UNIVERSITY OF CHICAGO
Booth School of Business

Bus 35120 – Portfolio Management — Prof. Lubos Pastor

Assignment #4
Solutions

B. DATA ANALYSIS.

The solutions given below were obtained in MATLAB using the program *hwk4_solutions.m*, which you can download from Canvas.

1.

The sample estimates of expected excess returns ($\hat{E}$) are

Exxon	0.0076
PG	0.0061
Pfizer	0.0078
Walmart	0.0140
Intel	0.0168

The sample estimate of the covariance matrix ($\hat{V}$):

Exxon	0.0025	0.0008	0.0011	0.0007	0.0016
PG	0.0008	0.0032	0.0015	0.0017	0.0016
Pfizer	0.0011	0.0015	0.0050	0.0014	0.0024
Walmart	0.0007	0.0017	0.0014	0.0081	0.0031
Intel	0.0016	0.0016	0.0024	0.0031	0.0143

The TP weights implied by these sample moments are as follows:

	weight:
Exxon	0.4620
PG	0.0534
Pfizer	0.0900
Walmart	0.2684
Intel	0.1261

These weights are fairly unbalanced; e.g., almost half of the portfolio is invested in a single stock, which is something that few money managers would be willing to do. Taking excessively large positions based entirely on past performance is generally not a good idea.

The minimum variance portfolio (MVP) weights are

	weight:
Exxon	0.5078
PG	0.3001
Pfizer	0.1153
Walmart	0.0887
Intel	-0.0118

Compared to the TP, PG looks a lot more attractive due to its low volatility, whereas Walmart looks less attractive. Intel, which is the most volatile, is shorted in a small amount.

The MVP has lower variance than the TP (0.0017 vs. 0.0023), which must be the case in sample, by construction. The TP has a higher expected excess return (0.0104 vs. 0.0077), which also makes sense – TP maximizes the risk-return tradeoff, whereas MVP only minimizes variance.

The TP weights are fairly sensitive to small changes in the expected return estimates. Rounding the estimates to two decimal places leads to the following weights:

	weight:
Exxon	0.4544
PG	0.2898
Pfizer	0.0752
Walmart	0.0485
Intel	0.1321

The differences are large for some assets; e.g., the weight in PG increases from 5% to 29%. The fact that rounding the imprecisely estimated expected return estimates produces such large differences in portfolio weights is an important cause for concern when using $\hat{E}$ and $\hat{V}$.

2.

When the identity matrix I substitutes for V, the tangency portfolio weights are:

	weight:
Exxon	0.1459
PG	0.1165
Pfizer	0.1499
Walmart	0.2672
Intel	0.3205

Relative to TP, Intel becomes more attractive and Exxon less attractive because their volatilities (high for Intel, low for Exxon) are ignored.

Rounding in this case has a substantially smaller impact on the weights than in part 1:

	weight:
Exxon	0.1667
PG	0.1667
Pfizer	0.1667
Walmart	0.1667
Intel	0.3333

This is because the inverse of the identity matrix (which is just I again) is "better behaved" than $\hat{V}^{-1}$, in that it is much farther away from singularity. In other words, expected return estimates are weighted by less extreme numbers when computing the weights.

3.

The CAPM-based estimates of expected excess monthly returns are given by $\beta \times$(market premium):

Exxon	0.0030
PG	0.0035
Pfizer	0.0060
Walmart	0.0045
Intel	0.0060

The CAPM-based optimal weights are:

	weight:
Exxon	0.2598
PG	0.1784
Pfizer	0.3908
Walmart	0.1151
Intel	0.0559

This is a more balanced portfolio than the one from part 1.

4.

