0.1 Basics

$\liminf E_n = \{x : x \in E_n \text{ for all but finitely many } n\}$

$\limsup E_n = \{x : x \in E_n \text{ for infinitely many } n\}$

Def: E_n a sequence of sets, $n \in \mathbb{N}$. Take the statement "$x \in E_n$ for all but finitely many n" to precicely mean "$\exists k \in \mathbb{N}$ s.t. $x \in \cap_{n=k}^{\infty} E_n$". Then, " $x \in E_n$ for infinitely many n" means "$x \in \cup_{n=k}^{\infty} E_n$, $\forall k \in \mathbb{N}$".

$$x \in \limsup E_n = \cap_{k=1}^{\infty} \cup_{n=k}^{\infty} E_n \Leftrightarrow (x \in \cup_{n=k}^{\infty} E_n,\ \forall k \in \mathbb{N})$$

$$x \in \liminf E_n = \cup_{k=1}^{\infty} \cap_{n=k}^{\infty} E_n \Leftrightarrow (x \in \cap_{n=k}^{\infty} E_n,\ \text{some } k \in \mathbb{N})$$

Chapter 1

Measure Theory

1.1 Measures