Market Price of Variance Risk and Performance of Hedge Funds

Oleg Bondarenko*

March 2004 (first version: June 2003)

Abstract

This paper implements a model-free approach to measure the market price of the variance risk. In this approach, the value of the variance contract is estimated from prices of traded options. We find that the variance risk is priced, its risk premium is negative and economically very large. In the application to hedge funds, we argue that the variance return is a key determinant in explaining performance of hedge funds. Most hedge funds exhibit negative exposure to the variance return, implying that they routinely "sell" the variance risk. The variance risk factor accounts for a considerable portion of hedge fund historical returns.

Keywords: Variance Risk, Option Valuation, Risk-Neutral Density, Stochastic Volatility, Hedge Funds

*Department of Finance (MC 168), University of Illinois at Chicago, 601 S. Morgan St., Chicago, IL 60607. Phone: (312) 996-2362. Email: olegb@uic.edu

1 Introduction

It is well-known that volatility of equity returns tends to change unpredictably over time. In particular, many empirical studies have documented that broad market indexes often experience large shifts in volatility. However, it is less-understood whether investors require compensation for the volatility risk and, if yes, to what extent. This issue is important for a number of asset pricing applications, including option pricing, risk management, performance evaluation, and others.

The issue is also a difficult one, because volatility is not a tradeable asset and its market price is not observable. The previous research has typically relied on strong parametric assumptions in order to infer the volatility risk premium from prices of traded options. As a result, those approaches may suffer from potential misspecification of the option pricing model and the data-generating process for the underlying asset.

Recently, there has been a growing demand for instruments to hedge shifts in volatility. Several derivative instruments have been introduced in over-the-counter market. One important example is variance swaps on various market indexes. A variance swap has a payoff, which is equal to the difference between the realized variance over a given period and the contract variance.[1] Trading in variance swaps has grown dramatically in the aftermath of the recent financial turmoils, such as the summer of 1997, the fall of 1998, and the fall of 2001. It seems likely that standardized variance swaps will soon be introduced on organized exchanges.

In principle, giving a history of prices for variance swaps, it would be relatively straightforward to estimate the market price of the variance risk. Unfortunately, this approach is not practical because the OTC market in variance swaps is still rather illiquid, it has a relatively short history, and its transaction prices are not readily available to researches. In this paper, we implement a different approach, where the market price of the variance contract is recovered from observed prices of exchange-listed options. The approach is motivated by the theoretical advances in Dupire (1993), Neuberger (1994), Carr and Madan (1998) and Demeterfi, Derman, Kamal, and Zou (1999).

The approach is model-free, in the sense that no parametric structure is imposed on the underlying price process. Its central idea can be explained as follows. Suppose that an underlying asset follows a continuous price process F_t. Let V_T denote the continuously-sampled variance from time-0 to time-T. Let U_0 be the time-0 market price of the variance contract, which at maturity-T has a payoff of V_T. The price U_0 reflects both the market expectations of the future variance and investor's preferences. By comparing U_0 with the ex post realized variance V_T, one can determine the market price of the variance risk. Specifically, if r_v denotes the realized return on the variance contract in excess of the risk free rate, then its sample average yields an estimate for the variance risk premium. The important insight in the above theoretical papers is that the price U_0 can be expressed in terms of the market price of the so-called log contract, which has a payoff $\log F_T$ at maturity-T. Although not traded in real markets, the log contract can nevertheless be synthesized from standard calls and puts with different strikes and the same maturity T.

The previous literature has three important limitations. First, it requires the price process F_t to be continuous. This is restrictive because the empirical literature presents ample evidence

[1] Both *variance* and *volatility* swaps are available over-the-counter, with volatility being defined as the square root of variance. Although the two types of swaps are closely related, variance swaps appear to be somewhat more popular. This is mainly due to the fact that variance swaps offer useful theoretical advantages and as such are easier to price and hedge.

of jumps in asset prices. Second, the literature assumes that continuous path monitoring is possible. In practice, of course, the variance must be computed from discretely-sampled data. Third, the dynamic replication of the variance payoff V_T requires *continuous* rebalancing, which is, again, impossible in practice because of transaction costs.

In this paper, we propose a way around this limitations. Specifically, we extend the above results to the case when the price process could be discontinuous, when it is monitored discretely (say, once a day), and when the variance payoff can be perfectly replicated without continuous rebalancing. To implement this approach, we use options on the S&P 500 index futures. Over the period from 1988 to 2000, we generate a series of monthly variance return r_v and document the following main results:

- We find that the variance risk is priced, its risk premium is negative and very large in economic terms. In particular, the variance risk premium is estimated as -26.66% per month.[2] For comparison, the risk premium on the market return r_m is 0.89% per month.

- It is well-documented in the empirical literature that the market variance is negatively correlated with the market return. That is, the variance tends to increase when the stock market falls. Consistent with the previous literature, we find that the variance return r_v and the market return r_m are negatively correlated. This raises an important question as to whether the variance risk is priced *beyond* its negative correlation with the market return. We show that the negative correlation accounts for only a small portion of the variance risk premium (-4.14% per month). The remaining portion (-22.52% per month) can be interpreted as the compensation for the "pure" variance risk.

- We argue that selling the variance contract would have resulted in very high profits over the 13-year period. The profits are high on the risk-adjusted basis, where the Sharpe ratio or other popular measures are used. In particular, the Sharpe ratio for selling the variance contract is more than twice that for the market itself.

- We find that the economic value to investors of being able to trade the variance contract could be very high. For example, consider an investor with the CRRA preferences who has $1 mln of the investable wealth. If the investor's risk aversion coefficient is 3 (10), then he is willing to pay up to $17,400 ($6,000) per month for being able to sell the variance contract. The economic value of introducing the variance contract is large even when the investor can also trade options on the market.

The fact that the variance risk is priced has important implications for explaining returns on various assets. If assets prices are sensitive to changes in the market variance, then assets expected returns must reflect the exposure to the variance risk. Even a slight exposure to the variance risk could have a non-trivial impact, because the variance risk premium is so substantial. To demonstrate this point, we present an application to hedge funds. The application is motivated by a popular claim that many hedge funds sell short market volatility. To our best knowledge, our paper is first to present a direct evidence for this claim. We find that returns of most hedge funds are indeed negatively correlated with the variance return.

[2] The issue whether and to what extent the variance risk is priced has been investigated in a number of recent papers, including Bakshi, Cao, and Chen (1997, 2000), Guo (1998), Bates (2000), Chernov and Ghysels (2000), Buraschi and Jackwerth (2001), Coval and Shumway (2001), Benzoni (2002), Pan (2002), Bakshi and Kapadia (2003a, 2003b). The related literature is reviewed in Section 2.5. Compared to the existing approaches, our approach is completely model-free and/or produces many new empirical insights.

This finding is important for hedge fund performance evaluation. Previous studies usually conclude that hedge funds, as a group, deliver superior risk-adjusted returns. That is, linear regressions of fund returns on various factors typically produce positive and statistically significant alpha coefficients. However, since these regressions do not include the variance return, the superior performance might be misleading when hedge funds have substantial negative exposures to the variance risk.

In our analysis, we focus on hedge fund indexes from two popular databases of the Hedge Fund Research (HFR) and the CSFB/Tremont. Both databases classify hedge funds into several categories according to their investment strategies. Empirical research on hedge funds is complicated by the fact that their returns are highly serially correlated. Asness, Krail, and Liew (2001) argue that hedge fund alpha coefficients are often overstated when serial correlation is not corrected for. Getmansky, Lo, and Makarov (2003) examines a number of potential explanations for serial correlation. They narrow down all explanations to 1) illiquidity of hedge fund assets, and 2) smoothing of returns (whether smoothing is deliberate or unintentional). To address serial correlation, we adopt the same econometric model as in Getmansky, Lo, and Makarov (2003). As a methodological contribution, we propose to estimate the model via a new maximum likelihood procedure, which extends the approach in Getmansky, Lo, and Makarov by enforcing additional restrictions on parameters implied by the model. The new procedure has important properties that it produces efficient estimates and is computationally simple.

The main empirical findings for hedge funds can be summarized as follows.

- For HFR database, the sensitivity to the variance return comes out negative and statistically significant for 11 of 15 individual fund categories. The largest "sellers" of the variance risk are Distressed Securities, Emerging Markets, Equity Non-Hedge, Event-Driven, Fixed Income: High Yield, and Macro. For these categories, the exposure to the variance risk accounts for 0.70%, 1.05%, 0.60%, 0.53%, 0.60%, and 0.48% per month, respectively. There are only three categories with positive (but not statistically significant) exposure to the variance risk. Those are Equity Market Neutral, Market Timing, and Short Selling.

- The inclusion of the variance return r_v in the model has effect of i) reducing the exposure to the market return r_m, and ii) reducing alpha coefficient. Again, the only exceptions are Equity Market Neutral, Market Timing, and Short Selling.

  For the above 6 fund categories, the change in the alpha coefficient due to the inclusion of the variance return is -0.56%, -0.85%, -0.47%, -0.42%, -0.48%, and -0.38% per month, respectively.

- Our analysis includes both equity and non-equity hedge funds. Although the variance return is derived exclusively from options on the equity index, we find that the variance return is an important explanatory variable even for *non-equity* funds.

- We check robustness of our results on CSFB/Tremont indexes, which are constructed using somewhat different methodology and strategy definitions. All findings remain qualitatively similar.

Overall, our results suggest that the variance risk accounts for a considerable portion of hedge fund average returns. On average, the hedge fund industry earns about 6.5% annually by short selling the variance risk. This implies that the variance risk accounted for roughly \$32.4 bln of hedge fund returns in 2000 alone.

The remainder of the paper is organized as follows. Section 2 presents the model-free methodology for estimating the variance return. Section 3 describes the option dataset of S&P 500 futures options and studies the statistical properties of the variance return. Section 4 considers the application to hedge funds. Section 5 concludes.

2 Theoretical Background

Our empirical study is based on options written on the S&P 500 futures. Therefore, let F_t denote the value of the S&P 500 futures contract expiring at some date T', and let $P_t(K) = P(K,T;F_t,t)$ and $C_t(K) = C(K,T;F_t,t)$ be the prices of European put and call with strike K and maturity $T \le T'$. To simplify exposition, we assume that the risk-free rate is zero.[3]

The option prices can be computed using the risk-neutral density (RND):

$$C_t(K) = \int_0^\infty (F_T - K)^+ \, h_t(F_T)\, dF_T, \qquad P_t(K) = \int_0^\infty (K - F_T)^+ \, h_t(F_T)\, dF_T,$$

where $h_t(F_T) = h(F_T,T;F_t,t)$ is RND. RND satisfies the relationship first discovered in Ross (1976), Breeden and Litzenberger (1978), Banz and Miller (1978):

$$h_t(F_T) = \left.\frac{\partial^2 C_t(K,T)}{\partial K^2}\right|_{K=F_T} = \left.\frac{\partial^2 P_t(K,T)}{\partial K^2}\right|_{K=F_T}. \tag{1}$$

2.1 The continuously-sampled variance contract

To present the main idea of our approach, we at first assume that F_t follows a continuous process. Continuity of F_t and the absence of arbitrage opportunities imply that there is a risk-neutral measure under which F_t evolves as

$$\frac{dF_t}{F_t} = \sqrt{v_t} dB_t^*, \tag{2}$$

where B_t^* is the standard Brownian motion and v_t is a strictly positive adapted process. The instantaneous variance v_t can be a very general stochastic process. In particular, it can have jumps. We only require that the following integral exists:

$$V(t_1,t_2) := \int_{t_1}^{t_2} \left(\frac{dF_t}{F_t}\right)^2 = \int_{t_1}^{t_2} v_t dt,$$

where $V(t_1,t_2)$ is the total continuously-sampled variance over the period $[t_1,t_2]$.

Suppose now that the holding period $[0,T]$ is fixed. We want to find the time-0 value of the *variance contract*. This contract pays at time-T the future variance $V_T = V(0,T)$. Its value U_0 is given by the expectation of the payoff under the risk-neutral measure:

$$U_0 = E_0^*[V_T],$$

[3] In reality, the risk-free rate is nonzero. However, in empirical tests, we convert *spot* prices of options into *forward* prices (for delivery at time-T). To obtain forward prices, spot prices are multiplied by $e^{r_f(T-t)}$, where r_f is the risk-free rate over $[t,T]$. For example, the forward put price is $P_t(K) = e^{r_f(T-t)} P_t^s(K)$, where $P_t^s(K)$ is the spot put price. A similar approach has been used in, for example, Dumas, Fleming, and Whaley (1998).

where $E_0^*[\cdot]$ denotes the risk-neutral expectation at time-0. Dupire (1993) and Neuberger (1994) independently demonstrate that the value of the variance contract is closely related to the value of the *log contract*.[4] The log contract has the final payoff of $\log F_T$ and can be statically replicated by a portfolio of standard call and put options that mature at time-T. Their argument goes as follows. Equation (2) and Ito's lemma imply that

$$d(\ln F_t) = \frac{dF_t}{F_t} - \frac{1}{2} v_t dt,$$

implying that

$$\frac{1}{2} V_T = \frac{1}{2} \int_0^T v_t dt = \int_0^T \frac{dF_t}{F_t} - \ln \frac{F_T}{F_0}. \tag{3}$$

Therefore,

$$\frac{1}{2} U_0 = \frac{1}{2} E_0^* \left[\int_0^T v_t dt \right] = -E_0^* \left[\ln \frac{F_T}{F_0} \right]. \tag{4}$$

Equation (4) says that the two payoffs, V_T and $-2 \ln F_T/F_0$, have the same market values at time-0. Note that it does *not* say that the two payoffs are the same at maturity-T. In fact, the two payoffs are quite different because V_T depends on the whole path of F_t over the period $[0, T]$, while $-2 \ln F_T/F_0$ only depends on the final value of the underlying at time-T. It is just an elegant mathematical identity that the *prices* of the two payoffs happen to coincide.

Equation (3) identifies the replication strategy for the variance contract. The first term on the right hand side represents continuous rebalancing in the underlying futures, where at each moment the position is long $1/F_t$ of the futures contract. The second term is a static short position in the log contract.

Although the log contract is not traded in real markets, the above theoretical papers demonstrate that its value can be inferred from prices of standard options. Formally, this is stated in the following proposition. (For completeness, the proof is provided in Appendix A.)

Proposition 1 *It holds that*

$$\begin{aligned} -E_0^* \left[\ln \frac{F_T}{F_0} \right] &= \ln F_0 - \int_0^\infty \ln F_T \, h_0(F_T) dF_T &(5) \\ &= \int_0^{F_0} \frac{P_0(K)}{K^2} dK + \int_{F_0}^\infty \frac{C_0(K)}{K^2} dK. &(6) \end{aligned}$$

Proof: See Appendix A.

Intuitively, the log contract is equivalent to the portfolio of standard calls and puts with different strikes, where portfolio weights are proportional to $1/K^2$. Note that in equation (6) the log contract is replicated with puts when $K \leq F_0$ and calls when $K \geq F_0$. This is because out-of-the-money (OTM) options are much more liquid than in-the-money (ITM) options. Therefore, the replicating portfolio consists of OTM puts for low strikes and OTM calls for high strikes.

Proposition 1 suggests two strategies to inferring the price of the log contract. In the first strategy, one estimates RND $h_0(F_T)$ and uses it to compute the expectation of the payoff

[4] See also Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999), and Britten-Jones and Neuberger (2000).

$\log F_T$ in (5). RND can be estimated from a cross-section of available calls with different strikes K and same maturity T using the relationship in (1). Several techniques have recently been developed to solve this problem. See Jackwerth (1999) for a survey of the literature. In the second strategy, one interpolates option prices with available strikes to obtain the call and put pricing functions $C_0(K)$ and $P_0(K)$ for a continuum of strikes. The two integrals in (6) are then evaluated by integrating $C_0(K)$ and $P_0(K)$ over K with the weight function $1/K^2$.

2.2 The discretely-sampled variance contract

The approach in the previous subsection has three important limitations. First, although the approach imposes essentially no structure on the variance process v_t, it does require the price process F_t to be continuous. This assumption is restrictive, since the empirical literature presents ample evidence of jumps in the S&P 500 index. Recent studies include Bakshi, Cao, and Chen (1997), Andersen, Benzoni, and Lund (2002), Pan (2002), Eraker, Johannes, and Polson (2003), Aït-Sahalia (2002), Carr and Wu (2003), among others.

Second, even under the assumption that the price path is continuous, computation of V_T would be problematic since it requires continuous path monitoring. In practice, of course, the variance must be computed from discretely-sampled data. Moreover, the discretely-sampled variance can differs considerably from its continuously-sampled counterpart.

Finally, the approach assumes that the dynamic replication of the payoff V_T in (3) is done *continuously*, which is, again, impossible in practice. Even though the underlying futures can be traded with low transaction costs, the overall costs of continuous rebalancing would infinite. On the other hand, if the replicating portfolio is rebalanced only at a finite number of dates, then perfect replication of the payoff V_T cannot be achieved.

We now propose a way around these limitations. Consider evenly spaced dates $0 = t_0 < t_1 < \ldots < t_n = T$ with $\Delta t = t_i - t_{i-1} = T/n$. In particular, our empirical implementation will use the holding period $T = 1$ month and $\Delta t = 1$ day. Define the return over $[t_{i-1}, t_i]$ as $x_i = F_{t_i}/F_{t_{i-1}} - 1$. Then the discretely-sampled variance can be defined in several ways:

$$\hat{V}_T^{(1)} := \sum_{i=1}^{n} [\ln(1 + x_i)]^2, \tag{7}$$

$$\hat{V}_T^{(2)} := \sum_{i=1}^{n} x_i^2. \tag{8}$$

Over-the-counter variance swaps all use daily sampling. Most of them are based on definition (7), although some are based on definition (8).[5] We also introduce the following non-standard specification for the discretely-sampled variance

$$\hat{V}_T^{(3)} := 2\sum_{i=1}^{n} (x_i - \ln(1 + x_i)). \tag{9}$$

For small returns x, the function $f^{(3)}(x) = 2(x - \ln(1 + x))$ in the definition of $\hat{V}_T^{(3)}$ is close to the functions $f^{(1)}(x) = [\ln(1 + x)]^2$ and $f^{(2)}(x) = x^2$ in the definitions of $\hat{V}_T^{(1)}$ and $\hat{V}_T^{(2)}$. (It is easy to see that the three functions differ only in terms of order $O(x^3)$.)

[5] In principle, the discretely-sampled variance can also be adjusted for the drift in the underlying. In this case, one subtracts from the expressions in (7)-(8) the adjustment term $\frac{1}{n}\left(\ln \frac{F_T}{F_0}\right)^2$. However, for typical variance swaps, such an adjustment is very small and is rarely used in practice.

Figure 1 plots $f^{(1)}(x)$, $f^{(2)}(x)$, and $f^{(3)}(x)$ for a range of returns x. The figure illustrates several points, which are also straightforward to verify analytically. First, the function $f^{(3)}(x)$ is positive for all returns x. Second, the three functions are very close to each other for the range of typical daily returns. In particular, the differences between them are virtually zero when $|x| \leq 5\%$ and very small when $|x| \leq 10\%$. Third, the function $f^{(3)}(x)$ always lies between $f^{(1)}(x)$ and $f^{(2)}(x)$. Also, the function $f^{(3)}(x)$ is closer to $f^{(1)}(x)$ than to $f^{(2)}(x)$.

What is so special about the non-standard definition in (9)? The answer is given in the following proposition. Define $U_0 := -2E_0^*[\ln F_T/F_0]$. When the price process F_t is continuous, U_0 can be interpreted as the market price of the continuously-sampled variance V_T. However, such an interpretation no longer holds when F_t could be discontinuous.

Proposition 2 *For any* $\Delta t > 0$,

$$E_0^*\left[\hat{V}_T^{(3)}\right] = U_0.$$

Proof: From definition (9),

$$\frac{1}{2}\hat{V}_T^{(3)} = \sum_{i=1}^{n}\left(\frac{F_{t_i} - F_{t_{i-1}}}{F_{t_{i-1}}}\right) - \ln\frac{F_T}{F_0}. \tag{10}$$

Because F_t is a martingale under the risk-neutral measure, we obtain that

$$\frac{1}{2}E_0^*\left[\hat{V}^{(3)}\right] = -E_0^*\left[\ln\frac{F_T}{F_0}\right].$$

□

Proposition 2 says that the market price of the discretely-sampled variance $\hat{V}_T^{(3)}$ is equal to U_0. This result may appear modest, but it has important implications. First, the result holds for completely general price processes, not only continuous ones. Second, the result holds for any sampling interval Δt, not necessarily a small one. Third, equation (10) implies that the replication strategy for $\hat{V}_T^{(3)}$ requires *discrete* rebalancing in the underlying futures. Specifically, the position in the futures contract $(1/F_t)$ is adjusted on the same dates t_i when the variance is sampled. Equation (10) is useful for traders because it identifies the exact timing for rebalancing. If the variance is calculated using daily closing prices, then rebalancing must be done at the end of each trading day. Only in this case will the payoff $\hat{V}_T^{(3)}$ be replicated perfectly. The second term in (10) again represents a static short position in the log contract.

