Неопределенный интеграл

ДЗ: 1808, 1822, 1837, 1842, 1867, 1870 (можно разложить квадратный трехчлен), 1882.

Задача 1. (**Д1808**)

$$\int x\ln\frac{1+x}{1-x}dx$$

□

$$\int x\ln\frac{1+x}{1-x}dx = \frac{x^2}{2}\ln\frac{1+x}{1-x} - \int\frac{x^2}{2}\frac{2}{1-x^2}dx \Rightarrow$$
$$\Rightarrow -\int\frac{x^2}{2}\frac{2}{1-x^2}dx = \int\frac{1-1-x^2}{1-x^2}dx = -\int\frac{1}{1-x^2}dx - x + C = -\frac{1}{2}\ln\frac{1+x}{1-x} - x + C \Rightarrow$$
$$\Rightarrow \int x\ln\frac{1+x}{1-x}dx = \frac{x^2}{2}\ln\frac{1+x}{1-x} - \frac{1}{2}\ln\frac{1+x}{1-x} - x + C$$

■

Задача 2. (**Д1822**)

$$\int e^{\sqrt{x}}dx$$

□

$$\int e^{\sqrt{x}}dx = \left|u=\sqrt{x},\, du=\frac{1}{2\sqrt{x}}dx\right| = \int e^u 2udu = 2ue^u - \int 2e^u du =$$
$$= 2ue^u - 2e^u + C = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

■

Задача 3. (**Д1837**)

$$\int\frac{dx}{x^2-x+2}$$

□

$$\int\frac{dx}{x^2-x+2} = \int\frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{7}{4}} = \int\frac{2d(2x-1)}{(2x-1)^2+7} = \int\frac{2du}{u^2+7} = \frac{2}{\sqrt{7}}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{7}}\right) + C$$

■

Задача 4. (**Д1842**)

$$\int\frac{x^3dx}{x^4-x^2+2}$$

□

$$\int\frac{x^3dx}{x^4-x^2+2} = |u=x^2,\, du=2xdx=2\sqrt{u}dx| = \frac{1}{2}\int\frac{udu}{u^2-u+2} = \frac{1}{2}\int\frac{4udu}{(2u-1)^2+7} =$$
$$= \frac{1}{2}\int\frac{(2u-1)+1}{(2u-1)^2+7}d(2u-1) = \frac{1}{2}\int\frac{vdv}{v^2+7} + \frac{1}{2\sqrt{7}}\operatorname{arctg}\left(\frac{2x^2-1}{\sqrt{7}}\right) =$$
$$= \frac{1}{4}\ln|v^2+7| + \frac{1}{2\sqrt{7}}\operatorname{arctg}\left(\frac{2x^2-1}{\sqrt{7}}\right) + C = \frac{1}{4}\ln|(2x^2-1)^2+7| + \frac{1}{2\sqrt{7}}\operatorname{arctg}\left(\frac{2x^2-1}{\sqrt{7}}\right) + C$$

■

Задача 5. (**Д1867**)

$$\int \frac{xdx}{(x+1)(x+2)(x+3)}$$

□

$$P(x) = x,\ Q(x) = (x+1)(x+2)(x+3),\ Q'(x) = (x+2)(x+3) + (x+1)(x+3) + (x+1)(x+2)$$

$$z_1 = -1,\ z_2 = -2,\ z_3 = -3,\ \deg P < \deg Q$$

$$\frac{P(z_1)}{Q'(z_1)} = \frac{-1}{2} = -\frac{1}{2},\ \frac{P(z_2)}{Q'(z_2)} = \frac{-2}{-1} = 2,\ \frac{P(z_3)}{Q'(z_3)} = \frac{-3}{2} = -\frac{3}{2}$$

$$\int \frac{xdx}{(x+1)(x+2)(x+3)} = \int -\frac{1}{2}\cdot\frac{1}{x+1} + 2\frac{1}{x+2} - \frac{3}{2}\cdot\frac{1}{x+3}dx =$$

$$= -\frac{1}{2}\ln(x+1) + 2\ln(x+2) - \frac{3}{2}\ln(x+3) + C = \frac{1}{2}\ln\left(\frac{(x+2)^4}{(x+1)(x+3)^3}\right) + C$$

■

Задача 6. (**Д1870**)

$$\int \frac{x^4dx}{x^4+5x^2+4}$$

□

$$\frac{x^4}{x^4+5x^2+4} = 1 - \frac{5x^2+4}{(x^2+1)(x^2+4)} = 1 - \frac{A}{x^2+1} - \frac{B}{x^2+4} = 1 - \frac{Ax^2+4A+Bx^2+B}{(x^2+1)(x^2+4)} \Rightarrow$$

$$\Rightarrow \begin{cases} A + B = 5 \\ 4A + B = 4 \end{cases} \Rightarrow A + 4 - 4A = 5 \Rightarrow \begin{cases} A = -\frac{1}{3} \\ B = \frac{16}{3} \end{cases}$$

$$\int \frac{x^4dx}{x^4+5x^2+4} = x + \frac{1}{3}\int\frac{dx}{x^2+1} - \frac{16}{3}\int\frac{dx}{x^2+4} = x + \frac{1}{3}\operatorname{arctg} x - \frac{8}{3}\operatorname{arctg}\left(\frac{x}{2}\right) + C$$

■

Задача 7. (**Д1882**)

$$\int \frac{xdx}{x^3-1}$$

□

$$\frac{x}{x^3-1} = \frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} = \frac{Ax^2+Ax+A+Bx^2-Bx+Cx-C}{(x-1)(x^2+x+1)} \Rightarrow$$

$$\Rightarrow \begin{cases} A + B = 0 \\ A - B + C = 1 \\ A - C = 0 \end{cases} \Rightarrow \begin{cases} A = -B \\ A = C \\ 3A = 1 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{3} \\ C = \frac{1}{3} \\ B = -\frac{1}{3} \end{cases}$$

$$\int\frac{xdx}{x^3-1} = \frac{1}{3}\int\frac{dx}{x-1} - \frac{1}{3}\int\frac{x-1}{x^2+x+1}dx = \frac{1}{3}\ln|x-1| - \frac{1}{6}\int\frac{2x+1}{x^2+x+1}dx + \frac{1}{2}\int\frac{dx}{x^2+x+1} =$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| + \frac{1}{2}\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{1}{3}\ln\left|\frac{x-1}{\sqrt{x^2+x+1}}\right| + \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}} + C$$

■