

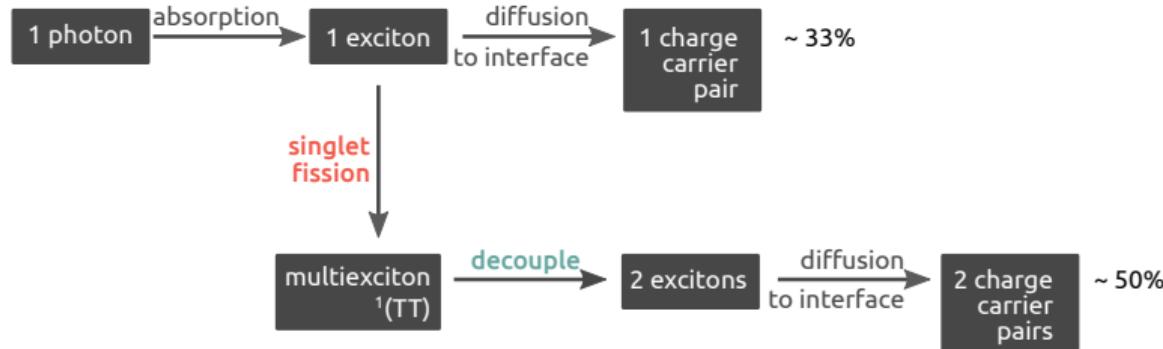
# Modeling singlet-fission biexciton states as an ab initio spin model: Justifications and applications

Vibin Abraham & Nick Mayhall  
Virginia Tech

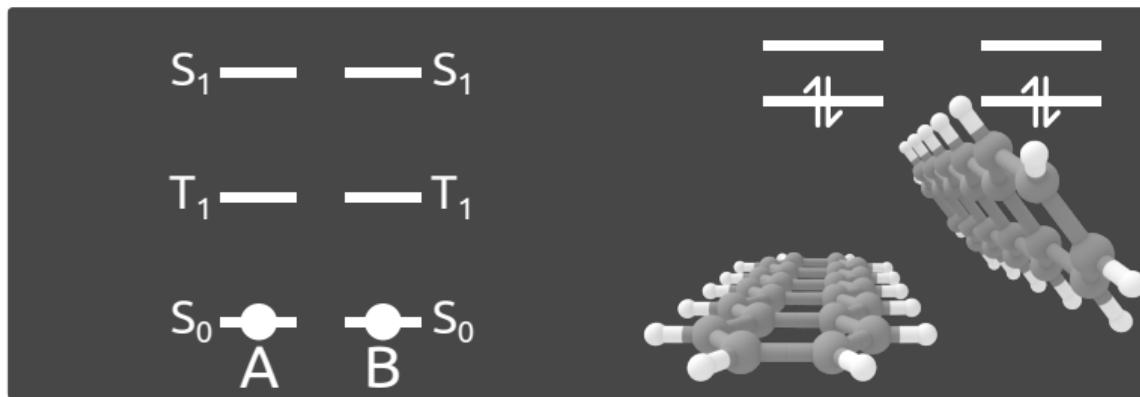
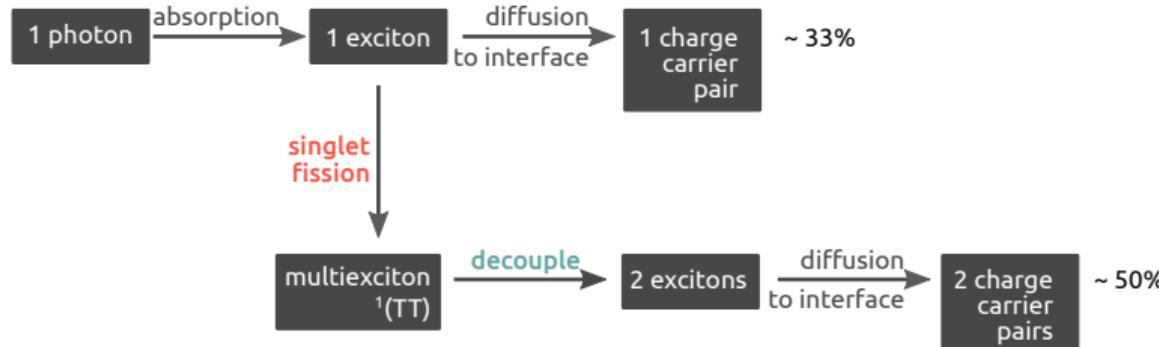
# Singlet Fission



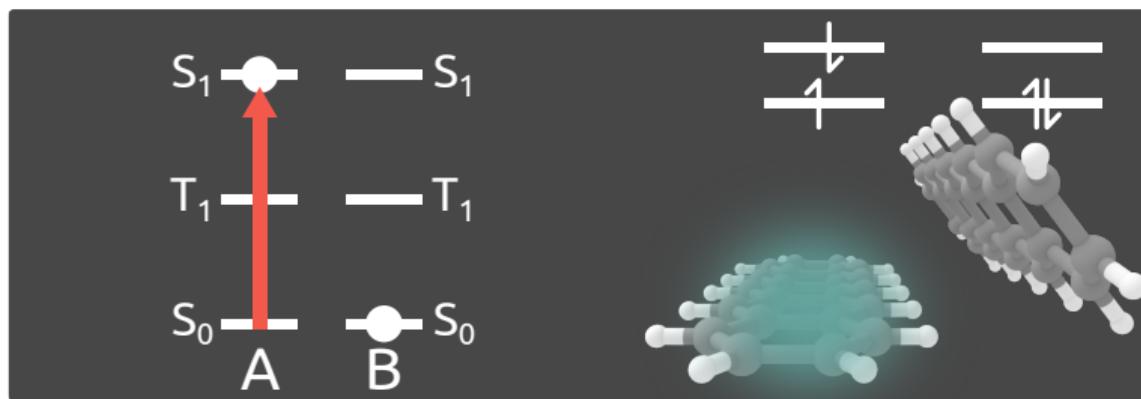
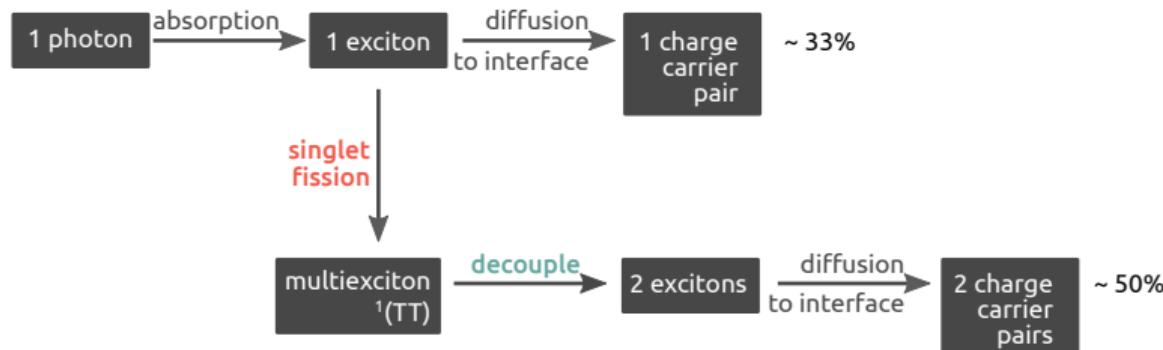
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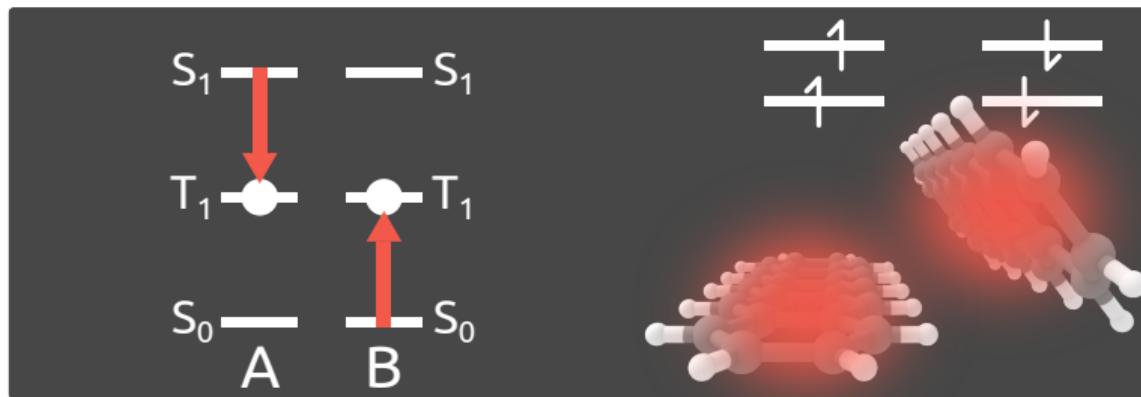
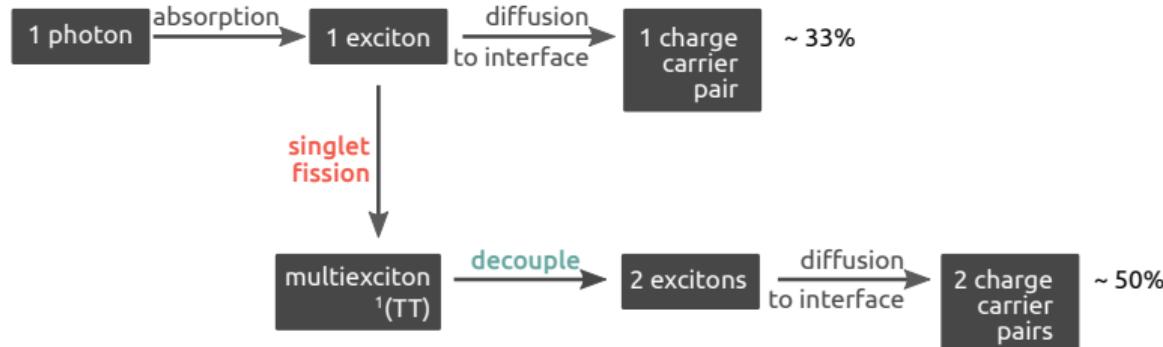
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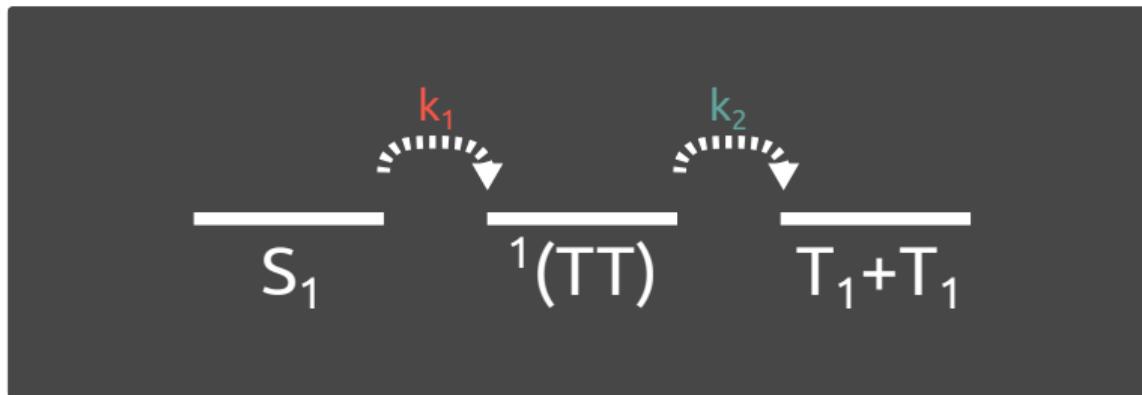
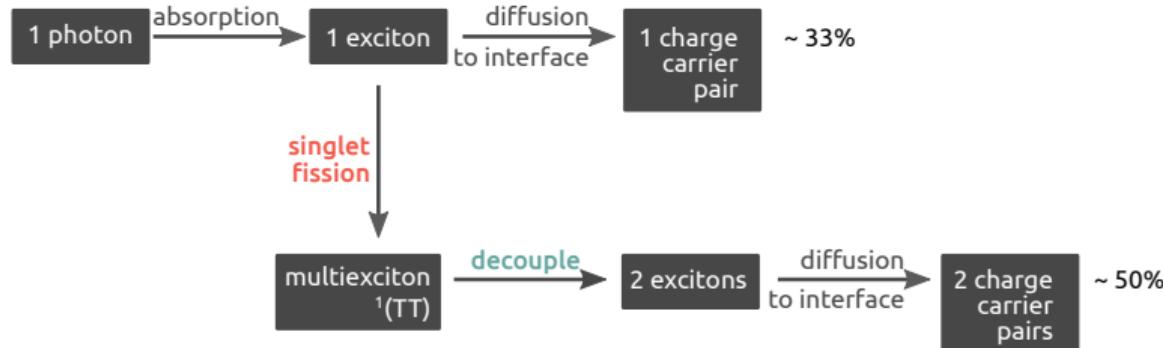
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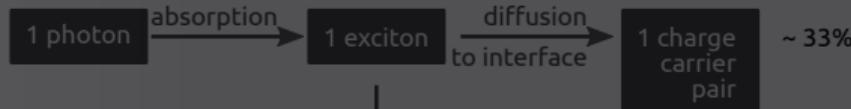
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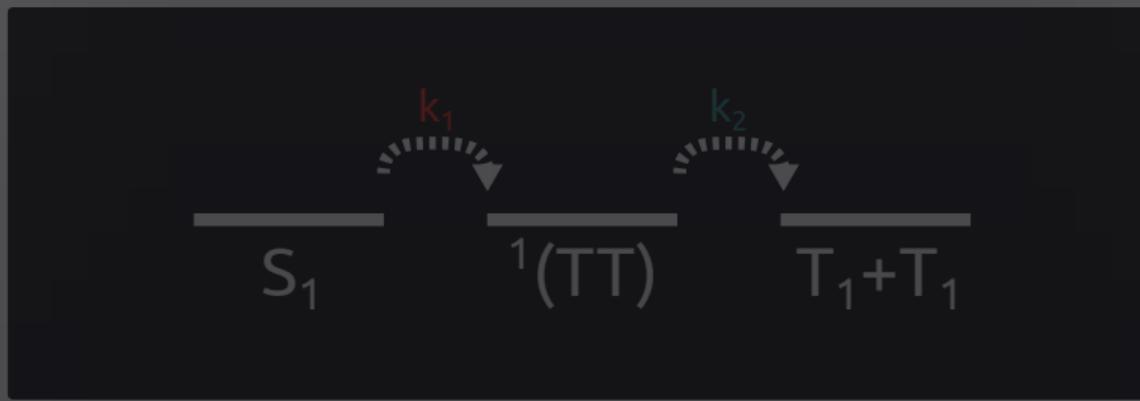
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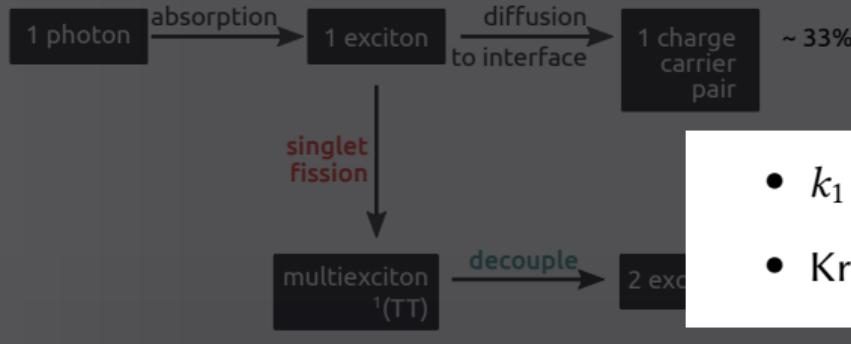
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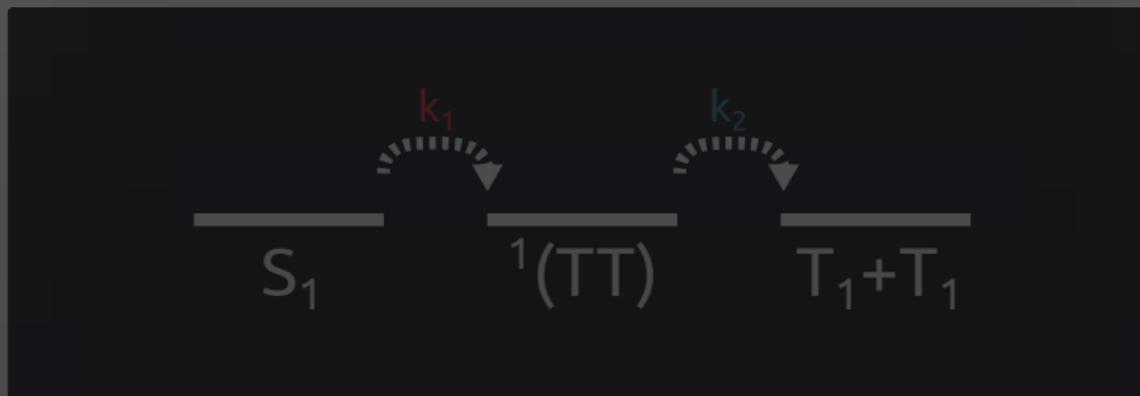
- $k_1$  has been studied extensively



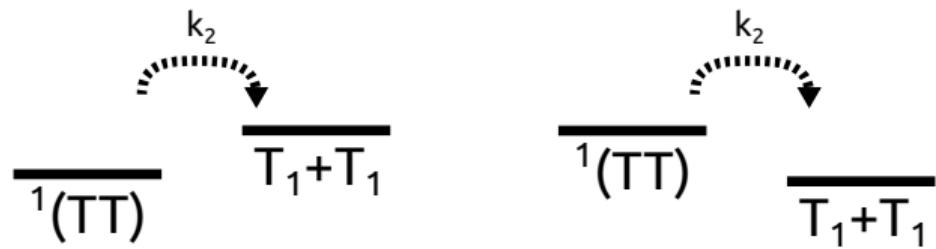
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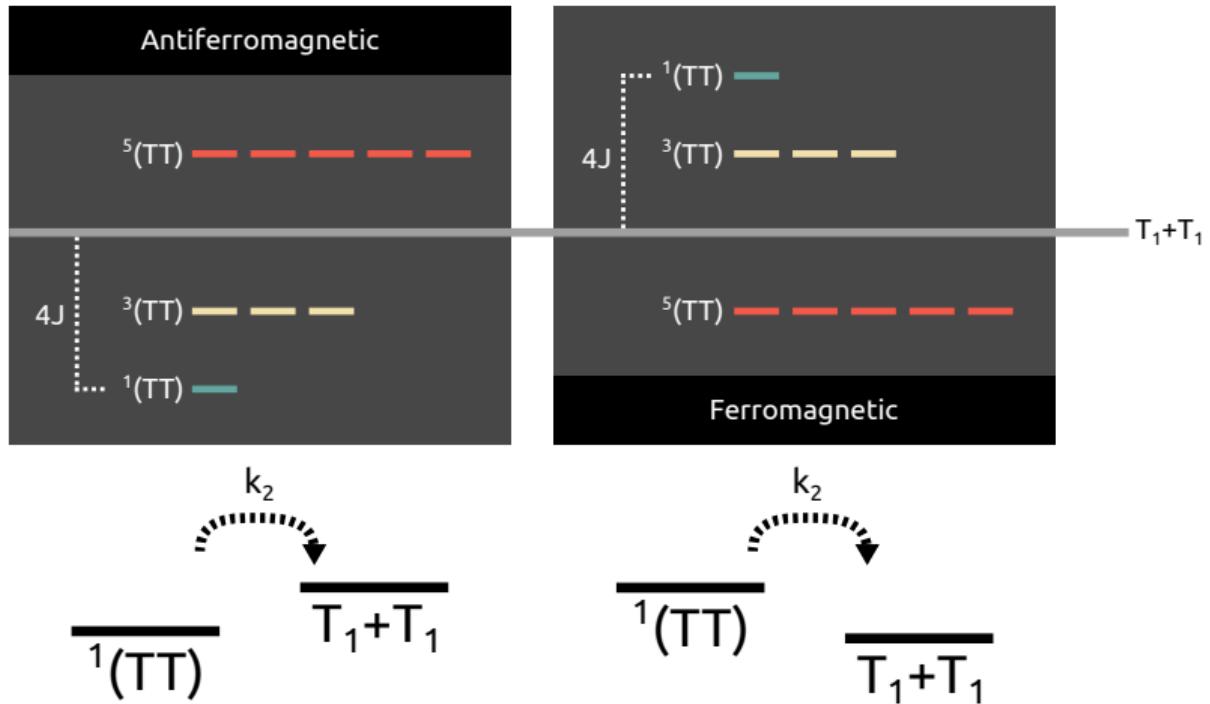
- $k_1$  has been studied extensively
- Krylov's kinetic model:  $k_2 \approx e^{\frac{E_b}{2k_B T}}$