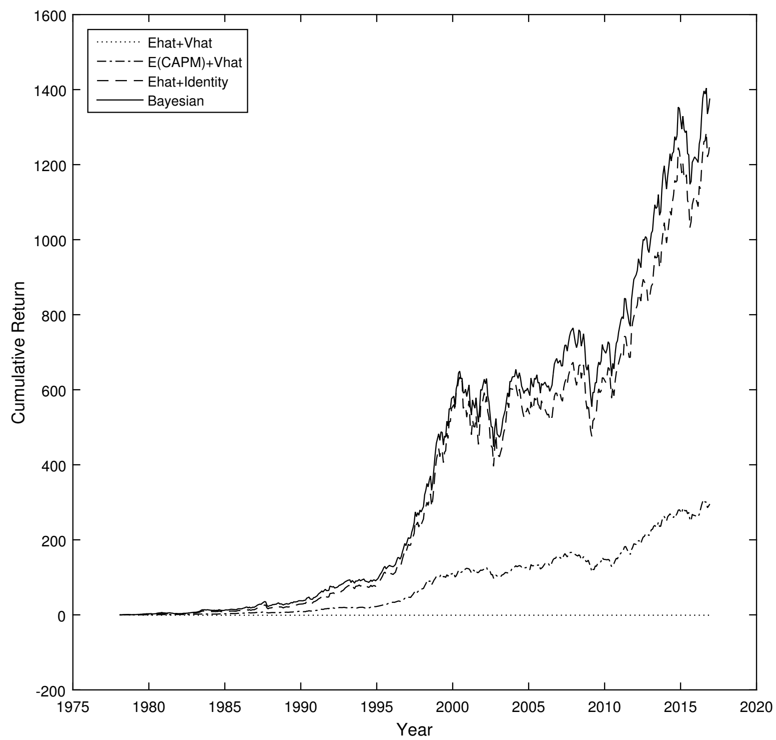
The shrinkage approach yields the following weights:

	weight:
Exxon	0.2163
PG	0.1344
Pfizer	0.2048
Walmart	0.2448
Intel	0.1998

This portfolio is even more equally balanced. Which set of weights works best is evaluated in the out-of-sample exercise below.

5.

The graph plots the cumulative returns from the four investment strategies.



The average monthly returns and the Sharpe ratios attained by each of the four strategies are summarized in the table below.

	Sample estimates	CAPM	Identity matrix	Bayesian
Average monthly return	0.0083	0.0132	0.0172	0.0174
Sharpe ratio	0.0160	0.2164	0.2207	0.2243

The Bayesian/shrinkage strategy has performed best, in terms of the total return as well as the Sharpe ratio. The strategy based on the identity matrix (which ignores variances and covariances) has performed almost as well. The worst-performing strategy by far is the one that relies on sample estimates, mostly due to occasional extreme portfolio weights.

C. EXAM-LIKE QUESTIONS.

1.

Let R denote the return on an arbitrary portfolio of the two assets, with $w \neq 0$ invested in the risky asset and $1 - w$ in the T-bill. Note that R is normally distributed for any w, and the portfolio's Sharpe ratio is 0.7 for any w. Denote the portfolio's expected return by E, its standard deviation by σ, and the T-bill rate by R_f. Then $\frac{R-E}{\sigma}$ follows a standard normal distribution.

$$\text{Prob}(R < R_f) \quad = \quad \text{Prob}(\frac{R-E}{\sigma} < \underbrace{\frac{R_f - E}{\sigma}}_{\text{-Sharpe ratio}}) = \text{Prob}(z < -0.7) = 0.242$$

for any w, so 24.2% is both the minimum and the maximum shortfall probability.

2.

(a)

Not clear; Fund 1 has a higher expected excess return but also higher volatility ($18\%/1.0 = 18\%$ vs $12\%/0.8 = 15\%$), so your choice depends on your risk aversion. Fund 1 has a higher Sharpe ratio, but if you are sufficiently risk-averse, you will prefer Fund 2.

You can actually calculate the risk aversion γ at which the investor with mean-variance utility is indifferent between the two funds: $\gamma = 12.12$. For $\gamma > 12.12$, you prefer Fund 2; otherwise you prefer Fund 1.

