

Definitions:

For a normed space X , write $\text{Ball}(X) := \{x \in X; \|x\| \leq 1\}$, and $\mathcal{U}_X := \{x \in X; \|x\| < 1\}$.

A function $f : X \rightarrow Y$, between topological spaces is continuous if $f^{-1}(U)$ is open for all open $U \subset Y$.

A function $f : X \rightarrow Y$, between topological spaces is an open map if $f(U)$ is open for all open $U \subset X$.

A homeomorphism or bicontinuous map is a bijective map which is also open, or f^{-1} is also continuous.

Prop: for a bijective map, $f : X \rightarrow Y$, f is an open map iff f^{-1} is continuous.

Pf: