Mutual fund performance and seemingly unrelated assets[*]

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Abstract

Estimates of standard performance measures can be improved by using returns on assets not used to define those measures. Alpha, the intercept in a regression of a fund's return on passive benchmark returns, can be estimated more precisely by using information in returns on non-benchmark passive assets, whether or not one believes those assets are priced by the benchmarks. A fund's Sharpe ratio can be estimated more precisely by using returns on other assets as well as the fund. New estimates of these performance measures for a large universe of equity mutual funds exhibit substantial differences from the usual estimates.

1. Introduction

A mutual fund's historical performance is often summarized by an estimate of its alpha or its Sharpe ratio. Alpha is defined as the intercept in a regression of the fund's excess return on the excess return of one or more passive benchmarks, while the Sharpe ratio is the fund's expected excess return divided by the standard deviation of the fund's return. These measures are usually estimated with historical returns on the assets that define them. That is, alpha is estimated using excess returns on the fund and the benchmarks, and the Sharpe ratio is estimated using the excess returns on the fund. This study demonstrates that an *estimate* of either performance measure can typically be made more precise by using historical returns on "seemingly unrelated" assets not used in the *definition* of that measure.

Alpha, for example, is usually estimated by applying ordinary-least-squares (OLS) to the regression

$$r_{A,t} = \alpha_A + \beta_A' r_{B,t} + \epsilon_{A,t}, \tag{1}$$

where $r_{A,t}$ is the fund's return in month t, $r_{B,t}$ is a $k \times 1$ vector containing the benchmark returns, and α_A denotes the fund's alpha. (Henceforth we use "returns" to denote rates of return in excess of a riskless interest rate or payoffs on zero-investment spread positions.) The choice of benchmarks is often guided by a pricing model, as in Jensen's (1969) pioneering use of the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) to investigate mutual fund alphas relative to a single market-index benchmark. Other studies, beginning with Lehmann and Modest (1987), examine fund alphas with respect to a set of multiple benchmarks viewed as the relevant factors for pricing in a multifactor model, such as the Arbitrage Pricing Theory of Ross (1976).

As one special case, assume that the benchmark assets used to define alpha do indeed exactly price other passive assets. Consider the regression of a non-benchmark passive return $r_{n,t}$ on the benchmark returns,

$$r_{n,t} = \alpha_n + \beta_n' r_{B,t} + \epsilon_{n,t}, \tag{2}$$

where the correlation between $\epsilon_{A,t}$ and $\epsilon_{n,t}$ is positive. If the benchmarks price other passive assets, then $\alpha_n = 0$. Now suppose that over the same sample period used to obtain the OLS estimate of α_A, the OLS estimate of α_n is less than zero. Since the true value of α_n is zero, the negative estimate of α_n is fully attributed to sampling error. Given the positive correlation between $\epsilon_{A,t}$ and $\epsilon_{n,t}$, the OLS estimate of α_A is expected to contain negative sampling error as well, and this additional information can be used in estimating α_A.

As another special case, assume that the benchmarks used to define alpha have no pricing ability. To see how non-benchmark assets provide additional information about α_A in this case, consider a fund whose available return history is shorter than the histories of $r_{n,t}$ and $r_{B,t}$. The explosive growth of the mutual fund industry in recent years presents investors with many funds that have relatively short histories. Suppose that the OLS estimate of α_n computed for the sample period of the fund's available history is less than the OLS estimate of α_n computed for a longer sample period. Since the latter estimate is more precise, the first estimate is more likely to be less than the true (unknown) value of α_n. Given the positive correlation between $\epsilon_{A,t}$ and $\epsilon_{n,t}$, the same can be said of the OLS estimate of α_A relative to its true value, and this information can be used in estimating α_A. The additional information comes not through a pricing model, as in the previous case, but through the longer histories of the passive asset returns.

In the two special cases described above, α_n is assumed to be either zero or completely unknown. One may well prefer an intermediate version in which the benchmarks are believed to be relevant for pricing other passive assets, but not without error. In that general case, which we implement in a Bayesian framework, non-benchmark assets play a role that combines aspects of both previous cases. Additional information about α_A is provided by the extent to which the short-history estimate of α_n differs from zero as well as from its long-history estimate. If the prior distribution for α_n is concentrated around zero, then most of the additional information is extracted from the difference of the short-history estimate from zero. As the prior spreads out, relatively more information is extracted from the difference between the short- and long-history estimates of α_n.

Similar arguments apply when estimating a fund's Sharpe ratio. That is, a more precise estimate can be obtained using historical returns on more than just the fund. Perhaps the simplest setting in which returns on other assets can help estimate the mean and standard deviation of the fund return is that of Stambaugh (1997), who shows how assets with longer histories provide information about the moments of short-history assets. That principle enters the methods developed here, but so does the role of a pricing model. Consider the expected return on the fund, which can be written as

$$E_A = \alpha_A + \beta'_A E_B, \tag{3}$$

where E_A and E_B denote the means of $r_{A,t}$ and $r_{B,t}$. The sample mean of the fund's return, the usual estimate of E_A, can be obtained by replacing α_A and β_A with their OLS estimates from (1) and by replacing E_B with the sample mean of the benchmarks over the same sample period. To obtain a more precise estimate of E_A, we essentially use (3) and couple a longer-

history estimate of E_B with the more precise estimate of α_A discussed above. The estimate of α_A, and hence the estimate of the fund's Sharpe ratio, relies on additional information provided by the return histories of the non-benchmark assets and incorporates beliefs about the degree to which those assets are priced by the benchmarks.

Our study does not recommend a particular set of benchmarks for defining alpha. Recent academic studies compute mutual fund alphas with respect to a single market benchmark (e.g., Malkiel (1995)) as well as sets of multiple benchmarks (e.g., Carhart (1997) and Elton, Gruber, and Blake (1996)). We compute alphas in both single-benchmark and multiple-benchmark settings. Alphas defined with respect to a single market benchmark may be of interest whether or not one believes in the CAPM. We offer just two of many examples of their use in practice: Morningstar, the leading provider of mutual fund information, reports alphas computed with respect to one of several broad market indexes; Capital Resource Advisors, one of the largest providers of performance information to institutional clients, reports alphas computed with respect to the S&P 500 Index. Our approach allows one to estimate alpha under various assumptions about whether the benchmarks that define alpha price other passive investments. A common interpretation of alpha, one that implicitly places confidence in the benchmarks' pricing ability, is that it represents the skill of the fund's manager in selecting mispriced securities. A more general interpretation is that a positive alpha simply indicates that an investor can combine the fund and the benchmarks to obtain a Sharpe ratio higher than what can obtained by combining the benchmarks alone.

We investigate the performance of a large sample of equity mutual funds and find that the additional information about a fund's alpha and Sharpe ratio provided by seemingly unrelated assets can be substantial. Suppose, for example, that one has no confidence in the CAPM's pricing ability but nevertheless wishes to report a small-company growth fund's traditional alpha defined with respect to a single market benchmark. The absolute difference between the OLS estimate and an alternative estimate that incorporates information in non-benchmark returns has a median value across such funds of 8.3% per annum. If instead one has complete confidence in the CAPM's pricing ability, then the median absolute difference in estimates is 7.2%. In both cases, the alternative estimate is about three times more precise than the OLS estimate for the median small-company growth fund.

Across all funds in our sample, the median Sharpe ratio estimated the usual way, using the return history of just the fund, is 0.68 (annualized). When estimated using the additional information in seemingly unrelated passive assets, the median Sharpe ratio is no more than half that value. The new Sharpe-ratio estimates are typically four to five times more precise

than the usual estimates. We also compare the rankings of funds based on the usual Sharpe-ratio estimates to the rankings based on the new estimates. Of the funds with return histories of at least three years, only about 2% enter the top decile in both rankings. Of the funds that rank in the top decile based on the usual estimates, about 30% fall into the bottom two-thirds of the rankings based on the new estimates.

A number of studies observe that OLS estimates of mutual fund alphas are sensitive to the specification of the benchmarks that define those alphas.[1] When the estimation of a fund's alpha incorporates non-benchmark assets, the definition of alpha typically becomes less important and, in some cases, even irrelevant. We estimate alphas defined with respect to the CAPM and with respect to the three Fama and French (1993) benchmark factors, which include size and value factors in addition to the market factor. When estimated using OLS, the median difference in alphas between the two models is 2.3% per annum for all funds and 8.1% for small-company growth funds. When the estimation incorporates non-benchmark assets but does not rely on the benchmarks to price them, those values fall to 1.2% and 2.0%.

If the benchmarks are assumed to price the non-benchmarks exactly, the estimates of a fund's alpha are identical under the CAPM and Fama-French models, even though the definitions of the alphas differ. This illustrates a general result. If alphas are defined with respect to different benchmarks but estimated using the same set of passive assets (benchmark and non-benchmark), then the estimates are identical if in each case the benchmarks are assumed to price the non-benchmark assets exactly. Loosely speaking, if you believe that *some* pricing model holds exactly and want a fund's alpha with respect to it, you need not identify the model. The appropriate estimate of alpha is then simply the estimated intercept in a regression of the fund's return on all of the passive assets. Such a regression can be likened to "style analysis," (e.g., Sharpe (1992)), in that the right-hand-side assets are included to capture multiple sources of variation (styles) in returns, regardless of whether only a subset of them might serve as the benchmarks in a pricing model. The intuition for the result is straightforward: adding to the right-hand side of the regression assets that are priced by others already included there lowers the residual standard deviation but leaves the true regression intercept unchanged.

As in numerous previous studies, we find that estimated alphas for the majority of equity mutual funds are negative.[2] For each investment objective and each age group, we find a

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[1]Examples include Lehmann and Modest (1987) and Grinblatt and Titman (1994). Roll (1978) provides a theoretical discussion of the potential sensitivity of alphas to benchmark specification.

[2]Grinblatt and Titman (1995) review the literature on mutual fund performance.

posterior probability near 100% that the average of the funds' CAPM alphas is negative when the non-benchmark assets are excluded. Alphas for most funds remain negative when defined with respect to multiple benchmarks as well as when the information in the non-benchmark assets is used the estimation.

Section 2 discusses the econometric issues involved in obtaining our estimates of a fund's alpha and Sharpe ratio. Section 3 then reports results from computing alternative estimates of those measures for 2,609 equity mutual funds. Section 4 briefly reviews our conclusions.

2.   Estimating performance measures

This section begins with some basic concepts underlying the use of seemingly unrelated assets to estimate a fund's performance measures. We then describe the details of our methodology, including our selection of the seemingly unrelated assets. Although the Bayesian framework we develop can accommodate informative prior beliefs about a fund's performance, all of the estimates we report in the next section are obtained using prior beliefs about a fund's performance that are "diffuse," or completely non-informative. That is, to be consistent with most of the academic literature as well as current practice, we allow the fund's track record to determine its estimated performance without any adjustment for prior beliefs about what one might think to be reasonable magnitudes for the performance measures. Such an approach is maintained here in order to focus on the contribution of seemingly unrelated assets. When considering fund performance in the context of fund selection or investment, an informative prior about performance is a sensible alternative. Baks, Metrick, and Wachter (2001) investigate the degree to which informative priors can preclude at least one actively managed fund from having a positive posterior mean for alpha and thereby looking attractive to an investor who can also invest in the passive benchmarks. Pástor and Stambaugh (2001) investigate the role of informative priors about fund performance, as well as pricing models, in selecting a portfolio of mutual funds.