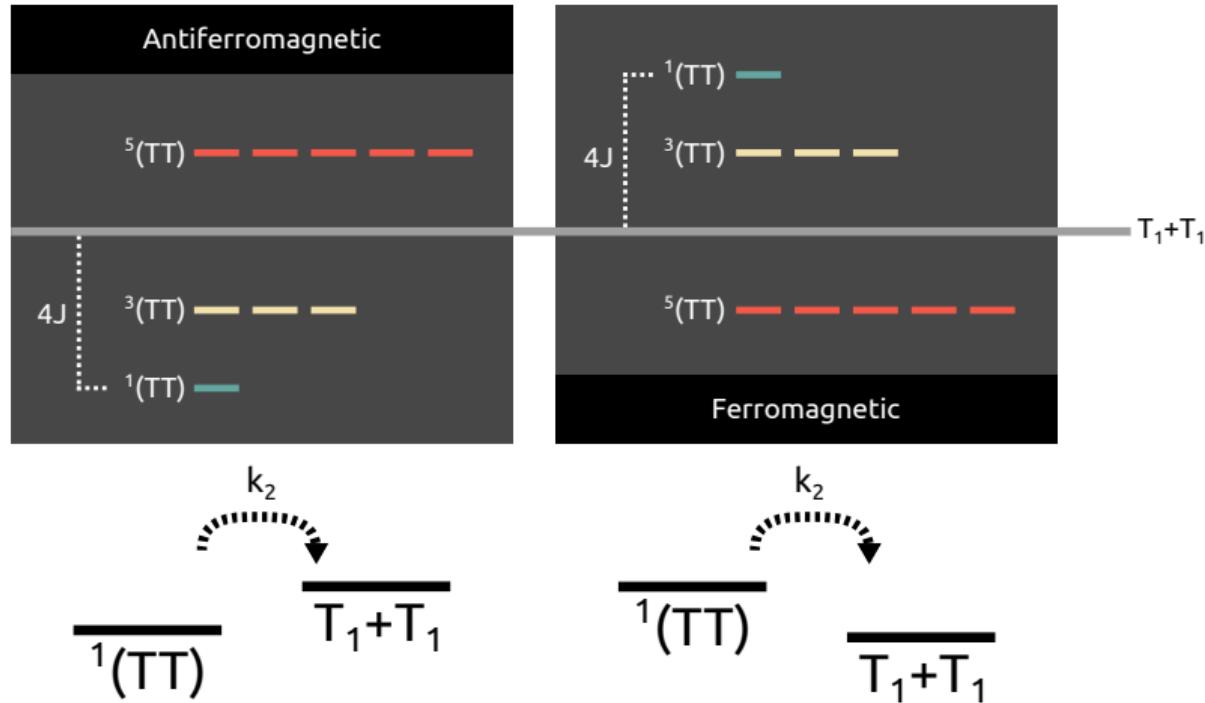
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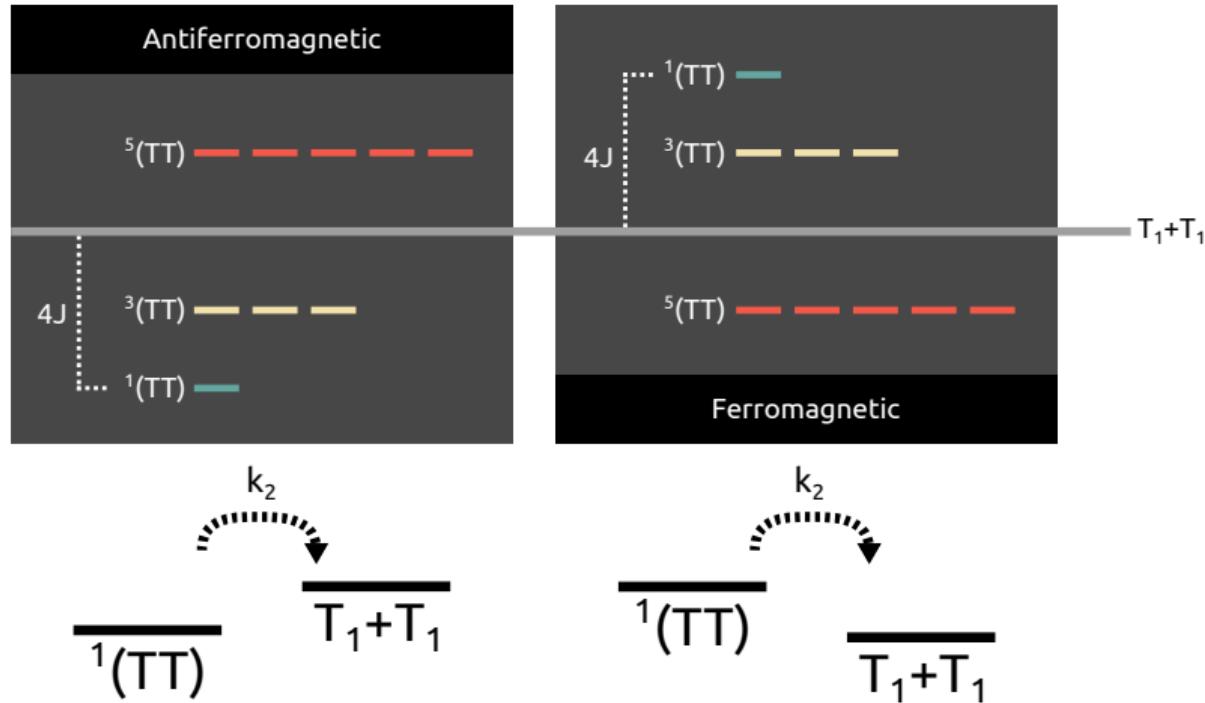


# Singlet Fission



- 2-Spin-flip methods (RAS-2SF) very well suited

# Singlet Fission



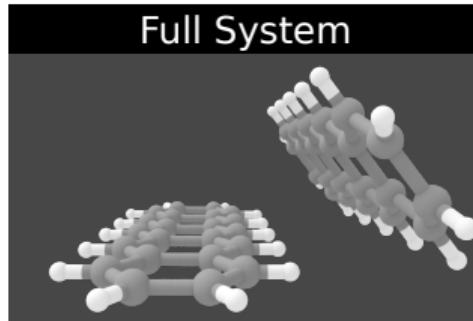
- 2-Spin-flip methods (RAS-2SF) very well suited
- However, the cost grows exponentially with active-space size

# Approximation: Spin lattice

- Make approximation:  $\hat{H} \approx -2 \sum_{AB} J_{AB} \hat{S}_A \hat{S}_B$
- Much simpler to solve
- Provides deeper conceptual insight
- We have recently mapped 1SF calculations to a spin-Hamiltonian

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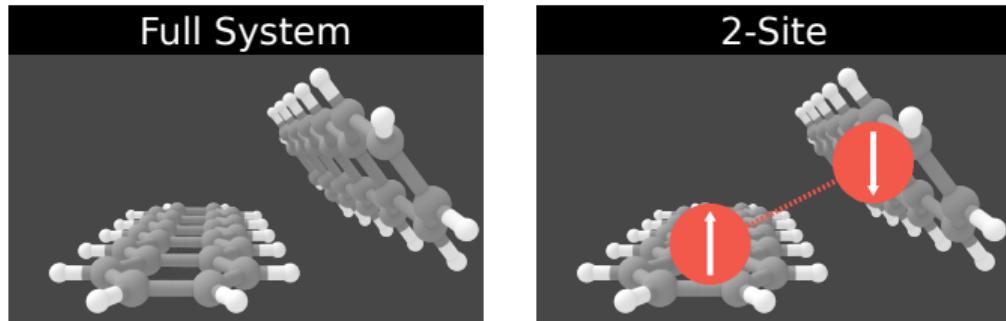
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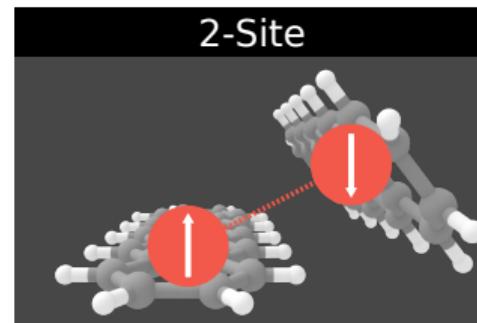
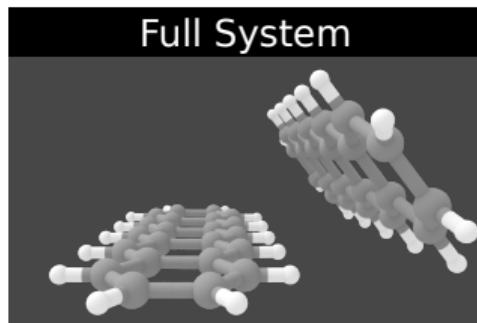
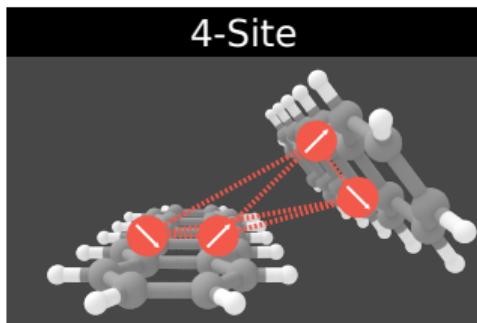
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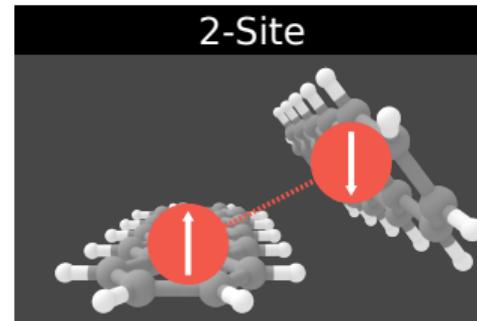
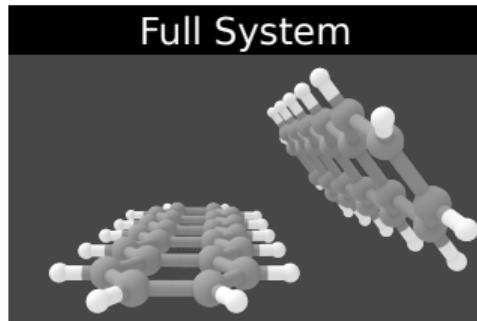
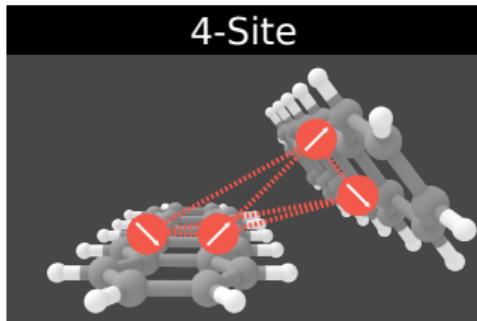
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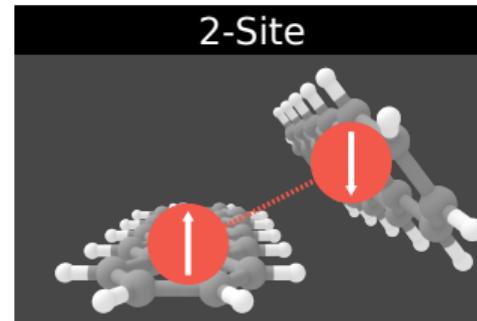
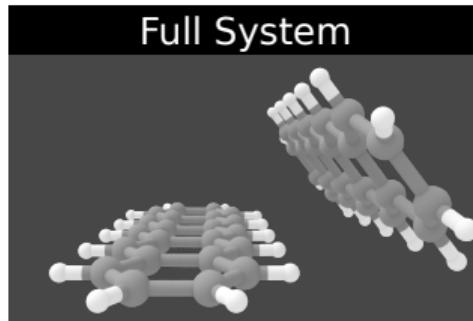
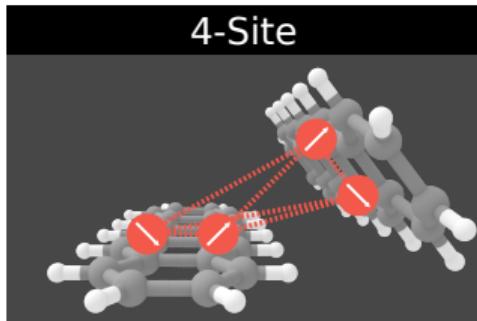
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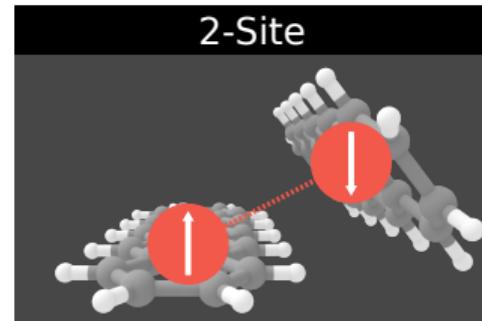
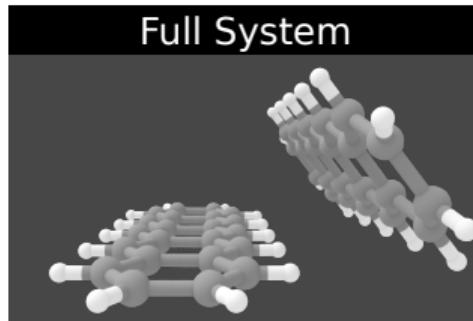
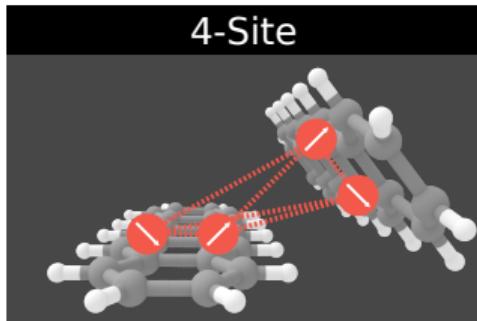
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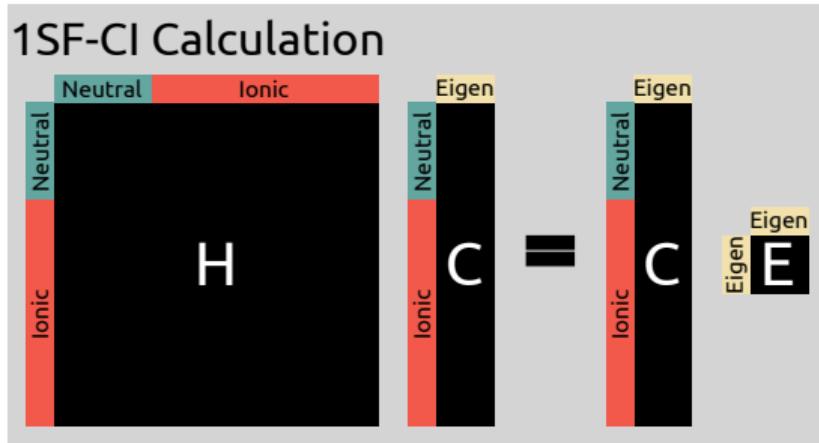
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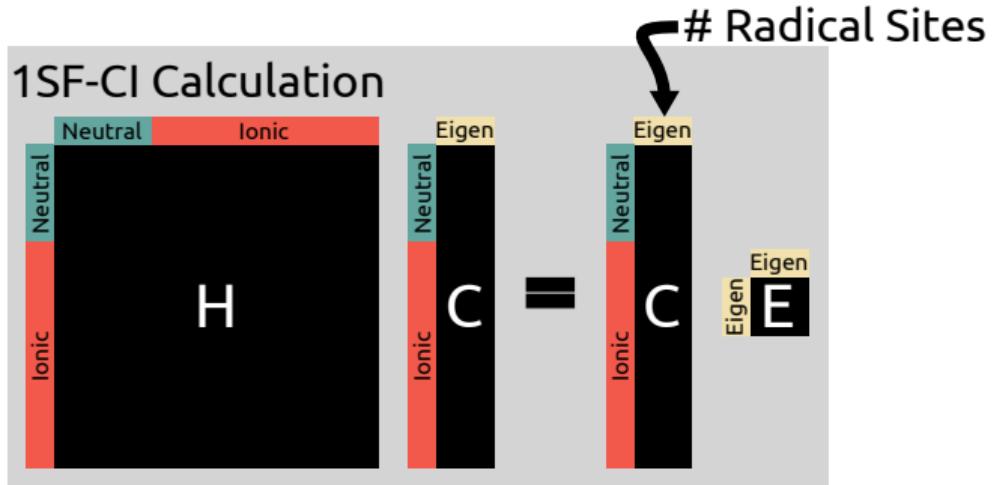


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# Extracting the Spin Hamiltonian



# Extracting the Spin Hamiltonian



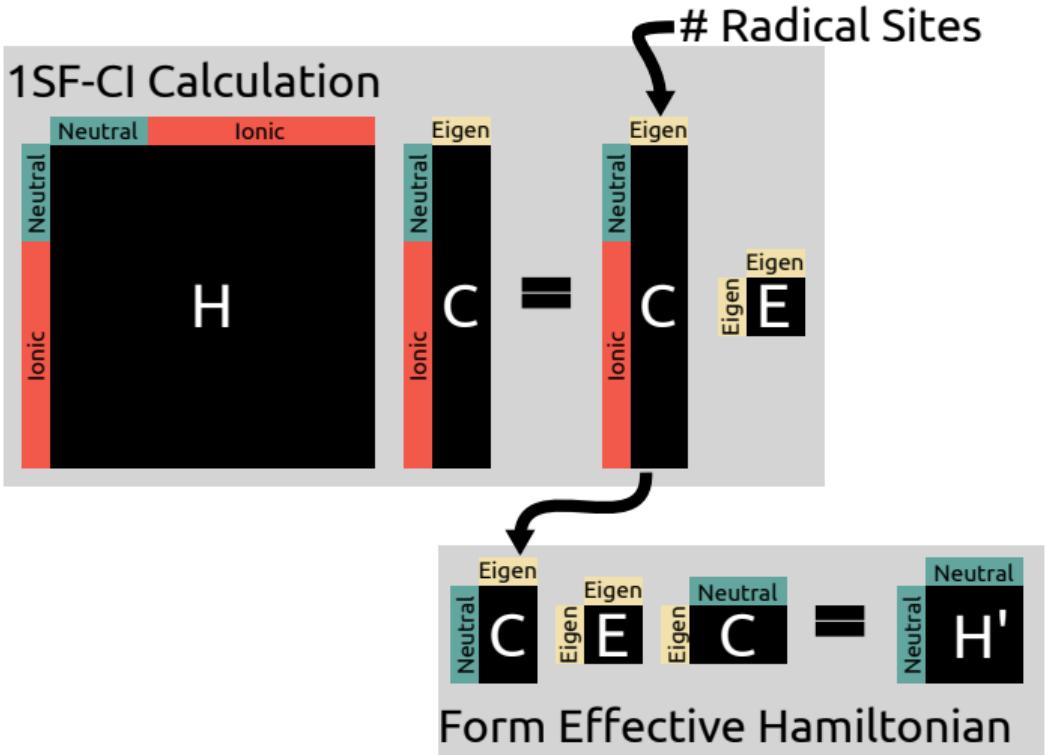
## Extracting the Spin Hamiltonian

The diagram illustrates the 1SF-CI calculation for three molecules: H, C, and E. Each molecule is represented by a vertical bar divided into three horizontal regions: Neutral (top), Ionic (middle), and Eigen (bottom). The Neutral region is teal, the Ionic region is red, and the Eigen region is yellow.

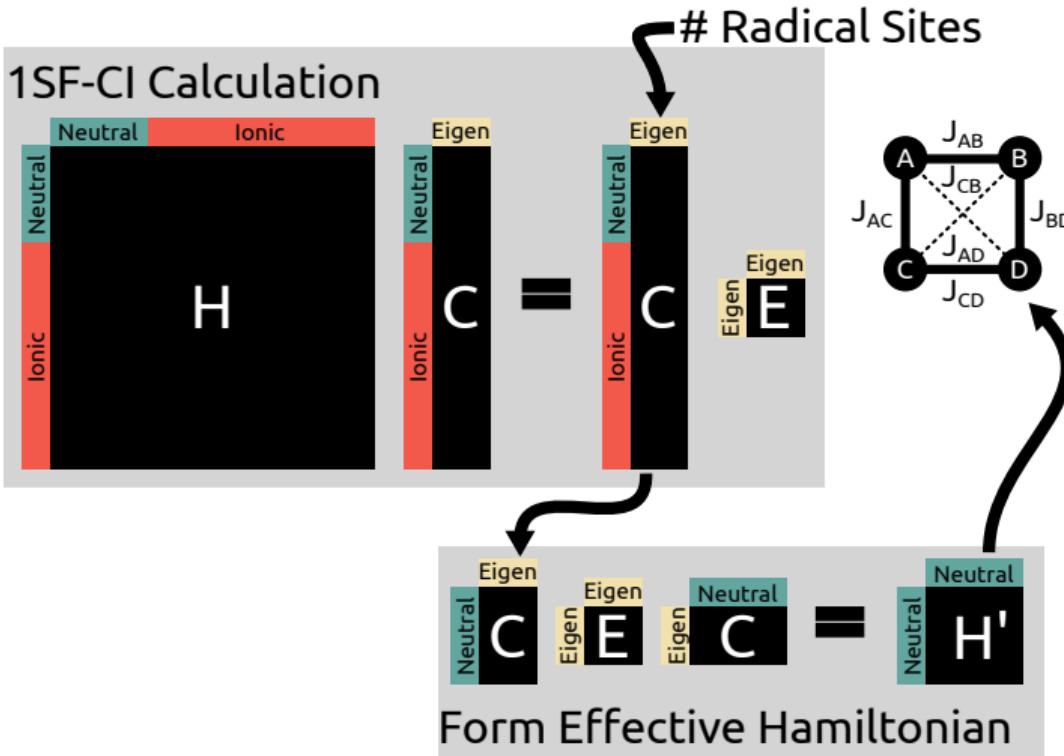
- Molecule H:** A single vertical bar representing hydrogen. It has a small teal Neutral region at the top, a large red Ionic region in the middle, and a small yellow Eigen region at the bottom. The letter 'H' is centered in the red Ionic region.
- Molecule C:** A vertical bar representing carbon. It has a small teal Neutral region at the top, a medium red Ionic region in the middle, and a large yellow Eigen region at the bottom. The letter 'C' is centered in the red Ionic region.
- Molecule E:** A vertical bar representing ethene. It has a small teal Neutral region at the top, a medium red Ionic region in the middle, and a large yellow Eigen region at the bottom. The letters 'E' and 'C' are stacked vertically in the yellow Eigen region.