(b)

Pick Fund 1, regardless of your risk aversion, because it has a higher Sharpe ratio. Since you can mix your fund with the T-bill, you can tailor the overall riskiness of your portfolio to your liking (by changing the weight in the T-bill) while keeping the same Sharpe ratio.

3.

The answer here is the minimum variance portfolio (MVP), whose weights are given by

$$w = \frac{V^{-1}\iota}{\iota' V^{-1}\iota},$$

where

$$V = \begin{pmatrix} 0.05 & 0.01 \\ 0.01 & 0.05 \end{pmatrix}$$

is the return covariance matrix and ι is 2×1 a vector of ones. The MVP weights are then proportional to

$$V^{-1}\iota = \begin{pmatrix} 0.05 & 0.01 \\ 0.01 & 0.05 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 20.83 & -4.17 \\ -4.17 & 20.83 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 16.67 \\ 16.67 \end{pmatrix}.$$

We then scale these weights to sum to one, and obtain

$$w = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

Alternatively, you could obtain the same result by computing

$$w^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} = \frac{0.05 - 0.01}{0.05 + 0.05 - 2(0.01)} = 0.5.$$

4.

Given that $U = E_P - \frac{6}{2}\sigma_P^2$, the investor's relative risk aversion is $\gamma = 6$. The optimal portfolio weight in the first asset is given by

$$w = \frac{(E_1 - E_2) - \gamma(\sigma_{12} - \sigma_2^2)}{\gamma(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})} = \frac{(0.12 - 0.07) - 6(0.01 - 0.05)}{6(0.05 + 0.05 - 2(0.01))} = 0.6042,$$

and the weight in the second asset is $1 - 0.6042 = 0.3958$.

5.

The optimal portfolio weights in the risky assets are given by

$$w = \frac{V^{-1}E}{\gamma} = \frac{1}{6} \begin{pmatrix} 0.05 & 0.01 \\ 0.01 & 0.05 \end{pmatrix}^{-1} \begin{pmatrix} 0.10 \\ 0.05 \end{pmatrix} = \begin{pmatrix} 0.3125 \\ 0.1042 \end{pmatrix}.$$

The remaining weight of $1 - 0.3125 - 0.1042 = 0.5833$ is put in the T-bill. (The 2×1 vector E of expected excess returns contains $0.12 - 0.02 = 0.10$ and $0.07 - 0.02 = 0.05$.)

6.

The future wealth of an investment banker moves more closely with the stock market than that of a tenured professor, so the banker should have a smaller allocation to stocks in order to decrease the overall riskiness of his/her total future wealth.

7.

The author thinks he criticizes the textbook concepts of diversification and the CAPM, whereas all he really criticizes is the blind use of historical correlations in estimating inputs to the construction of diversified portfolios and to the CAPM. Diversification, portfolio theory, and the CAPM are all perfectly fine, but their inputs (such as E, V, and β) must be estimated carefully.

The author is right in that using plain sample estimates (such as $\hat{E}$, $\hat{V}$, and $\hat{\beta}$) often works poorly – we talked about this in class. He is also right in that we often have additional information, such as the industry and the financial statements of the company.

As we discussed in class, this additional information can be incorporated using Bayesian techniques. For example, a priori, you might think that the correlation between Honda and Gillette is a lot lower than 0.8, and including this prior information would lead you to use a much lower correlation. Or you might think that Krispy Kreme is overvalued, in which case your prior belief about the firm's expected return is negative.

Another complication is that the inputs, especially volatilities and correlations, change over time. But this variation can be modeled separately in a realistic way. For example, you want a model in which volatilities and correlations go up when prices fall. Such models can be attached to portfolio theory as modules producing inputs that are more realistic than plain historical estimates.

Only a fool would criticize the concept of a car based on the fact that if you pour water instead of gas in your car's tank, the car won't run.