2.1.   The role of seemingly unrelated assets

Let $r_{N,t}$ denote the $m \times 1$ vector of returns in month t on m non-benchmark passive assets, so the multivariate version of the regression in (2) is written as

$$r_{N,t} = \alpha_N + B_N r_{B,t} + \epsilon_{N,t}, \tag{4}$$

where the variance-covariance matrix of $\epsilon_{N,t}$ is denoted by Σ. Let σ_ϵ^2 denote the variance of the disturbance $\epsilon_{A,t}$ in (1). Also define the regression of the fund's return on all $p \, (= m + k)$ passive assets,

$$r_{A,t} = \delta_A + c'_{AN} r_{N,t} + c'_{AB} r_{B,t} + u_{A,t}, \tag{5}$$

where the variance of $u_{A,t}$ is denoted by σ_u^2. Substituting the right-hand side of (4) for $r_{N,t}$ in (5) gives

$$r_{A,t} = \underbrace{\delta_A + c'_{AN} \alpha_N}_{\alpha_A} + \underbrace{(c'_{AN} B_N + c'_{AB})}_{\beta'_A} r_{B,t} + \underbrace{c'_{AN} \epsilon_{N,t} + u_{A,t}}_{\epsilon_{A,t}}. \tag{6}$$

That is, using (1) and the fact that $r_{B,t}$ is uncorrelated with both $\epsilon_{N,t}$ and $u_{A,t}$ gives

$$\alpha_A = \delta_A + c'_{AN} \alpha_N, \tag{7}$$

and

$$\beta_A = B'_N c_{AN} + c_{AB}. \tag{8}$$

As explained below, the equality in (7) provides the key to understanding how additional information about α_A is provided by the m non-benchmark assets, which are seemingly unrelated to α_A in that they are not required for its definition. Additional information about E_A and S_A is then provided, using (3), by the information about α_A as well as additional information about the expected returns of the k benchmark assets, which are seemingly unrelated to S_A.

2.1.1. Information about alpha: Intuition

To see how additional information about α_A is provided by non-benchmark assets, consider initially a simplified setting in which the second-moment parameters β_A, c_{AN}, and c_{AB} are viewed as known. Let S denote the number of observations in the fund's return history, and define estimators of the intercepts in (1), (4), and (5) as

$$\bar{\alpha}_A = (1/S) \sum_{t=1}^{S} (r_{A,t} - \beta'_A r_{B,t}), \tag{9}$$

$$\bar{\alpha}_N = (1/S) \sum_{t=1}^{S} (r_{N,t} - B_N r_{B,t}), \tag{10}$$

and

$$\bar{\delta}_A = (1/S) \sum_{t=1}^{S} (r_{A,t} - c'_{AN} r_{N,t} - c'_{AB} r_{B,t}). \tag{11}$$

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Note using (8) that $\bar{\alpha}_A$ is also equal to the result from substituting $\bar{\delta}_A$ and $\bar{\alpha}_N$ into the right-hand side of (7):

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\begin{aligned}
\bar{\delta}_A + c'_{AN}\bar{\alpha}_N &= (1/S)\sum_{t=1}^{S}(r_{A,t} - (B'_N c_{AN} + c_{AB})'r_{B,t}) \\
&= \bar{\alpha}_A.
\end{aligned}
\tag{12}
$$

Suppose first that α_N is treated as a vector of unknown parameters, so that the benchmarks have no assumed pricing ability. Then α_N can be estimated more precisely than in (10) if the available history of $r_{N,t}$ and $r_{B,t}$ is longer than the S observations in the fund's history. Substituting $\bar{\alpha}_N$ and $\bar{\delta}_A$ into the right-hand side of (7) gives $\bar{\alpha}_A$ as an estimator of α_A. Substituting the more precise estimator of α_N (along with $\bar{\delta}_A$) produces a more precise estimator of α_A, since $\bar{\delta}_A$ is uncorrelated with either estimator of α_N. (Note that, by construction, $u_{A,t}$ is uncorrelated with $\epsilon_{N,t}$.) Suppose instead that the benchmarks are assumed to price the non-benchmark assets exactly, so $\alpha_N = 0$ and thus $\alpha_A = \delta_A$. Then both $\bar{\alpha}_A$ and $\bar{\delta}_A$ are unbiased estimators of α_A, but the sampling variance of $\bar{\delta}_A$, σ_u^2/S, is less than or equal to the sampling variance of $\bar{\alpha}_A$, σ_ϵ^2/S. In this case, the non-benchmark asset returns explain additional variance of the fund's return and thereby provide a more precise estimator of its alpha.

The basic idea is that a more precise estimator of α_A is obtained by evaluating the right-hand side of (7) at $\bar{\delta}_A$ and a more precise estimator of α_N than $\bar{\alpha}_N$. A more precise estimator of α_N can be obtained by using a longer sample period, as in the case where the benchmarks are not assumed to have any pricing ability, or by simply setting $\alpha_N = 0$, as in the case where the benchmarks are assumed to price the non-benchmark assets perfectly. When $\epsilon_{A,t}$ is correlated with the elements of $\epsilon_{N,t}$ (i.e. when $c_{AN} \neq 0$), then the difference between $\bar{\alpha}_N$ and a more precise estimator of α_N supplies information about the likely difference between $\bar{\alpha}_A$ and α_A. When the more precise estimator of α_N relies on a longer history, the additional information about α_A is provided in essentially the same way that sample means of long-history assets provide information about expected returns on short-history assets, as in Stambaugh (1997).

2.1.2. Estimating alpha: General methodological issues

A Bayesian approach permits a range of prior beliefs about the ability of the k benchmark assets to price the m non-benchmark assets. A Bayesian setting is not required, however, to understand the basic issues governing the role of non-benchmark assets in estimating α_A.

We discuss here a number of those issues in the special cases where α_N is either restricted to be zero or left totally unrestricted.

Much of the intuition developed above when the slope coefficients β_A, c_{AN}, and c_{AB} are known extends to the actual setting in which those parameters must be estimated. Equation (7) also holds when all quantities are replaced by OLS estimators based on the sample of S observations. That is,

$$\hat{\alpha}_A = \hat{\delta}_A + \hat{c}'_{AN}\hat{\alpha}_N, \tag{13}$$

where $\hat{\alpha}_A$, $\hat{\alpha}_N$, and $\hat{\delta}_A$ are the OLS estimates of the intercepts in (1), (4), and (5), respectively, and $\hat{c}_{AN}$ is the OLS estimate in (5). As before, the information in non-benchmark assets is incorporated by replacing $\hat{\alpha}_N$ with a more precise estimator based either on a longer history or some degree of belief in a pricing model. When all parameters are unknown, substituting a more precise estimator of α_N can in some cases produce an estimator of α_A that is less precise than the usual estimator of the fund's alpha, $\hat{\alpha}_A$. For example, if one assumes that $\alpha_N = 0$ and substitutes that value into (13) in place of $\hat{\alpha}_N$, the resulting alternative estimator of α_A is simply $\hat{\delta}_A$. The mean of $\hat{\delta}_A$ is α_A, but the variance of $\hat{\delta}_A$ can exceed that of $\hat{\alpha}_A$. Since c_{AN} must be estimated and $\hat{\delta}_A$ and the elements of $\hat{c}_{AN}$ are correlated, replacing $\hat{\alpha}_N$ with a lower-variance quantity need not lower the variance of $\hat{\alpha}_A$. Such an outcome is most likely to occur as the number of non-benchmark assets increases without a sufficient increase in the R-squared in (5). In essence, the degrees-of-freedom effect can outweigh the additional explanatory power. We use between five and seven non-benchmark assets, depending on the number of benchmarks, and we find that the information provided by those assets produces a more precise estimate of α_A for most funds in our sample. In the Bayesian framework explained below, we also apply a moderate degree of shrinkage to the slope coefficients in (5) to increase their precision and thereby enhance the information provided by the non-benchmark assets. A potential direction for future research is the use of higher frequency data to increase the precision of the slope coefficients.

Suppose two researchers agree on an overall set of p passive assets to include when *estimating* α_A, but they disagree about the subset of those passive assets to designate as benchmarks for *defining* α_A. Their chosen benchmark subsets might not even have any members in common. Moreover, suppose each researcher believes his benchmarks price the remaining passive assets perfectly. Then those researchers' estimates of α_A will be identical, even though their definitions of α_A are not. That is, the definition of α_A is irrelevant to its estimation if, for whatever benchmarks might be designated for defining α_A, the remaining non-benchmark assets would be assumed to be priced exactly by those benchmarks. Perhaps ironically, if the benchmarks are not assumed to have perfect pricing ability, their designation

becomes relevant not only for defining α_A but also for estimating it.

To understand the above statements, consider first the maximum-likelihood estimator (MLE) of α_A under the restriction that $\alpha_N = 0$. If all regression disturbances are assumed to be normally distributed, independently and identically across t, then that estimator is given by $\hat{\delta}_A$, the OLS estimator of the intercept in (5), which does not depend on which of the p assets are designated as the benchmarks. To see this, note that the disturbances $\epsilon_{N,t}$ and $u_{A,t}$ are uncorrelated and, given the normality assumption, independent. The likelihood function can therefore be expressed as a product of two factors, one for each regression. The restriction on α_N does not affect the MLE of δ_A, which is $\hat{\delta}_A$, since α_N appears in the other factor. Substituting δ_A along with the restricted MLE of α_N (the zero vector) into the functional relation in (7) gives $\hat{\delta}_A$ as the MLE of α_A as well.

It can also be verified that $\hat{\delta}_A$ arises as the restricted estimator in a seemingly-unrelated-regression model, or SURM.[3] That is, let regressions (1) and (4) jointly constitute a SURM, and consider the estimation of the model subject to the restriction $\alpha_N = 0$. The restricted coefficient estimator requires the unknown joint covariance matrix of $(\epsilon_{A,t}\ \epsilon'_{N,t})$. If that matrix is replaced by the sample covariance matrix of the residuals from the first-pass unrestricted OLS estimation, the resulting "feasible" restricted SURM estimator of α_A is again simply $\hat{\delta}_A$. With no restriction on α_N, then of course both the MLE and SURM estimator of α_A is simply the usual estimator $\hat{\alpha}_A$. When shrinkage is applied to the slope coefficients in (5), as in the Bayesian setting described below, the same type of result obtains. That is, the assumption $\alpha_N = 0$ implies that the posterior mean of α_A is equal to the posterior mean of δ_A, which doesn't depend on the designation of the benchmarks.

The principles governing the role of non-benchmark assets also apply when α_A is estimated by the generalized method of moments (GMM) of Hansen (1982). Let the parameter vector γ contain the elements of δ_A, c_{AN}, c_{AB}, α_N, and B_N. The GMM estimator of γ is obtained by minimizing $g(\gamma)'Wg(\gamma)$, where $g(\gamma)$ denotes the vector of $(1+m+k)+m(1+k)$ moment conditions,

$$g(\gamma) \equiv \begin{pmatrix} \frac{1}{S}\sum_{t\in F}\left(r_{A,t} - \delta_A - c'_{AN}r_{N,t} - c'_{AB}r_{B,t}\right)\begin{pmatrix} 1 \\ r_{N,t} \\ r_{B,t} \end{pmatrix} \\ \text{vec}\left\{\frac{1}{T}\sum_{t=1}^{T}\left(r_{N,t} - \alpha_N - B_N r_{B,t}\right)\begin{pmatrix} 1 \\ r_{B,t} \end{pmatrix}'\right\} \end{pmatrix}, \qquad (14)$$

and F denotes the subset of the periods $\{1,\ldots,T\}$ representing the fund's return history of

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[3]Zellner (1962) introduces methods for estimating seemingly unrelated regressions. For a textbook treatment, including estimation under linear restrictions, see Theil (1971).

length S. The first set of moment conditions in (14) corresponds to the regression in (5), and the second set corresponds to the regression in (4). The weighting matrix W is block diagonal, since the disturbance in (4) is uncorrelated with that in (5). Consider the GMM estimates of the fund's alpha under the two cases discussed earlier. In the first case, with no restriction on α_N, the above moment conditions exactly identify γ. Using the GMM estimate $\check{\gamma}$, the fund's estimated alpha is then $\check{\alpha}_A = \check{\delta}_A + \check{c}'_{AN}\check{\alpha}_N$. Observe that $\check{\alpha}_N$ is based on the data for all T periods, whereas $\check{\delta}_A$ and $\check{c}_{AN}$ are based on only the observations corresponding to the fund's history. In the second case, with the pricing restriction $\alpha_N = 0$, the second set of moment conditions can be dropped and the fund's alpha is estimated simply as $\check{\alpha}_A = \check{\delta}_A$.

2.1.3. Estimating the Sharpe ratio

The k benchmark assets as well as the m non-benchmark assets are seemingly unrelated to a fund's Sharpe ratio, S_A, in that neither set of assets is related to the definition or usual estimate of S_A. The sample mean return on the fund obeys the relation,

$$\bar{r}_A = \hat{\alpha}_A + \hat{\beta}'_A \bar{r}_B, \tag{15}$$

where the OLS estimators $\hat{\alpha}_A$ and $\hat{\beta}_A$ and the sample mean vector of the benchmarks, $\bar{r}_B$, are computed using observations for the same S periods used to estimate $\bar{r}_A$. A more precise estimator of E_A can be obtained by replacing $\hat{\alpha}_A$ with the more precise estimator of α_A described above and by replacing $\bar{r}_B$ with the sample mean computed over a longer sample period. For example, when α_N is restricted to be zero, and hence $\alpha_A = \delta_A$, a simple alternative estimator of E_A is $\hat{\delta}_A + \hat{\beta}'_A \hat{E}_B$, where $\hat{E}_B$ is the sample mean vector of the benchmarks over a longer sample period of length T. In that case, additional information about E_A, and thus S_A, is provided both by the application of the pricing relation to the non-benchmark assets as well as by the longer history of the benchmark returns. This alternative estimator of E_A differs somewhat from the Bayesian estimator we actually use, which uses shrinkage techniques to obtain a more precise estimator of β_A, but it illustrates simply the main sources of additional precision in estimating E_A. When no restriction is placed on α_N, then the longer histories of both the benchmark and non-benchmark assets provide additional information about E_A in the same manner as in Stambaugh (1997), with the additional feature that shrinkage techniques are again applied to slope coefficients.

The denominator of a fund's Sharpe ratio is σ_A, the standard deviation of the fund's return. The seemingly unrelated assets provide additional information about that parameter as well. As in Stambaugh (1997), the longer histories of those assets provide information

about the volatility of the fund's return beyond what is provided by the fund's shorter return history. Although the Bayesian posterior mean of S_A cannot quite be viewed as a ratio of separate estimates of E_A and σ_A, the seemingly unrelated assets provide information about the latter parameters largely through the channels just described.

2.2. The benchmark and non-benchmark assets

Our set of benchmark and non-benchmark assets consists of eight portfolios constructed passively, in that their composition is determined using mechanical rules applied to simple, publicly available information. Monthly returns on these passive assets are constructed for the $35\frac{1}{2}$-year period from July 1963 through December 1998. The sample period for any given fund, typically much shorter, is a subset of that overall period. We specify up to three benchmark series, consisting of the three factors constructed by Fama and French (1993), updated through December 1998.[4] The first of these, MKT, is the excess return on a broad market index. The other two factors, SMB and HML, are payoffs on long-short spreads constructed by sorting stocks according to market capitalization and book-to-market ratio. We estimate "Fama-French" alphas, defined with respect to all three benchmarks, as well as "CAPM" alphas, defined with respect to just MKT.

When estimating CAPM alphas, SMB and HML become two of the non-benchmark series. Five additional non-benchmark series are used in the estimation of both CAPM and Fama-French alphas. The first of these, denoted as CMS, is the payoff on a characteristic-matched spread in which the long position contains stocks with low HML betas (in a multiple regression including MKT and SMB) and the short position contains stocks with high HML betas. The long and short positions are matched in terms of market capitalization and book-to-market ratio, and the overall spread position is formed from a set of triple-sorted equity portfolios constructed as in Pástor and Stambaugh (2000), who closely follow the procedures of Daniel and Titman (1997) and Davis, Fama, and French (2000). At the end of June of each year s, all NYSE, AMEX, and NASDAQ stocks in the intersection of the CRSP and Compustat files are sorted and assigned to three size categories and, independently, to three book-to-market categories. The nine groups formed by the intersection can be denoted by two letters, designating increasing values of size (S, M, B) and book-to-market (L, M, H). We then construct beta spreads within the four extreme groups of size and book-to-market: SL, SH, BL, and BH. The stocks within each group are sorted by their HML betas and assigned

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[4] We are grateful to Ken French for supplying these data.

to one of three value-weighted portfolios.[5] A spread within each group is constructed each month (from July of year s through June of year $s + 1$) by going long \$1 of the low-beta portfolio and short \$1 of the high-beta portfolio, and the value of CMS in month t is the equally weighted average of the four spread payoffs in month t.

The second non-benchmark series, denoted as MOM, is the "momentum" factor constructed by Carhart (1997). At the end of each month $t - 1$, all stocks in the CRSP file with return histories back to at least month $t - 12$ are ranked by their cumulative returns over months $t - 12$ through $t - 2$. The value of MOM in month t is the payoff on a spread consisting of a \$1 long position in an equally weighted portfolio of the top 30% of the stocks in that ranking and a corresponding \$1 short position in the bottom 30%.

The remaining three non-benchmark assets, whose returns are denoted as IP1, IP2, and IP3, are portfolios constructed from a universe of 20 value-weighted industry portfolios formed using the same classification scheme as Moskowitz and Grinblatt (1999). The three portfolios mimic the first three principal components of the disturbances in multiple regressions of the 20 industry returns on the other passive returns: MKT, SMB, HML, CMS, and MOM. The vector of weights for IP1 is proportional to the eigenvector for the largest eigenvalue of the sample covariance matrix of the residuals in those regressions, and the other two portfolios are similarly formed using eigenvectors for the second and third eigenvalues.

The specification of non-benchmark assets must be somewhat arbitrary, but our selection of the five described above is based on several considerations. Recall that non-benchmark assets supply information about α_A, the fund's alpha, when they explain additional variance of the fund's returns, i.e. when $c_{AN} \neq 0$. Also, except when the benchmarks are assumed to price the non-benchmark assets perfectly, the latter assets also provide information about α_A when they are mispriced by the benchmarks, i.e. when $\alpha_N \neq 0$. Our inclusion of the three industry portfolios is motivated primarily by the first consideration, explaining variance. Although we don't dismiss the possibility of their being mispriced, those portfolios are constructed to capture the most important sources of industry-related variation that is not accounted for by the other passive assets. On the other hand, our inclusion of CMS and MOM is motivated chiefly by the second consideration, mispricing. Evidence in other studies indicates that those spread positions may not be priced completely by the three benchmark

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[5]Using up to 60 months of data through December of year $s - 1$, the "pre-formation" HML betas are computed in a regression of the stock's excess returns on "fixed-weight" versions of the FF factors, which hold the weights on the constituent stocks constant at their June-end values of year s. Using the fixed-weight factors, as suggested by Daniel and Titman (1997), increases the dispersion in the "post-formation" betas of the resulting portfolios.

factors, MKT, SMB, and HML. For example, Daniel and Titman (1997) conclude that, during the post-1963 period, characteristic-matched spreads in HML beta produce nonzero alphas with respect to the Fama-French three-factor model.[6] Fama and French (1996) report a large three-factor alpha for the momentum strategy of Jegadeesh and Titman (1993). Of course, to be useful in estimating α_A, CMS and MOM also have to explain additional variance of the fund's returns, and we find that to be the case for many funds.

Parsimony is a consideration limiting our number of non-benchmark assets to five. As discussed earlier, the degrees-of-freedom effect in finite samples argues against indiscriminately specifying a large number of non-benchmark assets. One might instead include a larger number of the characteristic-matched spreads, say one for each size/book-to-market subgroup, or include all 20 industry portfolios instead of constructing the smaller set of three. We tried such alternatives and found that they quite often produce estimates of α_A similar to those obtained using the smaller set of five, but the precision of the estimates based on the larger set is lower. The OLS estimators of δ_A and c_{AN} are undefined, or essentially infinitely imprecise, when the total number of passive assets exceeds the length of the fund's history. The shrinkage estimator (explained below) can still be computed in that case, but it often yields a less precise inference than when fewer non-benchmark assets are used.

It is likely that future research could refine the selection of non-benchmark assets and further increase the precision of estimated alphas. A different set of non-benchmark assets could be specified for each fund, so that the assets have a high correlation with the specific fund at hand. With sector funds, for example, a passive index for the same sector could be included. A larger number of non-benchmark assets could be used for a fund with a longer history, since the degrees-of-freedom problem is then less severe. In general, some optimization over the set of non-benchmark assets would almost certainly increase the precision of our alpha estimates. Our specification of non-benchmark assets, motivated chiefly by simplicity, understates the potential gains from using non-benchmark assets to help estimate fund performance.

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[6]Davis, Fama, and French (2000) find that a hypothesis of zero mispricing for such spreads cannot be rejected within the longer 1929–97 period. In response to a referee's concern that some of the characteristic-related evidence could have been data-mined, we repeated our analysis with CMS excluded from the set of non-benchmark assets. The results are very close to those with CMS included.

2.3. The Bayesian framework

We compute the posterior means and variances of both α_A and S_A. The posterior moments of α_A can be obtained analytically, whereas the moments of S_A must be evaluated numerically be making repeated draws from the joint posterior distribution of the parameters. Derivations of the posterior moments of α_A as well as the details of computing the posterior moments of S_A are provided in the Appendix. The stochastic setting and specification of prior beliefs is discussed below.

The regression disturbances in (4) and (5) are assumed to be normally distributed, independently and identically across t. Recall from an earlier discussion that the likelihood function for each fund can then be expressed as a product of two factors, one for each regression. We assume that the disturbances in (5) are uncorrelated across funds, which implies that the likelihood functions across funds are independent.[7] Non-benchmark assets thus play yet another role, in that they account for covariance in fund returns that is not captured fully by the benchmarks. We also specify prior beliefs in which the parameters of the regression in (5) are independent across funds. The independence of the prior and the likelihood across funds allows us to conduct the analysis fund by fund.

First consider the parameters of the regression in (4). The prior distribution for Σ, the covariance matrix of $\epsilon_{N,t}$, is specified as inverted Wishart,

$$\Sigma^{-1} \sim W(H^{-1}, \nu). \tag{16}$$

We set the degrees of freedom $\nu = m + 3$, so that the prior contains very little information about Σ. From the properties of the inverted Wishart distribution (e.g., Anderson (1984)), the prior expectation of Σ equals $H/(\nu - m - 1)$. We specify $H = s^2(\nu - m - 1)I_m$, so that $\mathrm{E}(\Sigma) = s^2 I_m$. Following an "empirical Bayes" approach, the value of s^2 is set equal to the average of the diagonal elements of the sample estimate of Σ obtained using OLS. Conditional on Σ, the prior for α_N is specified as a normal distribution,

$$\alpha_N | \Sigma \sim N(0, \sigma_{\alpha_N}^2 (\frac{1}{s^2}\Sigma)). \tag{17}$$

Pástor and Stambaugh (1999) introduce the same type of prior for a single element of α_N, and Pástor (2000) and Pástor and Stambaugh (2000, 2001) apply the multivariate version in (17) to portfolio-choice problems. Having the conditional prior covariance matrix of α_N be

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[7]Our mutual fund data, discussed in Section 3, are consistent with this assumption. The average correlation of the disturbances in (5), calculated across all pairs of funds whose return histories overlap for at least 12 months, is less than 0.02.

proportional to Σ is motivated by the recognition that there exist portfolios of the passive assets with high Sharpe ratios if the elements of α_N are large when the elements of Σ are small (MacKinlay, 1995.) Such combinations receive lower prior probabilities under (17) than when each element of α_N has standard deviation σ_{α_N} but is distributed independently of Σ. The prior distribution for B_N is diffuse and independent of α_N and Σ.

Our earlier discussion focuses on the cases in which the benchmarks' ability to price the non-benchmark assets is assumed to be either perfect or nonexistent. That is, either α_N is set to the zero vector or the prior beliefs about α_N are diffuse. These two cases represent the opposite extremes on a continuum characterized by σ_{α_N}, the marginal prior standard deviation of each element in α_N. Specifying $\sigma_{\alpha_N} = 0$ is equivalent to setting $\alpha_N = 0$, corresponding to perfect confidence in the benchmarks' pricing ability. A diffuse prior for α_N corresponds to $\sigma_{\alpha_N} = \infty$. With a nonzero finite value of σ_{α_N}, prior beliefs are centered on the pricing restriction, but some degree of mispricing is entertained. We refer to σ_{α_N} as "mispricing uncertainty."