Arrows point from the Eigen regions of the C and E molecules to a separate vertical bar representing the radical form of carbon, labeled 'Eigen C'. This indicates that the radical sites are located in the Eigen regions of the neutral and ionic components of the molecule.

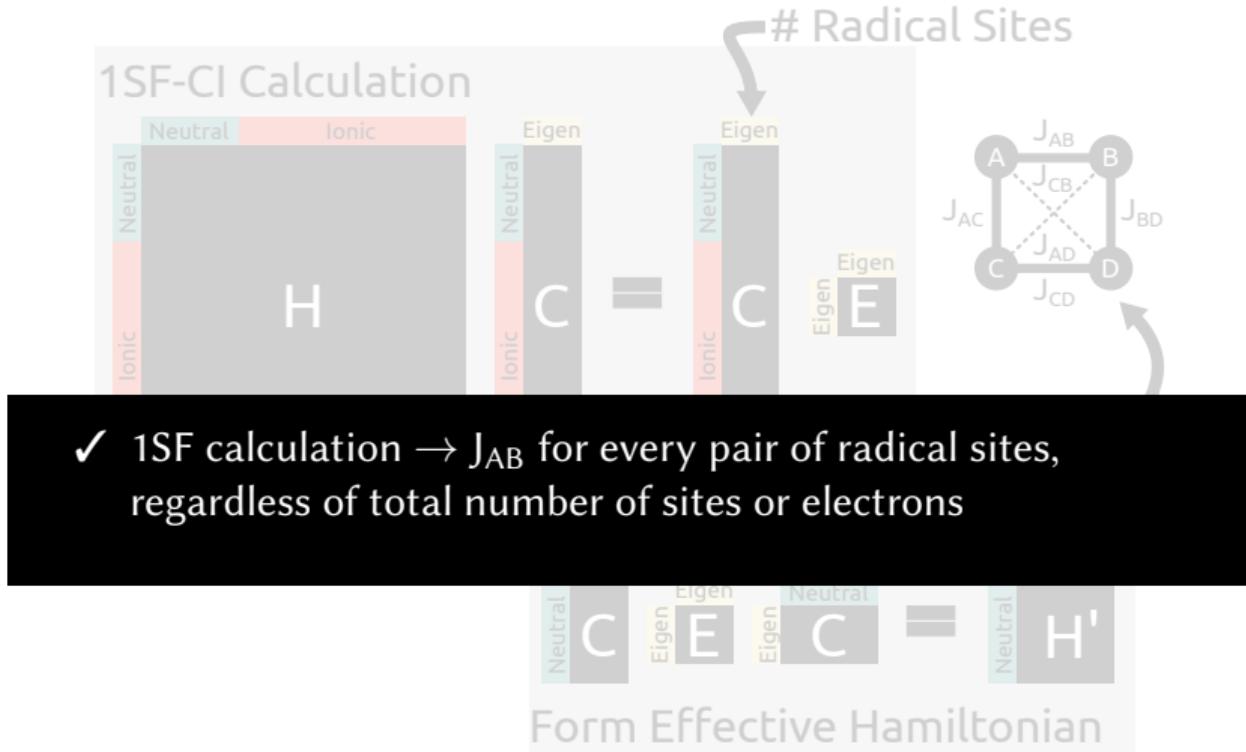
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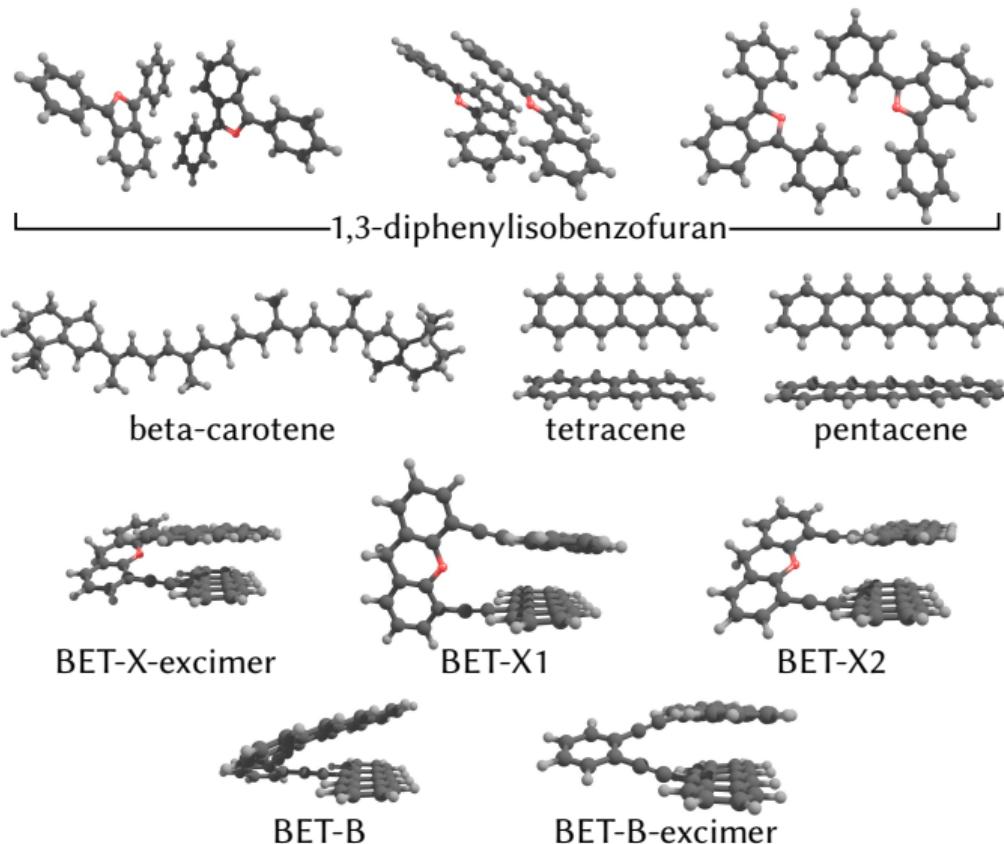
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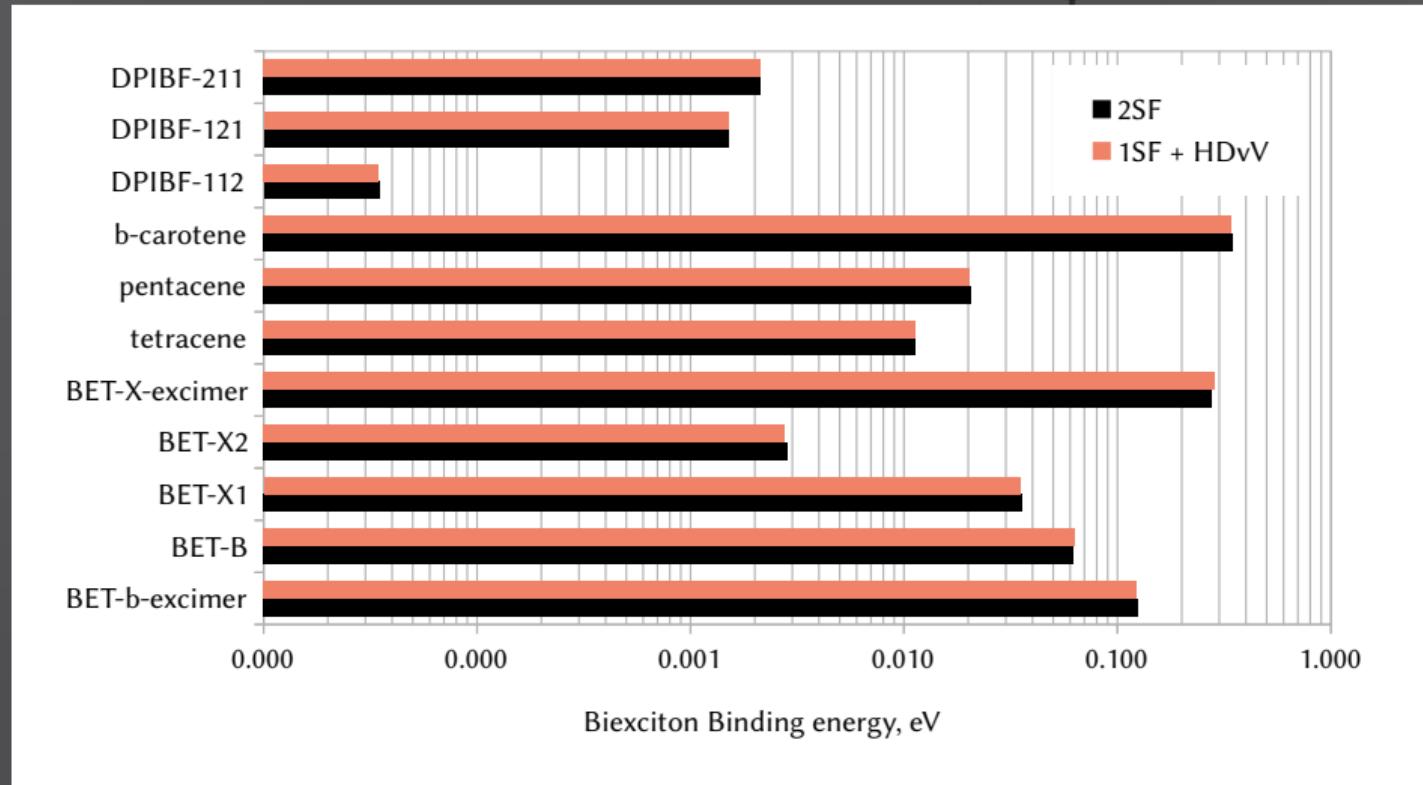
# Extracting the Spin Hamiltonian



Does this even work?

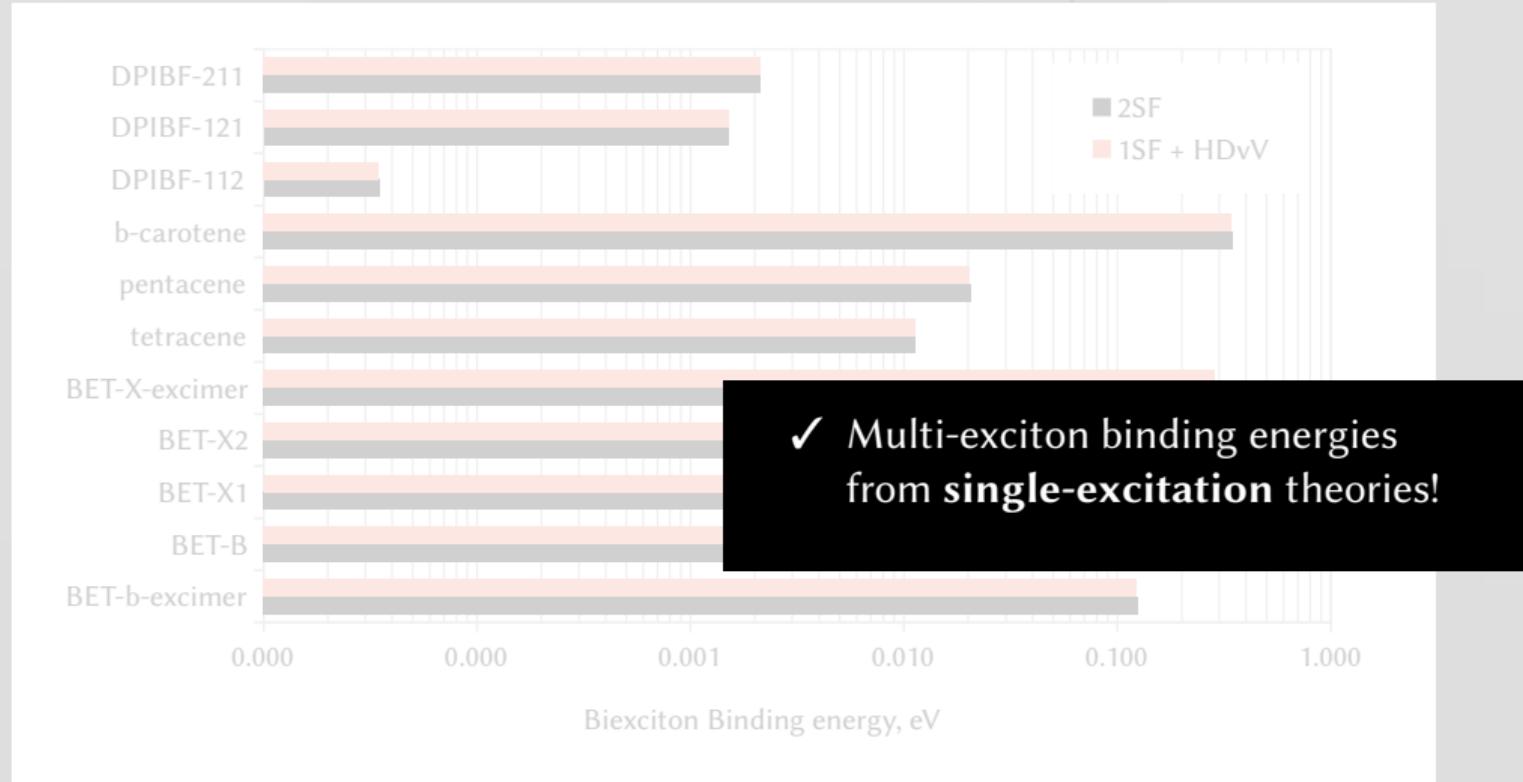


# Does this even work?



BET-B                    BET-B-excimer

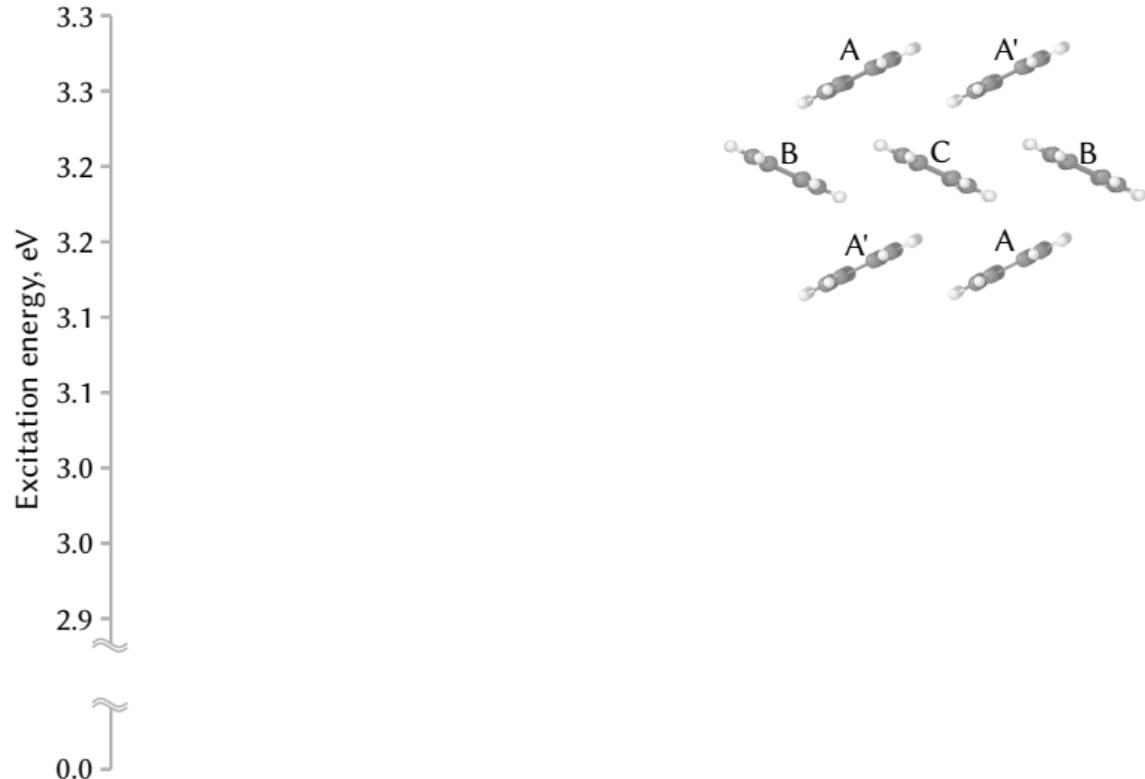
# Does this even work?



