." Read the case study and answer the following questions.
  - 1. Briefly summarize the key features of Vanguard's business philosophy.
  - 2. What are the two main theoretical arguments for reducing one's stock allocation as one grows older?
  - 3. Do you agree with those arguments? Why or why not?

Be prepared to discuss both cases in class.

- **B. DATA ANALYSIS.** The purpose of this assignment is to apply simulation techniques to make predictions about future asset returns. This exercise extends Assignment 1, using the same data.
  - 1. Assume that the stock and bond returns are normally distributed, at each return frequency. Recompute the answers to problem 6 in Assignment 1 by simulation. Simulate 10,000 stock (bond) returns from the appropriate normal distribution, and compute the fraction of the 10,000 simulations in which the return is less then k, for k = -20%, -10%, 0, 10%, 20%. Do you get the same answers as in Assignment 1? If not, what is the reason behind the difference?<sup>1</sup>

<sup>&</sup>lt;sup>1</sup> Matlab hint: Command randn simulates a normally distributed random variable with zero mean and unit variance. To simulate a random variable y with mean m and variance v, write y = m + sqrt(v) \* randn. Also, you don't want to hit enter 10,000 times. To loop over the 10,000 simulations, use the for command. Alternatively, randn(10000,1) generates a column of 10,000 random numbers.

2. Drop the assumption of normality, and recompute the answers to problem 6 in Assignment 1 using the resampling method. That is, randomly resample 10,000 stock (bond) returns from their empirical (i.e., actual, or historical) distributions, and compute the fraction of the 10,000 draws in which the return is less then k, for k = -20%, -10%, 0, 10%, 20%. Do your answers differ from those obtained in part 1? How good or bad is the assumption of return normality?<sup>2</sup>

The above two questions deal with predicting one-period returns. The following questions deal with multiperiod returns. Unless you are asked to use the resampling method, assume that the continuously compounded returns of stocks and bonds are i.i.d. normally distributed.<sup>3</sup>

- 3. Compute the probability that stocks (bonds) will yield a total cumulative return larger than 20% over the following 5 periods (days, months, years). Use the analytical approach discussed in class, not simulations.
- 4. Now answer the same question using the known-distribution simulation approach. That is, simulate 10,000 5-period paths of returns, compute the cumulative 5-period return for each path, and compute the fraction of the 10,000 simulations in which the cumulative stock (bond) return is larger than 20%. Do you get the same answers? If not, what is the reason behind the difference?
- 5. Now answer the same question using the resampling approach. That is, randomly resample 10,000 5-period paths of returns, compute the cumulative 5-period return for each path, and compute the fraction of the 10,000 draws in which the cumulative stock (bond) return is larger than 20%. Do you get the same answers as in question 3? If not, what is the reason behind the difference?
- 6. Compute the probability that stocks will underperform bonds over the following 30 periods (days, months, years). Use the analytical approach discussed in class.
- 7. Now answer the same question using the resampling approach. That is, randomly resample 10,000 30-period pairs of paths of stock and bond returns, compute the cumulative 30-period return for each pair of paths, and compute the fraction of the 10,000 draws in which the cumulative stock return is smaller than the cumulative bond return. Do you get the same answers as in question 6? If not, what is the reason behind the difference?<sup>4</sup>

## C. EXAM-LIKE QUESTIONS.

Note: Whenever convenient, you can assume that returns are normally distributed, to simplify the calculations. (In reality, we often use simulation techniques if we are worried about departures from normality, as discussed in class.)

<sup>&</sup>lt;sup>2</sup> Matlab hint: One way to draw a random integer y from the set  $\{1, 2, ..., N\}$  is the following: y = ceil(N\*rand). The rand command randomly draws a uniformly distributed variable, and the ceil command rounds a real number up to the nearest integer value (e.g., ceil(1.2) = 2).

<sup>&</sup>lt;sup>3</sup>Matlab hint: Command log computes the natural logarithm needed to convert the simple returns provided to you in the data files into continuously compounded returns.

<sup>&</sup>lt;sup>4</sup>While resampling, keep in mind that the pairs of stock and bond returns must be drawn from the same time period, to account for the stock-bond correlation.

