UNIVERSITY OF CHICAGO
Booth School of Business

Bus 35120 – Portfolio Management | Prof. Lubos Pastor

# Assignment #6
## Solutions

**B.**

The solutions given below were obtained in MATLAB using the program hwk6_solutions.m. You can download the program from Canvas into your current directory and run it by typing "hwk6_solutions" at the command line.

1.

The correlation between LIQ and VIX is -0.2562, or -25.6%.

This correlation is smaller in magnitude than the -57% correlation reported in Pastor and Stambaugh (2003). (On page 654 of their study, Pastor and Stambaugh report that "the within-month daily standard deviation of the value-weighted market return has a correlation of -.57 with the liquidity series.") The difference is in part due to different sample periods, but especially due to different measures of volatility. While Pastor and Stambaugh use realized volatility computed from daily returns throughout the month, this assignment uses implied volatility at the month-end. Any mid-month turbulence in the market is reflected in realized volatility but not necessarily in the month-end implied volatility (because the turbulence may be gone by the month-end). As a result, the correlation estimated here is less negative.

The correlation is negative because when volatility is high, market makers face more inventory risk, so they demand higher compensation for providing liquidity. The higher compensation comes in the form of higher price impact or higher bid-ask spreads faced by those who demand liquidity. Either way, liquidity is lower when volatility is higher.

2.

The correlation between LIQ and MKT is 0.3339, or 33.4%. It is positive because liquidity tends to dry up when the market goes down. When asset prices drop, financial intermediaries tend to lose capital. To maintain the same leverage (or risk exposure) as before with smaller capital, they have to cut back their asset holdings, including those from liquidity provision. The result is lower supply of liquidity in downmarkets.

This correlation is higher in downmarkets: 0.4329. It is virtually zero in upmarkets: 0.0081. This asymmetry is due to the fact that liquidity itself is asymmetric: it tends to evaporate occasionally and suddenly, mostly during financial crises and mini-crises, but it rarely surges. In addition, the economic mechanism described in the previous paragraph operates in downmarkets whereas its flipside in upmarkets—intermediaries providing more liquidity when they make money—is weaker.

3.

Yes, the results of Pastor and Stambaugh (2003) continue to hold through the present. The alphas of the 10 portfolios are

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10-1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Alpha (%/year) | -2.72 | -0.72 | -0.75 | 0.87 | -0.15 | 0.62 | 1.25 | 1.47 | 0.02 | 2.63 | 5.35 |
| $t$-statistic | -2.16 | -0.73 | -0.99 | 1.26 | -0.20 | 0.93 | 1.76 | 1.85 | 0.02 | 2.29 | 2.99 |

The Fama-French alphas largely increase across the ten portfolios indicating that stocks with higher liquidity betas have had higher returns, on average (even after adjusting for exposures to Fama-French factors). This evidence is consistent with the idea that investors demand compensation for liquidity risk in the form of higher average returns. The alpha of the 10-1 portfolio is positive and statistically significant.

The out-of-sample test produces similar results with the same conclusions:

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10-1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Alpha (%/year) | -6.30 | -0.19 | -2.54 | 2.17 | 2.24 | 1.29 | 3.06 | 1.74 | 2.24 | 2.14 | 8.44 |
| $t$-statistic | -2.80 | -0.11 | -1.62 | 1.63 | 1.52 | 1.00 | 2.56 | 1.06 | 1.22 | 0.98 | 2.44 |

This is reassuring. Not all results reported in the literature survive an out of sample test.

4.

The post-ranking (or future) liquidity betas of the ten portfolios in the full sample period are