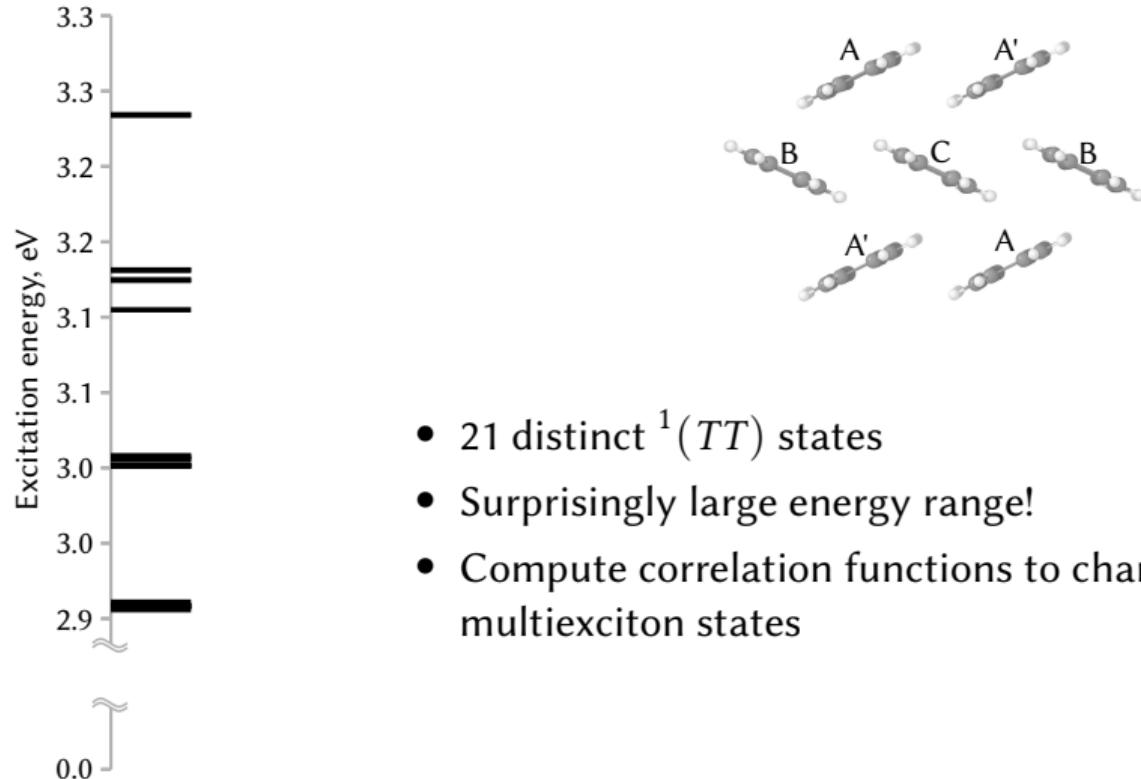
BET-B

BET-B-excimer

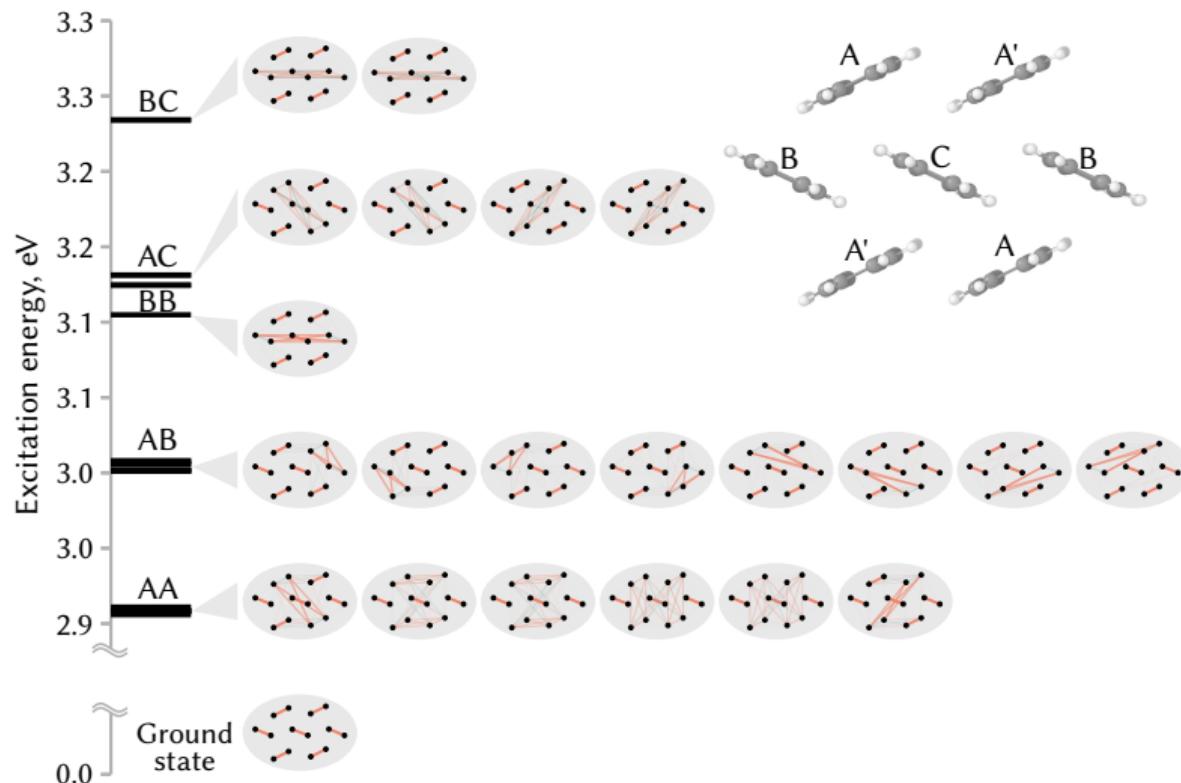
# Multiexcitons in molecular materials



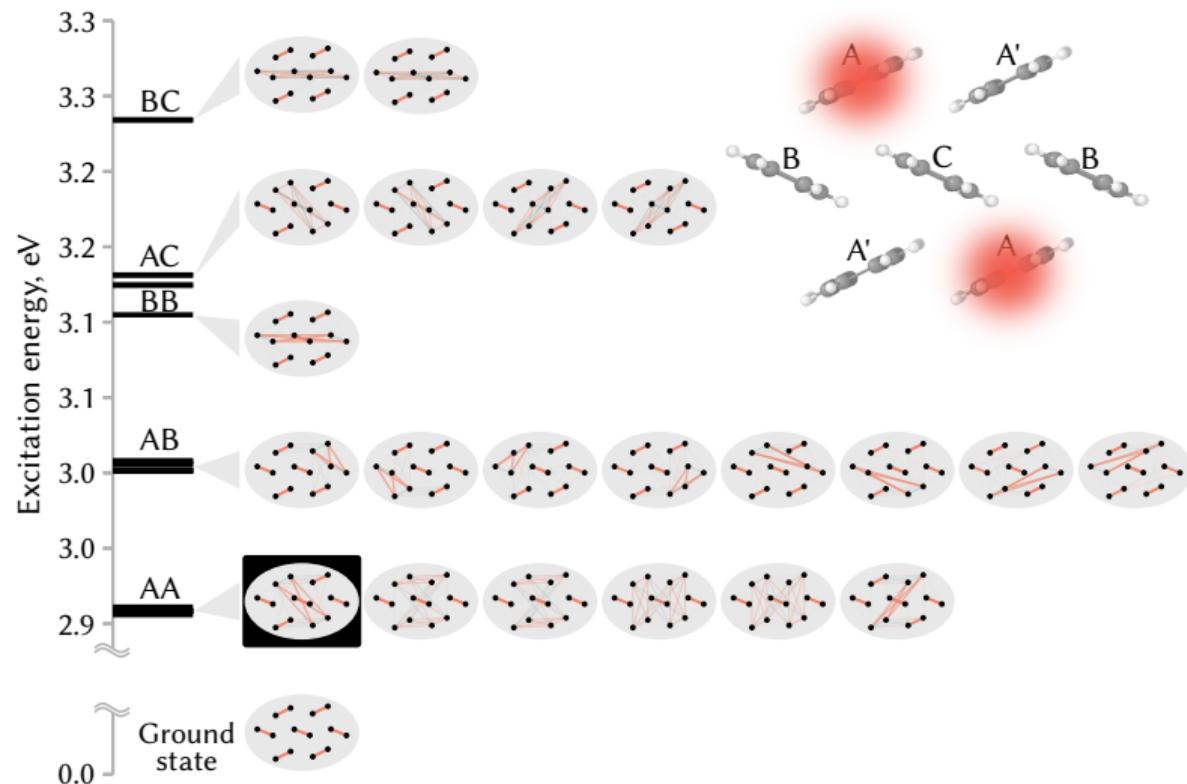
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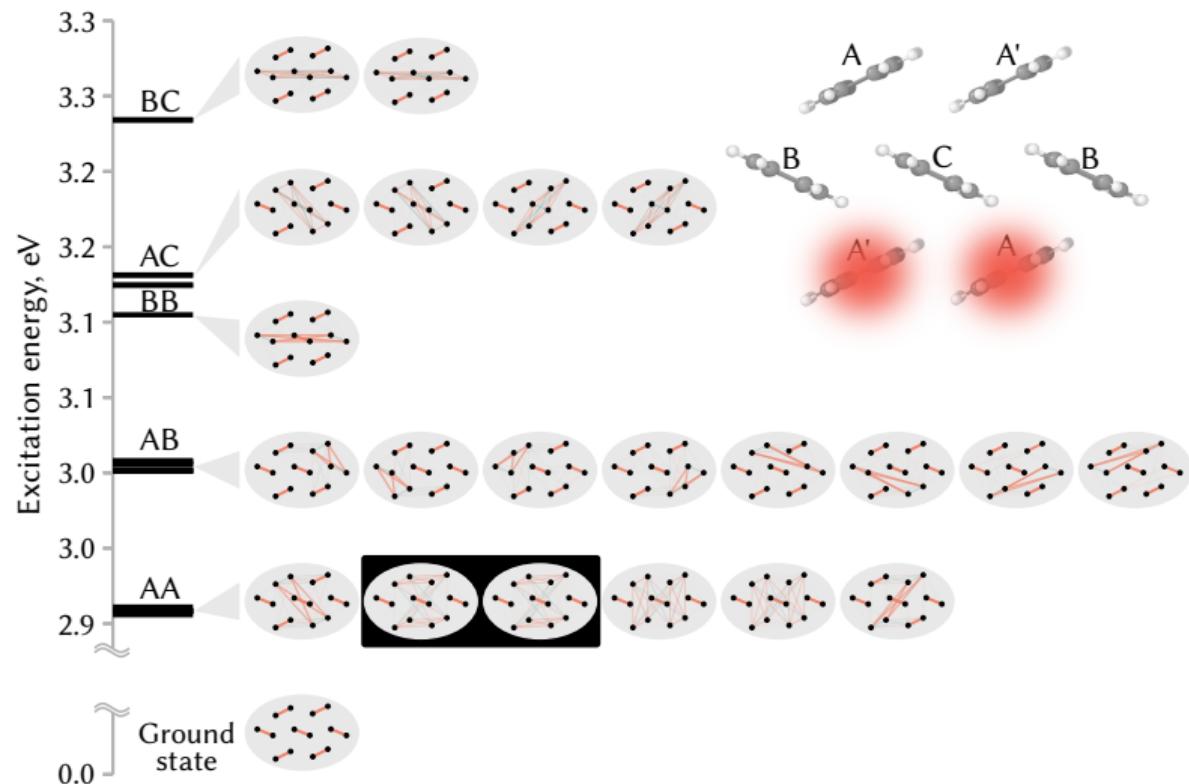
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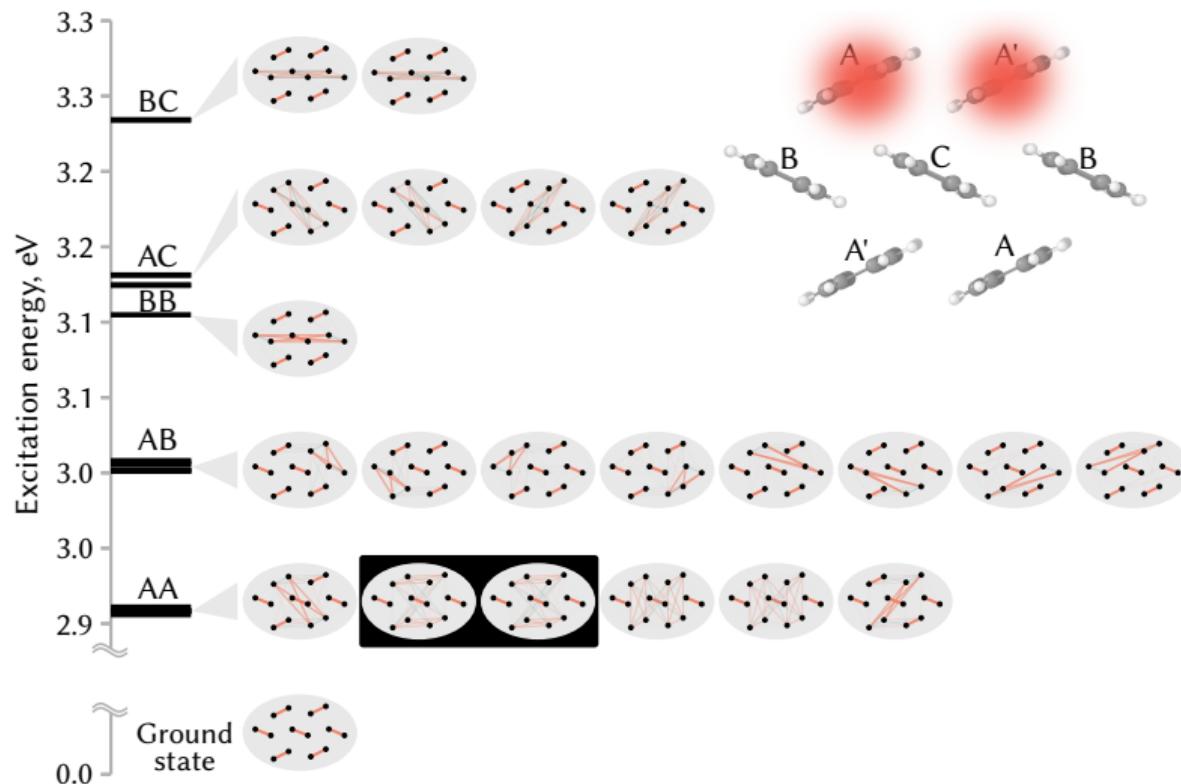
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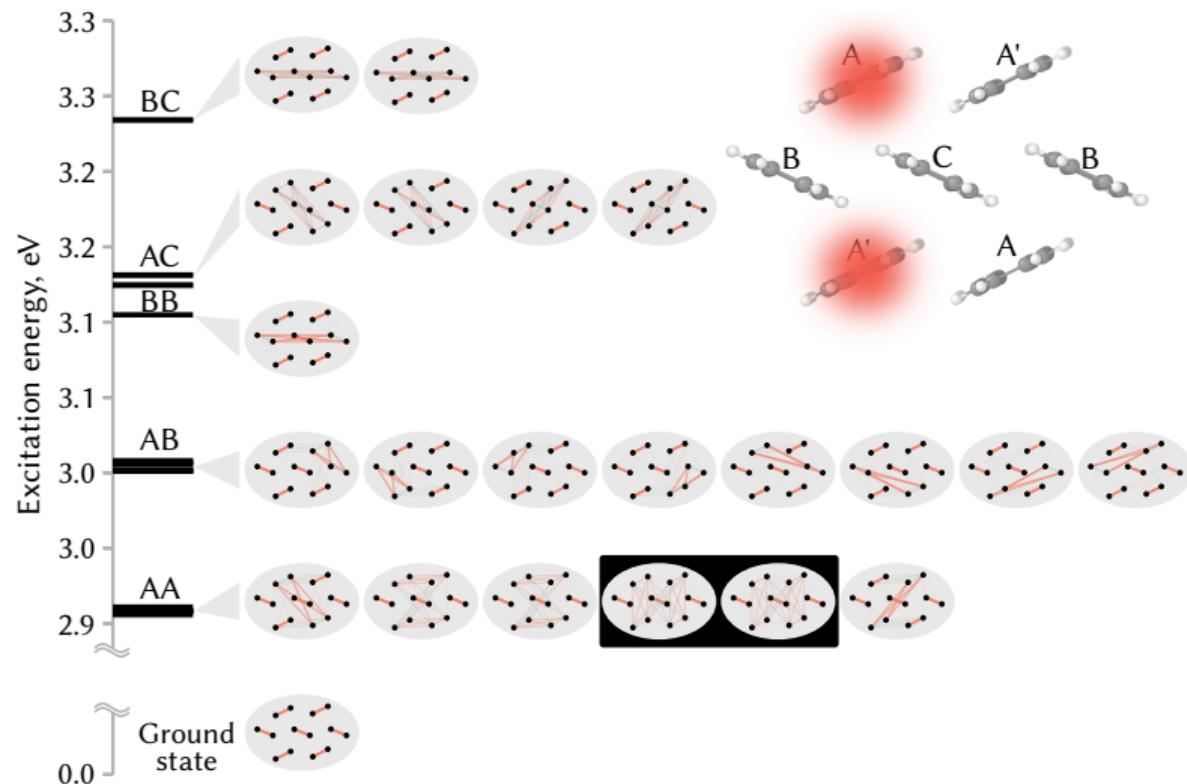
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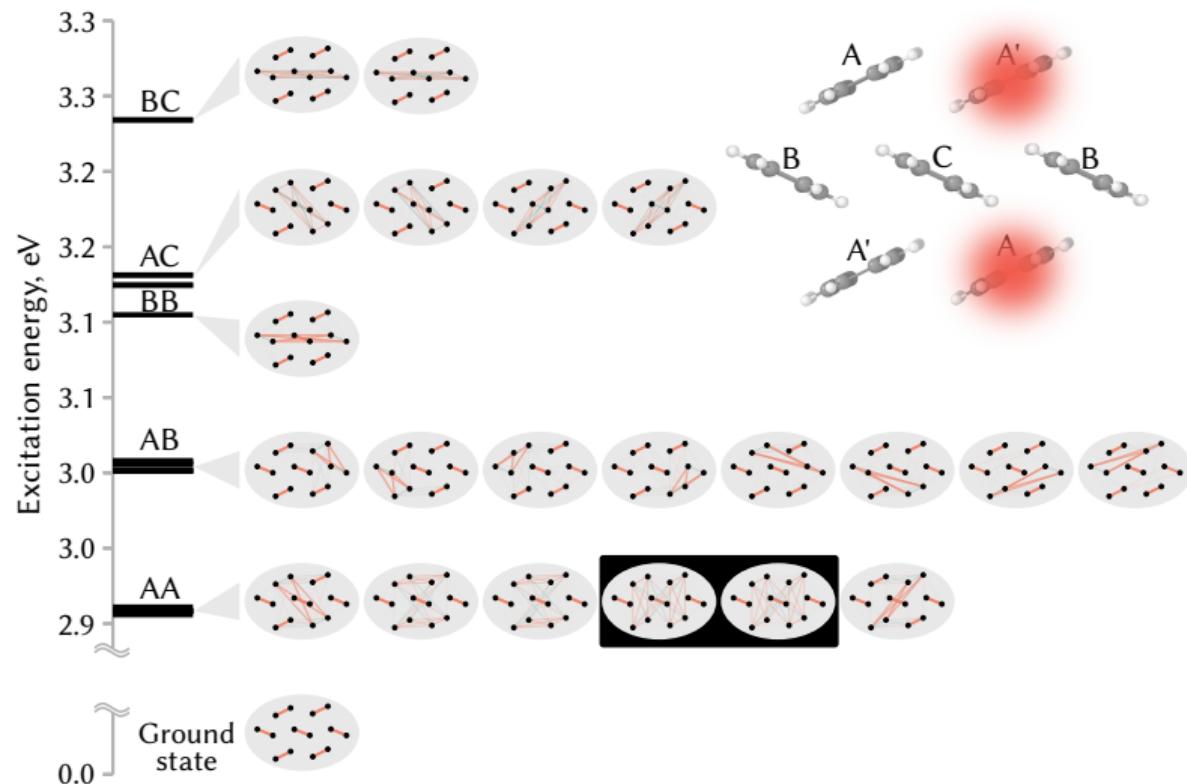
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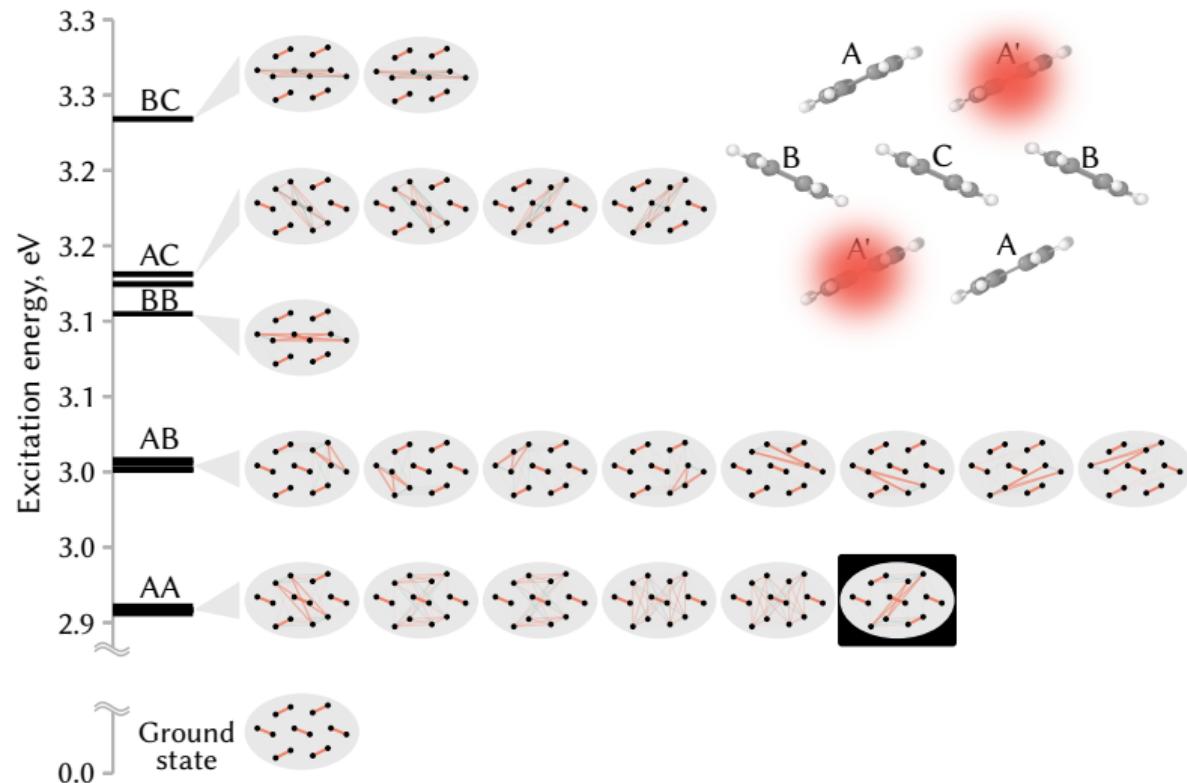
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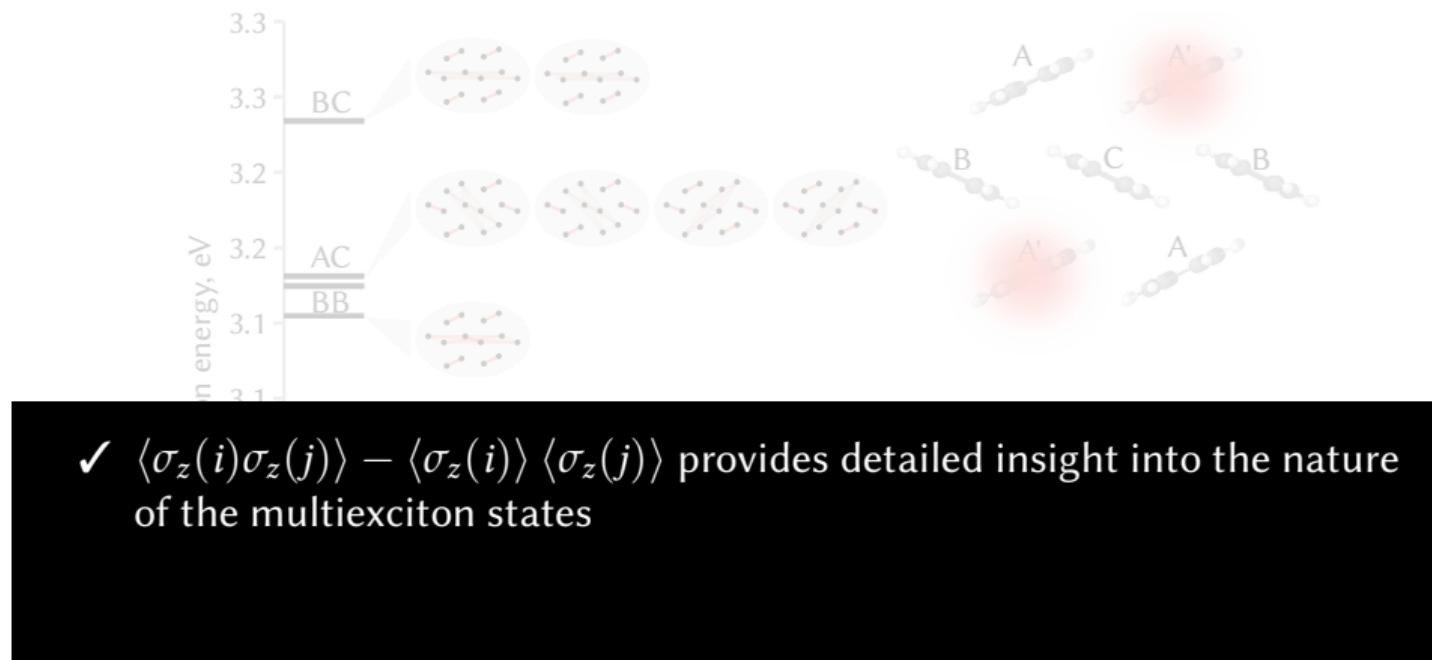
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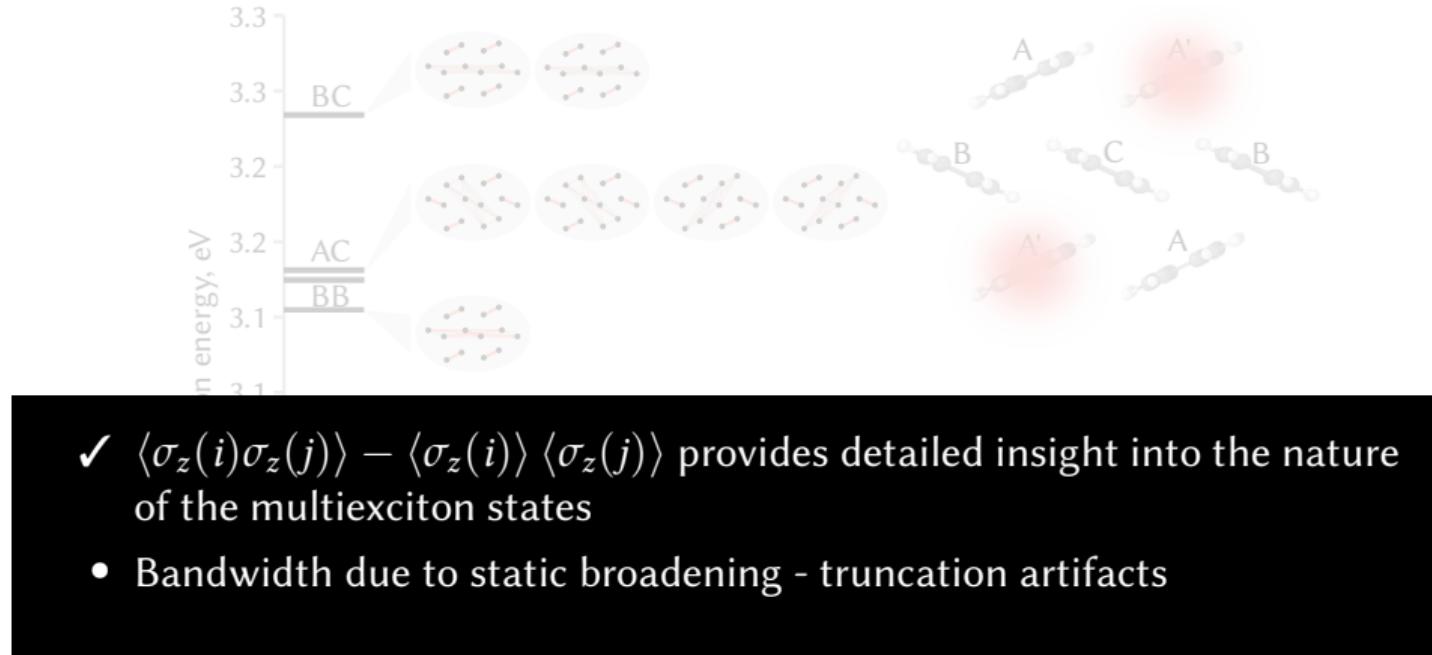
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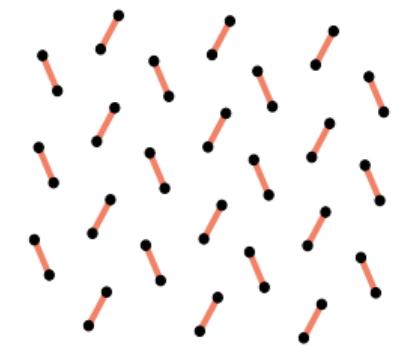