To illustrate the importance of the second implication, let us consider the simple case of the Black-Scholes model with the objective price process

$$\frac{dF_t}{F_t} = \mu dt + \sqrt{v}dB_t,$$

where μ and v are positive constants. In this model, the realized continuously-sampled variance is deterministic and $V_T = U_0 = vT$. The realized discretely-sampled variances $\hat{V}_T^{(1)}$, $\hat{V}_T^{(2)}$, and $\hat{V}_T^{(3)}$ are random variables. As Δt becomes smaller, the difference between the discretely-sampled variance $\hat{V}_T^{(1)}$ (or $\hat{V}_T^{(2)}$, or $\hat{V}_T^{(3)}$) and the continuously-sampled variance V_T tends to zero. For daily sampling, however, this difference can be substantial in some realizations. For example, suppose that the parameters are

$$\mu = 0.09, \qquad v = 0.02, \qquad T = 1/12, \qquad \Delta t = 1/252.$$

Consider the relative discrepancy $(\hat{V}_T^{(1)}/V_T - 1)$. Its mean and standard deviation can be computed explicitly as

$$\begin{aligned} \text{Mean} &= \frac{(\mu - 0.5v)^2}{v}\Delta t = 0.0013, \\ \text{Std. dev.} &= \sqrt{\frac{2\Delta t}{T}}\sqrt{1 + 2\frac{(\mu - 0.5v)^2}{v}\Delta t} = 0.31. \end{aligned}$$

Although the mean (or bias) is small, the standard deviation is considerable. (Intuitively, the discretely-sampled variance $\hat{V}_T^{(1)}$ deviates from the continuously-sampled variance V_T on average by about 31%. The results for $\hat{V}_T^{(2)}$, and $\hat{V}_T^{(3)}$ are similar.)

One way to interpret Proposition 2 is that, although the difference $\hat{V}_T^{(3)} - V_T$ could be large, the *market price* of that difference is zero. The result is exact for general price processes and for all Δt. The specification of the discretely-sampled variance $\hat{V}_T^{(3)}$ has a unique property that its market price is completely model-independent and invariant with respect to the sampling frequency. That is, whether Δt is small or large, this has no effect on the market price of $\hat{V}_T^{(3)}$.

In contrast, the market price for the discretely-sampled variance $\hat{V}_T^{(1)}$ $(\hat{V}_T^{(2)})$ depends on a specific model and the sampling frequency. It should be pointed out, however, that the difference between $\hat{V}_T^{(1)}$ $(\hat{V}_T^{(2)})$ and $\hat{V}_T^{(3)}$ is negligibly small in typical applications. In particular, when in Section 3 we compute the discretely-sampled variances for the S&P 500 futures, we find that the relative discrepancy $(\hat{V}_T^{(1)}/\hat{V}_T^{(3)} - 1)$ has mean of -0.000045 and standard deviation of 0.0035, while the relative discrepancy $(\hat{V}_T^{(2)}/\hat{V}_T^{(3)} - 1)$ has mean of 0.00023 and standard deviation of 0.0069. Alternatively, if $\hat{V}_T^{(1)}$ and $\hat{V}_T^{(2)}$ are regressed on $\hat{V}_T^{(3)}$ with no intercept, then the slope coefficients are

$$\begin{array}{lll} \hat{V}_T^{(1)}: & \text{Slope} = 1.004, & R^2 = 99.99\%, \\ \hat{V}_T^{(2)}: & \text{Slope} = 0.992, & R^2 = 99.97\%. \end{array}$$

This indicates that, for the daily frequency, the three discretely-sampled variances are essentially indistinguishable. For the empirical results reported in Sections 3 and 4, it makes no difference which one of the three definitions is used. In what follows, we measure the realized variance according to the definition $\hat{V}_T^{(3)}$ and, to simplify notation, we write $\hat{V}_T = \hat{V}_T^{(3)}$. In this case, the market price of the variance is precisely equal to U_0, regardless of the sampling frequency and whether the price process continuous or not.

By comparing U_0 with the realized variance $\hat{V}_T$, the variance risk premium can be determined. Specifically, let $r_v = \hat{V}_T/U_0 - 1$ denote the simple net return on the variance contract.[6] Its risk premium is $\pi_v = E_0[r_v] = E_0[\hat{V}_T]/U_0 - 1$, where $E_0[\cdot]$ is the objective expectation. Similarly, $r_m = F_T/F_0 - 1$ is excess return on the futures contract (interpreted in this paper as the market portfolio) and $\pi_m = E_0[r_m]$ is its risk premium.

[6] Recall that r_v is already defined in *excess* of the risk-free rate.

2.3 Estimation of the market value of the variance contract

To implement our approach, we also need the market value of the variance contract U_0. We estimate the market value of the variance contract U_0 using a new method developed in Bondarenko (2003a). The method is termed *Positive Convolution Approximation* (PCA) and it allows one to infer the conditional RND $\hat{h}_0(F_T)$ through the relationship in (1). It directly addresses the important limitations of option data that (a) options are only traded for a discrete set of strikes, as opposed to a continuum of strikes, (b) very low and very high strikes are usually unavailable, and (c) option prices are recorded with substantial measurement errors, which arise from nonsynchronous trading, price discreteness, and the bid-ask bounce. The PCA method is fully nonparametric, always produces arbitrage-free estimators, and controls against overfitting while allowing for small samples. In addition to the estimate for RND, the method also produces the estimates for the call and put pricing functions $\hat{C}_0(K)$ and $\hat{P}_0(K)$ for a continuum of strikes. Thus, we obtain an estimate $\hat{U}_0$ from (6) by numerically integrating $\hat{C}_0(K)$ and $\hat{P}_0(K)$ over the strike K with the weight function $1/K^2$.

To assess the empirical performance of the estimator $\hat{U}_0$, we conduct a Monte-Carlo experiment in conditions that approximate real data. The specific details are provided in Appendix C. In the experiment, we start with an analytical specification for RND $h_0(F_T)$, whose parameters are chosen to calibrate a typical cross-section of the S&P 500 options. Theoretical option prices $C_0(K_j)$ and $P_0(K_j)$ are computed for same strikes K_j which were actually available on the selected day. Then, we create simulated option prices by adding random noise to the theoretical prices. To model observational errors that arise from various market imperfections, we consider three cases: No noise, Moderate noise, and High noise.

Simulated option prices are generated 500 times. Each time, the estimator $\hat{U}_0$ is obtained via the PCA method. We measure its accuracy using the (normalized) RMSE criterion:

$$\text{RMSE}(\hat{U}_0) = \frac{1}{U_0}\left(E\left[\left(\hat{U}_0 - U_0\right)^2\right]\right)^{1/2}. \tag{11}$$

Then the results of the Monte-Carlo experiment can be summarized as follows:

	No noise	Moderate noise	High noise
RMSE($\hat{U}_0$):	0.007	0.012	0.020

Even when there is no noise in option prices, RMSE is strictly positive. This is mainly due to the fact that options are unavailable for very high or low strikes. See, for example, Figure 2. Naturally, the accuracy of the estimator declines when the level of noise increases. Nevertheless, we conclude that the procedure for estimating the market value of the variance contract is quite accurate, with RMSE being 1.2% (2.0%) for Moderate (High) noise.

2.4 Alternative Volatility Instruments

A unique feature of the variance contract is that it allows investors to obtain *pure* exposure to the future variance, which is independent of changes in the underlying price and other factors. Recently, several other instruments have been proposed with the objective of hedging the variance or volatility risk, including

- at-the-money option straddle – Coval and Shumway (2001),
- at-the-money-forward straddle – Brenner, Ou, and Zhang (2001),

- options on a volatility index, such as the CBOE Volatility Index (VIX) – Grunbichler and Longstaff (1996).

Common shortcomings of these alternative instruments are that 1) the sensitivity with respect to variance (or volatility) is nonlinear, 2) sensitivities to changes in other factors (the underlying price, volatility of volatility, etc.) are typically nonzero, and 3) sensitivities are model-specific and/or time-varying. As an example, consider the at-the-money straddle. The ATM straddle is a portfolio of call $C_0(K)$ and put $P_0(K)$, where the common strike is chosen so that $K \simeq F_0$. When the straddle is created, it is approximately delta-neutral and its price is primarily determined by the expected future variance. However, as the underlying drifts away from its initial level, the straddle is no longer delta-neutral and is now affected by directional movements of the underlying.

In contrast, the variance contract allows one to isolate the variance risk from other risks. In particular, it has zero sensitivity to changes in the level of the underlying F_t. This is important for empirical applications, because changes in the level and variance of the S&P 500 index tend to be strongly (negatively) correlated.

2.5 Related Literature

One of the objectives of this paper is to investigate whether and to what extent the variance risk is priced. Therefore, it may be useful to contrast it to other papers which have addressed this issue.

First, Bakshi, Cao, and Chen (2000) and Buraschi and Jackwerth (2001) present evidence that equity index options are non-redundant securities, suggesting that other risks (in addition to the underlying's price) are factored in option prices. One interpretation for their results is that the variance risk may be priced. See also Bondarenko (2003b). These approaches do not allow one to determine the sign and magnitude of the variance risk premium.

Second, Coval and Shumway (2001) study daily returns on the ATM zero-beta straddles and argue that the variance risk premium is negative. In their approach, some (weak) parametric assumptions on the option pricing model are needed (to compute option betas), and the sensitivity of the ATM zero-beta straddle to the variance risk is model-specific. Furthermore, the approach requires frequent trading in options, which could be unpractical because of substantial transaction costs.

Third, Bakshi and Kapadia (2003a, 2003b) investigate statistical properties of delta-hedged gains of equity call options, where a long option position is dynamically hedged with the underlying. In the stochastic volatility framework, they show that the sign of delta-hedged option portfolios corresponds to the sign of the variance risk premium. Empirically, they find that the variance risk is priced and is negative. In their approach, some (weak) parametric assumptions are required (to compute delta-hedges), and the magnitude of the variance risk premium cannot be determined.

Finally, there is an extensive strand of the literature, in which structural models, such as Hull and White (1987), Heston (1993), Bates (2000), and others, are calibrated to option data. Recent examples include Bakshi, Cao, and Chen (1997), Guo (1998), Bates (2000), Chernov and Ghysels (2000), Benzoni (2002), and Pan (2002), among others. These approaches make the strongest assumptions and are subject to model misspecification. In fact, since most popular structural models tend to be rejected by data, it is unclear how to interpret their estimates for the variance risk premium.

As argued earlier, the approach implemented in this paper is completely model-free. Importantly, the approach not only allows us to estimate the variance risk premium, but also generates time-series returns on the variance contract, which could be used as an additional risk factor in explaining asset returns.

3 Statistical Properties of the Variance Return

3.1 CME Options

Our data consist of daily prices of options on the S&P 500 futures traded on the Chicago Mercantile Exchange (CME) and the S&P 500 futures themselves. The data are obtained from the Futures Industry Institute. The sample period is from January 1988 through December 2000.[7] The S&P 500 futures have four different maturity months from the March quarterly cycle. The contract size is $250 times S&P 500 futures price (before November 1997, the contract size was $500 times S&P 500 futures price). On any trading day, the CME futures options are available for six maturity months: four months from the March quarterly cycle and two additional nearby months ("serial" options). The option contract size is one S&P 500 futures. The minimum price movement is 0.05. The strikes are multiples of 5 for near-term months and multiples of 25 for far months. If at any time the S&P 500 futures contract trades through the highest or lowest strike available, additional strikes are usually introduced.

The CME options on the S&P 500 futures and options on the S&P 500 Index itself, traded on the Chicago Board Option Exchange (CBOE), have been a focus of many empirical studies. For short maturities, prices of the CME and CBOE options are virtually indistinguishable. Nevertheless, there are a number of practical advantages in using the CME options:

- As well known, there is a 15-minute difference between the close of the CBOE markets and the NYSE, AMEX, and NASDAQ markets, where the S&P 500 components are traded. This difference leads to non-synchronicity biases between the recorded closing prices of the options and the level of the Index. In contrast, the CME options and futures close at the same time (3:15 pm CT).

- It is easier to hedge options using highly liquid futures as opposed to trading the 500 individual stocks. On the CME, futures and futures options are traded in pits side by side. This arrangement facilitates hedging, arbitrage, and speculation. It also makes the market more efficient. In fact, even traders of the CBOE options usually hedge their positions with the CME futures.

- Another complication is that the S&P 500 Index pays dividends. Because of this, to estimate the risk-neutral densities from the CBOE options, one needs to make some assumptions about the Index dividend stream. No such assumptions are needed in the case of the CME futures options.

A disadvantage of the CME options is their American-style feature. However, we conduct our empirical analysis in such a way that the effect of the early exercise is minimal.

[7]Data for earlier years are not used because of three main reasons: 1) the option market was considerably less liquid during its earlier years, 2) prior to September 1987, options had only 4 maturity months per year (as opposed to 12 thereafter), and 3) Jackwerth and Rubinstein (1996) argues that there was a structural break after the 1987 stock market crash.

To build as large a series of non-overlapping returns on the variance contract, we proceed as follows. Let j index different option maturities T_j, which partition the 13-year sample into $N = 156$ one-month periods. We compute the market return r_m^j and variance return r_v^j over holding periods $[T_j, T_{j+1}]$, $j = 1, \ldots, N$.[8] To obtain the variance returns r_v^j, we follow several steps, which are explained in Appendix B. In brief, these steps include 1) filtering out unreliable option data, 2) checking that option prices satisfy the theoretical no-arbitrage restrictions, 3) inferring forward prices of European puts and calls, 4) estimating RND, the put and call pricing functions for a continuum of strikes, and 5) estimating the market price of the variance contract using the relationship in (6).

3.2 The Market Price of the Variance Risk

Table 1 reports various statistics for monthly returns on the variance contract, the S&P 500 futures, and two additional portfolios of interest which will be discussed shortly. The average excess return (AR) on the variance contract is -26.66% per month, while AR for the market is about 0.89% per month. This indicates that the market price of the variance risk is priced and is negative.

The negative risk premium π_v on the variance contract return is partly due to negative correlation between r_v and r_m. To separate the effect of negative correlation, Panel B of Table 1 reports the Jensen's alpha and beta coefficients with respect to the market return. Specifically, for any return r_i, define the alpha and beta coefficients as

$$\alpha_i = E[r_i] - \beta_i E[r_m], \qquad \beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}.$$

The beta coefficient for r_v is -4.68, which accounts for about -4.14% of the risk-premium π_v; the alpha coefficient is -22.52% per month. High leverage of the variance contract complicates interpretation of its alpha coefficient. Therefore, Panel B of Table 1 reports several risk-adjusted measures that are unaffected by leverage:

- the Sharpe ratio, $SR := \frac{E[r_i]}{\sqrt{Var(r_i)}}$,
- the Treynor's measure, $TM := \frac{\alpha_i}{\beta_i}$, and
- M-squared of Modigliani and Modigliani (1997), $M^2 := \frac{E[r_i]}{\sqrt{Var(r_i)}}\sqrt{Var(r_m)}$.

In particular, the table demonstrates that selling the variance contract produces a rather high Sharpe ratio of 0.49, which is more than twice as large as the Sharpe ratio for the market. The M-squared statistics for selling the variance contract is 1.81% per month. Intuitively, this statistics shows the return that an investor would earn if the variance contract is de-leveraged with the risk-free asset to match the standard deviation on the market.

Additional insights are provided by Figures 3-5. The left panel of Figure 3 is the histogram of the variance return r_v. The histogram demonstrates that the variance return exhibits substantial positive skewness. The right panel of Figure 3 is a scatter plot of the variance

[8] In practice, maturity dates for options are set such that $\tau_j = T_{j+1} - T_j$ is always either 28 or 35 days (except for a few cases when a maturity date must be moved due to an exchange holiday). For simplicity, we refer to τ_j as one-month holding period.

return r_v versus the market return r_m, which confirms the fact that returns r_v and r_m are negatively correlated (the correlation coefficient is -0.32).

Figure 4 shows the distribution of the variance return over time. Also shown are time-series of the S&P 500 level and daily return on the underlying S&P 500 futures. Figure 5 contrasts time-series for the market price of the variance contract $U_t = E_t[\hat{V}(t,T)]$ and for the realized variance $\hat{V}(t,T)$. Both figures demonstrate that the variance return has a negative risk-premium and exhibits positive skewness.

Overall, the reported results suggest that selling the variance contract would have resulted in very high profits over the studied 13-year period. The profits are high on the risk-adjusted basis, where the Sharpe ratio and related risk measures are used. Table 1 analyzes monthly returns on two special portfolios, the *market-neutral portfolio* (MNP) and the *mean-variance portfolio* (MVP). In MNP, the short position in the variance contract is hedged with the S&P 500 futures in such a way that the sample correlation of the portfolio's and market returns is zero. That is, the MNP's return is given by

$$r_{mn} = -r_v + \beta_v r_m = -r_v - 4.68 r_m.$$

MVP is also formed from the variance contract and the S&P 500 futures. Now, however, the portfolio weights are chosen to achieve the highest possible Sharpe ratio. Its return is

$$r_{mn} = -0.25 r_v + 0.75 r_m.$$

For MVP, the Sharpe ratio is 0.50 and the M-squared measure is 1.84. Figure 6 compares cumulative returns on the four strategies that invest in 1) the S&P 500 futures, 2) the short variance contract, 3) MNP, and 4) MVP. To make comparison meaningful, the last three strategies are de-leveraged with the risk-free asset so that the standard deviations of their monthly returns over the sample period are the same as for the first strategy.

3.3 Comparison to Put and Call Options

One might expect a high correlation between returns on the variance contract and put options. Similar to the variance contract, puts earn considerable negative risk premiums and exhibit positive skewness. See, for example, Bondarenko (2003b). Still, there are substantial differences in returns on the variance contract and put options.

Table 2 reports various statistics for monthly excess returns of puts and call on the S&P 500 futures with different moneyness. At the beginning of each one-month period, we consider options with constant moneyness $k_t = K/F_t$ equal to 0.94, 0.96, 0.98, 1.00, 1.02, 1.04, and 1.06. For given moneyness, option prices are computed from the estimated RND. Table 2 shows that puts generally have a higher degree of leverage and generate more extreme returns. In particular, put returns have much larger standard deviations (except for the deep in-the-money put (ITM) with k=1.06). The Sharpe ratio, M^2, and the Treynor's measure are higher for selling the variance contract as opposed to selling the at-the-money put (ATM) with k=1.00. As k decreases and puts become more out-of-the-money (OTM), their returns become more skewed. The Sharpe ratio, M^2, and the Treynor's measure for selling puts are the higher, the lower k. Still, only the deep OTM put (k=0.94) has a higher Treynor's measure than the variance contract.

Table 2 indicates that correlations between the variance and put returns are not too high. The highest correlation (0.36) is between the variance contract and the ATM put. In general, of all levels of moneyness, the ATM put return comes closest to the variance return in terms of statistical properties. As expected, calls are negatively correlated with the variance contract. Correlations are typically smaller (in the absolute value) than those for puts.

All in all, the variance return seems to contain rather different information from that contained in option returns. To see this more clearly, consider any option that matures at time-T. The option payoff at maturity is *path-independent*, because the payoff is a function of the final value of the underlying F_T only. In contrast, the payoff of the variance contract is *path-dependent*. In other words, while F_T completely determines payoffs of all puts and calls that mature at time-T, one needs the whole history of F_t prior to T in order to determine the payoff of the variance contract.

Because the variance payoff is path-dependent, extreme returns on the variance contract do not necessarily correspond to extreme returns on S&P 500 and its options. This can immediately be seen from the following table, which looks at the three holding periods with the highest variance returns and shows the excess returns (in percents) for the variance contract, the S&P 500 futures, the ATM put (k=1.00), and the deep OTM put (k=0.96):

	Return on			
Holding period	Variance	S&P 500	ATM put	OTM put
1989: 09/15-10/20	354.80	0.23	-100.00	-100.00
1997: 10/17-11/21	224.46	2.21	-100.00	-100.00
1998: 08/21-09/18	142.27	-6.07	110.95	-95.26

Interestingly enough, the two highest variance returns are associated with rather ordinary, small *increases* in S&P 500 and *negative* put returns.

