Notes on Complex Measures, etc.

Nick Maxwell

Preliminaries

For $\mathcal{A}$ a σ-algebra, $E \in \mathcal{A}$, define $\mathcal{P}^*(E, \mathcal{A}) := \{\{E_k \in \mathcal{A}; k \in \mathbb{N}\}; E = \cup_k E_k, E_i \cap E_j = \phi \,\forall i \neq j\}$. Always $\{\phi, E\} \in \mathcal{P}^*(E, \mathcal{A})$, so $\mathcal{P}^*(E, \mathcal{A})$ is never empty, and also $\mathcal{P}^*(\phi, \mathcal{A}) = \{\phi\}$.

For $\mathcal{A}$ a σ-algebra, define

$$S^{\pm}(\mathcal{A}) := \{\sum_{k=1}^{n} c_k \,\mathbf{1}_{E_k}; \{c_k \in \mathbb{R}; c_i \neq c_j \,\forall i \neq j\}, \{E_k \in \mathcal{A}; E_i \cap E_j = \phi \,\forall i \neq j\}, n \in \mathbb{N}\} \tag{1}$$

$$S^{+}(\mathcal{A}) := \{\sum_{k=1}^{n} c_k \,\mathbf{1}_{E_k}; \{c_k \in [0, \infty], c_i \neq c_j \,\forall i \neq j\}, \{E_k \in \mathcal{A}; E_i \cap E_j = \phi \,\forall i \neq j\}, n \in \mathbb{N}\} \tag{2}$$

Main ideas