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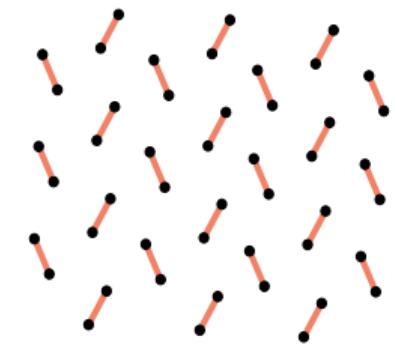
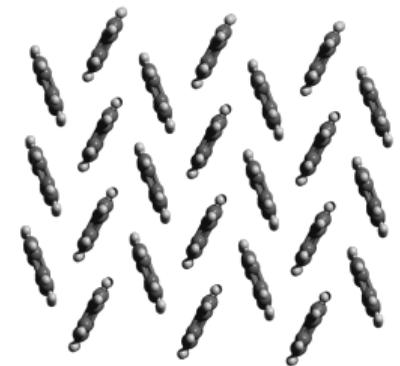
# Multiexcitons in molecular materials : Larger clusters

- Use ab initio parameters for systems too large for ab initio



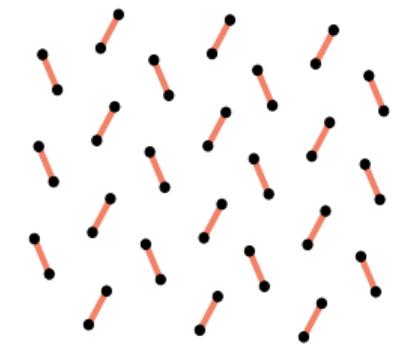
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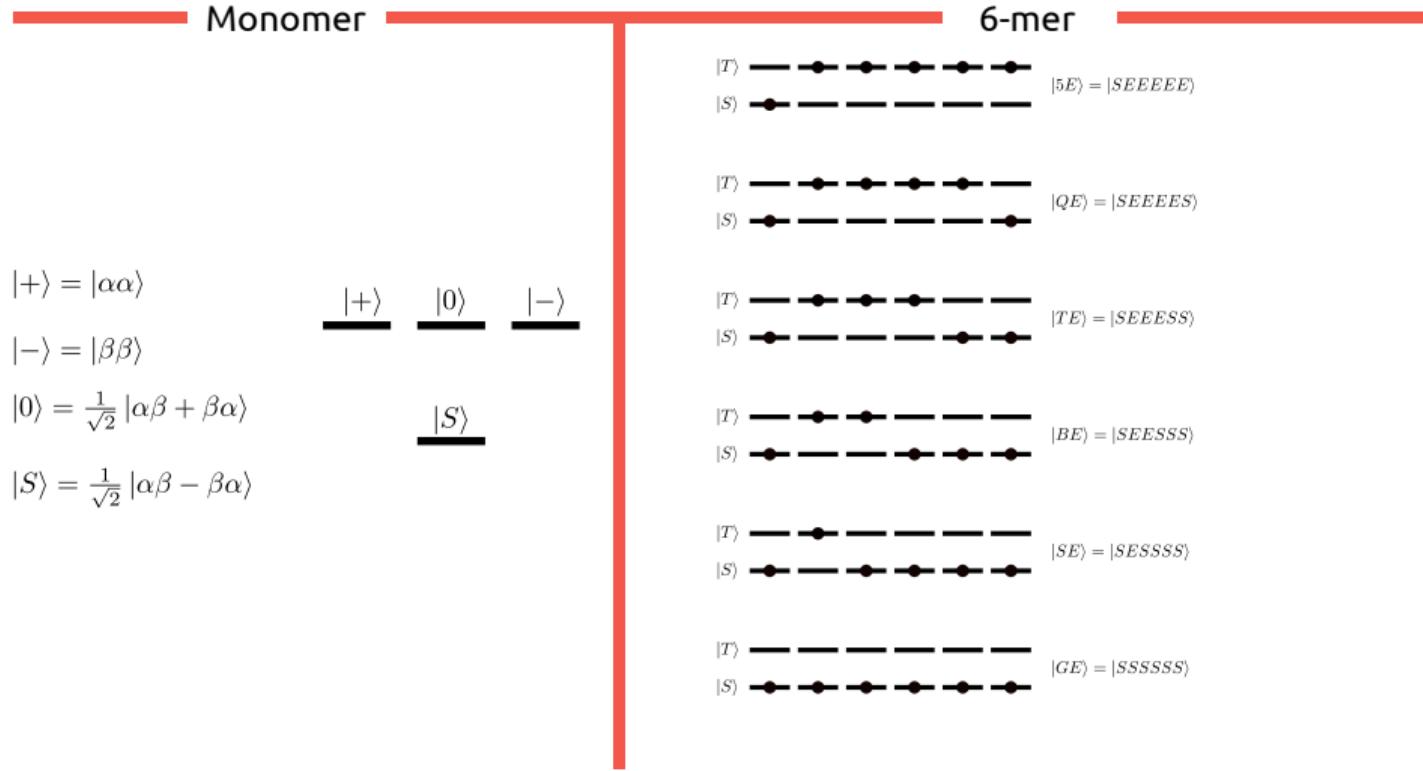


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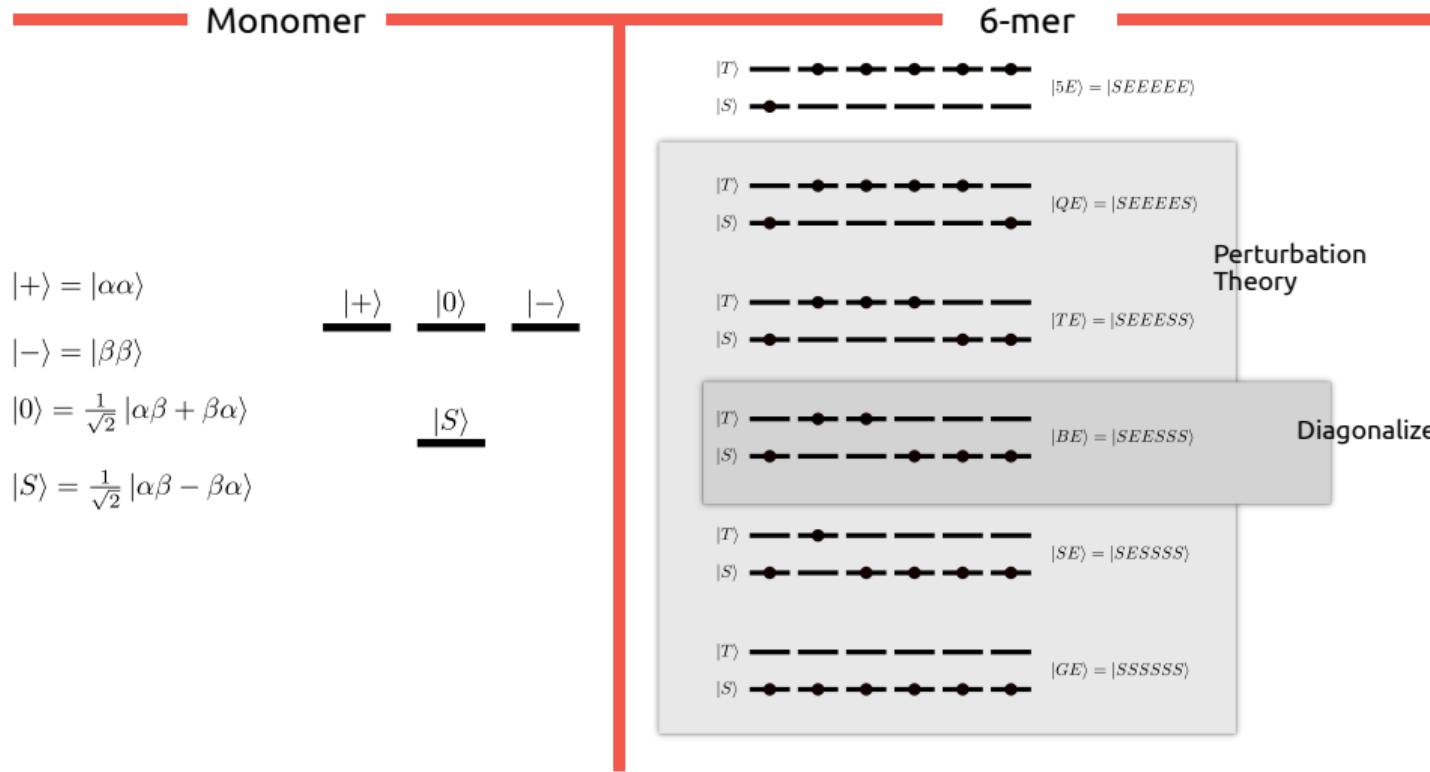
- Use ab initio parameters for systems too large for ab initio
- Spin Lattice *still* exponentially scaling
- Use our recently developed  $n$ -Body Tucker decomposition
  - Talk tomorrow: COMP 487



# Multiexcitons in molecular materials : Biexciton-Perturbation theory

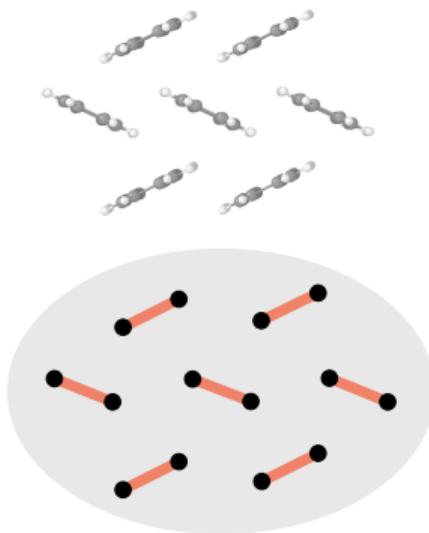


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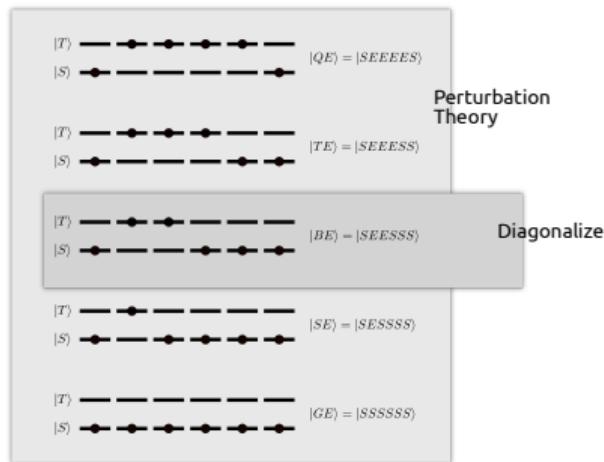
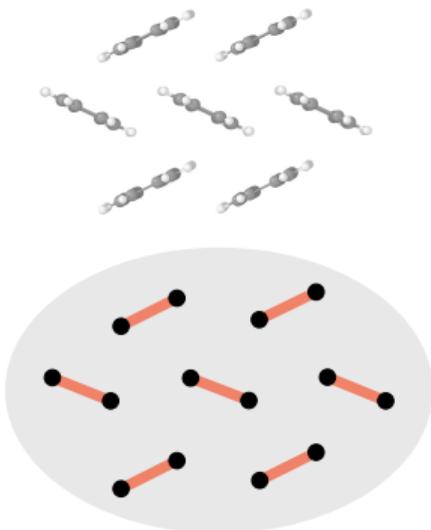
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7-mer



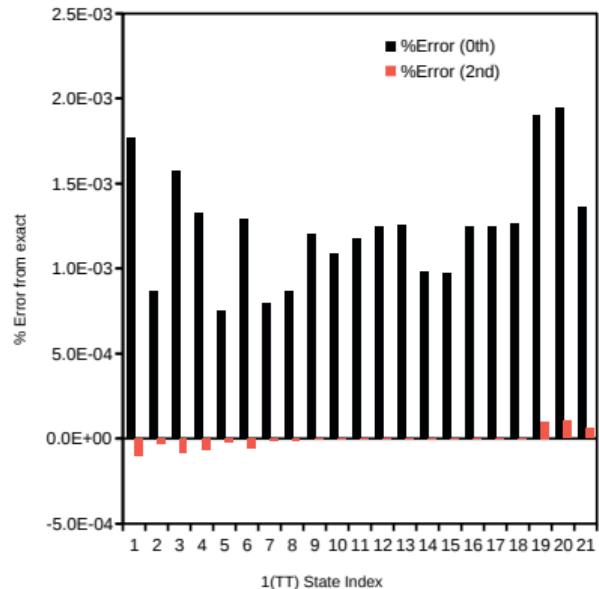
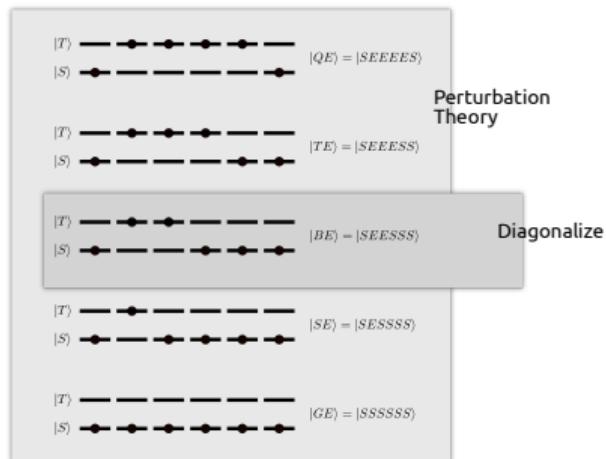
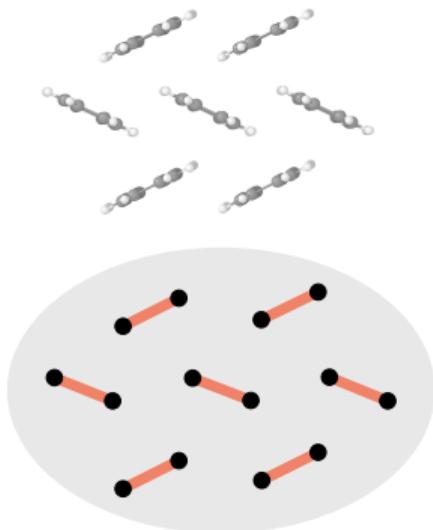
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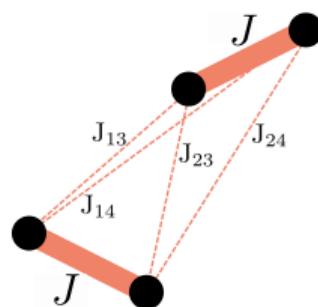


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7-mer



# Multiexcitons in molecular materials : Biexciton-Perturbation theory



# Multiexcitons in molecular materials : Biexciton-Perturbation theory

$$H = \begin{array}{c|ccc|ccc} \langle SS | & 6E & 0 & 0 & -t & t & -t \\ \langle S0 | & 0 & 2E & t & p & 0 & -p \\ \langle 0S | & 0 & t & 2E & q & 0 & -q \\ \hline \langle +- | & -t & p & q & -r - 2E & r & 0 \\ \langle 00 | & t & 0 & 0 & r & -2E & r \\ \hline \langle -+ | & -t & -p & -q & 0 & r & -r - 2E \end{array}$$

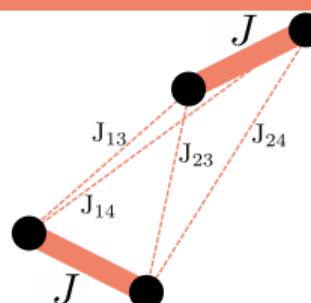
$$E = 0.5 \times J$$

$$r = 0.5 \times (J_{13} + J_{14} + J_{23} + J_{24})$$

$$t = 0.5 \times (J_{13} - J_{14} - J_{23} + J_{24})$$

$$p = 0.5 \times (J_{13} + J_{14} - J_{23} - J_{24})$$

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Block Diagonalize Biexcitons

$$H = \begin{array}{c|cc|ccc} \langle SS | & 6E & 0 & 0 & 0 & -\sqrt{3}t & 0 \\ \langle S0 | & 0 & 2E & t & 0 & 0 & -\sqrt{2}p \\ \langle 0S | & 0 & t & 2E & 0 & 0 & -\sqrt{2}q \\ \hline {}^5(\text{TT}) & 0 & 0 & 0 & r - 2E & 0 & 0 \\ {}^1(\text{TT}) & -\sqrt{3}t & 0 & 0 & 0 & -2E - 2r & 0 \\ {}^3(\text{TT}) & 0 & -\sqrt{2}p & -\sqrt{2}q & 0 & 0 & -r - 2E \end{array}$$

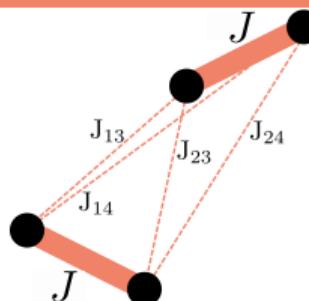
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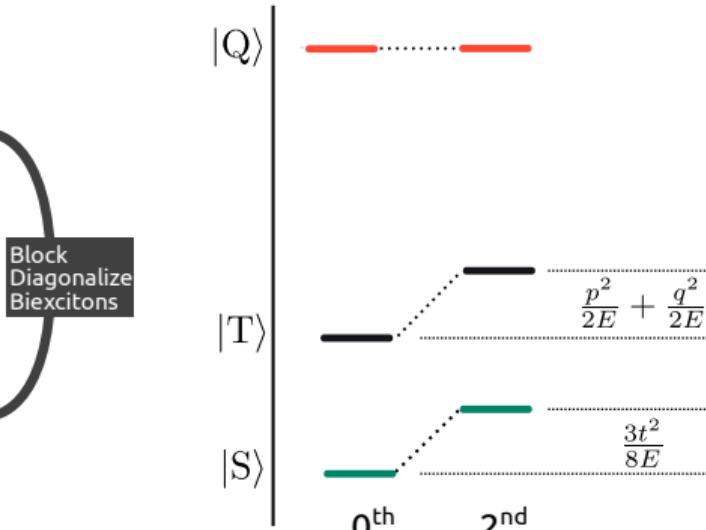
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$$q = 0.5 \times (J_{13} - J_{14} + J_{23} - J_{24})$$



# Multiexcitons in molecular materials : Biexciton-Perturbation theory

$$H = \begin{array}{|c|c|c|} \hline & \langle SS | & \langle S0 | & \langle 0S | \\ \hline \langle S0 | & 6E & 0 & 0 \\ \hline \langle 0S | & 0 & 2E & t \\ \hline \langle +- | & 0 & t & 2E \\ \hline \langle 00 | & -t & p & q \\ \hline \langle -+ | & t & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|c|} \hline & -t & t & -t \\ \hline \langle S0 | & p & 0 & -p \\ \hline \langle 0S | & q & 0 & -q \\ \hline \langle +- | & -r - 2E & r & 0 \\ \hline \langle 00 | & r & -2E & r \\ \hline \langle -+ | & 0 & r & -r - 2E \\ \hline \end{array}$$

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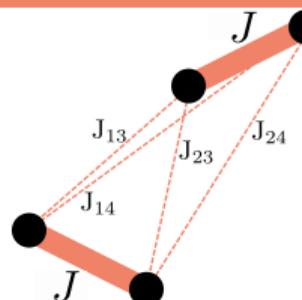
$$E = 0.5 \times J$$

$$r = 0.5 \times (J_{13} + J_{14} + J_{23} + J_{24})$$

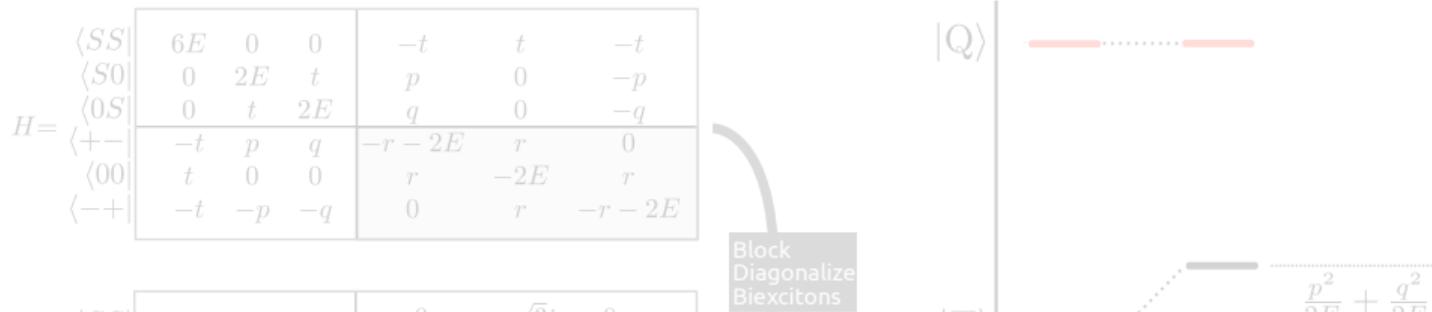
$$t = 0.5 \times (J_{13} - J_{14} - J_{23} + J_{24})$$

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# Multiexcitons in molecular materials : Biexciton-Perturbation theory

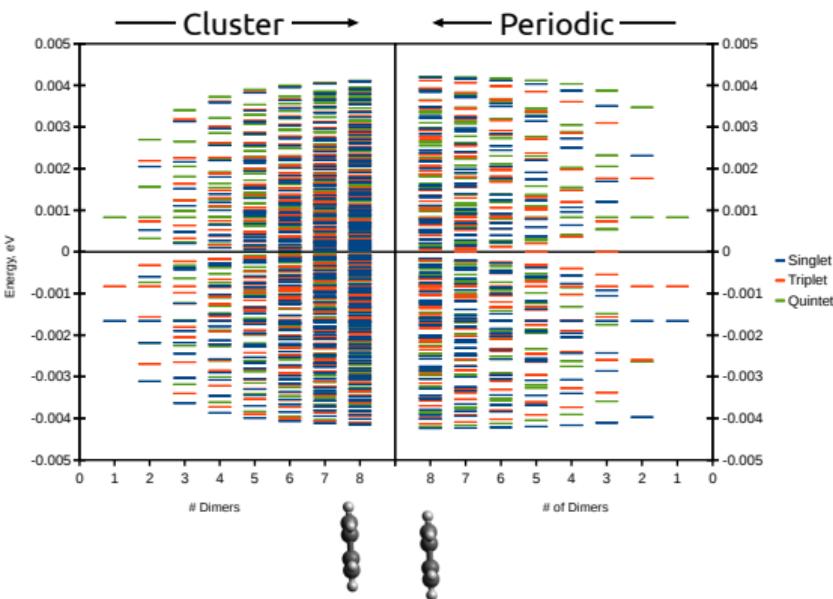
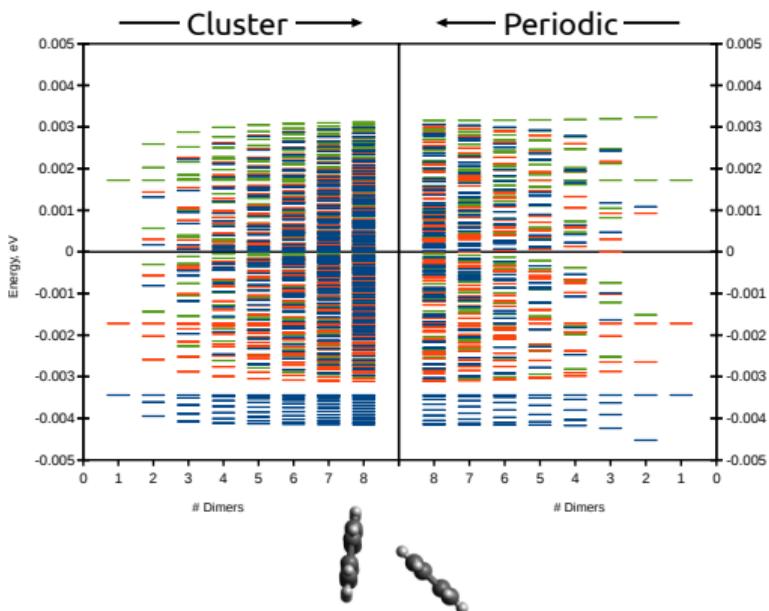


- Zeroth-order identical to simpler 2-site model
- 4-site model gives positional information
- We can understand larger clusters similarly:

$$H = \frac{|{}^1TT(AB)\rangle \quad |{}^1TT(AC)\rangle \quad |{}^1TT(BC)\rangle}{\langle {}^1TT(AB)| \quad -2E - 2r_{AB} \quad t_{BC} \quad t_{AC}} \begin{vmatrix} & & \\ \langle {}^1TT(AC)| & t_{BC} & -2E - 2r_{AC} & t_{AB} \\ \langle {}^1TT(BC)| & t_{AC} & t_{AB} & -2E - 2r_{BC} \end{vmatrix}$$

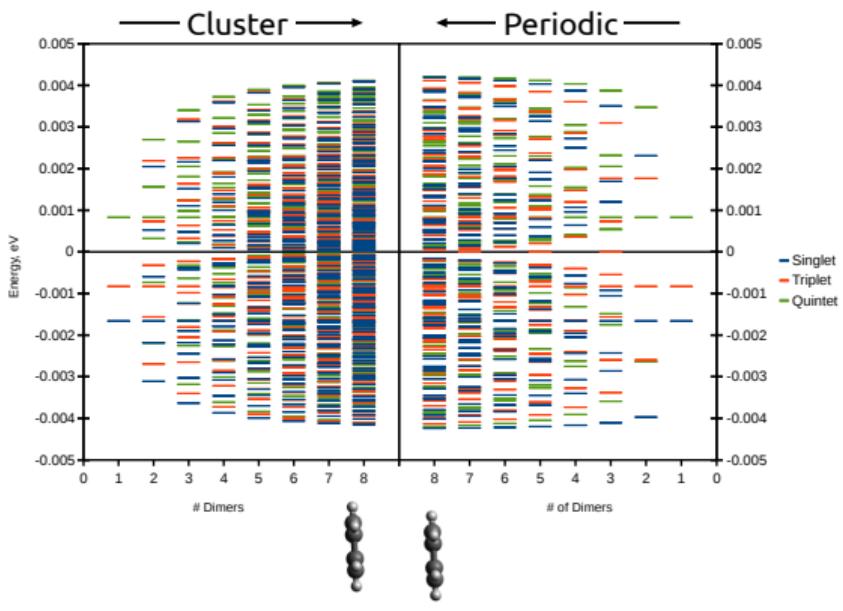
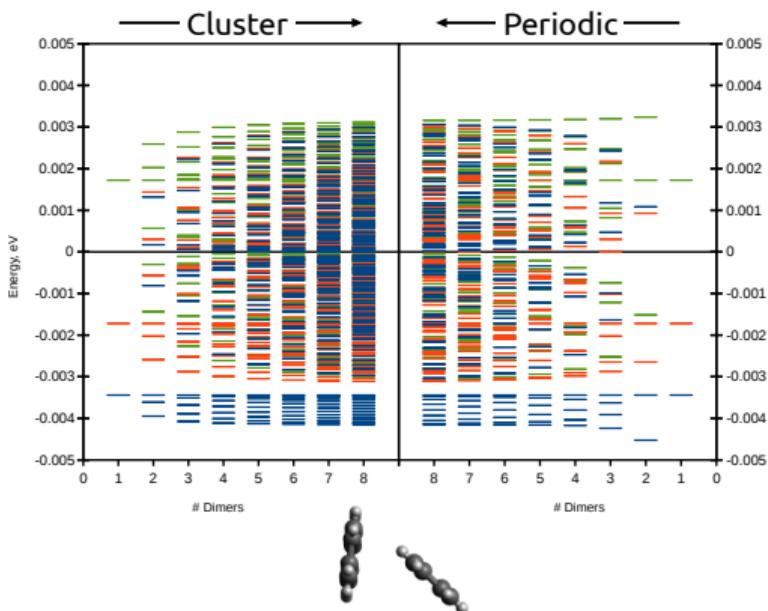
# Multiexcitons in molecular materials : Larger clusters

- Herringbone pattern produces a gapped singlet band at the bottom of the spectrum



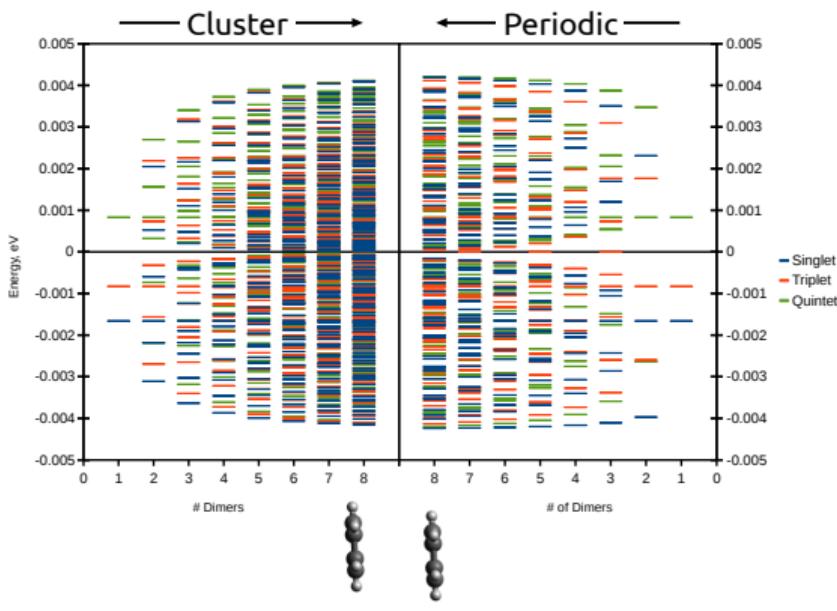
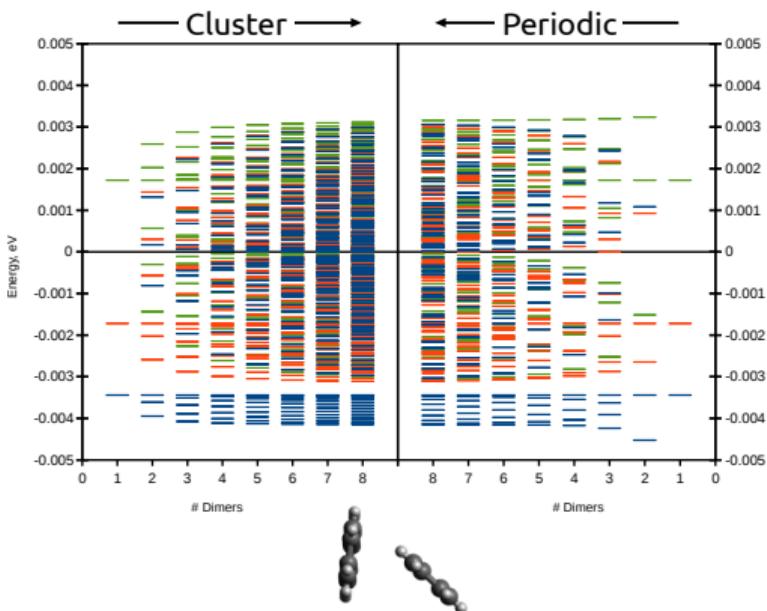
# Multiexcitons in molecular materials : Larger clusters

- Herringbone pattern produces a gapped singlet band at the bottom of the spectrum
- Open and Periodic boundary conditions converge



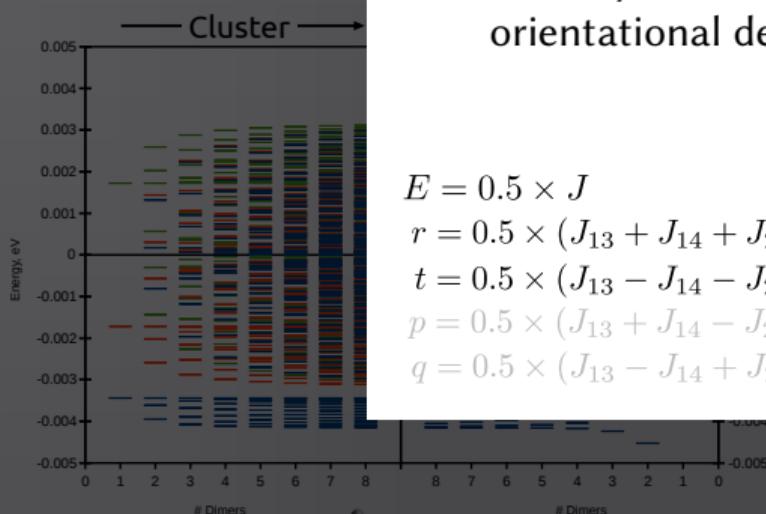
# Multiexcitons in molecular materials : Larger clusters

- Herringbone pattern produces a gapped singlet band at the bottom of the spectrum
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- Parallel alignment has significantly larger coupling/bandwidth



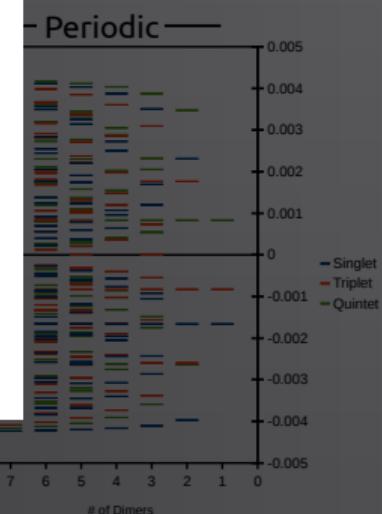
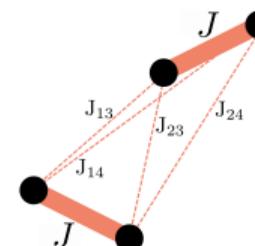
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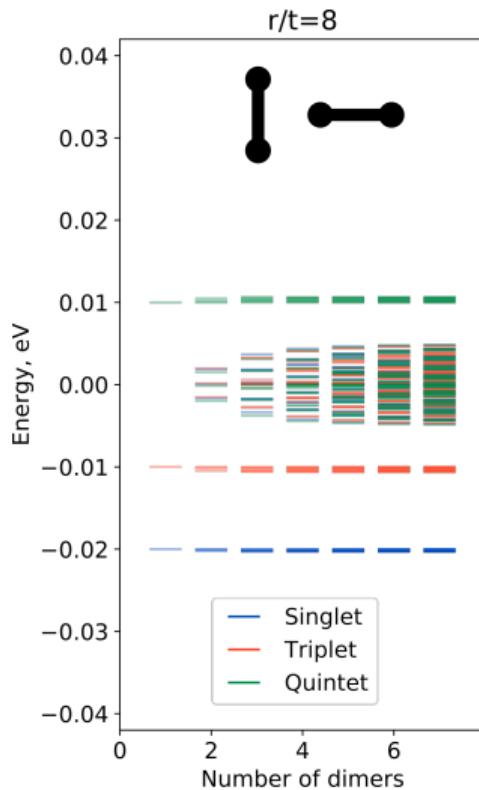
- $t$  is only zeroth-order quantity with orientational dependence

$$E = 0.5 \times J$$
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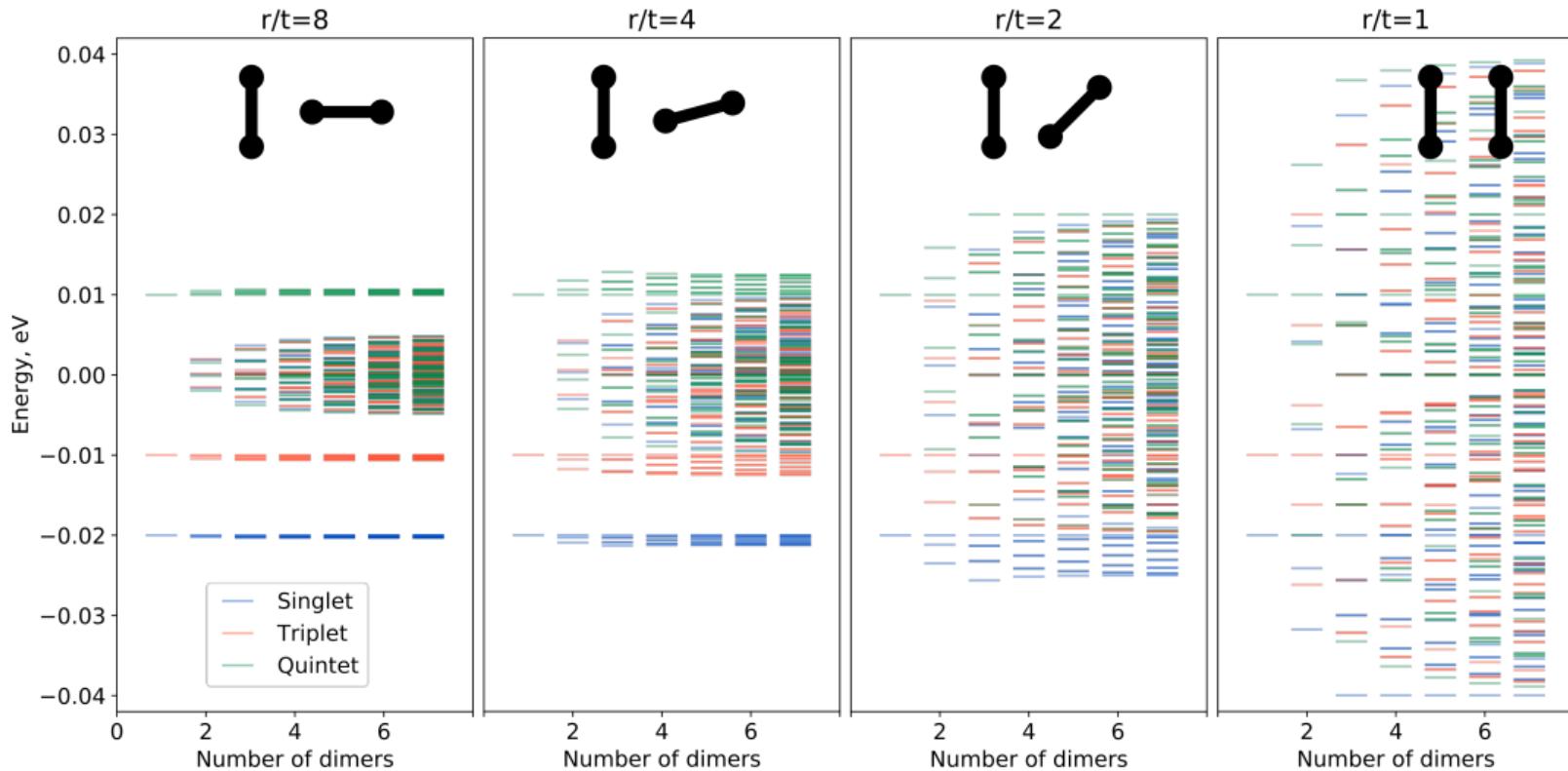
# Multiexcitons in molecular materials : Larger clusters