Next consider the parameters of the regression in (5). The prior for σ_u^2, the variance of $u_{A,t}$, is specified as inverted gamma, or

$$\sigma_u^2 \sim \frac{\nu_0 s_0^2}{\chi_{\nu_0}^2}, \tag{18}$$

where $\chi_{\nu_0}^2$ denotes a chi-square variate with ν_0 degrees of freedom. Define $c_A = (c_{AN}' \; c_{AB}')'$. Conditional on σ_u^2, the priors for δ_A and c_A are specified as normal distributions, independent of each other:

$$\delta_A | \sigma_u^2 \sim N(\delta_0, \left(\frac{\sigma_u^2}{\mathrm{E}(\sigma_u^2)}\right) \sigma_\delta^2), \tag{19}$$

and

$$c_A | \sigma_u^2 \sim N(c_0, \left(\frac{\sigma_u^2}{\mathrm{E}(\sigma_u^2)}\right) \Phi_c). \tag{20}$$

The marginal prior variance of δ_A is σ_δ^2, and the marginal prior covariance matrix of c_A is Φ_c. We set $\sigma_\delta^2 = \infty$ (or, computationally, a very large value), which implies that the prior for α_A is diffuse and that the prior mean δ_0 is irrelevant.[8]

Values for s_0, ν_0, c_0, and Φ_c in (18) through (20) are specified using an empirical-Bayes procedure. The basic idea is that a given fund is viewed as a draw from a cross-section of funds with the same investment objective, so the prior uncertainty about a parameter for the fund is governed by the cross-sectional dispersion of that parameter. The empirical-Bayes

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[8]In an analysis of mutual-fund investment, Pástor and Stambaugh (2001) set σ_δ^2 to finite values and specify δ_0 to reflect a fund's costs.

procedure uses the data to infer those properties of the cross-section. The prior mean and covariance matrix of c_A, denoted by c_0 and Φ_c, are set equal to the corresponding sample cross-sectional moments of $\hat{c}_A$, the OLS estimate of c_A, for all funds with at least 60 months of data and the same investment objective as the fund at hand. (The investment objectives are displayed in Table 1.) Setting Φ_c equal to the sample covariance matrix of $\hat{c}_A$, without adjusting for the sampling variation in those estimates, overstates the dispersion across funds in the true values of c_A. In that sense, our use of this empirical-Bayes procedure is conservative, in that it applies an intentionally modest degree of shrinkage toward the cross-sectional mean of $\hat{c}_A$ when computing the posterior moments of c_A for a given fund. With a diffuse prior on c_A, or no shrinkage, the estimator (posterior mean) of c_A is simply the OLS value $\hat{c}_A$. The degree of shrinkage applied here, albeit conservative, gives a more precise estimator of c_A, especially for a short-history fund, and thereby allows the non-benchmark assets to reveal more of their information about the fund's alpha.

The inverted gamma prior density for σ_u^2 implies (e.g., Zellner (1971, p. 372)),

$$\mathrm{E}(\sigma_u^2) = \frac{\nu_0 s_0^2}{\nu_0 - 2}, \tag{21}$$

and

$$\nu_0 = 4 + \frac{2(\mathrm{E}(\sigma_u^2))^2}{\mathrm{Var}(\sigma_u^2)}. \tag{22}$$

We substitute the cross-sectional mean and variance of $\hat{\sigma}_u^2$ for $\mathrm{E}(\sigma_u^2)$ and $\mathrm{Var}(\sigma_u^2)$ in (21) and (22). The value of ν_0 is set to the next largest integer of the resulting value on the right-hand side of (22), and then that value of ν_0 implies the value of s_0^2 using (21). Here again, using the cross-sectional variance of $\hat{\sigma}_u^2$ without adjusting for sampling error produces a conservative amount of shrinkage toward the cross-sectional mean of $\hat{\sigma}_u^2$ for funds with the same objective.

Our framework assumes fund managers have no ability to time the benchmark or non-benchmark assets. More generally, our framework models a fund's sensitivities to passive assets as constant over time. One way of relaxing this assumption is to model these coefficients as linear functions of state variables, as suggested by Shanken (1990) and applied by Ferson and Schadt (1996) in a mutual-fund context. In such a modification, passive asset returns scaled by the state variables can be viewed as returns on additional passive assets (dynamic passive strategies). The GMM formulation in (14) easily accommodates scaled returns, and the Bayesian approach developed here could be extended to such a setting as well. Another approach to dealing with temporal variation in parameters could employ data on fund holdings. Daniel, Grinblatt, Titman, and Wermers (1997) and Wermers (2000), for example, use such data in characteristic-based studies of fund performance.

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3. Empirical Analysis

The mutual-fund data come from the 1998 CRSP Survivor Bias Free Mutual Fund Database.[9] Our sample contains 2,609 domestic equity mutual funds with more than a year of available returns. Three quarters of our funds are still alive at the end of 1998. The funds are assigned to one of seven broad investment objectives, as described in the Appendix. Table 1 lists the number of funds in each objective, further classifying funds within each objective by the number of months in the fund's available return history. For each fund we compute the monthly return in excess of that on a one-month Treasury bill.

3.1. Estimates of funds' alphas.

Table 2 reports medians, within various fund classifications, of CAPM alphas (Panel A) and Fama-French alphas (Panel B). The posterior mean of α_A, denoted as $\tilde{\alpha}_A$, is computed for σ_{α_N} equal to zero, two percent (annualized), and infinity. Recall that the usual OLS estimator, denoted as $\hat{\alpha}_A$, makes no use of seemingly unrelated non-benchmark assets. Also reported are median absolute differences between $\hat{\alpha}_A$ and $\tilde{\alpha}_A$. Not surprisingly, non-benchmark assets play a greater role in the estimation of CAPM alphas, since two of the non-benchmark assets in that case, SMB and HML, are already included as benchmarks when estimating Fama-French alphas. Across all funds, the median value of $|\hat{\alpha}_A - \tilde{\alpha}_A|$ is two percent per annum for CAPM alphas but about one percent or less, depending on σ_{α_N}, for Fama-French alphas. Note also that $|\hat{\alpha}_A - \tilde{\alpha}_A|$ is typically smaller for the funds with longer histories. With a longer history, $\hat{\alpha}_A$ becomes more precise, so the additional information in non-benchmark returns has a smaller impact.

The manner by which non-benchmark assets provide information is illustrated most dramatically in the case of CAPM alphas for small-company growth funds. For such funds, incorporating the information in non-benchmark assets typically makes a difference of between 7.2% and 8.3% per annum when estimating the CAPM alpha, depending on σ_{α_N}. Nearly half of those 413 funds have track records of three years or less (see Table 1), and the bulk of their track records fall toward the end of the overall period. In recent years, small-firm indexes have underperformed their CAPM predictions, which is relevant when $\sigma_{\alpha_N} = 0$, and they have also underperformed their long-run historical averages, which is relevant when $\sigma_{\alpha_N} = \infty$. (Both statements are relevant when $\sigma_{\alpha_N} = 2\%$.) Incorporating that

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[9]CRSP, Center for Research in Security Prices, Graduate School of Business, The University of Chicago 1999, crsp.com. Used with permission. All rights reserved.

information is accomplished in either case largely by including as a non-benchmark asset the size factor SMB, which is positively correlated with the small-company growth funds' non-market-related returns.

An important issue in performance evaluation is whether the mutual fund industry adds value beyond standard passive benchmarks. We address this issue by computing posterior probabilities that average fund alphas within various fund classifications are negative. These probabilities are computed based on 100,000 draws of the average alpha from its posterior distribution. The probabilities are reported in Table 3, together with posterior means of average CAPM alphas (Panel A) and Fama-French alphas (Panel B). Some differences between the average alphas in Table 3 and the median alphas in Table 2 reflect skewness in the cross-sectional distribution of fund alphas. For example, the average of the OLS estimates of the CAPM alphas across all funds is -3.83%, compared to their median of -2.13%.

With few exceptions, Table 3 supports the inference that average fund alphas are negative. For example, for each investment objective and each age group, the average of the OLS estimates of the CAPM alphas is negative with 100% probability. The averages of the OLS estimates of the Fama-French alphas are mostly negative, although they are reliably positive for funds with histories longer than 10 years. When the non-benchmark assets are included, the average performance across all funds remains significantly negative, although the performance of long-history funds and aggressive growth funds improves with skeptical prior beliefs about pricing ($\sigma_{\alpha_N} = \infty$). The importance of beliefs about pricing can be illustrated by the average Fama-French alpha for small-cap growth funds. When the non-benchmark assets are not used, there is a 50% probability that the average alpha for those funds is negative. When the non-benchmark assets are included, the probability that the average alpha is negative rises to 100% when those assets are believed to be exactly priced by the benchmarks, but it drops to 9% when no pricing relation is used.

Our universe of funds includes those that ceased existence before the end of the overall sample period in December 1998 (a quarter of our funds). The alpha estimates in Tables 2 and 3 are generally higher for funds with longer histories. This age-related pattern is not surprising, since funds with poor track records are less likely to be long lived. One might therefore ask whether the estimate of a fund's alpha should be adjusted according to whether or not it survived to the end of the sample. The posterior mean of alpha is conditioned on the returns of the fund and the passive assets. If one assumes that, conditional on those realized returns, the probability of survival is unaffected by conditioning on the true parameters as well, then conditioning on survival has no incremental effect on the posterior mean of the

fund's alpha. This observation is made by Baks, Metrick and Wachter (2001), and their assumption that realized returns determine survival probabilities seems reasonable.[10] For this reason, a survival-based adjustment to our estimates is unnecessary.

When interpreting the age-related patterns in Tables 2 and 3, recall that our estimates are based on non-informative prior beliefs, so as to make our approach comparable to the standard practice in which a fund's performance measure is computed without an adjustment for prior beliefs about reasonable values. When the prior is non-informative, poor performance, whether or not it contributed to a fund's death, is translated fully to an inference that the fund's alpha is low. Readers who think that our estimated alphas for the short-history funds seem *too* low essentially have informative prior beliefs, which is a reasonable alternative. For example, with a prior that each fund's alpha is drawn from a distribution with a finite variance and a common mean, the alphas of short-history funds are shrunk more toward the grand mean than the alphas of long-history funds, thereby reducing the age-related differences. (The prior exerts more influence in a shorter sample.) That shrinkage is not an adjustment for survival, however. By the earlier argument, no such adjustment is necessary, whether or not the prior is informative.

To investigate whether including the non-benchmark assets leads to a more precise inference about a fund's alpha, in Table 4 we examine the ratio of two posterior variances. The numerator of the ratio is the posterior variance of α_A under our model in which non-benchmark assets are used and the prior variance for the elements of α_N is as given in the column heading. Recall that the posterior mean of α_A in that case is denoted as $\tilde{\alpha}_A$. The denominator of the ratio is the posterior variance of α_A when the non-benchmark assets are not used and diffuse priors are assigned to all parameters. The posterior mean of α_A in that case is the OLS estimate $\hat{\alpha}_A$. For ease of discussion, we commit a slight abuse of notation and refer to the posterior variances in the numerator and denominator as the "variances" of

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[10]Let R denote the returns over a given period, let s denote a random variable indicating whether or not a given fund dies at the end of that period, and let θ denote the vector of true (unknown) parameters of the return distribution. As observed by Baks, Metrick, and Wachter (2001), the assumption

$$p(s|R, \theta) = p(s|R)$$

implies, using Bayes' rule, that

$$p(\theta|R, s) = \frac{p(s|R, \theta)p(\theta|R)}{p(s|R)} = p(\theta|R).$$

Our assumption of independence across funds of the disturbances in (5) permits a fund-by-fund treatment, as discussed earlier, so the above holds when R includes the returns on all funds in our universe. Therefore, a fund's survival probability can depend not only on its performance relative to benchmarks but relative to other funds as well.

$\tilde{\alpha}_A$ and $\hat{\alpha}_A$. These variances reflect the precision of inferences about α_A in the sense generally associated with standard errors in a frequentist setting. In fact, the denominator of the ratio equals the squared standard error computed in the usual regression model.

