A Consumption-Based Explanation of Expected Stock Returns

MOTOHIRO YOGO*

ABSTRACT

When utility is nonseparable in nondurable and durable consumption and the elasticity of substitution between the two consumption goods is sufficiently high, marginal utility rises when durable consumption falls. The model explains both the cross-sectional variation in expected stock returns and the time variation in the equity premium. Small stocks and value stocks deliver relatively low returns during recessions, when durable consumption falls, which explains their high average returns relative to big stocks and growth stocks. Stock returns are unexpectedly low at business cycle troughs, when durable consumption falls sharply, which explains the countercyclical variation in the equity premium.

EXPLAINING THE VARIATION IN EXPECTED RETURNS across stocks and the variation in the equity premium over time as trade-offs between risk and return is a challenge for financial economists. In his review article on market efficiency, Fama (1991, p. 1610) concludes

> In the end, I think we can hope for a coherent story that (1) relates the cross-section properties of expected returns to the variation of expected returns through time, and (2) relates the behavior of expected returns to the real economy in a rather detailed way. Or we can hope to convince ourselves that no such story is possible.

This paper proposes a "coherent story" that satisfies both criteria.

A well-known empirical fact in finance is the high average returns of small stocks relative to big stocks (i.e., low relative to high market equity stocks) and of value stocks relative to growth stocks (i.e., high relative to low book-to-market equity stocks). The evidence suggests that there are size and value premia in the cross-section of expected stock returns. In an equilibrium asset

pricing model, cross-sectional variation in expected returns must be explained by cross-sectional variation in risk. The Capital Asset Pricing Model (CAPM), which measures risk by market beta, fails to explain the size and value premia (see Fama and French (1992) and references therein). The Consumption CAPM (CCAPM), which measures risk by nondurable consumption beta, also fails to explain the cross-section of expected stock returns (see Breeden, Gibbons, and Litzenberger (1989) and Mankiw and Shapiro (1986)).

Another well-known empirical fact is the predictability of stock returns by variables that are informative about the business cycle.[1] The evidence suggests that the equity premium is time varying, that is, it is higher at business cycle troughs than at peaks. In an equilibrium asset pricing model, time variation in the equity premium must be explained by time variation in the price or quantity of risk. Although there is some evidence for time variation in risk, it cannot be reconciled with the evidence for expected returns in a way that offers a consistent description of the time-varying trade-off between risk and return (see Harvey (1989) for evidence on the CAPM and Kandel and Stambaugh (1990) for the CCAPM).

This paper proposes a simple consumption-based explanation of both the cross-sectional variation in expected stock returns and the countercyclical variation in the equity premium. I use a representative household model, in which intraperiod utility is a constant elasticity of substitution (CES) function of nondurable and durable consumption. The household's intertemporal utility is Epstein and Zin's (1991) recursive function, which allows for the separation of the elasticity of intertemporal substitution (EIS) from risk aversion. The durable consumption model (as the model is referred to throughout the paper) nests the nonseparable expected utility model as a special case when EIS is the inverse of risk aversion (Dunn and Singleton (1986), Eichenbaum and Hansen (1990), Ogaki and Reinhart (1998)).

In the language of macroeconomics, the main findings can be summarized as follows. When the elasticity of substitution between nondurable and durable consumption is higher than the EIS, the marginal utility of consumption rises when durable consumption falls. First, small stocks and value stocks deliver low returns when marginal utility rises, that is, during recessions when durable consumption falls. Investors must therefore be rewarded with high expected returns to hold these risky stocks. Second, stocks deliver unexpectedly low returns when marginal utility rises sharply, that is, at business cycle troughs when durable consumption falls sharply relative to nondurable consumption. Investors must therefore be rewarded with high expected returns to hold stocks during recessions.

In the language of finance, the main findings can be summarized as follows. When utility is nonseparable in nondurable and durable consumption, optimal portfolio allocation implies a linear factor model in nondurable and durable consumption growth. The risk price for durable consumption is positive, provided that the elasticity of substitution between nondurable and durable goods is

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[1] See, for instance, Campbell (1987), Campbell and Shiller (1988b), Fama and French (1988, 1989), and Keim and Stambaugh (1986).

higher than the EIS. First, small stocks and value stocks have higher durable consumption betas than big stocks and growth stocks. Simply put, the returns on small stocks and value stocks are more procyclical, explaining their high average returns. Second, the covariance of stock returns with durable consumption growth is higher at business cycle troughs than at peaks. The equity premium is therefore countercyclical because the quantity of risk, measured by the conditional covariance of returns with durable consumption growth, is countercyclical.

Because both nondurable and durable consumption are smooth, the durable consumption model requires high risk aversion to fit the high level and volatility of expected stock returns. This paper shows that the model can successfully explain the cross-sectional and time variation in expected stock returns, conditional on an "equity premium puzzle" (Mehra and Prescott (1985)). The high risk aversion does not imply a "risk-free rate puzzle" (Weil (1989)) in the model because recursive utility allows the EIS to be higher than the inverse of risk aversion.

In related work, Pakoš (2004) considers a representative household model with nonhomothetic utility in nondurable and durable consumption. He focuses on the Leontief model, in which the elasticity of substitution between the two types of goods is zero. Since the consumption of durables relative to nondurables is procyclical, a low elasticity of substitution between the goods implies procyclical marginal utility. The Leontief model therefore cannot explain the value premium (since value stocks are more procyclical than growth stocks) or the countercyclical variation in the equity premium. In contrast, I estimate an elasticity of substitution between the goods that is higher than the EIS, implying countercyclical marginal utility.

The rest of the paper is organized as follows. Section I lays out the household's consumption and portfolio choice problem with a durable consumption good and derives the Euler equations. Section II describes the consumption data used in the empirical work. The service flow for durable goods (as defined in the national accounts) is more cyclical than the service flow for nondurable goods and services. The high cyclicality of the service flow, rather than durability of the good, is the key ingredient in explaining the known facts about expected stock returns.

In Section III, the durable consumption model is estimated and tested through its Euler equations. First, I test the model's unconditional moment restrictions using a large cross-section of stock returns. I find that the model explains the variation in average returns across the 25 Fama–French (1993) portfolios, portfolios sorted by book-to-market equity within industry, and portfolios sorted by risk (i.e., pre-formation betas). Second, I test the model's conditional moment restrictions using stock returns and instruments that predict returns. The test of overidentifying restrictions fails to reject the model. Both the additively separable model (Epstein and Zin (1991)) and the time separable model (Eichenbaum and Hansen (1987)) are rejected.

In Section IV, the unconditional Euler equation is approximated as a linear factor model to show that value stocks have higher durable consumption betas than growth stocks, explaining their relatively high average returns. In

Section V, the conditional Euler equation is approximated as a conditional factor model to show that much of the countercyclical variation in the equity premium is driven by countercyclical variation in the conditional covariance of returns with durable (rather than nondurable) consumption growth. Section VI concludes. The Appendices contain detailed descriptions of the data and lengthy derivations omitted in the main text.

I. Consumption and Portfolio Choice with a Durable Consumption Good

A. The Household's Optimization Problem

The consumption and portfolio choice problem of a household is as follows. In each period t, the household purchases C_t units of a nondurable consumption good and E_t units of a durable consumption good. P_t is the price of the durable good in units of the nondurable good. The nondurable good is entirely consumed in the period of purchase, whereas the durable good provides service flows for more than one period. The household's stock of the durable good D_t is related to its expenditure by the law of motion

$$D_t = (1 - \delta)D_{t-1} + E_t, \tag{1}$$

where $\delta \in (0, 1)$ is the depreciation rate.

There are $N + 1$ tradeable assets in the economy, indexed by $i = 0, \dots, N$. In period t, the household invests B_{it} units of wealth W_t in asset i, which realizes the gross rate of return $R_{i,t+1}$ in period $t + 1$. The household's total saving in assets satisfies the intraperiod identity

$$\sum_{i=0}^{N} B_{it} = W_t - C_t - P_t E_t. \tag{2}$$

The household's wealth in the subsequent period is given by the intertemporal budget constraint

$$W_{t+1} = \sum_{i=0}^{N} B_{it} R_{i,t+1}. \tag{3}$$

The household's intraperiod utility is specified by the CES function

$$u(C, D) = [(1 - \alpha)C^{1-1/\rho} + \alpha D^{1-1/\rho}]^{1/(1-1/\rho)}, \tag{4}$$

where $\alpha \in (0, 1)$ and $\rho \geq 0$ is the elasticity of substitution between the two consumption goods. The special case $\rho = 1$ is understood to be Cobb–Douglas intraperiod utility, $u(C, D) = C^{1-\alpha}D^{\alpha}$. Implicit in this specification is the assumption that the service flow from the durable good is linear in the stock of the durable good. I therefore use the words "stock" and "consumption" interchangeably in reference to the durable good, hopefully without confusion to the reader.

The household's intertemporal utility is specified by the recursive function

$$U_t = \left\{ (1 - \beta) u(C_t, D_t)^{1-1/\sigma} + \beta \left(\mathbf{E}_t \left[U_{t+1}^{1-\gamma} \right] \right)^{1/\kappa} \right\}^{1/(1-1/\sigma)}, \tag{5}$$

where $\kappa = (1 - \gamma)/(1 - 1/\sigma)$. The parameter $\beta \in (0, 1)$ is the household's subjective discount factor, $\sigma \geq 0$ is its EIS, and $\gamma > 0$ determines its relative risk aversion (see Epstein and Zin (1989, 1991) for further discussion of recursive utility).

There are two special cases of utility function (5) that have been used in previous work. First, when the elasticity of substitution across the two consumption goods is equal to the EIS (i.e., $\sigma = \rho$), utility is given by

$$U_t = \left\{ (1 - \beta) \left[(1 - \alpha) C_t^{1-1/\sigma} + \alpha D_t^{1-1/\sigma} \right] + \beta \left(\mathbf{E}_t \left[U_{t+1}^{1-\gamma} \right] \right)^{1/\kappa} \right\}^{1/(1-1/\sigma)}, \tag{6}$$

which is the additively separable model of Epstein and Zin (1991). Second, when the EIS is the inverse of risk aversion (i.e., $\sigma = 1/\gamma$), utility is given by

$$U_t^{1-\gamma} = (1 - \beta) \mathbf{E}_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}, D_{t+s})^{1-\gamma}, \tag{7}$$

which is the nonseparable expected utility model used in Dunn and Singleton (1986), Eichenbaum and Hansen (1990), and Ogaki and Reinhart (1998). The additively separable expected utility model, which is the leading model in macroeconomics and finance applications, is the special case in which all three parameters are related by $\sigma = 1/\gamma = \rho$.

Given the household's current level of wealth W_t and its stock of the durable good D_{t-1}, the household chooses consumption and saving $\{C_t, E_t, B_{0t}, \ldots, B_{Nt}\}$ to maximize its utility (5) subject to the constraints (1), (2), and (3).

B. Euler Equations

Let $R_{W,t+1}$ be the return on wealth from the optimal portfolio (see Appendix B for its relationship to the return on individual assets). Define the intertemporal marginal rate of substitution (IMRS) as

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\sigma} \left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)} \right)^{1/\rho - 1/\sigma} R_{W,t+1}^{1-1/\kappa} \right]^{\kappa}, \tag{8}$$

where

$$v\left(\frac{D}{C} \right) = \left[1 - \alpha + \alpha \left(\frac{D}{C} \right)^{1-1/\rho} \right]^{1/(1-1/\rho)}. \tag{9}$$

Note that $u(C, D) = Cv(D/C)$.

As shown in Appendix B, the household's first-order conditions (FOC) for the consumption and portfolio choice problem imply the Euler equation

$$\mathbf{E}_t [M_{t+1} R_{i,t+1}] = 1 \tag{10}$$

for all assets $i = 0, \ldots, N$. This implies that the excess return on asset i over asset 0 satisfies

$$\mathbf{E}_t[M_{t+1}(R_{i,t+1} - R_{0,t+1})] = 0. \tag{11}$$

Equation (10) is the basis for consumption-based asset pricing. Marginal utility is the appropriate measure of risk for an investor who cares about consumption. Assets that deliver low returns when marginal utility is high must have high expected returns to reward the investor for bearing risk. On the other hand, assets that deliver high returns when marginal utility is high provide a good hedge and consequently must have low expected returns.

Equation (10) is derived here in the context of a household optimization problem, but it holds more generally by a well-known existence theorem. In the absence of arbitrage, there exists a strictly positive stochastic discount factor (SDF) M_t, which satisfies equation (10) for all tradable assets $i = 0, 1, \ldots, N$ (see Cochrane (2001, Chapter 4.2)). Various asset pricing models correspond to particular forms of the SDF.

