MATH 4332
Homework 8, due Monday, April 6th

Q1. Show that the metric space (X, d) is complete if d is the discrete metric.

Q2. Let $d, \bar{d}$ be equivalent metrics on X, show that (X, d) is complete iff $(x, \bar{d})$ is complete (see exercises 7.1.7(9) for the definition of equivalent metrics).

Q3. Suppose that E, F are connected subsets of X. If $E \cap F \neq \emptyset$, need $E \cap F$ be connected?

Q4. Let $Y = \{0, 1\}$ with the discrete metric. Show that (X, d) is connected iff every continuous function $f : X \to Y$ is constant. Use this result to show that if (X_1, d_1), (X_2, d_2) are connected metric spaces then the product $(X_1 \times X_2, d)$ is connected (where we take the product metric $d((x_1, x_2), (y_1, y_2)) = \max\{d_1(x_1, y_1), d_2(x_2, y_2))\}$ on $X_1 \times X_2$).

Q5. Find an example of a sequence of closed non-compact connected subsets F_n of $\mathbb{R}^2$ such that $F_1 \supset F_2 \supset \cdots$ and $\cap_{n \geq 1} F_n$ is disconnected.
(Hint: If F_n is compact, then the intersection is connected. You will need to find a connected closed but unbounded subset of $\mathbb{R}^2$ that is disconnected by removing a countable number of pieces.)