3.4 Portfolio Choice Implications

To put the economic value of the variance contract in perspective, we consider an investor who maximizes the expected value of a Constant Relative Risk Aversion (CRRA) utility function

$$u(W_T) = \begin{cases} \frac{1}{1-\gamma} W_T^{1-\gamma} & \text{if } \gamma \neq 1 \\ \log W_T & \text{if } \gamma = 1, \end{cases}$$

where W_T is the final wealth and $\gamma > 0$ is the coefficient of risk aversion. Suppose that the investor can only invest in the market portfolio and the variance contract and let w_m and w_v denote the corresponding portfolio weights. Then, the final wealth can be represented as

$$W_T = (1 + r_f + w_m r_m + w_v r_v) W_0,$$

where r_f is the risk-free rate and W_0 is the initial wealth. (Recall that returns r_m and r_v are defined in excess of the risk-free rate.) The investor chooses the portfolio weights w_m and w_v to maximize $E[u(W_T)]$. Because of homotheticity of CRRA preferences, this is equivalent to maximizing $E[u(1 + r_f + w_m r_m + w_v r_v)]$.

Table 3, Panel A, computes the optimal portfolio weights using the sample moments. The investment horizon is one month and the coefficient of risk aversion γ is set to 1, 2, 3, 5, 10, 20, and 50. The portfolio weights are reported for two cases when the weight w_v is 1) constrained to zero and 2) unconstrained. By contrasting the two cases, we can quantify the economic value to the investor of being able to trade the variance contract. Specifically, we compute the certainty equivalent rate (CER), which measures how much the investor is willing to pay for the opportunity to trade the variance contract. In Table 3, CER is reported as a percentage of the investor's initial wealth.[9]

Table 3 shows that the investor with CRRA preferences is long the market and short the variance contract. The portfolio weight w_m is smaller for the unconstrained case than for the constrained case, which is due to negative correlation between the returns r_m and r_v. As risk aversion increases, the portfolio weights (in the absolute value) monotonically decline. The economic value of introducing the variance contract is considerable. For example, an investor with \$1 mln of the investable wealth and the risk aversion coefficient of 3 (10) is willing to pay up to about \$17,400 (\$6,000) per month for being able to sell the variance contract.

Next, we verify that the economic value of introducing the variance contract is large even when the investor can trade options on the market. In Panel B of Table 3, we repeat the previous analysis but now assume that the investor can also trade the ATM put option. That is, the final wealth is represented as

$$W_T = (1 + r_f + w_m r_m + w_p r_p + w_v r_v) W_0,$$

where w_p and r_p are the portfolio weight and return on the ATM put. In this case, the investor is short the market portfolio, the ATM put, and the variance contract. Shorting the market portfolio serves as a hedge for selling the ATM put. When the variance contract is available, the negative exposure to the market becomes larger, while the negative exposure to the ATM put becomes smaller. The weights w_v are very similar to those in Panel A. The economic value of introducing the variance contract is still considerable. An investor with \$1 mln of the investable wealth and the risk aversion coefficient of 3 (10) is now willing to pay about \$14,700 (\$4,900) per month to be able to sell the variance contract.[10]

4 Performance of Hedge Funds

In this section, we investigate how performance of hedge funds is related to the variance return. This application is motivated by a popular belief in the financial press that many hedge funds sell short market volatility. Our paper is first to present a direct evidence that returns of most hedge funds are negatively correlated with the variance return. This finding has important implications for hedge fund performance evaluation. Previous studies usually conclude that hedge funds, as a group, deliver superior risk-adjusted returns. That is, linear regressions of fund returns on various market factors often produce positive and statistically significant alpha coefficients. However, since these regressions do not include the variance return, the superior performance might be misleading when hedge funds have negative exposures to the variance risk. In this section, we argue that the variance risk often accounts for a very substantial portion of hedge fund average returns.

Several recent papers argue that equity-oriented hedge funds have nonlinear, option-like payoffs with respect to the market return. See Fung and Hsieh (2001), who study the "trend-following" strategy, Mitchell and Pulvino (2001), who analyze the "risk-arbitrage" strategy, and Agarwal and Naik (2004), who study a number of equity-oriented strategies. In particular,

[9] Here we follow Driessen and Maenhout (2003), who have used a similar approach to measure the economic value of introducing various option strategies to the investment opportunity set.

[10] When we repeat the analysis assuming that the investor can, in addition to the ATM put, also trade the OTM put with k=0.96, the results for CER are qualitatively similar.

Agarwal and Naik estimate a multi-factor model augmented with returns on S&P 500 options and find that several hedge fund categories exhibit returns similar to those from selling puts. We extend the existing literature by studying the exposure of hedge funds to the variance risk. Because the variance return contains information very different from option returns, it allows us to identify a new important determinant of fund returns. As Agarwal and Naik point out,

> "If one can locate or construct an instrument whose payoff is directly related to volatility of financial markets, then it would be interesting to include it as an additional asset class factor."

The variance contract is precisely the instrument that Agarwal and Naik are referring to. In our analysis, we consider both equity and non-equity hedge funds. Interestingly, we find that the variance return, which is derived exclusively from the equity market, is a very important explanatory variable even for *non-equity* funds.

4.1 How do hedge funds sell the variance risk?

Hedge funds can attain a negative exposure to the variance risk in several ways. First, many equity-oriented hedge funds sell the variance risk by trading in OTC variance swaps. These funds also routinely sell put and call options on market indexes and individual securities.[11] Selling options, however, contributes to shorting the variance risk.

Second, equity-oriented hedge funds often try to capture price movements generated by pending corporate deals such as mergers, spin-offs, takeovers, corporate restructuring, liquidation, bankruptcy, or reorganization. These strategies involve the risk of deal failure and are short the variance risk. This is because a larger fraction of deals fails or gets postponed in volatile markets than in normal markets. As an example, consider Merger Arbitrage, which is sometimes called Risk Arbitrage. After an announcement of a merger or acquisition, the target company's stock usually trades at a discount to the price offered by the acquiring company. A hedge fund attempts to profit from the arbitrage spread, or the difference between the offer price and the target's stock price. If the merger is successful, the fund captures the arbitrage spread. However, if the deal fails, the fund incurs a loss, which is typically much greater than the profit obtained if the deal succeeds.

Third, many non-equity hedge funds follow strategies that could be described as "convergence arbitrage" or "relative-value" trades. In those trades, a hedge fund attempts to take advantage of small differences in prices of related securities. The fund might construct a long/short, non-directional position by simultaneously buying one security, A, and selling another security, B. Securities A and B offer similar cash flows, however, security A is cheaper because it is exposed to some additional risks, such as liquidity, credit, currency, or other risks.

One example is where A is the off-the-run 29-year Treasury bond and B is the on-the-run 30-year Treasury bond.[12] Because A is slightly less liquid than B, the yield on A is a bit higher than that for B. The hedge fund amplifies the tiny spread between A and B through huge leverage. Over the long run, the liquidity spread between A and B will certainly disappear, and the hedge fund will realize a profit. In the short run, however, the spread could widen further if there is a major market disruption which causes a "flight to quality". Other examples

[11] For example, it has been publicized that the Long-Term Capital Management (LTCM) had a large short position in five-year equity options. For good accounts of the LTCM's meltdown, see Dunbar (1998), Lewis (1999), and Jorion (2000).

[12] This was another popular trade of LTCM.

of convergence trades are where A is a junk corporate bond, or a mortgage-backed security, or an emerging-market sovereign bond, and B is a matched Treasury bond. A common feature of all these strategies is that they usually make small profits in calm markets but take big losses during market disruptions.

4.2 HFR Indexes

Our application uses monthly returns of various hedge fund indexes. We first focus on indexes from the Hedge Fund Research (HFR) database, and then verify robustness of the results by using the CSFB/Tremont database.

For HFR database, Appendix E.1 provides a brief description of the sector indexes. We consider all main fund categories for which data are available from the database inception in 1990. Those include

- 11 equity categories: Convertible Arbitrage, Distressed Securities, Emerging Markets, Equity Hedge, Equity Market Neutral, Equity Non-Hedge, Event-Driven, Market Timing, Merger Arbitrage, Relative Value Arbitrage, and Short Selling;

- 4 non-equity categories: Fixed Income (Total), Fixed Income: Arbitrage, Fixed Income: High Yield, and Macro;

- 2 aggregate categories: Composite and Fund of Funds Composite.

Table 4 reports various summary statistics for monthly excess returns of HFR indexes, as well for the S&P 500 futures and the variance contract. To produce this table, the definition of the variance return must be modified. In Section 3, all monthly returns are computed from one option maturity to the next one (that is, roughly from the middle of one calendar month to the middle of the next one). However, since the standard reporting interval for hedge funds is a *calendar* month (from the end of one month to the end of the next one), the variance return must now be measured in the same way.

Let T be an option maturity date, and t_1 and t_2 are two trading dates such that $t_1 < t_2 \leq T$. The variance return over the period $[t_1, t_2]$ is computed as

$$r_v = \frac{\hat{V}(t_1, t_2) + E^*_{t_2}[\hat{V}(t_2, T)]}{E^*_{t_1}[\hat{V}(t_1, T)]} - 1. \tag{12}$$

When $t_2 = T$, the definition in (12) coincides with that used in Section 3. When $t_2 < T$, the nominator in (12) is equal to the realized variance over $[t_1, t_2]$ plus the market price of the remaining variance over $[t_2, T]$, while the denominator is equal to the market price of the variance over the whole period $[t_1, T]$. As before, the market price of the variance contract is estimated from options with different strikes and the same maturity T, on two trading dates t_1 and t_2. In our application, trading dates t_1 and t_2 are the last business days of two consecutive months preceeding the option maturity date T. (For example, when we compute the variance return over February, t_1 and t_2 are the last business days in January and February, and T is the option maturity date in March.)

The important observation from Table 4 is that hedge fund returns are highly serially correlated. This fact presents unique challenges for empirical research. Asness, Krail, and Liew (2001) argue that hedge fund alpha coefficients are usually overstated when serial correlation is not controlled for. Getmansky, Lo, and Makarov (2003) examines a number of potential explanations for serial correlation and narrows down all explanations to 1) illiquidity of hedge fund assets, and 2) smoothing of returns (whether smoothing is deliberate or unintentional). To address the issue of serial correlation, Getmansky, Lo, and Makarov propose the following econometric model. Let r_t denote the true excess return of a hedge fund in period t. The true return is not observed. Instead, r_t^o denotes the reported or observed return in period t, where

$$\begin{aligned} r_t^o &= \theta_0 r_t + \theta_1 r_{t-1} + \ldots + \theta_q r_{t-q}, && (13) \\ 1 &= \theta_0 + \theta_1 + \ldots + \theta_q, && (14) \\ \theta_j &\in [0,1], \quad j = 0, 1, \ldots, q. && (15) \end{aligned}$$

Specification (13)-(15) states that the observed return r_t^o is a weighted average of the fund true returns over the most recent $q+1$ period, which reflects the effects of illiquidity and return smoothing. Since most serial correlation in hedge fund returns is captured by the first two lags, we focus on the special case $q = 2$. This case is also studied in Getmansky, Lo, and Makarov (2003). The true return is assumed to satisfy

$$\begin{aligned} r_t &= \alpha + \beta r_{m,t} + \gamma r_{v,t} + \epsilon_t, && (16) \\ E[\epsilon_t] &= 0, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2). && (17) \end{aligned}$$

This implies that the hedge fund expected return $E[r]$ can be decomposed as

$$E[r] = \alpha + \beta E[r_m] + \gamma E[r_v]. \tag{18}$$

In what follows, we will contrast two cases: (1) the single-factor (market) model, in which γ is restricted to zero, and (2) the two-factor model, in which γ is unrestricted.

4.3 Least Squares Regression

One approach to estimate the model (13)-(17) is to run a least squares regression. Specifically, the model (13)-(17) can be re-expressed as

$$\begin{aligned} (1) \quad r_t^o &= \alpha + \beta_0 r_{m,t} + \beta_1 r_{m,t-1} + \beta_2 r_{m,t-2} + u_t, \qquad \text{or} \\ (2) \quad r_t^o &= \alpha + \beta_0 r_{m,t} + \beta_1 r_{m,t-1} + \beta_2 r_{m,t-2} + \gamma_0 r_{v,t} + \gamma_1 r_{v,t-1} + \gamma_2 r_{v,t-2} + u_t, \quad (19) \end{aligned}$$

where

$$u_t = \theta_0 \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}, \tag{20}$$

$$\beta_j = \beta \theta_j, \qquad \gamma_j = \gamma \theta_j, \qquad j = 0, 1, 2. \tag{21}$$

Specification (1) has been considered in Asness, Krail, and Liew (2001) and Getmansky, Lo, and Makarov (2003). Specification (2) extends specification (1) by introducing the contemporaneous and lagged variance returns. Table 5 reports the results for both specifications. For each fund category, it shows the OLS estimates $\hat{\alpha}$, $\hat{\beta}_j$, $\hat{\gamma}_j$ and their t-statistics. We use the Newey-West estimator to obtain heteroscedasticity and autocorrelation consistent standard errors. Also reported are $\hat{\beta} := \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2$, $\hat{\gamma} := \hat{\gamma}_0 + \hat{\gamma}_1 + \hat{\gamma}_2$, $\hat{\beta} E[r_m]$, and $\hat{\gamma} E[r_v]$. One can interpret $\hat{\beta}$ and $\hat{\gamma}$ as the overall exposures to the market and variance returns, while $\hat{\beta} E[r_m]$ and $\hat{\gamma} E[r_v]$ are the components of the expected return $E[r]$ in (18) attributable to the market and variance risk exposures.

Table 5 shows that the variance risk factor is indeed important for hedge fund returns. The sensitivity $\hat{\gamma}_0$ comes out statistically significant (at the five percent significance level) for all categories, except for Convertible Arbitrage, Equity Market Neutral, Market Timing, and Short Selling. The overall exposure to the variance return $\hat{\gamma}$ is negative for all categories, except for Equity Market Neutral, Market Timing, and Short Selling. Perhaps, it is not surprising that the three exceptions have positive (but not statistically significant) exposure to the variance return. Market Timing funds often follow "trend-following" strategies. Fung and Hsieh (1997, 2001) analyze these strategies and find that trend followers tend to have the highest returns during both the best and the worst performing months of the stock market, that is, when the variance return is typically high. In other words, trend followers are generally long the market variance. As expected, Short Selling funds have factor loadings which are opposite in sign to factor loadings of funds that are predominately long equities, such as Equity Non-Hedge funds. Also, as expected, Equity Market Neutral has virtually zero exposures to both the market and variance returns.

For most categories, the inclusion of the variance return in the OLS regression has the effect of i) reducing the overall exposure to the market risk, $\hat{\beta}$, and ii) reducing the alpha coefficient $\hat{\alpha}$. (Again, the only exceptions are Equity Market Neutral, Market Timing and Short Selling.) In other words, the OLS regression in specification (1) overstates both $\hat{\beta}$ and $\hat{\alpha}$.

Among individual categories, the largest "sellers" of the variance risk are Distressed Securities, Emerging Markets, Equity Non-Hedge, Event-Driven, Fixed Income (Total), Fixed Income: Arbitrage, Fixed Income: High Yield, and Macro. For these categories, the inclusion of the variance return reduces the alpha coefficient quite dramatically. The change in $\hat{\alpha}$ is -0.39%, -0.70%, -0.32%, -0.29%, -0.27%, -0.28%, -0.43%, and -0.42%, respectively. The variance risk component $\hat{\gamma}E[r_v]$ for these categories is substantial: 0.50%, 0.92%, 0.42%, 0.38%, 0.35%, 0.36%, 0.55%, and 0.55%, respectively. (Recall that all figures correspond to *monthly* returns.) The inclusion of the variance return has a particularly large effect on three fund categories: Emerging Markets, Fixed Income: High Yield, and Macro. For the aggregate index of all hedge funds (Composite), $\hat{\gamma}E[r_v]$ is 0.37% and the change in $\hat{\alpha}$ is -0.28%.

It is interesting to observe that several categories have a relatively low exposure to the equity market risk and a rather substantial exposure to the variance risk. Notable examples are Distressed Securities, Fixed Income (Total), Fixed Income: Arbitrage, Fixed Income: High Yield, and Macro. (Fund of Funds Composite also falls in this group.)

4.4 Maximum Likelihood Estimation

Although popular in the empirical literature, OLS estimation of the model in (13)-(17) has several important drawbacks.

First, because disturbances u_t are serially correlated, the OLS estimates are not efficient, although they are still consistent and can be useful as first approximations. Furthermore, the usual standard errors for the estimates, as well as based on them statistical inference, are incorrect. To account for serial correlation, t-statistics in Table 5 are computed using the Newey-West adjustment. Still, this approach does not fully exploit the explicit structure of the MA(2) specification in (20).

Second, the OLS regression does not enforce the restrictions in (21), which require that the OLS coefficients β_j and γ_j be proportional to the "weights" θ_j of the MA smoothing process. In particular, coefficients β_j and γ_j must satisfy:

$$\frac{\gamma_0}{\beta_0} = \frac{\gamma_1}{\beta_1} = \frac{\gamma_2}{\beta_2}.$$

Because coefficients β_j and γ_j are treated as independent parameters, the OLS regression tends to overfit the data. In general, suppose that one assumes MA(q) smoothing process (13)-(15) for the observed return r_t^o and specifies a J-factor linear model for the true return

$$r_t = \alpha + b_1 f_{1,t} + \ldots + b_J f_{J,t} + \epsilon_t, \tag{22}$$

where $f_{1,t}, \ldots, f_{J,t}$ are realizations of common factors at time-t. Then, the OLS regression produces $J^{ols} = 1 + qJ$ estimates, while the actual model has only $J^{act} = 1 + q + J$ independent parameters ($\alpha, b_1, \ldots, b_J$, and $\theta_1, \ldots, \theta_q$.)[13]

Finally, in the context of the OLS regression, the overall exposures to common factors is measured by $\hat{\beta} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2$ and $\hat{\gamma} = \hat{\gamma}_0 + \hat{\gamma}_1 + \hat{\gamma}_2$. However, the aggregate statistics $\hat{\beta}$ ($\hat{\gamma}$) may be difficult to interpret, especially when the individual sensitivities $\hat{\beta}_j$ ($\hat{\gamma}_j$) are not all statistically significant and when they are not of the same sign for different lags.

As an alternative to the OLS regression, we propose to estimate the model (19)-(21) via maximum likelihood (ML). The model can be viewed as a constrained ARMAX(0,2) model. This is because the statistical model includes a moving-average process of order 2, no autoregressive component, and additional regressors (also known as *impulses*) r_m, r_v, and their lags. The model is "constrained" because of the restrictions on the coefficients in (21) and the restrictions on the weights of the MA(2) smoothing process in (14)-(15).

Getmansky, Lo, and Makarov (2003) is the first paper that proposes ML to estimate the model (19)-(20) for hedge fund returns. However, their implementation is based on the *unconstrained* ARMAX(0,2) model, where the restrictions in (21) are not enforced. Thus, their approach only addresses the first drawback associated with the OLS regression.

In ML estimation, one searches for a set of parameters $\phi := (\alpha, \beta, \gamma, \sigma_\epsilon^2, \theta_1, \theta_2)$ which maximizes the log likelihood function:

$$\ln L(\phi) = -\frac{N}{2} \ln(2\pi\sigma_\epsilon^2) - \frac{1}{2} \ln |\Omega| - \frac{1}{\sigma_\epsilon^2} \mathbf{u}'\Omega^{-1}\mathbf{u}, \tag{23}$$

where

- N is the number of observations,
- $\mathbf{u} = (u_1, \ldots, u_N)'$ is a vector with elements

  $$u_t = r_t^o - (\alpha + \beta_0 r_{m,t} + \beta_1 r_{m,t-1} + \beta_2 r_{m,t-2} + \gamma_0 r_{v,t} + \gamma_1 r_{v,t-1} + \gamma_2 r_{v,t-2}),$$

  and

  $$\beta_j = \beta\theta_j, \qquad \gamma_j = \gamma\theta_j, \qquad j = 0, 1, 2, \qquad \theta_0 = 1 - \theta_1 - \theta_2,$$

- $\Omega = E[\mathbf{u}'\mathbf{u}]/\sigma_\epsilon^2$ is an $N \times N$ matrix with elements $\omega_{i,j}$:

  $$\omega_{i,j} = \begin{cases} \theta_0^2 + \theta_1^2 + \theta_2^2, & i = j \\ \theta_0\theta_1 + \theta_1\theta_2, & |i-j| = 1 \\ \theta_0\theta_2, & |i-j| = 2 \\ 0, & |i-j| \geq 3 \end{cases}$$

[13] As an illustration, consider the multi-factor model of Agarwal and Naik (2004). They use 12 market indicators and 4 additional option-based risk factors, so that $J = 16$. For the smoothing process (13)-(15) with $q = 2$, the difference between J^{ols} and J^{act} would be considerable. Since the data for hedge funds are only available for a relatively short period, the lack of parsimony of the OLS regression would be a serious shortcoming.