C. Intratemporal FOC

Let u_C and u_D denote the marginal utility of C and D, respectively. The marginal rate of substitution between the durable and nondurable consumption good is

$$\frac{u_D}{u_C} = \frac{\alpha}{1-\alpha} \left(\frac{D}{C}\right)^{-1/\rho}. \tag{12}$$

As shown in Appendix B, optimal consumption of the durable good requires an intratemporal FOC of the form

$$\frac{u_{Dt}}{u_{Ct}} = P_t - (1-\delta)\mathbf{E}_t[M_{t+1}P_{t+1}] = Q_t. \tag{13}$$

Since a unit of the durable consumption good costs P_t today and can be sold for $(1-\delta)P_{t+1}$ tomorrow, after depreciation, Q_t has a natural interpretation as the user cost of the service flow for the durable good. Equation (13) simply says that the marginal rate of substitution between the durable and nondurable consumption goods must equal the relative price of the durable good.[2]

II. Consumption Data

A. Source and Construction

Quarterly consumption data are from the U.S. national accounts. Following convention, nondurable consumption is measured as the sum of real personal

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[2] When $\delta = 1$ and $\rho = 1$, equation (13) reduces to $\alpha/(1-\alpha) = PD/C$, so α can be interpreted as the expenditure share of the "durable" good. When $\delta < 1$, α loses this economic interpretation since D is the stock, rather than the expenditure, of the durable good.

consumption expenditures (PCE) on nondurable goods and services.[3] Nondurable consumption includes food, clothing and shoes, housing, utilities, transportation, and medical care. Items such as clothing and shoes are durable at the quarterly frequency, but I include them as part of nondurable consumption to be consistent with previous studies of the CCAPM. Similarly, housing is the service flow imputed from the rental value of houses.

Durable consumption consists of items such as motor vehicles, furniture and appliances, and jewelry and watches. The Bureau of Economic Analysis (BEA) publishes year-end estimates of the chained quantity index for the net stock of consumer durable goods. Using quarterly data for real PCE on durable goods, I construct a quarterly series for the stock of durables by equation (1). Implicit in the data for the stock of durables are the depreciation rates used by the BEA for various components of durable goods. The implied depreciation rate for durable goods as a whole is approximately 6% per quarter.
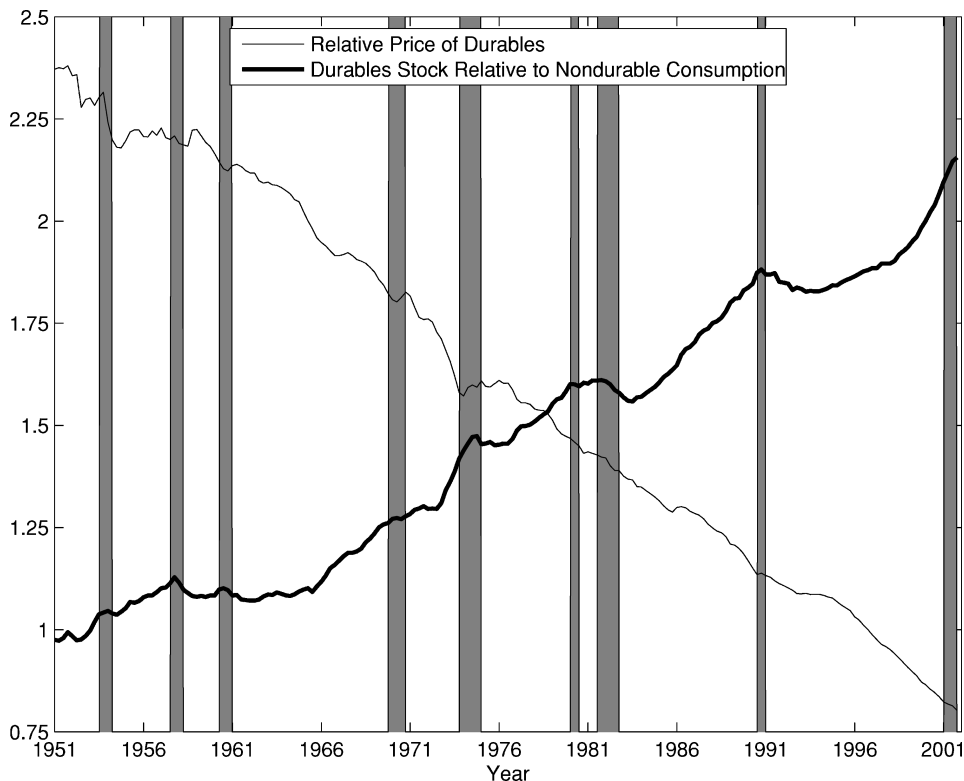
Both nondurable consumption and the stock of durables are divided by the population. The relative price of durables, that is P, is computed as the ratio of the price index for PCE on durable goods to the price index for PCE on nondurable goods and services. In matching consumption to returns data, I use the "beginning of period" timing convention, following Campbell (2003). In other words, the consumption data for each quarter are assumed to be the flow on the first, rather than the last, day of the quarter. Although quarterly consumption data are available since 1947, the period immediately after the war is associated with unusually high durable consumption growth due to the rapid restocking of durable goods. I therefore use data since 1951, following Ogaki and Reinhart (1998). The resulting sample period is 1951:1–2001:4.

B. How Does Durable Consumption Affect Marginal Utility?

Equation (8) reveals that when utility is not additively separable in the nondurable and durable consumption goods, marginal utility has an extra multiplicative term $v(D/C)^{\kappa(1/\rho-1/\sigma)}$. The effect of D/C on marginal utility depends on the relative magnitudes of σ and ρ. Suppose $\kappa > 0$ (i.e., $\sigma < 1$ and $\gamma > 1$), which is the empirically relevant case as discussed in Section III. Then for a given level of nondurable consumption, marginal utility decreases in D/C if $\sigma < \rho$. Intuitively, low nondurable consumption can be offset by high durable consumption provided that the elasticity of substitution between the two goods is sufficiently high. On the other hand, relatively high durable consumption increases marginal utility if the elasticity is low (i.e., $\sigma > \rho$). The additively separable model (i.e., $\sigma = \rho$) is the knife-edge case in which marginal utility is independent of durable consumption.

Figure 1 is a time-series plot of the ratio of the stock of durables to nondurable consumption, that is, D/C. The series has an upward trend in the post-war sample, which is consistent with the downward trend in the price of durables relative to nondurables. The shaded regions are recessions, from peak to trough,

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[3] See Whelan (2000) for issues concerning aggregation of chained national accounts data.

Figure 1.   Price and Stock of Durables Relative to Nondurables. The figure is a time-series plot of (1) the price of durables as a ratio of the price of nondurables and (2) the real stock of durables as a ratio of real nondurable consumption. The sample period is 1951:1–2001:4; the shaded regions are NBER recessions.

as defined by the National Bureau of Economic Research (NBER). The ratio D/C rises during booms and falls during recessions, which is the key empirical fact underlying the findings in this paper. Because D/C is strongly procyclical, the implied marginal utility is strongly countercyclical, provided that the elasticity of substitution between the goods is higher than the EIS (i.e., $\sigma < \rho$). Durable consumption is therefore a key ingredient in explaining stock returns because it makes marginal utility more countercyclical than the canonical CCAPM, in which only nondurable consumption enters marginal utility.

C. Basic Description of Consumption and Portfolio Data

Table I reports descriptive statistics for nondurable and durable consumption growth. (Note that the growth rate in the stock is the growth rate in the consumption of durable goods.) Nondurable consumption growth has mean 0.51% and standard deviation 0.54% per quarter. Durable consumption has mean 0.92% and standard deviation 0.54%. The correlation between them is 0.19.

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Table I

Descriptive Statistics

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The table reports the mean, standard deviation, and first-order autocorrelation of excess market return, SMB return, HML return, and nondurable and durable consumption growth. It also reports the correlations among these variables.

|	Mean	*SD*		Correlation			
Variable	(%)	(%)	Autocorrelation	Market	SMB	HML	Nondurables
Market	1.880	8.186	0.048				
SMB	0.508	5.580	−0.034	0.423			
HML	1.089	5.543	0.154	−0.386	−0.143		
Nondurables	0.513	0.542	0.282	0.281	0.130	0.004	
Durables	0.915	0.535	0.875	−0.110	−0.038	0.036	0.192

Durable consumption growth is much more persistent than nondurable consumption growth: The first-order autocorrelations are 0.88 and 0.28, respectively.

Table I also reports descriptive statistics for the three Fama–French factors, namely, excess returns on the market portfolio, returns on the SMB (Small Minus Big) portfolio, and returns on the HML (High Minus Low) portfolio. (See Appendix A for a detailed description of these factors.) The mean excess returns on the market portfolio, which can be interpreted as the equity premium, is 1.88% per quarter in this sample. Similarly, the mean returns on the SMB and HML portfolios suggest that the size and value premia are 0.51% and 1.09% per quarter, respectively. The Fama–French factors have a low correlation with the two consumption-based factors, especially with durable consumption growth.

D. Business Cycle Properties of Nondurable and Durable Consumption

Figure 2(a) is a time-series plot of the growth rates of nondurable and durable consumption in the post-war sample. Durable consumption growth is strongly procyclical, peaking during booms and hitting lows during recessions. It is therefore a good indicator variable for the business cycle. Nondurable consumption growth is also procyclical, but less so than durable consumption. It tends to fall sharply at the onset of recessions. Figure 2(b) is a time-series plot of nondurable consumption growth minus durable consumption growth. The growth rate of durable consumption generally exceeds that of nondurable consumption, except during and immediately after recessions. The series is strongly countercyclical, highest at business cycle troughs and lowest at business cycle peaks.

To examine the cyclical properties of nondurable consumption in further detail, Figure 3(a) shows the time series for nondurable consumption growth together with the growth rates of two of its components: (1) food and (2) housing. At the end of 2001, food (housing) accounted for 16% (17%) of consumption
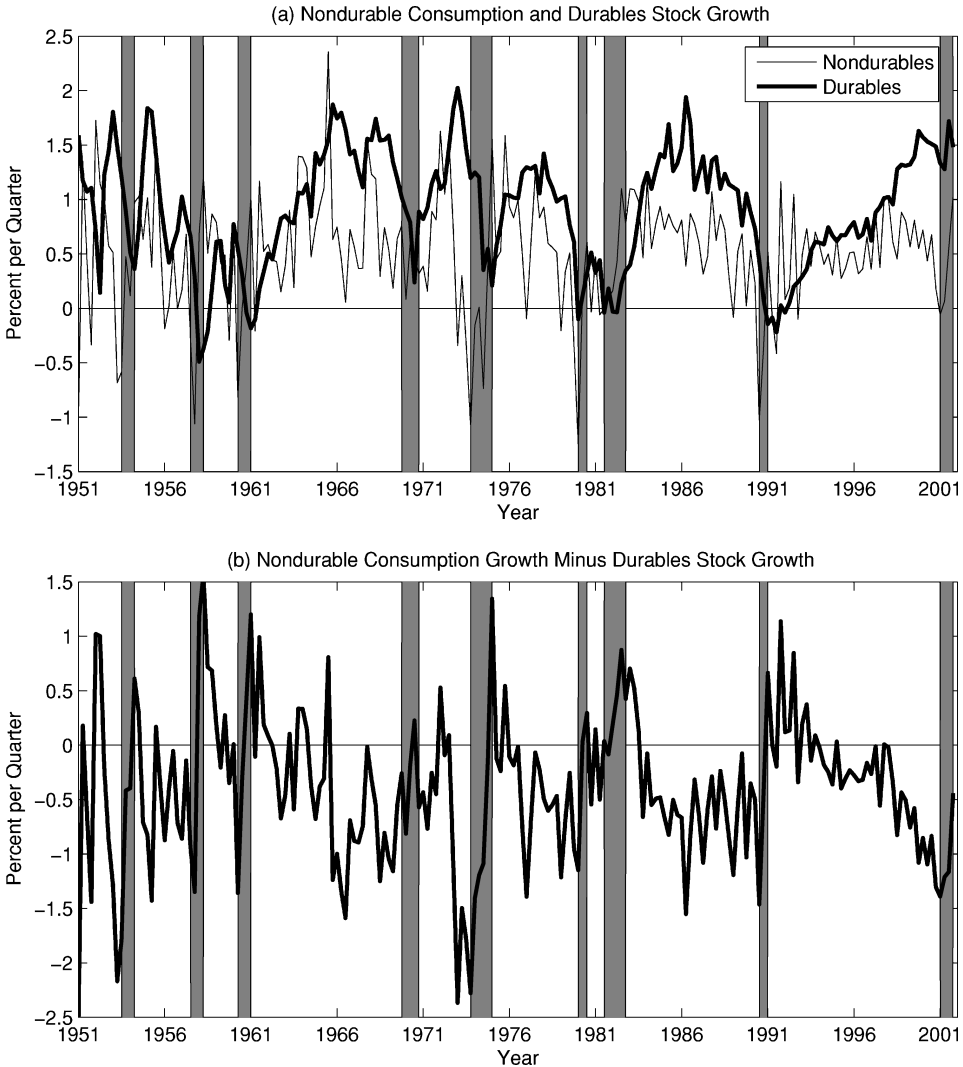
Figure 2. Nondurable and Durable Consumption Growth. The figure is a time-series plot of (a) the real growth rates of nondurable consumption and the stock of durables and (b) the difference in the growth rates. The sample period is 1951:1–2001:4; the shaded regions are NBER recessions.