- perpendicular( $t=0$ ): parallel( $t=\text{max}$ )



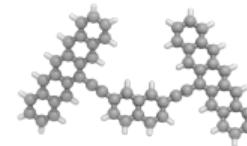
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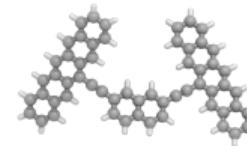
# An Ovchinnikov's Rule for excited states

- What about covalently-linked the dimers?
  - More control over dimer-coupling
  - Now both through-space and through-bond interactions
- Map onto Ising model:  $\hat{H} = -\sum_{ij} J_{ij} \hat{S}_i^z \hat{S}_j^z$
- Ovchinnikov's Rule :  $S = \frac{N_\alpha - N_\beta}{2}$



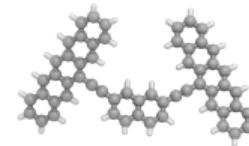
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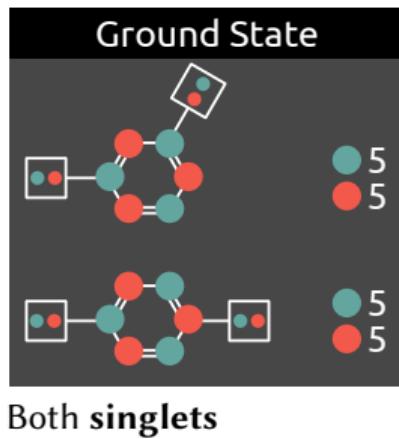
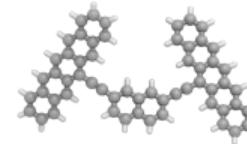
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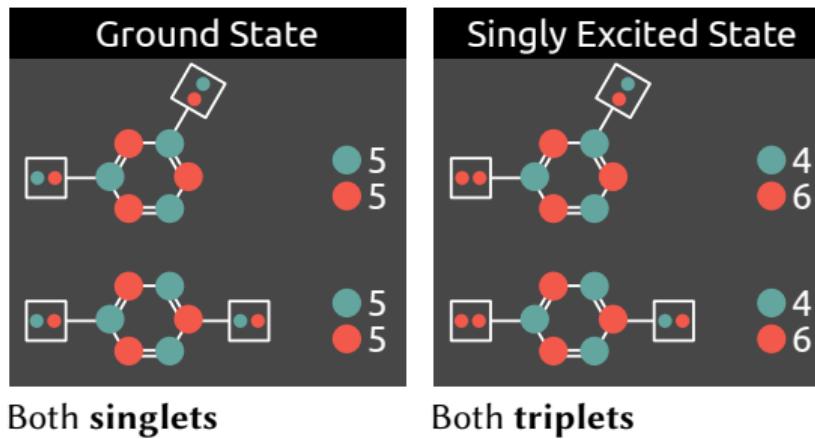
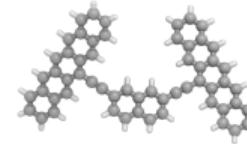
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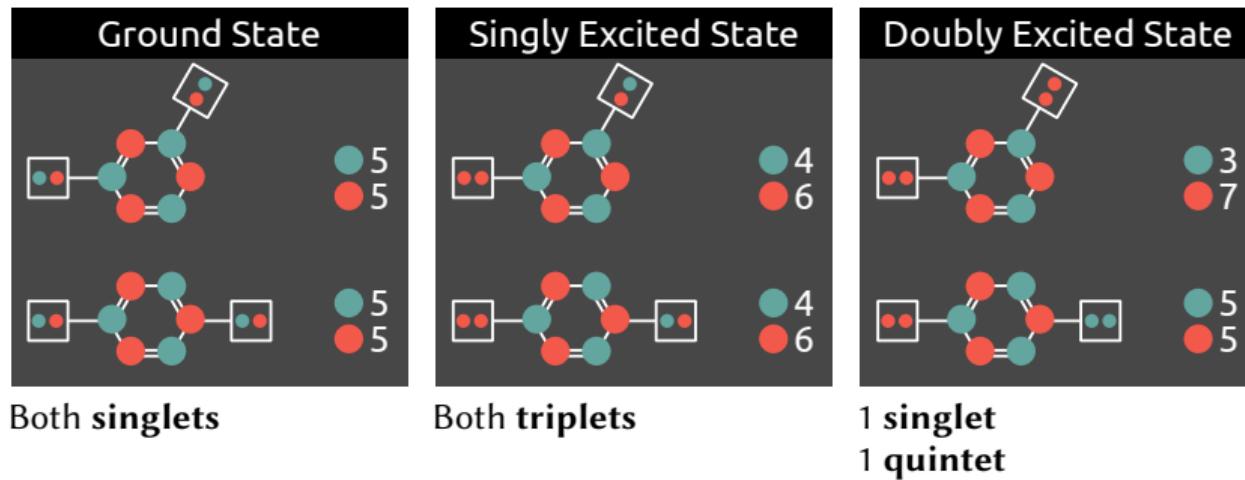
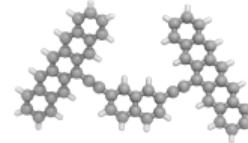
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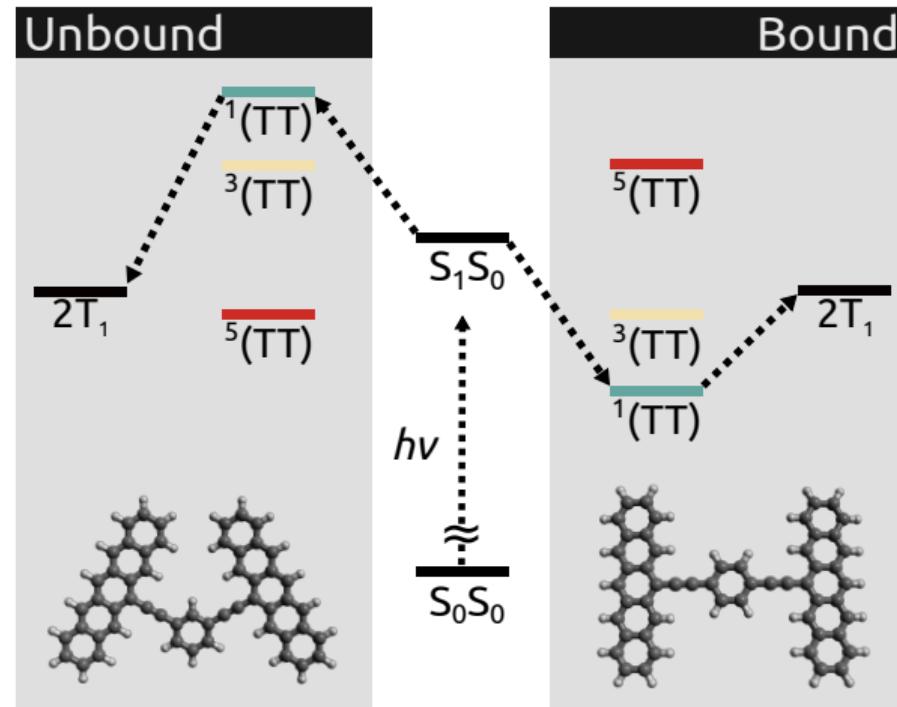


# An Ovchinnikov's Rule for excited states

- What about covalently-linked the dimers?
  - More control over dimer-coupling
  - Now both through-space and through-bond interactions
- Map onto Ising model:  $\hat{H} = -\sum_{ij} \gamma_{ij} \hat{\sigma}^z_i \hat{\sigma}^z_j$
- **Hypothesis:** *The through-bond ordering of multiexciton spin-states is predicted by parity of connection:*
  - even  $\rightarrow$  AF: Bound<sup>1</sup>(TT)
  - odd  $\rightarrow$  FM: Unbound<sup>5</sup>(TT)

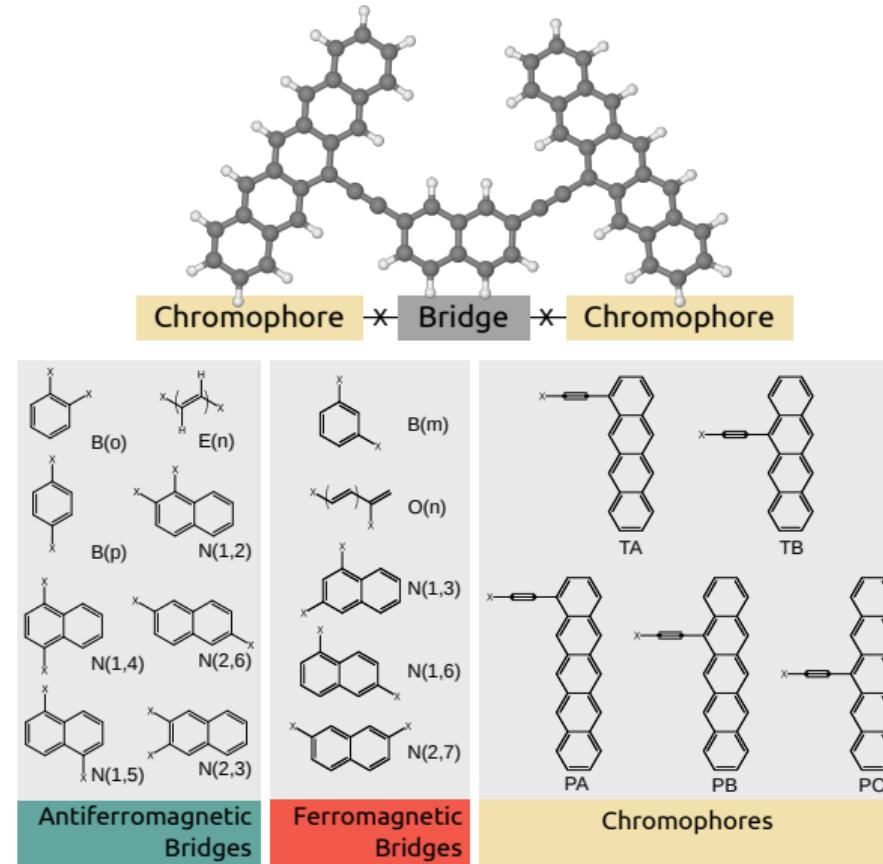


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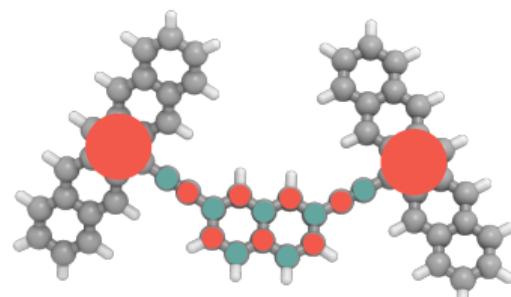
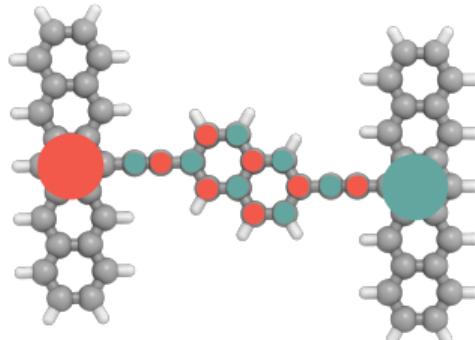
# An Ovchinnikov's Rule for excited states

- **Data set:** 125 molecules



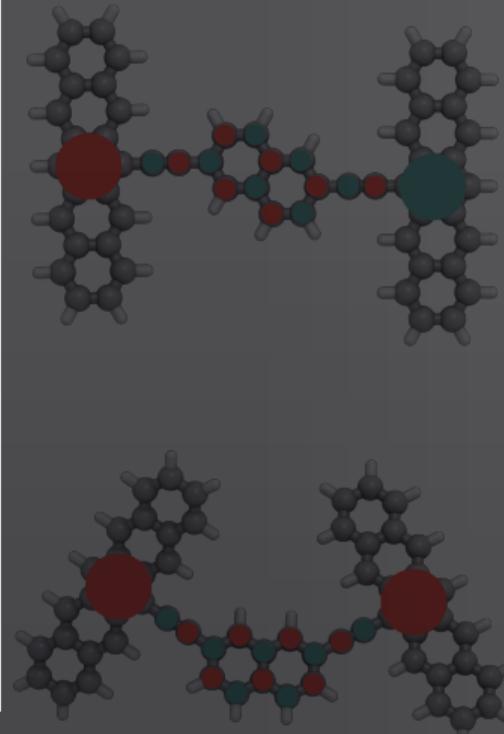
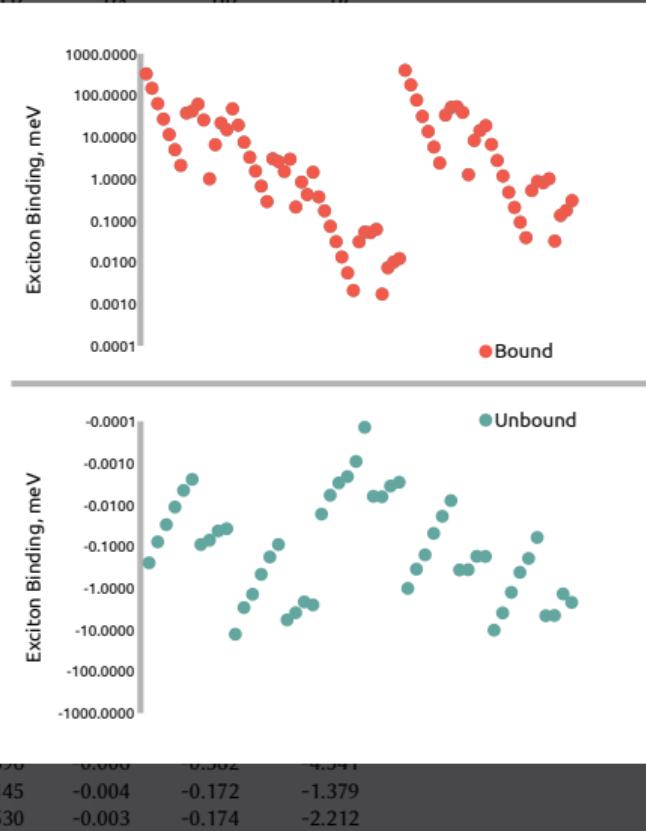
# An Ovchinnikov's Rule for excited states: Results

meV	TA	TB	PA	PB	PC
Antiferromagnetic (even) Bridges					
E(0)	6.700	399.164	0.369	47.762	328.825
E(1)	2.755	178.912	0.171	19.366	148.263
E(2)	1.172	77.129	0.073	7.558	63.429
E(3)	0.477	31.452	0.031	3.293	27.107
E(4)	0.207	13.693	0.013	1.539	11.582
E(5)	0.091	5.815	0.006	0.671	4.996
E(6)	0.039	2.397	0.002	0.286	2.105
B(o)	0.526	33.755	0.031	3.038	37.918
B(p)	0.869	51.799	0.053	2.596	42.247
N(1,2)	0.810	53.525	0.051	1.506	61.618
N(1,4)	1.008	39.231	0.062	2.962	25.767
N(1,5)	0.032	1.263	0.002	0.212	1.002
N(1,7)	0.133	8.299	0.007	0.838	6.596
N(2,3)	0.175	14.390	0.010	0.421	21.808
N(2,6)	0.301	18.846	0.012	1.443	15.102
Ferromagnetic (odd) Bridges					
O(0)	-0.248	-12.768	-0.017	-1.016	-10.211
O(1)	-0.078	-2.937	-0.006	-0.351	-3.944
O(2)	-0.030	-1.408	-0.003	-0.159	-1.263
O(3)	-0.011	-0.467	-0.002	-0.049	-0.142
O(4)	-0.005	-0.179	-0.001	-0.019	-0.194
O(5)	-0.002	-0.090	-0.003	-0.008	-0.061
B(m)	-0.090	-5.784	-0.006	-0.367	-4.597
N(1,3)	-0.071	-3.896	-0.006	-0.362	-4.541
N(1,6)	-0.043	-2.145	-0.004	-0.172	-1.379
N(2,7)	-0.038	-2.530	-0.003	-0.174	-2.212



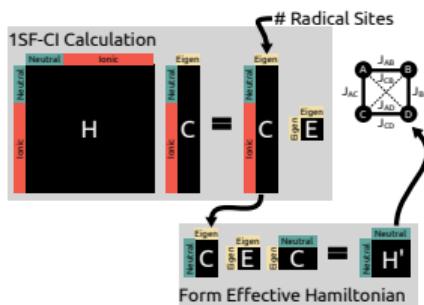
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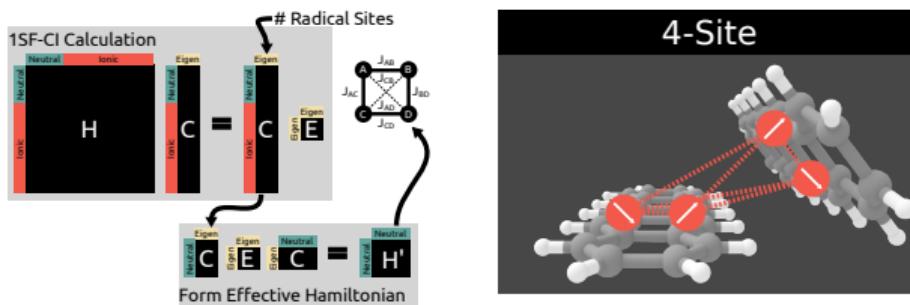
# In Summary

## 1. Bloch Effective Hamiltonian: 1 spin-flip $\rightarrow$ Heisenberg Hamiltonian



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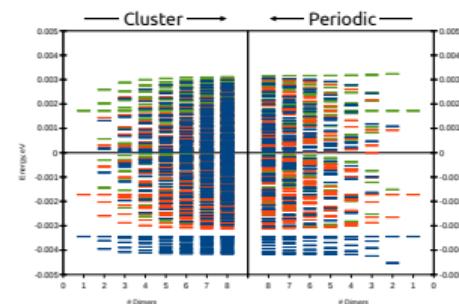
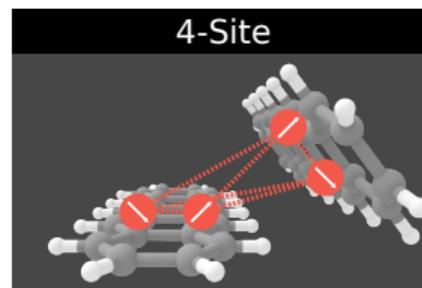
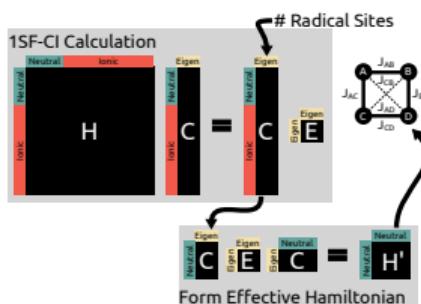