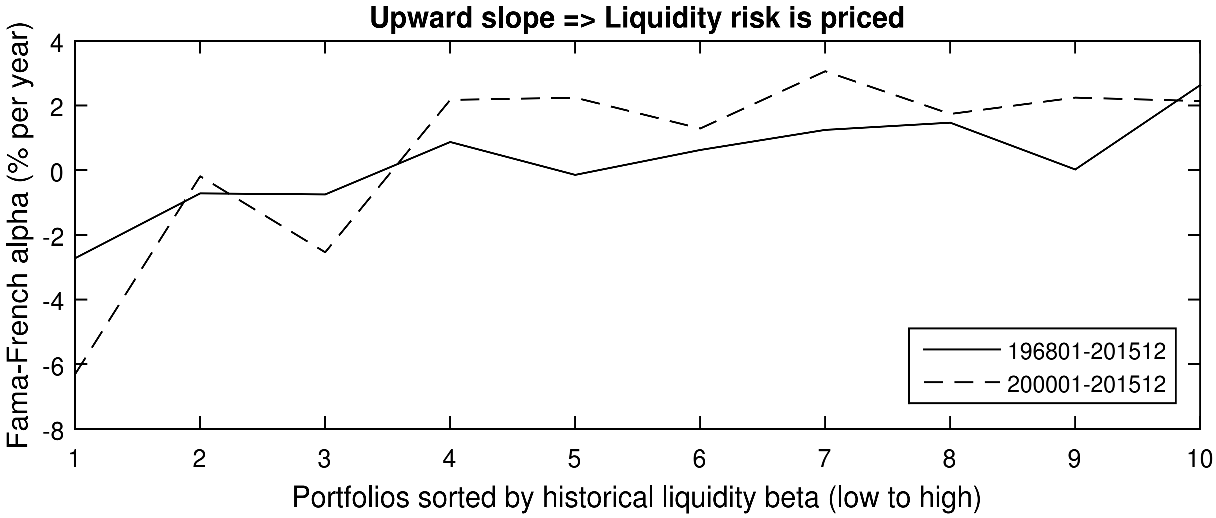
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10-1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LIQ beta | -0.06 | -0.02 | -0.02 | -0.01 | 0.01 | -0.01 | 0.01 | 0.03 | 0.04 | -0.00 | 0.06 |
| $t$-statistic | -3.09 | -1.63 | -1.47 | -0.87 | 0.64 | -0.54 | 0.79 | 2.54 | 2.86 | -0.13 | 2.09 |

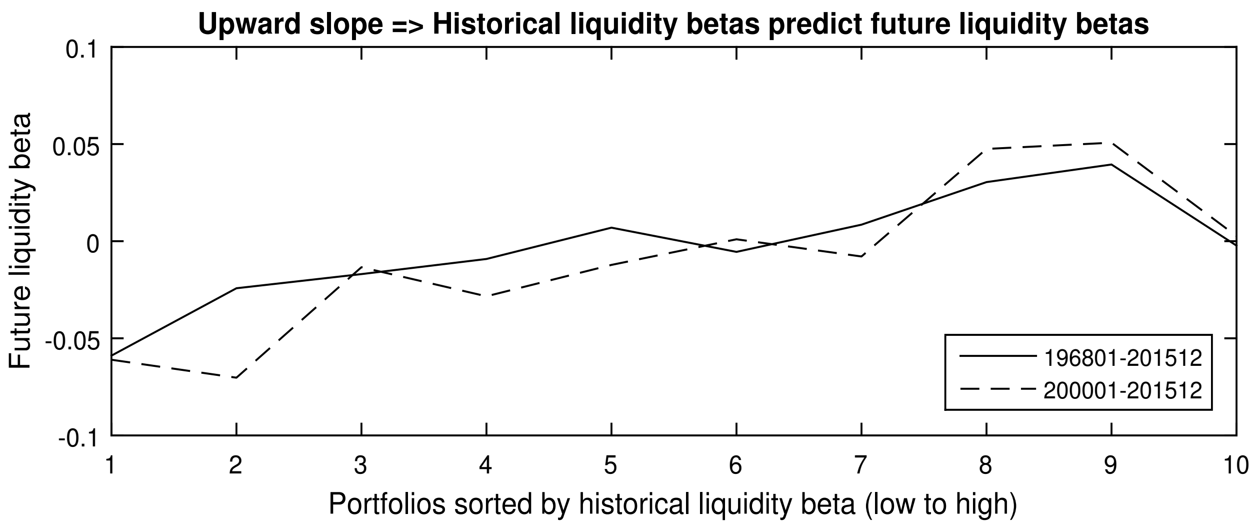
The future liquidity betas tend to increase across the portfolios, indicating that historical betas (based on which the portfolios are formed in real time) have decent power in predicting future betas. The pattern is not monotonic but that is to be expected given sampling error in the data. The 10-1 portfolio has a positive and significant liquidity beta.