expenditures on nondurables. The figure illustrates the fact that the components of nondurable consumption share the time-series properties of its aggregate, specifically, low volatility (compared to stock returns), low autocorrelation, and weak cyclicality. Although housing can be thought of as a durable good, its service flow is more similar to that of nondurable goods and services. Implicit in studies of the CCAPM is the assumption that the various components of nondurable consumption are perfect substitutes. This appears to be a reasonable
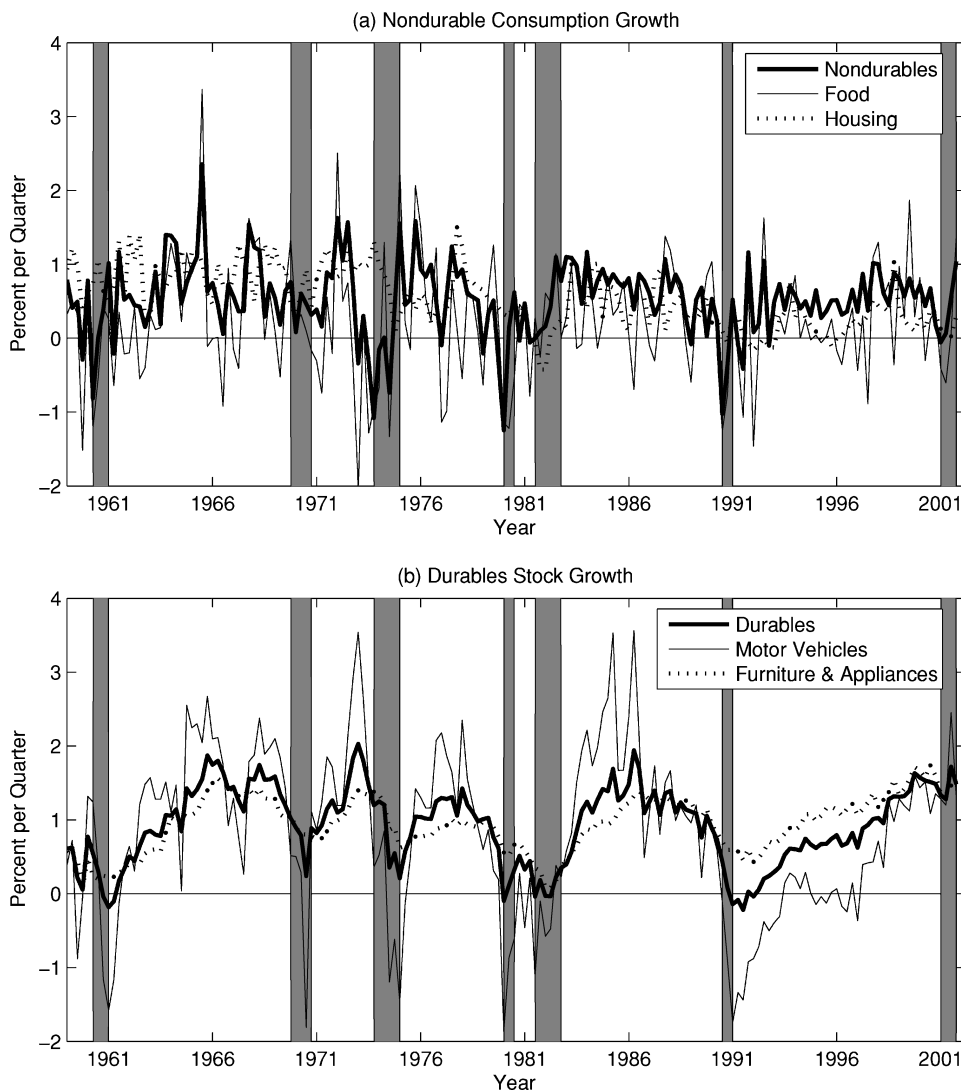
Figure 3. Components of Nondurable and Durable Consumption Growth. The figure is a time-series plot of (a) the real growth rates of nondurable, food, and housing consumption and (b) the real growth rates of the stock of durables, motor vehicles, and furniture and appliances. The sample period is 1959:1–2001:4; the shaded regions are NBER recessions.

assumption for the purposes of empirical work since the various components share similar time-series properties.

Figure 3(b) is a time-series plot of durable consumption growth together with the growth rates of two of its components: (1) motor vehicles and (2) furniture and appliances. At the end of 2001, motor vehicles (furniture and appliances) accounted for 30% (45%) of the stock of consumer durables. The figure illustrates

the fact that the components of durable consumption share the time-series properties of its aggregate, specifically, low volatility (compared to stock returns), high autocorrelation, and strong cyclicality. The consumption of motor vehicles is especially procyclical with sharp falls during recessions. The strong cyclicality of durable consumption is consistent with that of luxury goods (Aït-Sahalia, Parker, and Yogo (2004)).

E. *Estimating the Elasticity of Substitution through Cointegration*

Let lowercase letters denote the logs of the corresponding uppercase variables. Taking the log of both sides of equation (13),

$$\log\left(\frac{\alpha}{1-\alpha}\right) + \frac{1}{\rho}(c_t - d_t) - p_t = q_t - p_t. \tag{14}$$

Suppose the user cost and the spot price of the durable good are cointegrated so that $q_t - p_t$ is stationary. Then $c_t - d_t$ and p_t are cointegrated, with the cointegrating vector equal to $(1, -\rho)'$. Cointegration therefore provides a way to estimate the elasticity of substitution between the durable and nondurable consumption good without observations on the user cost of the durable good (Ogaki and Reinhart (1998)).

I estimate the elasticity of substitution by a dynamic ordinary least squares regression of $c_t - d_t$ on p_t with four leads and lags (Stock and Watson (1993)). For the full sample 1951:1–2001:4, I obtain an estimate of $\rho = 0.790$, with a heteroskedasticity- and autocorrelation-consistent (HAC) standard error of 0.082. Ogaki and Reinhart (1998, Table 2) estimate a higher elasticity of $\rho = 1.167$ with a standard error of 0.099. The difference can be accounted for by their shorter sample period, 1951:1–1983:4.

With homothetic preferences (4), the upward trend in the ratio of the stock of durables to nondurable consumption D/C is explained by the decline in the relative price of durables through the substitution effect (see Figure 1). The magnitude of the elasticity of substitution necessary to explain the trend is around [0.629, 0.950], which is the 95% confidence interval for ρ. Using nonhomothetic preferences, Pakoš (2004) finds that the upward trend in D/C can also be explained with a lower elasticity of substitution ρ as long as durables have a higher income elasticity than nondurables. Even with nonhomothetic preferences, the relationship $\sigma < \rho$ is necessary for generating countercyclical marginal utility because the stock of durables is procyclical (see Figure 2). See Yogo (2005) for a further discussion of this issue.

III. Estimation and Testing of the Durable Consumption Model

In this section, I estimate the preference parameters and test the model through the conditional moment restrictions (10) and (13). Let R_{0t} be the three-month T-bill rate, $R_{it}(i = 1, \ldots, N)$ be returns on N portfolios, and z_t be an $I \times 1$ vector of instrumental variables known at time t. Using the methodology developed by Hansen and Singleton (1982), I use the following moment restrictions for estimation and testing:

$$0 = \mathbf{E}[(M_{t+1}R_{0,t+1} - 1)z_t], \tag{15}$$

$$0 = \mathbf{E}[M_{t+1}(R_{i,t+1} - R_{0,t+1})z_t] \quad (i = 1, \ldots, N), \tag{16}$$

$$0 = \mathbf{E}\left[\left(1 - \frac{u_{Dt}}{P_t u_{Ct}} - (1 - \delta)M_{t+1}\frac{P_{t+1}}{P_t}\right)z_t\right]. \tag{17}$$

Equation (15) represents I moment restrictions implied by the Euler equation for the T-bill rate. Equation (16) represents NI moment restrictions implied by the Euler equations for N portfolio returns. Equation (17) represents I moment restrictions implied by the intratemporal FOC. There are five parameters (σ, γ, ρ, α, and β) to be estimated from a total of $(N + 2)I$ moment restrictions. The $(N + 2)I - 5$ overidentifying restrictions of the model can be tested through the J-test (Hansen (1982)).

The three-month T-bill rate used in the estimation is from the Center for Research in Security Prices (CRSP) Indices Database. All nominal returns are deflated by the price index for PCE on nondurable goods and services. In equation (17), I set $1 - \delta = 0.94$ since the depreciation rate is approximately 6% per quarter; the results are not sensitive to reasonable variation in this parameter. A test of the durable consumption model requires a proxy for the return to wealth (see equation (8)). Following Epstein and Zin (1991), I use the return on the CRSP value-weighted portfolio as the proxy, with the caveat that Roll's (1977) critique of the CAPM applies. For a more careful construction of the proxy that includes the return to human capital, see Bansal, Tallarini, and Yaron (2004).

Estimation is by a two-step (efficient) generalized method of moments (GMM), using the identity weighting matrix in the first stage. HAC standard errors are computed by the vector autoregressive HAC (VARHAC) procedure with automatic lag length selection by the Akaike information criteria (AIC) (see Den Haan and Levin (1997)).[4] Although the errors are in theory a martingale difference sequence, the maximum lag length is set to one quarter to account for the possibility of time aggregation in consumption data (see Hall (1988)).

Section A below focuses on cross-sectional tests based on a large number of portfolio returns and only a constant as the instrument. Section B below focuses on time-series tests based on a smaller number of portfolio returns and various instruments that predict returns.

A. Cross-Sectional Tests of the Model

Panel A of Table II reports four cross-sectional tests of the durable consumption model based on four sets of portfolio returns: 25 Fama–French portfolios

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[4] Den Haan and Levin (2000) find that the VARHAC covariance matrix estimator performs better than kernel-based estimators (e.g., Newey and West (1987) and Andrews (1991)) in various Monte Carlo setups.

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Table II

**Estimation of the Preference Parameters
through the Euler Equations**

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Panel A reports preference parameters for the durable consumption model estimated through the unconditional moment restrictions. From left to right, the test assets are 25 Fama–French portfolios sorted by size and book-to-market equity, 24 portfolios sorted by book-to-market equity within industry, 25 portfolios sorted by market and HML betas, and all 74 portfolios. Panel B reports preference parameters estimated through the conditional moment restrictions. The test assets are the market portfolio, SMB portfolio, and HML portfolio. The instruments are second lags of nondurable and durable consumption growth, dividend-price ratio, size spread, value spread, yield spread, and a constant. All estimates include the Euler equation for the three-month T-bill and the intratemporal FOC as additional moment restrictions. Estimation is by two-step GMM. HAC standard errors are in parentheses. The p-values for the Wald test for additive separability ($\sigma = \rho$), the Wald test for time separability ($\sigma = 1/\gamma$), and the J-test (test of overidentifying restrictions) are in parentheses.

	Panel A: Unconditional Moments				Panel B: Conditional Moments
Parameter	Fama–French	Industry & BE/ME	Beta-Sorted	All Portfolios	
σ	0.024	0.023	0.024	0.023	0.023
	(0.009)	(0.007)	(0.009)	(0.002)	(0.005)
γ	191.438	199.496	185.671	205.905	174.455
	(49.868)	(44.280)	(43.924)	(11.785)	(23.340)
ρ	0.520	0.554	0.870	0.700	0.554
	(0.544)	(0.604)	(1.955)	(0.247)	(0.026)
α	0.827	0.821	0.786	0.802	0.816
	(0.089)	(0.091)	(0.156)	(0.027)	(0.006)
β	0.900	0.935	0.926	0.939	0.884
	(0.055)	(0.054)	(0.057)	(0.018)	(0.030)
Test for $\sigma = \rho$	0.817	0.768	0.187	7.510	375.185
	(0.366)	(0.381)	(0.666)	(0.006)	(0.000)
Test for $\sigma = 1/\gamma$	5.594	8.424	4.637	140.620	12.385
	(0.018)	(0.004)	(0.031)	(0.000)	(0.000)
J-test	12.050	9.583	1.866	5.065	42.500
	(0.956)	(0.984)	(1.000)	(1.000)	(0.065)

sorted by size and book-to-market equity, 24 portfolios sorted by book-to-market within industry, 25 portfolios sorted by market and HML betas, and all 74 portfolios. See Appendix A for a detailed description of these portfolios.