For most funds, a more precise inference about alpha is obtained by including non-benchmark assets. Table 4 reports the median ratio of the variance of $\tilde{\alpha}_A$ to the variance of $\hat{\alpha}_A$. Also reported is the fraction of those ratios that are less than one. For the CAPM alpha estimates, the median variance ratio across all funds is approximately 0.7, and the ratio is less than one for roughly 90% of the funds. For Fama-French alphas, the ratio has a median of about 0.85 and is less than one for roughly 80% of the funds. In general, the median ratio is higher for the funds with longer histories. Note from Table 1 that funds with track records of at least 20 years represent only about 5% or our sample (139 out of 2609). For those funds, the OLS estimates of Fama-French alphas are typically about as precise as the estimates that incorporate the non-benchmark assets: the median variance ratios are 1.00 or just slightly less, and the ratios cluster fairly tightly around that value. At the other extreme lies the variance ratio associated with estimating CAPM alphas for small-company growth funds. That variance ratio has a median between 0.33 and 0.39, depending on σ_{α_N}, and the ratio is less than one for all such funds in our sample. Thus, for small-company growth funds in particular, not only is the CAPM $\tilde{\alpha}_A$ substantially higher than the CAPM $\hat{\alpha}_A$, it is also substantially more precise.

Recall that estimates of α_A are identical across different specifications of the benchmarks when one assumes the non-benchmark assets are priced exactly under each specification. In Table 2, note that the median values of $\tilde{\alpha}_A$ are indeed the same in Panels A and B when $\sigma_{\alpha_N} = 0$ (which sets $\alpha_N = 0$). Table 5 compares estimates of CAPM and Fama-French alphas when $\sigma_{\alpha_N} = 2\%$ (Panel A) and $\sigma_{\alpha_N} = \infty$ (Panel B). As expected, the median absolute differences between models are typically larger in the second case, but those differences are still substantially less than the median absolute differences between OLS estimates (Panel C). In other words, even when the non-benchmark assets are not believed to be priced whatsoever by either model's benchmarks, their presence in the estimation still makes the definition of α_A substantially less important than when they are not used at all. Across all funds, the median absolute difference between estimated CAPM and Fama-French alphas is 0.42% (per annum) under $\sigma_{\alpha_N} = 2\%$ and 1.24% under $\sigma_{\alpha_N} = \infty$, as compared to 2.28% for the OLS estimates. For small-company growth funds, the median difference is 0.69% under $\sigma_{\alpha_N} = 2\%$ and 2.03% under $\sigma_{\alpha_N} = \infty$, as compared to 8.07% for the OLS estimates.

Table 6 compares alphas defined for a given set of benchmarks and estimated with or

without the pricing restriction imposed on the non-benchmark assets. That is, for α_A defined with respect to a given pricing model, we compare estimates under $\sigma_{\alpha_N} = 0$ to estimates under $\sigma_{\alpha_N} = \infty$. Across all funds, the median difference is 0.94% for CAPM alphas and 0.68% for Fama-French alphas. Interestingly, the median differences display little if any relation to the length of the fund's history. Most of the median differences in the two-way sort (by objective and age) are 2% or less, with the exception of sector funds. For the funds with the longest histories, the effect of imposing the pricing restriction on the non-benchmark assets is often as large as the effect of not including the non-benchmark assets at all (shown earlier in Table 2). The latter effect is more important for history lengths of ten years or less when estimating CAPM alphas and five years or less when estimating Fama-French alphas. Note from Table 1 that about 85% of the equity funds in our sample have history lengths less than 10 years, and about 60% have histories of five years or less.

3.2. Estimates of funds' Sharpe ratios.

Table 7 reports medians of funds' Sharpe ratios, within various fund classifications, in a manner analogous to Table 2 for funds' alphas. The usual sample estimate of a fund's Sharpe ratio S_A, denoted as $\hat{S}_A$, is computed using only the fund's excess returns. In contrast, the posterior mean of S_A, denoted as $\tilde{S}_A$, also reflects information in returns of seemingly unrelated assets. The posterior means of S_A for the CAPM and the Fama-French model are reported for prior beliefs in which the pricing relations are assumed to hold exactly ($\sigma_{\alpha_N} = 0$). For the "No Model" results, prior beliefs in any pricing model are diffuse ($\sigma_{\alpha_N} = \infty$).

The median value of $\hat{S}_A$ across all funds is 0.68, but the median value of $\tilde{S}_A$ is only about half of that, ranging from 0.31 for the CAPM to 0.34 for "No Model". Since a typical fund in our sample has been around only during the bull market in the 1990s, the median $\hat{S}_A$ is quite high. The values of $\tilde{S}_A$ reflect additional information in the longer histories of returns on seemingly unrelated assets. Those returns are on average lower than the returns in the 1990s, resulting in lower values for $\tilde{S}_A$. Also note that $\tilde{S}_A$ is not very sensitive to the choice of the pricing model. The model benchmarks are not used to define the Sharpe ratio; the only role for the model is to determine the long-run expected returns on the non-benchmark assets. When estimating Sharpe ratios, including the long histories of seemingly unrelated assets is clearly more important than choosing a pricing model.

Table 7 also reports the median absolute differences between $\hat{S}_A$ and $\tilde{S}_A$. The median value of $|\hat{S}_A - \tilde{S}_A|$ across all funds is large, about 0.4. The median absolute differences are

the smallest for the longest-history funds, because, with a longer history, $\hat{S}_A$ becomes more precise and any additional information has a smaller effect. The long histories of returns on seemingly unrelated passive assets clearly contain a lot of information about the funds' Sharpe ratios, especially (but not exclusively) for funds with relatively short track records.

We examine the precision of the estimated Sharpe ratios in the same manner as reported previously in Table 4 for the precision of alphas. Table 8 reports the median ratio of the posterior variances of $\tilde{S}_A$ and $\hat{S}_A$ as well as the fraction of those ratios that are less than one. The numerator of the ratio is the posterior variance of S_A under our model in which returns on all passive assets are used, with prior beliefs corresponding either to exact pricing (CAPM, Fama-French models) or to no pricing ("No Model"). This posterior variance is computed from a large number of independent draws of each fund's Sharpe ratio from its posterior distribution, as described in the Appendix. The denominator of the ratio is the posterior variance of S_A when only the fund returns are used and diffuse priors are assigned to all parameters.

The results in Table 8 indicate that seemingly unrelated assets provide more precise Sharpe ratio estimates for over 99% of the funds in our universe. The median of the variance ratios across all funds is about 0.23, with little dependence on the choice of the pricing model. The precision of our Sharpe ratio estimates is therefore four to five times higher on average than the precision of the usual estimates. The median variance ratios increase uniformly with funds' history lengths, from about 0.16 for the funds with less than two years of data to about 0.88 for the funds with at least twenty years of data. Comparing the results in Table 8 to those in Table 4 reveals that seemingly unrelated assets help increase the precision of the funds' Sharpe ratios even more than the precision of their alphas. For Sharpe ratios, the set of seemingly unrelated assets includes all passive assets, not only non-benchmark assets, so the additional information contributed by seemingly unrelated assets is even more important.

Morningstar, Inc., a leading provider of mutual fund information, ranks funds into five categories (one to five stars) based on a risk-adjusted rating in which a measure of the fund's downside volatility is subtracted from a measure of the fund's average excess return. Although the Sharpe ratio and the Morningstar rating are defined differently, they share the same basic risk-adjustment concept and often provide similar rankings of funds, as demonstrated by Sharpe (1997, 1998). Moreover, the principles underlying the increase in precision in our Sharpe ratio estimates can clearly be used to improve the precision of Morningstar's

risk-adjusted ratings, especially the component based on the fund's average-excess return.[11] Since Morningstar rankings are known to be influential to mutual-fund investors, an important question is whether the fund rankings are affected by including seemingly unrelated assets. To explore this question, we examine rankings based on estimated Sharpe ratios. Specifically, we sort all 1,585 funds with at least three years of data into five categories, using the same breakpoints as Morningstar (top 10%, next 22.5%, next 35%, next 22.5%, bottom 10%). The funds are ranked first by their $\hat{S}_A$'s and then separately by their $\tilde{S}_A$'s, with the latter computed using prior beliefs that are noninformative about pricing.

Table 9 reports, in a five-by-five matrix, the numbers of funds in the intersection of the two independent rankings. For example, the $(1, 3)$ cell contains the number of funds that are ranked in the top group based on $\tilde{S}_A$ and in the third group based on $\hat{S}_A$. Large off-diagonal numbers in the matrix reveal that the $\tilde{S}_A$ ranking is quite different from the $\hat{S}_A$ ranking. At the extreme, there are three funds that are classified in the bottom 10% using the traditional $\hat{S}_A$ but in the top 10% using the more precise $\tilde{S}_A$. Only 526 out of the 1,585 funds are ranked in the same category under both $\hat{S}_A$ and $\tilde{S}_A$, and the rank correlation is only 0.53. With informative prior beliefs about pricing, the differences between the two rankings are even slightly larger. Clearly, the information in returns on seemingly unrelated assets has an important impact on fund rankings.

4.   Conclusions

This study develops and applies methods for improved estimation of performance measures using information in returns on "seemingly unrelated" assets not used to define those measures. A fund's alpha is defined relative to a set of passive benchmarks. The typically reported OLS estimate of alpha ignores information provided by returns on non-benchmark assets. The non-benchmark assets help estimate alpha if they are priced to some extent by the benchmarks or if their return histories are longer than the fund's. A fund's Sharpe ratio can be estimated more precisely by using information in returns on assets other than just the fund.

Using a sample of 2,609 U.S. equity mutual funds, we demonstrate that the returns on seemingly unrelated assets contain substantial information about fund performance. For

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[11]Whenever the average excess return on the fund's broad investment class is positive, the Morningstar rating uses essentially the same average excess fund return generally used to compute the fund's Sharpe ratio.

most funds, the estimates of alpha that incorporate the information in non-benchmark returns are more precise than the standard estimates. In the case of small-company growth funds, for example, the non-benchmark assets allow alphas defined with respect to the market benchmark to be estimated with only one third of the variance associated with the usual OLS estimates of those alphas. Across all funds, Sharpe ratios estimated using seemingly unrelated assets have a median only about half that of the usual estimates and are generally four to five times more precise. Moreover, fund rankings based on the improved Sharpe-ratio estimates differ substantially from those based on the usual estimates.

Compared to the usual estimates, the estimates of alpha that incorporate the information in the non-benchmark assets tend to exhibit less variation across different specifications of the benchmarks. At the extreme, if one believes that some subset of the passive assets used in the estimation prices the other passive assets exactly, then the estimate of alpha is the same regardless of which subset is designated as the benchmarks that define alpha. In other words, if one believes dogmatically in a pricing model, it does not matter which model that is when estimating alpha. For most funds, we find that including information in non-benchmark assets is more important than specifying the degree to which the non-benchmark assets are priced by the benchmarks. We also find that, across different beliefs about pricing, most funds have underperformed the CAPM and Fama-French benchmarks.

Appendix

This Appendix derives the posterior moments of α_A and S_A. The prior for the parameters of the regression in (5) is independent of the prior for the parameters of the regression in (4). In addition, both of those priors are assumed to be independent of the (diffuse) prior for E_B and V_{BB}, the mean and covariance matrix of the normal distribution for $r_{B,t}$. The independence of the priors and the independence of $u_{A,t}$ and $\epsilon_{N,t}$ imply that the posterior distributions for the parameters of the regression in (4) depend only on the sample of passive asset returns, not the fund returns. We first provide the moments of those distributions, relying on the derivations in Pástor and Stambaugh (2000), henceforth referred to as PS. Those moments are then combined with the posterior moments of δ_A and c_A to obtain the posterior moments of α_A. The posterior distribution of α_N is independent of δ_A and c_{AN}, so the posterior mean of α_A is obtained simply by evaluating the right-hand side of (7) at the posterior means of α_N, δ_A and c_{AN}. The posterior variances of α_A can also be computed using the posterior first and second moments of α_N, δ_A and c_{AN}. These results are then used to compute numerically, using simulation, the posterior moments of S_A. The Appendix concludes with an explanation of how funds are classified by investment objectives.