The out-of-sample test produces similar results:

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10-1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LIQ beta | -0.06 | -0.07 | -0.01 | -0.03 | -0.01 | 0.00 | -0.01 | 0.05 | 0.05 | 0.00 | 0.06 |
| $t$-statistic | -1.97 | -3.02 | -0.62 | -1.55 | -0.60 | 0.05 | -0.48 | 2.11 | 2.01 | 0.09 | 1.33 |

The plots of the Fama-French alphas and liquidity betas of the ten portfolios:




## C. EXAM-LIKE QUESTIONS.

1. Consider the standard market model regression

$$R_{SPAM,t} = \alpha + \beta R_{MKT,t} + e_t \ .$$

If SPAM marked to market properly, it would have $\alpha = 0$ and $\beta = 1$, which would properly indicate that SPAM is an index fund with no stock-picking skill. But since SPAM marks to market one month late and market returns are approximately uncorrelated month to month, we expect to find $\alpha = 1\%$ and $\beta = 0$, as if SPAM were a very successful market-neutral hedge fund. This is a problem! Fund managers have an incentive to hold illiquid assets to exploit the staleness of their prices. Keep this in mind next time a private equity manager boasts about having delivered a high risk-adjusted return with little risk.

To remove the stale price problem, we can adjust the standard regression by adding lagged market returns. In this specific case, adding just one lagged return will do the trick. If you estimate the regression

$$R_{SPAM,t} = \alpha + \beta_1 R_{MKT,t} + \beta_2 R_{MKT,t-1} + e_t \ ,$$

you will find $\alpha = 0$, $\beta_1 = 0$, and $\beta_2 = 1$. The adjusted alpha will thus be zero and the adjusted beta will be one, $\beta_1 + \beta_2 = 1$, properly reflecting SPAM's investment strategy.

2. The unsmoothing procedure produces the same expected return but higher variance.

   (a) Taking the expectations of both sides of equation (2) implies

   $$E(r) = \frac{1}{1-b}E(r^*) - \frac{b}{1-b}E(r^*) = E(r^*) \ .$$

   (b) First note from equation (1) that

   $$\begin{aligned} Cov(r_t^*, r_{t-1}^*) &= Cov(a + br_{t-1}^* + \epsilon_t, r_{t-1}^*) \\ &= bVar(r^*) \ , \end{aligned}$$

   where $Var(r^*) = Var(r_t^*) = Var(r_{t-1}^*)$ is the variance of reported returns. Taking variances of both sides of equation (2), we obtain

   $$\begin{aligned} Var(r) &= Var\left(\frac{1}{1-b}r_t^* - \frac{b}{1-b}r_{t-1}^*\right) \\ &= \frac{1}{(1-b)^2}Var\left(r_t^* - br_{t-1}^*\right) \\ &= \frac{1}{(1-b)^2}\left[Var(r_t^*) + b^2Var(r_{t-1}^*) - 2bCov(r_t^*, r_{t-1}^*)\right] \\ &= \frac{1}{(1-b)^2}\left[Var(r^*) + b^2Var(r^*) - 2b\left\{bVar(r^*)\right\}\right] \\ &= \frac{1}{(1-b)^2}\left[Var(r^*) - b^2Var(r^*)\right] \\ &= \frac{1-b^2}{(1-b)^2}Var(r^*) \\ &= \frac{1+b}{1-b}Var(r^*) \\ &> Var(r^*) \end{aligned}$$

   because $0 < b < 1$. Neat, isn't it? We just proved that the variance of true returns is always larger than the variance of reported returns. In other words, the volatility of reported returns is understated relative to true volatility.

(c) Since the expected values of $r_t$ and $r_t^*$ are the same but the variance of $r_t$ is higher, the true Sharpe ratio is always lower than the reported Sharpe ratio.

Lesson: Do not be dazzled by the high reported Sharpe ratios of PE funds or other funds holding illiquid assets. The true Sharpe ratios are lower, and potentially much lower if the funds' reported returns exhibit significant autocorrelation.

3. False. When people say that liquidity has dried up, they usually mean that price impact is large. In fact, trading volume is often large when liquidity is low. A good example is October 1987 when liquidity dried up but the share volume at the NYSE hit the record high.

4. False. They do tend to have higher spreads, but to compensate the *providers* of liquidity (market makers) for higher inventory risk.

5. True. Implementation shortfall is the difference between the pre-transaction-cost hypothetical returns on a strategy and the after-transaction-cost actual returns. Transaction costs are larger for small stocks.