A.1. Fama–French Portfolios

For the Fama–French portfolios, the estimate of the EIS is $\sigma = 0.024$ with a standard error of 0.009. The estimate of risk aversion is $\gamma = 191$ with a standard error of 50. The high risk aversion is a consequence of the low volatility of both nondurable and durable consumption (see Table I); the durable consumption model is therefore unable to resolve the equity premium puzzle. The Wald test for the hypothesis of time separability $\sigma = 1/\gamma$ rejects at the 5% level. This

is consistent with the empirical rejection of the nonseparable expected utility model when estimated with both T-bill and stock returns (Eichenbaum and Hansen (1987)). Intuitively, there is tension between the high risk aversion necessary to explain the equity premium and the "high" EIS (i.e., $\sigma > 1/\gamma$) necessary to explain the T-bill rate (see Yogo (2004, Chapter 1) for further discussion). This problem is resolved by preferences that separate the EIS from risk aversion.

The estimate of the elasticity of substitution between nondurables and durables is $\rho = 0.520$ with a standard error of 0.544. The estimate $\rho = 0.790$ based on cointegration is within a standard error of the point estimate (see Section II.E). The large standard error is evidence that the elasticity is not well identified based only on the cross-sectional moments from the Fama–French portfolios. (The elasticity is much better identified with additional portfolios or instruments, as shown below.) As a consequence, the Wald test for the hypothesis of additive separability $\sigma = \rho$ fails to reject at conventional significance levels.

The estimate of the "utility weight" of durables is $\alpha = 0.827$ with a standard error of 0.089. This indicates that durables have a relatively important role in explaining size and value premia. The intuition for the role of parameter α in explaining stock returns is more clear in the context of the linear factor model, which is discussed in Section IV.

The estimate of the subjective discount factor is $\beta = 0.900$ with a standard error of 0.055. This shows that the durable consumption model is able to explain the low average T-bill rate, despite the high risk aversion. This also explains the failure of the nonseparable expected utility model, in which high risk aversion necessarily implies a low EIS that is inconsistent with the T-bill rate. The expected utility model requires a negative rate of time preference (i.e., $\beta > 1$) to explain the low average T-bill rate, which is well known as the risk-free rate puzzle (see Eichenbaum and Hansen (1987) and Yogo (2004, Chapter 1)). By separating the EIS from risk aversion, the durable consumption model avoids the risk-free rate puzzle.

The J-test fails to reject the durable consumption model at conventional significance levels. In other words, the model successfully prices the 25 Fama–French portfolios. The economic mechanism behind this finding is discussed in Section IV.

A.2. Other Portfolios

Moving to the next two columns of Panel A, the durable consumption model also explains cross-sectional variation in returns across 24 portfolios sorted by book-to-market equity within industry as well as 25 portfolios sorted by market and HML betas. The estimates of the preference parameters are essentially the same as those estimated with the Fama–French portfolios.

The last column of Panel A reports an estimate of the durable consumption model using all 74 portfolios. While the point estimates of the parameters are similar to those from the individual sets of portfolios, the standard errors are

much smaller due to the gain in power. Importantly, the estimate of elasticity of substitution between nondurables and durables is now sufficiently precise to be able to reject the hypothesis of additive separability, $\sigma = \rho$. Because the ratio of the stock of durables to nondurable consumption is procyclical, $\sigma < \rho$ makes marginal utility more countercyclical than in the additively separable model (see Section II.B). Countercyclical marginal utility is the key ingredient in explaining the cross-section of expected stock returns.

B. Time-Series Tests of the Model

I now test the time-series implications of the durable consumption model through moment restrictions (15)–(17). In this section, I use a smaller number of portfolio returns (in order to keep the total number of moments manageable) and add nontrivial instruments that are informative about the state of the economy.

I focus on three portfolio returns that capture the common variation in returns across the 25 Fama–French portfolios, namely, excess returns on the market portfolio, returns on the SMB portfolio, and returns on the HML portfolio. In other words, I use the three Fama–French factors in moment restriction (16). In addition to a constant, I use the following six instruments: nondurable consumption growth, durable consumption growth, dividend-price ratio, size spread, value spread, and long-short yield spread. (See Appendix A for a detailed description of these instruments.) The instruments are lagged twice to account for time aggregation in consumption data (see Hall (1988)).

The result is reported in Panel B of Table II. The estimate of the EIS is $\sigma = 0.023$ with a standard error of 0.005. The estimate of risk aversion is $\gamma = 174$ with a standard error of 23. The Wald test for the hypothesis of time separability, $\sigma = 1/\gamma$, rejects strongly. In other words, the nonseparable expected utility model is rejected (Eichenbaum and Hansen (1987)). The estimate of the elasticity of substitution between nondurables and durables is $\rho = 0.554$ with a standard error of 0.026. The Wald test for the hypothesis of additive separability, $\sigma = \rho$, rejects strongly. In other words, the Epstein–Zin (1991) model is rejected. The estimate of the subjective discount factor is $\beta = 0.884$ with a standard error of 0.030. This shows that the durable consumption model avoids the risk-free rate puzzle. The J-test fails to reject the model at the 5% significance level.

The fact that the preference parameters reported in Panel A agree with those in Panel B deserves emphasis since it has important asset pricing implications. On the one hand, the estimates in Panel A are based on *unconditional* moments, using a large cross-section of portfolio returns. A successful fit of the model implies that the variation in average returns *across stocks* can be explained by the IMRS. Section IV develops this intuition in the context of the linear factor model. On the other hand, the estimates in Panel B are based on *conditional* moments, using instruments that are informative about the state of the economy. A successful fit of the model implies that the variation in average

stock returns *over time* can be explained by the IMRS. Section V develops this intuition in the context of the conditional factor model.

IV. Cross-Sectional Tests of the Linear Factor Model

In this section, I test the cross-sectional implications of the durable consumption model by approximating it as a linear factor model. The test assets are the 25 Fama–French portfolios (Section B) and portfolios sorted by book-to-market equity within industry (Section C). The main advantage of the linear model is that it makes transparent the central economic finding that small stocks and value stocks are procyclical. It also makes the results readily comparable to the large literature on cross-sectional asset pricing, which has focused on linear factor models.

A. *Approximating the Durable Consumption Model*

Appendix C shows that the unconditional Euler equation (11) can be approximated as a linear factor model

$$\mathbf{E}[R_{it} - R_{0t}] = b_1\text{Cov}(\Delta c_t, R_{it} - R_{0t}) + b_2\text{Cov}(\Delta d_t, R_{it} - R_{0t})$$
$$+ b_3\text{Cov}(r_{Wt}, R_{it} - R_{0t}), \tag{18}$$

where the risk prices are given by

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \kappa[1/\sigma + \alpha(1/\rho - 1/\sigma)] \\ \kappa\alpha(1/\sigma - 1/\rho) \\ 1 - \kappa \end{bmatrix}. \tag{19}$$

Suppose $\kappa > 0$ (i.e., $\sigma < 1$ and $\gamma > 1$), which is the empirically relevant case as shown in Table II. Equation (18) says that an asset with high nondurable consumption beta, $\text{Cov}(\Delta c_t, R_{it} - R_{0t})/\text{Var}(\Delta c_t)$, must have high expected returns. Similarly, an asset with high durable consumption beta, $\text{Cov}(\Delta d_t, R_{it} - R_{0t})/\text{Var}(\Delta d_t)$, must have high expected returns when $b_2 > 0$. The risk price of durable consumption is positive when $\rho > \sigma$, that is, when the elasticity of substitution between the two consumption goods is higher than the EIS. In equilibrium, differences in expected returns across assets must reflect differences in the quantity of risk across assets, measured by the covariance of returns with nondurable and durable consumption growth.

The linear factor model (18) nests the following models as important special cases.

1. The Epstein–Zin (1991) model, in which $\sigma = \rho$, $b_1 = \kappa/\sigma$, $b_2 = 0$, and $b_3 = 1 - \kappa$.
2. The CAPM (Sharpe (1964), Lintner (1965)), in which $\sigma = \rho$, $\sigma \to \infty$, $b_1 = b_2 = 0$, and $b_3 = \gamma$.

3. The nonseparable expected utility model, in which $\sigma = 1/\gamma, b_1 = \gamma + \alpha(1/\rho - \gamma), b_2 = \alpha(\gamma - 1/\rho)$, and $b_3 = 0$.
4. The CCAPM (Breeden (1979), Breeden and Litzenberger (1978), Rubinstein (1976)), in which $\sigma = 1/\gamma = \rho, b_1 = \gamma$, and $b_2 = b_3 = 0$.

B. Estimation with the Fama–French Portfolios

B.1. Estimation of Linear Factor Models

Table III reports estimates of the factor risk prices for the CAPM, the Fama–French three-factor model, the CCAPM, and the durable consumption model. Estimation is by two-step GMM; see Appendix C for details. HAC standard errors, reported in parentheses, are computed by the VARHAC procedure with automatic lag length selection by AIC. The maximum lag length is set to two quarters to account for autocorrelation; the results are not sensitive to allowing for longer lags. The correction for autocorrelation is especially important in estimating the durable consumption model due to the persistence of durable consumption growth.

Table III
Estimation of Linear Factor Models with the Fama–French Portfolios

The table reports the estimated factor risk prices for the CAPM, the Fama–French three-factor model, the CCAPM, and the durable consumption model. The test assets are the 25 Fama–French portfolios sorted by size and book-to-market equity. Estimation is by two-step GMM. HAC standard errors are in parentheses. The mean absolute pricing error (MAE) and R^2 are based on the first-stage estimate. The p-values for the J-test (test of overidentifying restrictions) are in parentheses.

Factor Price	CAPM	Fama–French	CCAPM	Durable Model
Market	4.268	4.632		0.659
	(0.510)	(0.841)		(0.849)
SMB		−0.860		
		(1.154)		
HML		6.072		
		(1.198)		
Nondurables			142.073	17.898
			(25.409)	(31.280)
Durables				170.569
				(15.561)
σ				0.002
				(0.004)
γ				189.127
				(35.259)
α				0.907
				(0.147)
MAE (%)	0.602	0.235	0.338	0.122
R^2	−0.620	0.716	0.350	0.935
J-test	72.414	54.920	46.785	23.170
	(0.000)	(0.000)	(0.004)	(0.392)

The CAPM has a positive and significant risk price on the market return. The mean absolute pricing error from the first stage is 0.60% per quarter. Instead of reporting the mean squared pricing error, I report one minus its ratio to the variance of average portfolio returns, which is referred to as the R^2 (see Campbell and Vuolteenaho (2004)). The R^2 for the CAPM is -62%, which implies that the model has less explanatory power than simply predicting constant average returns across the portfolios. The J-test, or the test of overidentifying restrictions, strongly rejects the model.

The Fama–French three-factor model is much more successful than the CAPM. (See Appendix C for a description of the three-factor model.) The mean absolute pricing error is 0.24%, and the R^2 is 72%. The risk price for SMB is not significantly different from zero, while the risk price for HML is significantly positive. Hence, the improvement over the CAPM is mostly captured by the explanatory power of HML. Although the first-stage measures of fit are much better than the CAPM, the J-test rejects the model.

For the CCAPM, the risk price for nondurable consumption is positive and significantly different from zero. The large point estimate of 142, which is a consequence of the low volatility of nondurable consumption, is consistent with the literature on the equity premium puzzle. The mean absolute pricing error is 0.34%, and the R^2 is 35%. Although the CCAPM has better first-stage measures of fit than the CAPM, it falls short of the three-factor model. Moreover, the J-test strongly rejects the model.

The last column of Table III reports an estimate of the durable consumption model (see equation (18)). The risk price for the market return is positive but not significantly different from zero. The risk price for nondurable consumption is 18, which is smaller than that estimated for the CCAPM, and not significantly different from zero. The risk price for durable consumption is larger at 171 and statistically significant. Therefore, the CCAPM, which is a restriction that the risk price on durable consumption be equal to zero, is strongly rejected. The mean absolute pricing error is 0.12%, and the R^2 is 94%. The J-test fails to reject the model at conventional significance levels.