A.1. Posterior distribution for the passive-asset parameters

Define $Y = (r_{N,1}, \ldots, r_{N,T})'$, $X = (r_{B,1}, \ldots, r_{B,T})'$, and $Z = (\iota_T \ X)$, where ι_T denotes a T-vector of ones. Also define the $(k+1) \times m$ matrix $G = (\alpha_N \ B_N)'$, and let $g = \text{vec}\,(G)$. For the T observations $t = 1, \ldots, T$, the regression model in (4) can be written as

$$Y = ZG + U, \quad \text{vec}\,(U) \sim N(0, \Sigma \otimes I_T), \tag{A.1}$$

where $U = (\epsilon_{N,t}, \ldots, \epsilon_{N,T})'$. Define the statistics $\hat{G} = (Z'Z)^{-1}Z'Y$, $\hat{g} = \text{vec}\,(\hat{G})$, $\hat{\Sigma} = (Y - Z\hat{G})'(Y - Z\hat{G})/T$, $\hat{E}_B = X'\iota_T/T$, and $\hat{V}_{BB} = (X - \iota_T \hat{E}_B')'(X - \iota_T \hat{E}_B')/T$. Let θ_P denote the parameters of the joint distribution of the passive asset returns (G, Σ, E_B, and V_{BB}), and define the $T \times p$ sample matrix of passive returns, $R_P = (X \ Y)$. The likelihood function for the passive returns can be factored as

$$p(R_P|\theta_P) = p(Y|\theta_P, X)\, p(X|\theta_P), \tag{A.2}$$

where

$$p(Y|\theta_P, X) \ \propto \ |\Sigma|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}\text{tr}\,(Y - ZG)'(Y - ZG)\Sigma^{-1}\right\} \tag{A.3}$$

$$p(X|\theta_P) \ \propto \ |V_{BB}|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}\text{tr}\,(X - \iota_T E_B')'(X - \iota_T E_B')V_{BB}^{-1}\right\}. \tag{A.4}$$

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The joint prior distribution of all passive-return parameters is

$$p(\theta_P) = p(\alpha_N|\Sigma)\, p(\Sigma)\, p(B_N)\, p(E_B)\, p(V_{BB}), \tag{A.5}$$

where

$$p(\alpha_N|\Sigma) \;\propto\; |\Sigma|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\alpha_N'(\frac{\sigma_{\alpha_N}^2}{s^2}\Sigma)^{-1}\alpha_N\right\} \tag{A.6}$$

$$p(\Sigma) \;\propto\; |\Sigma|^{-\frac{\nu+m+1}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr} H\Sigma^{-1}\right\} \tag{A.7}$$

$$p(B_N) \;\propto\; 1 \tag{A.8}$$

$$p(E_B) \;\propto\; 1 \tag{A.9}$$

$$p(V_{BB}) \;\propto\; |V_{BB}|^{-\frac{k+1}{2}}. \tag{A.10}$$

The priors of B_N, E_B, and V_{BB} are diffuse. The prior of Σ is inverted Wishart with a small number of degrees of freedom, so that it is essentially noninformative. The prior on α_N given Σ is normal and centered at the pricing restriction. Let D denote a $(k+1) \times (k+1)$ matrix whose $(1,1)$ element is $\frac{s^2}{\sigma_{\alpha_N}^2}$ and all other elements are zero. Also let $F = D + Z'Z$ and $Q = Z'(I_T - ZF^{-1}Z')Z$. Applying the analysis in PS, the likelihood and the prior are combined to obtain the posteriors of the parameters of interest. The posterior distributions of the slopes and the residual covariance matrix from regression (A.1) are

$$g|\Sigma, R_P \;\sim\; N(\tilde{g}, \Sigma \otimes F^{-1}) \tag{A.11}$$

$$\Sigma^{-1}|R_P \;\sim\; W((H + T\hat{\Sigma} + \hat{G}'Q\hat{G})^{-1}, T + \nu - k), \tag{A.12}$$

where W denotes the Wishart distribution, and the posterior moments are

$$\tilde{g} \;=\; \mathrm{E}(g|R_P) = (I_m \otimes F^{-1}Z'Z)\hat{g} \tag{A.13}$$

$$\tilde{\Sigma} \;=\; \mathrm{E}(\Sigma|R_P) = \frac{1}{T + \nu - m - k - 1}(H + T\hat{\Sigma} + \hat{G}'Q\hat{G}) \tag{A.14}$$

$$\mathrm{Var}(g|R_P) \;=\; \tilde{\Sigma} \otimes F^{-1}. \tag{A.15}$$

The posteriors of the mean and the covariance matrix of the benchmark returns are

$$E_B|V_{BB}, R_P \;\sim\; N(\hat{E}_B, \frac{1}{T}V_{BB}) \tag{A.16}$$

$$V_{BB}^{-1}|R_P \;\sim\; W((T\hat{V}_{BB})^{-1}, T - 1), \tag{A.17}$$

and the posterior moments are

$$\tilde{E}_B \;=\; \mathrm{E}(E_B|R_P) = \hat{E}_B \tag{A.18}$$

$$\tilde{V}_{BB} \;=\; \mathrm{E}(V_{BB}|R_P) = \frac{T}{T - k - 2}\hat{V}_{BB} \tag{A.19}$$

$$\mathrm{Var}(E_B|R_P) \;=\; \frac{1}{T - k - 2}\hat{V}_{BB}. \tag{A.20}$$

Posterior means are denoted using tildes for the remainder of the Appendix.

A.2. Posterior distribution for the parameters of (5)

Let r_A $(S \times 1)$ contain S observations of $r_{A,t}$, the return on a given fund A. We assume $S \leq T$ and that the (consecutive) months in which $r_{A,t}$ is observed form a subset of those in which $r_{N,t}$ and $r_{B,t}$ are observed. The assumption that the disturbances in (5) are independent across funds, coupled with the assumption that the priors of that regression's parameters are independent across funds, implies that the posterior distribution for a given fund's parameters of the regression in (5) does not depend on the observed returns of the other funds (conditional on the passive return sample R_P). Let θ_A denote the set of parameters δ_A, c_A, and σ_u^2. Our various modeling assumptions give

$$
\begin{aligned}
p(\theta_A, \theta_P | R_P, r_A) &\propto p(\theta_A, \theta_P) p(R_P, r_A | \theta_A, \theta_P) \\
&= p(\theta_A) p(\theta_P) p(r_A | R_P, \theta_A, \theta_P) p(R_P | \theta_A, \theta_P) \\
&= p(\theta_A) p(r_A | R_P, \theta_A) p(\theta_P) p(R_P | \theta_P) \\
&\propto p(\theta_A | R_P, r_A) p(\theta_P | R_P).
\end{aligned}
\tag{A.21}
$$

The second factor in (A.21) is the posterior derived previously. The first factor, the posterior for θ_A, combines the priors given in (18) through (20) with the likelihood,

$$
p(r_A | R_P, \theta_A) \propto \frac{1}{\sigma_u^T} \exp\left\{ -\frac{1}{2\sigma_u^2}(r_A - Z_A \phi_A)'(r_A - Z_A \phi_A) \right\},
\tag{A.22}
$$

where $R_{P,A}$ denotes the S rows of R_P corresponding to the months in which $r_{A,t}$ is observed, $Z_A = (\iota_S \ R_{P,A})$, and $\phi_A = (\delta_A \ c_A')'$. The prior densities corresponding to (18) through (20) are given by

$$
p(\sigma_u) \propto \frac{1}{\sigma_u^{\nu_0 + 1}} \exp\left\{ -\frac{\nu_0 s_0^2}{2\sigma_u^2} \right\}
\tag{A.23}
$$

and

$$
p(\phi_A | \sigma_u) \propto \frac{1}{\sigma_u^{p+1}} \exp\left\{ -\frac{1}{2\sigma_u^2}(\phi_A - \phi_0)' \Lambda_0 (\phi_A - \phi_0) \right\},
\tag{A.24}
$$

where $\phi_0 = (\delta_0 \ c_0')'$ and

$$
\Lambda_0 = \left(\frac{\nu_0 s_0^2}{\nu_0 - 2} \right) \begin{bmatrix} \sigma_\delta^2 & 0 \\ 0 & \Phi_c \end{bmatrix}^{-1}.
\tag{A.25}
$$

The product of (A.22), (A.23), and (A.24) gives, after simplifying,[12]

$$
\begin{aligned}
p(\phi_A, \sigma_u | R_P, r_A) &\propto \frac{1}{\sigma_u^{p+1}} \exp\left\{ -\frac{1}{2\sigma_u^2}(\phi_A - \tilde{\phi}_A)'(\Lambda_0 + Z_A' Z_A)(\phi_A - \tilde{\phi}_A) \right\} \\
&\quad \times \frac{1}{\sigma_u^{T+\nu_0+1}} \exp\left\{ -\frac{h_A}{2\sigma_u^2} \right\},
\end{aligned}
\tag{A.26}
$$

---

[12]See Zellner (1971, pp. 75-76) for a similar derivation.

where

$$\tilde{\phi}_A \ = \ (\Lambda_0 + Z_A'Z_A)^{-1}(\Lambda_0\phi_0 + Z_A'r_A) \tag{A.27}$$

$$h_A \ = \ \nu_0 s_0^2 + r_A'r_A + \phi_0'\Lambda_0\phi_0 - \tilde{\phi}_A'(\Lambda_0 + Z_A'Z_A)\tilde{\phi}_A. \tag{A.28}$$

It follows from (A.26) that

$$\phi_A|R_P, r_A, \sigma_u \ \sim \ N(\tilde{\phi}_A, \ \sigma_u^2(\Lambda_0 + Z_A'Z_A)^{-1}) \tag{A.29}$$

$$\sigma_u^2|R_P, r_A \ \sim \ \frac{h_A}{\chi_{T+\nu_0}^2}, \tag{A.30}$$

and hence

$$\tilde{\sigma}_u^2 = E(\sigma_u^2|R_P, r_A) \ = \ \frac{h_A}{T + \nu_0 - 2} \tag{A.31}$$

$$\mathrm{Var}(\phi_A|R_P, r_A) \ = \ \tilde{\sigma}_u^2(\Lambda_0 + Z_A'Z_A)^{-1}, \tag{A.32}$$

where the last equation follows from variance decomposition.

A.3. Posterior moments of a fund's alpha

Let $\tilde{\alpha}_N$ and V_{α_N} denote the posterior mean and covariance matrix of α_N, given by the appropriate submatrices of the moments in (A.13) and (A.15), and let V_{ϕ_A} denote the posterior covariance matrix of ϕ_A given in (A.32). From the previous discussion recall that the posteriors of ϕ_A and α_N are independent. Thus, from equation (7), the posterior mean of the fund's alpha is given by

$$\tilde{\alpha}_A = \tilde{\delta}_A + \tilde{c}_{AN}'\tilde{\alpha}_N, \tag{A.33}$$

where $\tilde{\delta}_A$ and $\tilde{c}_{AN}$ are subvectors of the posterior mean of ϕ_A given in (A.27).

To obtain the posterior variance of α_A, rewrite (7) as

$$\alpha_A = \phi_A'd, \tag{A.34}$$

where $d = (1 \ \alpha_N' \ 0)'$, and the posterior mean and covariance matrix of d are given by

$$\tilde{d} = \begin{bmatrix} 1 \\ \tilde{\alpha}_N \\ 0 \end{bmatrix}, \quad V_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & V_{\alpha_N} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A.35}$$

Let $\mathcal{D}$ denote the data, R_P and r_A, and note that by the variance-decomposition rule,

$$\mathrm{Var}(\alpha_A|\mathcal{D}) = \mathrm{E}[\mathrm{Var}(\phi_A'd|\mathcal{D}, d)|\mathcal{D}] + \mathrm{Var}[\mathrm{E}(\phi_A'd|\mathcal{D}, d)|\mathcal{D}]. \tag{A.36}$$

Using the independence of ϕ_A and d, the first term in (A.36) can expressed as

$$
\begin{aligned}
\mathrm{E}[\mathrm{Var}(\phi_A' d | \mathcal{D}, d) | \mathcal{D}] &= \mathrm{E}[d' V_{\phi_A} d | \mathcal{D}] \\
&= \mathrm{E}[\mathrm{tr}\,(V_{\phi_A} d d') | \mathcal{D}] \\
&= \mathrm{tr}\,[V_{\phi_A} \mathrm{E}(d d' | \mathcal{D})] \\
&= \mathrm{tr}\,(V_{\phi_A} V_d) + \tilde{d}' V_{\phi_A} \tilde{d},
\end{aligned}
\tag{A.37}
$$

and the second term can be expressed as

$$
\begin{aligned}
\mathrm{Var}[\mathrm{E}(\phi_A' d | \mathcal{D}, d) | \mathcal{D}] &= \mathrm{Var}[\tilde{\phi}_A d | \mathcal{D}] \\
&= \tilde{\phi}_A' V_d \tilde{\phi}_A.
\end{aligned}
\tag{A.38}
$$

A.4. Posterior distribution of a fund's Sharpe ratio

A fund's Sharpe ratio, S_A, is defined as

$$
S_A = \frac{E_A}{\sigma_A}.
\tag{A.39}
$$

The fund's expected return, E_A, is calculated from equation (5) as

$$
E_A = \delta_A + c_A' E_P = \phi_A' \begin{bmatrix} 1 \\ E_P \end{bmatrix},
\tag{A.40}
$$

where

$$
E_P = \begin{bmatrix} \alpha_N + B_N E_B \\ E_B \end{bmatrix}
\tag{A.41}
$$

is the vector of expected returns on all passive assets. The fund's standard deviation of returns, σ_A, is calculated from the same equation as

$$
\sigma_A^2 = c_A' V_P c_A + \sigma_u^2,
\tag{A.42}
$$

where

$$
V_P = \begin{bmatrix} B_N V_{BB} B_N' + \Sigma & B_N V_{BB} \\ V_{BB} B_N' & V_{BB} \end{bmatrix}
\tag{A.43}
$$

is the covariance matrix of returns on all passive assets. Independent posterior draws of E_A and σ_A are constructed by drawing parameters from their posterior distributions in equations (A.11), (A.12), (A.16), (A.17), (A.29), and (A.30). Using equation (A.39), these draws are then used to construct a large number of independent draws of S_A from its posterior distribution.

A.5. Fund Classification by Objectives

We select domestic equity funds from the 1998 CRSP Survivor Bias Free Mutual Fund
Database, based on the information that the database provides about fund holdings, fund
objectives, and fund names. In addition to international funds, money market funds, and
bond funds, we exclude balanced funds, flexible funds, and funds of funds. We also exclude
multiple share classes for the same fund, funds with at most 12 months of (continuous) return
data, funds with unknown objectives, funds with no expense, turnover, or load data, and a
small number of funds with apparent data errors.[13]

The resulting 2,609 domestic equity funds are assigned to one of seven broad investment
objectives, using the information that the CRSP database provides about classifications
by Wiesenberger ("OBJ"), ICDI ("ICDI_OBJ"), and Strategic Insight ("SI_OBJ"). Our
classification is as follows:[14]

1. Small company growth – OBJ: SCG; SI_OBJ: SCG

2. Other aggressive growth – OBJ: AGG; ICDI_OBJ: AG, AGG; SI_OBJ: AGG

3. Growth – OBJ: G, G-S, S-G, GRO, LTG; ICDI_OBJ: LG; SI_OBJ: GRO

4. Income – OBJ: I, I-S, IEQ, ING; ICDI_OBJ: IN; SI_OBJ: ING

5. Growth and income – OBJ: GCI, G-I, G-I-S, G-S-I, I-G, I-G-S, I-S-G, S-G-I, S-I-G,
   GRI; ICDI_OBJ: GI; SI_OBJ: GRI

6. Maximum capital gains – OBJ: MCG

7. Sector funds – OBJ: ENR, FIN, HLT, TCH, UTL; ICDI_OBJ: SF, UT; SI_OBJ: ENV,
   FIN, HLT, TEC, UTI, RLE, NTR, SEC

A fund is included in a particular category in any given year if at least one of the objective
classifications OBJ, ICDI_OBJ, SI_OBJ satisfies the above criterion. Funds are assigned to
the categories in ascending order, beginning with category one. For example, if a fund is

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[13]We made various corrections to the CRSP mutual fund data and also incorporated a number of corrections generously provided by Thomas Knox and the authors of Baks, Metrick, and Wachter (2001).

[14]Note that the Wiesenberger objective IEQ stands for "Income-Equity", not "International Equity" as stated in the 1998 CRSP manual. Also, the Wiesenberger objective LTG stands for "Long-term growth", not "Long-term government". The Strategic Insight codes "GMC" and "FOF", not mentioned in the manual, apparently denote "Growth: Mid-Cap" and "Fund of Funds", respectively. These corrections are easily confirmed by browsing the fund names.

classified as small-company growth by Wiesenberger and as aggressive growth by ICDI, it is counted as small company growth, since that category comes before the aggressive growth category. If a fund's objective is not the same in each year, the fund is assigned to the category into which it is classified most often.

Table 1

**Number of Equity Mutual Funds Classified by
History Length and Investment Objective**

The sample contains domestic equity mutual funds in the CRSP database with continuous return histories longer than one year. Multiple share classes for the same fund are excluded. Funds are assigned to one of seven broad investment objectives using information that the CRSP database provides about classifications by Wiesenberger, ICDI, and Strategic Insight.

Investment objective	Length of fund's return history (months)						
	13–23	24–35	36–59	60–119	120–239	≥ 240	All
Small-company growth	128	60	95	109	21	0	413
Other aggressive growth	40	30	41	32	10	0	153
Growth	213	130	226	251	92	60	972
Income	36	35	38	47	14	4	174
Growth and income	154	80	119	153	36	34	576
Maximum capital gains	9	10	17	16	13	41	106
Sector funds	61	37	45	68	4	0	215
All funds	641	382	581	676	190	139	2609

Table 2
Estimates of Alpha for Equity Mutual Funds

Each value in the table is the median across the set of designated funds, expressed in percent per annum. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N}, expressed in percent per annum, denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes. The prior for α_A is diffuse.

History length or investment objective	$\hat{\alpha}_A$	$\tilde{\alpha}_A$ for σ_{α_N} of			$\|\hat{\alpha}_A - \tilde{\alpha}_A\|$ for σ_{α_N} of		
		0	2%	∞	0	2%	∞
A. $r_{A,t} = \alpha_A + \beta_A \text{MKT}_t + \epsilon_{A,t}$							
13–23 months	-4.81	-2.07	-1.87	-1.34	3.27	3.29	3.33
24–35 months	-2.85	-1.64	-1.53	-1.17	2.53	2.47	2.72
36–59 months	-2.87	-1.61	-1.35	-1.13	2.44	2.24	2.10
60–119 months	-1.49	-0.97	-0.91	-0.56	1.35	1.29	1.42
120–239 months	-0.84	-0.09	-0.08	0.03	1.29	1.04	0.96
240 months and greater	-0.53	-0.17	-0.26	-0.14	0.70	0.53	0.17
Small-company growth	-8.45	-1.59	-0.97	-0.05	7.20	7.66	8.30
Other aggressive growth	-5.41	-0.97	-0.74	-1.06	4.80	4.65	4.58
Growth	-2.17	-0.97	-1.01	-1.17	1.64	1.48	1.52
Income	-0.39	-1.84	-1.40	-0.45	1.27	1.07	0.83
Growth and income	-0.51	-0.97	-0.87	-0.59	0.93	0.89	1.02
Maximum capital gains	-2.29	-1.47	-1.53	-1.95	2.16	1.75	1.34
Sector funds	-1.06	-3.96	-2.70	0.09	4.95	3.48	2.95
All funds	-2.13	-1.25	-1.07	-0.74	2.05	1.87	1.90
B. $r_{A,t} = \alpha_A + b_{A,1}\text{MKT}_t + b_{A,2}\text{SMB}_t + b_{A,3}\text{HML}_t + \eta_{A,t}$							
13–23 months	-1.68	-2.07	-1.96	-1.43	1.66	1.55	1.59
24–35 months	-1.63	-1.64	-1.52	-1.38	1.40	1.25	1.01
36–59 months	-1.29	-1.61	-1.46	-1.14	1.05	0.95	0.78
60–119 months	-0.92	-0.97	-0.94	-0.66	0.76	0.57	0.39
120–239 months	0.07	-0.09	-0.06	0.08	0.64	0.42	0.24
240 months and greater	0.12	-0.17	-0.13	0.17	0.76	0.50	0.05
Small-company growth	-0.41	-1.59	-1.16	-0.08	1.45	1.15	0.92
Other aggressive growth	-0.37	-0.97	-0.45	0.08	1.76	1.34	0.96
Growth	-0.88	-0.97	-0.86	-0.72	0.90	0.78	0.59
Income	-2.03	-1.84	-1.90	-1.74	0.74	0.61	0.47
Growth and income	-1.19	-0.97	-1.00	-1.11	0.79	0.68	0.44
Maximum capital gains	-0.28	-1.47	-1.32	-0.34	1.40	1.03	0.45
Sector funds	-1.84	-3.96	-3.51	-2.48	3.18	2.44	1.35
All funds	-1.07	-1.25	-1.14	-0.86	1.09	0.91	0.65

Table 3
Average Equity-Fund Alphas: Are They Negative?

The table reports the posterior mean of $\bar{\alpha}_A$, the average α_A across the set of designated funds, expressed in percent per annum. Also reported is the posterior probability (expressed in percent) that $\bar{\alpha}_A$ is negative. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N}, expressed in percent per annum, denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes. The prior for α_A is diffuse.

History length or investment objective	Posterior mean of $\bar{\alpha}_A$			Prob($\bar{\alpha}_A < 0$)		
		$\tilde{\alpha}_A$ for σ_{α_N} of			$\tilde{\alpha}_A$ for σ_{α_N} of	
	$\hat{\alpha}_A$	0	∞	$\hat{\alpha}_A$	0	∞
A. $r_{A,t} = \alpha_A + \beta_A MKT_t + \epsilon_{A,t}$						
13–23 months	-6.31	-1.67	-1.19	100	100	98
24–35 months	-4.77	-1.67	-1.29	100	100	98
36–59 months	-4.12	-1.78	-1.39	100	100	100
60–119 months	-2.13	-0.99	-0.66	100	100	94
120–239 months	-1.07	-0.08	0.11	100	72	42
240 months and greater	-0.50	-0.27	-0.26	100	99	70
Small-company growth	-9.29	-1.28	0.01	100	100	50
Other aggressive growth	-6.79	-0.99	-1.39	100	98	89
Growth	-3.06	-1.00	-1.27	100	100	100
Income	-0.84	-1.56	-0.36	100	100	80
Growth and income	-1.02	-1.25	-1.01	100	100	100
Maximum capital gains	-4.74	-2.85	-3.01	100	100	100
Sector funds	-4.14	-2.45	-0.54	100	100	78
All funds	-3.83	-1.33	-0.97	100	100	98
B. $r_{A,t} = \alpha_A + b_{A,1} MKT_t + b_{A,2} SMB_t + b_{A,3} HML_t + \eta_{A,t}$						
13–23 months	-1.54	-1.67	-1.06	100	100	100
24–35 months	-1.22	-1.67	-1.02	100	100	100
36–59 months	-1.32	-1.78	-1.10	100	100	100
60–119 months	-0.66	-0.99	-0.44	100	100	99
120–239 months	0.38	-0.08	0.46	0	72	1
240 months and greater	0.15	-0.27	0.21	10	99	17
Small-company growth	-0.00	-1.28	0.50	50	100	9
Other aggressive growth	0.27	-0.99	0.29	30	98	30
Growth	-0.78	-1.00	-0.61	100	100	99
Income	-2.04	-1.56	-1.64	100	100	100
Growth and income	-1.76	-1.25	-1.43	100	100	100
Maximum capital gains	-1.67	-2.85	-1.64	100	100	100
Sector funds	-1.43	-2.45	-1.25	99	100	99
All funds	-0.99	-1.33	-0.72	100	100	100

Table 4
Relative Precision of Estimates of Alpha for Equity Mutual Funds

Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N}, expressed in percent per annum, denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes. The prior for α_A is diffuse.

