Computational Astrophysics Exercises — Assignments Set 2

1. The vis-viva equation

We have shown in the lecture that the energy in the two body problem is conserved and it holds

$$\frac{1}{2}\dot{\mathbf{r}}^2 - \frac{\mu}{r} = \text{const.}, \tag{1}$$

where $\mu = G(m_1 + m_2)$ is the standard gravitational parameter. Show that the vis-viva equation

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \tag{2}$$

follows from the conservation of energy and the conservation of angular momentum. Here, a denotes the semi-major axis of the orbit and r is the distance between the two bodies. (Hint: Consider the energy at periapsis and apoapsis to calculate const. with the help of angular momentum.)

(0b100 points)

2. Natural units

Show that if the two-body problem is analyzed using units in which the gravitational constant, the sum of the masses of the bodies and the distance between these bodies are equal to 1, the orbital period of the two bodies around the center of mass is 2π.

(0b000 points)

3. Catastrophic cancellation

Consider the quadratic equation

$$ax^2 + bx + c = 0, \tag{3}$$

with the solutions

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{4}$$

Apparently, there is a problem with substractive cancellation for x_1 if $b^2 \gg 4ac$, when the nominator becomes $-b + \sqrt{b^2 - 4ac} \approx -b + \sqrt{b^2} = -b + b$.

a) Write a program that calculates all solutions for arbitrary values of a, b, and c.

b) Investigate how errors in your computed answers become large as the subtractive cancellation increases and relate this to the known machine precision. (Hint: A good test case is $a = b = 1$, $c = 10^{-n}$, $n = 1, 2, 3, \ldots$)

c) Implement an alternative way to calculate x_2, where substractive cancellation is avoided for $b^2 \gg 4ac$.

d) Modify your program from a) to automatically choose the implementation that gives the most accurate result.

(0b110 points)

in total 0b1010 points.

Deadline: 15 May 2025 at noon, via email (including code) to `ch.schaefer@uni-tuebingen.de`.