Equation (19) shows that the risk prices are related to the preference parameters by $\sigma = (1 - b_3)/(b_1 + b_3)$, $\gamma = b_1 + b_2 + b_3$, and $\alpha = b_2/[b_1 + b_2 + (b_3 - 1)/\rho]$. Note that ρ and α are not separately identified in the linear factor model. Table III reports the preference parameters implied by the estimates of the risk prices. I set $\rho = 0.790$ for the purposes of this calculation (see Section II.E). The implied parameters $\sigma = 0.002$, $\gamma = 189$, and $\alpha = 0.907$ are roughly consistent with those reported in Table II. Note that when $b_3 \approx 1$, $\alpha \approx b_2/(b_1 + b_2)$. Therefore, α can be interpreted as the importance of durables, relative to nondurables, as a factor in pricing assets. The large α suggests that durables are a relatively important factor in explaining the variation in returns across the Fama–French portfolios.

Figure 4(d) provides a visual summary of the empirical success of the durable consumption model. On the vertical axis is the realized average excess return. On the horizontal axis is the return predicted by the model, based on the first-stage estimates. The points represent the 25 Fama–French portfolios, and the
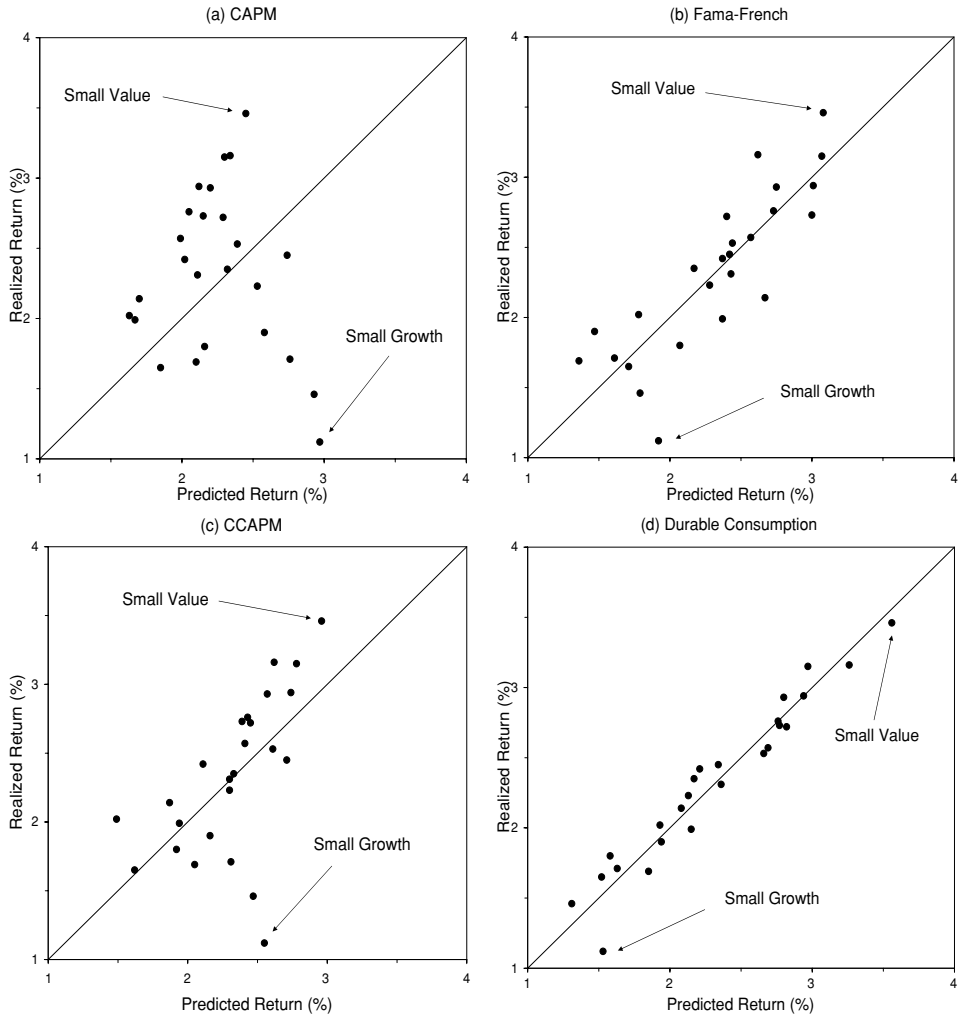
Figure 4. Realized versus Predicted Returns for the Fama–French Portfolios. The figure plots realized versus predicted excess returns (per quarter) for the 25 Fama–French portfolios sorted by size and book-to-market equity. The estimated models are (a) the CAPM, (b) the Fama–French three-factor model, (c) the CCAPM, and (d) the durable consumption model.

corresponding vertical distance to the diagonal line represents the pricing error. The pricing errors for the durable consumption model are much smaller than those for the CAPM (Panel a) and the CCAPM (Panel c). It even outperforms the Fama–French three-factor model (Panel b).

Figure 4 reveals that the small growth portfolio (i.e., the lowest quintile in both size and book-to-market equity) has the largest pricing error in all the linear factor models. For the durable consumption model, its pricing error is 0.41%. D'Avolio (2002) and Lamont and Thaler (2003) document limits to arbitrage,

due to short-sale constraints, for the types of stocks that are generally characterized as small growth. It is perhaps unsurprising then that these frictionless equilibrium models have difficulty explaining the small growth portfolio.

B.2. *Nondurable and Durable Consumption Betas*

To better understand the success of the durable consumption model, Table IV reports the nondurable and durable consumption betas implied by the first-stage GMM estimates. Panel A reports the average excess returns for the 25 Fama–French portfolios sorted by size and book-to-market equity. Reading down the columns of the panel, average returns decrease in size for a given

Table IV
Average Returns and Consumption Betas for the Fama–French Portfolios

Panel A reports average excess returns (per quarter) on the 25 Fama–French portfolios sorted by size and book-to-market equity. Panels B and C report nondurable and durable consumption betas, implied by the first-stage GMM estimate of the durable consumption model, respectively. The last row reports the difference between small and big stocks, and the last column reports the difference between high and low book-to-market stocks.

Size	Low	2	3	4	High	High–Low
	Book-to-Market Equity					
Panel A: Average Excess Return (%)						
Small	1.121	2.448	2.531	3.160	3.464	2.343
2	1.458	2.225	2.716	2.929	3.150	1.692
3	1.707	2.345	2.313	2.756	2.937	1.230
4	1.896	1.797	2.417	2.568	2.725	0.829
Big	1.686	1.652	2.015	1.987	2.140	0.454
Small−Big	−0.565	0.796	0.516	1.173	1.324	
Panel B: Nondurable Consumption Beta						
Small	6.512	6.126	5.814	5.438	6.216	−0.296
2	6.071	5.119	5.241	5.436	5.899	−0.172
3	5.457	5.142	5.057	5.159	5.926	0.469
4	4.923	4.302	4.465	5.225	5.061	0.137
Big	4.759	3.547	2.974	4.242	3.967	−0.792
Small−Big	1.754	2.578	2.841	1.196	2.249	
Panel C: Durable Consumption Beta						
Small	0.317	1.209	1.638	2.271	2.502	2.185
2	0.120	1.089	1.838	1.834	1.967	1.847
3	0.517	1.193	1.434	1.857	1.979	1.461
4	0.904	0.676	1.347	1.798	1.838	0.934
Big	0.956	0.750	1.268	1.396	1.325	0.368
Small−Big	−0.640	0.459	0.370	0.875	1.177	

book-to-market equity quintile. The only exception is for low book-to-market stocks, whose average returns roughly increase in size. Reading across the rows of the panel, average returns increase in book-to-market equity for a given size quintile. The table therefore confirms the well-known size and value premia.

Panel B of the table reports the nondurable consumption betas. Reading down the columns of the panel, nondurable consumption beta decreases in size for a given book-to-market equity quintile. This pattern is broadly consistent with the size premium. However, reading across the rows of the panel, nondurable consumption beta is not related to book-to-market equity in a consistent way for a given size quintile. Moreover, the variation in beta across book-to-market equity is relatively small compared to the variation across size. The difference in nondurable consumption beta between small and big stocks is at least 1.20 (for book-to-market quintile 4). On the other hand, the difference in beta between high and low book-to-market stocks is at most 0.47 (for size quintile 3). The relatively small variation in nondurable consumption beta across book-to-market equity is the reason why the CCAPM fails to explain the value premium.

Panel C of the table reports the durable consumption betas. Reading down the columns of the panel, durable consumption beta decreases in size for a given book-to-market equity quintile, with the exception of low book-to-market stocks. This is consistent with the pattern in average returns across the size quintiles. Moreover, durable consumption beta increases in book-to-market equity for a given size quintile, explaining the value premium. The difference in durable consumption beta between high and low book-to-market stocks is in general larger than that difference between small and big stocks. For instance, the difference in beta between high and low book-to-market stocks is 1.46 for the median size quintile. On the other hand, the difference in beta between small and big stocks is only 0.37 for the median book-to-market equity quintile.

Roughly speaking, durable consumption beta accounts for the variation in average returns across book-to-market equity (i.e., value premium), while nondurable consumption beta accounts for the variation in average returns across size (i.e., size premium).

B.3. A Discussion of Recent Macroeconomic Factor Models

The 25 Fama–French portfolios have been the focus of recent work on cross-sectional asset pricing because of the failure of the (C)CAPM in explaining their returns. Table V is a representative but not comprehensive summary of the success of recent macroeconomic factor models. The three criteria used in the comparison are whether the model (1) has a higher R^2 than the Fama–French three-factor model, (2) fits the risk-free rate, and (3) passes the J-test (test of overidentifying restrictions).

Several papers have found that the (C)CAPM can explain returns on the Fama–French portfolios if the market return or nondurable consumption growth is scaled by a conditioning variable (Lettau and Ludvigson (2001), Lustig and Van Nieuwerburgh (2005), Santos and Veronesi (2006)). A way to interpret these findings is that the (C)CAPM fails unconditionally, but works

<div align="center">

Table V

A Comparison of Recent Macroeconomic Factor Models
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The table summarizes the success of recent macroeconomic factor models in explaining the 25 Fama–French portfolios. The three criteria used in the comparison are whether the model (1) has a higher R^2 than the Fama–French three-factor model, (2) fits the risk-free rate, and (3) passes the J-test (test of overidentifying restrictions). Lettau and Ludvigson (2001, Table 3.A) is a conditional CCAPM with the consumption-wealth ratio as a conditioning variable. Lustig and Van Nieuwerburgh (2005, Tables 9 and 13) is a conditional CCAPM with the housing-human wealth ratio as a conditioning variable. Parker and Julliard (2005, Table 5.B) is a CCAPM with the three-year growth rate of nondurable consumption as a factor. Piazzesi, Schneider, and Tuzel (2003, Table 7) is a CCAPM with the growth rate of housing services as a factor. Santos and Veronesi (2006, Table 6.A) is a conditional CAPM with the labor income-consumption ratio as a conditioning variable.

Factor Model	Number of Factors	Sample	Higher R^2 than Fama–French	Fits Risk-Free Rate	Passes J-Test
Lettau and Ludvigson (2001)	3	Quarterly 1948–2001	No	No	Yes
Lustig and Van Nieuwerburgh (2005)	3	Annual 1926–2002	Yes	Yes	Yes
		Quarterly 1952–2002	No	Yes	Not Reported
Parker and Julliard (2005)	2	Quarterly 1947–1999	Yes	Yes	No
Piazzesi, Schneider, and Tuzel (2003)	3	Annual 1936–2001	No	No	Not Reported
Santos and Veronesi (2006)	4	Quarterly 1948–2001	No	No	Yes
This paper	3	Quarterly 1951–2001	Yes	Yes	Yes

conditionally. A common problem in tests of the conditional (C)CAPM is that the model is unable to explain the low risk-free rate. In terms of Figure 4, the (C)CAPM predicts the correct slope but the wrong intercept.

A different line of work finds that the long-run growth rate in nondurable consumption explains returns on the Fama–French portfolios (Bansal, Dittmar, and Lundblad (2005), Parker and Julliard (2005)). Piazzesi, Schneider, and Tuzel (2003) augment the CCAPM with the growth rate in housing services; this paper augments the CCAPM with the growth rate in durable consumption. To the extent that housing services and durables are a persistent source of risk (see Figure 2), these factors are related to the notion of long-run consumption risk.

C. Estimation with Portfolios Sorted by Book-to-Market Equity within Industry

To examine the value premium in more detail, I now test the durable consumption model on portfolios sorted by book-to-market equity within industry.

The question is whether value stocks, that is, stocks with high book-to-market equity relative to other stocks in the same industry, have high consumption betas that account for their premia.

C.1. Estimation of Linear Factor Models

Table VI reports estimates of linear factor models using the portfolios sorted by book-to-market equity within industry. For the durable consumption model, the point estimate of the risk price for durable consumption is 155, which is similar to that estimated using the Fama–French portfolios. Since the risk price is significantly different from zero, the CCAPM is rejected. The R^2 for the model is 71%, compared to 60% for the Fama–French three-factor model. The J-test rejects the model at the 5% level, but this must be due to approximation error since the nonlinear model is not rejected (see Panel A in Table II).

