Computational Astrophysics Exercises — Assignments Set 8

Linear Advection

Write a program to solve the 1D linear advection equation

$$u_t + a u_x = 0, \tag{1}$$

using the following methods: (i) centered differencing, (ii) upwind (iii) Lax-Wendroff, and periodic boundary conditions (see below).
Test your implementation on the following example: The initial condition is given by (see also Figure 1 on page 2).

$$u(x, t = 0) = \begin{cases} 1 & \text{for } |x| < 1/3 \\ 0 & \text{elsewhere} \end{cases}$$

The computational domain is $[x_{min}, x_{max}] = [-1, 1]$ and the boundaries are periodic, i.e. $u(-1, t) = u(1, t)$ for all times t. Define $\sigma = \frac{a\Delta t}{\Delta x}$ and use $\sigma = 0.8$ and $a = 1$, and calculate the solution for $t = 4$, $u(x, 4)$ using 400 gridpoints. To implement the periodic boundary conditions, you have to set the values to the grid cells on the boundaries at each time t^n after a completed timestep. Choose one grid point at each side as boundaries, i.e. x_0 and x_{N+1}, where the interior computation runs on gridpoints $i = 1$ to N, so the conditions are

$$u_0 = u_{N-1} \qquad u_{N+1} = u_1.$$

Alternatively, you can use the numpy function `np.roll` to implement the schemes. It applies periodic boundaries automatically.
Compare the obtained numerical solution with the analytical one and prepare plots where the analytical solution is overplotted with the numerical one. Describe the differences between the 1st order upwind method and the Lax-Wendroff method.

points in total (1370α)

Deadline:

Wed, 9 July at midnight via email (including code) to
`ch.schaefer@uni-tuebingen.de`.

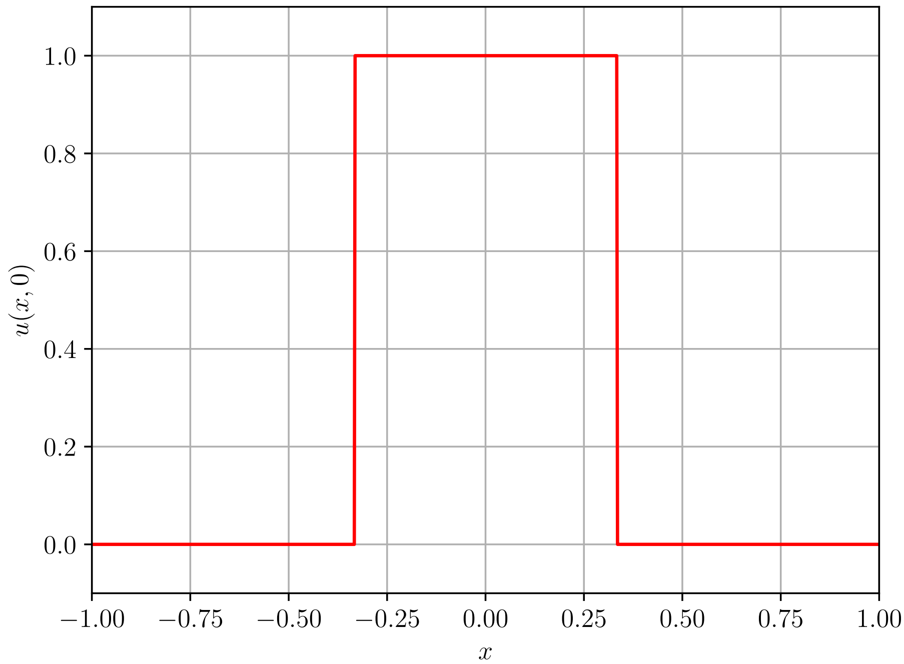

Figure 1: Initial condition for the test problem.