MATH 4331/2 & MATH 6312

Provisional Syllabus

Course:	**Introduction to Real Analysis**
Semester:	Fall 2008/Spring 2009
Meeting time	MW 4:00–5:30pm
Instructor:	Dr Michael Field
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WebPages:	www.math.uh.edu/~mike/MATH4332

Textbook: *Set Theory & Metric Spaces*, Kaplansky (AMS Chelsea). The text will be supplemented with extensive handouts.

COURSE OUTLINE

Introduction. Although there will be much common material between the course and what is described in the old syllabus for 4331/2, there will be some differences. Most notably: the order will be very different; Lebesgue integration will not be covered. As a practical matter, this class is very diverse. The text book - *Set Theory & Metric Spaces*, Kaplansky - will mainly be of use in semester 2 (for MATH 4332). However, the first two chapters do cover a lot of material we will be going over in the first two or three lectures. If you have not already done so, you are strongly advised to read chapter 1 of Kaplansky. For most of the remaining material covered in MATH 4331, I will provide notes. These will be available from my website and should be carefully read. I will also provide solutions for (most) homework questions. These too will be available online and should be carefully read. *Homework forms an essential component of the course and it is almost impossible to pass the course unless you do most of the homework.* (Specifically: 100% on the examination component of the course and no homework handed in gives a grade of **C**.) Finally, you are strongly advised to attend all lectures — the handouts complement the lectures and are not replacements for the lectures.

Outline syllabus. The outline covers the material that I will cover in MATH 4331 & MATH 4332. In general terms: MATH 4331 will cover 1 variable theory; MATH 4332 will mainly be about metric spaces and several variable theory. Now for a little more detail.

(1) Review of set theory and functions. Basic notations. Countable sets, non-countable sets, power set.

Date: December 12, 2008.

(2) Basic structure and properties of the real numbers. Basic notations. Real numbers not countable. Review (and proof) of two or three foundational results. Existence of Riemann integral (possibly later). ((1,2) will take about 4 to 5 lectures to cover.)
(3) Sequences, subsequences, convergence. Bolzano-Weierstrass theorem.
(4) Infinite series and tests for convergence (including Dirichlet and Abel).
(5) At this point we get to the main theme of semester 1: continuous functions. We start with continuous and smooth functions.
(6) Investigation of sequences and series of functions. Uniform convergence and uniform approximation play a major role here. Weierstrass Approximation theorem (this may get deferred till semester 2). Nowhere differentiable functions.
(7) Special series of functions: power series and Fourier series. Comparison with smooth functions. (Close to end of semester 1.)
(8) Infinite products, $\sin x = x\Pi_{n=1}^{\infty}(1 - \frac{x^2}{n^2\pi^2})$. The Γ function. Euler Maclaurin formula and applications (eg Stirlings formula approximating $n!$). Bernoulli numbers.
(9) Metric spaces. (Long section - emphasis on metric space properties of $\mathbb{R}^n$ and extensions of results of semester 1.) The approach will be along the lines of Kaplansky and *not* at all along the lines of the old text book Rudin.
(10) Contraction mapping lemma.
(11) Calculus on $\mathbb{R}^n$ and applications of contraction mapping lemma.

Some of the "big" theorems we prove: Bolzano-Weierstrass theorem, Weierstrass Approximation theorem, Fourier theorem on representation of sufficiently regular functions by Fourier series, Contraction Mapping Lemma, Existence theorem for ODEs, implicit function theorem.

Some of the main concepts: Structure and properties of $\mathbb{R}$, uniform convergence, approximation, derivative (as a linear map).

Some of the "new" stuff: C^∞-functions — construct your own smooth function, subtle tests for convergence, infinite products, the Γ function, the Euler-Maclaurin formula, representing $\sin x$ as an infinite product, countable and non-countable sets, Cantor sets, and lots more.

Some of the course goals: To acquire a deeper understanding of the real number system $\mathbb{R}$; to appreciate what analysis is about (it is has nothing to do with 'applying formulas'); to get a good overview of some of the beautiful results from classical one-variable analysis; to begin to acquire and appreciate some of the exciting modern contemporary tools and methods based on metrics and topology.