Call $C_c(X, Y)$ the space of all continuous functions on X, to Y, if Y is not specified, assume $Y = \mathbb{R}$. Write $||f||_u$ for the uniform norm of f, i.e., $||f||_u = \sup(\{|f(x)|; x \in X\})$ A linear functional I on $C_c(X)$ is positive is $I(f) \geq 0$ when $f \geq 0$. If U is open in X, and $f \in C_c(X)$, write $f \prec U$ to mean that $f(x) \in [0, 1]$ and $\text{supp}(f) \subset U$, this is a stronger statement than $0 \leq f \leq \mathbf{1}_U$, which only implies that $\text{supp}(f) \subset \overline{U}$.

Proposition (Folland 7.1), if I is a positive linear functional on $C_c(X)$, for each compact $K \subset X$, there is a constant, C_K, such that $|I(f)| \leq C_K||f||_u$ for all $f \in C_K(X)$ such that $\text{supp}(f) \subset K$.

If μ is a Borel measure on X such that $\mu(K) < \infty$ for every compact $K \subset X$, then $C_c(X) \subset L^1(\mu)$, so that $f \mapsto \int f\, d\mu$ is a positive linear functional on $C_c(X)$.

If I is a positive linear functional on $C_c(X)$, there is a unique Radon measure, μ on $\mathcal{B}\ell(X)$ such that $I(f) = \int_X f\, d\mu$ for all $f \in C_c(X)$. Moreover , μ satisfies

a) $\mu(U) = \sup(\{I(f); f \in C_c(X), f \prec U\})$ for all open $U \subset X$

b) $\mu(K) = \inf(\{I(f); f \in C_c(X), f \geq \mathbf{1}_K\})$ for all compact $K \subset X$.

Proof: ADD