- 1. You are 35 years old. You just received a \$500,000 bonus. You would like to buy a new car, and you are deciding between a \$100,000 Porsche, a \$200,000 Lamborghini, and a \$300,000 Maybach. At the same time, you would like to invest a sufficiently large part of your bonus in a retirement account (which earns continuously compounded annual returns with a mean of 10% and a 20% standard deviation) so that you have a 75% probability of having \$1 million available from this source for retirement at the age of 50. Which car(s) can you afford to buy?
- 2. Consider a bond portfolio whose continuously compounded monthly return has a mean of 0.3% and standard deviation of 1.5%.
  - (a) What is the probability that, over a horizon of 10 years, the bond portfolio will outperform a riskless T-bill earning the continuously compounded return of 0.3% per month?
  - (b) How does your answer in part (a) depend on the length of the investment horizon? How does it depend on the volatility of the bond portfolio?
- 3. What is the probability that investing in stocks will produce an investment value in T periods that is twice as large as investing in bonds? (Hint: Generalize our analysis of relative shortfall. Instead of computing  $Prob(V_{ST} < V_{BT})$ , show how we can compute  $Prob(\frac{V_{ST}}{V_{BT}} < K)$  for any positive K (not just K = 1).
- 4. The well known "rule of 72" can be used to determine the number of years until a riskless investment doubles in value: divide 72 by the percentage annual return on the asset. For example, an investment earning 8% per year doubles in about 9 years. Explain/derive the rule of 72.
- 5. (Not graded, but I ask that you write down your thoughts nonetheless.)
  - (a) Read the first article on the following page. Do you agree with Bill Gross that since stock prices have appreciated faster than GDP, "somehow stockholders must be skimming 3% off the top each and every year"?
  - (b) Read the second article on the following page. Do you agree with Nassim Taleb that value-at-risk is a "pure intellectual fraud" that should be banned? What do you think of the other rules he proposes for the world of finance?

### **Cult Figures**

By Bill Gross; PIMCO Investment Outlook, August 2012 (excerpt)

The cult of equity is dying. Like a once bright green aspen turning to subtle shades of yellow then red in the Colorado fall, investors' impressions of "stocks for the long run" or any run have mellowed as well.... [After mentioning the 6.6% historical average real return on U.S. stock S&P 500 index:] Yet the 6.6% real return belied a commonsensical flaw much like that of a chain letter or yes—a Ponzi scheme. If wealth or real GDP was only being created at an annual rate of 3.5% over the same period of time, then somehow stockholders must be skimming 3% off the top each and every year. If an economy's GDP could only provide 3.5% more goods and services per year, then how could one segment (stockholders) so consistently profit at the expense of the others (lenders, laborers and government)? The commonsensical "illogic" of such an arrangement when carried forward another century to 2112 seems obvious as well. If stocks continue to appreciate at a 3% higher rate than the economy itself, then stockholders will command not only a disproportionate share of wealth but nearly all of the money in the world! Owners of "shares" using the rather simple "rule of 72" would double their advantage every 24 years and in another century's time would have 16 times as much as the sceptics who decided to skip class and play hooky from the stock market...

#### From fat tails to Fat Tony

By Nassim Taleb; The Economist, The World In 2013 print edition (excerpt)

The world of finance needs four new rules, says Nassim Nicholas Taleb, professor of risk engineering at NYU-Poly (and a former derivatives trader).

... I have four suggestions, all simple measures I call "heuristics"—practical and solid rules that come from experience—that can decrease the fragility of the economic system, selected because they are both uncomplicated (except for academic economists) and highly effective.

My first suggestion aims to deter the "too big to fail" effect and prevent bonus-earners from taking advantage of the public. A company that is classified as a candidate for a taxpayer bail-out if it fails should not then be able to pay any of its staff more than a corresponding civil servant (since its employees have then become de facto civil servants). . . .

My second recommendation is to oblige those who start in public office to pledge never subsequently to earn from the private sector more than a set amount; the rest should go to the taxpayer. This will ensure sincerity in "service"—where employees are supposedly underpaid because of their emotional reward from serving society. It would prove that they are not in the public sector as an investment strategy. Currently, a civil servant can make rules that are friendly to an industry such as banking, and then go off to Goldman Sachs and recoup the difference between his or her current salary and the market rate. . . .

Third, we should force corporate managers to eat some of the losses. Contrary to public perception, corporate managers are no entrepreneurs, and hardly impressive agents of capitalism. Over the past 12 years in the United States, the stockmarket has lost its investors up to \$2 trillion (compared with leaving their funds in cash or Treasury bills). So one would think that since managers are paid on incentive, they would be hurt. Sadly, no: because of the options embedded in their profession, managers received more than \$400 billion in compensation. . . .

Finally, in finance, let's ban the risk-management method called "value-at-risk" currently used by banks. This is a pure intellectual fraud that allows banks to take more risks in the "tails". And the method is as much in use after the crisis as it was before: JPMorgan lost billions on trades in 2012 while the value-at-risk predicted very small tail exposures. Value-at-risk is not the only fraud: there are plenty of other contraptions of quantitative finance that continue simply because those who teach and practise them are themselves never harmed.

Should a single one of my four wishes come true, 2013 will be a good year. •