C.2. Nondurable and Durable Consumption Betas

Panel A of Table VII reports the average excess returns for the 24 portfolios. Reading across the rows of the panel, average returns increase in

Table VI
Estimation of Linear Factor Models with Portfolios Sorted by Book-to-Market Equity within Industry

The test assets are 24 portfolios sorted by book-to-market equity within industry. Portfolios are formed by first sorting stocks into eight industries, then by sorting into three levels of book-to-market equity (breakpoints of 30th and 70th percentiles) within each industry. See notes to Table III.

Factor Price	CAPM	Fama–French	CCAPM	Durable Model
Market	3.535	4.003		1.000
	(0.907)	(1.017)		(0.840)
SMB		0.565		
		(1.255)		
HML		5.153		
		(1.261)		
Nondurables			133.756	43.655
			(31.379)	(24.224)
Durables				154.500
				(25.705)
σ				0.000
				(0.004)
γ				199.155
				(30.818)
α				0.780
				(0.103)
MAE (%)	0.605	0.345	0.512	0.311
R^2	0.053	0.599	0.301	0.711
J-test	49.515	33.522	39.259	34.428
	(0.001)	(0.041)	(0.019)	(0.033)

Table VII

Average Returns and Consumption Betas for Portfolios Sorted by Book-to-Market Equity within Industry

Panel A reports average excess returns (per quarter) on 24 portfolios sorted by book-to-market equity (B/M) within industry. Panels B and C report nondurable and durable consumption betas, implied by the first-stage GMM estimate of the durable consumption model. See notes to Table VI for details on portfolio formation.

Industry	Panel A: Average Return (%)			Panel B: Nondurable Beta			Panel C: Durable Beta		
	Low B/M	Med B/M	High B/M	Low B/M	Med B/M	High B/M	Low B/M	Med B/M	High B/M
Manufacturing									
Nondurables	1.904	2.270	2.819	4.185	4.336	5.068	1.201	1.268	1.554
Durables	1.727	2.397	3.744	6.056	5.882	8.383	−0.050	0.770	2.796
Other	1.516	1.894	2.664	5.485	3.714	4.843	0.574	1.167	2.242
Retail									
Nondurables	1.961	2.627	2.522	6.035	4.465	4.918	0.397	0.894	1.035
Durables	2.259	2.052	3.480	5.732	6.309	5.552	0.375	−0.055	0.899
Services	1.670	1.298	2.182	4.381	3.406	5.815	−0.207	0.110	1.511
Finance	1.537	2.584	3.104	5.041	4.980	3.813	0.285	1.341	1.645
Natural resource	0.277	1.627	2.928	2.031	3.543	4.304	−0.334	1.255	2.782

book-to-market equity for each industry. In all industries, the high book-to-market portfolio has higher average returns than the low book-to-market portfolio. Interestingly, the high book-to-market portfolios in the durables manufacturing and durables retail industries have the highest average returns.

Panel B reports the nondurable consumption betas. Reading across the rows of the panel, nondurable consumption beta increases in book-to-market equity for only four out of eight industries. The relatively small variation in nondurable consumption beta across book-to-market equity is the reason why the CCAPM fails to explain the value premium. In constrast, durable consumption beta (Panel C) increases in book-to-market equity in all eight industries. Table VII makes clear the source of the value premia. In a given industry, high book-to-market stocks have returns that are more procyclical than low book-to-market stocks. Value stocks therefore carry a high premium to compensate the investor for bearing business cycle risk, measured by durable consumption growth.

V. Time Variation in Expected Stock Returns

Stock returns can be predicted by various financial variables such as valuation ratios and asset returns (see the references in the introduction). In a factor pricing model, time variation in expected returns must be explained by time variation in the quantity of risk, measured by the conditional covariance of the factors with returns. Therefore, the same variables that predict returns must predict the product of the innovation to returns with the factors. I now document this connection between risk and return for the durable consumption model.

A. Approximating the Durable Consumption Model

Appendix D shows that the conditional Euler equation (11) can be approximated as a conditional factor model

$$\mathbf{E}_{t-1}[R_{it} - R_{0t}] = b_1 \text{Cov}_{t-1}(\Delta c_t, R_{it} - R_{0t}) + b_2 \text{Cov}_{t-1}(\Delta d_t, R_{it} - R_{0t})$$
$$+ b_3 \text{Cov}_{t-1}(r_{Wt}, R_{it} - R_{0t}), \tag{20}$$

where the risk prices are given by equation (19).

Equation (20) says that the expected return on an asset is high when the covariance of its returns with nondurable consumption growth is high. Similarly, the expected return is high when the covariance of its returns with durable consumption growth is high, provided that $b_2 > 0$. In equilibrium, variation in expected returns over time must reflect variation in the quantity of risk over time, measured by the conditional covariance of returns with nondurable and durable consumption growth.

B. Predictability of Returns

As described in Appendix D, I estimate the conditional moments in equation (20) as linear functions of instruments known at $t - 1$. I use the same excess returns and instruments that are used in Section III.B for the test of the

conditional Euler equation. The returns are excess returns on the market portfolio, returns on the SMB portfolio, and returns on the HML portfolio. I report results for two sets of instruments. The first set consists of just nondurable and durable consumption growth. The second set also includes the dividend-price ratio, size spread, value spread, and long-short yield spread. I impose the risk prices implied by the estimated preference parameters reported in Panel B of Table II, namely, $b_1 = 39$ for nondurables, $b_2 = 139$ for durables, and $b_3 = -3.14$ for the market return.

Panel A of Table VIII reports estimates of a regression model for the conditional mean of returns (see equation (D6) in Appendix D). The discussion that follows focuses on the market portfolio since its evidence for predictability is much stronger than that for the SMB and HML portfolios. The coefficient on nondurable consumption growth is positive and significant, while the coefficient on durable consumption growth is negative and significant. In addition, the dividend-price ratio, size spread, and value spread predict returns. This implies that expected stock returns are high when nondurable consumption growth is high and durable consumption growth is low. As shown in Figure 2(b), nondurable consumption growth is high (low) relative to durable consumption growth at business cycle troughs (peaks). The coefficients therefore imply a countercyclical equity premium.

Panel B reports estimates of a regression model for the conditional covariance of returns with nondurable consumption growth (see equation (D7) in Appendix D). The dividend-price ratio, size spread, and value spread predict the product of the innovation to returns with nondurable consumption growth. This implies that the conditional covariance of stock returns with nondurable consumption growth is high when the dividend-price ratio, size spread, or value spread is high.

Panel C reports estimates of a regression model for the conditional covariance of returns with durable consumption growth. Nondurable consumption growth predicts the product of the innovation to returns with durable consumption growth positively, while durable consumption growth predicts it negatively. This implies that the conditional covariance of stock returns with durable consumption growth is high when nondurable consumption growth is high relative to durable consumption growth. In other words, the conditional covariance of stock returns with durable consumption growth is countercyclical.

To summarize, Table VIII uncovers some interesting facts about the predictability of stock returns. Nondurable and durable consumption growth predict returns because they predict durable (rather than nondurable) consumption risk, that is, the product of the innovation to returns with durable consumption growth. This is consistent with the implications of the conditional factor model (20); time variation in expected returns must be accounted for by time variation in the conditional covariance of returns with the factors.
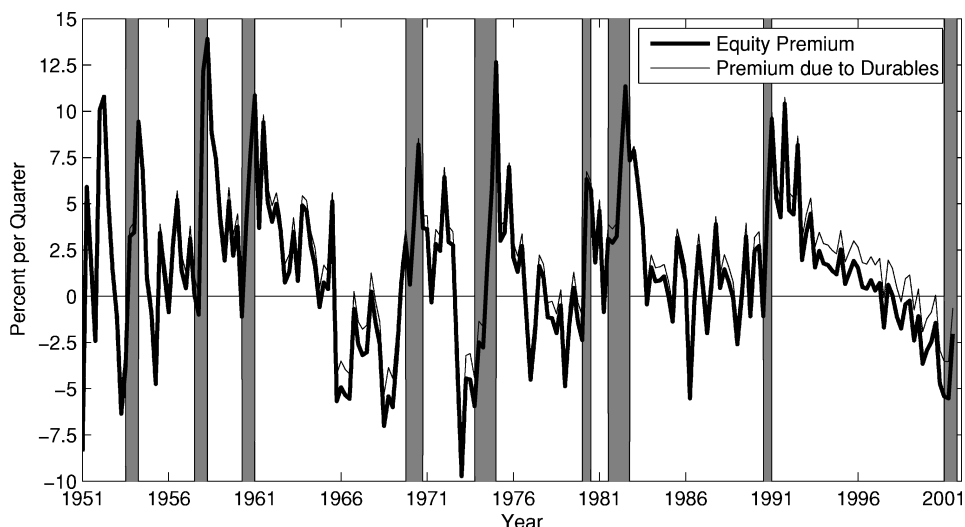
C. Implied Equity Premium

Figure 5 is a time-series plot of the equity premium, that is, expected excess returns on the market portfolio, implied by the estimates in Table VIII. The

Table VIII

Expected Return and Conditional Covariance

The table reports estimates of the regression model for expected returns in Panel A. The table also reports the regression models for the conditional covariance of returns with nondurable consumption growth (Panel B) and durable consumption growth (Panel C). The test assets are the market portfolio, SMB portfolio, and HML portfolio. The instruments are lags of nondurable and durable consumption growth, dividend-price ratio, size spread, value spread, yield spread, and a constant. Estimation is by two-step GMM. HAC standard errors in parentheses. Coefficients significant at the 5% level (i.e., *t*-statistic greater than 1.960) are in bold.

Instrument	Market	SMB	HML	Market	SMB	HML
			Panel A: Expected Return			
Nondurables	**2.961**	0.799	0.097	**4.596**	**1.999**	−0.645
	(0.654)	(0.648)	(0.455)	(0.730)	(0.582)	(0.472)
Durables	**−4.318**	−0.467	−0.316	**−5.660**	**−1.558**	0.052
	(0.665)	(0.537)	(0.468)	(0.749)	(0.480)	(0.422)
Dividend price				**0.045**	0.017	− 0.008
				(0.013)	(0.009)	(0.008)
Size spread				**0.054**	−0.008	0.008
				(0.025)	(0.019)	(0.017)
Value spread				**0.052**	0.027	−0.018
				(0.018)	(0.015)	(0.012)
Yield spread				0.374	−0.140	0.022
				(0.322)	(0.245)	(0.167)
			Panel B: Covariance with Nondurable Consumption			
Nondurables	−0.840	−0.163	0.510	−0.427	0.136	0.672
	(0.493)	(0.443)	(0.403)	(0.502)	(0.407)	(0.384)
Durables	−0.595	0.092	**−0.756**	**−0.840**	−0.027	−0.539
	(0.456)	(0.400)	(0.346)	(0.427)	(0.338)	(0.314)
Dividend price				**0.023**	0.013	**0.016**
				(0.009)	(0.007)	(0.006)
Size spread				**0.042**	−0.010	−0.012
				(0.017)	(0.014)	(0.011)
Value spread				**0.041**	0.019	−0.004
				(0.013)	(0.013)	(0.008)
Yield spread				−0.264	−0.040	−0.139
				(0.173)	(0.134)	(0.118)
			Panel C: Covariance with Durable Consumption			
Nondurables	**1.758**	0.496	0.179	**2.971**	**1.321**	−0.375
	(0.387)	(0.443)	(0.316)	(0.445)	(0.395)	(0.319)
Durables	**−2.812**	−0.291	−0.131	**−3.812**	**−1.114**	0.078
	(0.446)	(0.364)	(0.315)	(0.472)	(0.329)	(0.262)
Dividend price				**0.025**	0.007	−0.005
				(0.008)	(0.006)	(0.005)
Size spread				**0.033**	−0.004	0.009
				(0.017)	(0.013)	(0.012)
Value spread				**0.033**	0.017	− 0.013
				(0.012)	(0.010)	(0.009)
Yield spread				0.250	−0.124	0.004
				(0.185)	(0.161)	(0.108)

Figure 5. Time Variation in the Equity Premium. The figure is a time-series plot of expected excess returns on the market portfolio. The sample period is 1951:1–2001:3; the shaded regions are NBER recessions.

dark line represents the total equity premium, $\mathbf{E}_{t-1}[R_{it} - R_{0t}]$, and the light line represents the part due to durables, $b_2 \mathrm{Cov}_{t-1}(\Delta d_t, R_{it} - R_{0t})$. The difference, of course, is the premium due to nondurables and the market return. The plot reveals two interesting facts. First, the two lines tend to overlap, which implies that most of the time variation in the equity premium is driven by the time variation in durable consumption risk. This is the reason why the CCAPM fails to explain the time variation in expected returns; it misses an important component of the cyclical variation in expected returns by ignoring the durables premium. Second, the equity premium is strongly countercyclical, that is, highest at business cycle troughs and lowest at business cycle peaks.