In view of (15), the weights for the smoothing process must be constrained so that

$$0 \le \theta_1 \le 1, \qquad 0 \le \theta_2 \le 1, \qquad \theta_1 + \theta_2 \le 1. \tag{24}$$

To maximize the log likelihood function, one needs to solve a constrained nonlinear optimization problem with six parameters. Normally, such a problem would be computationally challenging. The objective function in (23) is not globally concave and there could be multiple local maxima. With a relatively large number of parameters, the search procedure can converge to a corner solution or fail to converge at all.

In our case, however, there is a simple way around those difficulties. The idea is to exploit the fact that the objective function in (23) is only nonlinear in θ_1 and θ_2, but not in other parameters. Specifically, suppose that $\theta := (\theta_0, \theta_1, \theta_2)$ is fixed and denote

$$\begin{aligned} r^{\theta}_{m,t} &:= \theta_0 r_{m,t} + \theta_1 r_{m,t-1} + \theta_0 r_{m,t-1}, \\ r^{\theta}_{v,t} &:= \theta_0 r_{v,t} + \theta_1 r_{v,t-1} + \theta_0 r_{v,t-1}. \end{aligned}$$

Then the remaining parameters α, β, γ, σ^2_ϵ can be estimated efficiently via a generalized least squares (GLS) regression:

$$r^o_t = \alpha + \beta r^{\theta}_{m,t} + \gamma r^{\theta}_{v,t} + u_t,$$

where the covariance matrix of disturbances is given by $E[\mathbf{u}'\mathbf{u}] = \sigma^2_\epsilon \Omega$ and the matrix $\Omega = \Omega(\theta)$ is known. Thus, a unique set of parameters $\alpha(\theta)$, $\beta(\theta)$, $\gamma(\theta)$, $\sigma^2_\epsilon(\theta)$ can be found by solving a linear system of equations. The problem of maximizing the log likelihood function in (23) effectively reduces to a nonlinear search in θ_1 and θ_2, which is a considerably simpler problem.

Intuitively, the optimization problem has been "decomposed" into two stages: 1) GLS estimation of α, β, γ, σ^2_ϵ for given θ_1 and θ_2, and 2) constrained nonlinear search in θ_1 and θ_2. This also means that one can apply the same solution strategy to estimate a more general multi-factor model in (22). Introducing additional factors only affects (very simple) stage 1) of the algorithm, but has no effect on stage 2). Specific details on ML estimation are collected in Appendix D, which states the conditions on θ_1 and θ_2 necessary for invertibility of the process r^o_t, as well as the asymptotic properties of the ML estimator.

The results of ML estimation are reported in Table 6. They are qualitatively similar to the results for the OLS regression. The main difference between the two approaches is that the variance return exposures are typically even more pronounced for ML estimation. Also, ML sometimes produces lower adjusted R^2 than those for the OLS regression. This is to be expected because the OLS regression does not enforce many restrictions of the model in (13)-(17) and, thus, tends to overfit the data. Table 6 can be summarized as follows:

- The sensitivity to the variance return $\hat{\gamma}$ is negative and statistically significant for all categories, except for Convertible Arbitrage, Equity Market Neutral, Market Timing, and Short Selling. (For the four exceptions, $\hat{\gamma}$ is not statistically significant.)
- As with the OLS regression, the inclusion of the variance return has effect of i) reducing the overall exposure to the market risk, $\hat{\beta}$, and ii) reducing the alpha coefficient $\hat{\alpha}$. The only exceptions are Equity Market Neutral, Market Timing, and Short Selling.

- Among individual categories, the largest "sellers" of the variance risk are Distressed Securities, Emerging Markets, Equity Non-Hedge, Event-Driven, Fixed Income: High Yield, and Macro. For these categories, the variance risk component $\hat{\gamma}E[r_v]$ is 0.70%, 1.05%, 0.60%, 0.53%, 0.60%, and 0.48%, respectively. Most of these figures are noticeably larger than those for the OLS regression. The change in $\hat{\alpha}$ due to the inclusion of the variance return is -0.56%, -0.85%, -0.47%, -0.42%, -0.48%, and -0.38%, respectively. The inclusion of the variance return has the largest effect on: Distressed Securities, Emerging Markets, Equity Non-Hedge, and Fixed Income: High Yield.

  For Composite index, $\hat{\gamma}E[r_v]$ is 0.47% and the change in $\hat{\alpha}$ is -0.38%.

- Table 6 also report estimates of the "smoothing index" $\xi := \theta_0^2 + \theta_1^2 + \theta_2^2$. This index has been introduced in Getmansky, Lo, and Makarov (2003) to measure the degree of smoothing in hedge fund returns. A lower value of ξ implies more smoothing, while the upper bound $\xi = 1$ implies no smoothing.

  Categories with the most smoothing are Convertible Arbitrage, Distressed Securities, Fixed Income (Total), and Fixed Income: High Yield (ξ ranges from 0.39 to 0.43). One can also see from Table 4 that these indexes produce the largest Q-statistics. Categories with the least smoothing are Equity Market Neutral, Market Timing, Merger Arbitrage, and Short Selling (ξ ranges from 0.76 to 0.87). As expected, these indexes have very low Q-statistics in Table 4.

The hedge fund expected return can be decomposed as $E[r] = \hat{\alpha}^{(1)} + \hat{\beta}^{(1)}E[r_m]$ and $E[r] = \hat{\alpha}^{(2)} + \hat{\beta}^{(2)}E[r_m] + \hat{\gamma}^{(2)}E[r_v]$ for cases (1) and (2) of the model in (16)-(17), respectively. The two decompositions are visualized in Figure 7. It is clear that the variance return has an important impact on hedge fund performance evaluation. As mentioned earlier, the inclusion of the variance return reduces alpha coefficients for most hedge funds. For a number of categories, positive alphas become negative. Some positive alphas change from statistically significant to statistically insignificant. The inclusion of the variance return also reduces the component due to the market risk $\hat{\beta}E[r_m]$ for most funds. Often, the component due to the variance risk $\hat{\gamma}E[r_v]$ noticeably exceeds the component due to the market risk $\hat{\beta}E[r_m]$. Categories which have a relatively low exposure to the market risk and a relatively high exposure to the variance risk include Distressed Securities, Event-Driven, Fixed Income: High Yield, Macro, and Fund of Funds Composite.

4.5 Choice of Database

To check the robustness of our results, we repeat the analysis on another popular source of hedge fund data, the CSFB/Tremont Hedge Fund Indexes. These indexes have been used in a number of studies, including Asness, Krail, and Liew (2001), Agarwal and Naik (2004), and Getmansky, Lo, and Makarov (2003).

There are several differences between the HFR and CSFB/Tremont indexes:

- While the HFR indexes are equally-weighted, the CSFB/Tremont indexes are asset-weighted. Therefore, the CSFB/Tremont indexes give more weight to the performance of larger funds.

- CSFB/Tremont uses a somewhat different classification of hedge fund strategies. The main sector indexes are Convertible Arbitrage, Dedicated Short Bias, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Global Macro, Long/Short Equity, and Managed Futures. (See Appendix E.2 for a brief description of the sector indexes.)

- HFR and CSFB/Tremont apply different criteria for including hedge funds in their indexes. In particular, CSFB/Tremont requires at least $10 million under management and a track record available for at least one year. In contrast, there is no required asset-size minimum and no required length of track record for fund inclusion in the HFR indexes.

  Compared to HFR, CSFB/Tremont selects a smaller number of funds and the selected ones on average have larger portfolios under management.[14]

- The CSFB/Tremont indexes are available for a shorter period, with the inception date being January 1994.

Table 7 provides the summary statistics for the CSFB/Tremont indexes. Table 8 and Figure 8 report the results of ML estimation. (To save on space, we omit the results for the OLS regression.) The main findings for the CSFB/Tremont indexes are as follows:

- The sensitivity to the variance return $\hat{\gamma}$ is negative and statistically significant for all categories, except for Convertible Arbitrage ($\hat{\gamma}$ is negative but not statistically significant), Dedicated Short Bias, Equity Market Neutral, and Managed Futures ($\hat{\gamma}$ is positive but not statistically significant).

- For most categories, the inclusion of the variance return has effect of i) reducing the overall exposure to the market risk, $\hat{\beta}$, and ii) reducing the alpha coefficient $\hat{\alpha}$. The only exceptions are Dedicated Short Bias, Equity Market Neutral, and Managed Futures.

- Among individual categories, the largest "sellers" of the variance risk are Emerging Markets, Event Driven, Global Macro, and Long/Short. For these categories, the variance risk component $\hat{\gamma}E[r_v]$ is 1.18%, 0.43%, 0.52%, and 0.55%, respectively. The change in $\hat{\alpha}$ due to the inclusion of the variance return is -0.87%, -0.31%, -0.35%, and -0.38%, respectively.

  For Aggregate Index, $\hat{\gamma}E[r_v]$ is 0.54% and the change in $\hat{\alpha}$ is -0.37%.

- Categories with the most smoothing are Convertible Arbitrage, Event Driven, and Fixed Income Arbitrage (the smoothing index ξ is 0.36 to 0.48), while categories with the least smoothing are Dedicated Short Bias and Managed Futures (ξ is 0.82 and 0.94).

Overall, the results for the CSFB/Tremont indexes are consistent with those for the HFR indexes. Comparable fund categories generally have very similar estimates for β, γ, and smoothing weights θ_j. (For example, compare HFR Emerging Markets and Macro to CSFB/Tremont Emerging Markets and Global Macro.)

There are two noticeable differences between the databases. First, the adjusted R^2's are somewhat lower for the CSFB/Tremont indexes. This could be due to differences in construction of the HFR and CSFB/Tremont indexes. In particular, the HFR indexes are equally-weighted, are based on a broader fund universe, and give more weight to performance of smaller funds. As a result, they better average out idiosyncratic components of hedge fund

[14] As of May 2003, there are 431 funds in the CSFB/Tremont fund universe and more than 1400 funds in the HFR fund universe.

returns. In contrast, the CSFB/Tremont indexes are asset-weighted and they are mostly determined by returns of a small number of larger funds, implying that the CSFB/Tremont indexes are less effective in diversifying away idiosyncratic noise. Second, estimated alpha coefficients for the CSFB/Tremont indexes are typically smaller than those for the HFR indexes. This fact has also been noted by Asness, Krail, and Liew (2001). Again, while many specifics of the two databases differ, one possible explanation is that smaller and newer hedge funds generally earn higher returns. See Peskin, Urias, Anjilvel, Boudreau (2000).[15]

4.6 Inclusion of Put Return

As mentioned earlier, Agarwal and Naik (2004) use returns on S&P 500 options to explain performance of equity-oriented hedge funds. They find that several fund categories exhibit returns similar to those from selling puts. That is, when the put return is included in multi-factor OLS regressions, its factor loading often comes out negative and statistically significant. Therefore, we verify robustness of our main findings by extending specification (16) with a more general specification

$$r_t = \alpha + \beta r_{m,t} + \gamma r_{v,t} + \delta r_{p,t} + \varepsilon_t, \tag{25}$$

where r_p denotes the return on the ATM put option. We estimate specification (25) via ML and find that the addition of the put return has a relatively minor effect on our earlier conclusions. The results for this specification (not reported) can be summarized as follows:

- The sensitivity to the variance return $\hat{\gamma}$ remains negative and significant for most fund categories, although its magnitude and statistical significance typically slightly decline.
- In particular, in Table 6 for HFR indexes, $\hat{\gamma}$ is negative and significant for 11 individual categories, as well as for Composite and Fund of Fund Composite. When the put return is added, $\hat{\gamma}$ remains negative and significant for all these categories except for two: Merger Arbitrage (t-statistics becomes -1.50 instead of -2.71) and Fixed Income: Arbitrage (t-statistics becomes -1.53 instead of -2.22). Of these 13 categories, $\hat{\gamma}$ actually becomes *more* negative for Equity Hedge, Equity Non-Hedge, and Fund of Funds Composite, while for Composite $\hat{\gamma}$ is unchanged.
- Similarly, in Table 8 for CSFB/Tremont indexes, $\hat{\gamma}$ is negative and significant for 5 individual categories, as well as Aggregate Index. When the put return is added, $\hat{\gamma}$ remains negative and significant for all these categories except for one: Global Macro (t-statistics becomes borderline insignificant -1.94 instead of -2.09). The sensitivity $\hat{\gamma}$ becomes more negative for Long/Short.
- For aggregate categories, the extension to specification (25) has essentially no affect on the decomposition of the expected excess return $E[r]$. This can be seen from the following table, where figures in parentheses correspond to specification (16):

Category	$\hat{\alpha}$	$\hat{\beta}E[r_m]$	$\hat{\gamma}E[r_v]$	$\hat{\delta}E[r_p]$
HFR Composite	0.13 (0.13)	0.33 (0.33)	0.47 (0.47)	0.00
HFR Fund of Funds Comp.	0.00 (-0.06)	0.17 (0.16)	0.49 (0.44)	-0.13
CSFB/Tremont Aggregate	-0.31 (-0.27)	0.35 (0.36)	0.52 (0.54)	0.06

[15] Peskin, Urias, Anjilvel, Boudreau (2000) also point out that because smaller funds are the most susceptible to reporting biases, such as backfill bias, their performance is more likely to be overstated.

Overall, we conclude that the variance return constitutes a truly new risk factor for hedge funds, separate from the put return. In fact, for most fund categories, the exposure to the variance return turns out to be much more important than the exposure to the put return.

5 Conclusion

This paper implements a new approach to measure the market price of the variance risk. We find that the variance risk is priced, its risk premium is negative and economically very large. In the application to hedge funds, we argue that the variance return goes a long way toward explaining hedge fund returns. Returns of most hedge funds exhibit negative exposure to the variance return, implying that funds routinely "sell" the variance risk. The exposure to the variance risk accounts for a considerable portion of hedge fund average returns.

To put in perspective the economic impact of the hedge fund exposure to the variance risk, we observe that the overall hedge fund industry on average earns about 6.5% annually by shorting the variance risk (using $\hat{\gamma}E[r_v]$ for asset-weighted CSFB/Tremont Aggregate Index in Table 8). In 2000, hedge funds had approximately \$500 bln in assets under management, implying that selling the variance risk accounted for roughly \$32.4 bln of hedge fund returns in this year alone! Given that the hedge fund industry continues to expand very rapidly, this figure will only grow larger in the future.

Our findings have important implications for hedge fund performance evaluation. When the variance risk exposure is not accounted for, many hedge fund categories appear to deliver superior risk-adjusted returns (i.e., they have positive and statistically significant alphas). However, after correcting for the variance risk exposure, the performance of most categories becomes much less impressive, with positive alphas often becoming negative or statistically insignificant. As a group, hedge funds no longer seem to "add value".

To be fair, however, we note that hedge funds could still be very useful to individual investors. As follows from Section 3.4, a typical CRRA investor would be willing to pay considerable amounts for being able to sell the variance contract. At present, there is no cheap and practical way for individual investors to trade the variance contract or related securities. In these circumstances, hedge funds are valuable because they offer investors an indirect way to attain negative exposure to the variance risk. Even though hedge funds charge very high fees, investing through them can greatly improve investors' utility. This situation, however, could change fundamentally in the near future. In spring 2004, CBOE began trading in futures on the volatility index VIX. It is also in CBOE's future plans to launch trading in another product, futures on the realized variance of S&P 500. Variance futures would offer an easy and direct way to manage the variance risk exposure. The analysis in this paper can help traders to better understand historical performance and risk characteristics of such a product.

One important conclusion of this paper is that the variance return should be considered as a new risk factor for various assets. In particular, we conjecture that this factor might come out significant for those assets and strategies that are exposed to the liquidity and/or credit risks. Extensions along these lines are left for future research.

Appendix

A Proof of Proposition 1

The proof of (6) follows Carr and Madan (1998) and Bakshi and Madan (2000). Let $g(F_T)$ denote a general payoff at time-T. If $g(F_T)$ is twice-continuously differentiable, then it can be represented as:

$$g(F_T) = g(x) + g'(x)(F_T - x) + \int_0^x g''(K)(K - F_T)^+ dK + \int_x^\infty g''(K)(F_T - K)^+ dK, \qquad (26)$$

for any $x \geq 0$.[16] When $g(F_T) = \ln F_T$ and $x = F_0$, equation (26) becomes

$$\ln F_T = \ln F_0 + \frac{1}{F_0}(F_T - F_0) - \int_0^{F_0} \frac{(K - F_T)^+}{K^2} dK - \int_{F_0}^\infty \frac{(F_T - K)^+}{K^2} dK.$$

Therefore,

$$-E_0^*\left[\ln \frac{F_T}{F_0}\right] = \int_0^{F_0} \frac{P_0(K,T)}{K^2} dK + \int_{F_0}^\infty \frac{C_0(K,T)}{K^2} dK,$$

which follows because

$$E_0^*[F_T] = F_0, \qquad E_0^*[(K - F_T)^+] = P_0(K,T), \qquad E_0^*[(F_T - K)^+] = C_0(K,T).$$

□

B Construction of Dataset

To construct our dataset, we follow several steps:

1. For both options and futures we use settlement prices. Settlement prices (as opposed to closing prices) do not suffer from nonsynchronous/stale trading of options and the bid-ask spreads. CME calculates settlement prices simultaneously for all options, based on their last bid and ask prices. Since these prices are used to determine daily margin requirements, they are carefully scrutinized by the exchange and closely watched by traders. As a result, settlement prices are less likely to suffer from recording errors and they rarely violate basic no-arbitrage restrictions. In contrast, closing prices are generally less reliable and less complete.

2. In the dataset, we match all puts and calls by trading date t, maturity T, and strike. For each pair (t, T), we drop very low (high) strikes for which put (call) price is less than 0.1. To convert spot prices to forward prices, we approximate the risk-free rate r_f over $[t, T]$ by the rate of Tbills.

3. Because the CME options are the American type, their prices $P_t^A(K)$ and $C_t^A(K)$ could be slightly higher than prices of the corresponding European options $P_t(K)$ and $C_t(K)$. The difference, however, is very small for short maturities that we focus on. This is particularly true for OTM an ATM options.[17]

To infer prices of European options $P_t(K)$ and $C_t(K)$, we proceed as follows. First, we discard all ITM options. That is, we use put prices for $K/F_t \leq 1.00$ and call prices for $K/F_t \geq 1.00$. Prices of

[16]One way to derive (26) is to first write the payoff $g(F_T)$ as

$$g(F_T) = \int_0^x g(K)\delta(K - F_T)dK + \int_x^\infty g(K)\delta(F_T - K)dK,$$

where $\delta(\cdot)$ is the Dirac delta function, and then integrate each integral by parts two times.

[17]As shown in Whaley (1986), the early exercise premium increases with the level of the risk-free rate, volatility, time to maturity, and degree to which an option is in-the-money.

OTM and ATM options are both more reliable and less affected by the early exercise feature. Second, we correct American option prices $P_t^A(K)$ and $C_t^A(K)$ for the value of the early exercise feature by using Barone-Adesi and Whaley (1987) approximation.[18] Third, we compute prices of ITM options through the put-call parity relationship

$$P_t(K) + F_t = C_t(K) + K.$$

4. We check option prices for violations of the no-arbitrage restrictions. To preclude arbitrage opportunities, call and put prices must be monotonic and convex functions of the strike. In particular, the call pricing function $C_t(K)$ must satisfy

$$(a) \quad C_t(K) \geq (F_t - K)^+, \qquad (b) \quad -1 \leq C_t'(K) \leq 0, \qquad (c) \quad C_t''(K) \geq 0.$$

The corresponding conditions for the put pricing function $P_t(K)$ follow from put-call parity. When restrictions (a)-(c) are violated, we enforce them by running the so-called *Constrained Convex Regression* (CCR). This procedure has been proposed in Bondarenko (1997) and also implemented in Bondarenko (2000). Intuitively, CCR searches for the smallest (in the sense of least squares) perturbation of option prices that restores the no-arbitrage restrictions. For most trading days, option settlement prices already satisfy the restrictions (a)-(c). Still, CCR is a useful procedure because it allows one to identify possible recording errors or typos. We eliminate an option cross-section if CCR detects substantial arbitrage violations, that is, if square root of mean squared deviation of option prices from the closest arbitrage-free prices is more than 0.1. (This filter eliminates less than 0.5% of trading days.)

5. For each pair (t, T), we estimate RND using the *Positive Convolution Approximation* (PCA) procedure of Bondarenko (2000, 2003a). Armed with RND, we obtain the put and call pricing functions. The market price of the variance contract is then computed via the relationship in (6).

