

# PREFACE

Remarkable progress has been made in both theory and applications of all important areas of control theory. Theory is rich and sophisticated. Some beautiful applications of control theory are presently being made in aerospace, biomedical engineering, industrial engineering, robotics, economics, power systems, etc. Unfortunately, the same assessment of progress does not hold in general for computations in control theory.

Many of the methods described in earlier control and systems theory text books were developed before the computer era and were based on approaches that are not numerically sound. Most of these methods, for example, require reduction of the system matrices to some condensed forms, such as a companion form or the Jordan canonical form, and it is well-known that these forms cannot, in general, be achieved in a numerically stable way.

The situation is, however, changing quite fast. In the last 20 years or so, *numerically viable algorithms have been developed for many of the common linear control problems*. Softwares based on these methods have been developed and are still being built.

Unfortunately, these methods and softwares do not seem to be widely known and easily accessible to broad groups of applied mathematicians, control theorists, and practicing control engineers. They are still largely confined in reprints and preprints (in this context it is noted that a reprint book on “*Numerical Linear Algebra Techniques for Systems and Control*” edited by R.V. Patel, A. Laub, and P. Vandooren containing a large number of important published papers in this area has recently been published by IEEE/CRC Press). The primary reason for the inaccessibility of these algorithms and the softwares, in my opinion, is that an understanding, efficient implementations, and making possible modifications of these methods needed for some applications of special interests, require an *interdisciplinary knowledge of linear algebra, numerical linear algebra, control theory, and computer science*; and such a combined expertise is hard to find.

*What is, therefore, needed is a book that makes these algorithms accessible to a wide variety of users, researchers, and students.*

For practicing users, it is important that the algorithms are described in a manner that is suitable for easy implementation on a wide range of computers, that important aspects of implementations are discussed, and a clear comparative study of one algorithm over the other for a given problem with respect to efficiency, storage, numerical stability, etc., is presented. The latter will help the users to choose the one most suitable for his or her applications. Furthermore, for the students and researchers, it is important that the mechanism of the development of the algorithms is clearly explained and aspects of perturbation analysis of the problems and round-off error analyses and convergence properties of the algorithms, whenever available, are included in some details.

Of course, all these need to be accomplished requiring a minimal amount of background in the areas mentioned above. This is certainly a difficult and an ambitious task. *But the present book aspires to do that and aims at reaching out to a broad spectrum of audience in a number of disciplines including mathematics, control and systems engineering, and other applications areas such as vibrations, aerospace, space-sciences, and structural and manufacturing engineering.*

The recent book on “Computational Methods for Linear Control Systems” by P. H. Petkov, N.D. Christov, and M. M. Konstantinov also aims to fulfill that need to some extent. The scope of this book is, however, much more limited than that of the present book.

The current book is an outgrowth of lecture notes compiled by the author over several years for a graduate course in *numerical methods in control theory* taught at Northern Illinois University (almost all students of this course have been mathematics students with no prior background in control theory). The book has also been used in several short courses given by the author including the SIAM short course on *Numerical Methods in Control, Signal, and Image Processing*, Seattle, August 15, 1993 and, the short course on *Numerical Methods for Linear Control and Systems* at the *International Conference on Mathematical Theory of Networks and Systems*, St. Louis, 1996. The audience of these short courses had varying backgrounds.

The book covers most important and relevant problems arising in control system design and analysis with a special emphasis on computational aspects. These include:

- Numerical solutions of state equations and frequency response computations
- Controllability, observability, and distance to controllability
- Stability, inertia, robust stability, and distance to instability
- Numerical solutions and conditioning of Lyapunov, Sylvester, and algebraic Riccati equations
- Numerical algorithms for feedback stabilization, eigenvalue and robust eigenvalue assignment and conditioning of the eigenvalue assignment problem

- Numerical algorithms for full-order and reduced-order observer design and Kalman filtering
- Realization and subspace algorithms for model identification
- Algorithms for balanced realization and model reduction
- Large-scale solutions of control problems
- $H_2$  and  $H_\infty$  control

The numerical algorithms described in the book have the following desirable features:

- **Efficiency.** Algorithms are of order  $O(n^3)$ .
- **Numerical Stability.** Algorithms are either numerically stable or composed of numerically stable computations.
- **State-of-the-art Algorithms.** The state-of-the-art algorithms for all problems have been included.
- **Comparative Study and Recommendations.** Whenever possible, a comparison of various algorithms for the same problem with respect to efficiency, numerical stability, and accuracy has been given and based on this comparative study, recommendation for practicing engineers has been made.
- **Step by Step Explanation.** All algorithms have been explained step by step with illustrative examples illustrating each step of the algorithm.
- **Software and Implementations.** Important selected software for each topic has been included.
- **MATLAB Toolkit.** There exists a MATLAB toolkit called **MATCONTROL**, implementing major algorithms in the book.
- **Algorithms for both Continuous-time and Discrete-time systems.** Algorithms are described both for continuous-time and discrete-time systems.

*The discussions on theoretical aspects of control theory have been kept to a minimum, only the relevant facts have been mentioned. However, the importance and applications of the problems have been discussed to an extent to motivate the readers in mathematics and other areas of science and engineering who are not familiar with control problems. Numerical Linear Algebra techniques needed to understand and implement the algorithms have been developed in the book itself in a concise manner without going into too much details and attempts have been made to make the techniques understandable to the readers who do not have a prior background in numerical linear algebra and numerical analysis. Of course, people having a background in numerical analysis or numerical algebra and/or control theory will have a definite advantage.*

*A special emphasis has been given to the clear understanding of the distinction between a “bad” algorithm and a “numerically effective” algorithm.*

Some discussions on *large-scale computing in control* have been included too. The research in this area is still in its infancy, but some aspects of current research have been included to give the readers a flavor. There is an urgent need for an expanded research in this area as outlined in the 1988 NSF panel report: **“Future Directions in Control Theory: A Mathematical Perspective.”** It is hoped our short coverage in this area will provide enough incentive and motivation to beginning researchers, both from control theory and applied and computational mathematics, to work in the area.

The MATLAB toolkit *MATCONTROL* will help the students and the users understand the merits and drawbacks of one algorithm over the others and possibly help a user to make a right decision in choosing an ideal algorithm for a particular application.

### Organization of the Book:

The book has **fifteen** chapters. These fifteen chapters have been organized into **four parts**; each part consisting of several chapters, grouped together (roughly) with a common theme.

#### Part I. REVIEW OF LINEAR AND NUMERICAL LINEAR ALGEBRA

- Chapter 2. A Review of Some Basic Concepts and Results from Theoretical Linear Algebra
- Chapter 3. Some Fundamental Tools and Concepts from Numerical Linear Algebra
- Chapter 4. Canonical Forms Obtained via Orthogonal Transformations

#### Part II. CONTROL SYSTEM ANALYSIS

- Chapter 5. Linear State Space Models and Solutions of the State Equations
- Chapter 6. Controllability, Observability and Distance to Uncontrollability
- Chapter 7. Stability, Inertia, and Robust Stability
- Chapter 8. Numerical Solutions and Conditioning of Lyapunov and Sylvester Equations

#### Part III. CONTROL SYSTEMS DESIGN

- Chapter 9. Realization and Subspace Identification
- Chapter 10. Feedback Stabilization, Eigenvalue Assignment, and Optimal Control
- Chapter 11. Numerical Methods and Conditioning of the Eigenvalue Assignment Problems
- Chapter 12. State Estimation: Observer and the Kalman Filter
- Chapter 13. Numerical Solutions and Conditioning of Algebraic Riccati Equations
- Chapter 14. Internal Balancing and Model Reduction

Part IV. SPECIAL TOPICS

Chapter 15. Large-scale Matrix Computations in Control: Krylov Subspace Methods

**Heading:      Intended Audience**

The book can be used as a textbook for an advanced graduate course in control engineering such as *Computational Methods for Control Systems Design and Analysis* and *Computer-aided Control System Design* or for an advanced graduate topic course on *Numerical Linear Algebra Techniques in Control and Systems* in applied mathematics and scientific computing. Far more material than can be covered in one semester has been included, so professors can tailor material to particular courses and develop their own course syllabi out of the book. *Above all, the book is intended to serve as a reference book for practicing engineers and applied scientists, researchers, and graduate students.* The book is also very suitable for **self-study**.