The plot of the equity premium resembles the plot of the difference between nondurable and durable consumption growth (see Figure 2(b)). During a recession, durable consumption falls sharply relative to nondurable consumption, causing the marginal utility of consumption to rise sharply. This causes the equity premium to rise sharply at the business cycle trough. As durable consumption rises relative to nondurable consumption during the subsequent boom, marginal utility falls gradually, and so does the equity premium. Time variation in the equity premium simply reflects time variation in risk, as measured by the marginal utility of consumption.

VI. Conclusion

The findings of this paper suggest that there is much empirical content in the theoretical paradigm of consumption-based asset pricing. The central insight of

the CCAPM is that the marginal utility of consumption is the relevant measure of risk for an investor. This paper shows that the marginal utility of consumption, when suitably modeled, can explain the trade-off between risk and return reflected in the size premium, the value premium, and the time-varying equity premium.

The central ingredient is a nonseparable utility function in nondurable and durable consumption, where the elasticity of substitution between the two types of goods is high relative to the additively separable case. Small stocks and value stocks deliver low returns when marginal utility rises, that is, during recessions when durable consumption falls. These stocks must therefore have high expected returns to reward the investor for bearing risk. In addition, stocks deliver unexpectedly low returns when marginal utility rises sharply, that is, at business cycle troughs when durable consumption falls sharply relative to nondurable consumption. The equity premium must therefore be high during recessions to reward the investor for bearing risk.

The mechanism through which the durable consumption model generates a countercyclical equity premium is similar to that of the external habit formation model (Campbell and Cochrane (1999)). In the Campbell–Cochrane model, the surplus consumption ratio is strongly procyclical and magnifies the countercyclicality of marginal utility relative to the canonical CCAPM. In the durable consumption model, the ratio of durable to nondurable consumption is strongly procyclical and magnifies the countercyclicality of marginal utility.

Although the durable consumption model can explain both the cross-section of expected stock returns and the time variation in the equity premium, it requires rather high risk aversion to do so because of the low volatility of both nondurable and durable consumption. The high risk aversion does not lead to a risk-free rate puzzle if preferences allow for the separation of the EIS and risk aversion. However, one may still "reject" the model on the grounds that high risk aversion is a priori unreasonable. The risk aversion implied by the Campbell–Cochrane model is also high, and in that model the risk-free rate puzzle is avoided by having intertemporal substitution exactly offset precautionary savings. I agree with the view that "high risk aversion is inescapable (or at least has not yet been escaped) in the class of identical-agent models that are consistent with the equity premium facts..." (Campbell and Cochrane (1999, p. 243)).

Regardless of whether one believes in the representative household model, this paper uncovers some intriguing facts about stock returns and the business cycle. Specifically,

1. Small stocks and value stocks have higher nondurable and durable consumption betas than big stocks and growth stocks. The returns on small stocks and value stocks are more procyclical than those on big stocks and growth stocks.
2. The expected stock return is high (low) when nondurable consumption growth is high (low) relative to durable consumption growth. The equity premium is strongly countercyclical.

3. The conditional covariance of stock returns with durable consumption growth is high (low) when nondurable consumption growth is high (low) relative to durable consumption growth. Stock returns tend to be unexpectedly low (high) during recessions (booms).

Appendix A: Data Used in Asset Pricing Tests

I. Portfolio Data

A. Fama–French Factors and Portfolios

The three Fama–French factors are excess returns on the market portfolio, returns on the SMB portfolio, and returns on the HML portfolio. The excess market return is the return on a value-weighted portfolio of NYSE, AMEX, and Nasdaq stocks minus the one-month T-bill rate. The SMB and HML portfolios are based on the six Fama–French benchmark portfolios sorted by size (breakpoint at the median) and book-to-market equity (breakpoints at the 30th and 70th percentiles). The SMB return is the difference in average returns between three small and three big stock portfolios. The HML return is the difference in average returns between two high and two low book-to-market portfolios.

The 25 Fama–French portfolios are constructed from an independent sort of all NYSE, AMEX, and Nasdaq stocks into quintiles based on size (i.e., market equity) and book-to-market equity. Data on the Fama–French factors and portfolios are obtained from Kenneth French's web page. See Fama and French (1993) for further details on the construction of the factors and portfolios.

B. Portfolios Sorted by Book-to-Market Equity within Industry

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. In June of each year t, stocks are sorted into eight industries based on their two-digit SIC codes: (1) nondurables manufacturing, (2) durables manufacturing, (3) other manufacturing, (4) nondurables retail, (5) durables retail, (6) services, (7) finance, and (8) natural resource. Within each industry, stocks are then sorted into three levels of book-to-market equity using breakpoints of 30th and 70th percentiles, based on their value in December of $t - 1$. Once the 24 portfolios are formed, their value-weighted returns are tracked from July of t through June of $t + 1$. Quarterly portfolio returns are computed by compounding monthly returns.

The industry definitions are designed to create variation in book-to-market equity that is independent of nondurable and durable consumption. See Table A1 for the SIC codes corresponding to each industry. The book equity data is a merge of historical data from Moody's Manuals (available from Kenneth French's web page) and COMPUSTAT. See Davis, Fama, and French (2000) for details on the computation of book equity.

Table A1
Industry Definitions

Industry	Two-Digit SIC Codes
Manufacturing	
Nondurables	20–23, 26–28, 31
Durables	25, 36, 37, 39
Other	15–19, 24, 29, 30, 32–35, 38
Retail	
Nondurables	51, 54, 56, 58, 59
Durables	50, 52, 53, 55, 57
Services	40–49, 70–99
Finance	60–69
Natural resource	1–14

C. Risk-Sorted Portfolios

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. In June of each year t, market and HML betas are computed for each stock using monthly returns from January of $t - 5$ through December of $t - 1$. Stocks with return data missing in any month are dropped from the sample. Then 25 portfolios are formed by independently sorting stocks into quintiles based on the market and HML betas. The value-weighted portfolio returns are then tracked from July of t through June of $t + 1$. Quarterly portfolio returns are computed by compounding monthly returns.

Risk-sorted portfolios provide a rigorous test for asset pricing models by creating a large spread in the post-formation betas. The sort based on the pre-formation market and HML betas works well in practice. Portfolios with high (low) pre-formation market betas have high (low) post-formation nondurable consumption betas, and portfolios with high (low) pre-formation HML betas have high (low) post-formation durable consumption betas. The reason for using the market and HML returns, rather than nondurable and durable consumption growth, in forming portfolios is that returns are much more noisy than consumption. Therefore, pre-formation consumption betas are too noisy and fail to create the desired spread in the post-formation betas.

II. Instruments Used in Time-Series Tests

A. Dividend-Price Ratio

The dividend-price ratio is constructed as the sum of dividends over the past four quarters divided by the current price for the CRSP NYSE–AMEX value-weighted portfolio. The dividend-price ratio is related, by a present value relationship, to the expectation of future returns and dividend growth, and therefore predicts returns (Campbell and Shiller (1988a)).

B. Size and Value Spread

Annual book equity and monthly market equity data for the six Fama–French benchmark portfolios is obtained from Kenneth French's web page. Following Cohen, Polk, and Vuolteenaho (2003), the book-to-market equity for each of the six portfolios is computed as the ratio of book equity to market equity as follows. The book-to-market equity in June of year t is the book equity in December of $t - 1$ divided by the market equity in June of t. The book-to-market equity in the subsequent months from July of t through May of $t + 1$ is the book equity in December of $t - 1$ divided by that month's market equity.

The value spread is the difference in average book-to-market equity between the two high and two low book-to-market portfolios. The value spread is related, by a present value relationship, to the expectation of future returns and profitability and therefore predicts HML returns (Cohen, Polk, and Vuolteenaho (2003)). The size spread is the difference in the average book-to-market equity between the three small and three big stock portfolios.

C. Long-Short Yield Spread

Following Fama and French (1989), the long yield used in computing the yield spread is Moody's Seasoned Aaa Corporate Bond Yield. The short rate used is the one-month T-bill rate from the CRSP Fama Risk-Free Rates Database. The yield spread "tends to be low near business cycle peaks and high near troughs" (Fama and French (1989, p. 30)), much like the difference in nondurable and durable consumption growth (see Figure 2(b)).

Appendix B: Derivation of the Euler Equations

Following Bansal, Tallarini, and Yaron (2004) and Cuoco and Liu (2000), I first simplify the consumption and portfolio choice problem with a durable consumption good through a change of variables. Define

$$\tilde{W}_t = W_t + (1 - \delta)P_t D_{t-1}, \tag{B1}$$

$$B_{N+1,t} = P_t D_t, \tag{B2}$$

$$R_{N+1,t+1} = (1 - \delta)\frac{P_{t+1}}{P_t}. \tag{B3}$$

Then the intraperiod identity (2) can be rewritten as

$$\sum_{i=0}^{N+1} B_{it} = \tilde{W}_t - C_t, \tag{B4}$$

and the intertemporal budget constraint (3) can be rewritten as

$$\tilde{W}_{t+1} = \sum_{i=0}^{N+1} B_{it} R_{i,t+1}. \tag{B5}$$

Define the portfolio shares $\omega_{it} = B_{it}/(\tilde{W}_t - C_t)$ for all $i = 0, \ldots, N + 1$. Then equations (B4) and (B5) are equivalent to

$$1 = \sum_{i=0}^{N+1} \omega_{it}, \tag{B6}$$

$$\tilde{W}_{t+1} = (\tilde{W}_t - C_t) \sum_{i=0}^{N+1} \omega_{it} R_{i,t+1}. \tag{B7}$$

Since $P_t D_t = \omega_{N+1,t}(\tilde{W}_t - C_t)$, D_t can be substituted out of intraperiod utility as

$$u(C_t, D_t) = C_t \left[1 - \alpha + \alpha \left(\frac{\omega_{N+1,t}(\tilde{W}_t/C_t - 1)}{P_t} \right)^{1-1/\rho} \right]^{1/(1-1/\rho)}$$

$$= C_t \tilde{v} \left(\frac{C_t}{\tilde{W}_t}, \omega_{N+1,t} \right). \tag{B8}$$

Finally, the household's problem can be restated as follows. Given its current level of wealth $\tilde{W}_t$, which includes the stock of the durable good, the household chooses its consumption and portfolio shares $\{C_t, \omega_{0t}, \ldots, \omega_{N+1,t}\}$ to maximize utility (5) subject to the constraints (B6) and (B7). The solution to this problem, that is the household's optimal consumption and portfolio shares, will be denoted by $\{C_t^*, \omega_{0t}^*, \ldots, \omega_{N+1,t}^*\}$.

The Bellman equation for the problem is given by

$$J_t(\tilde{W}_t) = \max_{C_t, \omega_{0t}, \ldots, \omega_{N+1,t}} \left\{ (1 - \beta) \left[C_t \tilde{v} \left(\frac{C_t}{\tilde{W}_t}, \omega_{N+1,t} \right) \right]^{1-1/\sigma} \right.$$

$$\left. + \beta \left(\mathbf{E}_t \left[J_{t+1}(\tilde{W}_{t+1})^{1-\gamma} \right] \right)^{1/\kappa} \right\}^{1/(1-1/\sigma)}. \tag{B9}$$

By the homogeneity of the optimization problem, the value function is proportional to wealth $J_t(\tilde{W}_t) = \phi_t \tilde{W}_t$. Using arguments similar to Epstein and Zin (1991), it can be shown that

$$\phi_t = \left[(1 - \beta)(1 - \alpha)\tilde{v} \left(\frac{C_t^*}{\tilde{W}_t}, \omega_{N+1,t}^* \right)^{1/\rho - 1/\sigma} \right]^{1/(1-1/\sigma)} \left(\frac{C_t^*}{\tilde{W}_t} \right)^{1/(1-\sigma)}. \tag{B10}$$

Let $R^*_{W,t+1} = \sum_{i=0}^{N+1} \omega^*_{it} R_{i,t+1}$ be the return on wealth from the optimal portfolio, and let

$$M^*_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\sigma} \left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)} \right)^{1/\rho-1/\sigma} R^{*1-1/\kappa}_{W,t+1} \right]^{\kappa}. \tag{B11}$$

After tedious algebra along the lines of Epstein and Zin (1991), the FOC with respect to C_t is given by

$$\mathbf{E}_t \left[M^*_{t+1} R^*_{W,t+1} \right] = \left(1 - \frac{\omega^*_{N+1,t} u_{Dt}}{P_t u_{Ct}} \right)^{\kappa}. \tag{B12}$$

Similarly, the FOC with respect to ω_{it} is given by

$$\mathbf{E}_t \left[M^*_{t+1} (R_{i,t+1} - R_{0,t+1}) \right] = 0 \tag{B13}$$

for all assets $i = 1, \ldots, N$. The FOC with respect to $\omega_{N+1,t}$, that is the fraction of wealth held in the durable good, is given by

$$\mathbf{E}_t \left[M^*_{t+1} (R_{0,t+1} - R_{N+1,t+1}) \right] = \frac{u_{Dt}}{P_t u_{Ct}} \left(1 - \frac{\omega^*_{N+1,t} u_{Dt}}{P_t u_{Ct}} \right)^{\kappa-1}. \tag{B14}$$

Straightforward algebra reveals that equations (B12), (B13), and (B14) imply

$$\mathbf{E}_t \left[M^*_{t+1} R_{i,t+1} \right] = \left(1 - \frac{\omega^*_{N+1,t} u_{Dt}}{P_t u_{Ct}} \right)^{\kappa-1} \tag{B15}$$

for all assets $i = 0, \ldots, N$.

Finally, equations (10) and (13) are obtained from equations (B14) and (B15) through the normalization

$$R_{W,t+1} = \left(1 - \frac{\omega^*_{N+1,t} u_{Dt}}{P_t u_{Ct}} \right)^{-1} R^*_{W,t+1}, \tag{B16}$$

$$M_{t+1} = \left(1 - \frac{\omega^*_{N+1,t} u_{Dt}}{P_t u_{Ct}} \right)^{1-\kappa} M^*_{t+1}. \tag{B17}$$

Appendix C: Linear Factor Model

Suppose there is a SDF M_t such that

$$\mathbf{E}[M_t (R_{it} - R_{0t})] = 0 \tag{C1}$$

for all assets $i = 1, \ldots, N$. Moreover, suppose that the SDF is linear in a vector f_t of F underlying factors, that is

$$-\frac{M_t}{\mathbf{E}[M_t]} = k + b' f_t. \tag{C2}$$

Let $\mu_f = \mathbf{E}[f_t]$, $\Sigma_{ff} = \mathbf{E}[(f_t - \mu_f)(f_t - \mu_f)']$, and $\Sigma_{fi} = \mathbf{E}[(f_t - \mu_f)(R_{it} - R_{0t})]$. Equation (C1) can then be written as a linear factor model

$$\mathbf{E}[R_{it} - R_{0t}] = b'\Sigma_{fi}. \qquad (C3)$$

This equation says that the premium on asset i is the price of risk b times its quantity of risk Σ_{fi}.

Define the "beta" of asset i as $\beta_i = \Sigma_{ff}^{-1}\Sigma_{fi}$, which can be interpreted as the coefficient vector in a multiple regression of R_{it} onto f_t. The linear factor model can be written as a beta pricing model

$$\mathbf{E}[R_{it} - R_{0t}] = \lambda'\beta_i, \qquad (C4)$$

where $\lambda = \Sigma_{ff}b$ is the factor risk premium.

I. Fama–French Three-Factor Model

In response to the failures of the CAPM and the CCAPM, Fama and French (1993) proposed an influential three-factor model. The three factors are excess returns on the market portfolio, returns on the SMB portfolio, and returns on the HML portfolio. The Fama–French three-factor model nests the CAPM as a special case, in which the risk prices for SMB and HML are restricted to zero.

Although the model is an empirical success, it falls short of a satisfactory understanding of the underlying risk reflected in stock returns. "Without a theory that specifies the exact form of the state variables or common factors in returns, the choice of any particular version of the factors is somewhat arbitrary" (Fama and French (1993, p. 53)). As emphasized by Cochrane (2001, Chapter 9), a satisfactory factor model must ultimately connect the factors to the marginal utility of consumption.

II. Approximating the Durable Consumption Model

Taking the log of both sides of (8) and approximating around the special case of Cobb–Douglas intraperiod utility (i.e., $\rho = 1$),

$$-m_t \approx -\kappa \log \beta + b_1 \Delta c_t + b_2 \Delta d_t + b_3 r_{Wt}. \qquad (C5)$$

(See equation (19) for b_1, b_2, and b_3.) The approximation is exact when $\rho = 1$.

A nonlinear SDF M_t can be approximated by first-order log-linear approximation as

$$\frac{M_t}{\mathbf{E}[M_t]} \approx 1 + m_t - \mathbf{E}[m_t]. \qquad (C6)$$

Using equation (C5), the SDF (8) of the durable consumption model can be approximated as

$$-\frac{M_t}{\mathbf{E}[M_t]} \approx k + b_1\Delta c_t + b_2\Delta d_t + b_3 r_{Wt}, \tag{C7}$$

where $k = -1 - b_1\mathbf{E}[\Delta c_t] - b_2\mathbf{E}[\Delta d_t] - b_3\mathbf{E}[r_{Wt}]$. This approximation results in the linear factor model (18).

III. GMM Estimation of Linear Factor Models

Since the linear factor model is a set of moment restrictions on asset returns, GMM is a natural way to estimate and test the model.[5] Since my focus is on consumption-based models, I base estimation on the covariance representation (C3), rather than the beta representation (C4) of the model. The coefficients b of the covariance representation are immediately interpretable as preference parameters, unlike the coefficients λ of the beta representation.

Define the parameter space $\Theta \subset \mathbb{R}^{2F}$ with a generic element $\theta = (b', \mu_f')'$. Let $R_{0t}, R_t = (R_{1t}, \ldots, R_{Nt})'$, and f_t be the time t observation of the reference return (e.g., T-bill rate), the vector of N test asset returns, and the vector of F factors, respectively. Stack the variables in a vector as $z_t = (R_{0t}, R_t', f_t')'$. Let ι be an $N \times 1$ vector of ones. Consider the $(N + F) \times 1$ moment function

$$e(z_t, \theta) = \begin{bmatrix} R_t - R_{0t}\iota - (R_t - R_{0t}\iota)(f_t - \mu_f)'b \\ f_t - \mu_f \end{bmatrix}. \tag{C8}$$

The moment function satisfies the moment restriction $\mathbf{E}[e(z_t, \theta_0)] = 0$, for some $\theta_0 \in \Theta$, through equation (C3). A necessary condition for identification is that $N \geq F$. A sufficient condition for identification is that the $F \times N$ matrix $[\Sigma_{f1} \cdots \Sigma_{fN}]$ has rank F. This condition assures that θ_0 is a unique solution to $\mathbf{E}[e(z_t, \theta)] = 0$, so that the key identification condition for GMM is satisfied (see Wooldridge (1994, Theorem 7.1)). Intuitively, the factors cannot be perfectly correlated in order for the factor risk prices to be identified.

In two-step GMM, a first-stage weighting matrix

$$W = \begin{bmatrix} kI_N & 0 \\ 0 & \hat{\Sigma}_{ff}^{-1} \end{bmatrix}, \tag{C9}$$

where $k > 0$ is a constant and $\hat{\Sigma}_{ff}$ is a consistent estimator of Σ_{ff}, puts an equal weight on the N moment restrictions for asset returns. The overidentifying restrictions of the model can be tested by Hansen's (1982) J-test. The degree of overidentification is $N - F$. The J-test tests the null hypothesis that the pricing errors are jointly zero across the N test assets. The test is conceptually similar to the GRS test (Gibbons, Ross, and Shanken (1989)) since the test statistic is a quadratic form in the vector of pricing errors (see Cochrane (2001, Chapters 12–13)).

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[5] See Cochrane (2001, Chapter 13) for a textbook treatment of GMM for linear factor models.

Appendix D: Conditional Factor Model

Suppose there is a SDF M_t such that

$$\mathbf{E}_{t-1}[M_t(R_{it} - R_{0t})] = 0 \tag{D1}$$

for all assets $i = 1, \ldots, N$. Moreover, suppose that the SDF is linear in a vector f_t of F underlying factors, that is

$$-\frac{M_t}{\mathbf{E}_{t-1}[M_t]} = k_{t-1} + b' f_t. \tag{D2}$$

Equation (D1) can then be written as a conditional factor model

$$\mathbf{E}_{t-1}[R_{it} - R_{0t}] = \sum_{j=1}^{F} b_j \operatorname{Cov}_{t-1}(f_{jt}, R_{it} - R_{0t}). \tag{D3}$$

This equation says that the premium on an asset is the price of risk b_j times the quantity of risk $\operatorname{Cov}_{t-1}(f_{jt}, R_{it} - R_{0t})$, summed over all factors $j = 1, \ldots, F$.

I. Approximating the Durable Consumption Model

A nonlinear SDF M_t can be approximated by first-order log-linear approximation as

$$\frac{M_t}{\mathbf{E}_{t-1}[M_t]} \approx 1 + m_t - \mathbf{E}_{t-1}[m_t]. \tag{D4}$$

Using equation (C5), the SDF (8) of the durable consumption model can be approximated as

$$-\frac{M_t}{\mathbf{E}_{t-1}[M_t]} \approx k_{t-1} + b_1 \Delta c_t + b_2 \Delta d_t + b_3 r_{Wt}, \tag{D5}$$

where $k_{t-1} = -1 - b_1 \mathbf{E}_{t-1}[\Delta c_t] - b_2 \mathbf{E}_{t-1}[\Delta d_t] - b_3 \mathbf{E}_{t-1}[r_{Wt}]$. This approximation results in the conditional factor model (20).

II. Estimation of Conditional Moments Using Instruments

I now describe a way to estimate the conditional moments of the conditional factor model (D3), using a vector x_{t-1} of I instrumental variables known at time $t-1$. The essential idea behind the method is that the representative household's information set can always be conditioned down to the econometrician's information set. Equation (D3) therefore holds even when the conditioning information is restricted to x_{t-1}. The methodology described here has been used previously in empirical work by Campbell (1987) and Harvey (1989).

Let $r_{it} = R_{it} - R_{0t}$ be the excess return on asset i. Consider the linear regression model

$$r_{it} = \Pi_i' x_{t-1} + \epsilon_{it} \quad (i = 1, \ldots, N), \tag{D6}$$

$$\epsilon_{it} f_{jt} = \Upsilon_{ij}' x_{t-1} + \eta_{ijt} \quad (i = 1, \ldots, N; j = 1, \ldots, F). \tag{D7}$$

Equation (D6) models the conditional mean excess returns. Equation (D7) models the conditional covariance of excess returns with the factors. The model (D6)–(D7) is exactly identified under the conditional moment restriction

$$\mathbf{E}[(\epsilon_{it}, \eta_{ijt})' \,|\, x_{t-1}] = 0 \quad \forall i, j. \tag{D8}$$

Define the matrices

$$\Pi = [\Pi_1 \cdots \Pi_N] \quad (I \times N), \tag{D9}$$

$$\Upsilon_j = [\Upsilon_{1j} \cdots \Upsilon_{Nj}] \quad (I \times N), \tag{D10}$$

$$\Upsilon = [\Upsilon_1 \cdots \Upsilon_F] \quad (I \times NF). \tag{D11}$$

The conditional factor model (D3) implies NI linear restrictions of the form

$$\Pi = \sum_{j=1}^{F} b_j \Upsilon_j. \tag{D12}$$

Using this equation to substitute out Π_i in equation (D6),

$$r_{it} = \left(\sum_{j=1}^{F} b_j \Upsilon_{ij} \right)' x_{t-1} + \epsilon_{it}. \tag{D13}$$

Define the parameter space $\Theta \subset \mathbb{R}^{NFI}$ with a generic element $\theta = \mathrm{vec}(\Upsilon)$. Let $r_t = (r_{1t}, \ldots, r_{Nt})'$ and f_t be the time t observation of the vector of N excess returns and the vector of F factors, respectively. Stack the variables and the instruments in a vector as $z_t = (r_t', f_t', x_{t-1})'$. The regression model (D7) and (D13) can be estimated through the $(N + NF)I \times 1$ moment function

$$e(z_t, \theta; b) = \begin{bmatrix} r_t - \left(\sum_{j=1}^{F} b_j \Upsilon_j \right)' x_{t-1} \\ \mathrm{vec}\left(\left[r_t - \left(\sum_{j=1}^{F} b_j \Upsilon_j \right)' x_{t-1} \right] f_t' \right) - \Upsilon' x_{t-1} \end{bmatrix} \otimes x_{t-1}. \tag{D14}$$

The moment function satisfies the moment restriction $\mathbf{E}[e(z_t, \theta_0; b)] = 0$, for some $\theta_0 \in \Theta$, through the conditional moment restriction (D8). If the vector of risk prices b is known, the model is overidentified by NI degrees. Otherwise, if the vector b is estimated together with Υ, the model is overidentified by $NI - F$ degrees.

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