C Monte-Carlo Experiment

The design of the Monte-Carlo experiment is similar to that in Bondarenko (2003a). We assume that the actual RND $h_0(F_T)$ is approximated by a flexible parametric specification. Specifically, let $h_0(F_T)$ be a mixture of three lognormals

$$h_0(F_T) = w_1 h_0^{BS}(F_T; X_1, \sigma_1) + w_2 h_0^{BS}(F_T; X_2, \sigma_2) + w_3 h_0^{BS}(F_T; X_3, \sigma_3), \tag{27}$$

$$w_1 + w_2 + w_3 = 1,$$

where $h_0^{BS}(F; X, \sigma)$ denotes the lognormal density corresponding to the Black-Scholes model:

$$h_0^{BS}(F; X, \sigma) := \frac{1}{\sqrt{2\pi}\sigma\sqrt{T}} \frac{1}{F} \exp\left\{ -\frac{(\ln F - \ln X - \frac{1}{2}\sigma^2 T)^2}{2\sigma^2 T} \right\}.$$

Under this specification,

$$F_0 = w_1 X_1 + w_2 X_2 + w_3 X_3,$$

$$U_0 = (w_1\sigma_1^2 + w_2\sigma_2^2 + w_3\sigma_3^2)T - 2(w_1 \ln X_1 + w_2 \ln X_2 + w_3 \ln X_3) - 2\ln F_0.$$

The parameters of the lognormals and the mixing probabilities are chosen to describe a typical cross-section of the S&P 500 futures options with about 1 month to maturity. To calibrate the RND $h_0(F_T)$, we assume that

- the initial date is November 15, 1996, when F_0=740.85;
- the maturity date is December 20, 1996, so that T=35 days;

[18] It is important to point out that this correction is always substantially smaller than typical bid-ask spreads. In particular, the correction generally does not exceed 0.2% of an option price.

- there are 42 strikes available, with $K_i = 590, 595, \ldots, 795$.

The fitted parameters of the mixture of three lognormals are found by minimizing the sum of squared errors. They are

$w_1 = 0.021$	$w_2 = 0.157$	$w_3 = 0.822$
$X_1 = 631.1$	$X_2 = 710.0$	$X_3 = 749.6$
$\sigma_1 = 0.273$	$\sigma_2 = 0.135$	$\sigma_3 = 0.089$

(28)

The resulting RND $h_0(F_T)$ is shown in the left panel of Figure 2. Using $h_0(F_T)$, we can compute the theoretical call and put prices $C_0(K_i)$ and $P_0(K_i)$. The right panel of this figure shows the Black-Scholes implied volatilities derived from market and fitted prices of options. To create simulated option prices, we add to the theoretical call and put prices random noise, which models observational errors that arise from nonsynchronicity, bid-ask spread, and other market imperfections. Specifically, for option prices $q \in \{C_0(K_i), P_0(K_i)\}$, we introduce measurement errors $\varepsilon(q)$, which are independent and for which $E[\varepsilon(q)] = 0$. The details of our specification for $\varepsilon(q)$ are explained below. We generate 500 simulated cross-sections of options. For each cross-section, an estimate $\hat{U}_0$ is obtained in the same way as with real data. Given 500 estimates $\hat{U}_0$, the RMSE measure is computed as in (11).

Error specification

To model the measurement errors $\varepsilon(q)$, we have used several approaches and have found that the relative performance of the estimator $\hat{U}_0$ is qualitatively very similar. Since CME calculates option settlement prices by using the last bid and ask prices, we assume that the introduced measurement error $\varepsilon(q)$ is uniformly distributed on $[-0.5s(q), 0.5s(q)]$, where $s(q)$ is the "effective spread" corresponding to the price q. Simple approaches (not reported) would be to assume that (i) the level of noise is the same across options in *absolute* terms, so that $s(q) = c$ for some constant c, or (ii) the level of noise is the same in *relative* terms, so that $s(q) = cq$ for some constant c. However, both approaches are not completely satisfactory.

The alternative approach is more realistic, although it is also more complex. We observe that some options exchanges set upper limits for bid-ask spreads. For example, the CBOE rules limit the maximum bid-ask spread for the S&P 500 options as:

	$q < 2$	$2 \le q < 5$	$5 \le q < 10$	$10 \le q < 20$	$20 \le q$
Maximum spread	1/4	3/8	1/2	3/4	1

To approximate these rules, we construct function $M(q)$, which represents the maximum spread for the option price q. Specifically, let

$$M(0) = \frac{1}{8}, \quad M(2) = \frac{1}{4}, \quad M(5) = \frac{3}{8}, \quad M(10) = \frac{1}{2}, \quad M(20) = \frac{3}{4}, \quad M(q) = 1, \quad q \ge 50,$$

and $M(q)$ is linearly interpolated for all other $q \in [0, 50]$. In particular, $M(q)$ is about 19% when $q = \$1$ and about 4% when $q = \$20$. Then we assume that $s(q)$ is proportional to the maximum spread, or $s(q) = cM(q)$. By varying constant c, the level of noise can be increased or decreased across all options. We report the results for three cases: 1) No noise, $c = 0$, 2) Moderate noise, $c = 0.5$ (the effective spreads are half of the exchange allowed maximum), and 3) High noise, $c = 1$ (the effective spreads are equal to the allowed maximum). The advantages of the proposed specification for $\varepsilon(q)$ are that (a) noise is smaller in the *absolute* terms for OTM options, (b) noise is larger in the *relative* terms for OTM options, and (c) simulated option prices are always nonnegative.

D Maximum Likelihood Estimation

This appendix states two useful results for the specification in (19)-(21) with constraints (14)-(15). These results follow immediately from Proposition 3 in Getmansky, Lo, and Makarov (2003).

1) The process r_t^o is invertible if and only if

$$\theta_1 < 1/2, \qquad \theta_1 < 1 - 2\theta_2. \tag{29}$$

2) Under the invertibility constraints in (29), the asymptotic distribution of the ML estimator $(\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2)$ is

$$\sqrt{N}(\hat{\theta}_j - \theta_j) \sim N\left[0, \Sigma_j\right], \qquad j = 0, 1, 2, \tag{30}$$

where

$$\begin{aligned}
\Sigma_0 &= (1 - \theta_1 - 2\theta_2)\left(2(1-\theta_1)(1-\theta_1-2\theta_2) + 2\theta_2^2\right), \\
\Sigma_1 &= (1 - \theta_1 - 2\theta_2)(1-\theta_1)(1-2\theta_1), \\
\Sigma_2 &= (1 - \theta_1 - 2\theta_2)\left((1-\theta_1-2\theta_2) + 2\theta_2^2\right).
\end{aligned}$$

We implement ML estimation in Matlab using Optimization Toolbox. The log likelihood objective function is maximized subject to the constraints in (24) and (29). The search procedure converges without difficulty for all categories of HFR and CSFB/Tremont indexes. The invertibility constraints in (29) are never binding. As for the constraints in (24), one category (HFR Equity Market Neutral) has $\hat{\theta}_1 = 0$ and two categories (HFR Short Selling, CSFB Managed Futures) have $\hat{\theta}_2 = 0$. We use the asymptotic distribution in (30) to compute standard errors for the estimator $(\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2)$ in Table 6 and Table 8.

E Hedge Fund Information

E.1 HFR Monthly Performance Indexes

Hedge Fund Research (HFR) Monthly Indexes are equally weighted performance indexes. The indexes are broken down into several categories by strategy, including the HFR Composite Index, which accounts for over 1,400 funds listed on the internal HFR Database. The following are some additional details on the HFR Indexes:

- All HFR Indexes are fund weighted (equal weighted).
- There is no required asset-size minimum for fund inclusion in the HFR Index.
- There is no required length of time a fund must be actively trading before inclusion in the HFR Index.
- The trailing 4 months are left as estimates and are subject to change. All performance prior to that is locked and is no longer subject to change.
- If a fund liquidates/closes, that fund's performance will be included in the HFR Index as of that fund's last reported performance update.
- The HFR Fund of Funds Composite Index is not included in the HFR Composite Index.
- Both domestic and offshore funds are included in the HFR Index.

Further information concerning the HFR Indexes can be found at www.hedgefundresearch.com. The following classification of hedge fund categories is reproduced from the HFR website.

Convertible Arbitrage involves purchasing a portfolio of convertible securities, generally convertible bonds, and hedging a portion of the equity risk by selling short the underlying common stock. Certain managers may also seek to hedge interest rate exposure under some circumstances. Most managers employ some degree of leverage, ranging from zero to 6:1. The equity hedge ratio may range from 30 to 100 percent. The average grade of bond in a typical portfolio is BB-, with individual ratings ranging from AA to CCC. However, as the default risk of the company is hedged by shorting the underlying common stock, the risk is considerably better than the rating of the unhedged bond indicates.

Distressed Securities funds invest in, and may sell short, the securities of companies where the security's price has been, or is expected to be, affected by a distressed situation. This may involve reorganizations, bankruptcies, distressed sales and other corporate restructurings. Depending on the manager's style, investments may be made in bank debt, corporate debt, trade claims, common stock, preferred stock and warrants. Strategies may be sub-categorized as "high-yield" or "orphan equities." Leverage may be used by some managers. Fund managers may run a market hedge using S&P put options or put options spreads.

Emerging Markets funds invest in securities of companies or the sovereign debt of developing or "emerging" countries. Investments are primarily long. "Emerging Markets" include countries in Latin America, Eastern Europe, the former Soviet Union, Africa and parts of Asia. Emerging Markets - Global funds will shift their weightings among these regions according to market conditions and manager perspectives. In addition, some managers invest solely in individual regions.

Equity Hedge funds invest in a core holding of long equities hedged at all times with short sales of stocks and/or stock index options. Some managers maintain a substantial portion of assets within a hedged structure and commonly employ leverage. Where short sales are used, hedged assets may be comprised of an equal dollar value of long and short stock positions. Other variations use short sales unrelated to long holdings and/or puts on the S&P 500 index and put spreads. Conservative funds mitigate market risk by maintaining market exposure from zero to 100 percent. Aggressive funds may magnify market risk by exceeding 100 percent exposure and, in some instances, maintain a short exposure. In addition to equities, some funds may have limited assets invested in other types of securities.

Equity Market Neutral funds seek to profit by exploiting pricing inefficiencies between related equity securities, neutralizing exposure to market risk by combining long and short positions. One example of this strategy is to build portfolios made up of long positions in the strongest companies in several industries and taking corresponding short positions in those showing signs of weakness.

Equity Non-Hedge funds are predominately long equities although they have the ability to hedge with short sales of stocks and/or stock index options. These funds are commonly known as "stock-pickers." Some funds employ leverage to enhance returns. When market conditions warrant, managers may implement a hedge in the portfolio. Funds may also opportunistically short individual stocks. The important distinction between equity non-hedge funds and equity hedge funds is equity non-hedge funds do not always have a hedge in place. In addition to equities, some funds may have limited assets invested in other types of securities.

Event-Driven is also known as "corporate life cycle" investing. This involves investing in opportunities created by significant transactional events, such as spin-offs, mergers and acquisitions, bankruptcy reorganizations, recapitalizations and share buybacks. The portfolio of some Event-Driven managers may shift in majority weighting between Risk Arbitrage and Distressed Securities, while others may take a broader scope. Instruments include long and short common and preferred stocks, as well as debt securities and options. Leverage may be used by some managers. Fund managers may hedge against market risk by purchasing S&P put options or put option spreads.

Market Timing funds allocate assets among investments by switching into investments that appear to be beginning an uptrend, and switching out of investments that appear to be starting a downtrend. This primarily consists of switching between mutual funds and money markets. Typically, technical trend-following indicators are used to determine the direction of a fund and identify buy and sell signals. In an up move "buy signal," money is transferred from a money market fund into a mutual fund in an attempt to capture a capital gain. In a down move "sell signal," the assets in the mutual fund are sold and moved back into the money market for safe keeping until the next up move. The goal is to avoid being invested in mutual funds during a market decline.

Merger Arbitrage, sometimes called **Risk Arbitrage**, involves investment in event-driven situations such as leveraged buy-outs, mergers and hostile takeovers. Normally, the stock of an acquisition target appreciates while the acquiring company's stock decreases in value. These strategies generate returns by purchasing stock of the company being acquired, and in some instances, selling short the stock of the acquiring company. Managers may employ the use of equity options as a low-risk alternative to the outright purchase or sale of common stock. Most Merger Arbitrage funds hedge against market risk by purchasing S&P put options or put option spreads.

Relative Value Arbitrage funds attempt to take advantage of relative pricing discrepancies between instruments including equities, debt, options and futures. Managers may use mathematical, fundamental, or technical analysis to determine misvaluations. Securities may be mispriced relative to the underlying security, related securities, groups of securities, or the overall market. Many funds use leverage and seek opportunities globally. Arbitrage strategies include dividend arbitrage, pairs trading, options arbitrage and yield curve trading.

Short Selling involves the sale of a security not owned by the seller; a technique used to take advantage of an anticipated price decline. To effect a short sale, the seller borrows securities from a third party in order to make delivery to the purchaser. The seller returns the borrowed securities to the lender by purchasing the securities in the open market. If the seller can buy that stock back at a lower price, a profit results. If the price rises, however, a loss results. A short seller must generally pledge other securities or cash with the lender in an amount equal to the market price of the borrowed securities. This deposit may be increased or decreased in response to changes in the market price of the borrowed securities.

Fixed Income: Arbitrage is a market neutral hedging strategy that seeks to profit by exploiting pricing inefficiencies between related fixed income securities while neutralizing exposure to interest rate risk. Fixed Income Arbitrage is a generic description of a variety of strategies involving investment in fixed income instruments, and weighted in an attempt to eliminate or reduce exposure to changes in the yield curve. Managers attempt to exploit relative mispricing between related sets of fixed income securities. The generic types of fixed income hedging trades include: yield-curve arbitrage, corporate versus Treasury yield spreads, municipal bond versus Treasury yield spreads and cash versus futures.

Fixed Income: High-Yield funds invest in non-investment grade debt. Objectives may range from high current income to acquisition of undervalued instruments. Emphasis is placed on assessing credit risk of the issuer. Some of the available high-yield instruments include extendible/reset securities, increasing-rate notes, pay-in-kind securities, step-up coupon securities, split-coupon securities and usable bonds.

Macro involves investing by making leveraged bets on anticipated price movements of stock markets, interest rates, foreign exchange and physical commodities. Macro managers employ a "top-down" global approach, and may invest in any markets using any instruments to participate in expected market movements. These movements may result from forecasted shifts in world economies, political fortunes or global supply and demand for resources, both physical and financial. Exchange-traded and over-the-counter derivatives are often used to magnify these price movements.

Fund of Funds Composite Index. Fund of Funds invest with multiple managers through funds or managed accounts. The strategy designs a diversified portfolio of managers with the objective of significantly lowering the risk (volatility) of investing with an individual manager. The Fund of Funds manager has discretion in choosing which strategies to invest in for the portfolio. A manager may allocate funds to numerous managers within a single strategy, or with numerous managers in multiple strategies. The minimum investment in a Fund of Funds may be lower than an investment in an individual hedge fund or managed account. The investor has the advantage of diversification among managers and styles with significantly less capital than investing with separate managers.

E.2 CSFB/Tremont Hedge Fund Indexes

The CSFB/Tremont Hedge Fund Index is the asset-weighted hedge fund index. To determine index constituents, CSFB/Tremont begins with funds in the TASS database which tracks about 3000 funds. Funds are separated into 9 primary categories based on their investment style. The Index in all cases represents at least 85% of the assets under management in the universe. The following are some specifications of the CSFB/Tremont Indexes:

- The index is asset-weighted. The index does not include managed accounts or funds of funds. The inception of the index is January 1994.

- To be included in the index, a fund must have at least \$10 million in assets, have audited financials, and meet the CSFB/Tremont Index LLC reporting requirements.
- In order to minimize survivorship bias, funds are not removed from the index until they are liquidated or fail to meet the financial reporting requirements. Thus, performance of a removed fund remains in the index for the period during which the fund was active. New funds are added to the index on a going forward basis only.
- The index is calculated on a monthly basis. However funds are reselected on a quarterly basis as necessary.

Further information concerning the CSFB/Tremont Indexes can be found at www.hedgeindex.com. The following classification of hedge fund categories is reproduced from the CSFB/Tremont website.

Convertible Arbitrage: This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.

Dedicated Short Bias: Dedicated short sellers were once a robust category of hedge funds before the long bull market rendered the strategy difficult to implement. A new category, short biased, has emerged. The strategy is to maintain net short as opposed to pure short exposure. Short biased managers take short positions in mostly equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.

Emerging Markets: This strategy involves equity or fixed income investing in emerging markets around the world. Because many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to hedge, emerging market investing often employs a long-only strategy.

Equity Market Neutral: This investment strategy is designed to exploit equity market inefficiencies and usually involves being simultaneously long and short matched equity portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both. Well-designed portfolios typically control for industry, sector, market capitalization, and other exposures. Leverage is often applied to enhance returns.

Event Driven: This strategy is defined as 'special situations' investing designed to capture price movement generated by a significant pending corporate event such as a merger, corporate restructuring, liquidation, bankruptcy or reorganization. There are three popular sub-categories in event-driven strategies: risk (merger) arbitrage, distressed/high yield securities, and Regulation D.

- **Risk (Merger) Arbitrage:** Specialists invest simultaneously long and short in the companies involved in a merger or acquisition. Risk arbitrageurs are typically long the stock of the company being acquired and short the stock of the acquirer. By shorting the stock of the acquirer, the manager hedges out market risk, and isolates his exposure to the outcome of the announced deal. In cash deals, the manager needs only long the acquired company. The principal risk is deal risk, should the deal fail to close. Risk arbitrageurs also often invest in equity restructurings such as spin-offs or 'stub trades'.
- **Distressed/High Yield Securities:** Fund managers in this non-traditional strategy invest in the debt, equity or trade claims of companies in financial distress or already in default. The securities of companies in distressed or defaulted situations typically trade at substantial discounts to par value due to difficulties in analyzing a proper value for such securities, lack of street coverage, or simply an inability on behalf of traditional investors to accurately value such claims or direct their legal interests during restructuring proceedings. Various strategies have been developed by which investors may take hedged or outright short positions in such claims, although this asset class is in general a long-only strategy.
- **Regulation D:** This sub-set refers to investments in micro and small capitalization public companies that are raising money in private capital markets. Investments usually take the form of a convertible security with an exercise price that floats or is subject to a look-back provision that insulates the investor from a decline in the price of the underlying stock.

Fixed Income Arbitrage: The fixed income arbitrageur aims to profit from price anomalies between related interest rate securities. Most managers trade globally with a goal of generating steady returns with low volatility. This category includes interest rate swap arbitrage, US and non-US government bond arbitrage, forward yield curve arbitrage, and mortgage-backed securities arbitrage. The mortgage-backed market is primarily US-based, over-the-counter and particularly complex.

Global Macro: Global macro managers carry long and short positions in any of the world's major capital or derivative markets. These positions reflect their views on overall market direction as influenced by major economic trends and or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

Long/Short Equity: This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short US or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock funds.

Managed Futures: This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as Commodity Trading Advisors, or CTAs. Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price and market specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

References

Aït-Sahalia, Y., 2002, "Telling from discrete data whether the underlying continuous-time model is a diffusion," *Journal of Finance* 96, 2075-2112.

Agarwal, V., and N. Naik, 2004, "Risks and Portfolio Decisions Involving Hedge Funds," *Review of Financial Studies* 17, 63-98.

Andersen, T., L. Benzoni, and J. Lund, 2002, "An Empirical Investigation of Continuous-Time Equity Returns Models," *Journal of Finance* 57, 1239-1284.

Asness, C., R. Krail, and J. Liew, 2001, "Do Hedge Funds Hedge?" *The Journal of Portfolio Management* 28, 6-19.

Bakshi, G., C. Cao, and Z. Chen, 1997, "Empirical Performance of Alternative Option Pricing Models," *Journal of Finance* 52, 2003-2049.

Bakshi, G., C. Cao, and Z. Chen, 2000, "Do Call Prices and the Underlying Stock Always Move in the Same Direction?" *Review of Financial Studies* 13, 549-584.

Bakshi, G., and N. Kapadia, 2003a, "Delta-Hedged Gains and the Negative Market Volatility Premium," *Review of Financial Studies* 16, 527-566.

Bakshi, G., and N. Kapadia, 2003b, "Volatility Risk Premium Embedded in Individual Equity Options: Some New Insights," working paper.

Bakshi, G., and D. Madan, 2000, "Spanning and Derivative-Security Valuation," *Journal of Financial Economics* 55, 205-238.

Bakshi, G., N. Kapadia, and D. Madan, 2003, "Stock Return Characteristics, Skew Laws, and Differential Pricing of Individual Equity Options," *Review of Financial Studies*, forthcoming.

Banz, R., and M. Miller, 1978, "Prices for State-Contingent Claims: Some Estimates and Applications," *Journal of Business* 51, 653-672.