History length or investment objective	$\sigma_{\alpha_N} = 0$ median	$\sigma_{\alpha_N} = 0$ % < 1	$\sigma_{\alpha_N} = 2\%$ median	$\sigma_{\alpha_N} = 2\%$ % < 1	$\sigma_{\alpha_N} = \infty$ median	$\sigma_{\alpha_N} = \infty$ % < 1
A. $r_{A,t} = \alpha_A + \beta_A \text{MKT}_t + \epsilon_{A,t}$						
13–23 months	0.62	83	0.62	83	0.63	82
24–35 months	0.63	88	0.62	88	0.63	89
36–59 months	0.65	90	0.65	92	0.68	91
60–119 months	0.70	93	0.69	97	0.73	96
120–239 months	0.76	93	0.77	97	0.84	98
240 months and greater	0.84	88	0.87	92	0.98	74
Small-company growth	0.33	100	0.34	100	0.39	100
Other aggressive growth	0.54	90	0.55	90	0.59	90
Growth	0.76	87	0.76	89	0.80	87
Income	0.75	84	0.75	89	0.77	89
Growth and income	0.77	83	0.77	85	0.80	85
Maximum capital gains	0.70	92	0.73	95	0.85	88
Sector funds	0.62	96	0.63	96	0.66	95
All funds	0.68	89	0.68	91	0.72	89
B. $r_{A,t} = \alpha_A + b_{A,1}\text{MKT}_t + b_{A,2}\text{SMB}_t + b_{A,3}\text{HML}_t + \eta_{A,t}$						
13–23 months	0.70	81	0.69	81	0.69	81
24–35 months	0.79	76	0.78	79	0.78	77
36–59 months	0.85	80	0.84	83	0.83	82
60–119 months	0.89	80	0.88	84	0.89	83
120–239 months	0.95	75	0.93	88	0.94	91
240 months and greater	1.00	53	0.98	71	0.99	83
Small-company growth	0.80	84	0.79	86	0.80	86
Other aggressive growth	0.81	86	0.79	88	0.78	88
Growth	0.88	78	0.86	82	0.88	82
Income	0.88	79	0.87	83	0.86	82
Growth and income	0.91	67	0.89	72	0.90	73
Maximum capital gains	0.96	66	0.94	75	0.96	83
Sector funds	0.74	95	0.74	95	0.76	95
All funds	0.86	78	0.84	82	0.85	82

Table 5

Comparison of Estimated CAPM and Fama-French Alphas under Alternative Roles for Non-Benchmark Assets

Each entry in Panels A and B is the median across funds of the absolute difference between posterior means of a fund's CAPM alpha and its Fama-French alpha, in percent per annum, under alternative prior beliefs about the degree to which each model prices the non-benchmark assets. Each entry in Panel C is the median absolute difference of the OLS alpha estimates. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (CAPM) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Fama-French). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on the benchmark and additional non-benchmark indexes. The quantity σ_{α_N} denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark returns, and the table compares $\tilde{\alpha}_A$ under a given non-zero σ_{α_N} but different specifications of the set of benchmark indexes. (The posterior means of the CAPM and Fama-French alphas are identical under $\sigma_{\alpha_N} = 0$.) The prior for α_A is diffuse.

	Length of fund's return history (months)						
Investment objective	13–23	24–35	36–59	60–119	120–239	≥ 240	All
A. Mispricing uncertainty for the non-benchmark assets (σ_{α_N}) equal to 2% per annum							
Small-company growth	0.58	1.01	0.74	0.67	0.70	na	0.69
Other aggressive growth	0.69	0.43	0.74	0.60	0.69	na	0.67
Growth	0.43	0.38	0.41	0.41	0.40	0.35	0.40
Income	0.48	0.38	0.47	0.49	0.57	0.41	0.47
Growth and income	0.18	0.20	0.26	0.19	0.33	0.23	0.20
Maximum capital gains	0.64	0.35	0.43	0.66	0.46	0.37	0.45
Sector funds	0.96	0.85	1.36	0.93	1.11	na	0.98
All funds	0.42	0.41	0.47	0.43	0.40	0.32	0.42
B. No reliance on the model to price the non-benchmark assets ($\sigma_{\alpha_N} = \infty$)							
Small-company growth	1.69	2.72	2.11	1.89	2.02	na	2.03
Other aggressive growth	2.10	1.29	2.38	1.75	2.20	na	1.89
Growth	1.25	1.11	1.18	1.13	1.13	1.02	1.14
Income	1.31	1.07	1.29	1.27	1.52	1.17	1.27
Growth and income	0.53	0.63	0.74	0.54	0.89	0.61	0.59
Maximum capital gains	2.00	1.16	1.64	2.15	1.30	1.12	1.34
Sector funds	2.43	2.41	3.40	2.37	2.91	na	2.47
All funds	1.18	1.18	1.38	1.26	1.20	0.99	1.24
C. No use of non-benchmark assets (OLS estimates)							
Small-company growth	11.21	11.33	7.98	4.72	3.96	na	8.07
Other aggressive growth	8.97	6.79	6.43	4.47	3.52	na	6.35
Growth	3.60	1.97	1.91	1.28	1.33	1.13	1.81
Income	1.80	1.43	1.30	1.44	1.62	0.93	1.44
Growth and income	1.79	1.73	1.54	1.01	0.85	0.68	1.25
Maximum capital gains	5.45	5.67	3.04	3.61	2.30	1.46	2.60
Sector funds	5.08	5.08	1.51	2.90	3.60	na	3.29
All funds	3.88	2.93	2.47	1.67	1.53	1.07	2.28

Table 6

Comparison of Estimated Alphas With and Without the Pricing-Model Restriction Applied to the Non-benchmark Assets

Each entry in the table is the median across funds of the absolute difference between posterior means of a fund's alpha, in percent per annum, under alternative prior beliefs about whether the pricing-model restriction applies to the non-benchmark assets. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N} denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes, and the table compares $\tilde{\alpha}_A$ under $\sigma_{\alpha_N} = 0$ versus $\sigma_{\alpha_N} = \infty$. The prior for α_A is diffuse.

Investment objective	Length of fund's return history (months)						
	13–23	24–35	36–59	60–119	120–239	$\geq$ 240	All
A. $r_{A,t} = \alpha_A + \beta_A MKT_t + \epsilon_{A,t}$							
Small-company growth	1.69	1.40	1.46	1.57	1.08	na	1.55
Other aggressive growth	0.72	0.67	0.88	0.82	0.92	na	0.77
Growth	0.78	0.75	0.85	0.84	0.91	0.83	0.83
Income	1.32	1.00	1.30	1.23	1.56	1.19	1.28
Growth and income	0.48	0.56	0.69	0.52	0.64	0.48	0.54
Maximum capital gains	0.85	0.93	1.36	1.35	1.45	0.95	1.07
Sector funds	3.31	2.94	6.22	3.61	4.78	na	4.31
All funds	0.90	0.89	1.01	1.00	0.99	0.80	0.94
B. $r_{A,t} = \alpha_A + b_{A,1} MKT_t + b_{A,2} SMB_t + b_{A,3} HML_t + \eta_{A,t}$							
Small-company growth	1.58	2.05	1.39	1.62	1.76	na	1.62
Other aggressive growth	1.88	1.20	1.90	1.78	1.19	na	1.63
Growth	0.56	0.63	0.55	0.57	0.65	0.67	0.59
Income	0.33	0.40	0.51	0.41	0.84	0.67	0.40
Growth and income	0.34	0.37	0.41	0.36	0.46	0.36	0.37
Maximum capital gains	1.49	1.33	1.91	1.41	0.89	1.30	1.29
Sector funds	1.25	1.30	2.85	2.16	2.01	na	1.93
All funds	0.66	0.66	0.68	0.66	0.73	0.74	0.68

Table 7
Estimates of Sharpe Ratios for Equity Mutual Funds

Each value in the table is the median across the set of designated funds. A fund's Sharpe ratio is denoted as S_A. The estimate $\hat{S}_A$ is based solely on the fund's available history. The posterior mean of S_A, denoted by $\tilde{S}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on benchmark and non-benchmark passive indexes. All results are reported in annualized terms. The results for the CAPM and the Fama-French model are reported for prior beliefs in which the pricing relations are assumed to hold exactly. For the "No Model" results, prior beliefs in any pricing model are diffuse.

History length or investment objective	$\hat{S}_A$	CAPM $\tilde{S}_A$	$\lvert\hat{S}_A - \tilde{S}_A\rvert$	Fama-French $\tilde{S}_A$	$\lvert\hat{S}_A - \tilde{S}_A\rvert$	No Model $\tilde{S}_A$	$\lvert\hat{S}_A - \tilde{S}_A\rvert$
13–23 months	0.59	0.26	0.39	0.29	0.37	0.30	0.37
24–35 months	0.72	0.28	0.46	0.30	0.45	0.31	0.43
36–59 months	0.82	0.29	0.56	0.31	0.52	0.32	0.52
60–119 months	0.79	0.33	0.47	0.35	0.43	0.35	0.43
120–239 months	0.61	0.38	0.23	0.39	0.20	0.39	0.21
240 months and greater	0.36	0.38	0.07	0.37	0.06	0.37	0.03
Small-company growth	0.51	0.29	0.23	0.27	0.25	0.36	0.17
Other aggressive growth	0.68	0.31	0.32	0.26	0.39	0.33	0.30
Growth	0.71	0.34	0.41	0.33	0.43	0.32	0.42
Income	0.87	0.26	0.61	0.41	0.48	0.36	0.53
Growth and income	0.91	0.34	0.58	0.42	0.51	0.36	0.58
Maximum capital gains	0.14	0.30	0.17	0.22	0.15	0.27	0.12
Sector funds	0.65	0.08	0.49	0.23	0.37	0.31	0.32
All funds	0.68	0.31	0.41	0.33	0.39	0.34	0.38

Table 8
Relative Precision of Estimates of Sharpe Ratios for Equity Mutual Funds

The table reports statistics for the ratio of variances, $\text{var}(\tilde{S}_A)/\text{var}(\hat{S}_A)$. A fund's Sharpe ratio is denoted as S_A. The estimate $\hat{S}_A$ is based solely on the fund's available history. The posterior mean of S_A, denoted by $\tilde{S}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on benchmark and non-benchmark passive indexes. The results for the CAPM and the Fama-French model are reported for prior beliefs in which the pricing relations are assumed to hold exactly. For the "No Model" results, prior beliefs in any pricing model are diffuse.

History length or investment objective	CAPM		Fama-French		No Model	
	median	% < 1	median	% < 1	median	% < 1
13–23 months	0.16	100	0.18	100	0.16	100
24–35 months	0.18	100	0.19	100	0.18	100
36–59 months	0.20	100	0.22	100	0.21	100
60–119 months	0.25	100	0.27	100	0.26	100
120–239 months	0.42	100	0.44	100	0.44	100
240 months and greater	0.87	89	0.88	91	0.88	93
Small-company growth	0.19	100	0.21	100	0.21	100
Other aggressive growth	0.23	100	0.24	100	0.24	100
Growth	0.23	100	0.25	100	0.23	100
Income	0.22	100	0.24	100	0.22	100
Growth and income	0.19	98	0.21	98	0.19	99
Maximum capital gains	0.47	100	0.46	100	0.46	99
Sector funds	0.33	100	0.37	100	0.35	100
All funds	0.22	99	0.24	99	0.23	100

Table 9
Comparison of Fund Rankings by Alternative Estimates of Sharpe Ratios

The table compares rankings for 1585 equity mutual funds with at least three years of return history. Each entry in the table is the total number of funds satisfying the given ranking classifications for both estimates. A fund's Sharpe ratio is denoted as S_A. The estimate $\hat{S}_A$ is based solely on the fund's available history. The posterior mean of S_A, denoted by $\tilde{S}_A$, is based on the fund's available history as well as the returns from July 1963 through December 1998 on benchmark and non-benchmark passive indexes. Prior beliefs in any pricing model are diffuse.

Rankings using $\tilde{S}_A$	Rankings using $\hat{S}_A$				
	Top 10%	Next 22.5%	Next 35%	Next 22.5%	Bottom 10%
Top 10%	35	54	63	4	3
Next 22.5%	75	93	142	44	2
Next 35%	43	160	192	142	18
Next 22.5%	6	47	135	120	49
Bottom 10%	0	2	23	47	86

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