1. Mayhall, JCTC, 12, 4263 (2016)

2. Abraham, Mayhall, JPCL, 8, 5472 (2017)

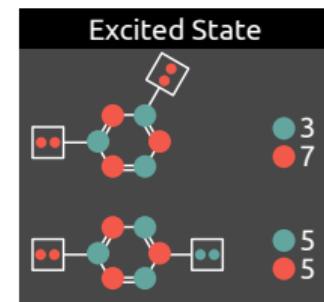
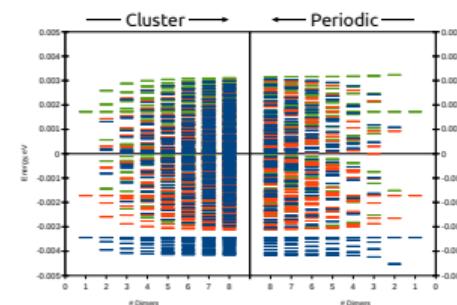
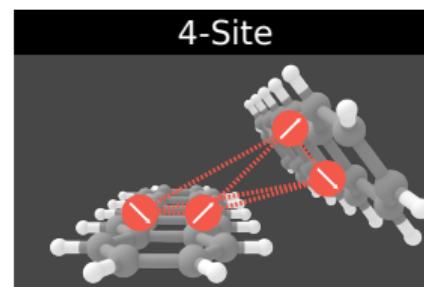
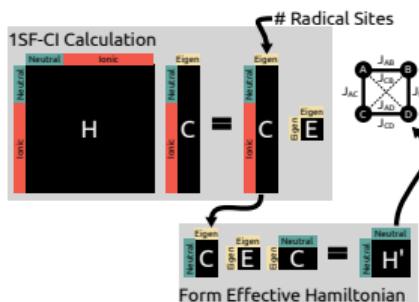
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4. Spin-lattice reveals new qualitative theory: **Excited state Ovchinnikov's rule**



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## Students

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Oinam Meitei

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Ed Barnes (VT Physics)  
Kyungwha Park (VT Physics)  
Dave Pappas (NIST/UC Boulder)

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DOE: DE-SC0018326  
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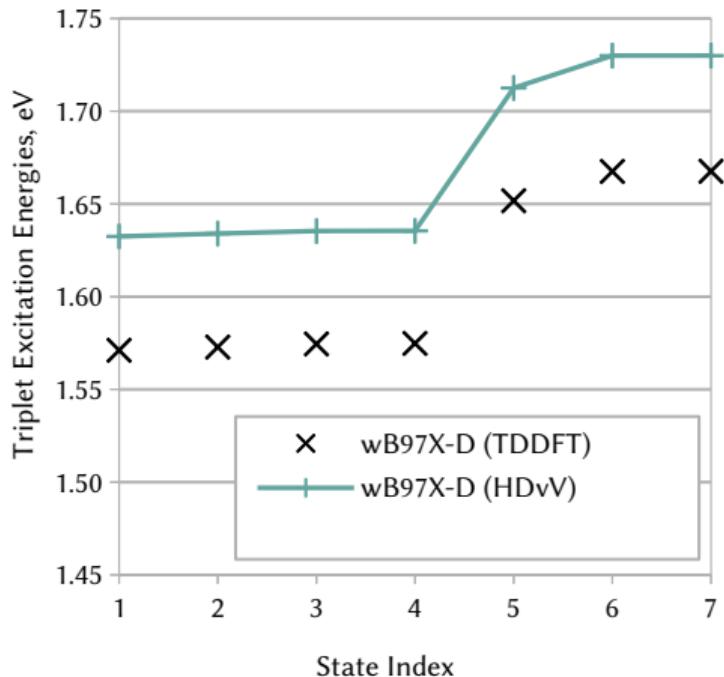
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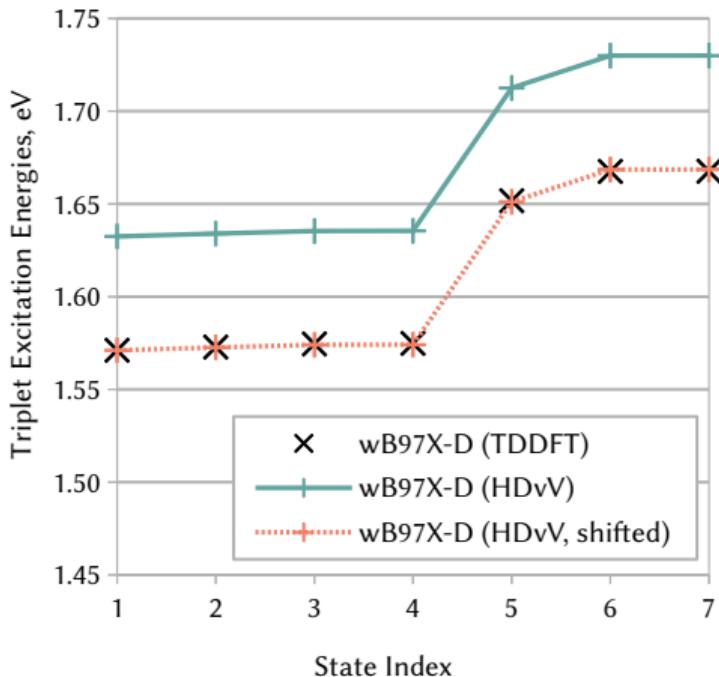
# Multiexcitons in molecular materials

- Compare single exciton triplet states - TDDFT vs. SF-TDDFT+HDvV



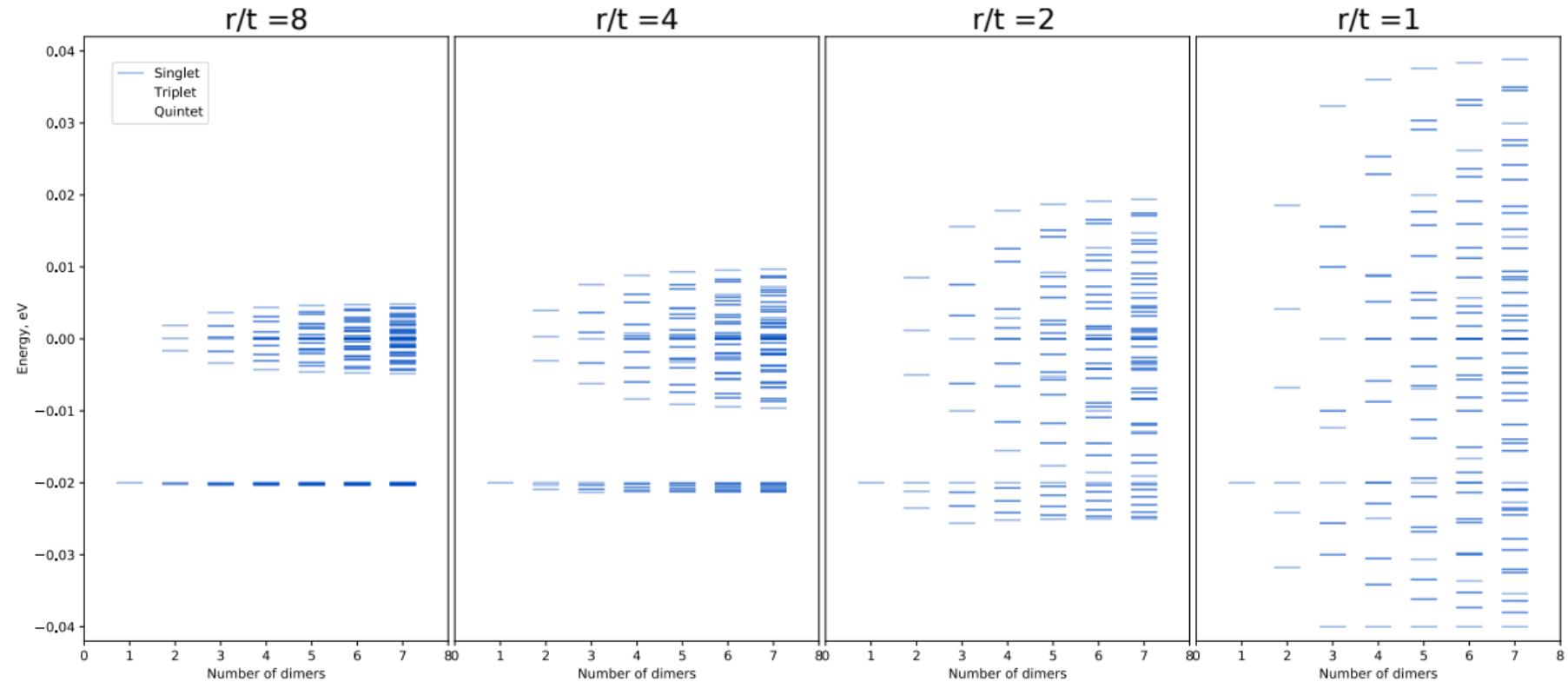
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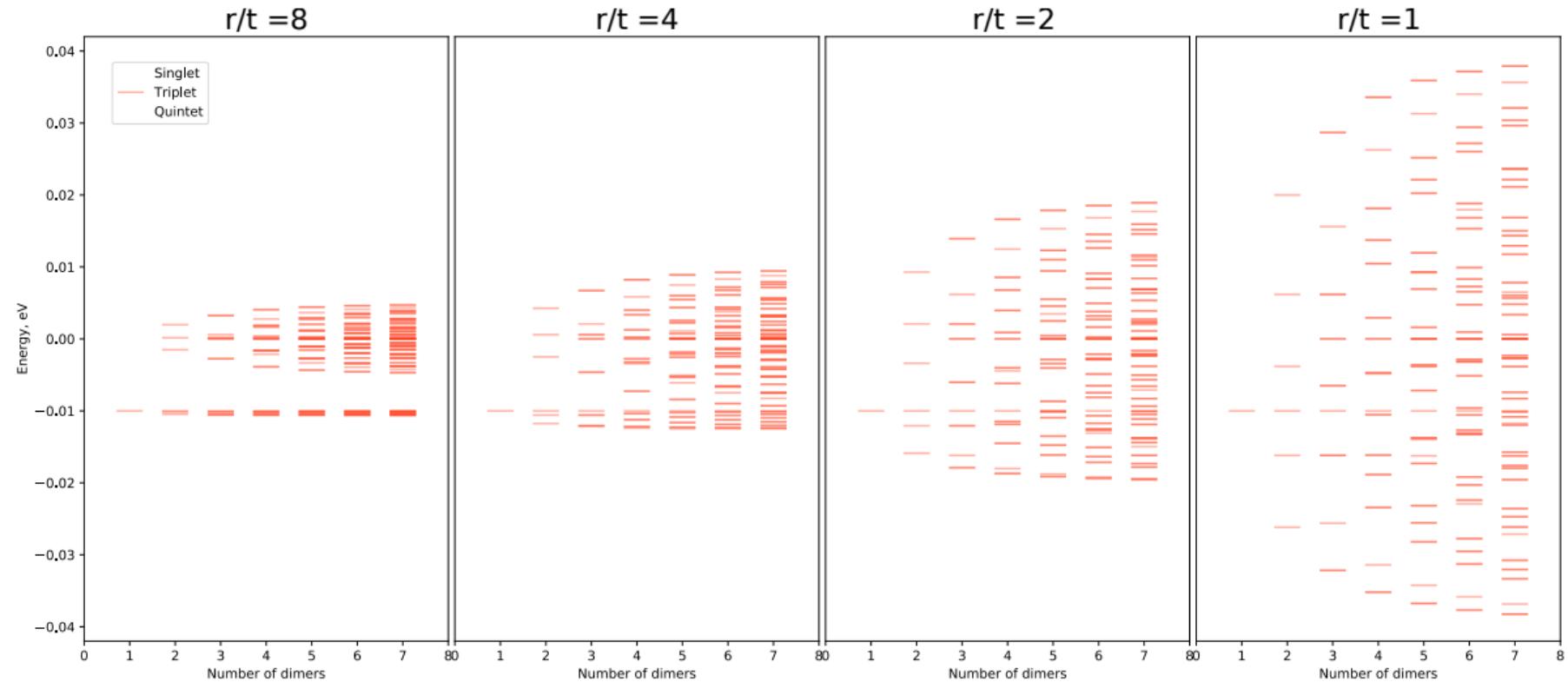
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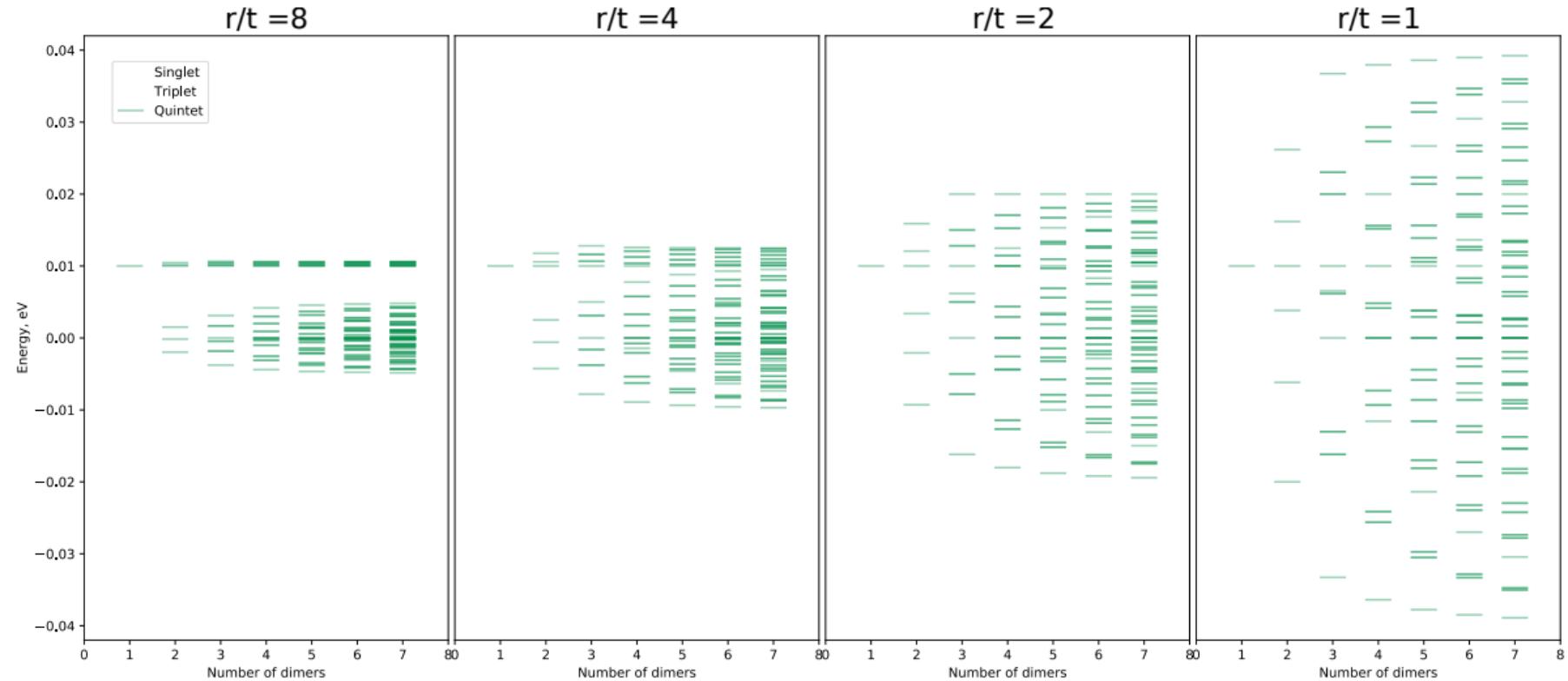
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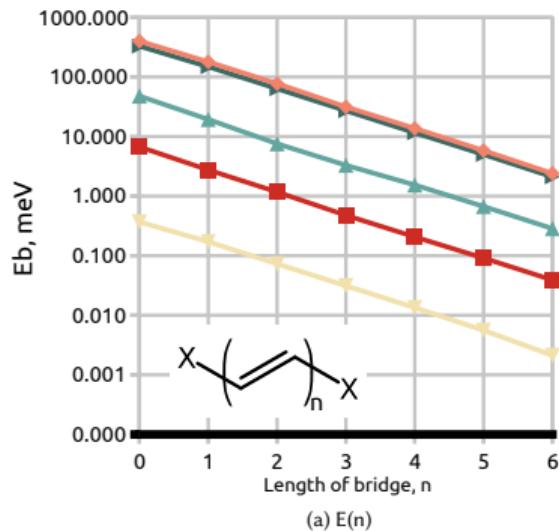


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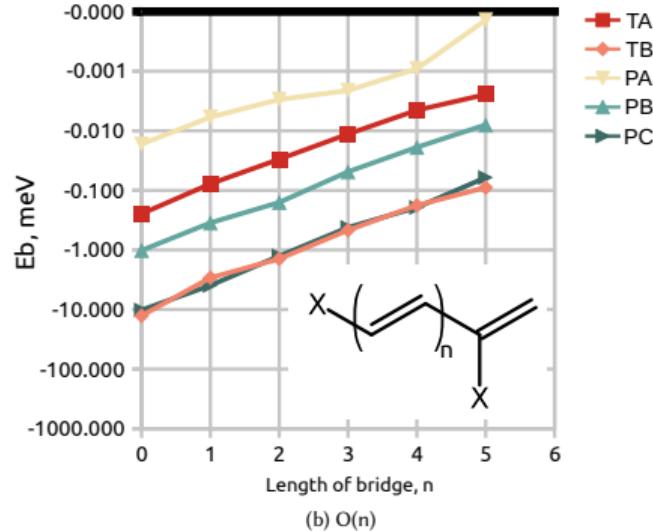
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# An Ovchinnikov's Rule for excited states: Distance decay

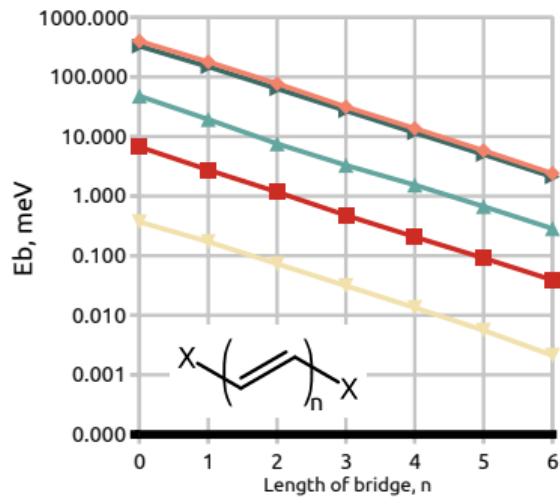


(a)  $E(n)$

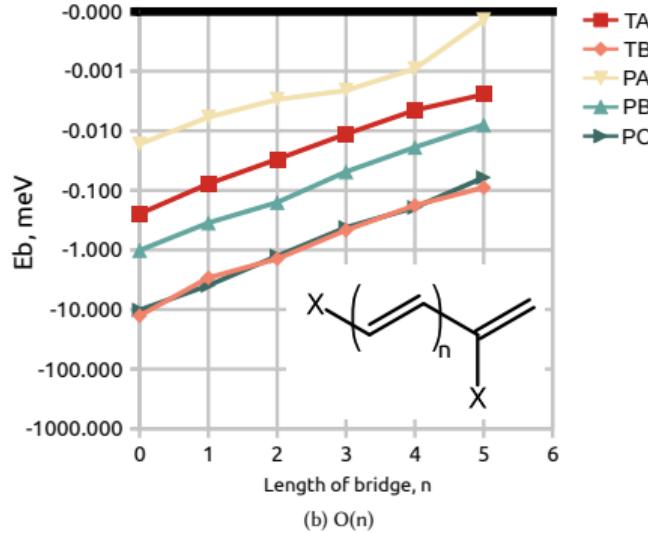


(b)  $O(n)$

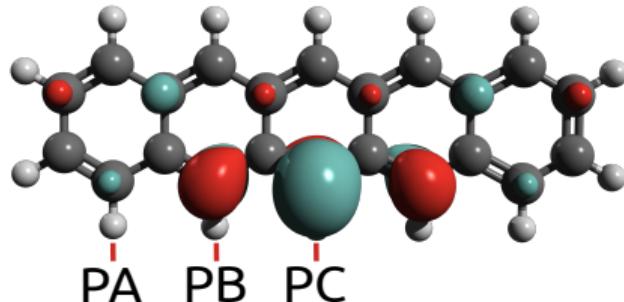
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(a)  $E(n)$

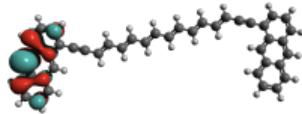
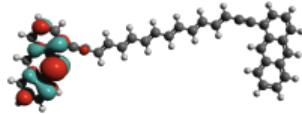
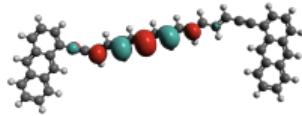
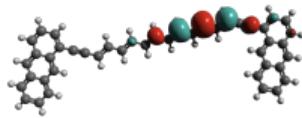


(b)  $O(n)$



# An Ovchinnikov's Rule for excited states: Exceptions

Bridges with  
low-lying triplet states



Non-alternate  
hydrocarbons