Barone-Adesi, G., and Whaley, R., 1987, "Efficient Analytical Approximation of American Option Values," *Journal of Finance* 41, 301-320.

Bates, D., 1996, "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options," *Review of Financial Studies* 9, 69-107.

Bates, D., 2000, "Post-'87 Crash Fears in the S&P 500 Futures Option Market," *Journal of Econometrics* 94, 181-238.

Benzoni, L., 2002, "Pricing Options under Stochastic Volatility: An Empirical Investigation," working paper, University of Minnesota.

Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, 637-654.

Bondarenko, O., 1997, "Testing Rationality of Financial Markets: An Application to S&P 500 Index Options," Caltech working paper.

Bondarenko, O., 2000, "Recovering Risk-Neutral Densities: A New Nonparametric Approach," UIC working paper.

Bondarenko, O., 2003a, "Estimation of Risk-Neutral Densities Using Positive Convolution Approximation," *Journal of Econometrics* 116, 85-112.

Bondarenko, O., 2003b, "Why are Put Options So Expensive?" UIC working paper.

Breeden, D., and R. Litzenberger, 1978, "Prices of State Contingent Claims Implicit in Options Prices," *Journal of Business* 51, 621-652.

Britten-Jones, M. and A. Neuberger, 2000, "Option Prices, Implied Price Processes, and Stochastic Volatility," *Journal of Finance* 55, 839-866.

Carr, P., and D. Madan, 1998, "Towards a Theory of Volatility Trading," *Volatility*, Risk Publications, R. Jarrow, Ed., 417-427.

Carr, P., and L. Wu, 2003, "What Type of Process Underlies Options? A Simple Robust Test," *Journal of Finance* 58, 2581-2610.

Chernov, M., and E. Ghysels, 2000, "A Study Towards a Unified Approach to the Joint Estimation of Objective and Risk Neutral Measures for the Purpose of Options Valuation," *Journal of Financial Economics* 56, 407-458.

Coval, J., and T. Shumway, 2001, "Expected Option Returns," *Journal of Finance* 56, 983-1009.

Demeterfi, K., E. Derman, M. Kamal, and J. Zou, 1999, "More Than You Ever Wanted to Know About Volatility Swaps," working paper, Goldman Sachs, http://www.gs.com/qs/doc/volswaps.pdf.

Driessen, J., P. Maenhout, "A Portfolio Perspective on Option Pricing Anomalies," working paper.

Duffie, D., K. Singleton, and J. Pan, "Transform Analysis and Asset Pricing for Affine Jump-Diffusions," *Econometrica* 68, 1343–1376.

Dumas, B., J. Fleming, and R. Whaley, 1998, "Implied Volatility Functions: Empirical Tests," *Journal of Finance* 53, 2059-2106.

Dunbar, N., 1998, "Meriwether's Meltdown," *Risk* (October) 32-36.

Dupire, B., 1993, "Model Art," *Risk* (September) 118-120.

Eraker, B., M. Johannes, and N. Polson, 2003, "The Impact of Jumps on Volatility and Returns," *Journal of Finance* 58, 1269-1300.

Fung, W., and D. Hsieh, 1997, "Survivorship Bias and Investment Style in the Returns of CTA," *Journal of Portfolio Management* 23, 30-41.

Fung, W., and D. Hsieh, 2001, "The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers," *Review of Financial Studies* 14, 313-341.

Getmansky, M., A. Lo, and I. Makarov, 2003, "An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns," *Journal of Financial Economics*, forthcoming.

Grunbichler, A., and F. Longstaff, 1996, "Valuing Futures and Options on Volatility," *Journal of Banking and Finance* 20, 985-1001.

Guo, D.,1998, "The Risk Premium of Volatility Implicit in Currency Options," *Journal of Business and Economic Statistics* 16, 498-507.

Heston, S., 1993, "A Closed-form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," *Review of Financial Studies* 6, 327-343.

Hull, J., and A. White, 1987, "The Pricing of Options on Assets with Stochastic Volatilities," *Journal of Finance* 42,

Jackwerth, J., 1999, "Option Implied Risk-Neutral Distributions and Implied Binomial Trees: A Literature Review," *Journal of Derivatives* 7, 66-82.

Jackwerth, J., 2000, "Recovering Risk Aversion from Option Prices and Realized Returns," *Review of Financial Studies* 13, 433-451.

Jorion, P., 1995, "Predicting Volatility in the Foreign Exchange Market," *Journal of Finance* 50, 507-528.

Jorion, P., 2000, "Risk Management Lessons from Long-Term Capital Management," *European Financial Management* 6(3), 277-300.

Lewis, M., 1999, "How the Eggheads Cracked," *New York Times Magazine*, January 24, 24-77.

Mitchell, M., and T. Pulvino, 2001, "Characteristics of Risk in Risk Arbitrage," *Journal of Finance* 56, 2135-2175.

Modigliani, F., and L. Modigliani, 1997, "Risk-Adjusted Performance," *Journal of Portfolio Management* 23 (2), 45-54.

Neuberger, A., 1994, "The Log Contract: A New Instrument to Hedge Volatility," *Journal of Portfolio Management*, Winter, 74-80.

Pan, J., 2002, "The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study," *Journal of Financial Economics* 63, 3-50.

Peskin, M., M. Urias, S. Anjilvel, and B. Boudreau, 2000, "Why Hedge Funds Make Sense," *Quantitative Strategies*, Global Equity and Derivative Markets, Morgan Stanley Dean Witter.

Ross, S., 1976, "Options and Efficiency," *Quarterly Journal of Economics* 90, 75-89.

Table 1: **Monthly Returns for Variance Contract, S&P 500, Market-Neutral Portfolio, and Mean-Variance Portfolio**

Panel A: Return Distribution

	Min.	1%	5%	10%	Med.	90%	95%	99%	Max.
VC	-86.17	-85.14	-73.96	-70.95	-39.48	25.48	50.22	219.53	354.80
S&P 500	-9.91	-8.84	-5.18	-3.11	1.06	5.23	7.22	10.58	13.54
MNP	-355.87	-227.54	-53.81	-22.37	34.34	64.70	74.62	83.61	85.37
MVP	-88.54	-53.60	-13.99	-5.03	10.87	19.94	21.81	24.05	27.75

Panel B: Risk Characteristics

	Mean	SD	Skew.	Kurt.	α	β	SR	TM	M^2
VC	-26.66	54.05	3.39	20.70	-22.52	-4.68	-0.49	4.81	-1.81
S&P 500	0.89	3.67	0.09	3.90	0	1.00	0.24	0	0.89
MNP	22.52	51.25	-3.67	24.37	22.52	0	0.44	n/a	1.61
MVP	7.33	14.62	-2.89	16.14	5.63	1.92	0.50	2.93	1.84

Notes: The sample period is 01/88–12/00. Statistics are reported for monthly excess returns of the variance contract (VC), the S&P 500 futures, the market-neutral portfolio (MNP), and the mean-variance portfolio (MVP). Returns are expressed in a percentage form. MNP is short the variance contract and hedged with the S&P 500 futures so that the sample correlation with the market return r_m is zero; its return $r_{mn} = -r_v - 4.68r_m$. MVP is the mean-variance optimal portfolio constructed using the variance contract and the S&P 500 futures; its return $r_{mv} = 0.75r_m - 0.25r_v$. Panel A shows minimum, median, maximum, and 1%, 5%, 10%, 90%, 95%, and 99% percentiles. Panel B shows mean, standard deviation (SD), skewness, kurtosis, α and β coefficients (with respect to S&P 500), Sharpe ratio (SR), Treynor's measure (TM), M-squared measure (M^2).

Table 2: **Risk Characteristics for Monthly Returns of S&P 500 Options**

Panel A: Put Returns

k	Mean	SD	Skew.	Kurt.	α	β	SR	TM	M^2	ρ
0.94	-83.30	128.28	9.70	102.63	-72.28	-12.43	-0.65	5.81	-2.38	0.26
0.96	-73.54	129.27	6.94	58.28	-58.38	-17.10	-0.57	3.41	-2.09	0.31
0.98	-63.25	119.51	4.63	27.96	-45.70	-19.81	-0.53	2.31	-1.94	0.35
1.00	-45.10	103.40	2.77	12.33	-25.94	-21.62	-0.44	1.20	-1.60	0.36
1.02	-30.33	85.51	1.53	5.74	-12.21	-20.44	-0.35	0.60	-1.30	0.35
1.04	-20.29	67.62	0.88	3.77	-4.89	-17.38	-0.30	0.28	-1.10	0.34
1.06	-14.34	53.47	0.49	3.27	-1.73	-14.23	-0.27	0.12	-0.98	0.32

Panel B: Call Returns

k	Mean	SD	Skew.	Kurt.	α	β	SR	TM	M^2	ρ
0.94	6.82	52.47	0.10	3.01	-5.68	14.10	0.13	-0.40	0.48	-0.30
0.96	6.93	66.74	0.32	2.69	-8.66	17.59	0.10	-0.49	0.38	-0.28
0.98	5.63	89.78	0.61	2.63	-14.45	22.66	0.06	-0.64	0.23	-0.25
1.00	4.27	125.73	1.11	3.49	-21.50	29.07	0.03	-0.74	0.12	-0.21
1.02	-2.26	184.78	2.34	8.56	-33.87	35.66	-0.01	-0.95	-0.04	-0.13
1.04	-17.89	274.90	4.63	27.48	-53.34	39.98	-0.07	-1.33	-0.24	-0.04
1.06	-31.31	359.28	7.28	63.14	-67.95	41.34	-0.09	-1.64	-0.32	0.00

Notes: The sample period is 01/88–12/00. Statistics are reported for monthly excess returns of puts and calls on the S&P 500 futures, for different moneyness k. Statistics include mean, standard deviation (SD), skewness, kurtosis, α and β coefficients (with respect to S&P 500), Sharpe ratio (SR), Treynor's measure (TM), M-squared measure (M^2), and correlation coefficient with the variance contract ρ. Returns are expressed in a percentage form.

Table 3: **Portfolio Weights and Certainty Equivalent Rate for CRRA Preferences**

Panel A: Market Portfolio and Variance Contract

	γ=1	γ=2	γ=3	γ=5	γ=10	γ=20	γ=50
w_m	6.143	3.263	2.202	1.331	0.669	0.336	0.136
w_m	4.350	2.146	1.391	0.808	0.391	0.191	0.075
w_v	-0.248	-0.174	-0.129	-0.085	-0.045	-0.024	-0.010
CER	3.632	2.388	1.742	1.123	0.596	0.312	0.134

Panel B: Market Portfolio, ATM Put, and Variance Contract

	γ=1	γ=2	γ=3	γ=5	γ=10	γ=20	γ=50
w_m	-1.866	-1.473	-1.110	-0.727	-0.386	-0.200	-0.082
w_p	-0.196	-0.139	-0.103	-0.067	-0.035	-0.018	-0.007
w_m	-2.997	-2.049	-1.495	-0.957	-0.500	-0.256	-0.104
w_p	-0.166	-0.117	-0.086	-0.056	-0.029	-0.015	-0.006
w_v	-0.278	-0.185	-0.135	-0.087	-0.046	-0.024	-0.010
CER	3.259	2.057	1.473	0.934	0.488	0.253	0.108

Notes: The sample period is 01/88–12/00. This table reports optimal portfolio weights for the investor with one month horizon and CRRA preferences (γ is the risk aversion coefficient). The investor can trade the market portfolio and the variance contract (Panel A), or the market portfolio, the ATM put, and the variance contract (Panel B). The top portions of both panels show portfolio weights when trading in the variance contract is not allowed. The certainty equivalent rate (CER) is the percent of wealth that investor is willing to pay each month to be able to trade the variance contract.

Table 4: **Summary Statistics for HFR Indexes**

Category	Mean	SD	Skew.	Kurt.	ρ_1	ρ_2	ρ_3	Qstat	p-val
Convertible Arbitrage	0.53	1.02	-1.40	5.89	0.57	0.28	0.00	56.81	0.00
Distressed Securities	0.80	1.92	-0.74	8.24	0.52	0.17	0.03	41.14	0.00
Emerging Markets	0.85	4.75	-0.78	6.24	0.33	0.11	0.04	16.63	0.00
Equity Hedge	1.32	2.72	0.04	4.25	0.09	0.05	-0.05	1.93	0.59
Equity Market Neutral	0.51	0.92	0.00	3.29	0.00	0.03	0.03	0.22	0.97
Equity Non-Hedge	1.14	4.20	-0.58	3.84	0.18	0.01	-0.15	6.42	0.09
Event-Driven	0.88	1.96	-1.57	8.85	0.29	0.06	0.02	11.28	0.01
Market Timing	0.83	1.99	0.10	2.46	-0.08	0.13	-0.05	2.86	0.41
Merger Arbitrage	0.63	1.30	-3.37	18.33	0.17	-0.03	0.02	3.08	0.38
Relative Value Arbitrage	0.71	1.17	-1.08	12.38	0.24	0.19	0.02	12.66	0.01
Short Selling	-0.09	6.66	0.13	4.24	0.07	-0.03	-0.02	0.57	0.90
Fixed Income (Total)	0.50	1.09	-0.49	6.95	0.46	0.29	0.03	40.04	0.00
Fixed Income: Arbitrage	0.32	1.43	-1.59	10.50	0.44	0.15	0.13	29.91	0.00
Fixed Income: High Yield	0.38	2.07	-0.79	8.25	0.45	0.24	0.00	36.00	0.00
Macro	1.07	2.69	0.18	2.99	0.18	0.02	-0.02	3.77	0.29
Composite	0.93	2.13	-0.80	6.07	0.24	0.09	-0.08	9.56	0.02
Fund of Funds Composite	0.54	1.81	-0.41	6.38	0.31	0.11	-0.02	14.40	0.00
S&P 500	0.82	3.95	-0.40	3.62	-0.12	0.04	-0.04	2.63	0.45
Variance Contract	-20.49	39.67	2.32	10.18	0.03	0.01	-0.10	1.13	0.77

Notes: The sample period is 01/90–12/00. Summary statistics are reported for monthly excess returns of HFR indexes, S&P 500 futures, and variance contract. Statistics include mean, standard deviation (SD), skewness, kurtosis, serial autocorrelation coefficients, Ljung-Box Q-statistic with first 3 lags, and its p-value.

Table 5: **Least Squares Regressions for HFR Indexes**

Category	$\hat{\alpha}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}_0 \times 10^2$	$\hat{\gamma}_1 \times 10^2$	$\hat{\gamma}_2 \times 10^2$	$\hat{\beta}$	$\hat{\gamma} \times 10^2$	$\hat{\beta}E[r_m]$	$\hat{\gamma}E[r_v]$	$\bar{R}^2$
Convertible Arbitrage	0.28	0.11	0.12	0.06				0.29		0.24		0.36
	(2.37)	(4.64)	(6.17)	(2.92)								
	0.16	0.10	0.11	0.04	-0.22	-0.14	-0.40	0.25	-0.76	0.20	0.16	0.37
	(1.02)	(5.15)	(6.14)	(2.24)	(-0.97)	(-0.72)	(-1.77)					
Distressed Securities	0.38	0.20	0.18	0.12				0.49		0.41		0.29
	(1.80)	(3.67)	(3.15)	(2.75)								
	0.00	0.11	0.14	0.10	-1.79	-0.59	-0.09	0.35	-2.46	0.28	0.50	0.39
	(0.00)	(2.61)	(2.63)	(2.14)	(-3.29)	(-1.61)	(-0.21)					
Emerging Markets	0.09	0.69	0.18	0.06				0.92		0.76		0.31
	(0.17)	(6.68)	(1.83)	(0.66)								
	-0.62	0.52	0.10	0.03	-3.06	-1.46	0.00	0.65	-4.51	0.54	0.92	0.36
	(-1.13)	(6.10)	(1.01)	(0.33)	(-2.98)	(-1.45)	(0.00)					
Equity Hedge	0.83	0.42	0.07	0.09				0.59		0.48		0.38
	(4.03)	(7.99)	(1.31)	(1.76)								
	0.72	0.35	0.08	0.11	-1.34	0.09	0.47	0.53	-0.77	0.44	0.16	0.40
	(2.54)	(6.61)	(1.22)	(1.74)	(-2.32)	(0.15)	(1.05)					
Equity Market Neutral	0.44	0.05	0.00	0.03				0.09		0.07		0.05
	(4.61)	(2.19)	(0.02)	(1.68)								
	0.44	0.06	0.01	0.03	0.07	0.13	-0.16	0.09	0.04	0.07	-0.01	0.03
	(3.50)	(2.74)	(0.28)	(1.32)	(0.28)	(0.67)	(-0.89)					
Equity Non-Hedge	0.26	0.81	0.18	0.06				1.05		0.86		0.57
	(1.00)	(13.33)	(2.55)	(1.00)								
	-0.05	0.69	0.18	0.05	-2.40	0.14	0.20	0.93	-2.06	0.76	0.42	0.60
	(-0.14)	(10.86)	(1.99)	(0.81)	(-3.44)	(0.16)	(0.29)					
Event-Driven	0.47	0.31	0.14	0.04				0.49		0.40		0.41
	(2.56)	(5.81)	(3.60)	(1.03)								
	0.18	0.22	0.13	0.03	-1.81	-0.17	0.12	0.37	-1.86	0.31	0.38	0.50
	(0.78)	(5.46)	(3.37)	(0.95)	(-3.45)	(-0.61)	(0.32)					
Market Timing	0.50	0.32	0.04	0.04				0.40		0.33		0.39
	(3.33)	(7.19)	(0.95)	(1.14)								
	0.58	0.33	0.03	0.07	0.30	-0.36	0.56	0.42	0.50	0.35	-0.10	0.39
	(3.26)	(7.13)	(0.58)	(1.84)	(0.66)	(-1.04)	(1.73)					
Merger Arbitrage	0.44	0.15	0.08	0.00				0.23		0.19		0.22
	(3.14)	(2.87)	(2.94)	(0.16)								
	0.40	0.10	0.10	0.01	-0.81	0.19	0.34	0.21	-0.27	0.18	0.06	0.26
	(2.34)	(2.78)	(4.17)	(0.68)	(-2.03)	(0.47)	(1.62)					
Relative Value Arbitrage	0.52	0.11	0.08	0.03				0.22		0.18		0.16
	(3.90)	(2.46)	(2.60)	(1.81)								
	0.39	0.06	0.09	0.02	-0.94	0.22	-0.12	0.17	-0.84	0.14	0.17	0.23
	(2.09)	(1.74)	(2.68)	(1.00)	(-2.21)	(1.25)	(-0.41)					
Short Selling	0.98	-1.13	-0.12	-0.05				-1.29		-1.06		0.43
	(2.27)	(-9.38)	(-0.85)	(-0.48)								
	1.14	-1.02	-0.10	-0.10	1.86	0.37	-1.12	-1.22	1.11	-1.00	-0.23	0.43
	(1.74)	(-6.77)	(-0.56)	(-0.81)	(1.07)	(0.25)	(-1.12)					
Fixed Income (Total)	0.28	0.12	0.08	0.06				0.26		0.22		0.26
	(2.36)	(3.98)	(3.55)	(1.68)								
	0.01	0.08	0.06	0.03	-0.81	-0.46	-0.42	0.17	-1.69	0.14	0.35	0.35
	(0.05)	(2.74)	(1.99)	(1.47)	(-3.40)	(-1.83)	(-1.32)					
Fixed Income: Arbitrage	0.23	-0.03	0.05	0.08				0.10		0.08		0.05
	(1.23)	(-0.76)	(1.10)	(1.66)								
	-0.05	-0.07	0.02	0.05	-0.72	-0.59	-0.43	0.00	-1.73	0.00	0.36	0.09
	(-0.18)	(-1.50)	(0.38)	(1.45)	(-2.48)	(-1.07)	(-0.92)					
Fixed Income: High Yield	-0.13	0.25	0.23	0.12				0.60		0.49		0.38
	(-0.60)	(4.37)	(4.39)	(2.46)								
	-0.56	0.18	0.19	0.08	-1.40	-0.68	-0.60	0.45	-2.68	0.37	0.55	0.45
	(-2.08)	(3.36)	(3.66)	(2.20)	(-3.06)	(-2.02)	(-1.27)					
Macro	0.72	0.31	0.07	0.03				0.41		0.34		0.19
	(2.81)	(5.83)	(1.54)	(0.58)								
	0.30	0.21	0.04	0.01	-1.97	-0.49	-0.24	0.25	-2.70	0.21	0.55	0.24
	(1.00)	(3.07)	(0.68)	(0.12)	(-3.06)	(-0.98)	(-0.40)					
Composite	0.48	0.37	0.11	0.06				0.54		0.45		0.47
	(2.71)	(8.62)	(2.96)	(1.62)								
	0.20	0.29	0.10	0.05	-1.59	-0.25	0.03	0.43	-1.81	0.36	0.37	0.53
	(0.92)	(8.23)	(2.16)	(1.31)	(-3.85)	(-0.59)	(0.09)					
Fund of Funds Composite	0.28	0.18	0.05	0.07				0.30		0.25		0.16
	(1.23)	(3.26)	(0.93)	(1.69)								
	0.03	0.12	0.03	0.05	-1.32	-0.29	-0.05	0.20	-1.66	0.17	0.34	0.21
	(0.07)	(2.34)	(0.62)	(1.45)	(-2.43)	(-0.60)	(-0.14)					

Notes: The sample period is 01/90–12/00. For each fund category, least squares estimates are reported for two regressions:

$$
\begin{aligned}
(1) \quad r_t^o &= \alpha + \beta_0 r_{m,t} + \beta_1 r_{m,t-1} + \beta_2 r_{m,t-2} + u_t, \\
(2) \quad r_t^o &= \alpha + \beta_0 r_{m,t} + \beta_1 r_{m,t-1} + \beta_2 r_{m,t-2} + \gamma_0 r_{v,t} + \gamma_1 r_{v,t-1} + \gamma_2 r_{v,t-2} + u_t,
\end{aligned}
$$

where r_m is the market return, and r_v is the variance return. The number of observations is 132. To correct for serial autocorrelation of disturbances, t-statistics (shown in parentheses) are computed using the Newey-West estimator with 2 lags. Also reported are $\hat{\beta} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2$, $\hat{\gamma} = \hat{\gamma}_0 + \hat{\gamma}_1 + \hat{\gamma}_2$, $\hat{\beta}E[r_m]$, $\hat{\gamma}E[r_v]$, and adjusted R^2.

Table 6: **Maximum Likelihood Estimation for HFR Indexes**

Category	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}\times 10^2$	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\beta}E[r_m]$	$\hat{\gamma}E[r_v]$	$\hat{\xi}$	$\bar{R}^2$
Convertible Arbitrage	0.36	0.19		0.51	0.31	0.19	0.15		0.39	0.18
	(2.78)	(5.72)		(-13.94)	(12.05)	(6.03)				
	0.26	0.16	-0.60	0.51	0.31	0.19	0.13	0.12	0.39	0.19
	(1.82)	(4.24)	(-1.64)	(-13.88)	(11.91)	(6.05)				
Distressed Securities	0.47	0.39		0.52	0.34	0.14	0.32		0.41	0.22
	(2.00)	(6.48)		(-11.85)	(13.42)	(3.92)				
	-0.08	0.21	-3.41	0.54	0.33	0.13	0.17	0.70	0.42	0.38
	(-0.37)	(3.54)	(-5.70)	(-11.00)	(12.44)	(3.63)				
Emerging Markets	-0.04	1.10		0.65	0.25	0.10	0.91		0.50	0.38
	(-0.09)	(9.29)		(-5.82)	(6.15)	(2.07)				
	-0.89	0.86	-5.10	0.65	0.26	0.09	0.70	1.05	0.50	0.45
	(-1.77)	(6.61)	(-3.93)	(-5.82)	(6.66)	(1.82)				
Equity Hedge	0.90	0.51		0.83	0.10	0.07	0.42		0.71	0.37
	(3.95)	(9.06)		(-1.89)	(1.54)	(1.04)				
	0.61	0.43	-1.74	0.83	0.10	0.07	0.35	0.36	0.71	0.40
	(2.49)	(6.72)	(-2.75)	(-1.89)	(1.61)	(0.98)				
Equity Market Neutral	0.46	0.06		0.92	0.00	0.08	0.05		0.85	0.04
	(5.30)	(2.83)		(-0.77)	(0.00)	(1.09)				
	0.47	0.06	0.03	0.92	0.00	0.08	0.05	-0.01	0.85	0.03
	(4.83)	(2.51)	(0.13)	(-0.77)	(0.00)	(1.09)				
Equity Non-Hedge	0.32	0.99		0.81	0.17	0.03	0.81		0.68	0.57
	(1.10)	(13.43)		(-2.20)	(2.90)	(0.40)				
	-0.15	0.83	-2.92	0.82	0.16	0.02	0.68	0.60	0.69	0.61
	(-0.47)	(10.41)	(-3.66)	(-2.05)	(2.78)	(0.32)				
Event-Driven	0.54	0.40		0.73	0.22	0.04	0.33		0.59	0.36
	(3.01)	(8.94)		(-3.55)	(4.74)	(0.70)				
	0.11	0.27	-2.59	0.73	0.23	0.04	0.22	0.53	0.59	0.48
	(0.64)	(5.88)	(-5.62)	(-3.51)	(4.88)	(0.59)				
Market Timing	0.53	0.37		0.86	0.02	0.12	0.30		0.76	0.38
	(3.35)	(9.29)		(-1.52)	(0.30)	(1.77)				
	0.57	0.38	0.23	0.86	0.02	0.11	0.31	-0.05	0.76	0.38
	(3.25)	(8.31)	(0.51)	(-1.49)	(0.30)	(1.73)				
Merger Arbitrage	0.50	0.15		0.87	0.13	0.00	0.12		0.77	0.14
	(4.13)	(5.02)		(-1.34)	(2.06)	(0.00)				
	0.35	0.11	-0.90	0.88	0.12	0.00	0.09	0.18	0.79	0.18
	(2.75)	(3.15)	(-2.71)	(-1.18)	(1.80)	(0.00)				
Relative Value Arbitrage	0.59	0.15		0.71	0.15	0.14	0.12		0.54	0.11
	(4.43)	(4.45)		(-4.49)	(3.07)	(2.74)				
	0.39	0.08	-1.20	0.73	0.14	0.13	0.07	0.25	0.57	0.18
	(2.88)	(2.35)	(-3.38)	(-3.89)	(2.63)	(2.44)				
Short Selling	0.90	-1.21		0.93	0.07	0.00	-0.99		0.86	0.43
	(1.88)	(-10.10)		(-0.68)	(1.01)	(0.00)				
	1.23	-1.10	2.01	0.93	0.07	0.00	-0.90	-0.41	0.87	0.43
	(2.34)	(-8.08)	(1.48)	(-0.65)	(0.96)	(0.00)				
Fixed Income (Total)	0.35	0.19		0.59	0.23	0.18	0.15		0.43	0.19
	(2.71)	(5.87)		(-8.96)	(6.57)	(4.71)				
	0.11	0.12	-1.45	0.59	0.25	0.16	0.10	0.30	0.43	0.28
	(0.81)	(3.40)	(-4.18)	(-8.71)	(7.12)	(4.15)				
Fixed Income: Arbitrage	0.35	-0.04		0.66	0.31	0.03	-0.03		0.54	-0.02
	(2.02)	(-0.91)		(-5.20)	(8.75)	(0.50)				
	0.17	-0.09	-1.08	0.66	0.31	0.03	-0.08	0.22	0.53	0.01
	(0.91)	(-1.89)	(-2.22)	(-5.19)	(8.52)	(0.56)				
Fixed Income: High Yield	0.00	0.43		0.54	0.28	0.17	0.35		0.41	0.23
	(0.01)	(6.59)		(-11.22)	(9.62)	(4.98)				
	-0.48	0.28	-2.93	0.55	0.28	0.17	0.23	0.60	0.41	0.32
	(-1.81)	(4.05)	(-4.28)	(-10.64)	(9.22)	(4.69)				
Macro	0.74	0.40		0.79	0.18	0.02	0.33		0.66	0.21
	(2.83)	(6.13)		(-2.44)	(3.34)	(0.37)				
	0.36	0.27	-2.36	0.81	0.18	0.01	0.22	0.48	0.70	0.26
	(1.34)	(3.87)	(-3.35)	(-2.06)	(3.14)	(0.10)				
Composite	0.51	0.51		0.72	0.19	0.09	0.42		0.56	0.48
	(2.78)	(11.27)		(-4.06)	(3.90)	(1.69)				
	0.13	0.40	-2.30	0.72	0.19	0.09	0.33	0.47	0.56	0.56
	(0.70)	(8.25)	(-4.78)	(-3.96)	(3.95)	(1.56)				
Fund of Funds Composite	0.29	0.30		0.67	0.22	0.10	0.25		0.51	0.20
	(1.46)	(6.01)		(-5.23)	(5.15)	(2.07)				
	-0.06	0.20	-2.13	0.67	0.23	0.10	0.16	0.44	0.51	0.28
	(-0.27)	(3.62)	(-3.85)	(-5.25)	(5.36)	(1.97)				

Notes: The sample period is 01/90–12/00. This table reports the maximum likelihood estimates of model $r_t^o = \theta_0 r_t + \theta_1 r_{t-1} + \theta_2 r_{t-2}$, subject to normalization $\theta_0 + \theta_1 + \theta_2 = 1$ and $\theta_j \in [0,1]$ $(j = 0,1,2)$, when the true return r_t satisfies either (1) $r_t = \alpha + \beta r_{m,t} + \epsilon_t$, or (2) $r_t = \alpha + \beta r_{m,t} + \gamma r_{v,t} + \epsilon_t$.
Shown in parentheses are t-statistics, computed from asymptotic standard errors with respect to the null hypothesis that the coefficient is 0, except for $\hat{\theta}_0$, for which the null hypothesis $\theta_0 = 1$ is used instead. Also reported are $\hat{\beta}E[r_m]$, $\hat{\gamma}E[r_v]$, the smoothing index $\hat{\xi} = \hat{\theta}_0^2 + \hat{\theta}_1^2 + \hat{\theta}_2^2$, and adjusted R^2.

Table 7: **Summary Statistics for CSFB/Tremont Indexes**

Category	Mean	SD	Skew.	Kurt.	ρ_1	ρ_2	ρ_3	Qstat	p-val
Aggregate Index	0.63	2.86	-0.05	3.50	0.10	0.03	-0.03	0.86	0.83
Convertible Arbitrage	0.44	1.45	-1.64	6.99	0.61	0.49	0.15	54.60	0.00
Dedicated Short Bias	-0.34	5.51	0.98	5.16	0.07	-0.04	-0.07	0.88	0.83
Emerging Markets	0.13	5.93	-0.45	5.19	0.30	0.02	-0.01	7.65	0.05
Equity Market Neutral	0.53	0.98	-0.01	2.74	0.29	0.16	0.05	9.27	0.03
Event Driven	0.55	1.91	-3.57	24.75	0.34	0.14	0.01	10.53	0.01
Fixed Income Arbitrage	0.13	1.25	-3.22	17.73	0.41	0.13	0.04	15.66	0.00
Global Macro	0.73	4.12	0.01	3.56	0.05	0.04	0.08	0.40	0.94
Long/Short	0.90	3.65	-0.01	5.00	0.13	0.03	-0.10	2.27	0.52
Managed Futures	0.08	3.31	0.18	4.15	0.03	-0.12	0.00	1.04	0.79
S&P 500	1.04	4.07	-0.60	3.47	-0.12	0.00	0.00	1.34	0.72
Variance Contract	-17.19	41.60	2.03	8.56	0.03	-0.03	-0.10	0.78	0.85

Notes: The sample period is 01/94–12/00. Summary statistics are reported for monthly excess returns of CSFB/Tremont indexes, S&P 500 futures, and variance contract. Statistics include mean, standard deviation (SD), skewness, kurtosis, serial autocorrelation coefficients, Ljung-Box Q-statistic with first 3 lags, and its p-value.

Table 8: **Maximum Likelihood Estimation for CSFB/Tremont Indexes**

Category	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}\times 10^2$	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\beta}E[r_m]$	$\hat{\gamma}E[r_v]$	$\hat{\xi}$	$\bar{R}^2$
Aggregate Index	0.10	0.52		0.77	0.12	0.12	0.54		0.62	0.29
	(0.28)	(6.17)		(-2.43)	(1.60)	(1.60)				
	-0.27	0.35	-3.15	0.77	0.13	0.10	0.36	0.54	0.62	0.37
	(-0.80)	(3.75)	(-3.48)	(-2.42)	(1.87)	(1.37)				
Convertible Arbitrage	0.31	0.11		0.46	0.26	0.28	0.12		0.36	0.00
	(1.24)	(1.81)		(-17.97)	(9.58)	(10.35)				
	0.19	0.06	-1.01	0.45	0.26	0.28	0.06	0.17	0.36	0.01
	(0.69)	(0.81)	(-1.44)	(-18.94)	(9.89)	(10.96)				
Dedicated Short Bias	0.87	-1.16		0.90	0.10	0.01	-1.20		0.82	0.58
	(1.97)	(-10.95)		(-0.78)	(1.10)	(0.06)				
	1.10	-1.05	1.98	0.90	0.10	0.00	-1.09	-0.34	0.82	0.59
	(2.39)	(-8.54)	(1.65)	(-0.77)	(1.13)	(0.02)				
Emerging Markets	-1.08	1.23		0.66	0.28	0.07	1.28		0.51	0.31
	(-1.37)	(6.49)		(-4.38)	(5.74)	(1.03)				
	-1.95	0.92	-6.84	0.64	0.30	0.07	0.95	1.18	0.50	0.39
	(-2.39)	(4.19)	(-3.22)	(-4.89)	(6.70)	(1.07)				
Equity Market Neutral	0.36	0.16		0.74	0.20	0.06	0.16		0.60	0.22
	(2.94)	(5.22)		(-2.70)	(3.18)	(0.76)				
	0.44	0.19	0.68	0.72	0.20	0.08	0.20	-0.12	0.57	0.24
	(3.35)	(5.50)	(1.96)	(-3.09)	(3.22)	(1.13)				
Event Driven	0.09	0.45		0.64	0.25	0.11	0.47		0.48	0.41
	(0.39)	(7.86)		(-5.00)	(5.03)	(1.91)				
	-0.21	0.32	-2.52	0.64	0.26	0.10	0.33	0.43	0.48	0.51
	(-0.92)	(5.15)	(-4.17)	(-5.10)	(5.54)	(1.74)				
Fixed Income Arbitrage	0.09	0.05		0.62	0.28	0.10	0.05		0.48	-0.02
	(0.42)	(1.05)		(-5.38)	(6.24)	(1.66)				
	-0.10	-0.03	-1.52	0.61	0.29	0.10	-0.03	0.26	0.46	0.05
	(-0.46)	(-0.49)	(-2.74)	(-5.95)	(7.01)	(1.80)				
Global Macro	0.26	0.46		0.81	0.08	0.11	0.48		0.68	0.10
	(0.49)	(3.57)		(-1.77)	(0.94)	(1.41)				
	-0.09	0.29	-3.00	0.82	0.08	0.10	0.30	0.52	0.69	0.14
	(-0.16)	(1.98)	(-2.09)	(-1.66)	(1.01)	(1.21)				
Long/Short	0.14	0.73		0.79	0.10	0.12	0.76		0.64	0.39
	(0.36)	(7.64)		(-2.15)	(1.26)	(1.58)				
	-0.23	0.56	-3.18	0.78	0.11	0.11	0.58	0.55	0.63	0.45
	(-0.58)	(5.21)	(-3.00)	(-2.21)	(1.52)	(1.42)				
Managed Futures	0.10	-0.02		0.96	0.04	0.00	-0.02		0.93	-0.04
	(0.27)	(-0.25)		(-0.25)	(0.36)	(0.00)				
	0.23	0.04	1.08	0.97	0.03	0.00	0.04	-0.19	0.94	-0.04
	(0.56)	(0.32)	(1.01)	(-0.22)	(0.32)	(0.00)				

Notes: The sample period is 01/94–12/00. This table reports maximum likelihood estimates of model $r_t^o = \theta_0 r_t + \theta_1 r_{t-1} + \theta_2 r_{t-2}$, subject to normalization $\theta_0 + \theta_1 + \theta_2 = 1$ and $\theta_j \in [0,1]$ $(j = 0, 1, 2)$, when the true return r_t satisfies either (1) $r_t = \alpha + \beta r_{m,t} + \epsilon_t$, or (2) $r_t = \alpha + \beta r_{m,t} + \gamma r_{v,t} + \epsilon_t$.

Shown in parentheses are t-statistics, computed from asymptotic standard errors with respect to the null hypothesis that the coefficient is 0, except for $\hat{\theta}_0$, for which the null hypothesis $\theta_0 = 1$ is used instead. Also reported are $\hat{\beta}E[r_m]$, $\hat{\gamma}E[r_v]$, the smoothing index $\hat{\xi} = \hat{\theta}_0^2 + \hat{\theta}_1^2 + \hat{\theta}_2^2$, and adjusted R^2.

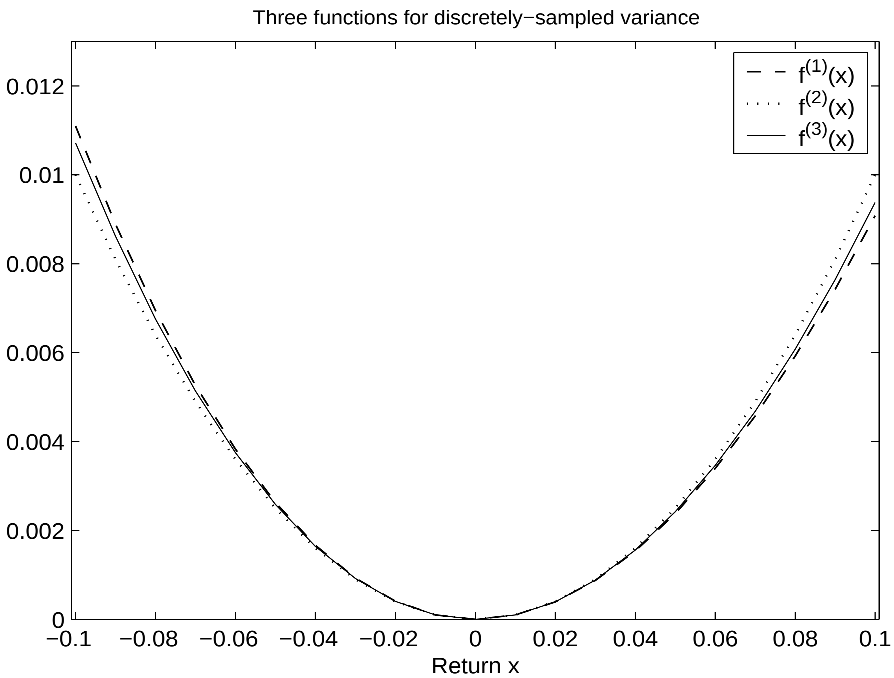

Figure 1: This figure shows behavior of functions $f^{(1)}(x) = [\ln(1+x)]^2$, $f^{(2)}(x) = x^2$, and $f^{(3)}(x) = 2\,(x - \ln(1+x))$ used in the definitions of the discretely-sampled variance $\hat{V}_T^{(1)}$, $\hat{V}_T^{(2)}$, and $\hat{V}_T^{(3)}$.

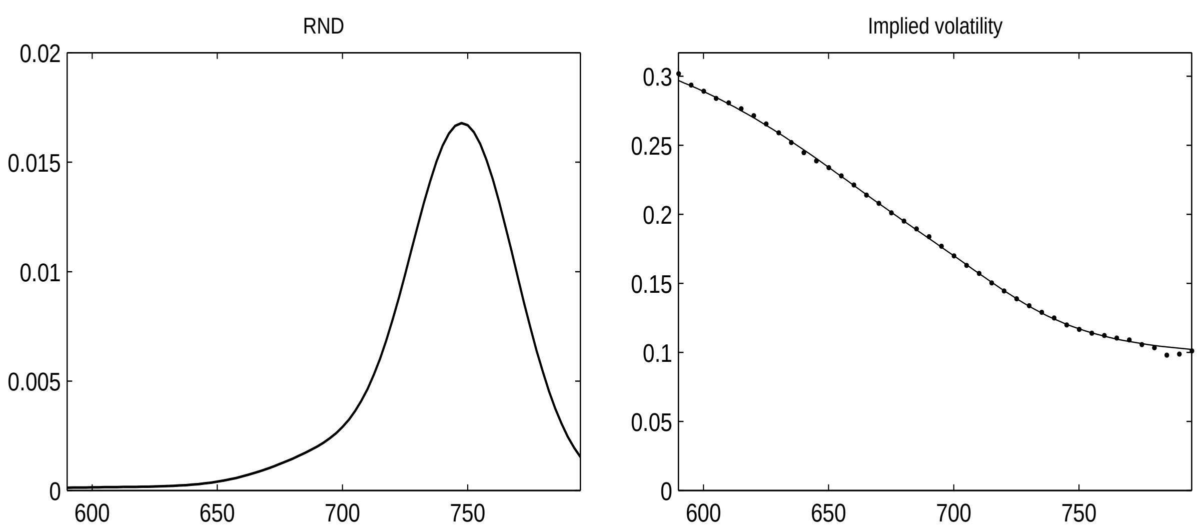

Figure 2: This figure illustrates the Monte-Carlo experiment using market prices on November 15, 1996 of options that mature on December 20, 1996. The left panel shows the fitted RND $h_0(F_T)$, which is approximated by a mixture of three lognormals as in (27). The right panel shows the option implied volatilities computed using market prices (the dots) and fitted prices (the solid line).

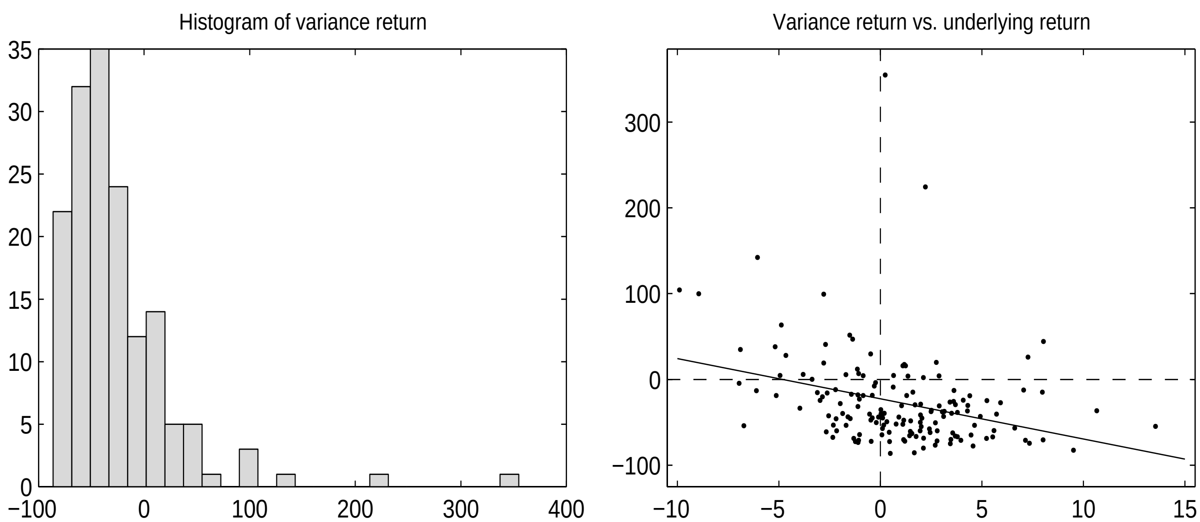

Figure 3: The left panel shows the histogram of the variance return. The right panel is a scatter plot of the variance return versus the return on the underlying S&P 500 futures, as well as the OLS regression line. All returns are monthly. The sample period is 01/88–12/00.

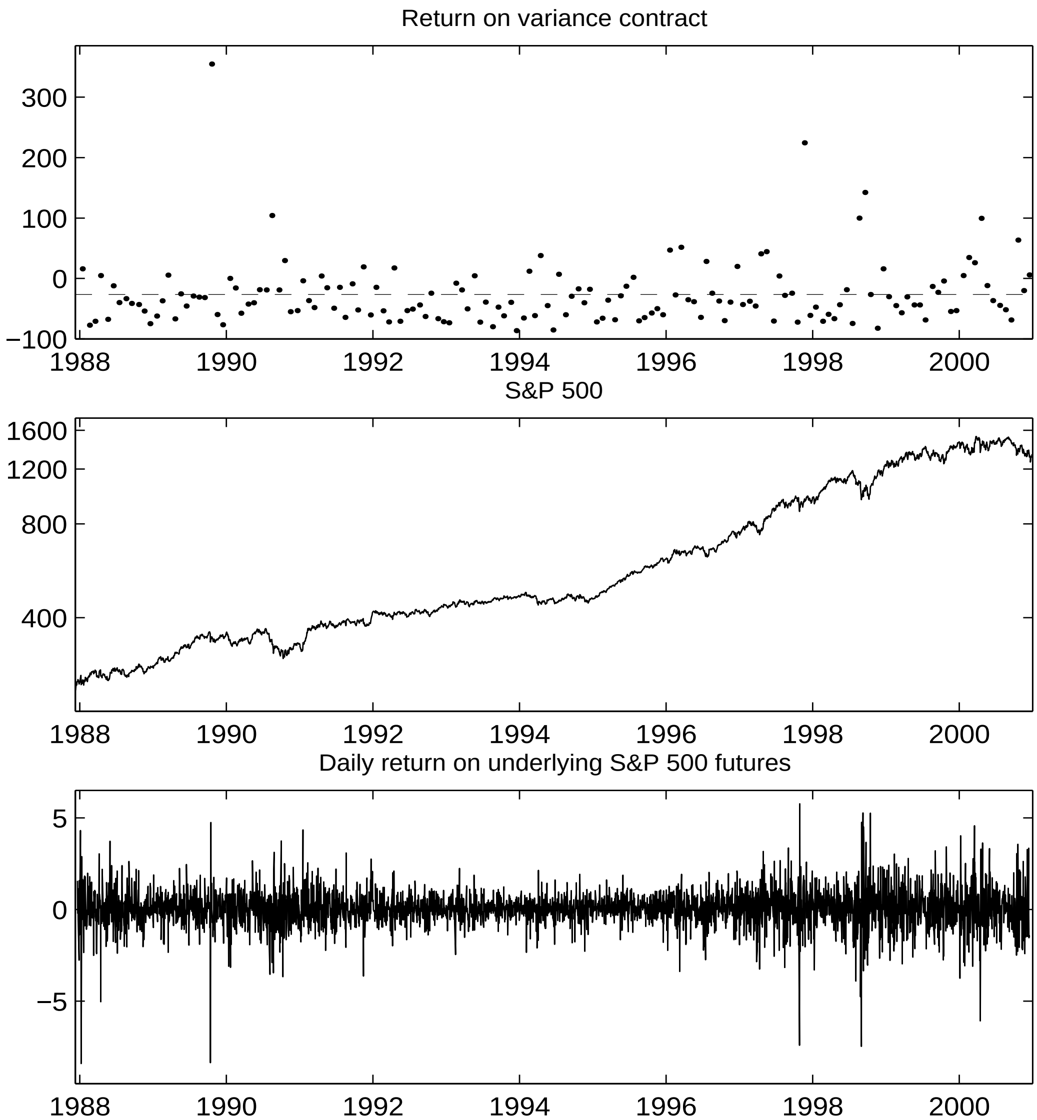

Figure 4: The top panel plots the monthly variance return from 1988 to 2000. Its sample mean is shown with the dashed line. The second and third panels plot the level of the S&P 500 Index and monthly realized variance of the underlying S&P 500 futures.

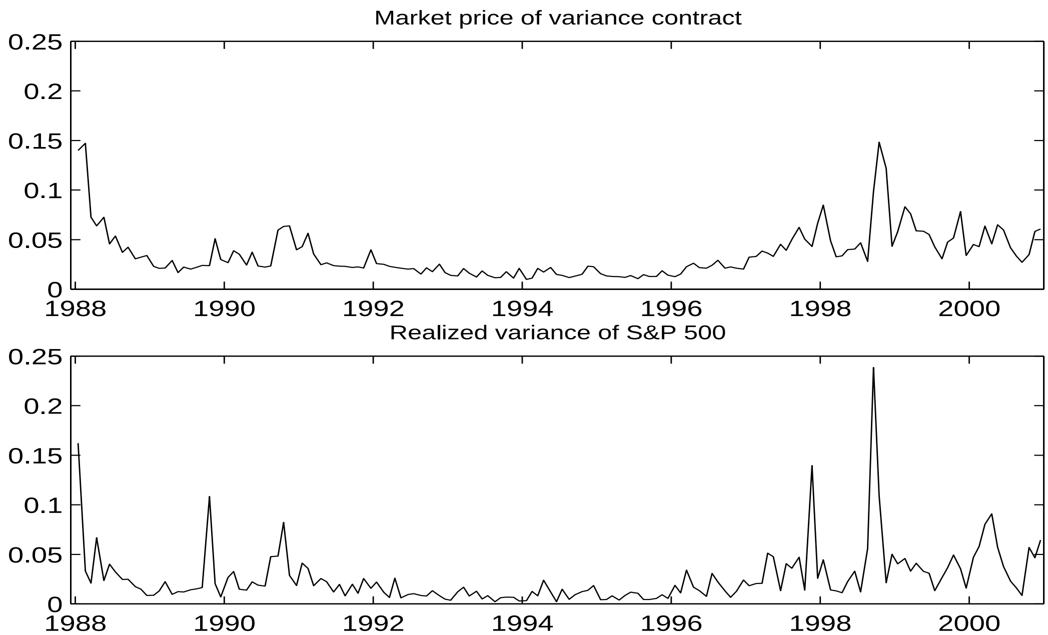

Figure 5: The top panel plots the market price of the variance contract and the bottom panel plots realized variance of S&P 500 futures. The holding period is one month. Both series are annualized.

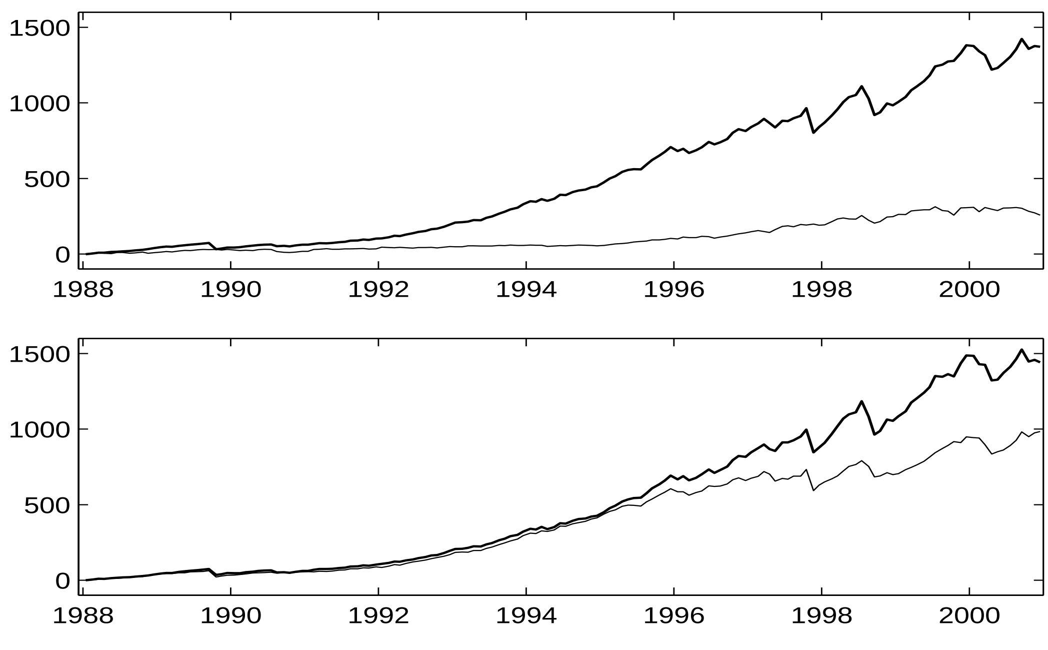

Figure 6: This figure shows the cumulative returns for four strategies that invest in 1) the S&P 500 futures, 2) the short variance contract, 3) the market-neutral portfolio MNP, and 4) the mean-variance portfolio MVP. Strategies 2)-4) are de-leveraged with the risk-free asset so that the standard deviations of their monthly returns over the sample period are the same as that for strategy 1). The top panel shows strategy 1) (the thin line) and strategy 2) (the thick line). The bottom panel shows strategy 3) (the thin line) and strategy 4) (the thick line).

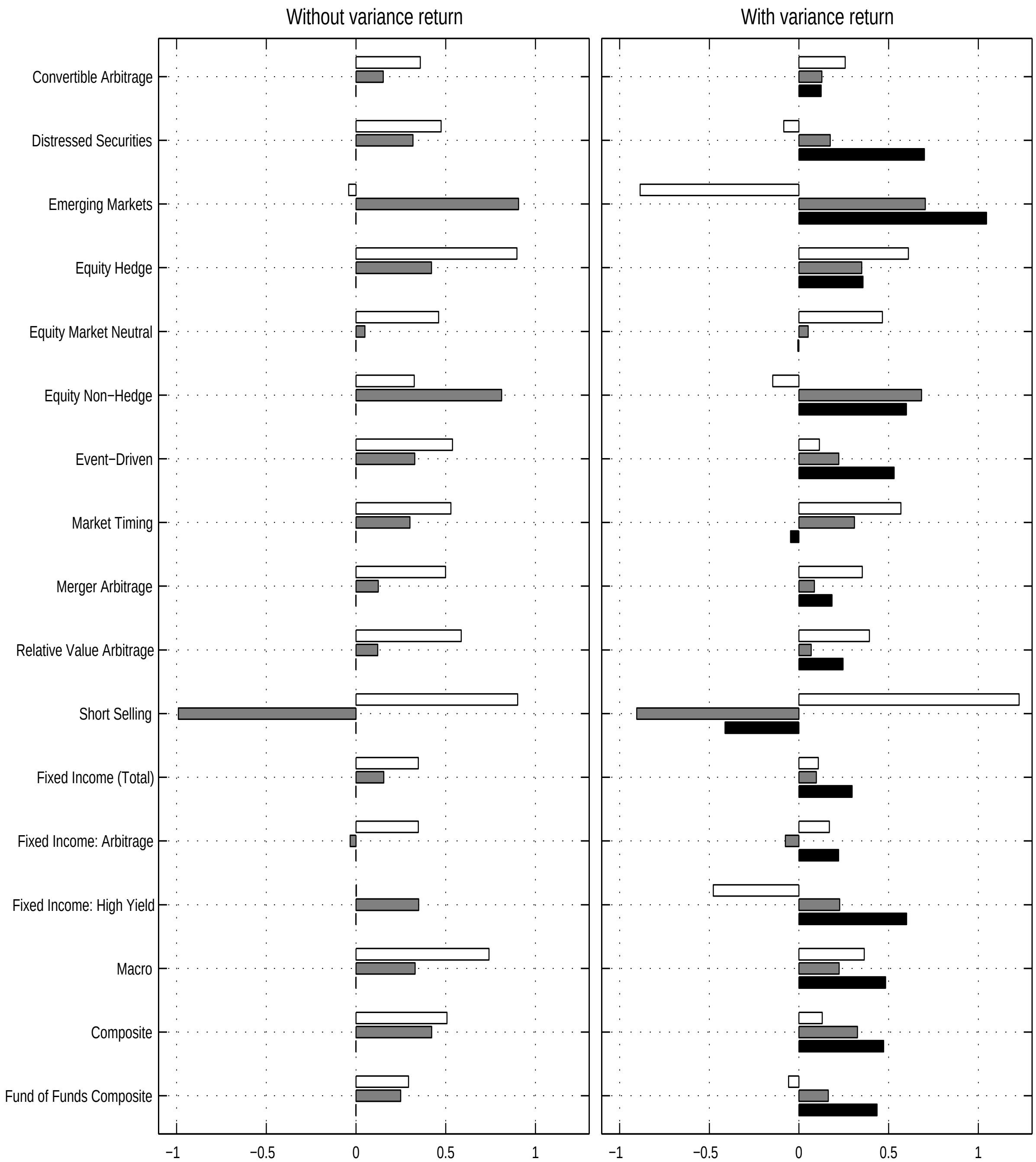

Figure 7: For HFR indexes, this figure plots decomposition of expected excess return $E[r] = \hat{\alpha}^{(1)} + \hat{\beta}^{(1)}E[r_m]$ (the left panel) and $E[r] = \hat{\alpha}^{(2)} + \hat{\beta}^{(2)}E[r_m] + \hat{\gamma}^{(2)}E[r_v]$ (the right panel). White bars show $\hat{\alpha}$, gray bars show $\hat{\beta}E[r_m]$, and black bars show $\hat{\gamma}E[r_v]$. Estimates are obtained via maximum likelihood for model $r_t^o = \theta_0 r_t + \theta_1 r_{t-1} + \theta_2 r_{t-2}$, subject to normalization $\theta_0 + \theta_1 + \theta_2 = 1$ and $\theta_j \in [0, 1]$ $(j = 0, 1, 2)$, when the true return r_t satisfies either (1) $r_t = \alpha + \beta r_{m,t} + \epsilon_t$, or (2) $r_t = \alpha + \beta r_{m,t} + \gamma r_{v,t} + \epsilon_t$. See also Table 6. The sample period is 01/90–12/00.

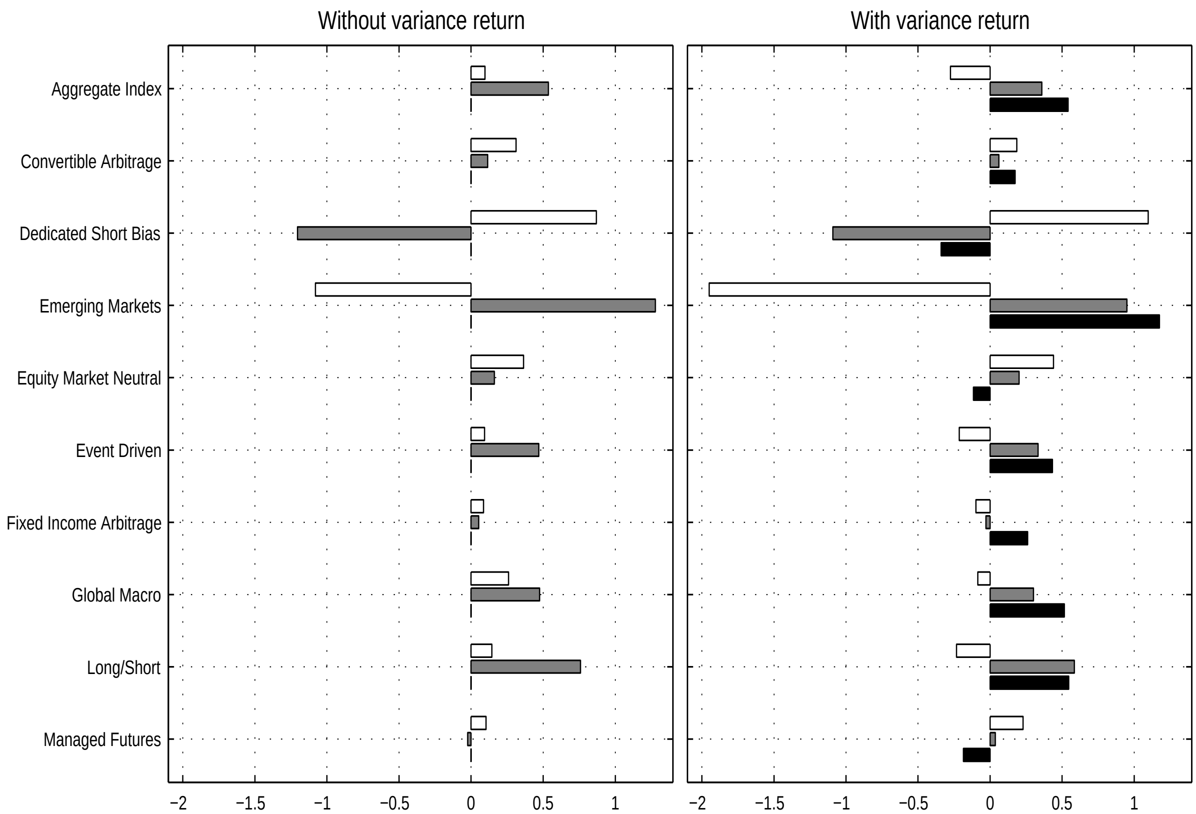

Figure 8: For CSFB indexes, this figure plots decomposition of expected excess return $E[r] = \hat{\alpha}^{(1)} + \hat{\beta}^{(1)}E[r_m]$ (the left panel) and $E[r] = \hat{\alpha}^{(2)} + \hat{\beta}^{(2)}E[r_m] + \hat{\gamma}^{(2)}E[r_v]$ (the right panel). White bars show $\hat{\alpha}$, gray bars show $\hat{\beta}E[r_m]$, and black bars show $\hat{\gamma}E[r_v]$. Estimates are obtained via maximum likelihood for model $r_t^o = \theta_0 r_t + \theta_1 r_{t-1} + \theta_2 r_{t-2}$, subject to normalization $\theta_0 + \theta_1 + \theta_2 = 1$ and $\theta_j \in [0, 1]$ $(j = 0, 1, 2)$, when the true return r_t satisfies either (1) $r_t = \alpha + \beta r_{m,t} + \epsilon_t$, or (2) $r_t = \alpha + \beta r_{m,t} + \gamma r_{v,t} + \epsilon_t$. See also Table 8. The sample period is 01/94–12/00.